Mathematica 9.01 results for Charlwood's integrals

1

In[42]:= `Integrate[ArcSin[x] Log[x], x]`

Out[42]= $-2\sqrt{1-x^2} + x\,\mathrm{ArcSin}[x]\,(-1+\mathrm{Log}[x]) + \left(-1+\sqrt{1-x^2}\right)\mathrm{Log}[x] + \mathrm{Log}\left[1+\sqrt{1-x^2}\right]$

2

In[43]:= $\mathbf{Integrate}\left[\frac{x\,\mathrm{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$

Out[43]= $x - \sqrt{1-x^2}\,\mathrm{ArcSin}[x]$

3

In[44]:= $\mathbf{Integrate}\left[\mathrm{ArcSin}\left[\sqrt{x+1} - \sqrt{x}\right], x\right]$

Out[44]= $-x\,\mathrm{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right] - \left((1+x)\left(1+2x-2\sqrt{x}\sqrt{1+x}\right)^2\left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}}\left(-3-2x+2\sqrt{x}\sqrt{1+x}\right) + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\,\mathrm{Log}\left[2\sqrt{-x+\sqrt{x}\sqrt{1+x}} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\right]\right)\right) \Big/ \left(8\sqrt{2}\left(-\sqrt{x}+\sqrt{1+x}\right)^3\left(1+x-\sqrt{x}\sqrt{1+x}\right)^2\right)$

4

In[45]:= `Integrate[Log[1 + x Sqrt[1 + x^2]], x]`

Out[45]=
$$-2x+\frac{\left(5+\sqrt{5}\right)\operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]}{\sqrt{10\left(1+\sqrt{5}\right)}}+\sqrt{\frac{2}{-1+\sqrt{5}}}\operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right]-\frac{\left(-5+\sqrt{5}\right)\operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\sqrt{\frac{2}{1+\sqrt{5}}}\operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right]+x\operatorname{Log}\left[1+x\sqrt{1+x^2}\right]$$

5

In[46]:= `Integrate[Cos[x]^2/Sqrt[Cos[x]^4 + Cos[x]^2 + 1], x]`

Out[46]=
$$-\left(2\,i\operatorname{Cos}[x]^2\operatorname{EllipticPi}\left[\frac{3}{2}+\frac{i\sqrt{3}}{2},\,i\operatorname{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{3}}}\operatorname{Tan}[x]\right],\,\frac{3\,i-\sqrt{3}}{3\,i+\sqrt{3}}\right]\sqrt{1-\frac{2\,i\operatorname{Tan}[x]^2}{-3\,i+\sqrt{3}}}\sqrt{1+\frac{2\,i\operatorname{Tan}[x]^2}{3\,i+\sqrt{3}}}\right)\Big/\left(\sqrt{-\frac{i}{-3\,i+\sqrt{3}}}\sqrt{15+8\operatorname{Cos}[2x]+\operatorname{Cos}[4x]}\right)$$

6

In[47]:= `Integrate[Tan[x] Sqrt[1 + Tan[x]^4], x]`

Out[47]=
$$\frac{1}{2}\sqrt{1+\operatorname{Tan}[x]^4}-\left(8\operatorname{Cos}\left[\frac{x}{2}\right]^4\operatorname{Cos}[x]^2\sqrt{-\frac{\left((-1+2\,i)+2\sqrt{-1-i}\right)\left(-i+\operatorname{Cos}[2x]\right)}{\left(i+\sqrt{-1-i}+\left((-1+i)+\sqrt{-1-i}\right)\operatorname{Cos}[x]\right)^2}}\right.$$
$$\left(\left((2+6\,i)-\frac{8}{\sqrt{-1-i}}-5\sqrt{-1+i}+(2+4\,i)\sqrt{2}\right)\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\ 4-2\sqrt{2}\right]+$$

$$\Big((-4-4\,i)-(3-5\,i)\sqrt{-1-i}+(5-3\,i)\sqrt{-1+i}+4\,(-1-i)^{3/2}\sqrt{-1+i}\Big)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\ 4-2\sqrt{2}\right]+$$

$$\Big((-4-4\,i)-(1-4\,i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2\,(-1-i)^{3/2}\sqrt{-1+i}\Big)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\ 4-2\sqrt{2}\right]\right)$$

$$\sqrt{\left(\left(\left(2\,i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left((1-2\,i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/\right.}$$

$$\left.\left(\left(-2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\right)$$

$$\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\left(\frac{2\,\text{Sec}[x]\,\text{Sin}[3\,x]}{\sqrt{3+\text{Cos}[4\,x]}}-\frac{2\,\text{Tan}[x]}{\sqrt{3+\text{Cos}[4\,x]}}\right)$$

$$\left.\sqrt{1+\text{Tan}[x]^4}\right)\Big/$$

$$\Big(\sqrt{-1+i}\left((-12+4\,i)+(7+8\,i)\sqrt{-1-i}\right)\left((2+2\,i)-(2-i)\sqrt{-1+i}\right)(3+\text{Cos}[4\,x])$$

$$
\left(-\left(16\,\text{Cos}\left[\frac{x}{2}\right]\sqrt{-\frac{\left((-1+2\,i)+2\sqrt{-1-i}\right)(-i+\text{Cos}[2\,x])}{\left(i+\sqrt{-1-i}+\left((-1+i)+\sqrt{-1-i}\right)\text{Cos}[x]\right)^{2}}}\right.\right.
$$

$$
\left(\left((2+6\,i)-\frac{8}{\sqrt{-1-i}}-5\sqrt{-1+i}+(2+4\,i)\sqrt{2}\right)\right.
$$

$$
\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]+
$$

$$
\left((-4-4\,i)-(3-5\,i)\sqrt{-1-i}+(5-3\,i)\sqrt{-1+i}+4\,(-1-i)^{3/2}\sqrt{-1+i}\right)
$$

$$
\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.
$$

$$
\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]+
$$

$$
\left((-4-4\,i)-(1-4\,i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2\,(-1-i)^{3/2}\sqrt{-1+i}\right)
$$

$$
\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.
$$

$$
\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}{\sqrt{-1+i}\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]\right)
$$

$$
\text{Sin}\left[\frac{x}{2}\right]\sqrt{\left(\left(\left(2\,i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left((1-2\,i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^{2}\right)\right)\Big/\right.}
$$

$$
\left.\left.\left(\left(-2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2\,i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^{2}\right)\right)\right)\right)
$$

