CHAPTER 2

Representation of Signals and Systems

Problem 2.1

(a) The half-cosine pulse g(t) of Fig. P2.1(a) may be considered as the product of the rectangular function rect(t/T) and the sinusoidal wave A cos(πt/T). Since

$$\mathrm{rect}(\frac{t}{T}) \rightleftharpoons T\,\mathrm{sinc}(fT)$$

$$A\cos(\frac{\pi t}{T}) \rightleftharpoons \frac{A}{2}[\delta(f-\frac{1}{2T}) + \delta(f+\frac{1}{2T})]$$

and multiplication in the time domain is transformed into convolution in the frequency domain, it follows that

$$G(f) = [T\,\mathrm{sinc}(fT)] \star \{\frac{A}{2}[\delta(f-\frac{1}{2T}) + \delta(f+\frac{1}{2T})]\}$$

where $\star$ denotes convolution. Therefore, noting that

$$\mathrm{sinc}(fT) \star \delta(f-\frac{1}{2T}) = \mathrm{sinc}[T(f-\frac{1}{2T})]$$

$$\mathrm{sinc}(fT) \star \delta(f+\frac{1}{2T}) = \mathrm{sinc}[T(f+\frac{1}{2T})]$$

we obtain the desired result

$$G(f) = \frac{AT}{2}[\mathrm{sinc}(fT-\frac{1}{2}) + \mathrm{sinc}(fT+\frac{1}{2})]$$

(b) The half-sine pulse of Fig. P2.1(b) may be obtained by shifting the half-cosine pulse to the right by T/2 seconds. Since a time shift of T/2 seconds is equivalent to multiplication by exp(-jπfT) in the frequency domain, it follows that the Fourier transform of the half-sine pulse is

$$G(f) = \frac{AT}{2}[\mathrm{sinc}(fT-\frac{1}{2}) + \mathrm{sinc}(fT+\frac{1}{2})]\exp(-j\pi fT)$$

(c) The Fourier transform of a half-sine pulse of duration aT is equal to

$$\frac{|a|AT}{2}[\mathrm{sinc}(afT-\frac{1}{2}) + \mathrm{sinc}(afT+\frac{1}{2})]\exp(-j\pi afT)$$

(d) The Fourier transform of the negative half-sine pulse shown in Fig. P2.1(c) is obtained from the result of part (c) by putting a= -1, and multiplying the result by -1, and so we find that its Fourier transform is equal to

$$= \frac{AT}{2}[\text{sinc}(fT+\frac{1}{2}) + \text{sinc}(fT-\frac{1}{2})]\exp(j\pi fT)$$

(e) The full-sine pulse of Fig. P2.1(d) may be considered as the superposition of the half-sine pulses shown in parts (b) and (c) of the figure. The Fourier transform of this pulse is therefore

$$G(f) = \frac{AT}{2}[\text{sinc}(fT-\frac{1}{2}) + \text{sinc}(fT+\frac{1}{2})][\exp(-j\pi fT)-\exp(j\pi fT)]$$

$$= -jAT[\text{sinc}(fT-\frac{1}{2}) + \text{sinc}(fT+\frac{1}{2})]\sin(\pi fT)$$

$$= -jAT\left[\frac{\sin(\pi fT-\frac{\pi}{2})}{\pi fT-\frac{\pi}{2}} + \frac{\sin(\pi fT+\frac{\pi}{2})}{\pi fT+\frac{\pi}{2}}\right]\sin(\pi fT)$$

$$= -jAT[-\frac{\cos(\pi fT)}{\pi fT-\frac{\pi}{2}} + \frac{\cos(\pi fT)}{\pi fT+\frac{\pi}{2}}]\sin(\pi fT)$$

$$= jAT[\frac{\sin(2\pi fT)}{2\pi fT-\pi} - \frac{\sin(2\pi fT)}{2\pi fT+\pi}]$$

$$= jAT[-\frac{\sin(2\pi fT-\pi)}{2\pi fT-\pi} + \frac{\sin(2\pi fT+\pi)}{2\pi fT+\pi}]$$

$$= jAT[\text{sinc}(2fT+1) - \text{sinc}(2fT-1)]$$

Problem 2.2

Consider next an exponentially damped sinusoidal wave defined by (see Fig. 1):

$$g(t) = \exp(-t)\sin(2\pi f_c t)u(t)$$

In this case, we note that

$$\sin(2\pi f_c t) = \frac{1}{2j}[\exp(j2\pi f_c t) - \exp(-j2\pi f_c t)]$$

Therefore, applying the frequency-shifting property to the Fourier transform pair we find that the Fourier transform of the damped sinusoidal wave of Fig. 1 is

$$G(f) = \frac{1}{2j}\left[\frac{1}{1+j2\pi(f-f_c)} - \frac{1}{1+j2\pi(f+f_c)}\right]$$

$$= \frac{2\pi f_c}{(1+j2\pi f)^2 + (2\pi f_c)^2}$$

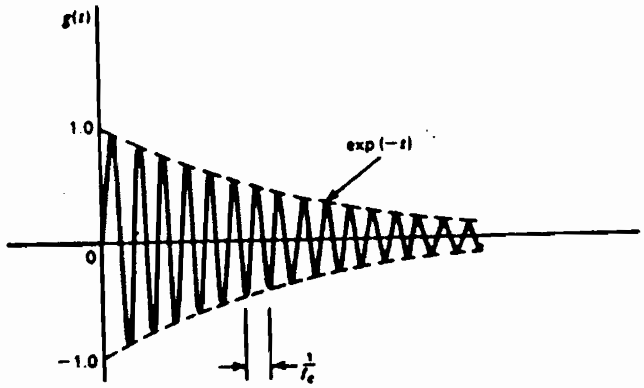


Figure 1 Damped sinusoidal wave.

