

HW 10
Electronic Communication Systems
Fall 2008
California State University, Fullerson

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1 Problem 3.24

3.24 Consider a composite wave obtained by adding a noncoherent carrier $A_c \cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t)m(t)$. This composite wave is applied to an ideal envelope detector. Find the resulting detector output for

(a) $\phi = 0$
 (b) $\phi \neq 0$ and $|m(t)| \ll A_c/2$

Figure 1: the Problem statement

$$s_1(t) = A_c \cos(\omega_c t + \phi)$$

DSB-SC signal is

$$s_2(t) = m(t) \cos(\omega_c t)$$

Hence by adding the above, we obtain

$$s(t) = m(t) \cos(\omega_c t) + A_c \cos(\omega_c t + \phi)$$

The above signal is applied to an ideal envelope detector. The output of an envelope detector is given by

$$a(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$$

Since $s(t)$ is a bandpass signal, we need to first write it in the canonical form $s_I(t) \cos(\omega_c t) - s_Q(t) \sin(\omega_c t)$

Using $\cos(A + B) = \cos A \cos B - \sin A \sin B$, then we have

$$\begin{aligned} s(t) &= m(t) \cos(\omega_c t) + A_c [\cos \omega_c t \cos \phi - \sin \omega_c t \sin \phi] \\ &= [m(t) + A_c \cos \phi] \cos(\omega_c t) - A_c \sin \omega_c t \sin \phi \end{aligned}$$

Hence we see that

$$\begin{aligned} s_I(t) &= m(t) + A_c \cos \phi \\ s_Q(t) &= A_c \sin \phi \end{aligned}$$

Now we can start answering parts (a) and (b)

1.1 Part(a)

When $\phi = 0$, then

$$\begin{aligned} s_I(t) &= m(t) + A_c \\ s_Q(t) &= 0 \end{aligned}$$

Hence

$$\begin{aligned} a(t) &= \sqrt{[m(t) + A_c]^2 + 0^2} \\ &= m(t) + A_c \end{aligned}$$

2 Part(b)

When $\phi \neq 0$ and $|m(t)| \ll \frac{A_c}{2}$

$$\begin{aligned} a(t) &= \sqrt{[m(t) + A_c]^2 + [A_c \sin \phi]^2} \\ &= \sqrt{[m^2(t) + A_c^2 + 2A_c m(t)] + [A_c^2 \sin^2 \phi]} \end{aligned}$$

Since $|m(t)| \ll \frac{A_c}{2}$, then $m^2(t) + A_c^2 + 2A_c m(t) \simeq A_c^2$ hence

$$\begin{aligned} a(t) &\simeq \sqrt{A_c^2 + A_c^2 \sin^2 \phi} \\ &= A_c \sqrt{1 + \sin^2 \phi} \end{aligned}$$

3 Problem 5.20

5-20 A modulated signal is described by the equation

$$s(t) = 10 \cos[(2\pi \times 10^8)t + 10 \cos(2\pi \times 10^3 t)]$$

Find each of the following:

- (a) Percentage of AM.
- (b) Normalized power of the modulated signal.
- (c) Maximum phase deviation.
- (d) Maximum frequency deviation.

Figure 2: the Problem statement

3.1 Part(a)

An AM signal is $s(t) = A_c [1 + \mu m(t)] \cos(2\pi f_c t + \theta(t))$. Now compare this form with the one given above, which is $s(t) = A_c \cos(2\pi f_c t + \theta(t))$. We see that $\mu = 0$, i.e. no message source exist. Hence percentage of modulation is zero.

3.2 Part(b)

$$P_{av} = \frac{1}{2} A_c^2$$

But $A_c = 10$, hence

$$\begin{aligned} P_{av} &= \frac{100}{2} \\ &= 50 \text{ watt} \end{aligned}$$

3.3 Part(c)

From the general form for angle modulated signal

$$s(t) = \cos(\omega_c t + \theta(t))$$

Looking at

$$s(t) = A_c \cos \left(\overbrace{\left(\underbrace{2\pi f_c}_{(2\pi \times 10^8)t} + \underbrace{\theta(t)}_{10 \cos(2\pi \times 10^3 t)} \right)}^{\text{Total Phase}} \right)$$

Phase deviation is

$$\theta(t) = 10 \cos(2\pi \times 10^3 t)$$

Which is maximum when $\cos(2\pi \times 10^3 t) = 1$ Hence maximum Phase deviation is 10 radians.

