# Charlwood Integration Problems

- **Problem #1**

$$\int \text{ArcSin}[x]\,\text{Log}[x]\,dx == -2\sqrt{1-x^2} + \text{ArcTanh}\left[\sqrt{1-x^2}\right] - x\,\text{ArcSin}[x]\,(1-\text{Log}[x]) + \sqrt{1-x^2}\,\text{Log}[x]$$

- **Problem #2**

$$\int \frac{x\,\text{ArcSin}[x]}{\sqrt{1-x^2}}\,dx == x - \sqrt{1-x^2}\,\text{ArcSin}[x]$$

- **Problem #3**

$$\int \text{ArcSin}\left[\sqrt{x+1} - \sqrt{x}\right]\,dx == \frac{\left(\sqrt{x} + 3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{4\sqrt{2}} - \left(\frac{3}{8} + x\right)\text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right]$$

- **Problem #4**

$$\int \text{Log}\left[1 + x\sqrt{1+x^2}\right]\,dx ==$$

$$-2x + \sqrt{2\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right] - \sqrt{2\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right] + x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

- **Problem #5**

$$\int \frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}\,dx == -\frac{\text{ArcSin}\left[\text{Cos}[x]^3\right]\sqrt{1-\text{Cos}[x]^6}\,\text{Csc}[x]}{3\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}$$

- **Problem #6**

$$\int \text{Tan}[x]\sqrt{1+\text{Tan}[x]^4}\,dx == -\frac{1}{2}\,\text{ArcSinh}\left[\text{Tan}[x]^2\right] - \frac{\text{ArcTanh}\left[\frac{1-\text{Tan}[x]^2}{\sqrt{2}\,\sqrt{1+\text{Tan}[x]^4}}\right]}{\sqrt{2}} + \frac{1}{2}\sqrt{1+\text{Tan}[x]^4}$$

■ **Problem #7**

$$\int \frac{\text{Tan}[x]}{\sqrt{1+\text{Sec}[x]^3}}\,dx = -\frac{2}{3}\,\text{ArcTanh}\left[\sqrt{1+\text{Sec}[x]^3}\right]$$

■ **Problem #8**

$$\int \sqrt{\text{Tan}[x]^2+2\,\text{Tan}[x]+2}\,dx =$$

$$\text{ArcSinh}[1+\text{Tan}[x]] + \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{-\sqrt{-1+\sqrt{5}}+\sqrt{1+\sqrt{5}}\,\text{Tan}[x]}{\sqrt{2}\,\sqrt{2+\text{Tan}[x]\,(2+\text{Tan}[x])}}\right] - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{5}}+\sqrt{-1+\sqrt{5}}\,\text{Tan}[x]}{\sqrt{2}\,\sqrt{2+\text{Tan}[x]\,(2+\text{Tan}[x])}}\right]$$

■ **Problem #9**

$$\int \text{Sin}[x]\,\text{ArcTan}\left[\sqrt{\text{Sec}[x]-1}\right]dx = \frac{1}{2}\,\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right] - \text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]\text{Cos}[x] + \frac{1}{2}\,\text{Cos}[x]\,\sqrt{-1+\text{Sec}[x]}$$

■ **Problem #10**

$$\int \frac{x^3\,e^{\text{ArcSin}[x]}}{\sqrt{1-x^2}}\,dx = \frac{1}{10}\,e^{\text{ArcSin}[x]}\left(3x+x^3-3\sqrt{1-x^2}-3x^2\sqrt{1-x^2}\right)$$

■ **Problem #11**

$$\int \frac{x\,\text{Log}\left[1+x^2\right]\text{Log}\left[x+\sqrt{1+x^2}\right]}{\sqrt{1+x^2}}\,dx = 4x-2\,\text{ArcTan}[x]-2\sqrt{1+x^2}\,\text{Log}\left[x+\sqrt{1+x^2}\right]+\text{Log}\left[1+x^2\right]\left(-x+\sqrt{1+x^2}\,\text{Log}\left[x+\sqrt{1+x^2}\right]\right)$$