Problem 2.3

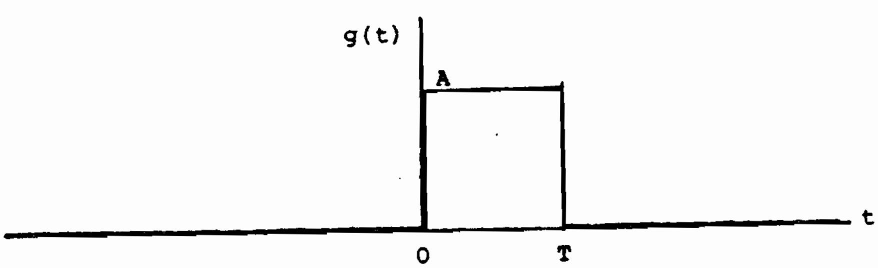
(a) The even part $g_e(t)$ of a pulse $g(t)$ is given by

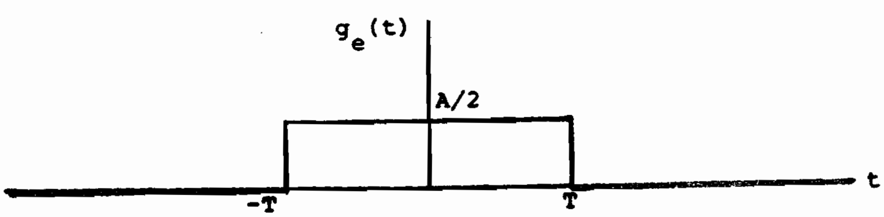
$$g_e(t) = \frac{1}{2}[g(t) + g(-t)]$$

Therefore, for $g(t) = A\ \mathrm{rect}(\frac{t}{T} - \frac{1}{2})$, we obtain

$$g_e(t) = \frac{A}{2}[\mathrm{rect}(\frac{t}{T} - \frac{1}{2}) + \mathrm{rect}(-\frac{t}{T} - \frac{1}{2})]$$
$$= \frac{A}{2}[\mathrm{rect}(\frac{t}{2T})]$$

which is shown illustrated below:

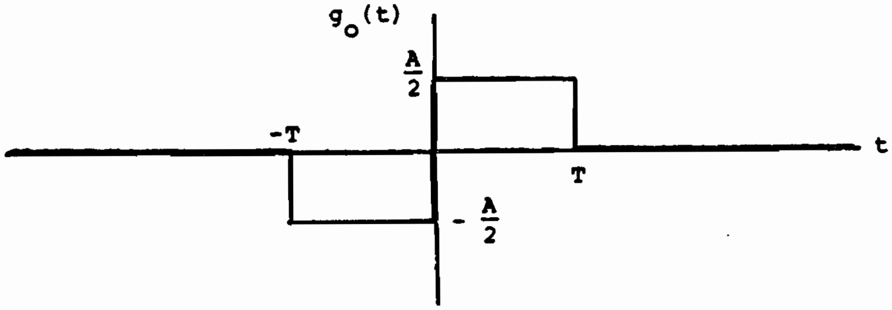




ne odd part of g(t) is defined by

$$g_o(t) = \frac{1}{2}[g(t) - g(-t)]$$
$$= \frac{A}{2}[\mathrm{rect}(\frac{t}{T} - \frac{1}{2}) - \mathrm{rect}(-\frac{t}{T} - \frac{1}{2})]$$

which is illustrated below:



(b) The Fourier transform of the even part is

$$G_e(f) = AT\,\mathrm{sinc}(2fT)$$

The Fourier transform of the odd part is

$$G_o(f) = \frac{AT}{2}\mathrm{sinc}(fT)\exp(-j\pi fT) - \frac{AT}{2}\mathrm{sinc}(fT)\exp(j\pi fT)$$
$$= \frac{AT}{j}\mathrm{sinc}(fT)\sin(\pi fT)$$

Problem 2.4

$$G(f) = \begin{cases} \exp(j\frac{\pi}{2}), & -W \le f \le 0 \\ \exp(-j\frac{\pi}{2}), & 0 \le f \le W \\ 0, & \text{otherwise} \end{cases}$$

Therefore, applying the formula for the inverse Fourier transform, we get

$$g(t) = \int_{-W}^{0} \exp(j\frac{\pi}{2})\exp(j2\pi ft)df + \int_{0}^{W} \exp(-j\frac{\pi}{2})\exp(j2\pi ft)dt$$

Replacing f with -f in the first integral and then interchanging the limits of integration:

$$g(t) = \int_0^W \exp(-j2\pi ft + j\frac{\pi}{2}) + \exp(j2\pi ft - j\frac{\pi}{2})]df$$

$$= 2\int_0^W \cos(2\pi ft - \frac{\pi}{2})df$$

$$= 2\int_0^W \sin(2\pi ft)df$$

$$= \left[-\frac{\cos(2\pi ft)}{\pi t}\right]_0^W$$

$$= \frac{1}{\pi t}[1-\cos(2\pi Wt)]$$

$$= \frac{2}{\pi t}\sin^2(\pi Wt)$$