$$\left((-1+2\,i)+2\sqrt{-1-i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\Bigg/\left(\sqrt{-1+i}\;\left((-12+4\,i)+(7+8\,i)\sqrt{-1-i}\right)\right.$$

$$\left.\left((2+2\,i)-(2-i)\sqrt{-1+i}\right)\sqrt{3+\mathrm{Cos}[4\,x]}\right)+$$

$$\left(16\,\mathrm{Cos}\left[\frac{x}{2}\right]^3\sqrt{-\frac{\left((-1+2\,i)+2\sqrt{-1-i}\right)\,(-i+\mathrm{Cos}[2\,x])}{\left(i+\sqrt{-1-i}+\left((-1+i)+\sqrt{-1-i}\right)\mathrm{Cos}[x]\right)^2}}\right.$$

$$\left(\left((2+6\,i)-\frac{8}{\sqrt{-1-i}}-5\sqrt{-1+i}+(2+4\,i)\sqrt{2}\right)\right.$$

$$\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\;\left((-1+2\,i)+2\sqrt{-1-i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]+$$

$$\left((-4-4\,i)-(3-5\,i)\sqrt{-1-i}+(5-3\,i)\sqrt{-1+i}+4\,(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\mathrm{EllipticPi}\left[\frac{2\sqrt{-1+i}\;\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\mathrm{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\;\left((-1+2\,i)+2\sqrt{-1-i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]+$$

$$\left((-4-4\,i)-(1-4\,i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2\,(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\mathrm{EllipticPi}\left[\frac{2\sqrt{-1+i}\;\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\left.\mathrm{ArcSin}\left[\frac{\sqrt{\frac{\left(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2\,i)+2\sqrt{-1+i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\;\left((-1+2\,i)+2\sqrt{-1-i}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],\,4-2\sqrt{2}\right]\right)$$

$$\text{Sin}\left[\frac{x}{2}\right]\sqrt{\left(\left(\left(2 i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left((1-2 i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/\left(\left(-2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\right)}$$

$$\left.\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\right)\Big/\left(\sqrt{-1+i}\left((-12+4 i)+(7+8 i)\sqrt{-1-i}\right)\right.$$

$$\left.\left((2+2 i)-(2-i)\sqrt{-1+i}\right)\sqrt{3+\text{Cos}[4 x]}\right)-$$

$$\left(4\,\text{Cos}\left[\frac{x}{2}\right]^4\left(\left((2+6 i)-\frac{8}{\sqrt{-1-i}}-5\sqrt{-1+i}+(2+4 i)\sqrt{2}\right)\right.\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],4-2\sqrt{2}\right]+$$

$$\left((-4-4 i)-(3-5 i)\sqrt{-1-i}+(5-3 i)\sqrt{-1+i}+4(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],4-2\sqrt{2}\right]+$$

$$\left((-4-4 i)-(1-4 i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],4-2\sqrt{2}\right]\right)$$

$$\Bigg(-\Big(2\,\big((-1+i)+\sqrt{-1-i}\big)\,\big((-1+2\,i)+2\,\sqrt{-1-i}\big)\,(-i+\text{Cos}[2\,x])\,\text{Sin}[x]\Big)\Big/\Big(i+\sqrt{-1-i}+$$

$$\big((-1+i)+\sqrt{-1-i}\big)\,\text{Cos}[x]\Big)^3+\frac{2\,\big((-1+2\,i)+2\,\sqrt{-1-i}\big)\,\text{Sin}[2\,x]}{\Big(i+\sqrt{-1-i}+\big((-1+i)+\sqrt{-1-i}\big)\,\text{Cos}[x]\Big)^2}\Bigg)$$

$$\sqrt{\Bigg(\bigg(\Big(\big(2\,i+\sqrt{-1-i}-\sqrt{-1+i}\big)\,\Big((1-2\,i)+2\,\sqrt{-1-i}-\text{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(\big(-2\,i+\sqrt{-1-i}+\sqrt{-1+i}\big)\,\Big((-1+2\,i)+2\,\sqrt{-1-i}+\text{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\bigg)\Bigg)$$

$$\Big((-1+2\,i)+2\,\sqrt{-1-i}+\text{Tan}\Big[\frac{x}{2}\Big]^2\Big)^2\Bigg)\Big/$$

$$\Bigg(\sqrt{-1+i}\,\big((-12+4\,i)+(7+8\,i)\,\sqrt{-1-i}\big)\,\big((2+2\,i)-(2-i)\,\sqrt{-1+i}\big)$$

$$\sqrt{-\frac{\big((-1+2\,i)+2\,\sqrt{-1-i}\big)\,(-i+\text{Cos}[2\,x])}{\Big(i+\sqrt{-1-i}+\big((-1+i)+\sqrt{-1-i}\big)\,\text{Cos}[x]\Big)^2}}\,\sqrt{3+\text{Cos}[4\,x]}\Bigg)-$$

$$\Bigg(16\,\text{Cos}\Big[\frac{x}{2}\Big]^4\,\sqrt{-\frac{\big((-1+2\,i)+2\,\sqrt{-1-i}\big)\,(-i+\text{Cos}[2\,x])}{\Big(i+\sqrt{-1-i}+\big((-1+i)+\sqrt{-1-i}\big)\,\text{Cos}[x]\Big)^2}}$$

$$\Bigg(\Big((2+6\,i)-\frac{8}{\sqrt{-1-i}}-5\,\sqrt{-1+i}+(2+4\,i)\,\sqrt{2}\Big)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\big(2\,i+\sqrt{-1-i}+\sqrt{-1+i}\big)\,\big((-1-2\,i)+2\,\sqrt{-1+i}+\text{Tan}[\frac{x}{2}]^2\big)}{\sqrt{-1+i}\,\big((-1+2\,i)+2\,\sqrt{-1-i}+\text{Tan}[\frac{x}{2}]^2\big)}}}{\sqrt{2}}\right],\,4-2\,\sqrt{2}\right]+$$

$$\Big((-4-4\,i)-(3-5\,i)\,\sqrt{-1-i}+(5-3\,i)\,\sqrt{-1+i}+4\,(-1-i)^{3/2}\,\sqrt{-1+i}\Big)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],4-2\sqrt{2}\right]+$$

$$\left((-4-4i)-(1-4i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],4-2\sqrt{2}\right]\right)$$

