# HW 3
# Electronic Communication Systems
# Fall 2008
# California State University, Fullerson

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# Contents

# 1 questions

443 HW #3 Sept. 17, 2008.

CH. 2 / SIGNALS AND SPECTRA

2-3 The voltage across a load is given by $v(t) = A_0 \cos \omega_0 t$, and the current through the load is a square wave,

$$i(t) = I_0 \sum_{n=-\infty}^{\infty} \left[ \Pi\left(\frac{t - nT_0}{T_0/2}\right) - \Pi\left(\frac{t - nT_0 - (T_0/2)}{T_0/2}\right) \right]$$

where $\omega_0 = 2\pi/T_0$, $T_0 = 1$ sec, $A_0 = 10$ V, and $I_0 = 5$ mA.

(a) Find the expression for the instantaneous power and sketch this result as a function of time.

(b) Find the value of the average power.

2-4 The voltage across a 50-Ω resistive load is the positive portion of a cosine wave That is,

$$v(t) = \begin{cases} 10 \cos \omega_0 t, & |t - nT_0| < T_0/4 \\ 0, & t \text{ elsewhere} \end{cases}$$

where $n$ is any integer.

(a) Sketch the voltage and current waveforms.

(b) Evaluate the dc values for the voltage and current.

(c) Find the rms values for the voltage and current.

(d) Find the total average power dissipated in the load.

2-5 For Prob. 2-4, find the energy dissipated in the load during a 1-hr interval if $T_0$ = 1 sec.

2-6 Determine whether each of the following signals is an energy signal or a pow signal and evaluate the normalized energy or power, as appropriate.

(a) $w(t) = \Pi(t/T_0)$.

(b) $w(t) = \Pi(t/T_0) \cos \omega_0 t$.

(c) $w(t) = \cos^2 \omega_0 t$.

✓ 2-7 An average reading power meter is connected to the output circuit of a transmitte The transmitter output is fed into a 75-Ω resistive load and the wattmeter rea 67 W.

(a) What is the power in dBm units?

(b) What is the power in dBk units?

(c) What is the value in dBmV units?

✓ 2-8 Assume that a waveform with a known rms value, $V_{rms}$, is applied across a 50- load. Derive a formula that can be used to compute the dBm value from $V_{rms}$.

✓ 2-9 An amplifier is connected to a 50-Ω load and driven by a sinusoidal current sour as shown in Fig. P2-9. The output resistance of the amplifier is 10 Ω and the in resistance is 2 kΩ. Evaluate the true decibel gain of this circuit.

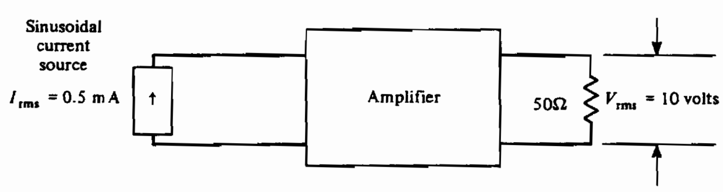



FIGURE P2-9

2-10 The voltage (rms) across the 300-$\Omega$ antenna input terminals of an FM receiver is 3.5 $\mu$V.
(a) Find the input power (watts).
(b) Evaluate the input power as measured in decibels below 1 mW (dBm).
(c) What would be the input voltage (in microvolts) for the same input power if the input resistance were 75 $\Omega$ instead of 300 $\Omega$?

2-11 What is the value for the phasor that corresponds to the voltage waveform $v(t) = 12\sin(\omega_0 t - 25°)$, where $\omega_0 = 2000\pi$?

2-12 A signal is $w(t) = 3\sin(100\pi t - 30°) + 4\cos(100\pi t)$. Find the corresponding phasor.

2-13 Evaluate the Fourier transform of

$$w(t) = \begin{cases} e^{-\alpha t}, & t \geq 1 \\ 0, & t < 1 \end{cases}$$

2-14 Find the spectrum for the waveform $w(t) = e^{-\pi(t/T)^2}$. What can we say about the width of $w(t)$ and $W(f)$ as $T$ increases? [*Hint:* Use (A-75).]

✓ 2-15 Using the convolution property, find the spectrum for

$$w(t) = \sin 2\pi f_1 t \cos 2\pi f_2 t$$

2-16 Find the spectrum (Fourier transform) of the triangle waveform

$$s(t) = \begin{cases} At, & 0 < t < T_0 \\ 0, & t \text{ elsewhere} \end{cases}$$

in terms of $A$ and $T_0$.

✓ 2-17 Find the spectrum for the waveform shown in Fig. P2-17.

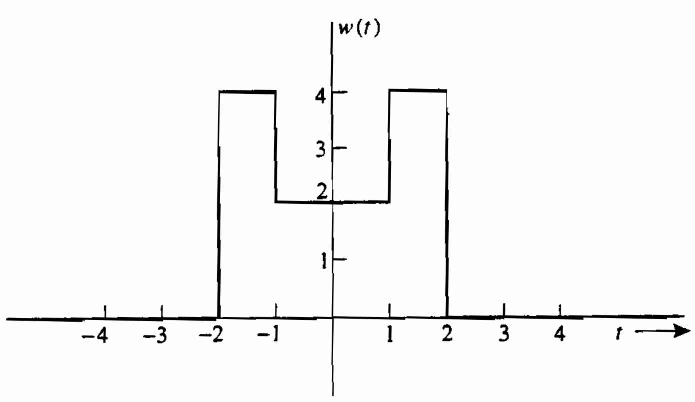



FIGURE P2-17

✓ 2-18 If $w(t)$ has the Fourier transform

$$W(f) = \frac{j2\pi f}{1 + j2\pi f}$$

find $X(f)$ for the following waveforms.
(a) $x(t) = w(2t + 2)$.

