HW 9
Electronic Communication Systems
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California State University, Fullerson

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Contents

1 Problem 5-5

5–5 A 50,000-W AM broadcast transmitter is being evaluated by means of a two-tone test. The transmitter is connected to a 50-Ω load, and $m(t) = A_1 \cos \omega_1 t + A_1 \cos 2\omega_1 t$, where $f_1 = 500$ Hz. Assume that a perfect AM signal is generated.

(a) Evaluate the complex envelope for the AM signal in terms of A_1 and ω_1.

(b) Determine the value of A_1 for 90% modulation.

(c) Find the values for the peak current and average current into the 50-Ω load for the 90% modulation case.

Figure 1: the Problem statement

1.1 part(a)

$$s(t) = \overbrace{A_c(1 + k_a m(t))}^{\text{in-phase component}} \cos \omega_c t$$

Assume $k_a = 1$ in this problem. $m(t) = A_1(\cos \omega_1 t + \cos 2\omega_1 t)$, then $s(t)$ becomes

$$s(t) = \overbrace{A_c(1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t))}^{\text{in-phase component}} \cos \omega_c t \tag{1}$$

But $s(t)$ can be written as

$$s(t) = s_I(t) \cos \omega_c t - s_Q(t) \sin \omega_c t \tag{2}$$

Where $s_I(t)$ is the inphase component and $s_Q(t)$ is the quadrature component of $s(t)$. Compare (1) to (2), we see that

$$s_I(t) = A_c[1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t)]$$
$$s_Q(t) = 0$$

Now, the complex envelope $\tilde{s}(t)$of $s(t)$ is given by

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$

Hence replacing the value found for $s_I(t)$ and $s_Q(t)$ we obtain

$$\tilde{s}(t) = A_c[1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t)] \tag{3}$$

Now, we can find A_c since the average power in the carrier signal is given as 50000 watt as follows

$$P_{\text{av\_carrier}} = \frac{A_c^2}{2(50)} = 50000$$

Hence

$$A_c = \sqrt{100 \times 50000} = 2236.1\text{volt}$$

Then (3) becomes

$$\tilde{s}(t) = 2236.1 \left[1 + A_1 \left(\cos \omega_1 t + \cos 2\omega_1 t\right)\right] \tag{4}$$

The above is the complex envelope in terms of A_1 and ω_1 only as required to show.

1.2 part(b)

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \tag{5}$$

Need to find angle at which $\cos \omega_1 t + \cos 2\omega_1 t$ is Max and at which it is min. then Let $\Delta = \cos \omega_1 t + \cos 2\omega_1 t$

We see that when $\omega_1 t = 2\pi$, then $\Delta = 1 + 1 = 2$, hence

$$A_{\max} = A_c \left(1 + 2A_1\right)$$

Need to find $A_{\min}$ hence we need to find $\Delta_{\min}$. For this case we must use calculus as it is not obvious where this is minimum

$$\begin{aligned}
\frac{\partial \Delta}{\partial t} &= 0 = -\omega_1 \sin \omega_1 t - 2\omega_1 \sin 2\omega_1 t \\
0 &= -\omega_1 \sin \omega_1 t - 2\omega_1 \left(2 \sin\left(\omega_1 t\right) \cos\left(\omega_1 t\right)\right) \\
&= -\omega_1 \sin \omega_1 t - 4\omega_1 \sin\left(\omega_1 t\right) \cos\left(\omega_1 t\right) \\
\frac{-1}{4} &= \cos\left(\omega_1 t\right)
\end{aligned}$$

Hence $\omega_1 t = \cos^{-1}\left(\frac{-1}{4}\right) \rightarrow \omega_1 t = 104.477^0$ (using calculator). hence

$$\begin{aligned}
\Delta_{\min} &= \cos\left(104.477^0\right) + \cos\left(2 \times 104.477^0\right) \\
&= -0.2499 - 0.875 \\
&= -1.1249
\end{aligned}$$

Then $A_{\min} = A_c \left(1 - 1.1249 A_1\right)$, so from (5) above

$$\begin{aligned}
\mu &= \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \\
0.9 &= \frac{A_c \left(1 + 2A_1\right) - A_c \left(1 - 1.1249 A_1\right)}{A_c \left(1 + 2A_1\right) + A_c \left(1 - 1.1249 A_1\right)} \\
&= \frac{\left(1 + 2A_1\right) - \left(1 - 1.1249 A_1\right)}{\left(1 + 2A_1\right) + \left(1 - 1.1249 A_1\right)} \\
&= \frac{1 + 2A_1 - 1 + 1.1249 A_1}{1 + 2A_1 + 1 - 1.1249 A_1} \\
&= \frac{3.1249 A_1}{2 + 0.8751 A_1}
\end{aligned}$$

Hence

$$\begin{aligned} 1.8 + 0.9\left(0.8751 A_1\right) - 3.9 A_1 &= 0 \\ 1.8 - 2.3 A_1 &= 0 \end{aligned}$$

Then

$$A_1 = 0.770$$

1.3 part(c)

Since

$$\begin{aligned} A_{\max} &= A_c\left(1 + 2A_1\right) \\ &= 2236.1\left(1 + 2 \times 0.770\,12\right) \\ &= 5680.2 \text{ volts} \end{aligned}$$

Then from Ohm's law, $V = RI$,

$$\begin{aligned} I_{\max} &= \frac{V_{\max}}{R} \\ &= \frac{5680.2}{50} \\ &= 113.6 \text{ amps} \end{aligned}$$

Since mean voltage is zero, then average current is zero.

2 Problem 5-8

5–8 Assume that transmitting circuitry restricts the modulated output signal to a certain peak value, say, A_p, because of power-supply voltages that are used and because of the peak voltage and current ratings of the components. If a DSB-SC signal with a peak value of A_p is generated by this circuit, show that the sideband power of this DSB-SC signal is four times the sideband power of a comparable AM signal having the same peak value A_p that could also be generated by this circuit.

