sums of sin or cosine to one sin using Mathematica

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Introduction

sums of cosine terms, all with same frequency but can have different amplitude or phases is converted to a single sin term. The first step is to use TrigExpand to break sin or cos with phase to pure sin or cos. Next step is to collect terms based on the pure sin and the pure cos terms. This can be done either use Collect (see appendix) or using CoefficientList.

The input to the function given below is the expression to reduce with the independent variable. Validation of expression is done using Mr.Wizard pattern test (see Appendix).

Table is given showing examples of results. The result is single sin which you can convert to cosine by adding Pi/2 to the phase.

Mathematica notebook

PDF file

Function

```
ClearAll["Global`*"]
```

```
In[10]:= sumsReduce[e_, x_] := Module[{e1 = e, p2, h, b, a, head},
  p2 = # | Verbatim[Plus][#] &[(_. (h : Sin | Cos)[_]) ..];
  head = Replace[e1, {p2 :> h, _ → False}];

  (*parsing is done, now check the result*)
  If[head === False,
   e1,

   e1 = TrigExpand[e1];
   b = Last@CoefficientList[e1, Cos[x], 2];
   a = Last@CoefficientList[e1, Sin[x], 2];
   (*watch out for the atan2. Mathematica
    uses atan2(x,y) while Wikipedia uses atan2(y,x) *)
   Sqrt[a^2 + b^2] Sin[x + ArcTan[a, b]]
  ]
 ]
```

exampes

```
ClearAll[a, b, c, d, e, f, g, h, x];
sumsReduce[a Cos[x] + b Cos[x + b], x] // TraditionalForm
```

$$\sqrt{(a+b\cos(b))^2+b^2\sin^2(b)}\ \sin\left(\tan^{-1}(-b\sin(b),\ a+b\cos(b))+x\right)$$

```
sumsReduce[a Cos[x + b] + c Cos[x + d] + e Cos[x - f], x] // TraditionalForm
```

$$\sqrt{(-a\sin(b)-c\sin(d)+e\sin(f))^2+(a\cos(b)+c\cos(d)+e\cos(f))^2}\ \sin\left(\tan^{-1}(-a\sin(b)-c\sin(d)+e\sin(f),\ a\cos(b)+c\cos(d)+e\cos(f))+x\right)$$

```
sumsReduce[a Sin[x + b] + c Sin[x] + e Sin[x - f], x] // TraditionalForm
```

$$\sqrt{(a\cos(b)+c+e\cos(f))^2+(a\sin(b)-e\sin(f))^2}\ \sin\left(\tan^{-1}(a\cos(b)+c+e\cos(f),\ a\sin(b)-e\sin(f))+x\right)$$

```
sumsReduce[a Sin[x + b] + c Sin[x] + e Sin[x - f] + Sin[x - 2 f] + 3 g Sin[x - 2 d], x] //
 TraditionalForm
```

$$\sqrt{\left(\left(a\cos(b)+c-3\,g\sin^2(d)+3\,g\cos^2(d)+e\cos(f)-\sin^2(f)+\cos^2(f)\right)^2+(a\sin(b)-6\,g\sin(d)\cos(d)-e\sin(f)-2\sin(f)\cos(f))^2\right)}\ \sin\left(\tan^{-1}\left(a\cos(b)+c-3\,g\sin^2(d)+3\,g\cos^2(d)+e\cos(f)-\sin^2(f)+\cos^2(f),\ a\sin(b)-6\,g\sin(d)\cos(d)-e\sin(f)-2\sin(f)\cos(f)\right)+x\right)$$