(b) $x(t) = \omega(t-1)e^{-jt}$

c) $x(t) = w(1-t)$

## 2 Problem 2.7

**Problem** An average reading power meter is connected to output of transmitter. Transmitter output is fed into 75$\Omega$ resistive load and the wattmeter read 67$W$

(a) What is power in dBm units?

(b) What is power in dBk units?

(c) What is the value in dBmV units?

### 2.1 part(a)

$$\begin{aligned} P_{dbm} &= 10\log_{10} P_m \\ &= 10\log_{10}(67000) \\ &= \boxed{48.2607}\ dbm \end{aligned}$$

(b)

$$\begin{aligned} P_{dbk} &= 10\log_{10} P_k \\ &= 10\log_{10}(0.067) \\ &= \boxed{-11.7393}\ dbk \end{aligned}$$

(c)

$$P = \frac{V^2}{R}$$

Hence

$$10\log_{10} P = 20\log_{10} V - 10\log_{10} R$$

Hence

$$20\log_{10} V = 10\log_{10} P + 10\log_{10} R$$

so

$$\begin{aligned} 20\log_{10} V &= 10\log_{10} 67000 + 10\log_{10} 75000 \\ &= \boxed{97.0114 \text{ dbmV}} \end{aligned}$$

## 3 Problem 2.8

Assume that a waveform with known rms value $V_{rms}$ is applied across a $50\Omega$ load. Derive a formula that can be used to computer the *dbm* value from $V_{rms}$

$$P\,(watt) = \frac{V_{rms}^2\,(V)}{R\,(\Omega)}$$

Hence

$$\begin{aligned} P_{dbm} &= 10\log_{10}\left(10^3 \times P_{watt}\right) \\ &= 10\log_{10}\frac{10^3 \times V_{rms}^2\,(V)}{R\,(\Omega)} \\ &= 10\left(\log_{10} 10^3 V_{rms}^2 - \log_{10} R\right) \\ &= 10\left(\log_{10} 10^3 + \log_{10} V_{rms}^2 - \log_{10} R\right) \\ &= 10\,(3 + 2\log_{10} V_{rms} - \log_{10} R) \end{aligned}$$

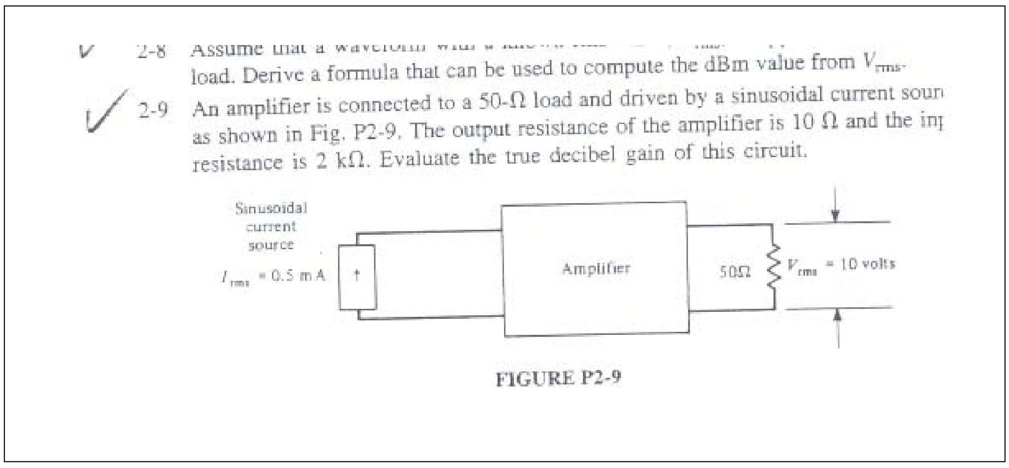
Hence

$$\boxed{P_{dbm} = 30 + 20\log_{10} V_{rms} - 10\log_{10} R}$$

When $R = 50\Omega$, we obtain

$$
\begin{aligned}
P_{dbm} &= 30 + 20\log_{10} V_{rms} - 10\log_{10} 50 \\
&= 30 + 20\log_{10} V_{rms} - 16.9897 \\
&= 13.0103 + 20\log_{10} V_{rms}
\end{aligned}
$$

# 4 Problem 2.9


2-8 Assume that a waveform ... load. Derive a formula that can be used to compute the dBm value from $V_{rms}$.

2-9 An amplifier is connected to a 50-Ω load and driven by a sinusoidal current soun as shown in Fig. P2-9. The output resistance of the amplifier is 10 Ω and the inp resistance is 2 kΩ. Evaluate the true decibel gain of this circuit.



FIGURE P2-9

Figure 1: the Problem statement

$$
\begin{aligned}
Gain(db) &= 10\log_{10}\frac{P_L}{P_i} \\
&= 10\log_{10}\frac{\left(\frac{V_{rms}^2}{R_L}\right)}{I_{rms}^2 R_{in}} \\
&= 10\log_{10}\frac{\left(\frac{10^2}{50}\right)}{\left(0.5\times 10^{-3}\right)^2 \times 2000} \\
&= 10\log_{10}\frac{10^5}{25} \\
&= 10\left(\log_{10} 10^5 - \log_{10} 25\right) \\
&= 10\left(5 - 1.39794\right) \\
&= 36.\,021
\end{aligned}
$$

# 5 Problem 2.15

Using the convolution property find the spectrum for $w(t) = \sin 2\pi f_1 t \;\; \cos 2\pi f_2 t$