3.4 part(d)

Now, we know that the instantaneous frequency f_i is given by

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi} \frac{d}{dt} (\text{total phase}) \\ &= \frac{1}{2\pi} \frac{d}{dt} [\omega_c t + \theta(t)] \\ &= \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + 10 \cos(2\pi \times 10^3 t)] \\ &= f_c - 10 (10^3) \sin(2\pi \times 10^3 t) \end{aligned}$$

The deviation of frequency is the difference between f_i and the carrier frequency f_c . Hence from the above we see that the frequency deviation is

$$\begin{aligned} \Delta f &= f_i - f_c \\ &= -10 (10^3) \sin(2\pi \times 10^3 t) \end{aligned}$$

So, maximum Δf occurs when $\sin(2\pi \times 10^3 t) = -1$, hence

$$\max(\Delta f) = 10^4 \text{ Hz}$$

4 Problem 5.22

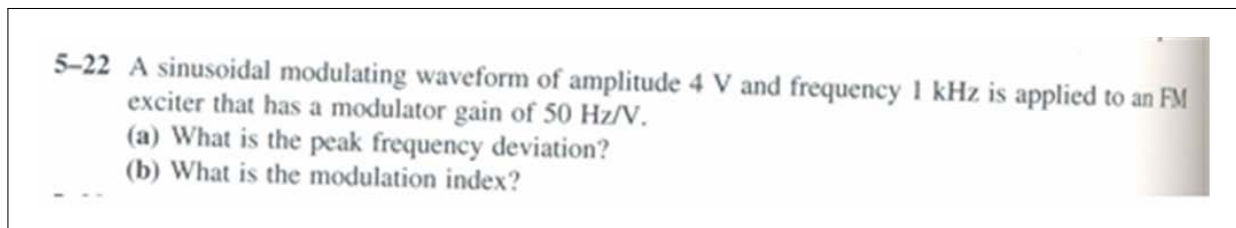


Figure 3: the Problem statement

The modulating waveform is $m(t)$ Hence (I am assuming it is cos since it said sinusoidal)

$$\begin{aligned} m(t) &= A_m \cos(2\pi f_m t) \\ &= 4 \cos(2000\pi t) \end{aligned}$$

Since it is an FM signal, then

$$s(t) = A_c \cos \left[\overbrace{\omega_c t + 2\pi k_f \int_0^t m(x) dx}^{\theta(t)} \right]$$

Where k_f is the frequency deviation constant in cycle per volt-second. The gain here means the frequency gain, which is the frequency deviation (deviation from the f_c frequency).

Let Δf be the frequency deviation in Hz, then

$$\begin{aligned}\Delta f &= f_i - f_c \\ &= \frac{1}{2\pi} \frac{d}{dt} \theta(t) \\ &= k_f m(t) \\ &= k_f [4 \cos(2000\pi t)]\end{aligned}$$

4.1 Part(a)

max Δf is

$$(\Delta f)_{\max} = 4k_f$$

But $k_f = 50$ hz/volt, hence

$$\begin{aligned}(\Delta f)_{\max} &= 4 \times 50 \\ &= 200\text{hz}\end{aligned}$$

4.2 Part(b)

Modulation index

$$\begin{aligned}\beta &= \frac{(\Delta f)_{\max}}{f_m} \\ &= \frac{200}{1000} \\ &= 0.2\end{aligned}$$

5 Problem 5.24

5-24 An FM transmitter has the block diagram shown in Fig. P5-24. The audio frequency response is flat over the 20-Hz-to-15-kHz audio band. The FM output signal is to have a carrier frequency of 103.7 MHz and a peak deviation of 75 kHz.

(a) Find the bandwidth and center frequency required for the bandpass filter.
 (b) Calculate the frequency f_0 of the oscillator.
 (c) What is the required peak deviation capability of the FM exciter?

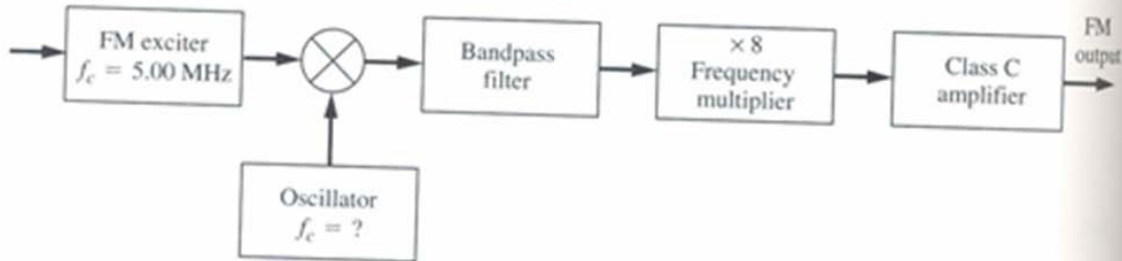


Figure P5-24

Figure 4: the Problem statement

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int_0^t m(x) dx \right)$$

We are told the carrier frequency has $f_c = 103.7$ Mhz, but there is a multiplier of 8, and hence the center frequency of the bandpass filter must be $\frac{1}{8}$ of the carrier frequency. i.e.

center frequency of the bandpass filter is $\frac{1}{8}103.7 = \frac{103.7}{8} = 12.963$

