Ten Hard Integrals

Nasser M. Abbasi

April 24, 2020

Contents

INTRODUCTION

These are first 10 integrals from Kevin Charlwood's 2008 article "Integration on Computer Algebra Systems" solved using different CAS systems.

The original post on this topic is sci.math.symbolic by Martin

These are the CAS systems used

1. Maple 2020 on windows 10 (64 bit)

2. Mathematica 12.1 on windows 10

3. Rubi 4.16.1

4. Maxima 5.43.2 on Linux

5. FriCAS 1.3.6 on Linux

6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.

7. mupad engine in Matlab 2016a symbolic toolbox

8. Giac/Xcas 1.5.0.87-1 on Linux.

Summary of result

The following is summary of results for each integral. Result with a () around it means the antiderivative contains either a complex number on non-elementary function.

system	1	2	3	4	5	6	7	8	9	10	score
FriCAS	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	100%
Maple	✓	✓	✓	✓	(✓)	✓	✓	✓	✓	✓	100%
Mathematica	✓	✓	✓	✓	(✓)	✓	✓	(✓)	(✓)	✓	100%
Rubi	✓	✓	✗	✓	(✓)	✓	✓	✓	✓	✓	90%
xCas	✓	✓	✗	✓	✗	✓	✓	(✓)	✓	✓	80%
Maxima	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
Sympy	✓	✓	✗	✗	✗	✗	✗	✗	✗	✓	30%
mupad 2016a	✗	✗	✗	✓	✗	✗	✗	✗	✓	✗	20%

Downloads, references and links

1. copy of Kevin Charlwood's 2008 paper in PDF

2. `CharlwoodIntegrationProblems.pdf` PDF file showing the 50 integrals and the best antiderivatives expected to each. By Albert Rich.

INTEGRALS

2.1 $\int \arcsin(x) \ln(x)\, dx$

2.1.1   Mathematica

```
Clear[x]
Integrate[ArcSin[x] Log[x], x]
```

$$-2\sqrt{1 - x^2} + \left(\sqrt{1 - x^2} - 1\right)\log(x) + \log\left(\sqrt{1 - x^2} + 1\right) + x(\log(x) - 1)\sin^{-1}(x)$$

2.1.2   Rubi

```
<< Rubi`
Clear[x]
Int[ArcSin[x] Log[x], x]
```

$$-2\sqrt{1-x^2} + \sqrt{1-x^2}\log(x) + \tanh^{-1}\left(\sqrt{1-x^2}\right) - x\sin^{-1}(x) + x\log(x)\sin^{-1}(x)$$

2.1.3   Maple

```
restart;
int(arcsin(x)*log(x),x);
```

$$2\,\frac{1}{1 + (\tan(1/2\,\arcsin(x)))^2}\left(\arcsin(x)\tan(1/2\,\arcsin(x))\ln\left(2\,\frac{\tan(1/2\,\arcsin(x))}{1 + (\tan(1/2\,\arcsin(x)))^2}\right) - (\tan(1/2)\right.$$
$$\left. - \ln\left(1 + \left(\tan\left(\frac{\arcsin(x)}{2}\right)\right)^2\right)\right)$$

2.1.4   Fricas

```
setSimplifyDenomsFlag(true)
ii:=integrate(asin(x)*log(x),x);
latex(ii)
```

$$\frac{\log\left(\sqrt{-x^2+1}+1\right) - \log\left(\sqrt{-x^2+1}-1\right) + (2\log(x)-4)\sqrt{-x^2+1} + 2\,x\arcsin(x)\log(x) - 2\,x\,\mathrm{arc}}{2}$$

2.1.5   Maxima

```
ii : integrate(asin(x)*log(x),x);
tex(ii);
```

$$\log\left(\frac{2\sqrt{1-x^2}}{|x|} + \frac{2}{|x|}\right) + \arcsin x\,(x\log x - x) + \sqrt{1-x^2}\log x - 2\sqrt{1-x^2}$$

2.1.6   XCAS

```
ii := integrate(asin(x)*log(x),x);
latex(ii);
```

$$\sqrt{-x^2+1}\ln x + \frac{2\ln\left(\sqrt{-x^2+1}+1\right)}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2} - \frac{2\ln|x|}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2}$$

$$+\, x\arcsin x \cdot \ln x - \frac{4x\arcsin x}{\left(\sqrt{-x^2+1}+1\right)\left(2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2\right)}$$

$$+\,\frac{4x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2} + \frac{2x^2\ln\left(\sqrt{-x^2+1}+1\right)\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2}$$

$$-\,\frac{2x^2\ln|x|\cdot\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2} - \frac{4}{2x^2\left(\frac{1}{\sqrt{-x^2+1}+1}\right)^2+2}$$

2.1.7   Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> from sympy import *
>>> x = symbols('x')
>>> ii = integrate(asin(x)*log(x),x)
>>> latex(ii)
```

$$x\log(x)\operatorname{asin}(x) - x\operatorname{asin}(x) + \sqrt{1-x^2}\log(x) - \sqrt{1-x^2}$$

$$-\begin{cases} -\dfrac{x}{\sqrt{-1+\frac{1}{x^2}}} - \operatorname{acosh}\left(\frac{1}{x}\right) + \dfrac{1}{x\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\[4mm] \dfrac{ix}{\sqrt{1-\frac{1}{x^2}}} + i\operatorname{asin}\left(\frac{1}{x}\right) - \dfrac{i}{x\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

2.1.8 MuPad