Solution:

$$
\mathcal{F}\left(w\left(t\right)\right) = \mathcal{F}\left(\sin 2\pi f_1 t\right) \otimes \mathcal{F}\left(\cos 2\pi f_2 t\right) \tag{1}
$$

But

$$
\begin{aligned}
\mathcal{F}\left(\sin 2\pi f_1 t\right) &= \frac{1}{2j}\left(\delta\left(f - f_1\right) - \delta\left(f + f_1\right)\right) \\
\mathcal{F}\left(\cos 2\pi f_2 t\right) &= \frac{1}{2}\left(\delta\left(f - f_2\right) + \delta\left(f + f_2\right)\right)
\end{aligned}
$$

Hence (1) becomes

$$
\begin{aligned}
\mathcal{F}\left(w\left(t\right)\right) &= \left\{\frac{1}{2j}\left(\delta\left(f - f_1\right) - \delta\left(f + f_1\right)\right)\right\} \otimes \left\{\frac{1}{2}\left(\delta\left(f - f_2\right) + \delta\left(f + f_2\right)\right)\right\} \\
&= \frac{1}{4j}\left\{\delta\left(f - f_1\right) - \delta\left(f + f_1\right)\right\} \otimes \left\{\delta\left(f - f_2\right) + \delta\left(f + f_2\right)\right\} \qquad (2)
\end{aligned}
$$

Applying the distributed property of convolution, i.e. $a \otimes (b+c) = a \otimes b + a \otimes c$ on equation (2) we obtain

$$4j\,F\left(w\left(t\right)\right) = \delta\left(f-f_1\right) \otimes \delta\left(f-f_2\right) + \delta\left(f-f_1\right) \otimes \delta\left(f+f_2\right) - \delta\left(f+f_1\right) \otimes \delta\left(f-f_2\right) - \delta\left(f+f_1\right) \otimes \delta\left(f+f_2 \tag{3}$$

Now

$$\begin{aligned}
\delta\left(f-f_1\right) \otimes \delta\left(f-f_2\right) &= \int_{-\infty}^{\infty} \delta\left(\lambda-f_1\right)\delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f+f_2-f_1\right)\int_{-\infty}^{\infty} \delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f+f_2-f_1\right)
\end{aligned} \tag{4}$$

And

$$\begin{aligned}
\delta\left(f-f_1\right) \otimes \delta\left(f+f_2\right) &= \int_{-\infty}^{\infty} \delta\left(\lambda-f_1\right)\delta\left(f-\left(\lambda+f_2\right)\right)d\lambda \\
&= \delta\left(f-f_2-f_1\right)\int_{-\infty}^{\infty} \delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f-f_2-f_1\right)
\end{aligned} \tag{5}$$

And

$$\begin{aligned}
\delta\left(f+f_1\right) \otimes \delta\left(f-f_2\right) &= \int_{-\infty}^{\infty} \delta\left(\lambda+f_1\right)\delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f+f_2+f_1\right)\int_{-\infty}^{\infty} \delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f+f_2+f_1\right)
\end{aligned} \tag{6}$$

And

$$\begin{aligned}
\delta\left(f+f_1\right) \otimes \delta\left(f+f_2\right) &= \int_{-\infty}^{\infty} \delta\left(\lambda+f_1\right)\delta\left(f-\left(\lambda+f_2\right)\right)d\lambda \\
&= \delta\left(f-f_2+f_1\right)\int_{-\infty}^{\infty} \delta\left(f-\left(\lambda-f_2\right)\right)d\lambda \\
&= \delta\left(f-f_2+f_1\right)
\end{aligned} \tag{7}$$

Substitute (4,5,6,7) into (3) we obtain

$$\boxed{F\left(w\left(t\right)\right) = \frac{1}{4j}\left[\delta\left(f+f_2-f_1\right) + \delta\left(f-f_2-f_1\right) - \delta\left(f+f_2+f_1\right) - \delta\left(f-f_2+f_1\right)\right]}$$

or

$$\boxed{F\left(w\left(t\right)\right) = \frac{1}{4j}\left[\delta\left(f+\left(f_2-f_1\right)\right) + \delta\left(f-\left(f_2+f_1\right)\right) - \delta\left(f+\left(f_2+f_1\right)\right) - \delta\left(f-\left(f_2-f_1\right)\right)\right]} \tag{8}$$

This problem can also be solved as follows

$$w\left(t\right) = \sin 2\pi f_1 t\ \cos 2\pi f_2 t$$

Using $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$, hence

$$\begin{aligned}
w(t) &= \frac{1}{2}(\sin(2\pi f_1 t - 2\pi f_2 t) + \sin(2\pi f_1 t + 2\pi f_2 t)) \\
&= \frac{1}{2}(\sin(2\pi(f_1 - f_2)t) + \sin(2\pi(f_1+f_2)t)) \\
&= \frac{1}{2}\left(\frac{1}{2j}(\delta(f-(f_1-f_2)) - \delta(f+(f_1-f_2))) + \frac{1}{2j}(\delta(f-(f_1+f_2)) - \delta(f+(f_1+f_2)))\right) \\
&= \frac{1}{4j}\{\delta(f-(f_1-f_2)) - \delta(f+(f_1-f_2)) + \delta(f-(f_1+f_2)) - \delta(f+(f_1+f_2))\} \\
&= \frac{1}{4j}\{\delta(f+(f_2-f_1)) + \delta(f-(f_2+f_1)) - \delta(f+(f_2+f_1)) - \delta(f-(f_2-f_1))\}
\end{aligned} \tag{9}$$

Compare (8) and (9) we see they are the same.

