

HW 3
EGEE 518 Digital Signal Processing I
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1 my solution

①

Definitions


① auto correlation $R_{xx}(n, n+m)$: Measures the similarity of R.P. $x(t)$ at time n and $x(t)$ at later time $n+m$.

$$R_{xx}(n, n+m) = E\{X(n) X^*(n+m)\}$$

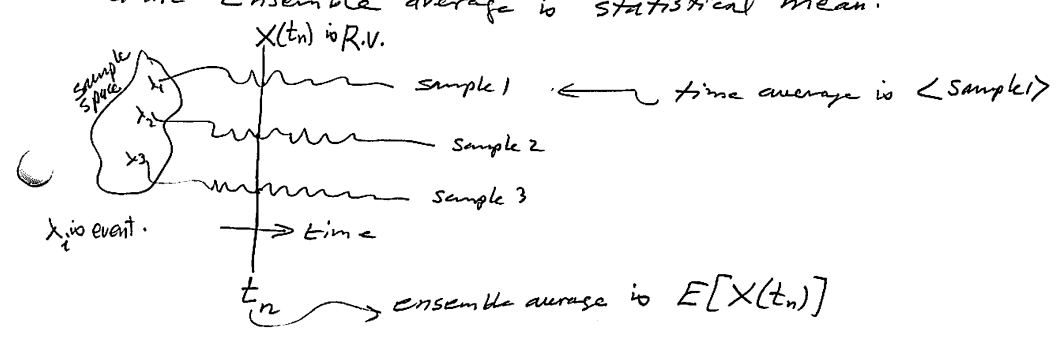
② stationary process.
This is a random process whose statistics do not change with shift in time origin.

③ Wide Sense Stationary process:
This is a random process $X(t)$ which satisfies the following conditions:

1. its mean is constant. i.e. $E[X] = \text{constant}$.
2. auto correlation depends only on time interval 'm'.
i.e. $R_{xx}(n, n+m) = R_{xx}(m)$.

Notice that stationary process is WSS, but WSS is not necessarily stationary. i.e. 

④ Time averages, Ensemble averages
Time averages is the average of the sample sequence, while Ensemble average is statistical mean.



$X(t_n)$ is R.V.

sample 1 ← time average is $\langle \text{sample} \rangle$

sample 2

sample 3

x_i is event.

time

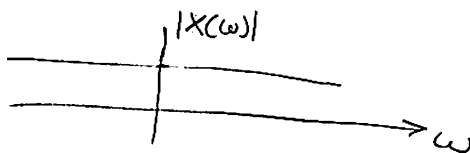
t_n → ensemble average is $E[X(t_n)]$

⑤ White Noise:

this is a R.P. whose power spectral density is constant.

i.e. power contained in a frequency bandwidth B is the same regardless of where this bandwidth is centered.

"flat" spectrum
implies $X(t)$ is
white noise process.



The above is a description in the frequency domain.

In the time domain, $\Phi_{XX}(m) = \delta(m)$ i.e.

the autocorrelation is non-zero only if time interval

is zero. i.e. $X(t)$ only correlates with itself at

zero time delay. So all R.V. that belong to a white noise process are uncorrelated with each other if time interval is non-zero.

⑥ Ergodic Process:

This is a R.P. whose statistics taken from the time samples are the same as statistics taken from Ensembles.

For example. We say a process is Ergodic in the mean, then

$$E\{X(t)\} = \langle X(t) \rangle$$

Statistical sample.
expected value of
R.V.

time average.
mean of a sample (or
time series)

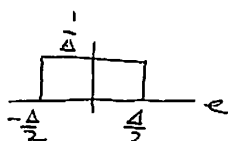
The above equality is in the limit, i.e. as the time series length increases, and the statistical mean is when the number of time series increases as well.

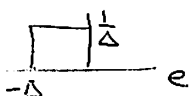
#3

(3)

$$y(n) = Q[x(n)] = x(n) + e(n) \rightarrow \text{quantization Error}$$

$e(n)$ is white noise.

Pdf for rounding is uniform 

Pdf for truncation is 

- a) Find mean and Variance due to rounding
 b) " " " " " " truncation.

