

Ten Hard Integrals

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March 5, 2014

Compiled on September 11, 2023 at 6:20pm

The first 10 integrals from Kevin Charlwood's 2008 article "Integration on Computer Algebra Systems" are solved using different CAS systems.

The original post on this topic is sci.math.symbolic by Martin

These are the CAS systems used

1. Maple 18 on windows 7 (64 bit)
2. Mathematica 9.01 on windows 7
3. Rubi 4.1 on Mathematica 9.01 on windows 7
4. Sage 5.4 using the Sage web server notebook interface
5. Fricas 1.2 on Linux using sbcl lisp
6. wxMaxima 12.04.0 (Maxima 5.28.02) on windows 7
7. Axiom on windows 7 (May 2012) welcome screen image
8. sympy on linux (python 2.7.3 full installation. sympy 0.7.1.rc1-3) starting image
9. reduce `reduce-windows64-20110414` help screen message with the `algint` package loaded.
10. mupad engine in Matlab 2013a symbolic toolbox
11. xcas 2013 January, version 1.0 on windows 7

Optimal answer to each intergal taken from Rich's referenced PDF below.

Downloads, references and links

1. Mathematica downloads
 - Mathematica notebook of the first 20 integrals
 - Mathematica PDF of the first 20 integrals
 - Mathematica HTML of the first 20 integrals
2. Rubi downloads
 - Rubi notebook for the first 20 integrals
 - Rubi PDF for the first 20 integrals

- Rubi HTML for the first 20 integrals

3. Maple downloads

- Maple worksheet in wm format
- HTML worksheet in HTML
- PDF worksheet in PDF

Maple Problem 10 trace entered as $\text{int}(x^3 \exp(1)^{\arcsin(x)} / \sqrt{1-x^2}, x);$

- HTML
- PDF
- mw Maple worksheet

Maple Problem 10 trace entered as $\text{int}(x^3 \exp(\arcsin(x)) / \sqrt{1-x^2}, x);$

- HTML
- PDF
- mw Maple worksheet

4. maxima.wxm Maxima notebook.

5. copy of Kevin Charlwood's 2008 paper in PDF

6. <http://www.apmaths.uwo.ca/~arich/CharlwoodIntegrationProblems.pdf> Albert Rich pdf file showing 50 integrals and the best antiderivatives expected

7. <http://www.apmaths.uwo.ca/~arich/CharlwoodProblems.m> The above is m format.

8. <http://www.math.utah.edu/faq/reduce/>

9. <http://www.reduce-algebra.com/packages.htm>

10. <http://reduce-algebra.sourceforge.net/>

11. <http://www.reduce-algebra.com/docs/reduce.pdf>

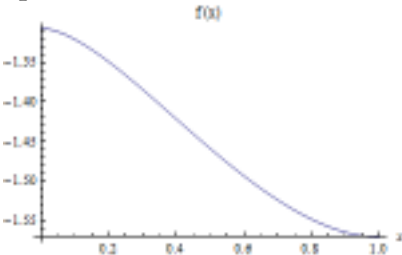
12. xCAS web page

13. <http://www.apmaths.uwo.ca/~arich/> Rubi Mathematica package home

The following is summary of results for each integral. Result with a () around it means the antiderivative contains nonelementary functions.

system	1	2	3	4	5	6	7	8	9	10	score
Mathematica 9.01	✓	✓	✓	✓	(✓)	(✓)	(✓)	(✓)	(✓)	✓	100%
Rubi 4.1	✓	✓	✗	✓	(✓)	✓	✓	✓	✓	✓	90%
Maple 18	✓	✓	✓	✓	(✓)	✓	✓	✓	✗	✓	90%
Axiom (May 2012)	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓	80%
FriCAS 1.2	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	80%
Sage 5.4	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
Maxima 5.28.02	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
xcas 1.0	✗	✗	✗	✓	✗	✓	✗	✓	✓	✗	40%
Sympy 0.7.2	✗	✓	✗	✗	✗	✗	✗	✗	✗	✓	20%
Reduce 2008	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗	20%
mupad 2013a	✗	✗	✗	✓	✗	✗	✗	✗	✓	✗	20%

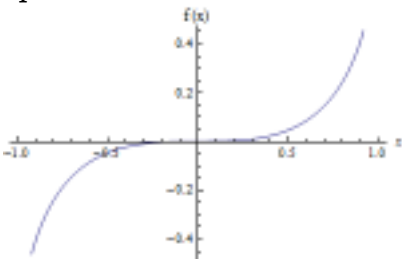
1 $\int \arcsin(x) \ln(x) dx$

<p>optimal</p> 	$-2\sqrt{1-x^2} + \sqrt{1-x^2} \log(x) + \tanh^{-1}(\sqrt{1-x^2}) - x(1-\log(x)) \sin^{-1}(x)$
<p>M 9.01</p>	<pre>Integrate[ArcSin[x] Log[x], x]</pre> $-2\sqrt{1-x^2} + x \operatorname{ArcSin}[x] (-1 + \operatorname{Log}[x]) + (-1 + \sqrt{1-x^2}) \operatorname{Log}[x] + \operatorname{Log}\left[1 + \sqrt{1-x^2}\right]$
<p>Rubi 4.1</p>	<pre>Int[ArcSin[x] Log[x], x]</pre> $-2\sqrt{1-x^2} - x \operatorname{ArcSin}[x] + \operatorname{ArcTanh}\left[\sqrt{1-x^2}\right] + \sqrt{1-x^2} \operatorname{Log}[x]$
<p>Maple 18</p>	<pre>> restart; int(arcsin(x)*log(x), x);</pre> $\frac{1}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2} \left(-2 \arcsin(x) \tan\left(\frac{1}{2} \arcsin(x)\right) - 2 \tan\left(\frac{1}{2} \arcsin(x)\right)^2 \ln\left(\frac{2 \tan\left(\frac{1}{2} \arcsin(x)\right)}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2}\right) + 2 \arcsin(x) \tan\left(\frac{1}{2} \arcsin(x)\right) \ln\left(\frac{2 \tan\left(\frac{1}{2} \arcsin(x)\right)}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2} - 4\right) - \ln\left(\dots\right) \right)$

Sage 5.4	<pre>integral(arcsin(x)*ln(x), x)</pre> $(x \log(x) - x) \arcsin(x) + \sqrt{-x^2 + 1} \log(x) - 2 \sqrt{-x^2}$
Fricas 1.2	<pre>(8) -> integrate(asin(x)*log(x), x)</pre> <p>(8)</p> $\frac{\log(2\sqrt{-x^2 + 1} + 2) - \log(-2\sqrt{-x^2 + 1} + 2) + (-2x \log(x) + 2x) \operatorname{atan}\left(\frac{x\sqrt{-x^2 + 1}}{x^2 - 1}\right) + (2\log(x) - 4)\sqrt{-x^2 + 1}}{2}$ <p>Type: Union(Expression(Integer), ...)</p>
sympy 0.7.1	<pre>>>> integrate(asin(x)*log(x), x) Integral(log(x)*asin(x), x) >>></pre>
Axiom 5/12	<pre>(5) -> integrate(asin(x)*log(x), x)</pre> <p>(5) -></p> <p>(5)</p> $\frac{\log(\sqrt{-x^2 + 1} + 1) - \log(\sqrt{-x^2 + 1} - 1) + (-x \log(x) + x) \operatorname{atan}\left(\frac{2x\sqrt{-x^2 + 1}}{2x^2 - 1}\right) + (2\log(x) - 4)\sqrt{-x^2 + 1}}{2}$ <p>Type: Union(Expression(Integer), ...)</p> <pre>(6) -></pre>