# 6 Problem 2.17

$$w(t) = 4\ rect\left(\frac{t}{4}\right) - 2\ rect\left(\frac{t}{2}\right)$$

By linearity of Fourier Transform

$$\digamma(w(t)) = 4 \times \digamma\left(rect\left(\frac{t}{4}\right)\right) - 2 \times \digamma\left(rect\left(\frac{t}{2}\right)\right) \tag{1}$$

Since

$$\digamma\left(rect\left(\frac{t}{4}\right)\right) = 4\operatorname{sinc}(4f)$$

and

$$\digamma\left(rect\left(\frac{t}{2}\right)\right) = 2\operatorname{sinc}(2f)$$

Then (1) becomes

$$\begin{aligned}
\digamma(w(t)) &= 4 \times 4\operatorname{sinc}(4f) - 2 \times 2\operatorname{sinc}(2f) \\
&= \boxed{16\operatorname{sinc}(4f) - 4\operatorname{sinc}(2f)}
\end{aligned}$$

Or in terms of just the sin function, the above becomes

$$\begin{aligned}
\digamma(w(t)) &= 16\frac{\sin(4\pi f)}{4\pi f} - 4\frac{\sin(2\pi f)}{2\pi f} \\
&= 4\frac{\sin(4\pi f)}{\pi f} - 2\frac{\sin(2\pi f)}{\pi f} \\
&= \boxed{\tfrac{4\sin(4\pi f) - 2\sin(2\pi f)}{\pi f}}
\end{aligned}$$

# 7 Problem 2.18

If $w(t)$ has the Fourier Transform $W(f) = \frac{j2\pi f}{1+j2\pi f}$ find $X(f)$ for the following waveforms

(a) $x(t) = w(2t+2)$

(b) $x(t) = w(t-1)e^{-jt}$

(c) $x(t) = w(1-t)$

Answer:

## 7.1 Part(a)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1 + j2\pi f}$$

Then

$$w(2t) \Leftrightarrow \frac{1}{2} X\left(\frac{f}{2}\right)$$
$$w(2t+2) \Leftrightarrow \frac{1}{2} X\left(\frac{f}{2}\right) e^{j2\pi\frac{f}{2}(2)}$$

Hence

$$w(2t+2) \Leftrightarrow \frac{1}{2}\left(\frac{j2\pi\frac{f}{2}}{1 + j2\pi\frac{f}{2}}\right) e^{j2\pi f}$$
$$\Leftrightarrow \frac{1}{2}\left(\frac{j\pi f}{1 + j\pi f}\right) e^{j2\pi f}$$

This can be simplified to

$$\boxed{w(2t+2) \Leftrightarrow \frac{\pi f}{2(\pi f - j)} e^{j2\pi f}}$$

## 7.2 Part(b)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1 + j2\pi f}$$

$$w(t-1) \Leftrightarrow X(f) e^{-j2\pi f(-1)}$$
$$w(t-1) \Leftrightarrow X(f) e^{j2\pi f}$$

Now Let $e^{-jt} = e^{-j2\pi f_0 t}$, hence $2\pi f_0 = 1$ or $f_0 = \frac{1}{2\pi}$, then

$$w(t-1) e^{-j2\pi f_0 t} \Leftrightarrow X(f + f_0) e^{j2\pi(f+f_0)}$$

Hence

$$w(t-1) e^{-jt} \Leftrightarrow \frac{j2\pi(f + f_0)}{1 + j2\pi(f + f_0)} e^{j2\pi(f+f_0)}$$
$$w(t-1) e^{-jt} \Leftrightarrow \frac{j2\pi\left(f + \frac{1}{2\pi}\right)}{1 + j2\pi\left(f + \frac{1}{2\pi}\right)} e^{j2\pi\left(f+\frac{1}{2\pi}\right)}$$
$$w(t-1) e^{-jt} \Leftrightarrow \frac{j2\pi(2\pi f + 1)}{2\pi + j2\pi(2\pi f + 1)} e^{j(2\pi f+1)}$$
$$w(t-1) e^{-jt} \Leftrightarrow \frac{j4\pi^2 f + j2\pi}{2\pi + j4\pi^2 f + j2\pi} e^{j2\pi f} e^{j}$$
$$w(t-1) e^{-jt} \Leftrightarrow \frac{2\pi f + 1}{-j + 2\pi f + 1} e^{j2\pi f} e^{j}$$

Hence

$$\boxed{w(t-1) e^{-jt} \Leftrightarrow \frac{2\pi f+1}{1-j+2\pi f} e^{j(2\pi f+1)}}$$

## 7.3 Part(c)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1 + j2\pi f}$$
$$w(-t) \Leftrightarrow X(-f)$$

Then

$$w(-t+1) \Leftrightarrow X(-f) e^{j2\pi f(1)}$$
$$w(1-t) \Leftrightarrow \frac{-j2\pi f}{1 - j2\pi f} e^{j2\pi f}$$

## 8 Key solution

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Key Solution.