Answer

$$a) m_e = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e \cdot f(e) de = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{e}{\Delta} de = \frac{1}{\Delta} \left(\frac{e^2}{2} \right)_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

$$= \frac{1}{2\Delta} \left[\left(\frac{\Delta}{2} \right)^2 - \left(-\frac{\Delta}{2} \right)^2 \right] = \frac{1}{2\Delta} (0) = \boxed{0}$$

$$E[e^2] = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^2 f(e) de = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^2 de = \frac{1}{\Delta} \left(\frac{e^3}{3} \right)_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

$$= \frac{1}{3\Delta} \left[\left(\frac{\Delta}{2} \right)^3 - \left(-\frac{\Delta}{2} \right)^3 \right] = \frac{1}{3\Delta} \left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right] = \frac{1}{3\Delta} \left[\frac{\Delta^3}{4} \right]$$

$$= \boxed{\frac{\Delta^2}{12}}$$

$$\text{so } \sigma^2 = E[e^2] - (E[e])^2 = \frac{\Delta^2}{12} - 0^2 = \boxed{\frac{\Delta^2}{12}}$$

$$b) m_e = \int_{-\Delta}^0 e f(e) de = \int_{-\Delta}^0 e \frac{1}{\Delta} de = \frac{1}{\Delta} \left(\frac{e^2}{2} \right)_{-\Delta}^0 = \frac{1}{2\Delta} (0^2 - (-\Delta)^2)$$

$$= \frac{1}{2\Delta} (0 - \Delta^2) = \boxed{-\frac{\Delta}{2}}$$

$$E[e^2] = \int_{-\Delta}^0 e^2 f(e) de = \int_{-\Delta}^0 e^2 \frac{1}{\Delta} de = \frac{1}{\Delta} \left[\frac{e^3}{3} \right]_{-\Delta}^0$$

$$= \frac{1}{3\Delta} [0^3 - (-\Delta)^3] = \boxed{\frac{\Delta^2}{3}}$$

$$\text{so } \sigma^2 = E[e^2] - (E[e])^2 = \frac{\Delta^2}{3} - \left(-\frac{\Delta}{2} \right)^2 = \frac{\Delta^2}{3} - \frac{\Delta^2}{4} = \frac{4\Delta^2 - 3\Delta^2}{12} = \boxed{\frac{\Delta^2}{12}}$$

4 let $e(n)$ white Noise sequence. Let $s(n)$ uncorrelated $\textcircled{4}$ sequence to $e(n)$. Show that $y(n) = s(n)e(n)$ is white. i.e. $E[y(n)y(n+m)] = A \delta(m)$.

Answer

$$\begin{aligned} E[y(n)y(n+m)] &= E[s(n)e(n)s(n+m)e(n+m)] \\ &= E[s(n)s(n+m)e(n)e(n+m)] \end{aligned}$$

Since $e(n)$ and $s(n)$ are uncorrelated, hence independent, then we can write the above as

$$= E[s(n)s(n+m)] E[e(n)e(n+m)]$$

but $e(n)$ is white. hence $E[e(n)e(n+m)] = \delta(m)$ by definition of white signal.

$$\text{hence } \phi_{yy}(n,m) = E[s(n)s(n+m)] \delta(m).$$

$$\text{Now, when } m=0, \phi_{yy}(n,m) = E[s(n)s(n)] \cdot 1.$$

Since $s(n)$ is uncorrelated with white Noise, then $m_s=0$ since $s(n)$ is also white.

$$\begin{aligned} \text{hence } E[s^2(n)] &= \text{Total average power in } s(n) \\ &= A \text{ some constant.} \end{aligned}$$

$$\text{hence when } m=0, \phi_{yy}(n,m) = A$$

$$\begin{aligned} \text{when } m \neq 0 \phi_{yy}(n,m) &= E[s(n)s(n+m)] \cdot 0 \\ &= 0 \end{aligned}$$

$$\text{Therefore } \boxed{\phi_{yy}(n,m) = A \delta(m)}$$

Since $\phi_{yy}(n,m)$ is function of only m , it is white signal.

#6 Consider 2 real stationary random processes $\{X_n\}$ and $\{Y_n\}$, with mean m_x, m_y , and variance σ_x^2, σ_y^2 .

