HW 4
Electronic Communication Systems
Fall 2008
California State University, Fullerson

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Contents

1 questions and hints

EE 443 HW # 4 page 1

Problem 1 Evaluate the transfer function of a linear system represented by the block diagram shown in Fig. P2.14.

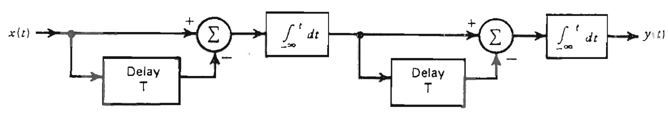


Figure P2.14

Problem 2.

(a) Determine the overall amplitude response of the cascade connection shown in P2.15, consisting of N identical stages, each with a time constant RC equal to τ_0.

(b) Show that as N approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp(-\frac{1}{2}f^2T^2)$, where for each value of N, the time constant τ_0 is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

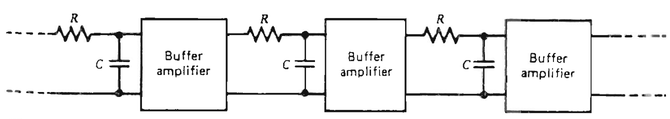


Figure P2.15

Problem 3. Determine the pre-envelope $g_+(t)$ corresponding to each of the following two signals:

(a) $g(t) = \text{sinc}(t)$ (Hint use $\frac{\sin t}{t} \overset{H.T.}{\longleftrightarrow} \frac{1-\cos t}{t}$ See prob # 4)

(b) $g(t) = [1 + k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

Prob # 4) Verify the following H.T :

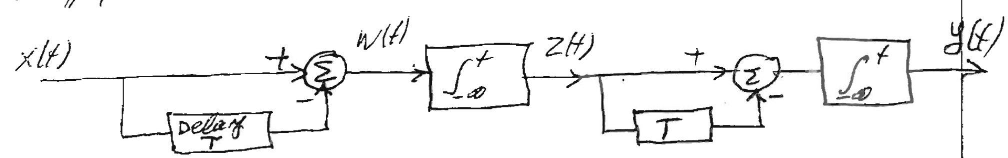
a) if $g(t) = \delta(t) \Rightarrow \hat{g}(t) = ?$

b) if $g(t) = \frac{\sin t}{t} \Rightarrow \hat{g}(t) = \frac{1-\cos t}{t}$

prob # 5) Respond prob. 2.44 of your book

EE 443 | HW # 4 (Hint) | page 1

Prob # 1



Method # 1

Consider the first half the system. The second half is identical to the first half. => If $H_1(f) = \frac{Z(f)}{X(f)}$,

then $H(f) = \frac{Y(f)}{X(f)} = H_1(f) \times H_1(f) = H_1^2(f)$

- write time domain equations, then take their F.T.

$w(t) = x(t) - x(t-T) \Rightarrow W(f) = X(f)\left[1 - e^{-j2\pi fT}\right]$ (1)

$z(t) = \int_{-\infty}^{t} w(t_1)\,dt_1 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} + \frac{1}{2}W(0)\delta(f)$ (2)

Find $W(0)$, Combine eq. (1) and (2) to find $H_1(f) = \frac{Z(f)}{X(f)}$

Method # 2

For first half find its impulse response $h_1(t)$

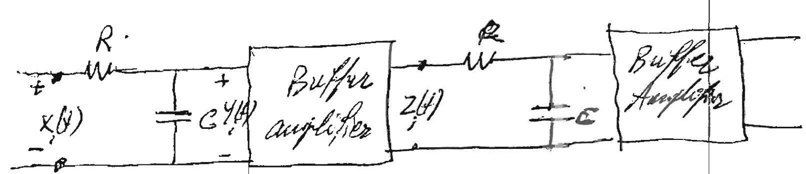
Then $H_1(f) = F.T[h_1(t)]$.

to find $h_1(t)$: if $x(t) = \delta(t)$, then $z(t) = h_1(t)$

That is $h_1(t) = \int_{-\infty}^{t}[\delta(t_1) - \delta(t_1 - T)]\,dt = \ ?$

Prob # 2

a)



Note: Buffer Amplifier has unity gain => $z_i(t) = y_i(t)$

Thus the transfer function of the ith stage is:

$$H_i(f) = \frac{Z_i(f)}{X_i(f)} = \frac{Y_i(f)}{X_i(f)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j2\pi fRC}$$

EE 443 Hw #4 (Hint) page (2)

prob #2 Cont'd)

$$H_i(f) = \frac{1}{1+j2\pi f\tau_0} \quad \text{where } \tau_0 \triangleq RC$$

- Find the overall $H(f)$, that is the transfer function of the Cascad of N identical systems.
- Find the amplitud response; that is $|H(f)| = ?$

b) Let $\tau_0^2 \triangleq \frac{\tau^2}{4\pi^2 N}$

Use the definition of e number; that use:

$$\lim_{x\to\infty} \left(1+\frac{\alpha}{x}\right)^{\pm\beta x} \triangleq \exp\left(\pm\frac{\alpha}{x}\beta x\right) = \exp(\alpha\beta)$$

and find $\lim_{N\to\infty} |H(f)| = ?$

prob # 3

a) $g(t) = \text{Sinc}(t)$, $\hat{g}(t) = ?$

I) Time domain approach

use $\frac{\sin t}{t} \xleftrightarrow{H.T} \frac{1-\cos t}{t}$ see prob #4)

Thus $\frac{\sin \pi t}{\pi t} \xleftrightarrow{H.T} \frac{1-\cos \pi t}{\pi t}$

use $g_+(t) = g(t) + j\,\hat{g}(t)$ $\Rightarrow$ {Ans $\hat{g}(t) = \text{Sinc}\left(\frac{t}{2}\right) e^{j\frac{\pi}{2}}$ verify.

II) Frequency domain approach:

Find $G(f)$, $G_+(f)$, $g_+(t)$ the find also Complex envelope $\tilde{g}(t) = g_+(t)\, e^{-j2\pi f t}$ and Envelope $a(t) = |\tilde{g}(t)| = |g_+(t)| = ?$

EE 443 | Hw #4 | page 3

4) a) $g(t) = \delta(t)$, $\hat{g}(t) = ?$

You may use time or frequency domain approach.

b) $g(t) = \dfrac{\sin t}{t}$ $\quad \hat{g}(t) = ?$ (Ans: $\hat{g}(t) = \frac{1}{t}(1-\cos t)$

Frequency domain approach: Remember

$\mathrm{rect}(t) \xleftrightarrow{F.T} \mathrm{sinc}(f)$

$\mathrm{sinc}(t) \xleftrightarrow{F.T.} \mathrm{rect}(-f) = \mathrm{rect}(f)$ Duality

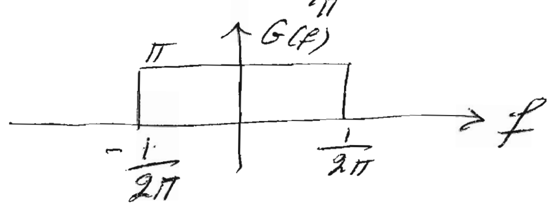
Thus: $\mathrm{sinc}(t) = \dfrac{\sin \pi t}{\pi t} \longleftrightarrow \mathrm{rect}(f)$

Using time scaling: if $x(t) \xleftrightarrow{F.T} X(f)$

then $x(at) \longleftrightarrow \dfrac{1}{|a|} X\left(\dfrac{f}{a}\right)$

in our case $a = \dfrac{1}{\pi}$:

$$g(t) = \frac{\sin t}{t} \xleftrightarrow{F.T} \frac{1}{1/\pi}\,\mathrm{rect}\left(\frac{f}{1/\pi}\right) = \pi\,\mathrm{rect}\left(\frac{f}{1/\pi}\right) \Longrightarrow$$



Now use:

$$\hat{G}(f) = -j\,\mathrm{sgn}(f)\,G(f) = -j\pi\,\mathrm{sgn}(f)\,\mathrm{rect}\left(\frac{f}{1/\pi}\right)$$

continue!

