Computer Algebra Report

Solving

Partial Differential Equations

Using

Maple 2021 and Mathematica 12.2

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Contents

CHAPTER 1

Introduction, Summary of results and lookup table

1.1 Introduction

This report gives the result of running a number of partial differential equations in Mathematica and Maple. This is work in progress as more PDE's are being added.

The following are the systems used

1. Mathematica 12.2 (64 bit).
2. Maple 2021 (64 bit) with Physics version 959.

The following are plain text files of the current collection of the PDE's used in this report.

Mathematica_PDE_IN_CAS_problems.txt

Maple_PDE_IN_CAS_problems.txt

10 minutes real time is used as the time limit to complete a problem. If CAS does not finish within this time limit, a failed score is given.

The PC used to run these tests is windows 10 professional 64 bit with 64 GB RAM running Intel core i7-8086K at 4 GHZ.

All possible options, assumptions and HINTS are tried to obtain a solution. The command `DSolve` is used in Mathematica and the command `pdsolve` in Maple.

In this version, all results are automatically simplified. This is done to reduce the size of the final solution.

It is possible I missed some option, assumption or HINT, which could help make the CAS able to solve a given PDE now marked as unsolved. Will correct such a case if found.

Most of the solutions returned are not verified. If a CAS returns a solution, it is assumed to be correct and that the problem was solved by the CAS.

These problems were collected from textbooks such as

1. Richard Haberman applied partial differential equations, 5th edition
2. David J Logan applied Partial differential equations.
3. Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.
4. Handbook of first order partial differential equations (HFOPDE), Volume 1, by Polyanin, Zaitsev, Moussiaux (2002).
5. Handbook of nonlinear partial differential equations (HNPDE), by Polyanin, Zaitsev (2004).
6. Nonlinear Partial Differential Equations by Lokenath Debnath, 3rd edition. (2012).
7. Introduction to Partial Differential Equations by Peter J. Olver, Springer. (2014).

PDE's from other text books will be added with time.

Some problems were also collected from Maple and Mathematica help pages, documentation and technical forums.

Some of these problems I solved by hand. I have tried to verify that all hand solutions are correct by verifying them using Maple's `pdetest()`. Will add more hand solutions in the future in order to compare with the computer solution.

The following are some PDF files used to collect some problems from

1. PDE_and_BC_during_december_2018.pdf
2. What_is_New_after_Maple_2018.pdf
3. Integral_Transforms_revamped_Oct_2019.pdf

1.2 Results

The current number of partial differential equations in this report is [2014].

Mathematica solved [1462] and Maple solved [1802].

Table 1.1: Percentage solved by each CAS for all problems

Mathematica	Maple
72.59%	89.47%

Table 1.2: Pecentage of solved by each CAS broken down by each chapter

chapter name	Number of Problems	Mathematica	Maple
Miscellaneous PDE's	142	73.94%	91.55%
Schrodinger PDE	8	100.%	87.5%
Parabolic PDE's (Diffusion)	131	91.6%	85.5%
Elliptic PDE's (Laplace, Poisson, Helmholtz)	53	71.7%	75.47%
Hyperbolic PDE's (Wave)	83	91.57%	85.54%
Handbook of first order partial differential equations	1585	70.28%	90.66%
Handbook of nonlinear partial differential equations	12	8.33%	41.67%

Table 1.3: Percentage of failed due to time out among all problems that could not be solved

Mathematica	Maple
7.25%	12.74%

Table 1.4: Total real time used to solve all problems

Mathematica	Maple
14.46 (hours)	9.07 (hours)

Table 1.5: Average real time used to solve one problem

Mathematica	Maple
25.851 (sec)	16.214 (sec)

List of problems not solved by Maple nor by Mathematica [175] (8.69%)

32, 48, 57, 61, 112, 119, 124, 128, 129, 130, 170, 171, 274, 279, 280, 281, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 334, 363, 364, 415, 456, 457, 458, 460, 476, 489, 490, 491, 494, 510, 518, 522, 523, 527, 528, 530, 532, 533, 545, 551, 559, 561, 563, 565, 566, 569, 622, 624, 625, 629, 630, 631, 652, 674, 675, 697, 700, 713, 714, 722, 725, 726, 737, 738, 748, 749, 750, 759, 760, 762, 763, 767, 768, 770, 772, 773, 774, 777, 778, 779, 781, 782, 785, 786, 787, 788, 789, 793, 804, 805, 810, 817, 818, 823, 824, 825, 826, 827, 828, 829, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 1031, 1174, 1181, 1196, 1368, 1379, 1380, 1382, 1385, 1386, 1387, 1388, 1389, 1415, 1418, 1419, 1423, 1430, 1431, 1452, 1453, 1454, 1455, 1459, 1464, 1465, 1557, 1576, 1579, 1580, 1630, 1631, 1634, 1728, 1738, 1750, 1751, 1802, 1803, 1908, 1909, 1916, 1917, 1918, 1920, 1926, 2003, 2005, 2010, 2011, 2012, 2013, 2014

List of problems solved by Maple but not by Mathematica [377] (18.72%)

16, 29, 34, 40, 67, 78, 93, 94, 95, 96, 97, 98, 100, 101, 109, 110, 111, 113, 114, 117, 120, 121, 123, 125, 126, 131, 133, 175, 183, 205, 215, 231, 299, 306, 312, 361, 396, 397, 410, 431, 451, 455, 459, 461, 462, 466, 467, 468, 471, 478, 482, 484, 486, 487, 488, 492, 495, 496, 501, 506, 507, 508, 509, 520, 521, 524, 526, 529, 531, 534, 539, 544, 553, 556, 560, 562, 572, 576, 578, 579, 580, 581, 585, 586, 591, 592, 593, 594, 595, 600, 601, 602, 603, 614, 617, 620, 623, 627, 636, 637, 638, 644, 645, 646, 648, 649, 650, 651, 654, 658, 659, 660, 661, 662, 663, 664, 665, 666, 670, 671, 673, 677, 678, 679, 680, 681, 684, 685, 686, 693, 698, 699, 701, 702, 703, 704, 707, 709, 711, 712, 719, 721, 723, 724, 731, 733, 735, 743, 745, 747, 755, 757, 758, 761, 771, 776, 780, 784, 790, 792, 794, 795, 797, 798, 800, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 819, 820, 821, 822, 831, 832, 877, 886, 887, 903, 904, 909, 919, 931, 996, 1003, 1006, 1020, 1021, 1038, 1039, 1059, 1060, 1067, 1068, 1081, 1086, 1109, 1180, 1182, 1183, 1197, 1210, 1211, 1212, 1213, 1236, 1237, 1264, 1269, 1281, 1354, 1362, 1367, 1376, 1377, 1383, 1384, 1392, 1412, 1416, 1422, 1424, 1425, 1427, 1438, 1439, 1443, 1448, 1449, 1456, 1457, 1458, 1461, 1463, 1470, 1471, 1477, 1482, 1483, 1488, 1489, 1492, 1493, 1494, 1495, 1513, 1514, 1525, 1529, 1531, 1560, 1561, 1565, 1577, 1578, 1588, 1609, 1610, 1620, 1632, 1650, 1652, 1657, 1666, 1668, 1669, 1673, 1680, 1686, 1691, 1697, 1699, 1700, 1701, 1702, 1705, 1706, 1711, 1713, 1716, 1717, 1721, 1722, 1729, 1731, 1732, 1733, 1734, 1735, 1739, 1740, 1742, 1744, 1746, 1747, 1748, 1749, 1759, 1780, 1781, 1785, 1791, 1793, 1801, 1805, 1811, 1816, 1820, 1822, 1826, 1828, 1831, 1837, 1838, 1839, 1842, 1846, 1847, 1849, 1853, 1855, 1860, 1865, 1867, 1868, 1869, 1870, 1873, 1874, 1876, 1879, 1880, 1882, 1885, 1886, 1890, 1891, 1900, 1901, 1902, 1903, 1904, 1907, 1911, 1913, 1915, 1919, 1933, 1940, 1950, 1953, 1960, 1962, 1965, 1966, 1968, 1971, 1972, 1976, 1981, 1986, 1991, 1995, 2000, 2001, 2006, 2007, 2008, 2009

List of problems solved by Mathematica but not by Maple [37] (1.84%)

87, 88, 148, 167, 204, 227, 265, 266, 267, 269, 270, 271, 272, 273, 277, 278, 302, 366, 395, 401, 403, 405, 411, 412, 413, 414, 1204, 1411, 1426, 1535, 1640, 1646, 1707, 1708, 1841, 1875, 1906

List of problems not solved by Mathematica [552] (27.41%)

16, 29, 32, 34, 40, 48, 57, 61, 67, 78, 93, 94, 95, 96, 97, 98, 100, 101, 109, 110, 111, 112, 113, 114, 117, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 133, 170, 171, 175, 183, 205, 215, 231, 274, 279, 280, 281, 299, 306, 309, 312, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 334, 361, 363, 364, 396, 397, 410, 415, 431, 451, 455, 456, 457, 458, 459, 460, 461, 462, 466, 467, 468, 471, 476, 478, 482, 484, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 506, 507, 508, 509, 510, 518, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 539, 544, 545, 551, 553, 556, 559, 560, 561, 562, 563, 565, 566, 569, 572, 576, 578, 579, 580, 581, 585, 586, 591, 592, 593, 594, 595, 600, 601, 602, 603, 614, 617, 620, 622, 623, 624, 625, 627, 629, 630, 631, 636, 637, 638, 644, 645, 646, 648, 649, 650, 651, 652, 654, 658, 659, 660, 661, 662, 663, 664, 665, 666, 670, 671, 673, 674, 675, 677, 678, 679, 680, 681, 684, 685, 686, 693, 697, 698, 699, 700, 701, 702, 703, 704, 707, 709, 711, 712, 713, 714, 719, 721, 722, 723, 724, 725, 726, 731, 733, 735, 737, 738, 743, 745, 747, 748, 749, 750, 755, 757, 758, 759, 760, 761, 762, 763, 767, 768, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 784, 785, 786, 787, 788, 789, 790, 792, 793, 794, 795, 797, 798, 800, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 877, 886, 887, 903, 904, 909, 919, 931, 996, 1003, 1006, 1020, 1021, 1031, 1038, 1039, 1059, 1060, 1067, 1068, 1081, 1086, 1109, 1174, 1180, 1181, 1182, 1183, 1196, 1197, 1210, 1211, 1212, 1213, 1236, 1237, 1264, 1269, 1281, 1354, 1362, 1367, 1368, 1376, 1377, 1379, 1380, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1392, 1412, 1415, 1416, 1418, 1419, 1422, 1423, 1424, 1425, 1427, 1430, 1431, 1438, 1439, 1443, 1448, 1449, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1461, 1463, 1464, 1465, 1470, 1471, 1477, 1482, 1483, 1488, 1489, 1492, 1493, 1494, 1495, 1513, 1514, 1525, 1529, 1531, 1557, 1560, 1561, 1565, 1576, 1577, 1578, 1579, 1580, 1588, 1609, 1610, 1620, 1630, 1631, 1632, 1634, 1650, 1652, 1657, 1666, 1668, 1669, 1673, 1680, 1686, 1691, 1697, 1699, 1700, 1701, 1702, 1705, 1706, 1711, 1713, 1716, 1717, 1721, 1722, 1728, 1729, 1731, 1732, 1733, 1734, 1735, 1738, 1739, 1740, 1742, 1744, 1746, 1747, 1748, 1749, 1750, 1751, 1759, 1780, 1781, 1785, 1791, 1793, 1801, 1802, 1803, 1805, 1811, 1816, 1820, 1822, 1826, 1828, 1831, 1837, 1838, 1839, 1842, 1846, 1847, 1849, 1853, 1855, 1860, 1865, 1867, 1868, 1869, 1870, 1873, 1874, 1876, 1879, 1880, 1882, 1885, 1886, 1890, 1891, 1900, 1901, 1902, 1903, 1904, 1907, 1908, 1909, 1911, 1913, 1915, 1916, 1917, 1918, 1919, 1920, 1926, 1933, 1940, 1950, 1953, 1960, 1962, 1965, 1966, 1968, 1971, 1972, 1976, 1981, 1986, 1991, 1995, 2000, 2001, 2003, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014

List of problems not solved by Maple [212] (10.53%)

32, 48, 57, 61, 87, 88, 112, 119, 124, 128, 129, 130, 148, 167, 170, 171, 204, 227, 265, 266, 267, 269, 270, 271, 272, 273, 274, 277, 278, 279, 280, 281, 302, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 334, 363, 364, 366, 395, 401, 403, 405, 411, 412, 413, 414, 415, 456, 457, 458, 460, 476, 489, 490, 491, 494, 510, 518, 522, 523, 527, 528, 530, 532, 533, 545, 551, 559, 561, 563, 565, 566, 569, 622, 624, 625, 629, 630, 631, 652, 674, 675, 697, 700, 713, 714, 722, 725, 726, 737, 738, 748, 749, 750, 759, 760, 762, 763, 767, 768, 770, 772, 773, 774, 777, 778, 779, 781, 782, 785, 786, 787, 788, 789, 793, 804, 805, 810, 817, 818, 823, 824, 825, 826, 827, 828, 829, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 1031, 1174, 1181, 1196, 1204, 1368, 1379, 1380, 1382, 1385, 1386, 1387, 1388, 1389, 1411, 1415, 1418, 1419, 1423, 1426, 1430, 1431, 1452, 1453, 1454, 1455, 1459, 1464, 1465, 1535, 1557, 1576, 1579, 1580, 1630, 1631, 1634, 1640, 1646, 1707, 1708, 1728, 1738, 1750, 1751, 1802, 1803, 1841, 1875, 1906, 1908, 1909, 1916, 1917, 1918, 1920, 1926, 2003, 2005, 2010, 2011, 2012, 2013, 2014

List of problems that are hand solved [195] (9.68%)

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24, 26, 27, 28, 29, 30, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 102, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 169, 172, 173, 176, 177, 178, 179, 181, 182, 184, 185, 186, 192, 195, 196, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 235, 236, 237, 238, 248, 253, 254, 259, 260, 263, 264, 265, 269, 270, 271, 272, 273, 274, 276, 277, 282, 283, 284, 289, 291, 292, 293, 294, 308, 309, 311, 312, 314, 317, 328, 330, 334, 335, 336, 337, 338, 339, 344, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 365, 366, 367, 374, 375, 384, 389, 390, 392, 393, 394, 396, 397, 398, 399, 403, 406, 408, 411, 412, 413, 414, 415, 420, 422, 424, 425, 426, 427, 428, 429, 430, 1369, 1370, 1371, 1372, 1516, 1517, 1586, 1587

List of problems that are animated [60] (2.98%)

2, 3, 4, 5, 6, 7, 8, 9, 10, 152, 153, 155, 160, 162, 164, 177, 182, 192, 200, 203, 209, 210, 211, 220, 223, 228, 230, 236, 238, 260, 264, 265, 270, 271, 273, 274, 276, 277, 336, 337, 338, 351, 356, 358, 359, 360, 365, 374, 375, 384, 389, 390, 392, 393, 397, 398, 399, 408, 413, 414

1.3 Result lookup table

1.3.1 Miscellaneous PDE's

Table 1.6: Miscellaneous PDE's breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1	General first order	Transport equation $u_t + u_x = 0$	✓	0.046	✓	0.014	Yes	
2	General first order	Transport equation $u_t - 3u_x = 0$ IC $u(0,x) = e^{-x^2}$. Peter Olver textbook, 2.2.2 (a)	✓	0.007	✓	0.061	Yes	Yes
3	General first order	Transport equation $u_t + 2u_x = 0$ IC $u(-1,x) = \frac{x}{1+x^2}$. Peter Olver textbook, 2.2.2 (b)	✓	0.006	✓	0.023	Yes	Yes
4	General first order	Transport equation $u_t + u_x + \frac{1}{2}u = 0$ IC $u(0,x) = \arctan(x)$. Peter Olver textbook, 2.2.2 (c)	✓	0.005	✓	0.047	Yes	Yes
5	General first order	Transport equation $u_t - 4u_x + u = 0$ IC $u(0,x) = \frac{1}{1+x^2}$. Peter Olver textbook, 2.2.2 (d)	✓	0.005	✓	0.023	Yes	Yes
6	General first order	Transport equation $u_t + 2u_x = \sin x$ IC $u(0,x) = \sin x$. Peter Olver textbook, 2.2.5	✓	0.073	✓	0.052	Yes	Yes
7	General first order	Transport equation $u_t + \frac{1}{1+x^2}u_x = 0$ IC $u(x,0) = \frac{1}{1+(3+x)^2}$. Peter Olver textbook, page 27	✓	0.023	✓	0.157	Yes	Yes
8	General first order	Transport equation $u_t - xu_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.17	✓	0.006	✓	0.107	Yes	Yes
9	General first order	Transport equation $u_t + (1-2t)u_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.29	✓	0.032	✓	0.213	Yes	Yes

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
10	General first order	Transport equation $u_t+\frac{1}{x^2+4}u_x=0$ IC $u(x,0)=e^{x^3+12x}$	✓	0.015	✓	0.085	Yes	Yes
11	General first order	$3u_x+5u_y=x$	✓	0.007	✓	0.012	Yes	
12	General first order	$xu_y+yu_x=-4xyu$ and $u(x,0)=e^{-x^2}$	✓	0.024	✓	0.155	Yes	
13	General first order	$u_t+u_x=0$ and $u(x,0)=\sin x$ and $u(0,t)=0$	✓	0.121	✓	0.374	Yes	
14	General first order	$u_t+cu_x=0$ and $u(x,0)=e^{-x^2}$	✓	0.005	✓	0.026	Yes	
15	General first order	(Haberman 12.2.2) $\omega_t-3\omega_x=0$ and $\omega(x,0)=\cos x$	✓	0.004	✓	0.021	Yes	
16	General first order	(Haberman 12.2.4) $\omega_t+c\omega_x=0$ and $\omega(x,0)=f(x)$ and $\omega(0,t)=h(t)$	✗	1.705	✓ Solution contains unresolved invlaplace calls	0.432	Yes	
17	General first order	(Haberman 12.2.5 (a)) $\omega_t+c\omega_x=e^{2x}$ and $\omega(x,0)=f(x)$	✓	0.048	✓	0.086	Yes	
18	General first order	(Haberman 12.2.5 (d)) $\omega_t+3t\omega_x=\omega(x,t)$ and $\omega(x,0)=f(x)$	✓	1.118	✓	0.096	Yes	
19	General first order	$2u_x+5u_y=u^2(x,y)+1$	✓	0.16	✓	0.038	Yes	
20	General first order	Clairaut equation $xu_x+yu_y+\frac{1}{2}((u_x)^2+(u_y)^2)=0$	✓	0.054	✓	0.191	Yes	
21	General first order	Clairaut equation. $xu_x+yu_y+\frac{1}{2}((u_x)^2+(u_y)^2)=0$ with $u(x,0)=\frac{1}{2}(1-x^2)$	✓	0.014	✓	0.813		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
22	General first order	Clairaut equation. $u = xu_x + yu_y + \sin(u_x + u_y)$	✓	0.054	✓	0.022		
23	General first order	Recover a function from its gradient vector	✓	0.032	✓	0.089		
24	General first order	$xf_y - f_x = \frac{g(x)}{h(y)} f^2$	✓	0.055	✓	0.04	Yes	
25	General first order	$f_x + (f_y)^2 = f(x,y,z) + z$	✓	0.132	✓	0.533		
26	General first order	$xu_x + yu_y = u$ (Example 3.5.1 in Lokenath Debnath)	✓	0.017	✓	0.011	Yes	
27	General first order	$xu_x + yu_y = nu$ Example 3.5.2 in Lokenath Debnath	✓	0.018	✓	0.01	Yes	
28	General first order	$x^2 u_x + y^2 u_y = (x+y)u$ Example 3.5.3 in Lokenath Debnath	✓	0.12	✓	0.027	Yes	
29	General first order	$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$ (Example 3.5.4 in Lokenath Debnath)	✗ (Timed out)	600.	✓	2.291	Yes	
30	General first order	$u(x+y)u_x + u(x-y)u_y = x^2 + y^2$ (Example 3.5.5 in Lokenath Debnath)	✓	0.489	✓	0.162	Yes	
31	General first order	$u_x - u_y = 1$ with $u(x,0) = x^2$ Example 3.5.6 in Lokenath Debnath	✓	0.005	✓	0.019		
32	General first order	$yu_x + xu_y = u$ with $u(x,0) = x^3$ and $u(0,y) = y^3$ Example 3.5.8 in Lokenath Debnath	✗	2.078	✗	0.545		
33	General first order	$xu_x + yu_y = xe^{-u}$ with $u = 0$ on $y = x^2$ Example 3.5.10 in Lokenath Debnath	✓	0.235	✓	0.063		
34	General first order	$u_t + uu_x = x$ with $u(x,0) = f(x)$ Example 3.5.11 in Lokenath Debnath.	✗	3.678	✓	0.256		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
35	General first order	$u_x = 0$ Problem 3.3(a) Lokenath Debnath	✓	0.004	✓	0.003		
36	General first order	$au_x + bu_y = 0$ Problem 3.3(b) Lokenath Debnath	✓	0.009	✓	0.01		
37	General first order	$u_x + yu_y = 0$ Problem 3.3(c) Lokenath Debnath	✓	0.029	✓	0.011		
38	General first order	$(1+x^2)u_x+u_y = 0$ Problem 3.3(d) Lokenath Debnath	✓	0.009	✓	0.01		
39	General first order	$2xyu_x + (x^2+y^2)u_y = 0$ Problem 3.3(e) Lokenath Debnath	✓	0.126	✓	0.033		
40	General first order	$(y+u)u_x + yu_y = x - y$ Problem 3.3(f) Lokenath Debnath	✗	105.34	✓	0.527		
41	General first order	$y^2u_x - xyu_y = x(u-2y)$ Problem 3.3(g) Lokenath Debnath	✓	0.137	✓	0.038		
42	General first order	$yu_y - xu_x = 1$ Problem 3.3(h) Lokenath Debnath	✓	0.02	✓	0.009		
43	General first order	$u_x + 2xy^2u_y = 0$ Problem 3.4 Lokenath Debnath	✓	0.126	✓	0.014		
44	General first order	$3u_x + 2u_y = 0$ with $u(x,0) = \sin x$. Problem 3.5(a) Lokenath Debnath	✓	0.005	✓	0.018		
45	General first order	$yu_x + xu_y = 0$ with $u(0,y) = e^{-y^2}$. Problem 3.5(b) Lokenath Debnath	✓	0.023	✓	0.033		
46	General first order	$xu_x + yu_y = 2xy$ with $u = 2$ on $y = x^2$. Problem 3.5(c) Lokenath Debnath	✓	0.017	✓	0.008		
47	General first order	$u_x + xu_y = 0$ with $u(0,y) = \sin y$. Problem 3.5(d) Lokenath Debnath	✓	0.006	✓	0.021		

Continued on next page

Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
48	General first order	$yu_x + xu_y = xy$ with $u(0,y) = e^{-y^2}, u(x,0) = e^{-x^2}$. Problem 3.5(e) Lokenath Debnath	✗	3.803	✗	0.523		
49	General first order	$u_x + xu_y = (y - \frac{1}{2}x^2)^2$ with $u(0,y) = e^y$. Problem 3.5(f) Lokenath Debnath	✓	0.012	✓	0.097		
50	General first order	$xu_x + yu_y = u + 1$ with $u = x^2$ on $y = x^2$ Problem 3.5(g) Lokenath Debnath	✓	0.021	✓	0.01		
51	General first order	$uu_x - uu_y = u^2 + (x+y)^2$ with $u(x,0) = 1$ Problem 3.5(h) Lokenath Debnath	✓	0.065	✓	0.065		
52	General first order	$xu_x + (x+y)u_y = u + 1$ with $u(x,0) = x^2$ Problem 3.5(i) Lokenath Debnath	✓	0.027	✓	0.07		
53	General first order	$xu_x + yu_y + zu_z = 0$ Problem 3.8(a) .Lokenath Debnath	✓	0.032	✓	0.014		
54	General first order	$x^2u_x + y^2u_y + z(x+y)u_z = 0$ Problem 3.8(b) Lokenath Debnath	✓	0.12	✓	0.019		
55	General first order	$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$ Problem 3.8(c) Lokenath Debnath	✓	0.049	✓	0.977		
56	General first order	$yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$ Problem 3.8(d) Lokenath Debnath	✓	0.114	✓	0.068		
57	General first order	$x(y^2-z^2)u_x + y(z^2-y^2)u_y + z(x^2-y^2)u_z = 0$ Problem 3.8(e) Lokenath Debnath	✗	45.677	✗	0.255		
58	General first order	$u_x + xu_y = y$ with $u(0,y) = y^2$ Problem 3.9(a) Lokenath Debnath	✓	0.007	✓	0.021		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
59	General first order	$u_x + xu_y = y$ with $u(1,y) = 2y$ Problem 3.9(b) Lokenath Debnath	✓	0.007	✓	0.009		
60	General first order	$(u_x+u_y)^2-u^2 = 0$. Problem 3.10 Lokenath Debnath	✓	0.011	✓	0.017		
61	General first order	$(y+u)u_x + yu_y = x-y$ with $u(x,1) = 1+x$. Problem 3.11 Lokenath Debnath	✗	211.318	✗	1.067		
62	General first order	$2xu_x + (x+1)u_y = y$ with $u(1,y) = 2y$. Problem 3.14(d) Lokenath Debnath	✓	0.021	✓	0.161		
63	General first order	$xu_x+yu_y = x^2+y^2$ with $u(x,1) = x^2$. Problem 3.14(e) Lokenath Debnath	✓	0.034	✓	0.073		
64	General first order	$y^2u_x + (xy)u_y = x$ with $u(x,1) = x^2$. Problem 3.14(f) Lokenath Debnath	✓	0.035	✓	0.05		
65	General first order	$xu_x + yu_y = xy$ with $u = \frac{x^2}{2}$ at $y = x$. Problem 3.14(g) Lokenath Debnath	✓	0.019	✓	0.016		
66	General first order	$u_x + uu_y = 1$ with $u(0,y) = ay$. Problem 3.16(a) Lokenath Debnath	✓	0.084	✓	0.026		
67	General first order	$(y+u)u_x+(x+u)u_y = x+y$. Problem 3.17(a) Lokenath Debnath	✗ (Timed out)	600.	✓	1.372		
68	General first order	$xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$. Problem 3.17(b) Lokenath Debnath	✓	0.065	✓	0.036		
69	General first order	$(x+y)u_x+(x-y)u_y = 0$. Problem 3.17(c) Lokenath Debnath	✓	0.057	✓	0.052		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
70	General first order	$yu_x - xu_y = e^u$ with $u(0,y) = y^2 - 1$	✓	0.163	✓	0.094	Yes	
71	General first order	$yu_x - xu_y = e^u$	✓	0.074	✓	0.001	Yes	
72	General first order	$u_t + xu_x = 0$ with $u(x,0) = x^2$. Math 5587	✓	0.007	✓	0.056	Yes	
73	General first order	$u_t + tu_x = 0$ with $u(x,0) = e^x$	✓	0.02	✓	0.202	Yes	
74	General first order	$2u_x + 3u_y = 1$	✓	0.008	✓	0.009	Yes	
75	General first order	$xu_t - tu_x = 0$	✓	0.027	✓	0.02	Yes	
76	General first order	$u_t + u_x = 0$ with $u(x,1) = \frac{x}{1+x^2}$	✓	0.005	✓	0.013	Yes	
77	General first order	$u_x u_y = 1$	✓	0.003	✓	0.02	Yes	
78	General first order	$u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$	✗	1.493	✓	0.226	Yes	
79	Solved by factoring into two transport equations	$u_{xx} + u_{xt} - 6u_{tt} = 0$	✓	0.012	✓	0.096	Yes	
80	Solved by factoring into two transport equations	$u_{xx} - u_{xt} - 12u_{tt} = 0$	✓	0.013	✓	0.276	Yes	
81	Solved by factoring into two transport equations	$u_{xx} - 3u_{xt} - 4u_{tt} = 0$	✓	0.013	✓	2.616	Yes	

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
82	Solved by factoring into two transport equations	$u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$	✓	0.018	✓	2.259		
83	Beam PDE	Beam PDE $u_{tt} + u_{xxxx} = 0$	✓	1.711	✓	0.205		
84	Burger's PDE	Inviscid Burgers $u_x + uu_y = 0$	✓ Implicit solution	0.028	✓	0.025	Yes	
85	Burger's PDE	Inviscid Burgers with I.C. $u_x + uu_y = 0$ and $u(x,0) = \frac{1}{x+1}$	✓	0.011	✓	0.039	Yes	
86	Burger's PDE	$u_t + uu_x = \mu u_{xx}$	✓	0.039	✓	0.075		
87	Burger's PDE	$u_t + uu_x + \mu u_{xx}$ with IC	✓	10.918	✗	0.556		
88	Burger's PDE	$u_t + uu_x + \mu u_{xx}$ IC as UnitBox	✓	39.853	✗	0.606		
89	Black Scholes PDE	classic Black Scholes model from finance, European call version	✓	3.061	✓	0.837		
90	Black Scholes PDE	Boundary value problem for the Black Scholes equation	✓	4.421	✓	2.12		
91	Korteweg-deVries PDE	$u_{xxx} + u_t - 6uu_x = 0$	✓	0.031	✓	0.186		
92	Tricomi PDE	$u_{xx} + yu_{yy} = 0$ with $u(x,0) = 0, u_y(x,0) = x^2$	✓	9.736	✓	3.436		
93	Tricomi PDE	$u_{xx} + xu_{yy} = 0$	✗	0.011	✓	2.783		
94	Keldysh equation	$xu_{xx} + u_{yy} = 0$	✗	0.007	✓	4.056		
95	Euler-Poisson-Darboux equation	$u_{xx} + u_{yy} + \frac{\beta}{x}u_x = 0$	✗	0.008	✓	0.139		
96	Euler-Poisson-Darboux equation	$u_{xx} - u_{yy} + \frac{\beta}{x}u_x = 0$	✗	0.008	✓	0.487		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
97	Euler-Poisson-Darboux equation	$u_{tt}-u_{xx}-\frac{2}{x}u_x=0$ with $u(x,0)=0, u_t(x,0)=g(x)$	✗	2.613	✓	4.295		
98	Chaplygin's equation	$u_{\theta\theta}+\frac{v^2}{1-\frac{v^2}{c^2}}u_{vv}+vu_v=0$	✗	0.027	✓	1.266		
99	Cauchy Riemann PDE's	Cauchy Riemann PDE with Prescribe the values of u and v on the x axis	✓	0.011	✓	0.168		
100	Cauchy Riemann PDE's	Cauchy Riemann PDE With extra term on right side	✗	0.003	✓	0.07		
101	Hamilton-Jacobi PDE	Hamilton-Jacobi type PDE	✗	0.01	✓	0.191		
102	Airy PDE	$u_t+u_{xxx}=0$	✓	0.059	✓	0.115	Yes	
103	Nonlinear PDE's	Bateman-Burgers $u_t+uu_x=\nu u_{xx}$	✓	0.03	✓	0.113		
104	Nonlinear PDE's	Benjamin Bona Mahony $u_t+u_x+uu+x-u_{xxt}=0$	✓	0.036	✓	0.115		
105	Nonlinear PDE's	Benjamin Ono $u_t+Hu_{xx}+uu_x=0$	✓	0.031	✓	0.106		
106	Nonlinear PDE's	Born Infeld $(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$	✓	0.014	✓	0.175		
107	Nonlinear PDE's	Boussinesq $u_{tt}-u_{xx}-u_{xxxx}-3(u^2)_{xx}=0$	✓	0.058	✓	0.117		
108	Nonlinear PDE's	Boussinesq type $u_{tt}-u_{xx}-2\alpha(uu_x)_x-\beta u_{xxtt}=0$	✓	0.048	✓	0.132		
109	Nonlinear PDE's	Buckmaster $u_t=(u^4)_{xx}+(u^3)_x$	✗	0.113	✓ Answer in terms of RootOf.	0.704		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
110	Nonlinear PDE's	Camassa Holm $u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}$	✗	0.199	✓ Answer in terms of RootOf.	1.45		
111	Nonlinear PDE's	Chaffee Infante $u_t = u_{xx} + \lambda(u^3 - u) = 0$	✗	0.105	✓	0.226		
112	Nonlinear PDE's	Clarke. $(\theta_t - \gamma e^\theta)_{tt} = (\theta_t - e^\theta)_{xx}$	✗	0.013	✗	0.05		
113	Nonlinear PDE's	Degasperis Procesi $u_t - u_{xxt} + 4uu_x = 3u_x u_x x + uu_{xxx}$	✗	0.195	✓ But still has unresolved ODE's in solution	0.621		
114	Nonlinear PDE's	Dym equation $u_t = u^3 u_{xxx}$	✗	0.096	✓ has RootOf	0.541		
115	Nonlinear PDE's	Estevez Mansfield Clarkson $u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$	✓	0.041	✓	0.158		
116	Nonlinear PDE's	Fisher's $u_t = u(1-u) + u_{xx}$	✓	0.063	✓	0.232		
117	Nonlinear PDE's	Hunter Saxton $(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$	✗	0.051	✓ with RootOf	0.123		
118	Nonlinear PDE's	Kadomtsev Petviashvili $(u_t + uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$	✓	0.073	✓	0.158		
119	Nonlinear PDE's	Klein Gordon $u_{xx} + u_{yy} + \lambda u^p = 0$	✗	0.006	✗	0.025		
120	Nonlinear PDE's	Klein Gordon $u_{xx} + u_{yy} + u^2 = 0$	✗	0.237	✓	0.408		
121	Nonlinear PDE's	Khokhlov Zabolotskaya $u_{xt} - (uu_x)_x = u_{yy}$	✗	0.073	✓	0.251		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
122	Nonlinear PDE's	Korteweg de Vries (KdV) $u_t + (u_x)^3 + 6uu_x = 0$	✓	0.038	✓	0.115		
123	Nonlinear PDE's	Lin Tsien $2u_{tx} + u_x u_{xx} - u_{yy} = 0$	✗	0.09	✓	0.277		
124	Nonlinear PDE's	Liouville $u_{xx} + u_{yy} + e^{\lambda u} = 0$	✗	0.006	✗	0.026		
125	Nonlinear PDE's	Plateau $(1+u_y^2)u_{xx} - 2u_x u_y y_{xy} + (1+u_x^2)u_{yy} = 0$	✗	0.039	✓	0.463		
126	Nonlinear PDE's	Rayleigh $u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$	✗	0.099	✓ Has RootOf	0.163		
127	Nonlinear PDE's	Sawada Kotera $u_t + 45u^2 u_x + 15u_x u_{xx} + 15uu_{xxx} + u_{xxxxx} = 0$	✓	0.093	✓	0.183		
128	Nonlinear PDE's	Sine Gordon $\phi_{tt} - \phi_{xx} + \sin\phi = 0$	✗	0.01	✗	0.025		
129	Nonlinear PDE's	Sinh Gordon $u_{xt} = \sinh u$	✗	0.01	✗	0.028		
130	Nonlinear PDE's	Sinh Poisson $u_{xx} + u_{yy} + \sinh u = 0$	✗	0.009	✗	0.026		
131	Nonlinear PDE's	Thomas equation $u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$	✗	0.074	✓	0.414		
132	Nonlinear PDE's	phi equation $\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$	✓	0.053	✓	0.138		
133	more miscellaneous	$SS_{xy} + S_x S_y = 1$	✗	0.04	✓	0.032		
134	more miscellaneous	$u_{rr} + u_{\theta\theta} = 0$	✓	30.952	✓	0.631		
135	more miscellaneous	$u_{xx} + yu_{yy} = 0$	✓	8.842	✓	2.995		
136	more miscellaneous	$u_t + u_{xxx} = 0$	✓	0.191	✓	5.69		
137	more miscellaneous	$u_{xy} = \sin(x)\sin(y)$	✓	5.14	✓	0.52		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
138	more miscellaneous	$w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3}$	✓	2.907	✓	0.745		
139	more miscellaneous	Linear PDE, initial conditions at $t = t_0$	✓	3.923	✓	0.718		
140	more miscellaneous	second order in time, Linear PDE, initial conditions at $t = t_0$	✓	2.226	✓	2.228		
141	more miscellaneous	Einstein-Weiner $u_t = -\beta u_x + D u_{xx}$	✓	0.046	✓	0.305		
142	more miscellaneous	Using integral transforms.	✓	40.116	✓	2.523		

1.3.2 Schrodinger PDE

Table 1.7: Schrodinger PDE breakdown of results. Time in seconds

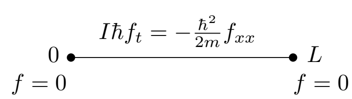
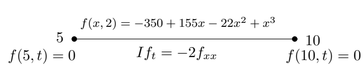
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
143	1D	 Logan textbook, page 30	✓	0.527	✓	0.474		
144	1D	 From Mathematica help pages	✓	0.756	✓	1.235		

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Table 1.7 – Schrodinger PDE. Continued from previous page

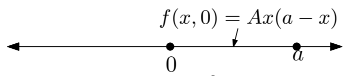
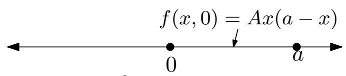
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
145	1D	 $I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$ $-\infty < x < \infty, t > 0$ David Griffiths, page 47	✓	30.831	✓	1.26		
146	1D	 $I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx} + V(x) f(x,t)$ $-\infty < x < \infty, t > 0$ David Griffiths, page 47	✓	0.678	✓	0.933		
147	1D	Deep well	✓ Does not handle $n = 2$ case correctly. Division by zero	47.385	✓	6.925		
148	1D	 $I f_t = -f_{xx} + 2x^2 f(x,t)$ $-\infty < x < \infty, t > 0$ From Mathematica help pages	✓	0.006	✗ Trivial solution. Maple does not support ∞ in boundary conditions	2.546		

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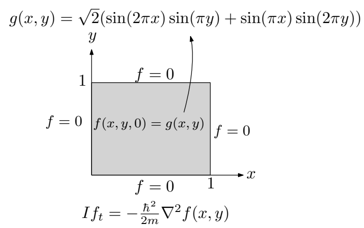
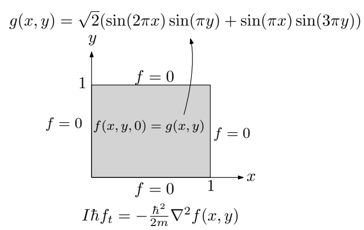
Table 1.7 – Schrodinger PDE. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
149	2D	 In a square, zero potential	✓	0.707	✓	4.08		
150	2D	 In a square	✓	0.648	✓	4.622		

1.3.3 Parabolic PDE's (Diffusion)

Table 1.8: Parabolic PDE's (Diffusion) breakdown of results. Time in seconds

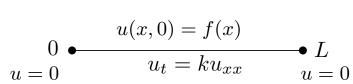
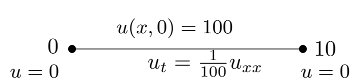
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
151	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 General initial conditions	✓	62.537	✓	1.31	Yes	
152	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Specific initial condition	✓	26.091	✓	0.619	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

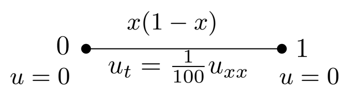
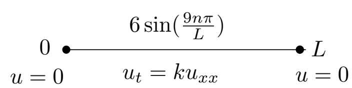
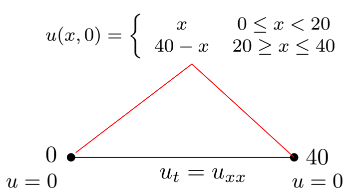
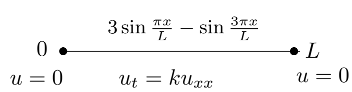
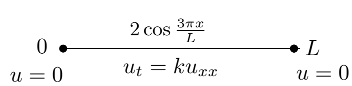
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
153	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 IC $u = x(1-x)$	✓	27.383	✓	0.611	Yes	Yes
154	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.3 (a)	✓	0.42	✓	3.055	Yes	
155	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 IC hat function	✓	35.813	✓	1.385	Yes	Yes
156	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.3 (b)	✓	1.236	✓	3.697	Yes	
157	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.3 (c)	✓ but $n=3$ should be special case	65.187	✓ handled $n=3$ case correctly.	5.432	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

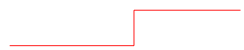
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
158	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=\begin{cases}1 & 0<x\le\frac{L}{2}\\ 2 & \frac{L}{2}<x\le L\end{cases}$  0 —— L; $u=0$, $u_t=ku_{xx}$, $u=0$ Haberman 2.3.3 (d)	✓	545.655	✓	26.013	Yes	
159	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$; -1 —— 1; $u=0$, $u_t=ku_{xx}$, $u=0$ domain from -1 to +1	✓	18.891	✓	1.119	Yes	
160	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$1-x^2$; -1 —— 1; $u=0$, $u_t=ku_{xx}$, $u=0$ domain from -1 to +1	✓	62.92	✓	0.753	Yes	Yes
161	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=f(x)$; 0 —— L; $u=0$, $u_t=ku_{xx}+Q(x)$, $u=0$ with source that depends on space only (general case)	✓	42.268	✓	1.606	Yes	
162	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=x(5-x)$; 0 —— 5; $u=0$, $u_t=\frac{1}{10}u_{xx}+x$, $u=0$ with source that depends on space only (special case)	✓	32.815	✓	2.285	Yes	Yes
163	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=f(x)$; 0 —— L; $u=0$, $u_t=ku_{xx}+Q(x,t)$, $u=0$ with source that depends on space and time (general case)	✓	71.123	✓	3.32	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
164	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=x(\pi-x)$; 0 to π; $u=0$, $u_t=ku_{xx}+e^{-t}\sin(3x)$, $u=0$ with source that depends on space and time (special case)	✓	35.558	✓	10.046	Yes	Yes
165	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0; 0 to π; $u=0$, $u_t=u_{xx}+t(\pi-x)$, $u=0$ Math 4567 Exam	✓	18.115	✓	1.938	Yes	
166	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$; 0 to 1; $u=0$, $u_t=ku_{xx}+Q(x,t)$, $u=0$ With source	✓	67.959	✓	6.367		
167	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$e^{\frac{45}{10}}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x))$; 0 to 1; $u=0$, $u_t=u_{xx}-9u_x$, $u=0$ special initial condition	✓	3.189	✗ (Timed out)	600.	Yes	
168	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$; 1 to b; $u=0$, $u_t=x^2u_{xx}+xu_x$, $u=0$ Diffusion Reaction and Euler-Cauchy Sturm-Liouville	✓	1.816	✓	8.987		
169	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=1$; 0 to 1; $u=0$, $u_t=\frac{1}{10}u_{xx}+ru$, $u=0$ Diffusion Reaction. Using growth form reaction term	✓	93.031	✓	0.83	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
170	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=1$; 0 to 1; $u=0$ at both ends; $u_t = \frac{1}{100}u_{xx} + \frac{1}{10}u(1-\frac{u}{10})$ Diffusion Reaction, using logistic form for reaction term	✗	4.709	✗	27.471		
171	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=1$; 0 to 1; $u=0$ at both ends; $u_t = \frac{1}{1000}u_{xx} + \frac{1}{100}u + \frac{1}{100}u^2 - \frac{5}{1000}u^3$ Diffusion Reaction, using Aleee form for reaction term	✗	7.192	✗	35.232		
172	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$; 0 to L; $u=0$ at both ends; $u_t = ku_{xx} - \alpha u$, $\alpha > 0$ Diffusion Reaction. Haberman 2.3.8	✓	34.92	✓	6.826	Yes	
173	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$; 0 to L; $u=0$ at both ends; $u_t = u_{xx} - u$ Diffusion Reaction	✓	34.502	✓	1.482	Yes	
174	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$\sin(2\pi x) - \sin(4\pi x)$; 0 to 1; $u=0$ at both ends; $u_t = 100u_{xx} - u$ Diffusion Reaction	✓	0.774	✓	1.497		
175	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0)=\sin(x)$; 0 to π; $u=0$ at both ends; $u_t = ku_{xx} - ux$ Diffusion Reaction	✗	43.242	✓	1.147		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

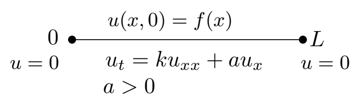
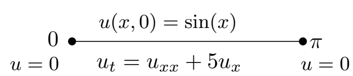
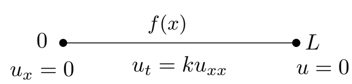
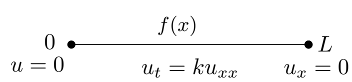
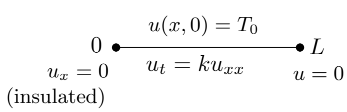
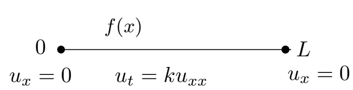
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
176	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Diffusion convection (general case)	✓	3.081	✓	2.102	Yes	
177	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Diffusion convection (special case)	✓	63.055	✓	1.978	Yes	Yes
178	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.4.2 (general case)	✓	30.562	✓	2.112	Yes	
179	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Left end zero, right end insulated, no source	✓	29.39	✓	2.041	Yes	
180	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 One end insulated	✓	32.45	✓	2.24		
181	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.7 (general case)	✓	32.108	✓	1.804	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

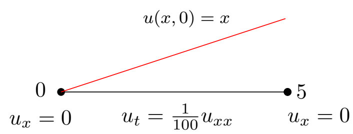
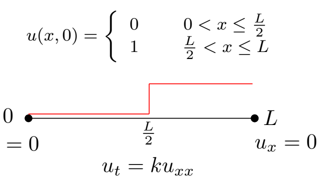
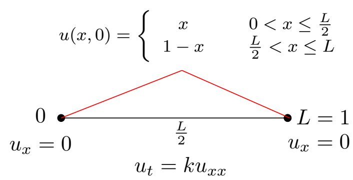
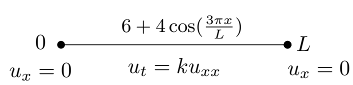
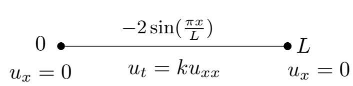
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
182	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 specific case	✓	12.122	✓	3.334	Yes	Yes
183	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.4.1 (a)	✗ (Timed out)	600.	✓	26.393		
184	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Both ends insulated, no source	✓	33.1	✓	2.065	Yes	
185	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.4.1 (b) (special case)	✓	1.606	✓	4.518	Yes	
186	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.4.1 (c) (special case)	✓	32.364	✓	6.848	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
187	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to L, IC: $-3\cos(\frac{8\pi x}{L})$; $u_x=0$, $u_t=ku_{xx}$, $u_x=0$ Haberman 2.4.1 (d)	✓	1.387	✓	4.021		
188	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to 1, IC: $\frac{x^2}{2}+x$; $u_x=0$, $u_t=13u_{xx}$, $u_x=0$ both ends insulated	✓	91.769	✓	4.442		
189	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to 1, IC: $1-\frac{x^3}{4}$; $u_x=0$, $u_t=ku_{xx}$, $u_x+u=0$ convection heat loss	✓	32.721	✓	3.185		
190	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to L, IC: x; $u_x=0$, $u_t=ku_{xx}+\cos(\omega t)$, $u_x=0$ Pinchover and Rubinstein 6.25	✓	42.65	✓	8.53		
191	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to L, IC: $f(x)$; $u_x=0$, $u_t=ku_{xx}+e^{ct}\sin(\frac{2\pi x}{L})$, $u_x=0$ external source	✓	18.726	✓	19.326		
192	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to π, $u(x,0)=x$; $u_x=0$, $u_t=ku_{xx}-\beta u$, $\beta>0$, $u_x=0$ Diffusion Reaction (general case)	✓	91.872	✓	0.938	Yes	Yes
193	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	0 to 1, IC: $3\cos(42\pi x)$; $u_x=0$, $u_t=13u_{xx}+g(x,t)$, $g(x,t)=e^{3t}\cos(17\pi x)$, $u_x=0$ Pinchover and Rubinstein 6.23	✓	64.245	✓	31.669		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
194	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$\pi\cos(2x)$ on bar from 0 to 1; $u_x=0$ at 0; $u_t=u_{xx}+g(x,t)$, $g(x,t)=t\cos(2001x)$; $u_x=0$ at 1. Pinchover and Rubinstein 6.21	✓	63.132	✓	3.556		
195	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$\ln x$ on bar from 1 to b; $u_x=0$ at 1; $u_t=x^2u_{xx}+xu_x$; $u_x+hu=0$ at b. Diffusion Reaction. Euler-Cauchy Sturm-Liouville	✓	2.684	✓	26.115	Yes	
196	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$ on bar from 0 to 1; $u=0$ at 0; $u_t=ku_{xx}$; $u_x+hu=0$, $h>0$ at 1. convection heat loss	✓	32.892	✓	1.936	Yes	
197	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$ on bar from 0 to L; $u_x+u=0$ at 0; $u_t=ku_{xx}$; $u_x+u=0$ at L. Mixed BC	✓	33.75	✓	4.725		
198	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$ on bar from 0 to π; $u_x=0$ at 0; $u_t=ku_{xx}+\sin(\frac{2\pi x}{L})$; $u_x=0$ at π. Haberman 8.2.1 (f)	✓	63.62	✓	4.279		
199	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	$u(x,0)=f(x)$ on bar from 0 to L; $u_x=0$ at 0; $u_t=ku_{xx}$; $u=T_0$ at L. left end insulated (general case)	✓	33.393	✓	4.627	Yes	
200	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	$u(x,0)=0$ on bar from 0 to 5; $u_x=0$ at 0; $u_t=\frac{1}{100}u_{xx}$; $u=10$ at 5. left end insulated (special case)	✓	7.485	✓	1.614	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

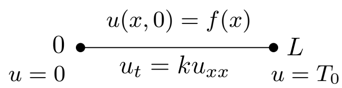
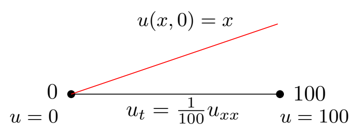
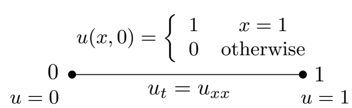
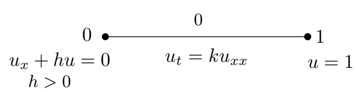
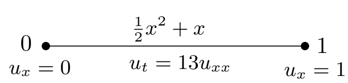
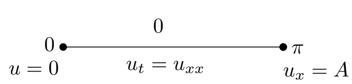
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
201	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC (general case)	✓	66.082	✓	2.418	Yes	
202	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC (special case)	✓	0.01	✓	3.152	Yes	
203	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC, special case	✓	10.202	✓	2.215	Yes	Yes
204	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 convection heat loss	✓	39.799	✗ (Timed out)	600.	Yes	
205	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 nonhomogeneous BC	✗	2.176	✓	9.692		
206	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 nonhomogeneous BC	✓	92.738	✓	1.904		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

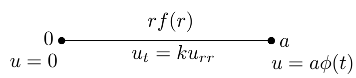
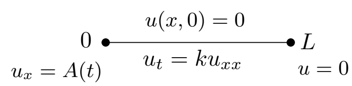
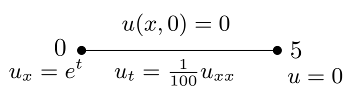
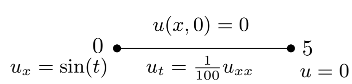
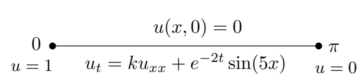
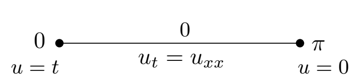
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
207	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 nonhomogeneous BC	✓	34.188	✓	15.72	Yes	
208	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	 left end BC depends on time (general case)	✓	66.569	✓	11.76	Yes	
209	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	 left end BC depends on time (special case)	✓	88.627	✓	15.321	Yes	Yes
210	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	 left end BC depends on time (special case)	✓	86.742	✓	13.846	Yes	Yes
211	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	 Haberman 8.3.6 (special case)	✓	95.024	✓	18.548	Yes	Yes
212	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	 BC depends on time (special case)	✓	12.162	✓	5.8		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

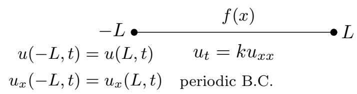
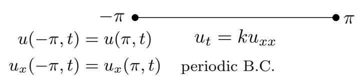
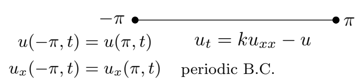
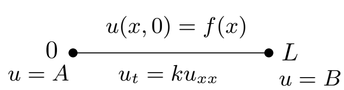
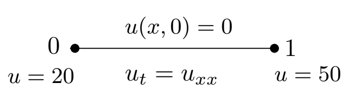
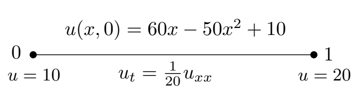
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
213	Diffusion in 1D Finite domain (bar), Periodic BC	 Periodic BC (general case). IC given	✓	15.368	✓	10.554		
214	Diffusion in 1D Finite domain (bar), Periodic BC	 Periodic BC (general case). No IC given	✓	0.014	✓	1.573	Yes	
215	Diffusion in 1D Finite domain (bar), Periodic BC	 Periodic BC (general case). Damped heat PDE. No IC given	✗	0.071	✓	1.459	Yes	
216	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 both ends non-homogeneous BC (general case)	✓	63.746	✓	4.237	Yes	
217	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 non-homogeneous BC (special case)	✓	16.942	✓	2.648	Yes	
218	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Articolo 8.4.1 (special case)	✓	63.188	✓	2.548	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
219	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A$, $u_t=ku_{xx}+Q(x)$, $u=B$. With source that depends on space only (general case)	✓	52.613	✓	6.341	Yes	
220	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=60-2x$; 0 to 30; $u=20$, $u_t=ku_{xx}+\frac{x}{10}$, $u=50$. With source that depends on space only (special case)	✓	94.816	✓	3.432	Yes	Yes
221	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A$, $u_t=ku_{xx}+Q(x,t)$, $u=B$. With source that depends on space and time only (general case)	✓	69.048	✓	9.333	Yes	
222	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A(t)$, $u_t=ku_{xx}$, $u=B(t)$. Both ends depend on time (general case)	✓	84.436	✓	17.345	Yes	
223	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=x$; 0 to 2; $u=\sin(t)$, $u_t=\frac{1}{10}u_{xx}$, $u=2\cos(t)$. Both ends depend on time (special case)	✓	67.824	✓	28.194	Yes	Yes
224	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$\sin x$; 0 to π; $u_x=1$, $u_t=u_{xx}$, $u_x=-1$. both ends nonhomogeneous	✓	63.852	✓	14.49	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

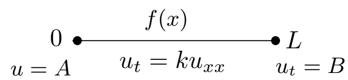
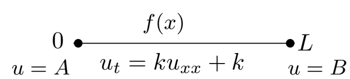
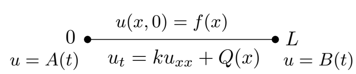
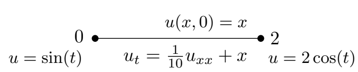
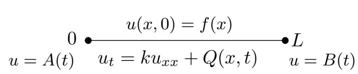
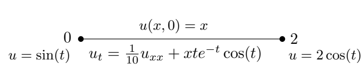
#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
225	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Haberman 8.2.1 (a) (general case)	✓	38.949	✓	6.474	Yes	
226	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Haberman 8.2.1 (d) (general solution)	✓	54.039	✓	5.482	Yes	
227	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source that depends on space only (general solution)	✓	115.005	✗ (Timed out)	600.	Yes	
228	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source that depends on space only (special case)	✓	104.862	✓	29.86	Yes	Yes
229	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source that depends on time and space (general solution)	✓	79.308	✓	16.293	Yes	
230	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source present (special case)	✓	117.548	✓	63.454	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
231	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	0 — $1+\cos(2x)$ — 1; $u_x=\sin(t)$, $u_t=u_{xx}+1+x\cos(t)$, $u_x=\sin(t)$ Pinchover and Rubinstein 6.17	✗	21.971	✓	11.401		
232	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	0 — $60-20x$ — π; $u_x=\frac{t\sin t}{5}$, $u_t=ku_{xx}+x$, $u_x=\frac{t\cos t}{10}$ nonhomogeneous BC	✓	111.737	✓	16.879		
233	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	0 — $f(x)$ — L; $u_x=A(t)$, $u_t=u_{xx}+Q(x,t)$, $u_x=B(t)$ Haberman 8.2.2. (a)	✓	122.354	✓	21.911	Yes	
234	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	0 — $\frac{-40x^2}{3}+\frac{45x}{2}+5$ — 1; $u=5$, $u_t=\frac{1}{20}u_{xx}+t$, $u_x+u=10$ Articolo 8.4.3	✓	5.486	✓	16.523		
235	Diffusion in 1D Semi-infinite domain	0 — $u(x,0)=0$ — ∞; $u=A$, $u_t=ku_{xx}$ left end constant (general case)	✓	60.035	✓	2.036	Yes	
236	Diffusion in 1D Semi-infinite domain	0 — $u(x,0)=0$ — ∞; $u=60$, $u_t=\frac{1}{10}u_{xx}$ left end constant (special case)	✓ It fail if assumption $x>0$ is given. A bug	31.691	✓	2.772	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
237	Diffusion in 1D Semi-infinite domain	0 ● → ∞, top: 0; $u = f(t)$, $u_t = u_{xx}$ Logan p. 76. Left end general function of time (general case)	✓	0.577	✓	6.391	Yes	
238	Diffusion in 1D Semi-infinite domain	0 ● → ∞, top: $u(x,0) = 0$; $u = \sin(t)$, $u_t = \frac{1}{10} u_{xx}$ Left end function of time (special case)	✓	60.047	✓	10.552	Yes	Yes
239	Diffusion in 1D Semi-infinite domain	0 ● → ∞, top: 0; $u = 1$, $u_t = k u_{xx}$ nonhomogeneous BC	✓	60.029	✓	2.204		
240	Diffusion in 1D Semi-infinite domain	$-x_0$ ● → ∞, top: $u(x,t_0) = 10$; $u = 0$, $u_t = \frac{1}{4} u_{xx}$ $x > \lvert x_0 \rvert, t > \lvert t_0 \rvert$ I.C. not zero	✓ due to IC/BC not zero	13.357	✓	2.941		
241	Diffusion in 1D Semi-infinite domain	0 ● → ∞, top: λ; $u = \mu$, $u_t = k u_{xx}$ $x > 0, t > 0$ nonhomogeneous BC	✓	60.074	✓	2.326		
242	Diffusion in 1D Semi-infinite domain	0 ● → ∞, top: $\cos x$; $u = 1$, $u_t = u_{xx}$ $x > 0, t > 0$ nonhomogeneous BC	✓	60.071	✓	6.131		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

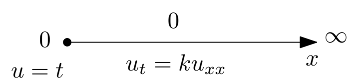
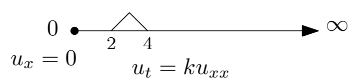
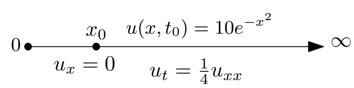
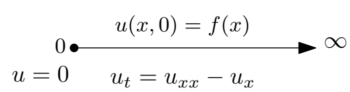
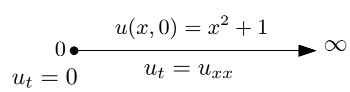
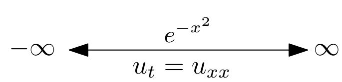
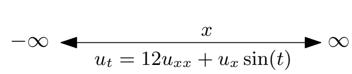
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
243	Diffusion in 1D Semi-infinite domain	 nonhomogeneous B.C.	✓	60.033	✓	2.684		
244	Diffusion in 1D Semi-infinite domain	 Unit triangle I.C.	✓	61.013	✓	2.839		
245	Diffusion in 1D Semi-infinite domain	 I.C. not at $t = 0$	✓	61.517	✓	4.622		
246	Diffusion in 1D Semi-infinite domain	 Diffusion with advection	✓	1.75	✓	4.813		
247	Diffusion in 1D Semi-infinite domain	 Practice exam problem	✓	60.108	✓	2.73		
248	Diffusion in 1D Infinite domain	 Inverse exponential I.C.	✓	0.869	✓	3.444	Yes	
249	Diffusion in 1D Infinite domain	 Advection term	✓	0.067	✓	2.516		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

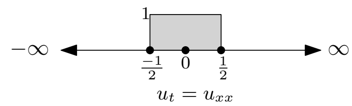
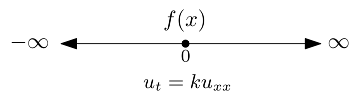
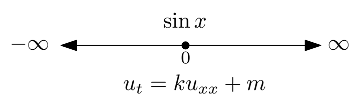
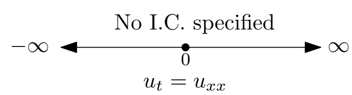
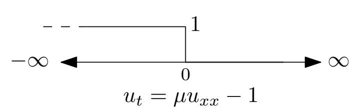
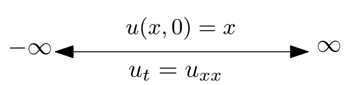
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
250	Diffusion in 1D Infinite domain	 UnitBox I.C.	✓	60.016	✓	4.32		
251	Diffusion in 1D Infinite domain	 No source	✓	0.4	✓	5.85		
252	Diffusion in 1D Infinite domain	 constant as source	✓	0.425	✓	2.873		
253	Diffusion in 1D Infinite domain	 No intial conditions	✓	0.004	✓	0.676	Yes	
254	Diffusion in 1D Infinite domain	 piecewise I.C.	✓ due to i.c. not at zero	52.493	✓	3.928	Yes	
255	Diffusion in 1D Infinite domain	 Practice exam problem	✓	0.065	✓	1.503		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

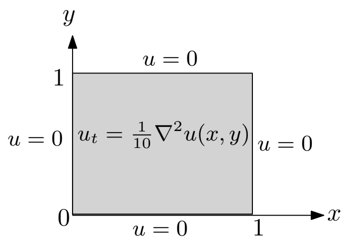
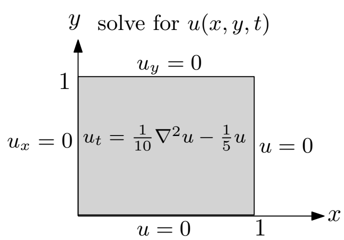
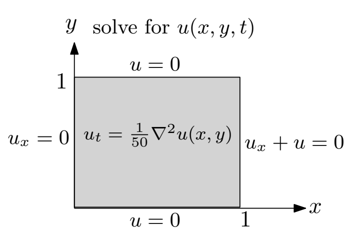
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
256	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 At $t = 0, u = x(1-x)(1-y)y$ No source	✓	4.249	✓	14.339		
257	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 At $t = 0, u = (1-x^2)(1-\frac{1}{2}y)y$ Internal source term	✓	10.246	✓	13.191		
258	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 At $t = 0, u = (1-\frac{x^2}{3})y(1-y)$ Articolo 6.6.3	✓	6.641	✓	20.238		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

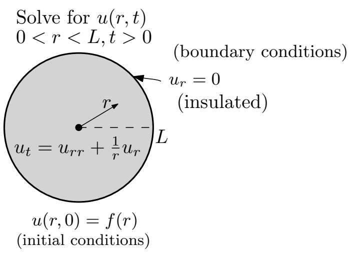
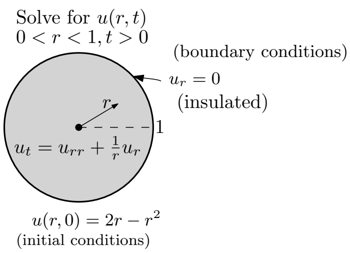
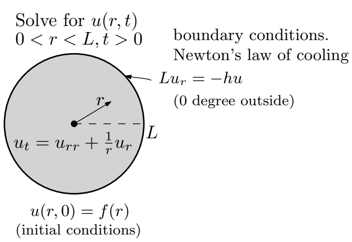
#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
259	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency, insulated (General solution)	✓	0.97	✓ Cant get series solution	51.553	Yes	
260	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency, insulated (Specific solution)	✓	2.743	✓ Do not understand Maple solution	17.502	Yes	Yes
261	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency	✓	0.333	✓	1.819		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

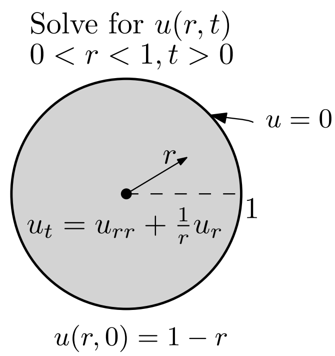
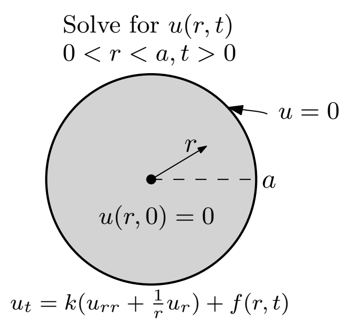
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
262	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency	✓	1.131	✓	1.511		
263	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 Haberman 8.3.5 (General solution)	✓	19.48	✓	5.353	Yes	

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

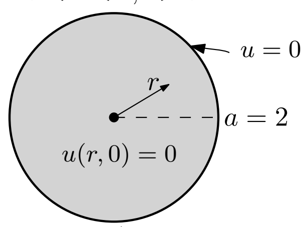
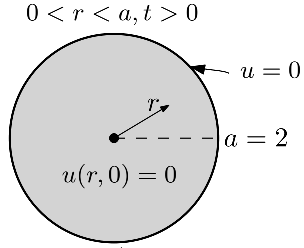
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
264	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,t)$ $0<r<a, t>0$  $u_t = k(u_{rr} + \frac{1}{r}u_r) + \sin(t)$ Where $k=\frac{1}{100}$ Specific example of the above	✓	3.516	✓	1.148	Yes	Yes
265	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,t)$ $0<r<a, t>0$  $u_t = k(u_{rr} + \frac{1}{r}u_r) + rte^{-t}$ Where $k=\frac{1}{100}$ Specific example of the above	✓	2.641	✗	1.184	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

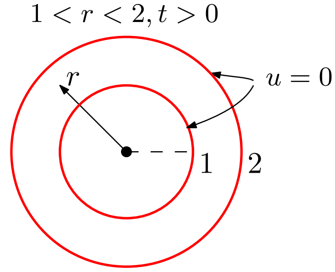
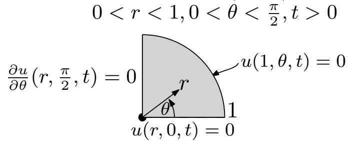
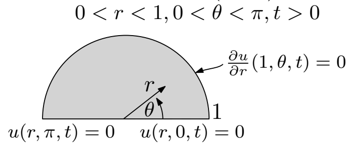
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
266	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,t)$ $1 < r < 2, t > 0$  $u_t = \frac{2}{r} u_r + u_{rr}$ $u(r,0) = -\sin(\pi r)$ Inside ring	✓ $n = 1$ causs division by zero	3.449	✗ (Timed out)	600.		
267	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0 < r < 1, 0 < \theta < \frac{\pi}{2}, t > 0$  I.C. $u(r,\theta,0) = (r - r^3)\sin\theta$ $u_t = \frac{1}{50}(u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta})$ Articolo 6.9.1	✓	3.973	✗	0.908		
268	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0 < r < 1, 0 < \theta < \pi, t > 0$  I.C. $u(r,\theta,0) = (r - \frac{r^3}{3})\sin\theta$ $u_t = \frac{1}{25}(u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta})$ Articolo 6.9.2	✓	4.38	✓	38.487		

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

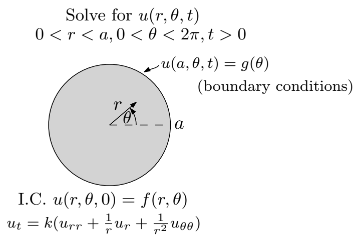
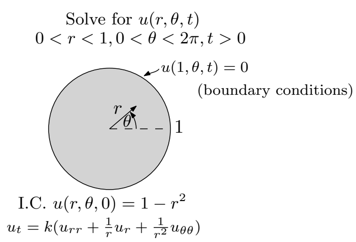
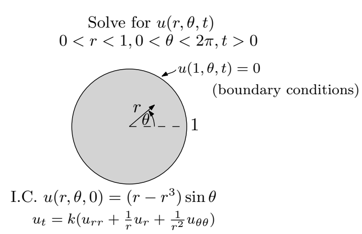
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
269	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 Haberman 8.2.5 with θ dependency (General case)	✓	310.649	✗	0.441	Yes	
270	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 With θ dependency (Specific example)	✓	8.757	✗	1.062	Yes	Yes
271	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 With θ dependency (specific example)	✓	10.1	✗	28.405	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

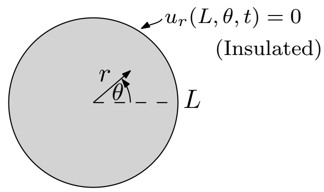
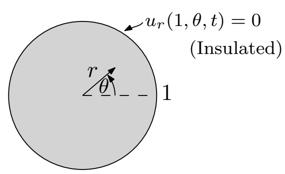
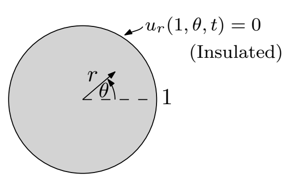
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
272	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<L, -\pi<\theta<\pi, t>0$  I.C. $u(r,\theta,0)=f(r,\theta)$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Insulated with θ dependency (General solution)	✓	4.65	✗	5.436	Yes	
273	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, -\pi<\theta<\pi, t>0$  I.C. $u(r,\theta,0)=(2r-r^2)\cos\theta\sin\theta$ $u_t=u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}$ Insulated with θ dependency (Specific example)	✓	16.146	✗	9.737	Yes	Yes
274	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, -\pi<\theta<\pi, t>0$  I.C. $u(r,\theta,0)=(2Lr-r^2)\theta\sin\theta e^{\cos\theta}$ $u_t=u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}$ Insulated with θ dependency (Specific example)	✗ (Timed out)	600.	✗	9.36	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

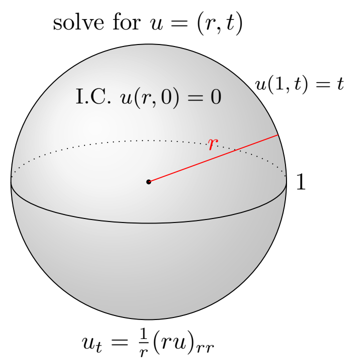
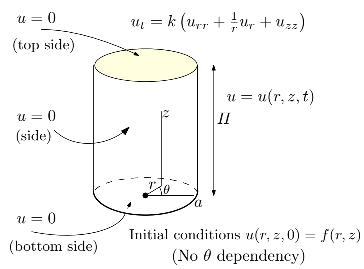
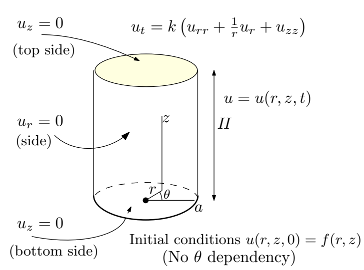
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
275	Diffusion in 3D Spherical coordinates	 No angle dependencies	✓	61.104	✓ Has unresolved Laplace integrals	3.518		
276	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.4 (a)	✓	1.883	✓	81.613	Yes	Yes
277	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.4 (b)	✓	6.469	✗	30.911	Yes	Yes

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Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

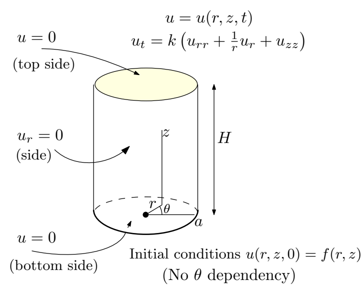
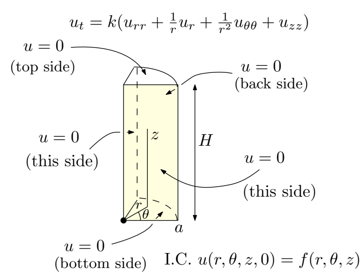
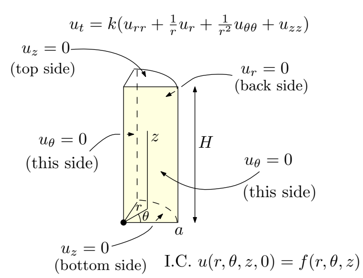
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
278	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.4 (c)	✓	3.162	✗	27.302		
279	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.3 (a)	✗	0.012	✗	4.145		
280	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.3 (b)	✗	0.008	✗	4.083		

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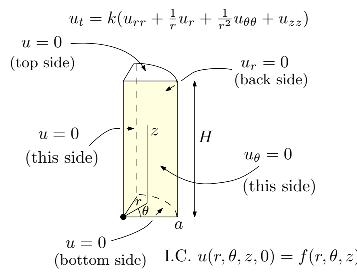
Table 1.8 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
281	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.3 (c)	✗	0.012	✗	4.082		

1.3.4 Elliptic PDE's (Laplace, Poisson, Helmholtz)

Table 1.9: Elliptic PDE's (Laplace, Poisson, Helmholtz) breakdown of results. Time in seconds

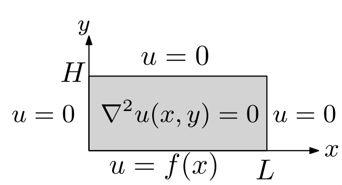
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
282	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, buttom edge not	✓	1.78	✓	79.735	Yes	

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

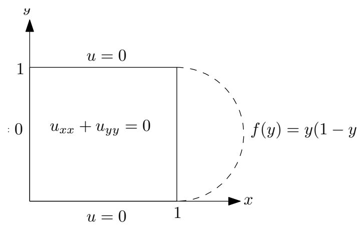
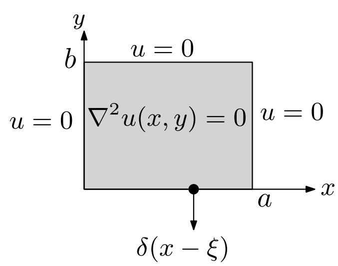
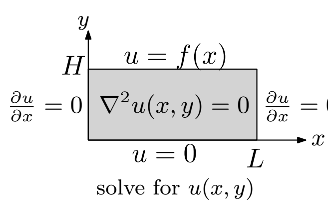
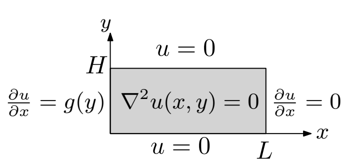
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
283	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, right edge not	✓	1.83	✓	8.797	Yes	
284	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, buttom edge has impulse	✓	4.12	✓	26.447	Yes	
285	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (a)	✓	0.988	✓	72.504		
286	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (b)	✓	0.944	✓	83.332		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

