MAE 185, UCI third project. Solution of Lotka-Volterra two species model

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Contents

1 User Guide For Running the program "Solution of Lotka-Volterra two species model"

To run the program, please copy the content of the floppy disk (which contains the required 2 matlab files, one is called nma_185_proj3.m, and the other is a utility support file called nma_inputNumeric.m) to your MATLAB *work* folder on your C: drive.

The MATLAB work folder will be located under the main MATLAB folder. The name of the MATLAB main folder depends on the version of MATLAB you have installed. For example, for MATLAB 6.5, it is called

```
C:\MATLAB6p5
```

Once the files are copied to the work folder below the above MATLAB main folder, then start matlab itself, and from the matlab console, type the command:

```
nma_185_proj3
```

The program now will start and asks for input. This is an example of the required input from one test run:

```
>> nma_185_proj3
Initial rabbits population ? >10
Initial fox population ? >5000
```

```
Number of steps ? >1000
Step size ? >1
>>
```

The program will then solve the problem and will display 3 different figure windows to show the results. It will display the state-space solution, the time-domain solution for each independent variable, both on the same plot and on separate plots. Please see test cases below for more examples.

2 Technical notes

2.1 Problem

Let prey by rabbits (R), and predators be foxes (F). Model is

$\frac{dR}{dt} = a\ R(t) - b\ R(t)\ \ F(t)$

$\frac{dF}{dt} = e\ b\ R(t)\ \ F(t) - c\ F(t)$

Where

a is natural growth of R in absence of F.

c is natural death rate of F in absence of its food R.

b is the death rate per encounter of R due to predation.

e is the efficiency of turning predated R into F.

Solve the couple ODE using Runge-Kutta 4th order classical method.

Use the following values for the above parameters:

$a = 0.04$

$c = 0.0005$

$b = 0.2$

$e = 0.1$

Try different initial conditions for R and F population.

2.2 Analysis and Solution

let $\frac{dR}{dt} = f(t, R, F)$

let $\frac{dF}{dt} = g(t, F, R)$

Since the rate of population is not explicitly given in terms of the independent variable t, I could write the above as

$\frac{dR}{dt} = f(R, F)$

$\frac{dF}{dt} = g(F, R)$

But for generality, I will keep the first form.

To solve using R-K 4th order method, then we write

$R_{i+1} = R_i + \frac{\Delta t}{6}(K_{1,R} + 2K_{2,R} + 2K_{3,R} + K_{4,R})$

$F_{i+1} = F_i + \frac{\Delta t}{6}(K_{1,F} + 2K_{2,F} + 2K_{3,F} + K_{4,F})$

Let $h = \Delta t$, the step size.

The only trick in these coupled ODE is the order in which we evaluate the K coefficients. This can be seen when we write the K down. When writing the K coefficients down, use this notation $K_{i,R}$ to mean the $i\ th$ K for rabbits. And $K_{i,F}$ to mean the $i\ th$ K for Foxes. This means the second subscript represents the independent variable. This will reduce confusion and mistakes.

$K_{1,R} = f(R_i, F_i)$

$K_{1,F} = g(F_i, R_i)$

$K_{2,R} = f\left(t + \frac{1}{2}h\ ,\quad R_i + \frac{1}{2}h\ K_{1,R},\quad F_i + \frac{1}{2}h\ K_{1,F}\right)$

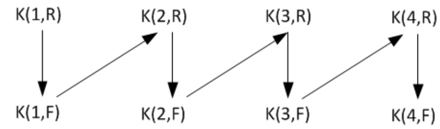
$K_{2,F} = g\left(t + \frac{1}{2}h,\quad F_i + \frac{1}{2}h\ K_{1,F}\ ,\quad R_i + \frac{1}{2}h\ K_{1,R}\right)$

$K_{3,1} = f\left(t + \frac{1}{2}h,\quad R_i + \frac{1}{2}h\ K_{2,R}\ ,\quad F_i + \frac{1}{2}h\ K_{2,F}\right)$

$K_{3,2} = g\left(t + \frac{1}{2}h,\quad F_i + \frac{1}{2}h\ K_{2,F},\quad R_i + \frac{1}{2}h\ K_{2,R}\right)$

$K_{4,1} = f(t + h,\quad R_i + h\ K_{3,R},\quad F_i + h\ K_{3,F})$

$K_{4,2} = g(t + h,\quad F_i + h\ K_{3,F},\quad R_i + h\ K_{3,R})$



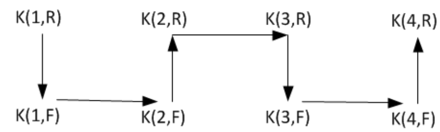


Figure 1: two possible ordering for K evaluation

Hence, looking at the dependency of the K above, we see some possible sequential ordering in which to evaluate those K for each step. See diagram 1

I will pick the first ordering sequence above for the implementation.