(a) $\gamma_{xx}(m)$. This is autocovariance .

$$\begin{aligned}\gamma_{xx}(m) &= E\{(X(n) - m_x)(X(n+m) - m_x)\} \\ &= E\{X(n)X(n+m) - m_x X(n) - m_x X(n+m) + m_x^2\} \\ &= E\{X(n)X(n+m)\} - m_x E\{X(n)\} - m_x E\{X(n+m)\} + m_x^2 \\ &= \phi_{xx}(n, n+m) - m_x^2 - m_x E\{X(n+m)\} + m_x^2 \\ &= \phi_{xx}(n, n+m) - m_x E\{X(n+m)\}.\end{aligned}$$

but $\{X_n\}$ is stationary, so its statistics do not change with shift of time origin. hence $E\{X(n+m)\} = E\{X(n)\} = m_x$.

so above becomes

$$\gamma_{xx}(m) = \phi_{xx}(n, n+m) - m_x^2.$$

but $\phi_{xx}(n, n+m) = \phi_{xx}(m)$ since stationary hence

$$\boxed{\gamma_{xx}(m) = \phi_{xx}(m) - m_x^2}$$

$$\begin{aligned}\gamma_{xy}(m) &= E\{(X(n) - m_x)(Y(n+m) - m_y)\} \\ &= E\{X(n)Y(n+m) - m_y X(n) - m_x Y(n+m) + m_y m_x\} \\ &= E\{X(n)Y(n+m)\} - m_y E\{X(n)\} - m_x E\{Y(n+m)\} + m_y m_x \\ \text{but due to stationarity, } E\{Y(n+m)\} &= m_y. \text{ so above becomes} \\ &= E\{X(n)Y(n+m)\} - m_y m_x - m_x m_y + m_y m_x \\ &= E\{X(n)Y(n+m)\} - m_y m_x.\end{aligned}$$

but $E\{X(n)Y(n+m)\} = \phi_{xy}(m)$ since stationary

$$\text{so } \boxed{\gamma_{xy}(m) = \phi_{xy}(m) - m_y m_x} \rightarrow$$

$$(b) \Phi_{XX}(0) = E\{X(n)X^*(n+m)\}$$

but $m=0$. hence

$$\Phi_{XX}(0) = E\{X(n)X^*(n)\} = E\{X^2(n)\}.$$

= mean square.

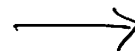
$$\gamma_{XX}(0) = E\{(X(n) - m_X)(X^*(n+m) - m_X^*)\}$$

but $m=0$ so

$$\begin{aligned} \gamma_{XX}(0) &= E\{(X(n) - m_X)(X^*(n) - m_X^*)\} \\ &= E\{X^2(n) - X(n)m_X - m_X X^*(n) + m_X^2\} \\ &= E\{X^2(n)\} - m_X E\{X(n)\} - m_X E\{X^*(n)\} + m_X^2 \\ &= E\{X^2(n)\} - m_X^2 - m_X^2 + m_X^2 \\ &= E\{X^2(n)\} - m_X^2 \end{aligned}$$

but this is the definition of σ_X^2 . hence

$$\boxed{\gamma_{XX}(0) = \sigma_X^2}$$



$$(c) \Phi_{XX}(m) = E\{x(n)x^*(n+m)\} = E\{x_{n+m}^*x_n\} = \left(E\{x_{n+m}x_n^*\}\right)^* \quad (7)$$

$$= \Phi_{XX}^*(-m)$$

if process is real, then $\Phi_{XX}^*(-m) = \Phi_{XX}(-m)$.

$$\text{i.e. } \Phi_{XX}(m) = \Phi_{XX}(-m)$$

$$\begin{aligned} \gamma_{XX}(m) &= E\{(x(n)-m_x)(x^*(n+m)-m_x^*)\} \\ &= \Phi_{XX}(m) - m_x m_x^* \quad (\text{From part (a)}) \quad (1) \\ &= \Phi_{XX}^*(-m) - m_x m_x^* \quad (\text{Using result above}) \\ &= \left(E\{x_{n+m}x_n^*\}\right)^* - m_x m_x^* \\ &= E\{x_{n+m}^*x_n\} - m_x m_x^* \\ &= \left(E\{x_{n+m}x_n^*\} - m_x^* m_x\right)^* \\ &= \gamma_{XX}^*(-m) \end{aligned}$$

if Real process, then $\gamma_{XX}^*(-m) = \gamma_{XX}(-m) \Rightarrow \boxed{\gamma_{XX}(m) = \gamma_{XX}(-m)}$