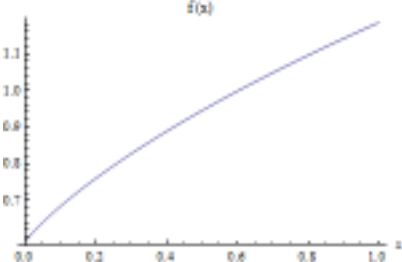
Maxima 5.28.02	<pre>integrate(asin(x)*log(x), x);</pre> $\log\left(\frac{2\sqrt{1-x^2}}{ x } + \frac{2}{ x }\right) + \operatorname{asin}(x)(x \log(x) - x) + \sqrt{1-x^2} \log(x) - 2\sqrt{1-x^2}$
reduce 2008	<pre>int(asin(x)*log(x), x);</pre> $\int \arcsin(x) \log(x) dx$ <p>HTML of trace PDF of trace</p>
mupad 2013a	<pre>evalin(symengine, 'int(asin(x)*log(x), x)')</pre> <pre>int(asin(x)*log(x), x)</pre>
xcas	<pre>int(asin(x)*log(x), x)</pre> <p>Simplification assuming x near 0+ Simplification assuming x near 0+</p> $\int \operatorname{asin}(x) \ln(x) dx$

$$2 \int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$$

<p>optimal</p> 	$x - \sqrt{1-x^2} \sin^{-1}(x)$
<p>M 9.01</p>	$\text{Integrate}\left[\frac{x \text{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$ $x - \sqrt{1-x^2} \text{ArcSin}[x]$
<p>Rubi 4.1</p>	$\text{Int}\left[\frac{x \text{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$ $x - \sqrt{1-x^2} \text{ArcSin}[x]$
<p>Maple 18</p>	<pre>> restart; int(x*arcsin(x)/sqrt(1-x^2), x);</pre> $x - \arcsin(x) \sqrt{-x^2 + 1}$
<p>Sage 5.4</p>	<pre>integral(x*arcsin(x)/sqrt(1-x^2), x)</pre> $-\sqrt{-x^2 + 1} \arcsin(x) + x$
<p>Fricas 1.2</p>	<pre>(9) -> integrate(x*asin(x)/sqrt(1-x^2), x)</pre> $(9) \sqrt{-x^2 + 1} \operatorname{atan}\left(\frac{x\sqrt{-x^2 + 1}}{x^2 - 1}\right) + x$ <pre>(10) -> █</pre> <p style="text-align: right;">Type: Union(Expression</p>

sympy 0.7.1	<pre>>>> integrate(x*asin(x)/sqrt(1-x**2),x) x - (-x**2 + 1)**(1/2)*asin(x) >>> >>></pre>
Axiom 5/12	<pre>-> -> integrate(x*asin(x)/sqrt(1-x^2),x) -> +-----+ 2 2x\ - x + 1 \ - x + 1 atan(-----) + 2x 2 2x - 1 (6) ----- 2 Type: Union(Expression(Integer),...) -> </pre>
Maxima 5.28.02	<pre>(%i5) integrate(x*asin(x)/sqrt(1-x^2),x); (%o5) x - sqrt(1-x^2) asin(x)</pre>
reduce 2008	<pre>int(x*asin(x)/sqrt(1-x^2),x); - sqrt(-x^2+1) arcsin(x) + x</pre>
mupad 2013a	<pre>evalin(symengine, 'int(x*asin(x)/sqrt(1-x^2),x)') int((x*asin(x))/(1 - x^2)^(1/2), x)</pre>
xcas	<pre>int(x*asin(x)/sqrt(1-x^2),x) Simplification assuming x near 0+ Simplification assuming x near 0+</pre> $\int \frac{x \operatorname{asin}(x)}{\sqrt{-(x^2-1)}} dx$

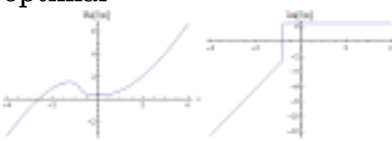
$$3 \int \arcsin(\sqrt{x+1} - \sqrt{x}) dx$$

<p>optimal</p> 	$\frac{(\sqrt{x+3\sqrt{x+1}})\sqrt{\sqrt{x}\sqrt{x+1}-x}}{4\sqrt{2}} - \left(x + \frac{3}{8}\right) \sin^{-1}(\sqrt{x} - \sqrt{x+1})$
<p>M 9.01</p>	$\begin{aligned} & \text{Integrate}[\text{ArcSin}[\sqrt{x+1} - \sqrt{x}], x] \\ & -x \text{ArcSin}[\sqrt{x} - \sqrt{1+x}] - \\ & \left((1+x) (1+2x-2\sqrt{x}\sqrt{1+x})^2 \left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}} \right. \right. \\ & \quad \left. \left. (-3-2x+2\sqrt{x}\sqrt{1+x}) + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \right) \right. \\ & \quad \left. \text{Log}\left[2\sqrt{-x+\sqrt{x}\sqrt{1+x}} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \right] \right. \\ & \quad \left. (8\sqrt{2}(-\sqrt{x}+\sqrt{1+x})^3(1+x-\sqrt{x}\sqrt{1+x})^2) \right) \end{aligned}$
<p>Rubi 4.1</p>	$\begin{aligned} & \text{Int}[\text{ArcSin}[\sqrt{x+1} - \sqrt{x}], x] \\ & -x \text{ArcSin}[\sqrt{x} - \sqrt{1+x}] + \frac{\text{Subst}\left[\text{Int}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}\right], \sqrt{2}\right]}{\sqrt{2}} \end{aligned}$

Maple 18	<pre>> restart; int(arcsin(sqrt(x+1)-sqrt(x)), x);</pre> $-\left(\frac{1}{16} \arcsin(-\sqrt{x+1} + \sqrt{x}) + \frac{3}{8} \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)\right)^3 - \frac{3}{8} (\sqrt{x})^5 - \frac{1}{8} \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)^7 + \frac{1}{8} \arcsin(-\sqrt{x+1} + \sqrt{x}) \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)^2 + \frac{9}{8} \arcsin(-\sqrt{x+1} + \sqrt{x}) (-\sqrt{x+1} + \sqrt{x})^4 + \frac{1}{8} \arcsin(-\sqrt{x+1} + \sqrt{x}) \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right) (-\sqrt{x+1} + \sqrt{x}) \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)^8 + \frac{1}{8} \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right) \left(\left(1 + \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)\right)^2\right)^2 \tan\left(\frac{1}{2} \arcsin(-\sqrt{x+1} + \sqrt{x})\right)$ <pre>=</pre>
Sage 5.4	<pre>integral(arcsin(sqrt(x+1)-sqrt(x)), x)</pre> <p>evaluate</p>
Fricas 1.2	<pre>(10) -> integrate(asin(sqrt(x+1)-sqrt(x)), x)</pre> $(10) \quad \frac{(8x + 3) \operatorname{atan}\left(\frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{2}\sqrt{x}\sqrt{x+1} - 2x}\right) + (3\sqrt{x+1} + \sqrt{x})\sqrt{2}\sqrt{x}}{8}$ <pre>----- Type: Union(Expression(Integer),...) (11) -> </pre>
sympy 0.7.1	<pre>>>> >>> integrate(asin(sqrt(x+1)-sqrt(x)), x) Integral(-asin(x**(1/2) - (x + 1)**(1/2)), x) >>> </pre>

Axiom 5/12	<pre>(7) -> (7) -> integrate(asin(sqrt(x+1)-sqrt(x)),x) 7) -> >> Error detected within library code: failed - cannot handle that integrand (7) -> </pre>
Maxima 5.28.02	<pre>(%i6) integrate(asin(sqrt(x+1)-sqrt(x)),x); (%o6) $\frac{\pi x}{2}$</pre>
reduce 2008	<pre>int(asin(sqrt(x+1)-sqrt(x)),x);</pre> $\int \arcsin(\sqrt{x+1} - \sqrt{x}) dx$
mupad 2013a	<pre>evalin(symengine,'int(asin(sqrt(x+1)-sqrt(x)),x)') -int(asin(x^(1/2) - (x + 1)^(1/2)), x)</pre>
xcas	<pre>int(asin(sqrt(x+1)-sqrt(x)),x)</pre> <p>Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0 Warning, choosing root of [1,0,%%%{-4,[1]%%}+%%{-2,[0]%%},0</p> <pre>sym2poly/r2sym(const gen & e,const index_m & i,const v</pre>

$$4 \int \ln(1 + x\sqrt{1+x^2}) dx$$

<p>optimal</p> 	$x \log(\sqrt{x^2+1}x+1) + \sqrt{2(1+\sqrt{5})} \tan^{-1}(\sqrt{\sqrt{5}-2}(\sqrt{x^2+1}x)) - \sqrt{2(\sqrt{5}-1)} \tanh^{-1}(\sqrt{2+\sqrt{5}}(\sqrt{x^2+1}x)) - 2x$
<p>M 9.01</p>	$\text{Integrate}[\text{Log}[1 + x\sqrt{1+x^2}], x]$ $-2x + \frac{(5 + \sqrt{5}) \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}} x\right]}{\sqrt{10(1+\sqrt{5})}} +$ $\sqrt{\frac{2}{-1+\sqrt{5}}} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right] - \frac{(-5 + \sqrt{5}) \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right]}{\sqrt{10(-1+\sqrt{5})}} +$ $\sqrt{\frac{2}{1+\sqrt{5}}} \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2}\right] + x \text{Log}[1 + x\sqrt{1+x^2}]$

