EE 443 HW # 3 page 1

Key Solution.

$\boxed{2-7.}$

(a) $dBm = 10\log\left(\frac{P_w}{0.001}\right) = 10\log\left(\frac{67}{0.001}\right) = \underline{\underline{48.26\,dBm}}$

(b) $dBk = 10\log\left(\frac{P_w}{1000}\right) = 10\log\left(\frac{67}{1000}\right) = \underline{\underline{-11.74\,dBk}}$

(c) $P = \frac{V_{rms}^2}{R}$

$\Rightarrow V_{rms} = \sqrt{PR} = \sqrt{(67)(75)} = 70.9\text{ volts}$

$\Rightarrow dBV = 20\log\left(\frac{70.9}{10^{-3}}\right) = \underline{\underline{97\,dBmV}}$

$\boxed{2-8.}$ $P = \frac{V_{rms}^2}{R} = \frac{V_{rms}^2}{50}$

$dBm = 10\log_{10}\left(\frac{P}{0.001}\right) = 10\log_{10}\left(\frac{V_{rms}^2}{0.050}\right) = 20\log_{10}(V_{rms}) - 10\log_{10}(0.050)$

$\Rightarrow \underline{\underline{dB_m = 20\log_{10}(V_{rms}) + 13}}$

$\boxed{2-9.}$ $P_{in} = I_{rms}^2 R_{in} = (0.5\times10^{-3})^2(2\times10^3) = 5.0\times10^{-4}\,W$

$P_{out} = \frac{V_{rms}^2}{R_{Load}} = \frac{100}{50} = 2\,W$

$dB = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{2}{5.0\times10^{-4}}\right) = \underline{\underline{36\,dB}}$

$\boxed{2-15.}$ $w(t) = \sin(2\pi f_1 t)\cos(2\pi f_2 t) = w_1(t)\,w_2(t)$

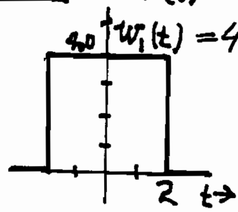
$\Rightarrow W(f) = W_1(f) * W_2(f) = \left[\frac{j}{2}\delta(f+f_1) - \frac{j}{2}\delta(f-f_1)\right] * \left[\frac{1}{2}\delta(f+f_2) + \frac{1}{2}\delta(f-f_2)\right]$

Aside: $\delta(f+f_1) * \delta(f+f_2) = \int_{-\infty}^{\infty}\delta(\lambda+f_1)\,\delta(f-\lambda+f_2)\,d\lambda = \delta(f+f_1+f_2)$

thus $\underline{\underline{W(f) = (j/4)\left[\delta(f+f_1+f_2) + \delta(f+f_1-f_2) - \delta(f-f_1+f_2) - \delta(f-f_1-f_2)\right]}}$

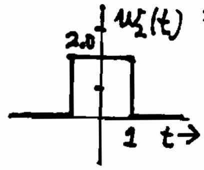
2-17. $w(t) = w_1(t) - w_2(t)$ where

$w_1(t) = 4\,\mathrm{rect}(\frac{t}{4})$



$$\longleftrightarrow \quad W_1(f) = 16\,\mathrm{Sinc}(4f)$$

$w_2(t) = 2\,\mathrm{rect}(\frac{t}{2})$



$$\longleftrightarrow \quad W_2(f) = 4\,\mathrm{Sinc}(2f)$$

$$\Rightarrow W(f) = W_1(f) - W_2(f) = \underline{\underline{16\,\mathrm{Sinc}(4f) - 4\,\mathrm{Sinc}(2f)}}$$

2-18.