Figure 2: the Problem statement

answer For normal modulation, let

$$s_{am}(t) = A_c\left(1 + m(t)\right)\cos\omega_c t$$

Maximum envelop is $2A_c$ (i.e. when $m_{\max}(t) = 1$), this means that $A_p = 2A_c$

But

$$s_{am}(t) = \overbrace{A_c \cos \omega_c t}^{\text{carrier}} + \overbrace{A_c m(t) \cos \omega_c t}^{\text{side band}}$$

So max of sideband is A_c or $\frac{A_p}{2}$. Hence maximum power of sideband is $\frac{1}{2}\left(\frac{A_p}{2}\right)^2 = \frac{A_p^2}{8}$ and for DSB-SC, where now use A_p in place of what we normally use A_c then we obtain

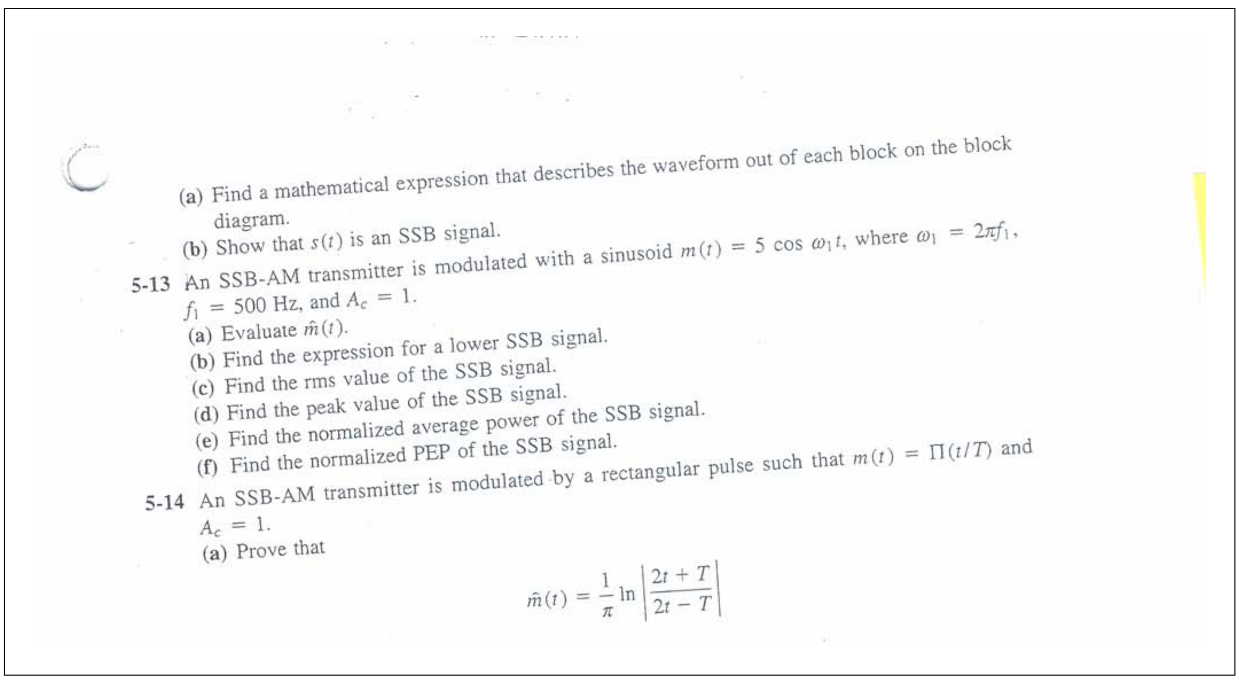
$$s(t) = A_p m(t) \cos \omega_c t$$

Hence maximum for sideband is $\frac{1}{2}A_p^2$

Hence we see that power of sideband of DSB-SC to the power of sideband of AM is

$$\frac{\frac{1}{2}A_p^2}{\frac{A_p^2}{8}} = 4$$

3 Problem 5-13



(a) Find a mathematical expression that describes the waveform out of each block on the block diagram.
(b) Show that $s(t)$ is an SSB signal.

5-13 An SSB-AM transmitter is modulated with a sinusoid $m(t) = 5 \cos \omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 500$ Hz, and $A_c = 1$.
(a) Evaluate $\hat{m}(t)$.
(b) Find the expression for a lower SSB signal.
(c) Find the rms value of the SSB signal.
(d) Find the peak value of the SSB signal.
(e) Find the normalized average power of the SSB signal.
(f) Find the normalized PEP of the SSB signal.

5-14 An SSB-AM transmitter is modulated by a rectangular pulse such that $m(t) = \Pi(t/T)$ and $A_c = 1$.
(a) Prove that

$$\hat{m}(t) = \frac{1}{\pi} \ln \left| \frac{2t + T}{2t - T} \right|$$

Figure 3: the Problem statement

3.1 part(a)

$$m(t) = 5 \cos \omega_1 t$$

$\hat{m}(t)$ is Hilbert transform of $m(t)$ defined as $\hat{m}(t) = \int_{-\infty}^{\infty} m(\tau) \frac{1}{t-\tau} d\tau$. Or we can use the frequency approach where $\hat{m}(t) = \mathcal{F}^{-1}[-j\ sign(f)\ M(f)]$ where $M(f)$ is the Fourier transform of $m(t)$. We can carry out this easily, but since this is a phase 90 change, and $m(t)$ is a cosine function, then

$$\hat{m}(t) = 5\sin\omega_1 t$$

3.2 part(b)

$$s_{SSB}(t) = A_c\left[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t\right]$$

Where the negative sign for upper sided band, and positive sign for the lower sided band, hence

$$\begin{aligned} s_{LSSB}(t) &= A_c\left[m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t\right] \\ &= 5A_c\left[\cos\omega_1 t\cos\omega_c t + \sin\omega_1 t\sin\omega_c t\right] \\ &= 5A_c\left[\cos(\omega_c - \omega_1)t\right] \end{aligned}$$

We can plug in numerical values given

$$s_{LSSB}(t) = 5\left[\cos(\omega_c - \omega_1)t\right]$$

3.3 Part(c)

To find the RMS value of the SSB, pick the above lower side band. First find P_{av}.