```
evalin(symengine,'int(asin(x)*log(x),x)')
```

<div align="center">did not solve</div>

2.2 $\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\, dx$

2.2.1 Mathematica

```
Clear[x]
Integrate[(x ArcSin[x])/Sqrt[1 - x^2], x]
```

$$x - \sqrt{1 - x^2}\sin^{-1}(x)$$

2.2.2 Rubi

```
<< Rubi`
Clear[x]
Int[(x ArcSin[x])/Sqrt[1 - x^2], x]
```

$$x - \sqrt{1 - x^2}\sin^{-1}(x)$$

2.2.3 Maple

```
restart;
integrand:=x*arcsin(x)/sqrt(1-x^2);
int(integrand,x);
latex(%)
```

$$x - \arcsin(x)\sqrt{-x^2 + 1}$$

2.2.4 Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
ii:=integrate(x*asin(x)/sqrt(1-x^2),x)
latex(ii)
```

$$-\arcsin\left(x\right)\sqrt{-x^2+1}+x$$

2.2.5 Maxima

```
ii : integrate(x*asin(x)/sqrt(1-x^2),x);
tex(ii);
```

$$x-\sqrt{1-x^2}\,\arcsin x$$

2.2.6 XCAS

```
ii := integrate(x*asin(x)/sqrt(1-x^2),x);
latex(ii)
```

$$-\sqrt{-x^2+1}\arcsin x+x$$

2.2.7 Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
ii=integrate(x*asin(x)/sqrt(1-x**2),x)
latex(ii)
```

$$x-\sqrt{1-x^2}\operatorname{asin}\left(x\right)$$

2.2.8   MuPad

```
evalin(symengine,'int(x*asin(x)/sqrt(1-x^2),x)')
```

did not solve

2.3 $\int -\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)\,dx$

2.3.1   Mathematica

```
ClearAll[x]
integrand = ArcSin[Sqrt[x + 1] - Sqrt[x]];
res = Integrate[integrand, x];
TeXForm[res]
```

$$-\frac{(x+1)\left(2x-2\sqrt{x+1}\sqrt{x}+1\right)^2\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}(-2x+2\sqrt{x+1}\sqrt{x}-3)+3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}}\right)}{8\sqrt{2}\left(\sqrt{x+1}-\sqrt{x}\right)^3\left(x-\sqrt{x+1}\sqrt{x}+1\right)}$$
$$-x\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$$

2.3.2   Rubi

```
<< Rubi`
ClearAll[x]
integrand = ArcSin[Sqrt[x + 1] - Sqrt[x]];
res = Int[integrand, x];
TeXForm[res]
```

$$\frac{\text{Subst}\left(\text{Int}\left(\sqrt{-x^2+\sqrt{x^2-1}x+1},x\right),x,\sqrt{x+1}\right)}{\sqrt{2}}-x\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$$

2.3.3 Maple

```
restart;
integrand:=arcsin(sqrt(x+1)-sqrt(x));
res:=int(integrand,x);
latex(res)
```

$$-\frac{1}{16}\left(\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\left(\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)\right)^{8}+2\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\left(\tan\right.$$

2.3.4 Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
ii:=integrate(asin(sqrt(x+1)-sqrt(x)),x)
latex(ii)
```

$$\frac{\left(3\sqrt{x+1}+\sqrt{x}\right)\sqrt{2\sqrt{x}\sqrt{x+1}-2\,x}+(8\,x+3)\arcsin\left(\sqrt{x+1}-\sqrt{x}\right)}{8}$$

2.3.5 Maxima

```
ii : integrate(asin(sqrt(x+1)-sqrt(x)),x);
tex(ii);
```

This reslult is wrong. bug.

$$\frac{\pi\,x}{2}$$

2.3.6   XCAS

```
ii :=   integrate(asin(sqrt(x+1)-sqrt(x)),x);

Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [92.1
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [53.1
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [5.38
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [6.79
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [84.3
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [77.6
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [72.5
Warning, need to choose a branch for the root of a polynomial with parameters. This might be
The choice was done assuming [t_nostep]=[0]
Warning, need to choose a branch for the root of a polynomial with parameters. This might be
The choice was done assuming [t_nostep]=[0]
Algebraic extensions not allowed in a rootof
Algebraic extensions not allowed in a rootof
Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters values [69.9

latex(ii)
```

did not solve

2.3.7   Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
ii=integrate(asin(sqrt(x+1)-sqrt(x)),x);
latex(ii)
```

did not solve

2.3.8 MuPad

```
evalin(symengine,'int(asin(sqrt(x+1)-sqrt(x)),x)')
```

did not solve

2.4 $\int \log\left(\sqrt{x^2+1}\,x+1\right)\,dx$

2.4.1 Mathematica