Table

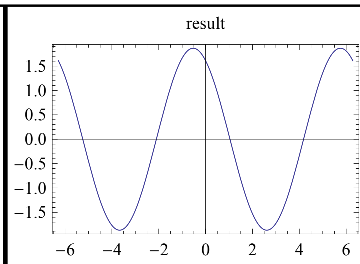
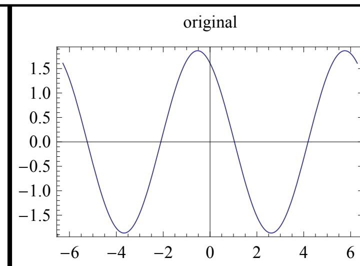
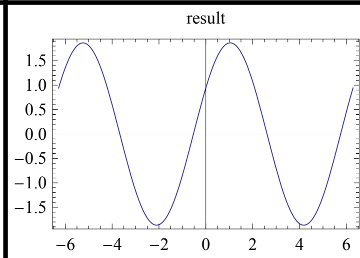
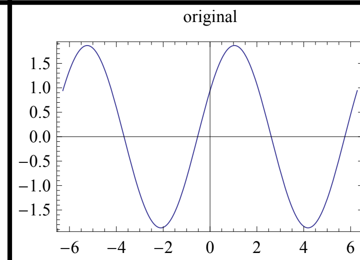
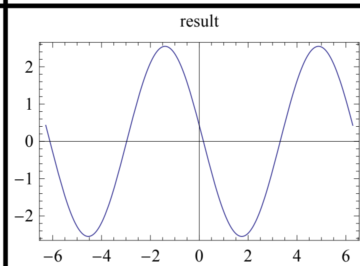
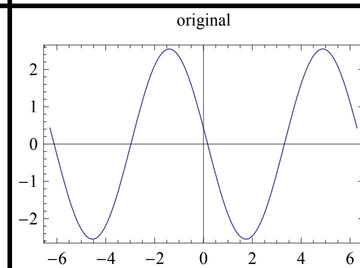
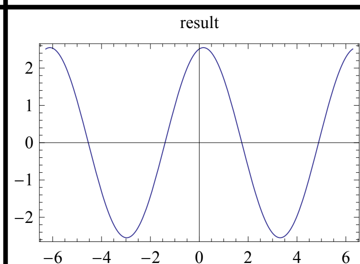
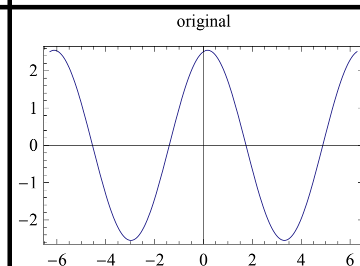
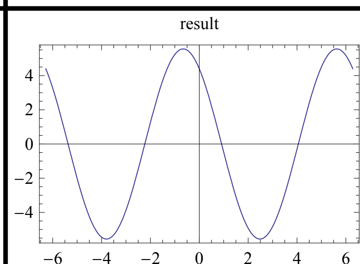
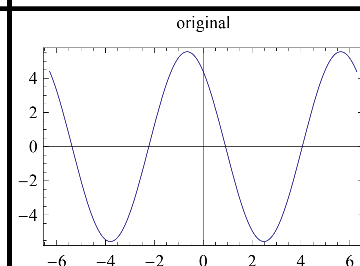
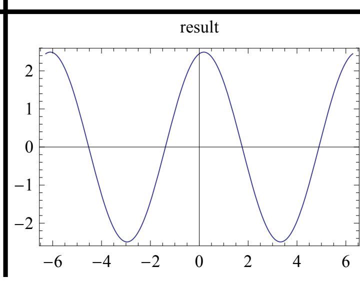
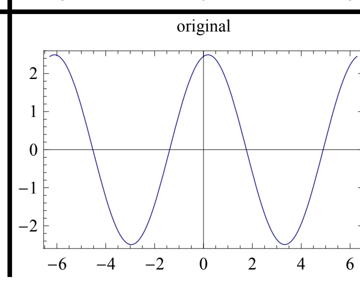
printout of table is here

```
In[11]:= parms = {a → 2. / 3, b → Pi / 4.};
Grid[{
  {
   Grid[{
     {expr = HoldForm[a Cos[x] + 2 a Cos[x + b]];
      Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
     {result = sumsReduce[ReleaseHold[expr], x];
      Row[{Style["reduced ", Bold, 14], result /. parms}]}
    }, Alignment → Left],
   Plot[result /. parms, {x, -2 Pi, 2 Pi},
    Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
   Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
    FrameLabel → {{None, None}, {None, "original"}}]
  },
  {
   Grid[{
     {expr = HoldForm[a Sin[x ] + 2 a Sin[x + b]];
      Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
     {result = sumsReduce[ReleaseHold[expr], x];
      Row[{Style["reduced ", Bold, 14], result /. parms}]}
    }, Alignment → Left],
   Plot[result /. parms, {x, -2 Pi, 2 Pi},
    Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
   Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
    FrameLabel → {{None, None}, {None, "original"}}]
  },
  {
   Grid[{
     {expr = HoldForm[a Cos[x ] + a Cos[x + b] + 2 a Cos[x + 2 b] + Cos[x + 3 b]];
      Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
     {result = sumsReduce[ReleaseHold[expr], x];
      Row[{Style["reduced ", Bold, 14], result /. parms}]}
    }, Alignment → Left],
   Plot[result /. parms, {x, -2 Pi, 2 Pi},
```

```
  Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
 Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
  FrameLabel → {{None, None}, {None, "original"}}]
},
{
 Grid[{
   {expr = HoldForm[a Sin[x] + a Sin[x + b] + 2 a Sin[x + 2 b] + Sin[x + 3 b]];
    Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
   {result = sumsReduce[ReleaseHold[expr], x];
    Row[{Style["reduced ", Bold, 14], result /. parms}]}
  }, Alignment → Left],
 Plot[result /. parms, {x, -2 Pi, 2 Pi},
  Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
 Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
  FrameLabel → {{None, None}, {None, "original"}}]
},
{
 Grid[{
   {expr = HoldForm[a Sin[x] + 2 a Sin[x + 2 b] + 3 a Sin[x + 3 b] + Sin[x - 4 b] - Sin[x - b] +
       2 a Sin[x - 5 b]]; Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
   {result = sumsReduce[ReleaseHold[expr], x];
    Row[{Style["reduced ", Bold, 14], result /. parms}]}
  }, Alignment → Left],
 Plot[result /. parms, {x, -2 Pi, 2 Pi},
  Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
 Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
  FrameLabel → {{None, None}, {None, "original"}}]
},
{
 Grid[{
   {expr = HoldForm[a Sin[x] - 2 a Sin[x - 2 b] + 4 a Sin[x - b / 10] + Sin[x - 4 b] - Sin[x - b / 2] +
       2 a Sin[x - 5 b]]; Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
   {result = sumsReduce[ReleaseHold[expr], x];
    Row[{Style["reduced ", Bold, 14], result /. parms}]}
  }, Alignment → Left],
 Plot[result /. parms, {x, -2 Pi, 2 Pi},
  Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
 Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
  FrameLabel → {{None, None}, {None, "original"}}]
},
{
 Grid[{
   {expr = HoldForm[a Cos[x] - 2 a Cos[x - 2 b] + 4 a Cos[x - b / 10] +
       Cos[x - 4 b] - Cos[x - b / 2] + 2 a Cos[x - 5 b] + Cos[x - 6 b] + 3 a Cos[x + 20 b]];
    Row[{Style["original ", Bold, 14], ReleaseHold@expr /. parms}]},
   {result = sumsReduce[ReleaseHold[expr], x];
    Row[{Style["reduced ", Bold, 14], result /. parms}]}
  }, Alignment → Left],
 Plot[result /. parms, {x, -2 Pi, 2 Pi},
  Frame → True, FrameLabel → {{None, None}, {None, "result"}}],
 Plot[ReleaseHold[expr] /. parms, {x, -2 Pi, 2 Pi}, Frame → True,
  FrameLabel → {{None, None}, {None, "original"}}]
```

```
  }
 }, Frame → All]
```