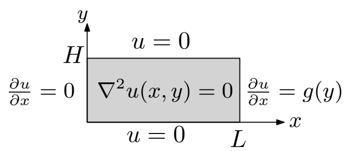
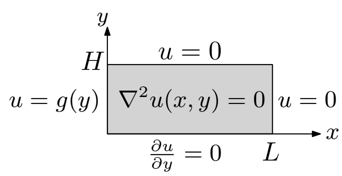
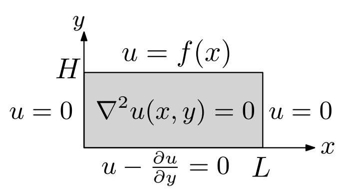
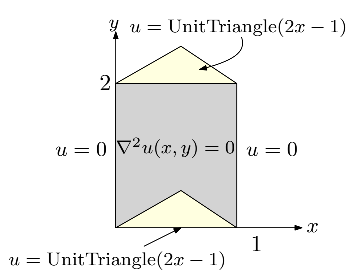
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
287	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (c)	✓	4.685	✓	58.768		
288	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (d)	✓	4.133	✓	83.677		
289	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (e)	✓	19.674	✓	73.192	Yes	
290	Laplace in 2D Cartesian coordinates	 Unit triangle B.C.	✓	0.985	✓	18.79		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

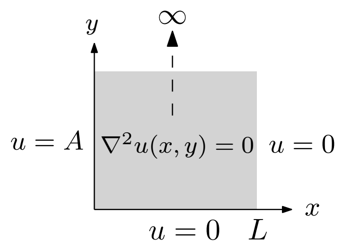
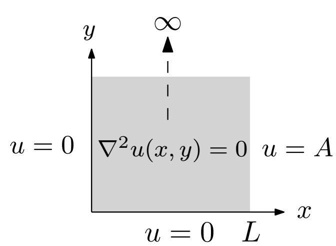
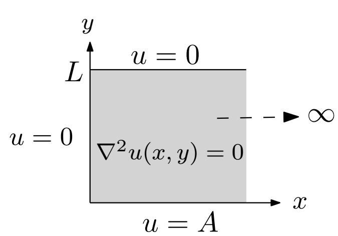
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
291	Laplace in 2D Cartesian coordinates	 Top edge at infinity	✓	3.586	✓	7.27	Yes	
292	Laplace in 2D Cartesian coordinates	 Top edge at infinity	✓	2.813	✓	9.875	Yes	
293	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	3.514	✓	7.235	Yes	

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

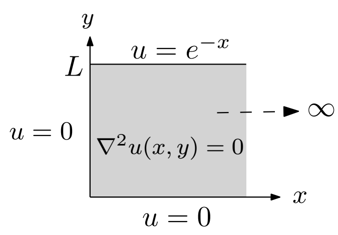
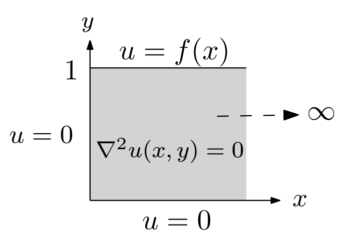
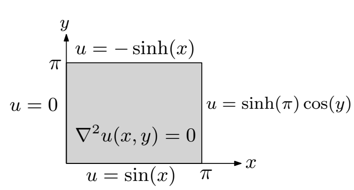
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
294	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	41.351	✓	41.03	Yes	
295	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	2.994	✓	32.109		
296	Laplace in 2D Cartesian coordinates	Laplace PDE in 2D Cartessian with boundary condition as Dirac function	✓	0.044	✓	3.511		
297	Laplace in 2D Cartesian coordinates	 One side homogeneous	✓	1.692	✓	51.323		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

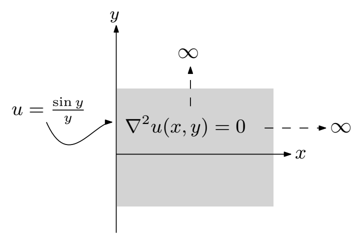
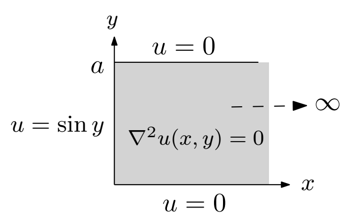
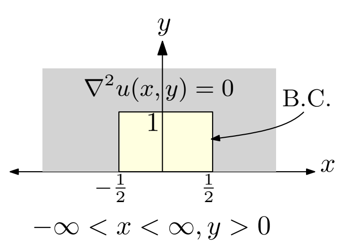
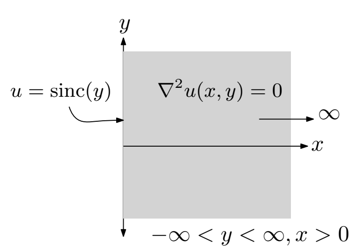
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
298	Laplace in 2D Cartesian coordinates	 In right half plane	✓	21.834	✓	4.04		
299	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✗	2.434	✓	23.138		
300	Laplace in 2D Cartesian coordinates	 Dirichlet problem Upper half	✓	1.371	✓	6.671		
301	Laplace in 2D Cartesian coordinates	 Right half-plane	✓	19.623	✓	4.481		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

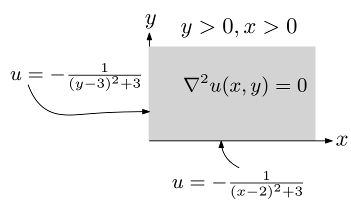
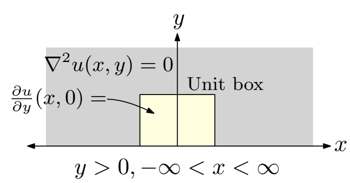
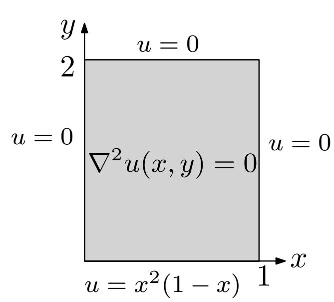
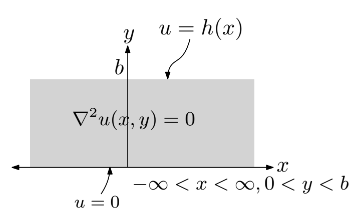
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
302	Laplace in 2D Cartesian coordinates	 First quadrant	✓	14.632	✗	31.473		
303	Laplace in 2D Cartesian coordinates	 Neumann problem upper half-plane	✓	4.77	✓ used convert(sol,Int).	8.997		
304	Laplace in 2D Cartesian coordinates	 Dirichlet problem in a rectangle	✓	2.303	✓	12.098		
305	Laplace in 2D Cartesian coordinates	 Strip in upper half	✓	60.258	✓	12.959		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

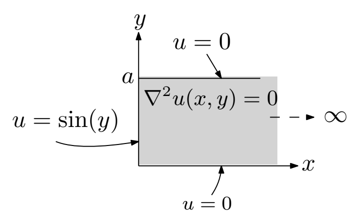
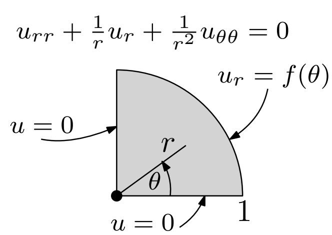
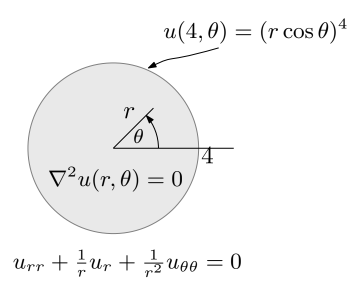
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
306	Laplace in 2D Cartesian coordinates	 in Rectangle, right edge at infinity	✗	2.353	✓	19.588		
307	Laplace in 2D Polar coordinates	 Laplace PDE inside quarter disk, Neumann BC at edge	✓	3.459	✓	2.842		
308	Laplace in 2D Polar coordinates	 $r=4$ and $u=x^4$ at boundary of disk	✓	150.086	✓	22.021	Yes	

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

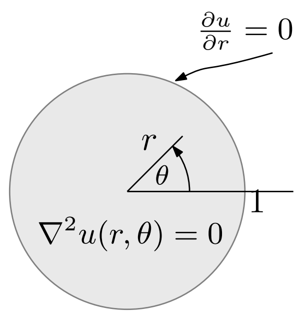
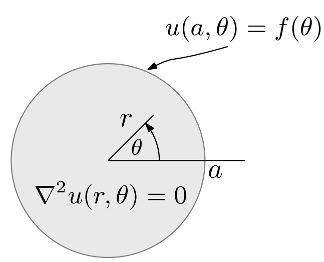
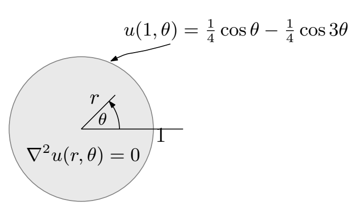
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
309	Laplace in 2D Polar coordinates	 $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$ $r = 1$ and $u_r = x$ at boundary of disk	✗	2.589	✗	1.888	Yes	
310	Laplace in 2D Polar coordinates	 $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$ Laplace inside disk. General solution	✓	0.624	✓	11.463		
311	Laplace in 2D Polar coordinates	 Laplace inside disk. Specific boundary conditions	✓	3.936	✓	13.769	Yes	

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

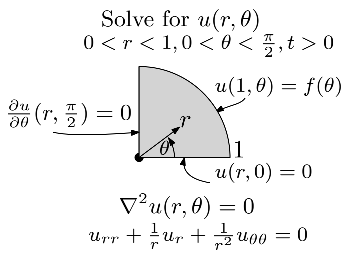
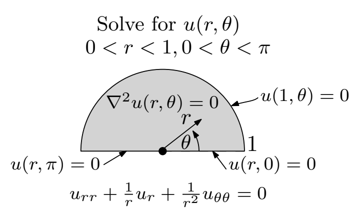
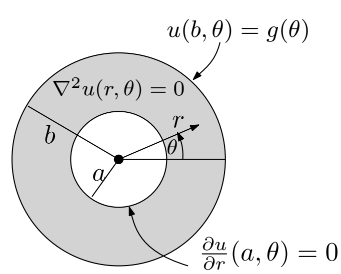
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
312	Laplace in 2D Polar coordinates	 Haberman 2.5.5 (c)	✗	3.989	✓	2.611	Yes	
313	Laplace in 2D Polar coordinates	 semi-circle	✓	2.08	✓	2.322		
314	Laplace in 2D Polar coordinates	 Haberman 2.5.8 (b)	✓	8.91	✓	361.819	Yes	

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

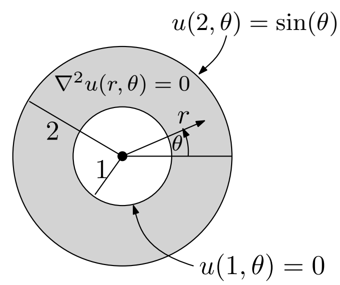
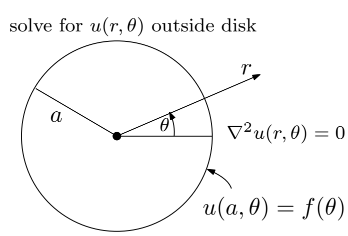
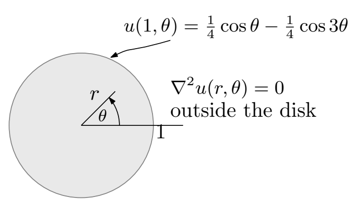
#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
315	Laplace in 2D Polar coordinates	 Circular annulus	✓	0.519	✓	9.499		
316	Laplace in 2D Polar coordinates	 Outside a disk	✓	6.915	✓	13.615		
317	Laplace in 2D Polar coordinates	 Outside a disk	✓	4.21	✓	15.946	Yes	
318	Laplace in 3D Spherical coordinates	In a sphere	✓	0.036	✓	3.401		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

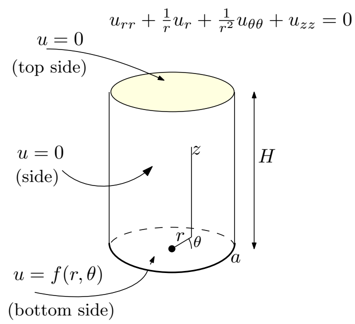
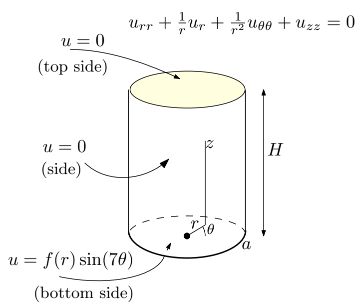
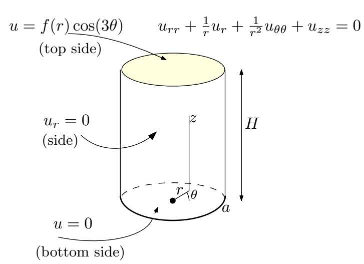
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
319	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (a)	✗	0.018	✗	3.513		
320	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (b)	✗	0.015	✗	3.773		
321	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (c)	✗	0.015	✗	3.813		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

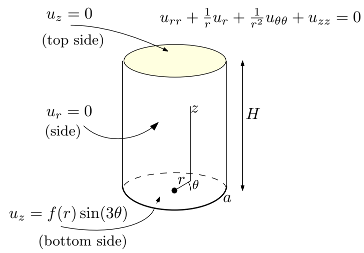
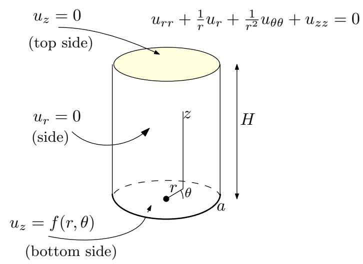
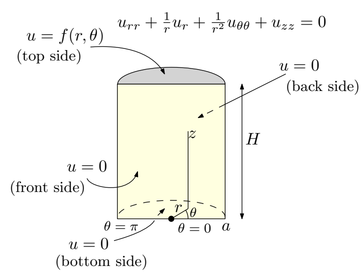
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
322	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (d)	✗	0.014	✗	3.881		
323	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (e)	✗	0.015	✗	3.827		
324	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (a)	✗	0.015	✗	4.865		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

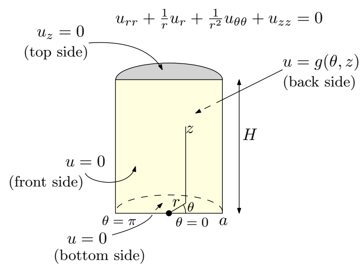
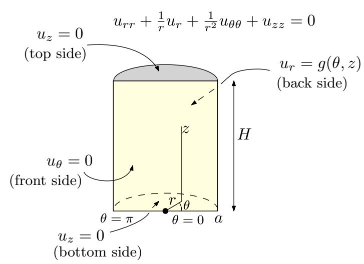
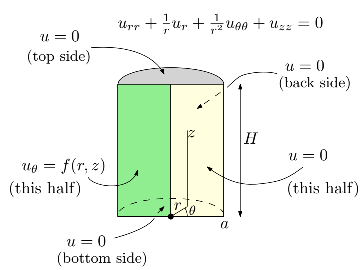
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
325	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (b)	✗	0.015	✗	39.671		
326	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (c)	✗	0.02	✗	53.644		
327	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (d)	✗	0.014	✗	4.81		

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Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

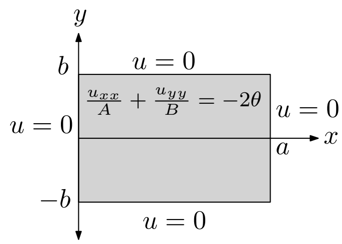
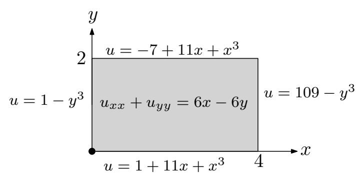
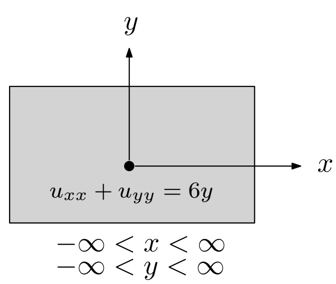
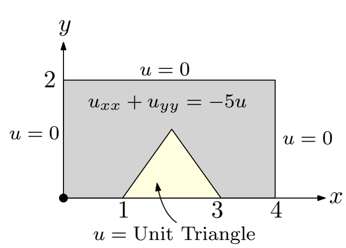
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
328	Poisson in 2D Cartesian coordinates	 All boundaries at zero	✗	0.015	✗	82.75	Yes	
329	Poisson in 2D Cartesian coordinates	 Dirichlet problem in a rectangle	✓	1.368	✓	1.599		
330	Poisson in 2D Cartesian coordinates	 Poisson PDE in whole 2D plane	✓	31.393	✓	0.066	Yes	
331	Helmholtz in 2D Cartesian coordinates	 In rectangle	✓	64.18	✓	25.968		

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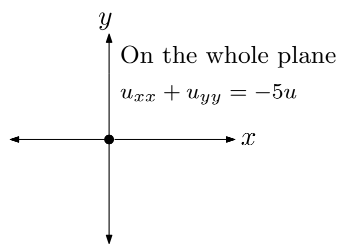
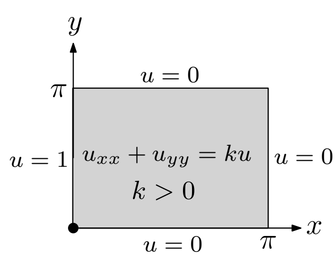
Table 1.9 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
332	Helmholtz in 2D Cartesian coordinates	 On whole plane	✓ why? It solved earlier with BC?	0.004	✓	0.289		
333	Helmholtz in 2D Cartesian coordinates	 Reduced Helmholtz Inside square	✓	62.146	✓	26.367		
334	Helmholtz in 3D Spherical coordinates	Chain reaction PDE	✗	0.123	✗ trivial solution	4.683	Yes	

1.3.5 Hyperbolic PDE's (Wave)

Table 1.10: Hyperbolic PDE's (Wave) breakdown of results. Time in seconds

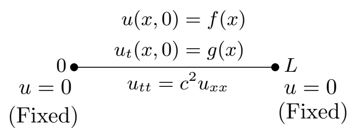
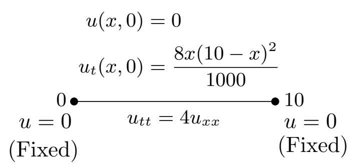
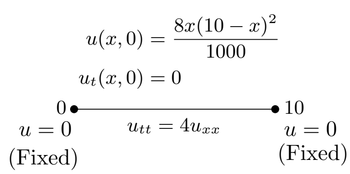
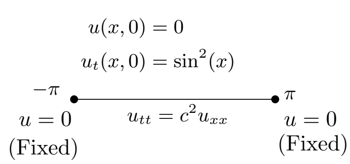
#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
335	Wave PDE in 1D Finite length string	 General solution for both ends fixed. Domain is $0 \ldots L$	✓	33.33	✓	26.077	Yes	
336	Wave PDE in 1D Finite length string	 both ends fixed, inital position zero (special case)	✓	3.324	✓	13.872	Yes	Yes
337	Wave PDE in 1D Finite length string	 both ends fixed, inital velocity zero (special case)	✓	2.518	✓	10.655	Yes	Yes
338	Wave PDE in 1D Finite length string	 both ends fixed but domain is $-\pi \ldots \pi$. zero intial position, non zero initial velocity	✓	6.978	✓	60.958	Yes	Yes

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

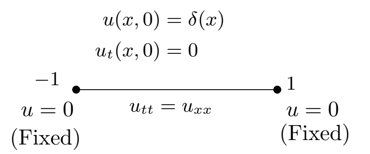
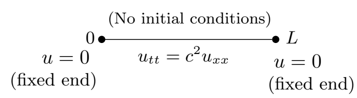
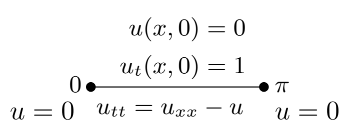
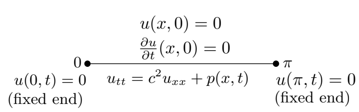
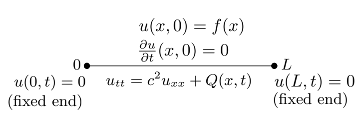
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
339	Wave PDE in 1D Finite length string	 both ends fixed but domain is $-1\dots1$. intial position is an impulse, zero initial velocity	✓	1.749	✓	23.098	Yes	
340	Wave PDE in 1D Finite length string	 Logan book, page 28. Both ends fixed	✓	0.974	✓	2.843		
341	Wave PDE in 1D Finite length string	 non-zero initial velocity. Both ends fixed	✓	86.507	✓	8.794		
342	Wave PDE in 1D Finite length string	 Logan book page 149)	✓	20.85	✓	15.192		
343	Wave PDE in 1D Finite length string	 Haberman 8.5.2 (a)	✓	23.852	✓	54.316		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
344	Wave PDE in 1D Finite length string	$u(x,0)=f(x)$, $\frac{\partial u}{\partial t}(x,0)=0$, $u_{tt}=c^2u_{xx}+g(x)\cos(\omega t)$, $0<x<L$, $u(0,t)=0$ (fixed end), $u(L,t)=0$ (fixed end). Haberman 8.5.2 (b)	✓	38.58	✓	36.438	Yes	
345	Wave PDE in 1D Finite length string	$v(x,0)=f(x)$, $\frac{\partial v}{\partial t}(x,0)=g(x)$, $v_{tt}=v_{xx}$, $0<x<1$, $v(0,t)=0$ (fixed end), $v(1,t)=0$ (fixed end). Both I.C. not zero	✓	22.264	✓	18.372		
346	Wave PDE in 1D Finite length string	$v(x,0)=f(x)$, $\frac{\partial v}{\partial t}(x,0)=g(x)$, $u_{tt}=c^2u_{xx}+1$, $0<x<L$, $u(0,t)=0$ (fixed end), $u(L,t)=0$ (fixed end). With constant source	✓	34.232	✓	30.72		
347	Wave PDE in 1D Finite length string	$u(x,0)=0$, $\frac{\partial u}{\partial t}(x,0)=0$, $u_{tt}=c^2u_{xx}+Ax$, $0<x<L$, $u(0,t)=0$ (fixed end), $u(L,t)=0$ (fixed end). Logan page 213	✓	4.763	✓	17.921		
348	Wave PDE in 1D Finite length string	$u(x,0)=f(x)$, $\frac{\partial u}{\partial t}(x,0)=0$, $u_{tt}+2u_t=c^2u_{xx}$, $0<x<\pi$, $u(0,t)=0$ (fixed end), $u(\pi,t)=0$ (fixed end). Telegraphy PDE	✓	86.878	✓	19.516		
349	Wave PDE in 1D Finite length string	$u(x,0)=f(x)$, $\frac{\partial u}{\partial t}(x,0)=0$, $u_{tt}+\gamma^2u=c^2u_{xx}$ ($\gamma^2 u$: dispersion term), $0<x<\pi$, $u(0,t)=0$ (Fixed), $u(\pi,t)=0$ (Fixed). Dispersion term present (general case)	✓ Due to adding dispersion term	25.235	✓	25.109	Yes	

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

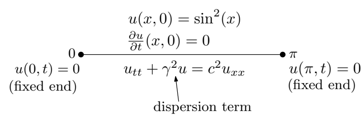
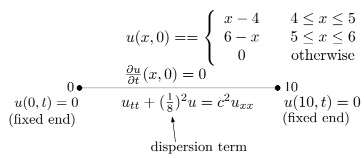
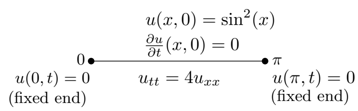
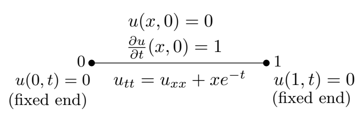
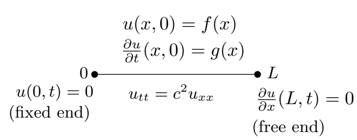
#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
350	Wave PDE in 1D Finite length string	 Dispersion term present	✓ Due to adding dispersion term	92.86	✓	89.473	Yes	
351	Wave PDE in 1D Finite length string	 Dispersion term present (specific case)	✓ Due to adding dispersion term	94.897	✓	22.849	Yes	Yes
352	Wave PDE in 1D Finite length string	 non-zero initial position	✓ But sum should not include $n=2$	7.991	✓ Handled $n=2$ case correctly	75.615		
353	Wave PDE in 1D Finite length string	 With source	✓	10.847	✓	79.233		
354	Wave PDE in 1D Finite length string	 Right end free (general case)	✓	25.353	✓	34.789	Yes	

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

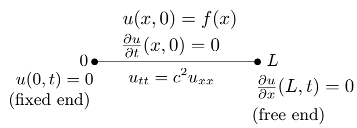
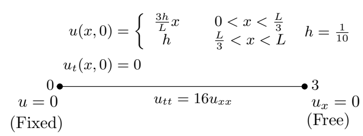
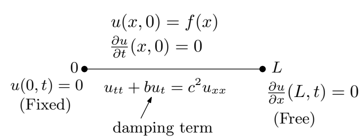
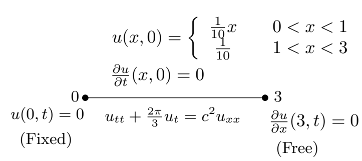
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
355	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity (general case)	✓	7.149	✓	25.612	Yes	
356	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity (special case)	✓	89.446	✓	15.144	Yes	Yes
357	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (general case)	✓	52.43	✓	33.813	Yes	
358	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (special case, underdamped)	✓	114.95	✓	29.971	Yes	Yes

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
359	Wave PDE in 1D Finite length string	$u(x,0)=\begin{cases}\frac{1}{10}x & 0<x<1\\ \frac{1}{10} & 1<x<3\end{cases}$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 … 3 $u(0,t)=0$ (Fixed) $u_{tt}+\frac{4\pi}{3}u_t=c^2u_{xx}$ $\frac{\partial u}{\partial x}(3,t)=0$ (Free) Right end free, zero initial velocity, damping present (special case, critical damped)	✓	114.627	✓	36.941	Yes	Yes
360	Wave PDE in 1D Finite length string	$u(x,0)=\begin{cases}\frac{1}{10}x & 0<x<1\\ \frac{1}{10} & 1<x<3\end{cases}$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 … 3 $u(0,t)=0$ (Fixed) $u_{tt}+\frac{4\pi}{3}u_t=c^2u_{xx}$ $\frac{\partial u}{\partial x}(3,t)=0$ (Free) Right end free, zero initial velocity, damping present (special case, over damped)	✓	116.345	✓	22.824	Yes	Yes
361	Wave PDE in 1D Finite length string	$u(x,0)=x^2-2x$ $u(x,1)=u(x,\frac{1}{2})+e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}}$ 0 … 1 $u=0$ (fixed end) $-u_{tt}+u=u_{xx}+2e^{-t}(x-\frac{1}{2}x^2+\frac{1}{2}t-1)$ $\frac{\partial u}{\partial x}(1,t)=0$ (free end) I.C. at different times, right end free, with source	✗	40.028	✓	85.616	Yes	
362	Wave PDE in 1D Finite length string	$u(x,0)=0$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 … π $u(0,t)=0$ (fixed end) $u_{tt}=4u_{xx}+(1+t)x$ $u(\pi,t)=\sin(t)$ (oscillating end) Right end oscillates	✓	66.861	✓	62.784		
363	Wave PDE in 1D Finite length string	I.C. $u(x,0)=x$ $u_t(x,0)=0$ $-\pi$ … π $u_{tt}=u_{xx}$ Periodic BC $u(-\pi,t)=u(\pi,t)$ $u_x(\pi,0)=u_x(\pi,t)$ Perioidic B.C.	✗	2.412	✗	4.542		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

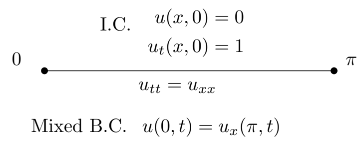
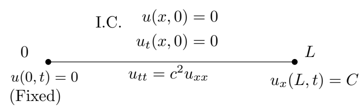
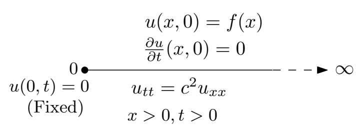
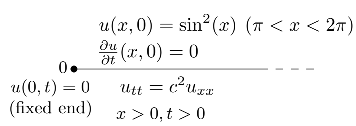
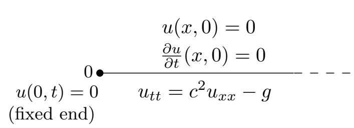
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
364	Wave PDE in 1D Finite length string	 Mixed B.C.	✗	15.603	✗	5.86		
365	Wave PDE in 1D Finite length string	 Left end fixed, right end non-homogeneous Neumann BC. Zero initial conditions	✓	63.343	✓	29.737	Yes	Yes
366	Wave PDE in 1D Semi-infinite domain	 Left end fixed, (general case)	✓	3.996	✗	5.185	Yes	
367	Wave PDE in 1D Semi-infinite domain	 Left end fixed with specific initial position	✓	15.596	✓	13.644	Yes	
368	Wave PDE in 1D Semi-infinite domain	 Logan page 115, left end fixed with source	✓	7.298	✓	5.306		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

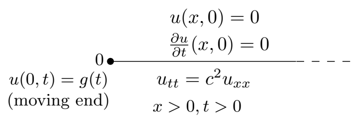
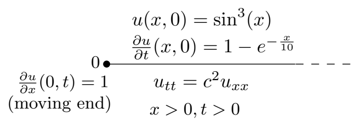
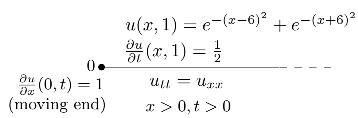
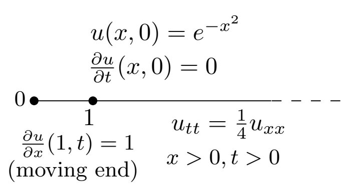
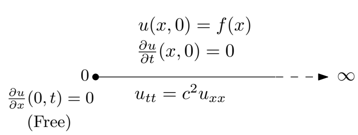
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
369	Wave PDE in 1D Semi-infinite domain	 Left moving boundary condition	✓	1.94	✓	4.509		
370	Wave PDE in 1D Semi-infinite domain	 moving Left end	✓	9.201	✓	6.497		
371	Wave PDE in 1D Semi-infinite domain	 I.C. at $t=1$	✓	9.147	✓	1.518		
372	Wave PDE in 1D Semi-infinite domain	 B.C. at $x=1$	✓	76.551	✓	2.819		
373	Wave PDE in 1D Semi-infinite domain	 Left end free. zero initial velocity (general solution)	✓	7.123	✓	1.703		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
374	Wave PDE in 1D Semi-infinite domain	$u(x,0)=\begin{cases}1 & 4\le x\le 5\\ 0 & \text{otherwise}\end{cases}$ $\frac{\partial u}{\partial t}(x,0)=0$ $0 \to \infty$ $\frac{\partial u}{\partial x}(0,t)=0$ (Free) $u_{tt}=c^2u_{xx}$ Left end free. zero initial velocity (Special solution)	✓	4.202	✓	6.313	Yes	Yes
375	Wave PDE in 1D Semi-infinite domain	$u(x,0)=\begin{cases}1 & 4\le x\le 5\\ 0 & \text{otherwise}\end{cases}$ $\frac{\partial u}{\partial t}(x,0)=0$ $0 \to \infty$ $u(0,t)=0$ (Fixed) $u_{tt}=c^2u_{xx}$ Left end fixed. zero initial velocity (Special solution)	✓	4.373	✓	8.789	Yes	Yes
376	Wave PDE in 1D Semi-infinite domain	$u(x,0)=0$ $\frac{\partial u}{\partial t}(x,0)=g(x)$ $0 \to \infty$ $\frac{\partial u}{\partial x}(0,t)=0$ (Free) $u_{tt}=c^2u_{xx}$ Left end free. zero initial position (general solution)	✓	8.691	✓	1.712		
377	Wave PDE in 1D Semi-infinite domain	$u(x,0)=f(x)$ $\frac{\partial u}{\partial t}(x,0)=g(x)$ $0 \to \infty$ $\frac{\partial u}{\partial x}(0,t)=0$ (Free) $u_{tt}=c^2u_{xx}$ Left end free. Non zero initial position and velocity (general solution)	✓	33.339	✓	1.483		
378	Wave PDE in 1D Semi-infinite domain	$u(x,0)=0$ $\frac{\partial u}{\partial t}(x,0)=x^3$ 0 $\frac{\partial u}{\partial x}(0,t)=0$ (free end) $u_{tt}=9u_{xx}+f(x,t)$ $x>0,t>0$ Left end free with source	✓	1.122	✓	1.232		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

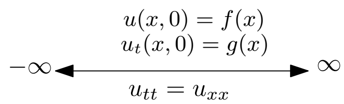
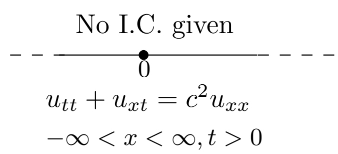
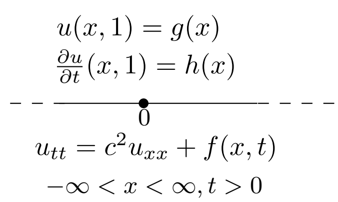
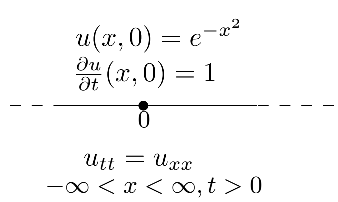
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
379	Wave PDE in 1D Infinite domain	 General case. $u_{tt} = u_{xx}$ with $u(x,0) = f(x), u_t(x,0) = g(x)$	✓	0.019	✓	0.69		
380	Wave PDE in 1D Infinite domain	 General case. No IC given. $u_{tt} + u_{xt} = c^2 u_{xx}$	✓	0.005	✓	0.55		
381	Wave PDE in 1D Infinite domain	 $u_{tt} = c^2 u_{xx} + f(x,t)$, IC at $t = 1$,$u(x,1) = g(x), u_t(x,1) = h(x)$	✓	0.158	✓	6.92		
382	Wave PDE in 1D Infinite domain	 No source. $u_{tt} = u_{xx}$, with $u(x,0) = e^{-x^2}, u_t(x,0) = 1$	✓	0.002	✓	0.307		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
383	Wave PDE in 1D Infinite domain	$u(x,0) = \sin x - \frac{\cos(3x)}{e^{\frac{\lvert x\rvert}{6}}}$ $\frac{\partial u}{\partial t}(x,0) = 0$ 0 $u_{tt} = u_{xx} + m$ $-\infty < x < \infty, t > 0$ With source term. $u_{tt} = u_{xx} + m$	✓	0.011	✓	0.444		
384	Wave PDE in 1D Infinite domain	No I.C. given 0 $u_t + 6u + u_{xxx} = 0$ $-\infty < x < \infty, t > 0$ non-linear (Solitons) $u_t + 6u(x,t)u_x + u_{xxx} = 0$	✓	0.028	✓	0.404	Yes	Yes
385	Wave PDE in 1D Infinite domain	No I.C. given 0 $u_{tt} = 3u_{xx} + u_{xt} - 1$ $-\infty < x < \infty, t > 0$ Inhomogeneous PDE $3u_{xx} - u_{tt} + u_{xt} = 1$	✓	0.003	✓	0.319		
386	Wave PDE in 1D Infinite domain	$u_t(x,0) = \cos(x)$ $u(x,0) = \sin(x)$ $-\infty$ … ∞ $u_{tt} = u_{xx}$ Practice exam problem Math 5587	✓	0.029	✓	0.365		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

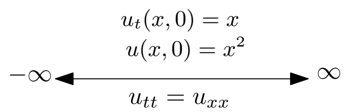
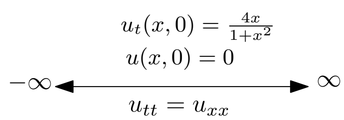
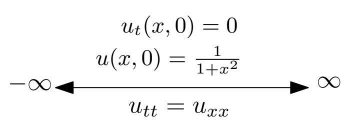
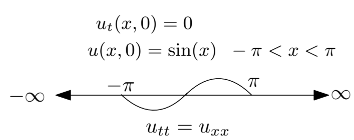
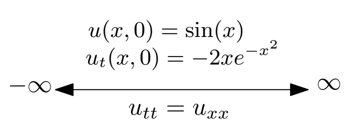
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
387	Wave PDE in 1D Infinite domain	 Practice exam problem Math 5587	✓	0.002	✓	0.29		
388	Wave PDE in 1D Infinite domain	 Practice exam problem Math 5587	✓	8.846	✓	0.491		
389	Wave PDE in 1D Infinite domain	 zero initial velocity	✓	0.002	✓	0.306	Yes	Yes
390	Wave PDE in 1D Infinite domain	 zero initial velocity	✓	0.004	✓	4.527	Yes	Yes
391	Wave PDE in 1D Infinite domain	 General case $u_{tt}=u_{xx}$ with $u(x,0)=\sin x, u_t(x,0)=-2xe^{-x^2}$	✓	0.1	✓	0.378		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

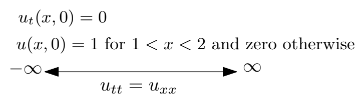
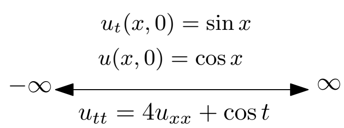
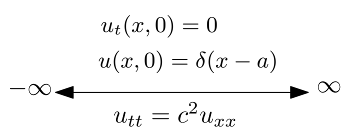
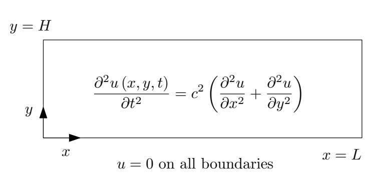
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
392	Wave PDE in 1D Infinite domain	 General case. $u_{tt}=u_{xx}$ dAlembert solution, box function as initial position	✓	0.003	✓	0.347	Yes	Yes
393	Wave PDE in 1D Infinite domain	 $u_{tt}=4u_{xx}+\cos(t)$ dAlembert solution with $u(x,0)=\sin x, u_t(x,0)=\cos x$	✓	0.096	✓	0.699	Yes	Yes
394	Wave PDE in 1D Infinite domain	 $u_{tt}=c^2u_{xx}$ dAlembert solution with $u(x,0)=\delta(x-a), u_t(x,0)=0$	✓	63.956	✓	0.695	Yes	
395	Wave PDE in 1D Infinite domain	system of 2 inhomogeneous linear hyperbolic system with constant coefficients	✓	0.267	✗	1.19		
396	Wave PDE in 2D Cartesian coordinates	 Rectangular membrane. Fixed on all edges, General solution	✗	2.271	✓	399.349	Yes	

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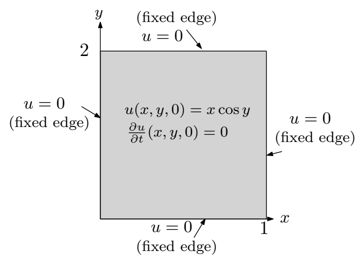
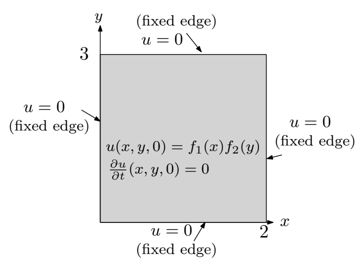
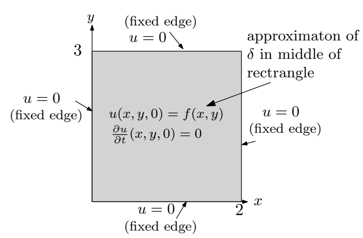
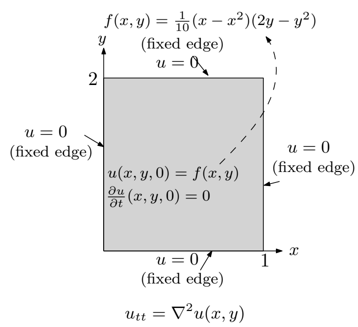
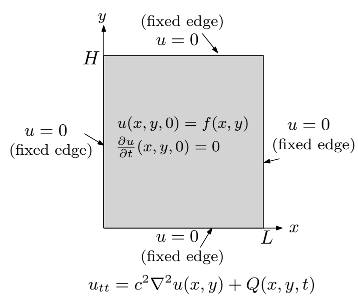
Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

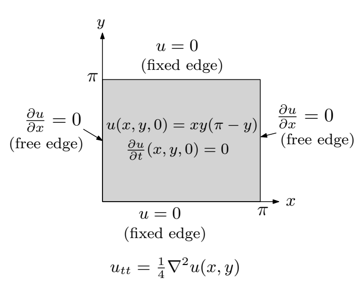
#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
397	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = (\frac{1}{10})^2 \nabla^2 u(x,y)$ Rectangular membrane. Fixed on all edges, zero velocity. Specific example	✗	0.424	✓	32.39	Yes	Yes
398	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = (\frac{1}{3})^2 \nabla^2 u(x,y)$ All 4 edges fixed, zero initial velocity, Specific example	✓	33.848	✓	57.503	Yes	Yes
399	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = (\frac{1}{3})^2 \nabla^2 u(x,y)$ All 4 edges fixed, zero initial velocity, Specific example, delta in center	✓	9.295	✓	54.177	Yes	Yes

Continued on next page

Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

<table>
<tr><th rowspan="2">#</th><th rowspan="2">PDE</th><th rowspan="2">description</th><th colspan="2">Mathematica</th><th colspan="2">Maple</th><th rowspan="2">hand solved?</th><th rowspan="2">animated?</th></tr>
<tr><th>result</th><th>time</th><th>result</th><th>time</th></tr>
<tr><td>400</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

All 4 edges fixed</td><td>✓</td><td>4.949</td><td>✓</td><td>32.94</td><td></td><td></td></tr>
<tr><td>401</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

All edges fixed (Haberman 8.5.5 (a)</td><td>✓</td><td>27.715</td><td>✗</td><td>3.128</td><td></td><td></td></tr>
<tr><td>402</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

2 edgs fixed, 2 free, zero initial velocity</td><td>✓</td><td>5.952</td><td>✓</td><td>63.49</td><td></td><td></td></tr>
</table>

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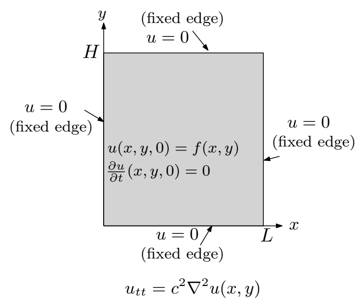
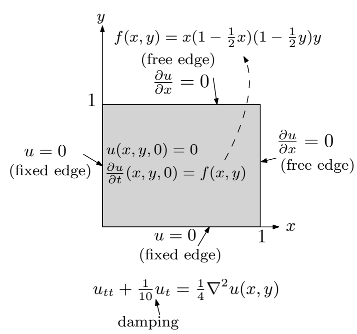
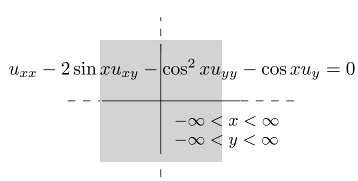
Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

<table>
<tr><th rowspan="2">#</th><th rowspan="2">PDE</th><th rowspan="2">description</th><th colspan="2">Mathematica</th><th colspan="2">Maple</th><th rowspan="2">hand solved?</th><th rowspan="2">animated?</th></tr>
<tr><th>result</th><th>time</th><th>result</th><th>time</th></tr>
<tr><td>403</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

All 4 edges fixed, zero initial velocity, general solution</td><td>✓</td><td>1.411</td><td>✗ (Timed out)</td><td>600.</td><td>Yes</td><td></td></tr>
<tr><td>404</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

With damping</td><td>✓</td><td>9.074</td><td>✓</td><td>88.935</td><td></td><td></td></tr>
<tr><td>405</td><td>Wave PDE in 2D Cartesian coordinates</td><td>

On the whole plane</td><td>✓</td><td>0.171</td><td>✗</td><td>0.388</td><td></td><td></td></tr>
</table>

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

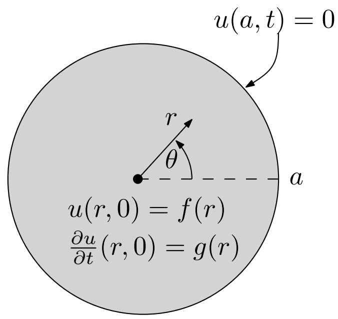
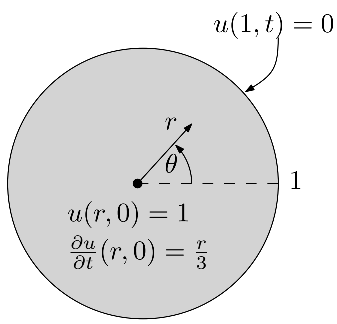
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
406	Wave PDE in 2D Polar coordinates	 $u_{tt}=c^2(u_{rr}+\frac{1}{r}u_r)$ no θ dependency, fixed boundary, general case	✓	3.01	✓ Has unresolved Invlaplace calls	8.424	Yes	
407	Wave PDE in 2D Polar coordinates	 $u_{tt}=c^2(u_{rr}+\frac{1}{r}u_r)$ no θ dependency. Specific example. Both initial conditions not zero	✓	2.684	✓ Has unresolved Invlaplace calls	73.886		

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

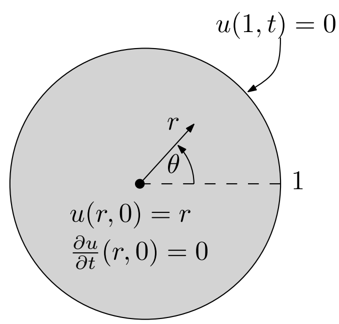
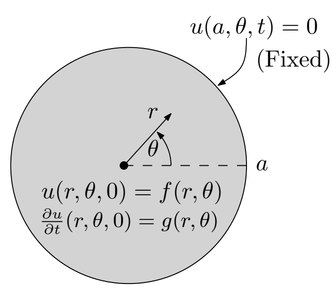
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
408	Wave PDE in 2D Polar coordinates	 $u_{tt} = c^2(u_{rr} + \frac{1}{r}u_r)$ $c = \frac{2}{10}$ no θ dependency. Specific example. Both initial conditions not zero	✓	2.549	✓ Has unresolved Invlaplace calls. How to get series solution?	68.303	Yes	Yes
409	Wave PDE in 2D Polar coordinates	no θ dependency. Using integral transforms. Source present. Specific example	✓	3.628	✓	29.343		
410	Wave PDE in 2D Polar coordinates	no θ dependency. Using integral transforms. Source present. Specific example	✗	2.511	✓	7.253		
411	Wave PDE in 2D Polar coordinates	 $u_t = c^2(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$ θ dependency, fixed on edges, general solution	✓	9.409	✗	16.112	Yes	

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

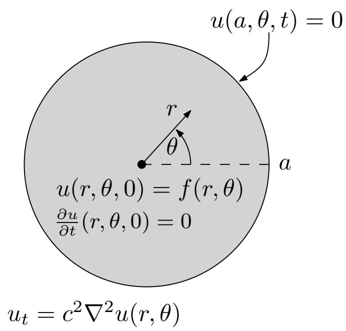
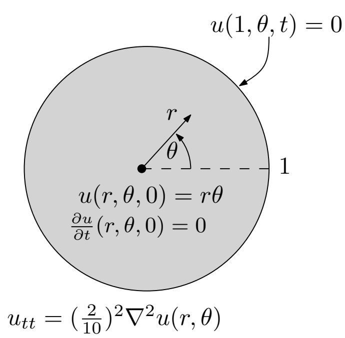
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
412	Wave PDE in 2D Polar coordinates	 θ dependency, fixed on edges, zero initial velocity, general solution	✓	4.687	✗	12.441	Yes	
413	Wave PDE in 2D Polar coordinates	 θ dependency, fixed on edges, zero initial velocity, specific example	✓	15.449	✗	10.359	Yes	Yes

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Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

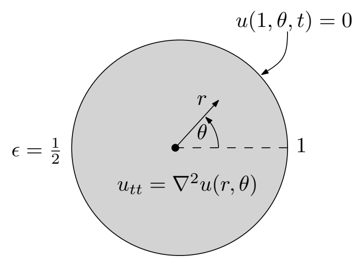
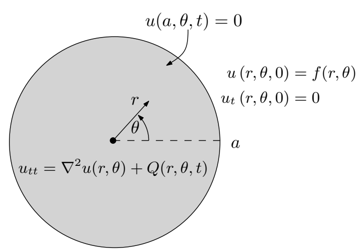
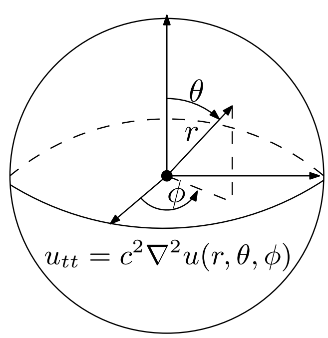
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
414	Wave PDE in 2D Polar coordinates	 $u(r,\theta,0)=0$ $u_t(r,\theta,0)=\begin{cases}\frac{1}{\pi\epsilon^2} & \text{if } r\leq\epsilon\\ 0 & \text{otherwise}\end{cases}$ θ dependency, fixed on edges, zero initial position, specific example	✓	7.747	✗	12.831	Yes	Yes
415	Wave PDE in 2D Polar coordinates	 θ dependency, fixed on edges, zero initial position with internal source (Haberman 8.5.5. (b)	✗	0.042	✗	4.539	Yes	
416	Wave PDE in 3D Spherical coordinates	 No I.C. no B.C.	✓	0.039	✓	6.306		

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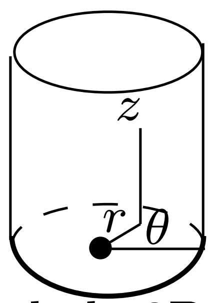
Table 1.10 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
417	Wave PDE in 3D Cylindrical coordinates	$$u_{tt} = \nabla^2 u$$  (whole 3D) No I.C. no B.C.	✓	0.006	✓	0.668		

