Homework 4 Solutions

3.2.34

$f'(x)$ is even.

3.2.37

(a) True. ★ (b) False. Only the restriction of $f(x)$ to $[-\pi, \pi]$ is odd. Its values outside that range are irrelevant as far as its periodic extension is concerned.

3.2.40a

(a) $\dfrac{1}{2} + \dfrac{2}{\pi} \displaystyle\sum_{j=0}^{\infty} \dfrac{(-1)^j \cos(2j+1)x}{2j+1}$; the Fourier series converges non-uniformly to the periodically extended box function, namely to 1 when $\left(2k - \frac{1}{2}\right)\pi < x < \left(2k + \frac{1}{2}\right)\pi$; to $\frac{1}{2}$ when $x = \left(k + \frac{1}{2}\right)\pi$; and to 0 when $\left(2k + \frac{1}{2}\right)\pi \le x \le \left(2k + \frac{3}{2}\right)\pi$ for $k = 0, \pm 1, \pm 2, \ldots$.

3.2.54

We substitute $x = \pi$ into the Fourier series (3.68) for e^x:

$$\tfrac{1}{2}(e^{\pi} + e^{-\pi}) = \frac{\sinh \pi}{2\pi} \sum_{k=-\infty}^{\infty} \frac{(-1)^k (1 + \mathrm{i}\, k)}{1 + k^2}\, e^{\mathrm{i}\, k \pi} = \frac{e^{\pi} - e^{-\pi}}{2\pi} \left(1 + \sum_{k=1}^{\infty} \frac{2}{1 + k^2} \right),$$

which gives the result.

3.2.60

$$\sum_{k=-\infty}^{\infty} c_k\, e^{\mathrm{i}\, k x} \sim f(x) = \overline{f(x)} \sim \sum_{k=-\infty}^{\infty} \overline{c_k}\, e^{-\mathrm{i}\, k x}$$ is real if and only if $c_{-k} = \overline{c_k}$.

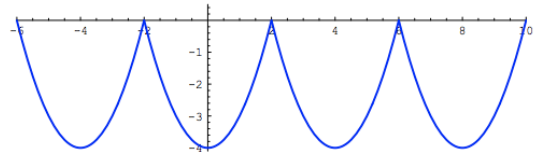
3.3.2

$$x^3 \sim \sum_{k=1}^{\infty} (-1)^k \left(\frac{12}{k^3} - \frac{2\pi^2}{6k} \right) \sin kx.$$

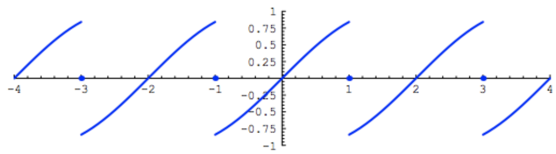
Differentiation does not produce the series for $3x^2$ because the periodic extension of x^3 is not continuous, and so Theorem 3.22 doesn't apply.

3.4.3 b,d

(b) $-\dfrac{8}{3}+\dfrac{16}{\pi^2}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\cos\frac{k\pi x}{2}$;



(d) $2\pi\sin 1\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{k\sin k\pi x}{1-k^2\pi^2}$;



3.4.4 (for 3.4.3 b,d)

.4. The differentiated Fourier series only converges when the periodic extension of the function is continuous:

(b) $\dfrac{8}{\pi}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\sin\frac{k\pi x}{2}$: converges to the 4–periodic extension of $2x$;

(d) $2\pi^2\sin 1\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{k^2}{1-k^2\pi^2}\cos k\pi x$:

does not converge to the 20–periodic extension of $\cos x$.

3.4.5 (for 3.4.3 b,d)

(b) $$\frac{x^3}{3}-4x \sim -\frac{8}{3}x+\frac{32}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^3}\sin\frac{k\pi x}{2} \sim \frac{32}{3\pi}\sum_{k=1}^{\infty}\left(\frac{\pi^2k^2+3}{\pi^2k^3}\right)\sin\frac{k\pi x}{2}$$

(d) $$\cos x \sim \sin 1 + 2\sin 1\sum_{k=1}^{\infty}(-1)^k\frac{\cos k\pi x}{1-k^2\pi^2}.$$

3.5.5a,f,i

(a) Pointwise, but not uniformly; ★ (*f*) neither; ★ (*i*) both.

3.5.7b,d,f

(*b*) pointwise; ★ (*d*) pointwise and uniformly; ★ (*f*) neither pointwise nor uniformly.