Math2520-01 **Assignment 2**

<u>INSTRUCTION</u>: Show all the necessary work. Write your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet as a PDF on D2L by Sunday June 13. Late June 14.

1. Let

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

Compute: a) $A + 2B$ b) A^T c) $A + C$ d) CB

If the expression is undefined explain why.

2. Determine the solution set to the given system. If using calculator, show three steps:
   1. Write the augmented matrix and determine the size of the matrix.
   2. Find the rref(augmented matrix)
   3. Write the solution.

a) $$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 3x_1 + 5x_2 + x_3 &= 3 \\ 2x_1 + 6x_2 + 7x_3 &= 1 \end{aligned}$$

b) $$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 &= 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 &= 2 \end{aligned}$$

If the system is inconsistent state why.

3. Given the following matrix function,

$$A(t) = \begin{bmatrix} -7 & t^2 \\ 1+t & \cos(\frac{\pi t}{2}) \end{bmatrix},$$

a) determine the derivative $\dfrac{dA}{dt}$,

b) determine $\int_0^1 A(t)dt$.

4. Write the system of equations with the given coefficient matrix and right-hand side vector.

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 1 & -2 & 6 \\ 3 & 1 & 4 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

5. Reduce to row-echelon form and determine the rank of the matrix.

a) $A = \begin{bmatrix} 2 & 1 & 4 \\ 2 & -3 & 4 \\ 3 & -2 & 6 \end{bmatrix}$ b) $B = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

6. Reduce the matrix to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix and list the pivot columns.

$$\begin{bmatrix} 1 & 3 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

7. Determine A^{-1}, if possible, using the Gauss-Jordan method.

a) $A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$ b) $B = \begin{bmatrix} 1 & 3 & -4 \\ -1 & -2 & 5 \\ 2 & 6 & 7 \end{bmatrix}$

8. Use A^{-1} to find the solution to the given system.

$$\begin{aligned} 6x_1 + 20x_2 &= -8 \\ 2x_1 + 7x_2 &= 4 \end{aligned}$$

9. Use the cofactor expansion theorem to evaluate the given determinant along the specified row or column.

a) $A = \begin{vmatrix} 2 & 1 & 4 \\ 2 & -3 & 4 \\ 3 & -2 & 6 \end{vmatrix}$, column 3 b) $B = \begin{bmatrix} 6 & -1 & 2 \\ -4 & 7 & 1 \\ 0 & 3 & 1 \end{bmatrix}$, second row

10. Find A^{-1} .

$$A = \begin{bmatrix} 3e^{t} & e^{2t} \\ 2e^{t} & 2e^{2t} \end{bmatrix}$$

11. Let A and B be 3×3 matrices with $\det(A) = 3$ and $\det(B) = -4$. Compute $\det(B^{-1}AB)^{2}$.

12. Use Cramer's rule to solve the given linear system.

$$\begin{aligned} 3x_1 - 2x_2 &= 6 \\ -5x_1 + 4x_2 &= 8 \end{aligned}$$