1.3.6 Handbook of first order partial differential equations

Table 1.11: Handbook of first order partial differential equations breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
418	chapter 1	problem number 1	✓	0.005	✓	0.072		
419	chapter 1	problem number 2	✓	0.004	✓	0.005		
420	chapter 1	problem number 3	✓	0.017	✓	0.006	Yes	
421	chapter 1	problem number 4	✓	0.016	✓	0.003		
422	chapter 1	problem number 5	✓	0.032	✓	0.031	Yes	
423	chapter 1	problem number 6	✓	0.03	✓	0.014		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
424	chapter 2 2.1	problem number 1	✓	0.006	✓	0.075	Yes	
425	chapter 2 2.1	problem number 2	✓	0.01	✓	0.072	Yes	
426	chapter 2 2.1	problem number 3	✓	0.054	✓	0.208	Yes	
427	chapter 2 2.1	problem number 4	✓	0.021	✓	0.171	Yes	
428	chapter 2 2.1	problem number 5	✓	0.024	✓	1.405	Yes	
429	chapter 2 2.1	problem number 6	✓	0.05	✓	1.384	Yes	
430	chapter 2 2.1	problem number 7	✓	0.055	✓	3.135	Yes	
431	chapter 2 2.1	problem number 8	✗	2.383	✓	18.946		
432	chapter 2 2.2	problem number 1	✓	0.009	✓	0.01		
433	chapter 2 2.2	problem number 2	✓	0.244	✓	0.582		
434	chapter 2 2.2	problem number 3	✓	0.106	✓	0.183		
435	chapter 2 2.2	problem number 4	✓	0.385	✓	0.413		
436	chapter 2 2.2	problem number 5	✓	0.25	✓	0.994		
437	chapter 2 2.2	problem number 6	✓	0.33	✓	0.435		
438	chapter 2 2.2	problem number 7	✓	0.353	✓	0.805		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
439	chapter 2 2.2	problem number 8	✓	0.511	✓	0.707		
440	chapter 2 2.2	problem number 9	✓	0.299	✓	0.525		
441	chapter 2 2.2	problem number 10	✓	0.171	✓	0.3		
442	chapter 2 2.2	problem number 11	✓	0.318	✓	2.757		
443	chapter 2 2.2	problem number 12	✓	0.822	✓	1.533		
444	chapter 2 2.2	problem number 13	✓	0.113	✓	0.3		
445	chapter 2 2.2	problem number 14	✓	0.27	✓	0.821		
446	chapter 2 2.2	problem number 15	✓	0.7	✓	1.048		
447	chapter 2 2.2	problem number 16	✓	1.564	✓	0.658		
448	chapter 2 2.2	problem number 17	✓	0.951	✓	0.828		
449	chapter 2 2.2	problem number 18	✓	0.292	✓	0.108		
450	chapter 2 2.2	problem number 19	✓	0.294	✓	0.793		
451	chapter 2 2.2	problem number 20	✗	0.261	✓	1.996		
452	chapter 2 2.2	problem number 21	✓	0.196	✓	2.307		
453	chapter 2 2.2	problem number 22	✓	0.348	✓	1.529		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
454	chapter 2 2.2	problem number 23	✓	0.438	✓	2.915		
455	chapter 2 2.2	problem number 24	✗	0.313	✓	0.734		
456	chapter 2 2.2	problem number 25	✗ (Timed out)	600.	✗ (Timed out)	600.		
457	chapter 2 2.2	problem number 26	✗	165.853	✗	45.839		
458	chapter 2 2.2	problem number 27	✗	101.598	✗	3.194		
459	chapter 2 2.2	problem number 28	✗	0.528	✓ solution contains RootOf	1.436		
460	chapter 2 2.2	problem number 29	✗	101.315	✗	5.214		
461	chapter 2 2.2	problem number 30	✗	0.305	✓	1.639		
462	chapter 2 2.2	problem number 31, Hesse's equation	✗	235.84	✓	10.262		
463	chapter 2 2.3	problem number 1	✓ But it can't solve it when assuming $b > 0$ which is strange.	0.491	✓	3.677		
464	chapter 2 2.3	problem number 2	✓	0.074	✓	0.278		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
465	chapter 2 2.3	problem number 3	✓	0.179	✓	2.572		
466	chapter 2 2.3	problem number 4	✗	0.277	✓	5.678		
467	chapter 2 2.3	problem number 5	✗	0.932	✓ Answer contains RootOf	0.536		
468	chapter 2 2.3	problem number 6	✗	0.232	✓ Answer contains RootOf	0.263		
469	chapter 2 2.3	problem number 7	✓	0.234	✓	0.239		
470	chapter 2 2.3	problem number 8	✓	0.265	✓	4.681		
471	chapter 2 2.3	problem number 9	✗ (Timed out)	600.	✓	0.102		
472	chapter 2 2.4	problem number 1	✓	0.037	✓	0.214		
473	chapter 2 2.4	problem number 2	✓	0.268	✓	1.645		
474	chapter 2 2.4	problem number 3	✓	0.196	✓	0.843		
475	chapter 2 2.4	problem number 4	✓	0.467	✓	6.135		
476	chapter 2 2.4	problem number 5	✗	0.53	✗ (Timed out)	600.		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
477	chapter 2 2.4	problem number 6	✓	1.952	✓	1.328		
478	chapter 2 2.4	problem number 7	✗	1.872	✓	65.123		
479	chapter 2 2.5	problem number 1	✓	0.058	✓	0.583		
480	chapter 2 2.5	problem number 2	✓	0.134	✓	1.028		
481	chapter 2 2.5	problem number 3	✓	0.198	✓	0.404		
482	chapter 2 2.5	problem number 4	✗	20.865	✓	4.145		
483	chapter 2 2.5	problem number 5	✓	0.933	✓	4.072		
484	chapter 2 2.5	problem number 6	✗	1.853	✓	9.467		
485	chapter 2 2.5	problem number 7	✓	0.458	✓	0.454		
486	chapter 2 2.5	problem number 8	✗	23.262	✓	12.719		
487	chapter 2 2.5	problem number 9	✗	1.15	✓	20.194		
488	chapter 2 2.5	problem number 10	✗	2.964	✓	17.552		
489	chapter 2 2.5	problem number 11	✗	26.182	✗	25.263		
490	chapter 2 2.5	problem number 12	✗	25.253	✗	21.109		
491	chapter 2 2.5	problem number 13	✗	30.158	✗	39.568		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
492	chapter 2 2.5	problem number 14	✗	0.36	✓ Solution contains RootOf	2.879		
493	chapter 2 2.5	problem number 15	✓	0.639	✓	5.063		
494	chapter 2 2.5	problem number 16	✗	78.509	✗	82.583		
495	chapter 2 2.5	problem number 17	✗	0.304	✓	6.927		
496	chapter 2 2.5	problem number 18	✗	0.578	✓	4.42		
497	chapter 2 2.5	problem number 19	✓	0.361	✓	3.761		
498	chapter 2 2.5	problem number 20	✓	0.298	✓	0.898		
499	chapter 2 2.5	problem number 21	✓	0.641	✓	0.808		
500	chapter 2 2.5	problem number 22	✓	0.59	✓	0.391		
501	chapter 2 2.5	problem number 23	✗	26.472	✓	0.487		
502	chapter 2 2.5	problem number 24	✓	0.331	✓	0.325		
503	chapter 2 2.5	problem number 25	✓	0.614	✓	0.282		
504	chapter 2 2.5	problem number 26	✓	0.86	✓	0.533		
505	chapter 2 2.5	problem number 27	✓	0.413	✓	3.769		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
506	chapter 2 2.5	problem number 28	✗	0.477	✓ Solution contains RootOf	4.192		
507	chapter 2 2.5	problem number 29	✗	0.365	✓ Solution contains RootOf	0.744		
508	chapter 2 2.5	problem number 30	✗	0.708	✓	9.119		
509	chapter 2 2.5	problem number 31	✗	18.477	✓	17.864		
510	chapter 2 2.5	problem number 32	✗	114.344	✗	35.89		
511	chapter 2 2.5	problem number 33	✓	0.562	✓	3.592		
512	chapter 2 2.5	problem number 34	✓	0.455	✓	2.213		
513	chapter 2 2.5	problem number 35	✓	0.242	✓	0.764		
514	chapter 2 2.5	problem number 36	✓	0.213	✓	1.009		
515	chapter 2 2.5	problem number 37	✓	0.479	✓	1.892		
516	chapter 2 2.5	problem number 38	✓	2.637	✓	1.227		
517	chapter 2 2.5	problem number 39	✓	0.182	✓	7.199		
518	chapter 2 2.5	problem number 40	✗	103.396	✗	26.251		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
519	chapter 2 2.5	problem number 41	✓	1.078	✓	4.981		
520	chapter 2 2.5	problem number 42	✗	4.276	✓	29.642		
521	chapter 2 2.5	problem number 43	✗	2.982	✓	11.045		
522	chapter 2 2.5	problem number 44	✗	31.136	✗	35.803		
523	chapter 2 2.5	problem number 45	✗	31.691	✗	43.207		
524	chapter 2 2.5	problem number 46	✗	12.09	✓	26.704		
525	chapter 2 2.5	problem number 47	✓	1.44	✓	0.553		
526	chapter 2 2.5	problem number 48	✗	33.94	✓	2.402		
527	chapter 2 2.5	problem number 49	✗	21.879	✗	28.604		
528	chapter 2 2.5	problem number 50	✗	51.312	✗	96.439		
529	chapter 2 2.5	problem number 51	✗	0.992	✓	9.535		
530	chapter 2 2.5	problem number 52	✗	11.804	✗	13.929		
531	chapter 2 2.5	problem number 53	✗	0.833	✓	13.071		
532	chapter 2 2.5	problem number 54	✗	0.956	✗	8.801		
533	chapter 2 2.5	problem number 55	✗	19.752	✗	18.898		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
534	chapter 2 2.5	problem number 56	✗	1.495	✓	12.736		
535	chapter 2 3.1	problem number 1	✓	0.021	✓	0.113		
536	chapter 2 3.1	problem number 2	✓	0.029	✓	0.012		
537	chapter 2 3.1	problem number 3	✓	0.215	✓	11.577		
538	chapter 2 3.1	problem number 4	✓	0.4	✓	14.391		
539	chapter 2 3.1	problem number 5	✗	1.053	✓	3.809		
540	chapter 2 3.1	problem number 6	✓	0.284	✓	1.342		
541	chapter 2 3.1	problem number 7	✓	0.235	✓	0.115		
542	chapter 2 3.1	problem number 8	✓	0.634	✓ Has RootOf	8.201		
543	chapter 2 3.1	problem number 9	✓	0.499	✓	12.494		
544	chapter 2 3.1	problem number 10	✗	0.666	✓	19.394		
545	chapter 2 3.1	problem number 11	✗	2.633	✗	7.209		
546	chapter 2 3.2	problem number 1	✓	0.786	✓	1.534		
547	chapter 2 3.2	problem number 2	✓	1.149	✓	0.585		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
548	chapter 2 3.2	problem number 3	✓	0.496	✓	0.203		
549	chapter 2 3.2	problem number 4	✓	0.625	✓	1.058		
550	chapter 2 3.2	problem number 5	✓	0.385	✓	0.275		
551	chapter 2 3.2	problem number 6	✗	23.202	✗	22.006		
552	chapter 2 3.2	problem number 7	✓	0.59	✓	0.79		
553	chapter 2 3.2	problem number 8	✗	1.051	✓	0.437		
554	chapter 2 3.2	problem number 9	✓	1.513	✓	2.165		
555	chapter 2 3.2	problem number 10	✓	1.289	✓	2.023		
556	chapter 2 3.2	problem number 11	✗	22.612	✓	2.587		
557	chapter 2 3.2	problem number 12	✓	3.572	✓	0.692		
558	chapter 2 3.2	problem number 13	✓	0.76	✓	0.284		
559	chapter 2 3.2	problem number 14	✗	24.234	✗	23.358		
560	chapter 2 3.2	problem number 15	✗	2.973	✓	4.565		
561	chapter 2 3.2	problem number 16	✗	22.908	✗	20.321		
562	chapter 2 3.2	problem number 17	✗	1.804	✓	4.062		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
563	chapter 2 3.2	problem number 18	✗	23.297	✗	16.888		
564	chapter 2 3.2	problem number 19	✓	0.513	✓	3.809		
565	chapter 2 3.2	problem number 20	✗	23.169	✗	14.983		
566	chapter 2 3.2	problem number 21	✗	25.852	✗	13.417		
567	chapter 2 3.2	problem number 22	✓	2.881	✓	0.766		
568	chapter 2 3.2	problem number 23	✓	0.787	✓	0.543		
569	chapter 2 3.2	problem number 24	✗	21.9	✗	17.232		
570	chapter 2 3.2	problem number 25	✓	0.485	✓	0.87		
571	chapter 2 3.2	problem number 26	✓	0.554	✓	0.625		
572	chapter 2 3.2	problem number 27	✗	1.139	✓ Solution contains RootOf	2.313		
573	chapter 2 3.2	problem number 28	✓	0.606	✓	4.207		
574	chapter 2 3.2	problem number 29	✓	0.41	✓	0.589		
575	chapter 2 3.2	problem number 30	✓	3.889	✓	0.849		
576	chapter 2 3.2	problem number 31	✗	50.01	✓	34.16		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
577	chapter 2 3.2	problem number 32	✓	0.28	✓	0.579		
578	chapter 2 3.2	problem number 33	✗	0.395	✓	2.288		
579	chapter 2 3.2	problem number 34	✗	1.469	✓	66.937		
580	chapter 2 3.2	problem number 35	✗	1.005	✓	41.681		
581	chapter 2 3.2	problem number 36	✗	1.093	✓	58.283		
582	chapter 2 4.1	problem number 1	✓	0.023	✓	0.143		
583	chapter 2 4.1	problem number 2	✓	0.14	✓	1.8		
584	chapter 2 4.1	problem number 3	✓	3.23	✓	15.25		
585	chapter 2 4.1	problem number 4	✗	43.098	✓	3.023		
586	chapter 2 4.1	problem number 5	✗	57.21	✓	19.35		
587	chapter 2 4.1	problem number 6	✓	0.22	✓	1.993		
588	chapter 2 4.1	problem number 7	✓	0.041	✓	0.237		
589	chapter 2 4.2	problem number 1	✓	0.019	✓	0.012		
590	chapter 2 4.2	problem number 2	✓	0.115	✓	1.313		
591	chapter 2 4.2	problem number 3	✗	61.929	✓	18.217		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
592	chapter 2 4.2	problem number 4	✗	64.619	✓	18.563		
593	chapter 2 4.2	problem number 5	✗	1.778	✓	9.164		
594	chapter 2 4.2	problem number 6	✗	2.923	✓	30.208		
595	chapter 2 4.2	problem number 7	✗	487.736	✓	8.38		
596	chapter 2 4.2	problem number 8	✓	0.212	✓	0.597		
597	chapter 2 4.3	problem number 1	✓	0.028	✓	0.015		
598	chapter 2 4.3	problem number 2	✓	0.185	✓	0.002		
599	chapter 2 4.3	problem number 3	✓	3.367	✓	4.988		
600	chapter 2 4.3	problem number 4	✗	43.425	✓	15.147		
601	chapter 2 4.3	problem number 5	✗	2.778	✓	9.777		
602	chapter 2 4.3	problem number 6	✗	4.636	✓	20.35		
603	chapter 2 4.3	problem number 7	✗	75.718	✓	7.616		
604	chapter 2 4.3	problem number 8	✓	0.341	✓	3.885		
605	chapter 2 4.4	problem number 1	✓	0.029	✓	0.155		
606	chapter 2 4.4	problem number 2	✓	0.189	✓	3.86		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
607	chapter 2 4.4	problem number 3	✓	3.102	✓	4.598		
608	chapter 2 4.4	problem number 4	✓	4.441	✓	3.854		
609	chapter 2 4.5	problem number 1	✓	0.28	✓	1.781		
610	chapter 2 4.5	problem number 2	✓	0.28	✓	0.664		
611	chapter 2 4.5	problem number 3	✓	2.838	✓	8.434		
612	chapter 2 4.5	problem number 4	✓	14.583	✓	13.157		
613	chapter 2 4.5	problem number 5	✓	0.192	✓	0.665		
614	chapter 2 4.5	problem number 6	✗	583.259	✓	16.266		
615	chapter 2 5.1	problem number 1	✓	0.061	✓	0.165		
616	chapter 2 5.1	problem number 3	✓	1.078	✓	0.109		
617	chapter 2 5.1	problem number 4	✗	3.213	✓	0.47		
618	chapter 2 5.2	problem number 1	✓	0.252	✓	0.659		
619	chapter 2 5.2	problem number 2	✓	0.271	✓	0.467		
620	chapter 2 5.2	problem number 3	✗	1.315	✓	1.175		
621	chapter 2 5.2	problem number 4	✓	0.838	✓	1.164		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
622	chapter 2 5.2	problem number 5	✗	23.747	✗	10.796		
623	chapter 2 5.2	problem number 6	✗	27.327	✓	9.23		
624	chapter 2 5.2	problem number 7	✗	27.996	✗	15.989		
625	chapter 2 5.2	problem number 8	✗	24.407	✗	14.186		
626	chapter 2 5.2	problem number 9	✓	0.937	✓	2.86		
627	chapter 2 5.2	problem number 10	✗	3.325	✓	4.48		
628	chapter 2 5.2	problem number 11	✓	0.218	✓	0.539		
629	chapter 2 5.2	problem number 12	✗	20.993	✗	9.505		
630	chapter 2 5.2	problem number 13	✗	23.817	✗	9.175		
631	chapter 2 5.2	problem number 14	✗	22.633	✗	10.421		
632	chapter 2 5.2	problem number 15	✓	0.486	✓	2.586		
633	chapter 2 5.2	problem number 16	✓	0.319	✓	0.405		
634	chapter 2 5.2	problem number 17	✓	1.122	✓	0.839		
635	chapter 2 5.2	problem number 18	✓	0.808	✓	3.392		
636	chapter 2 5.2	problem number 19	✗	4.562	✓	6.177		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
637	chapter 2 5.2	problem number 20	✗	4.582	✓	3.954		
638	chapter 2 5.2	problem number 21	✗	26.571	✓	3.814		
639	chapter 2 5.2	problem number 22	✓	0.721	✓	4.567		
640	chapter 2 5.2	problem number 23	✓	0.7	✓	2.847		
641	chapter 2 6.1	problem number 1	✓	0.497	✓	1.595		
642	chapter 2 6.1	problem number 2	✓ contains unre-solved integral	5.852	✓ contains unre-solved integral	4.149		
643	chapter 2 6.1	problem number 3	✓	1.43	✓ contains unre-solved integral	12.66		
644	chapter 2 6.1	problem number 4	✗	11.481	✓ contains unre-solved integral	7.566		
645	chapter 2 6.1	problem number 5	✗	42.587	✓	13.428		
646	chapter 2 6.1	problem number 6	✗	4.735	✓ contains unre-solved integrals	2.247		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
647	chapter 2 6.1	problem number 7	✓	19.363	✓	26.217		
648	chapter 2 6.1	problem number 8	✗	43.063	✓	3.993		
649	chapter 2 6.1	problem number 9	✗	44.379	✓	28.754		
650	chapter 2 6.1	problem number 10	✗	44.975	✓	14.128		
651	chapter 2 6.1	problem number 11	✗	37.431	✓	29.626		
652	chapter 2 6.1	problem number 12	✗	144.702	✗ (Timed out)	600.		
653	chapter 2 6.1	problem number 13	✓	0.761	✓	1.866		
654	chapter 2 6.1	problem number 14	✗	13.412	✓	13.095		
655	chapter 2 6.2	problem number 1	✓	0.529	✓ Contains unresolved integral	2.144		
656	chapter 2 6.2	problem number 2	✓	3.955	✓	5.842		
657	chapter 2 6.2	problem number 3	✓	0.819	✓	7.66		
658	chapter 2 6.2	problem number 4	✗	9.572	✓	4.918		
659	chapter 2 6.2	problem number 5	✗	42.291	✓	9.539		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
660	chapter 2 6.2	problem number 6	✗	42.82	✓	3.48		
661	chapter 2 6.2	problem number 7	✗	45.317	✓	16.95		
662	chapter 2 6.2	problem number 8	✗	45.734	✓	12.079		
663	chapter 2 6.2	problem number 9	✗	135.562	✓	25.044		
664	chapter 2 6.2	problem number 10	✗	2.281	✓	13.398		
665	chapter 2 6.2	problem number 11	✗	1.857	✓	42.632		
666	chapter 2 6.2	problem number 12	✗	53.846	✓	16.057		
667	chapter 2 6.3	problem number 1	✓	0.052	✓	0.143		
668	chapter 2 6.3	problem number 2	✓	0.434	✓	1.419		
669	chapter 2 6.3	problem number 3	✓	0.911	✓ Has unresolved integrals	7.417		
670	chapter 2 6.3	problem number 4	✗	44.497	✓	6.748		
671	chapter 2 6.3	problem number 5	✗	46.942	✓	3.165		
672	chapter 2 6.3	problem number 6	✓	2.57	✓	7.204		
673	chapter 2 6.3	problem number 7	✗	38.765	✓	18.134		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
674	chapter 2 6.3	problem number 8	✗	45.215	✗	87.83		
675	chapter 2 6.3	problem number 9	✗	166.671	✗ (Timed out)	600.		
676	chapter 2 6.3	problem number 10	✓	0.78	✓	2.565		
677	chapter 2 6.3	problem number 11	✗	94.742	✓	52.796		
678	chapter 2 6.3	problem number 12	✗	24.388	✓	14.816		
679	chapter 2 6.3	problem number 13	✗	2.08	✓	13.869		
680	chapter 2 6.3	problem number 14	✗	2.011	✓	16.572		
681	chapter 2 6.3	problem number 15	✗	27.354	✓	12.753		
682	chapter 2 6.4	problem number 1	✓	0.42	✓ Has unresolved integral	2.391		
683	chapter 2 6.4	problem number 2	✓	1.889	✓	3.04		
684	chapter 2 6.4	problem number 3	✗	10.658	✓	4.599		
685	chapter 2 6.4	problem number 4	✗	44.393	✓	3.522		
686	chapter 2 6.4	problem number 5	✗	46.966	✓	2.895		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
687	chapter 2 6.4	problem number 6	✓	0.674	✓	0.95		
688	chapter 2 6.4	problem number 7	✓	0.616	✓	11.699		
689	chapter 2 6.4	problem number 8	✓	0.557	✓	4.912		
690	chapter 2 6.4	problem number 9	✓	2.124	✓	2.5		
691	chapter 2 6.4	problem number 10	✓	2.42	✓	27.569		
692	chapter 2 6.4	problem number 11	✓	0.636	✓	4.846		
693	chapter 2 6.4	problem number 12	✗	7.997	✓	10.77		
694	chapter 2 6.5	problem number 1	✓	1.421	✓ Has unresolved integrals	4.171		
695	chapter 2 6.5	problem number 2	✓	2.541	✓	2.568		
696	chapter 2 6.5	problem number 3	✓	0.992	✓ Mathematica answer is simpler	5.158		
697	chapter 2 6.5	problem number 4	✗	26.076	✗	140.386		
698	chapter 2 6.5	problem number 5	✗	44.173	✓	4.745		
699	chapter 2 6.5	problem number 6	✗	67.024	✓	4.608		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
700	chapter 2 6.5	problem number 7	✗	134.91	✗	162.199		
701	chapter 2 6.5	problem number 8	✗	27.096	✓	5.187		
702	chapter 2 6.5	problem number 9	✗	25.423	✓	0.002		
703	chapter 2 6.5	problem number 10	✗	2.703	✓	12.582		
704	chapter 2 6.5	problem number 11	✗ (Timed out)	600.	✓	10.393		
705	chapter 2 7.1	problem number 1	✓	0.115	✓	0.542		
706	chapter 2 7.1	problem number 2	✓	3.243	✓	1.281		
707	chapter 2 7.1	problem number 3	✗	0.642	✓	1.933		
708	chapter 2 7.1	problem number 4	✓	0.166	✓	0.816		
709	chapter 2 7.1	problem number 5	✗	1.937	✓	2.717		
710	chapter 2 7.1	problem number 6	✓	0.546	✓	9.601		
711	chapter 2 7.1	problem number 7	✗	27.74	✓	20.897		
712	chapter 2 7.1	problem number 8	✗	1.547	✓	2.534		
713	chapter 2 7.1	problem number 9	✗	66.878	✗	84.036		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
714	chapter 2 7.1	problem number 10	✗	38.983	✗	72.918		
715	chapter 2 7.1	problem number 11	✓	2.214	✓	1.064		
716	chapter 2 7.1	problem number 12	✓	1.144	✓	2.67		
717	chapter 2 7.2	problem number 1	✓	0.111	✓	0.423		
718	chapter 2 7.2	problem number 2	✓	5.018	✓	1.151		
719	chapter 2 7.2	problem number 3	✗	15.525	✓	2.323		
720	chapter 2 7.2	problem number 4	✓	0.364	✓	1.302		
721	chapter 2 7.2	problem number 5	✗	6.465	✓	82.866		
722	chapter 2 7.2	problem number 6	✗	30.645	✗	23.073		
723	chapter 2 7.2	problem number 7	✗	47.941	✓	21.085		
724	chapter 2 7.2	problem number 8	✗	8.379	✓	74.218		
725	chapter 2 7.2	problem number 9	✗	41.156	✗	115.308		
726	chapter 2 7.2	problem number 10	✗	41.363	✗	140.156		
727	chapter 2 7.2	problem number 11	✓	2.468	✓	0.19		
728	chapter 2 7.2	problem number 12	✓	1.31	✓	1.005		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
729	chapter 2 7.3	problem number 1	✓	0.62	✓	0.185		
730	chapter 2 7.3	problem number 2	✓	4.24	✓	0.271		
731	chapter 2 7.3	problem number 3	✗	16.414	✓	0.457		
732	chapter 2 7.3	problem number 4	✓	0.477	✓	0.441		
733	chapter 2 7.3	problem number 5	✗	7.761	✓	1.876		
734	chapter 2 7.3	problem number 6	✓	2.485	✓	1.246		
735	chapter 2 7.3	problem number 7	✗	43.419	✓	21.22		
736	chapter 2 7.3	problem number 8	✓	0.31	✓	0.283		
737	chapter 2 7.3	problem number 9	✗	40.246	✗	18.052		
738	chapter 2 7.3	problem number 10	✗	43.105	✗ (Timed out)	600.		
739	chapter 2 7.3	problem number 11	✓	1.069	✓	0.248		
740	chapter 2 7.3	problem number 12	✓	0.923	✓	2.867		
741	chapter 2 7.4	problem number 1	✓	0.696	✓	0.185		
742	chapter 2 7.4	problem number 2	✓	7.007	✓	0.108		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
743	chapter 2 7.4	problem number 3	✗	21.042	✓	0.241		
744	chapter 2 7.4	problem number 4	✓	0.53	✓	0.264		
745	chapter 2 7.4	problem number 5	✗	7.845	✓	2.96		
746	chapter 2 7.4	problem number 6	✓	2.51	✓	1.598		
747	chapter 2 7.4	problem number 7	✗	43.597	✓	15.806		
748	chapter 2 7.4	problem number 8	✗	35.956	✗	10.949		
749	chapter 2 7.4	problem number 9	✗	40.981	✗	37.954		
750	chapter 2 7.4	problem number 10	✗	43.309	✗ (Timed out)	600.		
751	chapter 2 7.4	problem number 11	✓	1.151	✓	0.303		
752	chapter 2 7.4	problem number 12	✓	1.212	✓	5.337		
753	chapter 2 8.1	problem number 1	✓	0.089	✓	0.043		
754	chapter 2 8.1	problem number 2	✓	0.343	✓	0.175		
755	chapter 2 8.1	problem number 3	✗	0.635	✓	0.097		
756	chapter 2 8.1	problem number 4	✓	0.288	✓	0.095		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
757	chapter 2 8.1	problem number 5	✗	25.578	✓	0.725		
758	chapter 2 8.1	problem number 6	✗	0.951	✓	0.094		
759	chapter 2 8.1	problem number 7	✗	24.449	✗	4.		
760	chapter 2 8.1	problem number 8	✗	24.683	✗	4.237		
761	chapter 2 8.1	problem number 9	✗	0.98	✓	0.106		
762	chapter 2 8.1	problem number 10	✗	29.935	✗	15.819		
763	chapter 2 8.1	problem number 11	✗	28.621	✗	14.721		
764	chapter 2 8.1	problem number 12	✓	0.32	✓	0.134		
765	chapter 2 8.1	problem number 13	✓	2.757	✓	0.118		
766	chapter 2 8.2	problem number 1	✓	1.27	✓	0.478		
767	chapter 2 8.2	problem number 2	✗	22.195	✗	3.334		
768	chapter 2 8.2	problem number 3	✗	22.231	✗	3.281		
769	chapter 2 8.2	problem number 4	✓	0.356	✓	0.136		
770	chapter 2 8.2	problem number 5	✗	24.253	✗	4.21		
771	chapter 2 8.2	problem number 6	✗	1.4	✓	0.075		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
772	chapter 2 8.2	problem number 7	✗	27.397	✗	7.125		
773	chapter 2 8.2	problem number 8	✗	28.142	✗	6.062		
774	chapter 2 8.2	problem number 9	✗	24.18	✗	5.51		
775	chapter 2 8.2	problem number 10	✓	0.394	✓	0.145		
776	chapter 2 8.2	problem number 11	✗	0.398	✓	0.411		
777	chapter 2 8.3	problem number 1	✗	27.712	✗	16.778		
778	chapter 2 8.3	problem number 2	✗	30.952	✗	26.32		
779	chapter 2 8.3	problem number 3	✗	30.707	✗	11.393		
780	chapter 2 8.4	problem number 1	✗	22.104	✓	1.021		
781	chapter 2 8.4	problem number 2	✗	21.326	✗	2.787		
782	chapter 2 8.4	problem number 3	✗	21.383	✗	1.296		
783	chapter 2 8.4	problem number 4	✓	0.216	✓	0.072		
784	chapter 2 8.5	problem number 1	✗	27.023	✓	4.046		
785	chapter 2 8.5	problem number 2	✗	27.469	✗	18.031		
786	chapter 2 8.5	problem number 3	✗	27.157	✗	11.928		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
787	chapter 2 8.5	problem number 4	✗	30.787	✗	35.659		
788	chapter 2 8.5	problem number 5	✗	31.085	✗	16.26		
789	chapter 2 8.6	problem number 1	✗	20.868	✗	2.858		
790	chapter 2 8.6	problem number 2	✗	21.009	✓	0.521		
791	chapter 2 8.6	problem number 3	✓	0.157	✓	0.042		
792	chapter 2 8.6	problem number 4	✗	0.489	✓	0.42		
793	chapter 2 8.6	problem number 5	✗	21.358	✗	2.431		
794	chapter 2 8.6	problem number 6	✗	0.336	✓	0.091		
795	chapter 2 8.6	problem number 7	✗	0.389	✓	0.087		
796	chapter 2 8.6	problem number 8	✓	0.295	✓	0.188		
797	chapter 2 8.6	problem number 9	✗	0.785	✓	0.175		
798	chapter 2 8.6	problem number 10	✗	2.323	✓	0.384		
799	chapter 2 8.6	problem number 11	✓	0.396	✓	0.131		
800	chapter 2 8.6	problem number 12	✗	22.985	✓	0.996		
801	chapter 2 9.1	problem number 1	✓	0.087	✓	0.021		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
802	chapter 2 9.1	problem number 2	✗	0.2	✓	0.664		
803	chapter 2 9.1	problem number 3	✗	0.513	✓	0.656		
804	chapter 2 9.1	problem number 4	✗	0.96	✗	0.987		
805	chapter 2 9.1	problem number 5	✗	3.074	✗	2.858		
806	chapter 2 9.2	problem number 1	✗	0.254	✓	0.056		
807	chapter 2 9.2	problem number 2	✗	0.08	✓	0.093		
808	chapter 2 9.2	problem number 3	✗	0.449	✓	0.229		
809	chapter 2 9.2	problem number 4	✗	0.322	✓	0.143		
810	chapter 2 9.2	problem number 5	✗	0.666	✗	3.673		
811	chapter 2 9.2	problem number 6	✗	0.409	✓	0.148		
812	chapter 2 9.2	problem number 7	✗	0.585	✓	0.185		
813	chapter 2 9.2	problem number 8	✗	0.291	✓	0.151		
814	chapter 2 9.2	problem number 9	✗	0.349	✓	0.21		
815	chapter 2 9.2	problem number 10	✗	0.755	✓	1.118		
816	chapter 2 9.2	problem number 11	✗	0.404	✓	0.39		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
817	chapter 2 9.2	problem number 12	✗	1.715	✗	2.638		
818	chapter 2 9.2	problem number 13	✗	2.636	✗	2.017		
819	chapter 2 9.2	problem number 14	✗	1.018	✓	0.654		
820	chapter 2 9.2	problem number 15	✗	0.406	✓	0.897		
821	chapter 2 9.2	problem number 16	✗	0.578	✓	0.261		
822	chapter 2 9.3	problem number 1	✗	5.761	✓	5.146		
823	chapter 2 9.3	problem number 2	✗	5.171	✗	2.488		
824	chapter 2 9.3	problem number 3	✗	13.389	✗	6.826		
825	chapter 2 9.3	problem number 4	✗	2.527	✗	7.326		
826	chapter 2 9.3	problem number 5	✗	2.544	✗	5.814		
827	chapter 2 9.3	problem number 6	✗	23.376	✗	5.957		
828	chapter 2 9.3	problem number 7	✗	23.929	✗	4.987		
829	chapter 2 9.3	problem number 8	✗	13.317	✗	10.507		
830	chapter 2 9.3	problem number 9	✓	0.104	✓	0.045		
831	chapter 2 9.3	problem number 11	✗	0.382	✓	0.361		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
832	chapter 2 9.3	problem number 12	✗	3.96	✓	5.555		
833	chapter 2 9.3	problem number 13	✗	11.486	✗	3.845		
834	chapter 2 9.3	problem number 14	✗	11.034	✗	4.474		
835	chapter 2 9.3	problem number 15	✗	18.354	✗	4.669		
836	chapter 2 9.3	problem number 16	✗	18.094	✗	4.967		
837	chapter 2 9.3	problem number 17	✗	4.589	✗	2.686		
838	chapter 2 9.3	problem number 18	✗	28.101	✗	5.198		
839	chapter 2 9.3	problem number 19	✗	27.779	✗	4.121		
840	chapter 2 9.3	problem number 20	✗	34.12	✗	146.794		
841	chapter 2 9.3	problem number 21	✗	58.245	✗	41.75		
842	chapter 2 9.3	problem number 22	✗	12.827	✗	16.143		
843	chapter 2 9.3	problem number 23	✗	7.809	✗	48.678		
844	chapter 3 Examples	Example 1	✓	0.024	✓	0.035		
845	chapter 3 Examples	Example 2	✓	0.046	✓	0.046		
846	chapter 3 Examples	Example 3	✓	0.006	✓	0.01		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
847	chapter 3 2.1	Problem 1	✓	0.009	✓	0.011		
848	chapter 3 2.1	Problem 2	✓	0.025	✓	0.018		
849	chapter 3 2.1	Problem 3	✓	0.027	✓	0.019		
850	chapter 3 2.1	Problem 4	✓	0.007	✓	0.009		
851	chapter 3 2.1	Problem 5	✓	0.316	✓	0.064		
852	chapter 3 2.1	Problem 6	✓	0.187	✓	0.163		
853	chapter 3 2.1	Problem 7	✓	0.044	✓	0.046		
854	chapter 3 2.1	Problem 8	✓	0.067	✓	0.029		
855	chapter 3 2.2	Problem 1	✓	0.095	✓	0.022		
856	chapter 3 2.2	Problem 2	✓	0.22	✓	0.037		
857	chapter 3 2.2	Problem 3	✓	0.078	✓	0.03		
858	chapter 3 2.2	Problem 4	✓	0.395	✓	0.048		
859	chapter 3 2.2	Problem 5	✓	0.032	✓	0.027		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
860	chapter 3 2.2	Problem 6	✓	0.254	✓ Contains unresolved integral with RootOf	0.167		
861	chapter 3 2.2	Problem 7	✓	0.209	✓	0.024		
862	chapter 3 2.3	Problem 1	✓	0.026	✓	0.045		
863	chapter 3 2.3	Problem 2	✓	0.164	✓	0.049		
864	chapter 3 2.3	Problem 3	✓	0.151	✓	0.031		
865	chapter 3 2.3	Problem 4	✓	0.6	✓	0.05		
866	chapter 3 2.3	Problem 5	✓	0.024	✓	0.014		
867	chapter 3 2.3	Problem 6	✓	0.126	✓	0.146		
868	chapter 3 2.4	Problem 1	✓	0.133	✓	0.022		
869	chapter 3 2.4	Problem 2	✓	0.056	✓	0.021		
870	chapter 3 2.4	Problem 3	✓	0.031	✓	0.686		
871	chapter 3 2.4	Problem 4	✓	0.045	✓	0.036		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
872	chapter 3 2.4	Problem 5	✓	0.069	✓ Result has unresolved integral	0.084		
873	chapter 3 2.4	Problem 6	✓	0.117	✓ Result has unresolved integral	0.041		
874	chapter 3 2.4	Problem 7	✓	0.815	✓ Result has unresolved integral	1.062		
875	chapter 3 2.4	Problem 8	✓	2.198	✓ Result has unresolved integral	0.74		
876	chapter 3 2.4	Problem 9	✓	3.553	✓	0.439		
877	chapter 3 2.4	Problem 10	✗	2.222	✓	8.292		
878	chapter 3 2.4	Problem 11	✓	0.505	✓	1.082		
879	chapter 3 3.1	Problem 1	✓	0.078	✓	0.04		
880	chapter 3 3.1	Problem 2	✓	0.063	✓	0.028		
881	chapter 3 3.1	Problem 3	✓	0.272	✓	0.128		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
882	chapter 3 3.1	Problem 4	✓	0.292	✓	0.123		
883	chapter 3 3.1	Problem 5	✓	0.589	✓	0.119		
884	chapter 3 3.1	Problem 6	✓	0.732	✓	0.031		
885	chapter 3 3.1	Problem 7	✓	0.819	✓	0.125		
886	chapter 3 3.1	Problem 8	✗	3.636	✓	3.932		
887	chapter 3 3.1	Problem 9	✗	1.433	✓	1.319		
888	chapter 3 3.1	Problem 10	✓	1.86	✓	0.413		
889	chapter 3 3.1	Problem 11	✓	0.571	✓	0.605		
890	chapter 3 3.2	Problem 1	✓	0.195	✓	0.03		
891	chapter 3 3.2	Problem 2	✓	0.034	✓	0.067		
892	chapter 3 3.2	Problem 3	✓	0.095	✓	0.036		
893	chapter 3 3.2	Problem 4	✓	0.6	✓	0.256		
894	chapter 3 3.2	Problem 5	✓	0.911	✓	1.251		
895	chapter 3 3.2	Problem 6	✓	0.051	✓	0.025		
896	chapter 3 3.2	Problem 7	✓	0.053	✓	0.022		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
897	chapter 3 3.2	Problem 8	✓	0.436	✓	0.14		
898	chapter 3 3.2	Problem 9	✓	0.431	✓	0.633		
899	chapter 3 3.2	Problem 10	✓	0.402	✓	0.875		
900	chapter 3 3.2	Problem 11	✓	2.839	✓	0.353		
901	chapter 3 4.1	Problem 1	✓	0.119	✓	0.03		
902	chapter 3 4.1	Problem 2	✓	0.118	✓	0.022		
903	chapter 3 4.1	Problem 3	✗ (Timed out)	600.	✓	0.029		
904	chapter 3 4.1	Problem 4	✗ (Timed out)	600.	✓	6.531		
905	chapter 3 4.1	Problem 5	✓	16.09	✓	11.256		
906	chapter 3 4.2	Problem 1	✓	0.117	✓	0.087		
907	chapter 3 4.2	Problem 2	✓	0.113	✓	0.041		
908	chapter 3 4.2	Problem 3	✓	0.385	✓	0.038		
909	chapter 3 4.2	Problem 4	✗ (Timed out)	600.	✓	1.062		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
910	chapter 3 4.2	Problem 5	✓	153.043	✓	1.985		
911	chapter 3 4.3	Problem 1	✓	0.179	✓	0.028		
912	chapter 3 4.3	Problem 2	✓	0.097	✓	0.017		
913	chapter 3 4.3	Problem 3	✓	0.077	✓	0.018		
914	chapter 3 4.3	Problem 4	✓	538.891	✓	2.26		
915	chapter 3 4.3	Problem 5	✓	208.836	✓	1.6		
916	chapter 3 4.4	Problem 1	✓	0.271	✓	0.071		
917	chapter 3 4.4	Problem 2	✓	0.191	✓	0.017		
918	chapter 3 4.4	Problem 3	✓	0.093	✓	0.016		
919	chapter 3 4.4	Problem 4	✗ (Timed out)	600.	✓	2.255		
920	chapter 3 4.4	Problem 5	✓	253.475	✓	2.521		
921	chapter 3 4.5	Problem 1	✓	0.126	✓	0.049		
922	chapter 3 4.5	Problem 2	✓	0.275	✓	0.098		
923	chapter 3 4.5	Problem 3	✓	0.11	✓	0.058		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
924	chapter 3 4.5	Problem 4	✓	0.192	✓	0.195		
925	chapter 3 4.5	Problem 5	✓	0.177	✓	0.075		
926	chapter 3 5.1	Problem 1	✓	0.099	✓	0.024		
927	chapter 3 5.1	Problem 2	✓	0.029	✓	0.023		
928	chapter 3 5.1	Problem 3	✓	0.293	✓	0.67		
929	chapter 3 5.1	Problem 4	✓	1.482	✓	0.209		
930	chapter 3 5.1	Problem 5	✓	1.004	✓	0.244		
931	chapter 3 5.1	Problem 6	✗	0.326	✓	1.247		
932	chapter 3 5.2	Problem 1	✓	0.121	✓	0.023		
933	chapter 3 5.2	Problem 2	✓	0.32	✓ Result has unresolved integrals	0.806		
934	chapter 3 5.2	Problem 3	✓	0.149	✓	0.326		
935	chapter 3 5.3	Problem 4	✓	0.08	✓ Result has unresolved integrals	0.097		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
936	chapter 3 5.3	Problem 5	✓	0.09	✓	0.161		
937	chapter 3 5.3	Problem 6	✓	1.75	✓	1.455		
938	chapter 3 6.1	Problem 1	✓	0.129	✓	0.024		
939	chapter 3 6.1	Problem 2	✓	0.12	✓	0.024		
940	chapter 3 6.1	Problem 3	✓	0.087	✓	0.019		
941	chapter 3 6.1	Problem 4	✓	240.979	✓	3.825		
942	chapter 3 6.1	Problem 5	✓	92.342	✓ Result has unresolved integrals	5.837		
943	chapter 3 6.2	Problem 1	✓	0.131	✓	0.077		
944	chapter 3 6.2	Problem 2	✓	0.121	✓	0.042		
945	chapter 3 6.2	Problem 3	✓	0.085	✓	0.037		
946	chapter 3 6.2	Problem 4	✓	243.081	✓	2.164		
947	chapter 3 6.2	Problem 5	✓	53.544	✓	2.97		
948	chapter 3 6.3	Problem 1	✓	0.201	✓	0.028		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
949	chapter 3 6.3	Problem 2	✓	0.107	✓	0.018		
950	chapter 3 6.3	Problem 3	✓	0.085	✓	0.014		
951	chapter 3 6.3	Problem 4	✓	73.994	✓	2.963		
952	chapter 3 6.3	Problem 5	✓	30.294	✓	3.656		
953	chapter 3 6.4	Problem 1	✓	0.295	✓	0.074		
954	chapter 3 6.4	Problem 2	✓	0.179	✓	0.02		
955	chapter 3 6.4	Problem 3	✓	0.102	✓	0.02		
956	chapter 3 6.4	Problem 4	✓	86.433	✓	4.015		
957	chapter 3 6.4	Problem 5	✓	29.022	✓	3.298		
958	chapter 3 6.5	Problem 1	✓	0.152	✓	0.063		
959	chapter 3 6.5	Problem 2	✓	0.188	✓	0.068		
960	chapter 3 6.5	Problem 3	✓	0.417	✓	0.042		
961	chapter 3 6.5	Problem 4	✓	0.244	✓	0.412		
962	chapter 3 6.5	Problem 5	✓	0.309	✓	0.649		
963	chapter 3 6.5	Problem 6	✓	0.164	✓	0.21		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
964	chapter 3 7.1	Problem 1	✓	1.178	✓	0.071		
965	chapter 3 7.1	Problem 2	✓	0.455	✓	0.075		
966	chapter 3 7.1	Problem 3	✓	0.148	✓	0.024		
967	chapter 3 7.1	Problem 4	✓	8.92	✓	1.725		
968	chapter 3 7.1	Problem 5	✓	1.269	✓	1.04		
969	chapter 3 7.2	Problem 1	✓	1.238	✓	0.022		
970	chapter 3 7.2	Problem 2	✓	0.764	✓	0.016		
971	chapter 3 7.2	Problem 3	✓	0.205	✓	0.017		
972	chapter 3 7.2	Problem 4	✓	13.428	✓	1.386		
973	chapter 3 7.2	Problem 5	✓	1.478	✓	0.824		
974	chapter 3 7.3	Problem 1	✓	0.083	✓	0.195		
975	chapter 3 7.3	Problem 2	✓	0.106	✓	0.134		
976	chapter 3 7.3	Problem 3	✓	0.064	✓	0.047		
977	chapter 3 7.3	Problem 4	✓	0.548	✓	5.161		
978	chapter 3 7.3	Problem 5	✓	0.681	✓	1.393		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
979	chapter 3 7.4	Problem 1	✓	0.058	✓	0.048		
980	chapter 3 7.4	Problem 2	✓	0.122	✓	0.026		
981	chapter 3 7.4	Problem 3	✓	0.07	✓	0.035		
982	chapter 3 7.4	Problem 4	✓	0.609	✓	2.495		
983	chapter 3 7.4	Problem 5	✓	0.744	✓	1.138		
984	chapter 3 8.1	Problem 1	✓	0.01	✓	0.017		
985	chapter 3 8.1	Problem 2	✓	0.041	✓	0.028		
986	chapter 3 8.1	Problem 3	✓	0.032	✓	0.026		
987	chapter 3 8.1	Problem 4	✓	0.03	✓	0.096		
988	chapter 3 8.1	Problem 5	✓	0.034	✓	0.037		
989	chapter 3 8.1	Problem 6	✓	0.06	✓	0.018		
990	chapter 3 8.1	Problem 7	✓	0.082	✓	0.087		
991	chapter 3 8.1	Problem 8	✓	0.197	✓	0.044		
992	chapter 3 8.1	Problem 9	✓	0.185	✓	0.028		
993	chapter 3 8.1	Problem 10	✓	0.14	✓	0.013		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
994	chapter 3 8.1	Problem 11	✓	0.213	✓	0.328		
995	chapter 3 8.1	Problem 12	✓	0.554	✓	0.215		
996	chapter 3 8.1	Problem 13	✗	0.433	✓	0.522		
997	chapter 3 8.1	Problem 14	✓	0.241	✓	0.217		
998	chapter 3 8.1	Problem 15	✓	0.731	✓	0.441		
999	chapter 3 8.1	Problem 16	✓	0.084	✓	0.022		
1000	chapter 3 8.2	Problem 1	✓	0.018	✓	0.026		
1001	chapter 3 8.2	Problem 2	✓	0.019	✓	0.019		
1002	chapter 3 8.2	Problem 3	✓	0.06	✓	0.039		
1003	chapter 3 8.2	Problem 4	✗	0.092	✓	0.068		
1004	chapter 3 8.2	Problem 5	✓	0.57	✓	0.243		
1005	chapter 3 8.2	Problem 6	✓	0.061	✓	0.019		
1006	chapter 3 8.2	Problem 7	✗	0.187	✓	0.087		
1007	chapter 3 8.3	Problem 1	✓	0.037	✓	0.032		
1008	chapter 3 8.3	Problem 2	✓	0.018	✓	0.01		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1009	chapter 3 8.3	Problem 3	✓	0.037	✓	0.024		
1010	chapter 3 8.3	Problem 4	✓	0.061	✓	0.022		
1011	chapter 3 8.3	Problem 5	✓	0.066	✓	0.182		
1012	chapter 3 8.3	Problem 6	✓	0.094	✓	0.087		
1013	chapter 3 8.3	Problem 7	✓	0.048	✓	0.034		
1014	chapter 3 8.3	Problem 8	✓	0.216	✓	0.738		
1015	chapter 3 8.4	Problem 1	✓	0.01	✓	0.018		
1016	chapter 3 8.4	Problem 2	✓	0.054	✓	0.022		
1017	chapter 3 8.4	Problem 3	✓	0.078	✓	0.048		
1018	chapter 3 8.4	Problem 4	✓	0.193	✓	0.141		
1019	chapter 3 8.4	Problem 5	✓	0.523	✓	0.318		
1020	chapter 3 8.4	Problem 6	✗	0.478	✓	0.704		
1021	chapter 3 8.4	Problem 7	✗	0.159	✓ Contains RootOf	0.035		
1022	chapter 4 1.1	Example 1	✓	0.033	✓	0.034		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1023	chapter 4 1.1	Example 2	✓	0.102	✓	0.029		
1024	chapter 4 1.1	Example 3	✓	0.007	✓	0.015		
1025	chapter 4 2.1	Problem 1	✓	0.007	✓	0.017		
1026	chapter 4 2.1	Problem 2	✓	0.03	✓	0.017		
1027	chapter 4 2.1	Problem 3	✓	0.018	✓	0.01		
1028	chapter 4 2.1	Problem 4	✓	0.016	✓	0.029		
1029	chapter 4 2.1	Problem 5	✓	0.018	✓	0.01		
1030	chapter 4 2.1	Problem 6	✓	0.027	✓	0.013		
1031	chapter 4 2.1	Problem 7	✗	0.221	✗ (Timed out)	600.		
1032	chapter 4 2.2	Problem 1	✓	0.042	✓	0.059		
1033	chapter 4 2.2	Problem 2	✓	0.03	✓	0.033		
1034	chapter 4 2.2	Problem 3	✓	0.146	✓	0.048		
1035	chapter 4 2.2	Problem 4	✓	0.164	✓	0.068		
1036	chapter 4 2.2	Problem 5	✓	0.158	✓	0.093		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1037	chapter 4 2.2	Problem 6	✓	0.094	✓	0.073		
1038	chapter 4 2.2	Problem 7	✗	15.1	✓	1.808		
1039	chapter 4 2.2	Problem 8	✗	1.928	✓	1.122		
1040	chapter 4 2.3	Problem 1	✓	0.023	✓	0.05		
1041	chapter 4 2.3	Problem 2	✓	0.03	✓	0.031		
1042	chapter 4 2.3	Problem 3	✓	0.023	✓	0.027		
1043	chapter 4 2.3	Problem 4	✓	0.149	✓	0.101		
1044	chapter 4 2.3	Problem 5	✓	0.056	✓	0.186		
1045	chapter 4 2.3	Problem 6	✓	0.135	✓	0.141		
1046	chapter 4 2.4	Problem 1	✓	0.183	✓	0.033		
1047	chapter 4 2.4	Problem 2 case $n \neq -1, n \neq -2$	✓	0.049	✓	0.09		
1048	chapter 4 2.4	Problem 2 case $n = -1$	✓	0.058	✓	0.043		
1049	chapter 4 2.4	Problem 2 case $n = -2$	✓	0.098	✓	0.043		
1050	chapter 4 2.4	Problem 3	✓	0.032	✓	0.028		
1051	chapter 4 2.4	Problem 4	✓	0.046	✓	0.03		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1052	chapter 4 2.4	Problem 5	✓	0.071	✓	0.027		
1053	chapter 4 2.4	Problem 6	✓	0.121	✓	0.037		
1054	chapter 4 2.4	Problem 7	✓	0.814	✓	1.94		
1055	chapter 4 2.4	Problem 8	✓	1.868	✓	0.701		
1056	chapter 4 2.4	Problem 9	✓	3.649	✓	5.513		
1057	chapter 4 2.4	Problem 10	✓	1.421	✓	13.451		
1058	chapter 4 2.4	Problem 11	✓	0.509	✓	2.319		
1059	chapter 4 2.4	Problem 12	✗	0.506	✓	0.588		
1060	chapter 4 2.4	Problem 13	✗	0.512	✓	0.766		
1061	chapter 4 3.1	Problem 1	✓	0.159	✓	0.032		
1062	chapter 4 3.1	Problem 2	✓	0.124	✓	0.034		
1063	chapter 4 3.1	Problem 3	✓	0.281	✓	0.148		
1064	chapter 4 3.1	Problem 4	✓	0.377	✓	0.177		
1065	chapter 4 3.1	Problem 5	✓	0.588	✓	0.414		
1066	chapter 4 3.1	Problem 6	✓	4.021	✓	0.138		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1067	chapter 4 3.1	Problem 7	✗	2.408	✓	4.213		
1068	chapter 4 3.1	Problem 8	✗	1.317	✓	2.385		
1069	chapter 4 3.1	Problem 9	✓	1.984	✓	1.352		
1070	chapter 4 3.1	Problem 10	✓	0.398	✓	0.033		
1071	chapter 4 3.2	Problem 1	✓	0.572	✓	0.048		
1072	chapter 4 3.2	Problem 2	✓	0.078	✓	0.029		
1073	chapter 4 3.2	Problem 3	✓	0.052	✓	0.026		
1074	chapter 4 3.2	Problem 4	✓	0.381	✓	0.206		
1075	chapter 4 3.2	Problem 5	✓	0.444	✓	0.313		
1076	chapter 4 3.2	Problem 6	✓	0.406	✓	0.123		
1077	chapter 4 3.2	Problem 7	✓	2.94	✓	0.066		
1078	chapter 4 4.1	Problem 1	✓	0.483	✓	0.029		
1079	chapter 4 4.1	Problem 2	✓	0.518	✓	0.029		
1080	chapter 4 4.1	Problem 3	✓	0.364	✓	0.021		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1081	chapter 4 4.1	Problem 4	✗ (Timed out)	600.	✓	0.968		
1082	chapter 4 4.1	Problem 5	✓	281.809	✓	1.134		
1083	chapter 4 4.2	Problem 1	✓	0.546	✓	0.022		
1084	chapter 4 4.2	Problem 2	✓	0.509	✓	0.048		
1085	chapter 4 4.2	Problem 3	✓	0.381	✓	0.045		
1086	chapter 4 4.2	Problem 4	✗ (Timed out)	600.	✓	0.707		
1087	chapter 4 4.2	Problem 5	✓	151.99	✓	0.838		
1088	chapter 4 4.3	Problem 1	✓	0.195	✓	0.109		
1089	chapter 4 4.3	Problem 2	✓	0.098	✓	0.119		
1090	chapter 4 4.3	Problem 3	✓	0.094	✓	0.088		
1091	chapter 4 4.3	Problem 4	✓	575.866	✓	1.175		
1092	chapter 4 4.3	Problem 5	✓	204.54	✓	1.182		
1093	chapter 4 4.4	Problem 1	✓	0.417	✓	0.189		
1094	chapter 4 4.4	Problem 2	✓	0.237	✓	0.099		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1095	chapter 4 4.4	Problem 3	✓	0.125	✓	0.084		
1096	chapter 4 4.4	Problem 4	✓	578.971	✓	1.509		
1097	chapter 4 4.4	Problem 5	✓	225.853	✓	1.35		
1098	chapter 4 4.5	Problem 1	✓	0.534	✓	0.05		
1099	chapter 4 4.5	Problem 2	✓	0.401	✓	0.103		
1100	chapter 4 4.5	Problem 3	✓	0.163	✓	0.232		
1101	chapter 4 4.5	Problem 4	✓	0.084	✓	0.045		
1102	chapter 4 4.5	Problem 5	✓	0.28	✓	0.637		
1103	chapter 4 4.5	Problem 6	✓	0.823	✓	0.66		
1104	chapter 4 5.1	Problem 1	✓	0.1	✓	0.109		
1105	chapter 4 5.1	Problem 2	✓	0.041	✓	0.057		
1106	chapter 4 5.1	Problem 3	✓	1.38	✓	0.145		
1107	chapter 4 5.1	Problem 4	✓	0.783	✓	0.284		
1108	chapter 4 5.1	Problem 5	✓	0.215	✓	0.408		
1109	chapter 4 5.1	Problem 6	✗	0.356	✓	1.359		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1110	chapter 4 5.2	Problem 1	✓	0.236	✓	1.598		
1111	chapter 4 5.2	Problem 2	✓	0.625	✓	0.443		
1112	chapter 4 5.2	Problem 3	✓	0.155	✓	0.031		
1113	chapter 4 5.2	Problem 4	✓	0.08	✓	0.06		
1114	chapter 4 5.2	Problem 5	✓	0.076	✓	0.29		
1115	chapter 4 5.2	Problem 6	✓	1.543	✓	8.379		
1116	chapter 4 6.1	Problem 1	✓	0.485	✓	0.035		
1117	chapter 4 6.1	Problem 2	✓	0.278	✓	0.027		
1118	chapter 4 6.1	Problem 3	✓	0.307	✓	0.06		
1119	chapter 4 6.1	Problem 4	✓	242.628	✓	2.815		
1120	chapter 4 6.1	Problem 5	✓	87.276	✓	2.573		
1121	chapter 4 6.2	Problem 1	✓	0.509	✓	0.062		
1122	chapter 4 6.2	Problem 2	✓	0.522	✓	0.08		
1123	chapter 4 6.2	Problem 3	✓	0.378	✓	0.018		
1124	chapter 4 6.2	Problem 4	✓	240.874	✓	1.473		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1125	chapter 4 6.2	Problem 5	✓	51.635	✓	1.661		
1126	chapter 4 6.3	Problem 1	✓	0.105	✓	0.061		
1127	chapter 4 6.3	Problem 2	✓	0.209	✓	0.076		
1128	chapter 4 6.3	Problem 3	✓	0.089	✓	0.049		
1129	chapter 4 6.3	Problem 4	✓	73.537	✓	1.555		
1130	chapter 4 6.3	Problem 5	✓	30.743	✓	1.48		
1131	chapter 4 6.4	Problem 1	✓	0.256	✓	0.052		
1132	chapter 4 6.4	Problem 2	✓	0.443	✓	0.121		
1133	chapter 4 6.4	Problem 3	✓	0.135	✓	0.041		
1134	chapter 4 6.4	Problem 4	✓	83.174	✓	1.957		
1135	chapter 4 6.4	Problem 5	✓	28.145	✓	1.8		
1136	chapter 4 6.5	Problem 1	✓	0.217	✓	0.063		
1137	chapter 4 6.5	Problem 2	✓	0.081	✓	0.108		
1138	chapter 4 6.5	Problem 3	✓	0.21	✓	0.414		
1139	chapter 4 6.5	Problem 4	✓	0.217	✓	0.756		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1140	chapter 4 6.5	Problem 5	✓	0.797	✓	0.394		
1141	chapter 4 6.5	Problem 6	✓	0.201	✓	0.404		
1142	chapter 4 7.1	Problem 1	✓	1.176	✓	0.072		
1143	chapter 4 7.1	Problem 2	✓	0.431	✓	0.078		
1144	chapter 4 7.1	Problem 3	✓	0.492	✓	0.059		
1145	chapter 4 7.1	Problem 4	✓	0.094	✓	5.972		
1146	chapter 4 7.1	Problem 5	✓	1.293	✓	4.364		
1147	chapter 4 7.2	Problem 1	✓	1.358	✓	0.039		
1148	chapter 4 7.2	Problem 2	✓	0.704	✓	0.03		
1149	chapter 4 7.2	Problem 3	✓	0.647	✓	0.041		
1150	chapter 4 7.2	Problem 4	✓	0.105	✓	4.49		
1151	chapter 4 7.2	Problem 5	✓	0.382	✓	4.661		
1152	chapter 4 7.3	Problem 1	✓	0.174	✓	0.179		
1153	chapter 4 7.3	Problem 2	✓	0.12	✓	0.144		
1154	chapter 4 7.3	Problem 3	✓	0.435	✓	0.168		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1155	chapter 4 7.3	Problem 4	✓	0.133	✓	1.363		
1156	chapter 4 7.3	Problem 5	✓	0.409	✓	2.472		
1157	chapter 4 7.4	Problem 1	✓	0.146	✓	0.136		
1158	chapter 4 7.4	Problem 2	✓	0.133	✓	0.091		
1159	chapter 4 7.4	Problem 3	✓	0.345	✓	0.148		
1160	chapter 4 7.4	Problem 4	✓	0.138	✓	1.17		
1161	chapter 4 7.4	Problem 5	✓	0.408	✓	1.194		
1162	chapter 4 8.1	Problem 1	✓	0.01	✓	0.018		
1163	chapter 4 8.1	Problem 2	✓	0.041	✓	0.025		
1164	chapter 4 8.1	Problem 3	✓	0.032	✓	0.016		
1165	chapter 4 8.1	Problem 4	✓	0.031	✓	0.026		
1166	chapter 4 8.1	Problem 5	✓	0.035	✓	0.03		
1167	chapter 4 8.1	Problem 6	✓	0.058	✓	0.016		
1168	chapter 4 8.1	Problem 7	✓	0.084	✓	0.05		
1169	chapter 4 8.1	Problem 8	✓	0.193	✓	0.062		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1170	chapter 4 8.1	Problem 9	✓	0.19	✓	0.062		
1171	chapter 4 8.1	Problem 10	✓	0.142	✓	0.042		
1172	chapter 4 8.1	Problem 11	✓	0.215	✓	0.121		
1173	chapter 4 8.1	Problem 12	✓	0.549	✓	0.239		
1174	chapter 4 8.1	Problem 13	✗	23.95	✗	5.972		
1175	chapter 4 8.1	Problem 14	✓	0.247	✓	0.096		
1176	chapter 4 8.1	Problem 15	✓	0.242	✓	0.162		
1177	chapter 4 8.2	Problem 1	✓	0.018	✓	0.027		
1178	chapter 4 8.2	Problem 2	✓	0.019	✓	0.023		
1179	chapter 4 8.2	Problem 3	✓	0.058	✓	0.023		
1180	chapter 4 8.2	Problem 4	✗	0.085	✓ contains RootOf	0.047		
1181	chapter 4 8.2	Problem 5	✗	22.035	✗	2.423		
1182	chapter 4 8.2	Problem 6	✗	0.178	✓ has RootOf	0.097		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1183	chapter 4 8.2	Problem 7	✗	0.05	✓ has RootOf	0.042		
1184	chapter 4 8.3	Problem 1	✓	0.034	✓	0.031		
1185	chapter 4 8.3	Problem 2	✓	0.018	✓	0.02		
1186	chapter 4 8.3	Problem 3	✓	0.034	✓	0.02		
1187	chapter 4 8.3	Problem 4	✓	0.096	✓	0.313		
1188	chapter 4 8.3	Problem 5	✓	0.09	✓	0.038		
1189	chapter 4 8.3	Problem 6	✓	0.045	✓	0.033		
1190	chapter 4 8.3	Problem 7	✓	0.053	✓	1.953		
1191	chapter 4 8.4	Problem 1	✓	0.009	✓	0.013		
1192	chapter 4 8.4	Problem 2	✓	0.051	✓	0.015		
1193	chapter 4 8.4	Problem 3	✓	0.072	✓	0.021		
1194	chapter 4 8.4	Problem 4	✓	0.185	✓	0.028		
1195	chapter 4 8.4	Problem 5	✓	0.529	✓	0.267		
1196	chapter 4 8.4	Problem 6	✗	23.902	✗	5.206		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1197	chapter 4 8.4	Problem 7	✗	0.056	✓ has RootOf	0.092		
1198	chapter 5 2.1	Problem 1	✓	0.018	✓	0.026		
1199	chapter 5 2.1	Problem 2	✓	0.031	✓	0.02		
1200	chapter 5 2.1	Problem 3	✓	0.051	✓	0.042		
1201	chapter 5 2.1	Problem 4	✓	0.058	✓	0.024		
1202	chapter 5 2.1	Problem 5	✓	0.378	✓	0.065		
1203	chapter 5 2.1	Problem 6	✓	0.098	✓	0.064		
1204	chapter 5 2.1	Problem 7	✓	5.912	✗ (Timed out)	600.		
1205	chapter 5 2.1	Problem 8	✓	1.624	✓	0.798		
1206	chapter 5 2.2	Problem 1	✓	0.121	✓	0.042		
1207	chapter 5 2.2	Problem 2	✓	0.149	✓	0.034		
1208	chapter 5 2.2	Problem 3	✓	0.023	✓	0.029		
1209	chapter 5 2.2	Problem 4	✓	0.27	✓	0.052		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1210	chapter 5 2.2	Problem 5	✗ (Timed out)	600.	✓	24.969		
1211	chapter 5 2.2	Problem 6	✗ (Timed out)	600.	✓	7.573		
1212	chapter 5 2.2	Problem 7	✗	42.122	✓	0.978		
1213	chapter 5 2.2	Problem 8	✗	27.288	✓	28.928		
1214	chapter 5 2.3	Problem 1	✓	0.402	✓	0.287		
1215	chapter 5 2.3	Problem 2	✓	0.428	✓	0.087		
1216	chapter 5 2.3	Problem 3	✓	3.345	✓	0.153		
1217	chapter 5 2.3	Problem 4	✓	3.251	✓	0.003		
1218	chapter 5 2.3	Problem 5	✓	0.545	✓	0.657		
1219	chapter 5 2.3	Problem 6	✓	0.17	✓	0.162		
1220	chapter 5 2.3	Problem 7	✓	1.32	✓ contains RootOf	0.66		
1221	chapter 5 2.4	Problem 1	✓	0.544	✓	0.361		
1222	chapter 5 2.4	Problem 2	✓	0.148	✓	0.124		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1223	chapter 5 2.4	Problem 3	✓	0.049	✓	0.168		
1224	chapter 5 2.4	Problem 4	✓	0.141	✓	0.108		
1225	chapter 5 2.4	Problem 5	✓	0.63	✓	0.96		
1226	chapter 5 2.4	Problem 6	✓	0.492	✓	0.969		
1227	chapter 5 2.4	Problem 7	✓	0.295	✓	0.178		
1228	chapter 5 2.4	Problem 8	✓	0.274	✓	0.143		
1229	chapter 5 2.4	Problem 9	✓	6.684	✓	7.115		
1230	chapter 5 2.4	Problem 10	✓	2.088	✓	3.049		
1231	chapter 5 2.4	Problem 11	✓	4.096	✓	3.011		
1232	chapter 5 2.4	Problem 12	✓	1.276	✓	1.314		
1233	chapter 5 3.1	Problem 1	✓	0.371	✓	0.26		
1234	chapter 5 3.1	Problem 2	✓	0.56	✓	0.321		
1235	chapter 5 3.1	Problem 3	✓	1.446	✓	0.497		
1236	chapter 5 3.1	Problem 4	✗	2.4	✓	4.126		
1237	chapter 5 3.1	Problem 5	✗	1.468	✓	2.312		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1238	chapter 5 3.1	Problem 6	✓	5.785	✓	0.937		
1239	chapter 5 3.1	Problem 7	✓	0.319	✓	0.085		
1240	chapter 5 3.1	Problem 8	✓	0.333	✓	0.177		
1241	chapter 5 3.2	Problem 1	✓	0.176	✓	0.134		
1242	chapter 5 3.2	Problem 2	✓	0.157	✓	0.066		
1243	chapter 5 3.2	Problem 3	✓	0.131	✓	0.063		
1244	chapter 5 3.2	Problem 4	✓	0.787	✓	1.153		
1245	chapter 5 3.2	Problem 5	✓	0.398	✓	0.266		
1246	chapter 5 3.2	Problem 6	✓	0.289	✓	0.146		
1247	chapter 5 3.2	Problem 7	✓	3.02	✓	0.471		
1248	chapter 5 3.2	Problem 8	✓	0.094	✓	0.035		
1249	chapter 5 3.2	Problem 9	✓	0.605	✓	0.759		
1250	chapter 5 3.2	Problem 10	✓	0.658	✓	0.248		
1251	chapter 5 4.1	Problem 1	✓	9.188	✓	15.043		
1252	chapter 5 4.1	Problem 2	✓	267.327	✓	0.309		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1253	chapter 5 4.1	Problem 3	✓	73.281	✓	9.376		
1254	chapter 5 4.1	Problem 4	✓	3.952	✓	6.78		
1255	chapter 5 4.1	Problem 5	✓	0.726	✓	1.606		
1256	chapter 5 4.2	Problem 1	✓	9.015	✓	12.51		
1257	chapter 5 4.2	Problem 2	✓	136.984	✓	0.337		
1258	chapter 5 4.2	Problem 3	✓	74.284	✓	6.861		
1259	chapter 5 4.2	Problem 4	✓	0.151	✓	0.203		
1260	chapter 5 4.2	Problem 5	✓	3.934	✓	5.384		
1261	chapter 5 4.2	Problem 6	✓	2.066	✓	1.643		
1262	chapter 5 4.3	Problem 1	✓	10.527	✓	1.682		
1263	chapter 5 4.3	Problem 2	✓	82.418	✓	0.374		
1264	chapter 5 4.3	Problem 3	✗ (Timed out)	600.	✓	2.931		
1265	chapter 5 4.3	Problem 4	✓	5.011	✓	4.151		
1266	chapter 5 4.3	Problem 5	✓	1.458	✓	1.918		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1267	chapter 5 4.4	Problem 1	✓	10.298	✓	1.704		
1268	chapter 5 4.4	Problem 2	✓	80.641	✓	0.376		
1269	chapter 5 4.4	Problem 3	✗ (Timed out)	600.	✓	3.115		
1270	chapter 5 4.4	Problem 4	✓	5.027	✓	2.582		
1271	chapter 5 4.4	Problem 5	✓	1.452	✓	2.072		
1272	chapter 5 4.5	Problem 1	✓	1.022	✓	13.503		
1273	chapter 5 4.5	Problem 2	✓	7.489	✓	10.404		
1274	chapter 5 4.5	Problem 3	✓	0.451	✓	2.831		
1275	chapter 5 4.5	Problem 4	✓	15.107	✓	1.453		
1276	chapter 5 4.5	Problem 5	✓	4.184	✓	9.763		
1277	chapter 5 4.5	Problem 6	✓	3.921	✓	5.203		
1278	chapter 5 5.1	Problem 1	✓	0.253	✓	0.707		
1279	chapter 5 5.1	Problem 2	✓	0.087	✓	0.115		
1280	chapter 5 5.1	Problem 3	✓	0.604	✓	1.575		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1281	chapter 5 5.1	Problem 4	✗	0.14	✓	0.434		
1282	chapter 5 5.1	Problem 5	✓	0.458	✓	287.069		
1283	chapter 5 5.1	Problem 6	✓	0.192	✓	345.013		
1284	chapter 5 5.2	Problem 1	✓	0.653	✓	0.308		
1285	chapter 5 5.2	Problem 2	✓	0.565	✓	0.367		
1286	chapter 5 5.2	Problem 3	✓	0.324	✓	0.261		
1287	chapter 5 5.2	Problem 4	✓	0.611	✓	0.278		
1288	chapter 5 5.2	Problem 5	✓	0.205	✓	0.477		
1289	chapter 5 5.2	Problem 6	✓	1.445	✓	0.669		
1290	chapter 5 5.2	Problem 7	✓	0.307	✓	2.164		
1291	chapter 5 6.1	Problem 1	✓	0.588	✓	0.073		
1292	chapter 5 6.1	Problem 2	✓	1.067	✓	26.003		
1293	chapter 5 6.1	Problem 3	✓	3.414	✓	19.74		
1294	chapter 5 6.1	Problem 4	✓	1.999	✓	0.353		
1295	chapter 5 6.1	Problem 5	✓	4.365	✓	0.614		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1296	chapter 5 6.1	Problem 6	✓	1.809	✓	24.011		
1297	chapter 5 6.1	Problem 7	✓	0.835	✓	9.985		
1298	chapter 5 6.2	Problem 1	✓	0.569	✓	0.116		
1299	chapter 5 6.2	Problem 2	✓	1.338	✓	14.038		
1300	chapter 5 6.2	Problem 3	✓	3.409	✓	15.254		
1301	chapter 5 6.2	Problem 4	✓	1.509	✓	0.467		
1302	chapter 5 6.2	Problem 5	✓	4.631	✓	0.495		
1303	chapter 5 6.2	Problem 6	✓	1.8	✓	12.414		
1304	chapter 5 6.3	Problem 1	✓	2.393	✓	0.316		
1305	chapter 5 6.3	Problem 2	✓	5.764	✓	15.577		
1306	chapter 5 6.3	Problem 3	✓	2.392	✓	2.202		
1307	chapter 5 6.3	Problem 4	✓	0.926	✓	2.228		
1308	chapter 5 6.3	Problem 5	✓	5.744	✓	0.313		
1309	chapter 5 6.3	Problem 6	✓	1.841	✓	12.162		
1310	chapter 5 6.3	Problem 7	✓	1.21	✓	13.111		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1311	chapter 5 6.4	Problem 1	✓	5.518	✓	0.408		
1312	chapter 5 6.4	Problem 2	✓	6.402	✓	15.57		
1313	chapter 5 6.4	Problem 3	✓	2.301	✓	1.786		
1314	chapter 5 6.4	Problem 4	✓	1.288	✓	9.452		
1315	chapter 5 6.4	Problem 5	✓	7.149	✓	0.386		
1316	chapter 5 6.4	Problem 6	✓	1.823	✓	11.63		
1317	chapter 5 6.4	Problem 7	✓	1.218	✓	17.109		
1318	chapter 5 6.5	Problem 1	✓	1.143	✓	19.672		
1319	chapter 5 6.5	Problem 2	✓	3.027	✓	14.758		
1320	chapter 5 6.5	Problem 3	✓	1.456	✓	0.918		
1321	chapter 5 6.5	Problem 4	✓	1.893	✓	0.194		
1322	chapter 5 6.5	Problem 5	✓	1.882	✓	2.233		
1323	chapter 5 6.5	Problem 6	✓	1.729	✓	17.309		
1324	chapter 5 6.5	Problem 7	✓	1.887	✓	2.207		
1325	chapter 5 7.1	Problem 1	✓	1.11	✓	0.816		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1326	chapter 5 7.1	Problem 2	✓	1.478	✓	0.097		
1327	chapter 5 7.1	Problem 3	✓	2.364	✓	0.72		
1328	chapter 5 7.1	Problem 4	✓	0.48	✓	7.008		
1329	chapter 5 7.1	Problem 5	✓	0.119	✓	5.642		
1330	chapter 5 7.2	Problem 1	✓	1.366	✓	0.24		
1331	chapter 5 7.2	Problem 2	✓	1.493	✓	0.051		
1332	chapter 5 7.2	Problem 3	✓	4.798	✓	0.234		
1333	chapter 5 7.2	Problem 4	✓	0.463	✓	5.026		
1334	chapter 5 7.2	Problem 5	✓	0.131	✓	10.01		
1335	chapter 5 7.3	Problem 1	✓	2.42	✓	1.009		
1336	chapter 5 7.3	Problem 2	✓	1.782	✓	304.002		
1337	chapter 5 7.3	Problem 3	✓	2.437	✓	11.917		
1338	chapter 5 7.3	Problem 4	✓	0.431	✓	2.773		
1339	chapter 5 7.3	Problem 5	✓	0.162	✓	2.117		
1340	chapter 5 7.4	Problem 1	✓	2.588	✓	0.732		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1341	chapter 5 7.4	Problem 2	✓	1.769	✓	541.511		
1342	chapter 5 7.4	Problem 3	✓	3.26	✓	15.874		
1343	chapter 5 7.4	Problem 4	✓	0.477	✓	2.356		
1344	chapter 5 7.4	Problem 5	✓	0.166	✓	1.76		
1345	chapter 5 8.1	Problem 1	✓	0.039	✓	0.036		
1346	chapter 5 8.1	Problem 2	✓	0.085	✓	0.087		
1347	chapter 5 8.1	Problem 3	✓	0.1	✓	0.092		
1348	chapter 5 8.1	Problem 4	✓	0.08	✓	0.042		
1349	chapter 5 8.1	Problem 5	✓	0.153	✓	0.099		
1350	chapter 5 8.1	Problem 6	✓	0.059	✓	0.027		
1351	chapter 5 8.1	Problem 7	✓	0.229	✓	0.036		
1352	chapter 5 8.1	Problem 8	✓	0.213	✓	0.293		
1353	chapter 5 8.1	Problem 9	✓	0.616	✓	0.555		
1354	chapter 5 8.1	Problem 10	✗	0.487	✓	0.758		
1355	chapter 5 8.1	Problem 11	✓	0.268	✓	0.63		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1356	chapter 5 8.1	Problem 12	✓	0.335	✓	0.368		
1357	chapter 5 8.2	Problem 1	✓	0.044	✓	0.03		
1358	chapter 5 8.2	Problem 2	✓	0.179	✓	0.055		
1359	chapter 5 8.2	Problem 3	✓	0.044	✓	0.02		
1360	chapter 5 8.2	Problem 4	✓	0.104	✓	0.066		
1361	chapter 5 8.2	Problem 5	✓	0.071	✓	0.028		
1362	chapter 5 8.2	Problem 6	✗	0.087	✓	0.122		
1363	chapter 5 8.3	Problem 1	✓	0.043	✓	0.037		
1364	chapter 5 8.3	Problem 2	✓	0.101	✓	0.054		
1365	chapter 5 8.3	Problem 3	✓	0.062	✓	0.132		
1366	chapter 5 8.3	Problem 4	✓	0.217	✓	0.716		
1367	chapter 5 8.3	Problem 5	✗ (Timed out)	600.	✓	0.812		
1368	chapter 5 8.3	Problem 6	✗	24.019	✗	1.987		
1369	chapter 6 2.1	Problem 1	✓	0.009	✓	0.015	Yes	