The above gives all the setup I need to implement the algorithm. This is implemented in MATLAB function called nma_185_proj2.m. The function asks the user for the initial population of the F and R, and the number of time steps, and for the size of the time step and will display the solution obtained.

2.3 Algorithm

```
Read initial R and F and time step size and number of steps.

i=0
Loop
   i=i+1
   IF i greater than user supplied maximum number of steps THEN
      exit LOOP
   END IF

   Find the K's in the order shown above.
   Find R(i+1), F(i+1), use user supplied initial values for R,F for i=1
   Save these solutions in global solution matrix

End Loop
Plot the solution from the solution matrix.
```

2.4 Example outputs and results

```
>> nma_185_proj3
Initial rabbits population ? >400
Initial fox population ? >50
Number of steps ? >1000
Step size ? >1
```

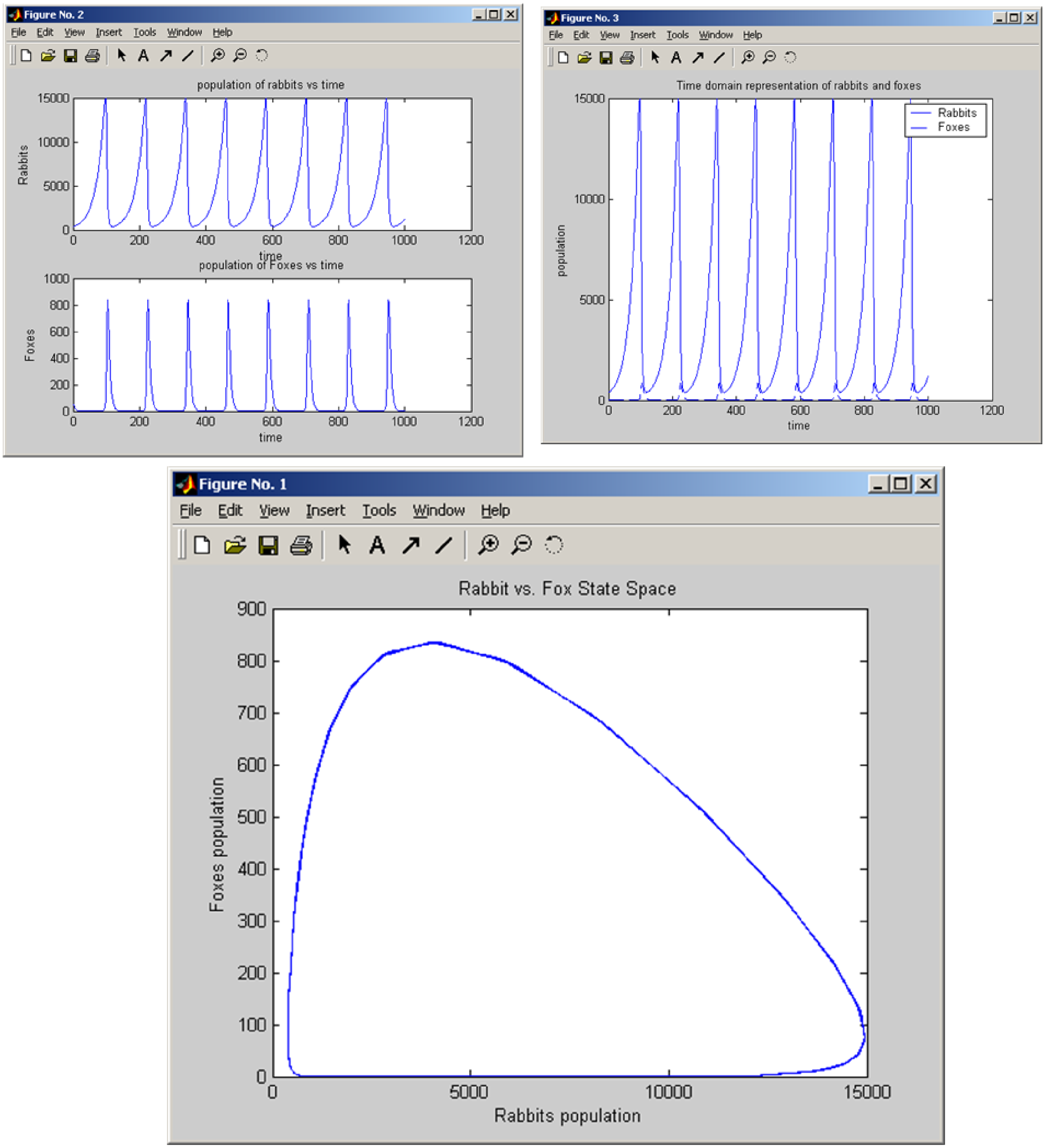


Figure 2: test1

```
>> nma_185_proj3
Initial rabbits population ? >1000
Initial fox population ? >10
Number of steps ? >1000
Step size ? >1
```