$\boxed{2\text{-}7.}$

(a) $dBm = 10\log\left(\frac{P_w}{0.001}\right) = 10\log\left(\frac{67}{0.001}\right) = \underline{\underline{48.26\,dBm}}$

(b) $dBk = 10\log\left(\frac{P_w}{1000}\right) = 10\log\left(\frac{67}{1000}\right) = \underline{\underline{-11.74\,dBk}}$

(c) $P = \frac{V_{rms}^2}{R}$

$\Rightarrow V_{rms} = \sqrt{PR} = \sqrt{(67)(75)} = 70.9\text{ volts}$

$\Rightarrow dBV = 20\log\left(\frac{70.9}{10^{-3}}\right) = \underline{\underline{97\,dBmV}}$

$\boxed{2\text{-}8.}$ $P = \frac{V_{rms}^2}{R} = \frac{V_{rms}^2}{50}$

$dBm = 10\log_{10}\left(\frac{P}{0.001}\right) = 10\log_{10}\left(\frac{V_{rms}^2}{0.050}\right) = 20\log_{10}(V_{rms}) - 10\log_{10}(0.050)$

$\Rightarrow \underline{\underline{dBm = 20\log_{10}(V_{rms}) + 13}}$

$\boxed{2\text{-}9.}$ $P_{in} = I_{rms}^2 R_{in} = (0.5\times10^{-3})^2(2\times10^3) = 5.0\times10^{-4}\,W$

$P_{out} = \frac{V_{rms}^2}{R_{Load}} = \frac{100}{50} = 2\,W$

$dB = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{2}{5.0\times10^{-4}}\right) = \underline{\underline{36\,dB}}$

$\boxed{2\text{-}15.}$ $w(t) = \sin(2\pi f_1 t)\cos(2\pi f_2 t) = w_1(t)\,w_2(t)$

$\Rightarrow W(f) = W_1(f) * W_2(f) = \left[\frac{j}{2}\delta(f+f_1) - \frac{j}{2}\delta(f-f_1)\right] * \left[\frac{1}{2}\delta(f+f_2) + \frac{1}{2}\delta(f-f_2)\right]$

Aside: $\delta(f+f_1) * \delta(f+f_2) = \int_{-\infty}^{\infty}\delta(\lambda+f_1)\,\delta(f-\lambda+f_2)\,d\lambda = \delta(f+f_1+f_2)$

thus $\underline{\underline{W(f) = (j/4)\left[\delta(f+f_1+f_2) + \delta(f+f_1-f_2) - \delta(f-f_1+f_2) - \delta(f-f_1-f_2)\right]}}$

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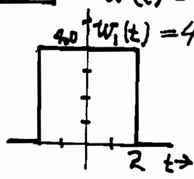
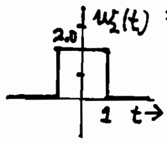
$\boxed{2\text{-}17.}$ $w(t) = w_1(t) - w_2(t)$ where

$w_1(t) = 4\,\text{rect}(\frac{t}{4})$




$\longleftrightarrow \quad W_1(f) = 16\,\text{Sinc}(4f)$

$w_2(t) = 2\,\text{rect}(\frac{t}{2})$




$\longleftrightarrow \quad W_2(f) = 4\,\text{Sinc}(2f)$

$\Rightarrow W(f) = W_1(f) - W_2(f) = \underline{\underline{16\,\text{Sinc}(4f) - 4\,\text{Sinc}(2f)}}$

$\boxed{2\text{-}18.}$ (a) $w(2t) \longleftrightarrow \frac{1}{2}\,\frac{j\pi f}{1+j\pi f}$

$\Rightarrow x(t) = w(2(t+1)) \longleftrightarrow \frac{j\pi f}{2(1+j\pi f)}e^{j2\pi f}$

(b) $w(t-1) \longleftrightarrow \frac{j2\pi f}{1+j2\pi f}e^{-j2\pi f}$

$x(t) = e^{-jt}w(t-1) \longleftrightarrow \frac{j2\pi(f+\frac{1}{2\pi})}{1+j2\pi(f+\frac{1}{2\pi})}e^{-j2\pi(f+\frac{1}{2\pi})}$

(c) $2\frac{dw(t)}{dt} \longleftrightarrow 2\,(j2\pi f\,W(f))$

$\Rightarrow x(t) \longleftrightarrow j4\pi f\left[\frac{j2\pi f}{1+j2\pi f}\right] = -\,\frac{8\pi^2 f^2}{1+j2\pi f}$

(d) $w(-t) \longleftrightarrow W(-f) = -\,\frac{j2\pi f}{1-j2\pi f}$

$\Rightarrow x(t) = w(-(t-1)) \longleftrightarrow \frac{-j2\pi f}{1-j2\pi f}e^{-j2\pi f}$

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2.17) $P = 67\ W$ and $R = 50\ \Omega$

a, b) $P_{dBW} \triangleq 10 \log_{10} P(w)$

$P_{dBm} = 10 \log_{10} P(mw)$ , $P_{dBK} = 10 \log_{10} P(KW)$

c) • For normalized case ($R = 1\ \Omega$), the average power P and rms voltage are related:

$$P_{av} = V_{rm}^2 = \frac{V_{peak}^2}{2} \Rightarrow V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

• For not normalized case ($R \neq 1\ \Omega$)

$$P_{av} = \frac{V_{rm}^2}{R} = \frac{V_{peak}^2}{2R}$$

if P and R are given => find $V_{rms}$.

