## Homework 10 - Due November 19

**Homework instructions:** Complete the assigned problems on your own paper. Once you are finished, scan or photograph your work and upload it to Gradescope. When prompted, tell Gradescope where to find each problem.
You are allowed (and in fact encouraged) to work with other students on homework assignments. If you do that, please indicate on each problem who you worked with. If you use sources other than your notes, the textbook, and any resources on Canvas for your homework, you must indicate the source on each problem. You are not permitted to view, request, or look for solutions to any of the homework problems from solutions manuals, homework help websites, online forums, other students, or any other sources.

**Textbook Problems:**

- §7.3: 4, 6, 8, 18, 38 (**do not** sketch direction fields or solution curves, **do** find particular solutions when asked for)

**Additional Problems:**

1. Consider the differential equation

$$x^{(3)} + x'' - 2x' = 0$$

   (a) Transform this into an equivalent system of first-order differential equations.

   (b) Write the system from (a) as $\mathbf{x}' = A\mathbf{x}$ (the matrix $A$ should be $3 \times 3$).

   (c) Use the eigenvalue method to solve the system in (b)

   (d) Using your solution to (c), what is the general solution $x(t)$ to the given differential equation?

2. Consider the following system of brine tanks:

   There are three tanks. Tank 1 contains 20 L of water, tank 2 contains 30 L of water, and tank 3 contains 60L of water. Fresh water is pumped into tank 1 at a rate of 120 L/min. The well-mixed solution is pumped from tank 1 to tank 2, from tank 2 to tank 3, and out of tank 3 all at a rate of 120 L/min.

   (a) Draw and label a diagram describing the system

   (b) Let $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$ be the vector function of the amount of salt in each tank at time $t$. Write a differential equation $\mathbf{x}' = A\mathbf{x}$ describing the system.

   (c) Find the general solution to the differential equation you wrote in (b) using the eigenvalue method

   (d) Initially, there is 100 kg of salt in tank 1 and 20 kg of salt in tank 2. Find the particular solution corresponding to these initial conditions.