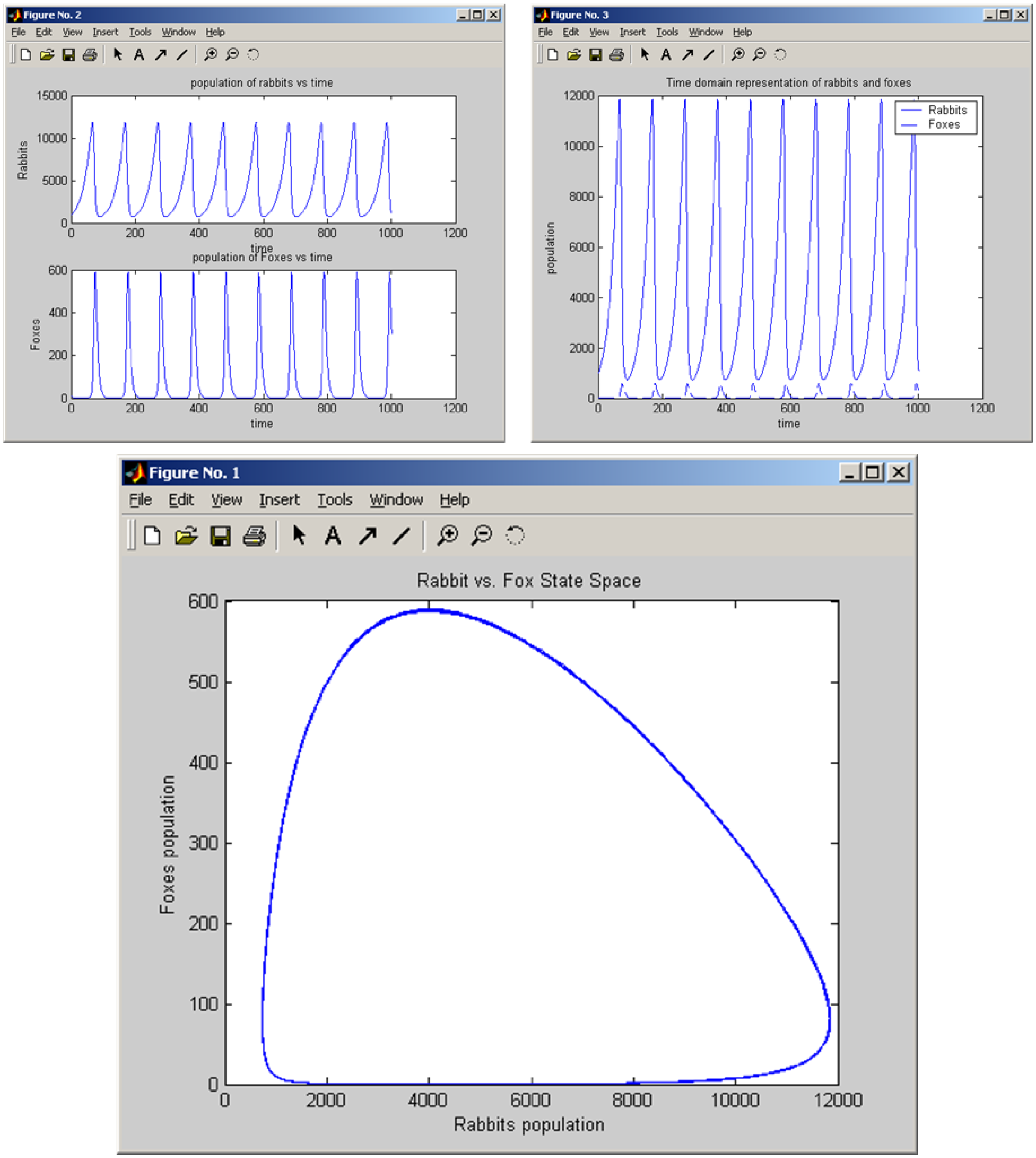


Figure 3: test2

```
>> nma_185_proj3
Initial rabbits population ? >0
Initial fox population ? >100
Number of steps ? >1000
Step size ? >1
```

