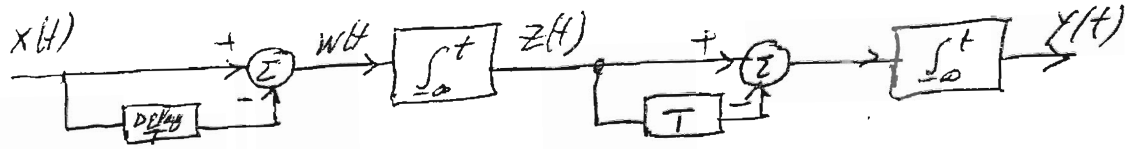


Problem 2.1

The first integrator input is equal to x(t)-x(t-T). The Fourier transform of this input signal is $[1-\exp(-j2\pi fT)]X(f)$. The value of this transform is zero at f=0. It follows therefore that the Fourier transform of the first integrator output is equal to

That is: $w(t)=x(t)-x(t-T) \Rightarrow W(f)=X(f)[1-e^{-j2\pi fT}]$

$$(*)\quad Z(f) = \frac{1}{j2\pi f}[1-\exp(-j2\pi fT)]X(f)$$

$z(t)=\int_{-\infty}^{t} w(t_1)\,dt_1 \Rightarrow Z(f)=\frac{W(f)}{j2\pi f}+\frac{1}{2}W(0)\delta(f)$

Since $W(0)=0 \Rightarrow Z(f)=\frac{W(f)}{j2\pi f} \rightarrow (*)$

The transfer function of the first stage of the system of Fig. P2.5 is therefore equal to $H_1(f)=\frac{Z(f)}{X(f)}$

$$\Rightarrow H_1(f) = \frac{1}{j2\pi f}[1-\exp(-j2\pi fT)] = \frac{e^{-j\pi fT}}{j2\pi f}\left[e^{j\pi fT}-e^{-j\pi fT}\right] = \frac{e^{-j\pi fT}}{\pi f}\sin(\pi fT) = Te^{-j\pi fT}\,\mathrm{Sinc}($$

The second stage of the system is identical to the first stage. The overall transfer function of the system is therefore: $H(f)=H_1(f)\times H_1(f)$

$$H(f) = \frac{1}{(j2\pi f)^2}[1-\exp(-j2\pi fT)]^2 = \frac{1+e^{-j4\pi fT}-2e^{-j2\pi fT}}{(j2\pi f)^2} = \frac{e^{-j2\pi fT}\left[e^{+j2\pi fT}+e^{-j2\pi fT}-2\right]}{(j2\pi fT)^2}$$

$$= \exp(-j2\pi fT)\left[\frac{\exp(j\pi fT)-\exp(-j\pi fT)}{j2\pi f}\right]^2$$

$$= \exp(-j2\pi fT)\left[\frac{\sin(\pi fT)}{\pi f}\right]^2$$

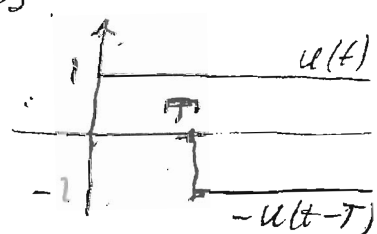
$$= T^2\,\mathrm{sinc}^2(fT)\exp(-j2\pi fT)$$

2° method

The impulse response of the first half:

if $x(t)=\delta(t)$, then $z(t)=h_1(t)$, Thus:

$$h_1(t)=\int_{-\infty}^{t}[\delta(t_1)-\delta(t_1-T)]\,dt_1 = u(t)-u(t-T)$$



$$\Rightarrow h_1(t) = \mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

$$\Rightarrow H_1(f) = F.T[h_1(t)] = T\,\mathrm{Sinc}(fT)\,e^{-j\pi fT}$$

$$H(f) = H_1^2(f) = T^2\,\mathrm{Sinc}^2(fT)\,e^{-j2\pi fT}$$

Problem 2.

(a) The transfer function of the ith stage of the system of Fig. P2.6 is

$$H_i(f) = \frac{1}{1+j2\pi fRC}$$

$$= \frac{1}{1+j2\pi f\tau_0}$$

where it is assumed that the buffer amplifier has a constant gain of one. The overall transfer function of the system is therefore

$$H(f) = \prod_{i=1}^{N} H_i(f)$$

$$= \frac{1}{(1+j2\pi f\tau_0)^N}$$

The corresponding amplitude response is

$$|H(f)| = \frac{1}{[1+(2\pi f\tau_0)^2]^{N/2}}$$

(b) Let

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

Then, we may rewrite the expression for the amplitude response as

$$|H(f)| = \left[1+\frac{1}{N}(fT)^2\right]^{-N/2}$$

In the limit, as N approaches infinity, we have

$$|H(f)| = \lim_{N\to\infty} [1+\frac{1}{N}(fT)^2]^{-N/2}$$

$$= \exp[-\frac{N}{2}\cdot\frac{1}{N}(fT)^2]$$

$$= \exp(-\frac{f^2T^2}{2})$$

Problem 3 .