$$s_{LSSB}(t) = 5\left[\cos(\omega_1 - \omega_c)t\right]$$

Hence

$$\begin{aligned} RMS \text{ value of signal} &= \frac{5}{\sqrt{2}} \\ &= 3.535\,5 \text{ volt} \end{aligned}$$

3.4 part(d)

Then maximum of $5\left[\cos(\omega_1 - \omega_c)t\right]$ is when $\cos(\omega_1 - \omega_c)t = 1$, hence

$$s_{LSSB_{\max}}(t) = 5\text{volt}$$

3.5 part(e)

$$\begin{aligned} P_{av} &= \frac{1}{2}A_c^2 \\ &= \frac{1}{2}\times 25 \\ &= 12.5\text{watt} \end{aligned}$$

3.6 Part(f)

$$
\begin{aligned}
PEP &= \frac{1}{2} s^2_{LSSB_{\max}}(t) \\
&= \frac{5^2}{2} \\
&= 12.5 \text{ watt}
\end{aligned}
$$

4 Problem 5-18

5-18 A phasing-type SSB-AM detector is shown in Fig. P5-18. This circuit is attached to the IF output of a conventional superheterodyne receiver to provide SSB reception.

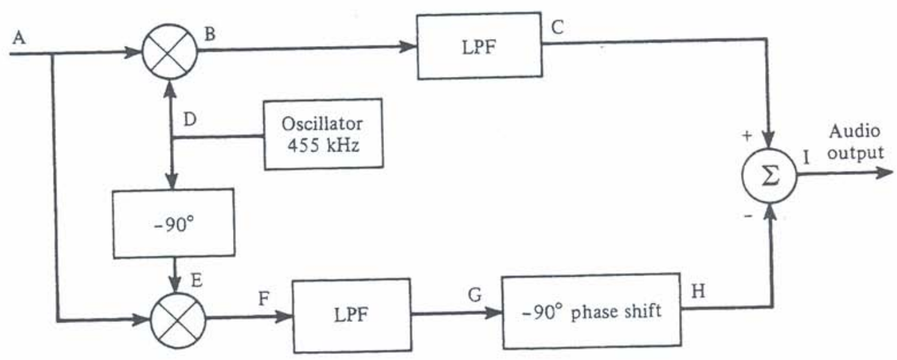


Figure P5-18

(a) Determine whether this detector is sensitive to LSSB or USSB signals. How would the detector be changed to receive SSB signals with alternate (opposite type of) sidebands?
(b) Assume that the signal at point *A* is a USSB signal with f_c = 455 kHz. Find the mathematical expressions for the signals at points *B* through *I*.
(c) Repeat part (b) for the case of an LSSB-AM signal at point *A*.
(d) Discuss the IF and LP filter requirements if the SSB signal at point *A* has a 3-kHz bandwidth.

Figure 4: the Problem statement

4.1 part(a)

This is a detector for USSB (Upper side band). i.e.

$$
s(t) = A_c \left(m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t \right)
$$

Note, I wrote A_c and not $\frac{A_c}{2}$ in the above. As long this is a constant, it gives the same analysis.

The reason is because at point H the signal is $-\frac{1}{2}m(t)$ and at the C point the signal is $+\frac{1}{2}m(t)$, hence due to subtraction at the audio output end we obtain $m(t)$. To receive LSSB, we should change the sign to positive at the audio output end.

4.2 part(b)

$$s(t) = A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)$$

at point B

$$\begin{aligned}
s_B(t) &= s(t) * \overbrace{A_c^{'}\cos\omega_c t}^{\text{local oscillator}} \\
&= A_c^{'}A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)\cos\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos^2\omega_c t - \hat{m}(t)\sin\omega_c t\cos\omega_c t\right) \\
&= A_c^{'}A_c\left(m(t)\left(\frac{1}{2}+\frac{1}{2}\cos 2\omega_c t\right) - \frac{1}{2}\hat{m}(t)\sin 2\omega_c t\right) \\
&= \overbrace{\frac{A_c^{'}A_c}{2}m(t)}^{\text{low pass}} + \overbrace{\frac{A_c^{'}A_c}{2}m(t)\cos 2\omega_c t}^{\text{high pass}} - \overbrace{\frac{A_c^{'}A_c}{2}\hat{m}(t)\sin 2\omega_c t}^{\text{high pass}}
\end{aligned}$$

at point C, after LPF we obtain

$$s_c(t) = A_c^{'}A_c\frac{m(t)}{2}$$

at point F we have

$$\begin{aligned}
s_f(t) &= s(t)\, A_c^{'}\sin\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)\sin\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos(\omega_c t)\sin(\omega_c t) - \hat{m}(t)\sin^2\omega_c t\right) \\
&= A_c^{'}A_c\left(m(t)\frac{1}{2}\sin(2\omega_c t) - \hat{m}(t)\left(\frac{1}{2}-\frac{1}{2}\cos 2\omega_c t\right)\right) \\
&= \frac{A_c^{'}A_c}{2}\left(m(t)\sin(2\omega_c t) - \hat{m}(t)\left(1-\cos 2\omega_c t\right)\right)
\end{aligned}$$

at point G after LPF

$$s_g(t) = -\frac{A_c^{'}A_c}{2}\hat{m}(t)$$

at point H after -90^0 phase shift

$$s_h(t) = +\frac{A_c^{'}A_c}{2}m(t)$$

at point I, we sum $s_h(t)$ and $s_c(t)$, hence $s_i(t) = A_c^{'}A_c\frac{m(t)}{2} + \frac{A_c^{'}A_c}{2}m(t) = A_c^{'}A_c m(t)$

4.3 Part(c)

$$s(t) = A_c(m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t)$$

This the same as part (b), except now since there is a sign difference, this carries all the way to point I, and then we obtain

$$s_i(t) = A_c' A_c \frac{m(t)}{2} - \frac{A_c' A_c}{2} m(t) = 0$$

This if this circuit is used as is to demodulate an LSSB AM signal, then the signal will be lost. So, instead of adding at point I we should now subtract to counter the effect of the negative sign.