At some point you may use $\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2}$

Prob. #5) Use $R_g(\tau) = F^{-1}T\left[S_g(f)\right]$

where $S_g(f) = \mathrm{rect}\left(\frac{f}{4}\right) + \mathrm{rect}\left(\frac{f}{2}\right)$

2 Problem 1

Solution Using transfer function cascading, then the overall transfer function for the system can be written as

$$H(f) = H_1(f)\,H_1(f) = [H_1(f)]^2 \tag{1}$$

Where

$$H_1(f) = \frac{Z(f)}{X(f)}$$

Where

$$\begin{aligned} Z(f) &= \mathcal{F}\left\{ \int_{-\infty}^{t} w(\tau)\, d\tau \right\} \\ &= \frac{1}{j2\pi f} W(f) + \frac{W(0)}{2}\delta(f) \end{aligned} \tag{2}$$

Where

$$\begin{aligned} W(f) &= \mathcal{F}\{x(t) - x(t-T)\} \\ &= X(f) - X(f)e^{-j2\pi fT} \\ &= X(f)\left[1 - e^{-j2\pi fT}\right] \end{aligned} \tag{3}$$

substitute (3) into (2) we obtain

$$\begin{aligned} Z(f) &= \frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi fT}\right] + \frac{X(0)\overbrace{\left[1 - e^{-j2\pi 0T}\right]}^{=0}}{2}\delta(f) \\ &= \frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi fT}\right] \end{aligned} \tag{4}$$

Hence

$$\begin{aligned} H_1(f) &= \frac{Z(f)}{X(f)} \\ &= \frac{\frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi fT}\right]}{X(f)} \end{aligned}$$

Hence

$$\boxed{H_1(f) = \tfrac{1}{j2\pi f}\left[1 - e^{-j2\pi fT}\right]}$$

Hence from (1)

$$\begin{aligned} H(f) &= \left(\frac{1}{j2\pi f}\left[1 - e^{-j2\pi fT}\right]\right)^2 \\ &= \frac{1}{-4\pi^2 f^2}\left[1 - e^{-j2\pi fT}\right]^2 \\ &= \frac{-1}{(2\pi f)^2}\left[1 - 2e^{-j2\pi fT} + e^{-j4\pi fT}\right] \end{aligned}$$

Hence

$$\boxed{H(f) = \tfrac{1}{(2\pi f)^2}\left[2e^{-j2\pi fT} - e^{-j4\pi fT} - 1\right]}$$

3 Problem 2

3.1 Part(a)

Transfer function for each stage is $H_i(f) = \frac{Y_i(f)}{X_i(f)} = \frac{1}{1+j2\pi fRC}$

Since $RC = \tau_0$, hence

$$H_i(f) = \frac{1}{1 + j2\pi f\tau_0}$$

Then, for N stages, the overall transfer function is

$$H(f) = H_1(f)\, H_2(f) \cdots H_N(f)$$

Since they are identical stages, then the transfer function of each stage is the same, and the above becomes

$$H(f) = \left(\frac{1}{1 + j2\pi f\tau_0}\right)^N$$

Hence the amplitude of the response is given by

$$\begin{aligned}|H(f)| &= \left(\frac{1}{|1+j2\pi f\tau_0|}\right)^N \\ &= \left(\frac{1}{\sqrt{1^2+(2\pi f\tau_0)^2}}\right)^N \\ &= \left(\frac{1}{(1+4\pi^2 f^2\tau_0^2)^{\frac{1}{2}}}\right)^N \\ &= \frac{1}{(1+4\pi^2 f^2\tau_0^2)^{\frac{N}{2}}}\end{aligned}$$

Let $\tau_0^2 = \frac{\tau^2}{4\pi^2 N}$, the above becomes

$$|H(f)| = \frac{1}{\left(1+\frac{f^2\tau^2}{N}\right)^{\frac{N}{2}}} \tag{1}$$

3.2 Part (b)

Let $\alpha = f^2\tau^2, \beta = \frac{1}{2}$, then (1) becomes

$$|H(f)| = \frac{1}{\left(1+\frac{\alpha}{N}\right)^{\beta N}}$$

But $\lim_{N\to\infty} \frac{1}{\left(1+\frac{\alpha}{N}\right)^{\beta N}} = e^{\alpha\beta}$,hence

$$\begin{aligned}|H(f)| &= \frac{1}{e^{\frac{f^2\tau^2}{2}}} \\ &= e^{-\frac{f^2\tau^2}{2}}\end{aligned}$$

Which is what we are asked to show.

4 Problem 3

4.1 Part(a)

(a) $g(t) = \text{sinc}(t)$

$$g_+(t) = g(t) + j\hat{g}(t) \tag{1}$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$

$$\begin{aligned}\hat{G}(f) &= -j\ sgn(f)\ \ G(f) \\ &= -j\ sgn(f)\ \ rect(f)\end{aligned}$$

Now find the inverse Fourier transform.

I derive the above to answer problem 4 part (b). The answer is the following (please see problem 4 part(b) for the derivation

$$\hat{g}(t) = \frac{1}{\pi t}(1-\cos\pi t)$$

In the above, I used $\text{sinc}(t) \equiv \frac{\sin \pi t}{\pi t}$. If one uses $\text{sinc}(t) \equiv \frac{\sin t}{t}$ then the answer becomes

$$\hat{g}(t) = \frac{1}{t}(1-\cos t) \tag{2}$$

The problem statement seems to want us to use the second definition of $\operatorname{sinc}(t)$, so I will continue the rest of the solution using (1).