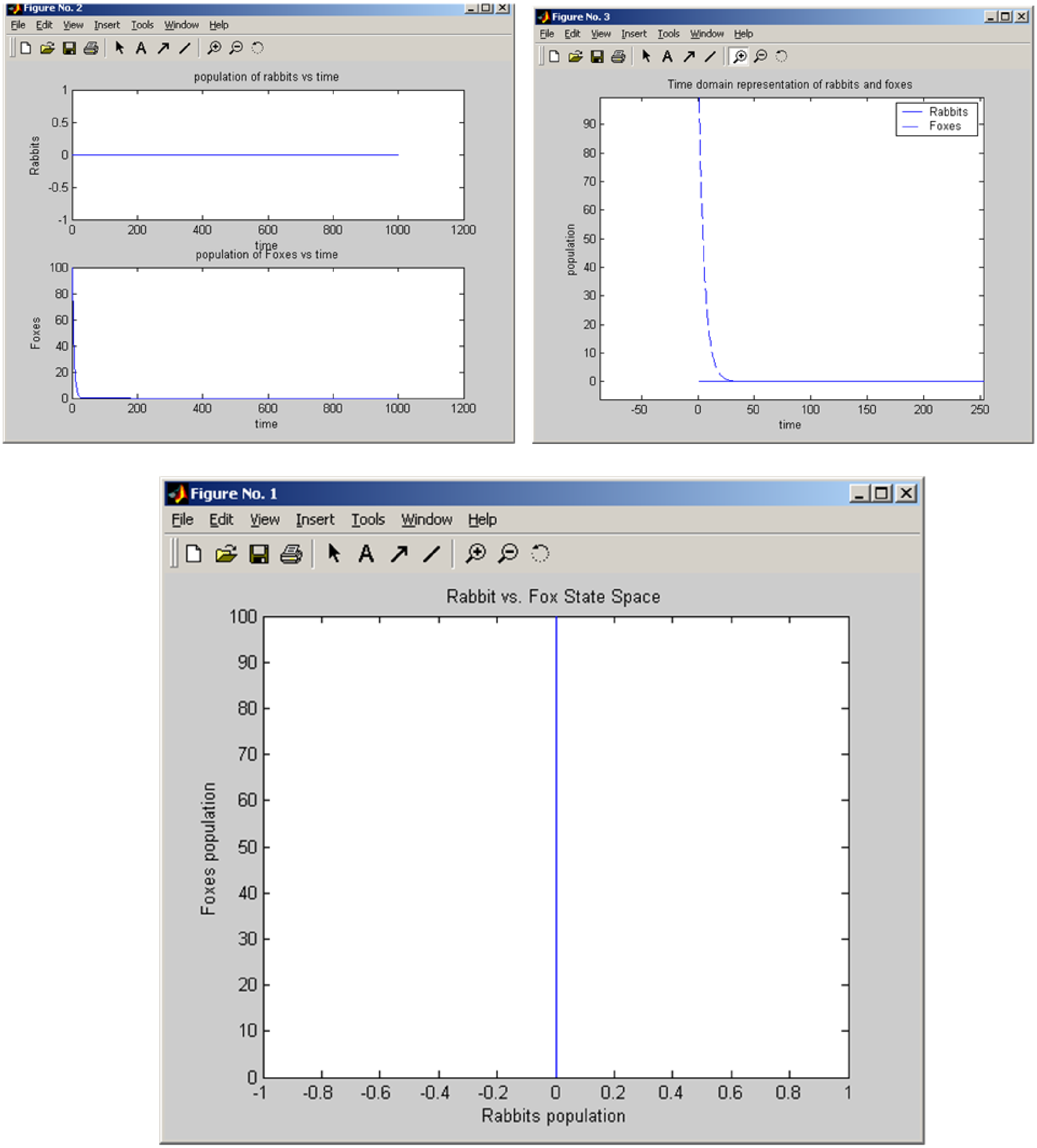


Figure 4: test3

```
>> nma_185_proj3
Initial rabbits population ? >5000
Initial fox population ? >60
Number of steps ? >1000
Step size ? >1
```