$$\text{Sin}[4x]\sqrt{\left(\left(\left(2i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left((1-2i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/\left(\left(-2i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\right)}$$

$$\left.\left((-1+2i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\right)\Big/\left(\sqrt{-1+i}\left((-12+4i)+(7+8i)\sqrt{-1-i}\right)\right.$$

$$\left.\left((2+2i)-(2-i)\sqrt{-1+i}\right)(3+\text{Cos}[4x])^{3/2}\right)-$$

$$\left(4\,\text{Cos}\left[\frac{x}{2}\right]^4\sqrt{-\frac{\left((-1+2i)+2\sqrt{-1-i}\right)(-i+\text{Cos}[2x])}{\left(i+\sqrt{-1-i}+\left((-1+i)+\sqrt{-1-i}\right)\text{Cos}[x]\right)^2}}\right.$$

$$\left(\left((2+6i)-\frac{8}{\sqrt{-1-i}}-5\sqrt{-1+i}+(2+4i)\sqrt{2}\right)\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2\sqrt{2}\right]+$$

$$\left((-4-4 i)-(3-5 i)\sqrt{-1-i}+(5-3 i)\sqrt{-1+i}+4(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left((-1+i)+\sqrt{-1-i}\right)}{\left((-1-i)+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right], 4-2\sqrt{2}\right]+$$

$$\left((-4-4 i)-(1-4 i)\sqrt{-1-i}+(4-i)\sqrt{-1+i}+2(-1-i)^{3/2}\sqrt{-1+i}\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-1+i}\left(i+\sqrt{-1-i}\right)}{\left(-i+\sqrt{-1+i}\right)\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)},\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1-2 i)+2\sqrt{-1+i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{-1+i}\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)}}}{\sqrt{2}}\right],$$

$$\left.\left.4-2\sqrt{2}\right]\right) \left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2$$

$$\left(-\left(\left(2 i+\sqrt{-1-i}-\sqrt{-1+i}\right)\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\left((1-2 i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/\right.$$

$$\left(\left(-2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2\right)-$$

$$\left(\left(2 i+\sqrt{-1-i}-\sqrt{-1+i}\right)\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]\right)\Big/$$

$$\left.\left(\left(-2 i+\sqrt{-1-i}+\sqrt{-1+i}\right)\left((-1+2 i)+2\sqrt{-1-i}+\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\right)\right)\Big/$$

$$\left(\sqrt{-1+i}\left((-12+4 i)+(7+8 i)\sqrt{-1-i}\right)\left((2+2 i)-(2-i)\sqrt{-1+i}\right)\right.$$

$$\sqrt{3+\text{Cos}[4 x]}\ \sqrt{\left(\left(\left(2 i+\sqrt{-1-i}-\sqrt{-1+i}\right)\left((1-2 i)+2\sqrt{-1-i}-\text{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/\right.}$$

$$\left(\left(-2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big)\Big)\Big)-$$

$$\Bigg(8\,\mathrm{Cos}\left[\frac{x}{2}\right]^4\sqrt{-\frac{\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}\right)(-\mathrm{i}+\mathrm{Cos}[2\,x])}{\left(\mathrm{i}+\sqrt{-1-\mathrm{i}}+\left((-1+\mathrm{i})+\sqrt{-1-\mathrm{i}}\right)\mathrm{Cos}[x]\right)^2}}$$

$$\sqrt{\Big(\Big(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}-\sqrt{-1+\mathrm{i}}\right)\left((1-2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}-\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\Big)\Big/}$$

$$\left(\left(-2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big)$$

$$\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2\Bigg(\Bigg(\left((2+6\,\mathrm{i})-\frac{8}{\sqrt{-1-\mathrm{i}}}-5\sqrt{-1+\mathrm{i}}+(2+4\,\mathrm{i})\sqrt{2}\right)$$

$$\left(\frac{\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}-\right.$$

$$\left(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]\left((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/$$

$$\left.\left(\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2\right)\right)\Bigg)\Big/$$

$$\Bigg(2\sqrt{2}\sqrt{\Big(\Big(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\left((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\Big)\Big/}$$

$$\left(\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big)$$

$$\sqrt{\Big(1-\Big(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\left((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\Big)\Big/}$$

$$\left(2\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big)$$

$$\sqrt{\Big(1-\Big(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\left(4-2\sqrt{2}\right)\left((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\Big)\Big/}$$

$$\left(2\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big)\Bigg)+$$

$$\Bigg(\Big((-4-4\,\mathrm{i})-(3-5\,\mathrm{i})\sqrt{-1-\mathrm{i}}+(5-3\,\mathrm{i})\sqrt{-1+\mathrm{i}}+4\,(-1-\mathrm{i})^{3/2}\sqrt{-1+\mathrm{i}}\Big)$$

$$\left(\frac{\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)}-\right.$$

$$\left(\left(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\right)\mathrm{Sec}\left[\frac{x}{2}\right]^2\mathrm{Tan}\left[\frac{x}{2}\right]\left((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\right)\Big/$$

$$\left.\left(\sqrt{-1+\mathrm{i}}\left((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2\right)\right)\Bigg)\Big/$$

$$\Bigg(2\sqrt{2}\,\sqrt{\Bigg(\Bigg(\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\Bigg(1-\Big(\big((-1+\mathrm{i})+\sqrt{-1-\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/$$

$$\Big(\big((-1-\mathrm{i})+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\sqrt{\Bigg(1-\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(2\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\sqrt{\Bigg(1-\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\big(4-2\sqrt{2}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(2\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)\Bigg)+$$

$$\Bigg(\Big((-4-4\,\mathrm{i})-(1-4\,\mathrm{i})\sqrt{-1-\mathrm{i}}+(4-\mathrm{i})\sqrt{-1+\mathrm{i}}+2\,(-1-\mathrm{i})^{3/2}\sqrt{-1+\mathrm{i}}\Big)$$

$$\Bigg(\frac{\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\mathrm{Sec}\big[\frac{x}{2}\big]^2\,\mathrm{Tan}\big[\frac{x}{2}\big]}{\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\big[\frac{x}{2}\big]^2\Big)}-$$

$$\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\mathrm{Sec}\Big[\frac{x}{2}\Big]^2\,\mathrm{Tan}\Big[\frac{x}{2}\Big]\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/$$

$$\Big(\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)^2\Big)\Bigg)\Bigg)\Big/$$

$$\Bigg(2\sqrt{2}\,\sqrt{\Bigg(\Bigg(\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\Bigg(1-\Big(\big(\mathrm{i}+\sqrt{-1-\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/$$