Substitute (2) into (1) we obtain

$$
\begin{aligned}
g_{+}(t) &= \operatorname{sinc}(t) + j\frac{1}{t}(1-\cos t)\\
&= \frac{\sin(t)}{t} + j\frac{1}{t}\left(1 - \frac{e^{jt}+e^{-jt}}{2}\right)\\
&= \frac{1}{t}\frac{e^{jt}-e^{-jt}}{2j} + \frac{1}{t}\left(j + \frac{e^{jt}+e^{-jt}}{2j}\right)\\
&= \frac{1}{t}\frac{e^{jt}-e^{-jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt}+e^{-jt}}{2j}\\
&= \frac{1}{t}\frac{e^{jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt}}{2j}
\end{aligned}
$$

Hence

$$\boxed{g_{+}(t) = \tfrac{1}{t}\left(j + e^{jt}\right)}$$

4.2 Part(b)

$$g(t) = \left[1 + k\cos 2\pi f_m t\right]\cos(2\pi f_c t)$$

$$g_{+}(t) = g(t) + j\hat{g}(t)$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$.

$$
G_{+}(f) = \begin{cases} 2G(f) & f > 0\\ G(0) & f = 0\\ 0 & f < 0 \end{cases}
$$

But

$$G(f) = \mathcal{F}\left[1 + k\cos 2\pi f_m t\right] \otimes \mathcal{F}\left[\cos(2\pi f_c t)\right] \tag{1}$$

But

$$\mathcal{F}\left[\cos(2\pi f_c t)\right] = \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

and

$$\mathcal{F}\left[1 + k\cos 2\pi f_m t\right] = \delta(f) + \frac{k}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$$

Hence (1) becomes

$$
\begin{aligned}
G(f) = &\left\{\delta(f) + \frac{k}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]\right\} \otimes \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]\\
&= \delta(f) \otimes \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]\\
&+ \frac{k}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right] \otimes \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]\\
\\
&= \frac{1}{2}\delta(f) \otimes \delta(f-f_c) +\\
&\frac{1}{2}\delta(f) \otimes \delta(f+f_c) +\\
&\frac{k}{4}\delta(f-f_m) \otimes \delta(f-f_c) +\\
&\frac{k}{4}\delta(f-f_m) \otimes \delta(f+f_c) +\\
&\frac{k}{4}\delta(f+f_m) \otimes \delta(f-f_c) +\\
&\frac{k}{4}\delta(f+f_m) \otimes \delta(f+f_c)
\end{aligned}
$$

Hence

$$\begin{aligned} G(f) = \frac{1}{2}\delta(f+f_c) + \\ \frac{1}{2}\delta(f-f_c) + \\ \frac{k}{4}\delta(f-f_m+f_c) + \\ \frac{k}{4}\delta(f-f_m-f_c) + \\ \frac{k}{4}\delta(f+f_m+f_c) + \\ \frac{k}{4}\delta(f+f_m-f_c) \end{aligned}$$

Hence for $f>0$,$G_+(f) = 2G(f)$ and we obtain

$G_+(f) = \delta(f-f_c) + \frac{k}{2}\left[\delta(f-f_m+f_c) + \delta(f-f_m-f_c) + \delta(f+f_m+f_c) + \delta(f+f_m-f_c)\right]$

Then (since carrier frequency $f_c > f_m$), we could simplify the above, by keeping positive frequencies f

$$G_+(f) = \delta(f-f_c) + \frac{k}{2}\left[\delta(f-f_m-f_c) + \delta(f+f_m-f_c)\right]$$

or

$$G_+(f) = \delta(f-f_c) + \frac{k}{2}\left[\delta(f-(f_m+f_c)) + \delta(f-(f_c-f_m))\right]$$

Hence

$$\begin{aligned} g_+(t) &= e^{j2\pi f_c t} + \frac{k}{2}\left(e^{j2\pi(f_m+f_c)t} + e^{j2\pi(f_c-f_m)t}\right) \\ &= e^{j2\pi f_c t} + \frac{k}{2}\left(e^{j2\pi f_m t}e^{j2\pi f_c t} + e^{j2\pi f_c t}e^{-j2\pi f_m t}\right) \\ &= e^{j2\pi f_c t}\left[1 + \frac{k}{2}\left(e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right)\right] \\ &= e^{j2\pi f_c t}\left[1 + \frac{k}{2}\left(2\cos(2\pi f_m t)\right)\right] \\ &= e^{j2\pi f_c t}\left[1 + k\cos(2\pi f_m t)\right] \end{aligned}$$

5 Problem 4

5.1 Part(a)

$g(t) = \delta(t)$

$$\begin{aligned} \hat{g}(t) &= g(t) \otimes \frac{1}{\pi t} \\ &= \frac{1}{\pi}\int_{-\infty}^{\infty} \delta(\tau)\frac{1}{t-\tau}d\tau \\ &= \frac{1}{\pi}\int_{-\infty}^{\infty} \delta(\tau)\frac{1}{t}d\tau \\ &= \frac{1}{\pi t}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau \\ &= \frac{1}{\pi t} \end{aligned}$$

5.2 Part(b)

And Since $sgn\left(f\right)=-1$ for $f<0$ and $sgn\left(f\right)=1$ for $f>0$ then

$$\hat{G}\left(f\right)=-j\left[-rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right)+rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right]$$

Hence

$$\hat{g}\left(t\right)=jF^{-1}\left[rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right)-rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right] \tag{1}$$

But $F^{-1}\left(rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right)\right)=\frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)e^{-j2\pi\frac{1}{4}t}$ and $F^{-1}\left(rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right)=\frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)e^{+j2\pi\frac{1}{4}t}$, hence (1) becomes

$$\begin{aligned}
\hat{g}\left(t\right) &= j\left[\frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)e^{-j2\pi\frac{1}{4}t}-\frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)e^{+j2\pi\frac{1}{4}t}\right]\\
&= \frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)\left[j\left(e^{-j2\pi\frac{1}{4}t}-e^{j2\pi\frac{1}{4}t}\right)\right]\\
&= \frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}t\right)\left[\frac{e^{-j\frac{\pi}{2}t}-e^{j\frac{\pi}{2}t}}{-j}\right]\\
&= \operatorname{sinc}\left(\frac{1}{2}t\right)\left[\frac{e^{j\frac{\pi}{2}t}-e^{-j\frac{\pi}{2}t}}{2j}\right]\\
&= \operatorname{sinc}\left(\frac{1}{2}t\right)\left[\sin\frac{\pi}{2}t\right]
\end{aligned}$$

But $\operatorname{sinc}\left(\frac{1}{2}t\right)=\frac{\sin\frac{\pi t}{2}}{\frac{\pi t}{2}}$ hence

$$\begin{aligned}
\hat{g}\left(t\right) &= \frac{\sin\frac{\pi t}{2}}{\frac{\pi t}{2}}\sin\frac{\pi}{2}t\\
&= \frac{2}{\pi t}\sin^2\frac{\pi}{2}t\\
&= \frac{2}{\pi t}\left(\frac{1}{2}-\frac{1}{2}\cos\pi t\right)\\
&= \frac{1}{\pi t}\left(1-\cos\pi t\right)
\end{aligned}$$

6 problem 5

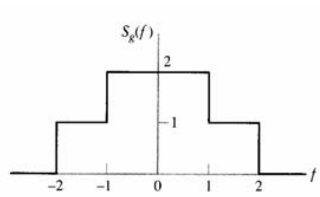


FIGURE 2.47
Problem 2.44

2.45 Consider the square wave $g(t)$ shown in Fig. 2.48. Find the power spectral density, average power, and autocorrelation function of this square wave. Does the wave have dc power? Explain your answer.