```
ClearAll[x]
integrand = Log[1 + x Sqrt[1 + x^2]];
res = Integrate[integrand, x];
TeXForm[res]
```

$$
x \log\left(\sqrt{x^2+1}\,x+1\right) - \frac{\sqrt{2\left(\sqrt{5}-1\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)}{1-\sqrt{5}}
$$

$$
-\sqrt{\frac{2}{1+\sqrt{5}}}\,\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right) - 2x
$$

$$
+\frac{\left(5+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} - \frac{\left(\sqrt{5}-5\right)\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}
$$

2.4.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Log[1 + x Sqrt[1 + x^2]]
res = Int[integrand, x];
TeXForm[res]
```

$$x \log \left(\sqrt{x^2 + 1}x + 1 \right) + \sqrt{\frac{2}{5} \left(\sqrt{5} - 1 \right)} \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}} \sqrt{x^2 + 1} \right)$$

$$+ \sqrt{\frac{2}{5 \left(\sqrt{5} - 1 \right)}} \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}} \sqrt{x^2 + 1} \right)$$

$$- \sqrt{\frac{2}{5} \left(1 + \sqrt{5} \right)} \tanh^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}} \sqrt{x^2 + 1} \right)$$

$$+ \sqrt{\frac{2}{5 \left(1 + \sqrt{5} \right)}} \tanh^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}} \sqrt{x^2 + 1} \right) - 2x$$

$$+ 2\sqrt{\frac{1}{5} \left(2 + \sqrt{5} \right)} \tan^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}}x \right) - \sqrt{\frac{1}{10} \left(1 + \sqrt{5} \right)} \tan^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}}x \right)$$

$$+ \sqrt{\frac{1}{10} \left(\sqrt{5} - 1 \right)} \tanh^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}}x \right)$$

$$+ 2\sqrt{\frac{1}{5} \left(\sqrt{5} - 2 \right)} \tanh^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}}x \right)$$

2.4.3 Maple

```
restart;
integrand:=ln(1 + x*sqrt(1 + x^2));
res:=int(integrand,x);
latex(res)
```

$$\ln\left(1 + x\sqrt{x^2+1}\right)x + \frac{1}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\,\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

$$+ \frac{\sqrt{5}}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\,\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

$$- \frac{1}{\sqrt{-2+2\sqrt{5}}}\operatorname{arctanh}\left(2\,\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)$$

$$+ \frac{\sqrt{5}}{\sqrt{-2+2\sqrt{5}}}\operatorname{arctanh}\left(2\,\frac{x}{\sqrt{-2+2\sqrt{5}}}\right) - 2x$$

$$- \frac{3\sqrt{5}}{10\sqrt{2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{1}{2\sqrt{2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{3\sqrt{5}}{10\sqrt{\sqrt{5}-2}}\operatorname{arctanh}\left(\frac{1}{\sqrt{\sqrt{5}-2}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$+ \frac{1}{2\sqrt{\sqrt{5}-2}}\operatorname{arctanh}\left(\frac{1}{\sqrt{\sqrt{5}-2}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{1}{2\sqrt{2+\sqrt{5}}}\operatorname{arctanh}\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{\sqrt{5}}{2\sqrt{2+\sqrt{5}}}\operatorname{arctanh}\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$+ \frac{1}{2\sqrt{\sqrt{5}-2}}\arctan\left(\frac{1}{\sqrt{\sqrt{5}-2}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{\sqrt{5}}{2\sqrt{\sqrt{5}-2}}\arctan\left(\frac{1}{\sqrt{\sqrt{5}-2}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$+ \frac{2\sqrt{2+\sqrt{5}}\sqrt{5}}{5}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)$$

$$- \frac{2\sqrt{\sqrt{5}-2}\sqrt{5}}{5}\operatorname{arctanh}\left(\frac{1}{\sqrt{\sqrt{5}-2}}\left(\sqrt{x^2+1}-x\right)\right)$$

2.4.4 Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand := log(1 + x*sqrt(1 + x^2));
ii:=integrate(integrand,x);
latex(ii)
```

$$-4 \sqrt{2} \sqrt{\sqrt{5}+1} \arctan \left(\frac{\left(\sqrt{2} \sqrt{x^2+1}+x \sqrt{2}\right) \sqrt{\sqrt{5}+1} \sqrt{\left(-16 x \sqrt{5}-32 x^3-16 x\right) \sqrt{x^2+1}+(16 x^2+8) \sqrt{5}+32 x^4+32 x^2+8-}}{8} \right.$$

2.4.5 Maxima

```
integrand : log(1 + x*sqrt(1 + x^2));
ii : integrate(integrand,x);
latex(ii)
```

did not solve

2.4.6 XCAS

```
integrand := log(1 + x*sqrt(1 + x^2));
ii := integrate(integrand,x);
latex(ii)
```

$$-\frac{1}{4}\sqrt{2\left(\sqrt{5}-1\right)}\ln\left|x-\sqrt{\frac{-1+\sqrt{5}}{2}}\right|+\frac{1}{4}\sqrt{2\left(\sqrt{5}-1\right)}\ln\left|x+\sqrt{\frac{-1+\sqrt{5}}{2}}\right|$$

$$+\frac{1}{2}\sqrt{2\left(\sqrt{5}+1\right)}\arctan\left(\frac{x}{\sqrt{-\frac{-1-\sqrt{5}}{2}}}\right)-2x$$

$$-2\left(-\frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\ln\left|\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}-\sqrt{\frac{4+4\sqrt{5}}{2}}\right|\right.$$

$$+\frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\ln\left(\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}+\sqrt{\frac{4+4\sqrt{5}}{2}}\right)$$

$$\left.-\frac{1}{4}\sqrt{2\left(\sqrt{5}+1\right)}\arctan\left(\frac{\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}}{\sqrt{-\frac{4-4\sqrt{5}}{2}}}\right)\right)+x\ln\left(1+x\sqrt{1+x^2}\right)$$

2.4.7   Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = log(1 + x*sqrt(1 + x**2));
ii = integrate(integrand,x);
latex(ii)
```