(a) $w(2t) \longleftrightarrow \frac{1}{2}\frac{j\pi f}{1+j\pi f}$

$$\Rightarrow x(t) = w(2(t+1)) \longleftrightarrow \frac{j\pi f}{2(1+j\pi f)}e^{j2\pi f}$$

(b) $w(t-1) \longleftrightarrow \frac{j2\pi f}{1+j2\pi f}e^{-j2\pi f}$

$$x(t) = e^{-jt}w(t-1) \longleftrightarrow \frac{j2\pi(f+\frac{1}{2\pi})}{1+j2\pi(f+\frac{1}{2\pi})}e^{j2\pi(f+\frac{1}{2\pi})}$$

(c.) $2\frac{dw(t)}{dt} \longleftrightarrow 2\,(j2\pi f\,W(f))$

$$\Rightarrow x(t) \longleftrightarrow j4\pi f\left[\frac{j2\pi f}{1+j2\pi f}\right] = -\frac{8\pi^2 f^2}{1+j2\pi f}$$

(d.) $w(-t) \longleftrightarrow W(-f) = -\frac{j2\pi f}{1-j2\pi f}$

$$\Rightarrow x(t) = w(-(t-1)) \longleftrightarrow \frac{-j2\pi f}{1-j2\pi f}e^{-j2\pi f}$$

2.17) $P = 67\ W$ and $R = 50\ \Omega$

a, b) $P_{dBW} \triangleq 10 \log_{10} P(w)$

$P_{dBm} = 10 \log_{10} P(mw)$, $P_{dBK} = 10 \log_{10} P(KW)$

c) • For normalized case ($R = 1\ \Omega$), the average power P and rms voltage are related:

$$P_{av} = V_{rm}^2 = \frac{V_{peak}^2}{2} \Rightarrow V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

• For not normalized case ($R \neq 1\ \Omega$)

$$P_{av} = \frac{V_{rm}^2}{R} = \frac{V_{peak}^2}{2R}$$

if P and R are given => find V_{rms}.

V_{rms} in dBmV is : $V_{rms}(dBmV) \triangleq 20 \log_{10} V_{rms}(mV)$

2.8) Given a Sine wave like:

$v(t) = V_{peak} \cos w_0 t$

the average power (normalized) of this periodic wave over one period, $T_0 = \frac{1}{f_0}$, is:

$$P_{av} \triangleq \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v^2(t)\, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} V_{peak}^2 \cos^2 w_0 t\, dt$$

$$= \frac{V_{peak}^2}{2T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2w_0 t)\, dt = \frac{V_{peak}^2}{2}$$

If $R \neq 1\ \Omega \Rightarrow P_{av} = \frac{V_{peak}^2}{2R} = \frac{V_{rm}^2}{R}$ watts

$$\Rightarrow P_{dBm} \triangleq 10 \log_{10} P(mw) = 10 \log \left[\frac{V_{rms}^2 \times 10^3}{R} \right] = \dots$$

2.9) The power gain is:

$$A_{p\,dB} \triangleq 10\log\frac{P_L}{P_{in}} \quad \text{where}, \quad P_L = \frac{V_{o(rms)}^2}{R_L}$$

P_L is the power transfered to the load and P_{in} is the input power supplied by the Source

2.17)
$w(t)$ may be expressed in two different way:

a) $w(t) = 4\,\text{rect}\left(\frac{t}{4}\right) - 2\,\text{rect}\left(\frac{t}{2}\right)$ or

b) $w(t) = 2\,\text{rect}\left(\frac{t}{2}\right) + 4\,\text{rect}\left(\frac{t+3/2}{1}\right) + 4\,\text{rect}\left(\frac{t-3/2}{1}\right)$

Find $W(f)$! The two answers should be the Same ~~If~~ you use the second method, you may after takin F.T use $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$

2.18)
if $w(t)$ has F.T, which is $W(f) = \dfrac{j2\pi f}{1+j2\pi f}$

a) Find the F.T of $x(t) = w(2t+2) = w(2(t+1))$

$$w(2t) \longleftrightarrow \frac{1}{2}\cdot\frac{j2\pi\left(\frac{f}{2}\right)}{1+j2\pi\left(\frac{f}{2}\right)} \quad \text{Scaling}$$

$$x(t) = w(2(t+1)) \longleftrightarrow \frac{j\pi f}{2(1+j\pi f)}\,e^{j2\pi f\cdot 1} \quad \text{time domain Shifting!}$$