Problem 1 Evaluate the transfer function of a linear system represented by the block diagram shown in Fig. P2.14.

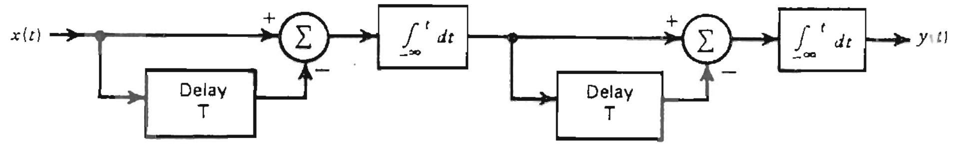


Figure P2.14

Problem 2.

(a) Determine the overall amplitude response of the cascade connection shown in P2.15, consisting of N identical stages, each with a time constant RC equal to τ_0.

(b) Show that as N approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp(-\frac{1}{2}f^2T^2)$, where for each value of N, the time constant τ_0 is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

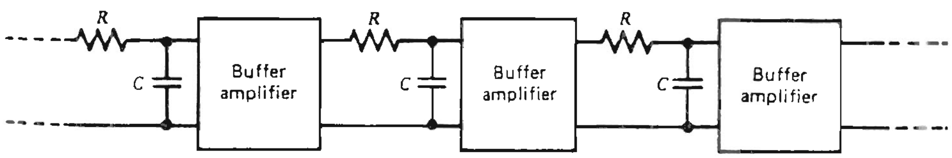


Figure P2.15

Problem 3. Determine the pre-envelope $g_+(t)$ corresponding to each of the following two signals:

(a) $g(t) = \text{sinc}(t)$ (Hint use $\frac{\sin t}{t} \overset{H.T.}{\longleftrightarrow} \frac{1-\cos t}{t}$ See prob # 4)

(b) $g(t) = [1 + k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

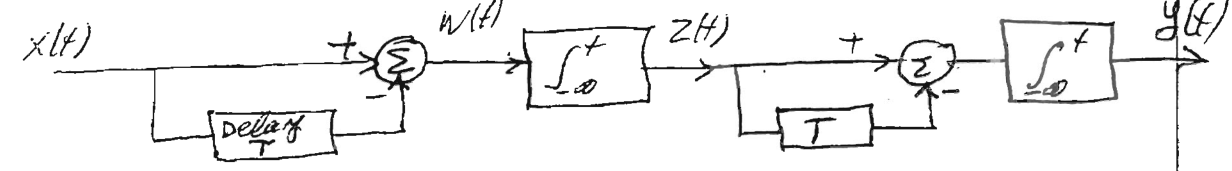
Prob # 4) Verify the following H.T:

a) if $g(t) = \delta(t) \Rightarrow \hat{g}(t) = ?$

b) if $g(t) = \frac{\sin t}{t} \Rightarrow \hat{g}(t) = \frac{1-\cos t}{t}$

prob # 5) Respond prob. 2.44 of your book

Prob # 1



Method # 1

Consider the first half the system. The second half is identical to the first half. => If $H_1(f) = \frac{Z(f)}{X(f)}$,

then $H(f) = \frac{Y(f)}{X(f)} = H_1(f) \times H_1(f) = H_1^2(f)$

- Write time domain equations, then take their F.T.

$w(t) = x(t) - x(t-T) \Rightarrow W(f) = X(f)\left[1 - e^{-j2\pi f T}\right]$ (1)

$z(t) = \int_{-\infty}^{t} w(t_1)dt_1 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} + \frac{1}{2}W(0)\delta(f)$ (2)

Find $W(0)$, Combine eq. (1) and (2) to find $H_1(f) = \frac{Z(f)}{X(f)}$

Method # 2

For first half find its impulse response $h_1(t)$

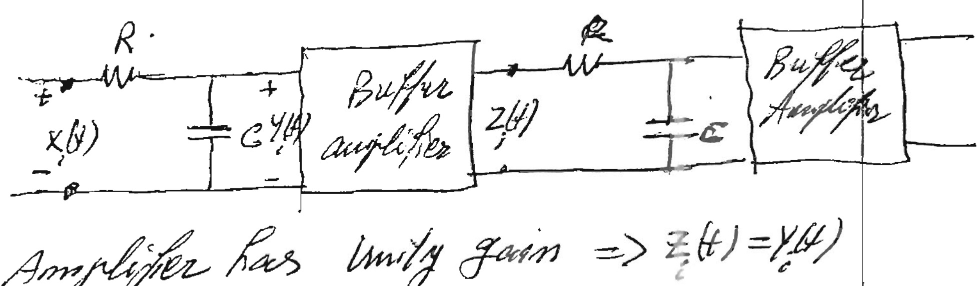
Then $H_1(f) = F.T[h_1(t)]$.