~~$$\begin{aligned} \Phi_{xy}(m) &= E\{(x(n)-m_x)(y(n+m)-m_y)\} \\ &= E\{x(n)y(n+m) - m_y x(n) - m_x y(n+m) + m_x m_y\} \\ &= E\{x(n)y(n+m)\} - m_y m_x - m_x m_y + m_x m_y \\ &= E\{x(n)y(n+m)\} - m_y m_x \end{aligned}$$~~

~~$$\begin{aligned} \text{But } \Phi_{yx}(-m) &= E\{(y(n)-m_y)(x(n-m)-m_x)\} \\ &= E\{y(n)x(n-m)\} - m_x m_y - m_y m_x + m_y m_x \\ &= E\{y(n)x(n-m)\} - m_x m_y \end{aligned}$$~~

Since sequences $x(n)$ and $y(n)$ are real, then $\Phi_{yx}^*(-m) = \Phi_{yx}(-m)$.

~~$$\text{So } \Phi_{yx}^*(-m) = E\{x(n-m)y(n)\} - m_x m_y \quad \rightarrow$$~~

part (c) cont.

show that $\Phi_{xy}(m) = \Phi_{yx}^*(-m)$.

$$\begin{aligned} \Phi_{xy}(m) &= E\{X_n y_{n+m}^*\} = E\{y_{n+m}^* X_n\} = \left(E\{y_{n+m} X_n^*\}\right)^* \\ &= \Phi_{yx}^*(-m) \end{aligned}$$

show that $\gamma_{xy}(m) = \gamma_{yx}^*(-m)$

$$\begin{aligned} \gamma_{xy}(m) &= E\{(X_n - m_x)(y_{n+m}^* - m_y^*)\} \\ &= E\{(y_{n+m}^* - m_y^*)(X_n - m_x)\} \\ &= \left(E\{(y_{n+m} - m_y)(X_n^* - m_x^*)\}\right)^* \\ &= \gamma_{yx}^*(-m) \end{aligned}$$

part (d)

(9)

show that $|\Phi_{xy}(m)| \leq \sqrt{\Phi_{xx}(0) \Phi_{yy}(0)}$

$$\Phi_{xy}(m) = E\{x_n y_{n+m}^*\}$$

$$\Phi_{xx}(0) = E\{x_n^2\}$$

$$\Phi_{yy}(0) = E\{y_n^2\}$$

we did this in class as follows:

$$\begin{aligned} 0 \leq E\{(x_n + a y_{n+m})^2\} &= E\{x_n^2 + a^2 y_{n+m}^2 + 2a x_n y_{n+m}\} \\ &= E(x_n^2) + a^2 E(y_{n+m}^2) + 2a E(x_n y_{n+m}) \end{aligned}$$

$$= \Phi_{xx}(0) + a^2 \Phi_{yy}(0) + 2a \Phi_{xy}(m) \quad (= Ax^2 + Bx + C)$$

this is a quadratic equation that is ≥ 0 always.

hence can't have 2 real roots. i.e.

discriminant ≤ 0 . i.e.

where $A = \Phi_{yy}(0)$, $B = 2\Phi_{xy}(m)$, $C = \Phi_{xx}(0)$.

but discriminant is $B^2 - 4AC$

$$\text{so } 4\Phi_{xy}^2(m) - 4\Phi_{yy}(0)\Phi_{xx}(0) \leq 0$$

i.e.

$$\Phi_{xy}^2(m) \leq \Phi_{yy}(0)\Phi_{xx}(0)$$

i.e.

$$\boxed{|\Phi_{xy}(m)| \leq \sqrt{\Phi_{yy}(0)\Phi_{xx}(0)}}$$

2 key solution

H.W. #3 Sol.