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1370	chapter 6 2.1	Problem 2	✓	0.01	✓	0.038	Yes	
1371	chapter 6 2.1	Problem 3	✓	0.054	✓	0.077	Yes	
1372	chapter 6 2.1	Problem 4	✓	0.123	✓	0.44	Yes	
1373	chapter 6 2.1	Problem 5	✓	0.033	✓	0.038		
1374	chapter 6 2.1	Problem 6	✓	0.123	✓	0.081		
1375	chapter 6 2.1	Problem 7	✓	0.077	✓	0.126		
1376	chapter 6 2.1	Problem 8	✗	0.031	✓	0.024		
1377	chapter 6 2.1	Problem 9	✗	0.025	✓	0.945		
1378	chapter 6 2.1	Problem 10	✓	0.192	✓	0.331		
1379	chapter 6 2.1	Problem 11	✗	6.979	✗	9.029		
1380	chapter 6 2.1	Problem 12	✗	3.292	✗	0.211		
1381	chapter 6 2.1	Problem 13	✓	0.041	✓	0.037		
1382	chapter 6 2.1	Problem 14	✗ (Timed out)	600.	✗	1.641		
1383	chapter 6 2.1	Problem 15	✗ (Timed out)	600.	✓	10.456		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1384	chapter 6 2.1	Problem 16	✗ (Timed out)	600.	✓	7.488		
1385	chapter 6 2.1	Problem 17	✗ (Timed out)	600.	✗	5.398		
1386	chapter 6 2.1	Problem 18	✗ (Timed out)	600.	✗	4.928		
1387	chapter 6 2.1	Problem 19	✗	2.034	✗	0.493		
1388	chapter 6 2.1	Problem 20	✗	232.551	✗	0.615		
1389	chapter 6 2.1	Problem 21	✗	0.093	✗	0.645		
1390	chapter 6 2.2	Problem 1	✓	0.3	✓	0.257		
1391	chapter 6 2.2	Problem 2	✓	0.193	✓	0.124		
1392	chapter 6 2.2	Problem 3	✗ (Timed out)	600.	✓	1.585		
1393	chapter 6 2.2	Problem 4	✓	0.257	✓	0.26		
1394	chapter 6 2.2	Problem 5	✓	0.043	✓	0.34		
1395	chapter 6 2.2	Problem 6	✓	0.04	✓	0.025		
1396	chapter 6 2.2	Problem 7	✓	0.028	✓	0.037		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1397	chapter 6 2.2	Problem 8	✓	0.039	✓	0.02		
1398	chapter 6 2.2	Problem 9	✓	0.225	✓	0.069		
1399	chapter 6 2.2	Problem 10	✓	0.036	✓	0.019		
1400	chapter 6 2.2	Problem 11	✓	0.156	✓	0.35		
1401	chapter 6 2.2	Problem 12	✓	0.012	✓	0.481		
1402	chapter 6 2.2	Problem 13	✓	0.034	✓	0.046		
1403	chapter 6 2.2	Problem 14	✓	0.136	✓	0.057		
1404	chapter 6 2.2	Problem 15	✓	0.044	✓	0.034		
1405	chapter 6 2.2	Problem 16	✓	0.036	✓	0.342		
1406	chapter 6 2.2	Problem 17	✓	0.041	✓	0.289		
1407	chapter 6 2.2	Problem 18	✓	0.041	✓	0.046		
1408	chapter 6 2.2	Problem 19	✓	0.062	✓	0.439		
1409	chapter 6 2.2	Problem 20	✓	0.042	✓	0.042		
1410	chapter 6 2.2	Problem 21	✓	0.047	✓	0.061		
1411	chapter 6 2.2	Problem 22	✓	0.047	✗	1.808		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1412	chapter 6 2.2	Problem 23	✗	9.231	✓	3.729		
1413	chapter 6 2.2	Problem 24	✓	0.083	✓	1.857		
1414	chapter 6 2.2	Problem 25	✓	0.594	✓	1.786		
1415	chapter 6 2.2	Problem 26	✗ (Timed out)	600.	✗	0.354		
1416	chapter 6 2.2	Problem 27	✗	0.769	✓	3.745		
1417	chapter 6 2.2	Problem 28	✓	1.628	✓	0.582		
1418	chapter 6 2.2	Problem 29	✗	0.423	✗	1.571		
1419	chapter 6 2.3	Problem 1	✗ (Timed out)	600.	✗	0.366		
1420	chapter 6 2.3	Problem 2	✓	0.082	✓	0.092		
1421	chapter 6 2.3	Problem 3	✓	0.03	✓	0.45		
1422	chapter 6 2.3	Problem 4	✗	0.024	✓	1.359		
1423	chapter 6 2.3	Problem 5	✗	56.178	✗	0.351		
1424	chapter 6 2.3	Problem 6	✗	0.548	✓	3.068		
1425	chapter 6 2.3	Problem 7	✗ (Timed out)	600.	✓	2.907		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
1426	chapter 6 2.3	Problem 8	✓	0.258	✗	0.454		
1427	chapter 6 2.3	Problem 9	✗	0.025	✓	1.342		
1428	chapter 6 2.3	Problem 10	✓	0.04	✓	0.029		
1429	chapter 6 2.3	Problem 11	✓	0.319	✓	0.752		
1430	chapter 6 2.3	Problem 12	✗	0.051	✗	2.088		
1431	chapter 6 2.3	Problem 13	✗	0.522	✗ (Timed out)	600.		
1432	chapter 6 2.4	Problem 1	✓	0.559	✓	6.854		
1433	chapter 6 2.4	Problem 2	✓	1.183	✓	1.		
1434	chapter 6 2.4	Problem 3	✓	0.231	✓	0.517		
1435	chapter 6 2.4	Problem 4	✓	0.755	✓	2.022		
1436	chapter 6 2.4	Problem 5	✓	0.729	✓	3.969		
1437	chapter 6 2.4	Problem 6	✓	0.408	✓	0.462		
1438	chapter 6 2.4	Problem 7	✗	0.583	✓	1.623		
1439	chapter 6 2.4	Problem 8	✗	0.031	✓	2.003		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1440	chapter 6 3.1	Problem 1	✓	0.192	✓	0.067		
1441	chapter 6 3.1	Problem 2	✓	0.212	✓	0.209		
1442	chapter 6 3.1	Problem 3	✓	0.258	✓	0.284		
1443	chapter 6 3.1	Problem 4	✗	0.971	✓	1.99		
1444	chapter 6 3.1	Problem 5	✓	0.492	✓	0.157		
1445	chapter 6 3.1	Problem 6	✓	0.45	✓	4.949		
1446	chapter 6 3.1	Problem 7	✓	1.037	✓	4.067		
1447	chapter 6 3.1	Problem 8	✓	1.052	✓	2.553		
1448	chapter 6 3.2	Problem 1	✗	0.61	✓	2.436		
1449	chapter 6 3.2	Problem 2	✗	0.399	✓	0.323		
1450	chapter 6 3.2	Problem 3	✓	1.88	✓	0.984		
1451	chapter 6 3.2	Problem 4	✓	1.624	✓	0.49		
1452	chapter 6 3.2	Problem 5	✗	38.384	✗	29.034		
1453	chapter 6 3.2	Problem 6	✗	37.789	✗	153.684		
1454	chapter 6 3.2	Problem 7	✗	13.661	✗	126.215		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1455	chapter 6 3.2	Problem 8	✗	23.146	✗	427.309		
1456	chapter 6 3.2	Problem 9	✗	8.759	✓	1.515		
1457	chapter 6 3.2	Problem 10	✗	2.909	✓	1.628		
1458	chapter 6 3.2	Problem 11	✗	2.154	✓	0.493		
1459	chapter 6 3.2	Problem 12	✗	28.853	✗ (Timed out)	600.		
1460	chapter 6 3.2	Problem 13	✓	1.63	✓	0.961		
1461	chapter 6 3.2	Problem 14	✗	1.396	✓	1.521		
1462	chapter 6 3.2	Problem 15	✓	1.847	✓	3.443		
1463	chapter 6 3.2	Problem 16	✗	3.102	✓	19.467		
1464	chapter 6 3.2	Problem 17	✗	31.866	✗	7.095		
1465	chapter 6 3.2	Problem 18	✗	21.55	✗	26.425		
1466	chapter 6 4.1	Problem 1	✓	0.032	✓	0.044		
1467	chapter 6 4.1	Problem 2	✓	0.192	✓	0.526		
1468	chapter 6 4.1	Problem 3	✓	0.252	✓	0.511		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1469	chapter 6 4.1	Problem 4	✓	0.461	✓	0.349		
1470	chapter 6 4.1	Problem 5	✗	11.171	✓	1.302		
1471	chapter 6 4.1	Problem 6	✗ (Timed out)	600.	✓	0.174		
1472	chapter 6 4.2	Problem 1	✓	0.032	✓	0.034		
1473	chapter 6 4.2	Problem 2	✓	0.048	✓	0.036		
1474	chapter 6 4.2	Problem 3	✓	0.188	✓	0.15		
1475	chapter 6 4.2	Problem 4	✓	0.248	✓	0.208		
1476	chapter 6 4.2	Problem 5	✓	0.491	✓	0.305		
1477	chapter 6 4.2	Problem 6	✗	176.322	✓	8.417		
1478	chapter 6 4.3	Problem 1	✓	0.269	✓	1.783		
1479	chapter 6 4.3	Problem 2	✓	0.085	✓	0.049		
1480	chapter 6 4.3	Problem 3	✓	0.743	✓	2.253		
1481	chapter 6 4.3	Problem 4	✓	0.91	✓	1.339		
1482	chapter 6 4.3	Problem 5	✗	2.001	✓	2.208		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1483	chapter 6 4.3	Problem 6	✗	0.65	✓	3.171		
1484	chapter 6 4.4	Problem 1	✓	0.258	✓	0.316		
1485	chapter 6 4.4	Problem 2	✓	0.086	✓	0.022		
1486	chapter 6 4.4	Problem 3	✓	0.782	✓	0.315		
1487	chapter 6 4.4	Problem 4	✓	0.837	✓	0.086		
1488	chapter 6 4.4	Problem 5	✗	4.917	✓	2.221		
1489	chapter 6 4.4	Problem 6	✗	0.649	✓	2.903		
1490	chapter 6 4.5	Problem 1	✓	0.502	✓	0.584		
1491	chapter 6 4.5	Problem 2	✓	0.294	✓	0.194		
1492	chapter 6 4.5	Problem 3	✗ (Timed out)	600.	✓	0.125		
1493	chapter 6 4.5	Problem 4	✗ (Timed out)	600.	✓	6.219		
1494	chapter 6 4.5	Problem 5	✗	0.703	✓	4.931		
1495	chapter 6 4.5	Problem 6	✗	0.668	✓	1.554		
1496	chapter 6 5.1	Problem 1	✓	0.24	✓	0.793		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1497	chapter 6 5.1	Problem 2	✓	0.01	✓	0.025		
1498	chapter 6 5.1	Problem 3	✓	0.283	✓	2.311		
1499	chapter 6 5.1	Problem 4	✓	0.216	✓	0.984		
1500	chapter 6 5.2	Problem 1	✓	0.041	✓	0.049		
1501	chapter 6 5.2	Problem 2	✓	0.215	✓	0.414		
1502	chapter 6 5.2	Problem 3	✓	0.263	✓	5.549		
1503	chapter 6 5.2	Problem 4	✓	0.309	✓	5.482		
1504	chapter 6 5.2	Problem 5	✓	0.121	✓	0.161		
1505	chapter 6 5.2	Problem 6	✓	0.595	✓	2.185		
1506	chapter 6 6.1	Problem 1	✓	0.182	✓	0.712		
1507	chapter 6 6.1	Problem 2	✓	0.2	✓	0.775		
1508	chapter 6 6.1	Problem 3	✓	0.159	✓	0.117		
1509	chapter 6 6.1	Problem 4	✓	0.46	✓	1.359		
1510	chapter 6 6.1	Problem 5	✓	3.111	✓	6.001		
1511	chapter 6 6.2	Problem 1	✓	0.464	✓	3.724		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1512	chapter 6 6.2	Problem 2	✓	0.443	✓	0.962		
1513	chapter 6 6.2	Problem 3	✗ (Timed out)	600.	✓	0.18		
1514	chapter 6 6.2	Problem 4	✗ (Timed out)	600.	✓	0.003		
1515	chapter 6 6.2	Problem 5	✓	3.096	✓	3.209		
1516	chapter 6 6.3	Problem 1	✓	0.209	✓	1.391	Yes	
1517	chapter 6 6.3	Problem 2	✓	0.671	✓	1.43	Yes	
1518	chapter 6 6.3	Problem 3	✓	0.96	✓	0.929		
1519	chapter 6 6.3	Problem 4	✓	0.214	✓	0.003		
1520	chapter 6 6.3	Problem 5	✓	1.441	✓	3.709		
1521	chapter 6 6.4	Problem 1	✓	0.214	✓	2.089		
1522	chapter 6 6.4	Problem 2	✓	0.664	✓	7.017		
1523	chapter 6 6.4	Problem 3	✓	0.836	✓	1.75		
1524	chapter 6 6.4	Problem 4	✓	0.257	✓	0.003		
1525	chapter 6 6.4	Problem 5	✗	1.799	✓	3.181		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1526	chapter 6 6.5	Problem 1	✓	162.609	✓	15.113		
1527	chapter 6 6.5	Problem 2	✓	0.096	✓	0.766		
1528	chapter 6 6.5	Problem 3	✓	0.528	✓	1.67		
1529	chapter 6 6.5	Problem 4	✗	67.888	✓	0.658		
1530	chapter 6 6.5	Problem 5	✓	0.494	✓	1.305		
1531	chapter 6 6.5	Problem 6	✗	20.608	✓	2.535		
1532	chapter 6 7.1	Problem 1	✓	0.325	✓	0.469		
1533	chapter 6 7.1	Problem 2	✓	0.392	✓	0.525		
1534	chapter 6 7.1	Problem 3	✓	0.087	✓	0.14		
1535	chapter 6 7.1	Problem 4	✓	0.252	✗ (Timed out)	600.		
1536	chapter 6 7.1	Problem 5	✓	0.29	✓	1.938		
1537	chapter 6 7.2	Problem 1	✓	0.287	✓	0.484		
1538	chapter 6 7.2	Problem 2	✓	0.403	✓	0.539		
1539	chapter 6 7.2	Problem 3	✓	0.073	✓	0.12		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1540	chapter 6 7.2	Problem 4	✓	0.242	✓	19.589		
1541	chapter 6 7.2	Problem 5	✓	0.274	✓	0.284		
1542	chapter 6 7.3	Problem 1	✓	0.789	✓	0.967		
1543	chapter 6 7.3	Problem 2	✓	0.689	✓	180.253		
1544	chapter 6 7.3	Problem 3	✓	0.398	✓	0.49		
1545	chapter 6 7.3	Problem 4	✓	0.215	✓	1.486		
1546	chapter 6 7.3	Problem 5	✓	0.586	✓	0.08		
1547	chapter 6 7.4	Problem 1	✓	0.939	✓	0.859		
1548	chapter 6 7.4	Problem 2	✓	0.723	✓	554.306		
1549	chapter 6 7.4	Problem 3	✓	0.427	✓	0.388		
1550	chapter 6 7.4	Problem 4	✓	0.199	✓	1.967		
1551	chapter 6 7.4	Problem 5	✓	0.558	✓	0.06		
1552	chapter 6 8.1	Problem 1	✓	0.01	✓	0.016		
1553	chapter 6 8.1	Problem 2	✓	0.193	✓	0.1		
1554	chapter 6 8.1	Problem 3	✓	0.099	✓	0.029		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1555	chapter 6 8.1	Problem 4	✓	0.306	✓	0.641		
1556	chapter 6 8.1	Problem 5	✓	0.189	✓	0.064		
1557	chapter 6 8.1	Problem 6	✗	0.006	✗	1.828		
1558	chapter 6 8.1	Problem 7	✓	4.141	✓	4.393		
1559	chapter 6 8.1	Problem 8	✓	0.845	✓	0.743		
1560	chapter 6 8.1	Problem 9	✗	0.567	✓	1.038		
1561	chapter 6 8.1	Problem 10	✗	0.734	✓	1.722		
1562	chapter 6 8.2	Problem 1	✓	0.045	✓	0.049		
1563	chapter 6 8.2	Problem 2	✓	0.476	✓	1.602		
1564	chapter 6 8.2	Problem 3	✓	3.228	✓	0.05		
1565	chapter 6 8.2	Problem 4	✗	0.239	✓	0.491		
1566	chapter 6 8.2	Problem 5	✓	0.157	✓	0.039		
1567	chapter 6 8.2	Problem 6	✓	0.191	✓	0.036		
1568	chapter 6 8.2	Problem 7	✓	0.151	✓	0.308		
1569	chapter 6 8.3	Problem 1	✓	0.021	✓	0.066		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1570	chapter 6 8.3	Problem 2	✓	0.028	✓	0.04		
1571	chapter 6 8.3	Problem 3	✓	0.039	✓	0.027		
1572	chapter 6 8.3	Problem 4	✓	0.069	✓	0.036		
1573	chapter 6 8.3	Problem 5	✓	0.072	✓	0.028		
1574	chapter 6 8.3	Problem 6	✓	0.47	✓	4.748		
1575	chapter 6 8.3	Problem 7	✓	2.493	✓	3.761		
1576	chapter 6 8.3	Problem 8	✗	133.566	✗ (Timed out)	600.		
1577	chapter 6 8.3	Problem 9	✗	0.351	✓	6.432		
1578	chapter 6 8.3	Problem 10	✗	0.27	✓	6.055		
1579	chapter 6 8.3	Problem 11	✗	45.363	✗	2.848		
1580	chapter 6 8.3	Problem 12	✗	1.548	✗	1.882		
1581	chapter 7 2.1	Problem 1	✓	0.041	✓	0.071		
1582	chapter 7 2.1	Problem 2	✓	0.154	✓	0.66		
1583	chapter 7 2.1	Problem 3	✓	0.038	✓	0.075		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1584	chapter 7 2.1	Problem 4	✓	0.583	✓	0.143		
1585	chapter 7 2.1	Problem 5	✓	0.116	✓	0.184		
1586	chapter 7 2.1	Problem 6	✓	0.125	✓	0.048	Yes	
1587	chapter 7 2.1	Problem 7	✓	0.173	✓	0.091	Yes	
1588	chapter 7 2.1	Problem 8	✗	0.023	✓	1.119		
1589	chapter 7 2.1	Problem 9	✓	0.861	✓	0.277		
1590	chapter 7 2.2	Problem 1	✓	0.093	✓	0.025		
1591	chapter 7 2.2	Problem 2	✓	0.033	✓	0.023		
1592	chapter 7 2.2	Problem 3	✓	0.136	✓	0.036		
1593	chapter 7 2.2	Problem 4	✓	0.078	✓	0.049		
1594	chapter 7 2.2	Problem 5	✓	0.2	✓	0.076		
1595	chapter 7 2.2	Problem 6	✓	0.22	✓	0.043		
1596	chapter 7 2.2	Problem 7	✓	0.147	✓	0.046		
1597	chapter 7 2.2	Problem 8	✓	0.148	✓	0.024		
1598	chapter 7 2.2	Problem 9	✓	0.175	✓	0.037		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1599	chapter 7 2.3	Problem 1	✓	0.024	✓	0.037		
1600	chapter 7 2.3	Problem 2	✓	0.036	✓	0.039		
1601	chapter 7 2.3	Problem 3	✓	0.06	✓	0.038		
1602	chapter 7 2.3	Problem 4	✓	0.114	✓	0.241		
1603	chapter 7 2.3	Problem 5	✓	0.187	✓	0.05		
1604	chapter 7 2.4	Problem 1	✓	0.325	✓	0.053		
1605	chapter 7 2.4	Problem 2	✓	0.196	✓	0.045		
1606	chapter 7 2.4	Problem 3	✓	0.101	✓	0.246		
1607	chapter 7 2.4	Problem 4	✓	0.161	✓	0.911		
1608	chapter 7 2.4	Problem 5	✓	0.091	✓	0.296		
1609	chapter 7 2.4	Problem 6	✗	0.026	✓	0.497		
1610	chapter 7 2.4	Problem 7	✗	1.871	✓	0.393		
1611	chapter 7 2.4	Problem 8	✓	3.208	✓	1.815		
1612	chapter 7 2.4	Problem 9	✓	1.135	✓	4.296		
1613	chapter 7 2.4	Problem 10	✓	0.766	✓	6.163		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	ani-mated?
1614	chapter 7 2.4	Problem 11	✓	10.748	✓	2.67		
1615	chapter 7 2.4	Problem 12	✓	0.05	✓	0.072		
1616	chapter 7 2.4	Problem 13	✓	0.409	✓	0.856		
1617	chapter 7 3.1	Problem 1	✓	0.044	✓	0.069		
1618	chapter 7 3.1	Problem 2	✓	0.477	✓	1.374		
1619	chapter 7 3.1	Problem 3	✓	2.421	✓	1.253		
1620	chapter 7 3.1	Problem 4	✗	0.906	✓	1.293		
1621	chapter 7 3.1	Problem 5	✓	0.56	✓	0.675		
1622	chapter 7 3.1	Problem 6	✓	0.363	✓	1.078		
1623	chapter 7 3.1	Problem 7	✓	1.107	✓	4.388		
1624	chapter 7 3.1	Problem 8	✓	1.064	✓	2.717		
1625	chapter 7 3.2	Problem 1	✓	0.988	✓	0.582		
1626	chapter 7 3.2	Problem 2	✓	0.86	✓	0.798		
1627	chapter 7 3.2	Problem 3	✓	0.365	✓	0.081		
1628	chapter 7 3.2	Problem 4	✓	0.373	✓	0.433		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1629	chapter 7 3.2	Problem 5	✓	0.382	✓	3.465		
1630	chapter 7 3.2	Problem 6	✗	37.92	✗	26.509		
1631	chapter 7 3.2	Problem 7	✗	14.713	✗	113.487		
1632	chapter 7 3.2	Problem 8	✗	23.568	✓	44.413		
1633	chapter 7 3.2	Problem 9	✓	1.885	✓	0.851		
1634	chapter 7 3.2	Problem 10	✗	15.035	✗	41.168		
1635	chapter 7 3.2	Problem 11	✓	2.318	✓	3.116		
1636	chapter 7 4.1	Problem 1	✓	0.553	✓	0.321		
1637	chapter 7 4.1	Problem 2	✓	30.778	✓	2.278		
1638	chapter 7 4.1	Problem 3	✓	1.61	✓	3.933		
1639	chapter 7 4.1	Problem 4	✓	19.909	✓	3.462		
1640	chapter 7 4.1	Problem 5	✓	30.965	✗ (Timed out)	600.		
1641	chapter 7 4.2	Problem 1	✓	0.557	✓	0.356		
1642	chapter 7 4.2	Problem 2	✓	26.371	✓	2.117		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1643	chapter 7 4.2	Problem 3	✓	1.649	✓	0.837		
1644	chapter 7 4.2	Problem 4	✓	14.672	✓	2.707		
1645	chapter 7 4.2	Problem 5	✓	0.32	✓	0.612		
1646	chapter 7 4.2	Problem 6	✓	27.166	✗ (Timed out)	600.		
1647	chapter 7 4.3	Problem 1	✓	0.429	✓	0.64		
1648	chapter 7 4.3	Problem 2	✓	142.248	✓	19.588		
1649	chapter 7 4.3	Problem 3	✓	0.974	✓	1.456		
1650	chapter 7 4.3	Problem 4	✗	0.846	✓	6.44		
1651	chapter 7 4.3	Problem 5	✓	1.292	✓	1.637		
1652	chapter 7 4.3	Problem 6	✗	2.011	✓	2.428		
1653	chapter 7 4.3	Problem 7	✓	43.572	✓	9.494		
1654	chapter 7 4.4	Problem 1	✓	0.448	✓	0.619		
1655	chapter 7 4.4	Problem 2	✓	140.61	✓	20.131		
1656	chapter 7 4.4	Problem 3	✓	0.984	✓	1.584		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
1657	chapter 7 4.4	Problem 4	✗	0.856	✓	4.569		
1658	chapter 7 4.4	Problem 5	✓	1.969	✓	1.72		
1659	chapter 7 4.4	Problem 6	✓	44.356	✓	6.568		
1660	chapter 7 4.5	Problem 1	✓	25.839	✓	12.969		
1661	chapter 7 4.5	Problem 2	✓	1.212	✓	0.705		
1662	chapter 7 4.5	Problem 3	✓	190.089	✓	2.147		
1663	chapter 7 4.5	Problem 4	✓	0.708	✓	1.007		
1664	chapter 7 4.5	Problem 5	✓	0.567	✓	1.168		
1665	chapter 7 5.1	Problem 1	✓	0.048	✓ Contains unresolve integral because maple can not integrate $\ln^n(x)$	0.035		
1666	chapter 7 5.1	Problem 2	✗	0.329	✓	0.863		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1667	chapter 7 5.1	Problem 3	✓	0.084	✓ Contains unresolve integral because maple can not integrate $\ln^n(x)$	0.098		
1668	chapter 7 5.1	Problem 4	✗	0.058	✓	1.542		
1669	chapter 7 5.1	Problem 5	✗	2.112	✓ Contains RootOf and unresolved integrals $\ln^n(x)$	1.558		
1670	chapter 7 5.2	Problem 1	✓	0.772	✓ Answer has unresolved integrals	11.423		
1671	chapter 7 5.2	Problem 2	✓	3.871	✓ Answer has unresolved integrals	4.288		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1672	chapter 7 5.2	Problem 3	✓ Generated internal errors from solve : inconsistent or redundant transcendental equation	0.099	✓ Answer has unresolved integrals and RootOf	1.314		
1673	chapter 7 5.2	Problem 4	✗	0.506	✓	1.146		
1674	chapter 7 5.2	Problem 5	✓	2.744	✓	6.178		
1675	chapter 7 6.1	Problem 1	✓	0.615	✓	1.349		
1676	chapter 7 6.1	Problem 2	✓	0.368	✓	1.107		
1677	chapter 7 6.1	Problem 3	✓	1.222	✓	2.978		
1678	chapter 7 6.1	Problem 4	✓	134.918	✓	12.812		
1679	chapter 7 6.1	Problem 5	✓	3.707	✓	3.915		
1680	chapter 7 6.1	Problem 6	✗	2.262	✓	24.67		
1681	chapter 7 6.2	Problem 1	✓	0.572	✓	0.966		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
1682	chapter 7 6.2	Problem 2	✓	1.449	✓	1.384		
1683	chapter 7 6.2	Problem 3	✓	1.267	✓	1.72		
1684	chapter 7 6.2	Problem 4	✓	118.275	✓	13.074		
1685	chapter 7 6.2	Problem 5	✓	2.903	✓	3.329		
1686	chapter 7 6.2	Problem 6	✗	2.192	✓	13.135		
1687	chapter 7 6.3	Problem 1	✓	0.439	✓	0.691		
1688	chapter 7 6.3	Problem 2	✓	0.478	✓	2.203		
1689	chapter 7 6.3	Problem 3	✓	0.865	✓	2.193		
1690	chapter 7 6.3	Problem 4	✓	39.998	✓	14.526		
1691	chapter 7 6.3	Problem 5	✗	1.544	✓	8.47		
1692	chapter 7 6.4	Problem 1	✓	0.438	✓	0.773		
1693	chapter 7 6.4	Problem 2	✓	0.706	✓	3.281		
1694	chapter 7 6.4	Problem 3	✓	0.859	✓	1.982		
1695	chapter 7 6.4	Problem 4	✓	44.145	✓	14.251		
1696	chapter 7 6.4	Problem 5	✓	3.04	✓	12.751		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1697	chapter 7 6.2	Problem 6	✗	1.563	✓	15.151		
1698	chapter 7 6.5	Problem 1	✓	1.505	✓	0.139		
1699	chapter 7 6.5	Problem 2	✗	68.895	✓	7.802		
1700	chapter 7 6.5	Problem 3	✗	33.256	✓	11.227		
1701	chapter 7 6.5	Problem 4	✗ (Timed out)	600.	✓	11.781		
1702	chapter 7 6.5	Problem 5	✗	12.338	✓	4.356		
1703	chapter 7 7.1	Problem 1	✓	0.102	✓	0.21		
1704	chapter 7 7.1	Problem 2	✓	0.754	✓	0.889		
1705	chapter 7 7.1	Problem 3	✗	1.071	✓	0.898		
1706	chapter 7 7.1	Problem 4	✗	0.818	✓	0.927		
1707	chapter 7 7.1	Problem 5	✓ Generates Solve::incnst: Inconsistent or redundant transcendental equation	2.22	✗ (Timed out)	600.		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1708	chapter 7 7.1	Problem 6	✓ Generates Solve::incnst: Inconsistent or redundant transcendental equation	0.714	✗ (Timed out)	600.		
1709	chapter 7 7.2	Problem 1	✓	0.097	✓ Answer contains unresolved integrals	0.184		
1710	chapter 7 7.2	Problem 2	✓	0.667	✓	0.043		
1711	chapter 7 7.2	Problem 3	✗	1.012	✓	0.39		
1712	chapter 7 7.2	Problem 4	✓	2.572	✓	20.171		
1713	chapter 7 7.2	Problem 5	✗	0.944	✓	0.646		
1714	chapter 7 7.3	Problem 1	✓	0.617	✓	0.376		
1715	chapter 7 7.3	Problem 2	✓	0.21	✓	0.568		
1716	chapter 7 7.3	Problem 3	✗	0.804	✓	0.505		
1717	chapter 7 7.3	Problem 4	✗	0.395	✓	184.541		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1718	chapter 7 7.3	Problem 5	✓	0.314	✓	1.917		
1719	chapter 7 7.4	Problem 1	✓	0.699	✓	0.692		
1720	chapter 7 7.4	Problem 2	✓	0.299	✓	0.169		
1721	chapter 7 7.4	Problem 3	✗	0.91	✓	0.532		
1722	chapter 7 7.4	Problem 4	✗	0.656	✓	0.987		
1723	chapter 7 7.4	Problem 5	✓	0.241	✓	0.451		
1724	chapter 7 8.1	Problem 1	✓	0.022	✓	0.205		
1725	chapter 7 8.1	Problem 2	✓	0.196	✓	0.115		
1726	chapter 7 8.1	Problem 3	✓	0.103	✓	0.024		
1727	chapter 7 8.1	Problem 4	✓	0.331	✓	2.214		
1728	chapter 7 8.1	Problem 5	✗	0.278	✗	2.091		
1729	chapter 7 8.1	Problem 6	✗	24.744	✓ Gives Warning: Incomplete separation	41.767		
1730	chapter 7 8.1	Problem 7	✓	0.78	✓	0.821		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica result	Mathematica time	Maple result	Maple time	hand solved?	animated?
1731	chapter 7 8.1	Problem 8	✗	0.614	✓	0.916		
1732	chapter 7 8.1	Problem 9	✗	0.764	✓	1.458		
1733	chapter 7 8.2	Problem 1	✗	0.174	✓	0.083		
1734	chapter 7 8.2	Problem 2	✗	0.233	✓	4.395		
1735	chapter 7 8.2	Problem 3	✗	0.207	✓	0.164		
1736	chapter 7 8.2	Problem 4	✓	0.465	✓	0.797		
1737	chapter 7 8.2	Problem 5	✓	1.238	✓	1.914		
1738	chapter 7 8.2	Problem 6	✗	38.56	✗	98.439		
1739	chapter 7 8.2	Problem 7	✗	0.346	✓	2.52		
1740	chapter 7 8.2	Problem 8	✗	0.25	✓	4.668		
1741	chapter 7 8.3	Problem 1	✓ Kernel message inconsistent or redundant transcendental equation	0.05	✓	0.11		
1742	chapter 7 8.3	Problem 2	✗	0.047	✓	0.115		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1743	chapter 7 8.3	Problem 3	✓	0.054	✓	0.068		
1744	chapter 7 8.3	Problem 4	✗	0.122	✓	0.088		
1745	chapter 7 8.3	Problem 5	✓	0.656	✓	3.107		
1746	chapter 7 8.3	Problem 6	✗ (Timed out)	600.	✓	5.109		
1747	chapter 7 8.3	Problem 7	✗	3.425	✓	7.692		
1748	chapter 7 8.3	Problem 8	✗	0.243	✓	21.24		
1749	chapter 7 8.3	Problem 9	✗	0.233	✓	16.869		
1750	chapter 7 8.3	Problem 10	✗	46.048	✗	8.941		
1751	chapter 7 8.3	Problem 11	✗	0.121	✗	2.766		
1752	chapter 8 2.1	Problem 1	✓	0.081	✓	0.24		
1753	chapter 8 2.1	Problem 2	✓	0.133	✓	0.985		
1754	chapter 8 2.1	Problem 3	✓	0.151	✓	0.192		
1755	chapter 8 2.1	Problem 4	✓	1.468	✓	0.418		
1756	chapter 8 2.1	Problem 5	✓	0.116	✓	0.173		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1757	chapter 8 2.1	Problem 6	✓	0.117	✓	0.228		
1758	chapter 8 2.1	Problem 7	✓	0.153	✓	0.22		
1759	chapter 8 2.1	Problem 8	✗	0.024	✓	1.782		
1760	chapter 8 2.1	Problem 9	✓	0.883	✓	0.315		
1761	chapter 8 2.2	Problem 1	✓	0.093	✓	0.072		
1762	chapter 8 2.2	Problem 2	✓	0.154	✓	0.035		
1763	chapter 8 2.2	Problem 3	✓	0.126	✓	0.03		
1764	chapter 8 2.2	Problem 4	✓	2.553	✓	0.654		
1765	chapter 8 2.2	Problem 5	✓	0.234	✓	0.283		
1766	chapter 8 2.2	Problem 6	✓	0.196	✓	0.366		
1767	chapter 8 2.2	Problem 7	✓	0.157	✓	0.133		
1768	chapter 8 2.2	Problem 8	✓	0.15	✓	0.052		
1769	chapter 8 2.2	Problem 9	✓	0.23	✓	0.096		
1770	chapter 8 2.3	Problem 1	✓	0.022	✓	0.111		
1771	chapter 8 2.3	Problem 2	✓	0.055	✓	0.13		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1772	chapter 8 2.3	Problem 3	✓	0.066	✓	0.062		
1773	chapter 8 2.3	Problem 4	✓	0.069	✓	0.788		
1774	chapter 8 2.3	Problem 5	✓	0.231	✓	0.11		
1775	chapter 8 2.4	Problem 1	✓	0.491	✓	0.089		
1776	chapter 8 2.4	Problem 2	✓	0.278	✓	0.072		
1777	chapter 8 2.4	Problem 3	✓	0.1	✓	0.411		
1778	chapter 8 2.4	Problem 4	✓	0.153	✓	0.928		
1779	chapter 8 2.4	Problem 5	✓	0.098	✓	0.434		
1780	chapter 8 2.4	Problem 6	✗	0.027	✓	0.246		
1781	chapter 8 2.4	Problem 7	✗	1.922	✓	0.542		
1782	chapter 8 2.4	Problem 8	✓	3.086	✓	3.025		
1783	chapter 8 2.4	Problem 9	✓	1.368	✓	5.376		
1784	chapter 8 2.4	Problem 10	✓	0.817	✓	2.639		
1785	chapter 8 2.4	Problem 11	✗	22.571	✓	10.807		
1786	chapter 8 2.4	Problem 12	✓	0.048	✓	0.104		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1787	chapter 8 2.4	Problem 13	✓	0.394	✓	1.158		
1788	chapter 8 3.1	Problem 1	✓	0.04	✓	0.107		
1789	chapter 8 3.1	Problem 2	✓	0.541	✓	0.131		
1790	chapter 8 3.1	Problem 3	✓ Generates Solve::incnst: Inconsistent or redundant transcendental equation	2.348	✓	0.598		
1791	chapter 8 3.1	Problem 4	✗	2.11	✓	2.632		
1792	chapter 8 3.1	Problem 5	✓	0.571	✓	0.909		
1793	chapter 8 3.1	Problem 6	✗	2.071	✓	5.061		
1794	chapter 8 3.1	Problem 7	✓	1.061	✓	6.09		
1795	chapter 8 3.1	Problem 8	✓	1.062	✓	3.043		
1796	chapter 8 3.2	Problem 1	✓	1.786	✓	1.841		
1797	chapter 8 3.2	Problem 2	✓	1.572	✓	0.143		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1798	chapter 8 3.2	Problem 3	✓	0.993	✓	0.527		
1799	chapter 8 3.2	Problem 4	✓	0.395	✓	0.323		
1800	chapter 8 3.2	Problem 5	✓	0.285	✓	3.192		
1801	chapter 8 3.2	Problem 6	✗	39.07	✓	197.072		
1802	chapter 8 3.2	Problem 7	✗	15.044	✗ (Timed out)	600.		
1803	chapter 8 3.2	Problem 8	✗	23.588	✗ (Timed out)	600.		
1804	chapter 8 3.2	Problem 9	✓	2.091	✓	0.758		
1805	chapter 8 3.2	Problem 10	✗	1.468	✓	1.946		
1806	chapter 8 3.2	Problem 11	✓	2.4	✓	7.545		
1807	chapter 8 4.1	Problem 1	✓	0.987	✓	0.438		
1808	chapter 8 4.1	Problem 2	✓	28.056	✓	0.578		
1809	chapter 8 4.1	Problem 3	✓	11.887	✓	0.378		
1810	chapter 8 4.1	Problem 4	✓	18.777	✓	0.837		
1811	chapter 8 4.1	Problem 5	✗	5.134	✓	2.452		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1812	chapter 8 4.2	Problem 1	✓	1.01	✓	0.286		
1813	chapter 8 4.2	Problem 2	✓	23.355	✓	0.364		
1814	chapter 8 4.2	Problem 3	✓	4.194	✓	0.003		
1815	chapter 8 4.2	Problem 4	✓	14.214	✓	0.217		
1816	chapter 8 4.2	Problem 5	✗	5.197	✓	2.203		
1817	chapter 8 4.3	Problem 1	✓	0.245	✓	0.58		
1818	chapter 8 4.3	Problem 2	✓	109.367	✓	3.671		
1819	chapter 8 4.3	Problem 3	✓	0.766	✓	0.026		
1820	chapter 8 4.3	Problem 4	✗	0.906	✓	8.303		
1821	chapter 8 4.3	Problem 5	✓	1.328	✓	2.086		
1822	chapter 8 4.3	Problem 6	✗	5.486	✓	6.848		
1823	chapter 8 4.4	Problem 1	✓	0.249	✓	0.845		
1824	chapter 8 4.4	Problem 2	✓	76.565	✓	5.044		
1825	chapter 8 4.4	Problem 3	✓	0.758	✓	0.067		
1826	chapter 8 4.4	Problem 4	✗	0.876	✓	1.853		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1827	chapter 8 4.4	Problem 5	✓	1.701	✓	1.928		
1828	chapter 8 4.4	Problem 6	✗	5.167	✓	3.711		
1829	chapter 8 4.5	Problem 1	✓	57.799	✓	0.163		
1830	chapter 8 4.5	Problem 2	✓	9.787	✓	0.128		
1831	chapter 8 4.5	Problem 3	✗	192.154	✓	1.449		
1832	chapter 8 4.5	Problem 4	✓	0.758	✓	0.031		
1833	chapter 8 4.5	Problem 5	✓	0.563	✓	1.388		
1834	chapter 8 5.1	Problem 1	✓	0.033	✓	0.051		
1835	chapter 8 5.1	Problem 2	✓	0.09	✓	0.139		
1836	chapter 8 5.1	Problem 3	✓	0.091	✓	0.074		
1837	chapter 8 5.1	Problem 4	✗	0.534	✓	0.463		
1838	chapter 8 5.1	Problem 5	✗	0.056	✓	1.276		
1839	chapter 8 5.2	Problem 1	✗	0.345	✓	0.946		
1840	chapter 8 5.2	Problem 2	✓	2.475	✓	0.151		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1841	chapter 8 5.2	Problem 3	✓	0.556	✗ (Timed out)	600.		
1842	chapter 8 5.2	Problem 4	✗	0.516	✓	1.512		
1843	chapter 8 5.2	Problem 5	✓	0.807	✓	15.274		
1844	chapter 8 6.1	Problem 1	✓	1.002	✓	1.484		
1845	chapter 8 6.1	Problem 2	✓	0.622	✓	1.318		
1846	chapter 8 6.1	Problem 3	✗	3.538	✓	7.16		
1847	chapter 8 6.1	Problem 4	✗	72.066	✓	16.824		
1848	chapter 8 6.1	Problem 5	✓	64.915	✓	7.193		
1849	chapter 8 6.1	Problem 6	✗	1.362	✓	15.069		
1850	chapter 8 6.2	Problem 1	✓	1.564	✓	0.877		
1851	chapter 8 6.2	Problem 2	✓	1.397	✓	6.867		
1852	chapter 8 6.2	Problem 3	✓	3.209	✓	0.138		
1853	chapter 8 6.2	Problem 4	✗	69.57	✓	19.99		
1854	chapter 8 6.2	Problem 5	✓	110.348	✓	10.691		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1855	chapter 8 6.2	Problem 6	✗	1.36	✓	11.584		
1856	chapter 8 6.3	Problem 1	✓	0.274	✓	0.934		
1857	chapter 8 6.3	Problem 2	✓	0.487	✓	2.384		
1858	chapter 8 6.3	Problem 3	✓	0.673	✓	0.111		
1859	chapter 8 6.3	Problem 4	✓	29.217	✓	8.885		
1860	chapter 8 6.3	Problem 5	✗	1.249	✓	6.61		
1861	chapter 8 6.4	Problem 1	✓	0.086	✓	0.98		
1862	chapter 8 6.4	Problem 2	✓	0.82	✓	3.789		
1863	chapter 8 6.4	Problem 3	✓	0.668	✓	0.082		
1864	chapter 8 6.4	Problem 4	✓	41.154	✓	31.016		
1865	chapter 8 6.4	Problem 5	✗	1.221	✓	8.908		
1866	chapter 8 6.5	Problem 1	✓	8.927	✓	0.138		
1867	chapter 8 6.5	Problem 2	✗	69.505	✓	17.781		
1868	chapter 8 6.5	Problem 3	✗	36.425	✓	12.36		
1869	chapter 8 6.5	Problem 4	✗	1.874	✓	13.761		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1870	chapter 8 6.5	Problem 5	✗	1.565	✓	8.911		
1871	chapter 8 7.1	Problem 1	✓	0.188	✓	0.216		
1872	chapter 8 7.1	Problem 2	✓	1.084	✓	0.987		
1873	chapter 8 7.1	Problem 3	✗	1.024	✓	0.821		
1874	chapter 8 7.1	Problem 4	✗	0.784	✓	1.072		
1875	chapter 8 7.1	Problem 5	✓	2.043	✗ (Timed out)	600.		
1876	chapter 8 7.1	Problem 6	✗	0.91	✓	1.972		
1877	chapter 8 7.2	Problem 1	✓	0.049	✓	0.235		
1878	chapter 8 7.2	Problem 2	✓	1.1	✓	0.168		
1879	chapter 8 7.2	Problem 3	✗	0.901	✓	0.409		
1880	chapter 8 7.2	Problem 4	✗	0.728	✓	0.456		
1881	chapter 8 7.2	Problem 5	✓	2.365	✓	1.668		
1882	chapter 8 7.2	Problem 6	✗	0.897	✓	0.179		
1883	chapter 8 7.3	Problem 1	✓	0.303	✓	0.365		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1884	chapter 8 7.3	Problem 2	✓	0.627	✓	0.947		
1885	chapter 8 7.3	Problem 3	✗	0.773	✓	0.194		
1886	chapter 8 7.3	Problem 4	✗	0.573	✓	416.475		
1887	chapter 8 7.3	Problem 5	✓	0.284	✓	1.052		
1888	chapter 8 7.4	Problem 1	✓	0.339	✓	0.456		
1889	chapter 8 7.4	Problem 2	✓	0.587	✓	0.135		
1890	chapter 8 7.4	Problem 3	✗	0.819	✓	0.073		
1891	chapter 8 7.4	Problem 4	✗	0.596	✓	0.049		
1892	chapter 8 7.4	Problem 5	✓	0.286	✓	2.223		
1893	chapter 8 8.1	Problem 1	✓	0.019	✓	0.133		
1894	chapter 8 8.1	Problem 2	✓	0.191	✓	0.146		
1895	chapter 8 8.1	Problem 3	✓	0.1	✓	0.026		
1896	chapter 8 8.1	Problem 4	✓	0.298	✓	0.23		
1897	chapter 8 8.1	Problem 5	✓	0.178	✓	0.79		
1898	chapter 8 8.1	Problem 6	✓	3.506	✓	1.536		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1899	chapter 8 8.1	Problem 7	✓	0.762	✓	0.851		
1900	chapter 8 8.1	Problem 8	✗	0.533	✓	1.046		
1901	chapter 8 8.1	Problem 9	✗	0.711	✓	1.634		
1902	chapter 8 8.2	Problem 1	✗	0.151	✓	0.225		
1903	chapter 8 8.2	Problem 2	✗	0.22	✓	0.452		
1904	chapter 8 8.2	Problem 3	✗	0.191	✓	0.114		
1905	chapter 8 8.2	Problem 4	✓	0.165	✓	0.553		
1906	chapter 8 8.2	Problem 5	✓	1.139	✗	17.637		
1907	chapter 8 8.2	Problem 6	✗	0.305	✓	2.655		
1908	chapter 8 8.2	Problem 7	✗	21.021	✗	5.711		
1909	chapter 8 8.2	Problem 8	✗	20.227	✗	4.512		
1910	chapter 8 8.3	Problem 1	✓	0.036	✓	0.089		
1911	chapter 8 8.3	Problem 2	✗	0.041	✓	0.079		
1912	chapter 8 8.3	Problem 3	✓	0.054	✓	0.062		
1913	chapter 8 8.3	Problem 4	✗	0.107	✓	0.076		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1914	chapter 8 8.3	Problem 5	✓	0.619	✓	3.907		
1915	chapter 8 8.3	Problem 6	✗ (Timed out)	600.	✓	5.979		
1916	chapter 8 8.3	Problem 7	✗	129.84	✗ (Timed out)	600.		
1917	chapter 8 8.3	Problem 8	✗	20.238	✗	5.535		
1918	chapter 8 8.3	Problem 9	✗	20.233	✗	15.995		
1919	chapter 8 8.3	Problem 10	✗	45.026	✓	3.161		
1920	chapter 8 8.3	Problem 11	✗	1.466	✗	4.557		
1921	chapter 9 2.1	Problem 1	✓	0.351	✓	0.154		
1922	chapter 9 2.1	Problem 2	✓	0.413	✓	1.898		
1923	chapter 9 2.1	Problem 3	✓	0.25	✓	0.22		
1924	chapter 9 2.1	Problem 4	✓	0.111	✓	0.136		
1925	chapter 9 2.1	Problem 5	✓	0.21	✓	0.233		
1926	chapter 9 2.1	Problem 6	✗ (Timed out)	600.	✗ (Timed out)	600.		
1927	chapter 9 2.1	Problem 7	✓	0.167	✓	0.337		

Continued on next page

Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1928	chapter 9 2.1	Problem 8	✓	0.123	✓	0.059		
1929	chapter 9 2.1	Problem 9	✓	0.272	✓	1.725		
1930	chapter 9 2.2	Problem 1	✓	0.3	✓	0.163		
1931	chapter 9 2.2	Problem 2	✓	0.083	✓	0.131		
1932	chapter 9 2.2	Problem 3	✓	0.257	✓	0.111		
1933	chapter 9 2.2	Problem 4	✗ (Timed out)	600.	✓	458.068		
1934	chapter 9 2.2	Problem 5	✓	0.053	✓	0.216		
1935	chapter 9 2.2	Problem 6	✓	0.05	✓	0.076		
1936	chapter 9 2.2	Problem 7	✓	0.317	✓	0.289		
1937	chapter 9 2.3	Problem 1	✓	0.133	✓	0.238		
1938	chapter 9 2.3	Problem 2	✓	0.139	✓	0.208		
1939	chapter 9 2.3	Problem 3	✓	0.144	✓	0.333		
1940	chapter 9 2.3	Problem 4	✗ (Timed out)	600.	✓	85.01		
1941	chapter 9 2.3	Problem 5	✓	0.304	✓	2.427		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1942	chapter 9 2.4	Problem 1	✓	0.185	✓	0.274		
1943	chapter 9 2.4	Problem 2	✓	0.177	✓	0.133		
1944	chapter 9 2.4	Problem 3	✓	0.267	✓	2.807		
1945	chapter 9 2.4	Problem 4	✓	0.143	✓	0.326		
1946	chapter 9 2.4	Problem 5	✓	0.044	✓	0.13		
1947	chapter 9 2.4	Problem 6	✓	0.142	✓	0.425		
1948	chapter 9 2.4	Problem 7	✓	3.95	✓	10.733		
1949	chapter 9 2.4	Problem 8	✓	2.038	✓	15.509		
1950	chapter 9 2.4	Problem 9	✗	23.655	✓	35.167		
1951	chapter 9 2.4	Problem 10	✓	0.074	✓	0.336		
1952	chapter 9 2.4	Problem 11	✓	0.185	✓	2.526		
1953	chapter 9 2.4	Problem 12	✗	0.031	✓	4.548		
1954	chapter 9 2.4	Problem 13	✓	2.958	✓	4.558		
1955	chapter 9 2.4	Problem 14	✓	0.537	✓	1.389		
1956	chapter 9 3.1	Problem 1	✓	0.073	✓	0.195		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1957	chapter 9 3.1	Problem 2	✓	0.102	✓	0.335		
1958	chapter 9 3.1	Problem 3	✓	0.226	✓	0.159		
1959	chapter 9 3.1	Problem 4	✓	0.194	✓	0.756		
1960	chapter 9 3.1	Problem 5	✗	2.677	✓	6.091		
1961	chapter 9 3.1	Problem 6	✓	1.455	✓	6.593		
1962	chapter 9 3.1	Problem 7	✗ (Timed out)	600.	✓	44.915		
1963	chapter 9 3.2	Problem 1	✓	0.078	✓	0.172		
1964	chapter 9 3.2	Problem 2	✓	0.104	✓	0.288		
1965	chapter 9 3.2	Problem 3	✗	0.167	✓	0.705		
1966	chapter 9 3.2	Problem 4	✗	2.52	✓	5.657		
1967	chapter 9 3.2	Problem 5	✓	2.575	✓	3.809		
1968	chapter 9 3.2	Problem 6	✗	2.641	✓	5.633		
1969	chapter 9 3.2	Problem 7	✓	0.347	✓	0.694		
1970	chapter 9 3.2	Problem 8	✓	1.351	✓	1.694		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1971	chapter 9 3.2	Problem 9	✗ (Timed out)	600.	✓	14.656		
1972	chapter 9 3.2	Problem 10	✗	1.863	✓	5.044		
1973	chapter 9 4.1	Problem 1	✓	127.643	✓	0.713		
1974	chapter 9 4.1	Problem 2	✓	3.164	✓	11.907		
1975	chapter 9 4.1	Problem 3	✓	0.162	✓	0.738		
1976	chapter 9 4.1	Problem 4	✗	167.253	✓	2.005		
1977	chapter 9 4.1	Problem 5	✓	150.807	✓	19.258		
1978	chapter 9 4.2	Problem 1	✓	130.254	✓	0.834		
1979	chapter 9 4.2	Problem 2	✓	3.226	✓	10.288		
1980	chapter 9 4.2	Problem 3	✓	0.143	✓	0.708		
1981	chapter 9 4.2	Problem 4	✗	176.196	✓	1.653		
1982	chapter 9 4.2	Problem 5	✓	155.26	✓	16.9		
1983	chapter 9 4.3	Problem 1	✓	74.218	✓	1.66		
1984	chapter 9 4.3	Problem 2	✓	3.212	✓	15.457		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
1985	chapter 9 4.3	Problem 3	✓	0.235	✓	1.266		
1986	chapter 9 4.3	Problem 4	✗	185.477	✓	10.98		
1987	chapter 9 4.3	Problem 5	✓	275.802	✓	6.223		
1988	chapter 9 4.4	Problem 1	✓	70.045	✓	1.968		
1989	chapter 9 4.4	Problem 2	✓	3.915	✓	5.489		
1990	chapter 9 4.4	Problem 3	✓	0.209	✓	0.819		
1991	chapter 9 4.4	Problem 4	✗	189.822	✓	3.596		
1992	chapter 9 4.4	Problem 5	✓	272.554	✓	14.666		
1993	chapter 9 4.5	Problem 1	✓	132.333	✓	0.26		
1994	chapter 9 4.5	Problem 2	✓	59.285	✓	1.175		
1995	chapter 9 4.5	Problem 3	✗	180.194	✓	0.703		
1996	chapter 9 4.5	Problem 4	✓	0.109	✓	0.691		
1997	chapter 9 4.5	Problem 5	✓	158.848	✓	5.229		
1998	chapter 9 5.1	Problem 1	✓	0.169	✓	0.236		
1999	chapter 9 5.1	Problem 2	✓	0.126	✓	0.496		

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Table 1.11 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
2000	chapter 9 5.1	Problem 3	✗	0.523	✓	0.63		
2001	chapter 9 5.1	Problem 4	✗	0.27	✓	2.477		
2002	chapter 9 5.1	Problem 5	✓	1.416	✓	349.608		

1.3.7 Handbook of nonlinear partial differential equations

Table 1.12: Handbook of nonlinear partial differential equations breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
2003	chapter 1 1.1	Problem 1	✗	0.361	✗	7.63		
2004	chapter 1 1.1	Problem 2	✓	0.108	✓	0.671		
2005	chapter 1 1.2	Problem 1	✗	0.08	✗	19.037		
2006	chapter 1 1.2	Problem 2	✗	0.094	✓	0.776		
2007	chapter 1 1.2	Problem 3	✗	0.182	✓	0.542		
2008	chapter 1 1.2	Problem 4	✗	0.175	✓	0.74		
2009	chapter 1 1.2	Problem 5	✗	0.891	✓	1.456		

Continued on next page

Table 1.12 – Handbook of nonlinear partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani- mated?
			result	time	result	time		
2010	chapter 1 1.3	Problem 1	✗	0.008	✗	0.093		
2011	chapter 1 1.3	Problem 2	✗	0.009	✗	0.018		
2012	chapter 1 1.3	Problem 3	✗	0.015	✗	0.198		
2013	chapter 1 1.3	Problem 4	✗	0.015	✗	0.025		
2014	chapter 1 1.4	Problem 1	✗	0.012	✗	0.018		

CHAPTER 2

Miscellaneous PDE's

Local contents

2.1 General first order

Local contents

2.1.1 Transport equation $u_t + u_x = 0$

problem number 1

Taken from Mathematica Symbolic PDE document

Solve for $u(x,t)$

$$u_t + u_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to c_1(t-x)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \_F1(t-x)$$

Hand solution

$$u_t + u_x = 0 \tag{1}$$

Let $u \equiv u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1) to (2) then we see that

$$\frac{du}{dt} = 0 \tag{3}$$
$$\frac{dx}{dt} = 1 \tag{4}$$

(3) says that u is constant. Since no initial conditions are given, let $u = F(x(0))$ where F is arbitrary function. To find $x(0)$ we solve (4). The solution to (4) is $x = x(0) + t$. Hence $x(0) = x - t$. Therefore

$$u(x,t) = F(x-t)$$

2.1.2 Transport equation $u_t - 3u_x = 0$ IC $u(0,x) = e^{-x^2}$. Peter Olver textbook, 2.2.2 (a)

problem number 2

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve for $u(t,x)$ in $u_t - 3u_x = 0$ with IC $u(0,x) = e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] -3* D[u[t, x], {x}] == 0;
ic = u[0,x]==Exp[-x^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x)\to e^{-(3t+x)^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) - 3*diff(u(t, x),x) =0;
ic:=u(0,x)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = \mathrm{e}^{-(3t+x)^2}$$

Hand solution

Solve

$$u_t - 3u_x = 0 \tag{1}$$

With initial conditions $u(x,0) = e^{-x^2}$

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = -3 \tag{4}$$

Eq (3) gives $u = u(x(0))$. Using the given initial conditions, this becomes

$$u = e^{-x(0)^2} \tag{5}$$

Eq (4) is now used to find $x(0)$. Soving (4) gives $x = x(0) - 3t$. Hence $x(0) = x - 3t$. Therefore (5) becomes

$$u(x,t) = e^{-(x-3t)^2}$$

The following is an animation of the solution

Out[ ]=

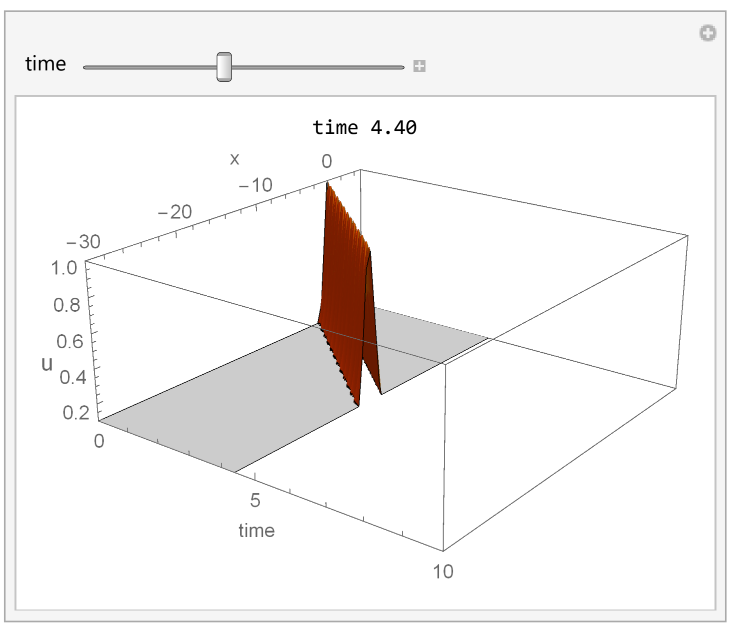


Figure 2.1: snap shot

Source code used for the above

```
(*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[-(x + 3 t)^2];
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
        {Quiet@Plot3D[u[x, t], {x, -20, 2}, {t, 0, time},
               PlotRange → {All, {0, 6}, {0.1, 1}},
               AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
               BaseStyle → 12, PerformanceGoal → "Quality",
               ImageSize → 400, PlotPoints → 40,
               PlotStyle → plotStyle, Mesh → None,
               ViewPoint → {-2.355, 1.645, 1.792},
               Boxed → False]
        }}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 6, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.2: Source code 3D

```
(*2D*)
u[x_, t_] := Exp[-(x + 3 t)^2];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
        {Quiet@Plot[u[x, time], {x, -20, 3},
               PlotRange → {All, {0, 1}},
               AxesLabel → {Style["x", 12], Style["u", 14]},
               BaseStyle → 12,
               ImageSize → 400
            ]
        }}
   ]
   ,
   {{time, 0, "time"}, 0, 6, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.3: Source code 2D

2.1.3 Transport equation $u_t + 2u_x = 0$ IC $u(-1,x) = \frac{x}{1+x^2}$. Peter Olver textbook, 2.2.2 (b)

problem number 3

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve for $u(t,x)$ in $u_t + 2u_x = 0$ with IC $u(-1,x) = \frac{x}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] +2* D[u[t, x], {x}] == 0;
ic = u[-1,x]==x/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to \frac{-2t+x-2}{4t^2-4t(x-2)+x^2-4x+5}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +2*diff(u(t, x),x) =0;
ic:=u(-1,x)=x/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = \frac{-2t+x-2}{(-2t+x-2)^2+1}$$

Hand solution

Solve

$$u_t + 2u_x = 0$$

With initial conditions $u(-1,x) = \frac{x}{1+x^2}$.

Solution

Let $u = u(x(t),t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = 2 \tag{4}$$

Eq (3) says that u is constant on the chataterstic lines, or $u = u(x(-1))$. Using the given initial conditions, this becomes

$$u(x(t)\,,t) = \frac{x(-1)}{1 + x\,(-1)^2} \tag{5}$$

Eq (4) is now used to find $x(-1)$. Soving (4) gives $x = x(0)+2t$. Hence $x(-1) = x(0)-2$ or $x(0) = x(-1) + 2$. Therefore

$$\begin{aligned} x &= x(-1) + 2 + 2t \\ x(-1) &= x - 2 - 2t \end{aligned}$$

Now that we found $x(-1)$, we substitute it in (5), giving the solution

$$u(x(t)\,,t) = \frac{x - 2 - 2t}{1 + (x - 2 - 2t)^2}$$

<u>Alternative method</u>. Using Lagrange-charpit method

$$\frac{dt}{1} = \frac{dx}{2} = \frac{du}{0}$$

Which implies that $du = 0$ or $u = C_1$. A constant. Integrating $\frac{dt}{1} = \frac{dx}{2}$ gives $t = \frac{1}{2}x + C_2$ or $C_2 = t - \frac{1}{2}x$. But $C_1 = F(C_2)$ always, where F is arbitrary function. Since $C_1 = u$ then

$$\begin{aligned} u &= F(C_2) \\ u &= F\left(t - \frac{1}{2}x\right) \end{aligned} \tag{1}$$

At $t = -1$ the above becomes

$$\frac{x}{1 + x^2} = F\left(-1 - \frac{1}{2}x\right)$$

Let $-1 - \frac{1}{2}x = z$ which implies $x = -2(1 + z)$ The above can be written as

$$\begin{aligned} \frac{-2(1 + z)}{1 + (-2\,(1 + z))^2} &= F(z) \\ F(z) &= -\frac{2(1 + z)}{4z^2 + 8z + 5} \end{aligned}$$

From the above then (1) can be written as

$$
\begin{aligned}
u(t,x) &= -\frac{2\left(1+\left(t-\frac{1}{2}x\right)\right)}{4\left(t-\frac{1}{2}x\right)^2+8\left(t-\frac{1}{2}x\right)+5}\\
&= \frac{x-2t-2}{4t^2-4tx+8t+x^2-4x+5}\\
&= \frac{x-2t-2}{1+(x-2-2t)^2}
\end{aligned}
$$

The following is an animation of the solution

Out[ ]=

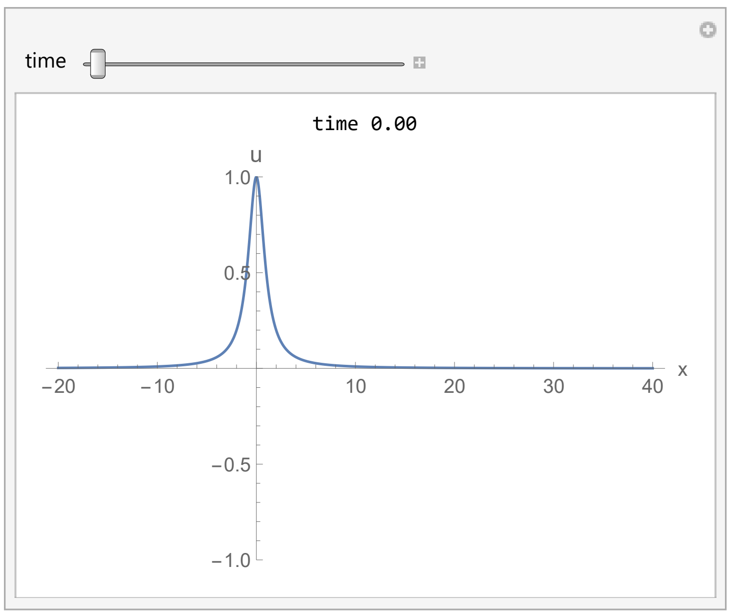


Figure 2.4: snap shot

Source code used for the above

```
(*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
         NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := (x - 2 t - 2)/(1 + (x - 2 - 2 t)^2);
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
         {
            Quiet@Plot3D[u[x, t], {x, -1, 30}, {t, 0, time},
                  PlotRange → {All, {0, 12}, {-.5, 1}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 75,
                  PlotStyle → plotStyle, Mesh → None,
                  ViewPoint → {1.765, 2.0924, 1.9887}, Boxed → True]
         }}
   ]
   ,
   {{time, 0.1, "time"}, 0.1, 12, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.5: Source code 3D

```
In[*]:= (*2D*)
u[x_, t_] := (x - 2 t - 2)/(1 + (x - 2 - 2 t)^2);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
         {Quiet@Plot[u[x, time], {x, -20, 40},
                  PlotRange → {All, {-1, 1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 12, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.6: Source code 2D

2.1.4 Transport equation $u_t + u_x + \frac{1}{2}u = 0$ IC $u(0,x) = \arctan(x)$. Peter Olver textbook, 2.2.2 (c)

problem number 4

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t + u_x + \frac{1}{2}u = 0$ with IC $u(0,x) = \arctan(x)$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] + D[u[t, x], {x}]+1/2*u[t,x] == 0;
ic = u[0,x]==ArcTan[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\{\{u(t,x) \to -e^{-t/2}\tan^{-1}(t-x)\}\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +diff(u(t, x),x) +1/2*u(t,x)=0;
ic:=u(0,x)=arctan(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = -\arctan(t-x)\,\mathrm{e}^{-\frac{t}{2}}$$

Hand solution

Solve

$$u_t + u_x + \frac{1}{2}u = 0$$

With initial conditions $u(x,0) = \arctan(x)$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = -\frac{1}{2}u \tag{3}$$

$$\frac{dx}{dt} = 1 \tag{4}$$

Solving (3) gives

$$\begin{aligned}\frac{du}{u} &= \frac{-1}{2}dt\\ \ln|u| &= -\frac{1}{2}t + c\\ u &= u(x(0))\, e^{\frac{-t}{2}}\end{aligned}$$

Using the given initial conditions, this becomes

$$u(x,t) = \arctan\left(x(0)\right) e^{\frac{-1}{2}t} \tag{5}$$

Now we just need to find $x(0)$. From (4)

$$\begin{aligned}x &= x(0) + t\\ x(0) &= x - t\end{aligned}$$

Substituting the above into (5) gives

$$u(x,t) = \arctan\left(x - t\right) e^{\frac{-1}{2}t}$$

The following is an animation of the solution

Out[ ]=

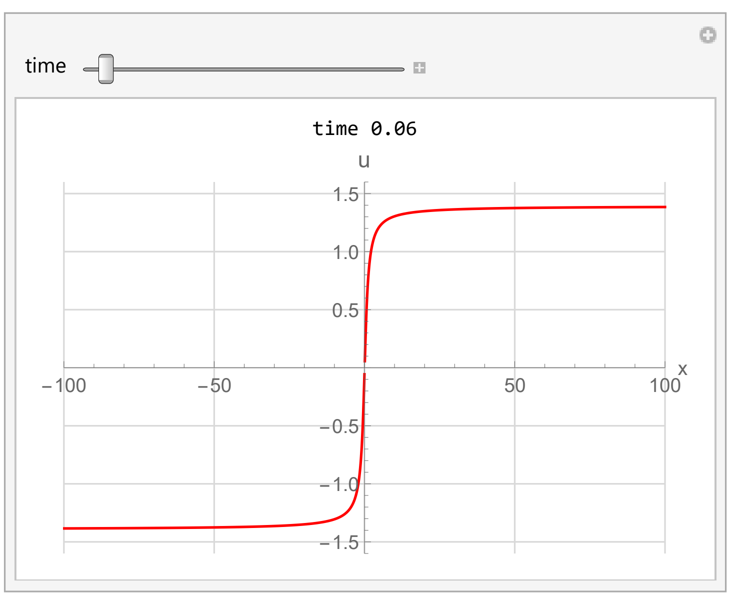


Figure 2.7: snap shot

Source code used for the above

```
In[ ]:= padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := ArcTan[x - t] Exp[-2 t];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
        {
          Quiet@Plot[u[x, time], {x, -100, 100},
               PlotRange → {{-100, 100}, {-1.6, 1.6}},
               AxesLabel → {Style["x", 12], Style["u", 14]},
               BaseStyle → 12, PerformanceGoal → "Quality",
               ImageSize → 400,
               PlotStyle → Red,
               GridLines → Automatic, GridLinesStyle → LightGray
             ]
        }}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 1.8, .05},
   TrackedSymbols :→ {time}
]
```

Figure 2.8: Source code

2.1.5 Transport equation $u_t - 4u_x + u = 0$ IC $u(0,x) = \frac{1}{1+x^2}$. Peter Olver textbook, 2.2.2 (d)

problem number 5

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t - 4u_x + u = 0$ with IC $u(0,x) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] -4*D[u[t, x], {x}]+u[t,x] == 0;
ic = u[0,x]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x)\to \frac{e^{-t}}{16t^2+8tx+x^2+1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) -4*diff(u(t, x),x) +u(t,x)=0;
ic:=u(0,x)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x)=\frac{e^{-t}}{(4t+x)^2+1}$$

Hand solution

$$u_t-4u_x+u=0 \tag{1}$$
$$u(x,0)=\frac{1}{1+x^2}$$

Solution

Let $u=u(x(t),t)$. Then

$$\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt}=-u \tag{3}$$
$$\frac{dx}{dt}=-4 \tag{4}$$

Solving (3) gives

$$\frac{du}{u}=-dt$$
$$\ln|u|=-t+c$$
$$u=u(x(0))\,e^{-t}$$

Using the given initial conditions, this becomes

$$u=\frac{1}{1+x(0)^2}e^{-t} \tag{5}$$

Now we just need to find $x(0)$. From (4)

$$x=-4t+x(0)$$
$$x(0)=x+4t$$

Substituting the above into (5) gives

$$u = \frac{e^{-t}}{1 + (x + 4t)^2}$$

The above is the method I prefer to solve these problems. Here are some alternative ways

Alternative approach

Let ξ be the characteristic variable defined such that $\xi = x - ct$. Where characteristic lines are given by $x = x_0 + ct$. But $c = -4$ in this problem. Hence characteristic lines are

$$x = x_0 - 4t$$

And

$$\xi = x + 4t$$

Then $u_t - 4u_x$ are transformed to $v(t,\xi)$ as was done in part (a) (will not be repeated) which results in

$$u_t - 4u_x = \frac{\partial v}{\partial t}$$

Substituting the above into (1) gives (where now v is used in place of u).

$$\frac{\partial v}{\partial t} + v = 0$$

This is now first order ODE since it only depends on t. Therefore $v' + v = 0$. This is linear in v. Hence the solution is $\frac{d}{dt}\left(ve^{\int dt}\right) = 0$ or $ve^t = F(\xi)$ where F is arbitrary function of ξ. Hence

$$v(t,\xi) = e^{-t}F(\xi)$$

Converting to $u(t,x)$ gives

$$u(t,x) = e^{-t}F(x + 4t) \tag{2}$$

At $u(0,x) = \frac{1}{1+x^2}$ the above becomes

$$\frac{1}{1 + x_0^2} = F(x_0)$$

From the above then (2) can be written as

$$u(t,x) = \frac{e^{-t}}{1 + (x + 4t)^2}$$

An alternative approach to solve transport PDE is by using Lagrange-charpit method

$$\frac{dt}{1} = -\frac{dx}{4} = \frac{-du}{u}$$

Integrating $\frac{dt}{1} = \frac{-dx}{4}$ gives $t = -\frac{1}{4}x + C_2$ or

$$C_2 = t + \frac{1}{4}x$$

Now either $\frac{dt}{1} = \frac{-du}{u}$ or $\frac{-dx}{4} = \frac{-du}{u}$ can be integrated. The choice is not important. Integrating $\frac{dt}{1} = \frac{-du}{u}$ gives $t = -\ln u + C_1$ or

$$C_1 = t + \ln u$$

But $C_1 = F(C_2)$ always, where F is arbitrary function therefore

$$\begin{aligned}
t + \ln u &= F(C_2) \\
t + \ln u &= F\left(t + \frac{1}{4}x\right) \\
\ln u &= F\left(t + \frac{1}{4}x\right) - t \\
u &= e^{-t} e^{F\left(t+\frac{1}{4}x\right)} \qquad (1)
\end{aligned}$$

At $u(0,x) = \frac{1}{1+x^2}$ the above becomes

$$\begin{aligned}
\frac{1}{1+x^2} &= e^{F\left(\frac{1}{4}x\right)} \\
F\left(\frac{x}{4}\right) &= \ln\left(\frac{1}{1+x^2}\right)
\end{aligned}$$

Let $z = \frac{x}{4}$, then $x = 4z$. The above becomes

$$F(z) = \ln\left(\frac{1}{1+(4z)^2}\right)$$

From the above then (1) can be written as

$$\begin{aligned}
u(t,x) &= e^{-t} e^{\ln\left(\frac{1}{1+\left(4\left(t+\frac{1}{4}x\right)\right)^2}\right)} \\
&= \frac{e^{-t}}{1+\left(4\left(t+\frac{1}{4}x\right)\right)^2} \\
&= \frac{e^{-t}}{1+(4t+x)^2}
\end{aligned}$$

The following is an animation of the solution

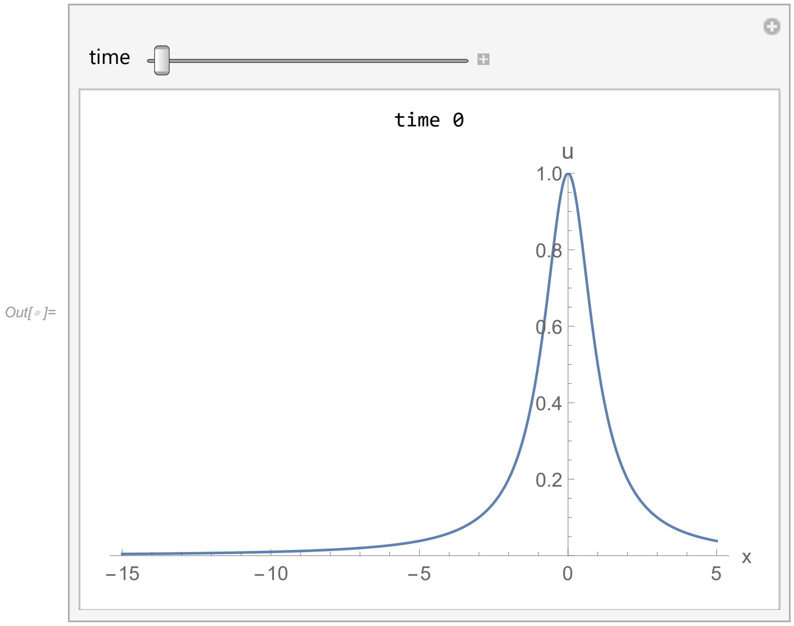


Figure 2.9: snap shot

Source code used for the above

```
In[∘]:= (*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[-t]/(1 + (4 t + x)^2);
plotStyle = Automatic; (*Directive[Orange,Specularity[White,20]];*)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {Quiet@Plot3D[u[x, t], {x, -7, 5}, {t, 0, time},
                  PlotRange → {{-7, 5}, {0, 2}, {0, 2}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 30, Mesh → None,
                  PlotStyle → plotStyle,
                  ViewPoint → {1.996, 2.148, 1.6889}]}}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 2, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.10: Source code 3D

In[ ]:=

```
(*2D*)
u[x_, t_] := Exp[-t]/(1 + (4 t + x)^2);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
         {
            Quiet@Plot[u[x, time], {x, -15, 5},
                   PlotRange → {All, {0, 1}},
                   AxesLabel → {Style["x", 12], Style["u", 14]},
                   BaseStyle → 12,
                   ImageSize → 400
                ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 3.5, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.11: Source code 2D

2.1.6 Transport equation $u_t + 2u_x = \sin x$ IC $u(0,x) = \sin x$. Peter Olver textbook, 2.2.5

problem number 6

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t + 2u_x = \sin x$ with IC $u(0,x) = \sin x$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] +2*D[u[t, x], {x}]== Sin[x];
ic = u[0,x]==Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to \frac{1}{2}(-2\sin(2t-x) + \cos(2t-x) - \cos(x))\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +2*diff(u(t, x),x) =sin(x);
ic:=u(0,x)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = -\frac{\cos(x)}{2} + \frac{\cos(2t-x)}{2} - \sin(2t-x)$$

Hand solution

Solve

$$u_t + 2u_x = \sin x \tag{1}$$

With initial conditions $u(x,0) = \sin x$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = \sin x(t) \tag{3}$$

$$\frac{dx}{dt} = 2 \tag{4}$$

Solving (3) gives

$$\int du = \int \sin x(t)\, dt \tag{3A}$$

From (4)

$$x = 2t + x(0)$$

Substituting the above into (3A) gives

$$\int du = \int \sin(2t + x(0))\, dt$$

$$u = \frac{-\cos(2t + x(0))}{2} + C \tag{3B}$$

At $t = 0$ the above becomes

$$\sin(x(0)) = \frac{-\cos(x(0))}{2} + C$$

$$C = \sin(x(0)) + \frac{\cos(x(0))}{2}$$

Hence (3B) becomes

$$u=\frac{-\cos\left(2t+x(0)\right)}{2}+\sin\left(x(0)\right)+\frac{\cos\left(x(0)\right)}{2}$$

But $x(0)=x-2t$, therefore

$$\begin{aligned}u(x,t)&=\frac{-\cos\left(2t+x-2t\right)}{2}+\sin\left(x-2t\right)+\frac{\cos\left(x-2t\right)}{2}\\&=\frac{-\cos\left(x\right)}{2}+\sin\left(x-2t\right)+\frac{\cos\left(x-2t\right)}{2}\end{aligned}$$

An alternative approach to solve transport PDE is by using Lagrange-charpit method

$$\frac{dt}{1}=\frac{dx}{2}=\frac{du}{\sin x}$$

$\frac{dt}{1}=\frac{dx}{2}$ gives $\frac{dx}{dt}=2$ or $x=2t+C_1$. Hence

$$C_1=x-2t$$

And $\frac{dx}{2}=\frac{du}{\sin x}$ gives $\frac{du}{dx}=\frac{1}{2}\sin x$. Integrating gives $u=\frac{-1}{2}\cos x+C_2$. Therefore

$$C_2=u+\frac{1}{2}\cos x$$

But $C_2=F(C_1)$ where F is arbitrary function. Therefore

$$\begin{aligned}u+\frac{1}{2}\cos x&=F(x-2t)\\u(t,x)&=F(x-2t)-\frac{1}{2}\cos x\end{aligned}\tag{1}$$

When $t=0$, $u(0,x)=\sin x$, therefore the above becomes

$$\begin{aligned}\sin x&=F(x)-\frac{1}{2}\cos x\\F(x)&=\sin x+\frac{1}{2}\cos x\\F(z)&=\sin z+\frac{1}{2}\cos z\end{aligned}$$

Therefore the solution (1) can now be written as

$$\begin{aligned}u(t,x)&=\left(\sin\left(x-2t\right)+\frac{1}{2}\cos\left(x-2t\right)\right)-\frac{1}{2}\cos x\\&=\sin\left(x-2t\right)+\frac{1}{2}\cos\left(x-2t\right)-\frac{1}{2}\cos x\end{aligned}$$

The following is an animation of the solution

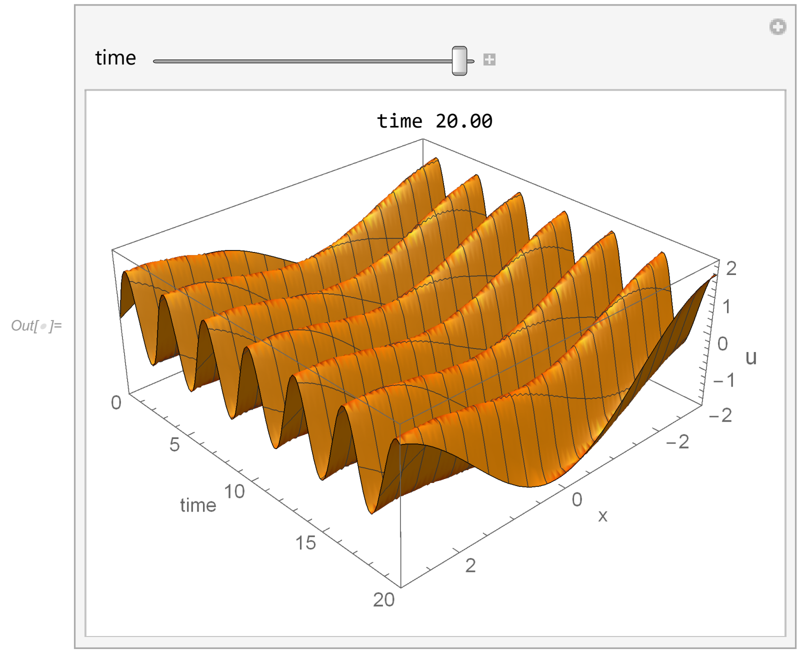


Figure 2.12: snap shot

Source code used for the above

```
In[ ]:= (*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
         NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Sin[x - 2 t] + 1 / 2 Cos[x - 2 t] - 1 / 2 Cos[x];
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
       {
         Quiet@Plot3D[u[x, t], {x, -3, 3}, {t, 0, time},
              PlotRange → {{-3, 3}, {0, 20}, {-2, 2}},
              AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
              BaseStyle → 12, PerformanceGoal → "Quality",
              ImageSize → 400, PlotPoints → 30,
              PlotStyle → plotStyle,
              ViewPoint → {1.996, 2.148, 1.6889}]
       }}
  ]
  ,
  {{time, 0.01, "time"}, 0.01, 20, .1},
  TrackedSymbols :→ {time}
]
```

Figure 2.13: Source code 3D

```
In[ ]:=
(*2D*)
u[x_, t_] := Sin[x - 2 t] + 1 / 2 Cos[x - 2 t] - 1 / 2 Cos[x];
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
       {
         Quiet@Plot[u[x, time], {x, -15, 15},
               PlotRange → {All, {-1.6, 1.6}},
               AxesLabel → {Style["x", 12], Style["u", 14]},
               BaseStyle → 12,
               ImageSize → 400
             ]
       }}
  ]
  ,
  {{time, 0, "time"}, 0, 15, .05},
  TrackedSymbols :→ {time}
]
```

Figure 2.14: Source code 2D

2.1.7 Transport equation $u_t + \frac{1}{1+x^2}u_x = 0$ IC $u(x,0) = \frac{1}{1+(3+x)^2}$. Peter Olver textbook, page 27

problem number 7

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. Example 2.4, page 27.

Solve $u_t + \frac{1}{1+x^2}u_x = 0$ with IC $u(x,0) = \frac{1}{1+(3+x)^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t],t] +1/(1+x^2)*D[u[x,t],x]== 0;
ic = u[x,0]==1/(1+(3+x)^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{}{\sqrt[3]{2}x^3\sqrt[3]{\sqrt{(-3t+x^3+3x)^2+4}-3t+x^3+3x}+3\sqrt[3]{2}x\sqrt[3]{\sqrt{(-3t+x^3+3x)^2+4}-3t+x}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +1/(1+x^2)*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+(3+x)^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{\left(\frac{\left(4x^3-12t+12x+4\sqrt{9\left(-\frac{1}{3}x^3+t-x\right)^2+4}\right)^{\frac{1}{3}}}{2} - \frac{2}{\left(4x^3-12t+12x+4\sqrt{9\left(-\frac{1}{3}x^3+t-x\right)^2+4}\right)^{\frac{1}{3}}}\right)^2 + 3\left(4x^3-12t+12x\right.}$$

<u>Hand solution</u>

Solve

$$u_t + \frac{1}{x^2+1}u_x = 0$$

With initial conditiomns $u(0,x) = \frac{1}{1+(x+3)^2}$

<u>Solution</u>

Let $u = u(x(t),t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = \frac{1}{x^2+1} \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1+(x(0)+3)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\begin{aligned} (x^2+1)\,dx &= dt \\ \frac{x^3}{3} + x &= t + C \end{aligned} \tag{6}$$

At $t = 0$

$$\frac{x(0)^3}{3} + x(0) = C$$

Hence (6) becomes

$$\frac{x^3}{3}+x-t=\frac{x(0)^3}{3}+x(0)$$

This is Cubic in $x(0)$. The solution is complicated and will not be given. All what is left is to substitute this solution back in (5) and this is what the computer did above.

The following is an animation of the solution obtained from CAS. Animation agrees with textbook screen shots.

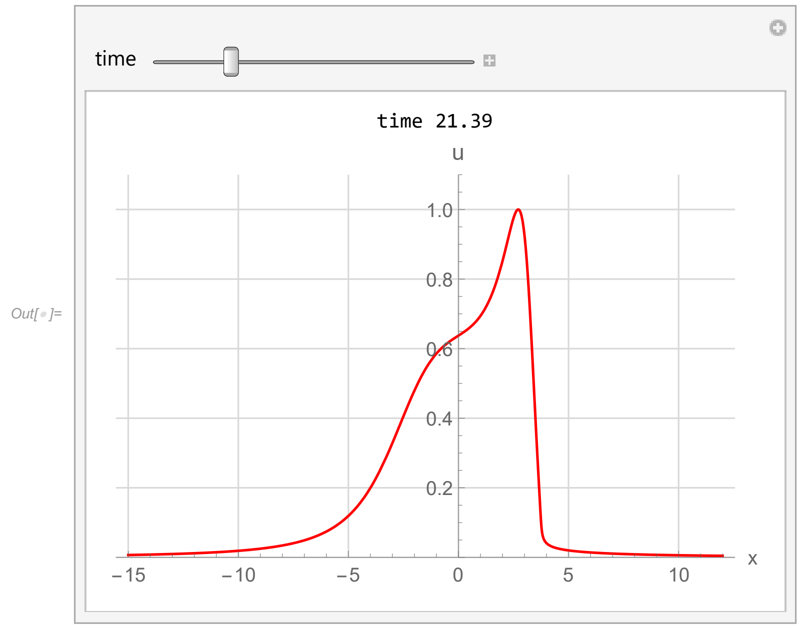


Figure 2.15: snap shot

Source code used for the above

```
In[ ]:=
u[x_, t_] := (2 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(2/3))/
    (2×2^(2/3) - 18×2^(2/3) t + 18×2^(2/3) x + 6×2^(2/3) x^3 + 6×2^(2/3) Sqrt[4 + (3 t - 3 x - x^3)^2] - 12×2^(1/3) (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) - 3×2^(1/3) t (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) +
      3×2^(1/3) x (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) + 2^(1/3) x^3 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) + 2^(1/3) Sqrt[4 + (3 t - 3 x - x^3)^2] (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) +
      16 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(2/3));
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
      {
        Quiet@Plot[u[x, time], {x, -15, 12},
            PlotRange -> {All, {0, 1.1}},
            AxesLabel -> {Style["x", 12], Style["u", 14]},
            BaseStyle -> 12,
            ImageSize -> 400,
            PlotPoints -> 40,
            GridLines -> Automatic,
            GridLinesStyle -> LightGray,
            PlotStyle -> Red
          ]
      }}
  ]
  ,
  {{time, 0, "time"}, 0, 100, .01},
  TrackedSymbols :> {time}
]
```

Figure 2.16: Source code

2.1.8 Transport equation $u_t - xu_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.17

problem number 8

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. problem 2.2.17

Solve $u_t - xu_x = 0$ with IC $u(x,0) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] -x*D[u[x,t], x]== 0;
ic = u[x,0]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{e^{2t}x^2+1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) -x*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{x^2 e^{2t} + 1}$$

Hand solution

Solve the initial value problem

$$u_t - xu_x = 0$$

With initial conditions $u(0,x) = \frac{1}{1+x^2}$

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$
$$\frac{dx}{dt} = -x \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1+x(0)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\begin{aligned} \ln|x| &= -t + C \\ x &= x(0)\,e^{-t} \\ x(0) &= xe^{t} \end{aligned} \tag{6}$$

Substituting (6) in (5) gives

$$u(x(t),t) = \frac{1}{1+x^2e^{2t}}$$

The following is an animation of the solution

Out[∘]=

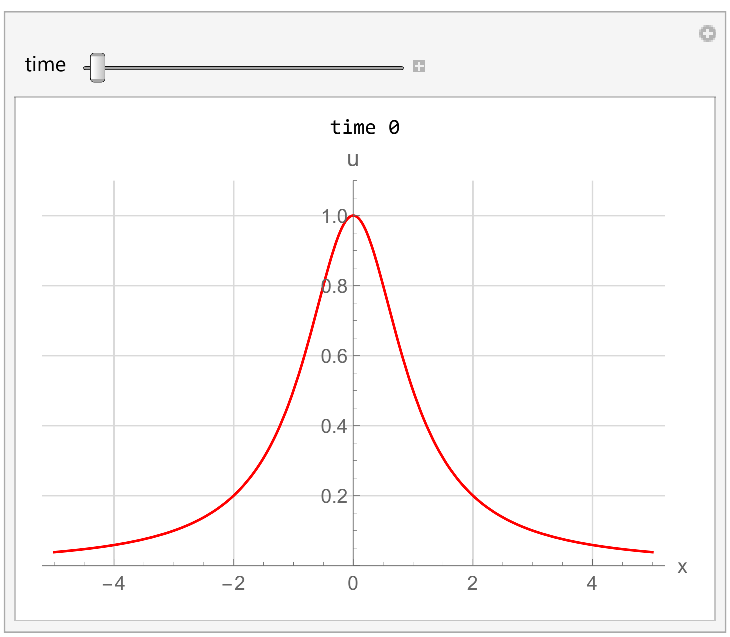


Figure 2.17: snap shot

Source code used for the above

```
In[∘]:=
(*2D*)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot[u[x, time], {x, -5, 5},
                  PlotRange → {All, {0, 1.1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 5, .01},
   TrackedSymbols :→ {time}
]
```

Figure 2.18: Source code 2D

2.1.9 Transport equation $u_t + (1-2t)u_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.29

problem number 9

Added Sept 15, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. problem 2.2.29

Solve $u_t + (1-2t)u_x = 0$ with IC $u(x,0) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] +(1-2*t)*D[u[x,t], x]== 0;
ic = u[x,0]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2t^2x + t^4 - 2t^3 + t^2 - 2tx + x^2 + 1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +(1-2*t)*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{\left(-t^2+t-x\right)^2+1}$$

Hand solution

Solve

$$u_t + (1-2t)\,u_x = 0$$

with initial conditions $u(0,x) = \frac{1}{1+x^2}$.

Solution

Let $u = u(x(t)\,,t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = (1-2t) \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1+x\,(0)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$x = t - t^2 + C$$

At $t = 0$

$$x(0) = C$$

Hence

$$\begin{aligned} x &= t - t^2 + x(0) \\ x(0) &= x - t + t^2 \end{aligned}$$

Substituting this back into (5) gives

$$u(x(t)\,,t) = \frac{1}{1+(x-t+t^2)^2}$$

The following is an animation of the solution

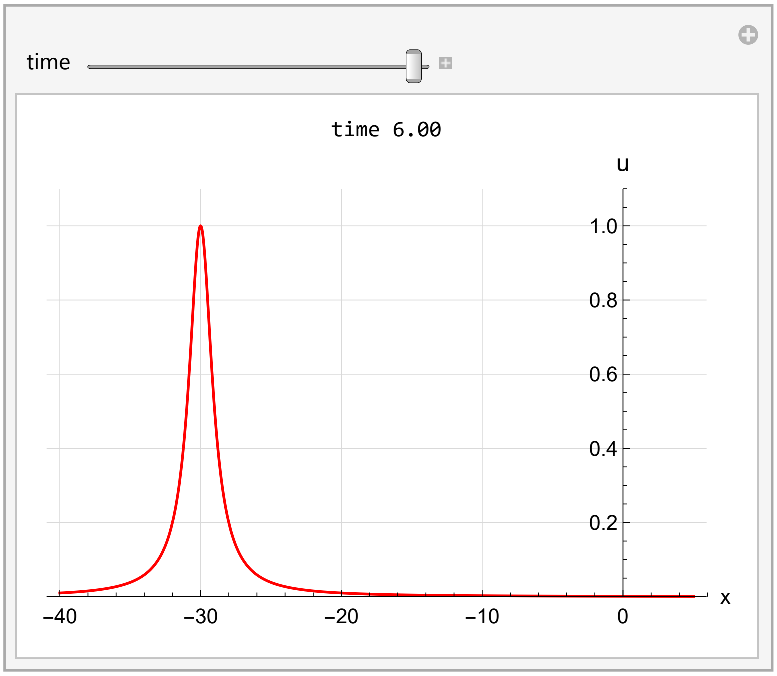


Figure 2.19: snap shot

Source code used for the above

```
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1/(1 + (x - t + t^2)^2)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {3, 2}]}]},
         {
            Quiet@Plot[u[x, time], {x, -40, 5},
                  PlotRange → {All, {0, 1.1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotPoints → 40
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 6, .01},
   TrackedSymbols :→ {time}
]
```

Figure 2.20: Source code 2D

2.1.10 Transport equation $u_t + \frac{1}{x^2+4} u_x = 0$ IC $u(x,0) = e^{x^3+12x}$

problem number 10

Added Oct 8, 2019.