4.4 part(d)

Since SSB has bandwidth of $3kHz$ then this means the width of upper (or lower) band is $3khz$. This means the signal has $3khz$ bandwidth. This diagram shows the LPF requirement

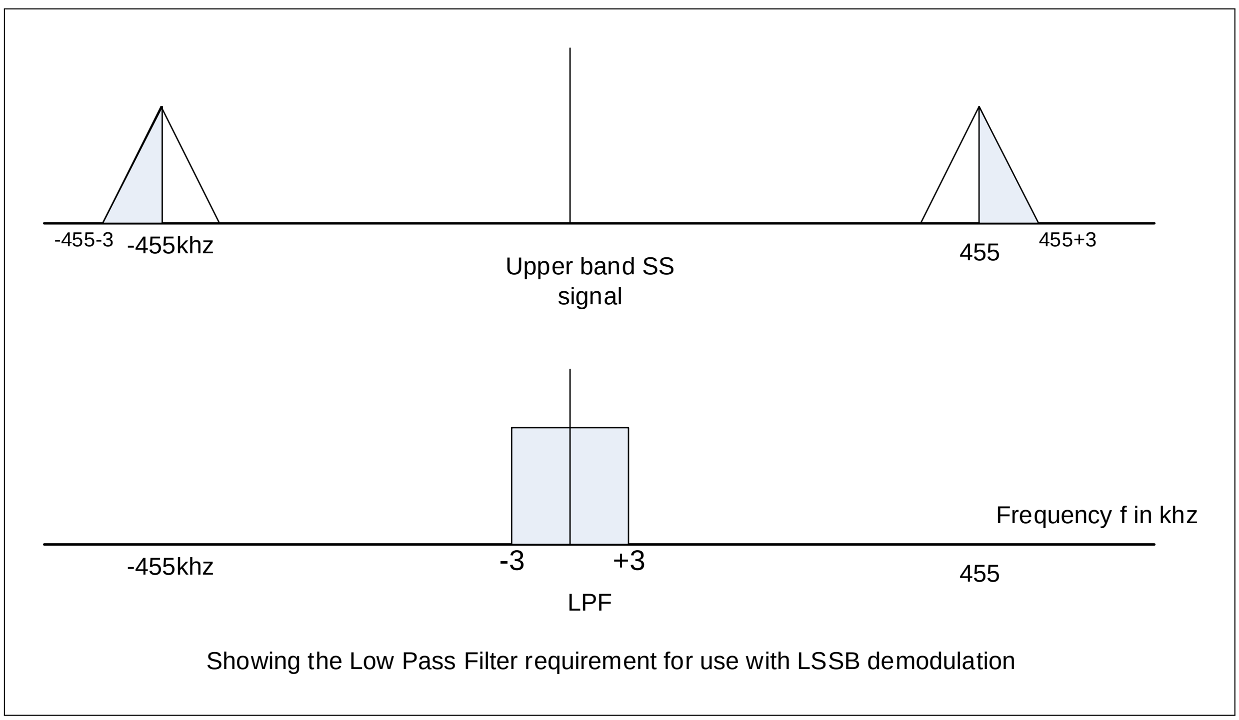


Figure 5: Low pass filter

Hence LPF is centered at zero frequency and have bandwidth of $3khz$ (may be make it a little over $3khz$ band width?)

The IF filter is centered at $455 + \left(\frac{3}{2}\right)$ for the upper band of the positive band, and centered at $-455 - \left(\frac{3}{2}\right)$ for the upper band of the negative band. (i.e. for the $USSB$).

For $LSSB$, IF should be centered at $455 - \left(\frac{3}{2}\right)$ for the lower band of the positive band, and centered at $-455 + \left(\frac{3}{2}\right)$ for the lower band of the negative band. (This works if there is a guard band around 455, small one, to make the design of IF possible).

5 Key solution

EE443 HW # 9. Key page 1

5-5. (a.) $50{,}000 = \dfrac{A_c^2}{2(50)} \Rightarrow A_c = 2236\ \text{V}$

$$g(t) = A_c\{1 + m(t)\}$$

$$= 2236\{1 + A_1(\cos\omega_1 t + \cos 2\omega_1 t)\}$$

(b.) to find $m(t)_{MIN}$: $x(\theta) = \cos\theta + \cos 2\theta$

$$0 = \frac{dx(\theta)}{d\theta} = -\sin\theta - 2\sin 2\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta \Rightarrow -\sin\theta = 4\sin\theta\cos\theta$

$$\theta = 104.5^\circ$$

$$x(104.5^\circ) = -1.125$$

$$A_{max} = 2236\{1 + 2A_1\}$$

$$A_{MIN} = 2236\{1 - 1.125A_1\}$$

$$.90 = \frac{A_{max} - A_{min}}{2A_c} = \frac{3.125}{2}A_1 \Rightarrow A_1 = .576$$

(c.) $A_{max} = 2236\{1 + 2(.576)\} = 4811.9$ volts

$$I_{max} = \frac{A_{max}}{50} = 96.238\ \text{Amps}$$

$$\langle s(t)\rangle = \langle 2236\{1 + .576(\cos\omega_1 t + \cos 2\omega_1 t)\} \cdot \cos\omega_c t\rangle$$

$= 0$ for $\omega_c \gg \omega_1$

$\therefore I_{AV} = 0$ Amps

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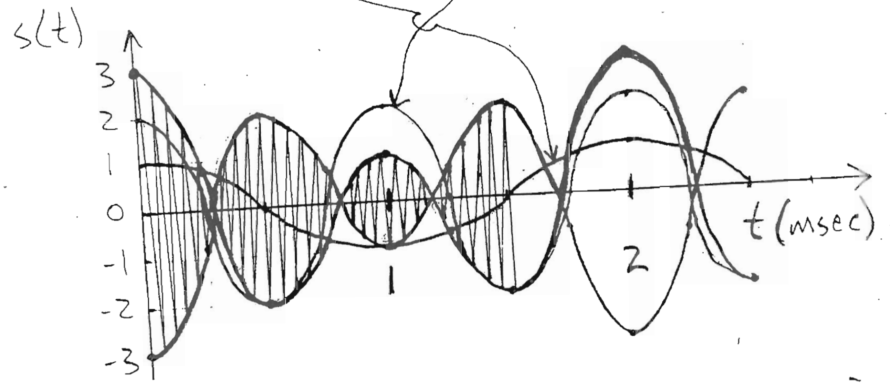
EE 443 HW # 9 page 2

