Math2520-01 **Assignment I**

INSTRUCTION: Show all the necessary work. Writ your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet on D2L by Monday June 6. Late June 7.

1. Determine the order of the differential equation.

   a) $\left(\frac{dy}{dx}\right)^3 + y^2 = \sin x$ b) $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin(t)$

2. Verify that for $t > 0$, $y(t) = \ln t$ is a solution to the differential equation

$$2\left(\frac{dy}{dt}\right)^3 = \frac{d^3y}{dt^3}.$$

3. Determine whether the differential equation is linear or nonlinear.

   a) $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \sin x\frac{dy}{dx} = xy^2 + \tan x.$ b) $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin(t)$

4. Prove (show) that the initial-value problem

$$y' = x\sin(x+y),\ \ y(0) = 1$$

   has a unique solution using the existence and uniqueness theorem.

5. Let

$$y' = (y-2)(y+1).$$

   a) Determine all equilibrium solutions.
   b) Determine the region in the xy-plane where the solutions are increasing, and where the solutions are decreasing.
   c)

6. Solve the following differential equations.

   a) $\frac{dy}{dx} = \frac{y}{x\ln x}$ b) $(x^2+1)y' + y^2 = -1,\ y(0) = 1$

7. Solve the following differential equations.

   a) $\frac{dy}{dx} + \frac{2}{x}y = 5x^2,\ x > 0.$ b) $t\frac{dx}{dt} + 2x = 4e^t,\ t > 0$

8. A container initially containing10 L of water in which there is 20 g of salt dissolved. A solution containing 4 g/L of salt is pumped into the container at a rate of 2 L/min, and the well-stilled mixture runs out at a rate of 1 L/min. How much salt is in the tank after 40 min?

9. Consider the RC circuit (See page 65 in the text) which has R = 5 Ω, C = $\frac{1}{50}$ F and $E(t) = 100$ V. If the capacitor is uncharged initially, determine the current in the circuit for $t \geq 0$.

10. Solve the initial-value problem.

$$\frac{dy}{dx} = \frac{2x - y}{x + 4y}, \quad y(1) = 1$$

11. Solve the given differential equation.

$$y' + 2x^{-1}y = 6y^2x^4$$

12. Determine whether the given differential equation is exact. Show the work.

$$2xe^y dx + (3y^2 + x^2e^y)dy = 0$$

13. Solve the given differential equation.

$$(y^2 + \cos x)dx + (2xy + \sin y)dy = 0$$

14. Determine an integrating factor for the given differential equation and hence find the general solution.

$$(xy - 1)dx + x^2 dy = 0$$