

RUBI 4.1 results for Charlwood's integrals

(*requires Rubi package loaded*)

1

$\text{Int}[\text{ArcSin}[x] \text{Log}[x], x]$

$$-2\sqrt{1-x^2} - x \text{ArcSin}[x] + \text{ArcTanh}[\sqrt{1-x^2}] + \sqrt{1-x^2} \text{Log}[x] + x \text{ArcSin}[x] \text{Log}[x]$$

2

$\text{Int}\left[\frac{x \text{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$

$$x - \sqrt{1-x^2} \text{ArcSin}[x]$$

3

$\text{Int}[\text{ArcSin}[\sqrt{x+1} - \sqrt{x}], x]$

$$-x \text{ArcSin}[\sqrt{x} - \sqrt{1+x}] + \frac{\text{Subst}\left[\text{Int}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}, x\right], x, \sqrt{1+x}\right]}{\sqrt{2}}$$

4

$$\begin{aligned} & \text{Int}\left[\text{Log}\left[1+x\sqrt{1+x^2}\right], x\right] \\ & -2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \\ & 2\sqrt{\frac{1}{5}(2+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] + \\ & \sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] + 2\sqrt{\frac{1}{5}(-2+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \\ & \sqrt{\frac{1}{10}(-1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \sqrt{\frac{2}{5(1+\sqrt{5})}} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] - \\ & \sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] + x \text{Log}\left[1+x\sqrt{1+x^2}\right] \end{aligned}$$

5

$$\begin{aligned} & \text{Int}\left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}, x\right] \\ & \left(\text{EllipticPi}\left[\frac{1}{2}(3-i\sqrt{3}), \text{ArcSin}\left[\sqrt{-\frac{2i}{3i+\sqrt{3}}}\text{Tan}[x]\right], \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right] (3+3\text{Tan}[x]^2 + \text{Tan}[x]^4)\right) / \\ & \left(3\sqrt{-\frac{2i}{3i+\sqrt{3}}}(1+\text{Tan}[x]^2)\sqrt{1+\frac{2\text{Tan}[x]^2}{3-i\sqrt{3}}}\sqrt{1+\frac{2\text{Tan}[x]^2}{3+i\sqrt{3}}}\sqrt{\frac{3+3\text{Tan}[x]^2 + \text{Tan}[x]^4}{(1+\text{Tan}[x]^2)^2}}\right) \end{aligned}$$

6

$$\begin{aligned} & \text{Int}\left[\text{Tan}[x]\sqrt{1+\text{Tan}[x]^4}, x\right] \\ & -\frac{1}{2}\text{ArcSinh}\left[\text{Tan}[x]^2\right] - \frac{\text{ArcTanh}\left[\frac{1-\text{Tan}[x]^2}{\sqrt{2}\sqrt{1+\text{Tan}[x]^4}}\right]}{\sqrt{2}} + \frac{1}{2}\sqrt{1+\text{Tan}[x]^4} \end{aligned}$$

7

$$\text{Int}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3 + 1}}, x\right]$$

$$-\frac{2}{3} \text{ArcTanh}\left[\sqrt{1 + \text{Sec}[x]^3}\right]$$

8

$$\text{Int}\left[\sqrt{\text{Tan}[x]^2 + 2 \text{Tan}[x] + 2}, x\right]$$

$$\text{ArcSinh}[1 + \text{Tan}[x]] - \frac{1}{2} i \sqrt{1 - 2 i} \text{ArcTanh}\left[\frac{(4 - 2 i) + (2 - 2 i) \text{Tan}[x]}{2 \sqrt{1 - 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}}\right] +$$

$$\frac{1}{2} i \sqrt{1 + 2 i} \text{ArcTanh}\left[\frac{(4 + 2 i) + (2 + 2 i) \text{Tan}[x]}{2 \sqrt{1 + 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}}\right]$$

9

$$\text{Int}\left[\text{Sin}[x] \text{ArcTan}\left[\sqrt{\text{Sec}[x] - 1}\right], x\right]$$

$$\frac{1}{2} \text{ArcTan}\left[\sqrt{-1 + \text{Sec}[x]}\right] - \text{ArcTan}\left[\sqrt{-1 + \text{Sec}[x]}\right] \text{Cos}[x] + \frac{1}{2} \text{Cos}[x] \sqrt{-1 + \text{Sec}[x]}$$