① a) Autocorrelation sequence: $\phi_{xx}(n, m)$ is defined by

$$\phi_{xx}(n, m) = E\{X_n X_m^*\} = \iint_{-\infty}^{\infty} X_n X_m^* P_{X_n X_m}(X_n, n, X_m, m) dX_n dX_m$$

b) A random process $\{X_n\}$ is a stationary process if its statistics are not affected by a shift in the time origin. i.e., X_n and X_m have the same statistics for all n and m

c) A stationary random process in the wide sense mean

(i) The mean is constant

(ii) the autocorrelation (2nd order statistics) depend only on the time difference between the random variables

d) Time average of a random process $\{X_n\}$ is defined as

$$\langle X_n \rangle = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N X_n$$

Ensemble average of a random process $\{X_n\}$ is defined as

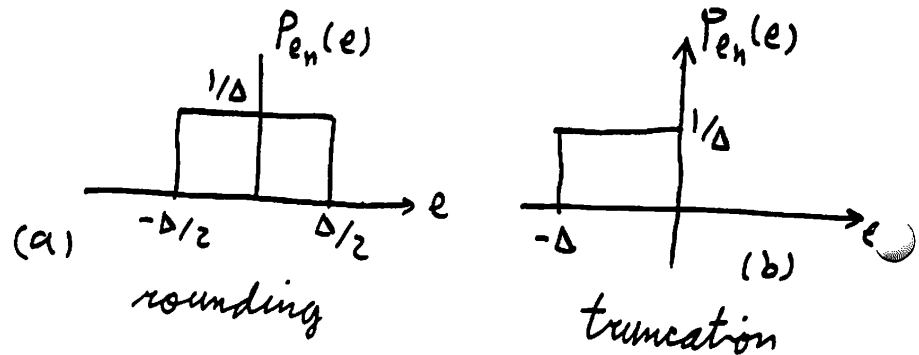
$$m_{X_n} = E\{X_n\} = \int_{-\infty}^{\infty} X P_{X_n}(X, n) dX$$

e) White noise is a random process in which all the random variables are independent with zero mean

$$\Phi_{xx}(m) = \sigma_x^2 \delta(m)$$

f) A random process for which the time averages equal the ensemble averages is called an ergodic process.

(2) 8.3



Prob. distribution

a) Mean & variance, rounding

$$m_e = \int_{-\infty}^{\infty} e P_e(e) de = \int_{-\Delta/2}^{\Delta/2} e \frac{1}{\Delta} de = \frac{1}{\Delta} \left. \frac{e^2}{2} \right|_{-\Delta/2}^{\Delta/2} = 0$$

$$\begin{aligned} \sigma_e^2 &= E\{e_n^2\} = \int_{-\infty}^{\infty} e^2 P_e(e) de = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} e^2 de \\ &= \frac{e^3}{3\Delta} \Big|_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta} \cdot 2 \cdot \frac{\Delta^3}{8} = \frac{\Delta^2}{12} \end{aligned}$$

b) For truncation

$$m_e = \frac{1}{\Delta} \int_{-\Delta}^0 e \, de = \frac{1}{\Delta} \left. \frac{e^2}{2} \right|_{-\Delta}^0 = -\frac{\Delta}{2}$$

$$\begin{aligned} \underline{\sigma_e^2} &= E \left\{ \left(e_n + \frac{\Delta}{2} \right)^2 \right\} = E \{ e_n^2 \} + \frac{\Delta^2}{4} + 2 \frac{\Delta}{2} E \{ e_n \} \\ &= E \{ e_n^2 \} + \frac{\Delta^2}{4} - \frac{\Delta^2}{2} = \underline{E \{ e_n^2 \} - \frac{\Delta^2}{4}} \end{aligned}$$

$$\sigma_e^2 = \frac{1}{\Delta} \int_{-\Delta}^0 e^2 \, de - \frac{\Delta^2}{4} = \frac{1}{\Delta} \left. \frac{e^3}{3} \right|_{-\Delta}^0 - \frac{\Delta^2}{4} = \frac{\Delta}{12}$$

③ 8.4 $e(n)$: white noise seq.
 $s(n)$: uncorrelated with $e(n)$

show $Y(n) = s(n) e(n)$ is white : i.e.

$$E \{ Y(n) Y(n+m) \} = A \delta(m)$$

Sol.