did not solve

2.4.8   MuPad

```
evalin(symengine,'int(log(1 + x*sqrt(1 + x^2)),x)')
```

$$\ln\left(1 + x\sqrt{x^2 + 1}\right)x - 2x$$

$$+ \frac{\frac{\sqrt{5}}{2} - \frac{5}{2}}{2\sqrt{1/2\sqrt{5} - 1/2} + 4\left(1/2\sqrt{5} - 1/2\right)^{3/2}} \ln\left(x - \frac{\sqrt{2}\sqrt{\sqrt{5} - 1}}{2}\right)$$

$$- \frac{\frac{\sqrt{5}}{2} - \frac{5}{2}}{2\sqrt{1/2\sqrt{5} - 1/2} + 4\left(1/2\sqrt{5} - 1/2\right)^{3/2}} \ln\left(x + \frac{\sqrt{2}\sqrt{\sqrt{5} - 1}}{2}\right)$$

$$- \frac{\frac{\sqrt{5}}{2} + \frac{5}{2}}{2\sqrt{-1/2\sqrt{5} - 1/2} + 4\left(-1/2\sqrt{5} - 1/2\right)^{3/2}} \ln\left(x - \frac{\sqrt{2}\sqrt{-\sqrt{5} - 1}}{2}\right)$$

$$+ \frac{\frac{\sqrt{5}}{2} + \frac{5}{2}}{2\sqrt{-1/2\sqrt{5} - 1/2} + 4\left(-1/2\sqrt{5} - 1/2\right)^{3/2}} \ln\left(x + \frac{\sqrt{2}\sqrt{-\sqrt{5} - 1}}{2}\right)$$

$$+ \frac{\sqrt{\frac{\sqrt{5}}{2} - \frac{1}{2}} + 2\left(1/2\sqrt{5} - 1/2\right)^{3/2}}{\left(2\sqrt{1/2\sqrt{5} - 1/2} + 4\left(1/2\sqrt{5} - 1/2\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2} + \frac{1}{2}}}\left(\ln\left(x - \frac{\sqrt{2}\sqrt{\sqrt{5} - 1}}{2}\right) - \ln\left(\frac{\sqrt{2}x\sqrt{\sqrt{5} - 1}}{2} + \right.\right.$$

$$+ \frac{\sqrt{\frac{\sqrt{5}}{2} - \frac{1}{2}} + 2\left(1/2\sqrt{5} - 1/2\right)^{3/2}}{\left(2\sqrt{1/2\sqrt{5} - 1/2} + 4\left(1/2\sqrt{5} - 1/2\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2} + \frac{1}{2}}}\left(\ln\left(x + \frac{\sqrt{2}\sqrt{\sqrt{5} - 1}}{2}\right) - \ln\left(\frac{\sqrt{2}\sqrt{\sqrt{5} + 1}}{2}\sqrt{x}\right.\right.$$

$$- \frac{\sqrt{-\frac{\sqrt{5}}{2} - \frac{1}{2}} + 2\left(-1/2\sqrt{5} - 1/2\right)^{3/2}}{\left(2\sqrt{-1/2\sqrt{5} - 1/2} + 4\left(-1/2\sqrt{5} - 1/2\right)^{3/2}\right)\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{2}}}\left(\ln\left(\frac{\sqrt{2}\sqrt{-\sqrt{5} + 1}}{2}\sqrt{x^2 + 1} - \frac{\sqrt{2}x\sqrt{-\sqrt{5}}}{2}\right.\right.$$

$$- \frac{\sqrt{-\frac{\sqrt{5}}{2} - \frac{1}{2}} + 2\left(-1/2\sqrt{5} - 1/2\right)^{3/2}}{\left(2\sqrt{-1/2\sqrt{5} - 1/2} + 4\left(-1/2\sqrt{5} - 1/2\right)^{3/2}\right)\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{2}}}\left(\ln\left(\frac{\sqrt{2}x\sqrt{-\sqrt{5} - 1}}{2} + \frac{\sqrt{2}\sqrt{-\sqrt{5} + 1}}{2}\sqrt{x^2}\right.\right.$$

2.5 $\int \frac{\cos^2(x)}{\sqrt{\cos^4(x) + \cos^2(x) + 1}}\, dx$

2.5.1 Mathematica

```
ClearAll[x]
integrand = Cos[x]^2/Sqrt[Cos[x]^4 + Cos[x]^2 + 1];
res = Integrate[integrand, x];
TeXForm[res]
```

$$
-\frac{2i\cos^2(x)\sqrt{1-\frac{2i\tan^2(x)}{\sqrt{3}-3i}}\sqrt{1+\frac{2i\tan^2(x)}{\sqrt{3}+3i}}\,\mathrm{EllipticPi}\left(\frac{3}{2}+\frac{i\sqrt{3}}{2},i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{3}-3i}}\tan(x)\right),\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}-3i}}\sqrt{8\cos(2x)+\cos(4x)+15}}
$$

2.5.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Cos[x]^2/Sqrt[Cos[x]^4 + Cos[x]^2 + 1];
res = Int[integrand, x];
TeXForm[res]
```

$$
-\frac{\left(1+\sqrt{3}\right)\cos^2(x)\left(\tan^2(x)+\sqrt{3}\right)\sqrt{\frac{\tan^4(x)+3\tan^2(x)+3}{\left(\tan^2(x)+\sqrt{3}\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\tan(x)}{\sqrt[4]{3}}\right),\frac{1}{4}\left(2-\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}
$$

$$
+\frac{\left(2+\sqrt{3}\right)\cos^2(x)\left(\tan^2(x)+\sqrt{3}\right)\sqrt{\frac{\tan^4(x)+3\tan^2(x)+3}{\left(\tan^2(x)+\sqrt{3}\right)^2}}\,\mathrm{EllipticPi}\left(\frac{1}{6}\left(3-2\sqrt{3}\right),2\tan^{-1}\left(\frac{\tan(x)}{\sqrt[4]{3}}\right),\frac{1}{4}\left(2-\sqrt{}\right)\right)}{4\sqrt[4]{3}\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}
$$

$$
+\frac{\cos^2(x)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^4(x)+3\tan^2(x)+3}}\right)\sqrt{\tan^4(x)+3\tan^2(x)+3}}{2\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}
$$

2.5.3 Maple

