

HW 7
EE 409 (Linear Systems), CSUF spring 2010
Spring 2010
CSUF

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- 6.3. A linear system is described by the following differential equation. This system is forced with an input as shown in the graph. Find the output of the system.

$$\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = u(t), \quad y(0) = 0, \quad y^{(1)}(0) = 1$$

$$\text{Answer: } (e^{-t} - e^{-2t})\xi(t) + \frac{1}{2}[1 - 2e^{-(t-1)} + e^{-2(t-1)}]\xi(t-1)$$

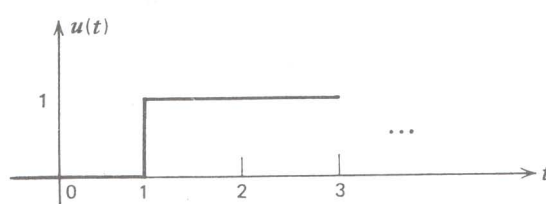


Figure 1: Problem description

$$y''(t) + 3y'(t) + 2y(t) = u(t)$$

Using the Laplace approach. First we note that the input is a delayed step input, hence $u(t) = \xi(t-1)$ where $\xi(t)$ is the unit step function. Laplace transform of a delayed unit step is

$$\int_0^{\infty} \xi(t-1) e^{-st} dt = \int_1^{\infty} e^{-st} dt = \frac{[e^{-st}]_1^{\infty}}{-s} = \frac{e^{-s}}{s}$$

Applying the Laplace transformation on the ODE gives

$$\begin{aligned} s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) &= \frac{e^{-s}}{s} \\ s^2Y(s) - 1 + 3sY(s) + 2Y(s) &= \frac{e^{-s}}{s} \\ Y(s)(s^2 + 3s + 2) - 1 &= \frac{e^{-s}}{s} \\ Y(s) &= \frac{1}{s^2 + 3s + 2} + \frac{e^{-s}}{s(s^2 + 3s + 2)} \end{aligned} \quad (1)$$

Considering the first term on the RHS of (1), calling it $Y_1(s) = \frac{1}{s^2+3s+2}$, and using partial fractions gives

$$\begin{aligned} Y_1(s) &= \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \\ A &= \lim_{s \rightarrow -1} \frac{1}{(s+2)} = 1 \\ B &= \lim_{s \rightarrow -2} \frac{1}{(s+1)} = -1 \end{aligned}$$

Hence

$$Y_1(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

Considering the second term on the RHS, calling it $Y_2(s) = \frac{e^{-s}}{s(s^2+3s+2)}$, and using partial fractions gives

$$\frac{Y_2(s)}{e^{-s}} = \frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A = \lim_{s \rightarrow 0} \frac{1}{(s+1)(s+2)} = \frac{1}{2}$$

$$B = \lim_{s \rightarrow -1} \frac{1}{s(s+2)} = -1$$

$$C = \lim_{s \rightarrow -2} \frac{1}{s(s+1)} = \frac{1}{2}$$

Hence

$$\frac{Y_2(s)}{e^{-s}} = \frac{1}{2} \frac{1}{s} - \frac{1}{s+1} + \frac{1}{2} \frac{1}{s+2}$$

Therefore

$$\begin{aligned} Y(s) &= Y_1(s) + Y_2(s) \\ &= \left(\frac{1}{s+1} - \frac{1}{s+2} \right) + \left(\frac{1}{2} \frac{e^{-s}}{s} - \frac{e^{-s}}{s+1} + \frac{1}{2} \frac{e^{-s}}{s+2} \right) \end{aligned} \quad (2)$$

The effect of e^{-as} is to cause a time delay when finding the inverse Laplace transform.

$$e^{-as}F(s) \rightarrow f(t-a)\xi(t-a)$$

Now, taking the inverse Laplace transform of (2) gives the solution

$$\begin{aligned} y(t) &= e^{-t}\xi(t) - e^{-2t}\xi(t) + \frac{1}{2}\xi(t-1) - e^{-(t-1)}\xi(t-1) + \frac{1}{2}e^{-2(t-1)}\xi(t-1) \\ &= (e^{-t} - e^{-2t})\xi(t) + \frac{1}{2}\left(1 - 2e^{-(t-1)} + e^{-2(t-1)}\right)\xi(t-1) \end{aligned}$$