Since peak deviation is 75kHz , which means the deviation from the central frequency has maximum of 75kHz , then

$$\frac{75}{8} = 9.375 \text{ kHz}$$

Hence bandwidth from center of frequency of bandwidth filter is 9.375 but we need to add frequency width of the audio which is $15000 - 20 = 14980$ Hz on both side, hence

Bandwidth of BPF is $9.375 \times 10^3 \pm 14980$

5.1 Part (b)

To do

6 Problem 5.26

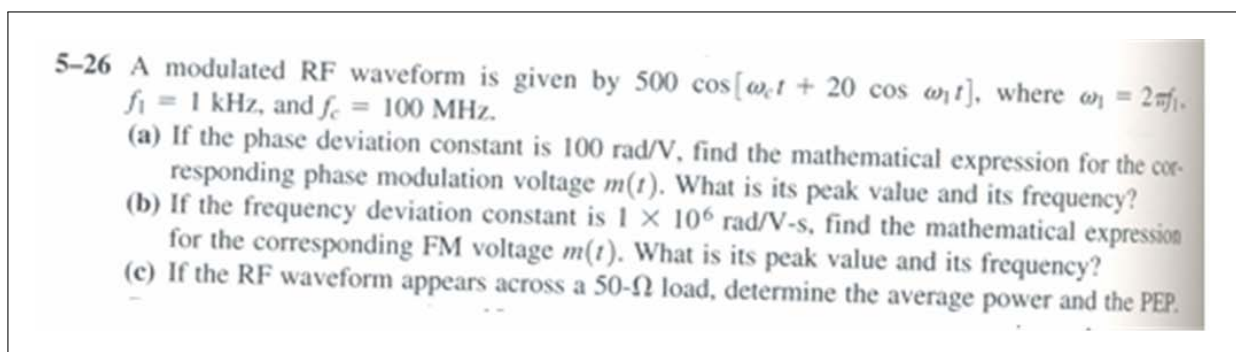


Figure 5: the Problem statement

$$s(t) = A_c \cos(\omega_c t + 20 \cos \omega_1 t)$$

where $A_c = 500$, $f_1 = 1\text{kHz}$, $f_c = 100\text{MHz}$

6.1 Part(a)

The general form of the above PM signal is

$$s(t) = A_c \cos\left(\omega_c t + \overbrace{k_p m(t)}^{\text{phase deviation}}\right)$$

Where $k_p m(t)$ is the phase deviation, and k_p is the phase deviation constant in radians per volt. Hence we write

$$k_p m(t) = 20 \cos \omega_1 t$$

Then

$$m(t) = \frac{20 \cos \omega_1 t}{k_p}$$

But we are given that $k_p = 100$ rad/voltage and $f_1 = 1000\text{Hz}$, then the above becomes

$$\begin{aligned} m(t) &= \frac{20 \cos(2000\pi t)}{100} \\ &= 0.2 \cos(2000\pi t) \end{aligned}$$

its frequency is 1kHz and its peak value is 0.2 volts

6.2 Part(b)

The general form of the above FM signal is

$$s(t) = A_c \cos \left(\omega_c t + k_f \int_0^t m(x) dx \right)$$

Where k_f is the frequency deviation constant in radians per volt-second

Hence

$$k_f \int_0^t m(x) dx = 20 \cos \omega_1 t$$

Solve for $m(t)$ in the above, given that $k_f = 10^6$ radians per volt-second, hence

$$\begin{aligned} k_f \int_0^t m(x) dx &= 20 \cos \omega_1 t \\ \int_0^t m(x) dx &= \frac{20 \cos (2000\pi t)}{10^6} \end{aligned}$$

Take derivative of both sides, we obtain

$$\begin{aligned} m(t) &= \frac{20}{10^6} [-\sin(2000\pi t) \times 2000\pi] \\ &= -\frac{20 \times 2000\pi}{10^6} \sin(2000\pi t) \\ &= -0.126 \sin(2000\pi t) \end{aligned}$$

Hence its peak value is 0.126 and its frequency is 1 khz

6.3 Part(c)

$$\begin{aligned} P_{av} &= \frac{\langle s^2(t) \rangle}{50} \\ &= \frac{\frac{1}{2} A_c^2}{50} \\ &= \frac{500^2}{100} \\ &= 2500 \text{ watt} \end{aligned}$$

PEP is average power obtained if the complex envelope is held constant at its maximum values. i.e. (the normalized PEP) is

$$PEP = \frac{1}{2} [\max(|\tilde{s}(t)|)]^2$$

Since

$$\begin{aligned} s(t) &= A_c \cos(\omega_c t + 20 \cos \omega_1 t) \\ &= A_c [\cos \omega_c t \cos(20 \cos \omega_1 t) - \sin \omega_c t \sin(20 \cos \omega_1 t)] \\ &= \underbrace{A_c \cos(20 \cos \omega_1 t)}_{s_I(t)} \cos \omega_c t - \underbrace{A_c \sin(20 \cos \omega_1 t)}_{s_Q(t)} \sin \omega_c t \end{aligned}$$