Figure 1: the Problem statement

$$S_g\left(f\right)=rect\left(\frac{f}{4}\right)+rect\left(\frac{f}{2}\right)$$

$$R_g\left(\tau\right)=F^{-1}\left(S_g\left(f\right)\right)$$

Hence

$$
\begin{aligned}
R_g(\tau) &= \mathcal{F}^{-1}\left(rect\left(\frac{f}{4}\right) + rect\left(\frac{f}{2}\right)\right) \\
&= \mathcal{F}^{-1}\left[rect\left(\frac{f}{4}\right)\right] + \mathcal{F}^{-1}\left[rect\left(\frac{f}{2}\right)\right] \\
&= 4\,\mathrm{sinc}\,(4t) + 2\,\mathrm{sinc}\,(2t)
\end{aligned}
$$

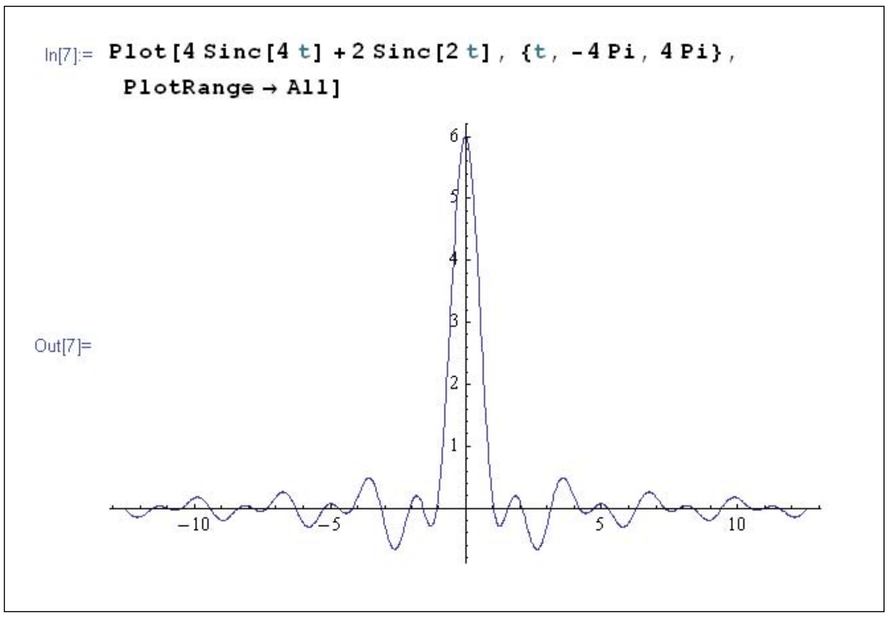
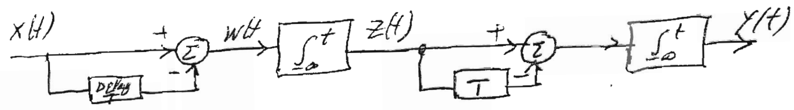


Figure 2: Plot for problem 5

7 Key solution

EE 443   HW#4 Key   page



Problem 2.

The first integrator input is equal to x(t)-x(t-T). The Fourier transform of this input signal is [1-exp(-j2πfT)]X(f). The value of this transform is zero at f=0. It follows therefore that the Fourier transform of the first integrator output is equal to

That is: $w(t) = x(t) - x(t-T) \Rightarrow W(f) = X(f)[1 - e^{-j2\pi f T}]$

$z(t) = \int_{-\infty}^{t} w(t_1)\,dt_1 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} + \frac{1}{2}W(0)\delta(f)$

Since $W(0) = 0 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} \rightarrow (*)$

$$(*)\quad Z(f) = \frac{1}{j2\pi f}[1-\exp(-j2\pi fT)]X(f)$$

The transfer function of the first stage of the system of Fig. P2.5 is therefore equal to $H_1(f) = \frac{Z(f)}{X(f)}$

$$\Rightarrow H_1(f) = \frac{1}{j2\pi f}[1-\exp(-j2\pi fT)] = \frac{e^{-j\pi fT}}{j2\pi f}\left[e^{j\pi fT} - e^{-j\pi fT}\right] = \frac{e^{-j\pi fT}}{\pi f}\sin(\pi fT) = T e^{-j\pi fT}\,\mathrm{sinc}($$

The second stage of the system is identical to the first stage. The overall transfer function of the system is therefore : $H(f) = H_1(f) \times H_1(f)$

$$H(f) = \frac{1}{(j2\pi f)^2}[1-\exp(-j2\pi fT)]^2 = \frac{1 + e^{-j4\pi fT} - 2e^{-j2\pi fT}}{(j2\pi f)^2} = \frac{e^{-j2\pi fT}\left[e^{+j2\pi fT} + e^{-j2\pi fT} - 2\right]}{(j2\pi fT)^2}$$

$$= \exp(-j2\pi fT)\left[\frac{\exp(j\pi fT)-\exp(-j\pi fT)}{j2\pi f}\right]^2$$

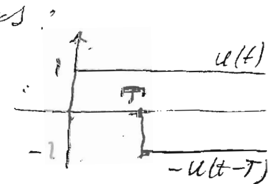
$$= \exp(-j2\pi fT)\left[\frac{\sin(\pi fT)}{\pi f}\right]^2$$

$$= T^2\,\mathrm{sinc}^2(fT)\exp(-j2\pi fT)$$

2° method

The impulse response of the first half:

if $x(t) = \delta(t)$, then $z(t) = h_1(t)$, Thus:



$$h_1(t) = \int_{-\infty}^{t} [\delta(t_1) - \delta(t_1 - T)]\,dt_1 = u(t) - u(t-T)$$

$$\Rightarrow h_1(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right)$$

$$\Rightarrow H_1(f) = F.T[h_1(t)] = T\,\mathrm{Sinc}(fT)\,e^{-j\pi fT}$$

$$H(f) = H_1^2(f) = T^2\,\mathrm{Sinc}^2(fT)\,e^{-j2\pi fT}$$

Problem 2.

(a) The transfer function of the ith stage of the system of Fig. P2.6 is

$$H_i(f) = \frac{1}{1+j2\pi f RC}$$

$$= \frac{1}{1+j2\pi f\tau_0}$$

where it is assumed that the buffer amplifier has a constant gain of one. The overall transfer function of the system is therefore

$$H(f) = \prod_{i=1}^{N} H_i(f)$$

$$= \frac{1}{(1+j2\pi f\tau_0)^N}$$

The corresponding amplitude response is

$$|H(f)| = \frac{1}{[1+(2\pi f\tau_0)^2]^{N/2}}$$

(b) Let

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

Then, we may rewrite the expression for the amplitude response as

$$|H(f)| = \left[1+\frac{1}{N}(fT)^2\right]^{-N/2}$$

In the limit, as N approaches infinity, we have

$$|H(f)| = \lim_{N\to\infty} [1+\frac{1}{N}(fT)^2]^{-N/2}$$

$$= \exp[-\frac{N}{2}\cdot\frac{1}{N}(fT)^2]$$

$$= \exp(-\frac{f^2T^2}{2})$$

Problem 3 ..