Exam problem. Math 5587, fall 2019. UMN

solve for $u(x,t)$ the PDE $u_t + \frac{1}{x^2+4} u_x = 0$ IC $u(x,0) = e^{x^3+12x}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] +1/(x^2+4)*D[u[x,t], x]== 0;
ic = u[x,0]==Exp[x^3+12*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{-3t+x^3+12x}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +1/(x^2+4)*diff(u(x,t),x) =0;
ic:=u(x,0)=exp(x^3+12*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
sol:=simplify(expand(sol));
```

$$u(x,t) = \mathrm{e}^{x^3-3t+12x}$$

Hand solution

Solve

$$u_t + \frac{1}{x^2+4}u_x = 0$$

with initial conditions $u(x,0) = e^{x^3+12x}$.

Solution

Let $u = u(x(t)\,,t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = \frac{1}{x^2+4} \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= e^{x(0)^3+12x(0)} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\frac{x^3}{3} + 4x = t + C \tag{6}$$

At $t = 0$

$$\frac{x(0)^3}{3} + 4x(0) = C$$

Hence (6) becomes

$$
\begin{aligned}
\frac{x^3}{3} + 4x &= t + \frac{x(0)^3}{3} + 4x(0) \\
x(0)^3 &= 3\left(\frac{x^3}{3} + 4x - t - 4x(0)\right) \\
&= x^3 + 12x - 3t - 12x(0)
\end{aligned}
$$

Substituting this back into (5) gives

$$
\begin{aligned}
u(x(t), t) &= \exp\left(x^3 + 12x - 3t - 12x(0) + 12x(0)\right) \\
&= e^{x^3+12x-3t}
\end{aligned}
$$

The following is an animation of the solution

Out[ ]=

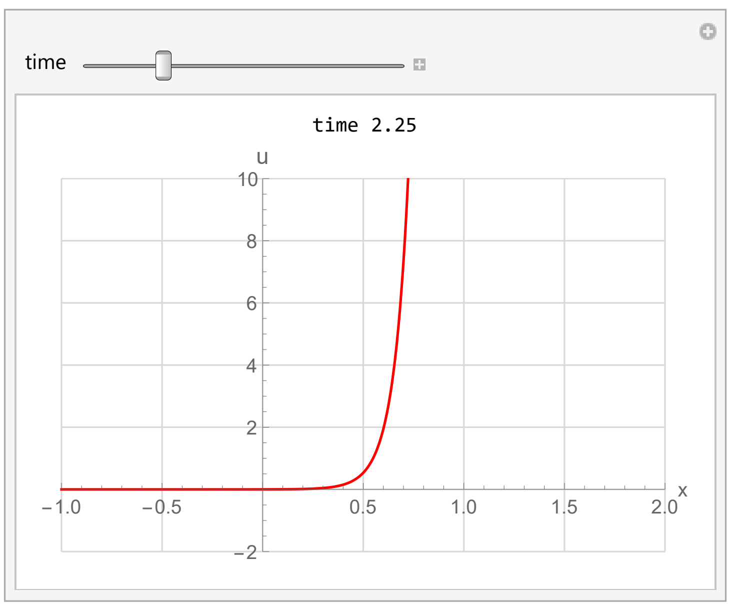


Figure 2.21: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[x^3 + 12 x - 3 t];

Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {3, 2}]}]},
        {
          Quiet@Plot[u[x, time], {x, -1, 2},
                PlotRange → {{-1, 2}, {-2, 10}},
                AxesLabel → {Style["x", 12], Style["u", 14]},
                BaseStyle → 12,
                ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotPoints → 40
              ]
        }}
  ]
  ,
  {{time, 0, "time"}, 0, 10, .01},
  TrackedSymbols :→ {time}
]
```

Figure 2.22: Source code 2D

2.1.11 $3u_x + 5u_y = x$

problem number 11

Taken from Mathematica help pages

Solve for $u(x, y)$

$$3u_x + 5u_y = x$$

Mathematica ✓

```
ClearAll["Global`*"];
sol = AbsoluteTiming[TimeConstrained[DSolve[3*D[u[x, y], x] + 5*D[u[x, y], y] == x, u[x, y],
```

$$\left\{\left\{u(x,y)\to \frac{x^2}{6}+c_1\left(y-\frac{5x}{3}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=3*diff(u(x, y), x) + 5*diff(u(x, y), y) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \frac{x^2}{6} + \_F1\left(-\frac{5x}{3} + y\right)$$

Hand solution

Solve

$$\begin{aligned} 3u_x + 5u_y &= x \\ u_x + \frac{5}{3}u_y &= \frac{x}{3} \qquad (1) \end{aligned}$$

Solution

Let $u = u(y(x), x)$. Then

$$\frac{du}{dx} = \frac{\partial u}{\partial y}\frac{dy}{dx} + \frac{\partial u}{\partial x} \qquad (2)$$

Comparing (1),(2) shows that

$$\frac{du}{dx} = \frac{x}{3} \qquad (3)$$

$$\frac{dy}{dx} = \frac{5}{3} \qquad (4)$$

Solving (3) gives

$$\begin{aligned} u &= \frac{x^2}{6} + C_1 \\ C_1 &= u - \frac{x^2}{6} \end{aligned}$$

From (4)

$$\begin{aligned} y &= \frac{5}{3}x + C_2 \\ C_2 &= y - \frac{5}{3}x \end{aligned}$$

Let $C_1 = F(C_2)$ where F is arbutrary function. This gives

$$\begin{aligned} u - \frac{x^2}{6} &= F\left(y - \frac{5}{3}x\right) \\ u(x,y) &= F\left(y - \frac{5}{3}x\right) + \frac{x^2}{6} \end{aligned}$$

2.1.12 $xu_y + yu_x = -4xyu$ and $u(x,0) = e^{-x^2}$

problem number 12

Taken from Mathematica help pages

Solve for $u(x,y)$

$$xu_y + yu_x = -4xyu$$

with initial value $u(x,0) = e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[u[x, y], y] + y*D[u[x, y], x] == -4*x*y*u[x, y];
ic  = u[x, 0] == Exp[-x^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{-x^2-y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(u(x, y), y) + y*diff(u(x, y), x) = -4*x*y*u(x, y);
ic  := u(x, 0) = exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, y))),outpu
```

$$u(x,y) = \mathrm{e}^{-x^2-y^2}$$

Hand solution

Solve

$$xu_y + yu_x = -4xyu$$

with $u(x,0) = e^{-x^2}$.

Solution

Let $u \equiv u(x(y), y)$. We've taken y as the independent variable for $x(y)$ here, since the initial conditions has $y(0)$ in it. The PDE can be written as

$$u_y + \frac{y}{x}u_x = -4yu \tag{1}$$

Then

$$\frac{du}{dy} = \frac{\partial u}{\partial x}\frac{dx}{dy} + \frac{\partial u}{\partial y} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dy} = -4yu \tag{3}$$

$$\frac{dx}{dy} = \frac{y}{x} \tag{4}$$

Solving (3) gives

$$\ln|u| = -\frac{4y^2}{2} + C_1$$

$$u = C_1 e^{-2y^2} \tag{5}$$

At $y = 0$, using initial conditions the above becomes

$$e^{-x(0)^2} = C_1$$

(5) becomes

$$u = e^{-x(0)^2} e^{-2y^2}$$

$$= e^{-x(0)^2 - 2y^2} \tag{5A}$$

All what is left is to find $x(0)$ to finish the solution. From (4)

$$\frac{x^2}{2} = \frac{y^2}{2} + C_2 \tag{6}$$

At $y = 0$

$$\frac{x(0)^2}{2} = C_2$$

Hence (6) becomes

$$\frac{x^2}{2} = \frac{y^2}{2} + \frac{x(0)^2}{2}$$

$$x(0)^2 = x^2 - y^2$$

Substituting the above in (5A) gives

$$u(x(y), x) = e^{-(x^2 - y^2) - 2y^2}$$

$$= e^{-x^2 - y^2}$$

The following is a plot of the above solution showing the initial conditions are red line

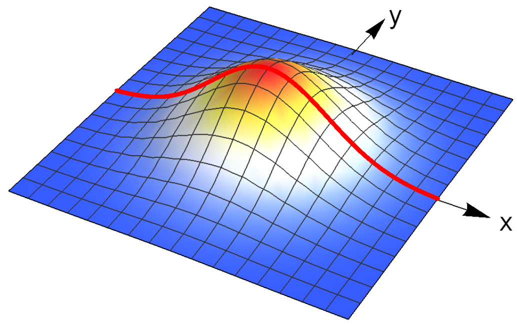


Figure 2.23: Solution $e^{-x^2-y^2}$

```
u[x_, y_] := Exp[-x^2 - y^2];
initialCurve = ParametricPlot3D[{x, 0, Exp[-x^2]}, {x, -2, 2},
      PlotStyle -> Red];
solution = Plot3D[u[x, y], {x, -2, 2}, {y, -2, 2},
      ColorFunction -> "TemperatureMap"];
Graphics3D[{
    First@solution,
    First@initialCurve,
    Arrow[{{0, 0, 0}, {2.6, 0, 0}}],
    Arrow[{{0, 0, 0}, {0, 2.8, 0}}],
    Text["x", {2.7, 0, 0}, {-1, 0}],
    Text["y", {0, 2.9, 0}, {-1, 0}]
  }, SphericalRegion -> True,
  Boxed → False, BaseStyle → 12,
  ImageSize → 300, PlotRange → All]
```

Figure 2.24: Code used for the plot

2.1.13 $u_t + u_x = 0$ and $u(x,0) = \sin x$ and $u(0,t) = 0$

problem number 13

Taken from Mathematica help pages

Solve for $u(x,t)$

$$u_t + u_x = 0$$

with initial value $u(x,0) = \sin x$ and boundary value $u(0,t) = 0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], x] == 0;
bc  = u[0, t] == 0;
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to (\theta(t-x)-1)\sin(t-x)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)=0;
bc  := u(0,t)=0;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = -\sin(t-x)\,\theta(-t+x)$$

Hand solution

Since initial and boundary conditions are given, the Laplace transform method will be used to solve this PDE. Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Applying Laplace transform to the PDE gives

$$sU - u(x,0) + \frac{dU}{dx} = 0$$
$$\frac{dU}{dx} + sU = \sin x$$

Integrating factor is $\mu = e^{\int s dx} = e^{sx}$. Multiplying the above by μ gives

$$\frac{d}{dx}(Ue^{sx}) = e^{sx}\sin x$$

Integrating

$$\begin{aligned} Ue^{sx} &= \int e^{sx}\sin x dx + C \\ &= \frac{e^{sx}(s\sin x - \cos x)}{1+s^2} + C \\ U(x,s) &= \frac{s\sin x - \cos x}{1+s^2} + Ce^{-sx} \end{aligned}$$

Applying boundary conditions $U(0,s)=0$ gives

$$0 = \frac{-1}{1+s^2} + C$$
$$C = \frac{1}{1+s^2}$$

Hence

$$\begin{aligned} U(x,s) &= \frac{s\sin x - \cos x}{1+s^2} + \frac{e^{-sx}}{1+s^2} \\ &= \frac{s\sin x}{1+s^2} - \frac{\cos x}{1+s^2} + \frac{e^{-sx}}{1+s^2} \end{aligned}$$

Applying inverse Laplace transform gives

$$\begin{aligned} u(x,t) &= \cos t \sin x - \cos x \sin t + \text{Heaviside}\,(t-x)\sin\,(t-x) \\ &= -\sin\,(t-x) + \text{Heaviside}\,(t-x)\sin\,(t-x) \\ &= (\text{Heaviside}\,(t-x) - 1)\sin\,(t-x) \end{aligned}$$

2.1.14 $u_t + cu_x = 0$ and $u(x,0) = e^{-x^2}$

problem number 14

Taken from Mathematica help pages

Solve for $u(x,t)$

$$u_t + cu_x = 0$$

With initial conditions $u(x,0) = e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
ic  = u[x, 0] == Exp[-x^2];
pde =  D[u[x, t], {t}] + c*D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{-(x-ct)^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) + c* diff(u(x, t),x) =0;
ic  := u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \mathrm{e}^{-(tc-x)^2}$$

Hand solution

Solve

$$u_t + cu_x = 0 \tag{1}$$

with initial conditions $u(x,0) = e^{-x^2}$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = c \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= e^{-x(0)^2} \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= ct + x(0) \\ x(0) &= x - ct \end{aligned}$$

Then (5) becomes

$$u(x(t), t) = e^{-(x-ct)^2}$$

2.1.15 (Haberman 12.2.2) $\omega_t - 3\omega_x = 0$ and $\omega(x,0) = \cos x$

problem number 15

Added Nov 25, 2018.

Problem 12.2.2 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t - 3\omega_x = 0$$

With initial conditions $\omega(x,0) = \cos x$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] - 3*D[w[x, t], x] == 0;
ic  = w[x, 0] == Cos[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x, t], {x, t}], 60*10]];
```

$$\{\{w(x,t) \to \cos(3t + x)\}\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)-3*diff(w(x,t),x)=0;
ic:=w(x,0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t) = \cos(3t + x)$$

Hand solution

Solve

$$w_t - 3w_x = 0 \tag{1}$$

With I.C. $w(x,0) = \cos x$

Solution

Let $w = w(x(t), t)$. Then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{dw}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = -3 \tag{4}$$

Solving (3) gives

$$\begin{aligned} w &= w(x(0)) \\ &= \cos(x(0)) \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= -3t + x(0) \\ x(0) &= x + 3t \end{aligned}$$

Hence (5) becomes

$$w(x(t)\,,t) = \cos(x + 3t)$$

2.1.16 (Haberman 12.2.4) $\omega_t + c\omega_x = 0$ and $\omega(x,0) = f(x)$ and $\omega(0,t) = h(t)$

problem number 16

Added Nov 25, 2018.

Problem 12.2.4 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + c\omega_x = 0$$

With $c > 0$. For $x > 0, t > 0$ if $\omega(x,0) = f(x)$ and $\omega(0,t) = h(t)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] + c*D[w[x, t], x] == 0;
ic  = w[x, 0] == f[x];
bc  = w[0, t] == h[t];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, w[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,t),t)+c*diff(w(x,t),x)=0;
ic:=w(x,0)=f(x);
bc:=w(0,t)=h(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],w(x,t)) assumi
```

$$w(x,t)=\frac{ch\left(t-\frac{x}{c}\right)\theta\left(t-\frac{x}{c}\right)+\mathcal{L}^{-1}\left(\left(\int \mathrm{e}^{\frac{sx}{c}} f(x)\,dx\right)\mathrm{e}^{-\frac{sx}{c}},s,t\right)-\mathcal{L}^{-1}\left(\left(\int^{0}\mathrm{e}^{\frac{-as}{c}} f(\_a)\,d\_a\right)\mathrm{e}^{-\frac{sx}{c}},s,t\right)}{c}$$

Solution contains unresolved invlaplace calls

Hand solution

$$\frac{\partial w}{\partial t}+c\frac{\partial w}{\partial x}=0 \tag{1}$$

Let

$$w\equiv w(x(t)\,,t)$$

Hence

$$\frac{dw}{dt}=\frac{\partial w}{\partial t}+\frac{\partial w}{\partial x}\frac{dx}{dt} \tag{2}$$

Comparing given (1) and (2), we see that if we let $\frac{dx}{dt}=c$ in (2), then we obtain (1). Hence we conclude that $\frac{dw}{dt}=0$. Therefore, $w(x(t)\,,t)$ is constant. At $t=0$, we are given that

$$w(x(t)\,,t)=f(x(0)) \qquad t=0 \tag{3}$$

We just now need to determine $x(0)$. This is found from $\frac{dx}{dt}=c$, which has the solution $x(t)=x(0)+ct$. Hence $x(0)=x(t)-ct$. Therefore (3) becomes

$$w(x,t)=f(x-ct)$$

This is valid for $x>ct$. We now start all over again, and look at Let

$$w\equiv w(x,t(x))$$

Hence

$$\frac{dw}{dx}=\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}\frac{dt}{dx} \tag{4}$$

Comparing (4) and (1), we see that if we let $\frac{dt}{dx}=\frac{1}{c}$ in (4), then we obtain (1). Hence we conclude that $\frac{dw}{dx}=0$. Therefore, $w(x,t(x))$ is constant. At $x=0$, we are given that

$$w(x,t(x))=h(t(0)) \qquad x=0 \tag{5}$$

We just now need to determine $t(0)$. This is found from $\frac{dt}{dx} = \frac{1}{c}$, which has the solution $t(x) = t(0) + \frac{1}{c}x$. Hence $t(0) = t(x) - \frac{1}{c}x$. Therefore (5) becomes

$$w(x,t) = h\left(t - \frac{1}{c}x\right)$$

Valid for $t > \frac{x}{c}$ or $x < ct$. Therefore, the solution is

$$w(x,t) = \begin{cases} f(x - ct) & x > ct \\ h\left(t - \frac{1}{c}x\right) & x < ct \end{cases}$$

2.1.17 (Haberman 12.2.5 (a)) $\omega_t + c\omega_x = e^{2x}$ and $\omega(x,0) = f(x)$

problem number 17

Added Nov 25, 2018.

Problem 12.2.5 (a) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + c\omega_x = e^{2x}$$

With $\omega(x,0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] + c*D[w[x, t], x] == Exp[2*x];
ic  = w[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}, Assumption
```

$$\left\{\left\{w(x,t) \to f(x-ct) + \frac{e^{2x}(1 - e^{-2ct})}{2c}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)+c*diff(w(x,t),x)=exp(2*x);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t)) assuming
```

$$w(x,t) = \frac{2cf(-tc+x) + e^{2x} - e^{-2tc+2x}}{2c}$$

<u>Hand solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = c \tag{2}$$
$$\frac{du}{ds} = e^{2x} \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = t_1, x(0) = t_2, u(0) = t_3$$

And $u(x,0) = f(x)$ becomes

$$t_3 = f(t_2), t_1 = 0 \tag{4}$$

Equation (1) gives

$$\begin{aligned} t &= s + t_1 \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives

$$x = cs + t_2 \tag{6}$$

From (5,6) solving for t_2 gives

$$\begin{aligned} t_2 &= x - cs \\ &= x - ct \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} du &= e^{2x} ds \\ &= e^{2(cs+t_2)} ds \end{aligned}$$

Integrating

$$u = \frac{e^{2(cs+t_2)}}{2c} + t_3$$

Using (7,4,5) in the above gives the solution

$$\begin{aligned} u(x,t) &= \frac{e^{2(ct+(x-ct))}}{2c} + f(x-ct) \\ &= \frac{1}{2c}e^{2x} + f(x-ct) \end{aligned}$$

My solution is not the same as CAS, but it was verified OK using Maple pdetest.

2.1.18 (Haberman 12.2.5 (d)) $\omega_t + 3t\omega_x = \omega(x,t)$ and $\omega(x,0) = f(x)$

problem number 18

Added Nov 25, 2018.

Problem 12.2.5 (d) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + 3t\omega_x = \omega(x,t)$$

with $\omega(x,0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] + 3*t*D[w[x, t], x] == w[x, t];
ic  = w[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}]], 60*10]];
```

$$\left\{\left\{w(x,t) \to e^t f\left(x - \frac{3t^2}{2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)+3*t*diff(w(x,t),x)=w(x,t);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t) = e^t f\left(-\frac{3t^2}{2} + x\right)$$

Hand solution

Solve

$$\frac{\partial w}{\partial t} + 3t\frac{\partial w}{\partial x} = w(x,t) \tag{1}$$

With initial conditions $w(x,0) = f(x)$

Solution

Let $w \equiv w(x(t), t)$ then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial t} \tag{2}$$

Comparing (1,2) shows that

$$\frac{dw}{dt} = w \tag{3}$$

$$\frac{dx}{dt} = 3t \tag{4}$$

Solving (3) gives

$$w = Ce^t$$

From initial conditions at $t = 0$, the above becomes $f(x(0)) = C$. Hence the above becomes

$$w(x,t) = f(x(0))\, e^t \tag{5}$$

From (4)

$$
\begin{aligned}
x &= \frac{3}{2}t^2 + x(0) \\
x(0) &= x - \frac{3}{2}t^2
\end{aligned}
$$

Substituting the above in (5) gives

$$
w(x(t),t) = f\left(x - \frac{3}{2}t^2\right)e^t
$$

<u>Alternative solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$
\begin{aligned}
\frac{dt}{ds} &= 1 \qquad (1) \\
\frac{dx}{ds} &= 3t \qquad (2) \\
\frac{dw}{ds} &= w \qquad (3)
\end{aligned}
$$

With initial conditions at $s = 0$

$$
t(0) = t_1, x(0) = t_2, w(0) = t_3
$$

And $w(x,0) = f(x)$ becomes

$$
t_3 = f(t_2), t_1 = 0 \qquad (4)
$$

Equation (1) gives

$$
\begin{aligned}
t &= s + t_1 \\
&= s \qquad (5)
\end{aligned}
$$

Equation (2) gives, after replacing t by s from (5)

$$
\begin{aligned}
\frac{dx}{ds} &= 3s \\
x &= \frac{3}{2}s^2 + t_2 \qquad (6)
\end{aligned}
$$

Solving for t_2 gives

$$
t_2 = x - \frac{3}{2}s^2 \qquad (7)
$$

Equation (3) gives

$$\begin{aligned}\ln w &= s + t_3\\ w &= t_3 e^s\\ &= f(t_2)\, e^s\end{aligned}$$

Using (7,5) in the above gives the solution

$$w(x,t) = f\left(x - \frac{3}{2}t^2\right) e^t$$

2.1.19 $2u_x + 5u_y = u^2(x,y) + 1$

problem number 19

Taken from Mathematica help pages

General solution for a quasilinear first-order PDE

Solve for $u(x,y)$

$$2u_x + 5u_y = u^2(x,y) + 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*D[u[x, y], x] + 5*D[u[x, y], y] == u[x, y]^2 + 1;
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \tan\left(\frac{x}{2} + c_1\left(y - \frac{5x}{2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := 2* diff(u(x, y), x) + 5*diff(u(x, y), y) = u(x, y)^2 + 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, y))),output='rea
```

$$u(x,y) = \tan\left(\frac{x}{2} + \frac{\_F1\left(-\frac{5x}{2} + y\right)}{2}\right)$$

Hand solution

Solve for $u(x,y)$ in $2u_x + 5u_y = u^2 + 1$. Using the Lagrange-charpit method, the characteristic equations are

$$\frac{dx}{2} = \frac{dy}{5} = \frac{du}{u^2+1}$$

From the first pair of equation we obtain

$$\begin{aligned} 5dx &= 2dy \\ 5x &= 2y + C_1 \\ C_1 &= 5x - 2y \end{aligned}$$

Now we can pick the pair $\frac{dy}{5} = \frac{du}{u^2+1}$ or $\frac{dx}{2} = \frac{du}{u^2+1}$ to solve for u. It does not matter which. Using

$$\frac{dx}{2} = \frac{du}{u^2+1}$$

Integrating gives

$$\begin{aligned} \frac{1}{2}x &= \arctan(u) + C_2 \\ C_2 &= \frac{1}{2}x - \arctan(u) \end{aligned}$$

C_1 and C_2 are always related by $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$\begin{aligned} \frac{1}{2}x - \arctan(u) &= F(5x - 2y) \\ \arctan(u) &= \frac{1}{2}x - F(5x - 2y) \\ u(x,y) &= \tan\left(\frac{1}{2}x - F(5x - 2y)\right) \end{aligned}$$

2.1.20 Clairaut equation $xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$

problem number 20

Taken from Mathematica Symbolic PDE document

nonlinear first-order PDE, the Clairaut equation

Solve for $u(x,y)$

$$xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1x+c_2y+\frac{1}{2}\left(c_1{}^2+c_2{}^2\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y)=-\frac{x^2}{2}-\frac{y^2}{2}-\_c_1\ln\left(x+\sqrt{x^2+2\_c_1}\right)+\_c_1\ln\left(y+\sqrt{y^2-2\_c_1}\right)+c_1+c_2-\frac{\sqrt{x^2+2\_c_1}\,x}{2}-\frac{\sqrt{y^2}}{}$$

Hand solution

Assuming the solution is $u(x,y)=X(x)+Y(y)$. Substituting this into the PDE gives

$$\begin{aligned}xX'+yY'+\frac{1}{2}\left((X')^2+(Y')^2\right)&=0\\ \frac{1}{2}(X')^2+xX'&=-\frac{1}{2}(Y')^2-yY'\end{aligned}$$

The above is possible when each side is equal to same constant, say C_1. This gives two ODE's

$$\frac{1}{2}(X')^2+xX'=C_1 \tag{1}$$

$$\frac{1}{2}(Y')^2+yY'=-C_1 \tag{2}$$

ODE (1) becomes

$$\begin{aligned}(X')^2+2xX'-2C_1&=0\\ X'&=\frac{-b}{2a}\pm\frac{1}{2a}\sqrt{b^2-4ac}\\ &=\frac{-2x}{2}\pm\frac{1}{2}\sqrt{4x^2+8C_1}\\ &=-x\pm\sqrt{x^2+2C_1}\end{aligned}$$

For the case $X' = -x + \sqrt{x^2 + 2C_1}$, the solution is

$$
\begin{aligned}
X(x) &= \int -x + \sqrt{x^2 + 2C_1} dx + C_2 \\
&= -\frac{x^2}{2} + \frac{x\sqrt{x^2 + 2C_1}}{2} + C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2
\end{aligned}
$$

For the case $X' = -x - \sqrt{x^2 + 2C_1}$, the solution is

$$
\begin{aligned}
X(x) &= \int -x - \sqrt{x^2 + 2C_1} dx + C_2 \\
&= -\frac{x^2}{2} - \frac{x\sqrt{x^2 + 2C_1}}{2} - C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2
\end{aligned}
$$

Combining the above two solutions to one gives

$$
X(x) = -\frac{x^2}{2} \pm \frac{x\sqrt{x^2 + 2C_1}}{2} \pm C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2 \tag{3}
$$

ODE (2) becomes

$$
\begin{aligned}
(Y')^2 + 2yY' + 2C_1 = 0 \\
Y' &= \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac} \\
&= \frac{-2y}{2} \pm \frac{1}{2}\sqrt{4y^2 - 8C_1} \\
&= -y \pm \sqrt{y^2 - 2C_1}
\end{aligned}
$$

For the case $Y' = -y + \sqrt{x^2 - 2C_1}$, the solution is

$$
\begin{aligned}
Y(y) &= \int -y + \sqrt{y^2 - 2C_1} dy + C_2 \\
&= \frac{-y^2}{2} + \frac{y\sqrt{y^2 - 2C_1}}{2} - C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3
\end{aligned}
$$

For the case $Y' = -y - \sqrt{y^2 - 2C_1}$, the solution is

$$
\begin{aligned}
Y(y) &= \int -y - \sqrt{y^2 + 2C_1} dy + C_2 \\
&= -\frac{y^2}{2} - \frac{y\sqrt{x^2 - 2C_1}}{2} + C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3
\end{aligned}
$$

Combining the above two solutions to one gives

$$
Y(x) = -\frac{y^2}{2} \pm \frac{y\sqrt{y^2 - 2C_1}}{2} \pm C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3 \tag{4}
$$

From (3,4) the final solution is

$$
\begin{aligned}
u(x,y) &= X(x) + Y(x) \\
&= \left(-\frac{x^2}{2} \pm \frac{x\sqrt{x^2+2C_1}}{2} \pm C_1 \ln\left(x+\sqrt{x^2+2C_1}\right) + C_2\right) + \left(-\frac{y^2}{2} \pm \frac{y\sqrt{y^2-2C_1}}{2} \pm C_1 \ln\left(y+\right.\right. \\
&= -\frac{x^2}{2} \pm \frac{x}{2}\sqrt{x^2+2C_1} \pm C_1 \ln\left(x+\sqrt{x^2+2C_1}\right) - \frac{y^2}{2} \pm \frac{y}{2}\sqrt{y^2-2C_1} \pm C_1 \ln\left(y+\sqrt{y^2-2C_1}\right)
\end{aligned}
$$

Where $C_4 = C_2 + C_3$.

2.1.21 Clairaut equation. $xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$ with $u(x,0) = \frac{1}{2}(1-x^2)$

problem number 21

Taken from Mathematica Symbolic PDE document

Clairaut equation with initial value

Solve for $u(x,y)$

$$xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$$

With $u(x,0) = \frac{1}{2}(1-x^2)$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x,
ic  = u[x, 0] == (1*(1 - x^2))/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -\frac{x^2}{2} + y + \frac{1}{2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
ic  := u(x,0)=1/2*(1-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\frac{(x-y+1)(x-y-1)}{2}$$
$$u(x,y) = -\frac{(x+y+1)(x+y-1)}{2}$$

2.1.22 Clairaut equation. $u = xu_x + yu_y + \sin(u_x + u_y)$

problem number 22

Taken from Mathematica DSolve help pages

Another example of nonlinear Clairaut equation

Solve for $u(x,y)$

$$u = xu_x + yu_y + \sin(u_x + u_y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = u[x, y] == x*D[u[x, y], x] + y*D[u[x, y], y] + Sin[D[u[x, y], x] + D[u[x, y], y]];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1 x + c_2 y + \sin(c_1 + c_2)\}\}$$

Maple ✓

```
restart;
pde := u(x,y)= x*diff(u(x,y),x)+y*diff(u(x,y),y)+sin(diff(u(x,y),x)+diff(u(x,y),y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = x\_c_1 + y\_c_2 + \sin(\_c_1 + \_c_2)$$

2.1.23 Recover a function from its gradient vector

problem number 23

Taken from Mathematica DSolve help pages

Solve for $f(x,y)$

$$\frac{\partial f}{\partial x} = xy\cos(xy) + \sin(xy)$$
$$\frac{\partial f}{\partial y} = -e^{-y} + x^2\cos(xy)$$

Mathematica ✓

```
ClearAll["Global`*"];
eq1 = D[f[x, y], x] == x*y*Cos[x*y] + Sin[x*y];
 eq2 = D[f[x, y], y] == -E^(-y) + x^2*Cos[x*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{eq1, eq2}, f[x, y], {x, y}], 60*10]];
```

$$\{\{f(x,y) \to x\sin(xy) + e^{-y} + c_1\}\}$$

Maple ✓

```
restart;
eq1:=diff(f(x,y),x)=x*y*cos(x*y)+sin(x*y);
eq2:=diff(f(x,y),y)=-exp(-y)+x^2*cos(x*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({eq1,eq2},f(x,y))),output=
```

$$\{f(x,y) = x\sin(xy) + c_1 + e^{-y}\}$$

2.1.24 $xf_y - f_x = \frac{g(x)}{h(y)} f^2$

problem number 24

Taken from Maple pdsolve help pages

General solution of a first order nonlinear PDE

Solve for $f(x,y)$

$$xf_y - f_x = \frac{g(x)}{h(y)} f^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[f[x, y], y] - D[f[x, y], x] == (f[x, y]^2*g[x])/h[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{f(x,y) \to -\frac{1}{\int_1^x -\frac{g(K[1])}{h\left(\frac{x^2}{2}-\frac{K[1]^2}{2}+y\right)} dK[1] + c_1\left(\frac{x^2}{2}+y\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(f(x,y),y)-diff(f(x,y),x)=f(x,y)^2*g(x)/h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y))),output='realt
```

$$f(x,y) = \frac{1}{\int^x \frac{g(\_a)}{h\left(-\frac{\_a^2}{2}+\frac{x^2}{2}+y\right)} d\_a + \_F1\left(\frac{x^2}{2}+y\right)}$$

<u>Hand solution</u>

Solve for $f(x,y)$ in $xf_y - f_x = \frac{g(x)}{h(y)} f^2$. Using the Lagrange-charpit method, the characteristic equations are

$$\frac{dy}{x} = \frac{-dx}{1} = \frac{df}{\frac{g(x)}{h(y)} f^2}$$

From the first pair of equation we obtain

$$\begin{aligned} dy &= -xdx \\ y &= -\frac{x^2}{2} + C_1 \\ C_1 &= y + \frac{x^2}{2} \end{aligned} \tag{1}$$

Using $-dx = \frac{df}{\frac{g(x)}{h(y)}f^2}$ as choice of the second pair of equations. Hence $\frac{df}{f^2} = -\frac{g(x)}{h(y)}dx$. But from (1) $y = C_1 - \frac{x^2}{2}$, therefore

$$\frac{df}{f^2} = -\frac{g(x)}{h\left(C_1 - \frac{x^2}{2}\right)}dx$$

Integrating gives

$$\begin{aligned} -\frac{1}{f} &= -\int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds + C_2 \\ C_2 &= -\frac{1}{f} + \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds \end{aligned}$$

But $C_2 = F(C_1)$ where F is arbitrary function. Therefore

$$\begin{aligned} -\frac{1}{f} + \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds = F\left(y + \frac{x^2}{2}\right) \\ -\frac{1}{f} &= F\left(\frac{1}{2}(2y + x^2)\right) - \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds \\ \frac{1}{f} &= \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds - F\left(\frac{1}{2}(2y + x^2)\right) \\ f &= \frac{1}{\int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)}ds - F\left(\frac{1}{2}\left(2y + x^2\right)\right)} \end{aligned}$$

But $C_1 = \frac{1}{2}(2y + x^2)$ and the above becomes

$$f(x,y) = \frac{1}{\int_0^x \frac{g(s)}{h\left(\frac{1}{2}(2y+x^2) - \frac{s^2}{2}\right)}ds - F\left(\frac{1}{2}\left(2y + x^2\right)\right)}$$

2.1.25 $f_x + (f_y)^2 = f(x,y,z) + z$

problem number 25

Taken from Maple pdsolve help pages, probem 5

Nonlinear first order PDE

Solve for $f(x,y,z)$

$$f_x + (f_y)^2 = f(x,y,z) + z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[f[x, y, z], x] + D[f[x, y, z], y]^2 == f[x, y, z] + z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y, z], {x, y, z}], 60*10]];
```

$$\left\{\left\{f(x,y,z) \to \frac{1}{4}\left((c_1(z))^2 \text{ProductLog}\left(-\frac{\exp\left(\frac{(x-1)c_1(z)+c_2(z)+y}{c_1(z)}\right)}{c_1(z)}\right)^2 + 2(c_1(z))^2 \text{ProductLog}\left(-\frac{\exp}{\quad}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(f(x,y,z),x) + (diff(f(x,y,z),y))^2 = f(x,y,z)+z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y,z),'build')),out
```

$$f(x,y,z) = \frac{\left(-c_5^2 z\, e^{-x} - c_3^2 e^x - c_5(c_3 y + c_4 z + c_1)\right) e^x}{c_5^2}$$

2.1.26 $xu_x + yu_y = u$ (Example 3.5.1 in Lokenath Debnath)

problem number 26

Added June 2, 2019.

From example 3.5.1, page 210 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = u$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[u[x, y], x] + y*D[u[x, y], y] ==u[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to xc_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=x\_F1\left(\frac{y}{x}\right)$$

Hand solution

Solve

$$xu_x+yu_y=u$$

Using the Lagrange-charpit method

$$\frac{dx}{x}=\frac{dy}{y}=\frac{du}{u}$$

The first pair of equations gives

$$x=C_1y$$

And $\frac{dx}{x}=\frac{du}{u}$ gives

$$x=C_2u$$

Since $C_2=G(C_1)$ then $\frac{x}{u}=G\left(\frac{x}{y}\right)$ or $u=xG^{-1}\left(\frac{x}{y}\right)$. Let $G^{-1}=F$. Then the solution

$$u(x,y)=xF\left(\frac{x}{y}\right)$$

2.1.27 $xu_x + yu_y = nu$ Example 3.5.2 in Lokenath Debnath

problem number 27

Added June 2, 2019.

From example 3.5.2, page 211 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = nu$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x, y], y] ==n*u[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to x^n c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=n*u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = x^n\_F1\left(\frac{y}{x}\right)$$

<u>Hand solution</u>

Solve

$$xu_x + yu_y = nu$$

Using the Lagrange-charpit method

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{nu}$$

The first pair of equations gives

$$x = C_1 y$$

And $\frac{dx}{x}=\frac{du}{nu}$ gives

$$
\begin{aligned}
\ln x &= \frac{1}{n}\ln u + C_2 \\
x &= C_2 u^{\frac{1}{n}} \\
x^n &= C_3 u
\end{aligned}
$$

Since $C_3 = G(C_1)$ then $\frac{x^n}{u} = G\left(\frac{x}{y}\right)$ or $u = x^n G^{-1}\left(\frac{x}{y}\right)$. Let $G^{-1}=F$. Then the solution

$$
u(x,y) = x^n F\left(\frac{x}{y}\right)
$$

2.1.28 $x^2u_x + y^2u_y = (x+y)u$ Example 3.5.3 in Lokenath Debnath

problem number 28

Added June 2, 2019.

From example 3.5.3, page 211 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$
x^2u_x + y^2u_y = (x+y)u
$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[u[x, y], x] + y^2*D[u[x, y], y] ==(x+y)*u[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$
\left\{\left\{u(x,y)\to xyc_1\left(\frac{1}{x}-\frac{1}{y}\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde :=x^2*diff(u(x,y),x)+y^2*diff(u(x,y),y)=(x+y)*u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$
u(x,y) = xy\_F1\left(\frac{x-y}{xy}\right)
$$

Hand solution

Solve

$$x^2 u_x + y^2 u_y = (x+y)\,u$$

Using the Lagrange-charpit method

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{du}{(x+y)\,u}$$

The first pair of equations gives

$$-\frac{1}{x} = -\frac{1}{y} + C_1 \tag{1}$$

And $\frac{dx}{x^2} = \frac{du}{(x+y)u}$ gives

$$\frac{(x+y)}{x^2}dx = \frac{du}{u}$$

But from (1) $\frac{1}{y} = \frac{1}{x} + C_1$ or $\frac{1}{y} = \frac{1+xC_1}{x}$ or $y = \frac{x}{1+xC_1}$ Therefore the above becomes

$$\begin{aligned}
\frac{\left(x + \frac{x}{1+xC_1}\right)}{x^2}dx &= \frac{du}{u} \\
\frac{(x(1+xC_1)+x)}{x^2\,(1+xC_1)}dx &= \frac{du}{u} \\
\frac{2x + x^2 C_1}{x^2 + x^3 C_1}dx &= \frac{du}{u} \\
\frac{2 + xC_1}{x + x^2 C_1}dx &= \frac{du}{u} \\
2\ln x - \ln(1 + C_1 x) &= \ln u + C_2 \\
\ln \frac{x^2}{1 + C_1 x} &= \ln u + C_2 \\
\frac{x^2}{1 + C_1 x} &= C_2 u
\end{aligned}$$

But $C_1 = \frac{1}{y} - \frac{1}{x}$ hence the above becomes

$$\begin{aligned}
\frac{x^2}{1 + \left(\frac{1}{y} - \frac{1}{x}\right) x} &= C_2 u \\
\frac{x^2}{1 + \left(\frac{x-y}{yx}\right) x} &= C_2 u \\
\frac{yx^2}{y + (x-y)} &= C_2 u \\
yx &= C_2 u
\end{aligned}$$

Since $C_2 = G(C_1)$ then $\frac{yx}{u} = G\left(\frac{1}{y} - \frac{1}{x}\right)$ or

$$
\begin{aligned}
u &= yxG^{-1}\left(\frac{1}{y} - \frac{1}{x}\right) \\
&= yxG^{-1}\left(\frac{y-x}{yx}\right)
\end{aligned}
$$

Let $G^{-1} = F$

$$u = yxF\left(\frac{y-x}{yx}\right)$$

Where F is arbitrary function.

2.1.29 $(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$ (Example 3.5.4 in Lokenath Debnath)

problem number 29

From example 3.5.4, page 212 nonlinear pde's by Lokenath Debnath, 3rd edition.

First order PDE of three unknowns. Solve for $u(x,y,z)$

$$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y - z)*D[u[x, y, z], x] + (z - x)*D[u[x, y, z], y] + (x - y)*D[u[x, y, z], z] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, z], {x, y, z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=(y-z)*diff(u(x,y,z),x)+(z-x)*diff(u(x,y,z),y)+(x-y)*diff(u(x,y,z),z)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z),'build')),out
```

$$u(x,y,z) = c_3c_4c_5 e^{c_1x} e^{c_1y} e^{c_1z} e^{\frac{c_2x^2}{2}} e^{\frac{c_2y^2}{2}} e^{\frac{c_2z^2}{2}}$$

Hand solution

Solve

$$(y-z)\,u_x + (z-x)\,u_y + (x-y)\,u_z = 0$$

The following method works only when the sum of the coefficients of u_x, u_y, u_z is zero. This is the case here. Hence we write

$$du = 0 \tag{1}$$
$$dx + dy + dz = (y - z) + (z - x) + (x - y) = 0 \tag{2}$$

We need one more equation which is

$$xdx + ydy + ydz = x(y - z) + y(z - x) + z(x - y) = 0 \tag{3}$$

Integrating (1,2,3) gives

$$\begin{aligned} u &= C_1 \\ x + y + z &= C_2 \\ x^2 + y^2 + z^2 &= C_3 \end{aligned}$$

And since we know that $C_1 = F(C_2, C_3)$ where F is arbitrary function, then the solution is

$$u(x, y, z) = F\left(x + y + z, x^2 + y^2 + z^2\right)$$

Again, the above method only worked because of the special value of the coefficients. If the PDE was $(2y - z)\, u_x + (z - x)\, u_y + (x - y)\, u_z = 0$ for example, then the above method will not work and we have to use the Lagrange-Charpit equations. But since the equations will be coupled, this would make the solution harder.

2.1.30 $u(x + y)u_x + u(x - y)u_y = x^2 + y^2$ (Example 3.5.5 in Lokenath Debnath)

problem number 30

Added June 2, 2019.

From example 3.5.5, page 214 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u(x + y)u_x + u(x - y)u_y = x^2 + y^2$$

with $u = 0$ on $y = 2x$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x,y]*(x+y)*D[u[x, y], x] + u[x,y]*(x-y)*D[u[x, y], y] ==x^2+y^2;
ic  = u[x,2*x]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y)\to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$

Maple ✓

```
restart;
pde :=u(x,y)*(x+y)*diff(u(x,y),x)+u(x,y)*(x-y)*diff(u(x,y),y)=x^2+y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = -\sqrt{-x^2+2xy+y^2}\sqrt{\frac{2xy+\_F1\left(-\frac{1}{\sqrt{-x^2+2xy+y^2}}\right)}{-x^2+2xy+y^2}}$$

Hand solution

Solve

$$u(x+y)\,u_x + u(x-y)\,u_y = x^2+y^2$$

With $u=0$ on $y=2x$. This solution follows the book method, but adds more details and more steps to make it more clear.

The following method works only when we can find two equations when the sum of the coefficients is zero. We see that

$$\begin{aligned} ydx + xdy - udu &= y(u(x+y)) + x(u(x-y)) - u(x^2+y^2) \\ &= yux + y^2u + x^2u - yxu - ux^2 - y^2u \\ &= 0 \end{aligned} \tag{1}$$

But we need one more equation. We see that

$$\begin{aligned} xdx - ydy - udu &= x(u(x+y)) - y(u(x-y)) - u(x^2+y^2) \\ &= ux^2 + uxy - yux + y^2u - ux^2 - y^2u \\ &= 0 \end{aligned} \tag{2}$$

Now we just Integrate (1,2). Rewriting (1) as

$$\begin{aligned} ydx + xdy - udu &= 0 \\ d\left(xy - \frac{1}{2}u^2\right) &= 0 \end{aligned} \tag{1A}$$

And rewriting (2) as

$$\begin{aligned} xdx - ydy - udu &= 0 \\ d\left(\frac{1}{2}(x^2 - y^2 - u^2)\right) &= 0 \end{aligned} \tag{2A}$$

Then integrating (1A,2A) now gives

$$xy - \frac{1}{2}u^2 = C_1 \tag{1C}$$

$$\frac{1}{2}(x^2 - y^2 - u^2) = C_2 \tag{2C}$$

Since $u = 0$ on $y = 2x$, then the above becomes

$$\begin{aligned} x(2x) &= C_1 \\ \frac{1}{2}(x^2 - (2x)^2) &= C_2 \end{aligned}$$

Or

$$\begin{aligned} 2x^2 &= C_1 \\ -\frac{3}{2}x^2 &= C_2 \end{aligned}$$

Or $\frac{C_1}{C_2} = \frac{2x^2}{-\frac{3}{2}x^2} = -\frac{4}{3}$. Hence $-4C_2 = 3C_1$. Using this on (1C,2C) gives

$$
\begin{aligned}
-4\left(\frac{1}{2}(x^2 - y^2 - u^2)\right) &= 3\left(xy - \frac{1}{2}u^2\right) \\
-2(x^2 - y^2 - u^2) &= 3xy - \frac{3}{2}u^2 \\
-2x^2 + 2y^2 + 2u^2 &= 3xy - \frac{3}{2}u^2 \\
u^2\left(2 + \frac{3}{2}\right) &= 3xy + 2x^2 - 2y^2 \\
\frac{7}{2}u^2 &= 3xy + 2x^2 - 2y^2
\end{aligned}
$$

Hence the solution is

$$
u(x, y) = \pm\sqrt{\frac{2}{7}(3xy + 2x^2 - 2y^2)}
$$

2.1.31 $u_x - u_y = 1$ with $u(x, 0) = x^2$ Example 3.5.6 in Lokenath Debnath

problem number 31

Added June 2, 2019.

From example 3.5.6, page 214 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$
u_x - u_y = 1
$$

with $u(x, 0) = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] - D[u[x, y], y] ==1;
ic  = u[x,0]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$
\{\{u(x, y) \to x^2 + 2xy + (y - 1)y\}\}
$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)-diff(u(x,y),y)=1;
ic  := u(x,2)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -y + (x+y-2)^2 + 2$$

2.1.32 $yu_x + xu_y = u$ with $u(x,0) = x^3$ and $u(0,y) = y^3$ Example 3.5.8 in Lokenath Debnath

problem number 32

Added June 2, 2019.

From example 3.5.8, page 216 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_x + xu_y = u$$

with with $u(x,0) = x^3$ and $u(0,y) = y^3$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] +x*D[u[x, y], y] ==u[x,y];
ic  = {u[x,0]==x^3,u[0,y]==y^3};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=y*diff(u(x,y),x)+x*diff(u(x,y),y)=u(x,y);
ic  := u(x,0)=x^3,u(0,y)=y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

sol=()

2.1.33 $xu_x + yu_y = xe^{-u}$ with $u = 0$ on $y = x^2$ Example 3.5.10 in Lokenath Debnath

problem number 33

Added June 2, 2019.

From example 3.5.10, page 218 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = xe^{-u}$$

with $u = 0$ on $y = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y] ==x*Exp[-u[x,y]];
ic  = u[x,x^2]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \log\left(-\frac{y}{x}+x+1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=x*exp(-u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \ln\left(x + \_F1\left(\frac{y}{x}\right)\right)$$

2.1.34 $u_t + uu_x = x$ with $u(x, 0) = f(x)$ Example 3.5.11 in Lokenath Debnath.

problem number 34

Added June 2, 2019.

From example 3.5.11, page 219 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_t + uu_x = x$$

with $u(x,0) = f(x)$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] +u[x,t]*D[u[x, t], x] ==x;
ic  = u[x,0]==f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)+u(x,t)*diff(u(x,t),x)=x;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \text{RootOf}\left((\_Z + f(\_Z))\left(\_Z\mathrm{e}^{2t} - 2x\,\mathrm{e}^{t} + \mathrm{e}^{2t}f(\_Z) + \_Z - f(\_Z)\right)\right)\mathrm{e}^{t} + \mathrm{e}^{t}f\left(\text{RootOf}\left((\_Z + f(\_\right.\right.$$

2.1.35 $u_x = 0$ Problem 3.3(a) Lokenath Debnath

problem number 35

Added June 2, 2019.

Problem 3.3(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(y)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1(y)$$

2.1.36 $au_x + bu_y = 0$ Problem 3.3(b) Lokenath Debnath

problem number 36

Added June 2, 2019.

Problem 3.3(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$au_x + bu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[u[x, y], x] + b*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(u(x,y),x)+b*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{ya-bx}{a}\right)$$

2.1.37 $u_x + yu_y = 0$ Problem 3.3(c) Lokenath Debnath

problem number 37

Added June 2, 2019.

Problem 3.3(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + yu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + y*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1\left(e^{-x}y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+y*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(y\,\mathrm{e}^{-x}\right)$$

2.1.38 $(1 + x^2)u_x + u_y = 0$ Problem 3.3(d) Lokenath Debnath

problem number 38

Added June 2, 2019.

Problem 3.3(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(1 + x^2)u_x + u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (1+x^2)*D[u[x, y], x] + D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(y - \tan^{-1}(x))\}\}$$

Maple ✓

```
restart;
pde :=(1+x^2)*diff(u(x,y),x)+diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1(y - \arctan(x))$$

2.1.39 $2xyu_x + (x^2 + y^2)u_y = 0$ Problem 3.3(e) Lokenath Debnath

problem number 39

Added June 2, 2019.

Problem 3.3(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$2xyu_x + (x^2 + y^2)u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*x*y*D[u[x, y], x] + (x^2+y^2)*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to c_1\left(\frac{y^2}{x} - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=2*x*y*diff(u(x,y),x)+(x^2+y^2)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{-x^2+y^2}{x}\right)$$

2.1.40 $(y+u)u_x + yu_y = x - y$ Problem 3.3(f) Lokenath Debnath

problem number 40

Added June 3, 2019.

Problem 3.3(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(y+u)u_x + yu_y = x - y$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] + y*D[u[x, y], y] ==x-y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x)+y*diff(u(x,y),y)=x-y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \frac{-c_1 y - y + \sqrt{c_1^2 x^2 + y^2 + c_1 (2xy + 2c_2)}}{c_1}$$

2.1.41 $y^2u_x - xyu_y = x(u-2y)$ Problem 3.3(g) Lokenath Debnath

problem number 41

Added June 3, 2019.

Problem 3.3(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$y^2u_x - xyu_y = x(u-2y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[u[x, y], x] - x*y*D[u[x, y], y] ==x*(u[x,y]-2*y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y)\to \boxed{\frac{\sqrt{y^2}c_1\left(\frac{1}{2}\left(x^2+y^2\right)\right)-x^2\sqrt{-y^2}}{\sqrt{-y^4}} \text{ if } x=0\lor y\geq 0}\right\}$$
$$\left\{u(x,y)\to \boxed{\frac{\sqrt{-y^2}x^2+\sqrt{y^2}c_1\left(\frac{1}{2}\left(x^2+y^2\right)\right)}{\sqrt{-y^4}} \text{ if } x=0\lor y\leq 0}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(u(x,y),x)- x*y*diff(u(x,y),y)=x*(u(x,y)-2*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = -\frac{x^2}{y} + \frac{\_F1(x^2+y^2)}{\sqrt{-y^2}}$$

2.1.42 $yu_y - xu_x = 1$ Problem 3.3(h) Lokenath Debnath

problem number 42

Added June 3, 2019.

Problem 3.3(h) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_y - xu_x = 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], y] - x*D[u[x, y], x] ==1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -\log(x) + c_1(xy)\}\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),y)- x*diff(u(x,y),x)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1(xy) - \ln(x)$$

2.1.43 $u_x + 2xy^2u_y = 0$ Problem 3.4 Lokenath Debnath

problem number 43

Added June 3, 2019.

Problem 3.4 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + 2xy^2u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + 2*x*y^2*D[u[x,y],y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1\left(-\frac{x^2y+1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ 2*x*y^2*diff(u(x,y),y)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{x^2y+1}{y}\right)$$

2.1.44 $3u_x + 2u_y = 0$ with $u(x,0) = \sin x$. Problem 3.5(a) Lokenath Debnath

problem number 44

Added June 3, 2019.

Problem 3.5(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$3u_x + 2u_y = 0$$

with $u(x,0) = \sin x$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  3*D[u[x, y], x] + 2*D[u[x,y],y] == 0;
ic  = u[x,0]==Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to\sin\left(x-\frac{3y}{2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=3*diff(u(x,y),x)+ 2*diff(u(x,y),y)= 0;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \sin\left(x - \frac{3y}{2}\right)$$

2.1.45 $yu_x + xu_y = 0$ with $u(0,y) = e^{-y^2}$. Problem 3.5(b) Lokenath Debnath

problem number 45

Added June 3, 2019.

Problem 3.5(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_x + xu_y = 0$$

with $u(0,y) = e^{-y^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] + x*D[u[x,y],y] == 0;
ic  = u[0,y]==Exp[-y^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{x^2-y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)+ x*diff(u(x,y),y)= 0;
ic  := u(0,y)=exp(-y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x, y) = e^{x^2 - y^2}$$

2.1.46 $xu_x + yu_y = 2xy$ with $u = 2$ on $y = x^2$. Problem 3.5(c) Lokenath Debnath

problem number 46

Added June 3, 2019.

Problem 3.5(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = 2xy$$

with with $u = 2$ on $y = x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x,y],y] == 0;
ic  = u[x,x^2]==2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to 2\}\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+ y*diff(u(x,y),y)= 0;
ic  := u(x,x^2)=2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x, y) = \_F1\left(\frac{y}{x}\right)$$

2.1.47 $u_x + x u_y = 0$ with $u(0,y) = \sin y$. Problem 3.5(d) Lokenath Debnath

problem number 47

Added June 3, 2019.

Problem 3.5(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + x u_y = 0$$

with $u(0,y) = \sin y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + x*D[u[x,y],y] == 0;
ic  = u[0,y]==Sin[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to -\sin\left(\frac{1}{2}(x^2-2y)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ x*diff(u(x,y),y)= 0;
ic  := u(0,y)=sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\sin\left(\frac{x^2}{2} - y\right)$$

2.1.48 $yu_x + xu_y = xy$ with $u(0, y) = e^{-y^2}, u(x, 0) = e^{-x^2}$. Problem 3.5(e) Lokenath Debnath

problem number 48

Added June 3, 2019.

Problem 3.5(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$yu_x + xu_y = xy$$

with $u(0, y) = e^{-y^2}, u(x, 0) = e^{-x^2}$ for $x > 0, y > 0$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] + x*D[u[x,y],y] == x*y;
ic  = {u[0,y]==Exp[-y^2],u[x,0]==Exp[-x^2]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y},Assumptions->{x>0,y>0
```

Failed

Maple ✗

```
restart;
pde :=y*diff(u(x,y),x)+ x*diff(u(x,y),y)= x*y;
ic  := u(0,y)=exp(-y^2),u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

sol=()

2.1.49 $u_x + xu_y = (y - \frac{1}{2}x^2)^2$ with $u(0, y) = e^y$. Problem 3.5(f) Lokenath Debnath

problem number 49

Added June 3, 2019.

Problem 3.5(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + xu_y = (y - \frac{x^2}{2})^2$$

with $u(0, y) = e^y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + x*D[u[x,y],y] == (y-x^2/2)^2;
ic  = u[0,y]==Exp[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{1}{4}x\left(x^2-2y\right)^2+e^{y-\frac{x^2}{2}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ x*diff(u(x,y),y)= (y-x^2/2)^2;
ic  := u(0,y)=exp(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y)=\frac{x^5}{4}-x^3y+x\,y^2+\mathrm{e}^{-\frac{x^2}{2}+y}$$

2.1.50 $xu_x + yu_y = u + 1$ with $u = x^2$ on $y = x^2$ Problem 3.5(g) Lokenath Debnath

problem number 50

Added June 3, 2019.

Problem 3.5(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = u + 1$$

with $u = x^2$ on $y = x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x,y],y] == u[x,y]+1;
ic  = u[x,x^2]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{x^2}{y}+y-1\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+ y*diff(u(x,y),y)= u(x,y)+1;
ic  := u(x,x^2)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y) = x\_F1\left(\frac{y}{x}\right) - 1$$

2.1.51 $uu_x - uu_y = u^2 + (x+y)^2$ with $u(x,0) = 1$ Problem 3.5(h) Lokenath Debnath

problem number 51

Added June 3, 2019.

Problem 3.5(h) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$uu_x - uu_y = u^2 + (x+y)^2$$

With $u(x,0) = 1$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x,y]*D[u[x, y], x] - u[x,y]*D[u[x, y], y]== u[x,y]^2+ (x+y)^2;
ic  = u[x,0]==1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to\sqrt{e^{-2y}\left(x^2+2xy+y^2+1\right)-(x+y)^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=u(x,y)*diff(u(x,y),x)- u(x,y)*diff(u(x,y),y)= u(x,y)^2+(x+y)^2;
ic  := u(x,0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=\sqrt{\left((x+y)^2+1\right)\mathrm{e}^{2x}\mathrm{e}^{-2x-2y}-(x+y)^2}$$

2.1.52 $xu_x+(x+y)u_y=u+1$ with $u(x,0)=x^2$ Problem 3.5(i) Lokenath Debnath

problem number 52

Added June 3, 2019.

Problem 3.5(i) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x+(x+y)u_y=u+1$$

With $u(x,0)=x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +(x+y)*D[u[x, y], y]== u[x,y]+1;
ic  = u[x,0]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to x^2e^{-\frac{y}{x}}+e^{\frac{y}{x}}-1\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+(x+y)*diff(u(x,y),y)= u(x,y)+1;
ic  := u(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=x^2\mathrm{e}^{-\frac{y}{x}}+\mathrm{e}^{\frac{y}{x}}-1$$

2.1.53 $xu_x+yu_y+zu_z=0$ Problem 3.8(a) .Lokenath Debnath

problem number 53

Added June 3, 2019.

Problem 3.8(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$xu_x+yu_y+zu_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y,z], x] +y*D[u[x, y,z], y]+z*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x,y,z)\to c_1\left(\frac{y}{x},\frac{z}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y,z),x)+diff(u(x,y,z),y)+diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x, y, z) = \_F1(y - \ln(x), z - \ln(x))$$

2.1.54 $x^2u_x + y^2u_y + z(x + y)u_z = 0$ Problem 3.8(b) Lokenath Debnath

problem number 54

Added June 3, 2019.

Problem 3.8(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y, z)$

$$x^2u_x + y^2u_y + z(x + y)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[u[x, y,z], x] +y^2*D[u[x, y,z], y]+z*(x+y)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x, y, z) \to c_1\left(\frac{1}{x} - \frac{1}{y}, \frac{z}{xy}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(u(x,y,z),x)+y^2*diff(u(x,y,z),y)+z*(x+y)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x, y, z) = \_F1\left(\frac{x - y}{xy}, \frac{z}{xy}\right)$$

2.1.55 $x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$ Problem 3.8(c) Lokenath Debnath

problem number 55

Added June 3, 2019.

Problem 3.8(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(y-z)*D[u[x, y,z], x] +y*(z-x)*D[u[x, y,z], y]+z*(x-y)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\{\{u(x,y,z) \to c_1(-xyz, x+y+z)\}\}$$

Maple ✓

```
restart;
pde :=x*(y-z)*diff(u(x,y,z),x)+y*(z-x)*diff(u(x,y,z),y)+z*(x-y)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z)),'build')),out
```

$$u(x,y,z) = \frac{c_4 c_5 x^{c_2} y^{c_2} z^{c_2} \mathrm{e}^{c_2} \mathrm{e}^{-c_1 x} \mathrm{e}^{-c_1 y} \mathrm{e}^{-c_1 z}}{c_3}$$

2.1.56 $yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$ Problem 3.8(d) Lokenath Debnath

problem number 56

Added June 3, 2019.

Problem 3.8(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*z*D[u[x, y,z], x] - x*z*D[u[x, y,z], y]+x*y*(x^2+y^2)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x,y,z)\to c_1\left(\frac{1}{2}\left(x^2+y^2\right),\frac{1}{2}\left(-x^2y^2-x^4+z^2\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*z*diff(u(x,y,z),x) - x*z*diff(u(x,y,z),y)+x*y*(x^2+y^2)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x,y,z)=\_F1\left(x^2+y^2,-x^4-x^2y^2+z^2\right)$$

2.1.57 $x(y^2-z^2)u_x+y(z^2-y^2)u_y+z(x^2-y^2)u_z=0$ Problem 3.8(e) Lokenath Debnath

problem number 57

Added June 3, 2019.

Problem 3.8(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$x(y^2-z^2)u_x+y(z^2-y^2)u_y+z(x^2-y^2)u_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(y^2-z^2)*D[u[x, y,z], x] +y*(z^2-y^2)*D[u[x, y,z], y]+z*(x^2-y^2)*D[u[x, y,z], z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=x*(y^2-z^2)*diff(u(x,y,z),x) + y*(z^2-y^2)*diff(u(x,y,z),y)+z*(x^2-y^2)*diff(u(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

sol=()

2.1.58 $u_x + xu_y = y$ with $u(0,y) = y^2$ Problem 3.9(a) Lokenath Debnath

problem number 58

Added June 3, 2019.

Problem 3.9(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + xu_y = y$$

With $u(0,y) = y^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +x*D[u[x, y], y]== y;
ic  = u[0,y]==y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to -x^2y+\frac{x^4}{4}-\frac{x^3}{3}+xy+y^2\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + x*diff(u(x,y),y)= y;
ic  := u(0,y)=y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=-\frac{x^3}{3}+xy+\left(-\frac{x^2}{2}+y\right)^2$$

2.1.59 $u_x + xu_y = y$ with $u(1, y) = 2y$ Problem 3.9(b) Lokenath Debnath

problem number 59

Added June 3, 2019.

Problem 3.9(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + xu_y = y$$

With $u(1, y) = 2y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +x*D[u[x, y], y]== y;
ic  = u[1,y]==2*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to -\frac{x^3}{3}-\frac{x^2}{2}+xy+y+\frac{5}{6}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + x*diff(u(x,y),y)= y;
ic  := u(1,y)=2*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + xy + y + \frac{5}{6}$$

2.1.60 $(u_x + u_y)^2 - u^2 = 0$. Problem 3.10 Lokenath Debnath

problem number 60

Added June 3, 2019.

Problem 3.10 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(u_x + u_y)^2 - u^2 = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (D[u[x, y], x] +D[u[x, y], y])^2-u[x,y]^2== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{u(x,y) \to e^{-x} c_1(y-x)\}$$
$$\{u(x,y) \to e^{x} c_1(y-x)\}$$

Maple ✓

```
restart;
pde :=(diff(u(x,y),x) + diff(u(x,y),y))^2-u(x,y)^2= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = c_1 \mathrm{e}^{\frac{y\_c_2+x}{-\_c_2+1}}$$

2.1.61 $(y + u)u_x + yu_y = x - y$ with $u(x, 1) = 1 + x$. Problem 3.11 Lokenath Debnath

problem number 61

Added June 3, 2019.

Problem 3.11 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(y + u)u_x + yu_y = x - y$$

With $u(x,1) = 1 + x$.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] +y*D[u[x, y], y]== x-y;
 ic=u[x,1]==1+x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x) + y*diff(u(x,y),y)= x-y;
ic  := u(x,1)=1+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

sol=()

2.1.62 $2xu_x + (x+1)u_y = y$ with $u(1,y) = 2y$. Problem 3.14(d) Lokenath Debnath

problem number 62

Added June 3, 2019.

Problem 3.14(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$2xu_x + (x+1)u_y = y$$

With $u(1,y) = 2y$ with $x > 0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*x*D[u[x, y], x] +(x+1)*D[u[x, y], y]== y;
 ic=u[1,y]==2*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}, Assumptions->x>0], 6
```

$$\left\{\left\{u(x,y)\to \frac{1}{8}\left(-2(x-2y+4)\log(x)-6x-\log^2(x)+16y+6\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=2*x*diff(u(x,y),x) + (x+1)*diff(u(x,y),y)= y;
ic  := u(1,y)=2*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

$$u(x,y) = -\frac{\ln(x)^2}{8} - \frac{3x}{4} + 2y + \frac{(-2x+4y-8)\ln(x)}{8} + \frac{3}{4}$$

2.1.63 $xu_x + yu_y = x^2 + y^2$ with $u(x,1) = x^2$. Problem 3.14(e) Lokenath Debnath

problem number 63

Added June 3, 2019.

Problem 3.14(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = x^2 + y^2$$

With $u(x,1) = x^2$ with $x > 0, y > 0$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y]== x^2+y^2;
 ic=u[x,1]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y},Assumptions->{x>0,y>0
```

$$\left\{\left\{u(x,y) \to \frac{x^2y^2 + x^2 + y^4 - y^2}{2y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x) + y*diff(u(x,y),y)= x^2+y^2;
ic  := u(x,1)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

$$u(x,y) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{x^2}{2y^2} - \frac{1}{2}$$

2.1.64 $y^2u_x + (xy)u_y = x$ with $u(x,1) = x^2$. Problem 3.14(f) Lokenath Debnath

problem number 64

Added June 3, 2019.

Problem 3.14(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$y^2u_x + (xy)u_y = x$$

With $u(x,1) = x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[u[x, y], x] +(x*y)*D[u[x, y], y]== x;
 ic=u[x,1]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to x^2 - y^2 + \frac{\log(y^2)}{2} + 1\right\}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(u(x,y),x) + (x*y)*diff(u(x,y),y)= x;
ic  := u(x,1)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y) = x^2 - y^2 + \frac{\ln(y^2)}{2} + 1$$

2.1.65 $xu_x + yu_y = xy$ with $u = \frac{x^2}{2}$ at $y = x$. Problem 3.14(g) Lokenath Debnath

problem number 65

Added June 3, 2019.

Problem 3.14(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = xy$$

With $u = \frac{x^2}{2}$ at $y = x$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y]== x*y;
 ic=u[x,x]==x^2/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{xy}{2} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x) + y*diff(u(x,y),y)= x*y;
ic  := u(x,x)=x^2/2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y) = \frac{xy}{2} + \_F1\left(\frac{y}{x}\right)$$

2.1.66 $u_x + uu_y = 1$ with $u(0,y) = ay$. Problem 3.16(a) Lokenath Debnath

problem number 66

Added June 3, 2019.

Problem 3.16(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + uu_y = 1$$

With $u(0,y) = ay$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +u[x,y]*D[u[x, y], y]== 1;
 ic=u[0,y]==a*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{ax^2 + 2ay + 2x}{2ax + 2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + u(x,y)*diff(u(x,y),y)=1;
ic  := u(0,y)=a*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y) = \frac{(x^2+2y)\,a+2x}{2ax+2}$$

2.1.67 $(y+u)u_x+(x+u)u_y=x+y$. Problem 3.17(a) Lokenath Debnath

problem number 67

Added June 3, 2019.

Problem 3.17(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(y+u)u_x+(x+u)u_y=x+y$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] +(x+u[x,y])*D[u[x, y], y]== x+y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x) + (x+u(x,y))*diff(u(x,y),y)=x+y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') ),outp
```

$$u(x,y) = \frac{-\operatorname{RootOf}\left(\left(c_1 e^{\frac{3c_1c_2}{y}}+e^{\frac{3c_1c_2}{y}}\right)\_Z^9 - c_1^3x^3 - 3c_1^2x^2y - 3c_1x\,y^2 + (3c_1^3x^3+9c_1^2x^2y+9c_1x\,y^2+3y}{(c_1x+y)^2}$$

2.1.68 $xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$. Problem 3.17(b) Lokenath Debnath

problem number 68

Added June 3, 2019.

Problem 3.17(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*u[x,y]*(u[x,y]^2+x*y)*D[u[x, y], x] -y*u[x,y]*(u[x,y]^2+x*y)*D[u[x, y], y]== x^4;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y)\to -\sqrt{-xy-\sqrt{4c_1(xy)+x^2y^2+x^4}}\right\}$$
$$\left\{u(x,y)\to \sqrt{-xy-\sqrt{4c_1(xy)+x^2y^2+x^4}}\right\}$$
$$\left\{u(x,y)\to -\sqrt{-xy+\sqrt{4c_1(xy)+x^2y^2+x^4}}\right\}$$
$$\left\{u(x,y)\to \sqrt{-xy+\sqrt{4c_1(xy)+x^2y^2+x^4}}\right\}$$

Maple ✓

```
restart;
pde :=x*u(x,y)*(u(x,y)^2+x*y)*diff(u(x,y),x) -y*u(x,y)*(u(x,y)^2+x*y)*diff(u(x,y),y)=x^4;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y)=\sqrt{-xy-\sqrt{x^4+x^2y^2+4\_F1(xy)}}$$

2.1.69 $(x+y)u_x+(x-y)u_y=0$. Problem 3.17(c) Lokenath Debnath

problem number 69

Added June 3, 2019.

Problem 3.17(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(x+y)u_x+(x-y)u_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x+y)*D[u[x, y], x] +(x-y)*D[u[x, y], y]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y)\to c_1\left(\log\left(-\sqrt{-x^2+2xy+y^2}\right)\right)\right\}$$
$$\left\{u(x,y)\to c_1\left(\frac{1}{2}\log\left(-x^2+2xy+y^2\right)\right)\right\}$$

Maple ✓

```
restart;
pde :=(x+y)*diff(u(x,y),x) +(x-y)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='real
```

$$u(x,y)=\_F1\left(-\frac{1}{\sqrt{-x^2+2xy+y^2}}\right)$$

2.1.70 $yu_x - xu_y = e^u$ with $u(0,y) = y^2 - 1$

problem number 70

Added May 21, 2019.

Characteristics, with IC

Taken from "the method of Characteristics" by Ryan C. Daileda. Page 16 http://ramanujan.math.trinity.edu/rdaileda/teach/s15/m3357/lectures/lecture_1_22_slides.pdf

Solve for $u(x,y)$

$$yu_x - xu_y = e^u$$

With initial conditions $u(0,y) = y^2 - 1$.

Mathematica ✓

```
ClearAll["Global`*"];
pde = y*D[u[x, y], x] - x*D[u[x, y], y] == Exp[u[x, y]];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, u[0, y] == y^2 - 1}, u[x, y], {x, y}], 60*
```

$$\left\{\left\{u(x,y) \to -\log\left(e^{-x^2-y^2+1} + \tan^{-1}\left(\frac{x}{\sqrt{y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)-x*diff(u(x,y),y)=exp(u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,u(0,y)=y^2-1],u(x,y))
```

$$u(x,y) = \ln\left(-\frac{1}{\arctan\left(\frac{x}{y}\right) - e^{-x^2-y^2+1}}\right)$$

Hand solution

Solve

$$\begin{aligned} yu_x - xu_y &= e^u \\ u(0,y) &= y^2 - 1 \end{aligned}$$

The characteristic equations (using s as the parameter) are

$$\frac{dx}{ds} = y \tag{1}$$
$$\frac{dy}{ds} = -x \tag{2}$$
$$\frac{du}{ds} = e^u \tag{3}$$

With initial point at $s = 0$ which can be written as $u(0,\xi) = \xi^2 - 1$. The idea of this method is to use (1,2) to obtain expressions for s,ξ. These are the unknowns. Then using these in (3) to obtain the final solution $u(x,y)$. One problem that we see right away, is that (1,2) are coupled. When this happens, we must decouple them first. Differentiating (1) gives $\frac{d^2x}{ds^2} = y' = -x$. Hence (1) becomes $x''(s) + x = 0$. This has solution

$$x(s) = c_1 \cos s + c_2 \sin s \tag{4}$$

But from (1), we see that $x'(s) = y$. Therefore

$$y(s) = -c_1 \sin s + c_2 \cos s \tag{5}$$

We made some progress. Found $x(s), y(s)$. But need to solve for c_1, c_2. This is done using the initial point, which is always at $s = 0$. When $s = 0$, using $u(0,\xi) = \xi^2 - 1$, which says $x(0) = 0$ and $y(0) = \xi$. Using $x(0) = 0$ when $s = 0$ then from (4) we obtain $0 = c_1$. Now the solutions becomes

$$x(s) = c_2 \sin s$$
$$y(s) = c_2 \cos s$$

Now we use the second initial condition on $y(s)$ which says $y(0) = \xi$. Hence from the second equation above, $\xi = c_2$. Therefore the solution now becomes

$$x(s) = \xi \sin s \tag{6}$$
$$y(s) = \xi \cos s \tag{7}$$

This is as far as we can go. Remembering that we are after expressions for s and ξ. Dividing (6/7) gives

$$\frac{x}{y} = \tan(s)$$
$$s = \arctan\left(\frac{x}{y}\right) \tag{8}$$

Good. We obtained relation for s in terms of x, y. What about ξ?. By equation (6) and (7) and adding them we obtain

$$\begin{aligned} x^2(s) + y^2(s) &= \xi^2 \\ \xi &= \sqrt{x^2 + y^2} \end{aligned} \tag{9}$$

Good. (8,9) is what we wanted. Equations (1,2) have done their job. We used them to find s, ξ. Now we move on to (3) which is

$$\frac{du}{ds} = e^u$$

Solving it gives

$$-e^{-u} = s + C$$

But at $s = 0$ we know that $u(0) = \xi^2 - 1$. $-e^{-(\xi^2-1)} = C$. Then the above becomes

$$-e^{-u} = s - e^{-(\xi^2-1)}$$

We are almost there. We just need now to go back to x, y from s, ξ. By using (8,9) the above becomes

$$\begin{aligned} -e^{-u} &= \arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)} \\ e^{-u} &= -\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)} \end{aligned}$$

We can stop here. But if assume $u > 0$ then the above can be simplified more

$$\begin{aligned} -u &= \ln\left(-\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)}\right) \\ u(x,y) &= -\ln\left(-\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)}\right) \\ &= -\ln\left(-\left(\arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)}\right)\right) \\ &= \ln\left(\frac{1}{-\left(\arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)}\right)}\right) \end{aligned}$$

I used document titled "The method of Characteristics" by Ryan C. Daileda for help which is a very useful document.

2.1.71 $yu_x - xu_y = e^u$

problem number 71

Added May 21, 2019.

This is same problem as above, but without I.C. given.

Solve for $u(x,y)$

$$yu_x - xu_y = e^u$$

No IC are given.

Mathematica ✓

```
ClearAll["Global`*"];
pde = y*D[u[x, y], x] - x*D[u[x, y], y] == Exp[u[x, y]];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to -\log\left(\tan^{-1}\left(\frac{x}{\sqrt{y^2}}\right)-c_1\left(\frac{1}{2}(x^2+y^2)\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)-x*diff(u(x,y),y)=exp(u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=\ln\left(-\frac{1}{\_F1\left(x^2+y^2\right)+\arctan\left(\frac{x}{y}\right)}\right)$$

Hand solution

Solve

$$yu_x - xu_y = e^u$$

Since no initial conditions are given, I find using Lagrange-charpit method is better here than using characteristic equations with parameter s. The Lagrange-charpit equations for the above PDE are

$$\frac{dx}{y}=\frac{-dy}{x}=\frac{du}{e^u}$$

The first two equations gives

$$
\begin{aligned}
xdx &= -ydy \\
\frac{x^2}{2} &= -\frac{y^2}{2} + C_1 \\
C_1 &= \frac{1}{2}(x^2 + y^2) \\
C_1 &= (x^2 + y^2) \qquad (1)
\end{aligned}
$$

Where the 2 is absorbed by the constant. We now need to decide to either solve $\frac{-dy}{x} = \frac{du}{e^u}$ together or $\frac{dx}{y} = \frac{du}{e^u}$. It does not matter which pair to pick. Using the second pair gives

$$
\frac{dx}{y} = \frac{du}{e^u}
$$

But from (1), $y = \sqrt{C_1 - x^2}$ (taking only the positive root) and the above equation now becomes

$$
\frac{dx}{\sqrt{C_1 - x^2}} = \frac{du}{e^u}
$$

Integrating gives

$$
\begin{aligned}
\arctan\left(\frac{x}{\sqrt{C_1 - x^2}}\right) &= -e^{-u} + C_2 \\
\arctan\left(\frac{x}{y}\right) &= -e^{-u} + C_2 \\
C_2 &= \arctan\left(\frac{x}{y}\right) + e^{-u}
\end{aligned}
$$

In this method, the constants C_1, C_2 are always related by $C_2 = F(C_1)$ where F is an arbitrary function. Hence we obtain

$$
\begin{aligned}
\arctan\left(\frac{x}{y}\right) + e^{-u} &= F(x^2 + y^2) \\
e^{-u} &= F(x^2 + y^2) - \arctan\left(\frac{x}{y}\right)
\end{aligned}
$$

For positive u the above simplifies to

$$-u = \ln\left(F(x^2+y^2) - \arctan\left(\frac{x}{y}\right)\right)$$
$$u(x,y) = -\ln\left(F(x^2+y^2) - \arctan\left(\frac{x}{y}\right)\right)$$

2.1.72 $u_t + xu_x = 0$ with $u(x,0) = x^2$. Math 5587

problem number 72

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 3.

Solve for $u(x,t)$

$$u_t + xu_x = 0$$

With with $u(x,0) = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + x*D[u[x, t], x] == 0;
ic  = u[x,0]==x^2;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to e^{-2t}x^2\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+x*diff(u(x,t),x)=0;
ic :=u(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x^2 e^{-2t}$$

Hand solution

Solve $u_t + xu_x = 0$ with $u(x,0) = x^2$. Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = x \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = 0, x(0) = \xi, u(0) = \xi^2$$

Equation (1) gives

$$\begin{aligned} t &= s + t(0) \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives

$$\begin{aligned} \ln x &= s + x(0) \\ x &= \xi e^s \end{aligned} \tag{6}$$

From (5,6) solving for ξ gives

$$\begin{aligned} \xi &= xe^{-s} \\ &= xe^{-t} \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} u &= u(0) \\ &= \xi^2 \\ &= x^2 e^{-2t} \end{aligned}$$

2.1.73 $u_t + tu_x = 0$ with $u(x,0) = e^x$

problem number 73

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 4.

Solve for $u(x,t)$

$$u_t + tu_x = 0$$

With with $u(x,0) = e^x$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + t*D[u[x, t], x] == 0;
ic  = u[x,0]==Exp[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{x-\frac{t^2}{2}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+t*diff(u(x,t),x)=0;
ic :=u(x,0)=exp(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \mathrm{e}^{-\frac{t^2}{2}+x}$$

Hand solution

Solve $u_t + xu_x = 0$ with $u(x,0) = e^x$. Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = t \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = 0, x(0) = \xi, u(0) = e^\xi$$

Equation (1) gives

$$\begin{aligned} t &= s + t(0) \\ &= s \end{aligned} \tag{5}$$

Equation (2) now becomes $\frac{dx}{ds} = s$, whose solution is

$$
\begin{aligned}
x &= \frac{s^2}{2} + x(0) \\
x &= \frac{s^2}{2} + \xi
\end{aligned} \tag{6}
$$

From (5,6) solving for ξ gives

$$
\begin{aligned}
\xi &= x - \frac{s^2}{2} \\
&= x - \frac{t^2}{2}
\end{aligned} \tag{7}
$$

Equation (3) gives

$$
\begin{aligned}
u &= u(0) \\
&= e^{\xi} \\
&= e^{\left(x - \frac{t^2}{2}\right)}
\end{aligned}
$$

2.1.74 $2u_x + 3u_y = 1$

problem number 74

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 5.

Solve for $u(x, y)$

$$
2u_x + 3u_y = 1
$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = 2 D[u[x, y], x] + 3*D[u[x, y], y] == 1;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$
\left\{\left\{u(x, y) \to \frac{x}{2} + c_1\left(y - \frac{3x}{2}\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde :=2*diff(u(x,y),x)+3*diff(u(x,y),y)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=\frac{x}{2}+\_F1\left(-\frac{3x}{2}+y\right)$$

Hand solution

Solve $2u_x+3u_y=1$. Using the method of characteristics

$$\frac{dx}{2}=\frac{dy}{3}=\frac{du}{1}$$

From the first pair of equations we obtain $\frac{1}{2}x=\frac{1}{3}y+C_1$ or $C_1=\frac{1}{2}x-\frac{1}{3}y$. From the pair $\frac{dx}{2}=\frac{du}{1}$ we obtain

$\frac{1}{2}x=u+C_2$ or $C_2=\frac{1}{2}x-u$. But $C_2=F(C_1)$ where F is arbitrary function. Hence

$$\frac{1}{2}x-u=F\left(\frac{1}{2}x-\frac{1}{3}y\right)$$
$$u=\frac{1}{2}x-F\left(\frac{1}{2}x-\frac{1}{3}y\right)$$

2.1.75 $xu_t-tu_x=0$

problem number 75

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 6.

Solve for $u(x,t)$ with $x>0,t>0$ and $u(x,0)=x^2$

$$xu_t-tu_x=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x D[u[x, t], t] - t*D[u[x, t], x] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to c_1\left(\frac{1}{2}\left(t^2+x^2\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,t),t)-t*diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \_F1(t^2 + x^2)$$

Hand solution

Solve $xu_t - tu_x = 0$. Using the method of characteristics

$$\frac{dt}{x} = -\frac{dx}{t} = \frac{du}{0}$$

From the first pair of equations we obtain $tdt = -xdx$ or $\frac{t^2}{2} = -\frac{x^2}{2} + C_1$ or $C_1 = \frac{t^2}{2} + \frac{x^2}{2}$. From $du = 0$ then $u = C_2$. But $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$u = F\left(\frac{t^2}{2} + \frac{x^2}{2}\right)$$

2.1.76 $u_t + u_x = 0$ with $u(x,1) = \frac{x}{1+x^2}$

problem number 76

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 3.

Solve for $u(x,t)$ with $x > 0, t > 0$ and initial conditions not zero $u(x,1) = \frac{x}{1+x^2}$

$$u_t + u_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + D[u[x, t], x] == 0;
ic  = u[x,1]== x/(1+1+x^2);
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{-t+x+1}{t^2 - 2t(x+1) + x^2 + 2x + 3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)=0;
ic  := u(x,1)= x/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{-t+x+1}{(-t+x+1)^2+1}$$

Hand solution

Solve $u_t + u_x = 0$ with initial conditions $u(x,1) = \frac{x}{1+x^2}$. The characteristic equations are

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = 1 \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

At $s = 0$ we have $x(0) = \xi, t(1) = 1, u(0) = \frac{\xi}{1+\xi^2}$. Solving (1) gives $t = s + t(1) = s + 1$. Solving (2) gives $x = s + x(0) = s + \xi$. From these solutions we solve for ξ, which gives $\xi = x - s = x - (t-1)$. Hence

$$\xi = x - t + 1$$

Equation (3) gives

$$\begin{aligned} u &= u(0) \\ &= \frac{\xi}{1+\xi^2} \end{aligned}$$

Hence

$$u(x,y) = \frac{x-t+1}{1+(x-t+1)^2}$$

2.1.77 $u_x u_y = 1$

problem number 77

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 5(a).

Solve for $u(x, y)$

$$u_x u_y = 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], x] * D[u[x, y], y] == 1;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x, y) \to \frac{x}{c_2} + c_2 y + c_1\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)*diff(u(x,y),y)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),HINT=`+`,'build
```

$$u(x, y) = y\_c_2 + c_1 + c_2 + \frac{x}{\_c_2}$$

<u>Hand solution</u>

Solve $u_x u_y = 1$. Let $u = f(x) + g(y)$. The pde becomes $f'(x)\, g'(y) = 1$ or $f'(x) = \frac{1}{g'(y)}$. Hence both are constant, say λ. Therefore $f'(x) = \lambda$ or $f(x) = \lambda x + C_1$ and similarly, $\frac{1}{g'(y)} = \lambda$ or $g'(y) = \frac{1}{\lambda}$ or $g(y) = \frac{y}{\lambda} + C_2$. Therefore the solution becomes

$$\begin{aligned} u(x, y) &= f(x) + g(y) \\ &= \lambda x + C_1 + \frac{y}{\lambda} + C_2 \end{aligned}$$

2.1.78 $u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$

problem number 78

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 5(b).