■ **Problem #12**

$$\int \text{ArcTan}\left[x+\sqrt{1-x^2}\right]dx =$$

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]+$$

$$x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\,\text{ArcTanh}\left[x\sqrt{1-x^2}\right]-\frac{1}{8}\,\text{Log}\left[1-x^2+x^4\right]$$

- **Problem #13**

$$\int \frac{x \operatorname{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx ==$$

$$-\frac{\operatorname{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\operatorname{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\operatorname{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\operatorname{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-$$

$$\sqrt{1-x^2}\,\operatorname{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\operatorname{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\operatorname{Log}\left[1-x^2+x^4\right]$$

- **Problem #14**

$$\int \frac{\operatorname{ArcSin}[x]}{1+\sqrt{1-x^2}}\,dx == \frac{\left(-1+\sqrt{1-x^2}\right)\operatorname{ArcSin}[x]}{x}+\frac{\operatorname{ArcSin}[x]^2}{2}-\operatorname{Log}\left[1+\sqrt{1-x^2}\right]$$

- **Problem #15**

$$\int \frac{\operatorname{Log}\left[x+\sqrt{1+x^2}\right]}{\left(1-x^2\right)^{3/2}}\,dx == -\frac{1}{2}\operatorname{ArcSin}\left[x^2\right]+\frac{x\operatorname{Log}\left[x+\sqrt{1+x^2}\right]}{\sqrt{1-x^2}}$$

- **Problem #16**

$$\int \frac{\operatorname{ArcSin}[x]}{\left(1+x^2\right)^{3/2}}\,dx == \frac{x\operatorname{ArcSin}[x]}{\sqrt{1+x^2}}-\frac{\operatorname{ArcSin}\left[x^2\right]}{2}$$

- **Problem #17**

$$\int \frac{\operatorname{Log}\left[x+\sqrt{x^2-1}\right]}{\left(1+x^2\right)^{3/2}}\,dx == -\frac{1}{2}\operatorname{ArcCosh}\left[x^2\right]+\frac{x\operatorname{Log}\left[x+\sqrt{-1+x^2}\right]}{\sqrt{1+x^2}}$$

- **Problem #18**

$$\int \frac{\operatorname{Log}[x]}{x^2\sqrt{x^2-1}}\,dx == \frac{\sqrt{-1+x^2}}{x}-\operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]+\frac{\sqrt{-1+x^2}\,\operatorname{Log}[x]}{x}$$

- **Problem #19**

$$\int \frac{\sqrt{1+x^3}}{x}\,dx == \frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\,\text{ArcTanh}\left[\sqrt{1+x^3}\right]$$

- **Problem #20**

$$\int \frac{x\,\text{Log}\left[x+\sqrt{x^2-1}\right]}{\sqrt{x^2-1}}\,dx == -x+\sqrt{-1+x^2}\;\text{Log}\left[x+\sqrt{-1+x^2}\right]$$

- **Problem #21**

$$\int \frac{x^3\,\text{ArcSin}[x]}{\sqrt{1-x^4}}\,dx == \frac{x\sqrt{1-x^4}}{4\sqrt{1-x^2}} - \frac{1}{2}\sqrt{1-x^4}\;\text{ArcSin}[x] + \frac{1}{4}\,\text{ArcTanh}\left[\frac{x\sqrt{1-x^2}}{\sqrt{1-x^4}}\right]$$

- **Problem #22**

$$\int \frac{x^3\,\text{ArcSec}[x]}{\sqrt{x^4-1}}\,dx == -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\;x} + \frac{1}{2}\sqrt{-1+x^4}\;\text{ArcSec}[x] + \frac{1}{2}\,\text{ArcTanh}\left[\frac{\sqrt{1-\frac{1}{x^2}}\;x}{\sqrt{-1+x^4}}\right]$$

- **Problem #23**

$$\int \frac{x\,\text{ArcTan}[x]\,\text{Log}\left[x+\sqrt{1+x^2}\right]}{\sqrt{1+x^2}}\,dx == -x\,\text{ArcTan}[x] + \frac{1}{2}\,\text{Log}\left[1+x^2\right] + \sqrt{1+x^2}\;\text{ArcTan}[x]\,\text{Log}\left[x+\sqrt{1+x^2}\right] - \frac{1}{2}\,\text{Log}\left[x+\sqrt{1+x^2}\right]^2$$