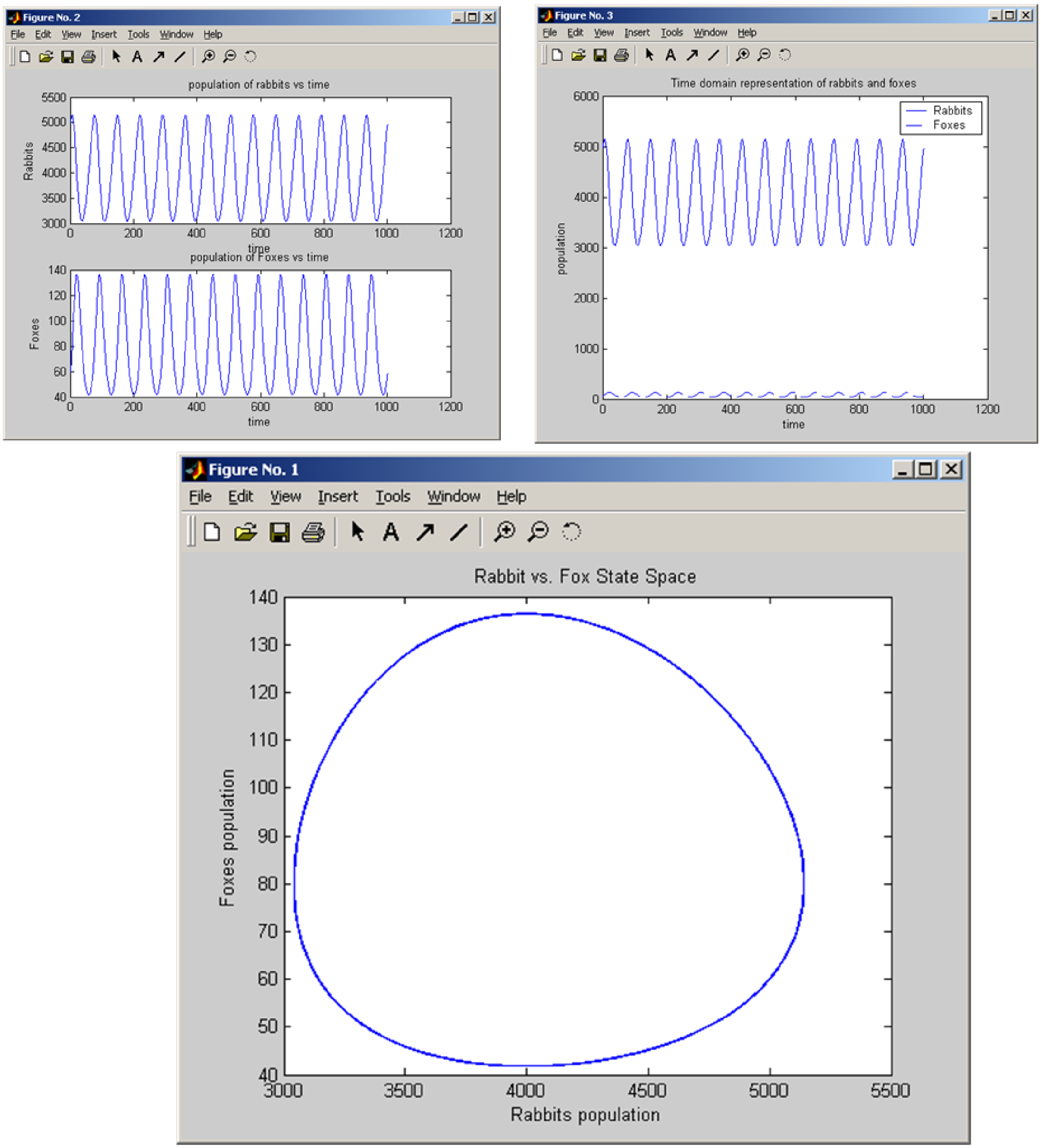
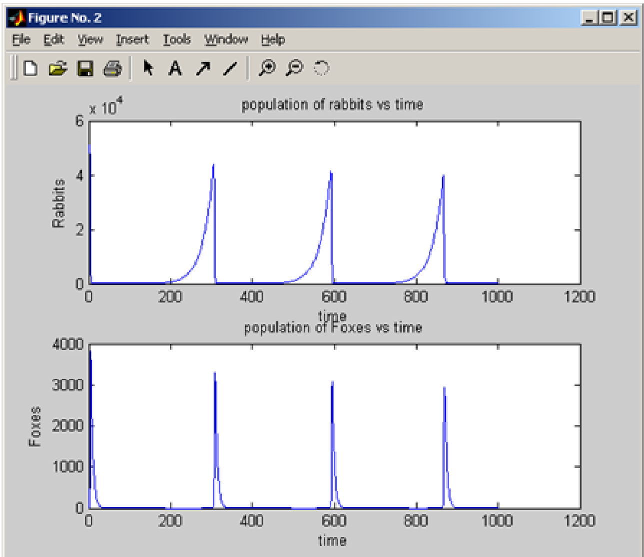
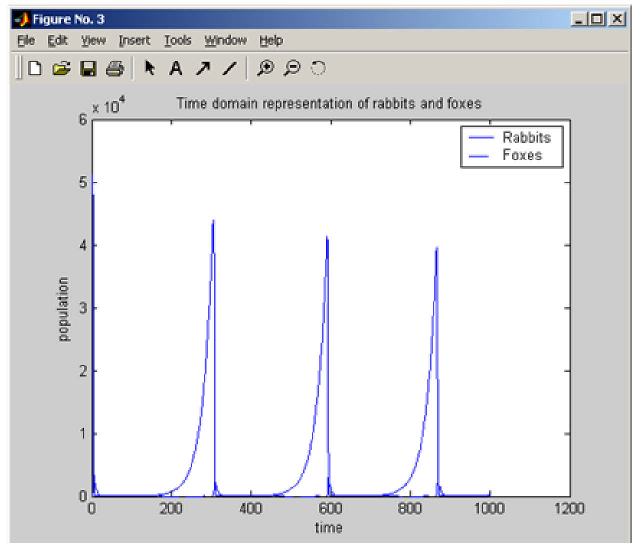


Figure 5: test4

```
>> nma_185_proj3
Initial rabbits population ? >50000
Initial fox population ? >10
Number of steps ? >1000
Step size ? >1
>>
```