Rubi 4.1

$$\text{Int}\left[\text{Log}\left[1+x\sqrt{1+x^2}\right], x\right]$$

$$-2x - \sqrt{\frac{1}{10}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] +$$

$$2\sqrt{\frac{1}{5}(2+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{2}{5(-1+\sqrt{5})}} \text{Ar}$$

$$\sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right] + 2\sqrt{\frac{1}{5}(-2$$

$$\sqrt{\frac{1}{10}(-1+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] + \sqrt{\frac{2}{5(1+\sqrt{5})}}$$

$$\sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right] + x \text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Maple 18

```
> restart;
int(log(1+x*sqrt(1+x^2)), x);
```

$$\ln(1+x\sqrt{x^2+1})x-2x+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2-2\sqrt{5}}}\right)}{\sqrt{-2-2\sqrt{5}}}$$

$$+\frac{\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}$$

$$-\frac{2}{5}\sqrt{\sqrt{5}-2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)+\frac{2}{5}\sqrt{2+\sqrt{5}}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)$$

$$-\frac{1}{2}\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}-\frac{1}{2}\frac{\sqrt{5}\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}$$

$$-\frac{3}{10}\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}+\frac{1}{2}\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}$$

$$-\frac{3}{10}\frac{\sqrt{5}\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}-\frac{1}{2}\frac{\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}$$

$$-\frac{1}{2}\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}+\frac{1}{2}\frac{\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}$$

Sage 5.4

```
integral(ln(1+x*sqrt(1+x^2)), x)
```

$$x \log\left(\sqrt{x^2+1}x+1\right)-2x+\operatorname{arctan}(x)+\int\frac{2x^2+1}{x^2+\sqrt{x^2+1}}$$

```
(12) -> integrate(log(1+x*sqrt(1+x^2)), x)
```

```
(12)
```

```

-
      +-----+
      | +-+
      4\|\|5 + 1
    *
      atan
      +-----+
      +-+ | 2 +-+ |
      ((\|5 - 1)\|x + 1 - x\|5 + x)\|\|
    /
      ROOT
      +-----+
      +-+ 3 | 2
      (- 8x\|5 - 16x - 8x)\|x +
    +
      2
      16x + 4
    +
      +-----+
      +-+ | 2 2 +-+
      - 2x\|2 \|x + 1 + (2x + 2)\|2
  +
  -
      +-----+
      | +-+
      \|\|5 - 1
    *
      log
      +-----+
      +-+ | 2 +-+ |
      ((\|5 + 1)\|x + 1 - x\|5 - x)\|\|
    +
      2 +-+
      (2x + 2)\|2
  +
  +-----+ +-----+
  | +-+ | +-+ +-+ +-+
  \|\|5 - 1 log(\|\|5 - 1 + x\|2 ) + 2x\|2 lo
  +
  +-----+ +-----+
  | +-+ | +-+ +-+
  - \|\|5 - 1 log(- \|\|5 - 1 + x\|2 )
  +
  +-----+
  |

```

Fricas 1.2

sympy 0.7.1

```
>>>
>>> integrate(log(1+x* sqrt(1+x**2)),x)
Integral(log(x*(x**2 + 1)**(1/2) + 1), x)
>>>
>>>
>>>
```


Axiom 5/12

```

(8) ->
(8) -> integrate(log(1+x*sqrt(1+x^2)),x)
(8) ->
(8)
-
      +-----+
      | +-+
      \|\|5 - 1
      *
      log
          +-----+
          +-+ | 2 +-+ | +-+
          ((\|5 + 1)\|x + 1 - x\|5 - x)\|\|5 - 1
          +
          2 +-+
          (2x + 2)\|2
+
      +-----+ +-----+
      | +-+ | +-+ +-+ +-+ | 2
      \|\|5 - 1 log(\|\|5 - 1 + x\|2 ) + 2x\|2 log(x\|x +
+
      +-----+ +-----+
      | +-+ | +-+ +-+
      - \|\|5 - 1 log(- \|\|5 - 1 + x\|2 )
+
      +-----+
      | +-+
      \|\|5 - 1
      *
      log
          +-----+
          +-+ | 2 +-+ | +-+
          ((- \|5 - 1)\|x + 1 + x\|5 + x)\|\|5 - 1 -
          +
          2 +-+
          (2x + 2)\|2
+
      +-----+ +-----+
      | +-+ | 2 +-+ | +-+
      2\|\|5 + 1 atan(-----)
          +-----+
          +-+ | 2 2 +-+
          2x\|2 \|x + 1 + (- 2x - 2)\|2
+
      +-----+ +-----+
      | +-+ | +-+
      - 2\|\|5 + 1 atan(-----) - 4x\|2
          +-+
          x\|2
/
      +-+
      2\|2
Type: Union(Expression Integer,...)

```

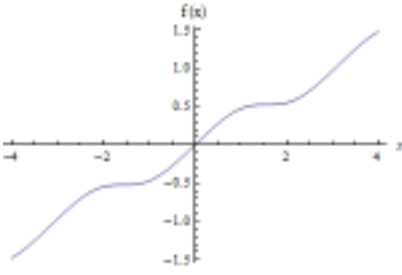
Maxima 5.28.02	<pre>integrate(log(1+x*sqrt(1+x^2)),x);</pre> $\int \frac{2x^2+1}{\sqrt{x^2+1}(x^3+x)+x^2+1} dx + x \log(x\sqrt{x^2+1}+1) + \operatorname{atan}(x) - 2$
reduce 2008	<pre>14: int(log(1+x*sqrt(1+x^2)),x);</pre> $\left(2\sqrt{\sqrt{5}+2}\sqrt{5} \arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right) - 2\sqrt{\sqrt{5}+2} \arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right) + \sqrt{\sqrt{5}-2} \right.$ $\left. \sqrt{\sqrt{5}-2} \log\left(\sqrt{x^2+1} - \sqrt{\sqrt{5}-2} + x\right) - \sqrt{\sqrt{5}-2} \log\left(\sqrt{x^2+1} + \sqrt{\sqrt{5}-2} + x\right) + 2 \right.$
mupad 2013a	<pre>evalin(symengine,'int(log(1+x*sqrt(1+x^2)),x)')</pre> <pre>result in mupad_4.txt</pre>

xcas

```
int(log(1+x*sqrt(1+x^2)),x)
```

$$\frac{\operatorname{rootof}([[1, -1, 3, -27], [1, 0, -6, -40, 29]]) \ln\left|x - \sqrt{x^2 - 1}\right|}{64} + \frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]]) \ln\left|x + \sqrt{x^2 - 1}\right|}{64} + \frac{\operatorname{rootof}([[1, -1, -9, -23], [1, 0, -14, -40, -11]]) \operatorname{atan}\left(\frac{\sqrt{x^2 - 1}}{x}\right)}{16} + \frac{\operatorname{rootof}([[-1, 2, 7], [1, 0, -14, -40, -11]]) \ln\left|\sqrt{x^2 - 1}\right|}{32} + \frac{\operatorname{rootof}([[1, -2, -7], [1, 0, -14, -40, -11]]) \ln\left|\sqrt{x^2 - 1}\right|}{32} + \frac{\operatorname{rootof}([[1, 3, -5, -39], [1, 0, -6, -40, 29]]) \operatorname{atan}\left(\frac{\sqrt{x^2 - 1}}{x}\right)}{64} - 2 \operatorname{atan}\left(\frac{\sqrt{x^2 - 1}}{x}\right)$$

