MATH 5587 (FALL2019): MIDTERM 1

PROFESSOR: SVITLANA MAYBORODA

Name (legibly!): _____

Problem 1 (20 points)

(a) Write down an explicit formula for the solution to the initial value problem:

$$\partial_t^2 u - 4\,\partial_x^2 u = 0, \quad u(0,x) = \sin x, \quad \partial_t u(0,x) = \cos x, \quad x \in \mathbb{R}, \quad t \geq 0.$$

(b) True or false: the solution is a periodic function of t. What is the period?

(c) Now solve the forced initial value problem

$$\partial_t^2 u - 4\,\partial_x^2 u = \cos t, \quad u(0,x) = \sin x, \quad \partial_t u(0,x) = \cos x, \quad x \in \mathbb{R}, \quad t \geq 0.$$

(d) True or false: the forced equation exhibits resonance. Explain.

Date: October 8, 2019.

Problem 2 (30 points)

(a) Find the (complex or real) Fourier series for the function $f(x) = x$

(b) Does the series converge to the same function, that is, $f(x) = x$, on $\mathbb{R}$? Whether the answer is yes or no, draw the function to which it converges below.

(c) Does the series converge (to the function you identified in (b)):

c1) pointwise? on which interval? Explain. Independently of your answer, write a definition of pointwise convergence.

c2) uniformly? on which interval? Explain. Independently of your answer, write a definition of uniform convergence.

c3) in norm? which norm? on which interval? Explain. Independently of your answer, write a definition of convergence in norm.

(d) Use the results above to find the (complex or real) Fourier series of $f(x) = x^2$

Problem 3 (15 points)

(a) Write down an explicit formula for the solution to the initial value problem. Show your work.

$$\partial_t u + \frac{1}{x^2+4}\,\partial_x u = 0, \quad u(0,x) = e^{x^3+12x}$$

(b) graph either some of the characteristic curves or the solution to the initial value problem for several values of t. You do not have to do both, just indicate which one you are graphing.