(a) $g(t) = \text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$

We note that

$$\hat{g}(t) = \frac{1-\cos(\pi t)}{\pi t}$$

Therefore,

$$g_+(t) = g(t) + j\hat{g}(t)$$

$$= \frac{\sin(\pi t)}{\pi t} + j\,\frac{1-\cos(\pi t)}{\pi t}$$

$$= \frac{j}{\pi t}[1 - \cos(\pi t) - j\sin(\pi t)]$$

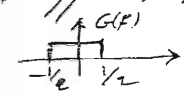
$$= \frac{j}{\pi t}[1 - \exp(j\pi t)] = \frac{1}{j\pi t}\left[e^{j\pi t} - 1\right] = \frac{1}{j\pi t}\left[\frac{e^{j\pi t} - 1}{e^{j\pi t/2}}\right]e^{j\pi t/2} = \frac{2}{\pi t}(\sin \pi t/2)\cdot e^{j\ldots}$$

$$= \text{Sinc}(t/2)\, e^{\ldots}$$

II° method:

using freq. domain approach:

$G(f) = \text{rect}(f)$



$G_+(f) = 2G(f) = 2\,\text{rect}\left(\frac{f - 1/4}{1/2}\right)$ for f...

$g_+(t) = 2\cdot\frac{1}{2}\,\text{Sinc}\left(\frac{t}{2}\right)\, e^{j2\pi\frac{1}{4}t}$

$\Rightarrow g_+(t) = \text{Sinc}(t/2)\; e^{j2\pi\frac{1}{4}t}$ $\quad f_c =$

$\tilde{g}(t) = g_+(t)\cdot e^{-j2\pi f_c t} = \text{Sinc}(t/2)$

$a(t) = |\tilde{g}(t)| = |\text{Sinc}(t/2)|$

(b) $g(t) = [1+k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

$$= \cos(2\pi f_c t) + \frac{k}{2}\cos[2\pi(f_c+f_m)t] + \frac{k}{2}\cos[2\pi(f_c-f_m)t]$$

Since the Hilbert transform of $\cos(2\pi ft)$ is equal to $\sin(2\pi ft)$, it follows that

$$\hat{g}(t) = \sin(2\pi f_c t) + \frac{k}{2}\sin[2\pi(f_c+f_m)t] + \frac{k}{2}\sin[2\pi(f_c-f_m)t]$$

where it is assumed that $f_c > f_m$. Therefore,

$$g_+(t) = \exp(j2\pi f_c t) + \frac{k}{2}\exp[j2\pi(f_c+f_m)t] + \frac{k}{2}\exp[j2\pi(f_c-f_m)t]$$

$$= [1 + \frac{k}{2}\exp(j2\pi f_m t) + \frac{k}{2}\exp(-j2\pi f_m t)]\exp(j2\pi f_c t)$$

$$= [1 + k\cos(2\pi f_m t)]\exp(j2\pi f_c t)$$

HW #. page

Problem 4

Prob #4 part b)

...ne domain approach:

(b) $g(t) = \frac{\sin t}{t}$

The Hilbert transform of sin t/t is

$$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau(t-\tau)}\, d\tau$$

$$= \frac{1}{\pi t}\int_{-\infty}^{\infty} \left(\frac{1}{\tau}+\frac{1}{t-\tau}\right)\sin\tau\, d\tau$$

$$= \frac{1}{\pi t}\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\, d\tau + \frac{1}{\pi t}\int_{-\infty}^{\infty} \frac{\sin\tau}{t-\tau}\, d\tau$$

We note that

$$\int_{-\infty}^{\infty} \mathrm{sinc}(t)\,dt = 1$$

Therefore,

$$\int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\, d\tau = \pi$$

$$\int_{-\infty}^{\infty} \frac{\sin\tau}{t-\tau}\, d\tau = \int_{-\infty}^{\infty} \frac{\sin(t-\tau)}{\tau}\, d\tau$$

$$= \sin t \underbrace{\int_{-\infty}^{\infty} \frac{\cos\tau}{\tau}\, d\tau}_{0 \text{ it is odd function}} - \cos t \int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\, d\tau$$

$$= -\pi \cos t$$

We thus obtain

$$\hat{g}(t) = \frac{1}{t}(1-\cos t)$$

Prob. #4 part b

II) Freq. domain approach

Remember: $\mathrm{rect}(t) \xleftrightarrow{F.T} \mathrm{Sinc}(f)$

using duality:

$\mathrm{Sinc}(t) \xleftrightarrow{F.T} \mathrm{rect}(-f) = \mathrm{rect}(f)$

Thus:

$\mathrm{Sinc}(t) = \frac{\sin \pi t}{\pi t} \longleftrightarrow \mathrm{rect}(f)$

$g(t) = \frac{\sin t}{t} = \frac{\sin(\pi t \cdot \frac{1}{\pi})}{(\pi t \cdot \frac{1}{\pi})} \longleftrightarrow \frac{1}{(\frac{1}{\pi})}\mathrm{rect}(\ldots$

where time scaling is used.

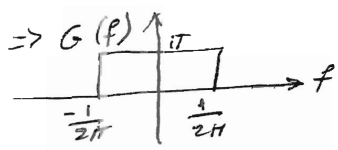
That is if $g(t) \longleftrightarrow G(f)$, then

$g(at) \longleftrightarrow \frac{1}{a} G(\frac{f}{a})$, Theref.

in our case:

$g(t) = \frac{\sin t}{t} \xleftrightarrow{F.T} \pi\,\mathrm{rect}\left(\frac{f}{\frac{1}{\pi}}\right)$

$\Rightarrow G(f)$



$\hat{G}(f) = -j\,\mathrm{sgn}(f)\, G(f) =$

$= -j\pi\,\mathrm{sgn}(f)\left[\mathrm{rect}\left(\frac{f-\frac{1}{4\pi}}{\frac{1}{2\pi}}\right) + \mathrm{rect}\left(\frac{f+\ldots}{\ldots}\right)\right]$

$= -j\pi\left[\mathrm{rect}\left(\frac{f-\frac{1}{4\pi}}{\frac{1}{2\pi}}\right) - \mathrm{rect}\left(\frac{f+\frac{1}{4\pi}}{\frac{1}{2\pi}}\right)\right]$

$\Rightarrow \hat{g}(t) = -j\pi\left[\frac{1}{2\pi}\,\mathrm{Sinc}\left(\frac{t}{2\pi}\right)\right]\left[e^{jt/2} - e^{-j\ldots}\right]$

$\hat{g}(t) = \mathrm{Sinc}\left(\frac{t}{2\pi}\right)\sin(t/2)$

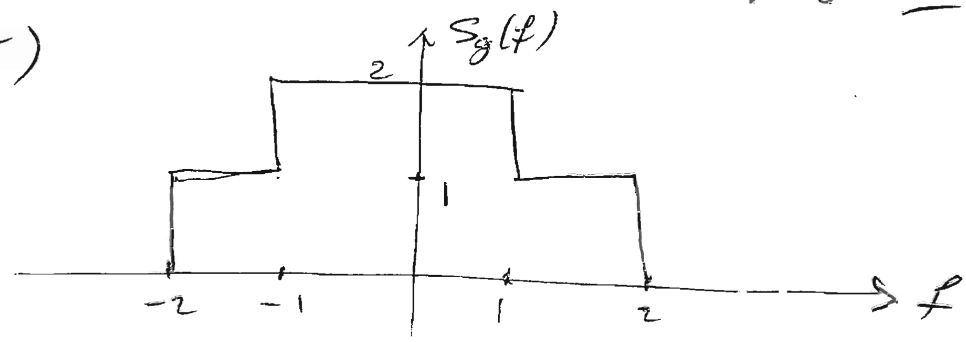
$= \frac{2}{t}\sin^2(t/2) = \frac{1}{t}(1-\cos t)$