```
restart;
integrand:= cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1);
res:=int(integrand,x);
latex(res)
```

$$
-2\,\frac{\sqrt{\left(\left(\cos\left(2x\right)\right)^2+4\cos\left(2x\right)+7\right)\left(\sin\left(2x\right)\right)^2\left(-3+i\sqrt{3}\right)\left(\cos\left(2x\right)+1\right)^2}}{\left(-1+i\sqrt{3}\right)\sqrt{\left(\cos\left(2x\right)-1\right)\left(\cos\left(2x\right)+1\right)\left(\cos\left(2x\right)+2+i\sqrt{3}\right)\left(i\sqrt{3}-\cos\left(2x\right)-2\right)}\sin\left(2x\right)\sqrt{\left(\mathrm{co}\right.}}
$$

2.5.4   Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand:= cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1);
res:=integrate(integrand,x);
latex(res)
```

$$\frac{\arctan\left(\frac{2\ \cos(x)^3\ \sin(x)\ \sqrt{\cos(x)^4+\cos(x)^2+1}}{2\ \cos(x)^6-1}\right)}{6}$$

2.5.5   Maxima

```
integrand : cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1);
res : integrate(integrand,x);
latex(res)
```

did not solve

2.5.6   XCAS

```
integrand := cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1);
res := integrate(integrand,x);
latex(res)
```

did not solve

2.5.7   Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand =  integrand = cos(x)**2/sqrt(cos(x)**4 + cos(x)**2 + 1);
```

```
res = integrate(integrand,x);
latex(res)
```

did not solve

2.5.8 MuPad

```
evalin(symengine,'int(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1),x)')
```

did not solve

2.6 $\int \tan(x)\sqrt{\tan^4(x)+1}\,dx$

2.6.1 Mathematica

```
ClearAll[x]
integrand = Tan[x] Sqrt[1 + Tan[x]^4];
res = Integrate[integrand, x];
TeXForm[res]
```

$$\frac{\sqrt{\tan^4(x)+1}\left(\sqrt{\cos(4x)+3}-2\sqrt{2}\cos^2(x)\sinh^{-1}(\cos(2x))-2\cos^2(x)\tanh^{-1}\left(\frac{2\sin^2(x)}{\sqrt{\cos(4x)+3}}\right)\right)}{2\sqrt{\cos(4x)+3}}$$

2.6.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Tan[x] Sqrt[1 + Tan[x]^4];
res = Int[integrand, x];
TeXForm[res]
```

$$\frac{1}{2}\sqrt{\tan^4(x)+1}-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{\tan^4(x)+1}}\right)}{\sqrt{2}}-\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right)$$

2.6.3 Maple

```
restart;
integrand := tan(x)*sqrt(1 + tan(x)^4);
res:=int(integrand,x);
latex(res)
```

$$\frac{1}{2}\sqrt{\left(1+(\tan(x))^2\right)^2 - 2\,(\tan(x))^2} - \frac{\operatorname{arcsinh}\left((\tan(x))^2\right)}{2}$$
$$- \frac{\sqrt{2}}{2}\operatorname{arctanh}\left(\frac{\left(-2\,(\tan(x))^2 + 2\right)\sqrt{2}}{4}\frac{1}{\sqrt{\left(1+(\tan(x))^2\right)^2 - 2\,(\tan(x))^2}}\right)$$

2.6.4 Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand := tan(x)*sqrt(1 + tan(x)^4);
res:=integrate(integrand,x);
latex(res)
```

$$\frac{2\log\left(\sqrt{\tan(x)^4+1} - \tan(x)^2\right) + \sqrt{2}\log\left(\frac{\left(2\,\sqrt{2}\,\tan(x)^2 - 2\,\sqrt{2}\right)\sqrt{\tan(x)^4+1} + 3\,\tan(x)^4 - 2\,\tan(x)^2 + 3}{\tan(x)^4 + 2\,\tan(x)^2 + 1}\right) + 2\,\sqrt{\tan}}{4}$$

2.6.5 Maxima

```
integrand : tan(x)*sqrt(1 + tan(x)^4);
res : integrate(integrand,x);
tex(res);
```

did not solve

2.6.6 XCAS

```
integrand := tan(x)*sqrt(1 + tan(x)^4);
res := integrate(integrand,x);
latex(res)
```

$$\frac{\sqrt{\tan^4 x + 1} + 2\left(\frac{\ln\left(\sqrt{\tan^4 x + 1} - \tan^2 x\right)}{2} + \frac{\ln\left(\frac{-2\left(\sqrt{\tan^4 x + 1} - \tan^2 x\right) + 2 + 2\sqrt{2}}{2\left(\sqrt{\tan^4 x + 1} - \tan^2 x\right) - 2 + 2\sqrt{2}}\right)}{\sqrt{2}}\right)}{2}$$

2.6.7 Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = tan(x)*sqrt(1 + tan(x)**4);
res = integrate(integrand,x);
latex(res)
```

did not solve

2.6.8 MuPad

```
evalin(symengine,'int(tan(x)*sqrt(1 + tan(x)^4),x)')
```

did not solve

2.7 $\int \dfrac{\tan(x)}{\sqrt{\sec^3(x)+1}}\, dx$

2.7.1 Mathematica

```
ClearAll[x]
integrand = Tan[x]/Sqrt[Sec[x]^3 + 1];
res = Integrate[integrand, x]
TeXForm[res]
```

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$$

2.7.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Tan[x]/Sqrt[Sec[x]^3 + 1];
res = Int[integrand, x]
TeXForm[res]
```

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$$

2.7.3 Maple