$V_{rms}$ in dBmV is : $V_{rms}(dBmV) \triangleq 20 \log_{10} V_{rms}(mV)$

2.8) Given a Sine wave like :

$v(t) = V_{peak} \cos w_0 t$

the average power (normalized) of this periodic wave over one period, $T_0 = \frac{1}{f_0}$ , is :

$$P_{av} \triangleq \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v^2(t)\, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} V_{peak}^2 \cos^2 w_0 t\, dt$$

$$= \frac{V_{pek}^2}{2T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2w_0 t)\, dt = \frac{V_{pek}^2}{2}$$

If $R \neq 1\ \Omega \Rightarrow P_{av} = \frac{V_{peak}^2}{2R} = \frac{V_{rm}^2}{R}$ watts

$$\Rightarrow P_{dBm} \triangleq 10 \log_{10} P(mw) = 10 \log \left[ \frac{V_{rms}^2 \times 10^3}{R} \right] = \ldots$$

2.9) The power gain is:

$$A_{PdB} \cong 10 \log \frac{P_L}{P_{in}} \quad \text{where}, \quad P_L = \frac{V_{O(rms)}^2}{R_L}$$

$P_L$ is the power transfered to the load and
$P_{in}$ is the input power supplied by the source

2.17)
w(t) may be expressed in two differet way:

a) $w(t) = 4\,\text{rect}\left(\frac{t}{4}\right) - 2\,\text{rect}\left(\frac{t}{2}\right)$ or

b) $w(t) = 2\,\text{rect}\left(\frac{t}{2}\right) + 4\,\text{rect}\left(\frac{t+3/2}{1}\right) + 4\,\text{rect}\left(\frac{t-3/2}{1}\right)$

Find W(f)! The two answers should be the same
If you use the second method, you may after takin F.T use $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$

2.18)
if w(t) has F.T, which is $W(f) = \dfrac{j2\pi f}{1+j2\pi f}$

a) Find the F.T of $x(t) = w(2t+2) = w(2(t+1))$

$$w(2t) \longleftrightarrow \frac{1}{2} \cdot \frac{j2\pi\left(\frac{f}{2}\right)}{1+j2\pi\left(\frac{f}{2}\right)} \qquad \text{Scaling}$$

$$x(t) = w(2(t+1)) \longleftrightarrow \frac{j\pi f}{2(1+j\pi f)}\, e^{j2\pi f\cdot 1} \qquad \text{time domain shifting!}$$

# 9 my graded HW

HW3, EGEE 443. CSUF, Fall 2008

Nasser Abbasi

September 22, 2008

13.5
30

# 1 Problem 2.7

**Problem** An average reading power meter is connected to output of transmitter. Transmitter output is fed into 75Ω resistive load and the wattmeter read $67W$

(a) What is power in dBm units?

(b) What is power in dBk units?

(c) What is the value in dBmV units?

## 1.1 part(a)

$$\begin{aligned} P_{dbm} &= 10\log_{10} P_m \\ &= 10\log_{10}(67000) \\ &= \boxed{48.2607}\ dbm \end{aligned}$$

(b)

$$\begin{aligned} P_{dbk} &= 10\log_{10} P_k \\ &= 10\log_{10}(0.067) \\ &= \boxed{-11.7393}\ dbk \end{aligned}$$

(c)

$$P = \frac{V^2}{R}$$

Hence

$$10\log_{10} P = 20\log_{10} V - 10\log_{10} R$$

Hence

$$20\log_{10} V = 10\log_{10} P + 10\log_{10} R$$

so

$$\begin{aligned} 20\log_{10} V &= 10\log_{10} 67000 + 10\log_{10} 75000 \\ &= \boxed{97.0114 \text{ dbmV}} \end{aligned}$$

## 2 Problem 2.8

Assume that a waveform with known rms value $V_{rms}$ is applied across a $50\Omega$ load. Derive a formula that can be used to computer the *dbm* value from $V_{rms}$

$$P\,(watt) = \frac{V_{rms}^2\,(V)}{R\,(\Omega)}$$

Hence

$$\begin{aligned}
P_{dbm} &= 10\log_{10}\left(10^3 \times P_{watt}\right) \\
&= 10\log_{10}\frac{10^3 \times V_{rms}^2\,(V)}{R\,(\Omega)} \\
&= 10\left(\log_{10} 10^3 V_{rms}^2 - \log_{10} R\right) \\
&= 10\left(\log_{10} 10^3 + \log_{10} V_{rms}^2 - \log_{10} R\right) \\
&= 10\left(3 + 2\log_{10} V_{rms} - \log_{10} R\right)
\end{aligned}$$

Hence

$$\boxed{P_{dbm} = 30 + 20\log_{10} V_{rms} - 10\log_{10} R}$$

When $R = 50\Omega$, we obtain

$$\begin{aligned}
P_{dbm} &= 30 + 20\log_{10} V_{rms} - 10\log_{10} 50 \\
&= 30 + 20\log_{10} V_{rms} - 16.9897 \\
&= 13.0103 + 20\log_{10} V_{rms}
\end{aligned}$$

# 3 Problem 2.9

2-8 Assume that a waveform with a ... load. Derive a formula that can be used to compute the dBm value from $V_{rms}$.

2-9 An amplifier is connected to a 50-Ω load and driven by a sinusoidal current source as shown in Fig. P2-9. The output resistance of the amplifier is 10 Ω and the input resistance is 2 kΩ. Evaluate the true decibel gain of this circuit.

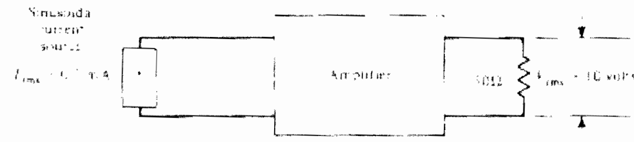

FIGURE P2-9

$$
\begin{aligned}
Gain(db) &= 10\log_{10}\frac{P_L}{P_i} \\
&= 10\log_{10}\frac{\left(\frac{V_{rms}^2}{R_L}\right)}{I_{rms}^2 R_{in}} \\
&= 10\log_{10}\frac{\left(\frac{10^2}{50}\right)}{(0.5\times10^{-3})^2\times 2000} \\
&= 10\log_{10}\frac{10^5}{25} \\
&= 10\left(\log_{10}10^5 - \log_{10}25\right) \\
&= 10\,(5 - 1.39794) \\
&= 36.021
\end{aligned}
$$