$$\Big(\big(-\mathrm{i}+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\sqrt{\Bigg(1-\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(2\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)$$

$$\sqrt{\Bigg(1-\Big(\big(2\,\mathrm{i}+\sqrt{-1-\mathrm{i}}+\sqrt{-1+\mathrm{i}}\big)\,\big(4-2\sqrt{2}\big)\,\Big((-1-2\,\mathrm{i})+2\sqrt{-1+\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Big/}$$

$$\Big(2\sqrt{-1+\mathrm{i}}\,\Big((-1+2\,\mathrm{i})+2\sqrt{-1-\mathrm{i}}+\mathrm{Tan}\Big[\frac{x}{2}\Big]^2\Big)\Big)\Bigg)\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{-1+i}\ \left((-12+4 i)+(7+8 i)\sqrt{-1-i}\right)\left((2+2 i)-(2-i)\sqrt{-1+i}\right)\sqrt{3+\text{Cos}[4 x]}\right)\Bigg)\Bigg)$$

7

In[49]:= **Integrate**$\left[\dfrac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3+1}}, x\right]$

Out[49]=
$$-\left(i\,\text{Cos}[x]^2\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{3}\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right]-\text{EllipticPi}\left[\frac{1}{6}\left(3+i\sqrt{3}\right), i\,\text{ArcSinh}\left[\sqrt{3}\sqrt{\frac{i\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right]\right)\text{Sec}\left[\frac{x}{2}\right]^4 \sqrt{(4+3\,\text{Cos}[x]+\text{Cos}[3 x])\,\text{Sec}[x]^3}\sqrt{\frac{\sqrt{3}-3 i\,\text{Tan}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3 i\,\text{Tan}\left[\frac{x}{2}\right]^2}{3 i+\sqrt{3}}}\right)\Bigg/\left(\sqrt{3}\sqrt{\frac{\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^2}{-3-i\sqrt{3}}}\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]^4\right)\right)$$

8

In[48]:= **Integrate**$\left[\sqrt{\text{Tan}[x]^2+2\,\text{Tan}[x]+2}, x\right]$

Out[48]=
$$-\Bigg(4\,\text{Cos}[x]\Bigg(\Bigg(\Bigg(\text{EllipticF}\Bigg[\text{ArcSin}\Bigg[\sqrt{\Big(\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 4]\big)\left(-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 1]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big/\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 1]-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 4]\big)\left(-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 2]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big)}\Bigg],$$
$$-\Big(\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 2]-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 3]\big)\big(\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 1]-\text{Root}[1+2\#1-2\#1^3+\#1^4\,\&, 4]\big)\Big)\Big/$$

```
((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
(1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) - EllipticPi[
  ((-1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
       Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) / ((-1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1])
     (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
  ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
        ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
  -((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
      (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
      (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]))
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))/
    ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2]))/
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))/
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))] /
```

$$\Big(\big(-1+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]\big)\,\big(1-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]\big)$$
$$\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]\big)$$
$$\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)$$
$$\sqrt{1+2\,\text{Tan}\left[\frac{x}{2}\right]-2\,\text{Tan}\left[\frac{x}{2}\right]^3+\text{Tan}\left[\frac{x}{2}\right]^4}\;\Big)-$$

$$\Big((2-i)\,\Big(\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{\Big(\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[}$$
$$1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\,\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big/$$
$$\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)$$
$$\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big)\Big],$$
$$-\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,3]\big)$$
$$\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big)\Big/$$
$$\Big(\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,3]\big)$$
$$\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big)\Big]$$
$$\big(i-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]\big)-\text{EllipticPi}\Big[$$
$$\Big(\big(-i+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]\big)\,\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+$$
$$\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big)\Big/\Big(\big(-i+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]\big)$$
$$\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big),$$
$$\text{ArcSin}\Big[\sqrt{\Big(\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)}$$
$$\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big/$$
$$\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)$$
$$\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big)\Big],$$
$$-\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,3]\big)$$
$$\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big)\Big/$$
$$\Big(\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,3]\big)$$
$$\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big)\Big]$$
$$\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]\big)\Big)$$
$$\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)$$
$$\sqrt{\Big(\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)}$$
$$\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big/$$
$$\Big(\big(\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)$$
$$\left(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Tan}\left[\frac{x}{2}\right]\right)\Big)\Big)$$

```
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2])) /
    ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]) /
 ((-I + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (I - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
   (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
   (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
   Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) -
((2 + I) (EllipticF[ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[
              1 + 2 #1 - 2 #1^3 + #1^4 &, 4]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
          ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
            (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
      -((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
          (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
        ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
          (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
    (-I - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) - EllipticPi[
     ((I + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
          Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) / ((I + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1])
        (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
     ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
            (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
          ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
```

```
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))))],
 -(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
     (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
     (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])))]
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]))
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
   ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]) /
((ⅈ + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-ⅈ - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
  (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
  Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) -
((EllipticF[ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
        (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
```

```
((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2]))))],
 -((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
(-1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) - EllipticPi[
 ((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]) (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] +
      Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) / ((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1])
   (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])),
 ArcSin[Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
     ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
      (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2]))))],
 -((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
 (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2]))
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
Sqrt[(((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2])) /
   ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
(-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2])) /
   ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
```

```
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))) /
        ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))) /
  ((1 + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1]) (-1 - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    Sqrt[1 + 2 Tan[x/2] - 2 Tan[x/2]^3 + Tan[x/2]^4]) +
 (2 EllipticF[ArcSin[Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
            (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))) /
          ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
            (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]],
      ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
         (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])) /
       ((Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
         (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4]))]
    (Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] - Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
    Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Tan[x/2]))) /
        ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
    (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])^2
    Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3] + Tan[x/2]))) /
        ((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 3])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2] + Tan[x/2])))]
    Sqrt[(((-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 1] + Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 2])
          (-Root[1 + 2 #1 - 2 #1^3 + #1^4 &, 4] + Tan[x/2]))) /
```

$$\Big(\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\Big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Tan}\Big[\frac{x}{2}\Big]\Big)\Big)\Big)\Big)\Big/$$

$$\Big(\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,1]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]\big)\big(-\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,2]+\text{Root}[1+2\,\#1-2\,\#1^3+\#1^4\,\&,4]\big)\sqrt{1+2\,\text{Tan}\Big[\frac{x}{2}\Big]-2\,\text{Tan}\Big[\frac{x}{2}\Big]^3+\text{Tan}\Big[\frac{x}{2}\Big]^4}\Big)\Big)$$

$$\sqrt{2+2\,\text{Tan}[x]+\text{Tan}[x]^2}\Big)\Big/\Big((1+\text{Cos}[x])\sqrt{\frac{3+\text{Cos}[2\,x]+2\,\text{Sin}[2\,x]}{(1+\text{Cos}[x])^2}}\Big)$$