Hence

$$\begin{aligned} \tilde{s}(t) &= s_I(t) + j s_Q(t) \\ &= A_c \cos(20 \cos \omega_1 t) + j A_c \sin(20 \cos \omega_1 t) \end{aligned}$$

Then

$$\begin{aligned} |\tilde{s}(t)| &= \sqrt{[A_c \cos(20 \cos \omega_1 t)]^2 + [A_c \sin(20 \cos \omega_1 t)]^2} \\ &= A_c \sqrt{\cos^2(20 \cos \omega_1 t) + \sin^2(20 \cos \omega_1 t)} \\ &= A_c \end{aligned}$$

Hence the non-normalized PEP is

$$\begin{aligned} PEP &= \frac{1}{2} [A_c]^2 \\ &= \frac{50}{50} \\ &= \frac{500^2}{100} \\ &= 2500 \text{ watt} \end{aligned}$$

ps. is there an easier or more direct way to find PEP than what I did? (assuming it is correct)

7 Key solution

EE 443 HW # 10 pages
and part of 9

5-18. (a.)

(A) $v_A(t) = m(t) \cos \omega_{IF} t + \hat{m}(t) \sin \omega_{IF} t$
↑ USSB
 ↓ LSSB

(B) $v_B(t) = \cos \omega_{IF} t$

(C) $v_C(t) = v_A(t) v_B(t) = m(t) \cos^2 \omega_{IF} t + \hat{m}(t) \sin \omega_{IF} t \cos \omega_{IF} t$
 $= \frac{m(t)}{2} (1 + \cos 2\omega_{IF} t) + \frac{\hat{m}(t)}{2} \sin 2\omega_{IF} t$

(D) $v_D(t) = \frac{m(t)}{2}$

(E) $v_E(t) = \sin \omega_{IF} t$

(F) $v_F(t) = v_A(t) v_E(t)$
 $= m(t) \sin \omega_{IF} t \cos \omega_{IF} t + \hat{m}(t) \sin^2 \omega_{IF} t$
 $= \frac{m(t)}{2} \sin 2\omega_{IF} t + \frac{\hat{m}(t)}{2} (1 - \cos 2\omega_{IF} t)$

(G) $v_G(t) = \frac{\hat{m}(t)}{2}$

(H) $v_H(t) = \pm \frac{m(t)}{2}$

(I) $v_I(t) = v_C(t) + v_H(t)$
 $= \frac{m(t)}{2} + \frac{m(t)}{2}$

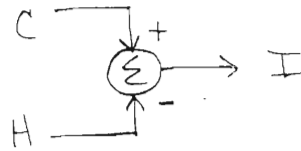
$v_I(t) = \left\{ \begin{array}{l} m(t), \text{ USSB} \\ \hat{m}(t), \text{ LSSB} \end{array} \right\}$

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page 2

5.18 cont'd) To receive LSSB signals, subtract
 $v_H(t)$ from $v_c(t)$ at the summer.



(b.) see part (a.)

(c.) see part (a.)

(d.) IF should be centered at $f_c \pm 1.5\text{kHz}$,
 have 3kHz BW and as small a roll-off factor as is economically feasible.

LPF should have 3kHz BW and as small a roll-off factor as is feasible, also.

$\sqrt{5-20}$ (a.) 0% AM

$$(b.) P_{\text{norm}} = A_c^2/2 = 10^2/2 = \underline{50\text{W}}$$

$$(c.) \Delta D_{\text{max}} = \underline{10 \text{ radians}}$$

$$(d.) \omega_d(t) = \frac{d\phi(t)}{dt} = -10(2000\pi)\sin(2000\pi t)$$

$$\Delta F = \frac{\Delta\omega_d}{2\pi} = \frac{10(2000\pi)}{2\pi} = 10^4 = \underline{10\text{kHz}}$$

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HW #

Page 3

$$\boxed{5-22.} \quad m(t) = A_m \cos(2\pi f_m t) = 4 \cos(2\pi \times 10^3 t)$$

$$(a.) \quad f_i(t) = f_c + \Delta F \cos(2\pi \times 10^3 t)$$

$$\Delta F = k_f A_m = \left(\frac{50 \text{ Hz}}{V}\right) (4V) = \underline{200 \text{ Hz}}$$

$$(b.) \quad \beta = \frac{\Delta F}{f_m} = \frac{200 \text{ Hz}}{1 \text{ kHz}} = \underline{0.2}$$

$$\sqrt{\boxed{5-24.}} \quad (a.) \quad f_{\text{BPF}} = \frac{103.7}{8} \text{ MHz} = \underline{\underline{12.96 \text{ MHz}}}$$

$$\Delta F_{\text{BPF}} = \frac{75 \text{ kHz}}{8} = 9.375 \text{ kHz}$$

$$\text{BW}_{\text{BPF}} = 2(\Delta F + f_m) = 2(9.375 + 15) \text{ kHz} \\ = \underline{\underline{48.75 \text{ kHz}}}$$

$$(b.) \quad f_{\text{BPF}} = f_c + f_o \Rightarrow f_o = 12.96 - 5 = \underline{\underline{7.96 \text{ MHz}}}$$

$$f_{\text{BPF}} = f_c - f_o \Rightarrow f_o = 12.96 + 5 = \underline{\underline{17.96 \text{ MHz}}}$$