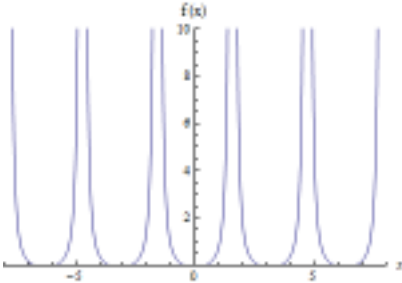
$$5 \int \frac{\cos^2(x)}{\sqrt{\cos^4(x) + \cos^2(x) + 1}} dx$$

<p>optimal</p> 	$\frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{\sin(x) \cos(x) (\cos^2(x)+1)}{\sqrt{\cos^4(x)+\cos^2(x)+1} \cos^2(x)+1} \right)$
<p>M 9.01</p>	$\text{Integrate} \left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}, x \right]$ $- \left(2 i \text{Cos}[x]^2 \text{EllipticPi} \left[\frac{3}{2} + \frac{i \sqrt{3}}{2}, i \text{ArcSinh} \left[\sqrt{-\frac{2 i}{-3 i + \sqrt{3}}} \text{Tan}[x] \right], \frac{3 i}{3 i + \sqrt{3}} \right] \right.$ $\left. \sqrt{1 - \frac{2 i \text{Tan}[x]^2}{-3 i + \sqrt{3}}} \sqrt{1 + \frac{2 i \text{Tan}[x]^2}{3 i + \sqrt{3}}} \right) / \left(\sqrt{-\frac{i}{-3 i + \sqrt{3}}} \sqrt{15 + 8 \text{Cos}[2 x]} \right)$
<p>Rubi 4.1</p>	$\text{Int} \left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}, x \right]$ $\text{EllipticPi} \left[\frac{1}{2} (3 - i \sqrt{3}), \text{ArcSin} \left[\sqrt{-\frac{2 i}{3 i + \sqrt{3}}} \text{Tan}[x] \right], \frac{3 i + \sqrt{3}}{3 i - \sqrt{3}} \right]$ $\frac{3 \sqrt{-\frac{2 i}{3 i + \sqrt{3}}} (1 + \text{Tan}[x]^2) \sqrt{1 + \frac{2 \text{Tan}[x]^2}{3 - i \sqrt{3}}} \sqrt{1 + \frac{2 \text{Tan}[x]^2}{3 + i \sqrt{3}}}}{\sqrt{15 + 8 \text{Cos}[2 x]}}$

Maple 18	<pre>> restart; int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x);</pre> $- \left(2\sqrt{-(\cos(2x)^2 + 4\cos(2x) + 7)(\cos(2x)^2 - 1)} (-3 + I\sqrt{3}) \sqrt{\frac{(-1 + I\sqrt{3})(\cos(2x) - 1)}{(-3 + I\sqrt{3})(\cos(2x) + 1)}} (\cos(2x) + 1) \right.$ $2 \sqrt{\frac{\cos(2x) + 2 + I\sqrt{3}}{(I\sqrt{3} + 3)(\cos(2x) + 1)}} \sqrt{\frac{I\sqrt{3} - \cos(2x) - 2}{(-3 + I\sqrt{3})(\cos(2x) + 1)}} \text{EllipticE}$ $\sqrt{\frac{(-1 + I\sqrt{3})(\cos(2x) - 1)}{(-3 + I\sqrt{3})(\cos(2x) + 1)}}, \frac{-3 + I\sqrt{3}}{-1 + I\sqrt{3}}, \sqrt{\frac{(1 + I\sqrt{3})(-3 + I\sqrt{3})}{(I\sqrt{3} + 3)(-1 + I\sqrt{3})}}$ $+ I\sqrt{3})$ $\sqrt{(\cos(2x) - 1)(\cos(2x) + 1)(\cos(2x) + 2 + I\sqrt{3})(I\sqrt{3} - \cos(2x))} \cdot$ $\sqrt{\cos(2x)^2 + 4\cos(2x) + 7})$
Sage 5.4	<pre>integral(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x)</pre> $\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$
Fricas 1.2	<pre>(16) -> integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1), x) >> Error detected within library code: Imaginary part is nonzero. Cannot retract. (16) -> </pre>
sympy 0.7.1	<pre>(waited one hr) >>> >>> integrate(cos(x)**2/sqrt(cos(x)**4 + cos(x)**2 + 1), x) ^C ^C ^C^C^C Killed > ></pre>

Axiom 5/12	<pre>(9) -> (9) -> integrate(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x) (9) -> (9) atan 5 4 3 (cos(x) + 2cos(x) + cos(x))sin(x) * +-----+ 4 2 16cos(x) + 16cos(x) + 16 ----- 4 3 2 \ cos(x) + 4cos(x) + 6cos(x) + 4cos(x) + 1 / 6 4cos(x) - 2 / 6 Type: Union(Expression Integer,...) (10) -> (10) -> █</pre>
Maxima 5.28.02	<pre>integrate(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x);</pre> $\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$
reduce 2008	<pre>int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x);</pre> $\int \frac{\sqrt{\cos(x)^4 + \cos(x)^2 + 1} \cos(x)}{\cos(x)^4 + \cos(x)^2 + 1} dx$
mupad 2013a	<pre>evalin(symengine,'int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x)')</pre> <pre>int(cos(x)^2/(cos(x)^4 + cos(x)^2 + 1)^(1/2), x)</pre>
xcas	<pre>int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x)</pre> $\int \frac{\cos(x)}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$

$$6 \quad \int \tan(x) \sqrt{1 + \tan^4(x)} dx$$

<p>optimal</p> 	$\frac{1}{2} \sqrt{\tan^4(x) + 1} - \frac{\tanh^{-1}\left(\frac{1 - \tan^2(x)}{\sqrt{2}\sqrt{\tan^4(x) + 1}}\right)}{\sqrt{2}} - \frac{1}{2} \sinh^{-1}(\tan^2(x))$
M 9.01	click to view
Rubi 4.1	$\text{Int}\left[\text{Tan}[x] \sqrt{1 + \text{Tan}[x]^4}, x\right]$ $-\frac{1}{2} \text{ArcSinh}[\text{Tan}[x]^2] - \frac{\text{ArcTanh}\left[\frac{1 - \text{Tan}[x]^2}{\sqrt{2} \sqrt{1 + \text{Tan}[x]^4}}\right]}{\sqrt{2}} + \frac{1}{2} \sqrt{1 + \text{Tan}[x]^4}$
Maple 18	<pre>> restart; int(tan(x)*sqrt(1+tan(x)^4), x);</pre> $\frac{1}{2} \sqrt{(1 + \tan(x)^2)^2 - 2 \tan(x)^2} - \frac{1}{2} \text{arcsinh}(\tan(x)^2)$ $- \frac{1}{2} \sqrt{2} \text{arctanh}\left(\frac{1}{4} \frac{(-2 \tan(x)^2 + 2) \sqrt{2}}{\sqrt{(1 + \tan(x)^2)^2 - 2 \tan(x)^2}}\right)$
Sage 5.4	<pre>integral(tan(x)*sqrt(1+tan(x)^4), x)</pre> $\int \sqrt{\tan(x)^4 + 1} \tan(x) dx$

Fricas 1.2	<pre>(21) -> integrate(tan(x)*sqrt(1+tan(x)^4), x) (21) +-----+ 4 2 2log(\ tan(x) + 1 - tan(x)) + +-----+ +-+ 2 +-+ 4 4 \ 2 log(-----) \ tan(x) + 1 + 3tan(x) - 2tan(x) 4 2 tan(x) + 2tan(x) + 1 + +-----+ 4 2\ tan(x) + 1 / 4 Type: Union(Expression(Integer (22) -> (23) -></pre>
sympy 0.7.1	<pre>>>> >>> integrate(tan(x)* sqrt(1+tan(x)**4),x) Integral((tan(x)**4 + 1)**(1/2)*tan(x), x) >>> >>></pre>