5-7. (a.) DSB-SC $m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

$$s(t) = [\cos\omega_1 t + 2\cos 2\omega_1 t]\cos\omega_c t$$

where $\omega_1 = 1000\pi$

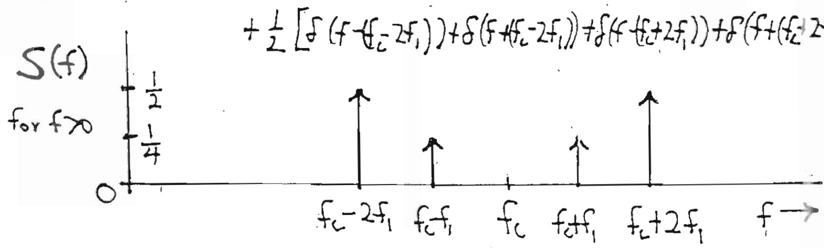


(b.) $s(t) = \frac{1}{2}[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t]$

$+ \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t$

5-7 (b.) Cont'd

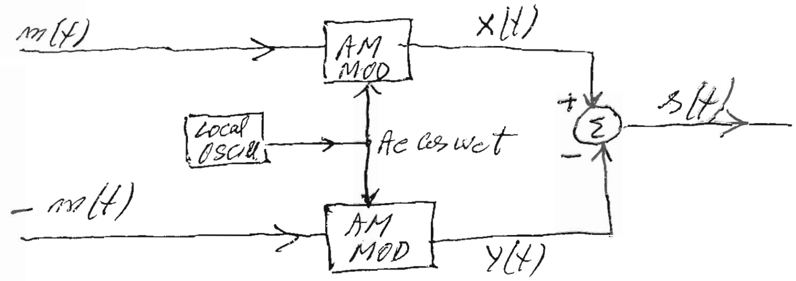
$$S(f) = \mathcal{F}[s(t)] = \frac{1}{4}[\delta(f-(f_c-f_1)) + \delta(f+(f_c-f_1)) + \delta(f-(f_c+f_1)) + \delta(f+(f_c+f_1))]$$
$$+ \frac{1}{2}[\delta(f-(f_c-2f_1)) + \delta(f+(f_c-2f_1)) + \delta(f-(f_c+2f_1)) + \delta(f+(f_c+2f_1))]$$



(c.) $P_{AV_{norm}} = \frac{1}{2}[(\frac{1}{2})^2 + (\frac{1}{2})^2 + (1)^2 + (1)^2] = \underline{1.25\ W}$

(d.) $A_{max} = 3 \Rightarrow PEP_{norm} = \frac{(3)^2}{2} = \underline{4.5\ W}$

Prob. 5.9 ✓



where:

$x(t) = A_c[1 + K_a m(t)] \cos w_c t$

$y(t) = A_c[1 - K_a m(t)] \cos w_c t$

$s(t) = x(t) - y(t) = 2 A_c K_a m(t) \cos w_c t$

s(t) is a DSBSC signal.

Prob. # 5.13

$s(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$

Assume $A_c = 1$, $m(t) = 5 \cos 2\pi f_1 t$

$f_1 = 500\,Hz$.

a) $\hat{m}(t) = H \cdot T[m(t)]$

$\hat{M}(f) = -j\,sgn(f) M(f) = -j\,sgn f \left\{ \frac{5}{2} [\delta(f - f_1) + \delta(f + f_1)] \right\}$

$= \frac{-5j}{2} [\delta(f - f_1) - \delta(f + f_1)]$

$= \frac{5}{2j} [\delta(f - f_1) - \delta(f + f_1)]$

$\hat{m}(t) = F^{-1}T[\hat{M}(f)] = 5 \sin 2\pi f_1 t$

prob # 5.13 cont'd)

b) $s_L(t) = \frac{5}{2}\cos\omega_1 t\cos\omega_c t + \frac{5}{2}\sin\omega_1 t\sin\omega_c t$ (1)

using $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
we have

$s_L(t) = \frac{5}{2}\cos[(\omega_c-\omega_1)t]$ (2)

c) $S_{rms} = \frac{S_{peak}}{\sqrt{2}} = \frac{5/2}{\sqrt{2}} = \frac{5}{2\sqrt{2}}$ volts

d) $S_{peak} = 5/2$ volts

e) $P_{av} = \frac{S_{peak}^2}{2} = \frac{(5/2)^2}{2} = \frac{25}{8}$ watts

f) PEP = ?
using eq. (1) find the envelope

$a(t) = \sqrt{S_I^2(t) + S_Q^2(t)} = \sqrt{\left(\frac{5}{2}\right)^2\cos^2\omega_1 t + \left(\frac{5}{2}\right)^2\sin^2\omega_1 t}$

$a(t) = 5/2$

$PEP = \frac{1}{2}\left[\text{Max } a(t)\right]^2 = \frac{(5/2)^2}{2} = \frac{25}{8}$ watt

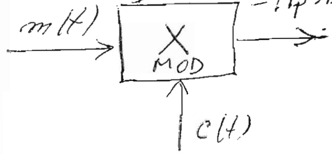
Note: S_{rms} can be obtained from:

