

## HOMWORK 9 - DUE NOVEMBER 12

**Homework instructions:** Complete the assigned problems on your own paper. Once you are finished, scan or photograph your work and upload it to Gradescope. When prompted, tell Gradescope where to find each problem.

You are allowed (and in fact encouraged) to work with other students on homework assignments. If you do that, please indicate on each problem who you worked with. If you use sources other than your notes, the textbook, and any resources on Canvas for your homework, you must indicate the source on each problem. You are not permitted to view, request, or look for solutions to any of the homework problems from solutions manuals, homework help websites, online forums, other students, or any other sources.

### Textbook Problems:

- §7.1: 8
- §7.2: 1, 5, 9, 15, 19, 24, 28

### Additional Problems:

1. There is a system of three brine tanks. Tanks 1 and 3 begin with 200 L of fresh water each and tank 2 begins with 100 L of water and 10 kg of salt.

Water containing 2 kg of salt per liter is pumped into tank 1 at a rate of 15 L/min. The well-mixed solution is pumped from tank 1 to tank 2 at a rate of 20 L/min, from tank 2 to tank 3 at a rate of 20 L/min, and from tank 3 to tank 1 at a rate of 5 L/min. The well-mixed solution is pumped out of tank 3 at a rate of 15 L/min.

- (a) Draw and label a picture that illustrates this situation
- (b) Let  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$  denote the amount of salt (in kilograms) in tanks 1, 2, and 3 respectively after  $t$  minutes. Write down differential equations for  $x'_1$ ,  $x'_2$ , and  $x'_3$ .
- (c) Write the system of differential equations in (b) as a matrix equation

$$\vec{x}' = P(t)\vec{x} + \vec{f}(t)$$

What are the initial conditions  $\vec{x}(0)$ ?