$$\begin{cases} e(n) \text{ white} \Rightarrow E \{ e(n) e(n+m) \} = \sigma_e^2 \delta(m) \\ \text{uncorrelated} \quad E \{ e(n) Y(m) \} = E \{ e(n) \} E \{ Y(m) \} \end{cases}$$

$$E \{ Y(n) Y(n+m) \} = E \{ s(n) e(n) s(n+m) e(n+m) \}$$

$$= E \{ s(n) s(n+m) e(n) e(n+m) \}$$

$$= E \{ s(n) s(n+m) \} E \{ e(n) e(n+m) \}$$

$$= E \{ s(n) s(n+m) \} \sigma_e^2 \delta(m)$$

$$= \sigma_s^2 \sigma_e^2 \delta(m)$$

assume
 $s(n)$ is WSS
 or white noise

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8.6 Consider the two real stationary random processes $\{x_n\}$ and $\{y_n\}$. with means m_x and m_y and variances σ_x^2 and σ_y^2 . show the following

$$(a) \delta_{xx}(m) = \phi_{xx}(m) - m_x^2 \quad \& \quad \gamma_{xy}(m) = \phi_{xy}(m) - m_x m_y$$

$$\begin{aligned} \delta_{xx}(m) &= E[(x_n - m_x)(x_{n+m} - m_x)] \\ &= E[x_n x_{n+m}] - m_x E[x_{n+m}] - m_x E[x_n] + m_x m_x \\ &= \phi_{xx}(m) - m_x m_x - m_x m_x + m_x m_x \\ &= \phi_{xx}(m) - m_x^2 \end{aligned}$$

$$\begin{aligned} \delta_{xy}(m) &= E[(x_n - m_x)(y_{n+m} - m_y)] \\ &= E[x_n y_{n+m}] - m_x m_y - m_y m_x + m_x m_y \\ &= \phi_{xy}(m) - m_x m_y \end{aligned}$$

$$(b) \phi_{xx}(0) = \text{mean square} \quad \& \quad \gamma_{xx}(0) = \sigma_x^2$$

$$\phi_{xx}(m) = E[x_n x_{n+m}] =$$

$$\phi_{xx}(0) = E[x_n x_n] = \text{mean square}$$

$$\delta_{xx}(m) = E[(x_n - m_x)(x_{n+m} - m_x)]$$

$$\delta_{xx}(0) = E[(x_n - m_x)^2] = \sigma_x^2$$

$$(c) \phi_{xx}(m) = \phi_{xx}(-m)$$

$$\phi_{xx}(-m) = (E[x_n x_{n-m}])$$

$$\text{let } n' = n - m$$

$$\phi_{xx}(-m) = (E[x_{n'+m} x_{n'}]) = E[x_{n'} x_{n'+m}]$$

$$= \phi_{xx}(m)$$

$$\delta_{xx}(m) = \delta_{xx}(-m)$$

$$\begin{aligned} \delta_{xx}(-m) &= (E[(x_n - m_x)(x_{n-m} - m_x)]) \\ &= (E[(x_{n'+m} - m_x)(x_{n'} - m_x)]) \\ &= E[(x_{n'} - m_x)(x_{n'+m} - m_x)] \\ &= \delta_{xx}(m) \end{aligned}$$

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$$\begin{aligned} \phi_{xy}(m) &= \phi_{yx}(-m) \\ \phi_{yx}(-m) &= (E[(y_n - m_y)(x_{n-m} - m_x)]) \\ &= (E[(y_{n'+m} - m_y)(x_{n'} - m_x)]) \\ &= E[(x_{n'} - m_x)(y_{n'+m} - m_y)] \\ &= \phi_{xy}(m) \end{aligned}$$

$$\begin{aligned} \delta_{xy}(m) &= \delta_{yx}(-m) \\ \delta_{yx}(-m) &= (E[(y_n - m_y)(x_{y-m} - m_x)]) \\ &= (E[(y_{n'+m} - m_y)(x_{n'} - m_x)]) \\ &= E[(x_{n'} - m_x)(y_{n'+m} - m_y)] \\ &= \delta_{xy}(m). \end{aligned}$$

(d) Consider the inequality

$$E \left\{ \left(\frac{x_n}{(E[x_n^2])^{1/2}} - \frac{y_{n+m}}{(E[y_{n+m}^2])^{1/2}} \right)^2 \right\} \geq 0$$

This is true since the quantity inside the brackets is > 0 for all m and n .