- **Problem #24**

$$\int \frac{x\,\text{Log}\left[1+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx == \sqrt{1-x^2} - \text{Log}\left[1+\sqrt{1-x^2}\right] - \sqrt{1-x^2}\;\text{Log}\left[1+\sqrt{1-x^2}\right]$$

- **Problem #25**

$$\int \frac{x\,\text{Log}\left[x+\sqrt{1+x^2}\right]}{\sqrt{1+x^2}}\,dx == -x+\sqrt{1+x^2}\;\text{Log}\left[x+\sqrt{1+x^2}\right]$$

- **Problem #26**

$$\int \frac{x \operatorname{Log}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2}+\frac{\operatorname{ArcTanh}\left[\sqrt{2}\,x\right]}{\sqrt{2}}-\frac{\operatorname{ArcTanh}\left[\sqrt{2}\,\sqrt{1-x^2}\right]}{\sqrt{2}}-\sqrt{1-x^2}\,\operatorname{Log}\left[x+\sqrt{1-x^2}\right]$$

- **Problem #27**

$$\int \frac{\operatorname{Log}[x]}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x}-\operatorname{ArcSin}[x]-\frac{\sqrt{1-x^2}\,\operatorname{Log}[x]}{x}$$

- **Problem #28**

$$\int \frac{x \operatorname{ArcTan}[x]}{\sqrt{1+x^2}}\,dx = -\operatorname{ArcSinh}[x]+\sqrt{1+x^2}\,\operatorname{ArcTan}[x]$$

- **Problem #29**

$$\int \frac{\operatorname{ArcTan}[x]}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}\,\operatorname{ArcTan}[x]}{x}-\operatorname{ArcTanh}\left[\sqrt{1-x^2}\right]+\sqrt{2}\,\operatorname{ArcTanh}\left[\frac{\sqrt{1-x^2}}{\sqrt{2}}\right]$$

- **Problem #30**

$$\int \frac{x \operatorname{ArcTan}[x]}{\sqrt{1-x^2}}\,dx = -\operatorname{ArcSin}[x]-\sqrt{1-x^2}\,\operatorname{ArcTan}[x]+\sqrt{2}\,\operatorname{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right]$$

- **Problem #31**

$$\int \frac{\operatorname{ArcTan}[x]}{x^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{1+x^2}\,\operatorname{ArcTan}[x]}{x}-\operatorname{ArcTanh}\left[\sqrt{1+x^2}\right]$$

- **Problem #32**

$$\int \frac{\operatorname{ArcSin}[x]}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}\,\operatorname{ArcSin}[x]}{x}+\operatorname{Log}[x]$$

- **Problem #33**

$$\int \frac{x \, \text{Log}[x]}{\sqrt{x^2-1}} \, dx = -\sqrt{-1+x^2} + \text{ArcTan}\left[\sqrt{-1+x^2}\right] + \sqrt{-1+x^2} \, \text{Log}[x]$$

- **Problem #34**

$$\int \frac{\text{Log}[x]}{x^2 \sqrt{1+x^2}} \, dx = -\frac{\sqrt{1+x^2}}{x} + \text{ArcSinh}[x] - \frac{\sqrt{1+x^2} \, \text{Log}[x]}{x}$$

- **Problem #35**

$$\int \frac{x \, \text{ArcSec}[x]}{\sqrt{x^2-1}} \, dx = \frac{\left(-1+x^2\right) \text{ArcSec}[x]}{\sqrt{-1+x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \, x \, \text{Log}[x]}{\sqrt{-1+x^2}}$$

- **Problem #36**

$$\int \frac{x \, \text{Log}[x]}{\sqrt{1+x^2}} \, dx = -\sqrt{1+x^2} + \text{ArcTanh}\left[\sqrt{1+x^2}\right] + \sqrt{1+x^2} \, \text{Log}[x]$$

- **Problem #37**

$$\int \frac{\text{Sin}[x]}{1+\text{Sin}[x]^2} \, dx = -\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

- **Problem #38**

$$\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}} \, dx = \frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

- **Problem #39**

$$\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}} \, dx = \frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{\sqrt{2}}$$