## 4 Problem 2.15

Using the convolution property find the spectrum for $w(t) = \sin 2\pi f_1 t \; \cos 2\pi f_2 t$

Solution:

$$F(w(t)) = F(\sin 2\pi f_1 t) \otimes F(\cos 2\pi f_2 t) \tag{1}$$

But

$$\begin{aligned} F(\sin 2\pi f_1 t) &= \frac{1}{2j}(\delta(f - f_1) - \delta(f + f_1)) \\ F(\cos 2\pi f_2 t) &= \frac{1}{2}(\delta(f - f_2) + \delta(f + f_2)) \end{aligned}$$

Hence (1) becomes

$$\begin{aligned} F(w(t)) &= \left\{\frac{1}{2j}(\delta(f - f_1) - \delta(f + f_1))\right\} \otimes \left\{\frac{1}{2}(\delta(f - f_2) + \delta(f + f_2))\right\} \\ &= \frac{1}{4j}\{\delta(f - f_1) - \delta(f + f_1)\} \otimes \{\delta(f - f_2) + \delta(f + f_2)\} \end{aligned} \tag{2}$$

Applying the distributed property of convolution, i.e. $a \otimes (b + c) = a \otimes b + a \otimes c$ on equation (2) we obtain

$$4j\,F(w(t)) = \delta(f - f_1)\otimes\delta(f - f_2) + \delta(f - f_1)\otimes\delta(f + f_2) - \delta(f + f_1)\otimes\delta(f - f_2) - \delta(f + f_1)\otimes\delta(f + f_2) \tag{3}$$

Now

$$\begin{aligned} \delta(f - f_1) \otimes \delta(f - f_2) &= \int_{-\infty}^{\infty} \delta(\lambda - f_1)\,\delta(f - (\lambda - f_2))\,d\lambda \\ &= \delta(f + f_2 - f_1) \int_{-\infty}^{\infty} \delta(f - (\lambda - f_2))\,d\lambda \\ &= \delta(f + f_2 - f_1) \end{aligned} \tag{4}$$

And

$$\begin{aligned} \delta(f - f_1) \otimes \delta(f + f_2) &= \int_{-\infty}^{\infty} \delta(\lambda - f_1)\,\delta(f - (\lambda + f_2))\,d\lambda \\ &= \delta(f - f_2 - f_1) \int_{-\infty}^{\infty} \delta(f - (\lambda - f_2))\,d\lambda \\ &= \delta(f - f_2 - f_1) \end{aligned} \tag{5}$$

And

$$
\begin{aligned}
\delta(f+f_1)\otimes\delta(f-f_2) &= \int_{-\infty}^{\infty}\delta(\lambda+f_1)\,\delta(f-(\lambda-f_2))\,d\lambda \\
&= \delta(f+f_2+f_1)\int_{-\infty}^{\infty}\delta(f-(\lambda-f_2))\,d\lambda \\
&= \delta(f+f_2+f_1) \qquad (6)
\end{aligned}
$$

And

$$
\begin{aligned}
\delta(f+f_1)\otimes\delta(f+f_2) &= \int_{-\infty}^{\infty}\delta(\lambda+f_1)\,\delta(f-(\lambda+f_2))\,d\lambda \\
&= \delta(f-f_2+f_1)\int_{-\infty}^{\infty}\delta(f-(\lambda-f_2))\,d\lambda \\
&= \delta(f-f_2+f_1) \qquad (7)
\end{aligned}
$$

Substitute (4,5,6,7) into (3) we obtain

$$\boxed{F(w(t)) = \tfrac{1}{4j}\left[\delta(f+f_2-f_1)+\delta(f-f_2-f_1)-\delta(f+f_2+f_1)-\delta(f-f_2+f_1)\right]}$$

or

$$\boxed{F(w(t)) = \tfrac{1}{4j}\left[\delta(f+(f_2-f_1))+\delta(f-(f_2+f_1))-\delta(f+(f_2+f_1))-\delta(f-(f_2-f_1))\right]} \qquad (8)$$

This problem can also be solved as follows

$$w(t) = \sin 2\pi f_1 t\ \cos 2\pi f_2 t$$

Using $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$, hence

$$
\begin{aligned}
w(t) &= \frac{1}{2}(\sin(2\pi f_1 t - 2\pi f_2 t)+\sin(2\pi f_1 t + 2\pi f_2 t)) \\
&= \frac{1}{2}(\sin(2\pi(f_1-f_2)t)+\sin(2\pi(f_1+f_2)t)) \\
&= \frac{1}{2}\left(\frac{1}{2j}(\delta(f-(f_1-f_2))-\delta(f+(f_1-f_2)))+\frac{1}{2j}(\delta(f-(f_1+f_2))-\delta(f+(f_1+f_2)))\right) \\
&= \frac{1}{4j}\{\delta(f-(f_1-f_2))-\delta(f+(f_1-f_2))+\delta(f-(f_1+f_2))-\delta(f+(f_1+f_2))\} \\
&= \frac{1}{4j}\{\delta(f+(f_2-f_1))+\delta(f-(f_2+f_1))-\delta(f+(f_2+f_1))-\delta(f-(f_2-f_1))\} \qquad (9)
\end{aligned}
$$

Compare (8) and (9) we see they are the same.