Thus $\hat{g}(t) = \frac{1}{t}(1-\cos t)$

a) $g(t) = \delta(t)$

$$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\delta(\tau)}{t-\tau}\, d\tau = \frac{1}{\pi t}\int_{-\infty}^{+\infty} \delta(\tau)\, d\tau = \frac{1}{\pi t}$$

prob #5)



$$S_g(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$$

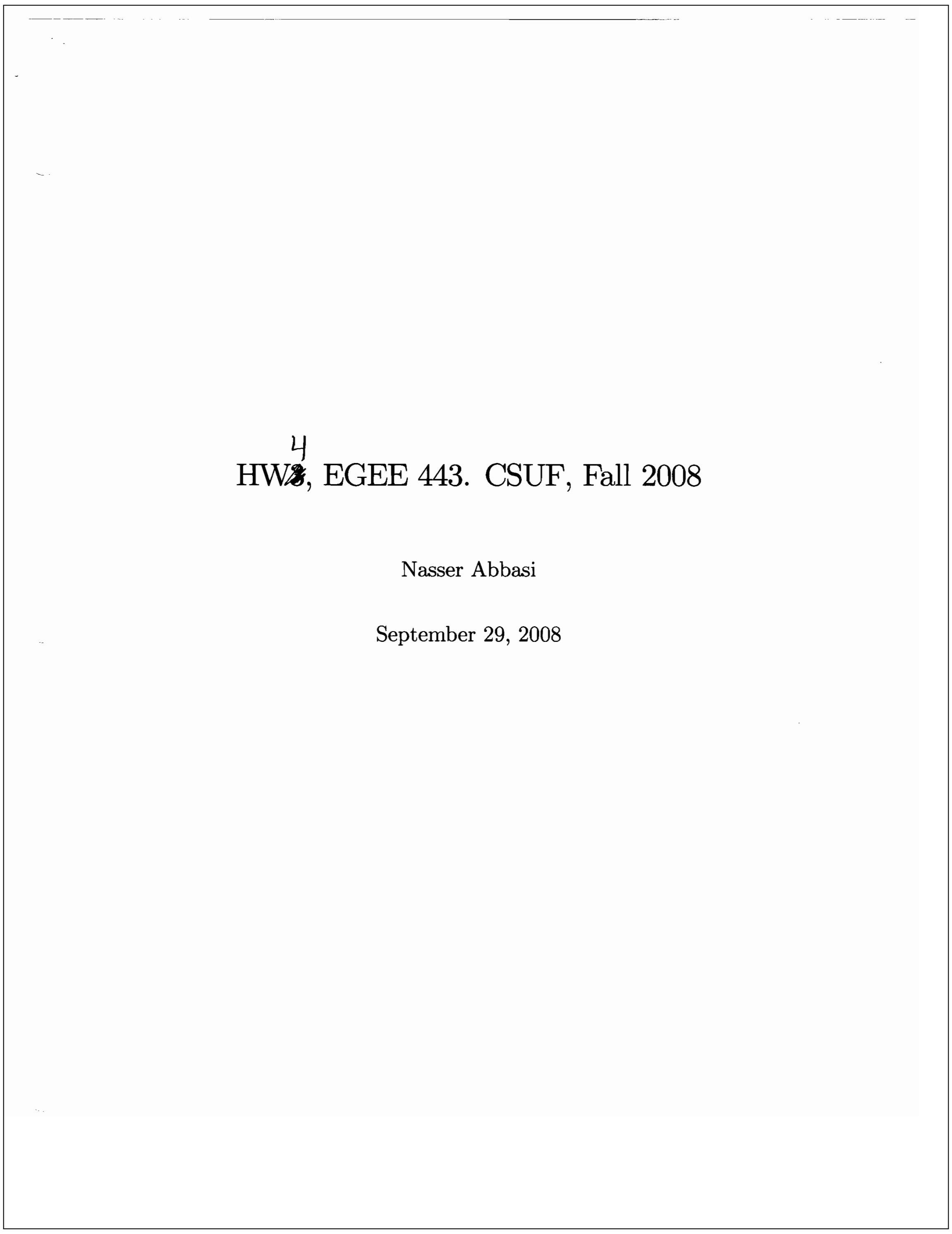
$$R_g(\tau) = F^{-1}\left[S_g(f)\right] = 4\,\text{sinc}(4\tau) + 2\,\text{sinc}(2\tau)$$

$$R_g(0) = P_{av} = 6 \text{ watts}$$

That is

$$R_g(0) = \int_{-\infty}^{+\infty} S_g(f)\,df = 6\ W \stackrel{\wedge}{=} P_{av}$$

8 my graded HW



HW~~3~~4, EGEE 443. CSUF, Fall 2008

Nasser Abbasi

September 29, 2008

17.5 / 20

1 Problem 1

Problem 1 Evaluate the transfer function of a linear system represented by the block diagram shown in Fig. P2.14.

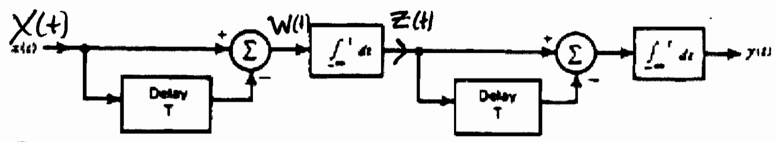


Figure P2.14

Solution Using transfer function cascading, then the overall transfer function for the system can be written as

$$H(f) = H_1(f)\, H_1(f) = [H_1(f)]^2 \tag{1}$$

Where

$$H_1(f) = \frac{Z(f)}{X(f)}$$

Where

$$\begin{aligned} Z(f) &= F\left\{ \int_{-\infty}^{t} w(\tau)\, d\tau \right\} \\ &= \frac{1}{j2\pi f} W(f) + \frac{W(0)}{2} \delta(f) \end{aligned} \tag{2}$$

Where

$$\begin{aligned} W(f) &= F\{x(t) - x(t-T)\} \\ &= X(f) - X(f)\, e^{-j2\pi f T} \\ &= X(f)\left[1 - e^{-j2\pi f T}\right] \end{aligned} \tag{3}$$

substitute (3) into (2) we obtain

$$\begin{aligned} Z(f) &= \frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi f T}\right] + \frac{X(0)\overbrace{\left[1 - e^{-j2\pi 0 T}\right]}^{=0}}{2} \delta(f) \\ &= \frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi f T}\right] \end{aligned} \tag{4}$$

Hence

$$\begin{aligned} H_1(f) &= \frac{Z(f)}{X(f)} \\ &= \frac{\frac{1}{j2\pi f} X(f)\left[1 - e^{-j2\pi f T}\right]}{X(f)} \end{aligned}$$

Hence

$$\boxed{H_1(f) = \frac{1}{j2\pi f}\left[1 - e^{-j2\pi fT}\right]}$$

Hence from (1)

$$\begin{aligned} H(f) &= \left(\frac{1}{j2\pi f}\left[1 - e^{-j2\pi fT}\right]\right)^2 \\ &= \frac{1}{-4\pi^2 f^2}\left[1 - e^{-j2\pi fT}\right]^2 \\ &= \frac{-1}{(2\pi f)^2}\left[1 - 2e^{-j2\pi fT} + e^{-j4\pi fT}\right] \end{aligned}$$

Hence

$$\boxed{H(f) = \frac{1}{(2\pi f)^2}\left[2e^{-j2\pi fT} - e^{-j4\pi fT} - 1\right]}$$

simplify it,
see sol.