Now

$$E \left[\frac{x_n^2}{E[x_n^2]} \right] + E \left[\frac{y_{n+m}^2}{E[y_{n+m}^2]} \right] - 2 \frac{E[x_n y_{n+m}]}{(E[x_n^2])^{1/2} (E[y_{n+m}^2])^{1/2}} \geq 0$$

This can be written as

$$\frac{\phi_{xx}(0)}{\phi_{xx}(0)} + \frac{\phi_{yy}(0)}{\phi_{yy}(0)} - \frac{2 \phi_{xy}(m)}{\phi_{xx}^{1/2}(0) \phi_{yy}^{1/2}(0)} \geq 0$$

$$\frac{\phi_{xy}(m)}{\phi_{xx}^{1/2}(0) \phi_{yy}^{1/2}(0)} \leq 1 \Rightarrow$$

$$\boxed{[\phi_{xx}(0) \phi_{yy}(0)]^{1/2} \geq \phi_{xy}(m)}$$

Now if we replace x_n by $(x_n - m_x)$ and y_{n+m} by $(y_{n+m} - m_y)$ in the inequality we can manipulate it in the same way to get

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$$[\gamma_{xx}(0) \gamma_{yy}(0)]^{1/2} \geq \gamma_{xy}(m)$$

Letting $y_m = x_m$ we can specialize these inequalities to

$$\frac{\phi_{xx}(0) \geq \phi_{xx}(m)}{\gamma_{xx}(0) \geq \gamma_{xx}(m)}$$

(e) Let $y_m = x_{m-m_0}$

$$\begin{aligned} \phi_{yy}(m) &= E[y_m y_{m+m}] \\ &= E[x_{m-m_0} x_{m+m-m_0}] \\ &= \phi_{xx}(m) \end{aligned}$$

Obviously $\gamma_{yy}(m) = \gamma_{xx}(m)$ for the same reasons.

(f) Let $\gamma_{xx}(m) \longleftrightarrow \Gamma_{xx}(z)$

$\gamma_{xy}(m) \longleftrightarrow \Gamma_{xy}(z)$

$$\Gamma_{xx}(z) \triangleq \sum_m \gamma_{xx}(m) z^{-m} \Rightarrow (1)$$

$$\gamma_{xx}(m) = \frac{1}{2\pi j} \oint_C \Gamma_{xx}(z) z^{m-1} dz$$

$$\gamma_{xx}(0) = \sigma_x^2 = \frac{1}{2\pi j} \oint_C \Gamma_{xx}(z) z^{-1} dz$$

(2) We have shown that $\gamma_{xx}(m) = \gamma_{xx}(-m)$

Therefore
$$\Gamma_{xx}(z) = \sum_{m=-\infty}^{\infty} \gamma_{xx}(m) z^{-m}$$

$$\begin{aligned} \Gamma_{xx}(z^{-1}) &= \sum_{m=-\infty}^{\infty} \gamma_{xx}(m) z^m = \sum_{p=-\infty}^{\infty} \gamma_{xx}(-p) z^p \\ &= \sum_{m=-\infty}^{\infty} \gamma_{xx}(m) z^{-m} = \Gamma_{xx}(z) \end{aligned}$$

$p \rightarrow m \Rightarrow$

use $\gamma_{xy}(m) = \gamma_{yx}^*(-m)$

$$\begin{aligned} \Gamma_{xy}(z) &= \sum_{m=-\infty}^{\infty} \gamma_{xy}(m) z^{-m} = \sum_{m=-\infty}^{\infty} \gamma_{yx}^*(-m) z^{-m} \\ &= \left(\sum_{l=-\infty}^{\infty} \gamma_{yx}(l) z^{*l} \right)^* \\ &= \left(\sum_{l=-\infty}^{\infty} \gamma_{yx}(l) (z^{*-1})^{-l} \right)^* = \Gamma_{yx}^*(1/z^*) \end{aligned}$$