# 5 Problem 2.17

2-17 Find the spectrum for the waveform shown in Fig. P2-17

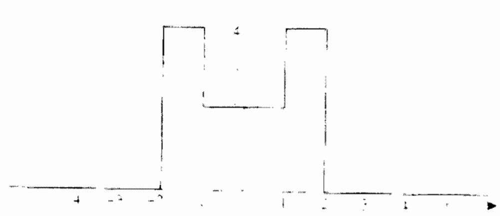

FIGURE P2-17

$$w(t) = 4\, rect\left(\frac{t}{4}\right) - 2\, rect\left(\frac{t}{2}\right)$$

By linearity of Fourier Transform

$$F(w(t)) = 4 \times F\left(rect\left(\frac{t}{4}\right)\right) - 2 \times F\left(rect\left(\frac{t}{2}\right)\right) \tag{1}$$

Since

$$F\left(rect\left(\frac{t}{4}\right)\right) = 4\,\mathrm{sinc}\,(4f)$$

and

$$F\left(rect\left(\frac{t}{2}\right)\right) = 2\,\mathrm{sinc}\,(2f)$$

Then (1) becomes

$$\begin{aligned} F(w(t)) &= 4 \times 4\,\mathrm{sinc}\,(4f) - 2 \times 2\,\mathrm{sinc}\,(2f) \\ &= \boxed{16\,\mathrm{sinc}\,(4f) - 4\,\mathrm{sinc}\,(2f)} \end{aligned}$$

Or in terms of just the sin function, the above becomes

$$\begin{aligned} F(w(t)) &= 16\frac{\sin(4\pi f)}{4\pi f} - 4\frac{\sin(2\pi f)}{2\pi f} \\ &= 4\frac{\sin(4\pi f)}{\pi f} - 2\frac{\sin(2\pi f)}{\pi f} \\ &= \boxed{\frac{4\sin(4\pi f) - 2\sin(2\pi f)}{\pi f}} \end{aligned}$$

# 6 Problem 2.18

If $w(t)$ has the Fourier Transform $W(f) = \frac{j2\pi f}{1+j2\pi f}$ find $X(f)$ for the following waveforms

(a) $x(t) = w(2t+2)$

(b) $x(t) = w(t-1)\,e^{-jt}$

(c) $x(t) = w(1-t)$

Answer:

## 6.1 Part(a)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1+j2\pi f}$$

Then

$$w(2t) \Leftrightarrow \frac{1}{2}X\left(\frac{f}{2}\right)$$
$$w(2t+2) \Leftrightarrow \frac{1}{2}X\left(\frac{f}{2}\right)e^{j2\pi\frac{f}{2}(2)}$$

Hence

$$w(2t+2) \Leftrightarrow \frac{1}{2}\left(\frac{j2\pi\frac{f}{2}}{1+j2\pi\frac{f}{2}}\right)e^{j2\pi f}$$
$$\Leftrightarrow \frac{1}{2}\left(\frac{j\pi f}{1+j\pi f}\right)e^{j2\pi f}$$

This can be simplified to

$$\boxed{w(2t+2) \Leftrightarrow \frac{\pi f}{2(\pi f - j)}e^{j2\pi f}}$$

### 6.2 Part(b)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1+j2\pi f}$$

$$w(t-1) \Leftrightarrow X(f)\, e^{-j2\pi f(-1)}$$
$$w(t-1) \Leftrightarrow X(f)\, e^{\cancel{j2\pi f}}$$

$w(t-1) \Leftrightarrow X(f) \cdot e^{-j2\pi f}$

Now Let $e^{-jt} = e^{-j2\pi f_0 t}$, hence $2\pi f_0 = 1$ or $f_0 = \frac{1}{2\pi}$, then

$$w(t-1)\, e^{-j2\pi f_0 t} \Leftrightarrow X(f+f_0)\, e^{j2\pi(f+f_0)}$$

Hence

$$w(t-1)\, e^{-jt} \Leftrightarrow \frac{j2\pi(f+f_0)}{1+j2\pi(f+f_0)} e^{j2\pi(f+f_0)}$$

$= \frac{j2\pi\left(f+\frac{1}{2\pi}\right)}{1+j2\pi\left(f+\frac{1}{2\pi}\right)} \cdot e^{-j2\pi\left(f+\frac{1}{2\pi}\right)}$

$$w(t-1)\, e^{-jt} \Leftrightarrow \frac{j2\pi\left(f+\frac{1}{2\pi}\right)}{1+j2\pi\left(f+\frac{1}{2\pi}\right)} e^{j2\pi\left(f+\frac{1}{2\pi}\right)}$$
$$w(t-1)\, e^{-jt} \Leftrightarrow \frac{j2\pi(2\pi f+1)}{2\pi+j2\pi(2\pi f+1)} e^{j(2\pi f+1)}$$
$$w(t-1)\, e^{-jt} \Leftrightarrow \frac{j4\pi^2 f+j2\pi}{2\pi+j4\pi^2 f+j2\pi} e^{j2\pi f} e^{j}$$
$$w(t-1)\, e^{-jt} \Leftrightarrow \frac{2\pi f+1}{-j+2\pi f+1} e^{j2\pi f} e^{j}$$

−1

Hence

$$\boxed{w(t-1)\, e^{-jt} \Leftrightarrow \frac{2\pi f+1}{1-j+2\pi f} e^{j(2\pi f+1)}}$$

### 6.3 Part(c)

$$w(t) \Leftrightarrow \frac{j2\pi f}{1+j2\pi f}$$
$$w(-t) \Leftrightarrow X(-f)$$

Then

$w(t-1) \Leftrightarrow X(f)\, e^{-j2\pi f}$

$$w(-t+1) \Leftrightarrow X(-f)\, e^{j2\pi f(1)}$$
$$w(1-t) \Leftrightarrow \frac{-j2\pi f}{1-j2\pi f} e^{j2\pi f}$$

−0.5