Solve for $u(x,y)$

$$u_x u_y = u$$

With $u(x,0) = 0, u(0,y) = 0$.

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, y], x] * D[u[x, y], y] == u[x,y];
bc={u[x,0]==0,u[0,y]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc},u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x)*diff(u(x,y),y)=u(x,y);
bc  := u(x,0)=0,u(0,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y),HINT=`*`))
```

$$u(x,y) = xy$$

Hand solution

Solve $u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$.

Let $u = X(x)Y(y)$. The pde becomes $(X'Y)(Y'X) = XY$ or $X'Y' = 1$ or $X' = \frac{1}{Y'}$. Hence both sides are constant, say λ. Therefore we have 2 differential equations to solve

$$X' = \lambda$$

And

$$\frac{1}{Y'} = \lambda$$

The solution to the first one is $X(x) = \lambda x + C_1$. The solution to the second equation is $Y(y) = \frac{1}{\lambda}y + C_2$.

Since $u = X(x)Y(y)$ then the solution is

$$\begin{aligned} u(x,y) &= (\lambda x + C_1)\left(\frac{1}{\lambda}y + C_2\right) \\ &= xy + C_2\lambda x + \frac{C_1}{\lambda}y + C_1C_2 \end{aligned}$$

Using the condition $u(x,0) = 0$ gives

$$0 = C_2\lambda x + C_1C_2$$

Differentiating gives

$$0 = C_2\lambda$$

But $\lambda \neq 0$ for nontrivial solution, hence $C_2 = 0$. Therefore (1) reduces to

$$u(x,y) = xy + \frac{C_1}{\lambda}y$$

Using the condition $u(0,y) = 0$ gives

$$0 = \frac{C_1}{\lambda}y$$

Hence $C_1 = 0$. Therefore the solution becomes

$$u(x,y) = xy$$

2.2 Solved by factoring into two transport equations

Local contents

2.2.1 $u_{xx} + u_{xt} - 6u_{tt} = 0$

problem number 79

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 8.

Solve for $u(x,t)$ with $u(x,0) = x$ and $u_t(x,0) = 0$ by factoring the PDE into two transport PDE

$$u_{xx} + u_{xt} - 6u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] + D[D[u[x,t],x],t] -  6*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==x, Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)+diff(diff(u(x,t),x),t) - 6 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=x, eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x$$

Hand solution

Solve $u_{xx}+u_{xt}-6u_{tt}=0$ with $u(x,0)=x$ and $u_t(x,0)=0$. Writing the PDE as

$$\left(\frac{\partial}{\partial x}+3\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x}-2\frac{\partial}{\partial t}\right)u=0$$

Let

$$\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial t}=w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x}+3\frac{\partial w}{\partial t}=0 \tag{2}$$

From (1),

$$w(x,0)=u_x(x,0)-2u_t(x,0)$$

But $u(x,0)=x$ hence $u_x(x,0)=1$, and $u_t(x,0)=0$. Therefore the above gives

$$w(x,0)=1$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds}&=1\\\frac{dt}{ds}&=3\\\frac{dw}{ds}&=0\end{aligned}$$

With $x(0)=\xi,t(0)=0,w(0)=1$. The above equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\t&=3s\\w&=u(0)=1\end{aligned}$$

Hence

$$w(x,y)=1$$

From (1)

$$\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial t}=1 \tag{3}$$

with $u(x,0)=x$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds}&=1\\\frac{dt}{ds}&=-2\\\frac{du}{ds}&=1\end{aligned}$$

With $x(0) = \xi, t(0) = 0, u(0) = \xi$. The above equations give

$$\begin{aligned} x &= s + x(0) = s + \xi \\ t &= -2s \\ u &= s + u(0) = s + \xi \end{aligned}$$

But $s = -\frac{t}{2}$ and $\xi = x - s = x - \left(-\frac{t}{2}\right) = x + \frac{t}{2}$, hence

$$\begin{aligned} u(x,t) &= -\frac{t}{2} + x + \frac{t}{2} \\ &= x \end{aligned}$$

2.2.2 $u_{xx} - u_{xt} - 12u_{tt} = 0$

problem number 80

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 9.

Solve for $u(x,t)$ with $u(x,0) = 0$ and $u_t(x,0) = x$ by factoring the PDE into two transport PDE

$$u_{xx} - u_{xt} - 12u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] - D[D[u[x,t],x],t] -  12*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==0, Derivative[0,1][u][x,0]==x};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to tx - \frac{t^2}{24}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)-diff(diff(u(x,t),x),t) - 12 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = -\frac{1}{24}t^2 + tx$$

Hand solution

Solve $u_{xx} - u_{xt} - 12u_{tt} = 0$ with $u(x,0) = x$ and $u_t(x,0) = x$. Writing the PDE as

$$\left(\frac{\partial}{\partial x} + 3\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - 4\frac{\partial}{\partial t}\right) u = 0$$

Let

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x} + 3\frac{\partial w}{\partial t} = 0 \tag{2}$$

From (1),

$$w(x,0) = u_x(x,0) - 4u_t(x,0)$$

But $u(x,0) = 0$ hence $u_x(x,0) = 0$, and $u_t(x,0) = x$. Therefore the above gives

$$w(x,0) = -4x$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds} &= 1\\ \frac{dt}{ds} &= 3\\ \frac{dw}{ds} &= 0\end{aligned}$$

With $x(0) = \xi, t(0) = 0, w(0) = -4\xi$. The above equations give

$$\begin{aligned} x &= s + x(0) = s + \xi\\ t &= 3s\\ w &= u(0) = -4\xi\end{aligned}$$

Solving for ξ, s from the first 2 equations. $\xi = x - s = x - \frac{t}{3}$. From the last equation above

$$w(x,y) = -4\left(x - \frac{t}{3}\right)$$

Using the above into (1) gives

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = -4\left(x - \frac{t}{3}\right) \tag{3}$$

with $u(x,0)=0$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=-4\\ \frac{du}{ds}&=-4\left(x-\frac{t}{3}\right)\end{aligned}$$

With $x(0)=\xi, t(0)=0, u(0)=0$. The above equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=-4s\end{aligned}$$

Solving the above for s,ξ gives $\xi=x-s=x+\frac{t}{4}$. Therefore $\frac{du}{ds}=-4\left(x-\frac{t}{3}\right)$ becomes

$$\begin{aligned}\frac{du}{ds}&=-4\left(s+\xi-\frac{(-4s)}{3}\right)\\ &=-4\left(s+\xi+\frac{4s}{3}\right)\\ &=-\frac{28}{3}s-4\xi\end{aligned}$$

Solving the above gives

$$\begin{aligned}u&=-\frac{28}{3}\frac{s^2}{2}-4\xi s+u(0)\\ &=-\frac{28}{3}\frac{s^2}{2}-4\xi s\end{aligned}$$

Converting to x,t

$$\begin{aligned}u(x,y)&=-\frac{28}{3}\frac{1}{2}\left(-\frac{t}{4}\right)^2-4\left(x+\frac{t}{4}\right)\left(-\frac{t}{4}\right)\\ &=-\frac{1}{24}\left(t^2-24xt\right)\\ &=-\frac{1}{24}t^2+xt\end{aligned}$$

2.2.3 $u_{xx} - 3u_{xt} - 4u_{tt} = 0$

problem number 81

Added May 25, 2019.

From HW 3, UMN Math 5587, Fall 2016, problem 2.

Solve for $u(x,t)$ with $u(x,0) = e^x$ and $u_t(x,0) = 0$ by factoring the PDE into two transport PDE

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] - 3*D[D[u[x,t],x],t] -  4*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==Exp[x], Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{5}\left(4e^{5t/4}+1\right)e^{x-t}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)-3*diff(diff(u(x,t),x),t) - 4 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=exp(x), eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{e^{-t+x}}{5} + \frac{4\,e^{\frac{t}{4}+x}}{5}$$

Hand solution

Solve $u_{xx} - 3u_{xt} - 4u_{tt} = 0$ with $u(x,0) = e^x$ and $u_t(x,0) = x$. Writing the PDE as

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - 4\frac{\partial}{\partial t}\right)u = 0$$

Let

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}=0 \tag{2}$$

From (1),

$$w(x,0)=u_x(x,0)-4u_t(x,0)$$

But $u(x,0)=e^x$ hence $u_x(x,0)=e^x$, and $u_t(x,0)=0$. Therefore the above gives

$$w(x,0)=e^x$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=1\\ \frac{dw}{ds}&=0\end{aligned}$$

With $x(0)=\xi, t(0)=0, w(0)=e^{\xi}$. The above equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=s\\ w&=u(0)=e^{\xi}\end{aligned}$$

Since $\xi=x-s=x-t$, then the last equation above gives

$$w(x,y)=e^{x-t}$$

Using the above into (1) gives

$$\frac{\partial u}{\partial x}-4\frac{\partial u}{\partial t}=e^{x-t} \tag{3}$$

with $u(x,0)=e^x$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=-4\\ \frac{du}{ds}&=e^{x-t}\end{aligned}$$

With $x(0)=\xi, t(0)=0, u(0)=e^{\xi}$. The above two equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=-4s\end{aligned}$$

Solving the above for s,ξ gives $\xi = x - s = x + \frac{t}{4}$. Therefore $\frac{du}{ds} = e^{x-t}$ becomes

$$\begin{aligned}\frac{du}{ds} &= e^{s+\xi-(-4s)}\\ &= e^{5s+\xi}\end{aligned}$$

Solving the above gives

$$\begin{aligned}u &= \frac{1}{5}e^{5s+\xi} + u(0)\\ &= \frac{1}{5}e^{5s+\xi} + e^{\xi}\end{aligned}$$

Converting to x,t, using $s = \frac{-t}{4}$ and $\xi = x + \frac{t}{4}$ gives

$$\begin{aligned}u(x,y) &= \frac{1}{5}e^{5\left(\frac{-t}{4}\right)+\left(x+\frac{t}{4}\right)} + e^{\left(x+\frac{t}{4}\right)}\\ &= \frac{1}{5}e^{x-t} + e^{\left(x+\frac{t}{4}\right)}\end{aligned}$$

2.2.4 $u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$

problem number 82

Added Oct 6, 2019.

Problem 2.4.19 Peter Olver, Into to Partial differential equations 4th edition

Solve $u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] - 2*D[D[u[x,t],x],t] -  3*D[u[x, t], {x,2}] == 0;
ic  = {u[x,0]==x^2, Derivative[0,1][u][x,0]==Exp[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to 3t^2 - \frac{e^{x-t}}{4} + \frac{1}{4}e^{3t+x} + x^2\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-2*diff(diff(u(x,t),x),t) - 3 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=x^2, D[2](u)(x,0)=exp(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x^2 + e^x - e^{-t+x}$$

2.3 Beam PDE

Local contents

2.3.1 Beam PDE $u_{tt} + u_{xxxx} = 0$

problem number 83

Added January 20, 2018.

Beam PDE with zero initial velocity. Solve

$$u_{tt} + u_{xxxx} = 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= -12t^2 \\ f(1,t) &= 1 - 12t^2 \\ \frac{\partial^2 u}{\partial x^2} u(0,t) &= 0 \\ \frac{\partial^2 u}{\partial x^2} u(1,t) &= 12 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^4 \\ \frac{\partial u}{\partial t} u(x,0) &= 0 \end{aligned}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + D[u[x, t], {x, 4}] == 0;
bc  = {u[0, t] == -12*t^2, u[1, t] == 1 - 12*t^2, Derivative[2, 0][u][0, t] == 0, Derivative
ic  = {u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+diff(u(x,t),x$4)=0;
bc  := u(0,t)=-12*t^2,
    u(1,t)=1-12*t^2,D[1,1](u)(0,t)=0,
    D[1,1](u)(1,t)=12;
ic  := u(x,0)=x^4,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT=`+
```

$$u(x,t) = x^4 - 12t^2$$

2.4 Burger's PDE

Local contents

2.4.1 Inviscid Burgers $u_x + uu_y = 0$

problem number 84

Taken from Mathematica Symbolic PDE document

quasilinear first-order PDE, scalar conservation law

Solve for $u(x, y)$

$$u_x + uu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Implicit solution

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
sol:=DEtools:-remove_RootOf(sol);
```

$$xu(x,y) - y + \_F1(u(x,y)) = 0$$

Hand solution

Solve for $u(x,y)$ in $u_x+u\,u_y=0$. Using the Lagrange-Charpit method, the characteristic equations are

$$\frac{dx}{1}=\frac{dy}{u}=\frac{du}{0}$$

From the first pair of equation we obtain

$$u=\frac{dy}{dx}$$

But $du=0$ or $u=C_2$. Hence the above becomes

$$\begin{aligned}\frac{dy}{dx}&=C_2\\ y&=xC_2+C_1\\ C_1&=y-xC_2\end{aligned}$$

Since $C_2=F(C_1)$ where F is arbitrary function, then

$$u(x,y)=F(y-ux)$$

2.4.2 Inviscid Burgers with I.C. $u_x+uu_y=0$ and $u(x,0)=\frac{1}{x+1}$

problem number 85

Taken from Mathematica Symbolic PDE document

quasilinear first-order PDE, scalar conservation law with initial value

Solve for $u(x,y)$

$$u_x+uu_y=0$$

With $u(x,0)=\frac{1}{x+1}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
ic  = u[x, 0] == 1/(x + 1);
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y)\to\frac{y+1}{x+1}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
ic  := u(x,0)=1/(x+1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \frac{y+1}{x+1}$$

<u>Hand solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dx}{ds} = 1 \tag{1}$$
$$\frac{dy}{ds} = u \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$x(0) = t_1, y(0) = t_2, u(0) = t_3$$

We are given that $u(x,0) = \frac{1}{1+x}$. This initial condition translates to

$$t_3 = \frac{1}{1+t_1}, t_2 = 0 \tag{4}$$

Equation (1) gives

$$x = s + t_1 \tag{5}$$

Equation (2) gives

$$\begin{aligned} y &= su + t_2 \\ &= su \end{aligned} \tag{7}$$

Equation (3) gives

$$u = t_3$$

Hence the solution is

$$
\begin{aligned}
u &= t_3 \\
&= \frac{1}{1+t_1} \\
&= \frac{1}{1+(x-s)} \\
&= \frac{1}{1+\left(x-\frac{y}{u}\right)}
\end{aligned}
$$

Solving for u gives

$$
\begin{aligned}
u\left(1+\left(x-\frac{y}{u}\right)\right) &= 1 \\
u+xu-y &= 1 \\
u(1+x) &= 1+y \\
u &= \frac{1+y}{1+x}
\end{aligned}
$$

2.4.3 $u_t + uu_x = \mu u_{xx}$

problem number 86

From Mathematica symbolic PDE document.

viscous fluid flow with no initial conditions

Solve for $u(x,t)$

$$
u_t + uu_x = \mu u_{xx}
$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to -2c_1\mu \tanh(c_2t + c_1x + c_3) - \frac{c_2}{c_1}\right\}\right\}
$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) + u(x, t)*diff(u(x, t), x) = mu* diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, t))),output='rea
```

$$u(x,t) = \frac{-2c_2^2\mu \tanh\left(c_3t + c_2x + c_1\right) - c_3}{c_2}$$

2.4.4 $u_t + uu_x + \mu u_{xx}$ with IC

problem number 87

From Mathematica symbolic PDE document.

Viscous fluid flow with initial conditions.

Solve for $u(x,t)$

$$u_t + uu_x + \mu u_{xx}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
ic  = u[x, 0] == Piecewise[{{1, x < 0}, {0, x >= 1}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> mu >
```

$$\left\{\left\{u(x,t) \to \frac{1}{\frac{e^{-\frac{t-2x}{4\mu}}\left(\text{erf}\left(\frac{x}{2\sqrt{\mu}\sqrt{t}}\right)+1\right)}{\text{erf}\left(\frac{t-x}{2\sqrt{\mu}\sqrt{t}}\right)+1} + 1}\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x))  = mu*(diff(u(x, t), x$2));
ic  := u(x, 0) =  piecewise(x>=0,0,x<0,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

2.4.5 $u_t + uu_x + \mu u_{xx}$ IC as UnitBox

problem number 88

From Mathematica DSolve help pages.

Viscous fluid flow with initial conditions as UnitBox

Solve for $u(x,t)$

$$u_t + uu_x = \mu u_{xx}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{e^{\frac{t+1}{4\mu}}\left(\operatorname{erf}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)-\operatorname{erf}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)\right)}{e^{\frac{t+1}{4\mu}}\left(\operatorname{Erfc}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)-\operatorname{Erfc}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)\right)+e^{\frac{x}{2\mu}}\left(\operatorname{Erfc}\left(\frac{1-2x}{4\sqrt{\mu t}}\right)+e^{\frac{1}{2}/\mu}\operatorname{Erfc}\left(\frac{2x+1}{4\sqrt{\mu t}}\right)\right)}\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic  := u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

2.5 Black Scholes PDE

Local contents

2.5.1 classic Black Scholes model from finance, European call version

problem number 89

From Mathematica symbolic PDE document.

Solve for $V(S,t)$ where V is the price of the option as a function of stock price S and time t. r is the risk-free interest rate, and σ is the volatility of the stock.

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}=rV-rS\frac{\partial V}{\partial S}$$

With boundary condition $V(S,T)=\max\{S-k,0\}$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation` See the European call version at bottom of the page.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1*sigma^2*D[u[x, t], {x, 2}])/2;
ic  = u[x, 0] == k*Exp[x - 1]*HeavisideTheta[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> k >
```

$$\left\{\left\{u(x,t)\to\frac{1}{2}ke^{\frac{\sigma^2 t}{2}+x-1}\left(\operatorname{erf}\left(\frac{\sigma^2 t+x}{\sqrt{2}\sqrt{t}|\sigma|}\right)+1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t) = 1/2*sigma^2*diff(u(x,t),x$2);
ic  := u(x, 0) = k*exp(x - 1)*Heaviside(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \mathrm{e}^{-1}\left(-i\mathcal{F}^{-1}\left(\frac{\mathrm{e}^{-\frac{s^2\sigma^2 t}{2}}}{s+i}, s, x\right) + \mathcal{F}^{-1}\left(\mathrm{e}^{-\frac{s^2\sigma^2 t}{2}}\mathcal{F}(\mathrm{e}^{x}, x, s), s, x\right)\right) k$$

2.5.2 Boundary value problem for the Black Scholes equation

problem number 90

From Mathematica DSolve help pages.

Solve for $V(t,s)$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2} + (r-q)s\frac{\partial v}{\partial s} - rv(t,s) = 0$$

With boundary condition $v(T,s) = \psi(s)$

Reference https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[v[t, s], t] + (1*sigma^2*s^2*D[v[t, s], {s, 2}])/2 + (r - q)*s*D[v[t, s], s] - r*v[
bc  = v[T, s] == psi[s];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, v[t, s], {t, s}], 60*10]];
```

$$\left\{\left\{v(t,s) \to \frac{e^{r(t-T)}\int_{-\infty}^{\infty}\psi\left(e^{K[1]}\right)\exp\left(-\frac{\left(-K[1]+\frac{1}{2}(t-T)(2q-2r+\sigma^2)+\log(s)\right)^2}{2\sigma^2(T-t)}\right)dK[1]}{\sqrt{2\pi}\sqrt{\sigma^2(T-t)}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(v(t, s), t) +s^2*(diff(v(t, s), s, s))/(2*sigma^2)+(r-q)*s*(diff(v(t, s), s))-r*
ic:=v(T, s) = psi(s);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],v(t,s))),output='
```

$$v(t, s) =$$

2.6 Korteweg-deVries PDE

Local contents

2.6.1 $u_{xxx} + u_t - 6uu_x = 0$

problem number 91

From Mathematica symbolic PDE document.

Korteweg-deVries (waves on shallow water surfaces) with no initial conditions

Solve for $u(x,t)$

$$u_{xxx} + u_t - 6uu_x = 0$$

Reference `https://en.wikipedia.org/wiki/Korteweg%E2%80%93de_Vries_equation`

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {x, 3}] + D[u[x, t], {t}] - 6*u[x, t]*D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{12{c_1}^3 \tanh^2(c_2 t + c_1 x + c_3) - 8{c_1}^3 + c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$3)+ diff(u(x,t),t)-6*u(x,t)* diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 2c_2^2\left(\tanh^2\left(c_3 t + c_2 x + c_1\right)\right) + \frac{-8c_2^3 + c_3}{6c_2}$$

2.7 Tricomi PDE

Local contents

2.7.1 $u_{xx} + yu_{yy} = 0$ with $u(x,0) = 0, u_y(x,0) = x^2$

problem number 92

From Mathematica DSolve helps pages.

Boundary value problem for the Tricomi equation.

Solve for $u(x,y)$

$$u_{xx} + yu_{yy} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -y(y - x^2)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ y*diff(u(x,y),y$2)=0;
bc  := u(x,0)=0,(D[2](u))(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = (x^2 - y)\,y$$

2.7.2 $u_{xx} + xu_{yy} = 0$

problem number 93

Added June 20, 2019

Taken from http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf

Solve for $u(x,y)$

$$u_{xx} + xu_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + x*D[u[x, y], {y, 2}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ x*diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \left(c_1 \operatorname{AiryAi}\left(-(-\_c_1)^{\frac{1}{3}} x\right) + c_2 \operatorname{AiryBi}\left(-(-\_c_1)^{\frac{1}{3}} x\right)\right) \left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\sqrt{\_c_1}\right)\right)$$

2.8 Keldysh equation

Local contents

2.8.1 $xu_{xx} + u_{yy} = 0$

problem number 94

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x, y)$

$$xu_{xx} + u_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \left(c_1 \operatorname{BesselJ}\left(1, 2\sqrt{-\_c_1}\,\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(1, 2\sqrt{-\_c_1}\,\sqrt{x}\right)\right)\left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\sqrt{\_c_1}\right)\right)$$

2.9 Euler-Poisson-Darboux equation

Local contents

2.9.1 $u_{xx} + u_{yy} + \frac{\beta}{x} u_x = 0$

problem number 95

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x,y)$

$$u_{xx} + u_{yy} + \frac{\beta}{x} u_x = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] + beta/x*D[u[x,y],x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}, Assumptions->beta>0], 60*
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2) + beta/x*diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') assumi
```

$$u(x,y) = \left(c_1 \operatorname{BesselJ}\left(\frac{\beta}{2} - \frac{1}{2}, \sqrt{-\_c_1}\, x\right) + c_2 \operatorname{BesselY}\left(\frac{\beta}{2} - \frac{1}{2}, \sqrt{-\_c_1}\, x\right)\right) \left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\right.\right.$$

2.9.2 $u_{xx} - u_{yy} + \frac{\beta}{x} u_x = 0$

problem number 96

Added June 20, 2019

Taken from http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf

Solve for $u(x, y)$

$$u_{xx} - u_{yy} + \frac{\beta}{x} u_x = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] - D[u[x, y], {y, 2}] + beta/x*D[u[x,y],x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}, Assumptions->beta>0], 60*
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)- diff(u(x,y),y$2) + beta/x*diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') assumi
```

$$u(x, y) = c_3 \beta^{\frac{\beta}{2}} \left(-\frac{1}{c_1 (2x + 2y) + 2c_2}\right)^{\frac{\beta}{2}} (c_1(x - y) - c_2)^{-\frac{\beta}{2}}$$

2.9.3 $u_{tt} - u_{xx} - \frac{2}{x} u_x = 0$ with $u(x, 0) = 0, u_t(x, 0) = g(x)$

problem number 97

Added Oct 6, 2019

Problem 2.4.18 from Peter Olver, Intoduction to Partial differential equations, 4th edition.

Solve for $u(x, t)$

$$u_{tt} - u_{xx} - \frac{2}{x} u_x = 0$$

With $u(x,0) = 0, u_t(x,0) = g(x)$. Note, in the book, it says to assume $g(x)$ is even function. In the code below, this assumption is not used. When I find the correct way to implement this assumption in CAS, will have to re-run these.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - 2/x*D[u[x,t],x] == 0;
ic = {u[x,0]==0,Derivative[0,1][u][x,0]==g[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)- diff(u(x,t),x$2) - 2/x*diff(u(x,t),x)=0;
ic:= u(x,0)=0, D[2](u)(x,0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) ),output=
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^{2n+1}(\mathbf{proc}(U)\mathbf{option}\, operator,\ arrow; \quad diff(diff(U,\ x),\ x) + 2 * x\hat{\ }{-1} * diff(U,\ x)\mathbf{end\ p}}{(2n+1)!}$$

2.10 Chaplygin's equation

Local contents

2.10.1 $u_{\theta\theta} + \frac{v^2}{1-\frac{v^2}{c^2}} u_{vv} + v u_v = 0$

problem number 98

Added June 20, 2019 From https://en.wikipedia.org/wiki/Chaplygin%27s_equation

Solve for $u(\theta, v)$

$$u_{\theta\theta} + \frac{v^2}{1 - \frac{v^2}{c^2}} u_{vv} + v u_v = 0$$

Here c is the speed of sound.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[theta, v], {theta, 2}] + v^2/(1-v^2/c^2)* D[u[theta,v],{v,2}]+v*D[u[theta,v],v]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[theta, v], {theta, v}, Assumptions->c>0]
```

Failed

Maple ✓

```
restart;
pde := diff(u(theta,v),theta$2)+ v^2/(1-v^2/c^2)* diff(u(theta,v),v$2)+v*diff(u(theta,v),v)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(theta,v),'build') as
```

$$u(\theta, v) = \frac{\left(c_1 e^{2\theta\sqrt{-c_1}} + c_2\right)\left(c_3 \operatorname{WhittakerM}\left(-\frac{-c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2}\right) + c_4 \operatorname{WhittakerW}\left(-\frac{-c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2}\right.\right.}{v}$$

2.11 Cauchy Riemann PDE's

Local contents

2.11.1 Cauchy Riemann PDE with Prescribe the values of u and v on the x axis

problem number 99

From Mathematica DSolve helps pages.

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

With boundary conditions

$$u(x,0) = x^3$$
$$v(x,0) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x];
bc  = {u[x, 0] == x^3, v[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2, bc}, {u[x, y], v[x, y]}, {x, y}],
```

$$\{\{u(x,y) \to x^3 - 3xy^2, v(x,y) \to 3x^2y - y^3\}\}$$

Maple ✓

```
restart;
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y);
bc  := u(x,0)=x^3,v(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2,bc],[u(x,y),v(x
```

$$\{u(x,y) = x^3 - 3x\,y^2, v(x,y) = -3x^2y + y^3\}$$

2.11.2 Cauchy Riemann PDE With extra term on right side

problem number 100

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} + y$$

Mathematica ✗

```
ClearAll["Global`*"];
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x] + y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2}, {u[x, y], v[x, y]}, {x, y}], 60*1
```

Failed

Maple ✓

```
restart;
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y)+y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2],[u(x,y),v(x,y)
```

$$\left\{u(x,y) = xy + c_1 - i\_F1(-ix+y) + i\_F2(ix+y)\,, v(x,y) = \frac{x^2}{2} + \_F1(-ix+y) + \_F2(ix+y)\right\}$$

2.12 Hamilton-Jacobi PDE

Local contents

2.12.1 Hamilton-Jacobi type PDE

problem number 101

Taken from Maple pdsolve help pages, which is taken from Landau, L.D. and Lifshitz, E.M. Translated by Sykes, J.B. and Bell, J.S. Mechanics. Oxford: Pergamon Press, 1969

Solve for $S(t, \xi, \eta, \phi)$

$$-\frac{\partial}{\partial t}S(t,\xi,\eta,\phi) = 1/2\,\frac{\left(\frac{\partial}{\partial \xi}S(t,\xi,\eta,\phi)\right)^2(\xi^2-1)}{\sigma^2 m\left(-\eta^2+\xi^2\right)} + 1/2\,\frac{\left(\frac{\partial}{\partial \eta}S(t,\xi,\eta,\phi)\right)^2(-\eta^2+1)}{\sigma^2 m\left(-\eta^2+\xi^2\right)} + 1/2\,\frac{\left(\frac{\partial}{\partial \phi}S(t,\xi,\eta\right.}{\sigma^2 m\left(\xi^2-1\right)(}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  -D[s[t, \[Zeta], \[Eta], \[Phi]], t] == ((\[Zeta]^2 - 1)*D[s[t, \[Zeta], \[Eta], \[Ph
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, s[t, \[Zeta], \[Eta], \[Phi]], {t, \[Zeta]
```

Failed

Maple ✓

```
restart;
pde := -diff(S(t,xi,eta,phi),t) =1/2*diff(S(t,xi,eta,phi),xi)^2*(xi^2-1)/sigma^2/m/(xi^2-eta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,'build')),output='real
```

$$S(t,\xi,\eta,\phi) = \phi\_c_4 + t\_c_1 + c_1 + c_2 + c_3 + c_4 - \left(\int \frac{\sqrt{(2\eta^2-2)\,m\,\sigma^2 b\,(\eta) - 2\,(\eta-1)\,(\eta+1)\,(\eta^2\_c_1 + \_c_3)\,m}}{\eta^2-1}\right.$$

2.13 Airy PDE

Local contents

2.13.1 $u_t + u_{xxx} = 0$

problem number 102

Added May 30, 2019.

Airy PDE

Solve for $u(x,t)$

$$u_t + u_{xxx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], {x,3}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to x^3(c_{11}t+c_4)+x^2(c_3-60c_6t)+x(c_2-24c_5t)-3t(c_{11}t+2c_4)-\frac{c_{11}x^6}{120}+c_6x^5+c_5x^4+c_1\right\}\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x$3)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t)=c_4\left(c_1\mathrm{e}^{-\frac{i\sqrt{3}\,x\_c_1^{\frac{1}{3}}}{2}}\mathrm{e}^{-\frac{x\_c_1^{\frac{1}{3}}}{2}}+c_2\mathrm{e}^{\frac{i\sqrt{3}\,x\_c_1^{\frac{1}{3}}}{2}}\mathrm{e}^{-\frac{x\_c_1^{\frac{1}{3}}}{2}}+c_3\mathrm{e}^{x\_c_1^{\frac{1}{3}}}\right)\mathrm{e}^{-t\_c_1}$$

Hand solution

Solve for $u_t + u_{xxx} = 0$ on the real line for $t > 0$. Let $u = T(t)\,X(t)$. The pde becomes

$$T'X + X'''T = 0$$
$$\frac{T'}{T} = -\frac{X'''}{X} = -\lambda$$

Hence $X''' + \lambda X = 0$. This ODE has solution

$$X(x) = C_1 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} - \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_2 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} + \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_3 e^{\lambda^{\frac{1}{3}}x}$$

The ODE $T' + \lambda T = 0$ has the solution $T(t) = C_4 e^{-\lambda t}$. Therefore the solution to the PDE is $T(t)\,X(t)$ given by

$$u(x,t) = C_4 e^{-\lambda t}\left(C_1 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} - \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_2 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} + \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_3 e^{\lambda^{\frac{1}{3}}x}\right)$$

2.14 Nonlinear PDE's

Local contents

2.14.1 Bateman-Burgers $u_t + uu_x = \nu u_{xx}$

problem number 103

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Bateman-Burgers.

Solve for $u(x,t)$

$$u_t + uu_x = \nu u_{xx}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + u[x, t]*D[u[x, t], x] == v*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -2c_1v\tanh(c_2t+c_1x+c_3)-\frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+u(x,t)*diff(u(x,t),x)=v*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\frac{-2c_2^2v\tanh\left(c_3t+c_2x+c_1\right)-c_3}{c_2}$$

2.14.2 Benjamin Bona Mahony $u_t + u_x + uu + x - u_{xxt} = 0$

problem number 104

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_t + u_x + uu + x - u_{xxt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], x] + u[x, t]*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to 12c_1c_2\tanh^2(c_2t+c_1x+c_3)-1-8c_1c_2-\frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)+u(x,t)*diff(u(x,t),x)-diff(u(x,t),x,x,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\frac{12c_3c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)-8c_3c_2^2-c_2-c_3}{c_2}$$

2.14.3 Benjamin Ono $u_t + Hu_{xx} + uu_x = 0$

problem number 105

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Benjamin Ono. Solve for $u(x,t)$

$$u_t + Hu_{xx} + uu_x = 0$$

Important note. H above is meant to be Hilbert transform. `https://en.wikipedia.org/wiki/Benjamin%E2%80%93Ono_equation` However, here in the code below it is taken as just a scalar. Need to correct this when I have time.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + h*D[u[x, t], {x, 2}] + u[x, t]*D[u[x, t], x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to 2c_1h\tanh(c_2t+c_1x+c_3)-\frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+H*diff(u(x,t),x$2)+u(x,t)*diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\frac{2Hc_2^2\tanh\left(c_3t+c_2x+c_1\right)-c_3}{c_2}$$

2.14.4 Born Infeld $(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$

problem number 106

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Born Infeld. Solve for $u(x,t)$

$$(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (1 - D[u[x, t], t]^2)*D[u[x, t], {x, 2}] + 2*D[u[x, t], x]*D[u[x, t], t]*D[D[u[x, t],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to c_1(t+x) + c_2(t-x)\}\}$$

Maple ✓

```
restart;
pde :=(1-diff(u(x,t),t)^2)*diff(u(x,t),x$2)+2*diff(u(x,t),x)*diff(u(x,t),t)*diff(u(x,t),x,t)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = c_7\left(\tanh^3\left(c_1 + c_2(-t+x)\right)\right) + c_5 \tanh\left(c_1 + c_2(-t+x)\right) + c_4$$

2.14.5 Boussinesq $u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$

problem number 107

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Boussinesq. Solve for $u(x,t)$

$$u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{6}\left(-12{c_1}^2 \tanh^2(c_2 t + c_1 x + c_3) - 1 + 8{c_1}^2 + \frac{{c_2}^2}{{c_1}^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)-diff(u(x,t),x$4)- 3 * diff( u(x,t)^2, x$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = -2c_2^2\left(\tanh^2\left(c_3t + c_2x + c_1\right)\right) + \frac{8c_2^4 - c_2^2 + c_3^2}{6c_2^2}$$

2.14.6 Boussinesq type $u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$

problem number 108

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Boussinesq type PDE. Solve for $u(x,t)$

$$u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{6}\left(-12{c_1}^2\tanh^2(c_2t + c_1x + c_3) - 1 + 8{c_1}^2 + \frac{{c_2}^2}{{c_1}^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)-2*alpha*diff( (u(x,t)*diff(u(x,t),x)) ,x) - beta*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{-12c_2^2c_3^2\beta\left(\tanh^2\left(c_3t + c_2x + c_1\right)\right) + \left(8c_3^2\beta - 1\right)c_2^2 + c_3^2}{2c_2^2\alpha}$$

2.14.7 Buckmaster $u_t = (u^4)_{xx} + (u^3)_x$

problem number 109

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Buckmaster. Solve for $u(x,t)$

$$u_t = (u^4)_{xx} + (u^3)_x$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t]^4, {x, 2}] + D[u[x, t]^3, x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t)^4,x$2)+diff(u(x,t)^3,x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \text{RootOf}\left(c_1x + c_2t + c_3 + c_4 + \int^{\_Z} \frac{4c_1^2\_f^3}{c_1\_f^3 + 4c_3c_1^2 - c_2\_f}d\_f\right)$$

Answer in terms of RootOf.

2.14.8 Camassa Holm $u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_{xxx}$

problem number 110

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Camassa Holm. Solve for $u(x,t)$

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 2*k*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] + 3*u[x, t]*D[u[x, t], x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)+2*k*diff(u(x,t),x)- diff(u(x,t),x,x,t)+3*u(x,t)*diff(u(x,t),x)=2*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{\mathrm{RootOf}\left(c_1c_5 - c_1x + c_1\left(\int^{-\frac{-\_Z^2+c_2}{c_1}} \frac{\sqrt{c_1\_f+c_2}}{\sqrt{-c_3c_1^3\_f+c_1\_f^3+2c_1\_f^2k-c_1^2c_2c_3+c_2\_f^2-c_4c_1^2}}d\_f\right) - c_2t - c_3\right)^2 -}{c_1}$$

Answer in terms of RootOf.

2.14.9 Chaffee Infante $u_t = u_{xx} + \lambda(u^3 - u) = 0$

problem number 111

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Chaffee Infante equation. Solve for $u(x,t)$

$$u_t = u_{xx} + \lambda(u^3 - u) = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] - D[u[x, t], {x, 2}] + lambda*(u[x, t]^3 - u[x, t]) == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)-diff(u(x,t),x$2)+lambda*(u(x,t)^3-u(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{\tanh\left(-\frac{3\lambda t}{4} + \frac{\sqrt{2}\,\sqrt{\lambda}\,x}{4} + c_1\right)}{2} - \frac{1}{2}$$

2.14.10 Clarke. $(\theta_t - \gamma e^{\theta})_{tt} = (\theta_t - e^{\theta})_{xx}$

problem number 112

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Clarke equation. Solve for $\theta(x,t)$

$$(\theta_t - \gamma e^{\theta})_{tt} = (\theta_t - e^{\theta})_{xx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[theta[x, t], t] - gamma*Exp[theta[x, t]], {t, 2}] == D[D[theta[x, t], t] - Exp[th
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, theta[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(diff(theta(x,t),t)-g*exp(theta(x,t)),t$2) = diff( diff(theta(x,t),t)-exp(theta(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,theta(x,t))),output='r
```

sol=()

2.14.11 Degasperis Procesi $u_t - u_{xxt} + 4uu_x = 3u_xu_xx + uu_{xxx}$

problem number 113

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Degasperis Procesi. Solve for $u(x,t)$

$$u_t - u_{xxt} + 4uu_x = 3u_xu_xx + uu_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] - D[D[u[x, t], {x, 2}], t] + 4*u[x, t]*D[u[x, t], x] == 3*D[u[x, t], x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)-diff(u(x,t),x,x,t)+4*u(x,t)*diff(u(x,t),x)=3*diff(u(x,t),x)*diff(u(x,t
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{\_F1(x)}{-\_c_2 t + c_2} \textbf{where} \left[\left\{\left\{\_F1(x) = \_a \textbf{where} \left[\left\{\frac{\_a\_b(\_a)^2 \left(\frac{d^2}{d\_a^2}\_\mathrm{b}(\_a)\right) + \_a\_b(\_a) \left(\frac{d}{d\_a-}\right.}{}\right.\right.\right.\right.\right.$$

But still has unresolved ODE's in solution

2.14.12 Dym equation $u_t = u^3 u_{xxx}$

problem number 114

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Dym equation. Solve for $u(x,t)$

$$u_t = u^3 u_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == u[x, t]^3*D[u[x, t], {x, 3}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)=u(x,t)^3 * diff(u(x,t),x$3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{\mathrm{RootOf}\left(c_3 + x - \left(\int^{\_Z} \frac{1}{\mathrm{RootOf}\left(c_2 + 2\left(\int^{\_Z} \frac{\_h}{\_h^2 + 2\, 2^{\frac{1}{3}} \left(-\_c_1^2\right)^{\frac{1}{3}} \mathrm{RootOf}\left(c_1 2^{\frac{1}{3}} \left(-\_c_1^2\right)^{\frac{1}{3}} \_h \, \mathrm{AiryBi}(\_Z) + 2c_1\_c_1 \, \mathrm{AiryBi}(1, \_Z)\right.}\right.\right.}\right.\right.}{\left(-3t\_c_1 + c_4\right)^{\frac{1}{3}}}$$

has RootOf

2.14.13 Estevez Mansfield Clarkson

$$u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$$

problem number 115

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Estevez Mansfield Clarkson equation. Solve for $u(x,y,t)$

$$u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], t], {y, 3}] + beta*D[u[x, y, t], y]*D[D[u[x, y, t], y], t] + beta*D[u
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to \frac{6c_1(x)\tanh\left(-4t(c_1(x))^3+yc_1(x)+c_3(x)\right)}{\beta}+c_4(x)\right\}\right\}$$

Maple ✓

```
restart;
beta='beta';
pde := diff(u(x,y,t),t,y,y,y)+ beta*diff(u(x,y,t),y)*diff(u(x,y,t),y,t) +  beta*diff(u(x,y,t
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t)=\frac{6c_3\tanh\left(-4c_3^3t+c_2x+c_3y+c_1\right)}{\beta}+c_5$$

2.14.14 Fisher's $u_t = u(1-u) + u_{xx}$

problem number 116

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Fisher's equation. Solve for $u(x,t)$

$$u_t = u(1-u) + u_{xx}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == u[x, t]*(1 - u[x, t]) + D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5t-\sqrt{6}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5t-i\sqrt{6}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5t-i\sqrt{6}x-12c_3\right)\right)\right)\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5t+i\sqrt{6}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5t+i\sqrt{6}x-12c_3\right)\right)\right)\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5t+\sqrt{6}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{5t}{12}-\frac{x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{5t}{12}-\frac{ix}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5t}{12}-\frac{ix}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{5t}{12}+\frac{ix}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5t}{12}+\frac{ix}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{5t}{12}+\frac{x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= u(x,t)*(1-u(x,t))+ diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,u(x,t)))
```

$$\{u(x,t)=1\}$$
$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$
$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$
$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$
$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$
$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$
$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$
$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$
$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$

2.14.15 Hunter Saxton $(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$

problem number 117

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Hunter Saxton. Solve for $u(x,t)$

$$(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[u[x, t], t] + u[x, t]*D[u[x, t], x], x] == (1*D[u[x, t], x]^2)/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff( (diff(u(x,t),t)+ u(x,t)* diff(u(x,t),x)) , x) = 1/2* (diff(u(x,t),x))^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{2\,\mathrm{RootOf}\left(-c_2\_c_1^3 - x\_c_1^3 + 2c_1^2 \ln\left(\sqrt{\_Z}\_c_1 + c_1\right) + \_Z\_c_1^2 - 2c_1\sqrt{\_Z}\_c_1\right)}{t\_c_1 + 2c_3}$$

with RootOf

2.14.16 Kadomtsev Petviashvili $(u_t + uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$

problem number 118

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Kadomtsev Petviashvili. Solve for $u(x,y,t)$

$$(u_t + uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], t] + u[x, y, t]*D[u[x, y, t], x] + eps^2*D[u[x, y, t], {x, 3}], t] +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to -\frac{12c_3{c_1}^3\text{eps}^2\tanh^2(c_3t+c_1x+c_2y+c_4)+c_3(c_3-8{c_1}^3\text{eps}^2)+{c_2}^2\lambda}{c_1c_3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff( diff(u(x,y,t),t)+u(x,y,t)*diff(u(x,y,t),x)+epsilon^2* diff(u(x,y,t),x$3),x)+ la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t)=\frac{-12c_2^4\epsilon^2\left(\tanh^2\left(c_4t+c_2x+c_3y+c_1\right)\right)+8c_2^4\epsilon^2-c_3^2\lambda-c_2c_4}{c_2^2}$$

2.14.17 Klein Gordon $u_{xx}+u_{yy}+\lambda u^p=0$

problem number 119

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Klein Gordon (nonlinear). Solve for $u(x,y)$

$$u_{xx}+u_{yy}+\lambda u^p=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + lambda*u[x, y]^p == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+lambda*u(x,y)^p=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

sol=()

2.14.18 Klein Gordon $u_{xx} + u_{yy} + u^2 = 0$

problem number 120

Added December 27, 2018.

Special case Klein Gordon (nonlinear). Solve for $u(x,y)$

$$u_{xx} + u_{yy} + u^2 = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + u[x, y]^2 == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+u(x,y)^2=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = -6\left(c_1^2 + c_2^2\right) \operatorname{WeierstrassP}\left(c_1x + c_2y + 2c_3, 0, c_4\right)$$

2.14.19 Khokhlov Zabolotskaya $u_{xt} - (uu_x)_x = u_{yy}$

problem number 121

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Khokhlov Zabolotskaya. Solve for $u(x,y,t)$

$$u_{xt} - (uu_x)_x = u_{yy}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], x], t] - D[u[x, y, t]*D[u[x, y, t], x], x] == D[u[x, y, t], {y, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y,t),x,t)- diff( (u(x,y,t)* diff(u(x,y,t),x)) ,x ) = diff(u(x,y,t),y$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = \frac{c_1c_3 - c_2^2 + \sqrt{c_1^2c_3^2 - 2c_1c_2^2c_3 + c_2^4 + 2c_4\left(c_3t + c_1x + c_2y + c_4\right)c_1^2 + 2c_5c_1^2}}{c_1^2}$$

2.14.20 Korteweg de Vries (KdV) $u_t + (u_x)^3 + 6uu_x = 0$

problem number 122

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Korteweg de Vries (KdV). Solve for $u(x,t)$

$$u_t + u_{xxx} + 6uu_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], {x,3}] + 6*u[x, t]*D[u[x, t], x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{12{c_1}^3\tanh^2(c_2t+c_1x+c_3)-8{c_1}^3+c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+ diff( u(x,t),x$3 ) + 6 * u(x,t)* diff(u(x,t),x) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)+\frac{8c_2^3-c_3}{6c_2}$$

2.14.21 Lin Tsien $2u_{tx}+u_xu_{xx}-u_{yy}=0$

problem number 123

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Lin Tsien equation. Solve for $u(x,y,t)$

$$2u_{tx}+u_xu_{xx}-u_{yy}=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[u[x, y, t], t, x] + D[u[x, y, t], x]*D[u[x, y, t], {x, 2}] - D[u[x, y, t], {y, 2}
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=2*diff(u(x,y,t),t,x)+ diff(u(x,y,t),x)* diff(u(x,y,t),x$2) - diff(u(x,y,t),y$2) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = c_4 + c_5(c_3t + c_1x + c_2y + c_4)$$

2.14.22 Liouville $u_{xx} + u_{yy} + e^{\lambda u} = 0$

problem number 124

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Liouville equation. Solve for $u(x,y)$

$$u_{xx} + u_{yy} + e^{\lambda u} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + Exp[lam*u[x, y]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)+exp(lambda*u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

2.14.23 Plateau $(1+u_y^2)u_{xx} - 2u_xu_yy_{xy} + (1+u_x^2)u_{yy} = 0$

problem number 125

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Plateau. Solve for $u(x,y)$

$$(1+u_y^2)u_{xx} - 2u_xu_yy_{xy} + (1+u_x^2)u_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (1 + D[u[x, y], y]^2)*D[u[x, y], {x, 2}] - 2*D[u[x, y], x]*D[u[x, y], y]*D[u[x, y], x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(1+diff(u(x,y),y)^2)* diff(u(x,y),x$2)-2* diff(u(x,y),x)*
       diff(u(x,y),y)*diff(u(x,y),x,y)+
       (1+diff(u(x,y),x)^2)*diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = c_7\left(\tanh^3\left(c_1 + c_2(-iy+x)\right)\right) + c_5 \tanh\left(c_1 + c_2(-iy+x)\right) + c_4$$

2.14.24 Rayleigh $u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$

problem number 126

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Rayleigh. Solve for $u(x,t)$

$$u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] == epsilon*(D[u[x, t], t] - D[u[x, t], t]^3);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)=epsilon*(diff(u(x,t),t)-diff(u(x,t),t)^3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{x^2\_c_1}{2} + c_1 x + c_2 + c_4 + \int \operatorname{RootOf}\left(c_3 + t + \int^{\_Z} \frac{1}{\_f^3\epsilon - \_f\epsilon - \_c_1} d\_f\right) dt$$

Has RootOf

2.14.25 Sawada Kotera

$$u_t + 45u^2 u_x + 15u_x u_{xx} + 15u u_{xxx} + u_{xxxxx} = 0$$

problem number 127

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sawada Kotera. Solve for $u(x,t)$

$$u_t + 45u^2 u_x + 15u_x u_{xx} + 15u u_{xxx} + u_{xxxxx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 45*u[x, t]^2*D[u[x, t], x] + 15*D[u[x, t], x]*D[u[x, t], {x, 2}] + 15
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{u(x,t)\to -\frac{4}{3}{c_1}^2\left(-2+3\tanh^2\left(-16{c_1}^5t+c_1x+c_3\right)\right)\right\}$$

$$\left\{u(x,t)\to \frac{-30{c_1}^{5/2}\tanh^2(c_2t+c_1x+c_3)+20{c_1}^{5/2}+\sqrt{20{c_1}^5-5c_2}}{15\sqrt{c_1}}\right\}$$

$$\left\{u(x,t)\to \frac{20{c_1}^{5/2}-\sqrt{20{c_1}^5-5c_2}}{15\sqrt{c_1}}-2{c_1}^2\tanh^2(c_2t+c_1x+c_3)\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+45* u(x,t)^2* diff(u(x,t),x)+ 15* diff(u(x,t),x)*diff(u(x,t),x$2)+15*u
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,u(x,t)))
```

$$\{u(x,t)=c_4\}$$

$$\left\{u(x,t)=-4c_2^2\left(\tanh^2\left(-16c_2^5t+c_2x+c_1\right)\right)+\frac{8c_2^2}{3}\right\}$$

$$\left\{u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)-\frac{-20c_2^3+\sqrt{20c_2^6-5c_3c_2}}{15c_2}\right\}$$

$$\left\{u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)+\frac{20c_2^3+\sqrt{20c_2^6-5c_3c_2}}{15c_2}\right\}$$

2.14.26 Sine Gordon $\phi_{tt} - \phi_{xx} + \sin\phi = 0$

problem number 128

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Sine Gordon. Solve for $u(x,t)$

$$\phi_{tt} - \phi_{xx} + \sin\phi = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[phi[x, t], {t, 2}] - D[phi[x, t], {x, 2}] + Sin[phi[x, t]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(phi(x,t),t$2)-diff(phi(x,t),x$2)+sin(phi(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,phi(x,t))),output='rea
```

sol=()

2.14.27 Sinh Gordon $u_{xt} = \sinh u$

problem number 129

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Sinh Gordon. Solve for $u(x,t)$

$$u_{xt} = \sinh u$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], x, t] == Sinh[u[x, t]];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,t),x,t)=sinh(u(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

sol=()

2.14.28 Sinh Poisson $u_{xx} + u_{yy} + \sinh u = 0$

problem number 130

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sinh Poisson. Solve for $u(x,t)$

$$u_{xx} + u_{yy} + \sinh u = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + Sinh[u[x, y]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+ sinh(u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

2.14.29 Thomas equation $u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$

problem number 131

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Thomas equation. Solve for $u(x,t)$

$$u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], x, y] + alpha*D[u[x, y], x] + beta*D[u[x, y], y] + nu*D[u[x, y], x]*D[u[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x,y)+alpha*diff(u(x,y),x)+beta*diff(u(x,y),y)
      +nu* diff(u(x,y),x)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \frac{-2\alpha y - 2\beta x - \ln\left(\frac{\alpha^2-2\alpha\beta+\beta^2-4\_c_1\nu}{\left(c_1 \mathrm{e}^{(x-y)\sqrt{\alpha^2-2\alpha\beta+\beta^2-4\_c_1\nu}}-c_2\right)^2\nu^2}\right) - \ln\left(\frac{\alpha^2+2\alpha\beta+\beta^2-4\_c_1\nu}{\left(c_3 \mathrm{e}^{(x+y)\sqrt{\alpha^2+2\alpha\beta+\beta^2-4\_c_1\nu}}-c_4\right)^2\nu^2}\right) + (}{2\nu}$$

2.14.30 phi equation $\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$

problem number 132

Added December 27, 2018.

Taken from **https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations**

phi equation. Solve for $phi(x,t)$

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[phi[x, t], t, t] - D[phi[x, t], x, x] - phi[x, t] + phi[x, t]^3 == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

$$\left\{\phi(x,t) \to -\tanh\left(c_2 t - \sqrt{\frac{1}{2} + c_2{}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to \tanh\left(c_2 t - \sqrt{\frac{1}{2} + c_2{}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to -\tanh\left(c_2 t + \sqrt{\frac{1}{2} + c_2{}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to \tanh\left(c_2 t + \sqrt{\frac{1}{2} + c_2{}^2} x + c_3\right)\right\}$$

Maple ✓

```
restart;
pde := diff(phi(x,t),t$2)-diff(phi(x,t),x$2) - phi(x,t) + phi(x,t)^3=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,phi(x,t)
```

$$\{\phi(x,t)=-1\}$$
$$\{\phi(x,t)=1\}$$
$$\left\{\phi(x,t)=-\tanh\left(c_2x+c_1-\frac{\sqrt{4c_2^2-2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t)=\tanh\left(c_2x+c_1+\frac{\sqrt{4c_2^2-2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t)=\tanh\left(c_2x+c_1-\frac{\sqrt{4c_2^2-2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t)=-\tanh\left(c_2x+c_1+\frac{\sqrt{4c_2^2-2}\,t}{2}\right)\right\}$$

2.15 more miscellaneous

Local contents

2.15.1 $SS_{xy} + S_x S_y = 1$

problem number 133

Taken from Maple pdsolve help pages, problem 4. A second order PDE

Solve for $S(x,y)$

$$SS_{xy} + S_x S_y = 1$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  s[x, y]*D[s[x, y], x, y] + D[s[x, y], x]*D[s[x, y], y] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, s[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := S(x,y)*diff(S(x,y),y,x) + diff(S(x,y),x)*diff(S(x,y),y) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,S(x,y),'build')),outpu
```

$$S(x,y) = \operatorname{RootOf}\left(\_Z^2 - 2xy - 2\_F1(y) - \_F2(x)\right)$$

2.15.2 $u_{rr} + u_{\theta\theta} = 0$

problem number 134

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Second order PDE in Polar coordinates. Solve for $u(r, \theta)$

$$u_{rr} + u_{\theta\theta} = 0$$

With boundary conditions

$$u(2, \theta) = 3\sin(2\theta) + 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + D[u[r, theta], {theta, 2}] == 0;
bc  = u[2, theta] == 3*Sin[2*theta] + 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta) \to \boxed{\left\{ \begin{array}{ccc} 6e^{4-2r}\cos(\theta)\sin(\theta)+1 & r \geq 2 & \text{if } r \neq 2 \\ \text{Indeterminate} & \text{True} & \end{array} \right.}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2)+diff(u(r, theta), theta$2) = 0;
bc  := u(2, theta) = 3*sin(2*theta)+1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r,theta),meth
```

$$u(r,\theta) = -\frac{3i\mathrm{e}^{2i\theta-2r+4}}{2} + \frac{3i\mathrm{e}^{-2i\theta+2r-4}}{2} + 1$$

2.15.3 $u_{xx} + yu_{yy} = 0$

problem number 135

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Laplace like PDE with polynomial solution. Solve for $u(x, y)$

$$u_{xx} + yu_{yy} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -y(y - x^2)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+y*(diff(u(x, y), y$2)) = 0;
bc  := u(x,0)=0, eval(diff(u(x,y),y),y=0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(x, y))),outpu
```

$$u(x,y) = (x^2 - y)\,y$$

2.15.4 $u_t + u_{xxx} = 0$

problem number 136

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Third oder PDE. Solve for $u(x,y)$

$$u_t + u_{xxx} = 0$$

With initial conditions

$$u(x,0) = f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == -D[u[x, t], {x, 3}];
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{\int_{-\infty}^{\infty} e^{iK[1](tK[1]^2+x)} \int_{-\infty}^{\infty} e^{-ixK[1]} f(x)\, dx dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=- diff(u(x, t), x$3);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t))),output=
```

$$u(x,t) = \frac{\int_{-\infty}^{\infty} \frac{4\pi\sqrt{-\frac{x+\zeta}{(-t)^{\frac{1}{3}}}}\,\mathrm{BesselK}\left(\frac{1}{3}, \frac{2\sqrt{3}\left(-\frac{x+\zeta}{(-t)^{\frac{1}{3}}}\right)^{\frac{3}{2}}}{9}\right) f(-\zeta)}{3(-t)^{\frac{1}{3}}} d\zeta}{4\pi^2}$$

2.15.5 $u_{xy} = \sin(x)\sin(y)$

problem number 137

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

PDE solved by Laplace transform. Solve for $u(x, y)$

$$u_{xy} = \sin(x)\sin(y)$$

With boundary conditions

$$u(x,0) = 1 + \cos(x)$$
$$\frac{\partial u}{\partial y}(0,y) = -2\sin y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], y, x] == Sin[x]*Sin[y];
bc  = {u[x, 0] == 1 + Cos[x], Derivative[0, 1][u][0, y] == -2*Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

$$\{\{u(x,y) \to (\cos(x)+1)\cos(y)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), y,x)=sin(x)*sin(y);
bc  := u(x,0)=1+cos(x),eval( diff(u(x,y),y),x=0)=-2*sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y))),output=
```

$$u(x,y) = (\cos(x)+1)\cos(y)$$

2.15.6 $w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3}$

problem number 138

Added December 20, 2018.

Example 25, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Linear PDE, initial conditions at $t = 1$. Solve for $w(x_1, x_2, x_3, t)$

$$w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3}$$

With initial condition $w(x_1, x_2, x_3, 1) = e^a x_1^2 + x_2x_3$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], {x1, 2}] + D[w[x1, x2, x3, t], {x2, 2}]
ic  = w[x1, x2, x3, 1] == Exp[a]*x1^2 + x2*x3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\{\{w(\text{x1}, \text{x2}, \text{x3}, t) \to e^a (2t + \text{x1}^2 - 2) + \text{x2x3}\}\}$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t) = diff(w(x1, x2, x3, t), x1$2)+diff(w(x1, x2, x3, t), x2$2)
ic  := w(x1, x2, x3, 1) = exp(a)*x1^2+x2*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = x2x3 + (x1^2 + 2t - 2)\,\mathrm{e}^a$$

2.15.7 Linear PDE, initial conditions at $t = t_0$

problem number 139

Added December 20, 2018.

Example 26, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w}{\partial t} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} + \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition $w(x_1, x_2, x_3, t_0) = e^{x_1} + x_2 - 3x_3$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1, x3] +
ic  = w[x1, x2, x3, t0] == Exp[x1] + x2 - 3*x3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\left\{\left\{w(\text{x1},\text{x2},\text{x3},t)\to e^{\text{x1}}+\text{x2}-3\text{x3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff(w
ic  := w(x1, x2, x3, t0) = exp(x1)+x2-3*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = x2 - 3x3 + e^{x1}$$

2.15.8 second order in time, Linear PDE, initial conditions at $t = t_0$

problem number 140

Added December 20, 2018.

Example 27, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w^2}{\partial t^2} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} - \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition

$$w(x_1,x_2,x_3,t_0)=x_1^3x_2^2+x_3$$
$$\frac{\partial w}{\partial t}(x_1,x_2,x_3,t_0)=-x_2x_3+x_1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], {t, 2}] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1,
ic  = {w[x1, x2, x3, t0] == x1^3*x2^2 + x3, Derivative[0, 0, 0, 1][w][x1, x2, x3, t0] == -(x
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\left\{\left\{w(\text{x1},\text{x2},\text{x3},t)\rightarrow t^2\left(3\text{t0}^2\text{x1}-\frac{\text{t0}}{2}+3\text{x1}^2\text{x2}\right)+t^3\left(\frac{1}{6}-2\text{t0x1}\right)+\frac{t^4\text{x1}}{2}+t\left(-2\text{t0}^3\text{x1}+\frac{\text{t0}^2}{2}-6\text{t0x1}^2\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t$2)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff
ic  := w(x1, x2, x3, t0) = x1^3*x2^2+x3, eval( diff( w(x1,x2,x3,t),t),t=t0)=-x2*x3+x1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1,x2,x3,t)=3t^2x1^2x2+\frac{t0^4x1}{2}+x1^3x2^2+\frac{t^3}{6}-tx2x3+\frac{(-12x1t-1)\,t0^3}{6}+\frac{(18x1\,t^2+18x1^2x2+3t)\,t0}{6}$$

2.15.9 Einstein-Weiner $u_t=-\beta u_x+Du_{xx}$

problem number 141

Added January 2, 2018.

Einstein-Weiner PDE. Solve for $u(x,t)$ with $x>0,t>0$

$$u_t=-\beta u_x+Du_{xx}$$

Assuming $\beta>0,D>0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == beta*D[u[x, t], x] + d*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}, Assumptions -> {beta > 0,
```

$$\{\{u(x,t) \to \cosh(c_2(\beta t + c_2 dt + x) + c_1) + \sinh(c_2(\beta t + c_2 dt + x) + c_1) + 1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=-beta*diff(u(x,t),x)+d*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t) = c_3 \left(c_1 e^{\frac{\sqrt{\beta^2 - 4d\_c_1}\, x}{2d}} + c_2 e^{-\frac{\sqrt{\beta^2 - 4d\_c_1}\, x}{2d}} \right) \sqrt{e^{\frac{\beta x}{d}}}\, e^{-t\_c_1}$$

2.15.10 Using integral transforms.

problem number 142

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$x^2 \frac{\partial^2}{\partial x^2} u(x,y) + x \frac{\partial}{\partial x} u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = 0$$

With boundary conditions

$$u(x,1) = \begin{cases} 1 & 0 \le x \text{ and } x < 1 \\ 0 & 1 < x \end{cases}$$

$$u(x,0) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x^2*D[u[x, y], {x, 2}] + x*D[u[x, y], x] + D[u[x, y], {y, 2}] ==0;
bc = {u[x, 0] == 0, u[x, 1] == Piecewise[{{1, 0 <= x < 1}, {0, 1 < x}}]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{\text{Integrate}\left[\frac{x^{-\frac{149}{33}-iK[1]}\csc\left(\frac{149}{33}+iK[1]\right)\sin\left(y\left(\frac{149}{33}+iK[1]\right)\right)}{\frac{149}{33}+iK[1]},\{K[1],-\infty,\infty\},\text{Assumptions}\to K[1]\in \mathbb{R}\right.}{2\pi}\right.\right.$$

Maple ✓

```
restart;
pde := x^2*diff(u(x, y), x, x) + x*diff(u(x, y), x) + diff(u(x, y), y, y) = 0;
iv := u(x, 0) = 0, u(x, 1) = piecewise(0 <= x and x < 1, 1, 1 < x, 0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(x,y))),output='
```

$$u(x,y) =$$

CHAPTER 3

Schrodinger PDE

Local contents

3.1 1D

Local contents

3.1.1 Logan textbook, page 30

problem number 143

From page 30, David J Logan textbook, applied PDE textbook.

Schrodinger PDE with zero potential (Logan p. 30)

Solve

$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,0) = 0$$

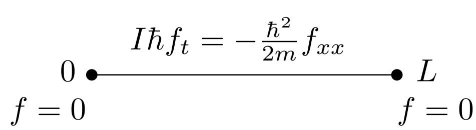


Figure 3.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
bc  = {f[0, t] == 0, f[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L >
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t)\to\sum_{n=1}^{\infty}e^{-\frac{ihn^2\pi^2t}{2L^2m}}c_n\sin\left(\frac{n\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t)) assuming
```

$$f(x,t)=\sum_{n=1}^{\infty}\_F1(n)\,\mathrm{e}^{-\frac{i\pi^2h\,n^2t}{2L^2m}}\sin\left(\frac{\pi n x}{L}\right)$$

3.1.2 From Mathematica help pages

problem number 144

Taken from Mathematica DSolve help pages

Initial value problem with Dirichlet boundary conditions. 1D, zero potential.

Solve for $f(x,t)$

$$If_t=-2f_{xx}$$

With boundary conditions

$$f(5,t)=0$$
$$f(10,t)=0$$

And initial conditions $f(x,2)=f(x)$ where $f(x)=-350+155x-22x^2+x^3$

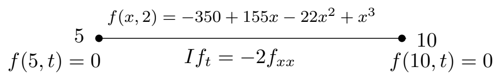


Figure 3.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, t], t] == -2*D[f[x, t], {x, 2}];
 g[x_] := -350 + 155*x - 22*x^2 + x^3;
ic  = f[x, 2] == g[x];
bc  = {f[5, t] == 0, f[10, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, f[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t)\to\sum_{n=1}^{\infty}\frac{100(7+8(-1)^n)\,e^{-\frac{2}{25}in^2\pi^2(t-2)}\sin\left(\frac{1}{5}n\pi(x-5)\right)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
pde :=I*diff(f(x,t),t)=-2*diff(f(x,t),x$2);
bc:=f(5,t)=0,f(10,t)=0;
g:=x->-350+155*x-22*x^2+x^3;
ic:=f(x,2)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],f(x,t))),outpu
```

$$f(x,t)=\sum_{n=1}^{\infty}\frac{(800(-1)^n+700)\,\mathrm{e}^{-\frac{2i\pi^2(t-2)n^2}{25}}\sin\left(\frac{\pi(x-5)n}{5}\right)}{\pi^3n^3}$$

3.1.3 David Griffiths, page 47

problem number 145

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47.

Solve for $f(x,t)$

$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$$

With initial conditions $f(x,0) = Ax(a-x)$ for $0 \leq x \leq a$ and zero otherwise.

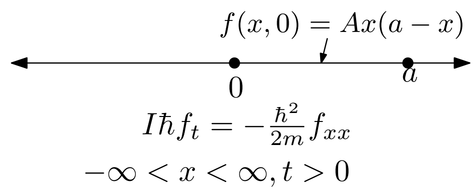


Figure 3.3: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = Piecewise[{{A*x*(a - x), 0 <= x <= a}, {0, True}}];
pde =  I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, f[x, 0] == ic}, f[x, t], {x, t}, Assumpti
```

$$\left\{\left\{f(x,t)\to \frac{\int_{-\infty}^{\infty} -\frac{A\exp\left(-\frac{iK[1](2am-2xm+htK[1])}{2m}\right)\left(aK[1]+e^{iaK[1]}(aK[1]+2i)-2i\right)}{K[1]^3}dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde :=I*h*diff(f(x,t),t) = -h^2/(2*m)*diff(f(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',dsolve([pde,ic],f(x,t)) assuming a
sol:=convert(sol,Int);
```

$$f(x,t) = -\frac{\left(a\left(\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-\frac{i\left(\frac{hst}{2}+(a-x)m\right)s}{m}}}{s^2}ds\right)+a\left(\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-\frac{ih\,s^2t}{2m}+isx}}{s^2}ds\right)-2i\left(\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-\frac{i\left(\frac{hst}{2}+(a-x)m\right)s}{m}}}{s^3}ds\right)+2i\left(\int_{-\infty}^{\infty}\right.\right.}{2\pi}$$

3.1.4 David Griffiths, page 47

problem number 146

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47. This is the same as the above problem but has an extra $V(x)f(x,t)$ terms where $V(x)$ is the infinite square well potential defined by $V(x) = 0$ if $0 \le x \le a$ and $V(x) = \infty$ otherwise.

Solve for $f(x,t)$

$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx} + V(x)f(x,t)$$

With initial conditions $f(x,0) = Ax(a-x)$ for $0 \le x \le a$ and zero otherwise.

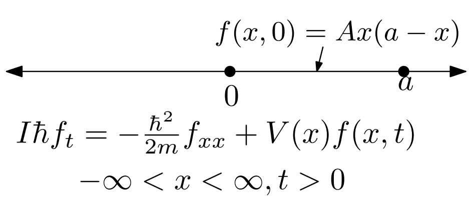


Figure 3.4: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[3*Pi*y]);
bc  = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
pde =  I*h*D[f[x, y, t], t] == -((h^2*(D[f[x, y, t], {x, 2}] + D[f[x, y, t], {y, 2}]))/(2*m)
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, f[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{f(x,y,t)\to\sqrt{2}e^{-\frac{5i\pi^2ht}{m}}\left(\sin(\pi x)\sin(3\pi y)+\sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2ht}{2m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
V:=x->piecewise(0<=x and x<=a,0,infinity);
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde :=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2) +V(x)*f(x,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],f(x,t)) assuming
```

Failed to convert to latex

3.1.5 Deep well

problem number 147

Added May 2, 2021. See my HW10. Physics 3041.

Consider $\psi(x,t)$ for $0\leq x\leq L$. Given $\psi(0,t)=\psi(L,t)=0$ and

$$\psi(x,0)=\begin{cases}A\sin\left(\frac{2\pi x}{L}\right) & 0\leq x\leq\frac{L}{2}\\ 0 & \frac{L}{2}\leq x\leq L\end{cases}$$

Find $\psi(x,t)$ that satisfies the following partial differential equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2\mu}\frac{\partial^2\psi}{\partial t^2}\tag{1}$$

Where $A,L,\hbar,\mu$ are positive constants.

Mathematica ✓

```
ClearAll["Global`*"];
pde = I*hbar*D[w[x, t], t] == -hbar^2/(2*mu)*D[w[x, t], {x, 2}];
bc = {w[0, t] == 0, w[L, t] == 0};
ic = w[x, 0] == Piecewise[{{A*Sin[2*Pi*x/L], 0 < x < L/2}, {0, L/2 < x < L}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, w[x, t], {x, t}, Assumptions ->
sol = sol /. K[1] -> n;
```

$$\left\{\left\{w(x,t)\to\sum_{n=1}^{\infty}-\frac{4Ae^{-\frac{i\text{hbar}n^2\pi^2t}{2L^2\mu}}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{L}\right)}{(n^2-4)\pi}\right\}\right\}$$

Does not handle $n=2$ case correctly. Division by zero

Maple ✓

```
restart;
pde:=I*hbar*diff(w(x,t),t) = -hbar^2/(2*mu)*diff(w(x,t),x$2);
bc:=w(0,t)=0,w(L,t)=0;
ic:=w(x,0)=piecewise(0<x and x<L/2,A*sin(2*Pi*x/L),L/2<x and x<L, 0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],w(x,t)) assumi
```

$$w(x,t)=\frac{A\,\mathrm{e}^{-\frac{2i\pi^2hbart}{L^2\mu}}\sin\left(\frac{2\pi x}{L}\right)}{2}+\frac{4A\,\mathrm{e}^{-\frac{i\pi^2hbart}{2L^2\mu}}\sin\left(\frac{\pi x}{L}\right)}{3\pi}+\sum_{n=3}^{\infty}\left(-\frac{4A\,\mathrm{e}^{-\frac{i\pi^2hbar\,n^2t}{2L^2\mu}}\sin\left(\frac{\pi nx}{L}\right)\sin\left(\frac{\pi n}{2}\right)}{\pi\left(n^2-4\right)}\right)$$

3.1.6 From Mathematica help pages

problem number 148

Taken from Mathematica DSolve help pages

Solve a Schrodinger equation with potential over the whole real line.

Solve for $f(x,t)$

$$If_t=-f_{xx}+2x^2f(x,t)$$

With boundary conditions

$$f(-\infty,t)=0$$
$$f(\infty,t)=0$$

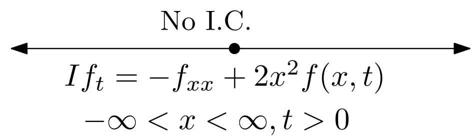


Figure 3.5: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, t], t] == -D[f[x, t], {x, 2}] + 2*x^2*f[x, t];
bc  = {f[-Infinity, t] == 0, f[Infinity, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t) \to \sum_{n=0}^{\infty} e^{-\frac{x^2+2i(2n+1)t}{\sqrt{2}}} c_n \,\text{HermiteH}\left(n, \sqrt[4]{2}x\right)\right\}\right\}$$

Maple ✗

```
restart;
pde :=I*diff(f(x,t),t)=-diff(f(x,t),x$2)+2*x^2*f(x,t);
bc:=f(-infinity ,t)=0,f(infinity,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t))),output='
```

$$f(x,t) = 0$$

Trivial solution. Maple does not support ∞ in boundary conditions

3.2 2D

Local contents

3.2.1 In a square, zero potential

problem number 149

With initial and boundary conditions. In a square, zero potential.

Solve for $f(x,y,t)$

$$If_t = -\frac{\hbar^2}{2m}\nabla^2 f(x,y)$$

With boundary conditions

$$\begin{aligned}
f(0,y,t) &= 0\\
f(1,y,t) &= 0\\
f(x,1,t) &= 0\\
f(x,0,t) &= 0
\end{aligned}$$

And initial conditions $f(x,y,0) = \sqrt{2}(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y))$

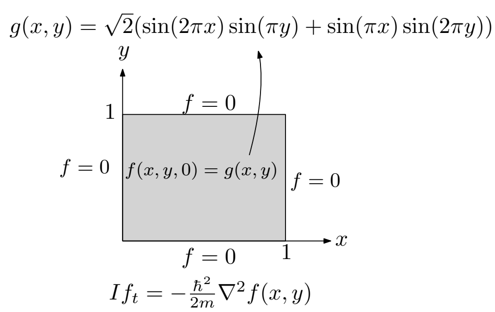


Figure 3.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, y, t], {t}] == -((hBar^2*Laplacian[f[x, y, t], {x, y}])/(2*m));
 initSum = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[2*Pi*y]);
 bcs = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcs, initSum}, f[x, y, t], {x, y, t}], 60
```

$$\left\{\left\{f(x,y,t)\to 2\sqrt{2}\sin(\pi x)\sin(\pi y)e^{-\frac{5i\pi^2\text{hBar}^2t}{2m}}(\cos(\pi x)+\cos(\pi y))\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  I* diff(f(x,y,t),t) = -hBar^2/(2*m) * (diff(f(x,y,t),x$2) +  diff(f(x,y,t),y$2));
ic  := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y) + sin(Pi*x)*sin(2*Pi*y));
bc  := f(0, y, t) = 0,
      f(1, y, t) = 0,
      f(x, 1, t) = 0,
      f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],f(x,y,t))),out
```

$$f(x,y,t)=\sqrt{2}\left(\sin\left(\pi x\right)\sin\left(2\pi y\right)+\sin\left(\pi y\right)\sin\left(2\pi x\right)\right)\mathrm{e}^{-\frac{5i\pi^2hBar^2t}{2m}}$$

3.2.2 In a square

problem number 150

Added December 20, 2018.