$S_{rms}^2 = P_{av} = \frac{(5/2)^2}{2} = \frac{25}{8}$

$S_{rms} = \sqrt{P_{av}} = \frac{5}{2\sqrt{2}}$

5.8)

a) DSBSC $s(t) = A_p\, m(t) \cos w_c t$

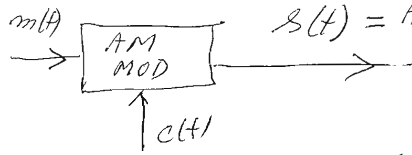


$P_{DSBSC} =$

$$P_{DSBSC} = \langle A_p^2\, m^2(t) \cos^2 w_c t \rangle = \langle A_p^2 \cos^2 w_c t \rangle \langle m^2(t) \rangle$$

$$= \frac{A_p^2}{2} \langle m^2(t) \rangle \qquad (1)$$

b) $s(t) = A_c [1 + m(t)] \cos w_c t$



Assume $Max[m(t)] = 1$ to avoide envelope distortion $|m(t)| \le 1$

The problem stated that:

$$s_{AM\,(peak)}(t) = A_c [1 + max(m(t))] = 2A_c \triangleq A_p$$

$$\Rightarrow A_c = \frac{A_p}{2}$$

Thus:

$$s_{AM}(t) = \frac{A_p}{2} [1 + m(t)] \cos w_c t$$

$$= \underbrace{\frac{A_p}{2} \cos w_c t}_{\text{Carrier}} + \underbrace{\frac{A_p}{2} m(t) \cos w_c t}_{\text{Sideband}}$$

$$P_{AM\,(SB)} = \langle \frac{A_p^2}{4} m^2(t) \cos^2 w_c t \rangle$$

$$= \frac{A_p^2}{4} \underbrace{\langle \cos^2 w_c t \rangle}_{1/2} \cdot \langle m^2(t) \rangle = \frac{A_p^2}{4 \times 2} \langle m^2(t) \rangle$$

Thus.

$$\frac{P_{DSBSC}}{P_{AM\,(SB)}} = \frac{\frac{A_p^2}{2} \langle m^2(t) \rangle}{\frac{A_p^2}{8} \langle m^2(t) \rangle} = 4 = 6\,dB$$

6 my graded HW

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HW9, EGEE 443. CSUF, Fall 2008 (5-5,5-8,5-13,5-18)

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November 20, 2008

14.5/16 17.5/20

1 Problem 5-5

1.1 part(a)

$$s(t) = \overbrace{A_c(1 + k_a m(t))}^{\text{in-phase component}} \cos \omega_c t$$

Assume $k_a = 1$ in this problem. $m(t) = A_1(\cos \omega_1 t + \cos 2\omega_1 t)$, then $s(t)$ becomes

$$s(t) = \overbrace{A_c(1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t))}^{\text{in-phase component}} \cos \omega_c t \tag{1}$$

But $s(t)$ can be written as

$$s(t) = s_I(t) \cos \omega_c t - s_Q(t) \sin \omega_c t \tag{2}$$

Where $s_I(t)$ is the inphase component and $s_Q(t)$ is the quadrature component of $s(t)$. Compare (1) to (2), we see that

$$s_I(t) = A_c[1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t)]$$
$$s_Q(t) = 0$$

Now, the complex envelope $\tilde{s}(t)$of $s(t)$ is given by

$$\tilde{s}(t) = s_I(t) + js_Q(t)$$

Hence replacing the value found for $s_I(t)$ and $s_Q(t)$ we obtain

$$\boxed{\tilde{s}(t) = A_c[1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t)]} \tag{3}$$

Now, we can find A_c since the average power in the carrier signal is given as 50000 watt as follows

$$P_{av\_carrier} = \frac{A_c^2}{2(50)} = 50000$$

Hence

$$A_c = \sqrt{100 \times 50000} = \boxed{2236.1 \text{ volt}}$$

Then (3) becomes

$$\boxed{\tilde{s}(t) = 2236.1[1 + A_1(\cos \omega_1 t + \cos 2\omega_1 t)]} \tag{4}$$

The above is the complex envelope in terms of A_1 and ω_1 only as required to show.

1.2 part(b)

$$\mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} \tag{5}$$

Need to find angle at which $\cos\omega_1 t + \cos 2\omega_1 t$ is Max and at which it is min. then Let $\Delta = \cos\omega_1 t + \cos 2\omega_1 t$

We see that when $\omega_1 t = 2\pi$, then $\Delta = 1 + 1 = 2$, hence

$$A_{\text{max}} = A_c(1 + 2A_1)$$

Need to find A_{min} hence we need to find Δ_{min}. For this case we must use calculus as it is not obvious where this is minimum

$$\begin{aligned}
\frac{\partial \Delta}{\partial t} &= 0 = -\omega_1 \sin\omega_1 t - 2\omega_1 \sin 2\omega_1 t \\
0 &= -\omega_1 \sin\omega_1 t - 2\omega_1 \left(2\sin(\omega_1 t)\cos(\omega_1 t)\right) \\
&= -\omega_1 \sin\omega_1 t - 4\omega_1 \sin(\omega_1 t)\cos(\omega_1 t) \\
\frac{-1}{4} &= \cos(\omega_1 t)
\end{aligned}$$

Hence $\omega_1 t = \cos^{-1}\left(\frac{-1}{4}\right) \to \omega_1 t = 104.477^0$ (using calculator). hence

$$\begin{aligned}
\Delta_{\text{min}} &= \cos\left(104.477^0\right) + \cos\left(2 \times 104.477^0\right) \\
&= -0.2499 - 0.875 \\
&= -1.1249
\end{aligned}$$

Then $A_{\text{min}} = A_c(1 - 1.1249 A_1)$, so from (5) above

$$\begin{aligned}
\mu &= \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} \\
0.9 &= \frac{A_c(1 + 2A_1) - A_c(1 - 1.1249A_1)}{A_c(1 + 2A_1) + A_c(1 - 1.1249A_1)} \\
&= \frac{(1 + 2A_1) - (1 - 1.1249A_1)}{(1 + 2A_1) + (1 - 1.1249A_1)} \\
&= \frac{1 + 2A_1 - 1 + 1.1249A_1}{1 + 2A_1 + 1 - 1.1249A_1} \\
&= \frac{3.1249A_1}{2 + 0.8751A_1}
\end{aligned}$$

Hence

1.64

$$\begin{aligned}
1.8 + 0.9(0.8751A_1) - 3.1249A_1 &= 0 \\
1.8 - 2.3373A_1 &= 0
\end{aligned}$$

Then

−0.5

$$\boxed{A_1 = 0.77012}$$ x 0.576

1.3 part(c)