10

$$\text{Int}\left[\frac{x^3 \text{Exp}[\text{ArcSin}[x]]}{\sqrt{1 - x^2}}, x\right]$$

$$\frac{3}{10} e^{\text{ArcSin}[x]} x + \frac{1}{10} e^{\text{ArcSin}[x]} x^3 - \frac{3}{10} e^{\text{ArcSin}[x]} \sqrt{1 - x^2} - \frac{3}{10} e^{\text{ArcSin}[x]} x^2 \sqrt{1 - x^2}$$

$$\text{Int}\left[\frac{\text{Sec}[x]}{\sqrt{1 + \text{Sin}[x]}}, x\right]$$

11

$$\text{Int}\left[(x * \text{Log}[1 + x^2] * \text{Log}[x + \text{Sqrt}[1 + x^2]]) / \text{Sqrt}[1 + x^2], x\right]$$

$$4 x - 2 \text{ArcTan}[x] - x \text{Log}[1 + x^2] - 2 \sqrt{1 + x^2} \text{Log}\left[x + \sqrt{1 + x^2}\right] + \sqrt{1 + x^2} \text{Log}[1 + x^2] \text{Log}\left[x + \sqrt{1 + x^2}\right]$$

12

$\text{Int}[\text{ArcTan}[x + \text{Sqrt}[1 - x^2]], x]$

$$\begin{aligned}
 & -\frac{\text{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{1 - 2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right]}{\sqrt{3}} + \\
 & \frac{1}{12}(3i - \sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right] + \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}{\sqrt{3}}\right]}{\sqrt{3}} - \\
 & \frac{1}{12}(3i + \sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}{\sqrt{1-x^2}}\right] + x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{8} \text{Log}[1 - x^2 + x^4]
 \end{aligned}$$

13

$\text{Int}[x * \text{ArcTan}[x + \text{Sqrt}[1 - x^2]] / \text{Sqrt}[1 - x^2], x]$

$$\begin{aligned}
 & -\frac{\text{ArcSin}[x]}{2} + \frac{1}{4}\sqrt{3} \text{ArcTan}\left[\frac{1 - 2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}\right]}{2\sqrt{3}} - \\
 & \frac{1}{12}(3i - \sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right] + \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}}{2\sqrt{3}}\right]}{2\sqrt{3}} + \\
 & \frac{1}{12}(3i + \sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}{\sqrt{1-x^2}}\right] - \sqrt{1-x^2} \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \frac{1}{8} \text{Log}[1 - x^2 + x^4]
 \end{aligned}$$

14

$\text{Int}[\text{ArcSin}[x] / (1 + \text{Sqrt}[1 - x^2]), x]$

$$-\frac{\text{ArcSin}[x]}{x} + \frac{\sqrt{1-x^2} \text{ArcSin}[x]}{x} + \frac{\text{ArcSin}[x]^2}{2} - \text{ArcTanh}\left[\sqrt{1-x^2}\right] - \text{Log}[x]$$

15

`Int[Log[x + Sqrt[1 + x^2]] / (1 - x^2)^(3/2), x]`

$$-\frac{1}{2} \operatorname{ArcSin}[x^2] + \frac{x \operatorname{Log}\left[x + \sqrt{1 + x^2}\right]}{\sqrt{1 - x^2}}$$

16

`Int[ArcSin[x] / (1 + x^2)^(3/2), x]`

$$\frac{x \operatorname{ArcSin}[x]}{\sqrt{1 + x^2}} - \frac{\operatorname{ArcSin}[x^2]}{2}$$

17

`Int[Log[x + Sqrt[x^2 - 1]] / (1 + x^2)^(3/2), x]`

$$-\frac{1}{2} \operatorname{ArcCosh}[x^2] + \frac{x \operatorname{Log}\left[x + \sqrt{-1 + x^2}\right]}{\sqrt{1 + x^2}}$$

18

`Int[Log[x] / (x^2 + Sqrt[x^2 - 1]), x]`

$$\frac{\sqrt{-1 + x^2}}{x} - \operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1 + x^2}}\right] + \frac{\sqrt{-1 + x^2} \operatorname{Log}[x]}{x}$$

19

`Int[Sqrt[1 + x^3] / x, x]`

$$\frac{2\sqrt{1 + x^3}}{3} - \frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1 + x^3}\right]$$

20

`Int[x * Log[x + Sqrt[x^2 - 1]] / Sqrt[x^2 - 1], x]`

$$-x + \sqrt{-1 + x^2} \operatorname{Log}\left[x + \sqrt{-1 + x^2}\right]$$