9

In[50]:= **Integrate[Sin[x] ArcTan[√Sec[x] - 1], x]**

$$\text{Out[50]=}\ -\text{ArcTan}\Big[\sqrt{(1-\text{Cos}[x])\,\text{Sec}[x]}\Big]\,\text{Cos}[x]+\frac{1}{2}\,\text{Cos}[x]\,\sqrt{(1-\text{Cos}[x])\,\text{Sec}[x]}\,-$$

$$\Big(\text{Cot}\Big[\frac{x}{4}\Big]\,\text{Cot}\Big[\frac{x}{2}\Big]\Big(\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\text{Tan}[\frac{x}{4}]}{\sqrt{3-2\sqrt{2}}}\Big],17-12\sqrt{2}\Big]+2\,\text{EllipticPi}\Big[-3+2\sqrt{2},-\text{ArcSin}\Big[\frac{\text{Tan}[\frac{x}{4}]}{\sqrt{3-2\sqrt{2}}}\Big],17-12\sqrt{2}\Big]\Big)$$

$$(-1+\text{Sec}[x])\sqrt{3-2\sqrt{2}-\text{Tan}\Big[\frac{x}{4}\Big]^2}\,\sqrt{1+\big(-3+2\sqrt{2}\big)\,\text{Tan}\Big[\frac{x}{4}\Big]^2}\Big)\Big/$$

$$\Big(8\Big(-\Big(\big(-3+2\sqrt{2}\big)\Big(\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\text{Tan}[\frac{x}{4}]}{\sqrt{3-2\sqrt{2}}}\Big],17-12\sqrt{2}\Big]+$$

$$2\,\text{EllipticPi}\Big[-3+2\sqrt{2},-\text{ArcSin}\Big[\frac{\text{Tan}[\frac{x}{4}]}{\sqrt{3-2\sqrt{2}}}\Big],17-12\sqrt{2}\Big]\Big)\,\text{Sec}\Big[\frac{x}{4}\Big]^2$$

$$\sqrt{-1+\mathrm{Sec}[x]}\;\sqrt{3-2\sqrt{2}-\mathrm{Tan}\left[\frac{x}{4}\right]^2}\Bigg)\Bigg/\left(8\sqrt{1+\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}\right)+$$

$$1\Bigg/\left(8\sqrt{3-2\sqrt{2}-\mathrm{Tan}\left[\frac{x}{4}\right]^2}\right)\Bigg(\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]+$$

$$2\,\mathrm{EllipticPi}\left[-3+2\sqrt{2},-\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]\Bigg)\mathrm{Sec}\left[\frac{x}{4}\right]^2\sqrt{-1+\mathrm{Sec}[x]}$$

$$\sqrt{1+\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}+\frac{1}{8}\mathrm{Csc}\left[\frac{x}{4}\right]^2\Bigg(\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]+$$

$$2\,\mathrm{EllipticPi}\left[-3+2\sqrt{2},-\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]\Bigg)$$

$$\sqrt{-1+\mathrm{Sec}[x]}\;\sqrt{3-2\sqrt{2}-\mathrm{Tan}\left[\frac{x}{4}\right]^2}\;\sqrt{1+\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}-$$

$$\frac{1}{2}\mathrm{Cot}\left[\frac{x}{4}\right]\sqrt{-1+\mathrm{Sec}[x]}\;\sqrt{3-2\sqrt{2}-\mathrm{Tan}\left[\frac{x}{4}\right]^2}\;\sqrt{1+\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}$$

$$\left(\frac{\mathrm{Sec}\left[\frac{x}{4}\right]^2}{4\sqrt{3-2\sqrt{2}}\;\sqrt{1-\frac{\mathrm{Tan}\left[\frac{x}{4}\right]^2}{3-2\sqrt{2}}}\;\sqrt{1-\frac{\left(17-12\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}{3-2\sqrt{2}}}}-\mathrm{Sec}\left[\frac{x}{4}\right]^2\Bigg/\Bigg(2\sqrt{3-2\sqrt{2}}\right.$$

$$\left.\sqrt{1-\frac{\mathrm{Tan}\left[\frac{x}{4}\right]^2}{3-2\sqrt{2}}}\;\sqrt{1-\frac{\left(17-12\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}{3-2\sqrt{2}}}\left(1-\frac{\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}{3-2\sqrt{2}}\right)\Bigg)\right)-$$

$$1\Bigg/\left(4\sqrt{-1+\mathrm{Sec}[x]}\right)\mathrm{Cot}\left[\frac{x}{4}\right]\Bigg(\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]+$$

$$2\,\mathrm{EllipticPi}\left[-3+2\sqrt{2},-\mathrm{ArcSin}\left[\frac{\mathrm{Tan}\left[\frac{x}{4}\right]}{\sqrt{3-2\sqrt{2}}}\right],17-12\sqrt{2}\right]\Bigg)$$

$$\mathrm{Sec}[x]\ \sqrt{3-2\sqrt{2}-\mathrm{Tan}\left[\frac{x}{4}\right]^2}\ \sqrt{1+\left(-3+2\sqrt{2}\right)\mathrm{Tan}\left[\frac{x}{4}\right]^2}\ \mathrm{Tan}[x]\Big)\Big)$$

10

In[51]:=

$$\mathbf{Integrate}\left[\frac{\mathbf{x^3\ Exp[ArcSin[x]]}}{\sqrt{\mathbf{1-x^2}}},\ \mathbf{x}\right]$$

Out[51]=

$$-\frac{1}{40}\ e^{\mathrm{ArcSin}[x]}\left(15\left(-x+\sqrt{1-x^2}\right)-3\,\mathrm{Cos}[3\,\mathrm{ArcSin}[x]]+\mathrm{Sin}[3\,\mathrm{ArcSin}[x]]\right)$$

11

In[52]:=

```
Integrate[(x * Log[1 + x^2] * Log[x + Sqrt[1 + x^2]]) / Sqrt[1 + x^2], x]
```

