Mathematica function showing Reduced Row Echelon Form (RREF) steps

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Contents

1 Introduction and how to use

This describes how to use `RREF.nb` which contains one Mathematica function called `displayRREF` which displays on the screen all the steps in the forward Gaussian elimination process and also in the backward phase to produced a reduced row-echelon form when applied on a matrix A. The matrix can be square or rectangular.

To use, download the `RREF.nb` from the link given at the top. Open the notebook using Mathematica, and evaluate the whole notebook using `Evaluation->Evaluate Notebook`. Now the function `displayRREF` is loaded and ready to be used.

Open a new notebook to use the function. Examples of usage are given below.

It has an option to normalize the pivot to 1 or not. Also, it has an option to display each step or just show the final result.

Note that, pivot has to be noramlized to one if we are to obtain an RREF form, since that is the definition of RREF. But this option is there if needed for some other reasons.

There are two functions in the notebook. `displayRREF` which does reduced RREF, and `displayREF` which does only the forward Gaussian elimination phase.

Any bugs please let me know.

2 Examples showing how to use displayRREF

More examples are shown in the notebook.

2.1 Example 1. RREF on square matrix

Given the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

The RREF is

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

To see the steps do

```
mat = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}
displaymat = True; normalizePivot = True;
{result, pivots} = displayRREF[mat, displaymat, normalizePivot]
```

Note that the default is to display the steps and to also normalized the pivot to one. So the above command can be reduced to

```
mat = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}
{result, pivots} = displayRREF[mat]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Pivot is A(1,1)

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Zeroing out element A(2,1) using row(2)=4*row(1)-row(2)

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{pmatrix}$$

Zeroing out element A(3,1) using row(3)=7*row(1)-row(3)

$$\begin{pmatrix} \boxed{1} & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

Pivot is A(2,2)

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & \boxed{-3} & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

Making the pivot 1 using using row(2)= row(2)/A(2,2)

Zeroing out element A(3,2) using row(3)=-6*row(2)-row(3)

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & \boxed{0} \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}}

Zeroing out element A(1,2) using row(1)=row(1)-A(1,2)*row(2)

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Figure 1: Steps displayed by Mathematica

2.2 Example 2. RREF on rectangular matrix

Given the matrix

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 5 \\ 2 & 6 & 5 & 6 \end{pmatrix}$$

The RREF is

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

To see the steps do

```
mat = {{1, 2, 2, 4}, {1, 3, 3, 5}, {2, 6, 5, 6}};
{result, pivots} = displayRREF[mat]
```

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 1 & 3 & 3 & 5 \\ 2 & 6 & 5 & 6 \end{pmatrix}$$

Pivot is A(1,1)

$$\begin{pmatrix} \boxed{1} & 2 & 2 & 4 \\ 1 & 3 & 3 & 5 \\ 2 & 6 & 5 & 6 \end{pmatrix}$$

Zeroing out element A(2,1) using row(2)=1*row(1)-row(2)

$$\begin{pmatrix} \boxed{1} & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 2 & 6 & 5 & 6 \end{pmatrix}$$

Zeroing out element A(3,1) using row(3)=2*row(1)-row(3)

$$\begin{pmatrix} \boxed{1} & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 1 & -2 \end{pmatrix}$$

Pivot is A(2,2)

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 2 & 1 & -2 \end{pmatrix}$$

Zeroing out element A(3,2) using row(3)=2*row(2)-row(3)

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & -1 & -4 \end{pmatrix}$$

Pivot is A(3,3)

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \boxed{-1} & -4 \end{pmatrix}$$

Making the pivot 1 using using row(3)= row(3)/A(3,3)

$$\begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \boxed{1} & 4 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Zeroing out element A(1,2) using row(1)=row(1)-A(1,2)*row(2)

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Zeroing out element A(2,3) using row(2)=row(2)-A(2,3)*row(3)

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & \boxed{1} & 4 \end{pmatrix}$$

Figure 2: Steps displayed by Mathematica

2.3 Example 3. RREF on rectangular matrix

Given the matrix

$$\begin{pmatrix} -7 & -6 & -12 & -33 \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

The RREF is

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

To see the steps do