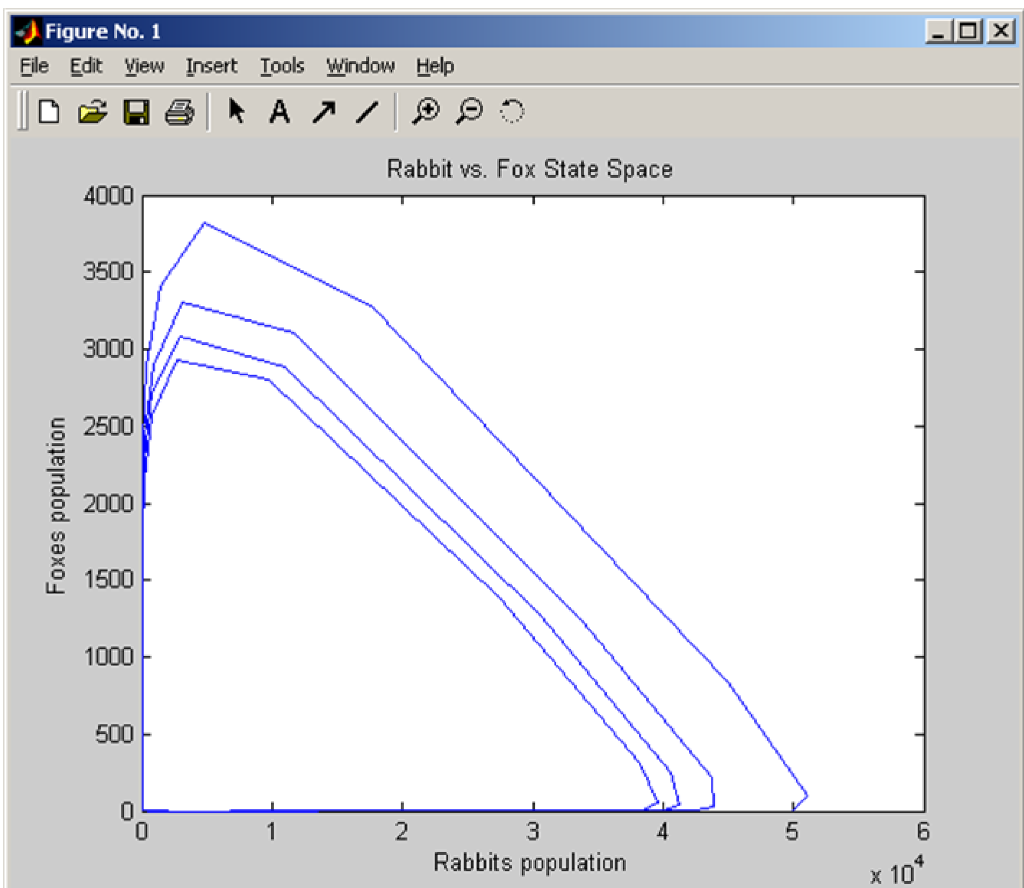


Figure 6: test5

```
>> nma_185_proj3
Initial rabbits population ? >70000
Initial fox population ? >10
Number of steps ? >1000
Step size ? >1
```

