HW2, EGEE 518. CSUF, Fall 2008

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1 Problem 1

Compute an appropriate sampling rate and DFT size $N = 2^v$ to analyze a single with no significant frequency content above $10khz$ and with a minimum resolution of $100hz$

Solution

From Nyquist sampling theory we obtain that sampling frequency is

$$f_s = 20000\ hz$$

Now, the frequency resolution is given by

$$\Delta f = \frac{f_s}{N}$$

where N is the number of FFT samples. Now since the minimum Δf is 100 hz then we write

$$\frac{f_s}{N} = \Delta f \geq 100$$

or

$$\frac{f_s}{N} \geq 100$$

Hence

$$\begin{aligned} N &\leq \frac{20,000}{100} \\ &\leq 200 \text{ samples} \end{aligned}$$

Therefore, we need the closest N below 200 which is power of 2, and hence

$$\boxed{N = 128}$$ 256

2 Problem 2

sketch the locus of points obtained using Chirp Z Transform in the Z plane for $M = 8, W_0 = 2, \phi_0 = \frac{\pi}{16}, A_0 = 2, \theta_0 = \frac{\pi}{4}$

Answer:

Chirp Z transform is defined as

$$X(z_k) = \sum_{n=0}^{N-1} x[n]\, z_k^{-n} \qquad k = 0, 1, \cdots, M-1 \tag{1}$$

Where

$$z_k = AW^{-k}$$

and $A = A_0 e^{j\theta_0}$ and $W = W_0 e^{-j\phi_0}$

Hence

$$\begin{aligned} z_k &= \left(A_0 e^{j\theta_0}\right)\left(W_0 e^{-j\phi_0}\right)^{-k} \\ &= \frac{A_0}{W_0^k} e^{j(\theta_0 + k\phi_0)} \end{aligned}$$

Hence

$$\begin{aligned} |z_k| &= \frac{A_0}{W_0^k} \\ &= \frac{2}{2^k} \end{aligned}$$

and

$$\begin{aligned} phase\ of\ z_k &= \theta_0 + k\phi_0 \\ &= \frac{\pi}{4} + k\frac{\pi}{16} \end{aligned}$$

Hence

k	$\lvert z_k\rvert = \frac{2}{2^k}$	$phase\ of\ z_k = \frac{\pi}{4} + k\frac{\pi}{16}$	$phase\ of\ z_k\ in$ degrees
0	$\frac{2}{1} = 2$	$\frac{\pi}{4} + 0 \times \frac{\pi}{16} = \frac{\pi}{4}$	45
1	$\frac{2}{2} = 1$	$\frac{\pi}{4} + 1 \times \frac{\pi}{16} = \frac{5}{16}\pi$	56.25
2	$\frac{2}{4} = \frac{1}{2}$	$\frac{\pi}{4} + 2 \times \frac{\pi}{16} = \frac{3}{8}\pi$	67.5
3	$\frac{2}{8} = \frac{1}{4}$	$\frac{\pi}{4} + 3 \times \frac{\pi}{16} = \frac{7}{16}\pi$	78.75
4	$\frac{2}{16} = \frac{1}{8}$	$\frac{\pi}{4} + 4 \times \frac{\pi}{16} = \frac{1}{2}\pi$	90
5	$\frac{2}{32} = \frac{1}{16}$	$\frac{\pi}{4} + 5 \times \frac{\pi}{16} = \frac{9}{16}\pi$	101.25
6	$\frac{2}{64} = \frac{1}{32}$	$\frac{\pi}{4} + 6 \times \frac{\pi}{16} = \frac{5}{8}\pi$	112.5
7	$\frac{2}{128} = \frac{1}{64}$	$\frac{\pi}{4} + 7 \times \frac{\pi}{16} = \frac{11}{16}\pi$	123.75

Below is plot of the above contour

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In[579]:= z[k_, W0_, A0_, θ0_, ϕ0_] := A0 Exp[I θ0] (W0 Exp[-I ϕ0])^-k
W0 = 2;
A0 = 2;
θ0 = Pi / 4;
ϕ0 = Pi / 16;
m = 8;
zValues = Table[z[k, W0, A0, θ0, ϕ0], {k, 0, m - 1}];
arg = Arg[zValues]
abs = Abs[zValues]
data = Transpose[{arg, abs}];
p1 = ListPolarPlot[data, AxesOrigin → {0, 0},
   PlotRange → All, Joined → False,
   PlotMarkers → {Automatic, Automatic}];
p2 = ListPolarPlot[data, AxesOrigin → {0, 0},
   PlotRange → All, Joined → True];
p3 = PolarPlot[1, {t, 0, 2 Pi}];
Show[p1, p2, p3]
```

Out[586]= $\left\{\frac{\pi}{4}, \frac{5\pi}{16}, \frac{3\pi}{8}, \frac{7\pi}{16}, \frac{\pi}{2}, \frac{9\pi}{16}, \frac{5\pi}{8}, \frac{11\pi}{16}\right\}$

Out[587]= $\left\{2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}\right\}$

Out[592]=