

```
ClearAll[x1, x2, θ1, θ2, t]
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$$T1 = \frac{1}{2} m1 (x1'[t]^2 + ((L + x1[t]) \theta1'[t])^2);$$

$$V1 = -m1 g (L + x1[t]) \text{Cos}[\theta1[t]] + 1/2 k1 x1[t]^2;$$

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T2 =
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$$\frac{1}{2} m2 ((x2'[t] + x1'[t] \text{Cos}[\theta1[t] - \theta2[t]])^2 + (x1'[t] \text{Sin}[\theta1[t] - \theta2[t]])^2) +$$

$$\frac{1}{2} m2 (((L + x2[t]) \theta2'[t] + ((L + x1[t]) \theta1'[t] \text{Cos}[\theta1[t] - \theta2[t])))^2 +$$

$$((L + x1[t]) \theta1'[t] \text{Sin}[\theta1[t] - \theta2[t]])^2);$$

$$V2 = -m2 g ((L + x1[t]) \text{Cos}[\theta1[t]] + (L + x2[t]) \text{Cos}[\theta2[t]]) + 1/2 k2 x2[t]^2;$$

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(*Lagrangian*)
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In[71]:= (lag = (T1 + T2) - (V1 + V2)) // Simplify
```

$$\text{Out[71]} = \frac{1}{2} \left( -k1 x1[t]^2 + 2 g m1 \text{Cos}[\theta1[t]] (L + x1[t]) - k2 x2[t]^2 + \right. \\ \left. 2 g m2 (\text{Cos}[\theta1[t]] (L + x1[t]) + \text{Cos}[\theta2[t]] (L + x2[t])) + \right. \\ \left. m2 (x1'[t]^2 + 2 \text{Cos}[\theta1[t] - \theta2[t]] x1'[t] x2'[t] + x2'[t]^2) + \right. \\ \left. m1 (x1'[t]^2 + (L + x1[t])^2 \theta1'[t]^2) + m2 (\text{Sin}[\theta1[t] - \theta2[t]]^2 (L + x1[t])^2 \theta1'[t]^2 + \right. \\ \left. (\text{Cos}[\theta1[t] - \theta2[t]] (L + x1[t]) \theta1'[t] + (L + x2[t]) \theta2'[t])^2 \right)$$

```
(*x1 *)
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In[72]:= (eq1 = D[D[lag, x1'[t]], t] - D[lag, x1[t]] == 0) // Simplify
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(*x2*)
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In[73]:= (eq2 = D[D[lag, x2'[t]], t] - D[lag, x2[t]] == 0) // Simplify
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```
(*theta 1*)
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In[75]:= (eq3 = D[D[lag, θ1'[t]], t] - D[lag, θ1[t]] == 0) // Simplify
```

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In[76]:= (eq4 = D[D[lag, θ2'[t]], t] - D[lag, θ2[t]] == 0) // Simplify
```

```
(*Numerically solve the equations of motion*)
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```
pars = {L -> 1, m1 -> 1, m2 -> 2, g -> 9.8, k1 -> 10, k2 -> 30};
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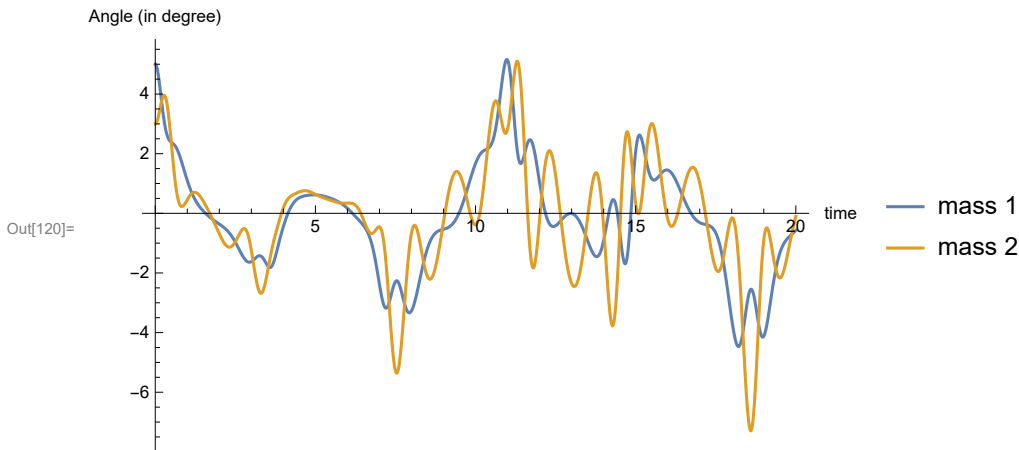
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ic = {θ1[0] == 5 Degree, θ1'[0] == 0, θ2[0] == 3 Degree,
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θ2'[0] == 0, x1[0] == 0, x1'[0] == 0, x2[0] == 0, x2'[0] == 0};
```

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eqs = Flatten[{eq1, eq2, eq3, eq4}] /. pars
```

```
In[122]:= numericalSolution = First@NDSolve[{eqs, ic}, {x1, x2, θ1, θ2}, {t, 0, 20}];
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```
In[120]:= Plot[Evaluate[({ $\theta_1[t]$ ,  $\theta_2[t]$ }) /. numericalSolution] * 180/Pi],  
  {t,  $\theta$ , 2 $\theta$ }, PlotRange -> All, AxesLabel -> {"time", "Angle (in degree)"},  
  ImageSize -> 400, PlotLegends -> {"mass 1", "mass 2"}]
```



```
In[121]:= Plot[Evaluate[{x1[t], x2[t]} /. numericalSolution], {t,  $\theta$ , 2 $\theta$ },  
  PlotRange -> All, AxesLabel -> {"time", "spring extensions in meters"},  
  ImageSize -> 400, PlotLegends -> {"mass 1", "mass 2"}]
```