\uparrow
 $f_c = 5 \text{ MHz}$

$$(c.) \quad \Delta F_{\text{FME}} = \frac{75 \text{ kHz}}{8} = \underline{\underline{9.38 \text{ kHz}}}$$

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HW #

page 4

$$\checkmark \quad \boxed{5-26.} \quad (a.) \quad \Theta(t) = D_p m_p(t) = 20 \cos \omega_c t$$

$$\Rightarrow m_p(t) = \frac{20}{D_p} \cos \omega_c t = \underline{\underline{0.2 \cos(2000\pi t)}}$$

$$m_p(t)_{\text{peak}} = \underline{\underline{0.2 \text{ V}}} \quad ; \quad f_m = \underline{\underline{1 \text{ kHz}}}$$

$$(b.) \quad \Theta(t) = D_f \int_{-\infty}^t m_f(\lambda) d\lambda = 20 \cos \omega_c t$$

$$\Rightarrow m_f(t) = \frac{20}{D_f} \frac{d}{dt} [\cos \omega_c t]$$

$$= \frac{-20}{10^6} (2000\pi) \sin \omega_c t$$

$$m_f(t) = \underline{\underline{-0.1257 \sin \omega_c t}}$$

$$m_f(t)_{\text{peak}} = \underline{\underline{0.1257 \text{ V}}} \quad ; \quad f_m = \underline{\underline{1 \text{ kHz}}}$$

$$(c.) \quad P_{AV} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{\underline{2.5 \text{ kW}}}$$

$$PEP = \underline{\underline{2.5 \text{ kW}}}$$

EE 443

HW # 8

Chapt. 3

page 5

3.24)



$$\begin{aligned}
 s(t) &= m(t) \cos 2\pi f_c t + A_c \cos(2\pi f_c t + \phi) \\
 &= m(t) \cos 2\pi f_c t + A_c \cos \phi \cos 2\pi f_c t - A_c \sin \phi \sin 2\pi f_c t \\
 &= \underbrace{(m(t) + A_c \cos \phi)}_{\text{In phase comp.}} \cos 2\pi f_c t - \underbrace{A_c \sin \phi}_{\text{Quadrature component}} \sin 2\pi f_c t
 \end{aligned}$$

$$\begin{aligned}
 a(t) &= \sqrt{(m(t) + A_c \cos \phi)^2 + A_c^2 \sin^2 \phi} \\
 &= \sqrt{m^2(t) + A_c^2 \cos^2 \phi + 2A_c \cos \phi m(t) + A_c^2 \sin^2 \phi} \\
 &= \sqrt{m^2(t) + A_c^2 + 2A_c \cos \phi m(t)} \quad (1)
 \end{aligned}$$

a) If $\phi = 0 \Rightarrow a(t) = \sqrt{m^2(t) + A_c^2 + 2A_c m(t)} = \sqrt{(m(t) + A_c)^2}$
 $\Rightarrow a(t) = |m(t) + A_c| = m(t) + A_c$ if $|m(t)| < A_c$

b) For $\phi \neq 0$ and $|m(t)| \ll A_c/2$ using eq (1)

$$\begin{aligned}
 a(t) &= A_c \sqrt{1 + \frac{2}{A_c} \cos \phi m(t) + \frac{m^2(t)}{A_c^2}} \\
 &\approx A_c \sqrt{1 + \frac{2}{A_c} \cos \phi m(t)} \\
 &= A_c \left[1 + \frac{1}{2} \cdot \frac{2}{A_c} \cos \phi m(t) \right] \\
 &= A_c + \cos \phi m(t)
 \end{aligned}$$

we can neglect $\frac{m^2(t)}{A_c^2}$ since $|m(t)| \ll A_c/2$
 using $(1+x)^\alpha \approx 1 + \alpha x$ if $x \ll 1$

8 my graded HW

HW10, EGEE 443. CSUF, Fall 2008 (3.24,
5-20,5-22,5-24,5-26)

Nasser Abbasi

December 4, 2008

1 Problem 3.24

18
20

3.24 Consider a composite wave obtained by adding a noncoherent carrier $A_c \cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t)m(t)$. This composite wave is applied to an ideal envelope detector. Find the resulting detector output for