Example 28, taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

In 2 space dimensions Solve for $f(x,y,t)$

$$I\hbar f_t=-\frac{\hbar^2}{2m}\nabla^2 f$$

With initial conditions $f(x,y,0)=\sqrt{2}(\sin(2\pi x)\sin(\pi y)+\sin(\pi x)\sin(3\pi y))$ and boundary conditions

$$
\begin{aligned}
f(0, y, t) &= 0 \\
f(1, y, t) &= 0 \\
f(x, 1, t) &= 0 \\
f(x, 0, t) &= 0
\end{aligned}
$$

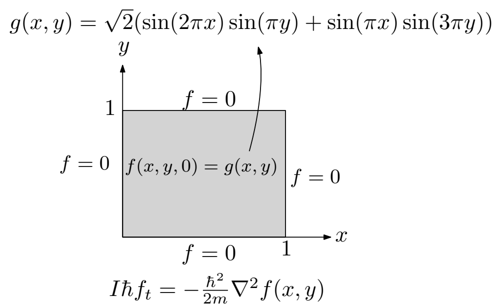


Figure 3.7: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y]+Sin[Pi*x]*Sin[3*Pi*y]);
bc  = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
pde =  I*h*D[f[x, y,t], t] == -h^2/(2*m)*(D[f[x, y, t], {x, 2}]+D[f[x, y, t], {y, 2}]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, f[x, y, t], {x, y, t}], 60*10]];
```

$$
\left\{\left\{f(x,y,t) \to \sqrt{2}e^{-\frac{5i\pi^2 ht}{m}}\left(\sin(\pi x)\sin(3\pi y) + \sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2 ht}{2m}}\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde := I*hbar* diff(f(x, y, t), t) = - hbar^2/(2*m)*  (diff(f(x, y, t), x$2)+diff(f(x, y, t)
ic  := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y)+sin(Pi*x)*sin(3*Pi*y));
bc  := f(0, y, t) = 0, f(1, y, t) = 0, f(x, 1, t) = 0, f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],f(x,y,t))),ou
```

$$
f(x,y,t) = \sqrt{2}\left(2\cos(\pi x)\,e^{-\frac{5i\pi^2 hbart}{2m}}\sin(\pi y) + e^{-\frac{5i\pi^2 hbart}{m}}\sin(3\pi y)\right)\sin(\pi x)
$$

CHAPTER 4

Parabolic PDE's (Diffusion)

Local contents

4.1 Diffusion in 1D

Local contents

4.1.1 Finite domain (bar), Both ends homogeneous BC

Local contents

4.1.1.1 [151] General initial conditions

problem number 151

Added June 9, 2019

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

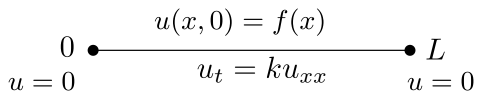


Figure 4.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. {K[1] -> n}
```

$$\left\{\left\{u(x,t)\to \sum_{n=1}^{\infty}\frac{2e^{-\frac{kn^2\pi^2t}{L^2}}\left(\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx\right)\sin\left(\frac{n\pi x}{L}\right)}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(L,t) = 0;
ic  := u(x,0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2 k\,n^2 t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L}$$

Hand solution

Solve

$$u_t = k u_{xx}$$

BC

$$u(0,t)=0 \qquad t>0$$
$$u(L,t)=0 \qquad t>0$$

Initial conditions

$$u(x,0) = f(x) \qquad 0 < x < L$$

Solution

Using separation of variables, let $u(x,t) = X(x)T(t)$. Substituting this back into the PDE gives

$$\begin{aligned} T'X &= kX''T \\ \frac{1}{k}\frac{T'}{T} &= \frac{X''}{X} = -\lambda \end{aligned}$$

Where the separation constant is some real value $-\lambda$. This gives the following two ODE's to solve

$$T' + \lambda k T = 0 \tag{1}$$
$$X'' + \lambda X = 0 \tag{2}$$

Starting with the spatial ODE in order to obtain the eigenvalues and eigenfunctions. The boundary conditions on the spatial ODE become

$$\begin{aligned} X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

There are three cases to consider, depending on if $\lambda < 0, \lambda = 0, \lambda > 0$. Each one of these cases give a different solution.

Case 1 Assuming $\lambda < 0$. Therefore $-\lambda$ is positive say μ. The solution (2) can now be written as

$$X(x) = c_1 \cosh\left(\sqrt{\mu} x\right) + c_2 \sinh\left(\sqrt{\mu} x\right) \tag{3B}$$

Applying left boundary conditions to (3B) gives

$$0 = c_1$$

The solution (3B) now reduces to

$$X(x) = c_2 \sinh\left(\sqrt{\mu} x\right)$$

Applying right side boundary conditions to the above results in

$$0 = c_2 \sinh\left(\sqrt{\mu} L\right)$$

But $\sinh\left(\sqrt{\mu} L\right) \neq 0$ since it was assumed μ is not zero and sinh is only zero when its argument is zero. The only possibility then is that $c_2 = 0$, which leads to trivial solution. Therefore $\lambda < 0$ is not an eigenvalue.

Case 2. Assuming $\lambda = 0$. The ODE becomes $X'' = 0$, which has the solution

$$X(x) = c_1 x + c_2$$

Applying left side B.C. gives

$$0 = c_2$$

The solution now reduces to

$$X(x) = c_1 x$$

Applying right side B.C. gives

$$0 = c_1 L$$

Leading to the trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case 3 Assuming $\lambda > 0$. In this case equation $\sqrt{-\lambda}$ is complex and solution to equation (2) can be expressed in terms of trig functions using Euler relation which results in

$$X(x) = c_1 \cos\left(\sqrt{\lambda} x\right) + c_2 \sin\left(\sqrt{\lambda} x\right) \tag{4}$$

Applying left side B.C. gives

$$0 = c_1$$

Solution (4) now reduces to

$$X(x) = C_2 \sin\left(\sqrt{\lambda} x\right) \tag{5}$$

Applying right side B.C. gives

$$0 = C_2 \sin\left(\sqrt{\lambda} L\right)$$

Non-trivial solution implies $\sin\left(\sqrt{\lambda} L\right) = 0$ or $\sqrt{\lambda} L = n\pi$ for $n = 1, 2, 3, \cdots$. Therefore the eigenvalues are

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunctions from (5) are

$$X_n(x) = C_n \sin\left(\sqrt{\lambda_n} x\right) \tag{6}$$

Now that the eigenvalues are known, the solution to the time ODE (1) can be found.

$$T' + \lambda_n k T = 0$$

This has the solution (using an integrating factor method)

$$T_n(t) = e^{-\lambda_n k t} \tag{7}$$

The constant of integration is not needed for (7) since it will be absorbed with the constant of integration coming from solution of the spatial ODE (6) when these solutions are multiplied with each others below. Therefore the fundamental solution is

$$u_n(x,t) = T_n(t)\, X_n(x)$$

Linear combination of fundamental solutions is also a solution (since this is a linear PDE). Therefore the general solution is given by

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} u_n \\ &= \sum_{n=1}^{\infty} T_n(t)\, X_n(x) \\ &= \sum_{n=1}^{\infty} C_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \end{aligned} \tag{8}$$

The initial condition is now used to determine c_n, . At $t=0$, $u(x,0) = f(x)$ and the above becomes

$$f(x) = \sum_{n=1}^{\infty} C_n \sin\left(\sqrt{\lambda_n} x\right)$$

Multiplying both sides of the above equation by eigenfunction $\sin\left(\sqrt{\lambda_m} x\right)$ and integrating over the domain of $f(x)$ gives

$$\int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx = \int_0^L \sum_{n=1}^{\infty} C_n \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

Interchanging the order of summation and integration gives

$$\int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx = \sum_{n=1}^{\infty} C_n \int_0^L \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

By the orthogonality of the sine functions, all terms in the right side vanish except when $n=m$, leading to

$$\begin{aligned} \int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx &= C_m \int_0^L \sin^2\left(\sqrt{\lambda_m} x\right) dx \\ &= C_m \frac{L}{2} \end{aligned}$$

Therefore (replacing m back to n now, since it is arbitrary)

$$C_n = \frac{2}{L} \int_0^L f(x) \sin\left(\sqrt{\lambda_n} x\right) dx \qquad n = 1,2,3,\cdots$$

Summary of solution

$$u(x,t)=\sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi}{L}\right)^2 \lambda_n t} \sin\left(\frac{n\pi}{L}x\right)$$

4.1.1.2 [152] Specific initial condition

problem number 152

Added June 20, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$ using the following specific values

$$f(x) = 100$$
$$k = \frac{1}{100}$$
$$L = 10$$

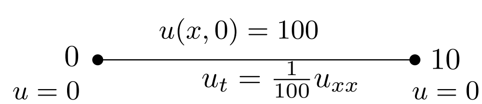


Figure 4.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
 L=10;
 f=100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n}
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{200(-1+(-1)^n)\,e^{-\frac{n^2\pi^2t}{10000}}\sin\left(\frac{n\pi x}{10}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
restart;
k:=1/100;
L:=10;
f:=100;
pde := diff(u(x,t), t) = k*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(L,t) = 0;
ic  := u(x,0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(-\frac{200((-1)^n-1)\,e^{-\frac{\pi^2n^2t}{10000}}\sin\left(\frac{\pi nx}{10}\right)}{\pi n}\right)$$

Hand solution

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}C_ne^{-\left(\frac{n\pi}{L}\right)^2\lambda_nt}\sin\left(\frac{n\pi}{L}x\right)$$

$$C_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\qquad n=1,2,3,\cdots$$

In this case $L=10, k=\frac{1}{100}, f(x)=100$. Hence the solution becomes

$$
\begin{aligned}
C_n &= \frac{2}{10}\int_0^{10} 100\sin\left(\frac{n\pi}{10}x\right)dx\\
&= \frac{200}{\pi n}(1-\cos(n\pi))\\
&= \frac{200}{\pi n}\left(1+(-1)^{n+1}\right)
\end{aligned}
$$

Hence

$$
u(x,t)=\frac{200}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(1+(-1)^{n+1}\right)e^{-\left(\frac{n\pi}{10}\right)^2\lambda_n t}\sin\left(\frac{n\pi}{10}x\right)
$$

The following is an animation of the solution

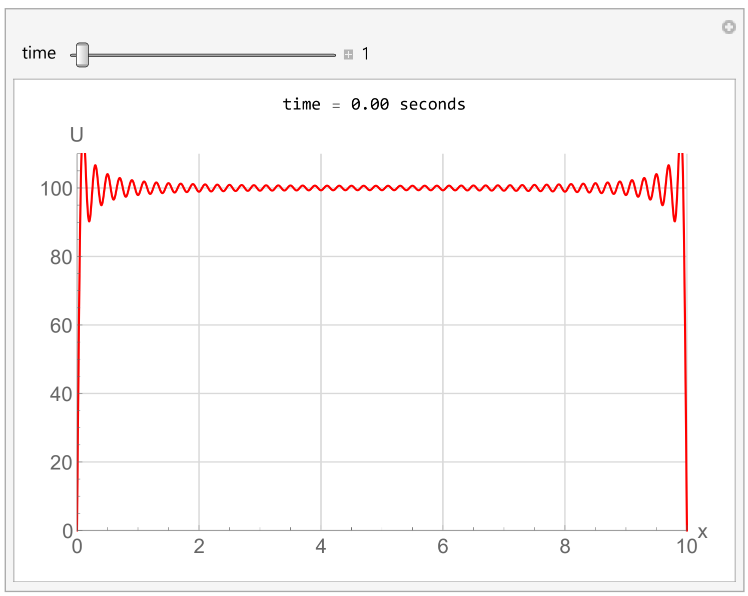


Figure 4.3: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, y, t, n, k]
L = 10;
k = 1 / 100;
f = 100;
numberOfTerms = 100;
mySol[x_, t_] = 200/π * Sum[1/n (1 + (-1)^(n+1)) Exp[-(n π/10)^2 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 4.4: Source code

```
In[•]:= tab = Table[
          Grid[{
            {Row[{"time = ", padIt2[t, {2, 2}], " seconds"}]},
            {
             Plot[Evaluate[mySol[x, t]], {x, 0, L},
              BaseStyle → 15,
              ImageMargins → 3,
              PerformanceGoal → "Quality",
              PlotRange → {{0, L}, {0, 110}},
              ImageSize → 500,
              AxesLabel → {"x", "U"},
              GridLines → Automatic,
              GridLinesStyle → LightGray,
              PlotStyle → Red
             ]

            }}],
          {t, 0, 50, 0.1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.5: Code for animation

4.1.1.3 [153] IC $u = x(1-x)$

problem number 153

Added June 11, 2019

Solve the heat equation

$$u_t = \frac{1}{100} u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = x(1-x)$

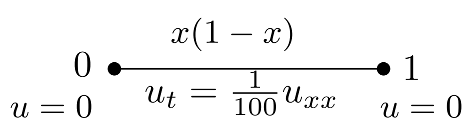


Figure 4.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 1/100*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == x*(1-x);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. {K[1] -> n}
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{4(-1+(-1)^n)\,e^{-\frac{1}{100}n^2\pi^2t}\sin(n\pi x)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = 1/100*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(1,t) = 0;
ic  := u(x,0) = x*(1-x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(-\frac{4((-1)^n-1)\,\mathrm{e}^{-\frac{\pi^2n^2t}{100}}\sin(\pi nx)}{\pi^3n^3}\right)$$

Hand solution

Solve $u_t=\frac{1}{100}u_{xx}$ with $0<x<1$ and initial conditions $u(x,0)=x(1-x)$

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}B_ne^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Here $L=1$ and $k=\frac{1}{100}$. Hence the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}B_ne^{-k(n\pi)^2t}\sin(n\pi x)$$

At $t=0$

$$x(1-x)=\sum_{n=1}^{\infty}B_n\sin(n\pi x)$$

Multiplying both sides by $\sin(m\pi x)$ and integrating gives

$$\int_0^1 x(1-x)\sin(m\pi x)\,dx = \int_0^1 \sum_{n=1}^{\infty} B_n \sin(n\pi x)\sin(m\pi x)\,dx$$

By orthogonality of sin function the above simplifies to

$$\begin{aligned}\int_0^1 x(1-x)\sin(n\pi x)\,dx &= B_n \int_0^1 \sin^2(n\pi x)\,dx\\ \int_0^1 x(1-x)\sin(n\pi x)\,dx &= \frac{1}{2}B_n\end{aligned}$$

But $\int_0^1 x(1-x)\sin(n\pi x)\,dx = \frac{2\left(1+(-1)^{n+1}\right)}{n^3\pi^3}$. Hence

$$B_n = \frac{4\left(1+(-1)^{n+1}\right)}{n^3\pi^3}$$

And the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4\left(1+(-1)^{n+1}\right)}{n^3\pi^3} e^{-k(n\pi)^2 t}\sin(n\pi x)$$

This is animation of the solution for 30 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
In[◦]:= ClearAll[x, t, n, lam]
L0 = 1;
k = 1 / 100;
pde = D[u[x, t], {t, 1}] == k * D[u[x, t], {x, 2}];
ic = u[x, 0] == x (1 - x);
bc = {u[0, t] == 0, u[L0, t] == 0};
maxTime = 30;
lam[n_] := (n Pi / L0) ^2;
numberOfTerms = 10;
mySol = Sum[(4 (1 + (-1)^(n+1)))/(n^3 π^3) Exp[- k lam[n] t] Sin[Sqrt[lam[n]] x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[ Evaluate[Quiet[N[mySol /. {x → x0, t → time}]]], {x0, 0, 1},
      PlotLabel → Row[{t, " sec,", " U at center=",
            padIt2@Evaluate[Quiet[N[mySol /. {x → 1 / 2, t → time}]]], {3, 2}]}],
      PlotRange → {{0, 1}, {0, .3}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
    ]
  ],
  {{time, 0, "time"}, 0, 35, .01, Appearance → "Labeled"}]

In[◦]:= ClearAll[x, t];
r = Table[
      Plot[ Evaluate[Quiet[mySol /. {x → x0, t → time}]], {x0, 0, 1},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " Temperature at center = ",
              padIt2@Evaluate[Quiet[mySol /. {x → 1 / 2, t → time}], {4, 3}]}],
        PlotRange → {{0, 1}, {0, .3}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
      ],
      {time, 0, 30, .08}];

In[◦]:= Export["anim.gif", r, "DisplayDurations" → Table[0.04, {Length@r}]]
```

Figure 4.7: Source code

4.1.1.4 [154] Haberman 2.3.3 (a)

problem number 154

This is problem 2.3.3, part (a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 6\sin\left(\frac{9\pi x}{L}\right)$

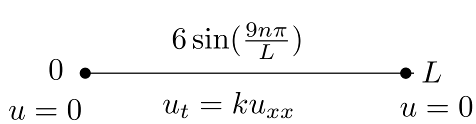


Figure 4.8: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 6*Sin[(9*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to 6e^{-\frac{81\pi^2kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=6*sin(9*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=6\,\mathrm{e}^{-\frac{81\pi^2kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)$$

Hand solution

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x,0) = 6\sin\left(\frac{9\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. At $t = 0$

$$6\sin\left(\frac{9\pi x}{L}\right)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)$$

For $n = 9$

$$6\sin\left(\frac{9\pi x}{L}\right)=B_9\sin\left(\frac{9\pi}{L}x\right)$$

Hence $B_9 = 6$ and all other terms are zero. Therefore the solution is

$$\begin{aligned} u(x,t) &= B_9 \sin\left(\sqrt{\lambda_9} x\right) e^{-k\lambda_9 t} \\ &= 6 \sin\left(\frac{9\pi}{L} x\right) e^{-k\left(\frac{9\pi}{L}\right)^2 t} \\ &= 6 \sin\left(\frac{9\pi}{L} x\right) e^{-k\frac{81\pi^2}{L^2} t} \end{aligned}$$

4.1.1.5 [155] IC hat function

problem number 155

Added Feb 10, 2018.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(40,t) &= 0 \end{aligned}$$

Initial condition is

$$u(x,0) = \begin{cases} x & 0 \leq x < 20 \\ 40 - x & 20 \geq x \leq 40 \end{cases}$$

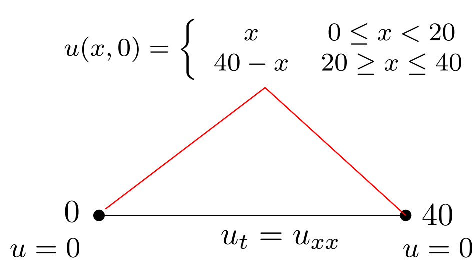


Figure 4.9: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f = Piecewise[{{x, Inequality[0, LessEqual, x, Less, 20]}, {40 - x, 20 <= x <= 40}}];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[40, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{640e^{-\frac{n^2\pi^2t}{1600}}\cos\left(\frac{n\pi}{4}\right)\sin^3\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{40}\right)}{n^2\pi^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
f:=piecewise(0<x and x<20,x,20<x and x<40,(40-x));
pde := diff(u(x,t),t)=diff(u(x,t),x$2);
ic  := u(x,0)=f;
bc  := u(0,t)=0,u(40,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{160\,\mathrm{e}^{-\frac{\pi^2n^2t}{1600}}\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi nx}{40}\right)}{\pi^2n^2}$$

Hand solution

Solve $u_t = ku_{xx}$ with $0 < x < 40$ and initial conditions $u(x,0) = f(x)$ as hat function.

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}B_ne^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Here $L=40$ and $k=1$. Hence the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}B_ne^{-\left(\frac{n\pi}{40}\right)^2t}\sin\left(\frac{n\pi}{40}x\right)$$

At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{40}x\right)$$

Multiplying both sides by $\sin(m\pi x)$ and integrating gives

$$\int_0^{40} f(x)\sin\left(\frac{m\pi}{40}x\right)dx = \int_0^{40}\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{40}x\right)\sin\left(\frac{m\pi}{40}x\right)dx$$

By orthogonality of sin function the above simplifies to

$$\int_0^{40} f(x)\sin\left(\frac{n\pi}{40}x\right)dx = B_n\int_0^{40}\sin^2\left(\frac{n\pi}{40}x\right)dx$$

$$\int_0^{20} x\sin\left(\frac{n\pi}{40}x\right)dx + \int_{20}^{40}(40-x)\sin\left(\frac{n\pi}{40}x\right)dx = 20B_n$$

But $\int_0^{20} x\sin\left(\frac{n\pi}{40}x\right)dx + \int_{20}^{40}(40-x)\sin\left(\frac{n\pi}{40}x\right)dx = \frac{3200}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$. Hence

$$B_n = \frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

And the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty}\frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)e^{-\left(\frac{n\pi}{40}\right)^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

When n is even the terms are zero Hence

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty}\frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)e^{-\left(\frac{n\pi}{40}\right)^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

Or

$$u(x,t) = \sum_{n=1}^{\infty}\frac{160}{(2n-1)^2\pi^2}\sin\left(\frac{(2n-1)\pi}{2}\right)e^{-\left(\frac{(2n-1)\pi}{40}\right)^2 t}\sin\left(\frac{(2n-1)\pi}{40}x\right)$$

This is animation of the solution for 400 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
ClearAll[x, t, n, lam]
L0 = 40;
k = 1;
pde = D[u[x, t], {t, 1}] == k*D[u[x, t], {x, 2}];

lam[n_] := ((2 n - 1) π / L0);
numberOfTerms = 15;
mySol = Sum[160/((2 n - 1)^2 π^2) Sin[(2 n - 1) π / 2] Exp[- k lam[n]^2 t] Sin[lam[n] x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[ Quiet[mySol /. {x → x0, t → time}], {x0, 0, L0},
      PlotLabel → Row[{padIt2[t, {4, 2}], " sec,", " U at center=",
          padIt2@Evaluate[Quiet[N[mySol /. {x → 20, t → time}]], {4, 3}]}],
      PlotRange → {{0, L0}, {0, 20}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
    ]
  ],
  {{time, 0, "time"}, 0, 400, .01, Appearance → "Labeled"}]

In[]:= ClearAll[x, t];
r = Table[
      Plot[Quiet[mySol /. {x → x0, t → time}], {x0, 0, L0},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " Temperature at center = ",
            padIt2@Evaluate[Quiet[mySol /. {x → L0 / 2, t → time}], {4, 3}]}],
        PlotRange → {{0, L0}, {0, L0 / 2}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
      ],
      {time, 0, 400, 1.0}];

In[]:= Export["anim.gif", r, "DisplayDurations" → Table[0.01, {Length@r}]]
```

Figure 4.10: Source code

4.1.1.6 [156] Haberman 2.3.3 (b)

problem number 156

This is problem 2.3.3, part (b) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

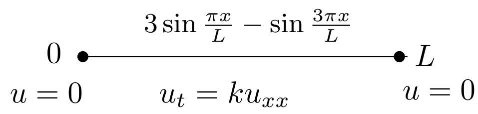


Figure 4.11: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 3*Sin[(Pi*x)/L] - Sin[(3*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{-\frac{9\pi^2kt}{L^2}}\sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2kt}{L^2}}-2\cos\left(\frac{2\pi x}{L}\right)-1\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = -\mathrm{e}^{-\frac{9\pi^2kt}{L^2}}\sin\left(\frac{3\pi x}{L}\right)+3\,\mathrm{e}^{-\frac{\pi^2kt}{L^2}}\sin\left(\frac{\pi x}{L}\right)$$

Hand solution

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x,0) = 3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where the eigenvalues are $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions.

Initial conditions are now applied. Setting $t = 0$, the above becomes

$$u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

As the series is unique, the terms coefficients must match for those shown only, and all other B_n terms vanish. This means that by comparing terms

$$3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right) = B_1 \sin\left(\frac{\pi x}{L}\right) + B_3 \sin\left(\frac{3\pi}{L}x\right)$$

Therefore

$$\begin{aligned} B_1 &= 3 \\ B_3 &= -1 \end{aligned}$$

And all other $B_n = 0$. The solution is

$$u(x,t) = 3\sin\left(\frac{\pi}{L}x\right) e^{-k\left(\frac{\pi}{L}\right)^2 t} - \sin\left(\frac{3\pi}{L}x\right) e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

4.1.1.7 [157] Haberman 2.3.3 (c)

problem number 157

This is problem 2.3.3, part (c) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 2\cos\frac{3\pi x}{L}$

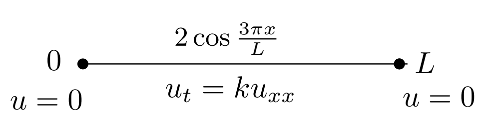


Figure 4.12: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 2*Cos[(3*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol/. K[1]->n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4(1+(-1)^n)\,e^{-\frac{kn^2\pi^2t}{L^2}}n\sin\left(\frac{n\pi x}{L}\right)}{(n^2-9)\,\pi}\right\}\right\}$$

but $n=3$ should be special case

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=2*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=-\frac{16\,\mathrm{e}^{-\frac{4\pi^2kt}{L^2}}\sin\left(\frac{2\pi x}{L}\right)}{5\pi}+\left(\sum_{n=4}^{\infty}\frac{4((-1)^n+1)\,n\,\mathrm{e}^{-\frac{\pi^2k\,n^2t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{\pi\,(n^2-9)}\right)$$

handled $n=3$ case correctly.

Hand solution

Solve $u_t=ku_{xx}$ with $0<x<L$ and initial conditions $u(x,0)=2\cos\left(\frac{3\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}B_ne^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t=0$, the above becomes

$$u(x,0)=2\cos\left(\frac{3\pi}{L}x\right)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating

$$\begin{aligned}\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx \\ &= \sum_{n=1}^{\infty} B_n \int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\end{aligned}$$

By orthogonality of sin functions the above simplifies to

$$\begin{aligned}\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= B_m \int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx \\ &= B_m \frac{L}{2} \\ B_m &= \frac{4}{L}\int_0^L \cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\end{aligned}$$

For $m=3, B_3=0$. For $m \neq 3$

$$\begin{aligned}B_m &= \frac{4}{L}\left(\frac{1+(-1)^m}{m^2-9}\frac{nL}{\pi}\right) \\ &= \frac{4n}{\pi}\left(\frac{1+(-1)^m}{m^2-9}\right)\end{aligned}$$

Hence the solution becomes

$$\begin{aligned}u(x,t) &= \sum_{n=1,n\neq 3}^{\infty} \frac{4n}{\pi}\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \\ &= \frac{4}{\pi}\sum_{n=1,n\neq 3}^{\infty} n\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\end{aligned}$$

When n is odd, all terms become zero, hence the above can be also be written as

$$u(x,t) = \frac{8}{\pi}\sum_{n=2,4,6,\cdots}^{\infty}\left(\frac{n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

4.1.1.8 [158] Haberman 2.3.3 (d)

problem number 158

This is problem 2.3.3, part (d) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = \begin{cases} 1 & 0 < x \leq \frac{L}{2} \\ 2 & \frac{L}{2} < x \leq L \end{cases}$

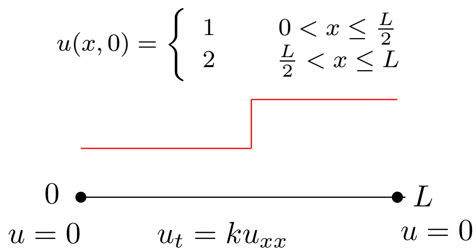


Figure 4.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == Piecewise[{{1, Inequality[0, Less, x, LessEqual, L/2]}, {2, L/2 < x < L}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4e^{-\frac{kn^2\pi^2 t}{L^2}}\left(4\cos\left(\frac{n\pi}{2}\right)+3\right)\sin^2\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{L}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{\left(-4(-1)^{n}+2\cos\left(\frac{\pi n}{2}\right)+2\right)\mathrm{e}^{-\frac{\pi^{2}k\,n^{2}t}{L^{2}}}\sin\left(\frac{\pi n x}{L}\right)}{\pi n}$$

Hand solution

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(x,t)=\sum_{n=1}^{\infty}B_{n}e^{-k\lambda_{n}t}\sin\left(\sqrt{\lambda_{n}}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t=0$, the above becomes

$$f(x)=\sum_{n=1}^{\infty}B_{n}\sin\left(\frac{n\pi}{L}x\right)\tag{3}$$

Where

$$f(x)=\begin{cases}1 & 0<x\leq\frac{L}{2}\\ 2 & \frac{L}{2}<x<L\end{cases}$$

Multiplying both sides of (3) by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating over the domain gives

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\int_{0}^{L}\left[\sum_{n=1}^{\infty}B_{n}\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)\right]dx$$

Interchanging the order of integration and summation

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\sum_{n=1}^{\infty}\left[B_{n}\left(\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx\right)\right]$$

But $\int_0^L\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx=0$ for $n\neq m$, hence only one term survives

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=B_{m}\int_{0}^{L}\sin^{2}\left(\frac{m\pi}{L}x\right)dx$$

Renaming m back to n and since $\int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$ the above becomes

$$
\begin{aligned}
\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx &= \frac{L}{2}B_n \\
B_n &= \frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx + \int_{\frac{L}{2}}^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx\right) \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) dx + 2\int_{\frac{L}{2}}^L \sin\left(\frac{n\pi}{L}x\right) dx\right) \\
&= \frac{2}{L}\left(\left.\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\right|_0^{\frac{L}{2}} + 2\left.\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\right|_{\frac{L}{2}}^{L}\right) \\
&= \frac{2}{n\pi}\left(\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_0^{\frac{L}{2}} + 2\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_{\frac{L}{2}}^{L}\right) \\
&= \frac{2}{n\pi}\left(\left[-\cos\left(\frac{n\pi}{L}\frac{L}{2}\right) + \cos(0)\right] + 2\left[-\cos(n\pi) + \cos\left(\frac{n\pi}{2}\right)\right]\right) \\
&= \frac{2}{n\pi}\left(-\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi) + 2\cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi)\right)
\end{aligned}
$$

Hence the solution is

$$
u(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}
$$

With

$$
\begin{aligned}
B_n &= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - 2\cos(n\pi) + 1\right) \\
&= \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(1 + 2(-1)^{n+1} + \cos\left(\frac{n\pi}{2}\right)\right)
\end{aligned}
$$

Therefore

$$
u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}
$$

4.1.1.9 [159] domain from -1 to +1

problem number 159

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = f(x)$

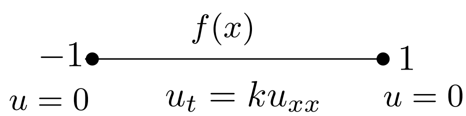


Figure 4.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[-1, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}2e^{-\frac{1}{4}\pi^2tK[1]^2}\sin\left(\frac{1}{2}\pi(x+1)K[1]\right)\int_{-1}^{1}\frac{1}{2}f(x)\sin\left(\frac{1}{2}\pi(x+1)K[1]\right)dx\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic  := u(x,0) = f(x);
bc  := u(-1,t)=0,  u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(\left(\int_{-1}^{1} f(x)\sin\left(\frac{\pi(x+1)\,n}{2}\right)dx\right)\mathrm{e}^{-\frac{\pi^2 n^2 t}{4}}\sin\left(\frac{\pi(x+1)\,n}{2}\right)\right)$$

Hand solution

Solve $u_t = ku_{xx}$ with $-1 < x < 1$ and initial conditions $u(x,0) = f(x)$ and B.C. $u(-1,t) = u(1,t) = 0$.

Let $\xi = x+1$ where $x = -1\cdots 1$ Therefore $\xi = 0\cdots 2$. This also implies mapping the initial temperature to $f(x+a)$. Now the ODE is solved

$$\begin{aligned}u_t &= u_{\xi\xi}\\ u(0,t) &= 0\\ u(2,t) &= 0\end{aligned}$$

The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$u(\xi,t)=\sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}\xi\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}\xi\right)$ are the eigenfunctions. Here $L = 2$. Hence the solution becomes

$$u(\xi,t)=\sum_{n=1}^{\infty} B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}\xi\right)$$

Changing back to x

$$u(x,t)=\sum_{n=1}^{\infty} B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

At $t=0$

$$f(x)=\sum_{n=1}^{\infty}B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2}(x+1)\right)$ and integrating gives

$$\int_{-1}^{1}f(x)\sin\left(\frac{m\pi}{2}(x+1)\right)dx=\int_{-1}^{1}\sum_{n=1}^{\infty}B_n\sin\left(\frac{m\pi}{2}(x+1)\right)\sin\left(\frac{n\pi}{2}(x+1)\right)dx$$

By orthogonality of sin function the above simplifies to

$$\begin{aligned}\int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx&=B_n\int_{-1}^{1}\sin^2\left(\frac{n\pi}{2}(x+1)\right)dx\\ \int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx&=B_n\end{aligned}$$

Hence

$$B_n=\int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx$$

And the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}\left(\int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx\right)e^{-k\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

4.1.1.10 [160] domain from -1 to +1

problem number 160

Solve the heat equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

For $-1<x<1$ and $t>0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0)=1-x^2$

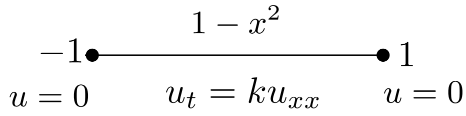


Figure 4.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == 1-x^2;
bc  = {u[-1, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \rightarrow \sum_{K[1]=1}^{\infty} -\frac{16\left(-1+(-1)^{K[1]}\right) e^{-\frac{1}{4}\pi^2 t K[1]^2} \sin\left(\frac{1}{2}\pi(x+1)K[1]\right)}{\pi^3 K[1]^3}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic  := u(x,0) = 1-x^2;
bc  := u(-1,t)=0,  u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(-\frac{16\left((-1)^n - 1\right) e^{-\frac{\pi^2 n^2 t}{4}} \sin\left(\frac{\pi(x+1)n}{2}\right)}{\pi^3 n^3}\right)$$

Hand solution

Solve $u_t = u_{xx}$ with $-1 < x < 1$ and initial conditions $u(x,0) = f(x) = 1 - x^2$ and B.C. $u(-1,t) = u(1,t) = 0$.

The basic solution for this type of PDE was already given in problem 4.1.1.9 on page 426 as

$$u(x,t)=\sum_{n=1}^{\infty}\left(\int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx\right)e^{-k\left(\frac{n\pi}{2}\right)^2t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

Replacing $f(x)=1-x^2$ then

$$\begin{aligned}\int_{-1}^{1}f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx &= \int_{-1}^{1}(1-x^2)\sin\left(\frac{n\pi}{2}(x+1)\right)dx\\ &= \frac{16\left(1+(-1)^{n+1}\right)}{n^3\pi^3}\end{aligned}$$

Hence the solution becomes, using $k=1$

$$u(x,t)=\sum_{n=1}^{\infty}\frac{16\left(1+(-1)^{n+1}\right)}{n^3\pi^3}e^{-\left(\frac{n\pi}{2}\right)^2t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

This is animation of the solution for 2 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
In[]:= ClearAll[x, t, n, lam]
L0 = 2;
k = 1;
pde = D[u[x, t], {t, 1}] == k*D[u[x, t], {x, 2}];

lam[n_] := (n π / L0);
numberOfTerms = 15;
mySol = Sum[(16 (1 + (-1)^(n+1)))/(n^3 π^3) Exp[- lam[n]^2 t] Sin[lam[n] (x + 1)], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[Quiet[mySol /. {x → x0, t → time}], {x0, -1, 1},
      PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " U at center = ",
          padIt2@Evaluate[Quiet[N[mySol /. {x → 0, t → time}]], {4, 3}]}],
      PlotRange → {{-1, 1}, {0, 1}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red, ImageSize → 300
    ]
  ],
  {{time, 0, "time"}, 0, 2, .01, Appearance → "Labeled"}]

In[]:= ClearAll[x, t];
r = Table[
      Plot[Quiet[mySol /. {x → x0, t → time}], {x0, -1, 1},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " U at center = ",
            padIt2@Evaluate[Quiet[N[mySol /. {x → 0, t → time}]], {4, 3}]}],
        PlotRange → {{-1, 1}, {0, 1}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red, ImageSize → 300
      ],
      {time, 0, 2, 0.1}];

In[]:= Export["anim.gif", r, "DisplayDurations" → Table[0.4, {Length@r}]]
```

Figure 4.16: Source code

4.1.1.11 [161] with source that depends on space only (general case)

problem number 161

Added July 3, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

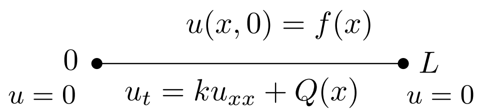


Figure 4.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\frac{\left(1-e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\right)\left(\int_0^L \frac{\sqrt{2}Q(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)L^2}{k\pi^2 K[1]^2} + e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\int_0^L \frac{\sqrt{2}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)\sin}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q(x);
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \frac{2Lk\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L \left(Lkf(\_a)+L\left(\int_0^{\_a}\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)-\_a\left(\int_0^L\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)\right)\sin\left(\frac{\pi\_an}{L}\right)d\_a\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}\sin\left(\frac{\pi\cdots}{\cdots}\right.}{L^2 k}\right.}{Lk}$$

<u>Hand solution</u>

Let the solution be

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x) \tag{1}$$

Where $\Phi_n(x)$ are the eigenfunctions of the eigenvalue problem for the pde $u_t = k u_{xx}$ with $u(0,x) = 0, u(L,x) = 0$. This was solved in problem 4.1.1.1 on page 402 and $\Phi_n(x)$ was found as

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\sin\left(\sqrt{\lambda_n}x\right) \tag{1A}$$

Substituting this back into the original given PDE gives

$$\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) = k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n\Phi_n(x)$$

Where $Q(x) = \sum_{n=1}^{\infty} q_n\Phi_n(x)$. Since $\Phi_n''(x) = -\lambda_n\Phi_n(x)$, then the above simplifies to

$$\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) = -k\sum_{n=1}^{\infty} a_n(t)\,\lambda_n\Phi_n(x) + \sum_{n=1}^{\infty} q_n\Phi_n(x)$$
$$a_n'(t) + k\lambda_n a_n(t) = q_n \tag{2}$$

We now need to find q_n. By orthogonality

$$\int_0^L Q(x)\,\Phi_n(x)\,dx = \int_0^L q_n \Phi_n^2(x)\,dx$$

But $\int_0^L \Phi_n^2(x)\,dx = \int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx = \frac{L}{2}$ and the above becomes

$$q_n = \frac{2}{L}\int_0^L Q(x)\,\Phi_n(x)\,dx$$

Using the above in (2) gives

$$a_n'(t) + k\lambda_n a_n(t) = \frac{2}{L}\int_0^L Q(x)\,\Phi_n(x)\,dx$$

The integrating factor is $I = e^{k\lambda_n t}$. Hence the above becomes

$$\begin{aligned}
\frac{d}{dt}\left(a_n(t)\,e^{k\lambda_n t}\right) &= \frac{2}{L}e^{k\lambda_n t}\int_0^L Q(x)\,\Phi_n(x)\,dx \\
a_n(t)\,e^{k\lambda_n t} &= \int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x)\,\Phi_n(x)\,dx\right)ds + a_n(0) \\
a_n(t) &= a_n(0)\,e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x)\,\Phi_n(x)\,dx\right)ds
\end{aligned}$$

Therefore the solution (1A) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left[a_n(0)\,e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x) \quad (3)$$

At $t = 0$

$$\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} a_n(0)\,\Phi_n(x) \\
\int_0^L f(x)\,\Phi_n(x)\,dx &= \int_0^L a_n(0)\,\Phi_n^2(x)\,dx \\
a_n(0) &= \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx
\end{aligned}$$

The solution (3) becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\left[\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x) \\
&= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

4.1.1.12 [162] with source that depends on space only (special case)

problem number 162

Added July 4, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$ using the following values

$$L = 5$$
$$k = \frac{1}{10}$$
$$f(x) = x(5-x)$$
$$Q(x) = x$$

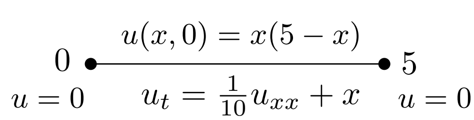


Figure 4.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5; k=1/10; f=x*(5-x); Q=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{100(-1+(-1)^n)\,e^{-\frac{1}{250}n^2\pi^2t}\sin\left(\frac{n\pi x}{5}\right)}{n^3\pi^3}+\sum_{n=1}^{\infty}-\frac{2500(-1)^n\left(1-e^{-\frac{1}{250}n^2\pi^2t}\right)\sin\left(\frac{n\pi x}{5}\right)}{n^3\pi^3}\right.\right.$$

Maple ✓

```
restart;
L=5;
k=1/10;
f=x*(5-x);
Q=x;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{(L-x)\,Qx+4k\left(\sum_{n=1}^{\infty}\frac{(-\pi^2fkn^2+L^2Q)((-1)^n-1)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{\pi^3kn^3}\right)}{2k}$$

<u>Hand solution</u>

Solve

$$\begin{aligned}u_t&=ku_{xx}+Q(x)\\u(0,t)&=0\\u(L,0)&=0\\u(x,0)&=f(x)\end{aligned}$$

With $k=\frac{1}{10},L=5,f(x)=x(5-x)\,,Q(x)=x.$

The general problem above was solved in 4.1.1.11 on page 431 and the solution is

$$u(x,t)=\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t\frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)$$

Where

$$\Phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n=\left(\frac{n\pi}{L}\right)^2\qquad n=1,2,3,\cdots$$

Substituting the specific values given above into this solution gives

$$\begin{aligned}u(x,t)&=\sum_{n=1}^{\infty}e^{-\frac{1}{10}\left(\frac{n\pi}{5}\right)^2 t}\sin\left(\frac{n\pi}{5}x\right)\left(\frac{2}{5}\int_0^5 s(5-s)\sin\left(\frac{n\pi}{5}s\right)ds\right)\\&+\sum_{n=1}^{\infty}e^{-\frac{1}{10}\left(\frac{n\pi}{5}\right)^2 t}\sin\left(\frac{n\pi}{5}x\right)\left(\int_0^t\frac{2}{5}e^{\frac{1}{10}\left(\frac{n\pi}{5}\right)^2\tau}\left(\int_0^5 s\sin\left(\frac{n\pi}{5}s\right)ds\right)d\tau\right)\end{aligned}$$

Animation is below

Out[ ]=

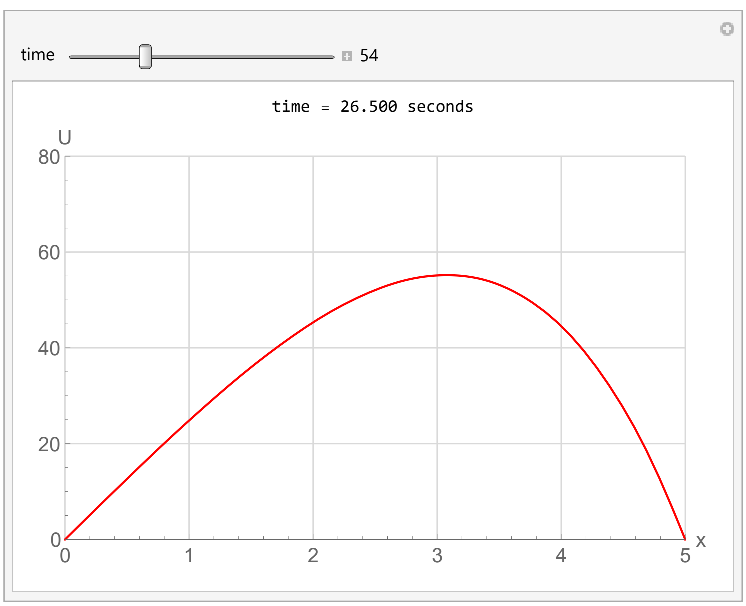


Figure 4.19: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
        L = 5;
        k = 1 / 10;
        f[x_] := x (5 - x);
        Q[x_] := x;
        ϕ[x_, n_] := Sin[√λ x];
        λ = (n π / L)^2;
        numberOfTerms = 15;
        padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[•]:= int1 = Integrate[Q[s] * ϕ[s, n], {s, 0, L}];
        int1 = Assuming[Element[n, Integers] && n > 0, Simplify[int1]];
        int2 = Integrate[2/L Exp[k λ τ] int1, {τ, 0, t}];
        int3 = Integrate[2/L f[s] × ϕ[s, n], {s, 0, L}];
        int3 = Assuming[Element[n, Integers] && n > 0, Simplify[int3]];
        mySol[x_, t_] = Chop@N@Sum[Exp[-k λ t] ϕ[x, n] int3, {n, 1, numberOfTerms}] + Chop@N@Sum[Exp[-k λ t] ϕ[x, n] int2, {n, 1, numberOfTerms}];
```

Figure 4.20: Source code

```
In[•]:= tab = Table[
           Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {0, 80}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
           {t, 0, 100, .5}];

In[•]:= Manipulate[tab⟦i⟧, {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.21: Code for animation

4.1.1.13 [163] with source that depends on space and time (general case)

problem number 163

Added June 23, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

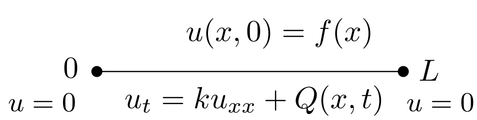


Figure 4.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2 K[1]^2 (t-K[2])}{L^2}} \text{Integrate}\left[\frac{\sqrt{2}Q(x,K[2]) \sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}, \{x,0,L\}, \text{Assumptions} \to k > \right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q(x,t);
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \int_0^t \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi n x}{L}\right) dx\right) e^{-\frac{\pi^2 (t-\tau) k n^2}{L^2}} \sin\left(\frac{\pi n x}{L}\right)}{L} \right) d\tau + \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right) dx\right) e}{L}\right.$$

Hand solution

Solving

$$
\begin{aligned}
u_t &= k v_{xx} + Q(x,t) \qquad (1)\\
u(0,t) &= 0\\
u(L,t) &= 0\\
u(x,0) &= f(x)
\end{aligned}
$$

Let the solution be

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x) \qquad (1)$$

Where $\Phi_n(x)$ are the eigenfunctions of the eigenvalue problem for the pde $u_t = k u_{xx}$ with $u(0,x) = 0, u(L,x) = 0$. This was solved in problem 4.1.1.1 on page 402 and $\Phi_n(x)$ was found as

$$
\begin{aligned}
\Phi_n(x) &= \sin\left(\sqrt{\lambda_n} x\right)\\
\lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots
\end{aligned}
$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t) \sin\left(\sqrt{\lambda_n} x\right) \qquad (1A)$$

Substituting this back into the original given PDE gives

$$\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) = k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)$$

Where $Q(x,t)=\sum_{n=1}^{\infty}q_n(t)\,\Phi_n(x)$. Since $\Phi_n''(x)=-\lambda_n\Phi_n(x)$, then the above simplifies to

$$\sum_{n=1}^{\infty}a_n'(t)\,\Phi_n(x)=-k\sum_{n=1}^{\infty}a_n(t)\,\lambda_n\Phi_n(x)+\sum_{n=1}^{\infty}q_n(t)\,\Phi_n(x)$$

$$a_n'(t)+k\lambda_n a_n(t)=q_n(t) \tag{2}$$

We now need to find $q_n(t)$. By orthogonality

$$\int_0^L Q(x,t)\,\Phi_n(x)\,dx=\int_0^L q_n(t)\,\Phi_n^2(x)\,dx$$

But $\int_0^L\Phi_n^2(x)\,dx=\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx=\frac{L}{2}$ and the above becomes

$$q_n(t)=\frac{2}{L}\int_0^L Q(x,t)\,\Phi_n(x)\,dx$$

Using the above in (2) gives

$$a_n'(t)+k\lambda_n a_n(t)=\frac{2}{L}\int_0^L Q(x,t)\,\Phi_n(x)\,dx$$

The integrating factor is $I=e^{k\lambda_n t}$. Hence the above becomes

$$\begin{aligned}\frac{d}{dt}\left(a_n(t)\,e^{k\lambda_n t}\right)&=\frac{2}{L}e^{k\lambda_n t}\int_0^L Q(x,t)\,\Phi_n(x)\,dx\\ a_n(t)\,e^{k\lambda_n t}&=\int_0^t\frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x,s)\,\Phi_n(x)\,dx\right)ds+a_n(0)\\ a_n(t)&=a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t\frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x,s)\,\Phi_n(x)\,dx\right)ds\end{aligned}$$

Therefore the solution (1A) becomes

$$u(x,t)=\sum_{n=1}^{\infty}\left[a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t\frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x) \tag{3}$$

At $t=0$

$$\begin{aligned}f(x)&=\sum_{n=1}^{\infty}a_n(0)\,\Phi_n(x)\\ \int_0^L f(x)\,\Phi_n(x)\,dx&=\int_0^L a_n(0)\,\Phi_n^2(x)\,dx\\ a_n(0)&=\frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx\end{aligned}$$

The solution (3) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left[\left(\frac{2}{L} \int_0^L f(s)\,\Phi_n(s)\,ds \right) e^{-k\lambda_n t} + e^{-k\lambda_n t} \int_0^t \frac{2}{L} e^{k\lambda_n \tau} \left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds \right) d\tau \right] \Phi_n(x)$$
$$= \sum_{n=1}^{\infty} e^{-k\lambda_n t} \Phi_n(x) \left(\frac{2}{L} \int_0^L f(s)\,\Phi_n(s)\,ds \right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t} \Phi_n(x) \left(\int_0^t \frac{2}{L} e^{k\lambda_n \tau} \left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds \right) d$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots$$

4.1.1.14 [164] with source that depends on space and time (special case)

problem number 164

Added July 5, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$. Using the following values

$$L = \pi$$
$$k = \frac{1}{300}$$
$$Q(x,t) = e^{-t}\sin(3x)$$
$$f(x) = x(\pi - x)$$

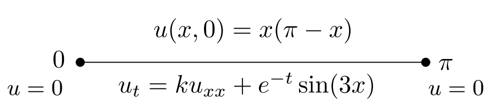


Figure 4.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=Pi; f=x*(Pi-x); k=1/300; Q=Exp[-t]*Sin[3*x];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol /. K[1] -> n
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{4(-1+(-1)^n)\,e^{-\frac{n^2t}{300}}\sin(nx)}{n^3\pi}+\frac{100}{97}e^{-t}\left(e^{97t/100}-1\right)\sin(3x)\right\}\right\}$$

Maple ✓

```
restart;
L:=Pi;
f:=x*(Pi-x);
k:=1/300;
Q:=exp(-t)*sin(3*x);
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t)=-\frac{100\,\mathrm{e}^{-t}\sin(3x)}{97}+\frac{100\,\mathrm{e}^{-\frac{3t}{100}}\sin(3x)}{97}-4\left(\sum_{n=1}^{\infty}\frac{((-1)^n-1)\,\mathrm{e}^{-\frac{n^2t}{300}}\sin(nx)}{\pi\,n^3}\right)$$

Hand solution

Solving

$$\begin{aligned}u_t &= ku_{xx}+Q(x,t)\\ u(0,t)&=0\\ u(L,0)&=0\\ u(x,0)&=f(x)\end{aligned}$$

With $k=\frac{1}{300}, L=\pi, f(x)=x(\pi-x)\,, Q(x)=e^{-t}\sin(3x)\,.$

The general problem above was solved in 4.1.6.4 on page 665 and the solution is

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)\\
&\quad+\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}
$$

Where

$$
\begin{aligned}
\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\
\lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots
\end{aligned}
$$

Substituting the specific values given above into this solution gives

$$
\begin{aligned}
\Phi_n(x) &= \sin(nx)\\
\lambda_n &= n^2 \qquad n=1,2,3,\cdots
\end{aligned}
$$

And

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} e^{-\frac{1}{300}n^2 t}\sin(nx)\left(\frac{2}{\pi}\int_0^{\pi} s(\pi-s)\sin(ns)\,ds\right)\\
&\quad+\sum_{n=1}^{\infty} e^{-\frac{1}{300}n^2 t}\sin(nx)\left(\int_0^t \frac{2}{\pi}e^{\frac{1}{300}n^2\tau}e^{-\tau}\left(\int_0^{\pi}\sin(3s)\sin(ns)\,ds\right)d\tau\right)
\end{aligned}
$$

But $\int_0^{\pi} s(\pi-s)\sin(ns)\,ds = \frac{2-2(-1)^n}{n^3}$ and $\int_0^{\pi}\sin(3s)\sin(ns)\,ds = 0$ when $n\neq 3$ and for $n=3$ it is $\frac{\pi}{2}$. Hence the above reduces to

$$
\begin{aligned}
u(x,t) &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx) + e^{-\frac{1}{300}(3)^2 t}\sin(3x)\left(\int_0^t \frac{2}{\pi}e^{\frac{1}{300}(9)^2\tau}e^{-\tau}\left(\frac{\pi}{2}\right)d\tau\right)\\
&= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx) + e^{-\frac{9t}{300}}\sin(3x)\left(\int_0^t e^{\frac{9\tau}{300}}e^{-\tau}d\tau\right)
\end{aligned}
$$

But $\int_0^t e^{\frac{9\tau}{300}}e^{-\tau}d\tau = \frac{100}{97} - \frac{100e^{\frac{-97}{100}t}}{97}$. Hence the above becomes

$$
\begin{aligned}
u(x,t) &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx) + e^{-\frac{9t}{300}}\sin(3x)\left(\frac{100}{97} - \frac{100e^{\frac{-97}{100}t}}{97}\right)\\
&= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx) + \sin(3x)\left(\frac{100}{97}e^{-\frac{3t}{100}} - \frac{100e^{-t}}{97}\right)\\
&= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx) - \frac{100e^{-t}}{97}\sin(3x) + \frac{100}{97}e^{-\frac{3t}{100}}\sin(3x)
\end{aligned}
$$

Animation is below

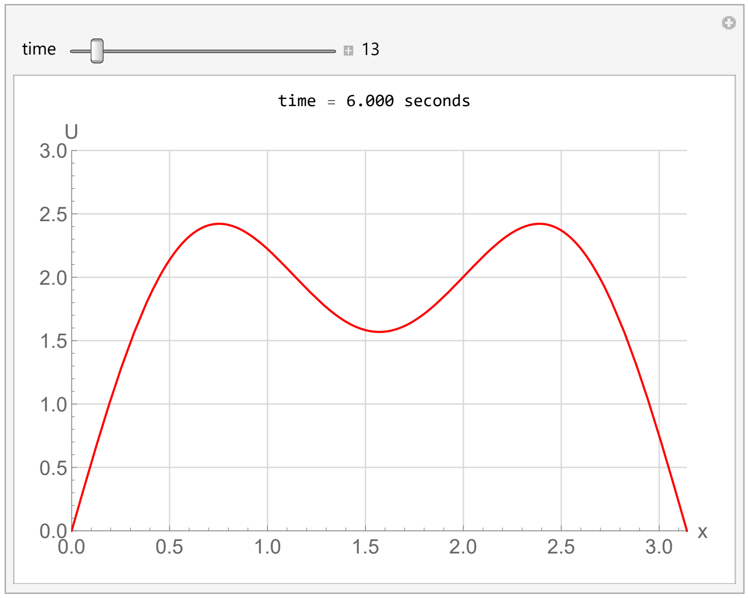


Figure 4.24: Initial state

Source code used for the above

```
ClearAll[x, t, n, f, A, B, s, mySol]
L = π;
k = 1 / 300;
f[x_] := x (π - x);
Q[x_, t_] := Exp[-t] Sin[3 x];
ϕ[x_, n_] := Sin[n x];
λ = n^2;
numberOfTerms = 15;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = N[4 * Sum[Exp[-k λ t] ϕ[x, n] ((1 - (-1)^n)/(π n^3)), {n, 1, numberOfTerms}]] - (100 Exp[-t])/97 Sin[3 x] + 100/97 Exp[(-3 t)/100] Sin[3 x];
```

Figure 4.25: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, L}, {0, 3}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 200, .5}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.26: Code for animation

4.1.1.15 [165] Math 4567 Exam

problem number 165

Added April 3, 2019.

Exam question. Math 4567 UMN. Spring 2019.

Solve the heat equation

$$u_t = u_{xx} + t(\pi - x)$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

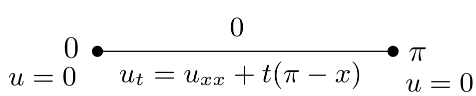


Figure 4.27: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + t*(Pi-x);
bc  = {u[0,t] == 0, u[Pi,t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> t
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{2\left(tn^2+e^{-n^2t}-1\right)\sin(nx)}{n^5}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+t*(Pi-x);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=0, u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=2\left(\sum_{n=1}^{\infty}\frac{\left(n^2t+e^{-n^2t}-1\right)\sin(nx)}{n^5}\right)$$

Hand solution

Let $u(x,t)=\sum_{n=1}^{\infty}B_n(t)\,X_n(x)$, where $X_n(x)$ are the eigenfunctions of the homogeneous PDE $u_t=u_{xx}$. These are known to be $X_n(x)=\sin\left(\sqrt{\lambda_n}x\right)$ where $\lambda_n=n^2, n=1,2,3,\cdots$. Hence

$$X_n(x)=\sin(nx)\qquad n=1,2,3,\cdots$$

Therefore the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}B_n(t)\sin(nx)\tag{A}$$

Substituting this into the PDE $u_t=u_{xx}+t(\pi-x)$ gives

$$\sum_{n=1}^{\infty}B_n'(t)\,X_n(x)=\sum_{n=1}^{\infty}B_n(t)\,X_n''(x)+\sum_{n=1}^{\infty}q_n(t)\,X_n(x)$$

Where $\sum_{n=1}^{\infty} q_n(t)\, X_n(x)$ is the Fourier series expansion of $t(\pi - x)$. But $X_n''(x) = -\lambda_n X_n(x) = -n^2 X_n(x)$. Hence the above becomes

$$\begin{aligned}
\sum_{n=1}^{\infty} B_n'(t)\, X_n(x) &= -\sum_{n=1}^{\infty} n^2 B_n(t)\, X_n(x) + \sum_{n=1}^{\infty} q_n(t)\, X_n(x) \\
B_n'(t)\, X_n(x) + n^2 B_n(t)\, X_n(x) &= q_n(t)\, X_n(x) \\
B_n'(t) + n^2 B_n(t) &= q_n(t) \qquad (1)
\end{aligned}$$

This is first order ODE which is now solved for B_n. But first $q_n(t)$ is found. Since

$$t(\pi - x) = \sum_{n=1}^{\infty} q_n(t)\, X_n(x)$$

Applying orthogonality. Integrating both sides and multiplying by $X_m(x)$ gives

$$\begin{aligned}
\int_0^{\pi} t(\pi - x)\, X_m(x)\, dx &= \sum_{n=1}^{\infty} q_n(t) \int_0^{\pi} X_n(x)\, X_m(x)\, dx \\
&= \sum_{n=1}^{\infty} q_n(t) \int_0^{\pi} \sin(nx) \sin(mx)\, dx \\
&= q_m(t) \int_0^{\pi} \sin^2(mx)\, dx \\
&= \frac{\pi}{2} q_m(t)
\end{aligned}$$

Therefore

$$\begin{aligned}
q_n(t) &= \frac{2}{\pi} \int_0^{\pi} t(\pi - x) \sin(nx)\, dx \\
&= \frac{2t(\pi n - \sin(n\pi))}{\pi n^2}
\end{aligned}$$

But n is integer, hence the above simplifies to

$$q_n(t) = \frac{2t}{n}$$

Hence (1) becomes

$$B_n'(t) + n^2 B_n(t) = \frac{2t}{n}$$

Integrating factor is $I = e^{\int n^2 dt} = e^{n^2 t}$. The above becomes

$$\frac{d}{dt}\left(B_n e^{n^2 t}\right) = \frac{2t}{n} e^{n^2 t}$$

Integrating

$$\begin{aligned} B_n e^{n^2 t} &= \frac{2}{n}\int_0^t s e^{n^2 s} ds + C \\ B_n(t) &= \frac{2}{n} e^{-n^2 t}\int_0^t s e^{n^2 s} ds + C e^{-n^2 t} \end{aligned} \tag{2}$$

At $t = 0, B_n(0) = 0$. The above reduces to

$$0 = C$$

Therefore (2) becomes

$$\begin{aligned} B_n(t) &= \frac{2}{n} e^{-n^2 t}\int_0^t s e^{n^2 s} ds \\ &= \frac{2}{n}\int_0^t s e^{n^2 (s-t)} ds \\ &= \frac{2\left(n^2 t + e^{-n^2 t} - 1\right)}{n^5} \end{aligned}$$

Hence Eq. (A) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\left(n^2 t + e^{-n^2 t} - 1\right)}{n^5} \sin(nx)$$

4.1.1.16 [166] With source

problem number 166

Taken from Maple PDE help pages

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= 0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$

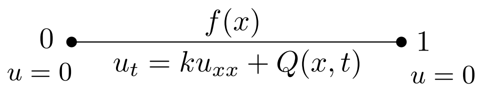


Figure 4.28: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {k
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\sqrt{2}\left(\int_0^t e^{-k\pi^2K[1]^2(t-K[2])}\text{Integrate}\left[\sqrt{2}Q(x,K[2])\sin(\pi xK[1]),\{x,0,1\},\text{Assumptions}\right.\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+Q(x, t);
bc  := u(0, t) = 0, u(l, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t)) as
```

$$u(x,t)=\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^l Q(x,\tau)\sin\left(\frac{\pi nx}{l}\right)dx\right)\mathrm{e}^{\frac{\pi^2(-t+\tau)k\,n^2}{l^2}}\sin\left(\frac{\pi nx}{l}\right)}{l}\right)d\tau+2\left(\sum_{n=1}^{\infty}\frac{\left(\int_0^l f(x)\sin\left(\frac{\pi nx}{l}\right)dx\right)\mathrm{e}^{-}}{l}\right.$$

4.1.1.17 [167] special initial condition

problem number 167

Added April 28, 2019.

Taken from `https://mathematica.stackexchange.com/questions/197155/solving-a-heat-equation-problem`

Solve $u(x,t)$

$$u_t = u_{xx} - 9u_x$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition $u(x,0) = e^{\frac{45}{10}}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$

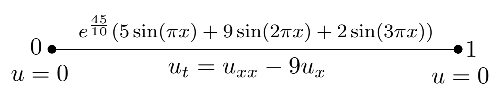


Figure 4.29: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - 9*D[u[x, t], x];
ic  = u[x, 0] == Exp[45/10 x]*(5 Sin[Pi*x] + 9 Sin[2*Pi*x] + 2 Sin[3*Pi*x]);
bc  = {u[0, t] == 0, u[1, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{\frac{9}{4}(2x-9t)} \sum_{K[1]=1}^{\infty} 0\right\}\right\}$$

Maple ✗

```
restart;
pde := diff(u(x,t),t)= diff(u(x, t), x$2) - 9*diff(u(x,t),x);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x, 0) = exp(45/10*x)*(5*sin(Pi*x) + 9*sin(2*Pi*x) + 2*sin(3*Pi*x));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

time expired
Hand solution

Solve

$$u_t = u_{xx} - 9u_x$$

IC

$$u(x,0) = e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$$

BC

$$u(0,x) = 0$$
$$u(1,x) = 0$$

Let $u = X(x)T(t)$, the PDE becomes

$$T'X = X''T - 9X'T$$
$$\frac{T'}{T} = \frac{X''}{X} - 9\frac{X'}{X} = -\lambda$$

Where λ is the seperation constant. From B.C. we know $\lambda > 0$. Hence the eigenvalue ODE is

$$X'' - 9X' + \lambda X = 0$$

The solution to the above is

$$X(x) = C_1 e^{\frac{1}{2}(9-\sqrt{81-4\lambda})x} + C_2 e^{\frac{1}{2}(9+\sqrt{81-4\lambda})x}$$
$$= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)$$

At $X(0) = 0$ this gives

$$0 = C_1 + C_2$$

And at $X(1) = 0$

$$
\begin{aligned}
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= C_1 e^{\frac{9}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)
\end{aligned}
$$

For nontrivial solution we want

$$
\begin{aligned}
e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}} &= 0\\
e^{-\frac{1}{2}\sqrt{81-4\lambda}} &= e^{\frac{1}{2}\sqrt{81-4\lambda}} \qquad (1)
\end{aligned}
$$

<u>Case $81 - 4\lambda > 0$</u>

This means $81 - 4\lambda$ must be zero or

$$
\lambda = \frac{81}{4}
$$

But using this eigenvalue makes the eigenfunction zero as shown below

$$
\begin{aligned}
X(x) &= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}(1-1)\\
&= 0
\end{aligned}
$$

Therefore $\lambda = \frac{81}{4}$ can not be used as eigenfunction.

<u>Case $81 - 4\lambda < 0$</u>

Then (1) becomes

$$
\begin{aligned}
e^{-\frac{i}{2}\sqrt{4\lambda-81}} &= e^{\frac{i}{2}\sqrt{4\lambda-81}}\\
\cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) - i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= \cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) + i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right)\\
2i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\frac{1}{2}\sqrt{4\lambda-81} &= n\pi \qquad n = 1,2,\cdots
\end{aligned}
$$

Hence

$$\begin{aligned}\frac{1}{4}(4\lambda-81)&=n^2\pi^2\\4\lambda&=81+4n^2\pi^2\\\lambda_n&=\frac{81}{4}+n^2\pi^2\end{aligned}$$

The corresponding eigenfunctions are (and since $C_2=-C_1$) then

$$\begin{aligned}X_n(x)&=C_n\left(e^{\frac{1}{2}(9-\sqrt{81-4\lambda_n})x}-e^{\frac{1}{2}(9+\sqrt{81-4\lambda_n})x}\right)\\&=C_n\left(e^{\frac{1}{2}(9-i\sqrt{4\lambda_n-81})x}-e^{\frac{1}{2}(9+i\sqrt{4\lambda_n-81})x}\right)\\&=C_ne^{\frac{9x}{2}}\left(e^{-\frac{i}{2}\sqrt{4\lambda_n-81}x}-e^{\frac{i}{2}\sqrt{4\lambda_n-81}x}\right)\\&=C_ne^{\frac{9x}{2}}\left(\cos\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)-i\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)-\cos\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)-\sin\left(\frac{1}{2}\sqrt{4\lambda_n-\right.\right.\\&=C_ne^{\frac{9x}{2}}\left(-2i\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)\right)\\&=A_ne^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)\end{aligned}$$

Hence the solution is

$$u(x,t)=\sum_{n=1}^{\infty}X_n(t)\,T_n(t)$$

But $T'+\lambda_nT=0$ has solution $T=e^{-\lambda_nt}$. Therefore the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}A_ne^{-\lambda_nt}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)$$

At $t=0$

$$e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x))=\sum_{n=1}^{\infty}A_ne^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)$$

But $\lambda_n=\frac{81}{4}+n^2\pi^2$. The above becomes

$$\begin{aligned}e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x))&=A_1e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+\pi^2\right)-81x}\right)\\&+A_2e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+4\pi^2\right)-81x}\right)\\&+A_3e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+9\pi^2\right)-81x}\right)\\&+\cdots\end{aligned}$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\pi^2}x\right)\\
&+ A_2 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{16\pi^2}x\right)\\
&+ A_3 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{36\pi^2}x\right)\\
&+\cdots
\end{aligned}
$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin(\pi x)\\
&+ A_2 e^{\frac{9x}{2}}\sin(2\pi x)\\
&+ A_3 e^{\frac{9x}{2}}\sin(3\pi x)\\
&+\cdots
\end{aligned}
$$

By comparing coefficients, we see that $A_1 e^{\frac{9x}{2}} = 5e^{\frac{45}{10}x}$ or $A_1 = e^{\left(\frac{45}{10}-\frac{9}{2}\right)x} = 5$ and $A_2 = 9$ and $A_3 = 2$ and and all other A_n for $n > 3$ are zero. Hence the solution becomes

$$
\begin{aligned}
u(x,t) &= 5e^{-\lambda_1 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_1-81}x\right)+9e^{-\lambda_2 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_2-81}x\right)+2e^{-\lambda_3 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_3-81}x\right)\\
&= e^{-\left(\frac{81}{4}+\pi^2\right)t}e^{\frac{9x}{2}}\sin(\pi x)+e^{-\left(\frac{81}{4}+4\pi^2\right)t}e^{\frac{9x}{2}}\sin(2\pi x)+e^{-\left(\frac{81}{4}+9\pi^2\right)t}e^{\frac{9x}{2}}\sin(3\pi x)\\
&= e^{-\frac{81}{4}t+\frac{9}{2}x}\left(5e^{-\pi^2 t}\sin(\pi x)+9e^{-4\pi^2 t}\sin(2\pi x)+2e^{-9\pi^2 t}\sin(3\pi x)\right)
\end{aligned}
$$

4.1.1.18 [168] Diffusion Reaction and Euler-Cauchy Sturm-Liouville

problem number 168

Added May 5, 2019.

Solve $u(x,t)$

$$u_t = x^2 u_{xx} + x u_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(1,t) &= 0\\
u(b,t) &= 0
\end{aligned}
$$

Initial condition $u(x,0) = f(x)$

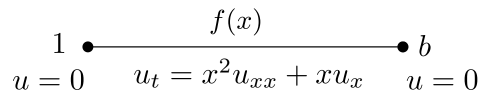


Figure 4.30: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic  = u[x, 0] == f[x];
bc  = {u[1, t] == 0, u[b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> b
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\frac{2e^{-\frac{\pi^2 tK[1]^2}{\log^2(b)}}\left(\int_1^b \frac{f(x)\sin\left(\frac{\pi K[1]\log(x)}{\log(b)}\right)}{x}dx\right)\sin\left(\frac{\pi K[1]\log(x)}{\log(b)}\right)}{\log(b)}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff( u(x,t),t)= x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc  := u(1,t)=0,u(b,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_1^b \frac{f(x)\sin\left(\frac{\pi n\ln(x)}{\ln(b)}\right)}{x}dx\right)\mathrm{e}^{-\frac{\pi^2 n^2 t}{\ln(b)^2}}\sin\left(\frac{\pi n\ln(x)}{\ln(b)}\right)}{\ln(b)}$$

4.1.1.19 [169] Diffusion Reaction. Using growth form reaction term

problem number 169

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = k u_{xx} + r u$$

with $k = \frac{1}{10}, r = 1$ and $0 < x < 1$ and $t > 0$. With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x,0) = 1$.

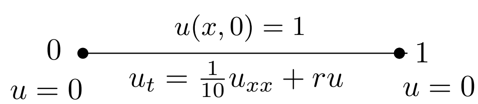


Figure 4.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/10;
 r = 1;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} -\frac{2\left(-1+(-1)^{K[1]}\right) e^{t-\frac{1}{10}\pi^2 t K[1]^2} \sin(\pi x K[1])}{\pi K[1]}\right\}\right\}$$

Maple ✓

```
restart;
k:=1/10;
r:=1;
pde := diff(u(x,t), t) = k*diff(u(x, t), x$2) + r*u(x,t);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(-\frac{2((-1)^n-1)\,\mathrm{e}^{-\frac{(\pi^2n^2-10)t}{10}}\sin(\pi n x)}{\pi n}\right)$$

Hand solution

Solution added 4/3/2019.

$$\begin{aligned} u_t &= ku_{xx}+ru \qquad t>0, 0<x<1\\ u(0,t)&=0\\ u(1,t)&=0\\ u(x,0)&=1 \end{aligned}$$

Let $u=ve^{rt}$. Hence $u_t=v_te^{rt}+vre^{rt}$ and $u_{xx}=v_{xx}e^{rt}$. Hence the PDE becomes $v_te^{rt}+vre^{rt}=v_{xx}e^{rt}+rve^{rt}$ which simplifies to

$$\begin{aligned} v_t &= kv_{xx} \qquad t>0, 0<x<1\\ v(0,t)&=0\\ v(1,t)&=0 \end{aligned}$$

The above is now in canonical form, it is standard heat PDE with homogeneous B.C. This has the solution

$$v(x,t)=\sum_{n=1}^{\infty}B_ne^{-n^2\pi^2t}\sin(n\pi x)$$

Therefore

$$\begin{aligned} u&=ve^{rt}\\ u&=\sum_{n=1}^{\infty}B_ne^{-t(n^2\pi^2-r)}\sin(n\pi x) \end{aligned}$$

At $t = 0$ the above becomes

$$1 = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

Hence B_n are sin Fourier coefficient of 1 which is

$$\begin{aligned} B_n &= 2 \int_0^1 \sin(n\pi x)\, dx \\ &= 2\left(-\frac{1}{n\pi}\right)(\cos n\pi x)_0^1 \\ &= \frac{-2}{n\pi}((-1)^n - 1) \end{aligned}$$

Hence the solution becomes

$$u = \sum_{n=1}^{\infty} \frac{-2}{n\pi}((-1)^n - 1)\, e^{-t(n^2\pi^2 - r)} \sin(n\pi x)$$

But $r = \frac{1}{10}$, therefore

$$u(x,t) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{((-1)^n - 1)}{n} e^{-t\left(n^2\pi^2 - \frac{1}{10}\right)} \sin(n\pi x)$$

4.1.1.20 [170] Diffusion Reaction, using logistic form for reaction term

problem number 170

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = k u_{xx} + r u\left(1 - \frac{u}{\alpha}\right)$$

with $k = \frac{1}{100}, r = \frac{1}{10}, \alpha = 10$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= 0 \end{aligned}$$

And initial conditions $u(x,0) = 1$.