Axiom 5/12


```

(10) ->
(10) -> integrate( tan(x)*sqrt(1+tan(x)^4),x)
(10) ->
(10)
-
      +-+
      \|2
      *
      log
      +-----+
      +-+      4      +-+      2      | 2cos(x)  - 2cos
      (4\|2 cos(x)  - 2\|2 cos(x) )  |-----+
      \|                               |
      +
      4      2
      8cos(x)  - 8cos(x)  + 3
      +
      log
      +-----+
      4      2      | 2cos(x)  - 2cos(x)  + 1
      (2cos(x)  - 2cos(x) )  |-----+
      \|                               |
      4
      cos(x)
      +
      2
      - 4cos(x)  + 2
      /
      4
      cos(x)
      +
      +-----+
      4      2      | 2cos(x)  - 2cos(x)  + 1
      |-----+
      4
      cos(x)
      /
      4
Type: Union(Expression Integer,...)
(11) ->
(11) ->
(11) ->

```

Maxima 5.28.02	<pre>integrate(tan(x)*sqrt(1+tan(x)^4),x);</pre> $\int \tan(x) \sqrt{\tan(x)^4 + 1} dx$
reduce 2008	<pre>int(tan(x)*sqrt(1+tan(x)^4),x);</pre> $\int \sqrt{\tan(x)^4 + 1} \tan(x)$
mupad 2013a	<pre>evalin(symengine,'int(tan(x)*sqrt(1+tan(x)^4),x)')</pre> <pre>int(tan(x)*(tan(x)^4 + 1)^(1/2), x)</pre>
xcas	<pre>int(tan(x)*sqrt(1+tan(x)^4),x)</pre> $\frac{\sqrt{4*\left(\frac{\tan(x)}{2}\right)^2 + 1}}{2} + 2*\left(\frac{\ln\left(\sqrt{4*\left(\frac{\tan(x)}{2}\right)^2 + 1} - \frac{2*\tan(x)}{2}\right)}{4} + \frac{1}{2}\right)$

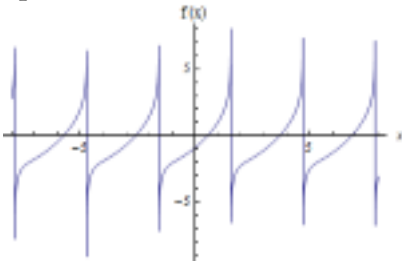
$$7 \int \frac{\tan(x)}{\sqrt{\sec^3(x) + 1}} dx$$

<p>optimal</p> 	$-\frac{2}{3} \tanh^{-1} \left(\sqrt{\sec^3(x) + 1} \right)$
<p>M 9.01</p>	<pre>In[23]:= Integrate[$\frac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3 + 1}}$, x]</pre> $\text{Out[23]} = - \left(i \cos[x]^2 \left(\text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{3} \sqrt{\frac{i \cos[x] \sec\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}} \right], \frac{3i - \sqrt{3}}{3i + \sqrt{3}} \right] \right. \right.$ $\left. \left. \text{EllipticPi} \left[\frac{1}{6} (3 + i\sqrt{3}), i \text{ArcSinh} \left[\sqrt{3} \sqrt{\frac{i \cos[x] \sec\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}} \right] \right] \right.$ $\left. \sqrt{(4 + 3 \cos[x] + \cos[3x]) \sec[x]^3} \sqrt{\frac{\sqrt{3} - 3i \tan\left[\frac{x}{2}\right]^2}{-3i + \sqrt{3}}} \sqrt{\frac{\sqrt{3} + 3i}{3i}} \right.$ $\left. \left. \left(\sqrt{3} \sqrt{\frac{\cos[x] \sec\left[\frac{x}{2}\right]^2}{-3 - i\sqrt{3}}} \left(1 + 3 \tan\left[\frac{x}{2}\right]^4 \right) \right) \right)$
<p>Rubi 4.1</p>	<pre>In[22]:= Int[$\frac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3 + 1}}$, x]</pre> $\text{Out[22]} = -\frac{2}{3} \text{ArcTanh} \left[\sqrt{1 + \text{Sec}[x]^3} \right]$
<p>Maple 18</p>	<pre>restart; int(tan(x)/sqrt(sec(x)^3+1), x);</pre> $-\frac{2}{3} \text{arctanh} \left(\sqrt{\sec(x)^3 + 1} \right)$

Sage 5.4	<pre>integral(tan(x)/sqrt(sec(x)^3+1), x)</pre> $\frac{1}{3} \log\left(\sqrt{\frac{1}{\cos(x)^3} + 1} - 1\right) - \frac{1}{3} \log\left(\sqrt{\frac{1}{\cos(x)^3} + 1} + 1\right)$
Fricas 1.2	<pre>(17) -> (17) -> integrate(tan(x)/sqrt(sec(x)^3+1), x)</pre> $\log\left(\frac{-\cos(x)^3 - \cos(x)^2 \sqrt{4\cos(x)^2 - 4\cos(x) + 4}}{\cos(x)^2 + \cos(x)}\right)$ <pre>(17) ----- Type: Union(Expression(Integer), ...)</pre> <pre>(18) -> █</pre>
sympy 0.7.1	<pre>>>> >>> integrate(tan(x)/sqrt(sec(x)**3+1),x) Traceback (most recent call last): File "<stdin>", line 1, in <module> NameError: name 'sec' is not defined >>></pre>
Axiom 5/12	<pre>(11) -> (11) -> integrate(tan(x)/sqrt(sec(x)^3+1),x) (11) -> (11)</pre> $\frac{\log\left(\frac{(-4\cos(x)^6 - 4\cos(x)^5 - 2\cos(x)^3 - 2\cos(x)^2) \sqrt{4\cos(x)^2 - 4\cos(x) + 4}}{8\cos(x)^6 + 8\cos(x)^3 + 1}\right)}{6}$ <pre>(12) -> █</pre>

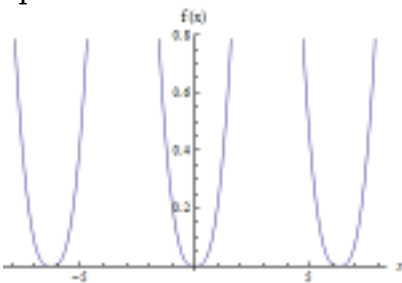
Maxima 5.28.02	<pre>integrate(tan(x)/sqrt(sec(x)^3+1), x);</pre> $\frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}-1\right)}{3} - \frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)}{3}$
reduce 2008	<pre>int(tan(x)/sqrt(sec(x)^3+1), x);</pre> $\int \frac{\sqrt{\sec(x)^3 + 1} \tan(x)}{\sec(x)^3 + 1} dx$
mupad 2013a	<pre>evalin(symengine, 'int(tan(x)/sqrt(sec(x)^3+1), x)')</pre> $\text{int}(\tan(x)/(1/\cos(x)^3 + 1)^{(1/2)}, x)$
xcas	<pre>int(tan(x)/sqrt(sec(x)^3+1), x)</pre> <p>Warning, integration of abs or sign assumes constant sign by intervals (</p> <p>Check Vector [abs(cos(x))]</p> <p>Discontinuities at zeroes of cos(x) were not checked</p> $\int \frac{1}{\sqrt{\cos(x)^4 + \cos(x)}} * \left(\frac{1}{\cos(x)}\right)^2 * \left(\frac{1}{\sin(x)}\right)$