```
restart;
integrand := tan(x)/sqrt(sec(x)^3 + 1);
res:=int(integrand,x);
latex(res)
```

$$-\frac{2}{3}\operatorname{arctanh}\left(\sqrt{(\sec(x))^3+1}\right)$$

2.7.4   Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand := tan(x)/sqrt(sec(x)^3 + 1);
res:=integrate(integrand,x);
latex(res)
```

$$\frac{\log\left(2\,\cos\left(x\right)^3\,\sqrt{\frac{\cos(x)^3+1}{\cos(x)^3}}-2\,\cos\left(x\right)^3-1\right)}{3}$$

2.7.5   Maxima

```
integrand : tan(x)/sqrt(sec(x)^3 + 1);
res : integrate(integrand,x);
tex(res)
```

$$\frac{\log\left(\sqrt{\frac{1}{\cos^3 x}+1}-1\right)}{3}-\frac{\log\left(\sqrt{\frac{1}{\cos^3 x}+1}+1\right)}{3}$$

2.7.6   XCAS

```
integrand := tan(x)/sqrt(sec(x)^3 + 1);
res := integrate(integrand,x);
latex(res)
```

$$2\left(-\frac{\ln\left(\sqrt{\left(\frac{1}{\cos x}\right)^3+1}+1\right)}{6}+\frac{\ln\left|\sqrt{\left(\frac{1}{\cos x}\right)^3+1}-1\right|}{6}\right)$$

2.7.7 Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = tan(x)/sqrt(sec(x)**3 + 1);
res = integrate(integrand,x);
latex(res)
```

did not solve

2.7.8 MuPad

```
evalin(symengine,'int(tan(x)/sqrt(sec(x)^3 + 1),x)')
```

did not solve

2.8 $\int \sqrt{\tan^2(x) + 2\tan(x) + 2}\,dx$

2.8.1 Mathematica

```
ClearAll[x];
integrand = Sqrt[Tan[x]^2 + 2 Tan[x] + 2];
res = Integrate[integrand, x];
TeXForm[res]
```

$$\sinh^{-1}(\tan(x) + 1) + \frac{1}{2}i\left(\sqrt{1+2i}\,\tanh^{-1}\left(\frac{(1+i)\tan(x) + (2+i)}{\sqrt{1+2i}\,\sqrt{\tan^2(x) + 2\tan(x) + 2}}\right)\right.$$
$$\left. - \sqrt{1-2i}\,\tanh^{-1}\left(\frac{(2-2i)\tan(x) + (4-2i)}{2\sqrt{1-2i}\,\sqrt{\tan^2(x) + 2\tan(x) + 2}}\right)\right)$$

2.8.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Sqrt[Tan[x]^2 + 2 Tan[x] + 2];
res = Int[integrand, x];
TeXForm[res]
```

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)$$

$$-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)\tan(x)+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)$$

$$+\sinh^{-1}(\tan(x)+1)$$

2.8.3 Maple

```
restart;
integrand := sqrt(tan(x)^2 + 2*tan(x) + 2);
res:=simplify(int(integrand,x));
latex(res)
```

$$-10\,\frac{1}{\sqrt{-10+10\sqrt{5}}\cos\left(x\right)\left(\sqrt{5}-5\right)}\left(-\sqrt{2}\sqrt{\frac{2\sin\left(x\right)\cos\left(x\right)+\left(\cos\left(x\right)\right)^2+1}{\cos\left(x\right)\left(2\sin\left(x\right)+\cos\left(x\right)\right)\sqrt{5}+\left(\cos\left(x\right)\right)^2+2\sin\left(x\right)\cos}}\right.$$

2.8.4 Fricas

```
integrand := sqrt(tan(x)^2 + 2*tan(x) + 2);
res:=integrate(integrand,x);
latex(res)
```

$$8 \sqrt[4]{5} \arctan\left(\cfrac{}{\left(\sqrt{5}-1\right) \sqrt{\frac{\sqrt{5}-5}{\sqrt{5}-3}} \sqrt{\left(\left(-375 \sqrt{5}+625\right) \sqrt[4]{5} \cos(x) \sin(x)+\left(125 \sqrt{5}-625\right) \sqrt[4]{5} \cos(x)^2\right) \sqrt{\frac{\sqrt{5}-5}{\sqrt{5}-3}} \sqrt{\frac{2 \cos(x) \sin(x)+\cos(x)^2+}{\cos(x)^2}}}} \right.$$