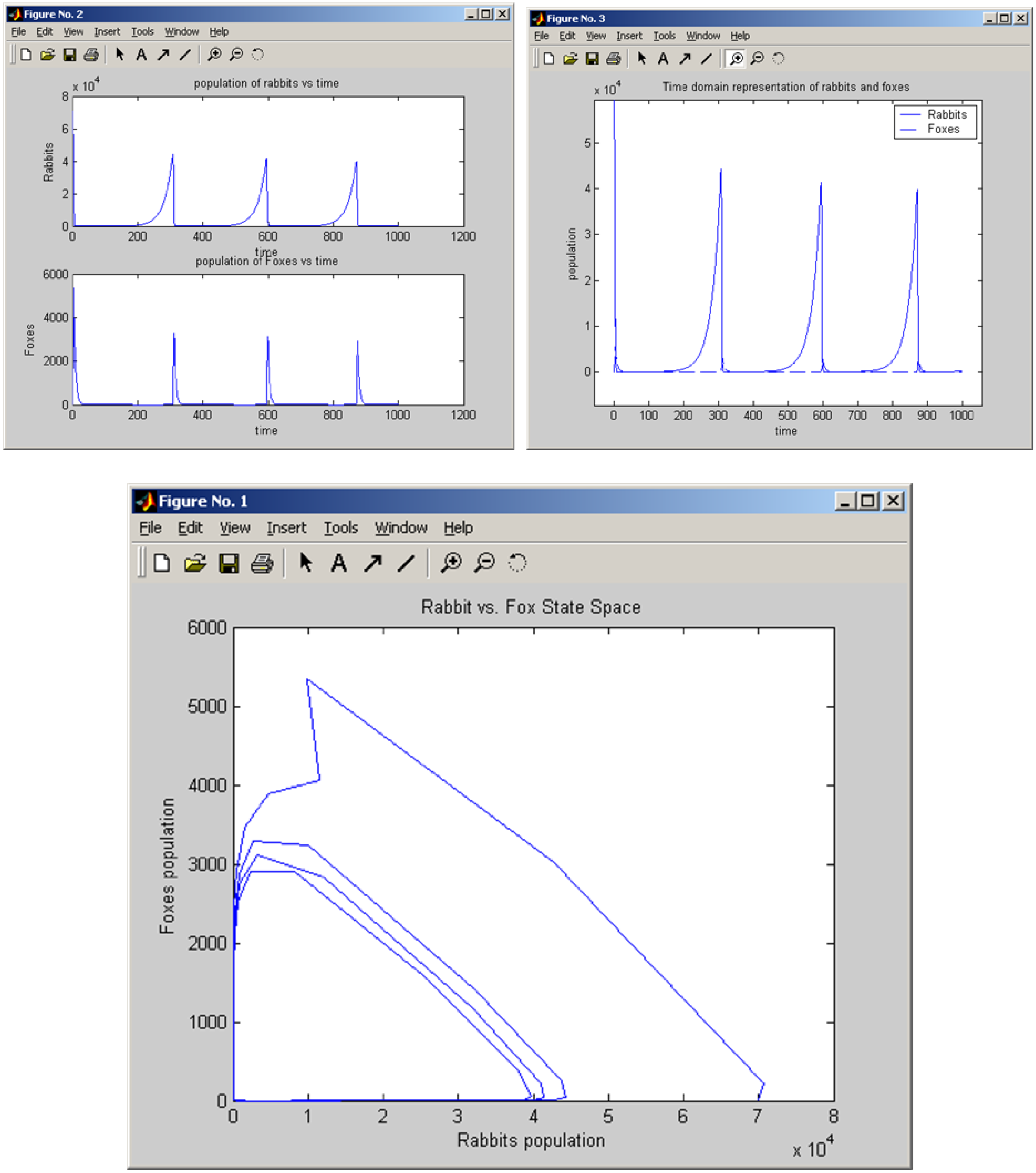


Figure 7: test6

```
>> nma_185_proj3
Initial rabbits population ? >10
Initial fox population ? >5000
Number of steps ? >1000
Step size ? >1
>>
```