$$8 \int \sqrt{\tan^2(x) + 2 \tan(x) + 2} dx$$

<p>optimal</p> 	$\sqrt{\frac{1}{2}(1 + \sqrt{5})} \tan^{-1} \left(\frac{\sqrt{1 + \sqrt{5}} \tan(x) - \sqrt{\sqrt{5} - 1}}{\sqrt{2} \sqrt{\tan(x)(\tan(x) + 2) + 2}} \right) - \sqrt{\frac{1}{2}(\sqrt{5} - 1)} \tanh^{-1} \left(\frac{\sqrt{\sqrt{5} - 1}}{\sqrt{2} \sqrt{\tan(x)(\tan(x) + 2) + 2}} \right) + \sinh^{-1}(\tan(x) + 1)$
M 9.01	click to view
Rubi 4.1	$\text{Int} \left[\sqrt{\text{Tan}[x]^2 + 2 \text{Tan}[x] + 2}, x \right]$ $\text{ArcSinh}[1 + \text{Tan}[x]] - \frac{1}{2} i \sqrt{1 - 2 i} \text{ArcTanh} \left[\frac{(4 - 2 i) + \sqrt{1 - 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}}{2 \sqrt{1 - 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}} \right] + \frac{1}{2} i \sqrt{1 + 2 i} \text{ArcTanh} \left[\frac{(4 + 2 i) + (2 + 2 i) \text{Tan}[x]}{2 \sqrt{1 + 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}} \right]$
Maple 18	image
Sage 5.4	<pre>integral(sqrt(tan(x)^2+2*tan(x)+2), x) Traceback (click to the left of this b ... RuntimeError: ECL says: Error executin cannot be imaginary; found %i</pre>
Fricas 1.2	<pre>(18) -> (18) -> integrate(sqrt(tan(x)^2+2*tan(x)+2), x) x +-----+ ++ 2 (18) \ tan(%A) + 2tan(%A) + 2 d%A ++ Type: Union(Expression(Integer),...) (19) -> █</pre>

sympy 0.7.1	<pre>>>> >>> integrate(sqrt(tan(x)**2 + 2 *tan(x)+2),x) Integral((tan(x)**2 + 2*tan(x) + 2)**(1/2), x) >>> █</pre>
Axiom 5/12	HTML
Maxima 5.28.02	<pre>(%i12) integrate(sqrt(tan(x)^2+2*tan(x)+2),x); sign: argument cannot be imaginary; found %i -- an error. To debug this try: debugmode(true)</pre>
reduce 2008	<pre>int(sqrt(tan(x)^2+2*tan(x)+2),x);</pre> $\int \sqrt{\tan(x)^2 + 2\tan(x) + 2} dx$
mupad 2013a	<pre>evalin(symengine,'int(sqrt(tan(x)^2+2*tan(x)+2),x)') int((2*tan(x) + tan(x)^2 + 2)^(1/2), x)</pre>
xcas	<pre>int(sqrt(tan(x)^2+2*tan(x)+2),x)</pre> $\frac{\left(8^{\frac{1}{4}} + 8 \cdot 8^{\frac{1}{4}}\right) \ln\left(\frac{\sqrt{\tan(x)^2 + 2\tan(x) + 2} - \tan(x)}{2}\right) + \left(-8^{\frac{1}{4}}\right) \cdot \left(\sqrt{2}\right)}{64}$ $\frac{\left(-8^{\frac{1}{4}}\right) - 8 \cdot \left(\sqrt{2}\right) + 8 \cdot 8^{\frac{1}{4}}}{32} \operatorname{atan}\left(\frac{\sqrt{\tan(x)^2 + 2\tan(x) + 2} - \tan(x)}{8^{\frac{1}{4}}}\right)$ $\frac{\left(-8^{\frac{1}{4}}\right) - 8 \cdot 8^{\frac{1}{4}}}{64} \ln\left(\frac{\sqrt{\tan(x)^2 + 2\tan(x) + 2} - \tan(x)}{2}\right) + 8^{\frac{1}{4}} \cdot \left(\sqrt{2}\right)$ $\frac{\left(-8^{\frac{1}{4}}\right) + 8 \cdot \left(\sqrt{2}\right) + 8 \cdot 8^{\frac{1}{4}}}{32} \operatorname{atan}\left(\frac{\sqrt{\tan(x)^2 + 2\tan(x) + 2} - \tan(x)}{8^{\frac{1}{4}}}\right)$ $2 \cdot \left(\frac{\left(-8^{\frac{1}{4}}\right) + 8 \cdot \left(\sqrt{2}\right) + 8 \cdot 8^{\frac{1}{4}}}{32}\right)$

$$9 \quad \int \sin(x) \arctan \left(\sqrt{\sec(x) - 1} \right) dx$$

<p>optimal</p> 	$\frac{1}{2} \cos(x) \sqrt{\sec(x) - 1} + \frac{1}{2} \tan^{-1} \left(\sqrt{\sec(x) - 1} \right) - \cos(x) \tan^{-1} \left(\sqrt{\sec(x) - 1} \right)$
M 9.01	click to view
Rubi 4.1	<pre>Int[Sin[x] ArcTan[Sqrt[Sec[x] - 1]], x]</pre> $\frac{1}{2} \text{ArcTan} \left[\sqrt{-1 + \text{Sec}[x]} \right] - \text{ArcTan} \left[\sqrt{-1 + \text{Sec}[x]} \right] \text{Cos}[x] +$
Maple 18	<pre>restart; int(sin(x)*arctan(sqrt(sec(x)-1)), x);</pre> $\int \sin(x) \arctan(\sqrt{\sec(x) - 1}) dx$
Sage 5.4	<pre>integral(sin(x)*arctan(sqrt(sec(x)-1)), x)</pre> $-\cos(x) \arctan \left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}} \right) - \frac{\sqrt{-\frac{\cos(x)-1}{\cos(x)}}}{2 \left(\frac{\cos(x)-1}{\cos(x)} - 1 \right)} + \frac{1}{2} \arctan \left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}} \right)$

Fricas 1.2	<pre>(19) -> (19) -> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) >> Error detected within library code: failed - cannot handle that integrand (19) -> █</pre> <p>update: per post on sci.math.symbolic on June BTW: Current development FriCAS can also do #9:</p> <pre>(3) -> integrate(sin(x)*atan(sqrt(sec(x) - 1)), x) (3)</pre> $\frac{\cos(x) \sqrt{\cos(x)} \operatorname{atan}\left(\sqrt{\sec(x) - 1}\right) + \operatorname{atan}\left(\frac{\sqrt{\cos(x)}}{\cos(x) - 1}\right) + \cos(x) \operatorname{atan}\left(\frac{\sqrt{\cos(x) + 1}}{\sqrt{\cos(x)}}\right)}{2}$ <p style="text-align: right;">Type: Union(Expression Integer,...)</p>
sympy 0.7.1	<pre>>>> >>> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) Traceback (most recent call last): File "<stdin>", line 1, in <module> NameError: name 'sec' is not defined >>></pre>
Axiom 5/12	<pre>(13) -> (13) -> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) (13) -> >> Error detected within library code: failed - cannot handle that integrand (13) -> █</pre>

Maxima 5.28.02	<pre>integrate(sin(x)*atan(sqrt(sec(x)-1)),x);</pre> $-\cos(x) \operatorname{atan}\left(\sqrt{\frac{\cos(x)-1}{\cos(x)}}\right) + \frac{\operatorname{atan}\left(\sqrt{\frac{\cos(x)-1}{\cos(x)}}\right)}{2} + \frac{\sqrt{\frac{\cos(x)-1}{\cos(x)}}}{2 - \frac{\cos(x)-1}{\cos(x)}}$
reduce 2008	<pre>int(sin(x)*atan(sqrt(sec(x)-1)),x);</pre> $\int \arctan\left(\sqrt{\sec(x)-1}\right) dx$
mupad 2013a	<pre>evalin(symengine,'int(sin(x)*atan(sqrt(sec(x)-1)),x)') pretty(ans)</pre> $-\operatorname{atan}\left(\sqrt{\frac{1}{\cos(x)}-1}\right) + \frac{\sqrt{\frac{1}{\cos(x)}-1}}{2\cos(x)} + \frac{\operatorname{asin}\left(\sqrt{\frac{1}{\cos(x)}-1}\right)}{3\cos(x)}$

xcas

```
int(sin(x)*atan(sqrt(sec(x)-1)) , x)
```

Warning, integration of abs or sign assumes constant sign by intervals (

Check Vector [abs(cos(x))]