```
mat = {{-7, -6, -12, -33}, {5, 5, 7, 24}, {1, 0, 4, 5}};
displaymat = True; normalizePivot = True;
{result, pivots} = displayRREF[mat, displaymat, normalizePivot]
```

$$\begin{pmatrix} -7 & -6 & -12 & -33 \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Pivot is A(1,1)

$$\begin{pmatrix} \boxed{-7} & -6 & -12 & -33 \\ 5 & 5 & 7 & 24 \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Making the pivot 1 using using row(1)= row(1)/A(1,1)

Zeroing out element A(2,1) using row(2)=5*row(1)-row(2)

$$\begin{pmatrix} \boxed{1} & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \frac{5}{7} & -\frac{11}{7} & \frac{3}{7} \\ 1 & 0 & 4 & 5 \end{pmatrix}$$

Zeroing out element A(3,1) using row(3)=1*row(1)-row(3)

$$\begin{pmatrix} \boxed{1} & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \frac{5}{7} & -\frac{11}{7} & \frac{3}{7} \\ 0 & -\frac{6}{7} & \frac{16}{7} & \frac{2}{7} \end{pmatrix}$$

Pivot is A(2,2)

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \boxed{\frac{5}{7}} & -\frac{11}{7} & \frac{3}{7} \\ 0 & -\frac{6}{7} & \frac{16}{7} & \frac{2}{7} \end{pmatrix}$$

Making the pivot 1 using using row(2)= row(2)/A(2,2)

Zeroing out element A(3,2) using row(3)=$-\frac{6}{7}$*row(2)-row(3)

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & \boxed{1} & -\frac{11}{5} & \frac{3}{5} \\ 0 & 0 & \frac{2}{5} & \frac{4}{5} \end{pmatrix}$$

Pivot is A(3,3)

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & 1 & -\frac{11}{5} & \frac{3}{5} \\ 0 & 0 & \boxed{\frac{2}{5}} & \frac{4}{5} \end{pmatrix}$$

Making the pivot 1 using using row(3)= row(3)/A(3,3)

$$\begin{pmatrix} 1 & \frac{6}{7} & \frac{12}{7} & \frac{33}{7} \\ 0 & 1 & -\frac{11}{5} & \frac{3}{5} \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix}$$

>>>>>>Starting backward elimination phase. The pivots are {{1, 1}, {2, 2}, {3, 3}}

Zeroing out element A(1,2) using row(1)=row(1)-A(1,2)*row(2)

$$\begin{pmatrix} 1 & 0 & \frac{18}{5} & \frac{21}{5} \\ 0 & \boxed{1} & -\frac{11}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Zeroing out element A(2,3) using row(2)=row(2)-A(2,3)*row(3)

$$\begin{pmatrix} 1 & 0 & \frac{18}{5} & \frac{21}{5} \\ 0 & 1 & 0 & 5 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix}$$

Zeroing out element A(1,3) using row(1)=row(1)-A(1,3)*row(3)

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix}$$

Figure 3: Steps displayed by Mathematica

2.4 Example 4. RREF on symbolic matrix

Given A as

$$\begin{pmatrix} s & \sqrt{s} & 3 & 10 & s^2 \\ 1 & s & 2s & 10 & 1 \\ 0 & 2 & 3 & 10 & s^9 \\ \frac{1}{s} & 5 & 3 & 8 & s+2 \end{pmatrix}$$

The RREF is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{s\left(8s^{21/2}-15s^{19/2}-10s^{5/2}-5s^{3/2}-47s^{10}+75s^9+31s^3-25s^2+10s+33\sqrt{s}-66\right)}{-7s^{3/2}+31s^3-45s^2+14s+15\sqrt{s}-30} \\ 0 & 1 & 0 & 0 & -\frac{s^2\left(8s^{10}-15s^9-7s^8+15s^7-10s^2+2s+18\right)}{-7s^{3/2}+31s^3-45s^2+14s+15\sqrt{s}-30} \\ 0 & 0 & 1 & 0 & \frac{-4s^{21/2}+5s^{19/2}+5s^{5/2}+10s^{3/2}+4s^{12}-25s^{11}+20s^{10}-5s^4-12s^3+17s^2-20s-5\sqrt{s}+10}{-7s^{3/2}+31s^3-45s^2+14s+15\sqrt{s}-30} \\ 0 & 0 & 0 & 1 & \frac{-s^{21/2}+3s^{5/2}+6s^{3/2}-7s^{12}+12s^{10}+s^4-8s^3+3s^2-12s-3\sqrt{s}+6}{14s^{3/2}-62s^3+90s^2-28s-30\sqrt{s}+60} \end{pmatrix}$$

To see the steps do

```
mat = {{s, Sqrt[s], 3, 10, s^2},
       {1, s, 2*s, 10, 1},
       {0, 2, 3, 10, s^9},
       {1/s, 5, 3, 8, 2 + s}};
displaymat = True; normalizePivot = True;
{result, pivots} = displayRREF[mat, displaymat, normalizePivot]
```

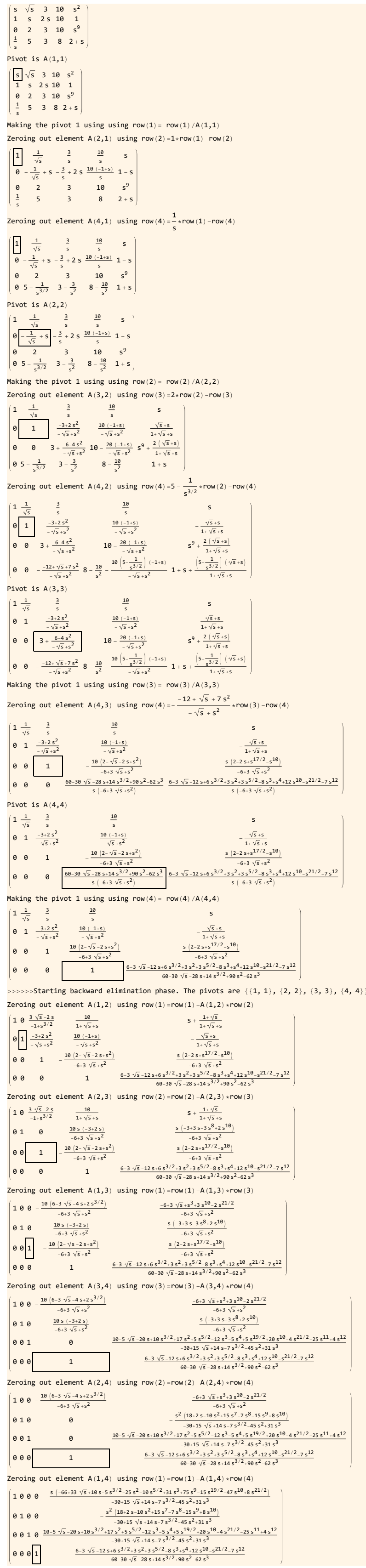

Figure 4: Steps displayed by Mathematica