

HW 5  
EE 409 (Linear Systems), CSUF spring 2010  
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CSUF

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## 1 Problem 3.23 (a)

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- 3.27. For the block diagram systems shown below, find
- The matrices  $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$  of the state-variable description.
  - The matrix  $e^{\mathbf{A}t}$ .
  - The matrix  $(j\omega\mathbf{I} - \mathbf{A})^{-1}$ .
  - The frequency-response function, with a sketch of the amplitude and phase responses.
  - The impulse-response function, with a sketch.

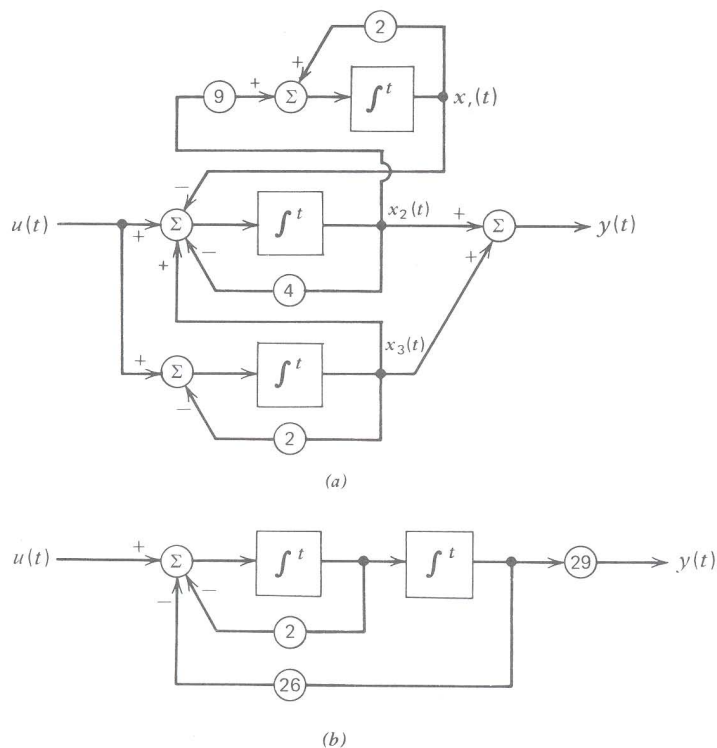


Figure 1: Problem description

## 1.1 Part(a)

Labeling the output from the branches as follows

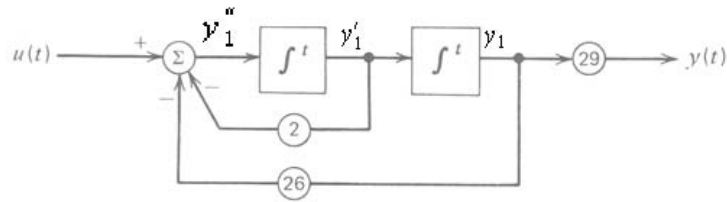


Figure 2: Problem description part(a) labeled

Then the differential equation becomes

$$y_1'' = u - 2y_1' - 26y_1$$

While the output equation become

$$y = 29y_1$$

Let  $x_1 = y_1$

$$\left. \begin{array}{l} x_1 = y_1 \\ x_2 = y_1' \end{array} \right\} \rightarrow \left. \begin{array}{l} x_1' = y_1' = x_2 \\ x_2' = y_1'' = u - 2y_1' - 26y_1 = u - 2x_2 - 26x_1 \end{array} \right\}$$

Hence

$$\begin{aligned} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} &= \overbrace{\begin{pmatrix} 0 & 1 \\ -26 & -2 \end{pmatrix}}^A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}^B u(t) \\ y(t) &= \overbrace{\begin{pmatrix} 29 & 0 \end{pmatrix}}^C \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{(0)}^D u(t) \end{aligned}$$

## 1.2 Part b

To find  $e^{At}$  use the eigenvalue approach. Find find  $|A - \lambda I|$

$$|A - \lambda I| = \left| \begin{pmatrix} 0 & 1 \\ -26 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = \left| \begin{pmatrix} -\lambda & 1 \\ -26 & -2 - \lambda \end{pmatrix} \right| = -\lambda(-2 - \lambda) + 26$$