(a) $g(t) = \text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$

We note that

$$\hat{g}(t) = \frac{1-\cos(\pi t)}{\pi t}$$

Therefore,

$$g_+(t) = g(t) + j\hat{g}(t)$$

$$= \frac{\sin(\pi t)}{\pi t} + j\,\frac{1-\cos(\pi t)}{\pi t}$$

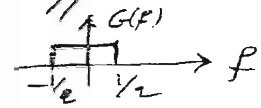
$$= \frac{j}{\pi t}[1 - \cos(\pi t) - j\sin(\pi t)]$$

$$= \frac{j}{\pi t}[1 - \exp(j\pi t)] = \frac{1}{j\pi t}\left[e^{j\pi t} - 1\right] = \frac{1}{j\pi t}\left[\frac{e^{j\pi t} - 1}{e^{j\pi t/2}}\right]e^{j\pi t/2} = \frac{2}{\pi t}(\sin \pi t/2)\cdot e^{j\ldots} = \text{sinc}(t/2)\,e^{j\ldots}$$

II° method:

using freq. domain approach:

$G(f) = \text{rect}(f)$



$G_+(f) = 2G(f) = 2\,\text{rect}\left(\frac{f-1/4}{1/2}\right)$ for f

$g_+(t) = 2\cdot\frac{1}{2}\,\text{sinc}\left(\frac{t}{2}\right)\,e^{j2\pi\frac{1}{4}t}$

$\Rightarrow g_+(t) = \text{sinc}(t/2)\,e^{j2\pi\frac{1}{4}t}$ $\quad f_c =$

$\tilde{g}(t) = g_+(t)\cdot e^{-j2\pi f_c t} = \text{sinc}(t/2)$

$a(t) = |\tilde{g}(t)| = |\text{sinc}(t/2)|$

(b) $g(t) = [1+k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

$$= \cos(2\pi f_c t) + \frac{k}{2}\cos[2\pi(f_c+f_m)t] + \frac{k}{2}\cos[2\pi(f_c-f_m)t]$$

Since the Hilbert transform of $\cos(2\pi ft)$ is equal to $\sin(2\pi ft)$, it follows that

$$\hat{g}(t) = \sin(2\pi f_c t) + \frac{k}{2}\sin[2\pi(f_c+f_m)t] + \frac{k}{2}\sin[2\pi(f_c-f_m)t]$$

where it is assumed that $f_c > f_m$. Therefore,

$$g_+(t) = \exp(j2\pi f_c t) + \frac{k}{2}\exp[j2\pi(f_c+f_m)t] + \frac{k}{2}\exp[j2\pi(f_c-f_m)t]$$

$$= [1 + \frac{k}{2}\exp(j2\pi f_m t) + \frac{k}{2}\exp(-j2\pi f_m t))]\exp(j2\pi f_c t)$$

$$= [1 + k\cos(2\pi f_m t)]\exp(j2\pi f_c t)$$

HW #1 page

Problem 4

Prob #4 part b)

time domain approach:

(b) $g(t) = \frac{\sin t}{t}$

The Hilbert transform of sin t/t is

$$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\,d\tau$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau(t-\tau)}\,d\tau$$

$$= \frac{1}{\pi t}\int_{-\infty}^{\infty} \left(\frac{1}{\tau} + \frac{1}{t-\tau}\right)\sin\tau\,d\tau$$

$$= \frac{1}{\pi t}\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\,d\tau + \frac{1}{\pi t}\int_{-\infty}^{\infty} \frac{\sin\tau}{t-\tau}\,d\tau$$

We note that

$$\int_{-\infty}^{\infty} \mathrm{sinc}(t)\,dt = 1$$

Therefore,

$$\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\,d\tau = \pi$$

$$\int_{-\infty}^{\infty} \frac{\sin\tau}{t-\tau}\,d\tau = \int_{-\infty}^{\infty} \frac{\sin(t-\tau)}{\tau}\,d\tau$$

Prob. #4 part b

II) Freq. domain approach

Remember: $\mathrm{rect}(t) \xleftrightarrow{F.T} \mathrm{Sinc}(f)$

using duality:

$\mathrm{Sinc}(t) \xleftrightarrow{F.T} \mathrm{rect}(-f) = \mathrm{rect}(f)$

Thus:

$\mathrm{Sinc}(t) = \frac{\sin \pi t}{\pi t} \longleftrightarrow \mathrm{rect}(f)$

$g(t) = \frac{\sin t}{t} = \frac{\sin(\pi t \cdot \frac{1}{\pi})}{(\pi t \cdot \frac{1}{\pi})} \longleftrightarrow \frac{1}{(1/\pi)}\mathrm{rect}(-$

where time scaling is used.