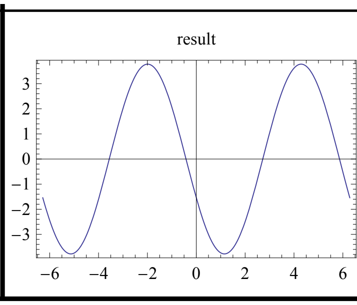
Out[12]=

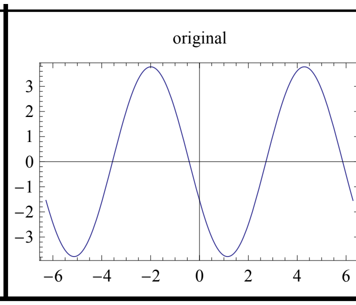
original 0.666667 Cos[x] + 1.33333 Cos[0.785398 + x] **reduced** 1.86529 Sin[2.1007 + x]		
original 0.666667 Sin[x] + 1.33333 Sin[0.785398 + x] **reduced** 1.86529 Sin[0.529903 + x]		
original 0.666667 Cos[x] + 0.666667 Cos[0.785398 + x] + 1.33333 Cos[1.5708 + x] + Cos[2.35619 + x] **reduced** 2.54855 Sin[2.97167 + x]		
original 0.666667 Sin[x] + 0.666667 Sin[0.785398 + x] + 1.33333 Sin[1.5708 + x] + Sin[2.35619 + x] **reduced** 2.54855 Sin[1.40088 + x]		
original Sin[0.785398 − x] − Sin[3.14159 − x] − 1.33333 Sin[3.92699 − x] + 0.666667 Sin[x] + 1.33333 Sin[1.5708 + x] + 2. Sin[2.35619 + x] **reduced** 5.55702 Sin[2.2286 + x]		
original −2.66667 Sin[0.0785398 − x] + Sin[0.392699 − x] + 1.33333 Sin[1.5708 − x] − Sin[3.14159 − x] − 1.33333 Sin[3.92699 − x] + 0.666667 Sin[x] **reduced** 2.49213 Sin[1.38579 + x]		

original

```
2.66667 Cos[0.0785398 - x] -
 Cos[0.392699 - x] -
 1.33333 Cos[1.5708 - x] +
 Cos[3.14159 - x] +
 1.33333 Cos[3.92699 - x] +
 Cos[4.71239 - x] + 0.666667 Cos[x] +
 2. Cos[15.708 + x]
```

reduced `-3.77839 Sin[2.72133 - x]`





References

1. Mr Wizard’s pattern matcher used in the above function http://mathematica.stackexchange.com/questions/30588/using-matchq-or-other-means-to-parse-an-expression-using-sums-of-cos-or-sin-co

2. Wikepedia article http://en.wikipedia.org/wiki/List_of_trigonometric _identities

3. Ray Koopman implementation using Collect http://forums.wolfram.com/mathgroup/archive/2009/Jun/msg00479.html