Now solve  $-\lambda(-2 - \lambda) + 26 = 0$  or  $\lambda^2 + 2\lambda + 26 = 0$ , which has solutions

$$\lambda_1 = -1 + 5j$$

$$\lambda_2 = -1 - 5j$$

Hence we have the following 2 equations to solve for  $\beta_0$  and  $\beta_1$

$$e^{\lambda_1 t} = \beta_0 + \lambda_1 \beta_1$$

$$e^{\lambda_2 t} = \beta_0 + \lambda_2 \beta_1$$

Solving we find

$$\beta_0 = e^{-t} \left( \cos 5t + \frac{1}{5} \sin 5t \right)$$

$$\beta_1 = \frac{1}{5} e^{-t} \sin 5t$$

Hence

$$\begin{aligned} e^{At} &= \beta_0 + \beta_1 A \\ &= e^{-t} \left( \cos 5t + \frac{1}{5} \sin 5t \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{5} e^{-t} \sin 5t \begin{pmatrix} 0 & 1 \\ -26 & -2 \end{pmatrix} \\ &= e^{-t} \begin{pmatrix} \cos 5t + \frac{1}{5} \sin 5t & \frac{1}{5} \sin 5t \\ \frac{-26}{5} \sin 5t & \cos 5t - \frac{1}{5} \sin 5t \end{pmatrix} \end{aligned}$$

### 1.3 Part c

To find matrix  $(j\omega I - A)^{-1}$

$$\begin{aligned} j\omega I - A &= j\omega \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -26 & -2 \end{pmatrix} \\ &= \begin{pmatrix} j\omega & 0 \\ 0 & j\omega \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -26 & -2 \end{pmatrix} \\ &= \begin{pmatrix} j\omega & -1 \\ 26 & j\omega + 2 \end{pmatrix} \end{aligned}$$

Hence

$$\begin{aligned} \begin{pmatrix} j\omega & -1 \\ 26 & j\omega + 2 \end{pmatrix}^{-1} &= \frac{\begin{pmatrix} j\omega + 2 & 1 \\ -26 & j\omega \end{pmatrix}}{(j\omega)(j\omega + 2) + 26} = \frac{\begin{pmatrix} j\omega + 2 & 1 \\ -26 & j\omega \end{pmatrix}}{-\omega^2 + 2j\omega + 26} \\ &= \frac{1}{-\omega^2 + 2j\omega + 26} \begin{pmatrix} j\omega + 2 & 1 \\ -26 & j\omega \end{pmatrix} \end{aligned}$$

### 1.4 Part d

To find the frequency response function. Assuming zero initial conditions, from equation 3.10.4 in the book

$$\begin{aligned}
 H(j\omega) &= C(j\omega I - A)^{-1} B \\
 &= \begin{pmatrix} 29 & 0 \end{pmatrix} \frac{1}{-\omega^2 + 2j\omega + 26} \begin{pmatrix} j\omega + 2 & 1 \\ -26 & j\omega \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
 &= \frac{1}{-\omega^2 + 2j\omega + 26} \begin{pmatrix} 29 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ j\omega \end{pmatrix} \\
 &= \frac{29}{-\omega^2 + 2j\omega + 26}
 \end{aligned}$$

Hence

$$|H(j\omega)| = \frac{29}{|-\omega^2 + 2j\omega + 26|} = \frac{29}{\sqrt{(26 - \omega^2)^2 + 4\omega^2}}$$

And phase is

$$\begin{aligned}
 \arg(H(j\omega)) &= \arg(29) - \arg(-\omega^2 + 2j\omega + 26) \\
 &= -\tan^{-1} \frac{2\omega}{26 - \omega^2}
 \end{aligned}$$

### 1.5 Part e

The state solution is

$$x(t) = \int_0^t e^{A\tau} B u(\tau) d\tau$$

and

$$y(t) = Cx(t) = \int_0^t C e^{A\tau} B u(\tau) d\tau$$

Hence, let  $u(\tau) = \delta(t)$ , then

$$\begin{aligned}
 h(t) &= C e^{At} B \\
 &= \begin{pmatrix} 29 & 0 \end{pmatrix} e^{-t} \begin{pmatrix} \cos 5t + \frac{1}{5} \sin 5t & \frac{1}{5} \sin 5t \\ \frac{-26}{5} \sin 5t & \cos 5t - \frac{1}{5} \sin 5t \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
 &= e^{-t} \begin{pmatrix} 29 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{5} \sin 5t \\ \cos 5t - \frac{1}{5} \sin 5t \end{pmatrix} \\
 &= e^{-t} \left( \frac{29}{5} \sin 5t \right) \xi(t)
 \end{aligned}$$