Homework 1 – Solutions

These solutions demonstrate one way to approach each of the homework problems. In many cases, there are other correct solutions. If you would like to discuss alternative solutions or the grading of your assignment, please see me during office hours or send me an email.

Textbook Problems:

3.1.11 Our original system is

$$\begin{aligned} 2x + 7y + 3z &= 11 \\ x + 3y + 2z &= 2 \\ 3x + 7y + 9z &= -12 \end{aligned}$$

We add (-2) times the second equation to the first and (-3) times the second equation to the third.

$$\begin{aligned} y - z &= 7 \\ x + 3y + 2z &= 2 \\ -2y + 3z &= -18 \end{aligned}$$

Now, add 2 times the first equation to the third.

$$\begin{aligned} y - z &= 7 \\ x + 3y + 2z &= 2 \\ z &= -4 \end{aligned}$$

At this point, back substitution gives us the unique solution $x = 1, y = 3, z = -4$. The system is consistent.

3.1.15 Our original system is

$$\begin{aligned} x + 3y + 2z &= 5 \\ x - y + 3z &= 3 \\ 3x + y + 8z &= 10 \end{aligned}$$

Add (-1) times the first equation to the second and (-3) times the first equation to the third.

$$\begin{aligned} x + 3y + 2z &= 5 \\ -4y + z &= -2 \\ -8y + 2z &= -5 \end{aligned}$$

Add (-2) times the second equation to the third.

$$
\begin{aligned}
x + 3y + 2z &= 5 \\
-4y + z &= -2 \\
0 &= -1
\end{aligned}
$$

Our third equation is contradictory, so the system has no solutions and is inconsistent.

3.1.17 Our original system is

$$
\begin{aligned}
2x - y + 4z &= 7 \\
3x + 2y - 2z &= 3 \\
5x + y + 2z &= 15
\end{aligned}
$$

Add (-1) times the first equation to the second.

$$
\begin{aligned}
2x - y + 4z &= 7 \\
x + 3y - 6z &= -4 \\
5x + y + 2z &= 15
\end{aligned}
$$

Add (-2) times the second equation to the first and (-5) times the second equation to the third.

$$
\begin{aligned}
-7y + 16z &= 15 \\
x + 3y - 6z &= -4 \\
-14y + 32z &= 35
\end{aligned}
$$

Add (-2) times the first equation to the third.

$$
\begin{aligned}
-7y + 16z &= 15 \\
x + 3y - 6z &= -4 \\
0 &= 5
\end{aligned}
$$

Our third equation is contradictory, so the system has no solutions and is inconsistent.

3.2.7 Our system is

$$
\begin{aligned}
x_1 + 2x_2 + 4x_3 - 5x_4 &= 17 \\
x_2 - 2x_3 + 7x_4 &= 7
\end{aligned}
$$

Our free variables are x_3 and x_4, so we set them equal to parameters: $x_3 = s, x_4 = t$. Solving for the other two variables gives $x_2 = 7 + 2s - 7t$ and $x_1 = 3 - 8s + 19t$.

3.2.9 Our system is

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 &= 6 \\ 3x_2 - x_3 - 2x_4 &= 2 \\ 3x_3 + 4x_4 &= 9 \\ x_4 &= 6 \end{aligned}$$

Working from the bottom up, we solve for each variable to get $x_4 = 6, x_3 = -5, x_2 = 3$, and $x_1 = 1$.

3.2.15 We do row operations to the augmented coefficient matrix. For brevity, we may do more than one row operation in a step.

$$\begin{aligned} \begin{bmatrix} 3 & 1 & -3 & -4 \\ 1 & 1 & 1 & 1 \\ 5 & 6 & 8 & 8 \end{bmatrix} &\xrightarrow[(-5)R_2+R_3]{(-3)R_2+R_1} \begin{bmatrix} 0 & -2 & -6 & -7 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \end{bmatrix} \\ &\xrightarrow{(2)R_3+R_1} \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \end{bmatrix} \\ &\xrightarrow{SWAP(R_1,R_2)} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 3 & 3 \end{bmatrix} \\ &\xrightarrow{SWAP(R_2,R_3)} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \end{aligned}$$

We continued with our reduction to reach echelon form, but notice that after the second step our first row corresponds to the equation $0 = 11$, a contradiction. So the system has no solutions.

3.3.8

$$\begin{bmatrix}1 & -4 & -5\\ 3 & -9 & 3\\ 1 & -2 & 3\end{bmatrix} \xrightarrow[(-3)R_1+R_2]{(-1)R_1+R_3} \begin{bmatrix}1 & -4 & -5\\ 0 & 3 & 18\\ 0 & 2 & 8\end{bmatrix}$$

$$\xrightarrow[(\frac{1}{3})R_2]{(\frac{1}{2})R_3} \begin{bmatrix}1 & -4 & -5\\ 0 & 1 & 6\\ 0 & 1 & 4\end{bmatrix}$$

$$\xrightarrow{(-1)R_2+R_3} \begin{bmatrix}1 & -4 & -5\\ 0 & 1 & 6\\ 0 & 0 & -2\end{bmatrix}$$

$$\xrightarrow{(-\frac{1}{2})R_3} \begin{bmatrix}1 & -4 & -5\\ 0 & 1 & 6\\ 0 & 0 & 1\end{bmatrix}$$

$$\xrightarrow[(5)R_3+R_1]{(-6)R_3+R_2} \begin{bmatrix}1 & -4 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\xrightarrow{(4)R_2+R_1} \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

3.3.11

$$\begin{bmatrix}3 & 9 & 1\\ 2 & 6 & 7\\ 1 & 3 & -6\end{bmatrix} \xrightarrow[(-2)R_3+R_2]{(-3)R_3+R_1} \begin{bmatrix}0 & 0 & 19\\ 0 & 0 & 19\\ 1 & 3 & -6\end{bmatrix}$$

$$\xrightarrow{(-1)R_2+R_1} \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 19\\ 1 & 3 & -6\end{bmatrix}$$

$$\xrightarrow[(\frac{1}{19})R_2]{SWAP(R_1,R_3)} \begin{bmatrix}1 & 3 & -6\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

$$\xrightarrow{(6)R_2+R_1} \begin{bmatrix}1 & 3 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

Additional Problems:

1. Check the class notes for sketches of each case.

   Unique solution – two intersecting lines

   No solutions – two parallel lines

   Infinitely many solutions – two identical lines

2. There are many possibilities. One option: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

3. There are multiple possibilities here, but all correct answers will have their nonzero entries in the same places. One option: $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$