Out[52]=

$$4\,x-2\,\mathrm{ArcTan}[x]-2\sqrt{1+x^2}\ \mathrm{Log}\left[x+\sqrt{1+x^2}\right]+\mathrm{Log}\left[1+x^2\right]\left(-x+\sqrt{1+x^2}\ \mathrm{Log}\left[x+\sqrt{1+x^2}\right]\right)$$

12

In[53]:=

```
Integrate[ArcTan[x + Sqrt[1 - x^2]], x]
```

Out[53]=

$$x\,\mathrm{ArcTan}\left[x+\sqrt{1-x^2}\right]+$$

$$\frac{1}{16}\Big(-8\,\mathrm{ArcSin}[x]+2\sqrt{2+2\,\mathrm{i}\sqrt{3}}\ \mathrm{ArcTan}\Big[\Big(\left(1+\mathrm{i}\sqrt{3}-2\,x^2\right)\left(-1+x^2\right)\Big)\Big/\Big(-3\,\mathrm{i}-\sqrt{3}+2\sqrt{3}\,x^4+$$

$$x^3\left(-6-2\,\mathrm{i}\sqrt{3}-2\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+x\left(6+2\,\mathrm{i}\sqrt{3}-2\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+$$

$$x^2\left(3\,\mathrm{i}-\sqrt{3}+2\sqrt{6-6\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)\Big)\Big]-2\sqrt{2+2\,\mathrm{i}\sqrt{3}}\ \mathrm{ArcTan}\Big[$$

$$\Big(\left(1+\mathrm{i}\sqrt{3}-2\,x^2\right)\left(-1+x^2\right)\Big)\Big/\Big(-3\,\mathrm{i}-\sqrt{3}+2\sqrt{3}\,x^4+2\,x\left(-3-\mathrm{i}\sqrt{3}+\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+$$

$$2\,x^3\left(3+\mathrm{i}\sqrt{3}+\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+x^2\left(3\,\mathrm{i}-\sqrt{3}+2\sqrt{6-6\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)\Big)\Big]-$$

$$2\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \mathrm{ArcTan}\Big[\Big(\left(-1+x^2\right)\left(-1+\mathrm{i}\sqrt{3}+2\,x^2\right)\Big)\Big/$$

$$\Big(3\,\mathrm{i}-\sqrt{3}+2\sqrt{3}\,x^4+x\left(6-2\,\mathrm{i}\sqrt{3}-2\sqrt{2+2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+$$

$$x^3\left(-6+2\,\mathrm{i}\sqrt{3}-2\sqrt{2+2\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)+x^2\left(-3\,\mathrm{i}-\sqrt{3}+2\sqrt{6+6\,\mathrm{i}\sqrt{3}}\ \sqrt{1-x^2}\right)\Big)\Big]+$$

$$2\sqrt{2-2\,\mathrm{i}\sqrt{3}}\ \mathrm{ArcTan}\Big[\Big(\left(-1+x^2\right)\left(-1+\mathrm{i}\sqrt{3}+2\,x^2\right)\Big)\Big/$$

$$\left(3\,\mathrm{i} - \sqrt{3} + 2\sqrt{3}\,x^4 + 2\,x^3 \left(3 - \mathrm{i}\sqrt{3} + \sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) +\right.$$

$$\left. 2\,x\left(-3 + \mathrm{i}\sqrt{3} + \sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) + x^2\left(-3\,\mathrm{i} - \sqrt{3} + 2\sqrt{6 + 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right)\right)] -$$

$$2\,\text{Log}\left[-\frac{1}{2} - \frac{\mathrm{i}\sqrt{3}}{2} + x^2\right] + 2\,\mathrm{i}\sqrt{3}\,\text{Log}\left[-\frac{1}{2} - \frac{\mathrm{i}\sqrt{3}}{2} + x^2\right] - 2\,\text{Log}\left[\frac{1}{2}\,\mathrm{i}\left(\mathrm{i} + \sqrt{3}\right) + x^2\right] -$$

$$2\,\mathrm{i}\sqrt{3}\,\text{Log}\left[\frac{1}{2}\,\mathrm{i}\left(\mathrm{i} + \sqrt{3}\right) + x^2\right] -$$

$$\mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\left[16\left(1 + \sqrt{3}\,x + x^2\right)^2\right] +$$

$$\mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\left[16\left(1 + \sqrt{3}\,x + x^2\right)^2\right] +$$

$$\mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\left[\left(4 - 4\sqrt{3}\,x + 4\,x^2\right)^2\right] -$$

$$\mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\left[\left(4 - 4\sqrt{3}\,x + 4\,x^2\right)^2\right] -$$

$$\mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\Big[3\,\mathrm{i} + \sqrt{3} - \left(-\mathrm{i} + \sqrt{3}\right)x^4 + 2\,\mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2} +$$

$$5\,\mathrm{i}\,x^2\left(2 + \sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) + x\left(3 + 5\,\mathrm{i}\sqrt{3} + 3\,\mathrm{i}\sqrt{6 - 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) +$$

$$\mathrm{i}\,x^3\left(3\,\mathrm{i} + 3\sqrt{3} + \sqrt{6 - 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right)\Big] + \mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}$$

$$\text{Log}\Big[3\,\mathrm{i} + \sqrt{3} - \left(-\mathrm{i} + \sqrt{3}\right)x^4 + 2\,\mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2} + 5\,\mathrm{i}\,x^2\left(2 + \sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) +$$

$$x^3\left(3 - 3\,\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{6 - 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) - \mathrm{i}\,x\left(-3\,\mathrm{i} + 5\sqrt{3} + 3\sqrt{6 - 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right)\Big] +$$

$$\mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}\,\text{Log}\Big[-3\,\mathrm{i} + \sqrt{3} - \left(\mathrm{i} + \sqrt{3}\right)x^4 - 2\,\mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2} -$$

$$5\,\mathrm{i}\,x^2\left(2 + \sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) + x\left(3 - 5\,\mathrm{i}\sqrt{3} - 3\,\mathrm{i}\sqrt{6 + 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) -$$

$$\mathrm{i}\,x^3\left(-3\,\mathrm{i} + 3\sqrt{3} + \sqrt{6 + 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right)\Big] - \mathrm{i}\sqrt{2 - 2\,\mathrm{i}\sqrt{3}}$$

$$\text{Log}\Big[-3\,\mathrm{i} + \sqrt{3} - \left(\mathrm{i} + \sqrt{3}\right)x^4 - 2\,\mathrm{i}\sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2} - 5\,\mathrm{i}\,x^2\left(2 + \sqrt{2 + 2\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) +$$

$$x^3\left(3 + 3\,\mathrm{i}\sqrt{3} + \mathrm{i}\sqrt{6 + 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right) + \mathrm{i}\,x\left(3\,\mathrm{i} + 5\sqrt{3} + 3\sqrt{6 + 6\,\mathrm{i}\sqrt{3}}\,\sqrt{1 - x^2}\right)\Big]\Big)$$