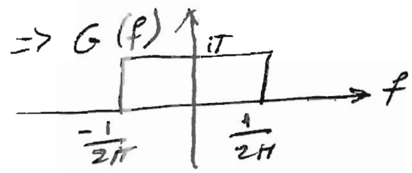
That is if $g(t) \longleftrightarrow G(f)$, then

$g(at) \longleftrightarrow \frac{1}{a} G(f/a)$, Theref.

in our case:

$g(t) = \frac{\sin t}{t} \xleftrightarrow{F.T} \pi\,\mathrm{rect}\left(\frac{f}{1/\pi}\right)$

$\Rightarrow G(f)$



$\hat{G}(f) = -j\,\mathrm{sgn}(f)\,G(f) =$

$= -j\pi\,\mathrm{sgn}(f)\left[\mathrm{rect}\left(\frac{f-\frac{1}{4\pi}}{1/2\pi}\right) + \mathrm{rect}\left(\frac{f+\frac{1}{4\pi}}{1/2\pi}\right)\right]$

$= -j\pi\left[\mathrm{rect}\left(\frac{f-\frac{1}{4\pi}}{1/2\pi}\right) - \mathrm{rect}\left(\frac{f+\frac{1}{4\pi}}{1/2\pi}\right)\right]$

$\Rightarrow \hat{g}(t) = -j\pi\left[\frac{1}{2\pi}\mathrm{Sinc}\left(\frac{t}{2\pi}\right)\right]\left[e^{jt/2} - e^{-jt/2}\right]$

$\hat{g}(t) = \mathrm{Sinc}\left(\frac{t}{2\pi}\right)\sin(t/2)$

$= \frac{2}{t}\sin^2(t/2) = \frac{1}{t}(1-\cos t)$

$$= \sin t \underbrace{\int_{-\infty}^{\infty} \frac{\cos\tau}{\tau}\,d\tau}_{0} - \cos t \int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\,d\tau$$

it is odd function

$$= -\pi \cos t$$

We thus obtain

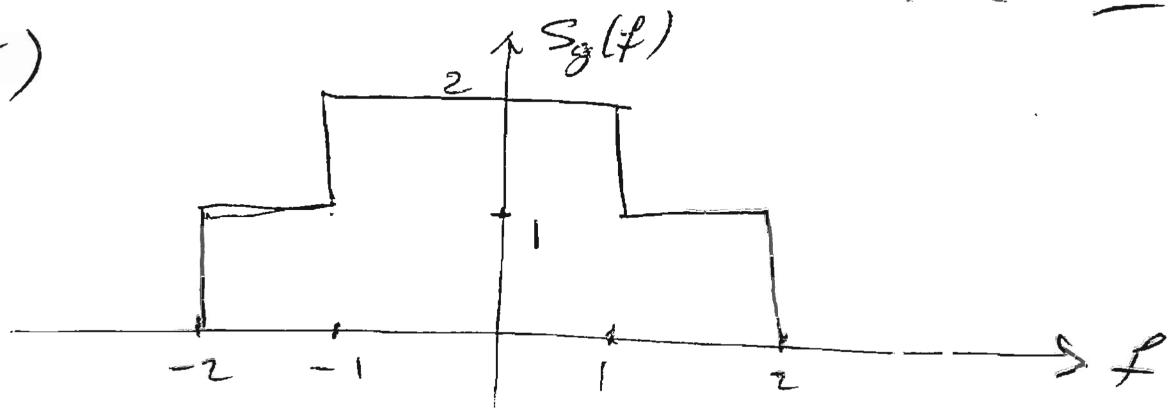
$$\hat{g}(t) = \frac{1}{t}(1-\cos t)$$

Thus $\hat{g}(t) = \frac{1}{t}(1-\cos t)$

a) $g(t) = \delta(t)$

$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\delta(\tau)}{t-\tau}\,d\tau = \frac{1}{\pi t}\int_{-\infty}^{+\infty} \delta(\tau)\,d\tau = \frac{1}{\pi t}$

prob #5)



$$S_g(f) = \mathrm{rec}\left(\frac{f}{4}\right) + \mathrm{rect}\left(\frac{f}{2}\right)$$

$$R_g(\tau) = F^{-1}\left[S_g(f)\right] = 4\,\mathrm{Sinc}(4\tau) + 2\,\mathrm{Sinc}(2\tau)$$

$$R_g(0) = P_{av} = 6 \text{ watts}$$

That is

$$R_g(0) = \int_{-\infty}^{+\infty} S_g(f)\,df = 6\,W \triangleq P_{av}$$