

1. (30 points) Suppose both $f(x) = \sin(2\pi x)$ and $g(x) = \cos(2\pi x) + c$ are eigenfunctions corresponding to distinctive eigenvalues to the following Sturm-Liouville problem, where r , r' and q are all assumed to be continuous on $[0, 1]$. Find constant c .

$$[r(x)X'(x)]' + [q(x) + \lambda(x+1)]X(x) = 0, \quad 0 < x < 1;$$

$$X(0) = X(1), \quad X'(0) = X'(1).$$

2. (30 points) Solve for the eigenvalues and normalized eigenfunctions.

$$X'' + \lambda X = 0, \quad 0 < x < 1;$$

$$X(0) - X'(0) = 0, \quad X(1) + X'(1) = 0.$$

3. (40 points) Solve the boundary value problem

$$(1+t)u_t(x,t) = u_{xxx}(x,t) \quad (0 < x < 1, t > 0).$$

$$u_x(0,t) = -1, \quad u(1,t) = 0, \quad \underline{u(x,0) = 0.}$$