2 Problem 2

Problem 2.

(a) Determine the overall amplitude response of the cascade connection shown in P2.15, consisting of N identical stages, each with a time constant RC equal to τ_0.
(b) Show that as N approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp(-\frac{1}{2}f^2T^2)$, where for each value of N, the time constant τ_0 is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

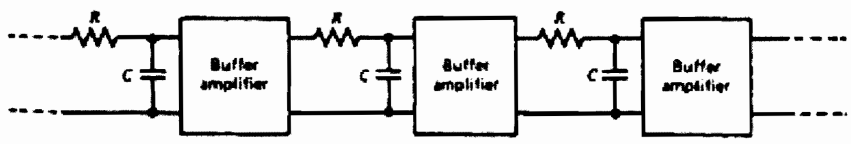


Figure P2.15

2.1 Part(a)

Transfer function for each stage is $H_i(f) = \frac{Y_i(f)}{X_i(f)} = \frac{1}{1+j2\pi fRC}$
Since $RC = \tau_0$, hence

$$H_i(f) = \frac{1}{1+j2\pi f\tau_0}$$

Then, for N stages, the overall transfer function is

$$H(f) = H_1(f)H_2(f)\cdots H_N(f)$$

Since they are identical stages, then the transfer function of each stage is the same, and the above becomes

$$H(f) = \left(\frac{1}{1+j2\pi f\tau_0}\right)^N$$

Hence the amplitude of the response is given by

$$\begin{aligned}
|H(f)| &= \left(\frac{1}{|1+j2\pi f\tau_0|}\right)^N \\
&= \left(\frac{1}{\sqrt{1^2+(2\pi f\tau_0)^2}}\right)^N \\
&= \left(\frac{1}{(1+4\pi^2f^2\tau_0^2)^{\frac{1}{2}}}\right)^N \\
&= \frac{1}{(1+4\pi^2f^2\tau_0^2)^{\frac{N}{2}}}
\end{aligned}$$

Let $\tau_0^2 = \frac{\tau^2}{4\pi^2 N}$, the above becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{f^2\tau^2}{N}\right)^{\frac{N}{2}}} \tag{1}$$

2.2 Part (b)

Let $\alpha = f^2\tau^2, \beta = \frac{1}{2}$, then (1) becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{\alpha}{N}\right)^{\beta N}}$$

But $\lim_{N\to\infty} \frac{1}{\left(1+\frac{\alpha}{N}\right)^{\beta N}} = e^{\alpha\beta}$,hence

$$|H(f)| = \frac{1}{e^{\frac{f^2\tau^2}{2}}}$$
$$= e^{-\frac{f^2\tau^2}{2}}$$

Which is what we are asked to show.

3 Problem 3

Problem 3. Determine the pre-envelope $g_+(t)$ corresponding to each of the following two signals:

(a) $g(t) = \text{sinc}(t)$ (Hint use $\frac{\sin t}{t} \overset{H.T.}{\longleftrightarrow} \frac{1-\cos t}{t}$ See prob # 4)

(b) $g(t) = [1 + k\cos(2\pi f_m t)]\cos(2\pi f_c t)$

(a) $g\left(t\right) = \text{sinc}\left(t\right)$

$$g_+\left(t\right) = g\left(t\right) + j\hat{g}\left(t\right) \tag{1}$$

Where $\hat{g}\left(t\right)$ is Hilbert transform of $g\left(t\right)$ defined as $\hat{g}\left(t\right) = g\left(t\right) \otimes \frac{1}{\pi t}$

$$\begin{aligned}\hat{G}\left(f\right) &= -j\ sgn\left(f\right)\ G\left(f\right) \\ &= -j\ sgn\left(f\right)\ rect\left(f\right)\end{aligned}$$

Now find the inverse Fourier transform.

I derive the above to answer problem 4 part (b). The answer is the following (please see problem 4 part(b) for the derivation

$$\hat{g}\left(t\right) = \frac{1}{\pi t}\left(1 - \cos \pi t\right)$$

In the above, I used $\text{sinc}\left(t\right) \equiv \frac{\sin \pi t}{\pi t}$. If one uses $\text{sinc}\left(t\right) \equiv \frac{\sin t}{t}$ then the answer becomes

$$\hat{g}\left(t\right) = \frac{1}{t}\left(1 - \cos t\right) \quad ? \tag{2}$$

The problem statement seems to want us to use the second definition of $\text{sinc}\left(t\right)$, so I will continue the rest of the solution using (1).

Substitute (2) into (1) we obtain

$$\begin{aligned}g_+\left(t\right) &= \text{sinc}\left(t\right) + j\frac{1}{\pi t}\left(1 - \cos t\right) \\ &= \frac{\sin\left(t\right)}{t} + j\frac{1}{t}\left(1 - \frac{e^{jt} + e^{-jt}}{2}\right) \\ &= \frac{1}{t}\frac{e^{jt} - e^{-jt}}{2j} + \frac{1}{t}\left(j + \frac{e^{jt} + e^{-jt}}{2j}\right) \\ &= \frac{1}{t}\frac{e^{jt} - e^{-jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt} + e^{-jt}}{2j} \\ &= \frac{1}{t}\frac{e^{jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt}}{2j}\end{aligned}$$

-1

Hence

$$\boxed{g_+\left(t\right) = \frac{1}{t}\left(j + e^{jt}\right)}$$

X See Sol.

