Due Tue Apr 19.

A cube in $D-$dimensional space, centered at the origin, has sides of length $L$, so each corner has a coordinate $(\pm L/2, \ \pm L/2, ...)$.

Intersecting that cube is a plane given by the equation $\mathbf{n} \cdot \mathbf{x} = d$. If $||\mathbf{n}||_2 = 1$, then this is called the Hessian normal form of the plane, and $|d|$ is the smallest distance from the origin to the plane.

Let

$$\mathbf{n} = \sqrt{\frac{4}{5}} \begin{pmatrix} 1 \\ 1/2 \end{pmatrix} \qquad \text{2D}$$

$$\sqrt{\frac{16}{21}} \begin{pmatrix} 1 \\ 1/2 \\ 1/4 \end{pmatrix} \qquad \text{3D}$$

$$\sqrt{\frac{64}{85}} \begin{pmatrix} 1 \\ 1/2 \\ 1/4 \\ 1/8 \end{pmatrix} \qquad \text{4D}$$

$$\vdots$$

and

$$d = \frac{2L}{3}.$$

For $D = 2, 3, ..., 10$, find the smallest distance from the origin to that part of the plane that is inside the cube, and give the coordinate of this point of closest approach.