Math 121a Final, Spring 2004, F.Rezakhanlou

1. (3 points) Find the Lagrange's equation in polar coordinates for a particle moving in a plane if the potential energy is $V = r^{-1}$.

2. (3 points) Use the cylindrical coordinates to find the equation of the shortest path connecting two points on a circular cylinder.

$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$

3. (3 points) Solve the Euler equation corresponding to the action

$$\int_a^b \frac{\sqrt{1+y'^2}}{1+y}\,dx.$$

4. (3 points) Find the inverse Laplace transform of $\tilde{f}(z) = (z-1)^{-2}(z^2+4)^{-1}$.

5. (3 points)Find the exponential and sine Fourier transform of a function

$$f(x) = \begin{cases} 1 \text{ if } 0 < x \leq 2 \\ -1 \text{ if } -2 \leq x \leq 0 \\ 0 \text{ otherwise.} \end{cases}$$

Use this to evaluate

$$\int_0^\infty \frac{(\cos 2y - 1)\sin y}{y} dy.$$

✓ 6. (3 points) Solve $(x^2+1)y'' - 2xy' + 2y = (x^2+1)^2$ using the fact that x and $1-x^2$ are solutions to the homogeneous equations.

7. (3 points) Find $y = y(x)$ such that $y(0) = y'(0) = 0$ and $4y'' + 4y' + 10y = \delta(x - x_0)$.

8. (3 points) Given $f(x) = |x|$ on $(-\pi, \pi)$, expand f in an appropriate Fourier series of period 2π. To what value does the series converges at π?

9. (3 points) Evaluate $\int_0^\infty \frac{\cos x}{1+x^4}\,dx$.

10. (3 point) Use the transformation $f(z) = z^{-1}$ to find the temperature distribution T in the region

$$\{(x, y) : (x-1)^2 + y^2 > 1, x > 0\}$$

provided that $T(x, y) = 20$ if $(x-1)^2 + y^2 = 1$ and $T(0, y) = 10$.

11. (2 points) Evaluate

$$\frac{d^2}{dx^2}\int_0^x\int_0^x f(s,t)\,ds\,dt.$$

12. (3 points)If $z = xy$, $2x^3 + 2y^3 = 3t^2$ and $3x^2 + 3y^2 = 6t$, find $\frac{dz}{dt}$.