Math2520-01 **Assignment 4**

INSTRUCTION: ***Show all the necessary work***. Write your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet on D2L by Sunday June 27. In this section you may need to remember the theorems and how to apply them to answer the questions.

1. Determine the null space of A and verify the Rank-Nullity Theorem.

$$A=\begin{bmatrix} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{bmatrix}$$

2. Using the definition of linear transformation, verify that the given transformation is linear.

$$T: R^2 \to R^2 \text{ defined by } T(x,y)=(x+2y, 2x-y).$$

3. Determine the matrix of the given linear transformation.

$$T: R^3 \to R^2 \text{ defined by } T(x,y,z)=(x-y+z, z-x)$$

4. Let $T: R^2 \to R^2$ be a linear transformation that maps $u=(5,2)$ into $(2,1)$ and $v=(1,3)$ into $(-1,3)$. Use the fact that *T* is linear to find the image under *T* of $3u+2v$.

5. Assume that *T* defines a linear transformation and use the given information to find the matrix of *T*.

   $T: R^2 \to R^4$ such that $T(0,1)=(1,0,-2,2)$ and $T(1,2)=(-3,1,1,1)$.

6. Find the Ker(T) and Rng(T) and their dimensions.

   $T: R^3 \to R^2$ defined by $T(x)=Ax$, where

$$A=\begin{bmatrix} 1 & -1 & 2 \\ -3 & 3 & -6 \end{bmatrix}.$$

7. Let $T: R^3 \to R^3$ be a linear transformation defined by $Tx = Ax$ where

$$A = \begin{bmatrix} 3 & 5 & 1 \\ 1 & 2 & 1 \\ 2 & 6 & 7 \end{bmatrix}.$$

Show that T is both one-to-one and onto.

8. Determine all eigenvalues and corresponding eigenvectors of the given matrix.

i) $A = \begin{bmatrix} 5 & -4 \\ 8 & -7 \end{bmatrix}.$

ii) $A = \begin{bmatrix} 7 & 4 \\ -1 & 3 \end{bmatrix}.$

iii) $A = \begin{bmatrix} 7 & 3 \\ -6 & 1 \end{bmatrix}$

9. If $v_1 = (1,-1)$ and $v_2 = (2,1)$ be eigenvectors of the matrix A corresponding to the eigenvalues $\lambda_1 = 2, \lambda_2 = -3$, respectively find $A(3v_1 - v_2)$.

10. Determine the multiplicity of each eigenvalue and a basis for each eigenspace of the given matrix A. Determine the dimension of each eigenspce and state whether the matrix is defective or nondefective.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

11. Determine whether the given matrix A is diagonalizable.

$$A = \begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix}$$

12. Determine the general solution to the given differential equation.

a) $y'' - y' - 2y = 0$

b) $y'' + 10y' + 25y = 0$

c) $y'' + 6y' + 11y = 0$