2.8.5 Maxima

```
integrand : sqrt(tan(x)^2 + 2*tan(x) + 2);
res : integrate(integrand,x);
```

did not solve

2.8.6 XCAS

```
integrand := sqrt(tan(x)^2 + 2*tan(x) + 2);
res := integrate(integrand,x);
latex(res)
```

$$2\left(-\frac{\ln\left(\sqrt{\tan^2 x + 2\tan x + 2} - \tan x - 1\right)}{2} + \frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\left(1\right.\right.$$

$$\left.-\frac{32i}{16\sqrt{5}-16}\right)\ln\left(-16\left(1+\frac{512i}{512\sqrt{5}-1024}\right)\sqrt{\sqrt{5}-2} + (-16+16i)\right.$$

$$+ (16+16i)\left(\sqrt{\tan^2 x + 2\tan x + 2} - \tan x\right)\right) - \frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\left(1\right.$$

$$\left.-\frac{32i}{16\sqrt{5}-16}\right)\ln\left(16\left(1+\frac{512i}{512\sqrt{5}-1024}\right)\sqrt{\sqrt{5}-2} + (-16+16i)\right.$$

$$+ (16+16i)\left(\sqrt{\tan^2 x + 2\tan x + 2} - \tan x\right)\right) + \frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\left(1\right.$$

$$\left.+\frac{32i}{16\sqrt{5}-16}\right)\ln\left(-16\left(1+\frac{512i}{512\sqrt{5}+1024}\right)\sqrt{\sqrt{5}+2} + (16-16i)\right.$$

$$+ (16+16i)\left(\sqrt{\tan^2 x + 2\tan x + 2} - \tan x\right)\right) - \frac{1}{8}\sqrt{2\left(\sqrt{5}-1\right)}\left(1\right.$$

$$\left.+\frac{32i}{16\sqrt{5}-16}\right)\ln\left(16\left(1+\frac{512i}{512\sqrt{5}+1024}\right)\sqrt{\sqrt{5}+2}\right.$$

$$\left.\left.\left.+ (16-16i) + (16+16i)\left(\sqrt{\tan^2 x + 2\tan x + 2} - \tan x\right)\right)\right)\right)$$

2.8.7 Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = sqrt(tan(x)**2 + 2*tan(x) + 2);
res = integrate(integrand,x);
latex(res)
```

did not solve

2.8.8 MuPad

```
evalin(symengine,'int(sqrt(tan(x)^2 + 2*tan(x) + 2),x)')
```

<div align="center">did not solve</div>

2.9 $\int \sin(x) \tan^{-1}\left(\sqrt{\sec(x)-1}\right)\, dx$

2.9.1 Mathematica

```
ClearAll[x];
integrand = Sin[x] ArcTan[Sqrt[Sec[x] - 1]];
res = Integrate[integrand, x]
TeXForm[res]
```

$$-\frac{1}{2}\left(-3-2\sqrt{2}\right)\left(\left(\sqrt{2}-2\right)\cos\left(\frac{x}{2}\right)-\sqrt{2}\right.$$

$$\left.+1\right)\cos^2\left(\frac{x}{4}\right)\sqrt{-\tan^2\left(\frac{x}{4}\right)-2\sqrt{2}+3}\sqrt{\left(2\sqrt{2}-3\right)\tan^2\left(\frac{x}{4}\right)+1}\cot\left(\frac{x}{4}\right)\sqrt{\sec(x)-1}\sec(x)\sqrt{\left(\left(10-\right.\right.}$$

$$\left.-12\sqrt{2}\right)+2\text{EllipticPi}\left(2\sqrt{2}-3,-\sin^{-1}\left(\frac{\tan\left(\frac{x}{4}\right)}{\sqrt{3-2\sqrt{2}}}\right),17-12\sqrt{2}\right)\right)$$

$$+\frac{1}{2}\cos(x)\sqrt{\sec(x)-1}-\cos(x)\tan^{-1}\left(\sqrt{\sec(x)-1}\right)$$

2.9.2 Rubi

```
<< Rubi`
ClearAll[x]
integrand = Sin[x] ArcTan[Sqrt[Sec[x] - 1]];
res = Int[integrand, x];
TeXForm[res]
```

$$\frac{1}{2}\cos(x)\sqrt{\sec(x)-1}+\frac{1}{2}\tan^{-1}\left(\sqrt{\sec(x)-1}\right)-\cos(x)\tan^{-1}\left(\sqrt{\sec(x)-1}\right)$$

2.9.3   Maple

```
restart;
integrand := sin(x)*arctan(sqrt(sec(x) - 1));
res:=int(integrand,x);
latex(res)
```

$$-\frac{1}{\sec(x)}\arctan\left(\sqrt{-\left(-1+\left(\sec(x)\right)^{-1}\right)\sec(x)}\right)$$
$$+\frac{1}{2\sec(x)}\sqrt{\sec(x)-1}+\frac{1}{2}\arctan\left(\sqrt{\sec(x)-1}\right)$$

2.9.4   Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand := sin(x)*atan(sqrt(sec(x) - 1));
res:=integrate(integrand,x);
latex(res)
```

$$\frac{\left(-2\cos(x)+1\right)\arctan\left(\sqrt{\sec(x)-1}\right)+\cos(x)\sqrt{\frac{-\cos(x)+1}{\cos(x)}}}{2}$$

2.9.5   Maxima

```
integrand : sin(x)*atan(sqrt(sec(x) - 1));
res : integrate(integrand,x);
tex(res);
```

$$-\cos x\arctan\left(\frac{\sqrt{1-\cos x}}{\sqrt{\cos x}}\right)+\frac{\arctan\left(\frac{\sqrt{1-\cos x}}{\sqrt{\cos x}}\right)}{2}+\frac{\sqrt{1-\cos x}}{\left(2-\frac{2\left(\cos x-1\right)}{\cos x}\right)\sqrt{\cos x}}$$