- (a) $\phi = 0$
 (b) $\phi = 0$ and $m(t) = A_m \cos 2\pi f_m t$

$$s_1(t) = A_c \cos(\omega_c t + \phi)$$

DSB-SC signal is

$$s_2(t) = m(t) \cos(\omega_c t)$$

Hence by adding the above, we obtain

$$s(t) = m(t) \cos(\omega_c t) + A_c \cos(\omega_c t + \phi)$$

The above signal is applied to an ideal envelope detector. The output of an envelope detector is given by

$$a(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$$

Since $s(t)$ is a bandpass signal, we need to first write it in the canonical form $s_I(t) \cos(\omega_c t) - s_Q(t) \sin(\omega_c t)$

Using $\cos(A + B) = \cos A \cos B - \sin A \sin B$, then we have

$$\begin{aligned} s(t) &= m(t) \cos(\omega_c t) + A_c [\cos \omega_c t \cos \phi - \sin \omega_c t \sin \phi] \\ &= [m(t) + A_c \cos \phi] \cos(\omega_c t) - A_c \sin \omega_c t \sin \phi \end{aligned}$$

Hence we see that

$$\begin{aligned} s_I(t) &= m(t) + A_c \cos \phi \\ s_Q(t) &= A_c \sin \phi \end{aligned}$$

Now we can start answering parts (a) and (b)

1.1 Part(a)

When $\phi = 0$, then

$$\begin{aligned} s_I(t) &= m(t) + A_c \\ s_Q(t) &= 0 \end{aligned}$$

Hence

$$\begin{aligned} a(t) &= \sqrt{[m(t) + A_c]^2 + 0^2} && - 0.5 \\ &= |m(t) + A_c| && \leftarrow \text{don't miss the absolute value} \\ &= m(t) + A_c, \text{ if } |m(t)| < A_c \end{aligned}$$

1.2 Part(b)

When $\phi \neq 0$ and $|m(t)| \ll \frac{A_c}{2}$

$$\begin{aligned} a(t) &= \sqrt{[m(t) + A_c]^2 + [A_c \sin \phi]^2} \\ &= \sqrt{m^2(t) + A_c^2 + 2A_c m(t) + [A_c^2 \sin^2 \phi]} \end{aligned}$$

Since $|m(t)| \ll \frac{A_c}{2}$, then $m^2(t) + A_c^2 + 2A_c m(t) \simeq A_c^2$ hence

$$\begin{aligned} a(t) &\simeq \sqrt{A_c^2 + A_c^2 \sin^2 \phi} \\ &= A_c \sqrt{1 + \sin^2 \phi} \quad \text{ok. See sol.} \end{aligned}$$

2 Problem 5.20

5-20 A modulated signal is described by the equation

$$s(t) = 10 \cos[(2\pi \times 10^8)t + 10 \cos(2\pi \times 10^3t)]$$

Find each of the following:

- (a) Percentage of AM.
- (b) Normalized power of the modulated signal.
- (c) Maximum phase deviation.
- (d) Maximum frequency deviation.

2.1 Part(a)

An AM signal is $s(t) = A_c [1 + \mu m(t)] \cos(2\pi f_c t + \theta(t))$. Now compare this form with the one given above, which is $s(t) = A_c \cos(2\pi f_c t + \theta(t))$. We see that $\mu = 0$, i.e. no message source exist. Hence percentage of modulation is zero.

2.2 Part(b)

$$P_{av} = \frac{1}{2} A_c^2$$

But $A_c = 10$, hence

$$\begin{aligned} P_{av} &= \frac{100}{2} \\ &= \boxed{50 \text{ watt}} \end{aligned}$$

2.3 Part(c)

From the general form for angle modulated signal

$$s(t) = \cos(\omega_c t + \theta(t))$$

Looking at

$$s(t) = A_c \cos \left(\overbrace{\left(\underbrace{2\pi f_c}_{(2\pi \times 10^8)} t + \underbrace{10 \cos(2\pi \times 10^3 t)}_{\theta(t)} \right)}^{\text{Total Phase}} \right)$$

Phase deviation is

$$\theta(t) = 10 \cos(2\pi \times 10^3 t)$$

Which is maximum when $\cos(2\pi \times 10^3 t) = 1$ Hence maximum Phase deviation is 10 radians.

2.4 part(d)

Now, we know that the instantaneous frequency f_i is given by

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi} \frac{d}{dt} (\text{total phase}) \\ &= \frac{1}{2\pi} \frac{d}{dt} [\omega_c t + \theta(t)] \\ &= \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + 10 \cos(2\pi \times 10^3 t)] \\ &= f_c - 10(10^3) \sin(2\pi \times 10^3 t) \end{aligned}$$

The deviation of frequency is the difference between f_i and the carrier frequency f_c . Hence from the above we see that the frequency deviation is

$$\begin{aligned} \Delta f &= f_i - f_c \\ &= -10(10^3) \sin(2\pi \times 10^3 t) \end{aligned}$$

So, maximum Δf occurs when $\sin(2\pi \times 10^3 t) = -1$, hence

$$\max(\Delta f) = 10^4 \text{ Hz}$$

3 Problem 5.22

5-22 A sinusoidal modulating waveform of amplitude 4 V and frequency 1 kHz is applied to an FM exciter that has a modulator gain of 50 Hz/V.
(a) What is the peak frequency deviation?
(b) What is the modulation index?