## Problem #40

$$\int \frac{\text{Log}[\text{Sin}[x]]}{1+\text{Sin}[x]}\,dx == -x - \text{ArcTanh}[\text{Cos}[x]] - \frac{\text{Cos}[x]\,\text{Log}[\text{Sin}[x]]}{1+\text{Sin}[x]}$$

## Problem #41

$$\int \text{Log}[\text{Sin}[x]]\,\sqrt{1+\text{Sin}[x]}\,dx == \frac{4\,\text{Cos}[x]}{\sqrt{1+\text{Sin}[x]}} - \frac{2\,\text{Cos}[x]\,\text{Log}[\text{Sin}[x]]}{\sqrt{1+\text{Sin}[x]}} - 4\,\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{1+\text{Sin}[x]}}\right]$$

## Problem #42

$$\int \frac{\text{Sec}[x]}{\sqrt{\text{Sec}[x]^4-1}}\,dx == -\frac{\text{ArcTanh}\left[\frac{\sqrt{\text{Sec}[x]^4-1}}{\sqrt{2}\,\text{Sec}[x]\,\text{Tan}[x]}\right]}{\sqrt{2}}$$

## Problem #43

$$\int \frac{\text{Tan}[x]}{\sqrt{1+\text{Tan}[x]^4}}\,dx == -\frac{\text{ArcTanh}\left[\frac{1-\text{Tan}[x]^2}{\sqrt{2}\,\sqrt{1+\text{Tan}[x]^4}}\right]}{2\sqrt{2}}$$

## Problem #44

$$\int \frac{\text{Sin}[x]}{\sqrt{1-\text{Sin}[x]^6}}\,dx == \frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\text{Cos}[x]\,\left(1+\text{Sin}[x]^2\right)}{2\sqrt{1-\text{Sin}[x]^6}}\right]}{2\sqrt{3}}$$

- **Problem #45**

$$\int \sqrt{\sqrt{\text{Sec}[x]+1}-\sqrt{\text{Sec}[x]-1}}\,dx =$$

$$\sqrt{2}\,\text{Cot}[x]\sqrt{-1+\text{Sec}[x]}\,\sqrt{1+\text{Sec}[x]}\left(\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]\right.$$

$$-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]-\sqrt{1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]+$$

$$\left.\sqrt{-1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]\right)$$

- **Problem #46**

$$\int x\,\text{Log}\left[x^2+1\right]\text{ArcTan}[x]^2\,dx = x\,\text{ArcTan}[x]\left(3-\text{Log}\left[1+x^2\right]\right)-\frac{1}{4}\left(6-\text{Log}\left[1+x^2\right]\right)\text{Log}\left[1+x^2\right]-\frac{1}{2}\text{ArcTan}[x]^2\left(3+x^2-\left(1+x^2\right)\text{Log}\left[1+x^2\right]\right)$$

- **Problem #47**

$$\int \text{ArcTan}\left[x\sqrt{1+x^2}\,\right]dx = \frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{1+x^2}}{x^2}\right]+x\,\text{ArcTan}\left[x\sqrt{1+x^2}\,\right]+\frac{1}{2}\sqrt{3}\,\text{ArcTanh}\left[\frac{\sqrt{3}\,\sqrt{1+x^2}}{2+x^2}\right]$$

- **Problem #48**

$$\int \text{ArcTan}\left[\sqrt{x+1}-\sqrt{x}\,\right]dx = \frac{\sqrt{x}}{2}+(1+x)\,\text{ArcTan}\left[\sqrt{1+x}-\sqrt{x}\,\right]$$

- **Problem #49 (Note: The problem was altered so as to have an answer involving only elementary functions and operators.)**

$$\int \text{ArcSin}\left[\frac{x}{\sqrt{1-x^2}}\right]dx = x\,\text{ArcSin}\left[\frac{x}{\sqrt{1-x^2}}\right]+\text{ArcTan}\left[\sqrt{1-2x^2}\,\right]$$

- **Problem #50**

$$\int \text{ArcTan}\left[x\sqrt{1-x^2}\,\right]dx = x\,\text{ArcTan}\left[x\sqrt{1-x^2}\,\right]-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\sqrt{1-x^2}\,\right]+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\sqrt{1-x^2}\,\right]$$