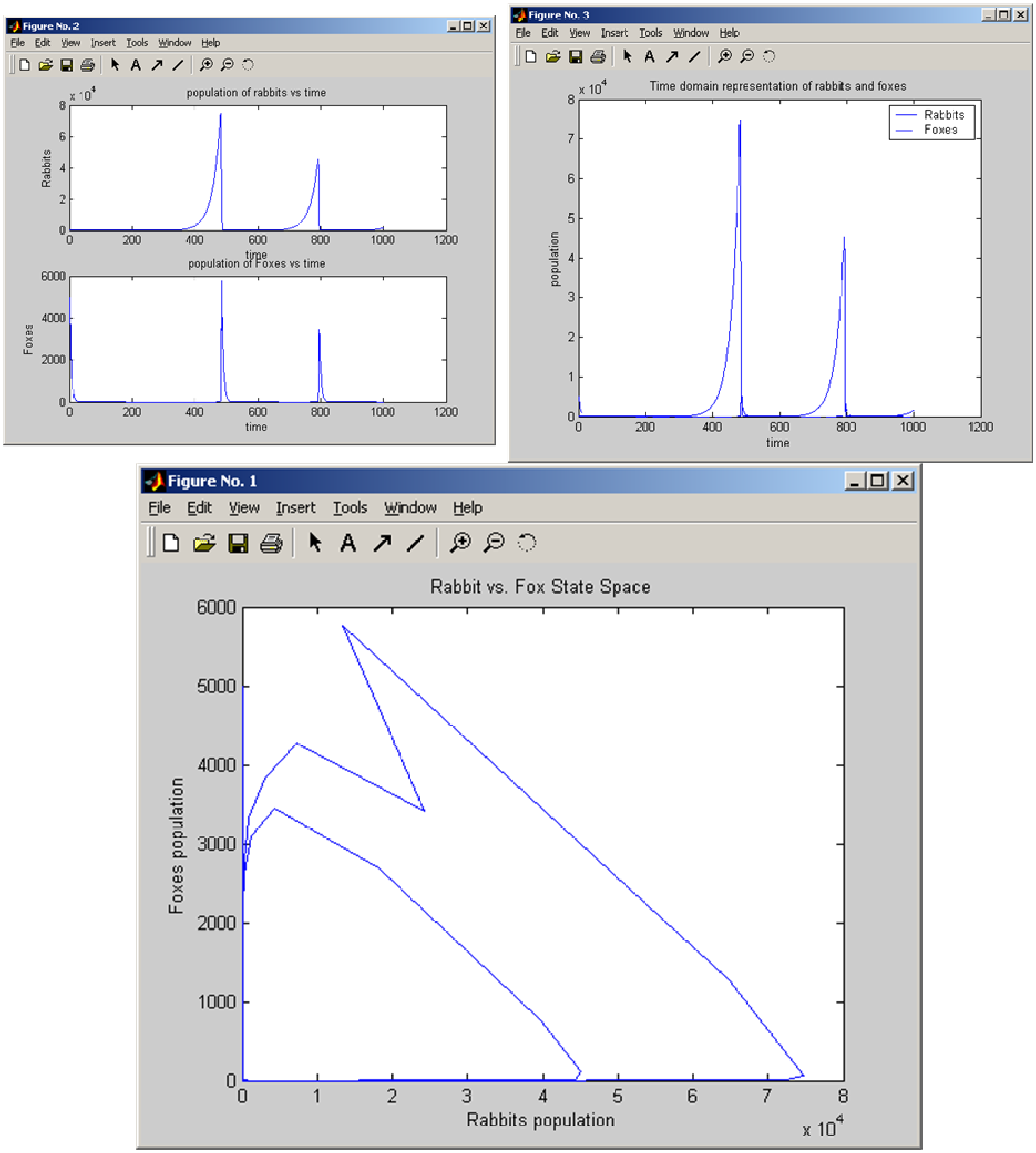


Figure 8: test7

```
>> nma_185_proj3
Initial rabbits population ? >40
Initial fox population ? >1000
Number of steps ? >1000
Step size ? >1
>>
```

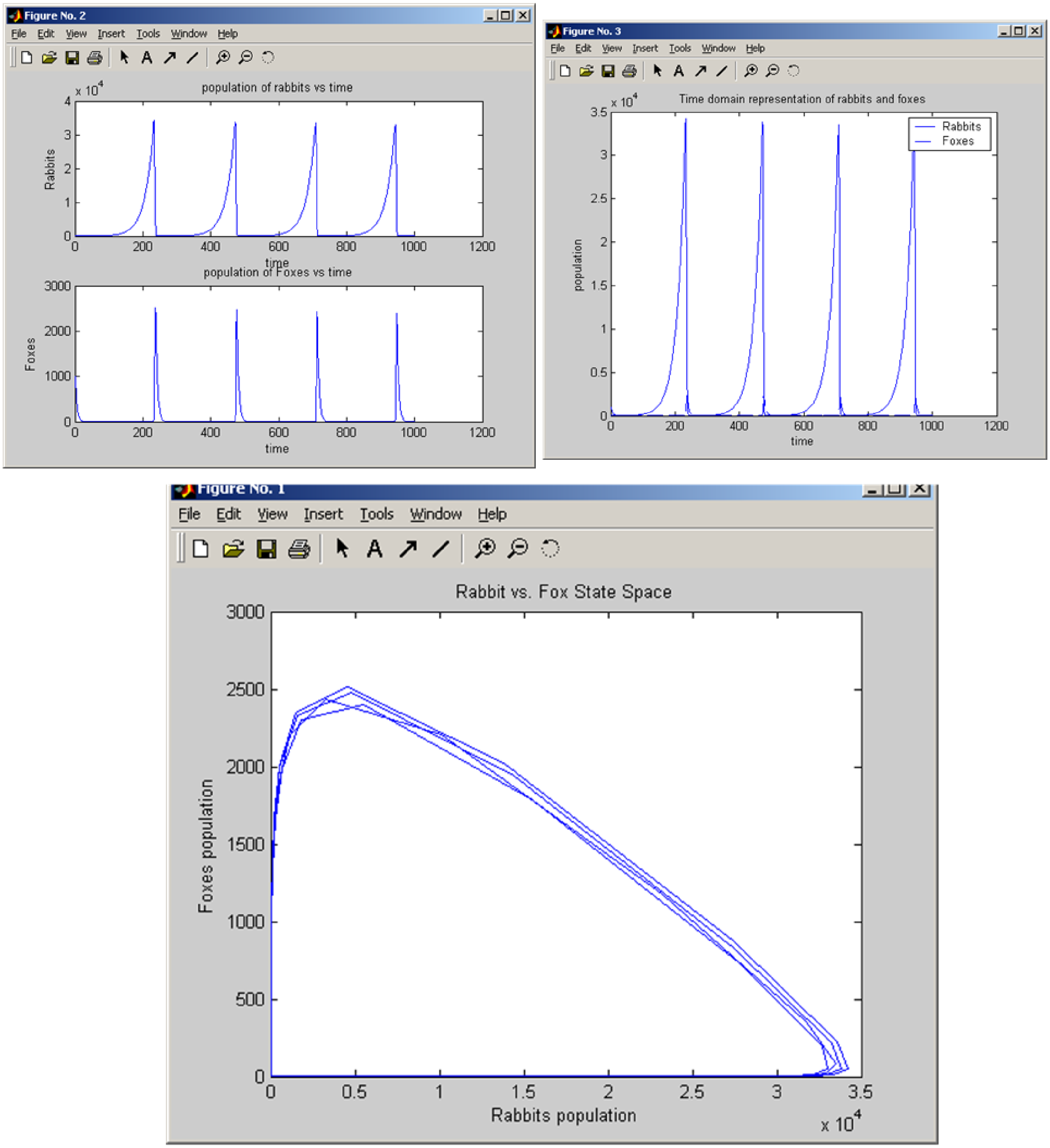


Figure 9: test8

2.5 Source code listing

The file `nma_185_proj3.m` is moved to my main matlab functions page here