The modulating waveform is $m(t)$ Hence (I am assuming it is cos since it said sinusoidal)

$$\begin{aligned} m(t) &= A_m \cos(2\pi f_m t) \\ &= 4 \cos(2000\pi t) \end{aligned}$$

Since it is an FM signal, then

$$s(t) = A_c \cos \left[\overbrace{\omega_c t + 2\pi k_f \int_0^t m(x) dx}^{\theta(t)} \right]$$

Where k_f is the frequency deviation constant in cycle per volt-second. The gain here means the frequency gain, which is the frequency deviation (deviation from the f_c frequency). Let Δf be the frequency deviation in Hz, then

$$\begin{aligned} \Delta f &= f_i - f_c \\ &= \frac{1}{2\pi} \frac{d}{dt} \theta(t) \\ &= k_f m(t) \\ &= k_f [4 \cos(2000\pi t)]_{\max} \\ &= \frac{50}{V} \times 4V = 200 \text{ Hz} \end{aligned}$$

3.1 Part(a)

max Δf is

$$(\Delta f)_{\max} = 4k_f$$

But $k_f = 50 \text{ hz/volt}$, hence

$$\begin{aligned} (\Delta f)_{\max} &= 4 \times 50 \\ &= \boxed{200 \text{ hz}} \end{aligned}$$

3.2 Part(b)

Modulation index

$$\begin{aligned}\beta &= \frac{(\Delta f)_{\max}}{f_m} \\ &= \frac{200}{1000} \\ &= \boxed{0.2}\end{aligned}$$

4 Problem 5.24

5-24 An FM transmitter has the block diagram shown in Fig. P5-24. The audio frequency response is flat over the 20-Hz-to-15-kHz audio band. The FM output signal is to have a carrier frequency of 103.7 MHz and a peak deviation of 75 kHz.

(a) Find the bandwidth and center frequency required for the bandpass filter.

(b) Calculate the frequency f_0 of the oscillator.

(c) What is the required peak deviation capability of the FM exciter?

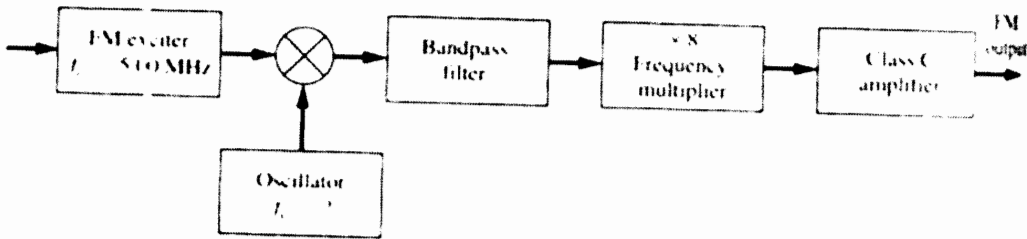


Figure P5-24

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int_0^t m(x) dx \right)$$

We are told the carrier frequency has $f_c = 103.7$ MHz, but there is a multiplier of 8, and hence the center frequency of the bandpass filter must be $\frac{1}{8}$ of the carrier frequency. i.e.

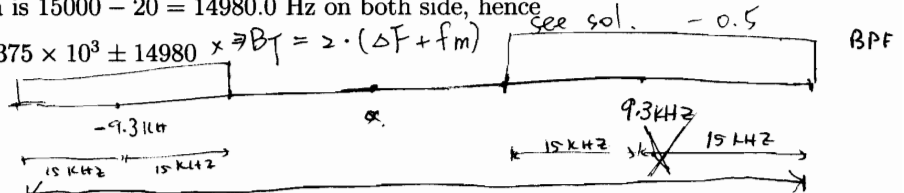
center frequency of the bandpass filter is $\frac{1}{8} 103.7 = \frac{103.7}{8} = 12.963$ MHz

Since peak deviation is 75 kHz, which means the deviation from the central frequency has maximum of 75 kHz, then

$$\frac{75}{8} = 9.375 \text{ kHz}$$

Hence bandwidth from center of frequency of bandwidth filter is 9.375 but we need to add frequency width of the audio which is $15000 - 20 = 14980.0$ Hz on both side, hence

Bandwidth of BPF is $9.375 \times 10^3 \pm 14980 \Rightarrow BT = 2 \cdot (\Delta F + f_m)$



4.1 Part (b)

~~need more time. sorry. I run out of time to finish. not sure how to finish now.~~

$$f_0 = \frac{f_c}{2} = 2.5 \text{ MHz} \quad ?$$

~~$$f_0 = f_c \pm f_{BPF}$$~~

$$f_0 = f_{BPF} \pm f_c$$

$$= 12.963 \pm 5 \text{ (MHz)}$$

Part (c)