Since

$$\begin{aligned} A_{\max} &= A_c(1 + 2A_1) \\ &= 2236.1\,(1 + 2 \times 0.770\,12) \\ &= 5680.2 \text{ volts} \end{aligned}$$

X 4811.9

Then from Ohm's law, $V = RI$,

— 1.

$$\begin{aligned} I_{\max} &= \frac{V_{\max}}{R} \\ &= \frac{5680.2}{50} \\ &= 113.6 \text{ amps} \end{aligned}$$

X. 96.238

Since mean voltage is zero, then average current is zero. ✓

2 Problem 5-8

answer For normal modulation, let

$$s_{am}(t) = A_c (1 + m(t)) \cos \omega_c t$$

Maximum envelop is $2A_c$ (i.e. when $m_{\max}(t) = 1$), this means that $A_p = 2A_c$
But

$$s_{am}(t) = \overbrace{A_c \cos \omega_c t}^{\text{carrier}} + \overbrace{A_c m(t) \cos \omega_c t}^{\text{side band}}$$

So max of sideband is A_c or $\frac{A_p}{2}$. Hence maximum power of sideband is $\frac{1}{2}\left(\frac{A_p}{2}\right)^2 = \boxed{\frac{A_p^2}{8}}$ and for DSB-SC, where now use A_p in place of what we normally use A_c then we obtain

$$s(t) = A_p m(t) \cos \omega_c t$$

Hence maximum for sideband is $\boxed{\frac{1}{2}A_p^2}$ ok. See sol.

Hence we see that power of sideband of DSB-SC to the power of sideband of AM is

$$\frac{\frac{1}{2}A_p^2}{\frac{A_p^2}{8}} = \boxed{4}$$

3 Problem 5-13

(a) Find a mathematical expression that describes the waveform out of each block on the block diagram.

(b) Show that $s(t)$ is an SSB signal.

5-13 An SSB-AM transmitter is modulated with a sinusoid $m(t) = 5 \cos \omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 500$ Hz, and $A_c = 1$.

(a) Evaluate $\hat{m}(t)$.

(b) Find the expression for a lower SSB signal.

(c) Find the rms value of the SSB signal.

(d) Find the peak value of the SSB signal.

(e) Find the normalized average power of the SSB signal.

(f) Find the normalized PEP of the SSB signal.

5-14 An SSB-AM transmitter is modulated by a rectangular pulse such that $m(t) = \Pi(t/T)$ and $A_c = 1$.

(a) Prove that

$$\hat{m}(t) = \frac{1}{\pi} \ln \left| \frac{2t + T}{2t - T} \right|$$

3.1 part(a)

$$m(t) = 5 \cos \omega_1 t$$

$\hat{m}(t)$ is Hilbert transform of $m(t)$ defined as $\hat{m}(t) = \int_{-\infty}^{\infty} m(\tau) \frac{1}{t-\tau} d\tau$. Or we can use the frequency approach where $\hat{m}(t) = F^{-1}[-j\ sign(f)\ M(f)]$ where $M(f)$ is the Fourier transform of $m(t)$. We can carry out this easily, but since this is a phase 90 change, and $m(t)$ is a cosine function, then

$$\boxed{\hat{m}(t) = 5 \sin \omega_1 t}$$

3.2 part(b)

$$s_{SSB}(t) = A_c [m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t]$$

Where the negative sign for upper sided band, and positive sign for the lower sided band, hence

$$\begin{aligned} s_{LSSB}(t) &= A_c [m(t) \cos \omega_c t + \hat{m}(t) \sin \omega_c t] \\ &= 5A_c [\cos \omega_1 t \cos \omega_c t + \sin \omega_1 t \sin \omega_c t] \\ &= 5A_c [\cos(\omega_c - \omega_1) t] \end{aligned}$$

We can plug in numerical values given

$$\boxed{s_{LSSB}(t) = 5 [\cos(\omega_c - \omega_1) t]}$$

3.3 Part(c)

To find the RMS value of the SSB, pick the above lower side band. First find P_{av}.

$$s_{LSSB}(t) = 5[\cos(\omega_1 - \omega_c)t]$$

Hence

$$\begin{aligned} RMS \text{ value of signal} &= \frac{5}{2\sqrt{2}} \\ &= 3.5355 \text{ volt} \end{aligned}$$

X

3.4 part(d)

Then maximum of $5[\cos(\omega_1 - \omega_c)t]$ is when $\cos(\omega_1 - \omega_c)t = 1$, hence

$$s_{LSSB_{\max}}(t) = \boxed{5/2 \text{ volt}}$$

3.5 part(e)

$$\begin{aligned} P_{av} &= \frac{1}{2}A_c^2 \\ &= \frac{1}{2} \times 25 \\ &= \boxed{12.5 \text{ watt}} \end{aligned}$$

Speak 2

-1

X.

3.6 Part(f)

see sol.

$$\begin{aligned} PEP &= \frac{1}{2}s_{LSSB_{\max}}^2(t) \\ &= \frac{5^2}{2} \\ &= 12.5 \text{ watt} \end{aligned}$$

X

4 Problem 5-18

5-18 A phasing-type SSB-AM detector is shown in Fig. P5-18. This circuit is attached to the IF output of a conventional superheterodyne receiver to provide SSB reception.

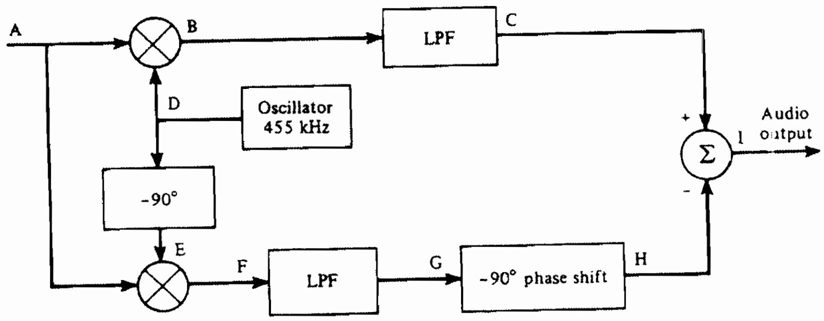


Figure P5-18

(a) Determine whether this detector is sensitive to LSSB or USSB signals. How would the detector be changed to receive SSB signals with alternate (opposite type of) sidebands?
(b) Assume that the signal at point A is a USSB signal with f_c = 455 kHz. Find the mathematical expressions for the signals at points B through I.
(c) Repeat part (b) for the case of an LSSB-AM signal at point A.
(d) Discuss the IF and LP filter requirements if the SSB signal at point A has a 3-kHz bandwidth.