to find $h_1(t)$: if $x(t) = \delta(t)$, then $z(t) = h_1(t)$

That is $h_1(t) = \int_{-\infty}^{t} [\delta(t_1) - \delta(t_1 - T)]dt = ?$

Prob # 2

a)



Note: Buffer Amplifier has unity gain => $z_i(t) = y_i(t)$

Thus the transfer function of the ith stage is:

$$H_i(f) = \frac{Z_i(f)}{Y_i(f)} = \frac{Y_i(f)}{X_i(f)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j2\pi f RC}$$

prob #2 cont'd)

$$H_i(f) = \frac{1}{1 + j2\pi f \tau_0} \quad \text{where } \tau_0 \triangleq RC$$

- Find the overall $H(f)$, that is the transfer function of the cascad of N identical systems.
- Find the amplitud response; that is $|H(f)| = ?$

b) Let $\tau_0^2 \triangleq \dfrac{\tau^2}{4\pi^2 N}$

Use the definition of e number; that use:

$$\lim_{X \to \infty} \left(1 + \frac{\alpha}{X}\right)^{\pm \beta X} \triangleq \exp\left(\pm\frac{\alpha}{X}\beta X\right) = \exp(\alpha\beta)$$

and find $\lim_{N \to \infty} |H(f)| = ?$

prob # 3

a) $g(t) = \text{Sinc}(t)$, $\hat{g}(t) = ?$

I) Time domain approach

Use $\dfrac{\sin t}{t} \xleftrightarrow{H.T} \dfrac{1 - \cos t}{t}$ see prob #4)

Thus $\dfrac{\sin \pi t}{\pi t} \xleftrightarrow{H.T} \dfrac{1 - \cos \pi t}{\pi t}$

use $g_+(t) = g(t) + j\,\hat{g}(t)$ $\Rightarrow$ {Ans $\hat{g}(t) = \text{Sinc}\left(\frac{t}{2}\right) e^{j\frac{\pi}{2}}$, verify.

II) Frequency domain approach:

Find $G(f)$, $G_+(f)$, $g_+(t)$ the find also complex envelope $\tilde{g}(t) = g_+(t)\, e^{-j2\pi f t}$ and

Envelope $a(t) = |\tilde{g}(t)| = |g_+(t)| = ?$

4) a) $g(t) = \delta(t)$, $\hat{g}(t) = ?$

You may use time or frequency domain approach.

b) $g(t) = \frac{\sin t}{t}$ $\hat{g}(t) = ?$ (Ans: $\hat{g}(t) = \frac{1}{t}(1-\cos t)$

Frequency domain approach: Remember

$\text{rect}(t) \xleftrightarrow{F.T} \text{Sinc}(f)$

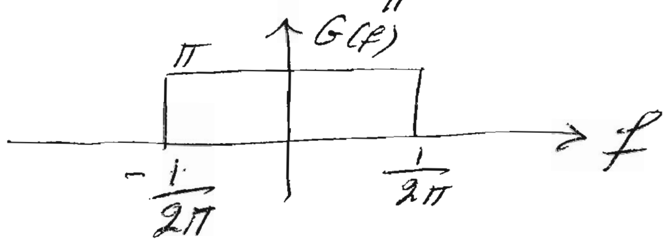
$\text{Sinc}(t) \xleftrightarrow{F.T.} \text{rect}(-f) = \text{rec}(f)$ Duality

Thus: $\text{Sinc}(t) = \frac{\sin \pi t}{\pi t} \longleftrightarrow \text{rect}(f)$

Using time scaling: if $x(t) \xleftrightarrow{F.T} X(f)$
then $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$

in our case $a = \frac{1}{\pi}$:

$$g(t) = \frac{\sin t}{t} \xleftrightarrow{F.T} \frac{1}{1/\pi} \text{rec}\left(\frac{f}{1/\pi}\right) = \pi \,\text{rec}\left(\frac{f}{1/\pi}\right) \Rightarrow$$



Now use:

$$\hat{G}(f) = -j\,\text{Sgn}(f)\,G(f) = -j\pi\,\text{Sgn}(f)\,\text{rect}\left(\frac{f}{1/\pi}\right)$$

continue !

At some point you may use $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

Prob. #5) Use $R_g(\tau) = F^{-1}T\left[S_g(f)\right]$

where $S_g(f) = \text{rec}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$

... $S_g(f)$ curve. !