3.1 Part(b)

$$g(t) = [1 + k\cos 2\pi f_m t]\cos(2\pi f_c t)$$

$$g_+(t) = g(t) + j\hat{g}(t)$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$.

$$G_+(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0 \end{cases}$$

But

$$G(f) = \mathcal{F}[1 + k\cos 2\pi f_m t] \otimes \mathcal{F}[\cos(2\pi f_c t)] \tag{1}$$

But

$$\mathcal{F}[\cos(2\pi f_c t)] = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

and

$$\mathcal{F}[1 + k\cos 2\pi f_m t] = \delta(f) + \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)]$$

Hence (1) becomes

$$\begin{aligned} G(f) &= \left\{\delta(f) + \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)]\right\} \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ &= \delta(f) \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ &+ \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)] \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}\delta(f) \otimes \delta(f - f_c) + \\ &\frac{1}{2}\delta(f) \otimes \delta(f + f_c) + \\ &\frac{k}{4}\delta(f - f_m) \otimes \delta(f - f_c) + \\ &\frac{k}{4}\delta(f - f_m) \otimes \delta(f + f_c) + \\ &\frac{k}{4}\delta(f + f_m) \otimes \delta(f - f_c) + \\ &\frac{k}{4}\delta(f + f_m) \otimes \delta(f + f_c) \end{aligned}$$

Hence

$$
\begin{aligned}
G(f) = &\frac{1}{2}\delta(f+f_c)+\\
&\frac{1}{2}\delta(f-f_c)+\\
&\frac{k}{4}\delta(f-f_m+f_c)+\\
&\frac{k}{4}\delta(f-f_m-f_c)+\\
&\frac{k}{4}\delta(f+f_m+f_c)+\\
&\frac{k}{4}\delta(f+f_m-f_c)
\end{aligned}
$$

Hence for $f>0$,$G_+(f)=2G(f)$ and we obtain

$$
G_+(f)=\delta(f-f_c)+\frac{k}{2}\left[\delta(f-f_m+f_c)+\delta(f-f_m-f_c)+\delta(f+f_m+f_c)+\delta(f+f_m-f_c)\right]
$$

Then (since carrier frequency $f_c>f_m$), we could simplify the above, by keeping positive frequencies f

$$
G_+(f)=\delta(f-f_c)+\frac{k}{2}\left[\delta(f-f_m-f_c)+\delta(f+f_m-f_c)\right]
$$

or

$$
G_+(f)=\delta(f-f_c)+\frac{k}{2}\left[\delta(f-(f_m+f_c))+\delta(f-(f_c-f_m))\right]
$$

Hence

$$
\begin{aligned}
g_+(t) &= e^{j2\pi f_c t}+\frac{k}{2}\left(e^{j2\pi(f_m+f_c)t}+e^{j2\pi(f_c-f_m)t}\right)\\
&= e^{j2\pi f_c t}+\frac{k}{2}\left(e^{j2\pi f_m t}e^{j2\pi f_c t}+e^{j2\pi f_c t}e^{-j2\pi f_m t}\right)\\
&= e^{j2\pi f_c t}\left[1+\frac{k}{2}\left(e^{j2\pi f_m t}+e^{-j2\pi f_m t}\right)\right]\\
&= e^{j2\pi f_c t}\left[1+\frac{k}{2}\left(2\cos(2\pi f_m t)\right)\right]\\
&= e^{j2\pi f_c t}\left[1+k\cos(2\pi f_m t)\right]
\end{aligned}
$$

4 Problem 4

Prob # 4) Verify the following H.T :

a) if $g(t) = \delta(t) \Rightarrow \hat{g}(t) = ?$

b) if $g(t) = \frac{\sin t}{t} \Rightarrow \hat{g}(t) = \frac{1-\cos t}{t}$

4.1 Part(a)

$g(t) = \delta(t)$

$$\begin{aligned}
\hat{g}(t) &= g(t) \otimes \frac{1}{\pi t} \\
&= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t-\tau} d\tau \\
&= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t} d\tau \\
&= \frac{1}{\pi t} \int_{-\infty}^{\infty} \delta(\tau) d\tau \\
&= \frac{1}{\pi t}
\end{aligned}$$

4.2 Part(b)

$g(t) = \frac{\sin(\pi t \cdot \frac{1}{2})}{\pi t \cdot \frac{1}{2}} \leftrightarrow \frac{1}{\frac{1}{2}} \cdot rect()$

And Since $sgn(f) = -1$ for $f < 0$ and $sgn(f) = 1$ for $f > 0$ then

$$\hat{G}(f) = -\cancel{j}\left[-rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right) \overset{-0.5}{+} rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right]$$

Hence

$$\hat{g}(t) = jF^{-1}\left[rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right) - rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right] \tag{1}$$

But $F^{-1}\left(rect\left(\frac{f+\frac{1}{4}}{\frac{1}{2}}\right)\right) = \frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)e^{-j2\pi\frac{1}{4}t}$ and $F^{-1}\left(rect\left(\frac{f-\frac{1}{4}}{\frac{1}{2}}\right)\right) = \frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)e^{+j2\pi\frac{1}{4}t}$, hence (1) becomes

$$\begin{aligned}
\hat{g}(t) &= j\left[\frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)e^{-j2\pi\frac{1}{4}t} - \frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)e^{+j2\pi\frac{1}{4}t}\right] \\
&= \frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)\left[j\left(e^{-j2\pi\frac{1}{4}t} - e^{j2\pi\frac{1}{4}t}\right)\right] \\
&= \frac{1}{2}\,\text{sinc}\left(\frac{1}{2}t\right)\left[\frac{e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}}{-j}\right] \\
&= \text{sinc}\left(\frac{1}{2}t\right)\left[\frac{e^{j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t}}{2j}\right] \\
&= \text{sinc}\left(\frac{1}{2}t\right)\left[\sin\frac{\pi}{2}t\right]
\end{aligned}$$

But $\text{sinc}\left(\frac{1}{2}t\right) = \frac{\sin\frac{\pi t}{2}}{\frac{\pi t}{2}}$ hence

$$\begin{aligned}
\hat{g}(t) &= \frac{\sin\frac{\pi t}{2}}{\frac{\pi t}{2}}\sin\frac{\pi}{2}t \\
&= \frac{2}{\pi t}\sin^2\frac{\pi}{2}t \\
&= \frac{2}{\pi t}\left(\frac{1}{2} - \frac{1}{2}\cos\pi t\right) \\
&= \frac{1}{\pi t}(1 - \cos\pi t)
\end{aligned}$$

See sol.

5 problem 5

prob #5) Respond prob 2.44 of your book

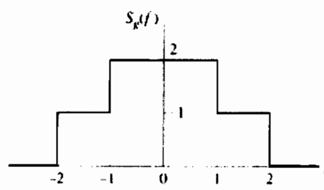


FIGURE 2.47
Problem 2.44

2.45 Consider the square wave $g(t)$ shown in Fig. 2.48. Find the power spectral density, average power, and autocorrelation function of this square wave. Does the wave have dc power? Explain your answer.

?

$$S_g(f) = rect\left(\frac{f}{4}\right) + rect\left(\frac{f}{2}\right)$$

$$R_g(\tau) = F^{-1}(S_g(f))$$

Rg(0) = ?

Hence

$$\begin{aligned} R_g(\tau) &= F^{-1}\left(rect\left(\frac{f}{4}\right) + rect\left(\frac{f}{2}\right)\right) \\ &= F^{-1}\left[rect\left(\frac{f}{4}\right)\right] + F^{-1}\left[rect\left(\frac{f}{2}\right)\right] \\ &= 4\,\mathrm{sinc}\,(4t) + 2\,\mathrm{sinc}\,(2t) \end{aligned}$$

```
In[7]:= Plot[4 Sinc[4 t] + 2 Sinc[2 t], {t, -4 Pi, 4 Pi},
        PlotRange → All]
```

Out[7]=