Answer

4.1 part(a)

This is a detector for USSB (Upper side band). i.e.

$$s(t) = A_c \left(m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t \right)$$

ok. See sol.

Note, I wrote A_c and not $\frac{A_c}{2}$ in the above. As long this is a constant, it gives the same analysis.

The reason is because at point H the signal is $-\frac{1}{2} m(t)$ and at the C point the signal is $+\frac{1}{2} m(t)$, hence due to subtraction at the audio output end we obtain $m(t)$. To receive LSSB, we should change the sign to positive at the audio output end.

4.2 part(b)

$$s(t) = A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)$$

at point B

$$\begin{aligned}
s_B(t) &= s(t) * \overbrace{A_c^{'}\cos\omega_c t}^{\text{local oscillator}} \\
&= A_c^{'}A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)\cos\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos^2\omega_c t - \hat{m}(t)\sin\omega_c t\cos\omega_c t\right) \\
&= A_c^{'}A_c\left(m(t)\left(\frac{1}{2}+\frac{1}{2}\cos 2\omega_c t\right) - \frac{1}{2}\hat{m}(t)\sin 2\omega_c t\right) \\
&= \overbrace{\frac{A_c^{'}A_c}{2}m(t)}^{\text{low pass}} + \overbrace{\frac{A_c^{'}A_c}{2}m(t)\cos 2\omega_c t}^{\text{high pass}} - \overbrace{\frac{A_c^{'}A_c}{2}\hat{m}(t)\sin 2\omega_c t}^{\text{high pass}}
\end{aligned}$$

at point C, after LPF we obtain

$$s_c(t) = A_c^{'}A_c\frac{m(t)}{2}$$

at point F we have

$$\begin{aligned}
s_f(t) &= s(t)\,A_c^{'}\sin\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right)\sin\omega_c t \\
&= A_c^{'}A_c\left(m(t)\cos(\omega_c t)\sin(\omega_c t) - \hat{m}(t)\sin^2\omega_c t\right) \\
&= A_c^{'}A_c\left(m(t)\frac{1}{2}\sin(2\omega_c t) - \hat{m}(t)\left(\frac{1}{2}-\frac{1}{2}\cos 2\omega_c t\right)\right) \\
&= \frac{A_c^{'}A_c}{2}\left(m(t)\sin(2\omega_c t) - \hat{m}(t)\left(1-\cos 2\omega_c t\right)\right)
\end{aligned}$$

at point G after LPF

$$s_g(t) = -\frac{A_c^{'}A_c}{2}\hat{m}(t)$$

at point H after -90^0 phase shift

$$s_h(t) = +\frac{A_c^{'}A_c}{2}m(t)$$

at point I, we sum $s_h(t)$ and $s_c(t)$, hence $s_i(t) = A_c^{'}A_c\frac{m(t)}{2} + \frac{A_c^{'}A_c}{2}m(t) = A_c^{'}A_c m(t)$ ✓

4.3 Part(c)

$$s(t) = A_c (m(t) \cos \omega_c t + \hat{m}(t) \sin \omega_c t)$$

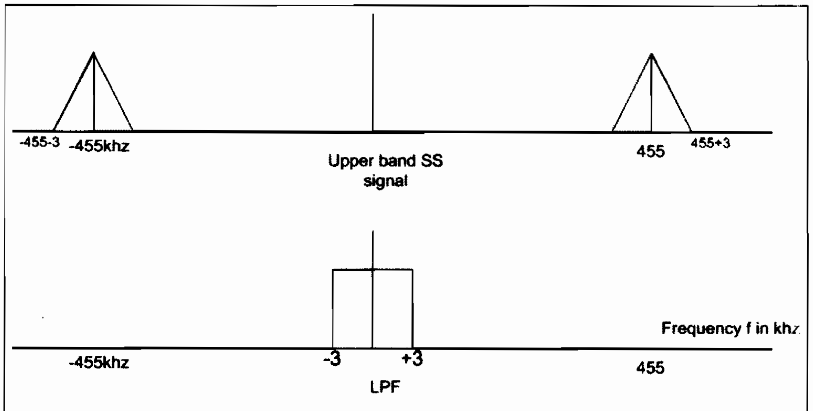
This the same as part (b), except now since there is a sign difference, this carries all the way to point I, and then we obtain

$$s_i(t) = A_c^{'} A_c \frac{m(t)}{2} - \frac{A_c^{'} A_c}{2} m(t) = 0$$

This if this circuit is used as is to demodulate an LSSB AM signal, then the signal will be lost. So, instead of adding at point I we should now subtract to counter the effect of the negative sign.

4.4 part(d)

Since SSB has bandwidth of $3kHz$ then this means the width of upper (or lower) band is $3khz$. This means the signal has $3khz$ bandwidth. This diagram shows the LPF requirement



Showing the Low Pass Filter requirement for use with LSSB demodulation

Hence LPF is centered at zero frequency and have bandwidth of $3khz$ (may be make it a little over $3khz$ band width?)

The IF filter is centered at $455 + \left(\frac{3}{2}\right)$ for the upper band of the positive band, and centered at $-455 - \left(\frac{3}{2}\right)$ for the upper band of the negative band. (i.e. for the $USSB$).

For $LSSB$, IF should be centered at $455 - \left(\frac{3}{2}\right)$ for the lower band of the positive band, and centered at $-455 + \left(\frac{3}{2}\right)$ for the lower band of the negative band. (This works if there is a guard band around 455, small one, to make the design of IF possible).