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```
Integrate[x * ArcTan[x + Sqrt[1 - x^2]] / Sqrt[1 - x^2], x]
```

$$-\frac{\text{ArcSin}[x]}{2} - \sqrt{1 - x^2}\,\text{ArcTan}\left[x + \sqrt{1 - x^2}\right] +$$

$$\frac{1}{4\sqrt{6\left(1 - \mathrm{i}\sqrt{3}\right)}}\left(-3\,\mathrm{i} + \sqrt{3}\right)\text{ArcTan}\Big[\Big(3 - \mathrm{i}\sqrt{3} - 12\,\mathrm{i}\,x + 4\sqrt{3}\,x -$$

$$12\, i \sqrt{3}\, x^2 - 12\, i\, x^3 - 4\sqrt{3}\, x^3 - 3\, x^4 - i\sqrt{3}\, x^4 - 2\, i \sqrt{2\,(1 - i\sqrt{3})}\; x \sqrt{1 - x^2} -$$

$$2\, i \sqrt{6\,(1 - i\sqrt{3})}\; x^2 \sqrt{1 - x^2} - 2\, i \sqrt{2\,(1 - i\sqrt{3})}\; x^3 \sqrt{1 - x^2}\,\Big) \Big/$$

$$\big(i - \sqrt{3} - 6\, x + 6\, i\sqrt{3}\, x + 30\, i\, x^2 - 2\sqrt{3}\, x^2 + 6\, x^3 + 18\, i\sqrt{3}\, x^3 + 11\, i\, x^4 + 3\sqrt{3}\, x^4\big)\Big] -$$

$$\frac{1}{4\sqrt{6\,(1 - i\sqrt{3})}}\,\big(-3\, i + \sqrt{3}\big)\,\mathrm{ArcTan}\Big[\Big(3 - i\sqrt{3} + 12\, i\, x - 4\sqrt{3}\, x - 12\, i\sqrt{3}\, x^2 +$$

$$12\, i\, x^3 + 4\sqrt{3}\, x^3 - 3\, x^4 - i\sqrt{3}\, x^4 + 2\, i \sqrt{2\,(1 - i\sqrt{3})}\; x \sqrt{1 - x^2} -$$

$$2\, i \sqrt{6\,(1 - i\sqrt{3})}\; x^2 \sqrt{1 - x^2} + 2\, i \sqrt{2\,(1 - i\sqrt{3})}\; x^3 \sqrt{1 - x^2}\,\Big) \Big/$$

$$\big(i - \sqrt{3} + 6\, x - 6\, i\sqrt{3}\, x + 30\, i\, x^2 - 2\sqrt{3}\, x^2 - 6\, x^3 - 18\, i\sqrt{3}\, x^3 + 11\, i\, x^4 + 3\sqrt{3}\, x^4\big)\Big] -$$

$$\frac{1}{4\sqrt{6\,(1 + i\sqrt{3})}}\,\big(3\, i + \sqrt{3}\big)\,\mathrm{ArcTan}\Big[\Big(-3 - i\sqrt{3} - 12\, i\, x - 4\sqrt{3}\, x - 12\, i\sqrt{3}\, x^2 -$$

$$12\, i\, x^3 + 4\sqrt{3}\, x^3 + 3\, x^4 - i\sqrt{3}\, x^4 - 2\, i \sqrt{2\,(1 + i\sqrt{3})}\; x \sqrt{1 - x^2} -$$

$$2\, i \sqrt{6\,(1 + i\sqrt{3})}\; x^2 \sqrt{1 - x^2} - 2\, i \sqrt{2\,(1 + i\sqrt{3})}\; x^3 \sqrt{1 - x^2}\,\Big) \Big/$$

$$\big(-i - \sqrt{3} - 6\, x - 6\, i\sqrt{3}\, x - 30\, i\, x^2 - 2\sqrt{3}\, x^2 + 6\, x^3 - 18\, i\sqrt{3}\, x^3 - 11\, i\, x^4 + 3\sqrt{3}\, x^4\big)\Big] +$$

$$\frac{1}{4\sqrt{6\,(1 + i\sqrt{3})}}\,\big(3\, i + \sqrt{3}\big)\,\mathrm{ArcTan}\Big[\Big(-3 - i\sqrt{3} + 12\, i\, x + 4\sqrt{3}\, x - 12\, i\sqrt{3}\, x^2 +$$

$$12\, i\, x^3 - 4\sqrt{3}\, x^3 + 3\, x^4 - i\sqrt{3}\, x^4 + 2\, i \sqrt{2\,(1 + i\sqrt{3})}\; x \sqrt{1 - x^2} -$$

$$2\, i \sqrt{6\,(1 + i\sqrt{3})}\; x^2 \sqrt{1 - x^2} + 2\, i \sqrt{2\,(1 + i\sqrt{3})}\; x^3 \sqrt{1 - x^2}\,\Big) \Big/$$

$$\big(-i - \sqrt{3} + 6\, x + 6\, i\sqrt{3}\, x - 30\, i\, x^2 - 2\sqrt{3}\, x^2 - 6\, x^3 + 18\, i\sqrt{3}\, x^3 - 11\, i\, x^4 + 3\sqrt{3}\, x^4\big)\Big] -$$

$$\frac{i\,\big(-3\, i + \sqrt{3}\big)\,\mathrm{Log}\Big[\big(-i + \sqrt{3} - 2\, x\big)^2 \big(i + \sqrt{3} - 2\, x\big)^2\Big]}{8\sqrt{6\,(1 - i\sqrt{3})}} +$$

$$\frac{i\,\big(3\, i + \sqrt{3}\big)\,\mathrm{Log}\Big[\big(-i + \sqrt{3} - 2\, x\big)^2 \big(i + \sqrt{3} - 2\, x\big)^2\Big]}{8\sqrt{6\,(1 + i\sqrt{3})}} +$$

$$\frac{i\left(-3 i+\sqrt{3}\right) \text{Log}\left[\left(-i+\sqrt{3}+2 x\right)^2 \left(i+\sqrt{3}+2 x\right)^2\right]}{8 \sqrt{6 \left(1-i \sqrt{3}\right)}} -$$

$$\frac{i\left(3 i+\sqrt{3}\right) \text{Log}\left[\left(-i+\sqrt{3}+2 x\right)^2 \left(i+\sqrt{3}+2 x\right)^2\right]}{8 \sqrt{6 \left(1+i \sqrt{3}\right)}} +$$

$$\frac{\left(3 i+\sqrt{3}\right) \text{Log}\left[-\frac{1}{2}-\frac{i \sqrt{3}}{2}+x^2\right]}{8 \sqrt{3}} +$$

$$\frac{\left(-3 i+\sqrt{3}\right) \text{Log}\left[-\frac{1}{2}+\frac{i \sqrt{3}}{2}+x^2\right]}{8 \sqrt{3}} +$$

$$\frac{1}{8 \sqrt{6 \left(1-i \sqrt{3}\right)}}$$

$$i\left(-3 i+\sqrt{3}\right) \text{Log}\Big[3 i+\sqrt{3}-3 x-5 i \sqrt{3} x+10 i x^2+3 x^3-3 i \sqrt{3} x^3+$$
$$i x^4-\sqrt{3} x^4+2 i \sqrt{2 \left(1-i \sqrt{3}\right)} \sqrt{1-x^2}-3 i \sqrt{6 \left(1-i \sqrt{3}\right)} x \sqrt{1-x^2}+$$
$$5 i \sqrt{2 \left(1-i \sqrt{3}\right)} x^2 \sqrt{1-x^2}-i \sqrt{6 \left(1-i \sqrt{3}\right)} x^3 \sqrt{1-x^2}\Big] -$$

$$\frac{1}{8 \sqrt{6 \left(1-i \sqrt{3}\right)}} i\left(-3 i+\sqrt{3}\right) \text{Log}\Big[3 i+\sqrt{3}+3 x+5 i \sqrt{3} x+10 i x^2-3 x^3+3 i \sqrt{3} x^3+$$
$$i x^4-\sqrt{3} x^4+2 i \sqrt{2 \left(1-i \sqrt{3}\right)} \sqrt{1-x^2}+3 i \sqrt{6 \left(1-i \sqrt{3}\right)} x \sqrt{1-x^2}+$$
$$5 i \sqrt{2 \left(1-i \sqrt{3}\right)} x^2 \sqrt{1-x^2}+i \sqrt{6 \left(1-i \sqrt{3}\right)} x^3 \sqrt{1-x^2}\Big] +$$

$$\frac{1}{8 \sqrt{6 \left(1+i \sqrt{3}\right)}} i\left(3 i+\sqrt{3}\right) \text{Log}\Big[-3 i+\sqrt{3}+3 x-5 i \sqrt{3} x-10 i x^2-3 x^3-3 i \sqrt{3} x^3-$$
$$i x^4-\sqrt{3} x^4-2 i \sqrt{2 \left(1+i \sqrt{3}\right)} \sqrt{1-x^2}-3 i \sqrt{6 \left(1+i \sqrt{3}\right)} x \sqrt{1-x^2}-$$
$$5 i \sqrt{2 \left(1+i \sqrt{3}\right)} x^2 \sqrt{1-x^2}-i \sqrt{6 \left(1+i \sqrt{3}\right)} x^3 \sqrt{1-x^2}\Big] -$$

$$\frac{1}{8 \sqrt{6 \left(1+i \sqrt{3}\right)}} i\left(3 i+\sqrt{3}\right) \text{Log}\Big[-3 i+\sqrt{3}-3 x+5 i \sqrt{3} x-10 i x^2+3 x^3+3 i \sqrt{3} x^3-$$

$$i\, x^4 - \sqrt{3}\, x^4 - 2\, i \sqrt{2\left(1 + i\sqrt{3}\right)}\ \sqrt{1-x^2} + 3\, i \sqrt{6\left(1 + i\sqrt{3}\right)}\ x \sqrt{1-x^2} -$$
$$5\, i \sqrt{2\left(1 + i\sqrt{3}\right)}\ x^2 \sqrt{1-x^2} + i \sqrt{6\left(1 + i\sqrt{3}\right)}\ x^3 \sqrt{1-x^2}\ \Big]$$

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```
Integrate[ArcSin[x] / (1 + Sqrt[1 - x^2]), x]
```