5 Problem 5.26

- 5-26** A modulated RF waveform is given by $500 \cos \omega_c t + 20 \cos \omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 1 \text{ kHz}$, and $f_c = 100 \text{ MHz}$.
- (a) If the phase deviation constant is 100 rad/V , find the mathematical expression for the corresponding phase modulation voltage $m(t)$. What is its peak value and its frequency?
- (b) If the frequency deviation constant is $1 \times 10^6 \text{ rad/V-s}$, find the mathematical expression for the corresponding FM voltage $m(t)$. What is its peak value and its frequency?
- (c) If the RF waveform appears across a $50\text{-}\Omega$ load, determine the average power and the PEP.

$$s(t) = A_c \cos(\omega_c t + 20 \cos \omega_1 t)$$

where $A_c = 500$, $f_1 = 1 \text{ kHz}$, $f_c = 100 \text{ MHz}$

5.1 Part(a)

The general form of the above PM signal is

$$s(t) = A_c \cos \left(\omega_c t + \overbrace{k_p m(t)}^{\text{phase deviation}} \right)$$

Where $k_p m(t)$ is the phase deviation, and k_p is the phase deviation constant in radians per volt. Hence we write

$$k_p m(t) = 20 \cos \omega_1 t$$

Then

$$m(t) = \frac{20 \cos \omega_1 t}{k_p}$$

But we are given that $k_p = 100 \text{ rad/voltage}$ and $f_1 = 1000 \text{ Hz}$, then the above becomes

$$\begin{aligned} m(t) &= \frac{20 \cos(2000\pi t)}{100} \\ &= \boxed{0.2 \cos(2000\pi t)} \end{aligned}$$

its frequency is 1 kHz and its peak value is 0.2 volts

5.2 Part(b)

The general form of the above FM signal is

$$s(t) = A_c \cos \left(\omega_c t + k_f \int_0^t m(x) dx \right)$$

Where k_f is the frequency deviation constant in radians per volt-second

Hence

$$k_f \int_0^t m(x) dx = 20 \cos \omega_1 t$$

Solve for $m(t)$ in the above, given that $k_f = 10^6$ radians per volt-second, hence

$$\begin{aligned} k_f \int_0^t m(x) dx &= 20 \cos \omega_1 t \\ \int_0^t m(x) dx &= \frac{20 \cos(2000\pi t)}{10^6} \end{aligned}$$

Take derivative of both sides, we obtain

$$\begin{aligned} m(t) &= \frac{20}{10^6} [-\sin(2000\pi t) \times 2000\pi] \\ &= -\frac{20 \times 2000\pi}{10^6} \sin(2000\pi t) \\ &= \boxed{-0.12566 \sin(2000\pi t)} \end{aligned}$$

Hence its peak value is 0.12566 and its frequency is 1 kHz

5.3 Part(c)

$$\begin{aligned} P_{av} &= \frac{\langle s^2(t) \rangle}{50} \\ &= \frac{\frac{1}{2} A_c^2}{50} \\ &= \frac{500^2}{100} \\ &= \boxed{2500 \text{ watt}} \end{aligned}$$

PEP is average power obtained if the complex envelope is held constant at its maximum values. i.e. (the normalized PEP) is

$$PEP = \frac{1}{2} [\max(|\tilde{s}(t)|)]^2$$

Since

$$\begin{aligned} s(t) &= A_c \cos(\omega_c t + 20 \cos \omega_1 t) \\ &= A_c [\cos \omega_c t \cos(20 \cos \omega_1 t) - \sin \omega_c t \sin(20 \cos \omega_1 t)] \\ &= \underbrace{A_c \cos(20 \cos \omega_1 t)}_{s_I(t)} \cos \omega_c t - \underbrace{A_c \sin(20 \cos \omega_1 t)}_{s_Q(t)} \sin \omega_c t \end{aligned}$$

Hence

$$\begin{aligned} \tilde{s}(t) &= s_I(t) + js_Q(t) \\ &= A_c \cos(20 \cos \omega_1 t) + jA_c \sin(20 \cos \omega_1 t) \end{aligned}$$

Then

$$\begin{aligned} |\tilde{s}(t)| &= \sqrt{[A_c \cos(20 \cos \omega_1 t)]^2 + [A_c \sin(20 \cos \omega_1 t)]^2} \\ &= A_c \sqrt{\cos^2(20 \cos \omega_1 t) + \sin^2(20 \cos \omega_1 t)} \\ &= A_c \end{aligned}$$

Hence the non-normalized PEP is

$$\begin{aligned} PEP &= \frac{\frac{1}{2} [A_c]^2}{50} \\ &= \frac{500^2}{100} \\ &= \boxed{2500 \text{ watt}} \end{aligned}$$

ps. is there an easier or more direct way to find PEP than what I did? (assuming it is correct)

It's good!

* in angle Mod, $a(t) = A_c$ * $\therefore PEP = \frac{(A_c)^2}{2R}$