2.9.6 XCAS

```
integrand := sin(x)*atan(sqrt(sec(x) - 1));
res := integrate(integrand,x);

Warning, integration of abs or sign assumes constant sign by intervals (correct if the argum
Check [abs(cos(x))]
Discontinuities at zeroes of cos(x) were not checked

latex(res)
```

Unable to conver to latex. Here is the raw output

```
expr:=(1/2*asin(2*cos(x)-1)/(sign(cos(x))^2-1)+sign(cos(x))^2*
   atan(1/2*(sign(cos(x))^2+(-2*sqrt(-cos(x)^2+cos(x))+1)/(-2*cos(x)+1)
   +(-2*sqrt(-cos(x)^2+cos(x))+1)*sign(cos(x))^2/(-2*cos(x)+1)-1)/sign(cos(x)))/
   ((sign(cos(x))^2-1)*sign(cos(x))))*sign(cos(x))
   -cos(x)*atan(sqrt(-cos(x)^2+cos(x))*sign(cos(x))/cos(x));
```

2.9.7 Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = sin(x)*atan(sqrt(sec(x) - 1));
res = integrate(integrand,x);
latex(res)
```

did not solve

2.9.8   MuPad

```
evalin(symengine,'int(sin(x)*arctan(sqrt(sec(x) - 1)),x)')
```

$$-\arctan\left(\sqrt{\left(\cos\left(x\right)\right)^{-1}-1}\right)\cos\left(x\right)$$
$$-\frac{\cos\left(x\right)}{3}\left(\frac{3}{2}\arcsin\left(\sqrt{\cos\left(x\right)}\right)\left(\cos\left(x\right)\right)^{-\frac{3}{2}}-\frac{3}{2\cos\left(x\right)}\sqrt{1-\cos\left(x\right)}\right)\sqrt{1-\cos\left(x\right)}\frac{1}{\sqrt{\left(\cos\left(x\right)\right)^{-1}-1}}$$

2.10 $\int \frac{x^3 e^{\sin^{-1}(x)}}{\sqrt{1-x^2}}\,dx$

2.10.1   Mathematica

```
ClearAll[x];
integrand = (x^3 Exp[ArcSin[x]])/Sqrt[1 - x^2];
res = Integrate[integrand, x]
TeXForm[res]
```

$$-\frac{1}{40}e^{\sin^{-1}(x)}\left(15\left(\sqrt{1-x^2}-x\right)+\sin\left(3\sin^{-1}(x)\right)-3\cos\left(3\sin^{-1}(x)\right)\right)$$

2.10.2   Rubi

```
<< Rubi`
ClearAll[x]
integrand = (x^3 Exp[ArcSin[x]])/Sqrt[1 - x^2];
res = Int[integrand, x];
TeXForm[res]
```

$$\frac{1}{10}x^3 e^{\sin^{-1}(x)}-\frac{3}{10}\sqrt{1-x^2}x^2 e^{\sin^{-1}(x)}-\frac{3}{10}\sqrt{1-x^2}e^{\sin^{-1}(x)}+\frac{3}{10}xe^{\sin^{-1}(x)}$$

2.10.3 Maple

```
restart;
integrand := (x^3*exp(1)^arcsin(x))/sqrt(1 - x^2);
res:=int(integrand,x);
latex(res)
```

$$\frac{e^{\arcsin(x)}x^2}{10}\left(x - 3\sqrt{-x^2+1}\right) + \frac{3\,e^{\arcsin(x)}}{10}\left(x - \sqrt{-x^2+1}\right)$$

2.10.4 Fricas

```
set output tex off
setSimplifyDenomsFlag(true)
integrand := (x^3*exp(asin(x)))/sqrt(1 - x^2);
res:=integrate(integrand,x);
latex(res)
```

$$\frac{\left(\left(-3\,x^2 - 3\right)\sqrt{-x^2+1} + x^3 + 3\,x\right)e^{\arcsin(x)}}{10}$$

2.10.5 Maxima

```
integrand : (x^3*exp(asin(x)))/sqrt(1 - x^2);
res : integrate(integrand,x);
tex(res);
```

did not solve

2.10.6 XCAS

```
integrand := (x^3*exp(asin(x)))/sqrt(1 - x^2);
res := integrate(integrand,x);
latex(res)
```

$$\frac{2}{5}xe^{\arcsin x} - \frac{3}{5}\sqrt{-x^2+1}e^{\arcsin x} + \frac{3}{10}\sqrt{-x^2+1}\left(-x^2+1\right)e^{\arcsin x} - \frac{1}{10}x\left(-x^2+1\right)e^{\arcsin x}$$

2.10.7   Sympy

```
>python
Python 3.7.3 (default, Mar 27 2019, 22:11:17)
[GCC 7.3.0] :: Anaconda, Inc. on linux

from sympy import *
x = symbols('x')
integrand = (x**3*exp(asin(x)))/sqrt(1 - x**2);
res = integrate(integrand,x);
latex(res)
```

$$\frac{x^3 e^{\operatorname{asin}(x)}}{10} - \frac{3x^2\sqrt{1-x^2}e^{\operatorname{asin}(x)}}{10} + \frac{3x e^{\operatorname{asin}(x)}}{10} - \frac{3\sqrt{1-x^2}e^{\operatorname{asin}(x)}}{10}$$

2.10.8   MuPad

```
evalin(symengine,'int( (x^3*exp(arcsin(x)))/sqrt(1 - x^2),x)')
```

did not solve