$$\frac{\left(-1+\sqrt{1-x^2}\right)\text{ArcSin}[x]}{x} + \frac{\text{ArcSin}[x]^2}{2} - \text{Log}\left[1+\sqrt{1-x^2}\right]$$

15

```
Integrate[Log[x + Sqrt[1 + x^2]] / (1 - x^2)^(3 / 2), x]
```

$$\frac{1}{2}\sqrt{1-x^2}\left(-\frac{\sqrt{1+x^2}\ \text{ArcSin}\left[x^2\right]}{\sqrt{1-x^4}} - \frac{2\, x\, \text{Log}\left[x+\sqrt{1+x^2}\right]}{-1+x^2}\right)$$

16

```
Integrate[ArcSin[x] / (1 + x^2)^(3 / 2), x]
```

$$\frac{x\, \text{ArcSin}[x]}{\sqrt{1+x^2}} - \frac{\text{ArcSin}\left[x^2\right]}{2}$$

17

```
Integrate[Log[x + Sqrt[x^2 - 1]] / (1 + x^2)^(3 / 2), x]
```

$$\frac{2\, x\, \text{Log}\left[x+\sqrt{-1+x^2}\right] - \frac{\sqrt{-1+x^2}\ \left(1+x^2\right)\text{Log}\left[x^2+\sqrt{-1+x^4}\right]}{\sqrt{-1+x^4}}}{2\sqrt{1+x^2}}$$

18

```
Integrate[Log[x] / (x^2 * Sqrt[x^2 - 1]), x]
```

$$\frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2}\ \text{Log}[x]}{x} - \text{Log}\left[x+\sqrt{-1+x^2}\right]$$

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$$\frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]$$

20

```
Integrate[x * Log[x + Sqrt[x^2 - 1]] / Sqrt[x^2 - 1], x]
```

$$-x + \sqrt{-1+x^2}\,\text{Log}\left[x + \sqrt{-1+x^2}\right]$$