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

Simplification assuming x near 0+

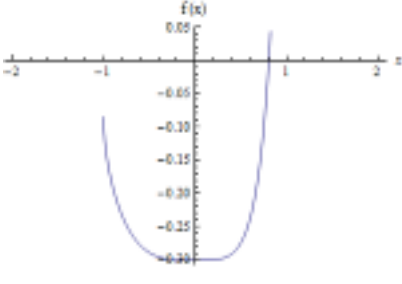
Simplification assuming x near 0+

Discontinuities at zeroes of cos(x) were not checked

Evaluation time: 11.933

$$\begin{aligned}
 & -\operatorname{atan}\left(\frac{\sqrt{-\cos(x)^2+\cos(x)\operatorname{sign}(\cos(x))\cos(x)}}{\cos(x)^2}\right)\cos(x) + 2(-\operatorname{sign}(\cos(x))) \\
 & \frac{1}{(\operatorname{sign}(\cos(x)))^2-1} \cdot \frac{1}{2} \cdot \frac{1}{2}\operatorname{asin}(2\cdot-\cos(x)+1) + 2(\operatorname{sign}(\cos(x)))^3 \cdot \frac{1}{(\operatorname{sign}(\cos(x)))^2-1} \\
 & \frac{1}{\operatorname{abs}(\operatorname{sign}(\cos(x)))}\operatorname{atan}\left(\frac{(\operatorname{sign}(\cos(x)))^2(2\sqrt{-\cos(x)^2+\cos(x)-1})-(\operatorname{sign}(\cos(x)))^2+2\sqrt{-\cos(x)^2+\cos(x)-1}}{-2\cdot-\cos(x)-1}\right) \frac{1}{\operatorname{abs}(\operatorname{sign}(\cos(x)))\cdot 2}
 \end{aligned}$$

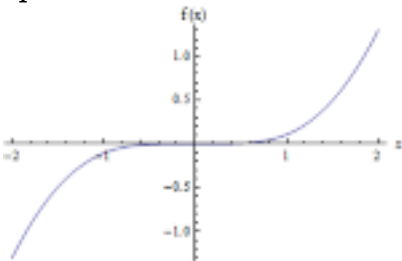
$$10 \int \frac{x^3 e^{\arcsin(x)}}{\sqrt{1-x^2}} dx$$

<p>optimal</p> 	$\frac{1}{10} (x^3 - 3\sqrt{1-x^2}x^2 - 3\sqrt{1-x^2} + 3x) e^{\sin^{-1}(x)}$
<p>M 9.01</p>	$\text{Integrate} \left[\frac{x^3 \text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}, x \right]$ $-\frac{1}{40} e^{\text{ArcSin}[x]} \left(15 \left(-x + \sqrt{1-x^2} \right) - 3 \text{Cos}[3 \text{ArcSin}[x]] \right) + \text{Sin}$
<p>Rubi 4.1</p>	$\text{Int} \left[\frac{x^3 \text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}, x \right]$ $\frac{3}{10} e^{\text{ArcSin}[x]} x + \frac{1}{10} e^{\text{ArcSin}[x]} x^3 - \frac{3}{10} e^{\text{ArcSin}[x]} \sqrt{1-x^2} - \frac{3}{10} e^{\text{ArcSin}[x]}$
<p>Maple 18</p>	<pre>> restart; int(x^3*exp(1)^arcsin(x)/sqrt(1-x^2), x); 1/10 (x - 3*sqrt(-x^2+1)) e^arcsin(x) x^2 + 3/10 e^arcsin(x) (x - sqrt(1-x^2))</pre>
<p>Sage 5.4</p>	<pre>integral(x^3*exp(arcsin(x))/sqrt(1-x^2),</pre> $\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{-x^2+1}} dx$

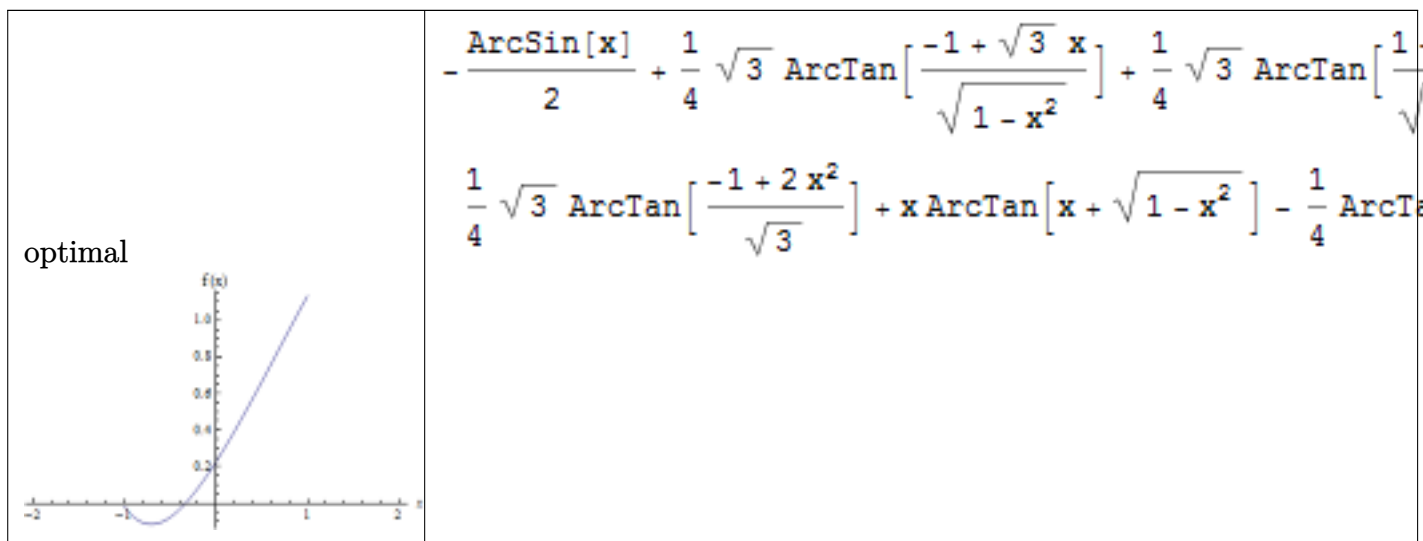
Fricas 1.2	<pre>(19) -> (19) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x) +-----+ 2 x\ - x + 1 - atan(-----) 2 x - 1 (19) ----- 2 2 3 ((- 3x - 3)\ - x + 1 + x + 3x)%e ----- 10 Type: Union(Expression(Integer),...) (20) -> (20) -> █</pre>
sympy 0.7.1	<pre>>>> >>> integrate(x**3*exp(asin(x))/sqrt(1-x**2),x) x**3*exp(asin(x))/10 - 3*x**2*(-x**2 + 1)**(1/2)*exp(asin(x))/10 + 3*x*exp(as)**(1/2)*exp(asin(x))/10 >>> >>></pre>
Axiom 5/12	<pre>(13) -> (13) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x) (13) -> +-----+ 2 2x\ - x + 1 atan(-----) 2 2x - 1 (13) ----- 2 2 3 ((- 3x - 3)\ - x + 1 + x + 3x)%e ----- 10 Type: Union(Expression(Integer),...) (14) -> █</pre>
Maxima 5.28.02	<pre>integrate(x^3*exp(asin(x))/sqrt(1-x^2),x);</pre> $\int \frac{x^3 e^{\operatorname{asin}(x)}}{\sqrt{1-x^2}} dx$
reduce 2008	<pre>int(x^3*exp(asin(x))/sqrt(1-x^2),x);</pre> $\int \frac{e^{\operatorname{arcsin}(x)} x^3}{\sqrt{-x^2+1}} dx$
mupad 2013a	<pre>evalin(symengine,'int(x^3*exp(asin(x))/sqrt(1-x^2),x)')</pre> $\operatorname{int}((x^3 \exp(\operatorname{asin}(x))) / (1 - x^2)^{(1/2)}, x)$

xcas	<pre>int(x^3*exp(asin(x))/sqrt(1-x^2),x)</pre> <p>Simplification assuming x near 0+</p> <p>Simplification assuming x near 0+</p> $\int x^3 * \frac{1}{\sqrt{1-x^2}}$
------	--

$$11 \quad \int \frac{x \log(1+x^2) \log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

<p>optimal</p> 	$4x - 2 \operatorname{ArcTan}[x] - 2\sqrt{1+x^2} \operatorname{Log}[x + \sqrt{1+x^2}] + \operatorname{Log}[1+x^2]$
M 9.01	<pre>Integrate[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]])/Sqrt[1+x^2], x]</pre> $4x - 2 \operatorname{ArcTan}[x] - 2\sqrt{1+x^2} \operatorname{Log}[x + \sqrt{1+x^2}] + \operatorname{Log}[1+x^2] \left(-x + \sqrt{1+x^2} \operatorname{Log}[x + \sqrt{1+x^2}] \right)$
Rubi 4.1	<pre>Int[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]])/Sqrt[1+x^2], x]</pre> $4x - 2 \operatorname{ArcTan}[x] - x \operatorname{Log}[1+x^2] - 2\sqrt{1+x^2} \operatorname{Log}[x + \sqrt{1+x^2}] + \sqrt{1+x^2} \operatorname{Log}[1+x^2] \operatorname{Log}[x + \sqrt{1+x^2}]$
Maple 18	<pre>> restart; intg:= x*log(1+x^2)*log(x+sqrt(1+x^2))/sqrt(1+x^2); intg := $\frac{x \ln(x^2 + 1) \ln(x + \sqrt{x^2 + 1})}{\sqrt{x^2 + 1}}$ > int(intg, x); $\int \frac{x \ln(x^2 + 1) \ln(x + \sqrt{x^2 + 1})}{\sqrt{x^2 + 1}} dx$</pre>

$$12 \quad \int \tan^{-1} \left(\sqrt{1-x^2} + x \right) dx$$



M 9.01

Integrate[ArcTan[x + Sqrt[1 - x^2]], x]

$$\begin{aligned}
& x \operatorname{ArcTan}\left[x + \sqrt{1 - x^2}\right] + \frac{1}{16} \left(-8 \operatorname{ArcSin}[x] + 2 \sqrt{2 + 2i\sqrt{3}} \right. \\
& \operatorname{ArcTan}\left[\frac{(1 + i\sqrt{3} - 2x^2)(-1 + x^2)}{(-3i - \sqrt{3} + 2\sqrt{3}x^4 + x^3(-6 - 2i\sqrt{3} - 2\sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}) + \right. \\
& \quad \left. x(6 + 2i\sqrt{3} - 2\sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}) + x^2(3i - \sqrt{3} + 2\sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2})\right)}] - 2\sqrt{2 + 2i\sqrt{3}} \\
& \operatorname{ArcTan}\left[\frac{(1 + i\sqrt{3} - 2x^2)(-1 + x^2)}{(-3i - \sqrt{3} + 2\sqrt{3}x^4 + 2x(-3 - i\sqrt{3} + \sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}) + \right. \\
& \quad \left. 2x^3(3 + i\sqrt{3} + \sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}) + x^2(3i - \sqrt{3} + 2\sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2})\right)}] - 2\sqrt{2 - 2i\sqrt{3}} \\
& \operatorname{ArcTan}\left[\frac{(-1 + x^2)(-1 + i\sqrt{3} + 2x^2)}{(3i - \sqrt{3} + 2\sqrt{3}x^4 + x(6 - 2i\sqrt{3} - 2\sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}) + x^3 \right. \\
& \quad \left. (-6 - 2i\sqrt{3} - 2\sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}) + x^2(-3i - \sqrt{3} + 2\sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2})\right)}] + 2\sqrt{2 - 2i\sqrt{3}} \\
& \operatorname{ArcTan}\left[\frac{(-1 + x^2)(-1 + i\sqrt{3} + 2x^2)}{(3i - \sqrt{3} + 2\sqrt{3}x^4 + 2x^3(3 - i\sqrt{3} + \sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}) + \right. \\
& \quad \left. 2x(-3 - i\sqrt{3} + \sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}) + x^2(-3i - \sqrt{3} - 2\sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2})\right)}] - \\
& 2 \operatorname{Log}\left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right] + 2i\sqrt{3} \operatorname{Log}\left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right] - 2 \operatorname{Log}\left[\frac{1}{2}i(i + \sqrt{3}) + x^2\right] - \\
& 2i\sqrt{3} \operatorname{Log}\left[\frac{1}{2}i(i + \sqrt{3}) + x^2\right] - \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log}\left[16(1 + \sqrt{3}x + x^2)^2\right] + \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log}\left[16(1 + \sqrt{3}x + x^2)^2\right] + \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log}\left[(4 - 4\sqrt{3}x + 4x^2)^2\right] - \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log}\left[(4 - 4\sqrt{3}x + 4x^2)^2\right] - \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log}\left[3i + \sqrt{3} - (-i + \sqrt{3})x^4 + 2i\sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2} + 5ix^2\left(2 + \sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}\right) + \right. \\
& \quad \left. x\left(3 + 5i\sqrt{3} - 3i\sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2}\right) + ix^3\left(3i + 3\sqrt{3} + \sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2}\right)\right] + \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log}\left[3i + \sqrt{3} - (-i + \sqrt{3})x^4 - 2i\sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2} + 5ix^2\left(2 + \sqrt{2 - 2i\sqrt{3}}\sqrt{1 - x^2}\right) + \right. \\
& \quad \left. x^3\left(3 - 3i\sqrt{3} - i\sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2}\right) - ix\left(-3i + 5\sqrt{3} + 3\sqrt{6 - 6i\sqrt{3}}\sqrt{1 - x^2}\right)\right] + \\
& i\sqrt{2 - 2i\sqrt{3}} \operatorname{Log}\left[-3i + \sqrt{3} - (i + \sqrt{3})x^4 - 2i\sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2} - 5ix^2\left(2 + \sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}\right) + \right. \\
& \quad \left. x\left(3 - 5i\sqrt{3} - 3i\sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2}\right) - ix^3\left(-3i + 3\sqrt{3} + \sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2}\right)\right] - \\
& i\sqrt{2 + 2i\sqrt{3}} \operatorname{Log}\left[-3i + \sqrt{3} - (i + \sqrt{3})x^4 - 2i\sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2} - 5ix^2\left(2 + \sqrt{2 + 2i\sqrt{3}}\sqrt{1 - x^2}\right) + \right. \\
& \quad \left. x^3\left(3 + 3i\sqrt{3} + i\sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2}\right) + ix\left(3i + 5\sqrt{3} + 3\sqrt{6 + 6i\sqrt{3}}\sqrt{1 - x^2}\right)\right] \Big)
\end{aligned}$$

Rubi 4.1	$\text{Int}[\text{ArcTan}[x + \text{Sqrt}[1 - x^2]], x]$ $-\frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{1 - 2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}} +$ $\frac{1}{12} (3i - \sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right] + \frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right]}{\sqrt{3}} -$ $\frac{1}{12} (3i + \sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right] + x \text{ArcTan}[x + \sqrt{1-x^2}] - \frac{1}{8} \text{Log}[1-x^2]$
Maple 18	<pre>> restart; intg:= arctan(sqrt(1-x^2)+x); int(intg,x);</pre> $\text{intg} := \arctan(\sqrt{-x^2+1} + x)$ $x \arctan(\sqrt{-x^2+1} + x) - \frac{1}{8} \ln(x^4 - x^2 + 1) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3}(2x^2 - 1)\sqrt{3}\right) + \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}\right)$ $+ \frac{(-1 - I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1) - \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{(1 + I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1\right)$ $+ \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{(-1 + I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1\right) - \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}\right)$ $+ \frac{(-1 + I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1) - \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{(1 + I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1\right)$ $+ \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{(1 - I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1\right) - \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2}\right)$ $+ \frac{(1 - I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1) + \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{(-1 - I\sqrt{3})(\sqrt{-x^2+1}-1)}{x} - 1\right)$ $+ \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$