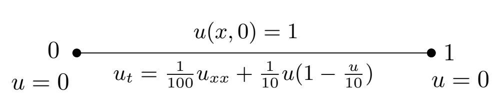


Figure 4.32: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
k = 1/100;
 r = 1/10;
 alpha = 10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t]*(1 - u[x, t]/alpha);
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
k := 1/100;
r := 1/10;
alpha := 10;
pde := diff(u(x, t), t)=   k*diff(u(x, t),x$2) + r*u(x, t)*(1- u(x,t)/alpha);
bc  := u(0, t) = 0, u(1, t) = 0;
ic  := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc], u(x, t))),o
```

sol=()

4.1.1.21 [171] Diffusion Reaction, using Aleee form for reaction term

problem number 171

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = ku_{xx} + \alpha u + \beta u^2 - \gamma u^3$$

with $k = \frac{1}{1000}, \alpha = \frac{1}{100}, \beta = \frac{1}{100}, \gamma = \frac{5}{1000}$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x,0) = 1$.

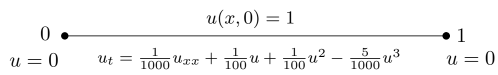


Figure 4.33: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
k = 1/1000;
 alpha = 1/10;
 beta = 1/100;
 gamma = 5/1000;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + alpha*u[x, t] + beta*u[x, t]^2 - gamma*u[x, t
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
k := 1/100;
alpha:=1/100;
beta:=1/1000;
g:=5/1000;
pde := diff(u(x, t), t)= k*diff(u(x, t),x$2) +
        alpha*u(x,t)+ beta*u(x,t)^2 - g*u(x,t)^3;
bc  := u(0, t) = 0, u(1, t) = 0;
ic  := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

sol=()

4.1.1.22 [172] Diffusion Reaction. Haberman 2.3.8

problem number 172

This is problem 2.3.8, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx} - \alpha u$$

This corresponds to a one-dimentional rod either with heat loss through the lateral sides with outside temperature zero degrees ($\alpha > 0$) or with insulated sides with a heat sink propertional to the temperature.

Suppose the boundary conditions are $u(0,t) = 0, u(L,t) = 0$, solve with the temperature initially $u(x,0) = f(x)$ if $\alpha > 0$

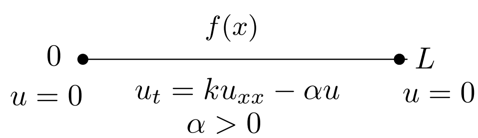


Figure 4.34: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - alpha*u[x, t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\, e^{-t\left(\frac{k\pi^2 K[1]^2}{L^2}+\alpha\right)}\left(\int_0^L \frac{\sqrt{2} f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-alpha*u(x,t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\left(\int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) dx\right) e^{-\frac{\left(L^2\alpha+\pi^2 k n^2\right)t}{L^2}} \sin\left(\frac{\pi n x}{L}\right)}{L}$$

Hand solution

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - \alpha u$$
$$\frac{\partial u}{\partial t} + \alpha u = k\frac{\partial^2 u}{\partial x^2}$$

Assuming $u(x,t) = X(x)T(t)$ and substituting in the above gives

$$XT' + \alpha XT = kTX''$$

Dividing by $kXT \neq 0$

$$\frac{T'}{kT} + \frac{\alpha}{k} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T} + \frac{\alpha}{k} = \frac{X''}{X} = -\lambda$$

The two ODE's are

$$\frac{1}{k}\frac{T'}{T} + \frac{\alpha}{k} = -\lambda$$
$$\frac{X''}{X} = -\lambda$$

Or

$$T' + (\alpha + \lambda k)T = 0$$
$$X'' + \lambda X = 0$$

The solution to the space ODE is the familiar (where $\lambda > 0$ is only possible case, As found in Haberman problem 2.3.3, part d. Since it has the same B.C.)

$$X_n = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2$. The time ODE is now solved.

$$\frac{dT_n}{dt} + (\alpha + \lambda_n k)\, T_n = 0$$

This has the solution

$$\begin{aligned} T_n(t) &= e^{-(\alpha+\lambda_n k)t} \\ &= e^{-\alpha t} e^{-\left(\frac{n\pi}{L}\right)^2 kt} \end{aligned}$$

For the same eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the B_n when combined in the following step below. Therefore the solution to the PDE is

$$u_n(x,t) = T_n(t)\, X_n(x)$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} u_n(x,t) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\alpha t} e^{-\left(\frac{n\pi}{L}\right)^2 kt} \\ &= e^{-\alpha t} \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt} \end{aligned}$$

Where $e^{-\alpha t}$ was moved outside since it does not depend on n. From initial condition

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

Applying orthogonality of sin as before to find B_n results in

$$B_n = \frac{2}{L} \int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx$$

Hence the solution becomes

$$\begin{aligned} u(x,t) &= \frac{2}{L} e^{-\alpha t} \left(\sum_{n=1}^{\infty} \left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx \right) \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt} \right) \\ &= \frac{2}{L} \sum_{n=1}^{\infty} \left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx \right) \sin\left(\frac{n\pi}{L}x\right) e^{-t\left(\frac{n^2\pi^2 k + \alpha L^2}{L^2}\right)} \end{aligned}$$

Hence it is clear that in the limit as t becomes large $u(x,t) \to 0$ since $\alpha > 0$ and

$$\lim_{t\to\infty} u(x,t) = 0$$

4.1.1.23 [173] Diffusion Reaction

problem number 173

Solve the heat equation

$$u_t = u_{xx} - u(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

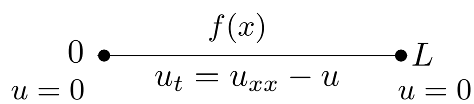


Figure 4.35: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + u[x, t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \sqrt{2}e^{t\left(-\frac{\pi^2 K[1]^2}{L^2}-1\right)}\sqrt{\frac{1}{L}}\left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)dx\right)\sin\left(\frac{\pi x K[1]}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)+u(x,t)=diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\left(L^2+\pi^2 n^2\right)t}{L^2}}\sin\left(\frac{\pi n x}{L}\right)}{L}$$

Hand solution

$$
\begin{aligned}
u_t &= u_{xx}-u\\
u(0,t)&=0\\
u(L,t)&=0\\
u(x,0)&=f(x)
\end{aligned}
$$

Let $u(x,t)=v(x,t)\,e^{-t}$, hence $u_t=v_te^{-t}-ve^{-t}$ and $u_{xx}=v_{xx}e^{-t}$. Therefore the above PDE becomes

$$
\begin{aligned}
v_te^{-t}-ve^{-t}&=v_{xx}e^{-t}-ve^{-t}\\
v_t&=v_{xx}
\end{aligned}
$$

With boundary conditions

$$
\begin{aligned}
v(0,t)&=0\\
v(L,t)&=0
\end{aligned}
$$

The solution to this PDE is known, since it has homogenouse BC and it is in standard form. The solution is

$$v(x,t)=\sum_{n=1}^{\infty}B_ne^{-\left(\frac{n\pi}{L}\right)^2t}\sin\left(\frac{n\pi}{L}x\right)$$

Hence

$$
\begin{aligned}
u(x,t) &= e^{-t}\sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t-t}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2}{L^2}+1\right)}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\sin\left(\frac{n\pi}{L}x\right) \qquad (1)
\end{aligned}
$$

Applying initial conditions $u(x,0)=f(x)$ gives

$$
f(x)=\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)
$$

Hence B_n are the Fourier sine coefficients of $f(x)$

$$
B_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx
$$

Therefore the solution in (1) becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\frac{2}{L}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\sin\left(\frac{n\pi}{L}x\right)\\
&= \frac{2}{L}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{L}x\right)e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)
\end{aligned}
$$

4.1.1.24 [174] Diffusion Reaction

problem number 174

Added Feb 10, 2018.

Solve the heat equation

$$
u_t+u(x,t)=100u_{xx}
$$

For $0<x<1$ and $t>0$. The boundary conditions are

$$
\begin{aligned}
u(0,t)&=0\\
u(1,t)&=0
\end{aligned}
$$

Initial condition is $u(x,0) = \sin(2\pi x) - \sin(5\pi x)$

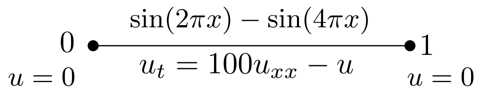


Figure 4.36: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 100*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == Sin[2*Pi*x] - Sin[5*Pi*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{-400\pi^2 t}\sin(2\pi x) - e^{-2500\pi^2 t}\sin(5\pi x)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=100*diff(u(x,t),x$2);
ic  := u(x,0)=sin(2*Pi*x)-sin(5*Pi*x);
bc  := u(0,t)=0,u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = -e^{-2500\pi^2 t}\sin(5\pi x) + e^{-400\pi^2 t}\sin(2\pi x)$$

4.1.1.25 [175] Diffusion Reaction

problem number 175

Added June 23, 2019.

Solve the heat equation

$$u_t = ku_{xx} - ux$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= 0 \\
u(\pi,t) &= 0
\end{aligned}
$$

Initial condition $u(x,0) = \sin(x)$

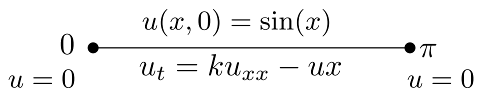


Figure 4.37: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - u[x,t]*x;
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)- u(x,t)*x;
ic  := u(x,0)=sin(x);
bc  := u(0,t)=0,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$
u(x,t) = \sum_{n=0}^{\infty} \frac{\left(\cosh\left(t\lambda_n\right) + \sinh\left(t\lambda_n\right)\right)\left(-\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)\operatorname{AiryBi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right) + \operatorname{AiryAi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right)\operatorname{AiryBi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)}{\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)^2\left(\int_0^{\pi}\operatorname{AiryBi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right)^2 dx\right) - 2\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)\operatorname{AiryBi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)}
$$

4.1.1.26 [176] Diffusion convection (general case)

problem number 176

Added June 23, 2019.

Solve the heat equation

$$u_t = k u_{xx} + a u_x$$

Where $a > 0$. For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition $u(x,0) = f(x)$

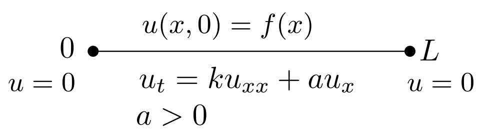


Figure 4.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + a*D[u[x, t], x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to e^{-\frac{a(at+2x)}{4k}} \sum_{K[1]=1}^{\infty} \frac{2e^{-\frac{k\pi^2 t K[1]^2}{L^2}} \left(\int_0^L e^{\frac{ax}{2k}} f(x) \sin\left(\frac{\pi x K[1]}{L}\right) dx\right) \sin\left(\frac{\pi x K[1]}{L}\right)}{L}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+ a*diff(u(x,t),x);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\cosh\left(\frac{ax}{2k}\right)-\sinh\left(\frac{ax}{2k}\right)\right)\left(\int_0^L\left(\cosh\left(\frac{ax}{2k}\right)-\sinh\left(\frac{ax}{2k}\right)\right)\mathrm{e}^{\frac{ax}{k}}f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi nx}{L}\right)}{\left(\cosh\left(\frac{(L^2a^2+4\pi^2k^2n^2)t}{4L^2k}\right)+\sinh\left(\frac{(L^2a^2+4\pi^2k^2n^2)t}{4L^2k}\right)\right)L}$$

Hand solution

Solve

$$\begin{aligned}u_t&=ku_{xx}+au_x\\u(0,t)&=0\\u(L,0)&=0\\u(x,0)&=f(x)\end{aligned}$$

Trying separation of variables. Let $u=XT$, then the PDE becomes

$$\begin{aligned}T'X&=kX''T+aX'T\\\frac{1}{k}\frac{T'}{T}&=\frac{X''}{X}+\frac{a}{k}\frac{X'}{X}=-\lambda\end{aligned}$$

Where λ is separation constant. Hence

$$T'+k\lambda T=0$$

Which has solution $T(t)=e^{-k\lambda t}$ (the constant of integration is not needed, it will be combined with constant coming from the spatial ODE), and the spatial ODE is

$$\begin{aligned}\frac{X''}{X}+\frac{a}{k}\frac{X'}{X}&=-\lambda\\X''+\frac{a}{k}X'+\lambda X&=0\\X(0)&=0\\X(L)&=0\end{aligned}$$

The characteristic equation is $r^2 + \frac{a}{k}r + \lambda = 0$.

Case $\lambda = 0$ Then $r^2 + \frac{a}{k}r = 0$ or $r\left(r + \frac{a}{k}\right) = 0$. Hence $r = 0$ or $r = -\frac{a}{k}$. So the solution is

$$X(x) = c_1 + c_2 e^{-\frac{a}{k}x}$$

At $X(0) = 0$ the above gives $0 = c_1 + c_2$. Therefore the solution is $X(x) = c_1\left(1 - e^{\frac{-a}{k}x}\right)$. At $X(L) = 0$ then $c_1\left(1 - e^{\frac{a}{k}L}\right) = 0$. This means this for non-trivial solution $e^{\frac{-a}{k}L} = 1$ or $\frac{-a}{k}L = 0$. Hence $\lambda = 0$ is not an eigenvalue.

Case $\lambda < 0$ Then $r^2 + \frac{a}{k}r + \lambda = 0$ or $r = \frac{-b}{2} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-a}{2k} \pm \frac{\sqrt{\frac{a^2}{k^2}-4\lambda}}{2}$. Since $\lambda < 0$ then the term under the sqrt root is positive. Hence the solution will have real roots. Not complex conjugate. Hence the solution is pure exponentials. Let $\frac{a^2}{k^2} - 4\lambda = \alpha$ and $\frac{a}{2k} = \beta$ then

$$\begin{aligned} r_1 &= -\beta + \sqrt{\alpha} \\ r_2 &= -\beta - \sqrt{\alpha} \end{aligned}$$

Then $X(x) = c_1 \cosh\left(\left(-\beta + \alpha\right)x\right) + c_2 \sinh\left(\left(-\beta - \alpha\right)x\right)$. At $X(0) = 0$ this gives $0 = c_1$. Hence solution is $X(x) = c_2 \sinh\left(\left(-\beta - \alpha\right)x\right)$. At $X(L) = 0$, this gives

$$0 = c_2 \sinh\left(\left(-\beta - \alpha\right)L\right)$$

For non-trivial solution, $\sinh\left(\left(-\beta - \alpha\right)L\right) = 0$. But this is zero only when $\left(-\beta - \alpha\right)L$ or $\beta = -\alpha$. Or $\frac{-a}{2k} = -\sqrt{\frac{a^2}{k^2} - 4\lambda}$. Or $\frac{a^2}{4k^2} = \frac{a^2}{k^2} - 4\lambda$ or

$$\begin{aligned} 4\lambda &= \frac{a^2}{k^2} - \frac{a^2}{4k^2} \\ \lambda &= \frac{3}{16}\frac{a^2}{k^2} \end{aligned}$$

But λ was assumed negative. Hence this is not possible. $\lambda < 0$ is not eigenvalue.

Case $\lambda > 0$ Then $r^2 + \frac{a}{k}r + \lambda = 0$ or $r = \frac{-a}{2k} \pm \frac{1}{2}\sqrt{\frac{a^2}{k^2} - 4\lambda} = \frac{-a}{2k} \pm \sqrt{\frac{a^2}{4k^2} - \lambda}$. Since $\lambda > 0$ then the term under the sqrt root can be negative. Then only when $\lambda > \frac{a^2}{4k^2}$ will there be complex roots. Therefore assuming $\lambda > \frac{a^2}{4k^2}$, then

$$\begin{aligned} r_1 &= \frac{-a}{2k} + i\sqrt{\lambda - \frac{a^2}{4k^2}} \\ r_2 &= \frac{-a}{2k} - i\sqrt{\lambda - \frac{a^2}{4k^2}} \end{aligned}$$

The solution is

$$X(x) = e^{\frac{-a}{2k}x}\left(c_1 \cos\left(\sqrt{\lambda - \frac{a^2}{4k^2}}x\right) + c_2 \sin\left(\sqrt{\lambda - \frac{a^2}{4k^2}}x\right)\right)$$

At $X(0) = 0$ this gives $0 = c_1$. Hence solution is

$$X(x) = e^{\frac{-a}{2k}x} c_2 \sin\left(\sqrt{\lambda - \frac{a^2}{4k^2}}x\right)$$

At $X(L) = 0$, this gives

$$0 = e^{\frac{-a}{2k}\pi} c_2 \sin\left(\sqrt{\lambda - \frac{a^2}{4k^2}}L\right)$$

For non-trivial solution, $\sin\left(\sqrt{\lambda - \frac{a^2}{4k^2}}L\right) = 0$ or $\sqrt{\lambda - \frac{a^2}{4k^2}}L = n\pi$. Hence

$$\lambda - \frac{a^2}{4k^2} = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 + \frac{a^2}{4k^2} \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunction is

$$X_n(x) = e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Therefore the solution is

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-k\lambda_n t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

c_n is found from initial conditions. At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} c_n e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

$$e^{\frac{a}{2k}x} f(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{L}x\right)$$

Therefore, by orthogonality

$$\int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx = c_n \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$\int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx = c_n \frac{L}{2}$$

$$c_n = \frac{2}{L}\int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

Hence the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-k\lambda_n t}e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2+\frac{a^2}{4k^2}\qquad n=1,2,3,\cdots$

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-k\left(\frac{n^2\pi^2}{L^2}+\frac{a^2}{4k^2}\right)t}e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)$$

4.1.1.27 [177] Diffusion convection (special case)

problem number 177

Added June 23, 2019.

Solve the heat equation

$$u_t=ku_{xx}+au_x$$

Where $a>0$. For $0<x<L$ and $t>0$. The boundary conditions are

$$u(0,t)=0$$
$$u(L,t)=0$$

Initial condition $u(x,0)=f(x)$ using the following values

$$f(x)=\sin(x)$$
$$k=1$$
$$a=5$$
$$L=\pi$$

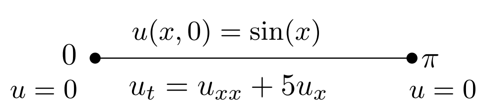


Figure 4.39: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f=Sin[x];
L=Pi;
k=1;
a=5;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + a*D[u[x, t], x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{-\text{Integrate}\left[\frac{(1+e^{-i\pi K})e^{K(-Kt+5it+ix)}}{1-K^2},\{K,-\infty,\infty\},\text{Assumptions}\to K\in\mathbb{C}\right]+\int_{\infty}^{-\infty}\frac{\exp\left(it\sqrt{4}\right.}{}}{}\right.\right.$$

Maple ✓

```
restart;
f:=sin(x);
L:=Pi;
k:=1;
a:=5;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+ a*diff(u(x,t),x);
ic  := u(x,0)=f;
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{80i\left(2in+\left(-\frac{1}{5}n^2+in+\frac{29}{20}\right)e^{-i\pi n+\frac{5}{2}\pi}+\left(\frac{1}{5}n^2+in-\frac{29}{20}\right)e^{i\pi n+\frac{5}{2}\pi}\right)\left(\cosh\left(n^2t+\frac{25}{4}t\right)-\sinh\left(\right.\right.}{\pi\left(16n^4+168n^2+841\right)}$$

Hand solution

Solve

$$\begin{aligned} u_t &= k u_{xx} + a u_x \\ u(0,t) &= 0 \\ u(L,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

With $a = 5, k = 1, L = \pi, f(x) = \sin(x)$. The above problem was solved in 4.1.1.26 on page 469 and the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\lambda_n t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2 + \frac{a^2}{4k^2} \qquad n = 1, 2, 3, \cdots$

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

Substituting for $f(x) = \sin(x)$ the above becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} \sin(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

But

$$\int_0^{\pi} e^{\frac{a}{2k}x} \sin(x) \sin(nx)\, dx = \frac{-16nak^3\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2-1\right)^2 + 8a^2k^2\left(1+n^2\right)}$$

Hence the solution becomes

$$u(x,t) = \frac{-32ak^3}{L} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2-1\right)^2 + 8a^2k^2\left(1+n^2\right)} \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

But $L = \pi$, hence

$$u(x,t) = \frac{-32ak^3}{\pi} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2-1\right)^2 + 8a^2k^2\left(1+n^2\right)} \right) e^{-k\left(n^2 + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin(nx)$$

And $a = 5$

$$u(x,t) = \frac{-160k^3}{\pi} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{5\pi}{2k}}\right)}{625 + 16k^4\left(n^2-1\right)^2 + 200k^2\left(1+n^2\right)} \right) e^{-k\left(n^2 + \frac{25}{5k^2}\right)t} e^{\frac{-5}{2k}x} \sin(nx)$$

And $k = 1$

$$u(x,t)=\frac{-160}{\pi}\sum_{n=1}^{\infty}\left(\frac{n\left(1+(-1)^{n}e^{\frac{5\pi}{2}}\right)}{625+16\left(n^{2}-1\right)^{2}+200\left(1+n^{2}\right)}\right)e^{-\left(n^{2}+\frac{25}{4}\right)t}e^{\frac{-5}{2}x}\sin\left(nx\right)$$

Animation is below

Out[ ]=

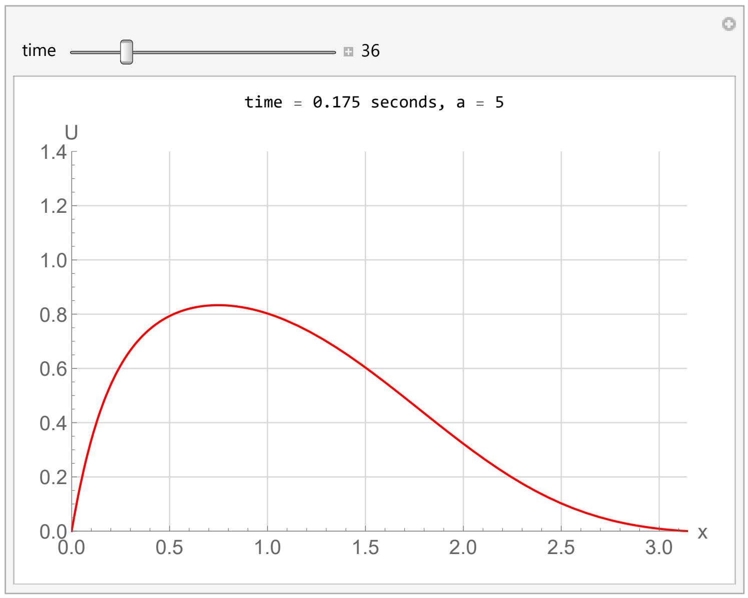


Figure 4.40: Initial state

Source code used for the above

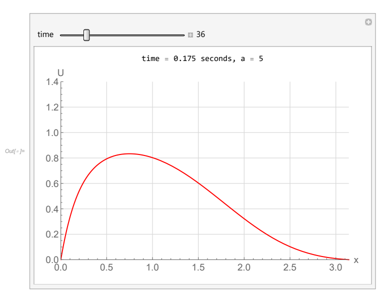
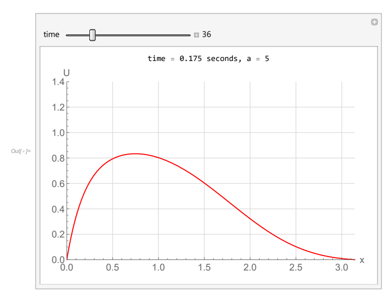
```
In[ ]:= ClearAll[x, y, t, n, k]
L = Pi;
numberOfTerms = 100;
a = 5;
k = 1;
mySol[x_, t_] = (-32 a k^3)/Pi Sum[(n (1 + (-1)^n Exp[(Pi a)/(2 k)]))/(a^4 + 16 k^4 (-1 + n^2)^2 + 8 a^2 k^2 (1 + n^2)) Exp[-k (n^2 + a^2/(4 k^2)) t] Exp[(-a)/(2 k) x] Sin[n x], {n, 1, numberOfTerms}];

padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 4.41: Source code

```
In[∘]:= tab = Table[
          Grid[{
            {Row[{"time = ", padIt2[t, {4, 3}], " seconds, a = ", 5}]},
            {
              Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {0, 1.4}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

            }}],
          {t, 0, 0.95, .005}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[∘]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.42: Code for animation

The following animations for $a = 1, a = 7$. Showing the effect of increasing the convection factor a on the solution.

$a = 1$	$a = 7$
	

4.1.1.28 [178] Haberman 2.4.2 (general case)

problem number 178

This is problem 2.4.2 from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

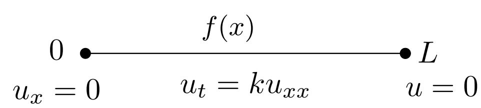


Figure 4.43: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=0}^{\infty} e^{-\frac{k(2\pi n+\pi)^2 t}{4L^2}} \cos\left(\frac{(2n+1)\pi x}{2L}\right)\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right) f(x)\,dx}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right) f(x)\,dx\right)\cos\left(\frac{(2n+1)\pi x}{2L}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2kt}{4L^2}}}{L}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t}=\kappa\frac{\partial^2 u}{\partial x^2}$$

Let $u(x,t)=T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{\kappa}T'X=X''T$$

Dividing by XT

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is real.

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

The two ODE's are

$$T'+k\lambda T=0 \tag{1}$$
$$X''+\lambda X=0 \tag{2}$$

Per problem statement, $\lambda\geq 0$, so only two cases needs to be examined.

Case $\lambda=0$

The space equation becomes $X''=0$ with the solution

$$X=Ax+b$$

Hence left B.C. implies $X'(0) = 0$ or $A = 0$. Therefore the solution becomes $X = b$. The right B.C. implies $X(L) = 0$ or $b = 0$. Therefore this leads to $X = 0$ as the only solution. This results in trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$

Starting with the space ODE, the solution is

$$\begin{aligned} X(x) &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\ X'(x) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right) \end{aligned}$$

Left B.C. gives

$$\begin{aligned} 0 &= X'(0) \\ &= B\sqrt{\lambda} \end{aligned}$$

Hence $B = 0$ since it is assumed $\lambda \neq 0$ and $\lambda > 0$. Solution becomes

$$X(x) = A\cos\left(\sqrt{\lambda}x\right)$$

Applying right B.C. gives

$$\begin{aligned} 0 &= X(L) \\ &= A\cos\left(\sqrt{\lambda}L\right) \end{aligned}$$

$A = 0$ leads to trivial solution. Therefore $\cos\left(\sqrt{\lambda}L\right) = 0$ or

$$\begin{aligned} \sqrt{\lambda} &= \frac{n\pi}{2L} \qquad n = 1,3,5,\cdots \\ &= \frac{(2n-1)\pi}{2L} \qquad n = 1,2,3\cdots \end{aligned}$$

Hence

$$\begin{aligned} \lambda_n &= \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots \\ &= \frac{(2n-1)^2\pi^2}{4L^2} \qquad n = 1,2,3\cdots \end{aligned}$$

Therefore

$$X_n(x) = A_n\cos\left(\frac{n\pi}{2L}x\right) \qquad n = 1,3,5,\cdots$$

And the corresponding time solution

$$T_n = e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \qquad n = 1,3,5,\cdots$$

Hence

$$
\begin{aligned}
u_n(x,t) &= X_n T_n \\
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \sum_{n=1}^{\infty} A_n \cos\left(\frac{(2n-1)\pi}{2L}x\right) e^{-k\left(\frac{(2n-1)\pi}{2L}\right)^2 t}
\end{aligned}
$$

From initial conditions

$$
f(x) = \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right)
$$

Multiplying both sides by $\cos\left(\frac{m\pi}{2L}x\right)$ and integrating

$$
\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx = \int \left(\sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{m\pi}{2L}x\right)\cos\left(\frac{n\pi}{2L}x\right)\right) dx
$$

Interchanging order of summation and integration and applying orthogonality results in

$$
\begin{aligned}
\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx &= A_m \frac{L}{2} \\
A_n &= \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx
\end{aligned}
$$

Therefore the solution is

$$
\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty} \left[\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx\right] \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \frac{2}{L}\sum_{n=0}^{\infty} \left(\int_0^L f(x)\cos\left(\frac{(2n+1)\pi}{2L}x\right) dx\right) \cos\left(\frac{(2n+1)\pi}{2L}x\right) e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}
\end{aligned}
$$

4.1.1.29 [179] Left end zero, right end insulated, no source

problem number 179

Added January 13, 2020.

Problem 4.1.4, Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

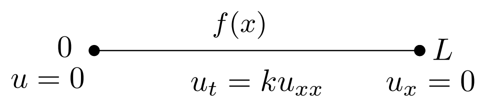


Figure 4.44: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty} e^{-\frac{k(1-2n)^2\pi^2 t}{4L^2}}\left(\int_0^L f(x)\sin\left(\frac{(2n-1)\pi x}{2L}\right)dx\right)\sin\left(\frac{(2n-1)\pi x}{2L}\right)}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2 kt}{4L^2}}\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}$$

Hand solution

The problem to solve is to solve for $u(x,t)$ in

$$u_t = k u_{xx} \qquad 0 < x < L, t > 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u_x(L,t) &= 0 \end{aligned}$$

And initial conditions

$$u(x,0) = f(x)$$

Let $u(x,t) = T(t)\,X(x)$, then the PDE becomes

$$T'X = kX''T$$

Dividing by XT

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's are

$$T' + \lambda k T = 0 \tag{1}$$

And the eigenvalue ODE

$$\begin{aligned} X'' + \lambda X &= 0 \qquad (2) \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Now we solve (2) to find the eigenvalues and eigenfunctions.

<u>Case $\lambda < 0$</u>

Let $-\lambda = \omega^2$. Hence the ODE is $X'' - \omega^2 X = 0$ and the solution becomes

$$X(x) = C_1 \cosh(\omega x) + C_2 \sinh(\omega x)$$

At $x = 0$ the above gives

$$0 = C_1$$

Hence the solution now becomes

$$X(x) = C_2 \sinh(\omega x)$$

Taking derivative gives

$$X'(x) = \omega C_2 \sinh(\omega x)$$

At $x = L$

$$0 = \omega C_2 \cosh(\omega L)$$

But $\cosh(\omega L)$ is never zero. Therefore $C_2 = 0$ which leads to trivial solution. Therefore $\lambda < 0$ is not eigenvalue.

Case $\lambda = 0$

The space equation becomes $X'' = 0$ with the solution

$$X = Ax + B$$

At $x = 0$ the above gives $0 = B$. Therefore the solution is $X = Ax$. Taking derivative gives $X' = A$. At $x = L$ this gives $0 = A$. Which leads to trivial solutions. Therefore $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$

Starting with the space ODE, the solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Left B.C. gives

$$0 = A$$

The solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivative gives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Applying right B.C. gives

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

For non trivial solution we want $\cos\left(\sqrt{\lambda}L\right) = 0$ or

$$\sqrt{\lambda} = \frac{n\pi}{2L} \qquad n = 1, 3, 5, \cdots$$

Hence the eigenvalues are

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1, 3, 5, \cdots$$

Therefore the eigenfunctions are

$$X_n(x) = \sin\left(\frac{n\pi}{2L}x\right) \qquad n = 1,3,5,\cdots$$

Now that we found the eigenvalues, we can solve the time ODE (1).

$$\begin{aligned} T_n' + k\lambda_n T &= 0 \\ T_n &= B_n e^{-k\lambda_n t} \\ &= B_n e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \end{aligned}$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,t) &= X_n T_n \\ u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \end{aligned} \tag{3}$$

From initial conditions

$$f(x) = \sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{n\pi}{2L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2L}x\right)$ and integrating

$$\int_0^L f(x)\sin\left(\frac{m\pi}{2L}x\right)dx = \int_0^L \left(\sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{m\pi}{2L}xx\right)\sin\left(\frac{n\pi}{2L}x\right)\right)dx$$

Interchanging order of summation and integration and applying orthogonality between cos functions results in

$$\begin{aligned} \int_0^L f(x)\sin\left(\frac{m\pi}{2L}x\right)dx &= \int_0^L B_m \sin^2\left(\frac{m\pi}{2L}x\right)dx \\ &= B_m \frac{L}{2} \end{aligned}$$

Therefore

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx$$

Therefore the solution is (3) becomes

$$\begin{aligned} u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\sin\left(\frac{n\pi}{2L}x\right)e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\ &= \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)e^{-\left(\frac{(2n+1)\pi}{2L}\right)^2 t} \end{aligned}$$

4.1.1.30 [180] One end insulated

problem number 180

Added June 9, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Where $T_0 > 0$ and initial condition is $u(x,0) = 0$

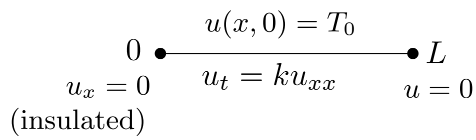


Figure 4.45: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == 0, u[1, t] == T0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{T0
 sol= sol/.{K[1]->n};
```

$$\left\{\left\{u(x,t) \to 2\sum_{n=0}^{\infty} -\frac{2e^{-\frac{1}{4}k(2\pi n+\pi)^2 t}\text{T0}\cos(n\pi)\cos\left(\frac{1}{2}(2n+1)\pi x\right)}{2\pi n+\pi} + \text{T0}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =0,u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \mathit{T0} - 4\left(\sum_{n=0}^{\infty} \frac{\mathit{T0}(-1)^n \cos\left(\frac{(2n+1)\pi x}{2L}\right) \mathrm{e}^{-\frac{\pi^2(2n+1)^2 kt}{4L^2}}}{(2n+1)\,\pi}\right)$$

4.1.1.31 [181] Haberman 2.3.7 (general case)

problem number 181

This is problem 2.3.7, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = ku_{xx}$$

Subject to boundary conditions $u_x(0,t) = 0$, $u_x(L,t) = 0$ with initial conditions $u(x,0) = f(x)$

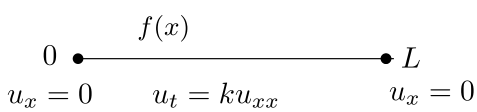


Figure 4.46: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty} e^{-\frac{kn^2\pi^2 t}{L^2}} \cos\left(\frac{n\pi x}{L}\right)\int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx + \int_0^L f(x)\,dx}{L}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L \cos\left(\frac{\pi n x}{L}\right) f(x)\,dx\right)\cos\left(\frac{\pi n x}{L}\right) e^{-\frac{\pi^2 k n^2 t}{L^2}}}{L}\right) + \frac{\int_0^L f(x)\,dx}{L}$$

Hand solution

$$\begin{aligned}\frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2}\\ u_x(0,t) &= 0\\ u_x(L,t) &= 0\\ u(x,0) &= f(x)\end{aligned}$$

Let $u(x,t) = T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{k}T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's generated are

$$T' + k\lambda T = 0 \tag{1}$$
$$X'' + \lambda X = 0 \tag{2}$$

Starting with the space ODE equation (2), with corresponding boundary conditions $\frac{dX}{dx}(0) = 0$, $\frac{dX}{dx}(L) = 0$. Assuming the solution is $X(x) = e^{rx}$, Then the characteristic equation is

$$\begin{aligned} r^2 + \lambda &= 0 \\ r^2 &= -\lambda \\ r &= \pm\sqrt{-\lambda} \end{aligned}$$

The following cases are considered.

<u>case $\lambda < 0$</u> In this case, $-\lambda$ and also $\sqrt{-\lambda}$ are positive. Hence the roots $\pm\sqrt{-\lambda}$ are both real. Let

$$\sqrt{-\lambda} = s$$

Where $s > 0$. This gives the solution

$$\begin{aligned} X(x) &= A\cosh(sx) + B\sinh(sx) \\ \frac{dX}{dx} &= A\sinh(sx) + B\cosh(sx) \end{aligned}$$

Applying the left B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(0) \\ &= B\cosh(0) \\ &= B \end{aligned}$$

The solution becomes $X(x) = A\cosh(sx)$ and hence $\frac{dX}{dx} = A\sinh(sx)$. Applying the right B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(L) \\ &= A\sinh(sL) \end{aligned}$$

$A=0$ result in trivial solution. Therefore assuming $\sinh(sL)=0$ implies $sL=0$ which is not valid since $s>0$ and $L\neq 0$. Hence only trivial solution results from this case. $\lambda<0$ is not an eigenvalue.

case $\lambda=0$

The ODE becomes

$$\frac{d^2X}{dx^2}=0$$

The solution is

$$\begin{aligned}X(x)&=c_1x+c_2\\ \frac{dX}{dx}&=c_1\end{aligned}$$

Applying left boundary conditions gives

$$\begin{aligned}0&=\frac{dX}{dx}(0)\\ &=c_1\end{aligned}$$

Hence the solution becomes $X(x)=c_2$. Therefore $\frac{dX}{dx}=0$. Applying the right B.C. provides no information.

Therefore this case leads to the solution $X(x)=c_2$. Associated with this one eigenvalue, the time equation becomes $\frac{dT_0}{dt}=0$ hence T_0 is constant, say α. Hence the solution $u_0(x,t)$ associated with this $\lambda=0$ is

$$\begin{aligned}u_0(x,t)&=X_0T_0\\ &=c_2\alpha\\ &=A_0\end{aligned}$$

where constant $c_2\alpha$ was renamed to A_0 to indicate it is associated with $\lambda=0$. $\lambda=0$ is an eigenvalue.

case $\lambda>0$

Hence $-\lambda$ is negative, and the roots are both complex.

$$r=\pm i\sqrt{\lambda}$$

The solution is

$$\begin{aligned}X(x)&=A\cos\left(\sqrt{\lambda}x\right)+B\sin\left(\sqrt{\lambda}x\right)\\ \frac{dX}{dx}&=-A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)+B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right)\end{aligned}$$

Applying the left B.C. gives

$$\begin{aligned}0 &= \frac{dX}{dx}(0)\\ &= B\sqrt{\lambda}\cos(0)\\ &= B\sqrt{\lambda}\end{aligned}$$

Therefore $B = 0$ as $\lambda > 0$. The solution becomes $X(x) = A\cos\left(\sqrt{\lambda}x\right)$ and $\frac{dX}{dx} = -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)$. Applying the right B.C. gives

$$\begin{aligned}0 &= \frac{dX}{dx}(L)\\ &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}L\right)\end{aligned}$$

$A = 0$ gives a trivial solution. Selecting $\sin\left(\sqrt{\lambda}L\right) = 0$ gives

$$\sqrt{\lambda}L = n\pi \qquad n = 1, 2, 3, \cdots$$

Or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Therefore the space solution is

$$X_n(x) = A_n\cos\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time solution is found by solving

$$\frac{dT_n}{dt} + k\lambda_n T_n = 0$$

This has the solution

$$\begin{aligned}T_n(t) &= e^{-k\lambda_n t}\\ &= e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n = 1, 2, 3, \cdots\end{aligned}$$

For the same set of eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the A_n when combined in the following step below. Since for $\lambda = 0$ the time solution was found to be constant, and for $\lambda > 0$ the time solution is $e^{-k\left(\frac{n\pi}{L}\right)^2 t}$, then no time solution will grow with time. Time solutions always decay with time as the exponent $-k\left(\frac{n\pi}{L}\right)^2 t$ is negative quantity. The solution to the PDE for $\lambda > 0$ is

$$u_n(x,t) = T_n(t)\, X_n(x) \qquad n = 0, 1, 2, 3, \cdots$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned}u(x,t) &= u_{\lambda=0}(x,t) + \sum_{n=1}^{\infty} u_n(x,t)\\ &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}\end{aligned}$$

From the solution found above, setting $t=0$ gives

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides with $\cos\left(\frac{m\pi}{L}x\right)$ where in this problem $m=0,1,2,\cdots$ (since there was an eigenvalue associated with $\lambda=0$), and integrating over the domain gives

$$\begin{aligned}\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \cos\left(\frac{m\pi}{L}x\right)\left(A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)\right)dx\\ &= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right)dx + \int \cos\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)dx\\ &= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right)dx + \int_0^L \sum_{n=1}^{\infty} A_n \cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx\end{aligned}$$

Interchanging the order of summation and integration

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx = \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right)dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx \tag{1}$$

case $m=0$

When $m=0$ then $\cos\left(\frac{m\pi}{L}x\right)=1$ and the above simplifies to

$$\int_0^L f(x)\,dx = \int_0^L A_0 dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{n\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{n\pi}{L}x\right)dx = 0$ and the above becomes

$$\begin{aligned}\int_0^L f(x)\,dx &= \int_0^L A_0 dx\\ &= A_0 L\end{aligned}$$

Therefore

$$A_0 = \frac{1}{L}\int_0^L f(x)\,dx$$

case $m > 0$

From (1), one term survives in the integration when only $n = m$, hence

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx = A_0\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx + A_m\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx = 0$ and the above becomes

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx = A_m\frac{L}{2}$$

Therefore

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

For $n = 1, 2, 3, \cdots$

Therefore the solution is

$$\begin{aligned}
u(x,t) &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\\
&= \frac{1}{L}\int_0^L f(x)\,dx + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}
\end{aligned}$$

In the limit as $t \to \infty$ the term $e^{-k\left(\frac{n\pi}{L}\right)^2 t} \to 0$. What is left is A_0. But $A_0 = \frac{1}{L}\int_0^L f(x)\,dx$ from above. This quantity is the average of the initial temperature.

4.1.1.32 [182] specific case

problem number 182

Added June 21, 2019

Solve

$$u_t = k u_{xx}$$

Subject to boundary conditions $u_x(0,t) = 0$, $u_x(L,t) = 0$ with initial conditions $u(x,0) = f(x)$ using the following values

$$\begin{aligned}
L &= 5\\
k &= \frac{1}{100}\\
f(x) &= x
\end{aligned}$$

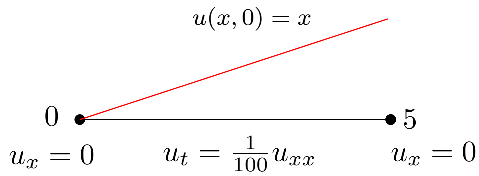


Figure 4.47: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
L=5;
f=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t)\to\frac{2}{5}\sum_{n=1}^{\infty}\frac{25(-1+(-1)^n)\,e^{-\frac{n^2\pi^2t}{2500}}\cos\left(\frac{n\pi x}{5}\right)}{n^2\pi^2}+\frac{5}{2}\right\}\right\}$$

Maple ✓

```
restart;
L:=5;
k:=1/100;
f:=x;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=10\left(\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)\cos\left(\frac{\pi nx}{5}\right)\mathrm{e}^{-\frac{\pi^2n^2t}{2500}}}{\pi^2n^2}\right)+\frac{5}{2}$$

Hand solution

The general solution for this type of PDE is given in problem 4.1.1.31 on page 487 as

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

In this problem $u(x,0) = f(x) = x, L = 5, k = \frac{1}{100}$. Hence the above becomes

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{5}x\right) e^{-\left(\frac{1}{100}\left(\frac{n\pi}{5}\right)^2 t\right)} \tag{2}$$

At $t = 0$ the above becomes

$$x = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{5}x\right)$$

For $n = 0$ orthogonality gives

$$\begin{aligned}\int_0^5 x dx &= \int_0^5 A_0 dx \\ \frac{25}{2} &= 5A_0 \\ A_0 &= \frac{5}{2}\end{aligned}$$

For $n > 0$

$$\begin{aligned}\int_0^5 x \cos\left(\frac{n\pi}{5}x\right) dx &= \int_0^5 A_n \cos^2\left(\frac{n\pi}{5}x\right) dx \\ \frac{25}{n^2\pi^2}(-1+(-1)^n) &= \frac{5}{2}A_n \\ A_n &= \frac{10}{n^2\pi^2}(-1+(-1)^n)\end{aligned}$$

Hence the solution (2) becomes

$$u(x,t) = \frac{5}{2} + \frac{10}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2}(-1+(-1)^n) \cos\left(\frac{n\pi}{5}x\right) e^{-\frac{n^2\pi^2}{2500}t}$$

The following is an animation of the solution

Out[∘]=

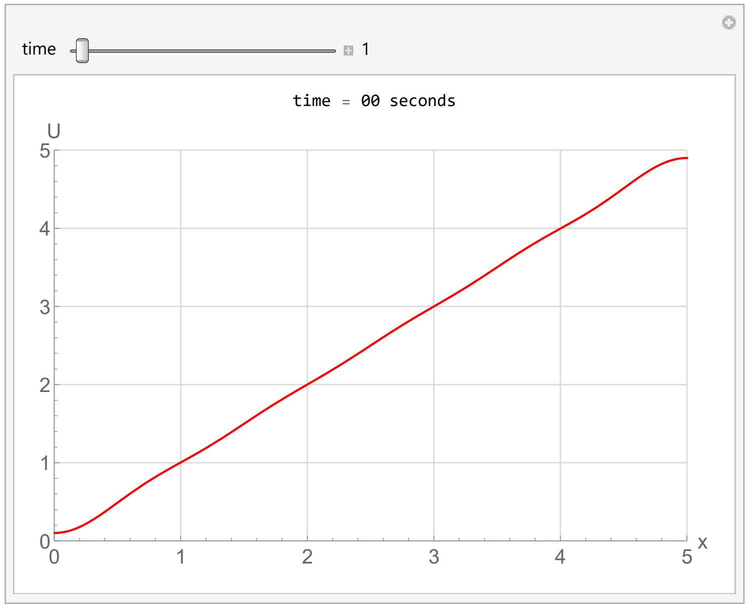


Figure 4.48: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, y, t, n, k]
L = 5;
k = 1 / 100;
f = x;
numberOfTerms = 10;
mySol[x_, t_] = 5/2 + 10/π^2 Sum[1/n^2 (-1 + (-1)^n) Exp[-(n^2 π^2/2500) t] Cos[n π/5 x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 4.49: Source code

```
In[]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {2, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {0, 5}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 700, 2}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.50: Code for animation

4.1.1.33 [183] Haberman 2.4.1 (a)

problem number 183

This is problem 2.4.1 part(a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = ku_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$. Initial conditions

$$u(x,0) = \begin{cases} 0 & x < \frac{L}{2} \\ 1 & x > \frac{L}{2} \end{cases}$$

$$u(x,0)=\begin{cases}0 & 0<x\leq \frac{L}{2}\\ 1 & \frac{L}{2}<x\leq L\end{cases}$$

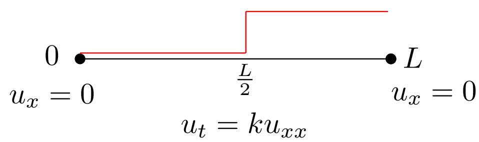


Figure 4.51: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n};
```

$Aborted

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=-2\left(\sum_{n=1}^{\infty}\frac{\cos\left(\frac{\pi n x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}\sin\left(\frac{\pi n}{2}\right)}{\pi n}\right)+\frac{1}{2}$$

4.1.1.34 [184] Both ends insulated, no source

problem number 184

Added January 13, 2020

This is problem 4.1.7 Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$. Initial conditions

$$u(x,0) = \begin{cases} x & 0 < x < \frac{L}{2} \\ 1-x & \frac{L}{2} < x < L \end{cases} \quad Use\text{L=1.}$$

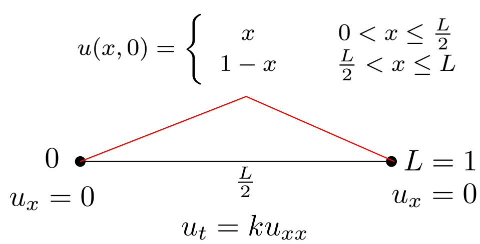


Figure 4.52: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=1;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == Piecewise[{{x, 0<x < L/2}, {1-x, L/2<=x<L}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to 2\sum_{n=1}^{\infty} \frac{4e^{-kn^2\pi^2 t} \cos\left(\frac{n\pi}{2}\right) \cos(n\pi x) \sin^2\left(\frac{n\pi}{4}\right)}{n^2\pi^2} + \frac{1}{4}\right\}\right\}$$

Maple ✓

```
restart;
L:=1;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,x,L/2<x and x<L,1-x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 2\left(\sum_{n=1}^{\infty}\left(-\frac{\left((-1)^n - 2\cos\left(\frac{\pi n}{2}\right)+1\right)\cos(\pi n x)\,\mathrm{e}^{-\pi^2 k\, n^2 t}}{\pi^2 n^2}\right)\right)+\frac{1}{4}$$

Hand solution

Let $u(x,t) = T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{k}T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's generated are

$$T' + k\lambda T = 0 \tag{1}$$

And the eigenvalue ODE

$$X'' + \lambda X = 0 \tag{2}$$
$$X'(0) = 0$$
$$X'(L) = 0$$

Starting with the eigenvalue ODE equation (2). The following cases are considered.

case $\lambda < 0$

In this case, $-\lambda$ is positive. Let $-\lambda = \omega^2$. Hence the ODE is $X'' - \omega^2 X = 0$ and the solution becomes

$$X(x) = C_1\cosh(\omega x) + C_2\sinh(\omega x)$$

Therefore

$$X' = C_1 \sinh(\omega x) + C_2 \cosh(\omega x)$$

Applying the left B.C. gives

$$0 = C_2$$

Therefore the solution becomes $X(x) = C_1 \cosh(\omega x)$ and $X'(x) = C_1 \sinh(\omega x)$. Applying the right B.C. gives

$$0 = C_1 \sinh(\omega L)$$

For non-trivial solution we want $\sinh(\omega L) = 0$. But this is not possible since sinh is zero when its argument is zero, which is not the case here. Hence only trivial solution results from this case. $\lambda < 0$ is not an eigenvalue.

case $\lambda = 0$

The solution is

$$\begin{aligned} X(x) &= c_1 x + c_2 \\ X'(x) &= c_1 \end{aligned}$$

Applying left boundary conditions gives

$$0 = c_1$$

Hence the solution becomes $X(x) = c_2$. Therefore $\frac{dX}{dx} = 0$. Applying the right B.C. provides no information. Any c_2 will work. Therefore this case leads to the solution $X(x) = c_2$. Associated with this one eigenvalue, the time equation becomes $T_0'(t) = 0$ hence $T_0(t)$ is a constant. Hence the solution $u_0(x,t)$ associated with this $\lambda = 0$ is

$$\begin{aligned} u_0(x,t) &= X_0 T_0 \\ &= A_0 \end{aligned}$$

where constant $c_2 T_0$ was renamed to $\frac{A_0}{2}$ to indicate it is associated with $\lambda = 0$. $\lambda = 0$ is an eigenvalue with eigenfunction constant $\frac{A_0}{2}$.

case $\lambda > 0$

The solution is

$$\begin{aligned} X(x) &= c_1 \cos\left(\sqrt{\lambda} x\right) + c_2 \sin\left(\sqrt{\lambda} x\right) \\ X'(x) &= -c_1 \sqrt{\lambda} \sin\left(\sqrt{\lambda} x\right) + c_2 \sqrt{\lambda} \cos\left(\sqrt{\lambda} x\right) \end{aligned}$$

Applying the left B.C. gives

$$0 = c_2 \sqrt{\lambda}$$

Therefore $c_2 = 0$ as $\lambda > 0$. The solution becomes

$$X(x) = c_1 \cos\left(\sqrt{\lambda} x\right)$$

And $X'(x) = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)$. Applying the right B.C. gives

$$0 = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}L\right)$$

$c_1 = 0$ gives a trivial solution. Selecting $\sin\left(\sqrt{\lambda}L\right) = 0$ gives

$$\sqrt{\lambda}L = n\pi \qquad n = 1, 2, 3, \cdots$$

Or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Therefore the eigenfunctions are

$$X_n(x) = \cos\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time solution is found by solving

$$T_n'(t) + k\lambda_n T_n(t) = 0$$

This has the solution

$$\begin{aligned} T_n(t) &= A_n e^{-k\lambda_n t} \\ &= A_n e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n = 1, 2, 3, \cdots \end{aligned}$$

The solution to the PDE is

$$u_n(x,t) = T_n(t)\, X_n(x) \qquad n = 0, 1, 2, 3, \cdots$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned} u(x,t) &= u_0(x,t) + \sum_{n=1}^{\infty} u_n(x,t) \\ &= \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned} \tag{1}$$

From the solution found above, setting $t = 0$ gives

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Hence A_0, A_n are the Fourier cos coefficients for the function $f(x)$. Doing an even extension of $f(x)$ from $[-L, L]$, then $\frac{A_0}{2}$ is the average of the function $f(x)$ over $[-L, L]$. But this average is seen as $\frac{2\left(\frac{1}{2}\times\frac{1}{2}\right)}{2} = \frac{1}{4}$. The term $\frac{1}{2}\times\frac{1}{2}$ is the area of $f(x)$ from $[0, L]$.

$$\frac{A_0}{2} = \frac{1}{4}$$

For A_n

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

Replacing $L = 1$ and using the definition of $f(x)$ given above gives

$$A_n = \int_{-1}^{1} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

But $f(x)$ is even (after even extending) and cos is even, hence the above becomes

$$\begin{aligned}
A_n &= 2\int_0^1 f(x)\cos(n\pi x)\,dx \\
&= 2\left(\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx + \int_{\frac{1}{2}}^1 (1-x)\cos(n\pi x)\,dx\right) \\
&= 2\left(\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx + \int_{\frac{1}{2}}^1 \cos(n\pi x)\,dx - \int_{\frac{1}{2}}^1 x\cos(n\pi x)\,dx\right) \qquad (2)
\end{aligned}$$

But

$$\begin{aligned}
\int_a^b x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_a^b - \frac{1}{n\pi}\int_a^b \sin(n\pi x)\,dx \\
&= \frac{1}{n\pi}[x\sin(n\pi x)]_a^b + \frac{1}{n^2\pi^2}[\cos(n\pi x)]_a^b \qquad (3)
\end{aligned}$$

When $a = 0, b = \frac{1}{2}$ the above gives

$$\begin{aligned}
\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_0^{\frac{1}{2}} + \frac{1}{n^2\pi^2}[\cos(n\pi x)]_0^{\frac{1}{2}} \\
&= \frac{1}{n\pi}\left(\frac{1}{2}\sin\left(\frac{n\pi}{2}\right)\right) + \frac{1}{n^2\pi^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right) \\
&= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right) \\
&= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2} \qquad (4)
\end{aligned}$$

And when $a = \frac{1}{2}, b = 1$ (3) gives

$$\begin{aligned}\int_{\frac{1}{2}}^{1} x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_{\frac{1}{2}}^{1} + \frac{1}{n^2\pi^2}[\cos(n\pi x)]_{\frac{1}{2}}^{1}\\ &= \frac{1}{n\pi}\left[\sin(n\pi) - \frac{1}{2}\sin\left(\frac{n\pi}{2}\right)\right] + \frac{1}{n^2\pi^2}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right]\\ &= -\frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos(n\pi) - \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) \qquad (5)\end{aligned}$$

Substituting (4,5) into (2) gives

$$\begin{aligned}\frac{A_n}{2} &= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2}\\ &\quad + \int_{\frac{1}{2}}^{1}\cos(n\pi x)\,dx\\ &\quad - \left(-\frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos(n\pi) - \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)\right)\end{aligned}$$

Or

$$\begin{aligned}\frac{A_n}{2} &= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2}\\ &\quad + \frac{1}{n\pi}\overbrace{\sin(n\pi)}^{0} - \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)\\ &\quad + \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2}\cos(n\pi) + \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)\end{aligned}$$

Or

$$\begin{aligned}\frac{A_n}{2} &= \left(\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2}\right) - \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2}\cos(n\pi)\\ &= \frac{2\cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n}{n^2\pi^2}\end{aligned}$$

Therefore the solution (1) becomes, after replacing $L = 1$

$$u(x,t) = \frac{1}{4} + 2\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n}{n^2\pi^2}\cos(n\pi x)\,e^{-kn^2\pi^2 t}$$

4.1.1.35 [185] Haberman 2.4.1 (b) (special case)

problem number 185

This is problem 2.4.1 part(b) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = 6 + 4\cos\left(\frac{3\pi x}{L}\right)$

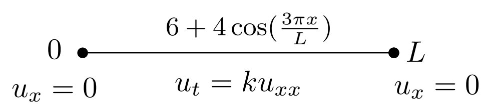


Figure 4.53: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == 6 + 4*Cos[(3*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to 4e^{-\frac{9\pi^2 kt}{L^2}} \cos\left(\frac{3\pi x}{L}\right) + 6\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0 and k>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=6+4*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 4\cos\left(\frac{3\pi x}{L}\right)\mathrm{e}^{-\frac{9\pi^2 kt}{L^2}} + 6$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 4.1.1.31 on page 487 as

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

In this example $u(x,0) = f(x) = 6 + 4\cos\frac{3\pi x}{L}$. Hence at $t = 0$ the above becomes

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$
$$6 + 4\cos\frac{3\pi x}{L} = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Comparing terms shows that

$$A_0 = 6$$
$$A_3 = 4$$

And all other $A_n = 0$. Hence the solution (1) is

$$u(x,t) = 6 + 4\cos\left(\frac{3\pi}{L}x\right) e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

4.1.1.36 [186] Haberman 2.4.1 (c) (special case)

problem number 186

This is problem 2.4.1 part(c) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = -2\sin\frac{\pi x}{L}$

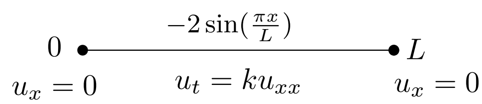


Figure 4.54: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == -2*Sin[(Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty}\frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L}-\frac{4}{\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0 and k>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=-2*sin(Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\left(\sum_{n=2}^{\infty}\frac{4((-1)^n+1)\cos\left(\frac{\pi n x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}}{\pi\left(n^2-1\right)}\right)-\frac{4}{\pi}$$

Hand solution

The general solution for this type of PDE is given in problem 4.1.1.31 on page 487 as

$$u(x,t)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

At $t=0$ the above becomes

$$\begin{aligned}
f(x)&=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\\
-2\sin\frac{\pi x}{L}&=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)
\end{aligned}$$

Multiplying both sides by $\cos\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}
-2\int_0^L\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx&=\int_0^L\left(A_0\cos\left(\frac{m\pi}{L}x\right)+\cos\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\right)dx\\
&=\int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\int_0^L\sum_{n=1}^{\infty}A_n\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx
\end{aligned}$$

Interchanging the order of integration and summation

$$\int_0^L-2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx=\int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\sum_{n=1}^{\infty}A_n\int_0^L\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

Case $m=0$

The above becomes

$$-2\int_0^L \sin\left(\frac{\pi x}{L}\right)dx = \int_0^L A_0 dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{n\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{n\pi}{L}x\right)dx = 0$ hence

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)dx = \int_0^L A_0 dx$$

$$\begin{aligned} A_0 L &= -2\int_0^L \sin\left(\frac{\pi x}{L}\right)dx \\ A_0 L &= -2\left(-\frac{\cos\left(\frac{\pi x}{L}\right)}{\frac{\pi}{L}}\right)_0^L \\ &= -\frac{2L}{\pi}\left(-\cos\left(\frac{\pi L}{L}\right) + \cos\left(\frac{\pi 0}{L}\right)\right) \\ &= -\frac{2L}{\pi}(-(-1)+1) \\ &= -\frac{4L}{\pi} \end{aligned}$$

Hence

$$A_0 = \frac{-4}{\pi}$$

<u>Case $m > 0$</u>

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

One term survives the summation resulting in

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \frac{-4}{\pi}\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx + A_m \int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx = 0$ and $\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$, therefore

$$\begin{aligned} \int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx &= A_m \frac{L}{2} \\ A_n &= \frac{-4}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx \end{aligned}$$

But

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{-L(1+\cos(n\pi))}{\pi(n^2-1)}$$

Therefore

$$\begin{aligned} A_n &= 4\frac{(1+\cos(n\pi))}{\pi(n^2-1)} \\ &= 4\frac{(-1)^n+1}{\pi(n^2-1)} \qquad n=1,2,3,\cdots \end{aligned}$$

Hence the solution becomes

$$u(x,t) = \frac{-4}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n+1}{(n^2-1)}\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

4.1.1.37 [187] Haberman 2.4.1 (d)

problem number 187

This is problem 2.4.1 part(d) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t)=0$ and $\frac{\partial u}{\partial x}(L,t)=0$ with the temperature initially $u(x,0) = -3\cos\frac{8\pi x}{L}$

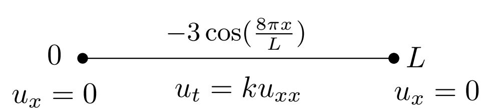


Figure 4.55: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == -3*Cos[(8*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -3e^{-\frac{64\pi^2 kt}{L^2}}\cos\left(\frac{8\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0 and k>0);
ic  := u(x,0)=-3*cos(8*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = -3\cos\left(\frac{8\pi x}{L}\right) e^{-\frac{64\pi^2 kt}{L^2}}$$

4.1.1.38 [188] both ends insulated

problem number 188

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve the heat equation for $u(x,t)$

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$

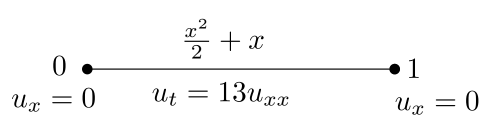


Figure 4.56: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 13*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
ic  = u[x, 0] == (1*x^2)/2 + x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{1}{2}\left(2\sum_{K[1]=1}^{\infty}\frac{2\left(-1+(-1)^{K[1]}\right)e^{-13\pi^2tK[1]^2}\cos(\pi xK[1])}{\pi^2K[1]^2}+26t+x^2+1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0 , eval( diff(u(x,t),x),x=1)=1;
ic  := u(x,0)=1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t))),out
```

$$u(x,t)=\frac{x^2}{2}+13t+2\left(\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)\cos\left(\pi nx\right)\mathrm{e}^{-13\pi^2n^2t}}{\pi^2n^2}\right)+\frac{1}{2}$$

4.1.1.39 [189] convection heat loss

problem number 189

This problem is taken from Maple primes post

Left end insulated, right end has convection heat loss `https://www.mapleprimes.com/posts/209681-Solving-PDEs-With-Initial-And-Boundary`

Solve the heat equation

$$u_t=ku_{xx}$$

The boundary conditions are, on the left end $\frac{\partial u}{\partial x}(0,t)=0$ and on the right end $\frac{\partial u}{\partial x}(1,t)=-u(1,t)$ with the temperature initially $u(x,0)=1-\frac{1}{4}x^3$

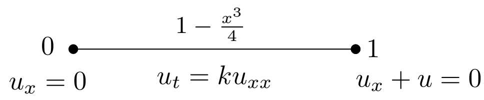


Figure 4.57: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == 1 - (1*x^3)/4;
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == -u[1, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty} -\frac{e^{-ktK[2,K[1]]}\cos\left(x\sqrt{K[2,K[1]]}\right)\left(3\cos\left(\sqrt{K[2,K[1]]}\right)(K[2,K[1]]-2)-3\sqrt{K[2,K[1]]}(K[2,K[1]]+2)\sin\left(\sqrt{\cdots}\right)\right.}{\sqrt{2}\sqrt{3-\cos\left(2\sqrt{K[2,K[1]]}\right)}K[2,K[1]]^2\sqrt{\sin^2\left(\sqrt{K[2,K[1]]}\right)+1}}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic  := u(x,0) = 1-(1/4)*x^3;
bc  := eval(diff(u(x,t), x), x = 0) = 0, eval(diff(u(x,t), x), x = 1)+u(1,t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 3\left(\sum_{n=0}^{\infty} \frac{\left(\lambda_n^3 \sin(\lambda_n) - \lambda_n^2 \cos(\lambda_n) + 2\lambda_n \sin(\lambda_n) + 2\cos(\lambda_n) - 2\right)\cos(x\lambda_n)\,\mathrm{e}^{-kt\lambda_n^2}}{\left(2\lambda_n + \sin(2\lambda_n)\right)\lambda_n^3}\right) \textbf{where}\{\lambda_n \text{ ta}$$

4.1.1.40 [190] Pinchover and Rubinstein 6.25

problem number 190

Added July 2, 2018. Taken from Maple 2018.1 document, originally exercise 6.25 from Pinchover and Rubinstein.

Solve the heat equation

$$u_t = k u_{xx} + \cos(wt)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = x$.

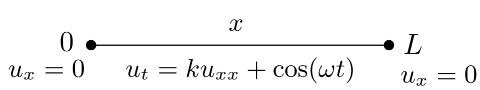


Figure 4.58: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Cos[w*t];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{2\left(-1+(-1)^{K[1]}\right) e^{-\frac{k\pi^2 t K[1]^2}{L^2}} L \cos\left(\frac{\pi x K[1]}{L}\right)}{\pi^2 K[1]^2} + \frac{L}{2} + \frac{\sin(tw)}{w}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+cos(w*t);
bc  := (D[1](u))(L, t) = 0, (D[1](u))(0, t) = 0;
ic  := u(x, 0) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{L}{2}+\left(\sum_{n=1}^{\infty}\frac{2((-1)^n-1)\,L\cos\left(\frac{\pi n x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k\,n^2 t}{L^2}}}{\pi^2 n^2}\right)+\frac{\sin(tw)}{w}$$

4.1.1.41 [191] external source

problem number 191

Added March 18, 2018.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+\left(e^{-ct}\sin\left(\frac{2\pi x}{L}\right)\right)$$

For $0<x<L$ and $t>0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t)=0$$
$$\frac{\partial u}{\partial x}(L,t)=0$$

Initial condition is $u(x,0)=f(x)$.

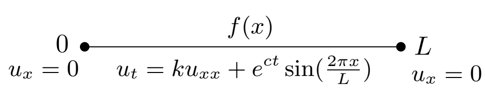


Figure 4.59: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Exp[-(c*t)]*Sin[(2*Pi*x)/L];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\int_0^L\frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)f(x)}{\sqrt{L}}\,dx+\left(\begin{cases}\frac{2\sqrt{2}(-1+(-1)^{K[1]})e^{-t\left(\frac{k\pi^2K[1]^2}{L^2}\right.}}{\pi(K[1]^2-4)(k\pi}\\ 0\end{cases}\right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+(exp(-c*t)*sin(2*Pi*x/L));
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L\cos\left(\frac{\pi n x}{L}\right)f(x)\,dx\right)\cos\left(\frac{\pi n x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k\,n^2 t}{L^2}}}{L}\right)+\frac{\int_0^L f(x)\,dx}{L}+\frac{-\frac{8L^2\cos\left(\frac{\pi x}{L}\right)\mathrm{e}^{-ct}}{3}+\frac{8L^2\cos\left(\frac{\pi x}{L}\right)\mathrm{e}^{-}}{3}}{}$$

4.1.1.42 [192] Diffusion Reaction (general case)

problem number 192

Added June 9, 2019

Consider the heat equation

$$u_t = k u_{xx} - \beta u$$

Suppose the boundary conditions are $u_x(0,t) = 0, u_x(\pi,t) = 0$, solve with the temperature initially $u(x,0) = x$

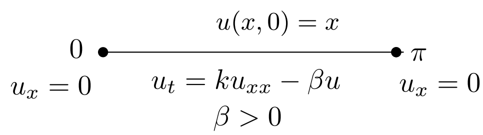


Figure 4.60: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - beta*u[x, t];
bc  = {Derivative[1,0][u][0, t] == 0, Derivative[1,0][u][Pi, t] == 0};
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{2\left(-1+(-1)^{K[1]}\right) e^{-t\left(kK[1]^2+\beta\right)} \cos(xK[1])}{\pi K[1]^2} + \frac{1}{2}\pi e^{-\beta t}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-beta*u(x,t);
bc:=eval(diff(u(x,t),x),x=0)=0,eval(diff(u(x,t),x),x=Pi)=0;
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 2\left(\sum_{n=1}^{\infty} \frac{((-1)^n - 1)\cos(nx)\,\mathrm{e}^{-(k\,n^2+\beta)t}}{\pi\,n^2}\right) + \frac{\pi\,\mathrm{e}^{-\beta t}}{2}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - \beta u \qquad 0 < x < \pi, t > 0$$

with, $k > 0, \beta > 0$

$$\frac{\partial u(0,t)}{\partial x} = 0$$
$$\frac{\partial u(\pi,t)}{\partial x} = 0$$

And initial conditions

$$u(x,0) = x$$

Solution

Let $u = X(x)T(t)$. Substituting into the PDE gives

$$T'X = kX''T - \beta XT$$

Dividing by $XT \neq 0$ gives

$$\frac{T'}{T} = \frac{kX''}{X} - \beta$$
$$\frac{T'}{T} + \beta = \frac{kX''}{X}$$
$$\frac{T'}{kT} + \frac{\beta}{k} = \frac{X''}{X}$$

Since each sides depends on different variable and both are equal, they must be equal to same constant, say $-\lambda$

$$\frac{T'}{\alpha T} + \frac{\beta}{\alpha} = \frac{X''}{X} = -\lambda$$

This gives two ODE's to solve

$$
\begin{aligned}
X'' + \lambda X &= 0 \qquad (1)\\
X'(0) &= 0\\
X'(\pi) &= 0
\end{aligned}
$$

And

$$
\begin{aligned}
\frac{T'}{kT} + \frac{\beta}{k} &= -\lambda \qquad (2)\\
T' + \beta T &= -\lambda k T\\
T' + T(\lambda k + \beta) &= 0
\end{aligned}
$$

Starting with (1).

Assuming $\lambda < 0$, the solution is

$$
\begin{aligned}
X(x) &= A\cosh\left(\sqrt{-\lambda}x\right) + B\sinh\left(\sqrt{-\lambda}x\right)\\
X' &= \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}x\right) + \sqrt{-\lambda}B\cosh\left(\sqrt{-\lambda}x\right)
\end{aligned}
$$

Applying first B.C. gives $0 = \sqrt{-\lambda}B$, hence $B = 0$. Therefore the solution becomes $X(x) = A\cosh\left(\sqrt{-\lambda}x\right)$ and $X' = \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}x\right)$. Applying second B.C. gives

$$
0 = \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}\pi\right)
$$

But $\sinh\left(\sqrt{-\lambda}\pi\right) = 0$ only when its argument is zero, which is not the case here. This means $A = 0$, leading to trivial solution. Therefore $\lambda < 0$ is not eigenvalue.

Assuming $\lambda = 0$. The solution is $X = Ax + B$. Hence $X' = A$. Applying first B.C. Gives $A = 0$. Hence solution is $X = B$. Second boundary condition gives no additional information. Therefore $X = 1$ (constant) is the eigenfunction associated with $\lambda = 0$.

Assuming $\lambda > 0$. The solution is

$$
\begin{aligned}
X &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)\\
X' &= -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}x\right) + \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)
\end{aligned}
$$

Applying first B.C. gives $0 = \sqrt{\lambda}B$, hence $B = 0$. and the solution becomes $X = A\cos\left(\sqrt{\lambda}x\right)$, $X' = -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}x\right)$. Applying second B.C. gives

$$
0 = -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}\pi\right)
$$

Therefore

$$
\begin{aligned}
\sin\left(\sqrt{\lambda}\pi\right) &= 0 \\
\sqrt{\lambda}\pi &= n\pi \qquad n = 1,2,3,\cdots \\
\lambda &= n^2 \qquad n = 1,2,3,\cdots
\end{aligned}
$$

Therefore the space solution is

$$
X(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) \tag{3}
$$

Now the time ODE (2) is solved.

$$
T' + T(\lambda k + \beta) = 0
$$

Integrating factor is $e^{\int \lambda k+\beta dt} = e^{(\lambda k+\beta)t}$. Hence $\frac{d}{dt}\left(Te^{(\lambda k+\beta)t}\right) = 0$ or $Te^{(\lambda k+\beta)t} = c$ where c is constant. Therefore

$$
T(t) = ce^{-(\lambda k+\beta)t}
$$

For $\lambda = 0$, the solution is

$$
T_0(t) = c_0 e^{-\beta t}
$$

and for $\lambda > 0$, the solution is

$$
T_n(t) = c_n e^{-(n^2 k+\beta)t}
$$

Hence the time domain solution is

$$
T(t) = c_0 e^{-\beta t} + \sum_{n=1}^{\infty} c_n e^{-(n^2 k+\beta)t} \tag{4}
$$

Combining (3,4) the solution is

$$
\begin{aligned}
u_n &= X_n T_n \\
u(x,t) &= \sum_{n=0}^{\infty} X_n T_n \\
&= c_0 e^{-\beta t} + \sum_{n=1}^{\infty} b_n \cos(nx)\, e^{-(n^2 k+\beta)t}
\end{aligned} \tag{5}
$$

where both constants from space and time eigenfunctions are combined into one constant b_n above in the sum. Now initial conditions are used to find coefficients. At $t = 0$

$$
\begin{aligned}
u(x,0) &= c_0 + \sum_{n=1}^{\infty} b_n \cos(nx) \\
x &= c_0 + \sum_{n=1}^{\infty} b_n \cos(nx)
\end{aligned}
$$

Multiplying both sides by $\cos(mx)$ and integrating gives

$$\int_0^\pi x\cos mx dx = \int_0^\pi \left(c_0 \cos mx + \sum_{n=1}^\infty b_n \cos mx \cos(nx) \right) dx$$

For $m = 0$, all terms in the sum in RHS vanish, and only c_0 left

$$\begin{aligned}\int_0^\pi x dx &= \int_0^\pi c_0 dx \\ &= c_0 \pi\end{aligned}$$

Hence

$$\begin{aligned}c_0 &= \frac{1}{\pi}\int_0^\pi x dx \\ &= \frac{1}{\pi}\frac{\pi^2}{2} \\ &= \frac{\pi}{2}\end{aligned} \tag{6}$$

For $n > 0$

$$\begin{aligned}\int_0^\pi x\cos mx dx &= \int_0^\pi \sum_{n=1}^\infty b_n \cos mx \cos(nx) dx \\ &= \sum_{n=1}^\infty b_n \int_0^\pi \cos mx \cos(nx) dx\end{aligned}$$

All terms in the sum vanish except for $n = m$. The above becomes

$$\begin{aligned}\int_0^\pi x\cos mx dx &= b_n \frac{\pi}{2} \\ b_n &= \frac{2}{\pi}\int_0^\pi x\cos mx dx\end{aligned} \tag{7}$$

Using integration by part, $\int_0^\pi x\cos nx dx$, let $u = x, dv = \cos nx, \rightarrow du = 1, v = \frac{\sin nx}{n}$, therefore

$$\begin{aligned}\int_0^\pi x\cos nx dx &= \left(x\frac{\sin nx}{n} \right)_0^\pi - \frac{1}{n}\int_0^\pi \sin nx dx \\ &= \frac{1}{n}(\pi \sin n\pi - 0) + \frac{1}{n}\left(\frac{\cos nx}{n} \right)_0^\pi \\ &= 0 + \frac{1}{n^2}(\cos nx)_0^\pi \\ &= \frac{1}{n^2}(\cos n\pi - 1)\end{aligned}$$

Hence

$$\begin{aligned} b_n &= \frac{2}{\pi}\int_0^{\pi} x\cos mx dx \\ &= \frac{2}{\pi}\frac{(-1^n - 1)}{n^2} \end{aligned}$$

Therefore the final solution is, from (5), by combing all above results, becomes

$$\begin{aligned} u(x,t) &= c_0 e^{-\beta t} + \sum_{n=1}^{\infty} b_n \cos(nx)\, e^{-(n^2 k+\beta)t} \\ &= \frac{\pi}{2}e^{-\beta t} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1^n - 1)}{n^2}\cos(nx)\, e^{-(n^2 k+\beta)t} \end{aligned}$$

This is animation of the solution for $k = 1, \beta = 2$ for 2 seconds. (Animation will only show in the HTML version)

Out[∘]=

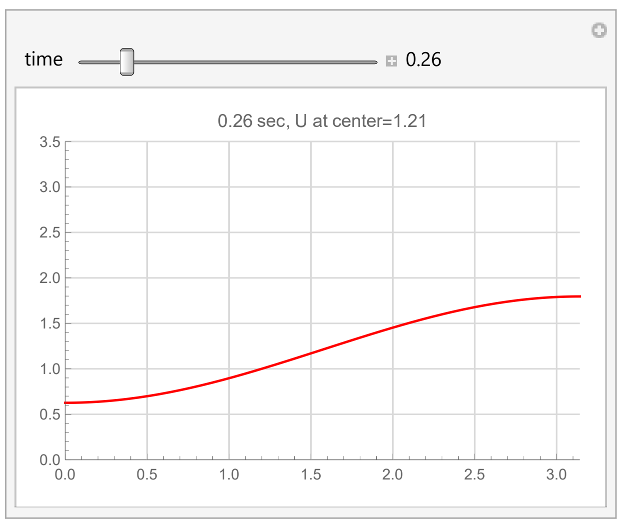


Figure 4.61: Screen shot

Source code used for the above

```
beta = 1; k = 2;
T0[x_, t_, m_] := Quiet[Pi/2 Exp[-beta t] + 2/Pi Sum[((-1)^n - 1)/n^2 Cos[n x] Exp[-(n^2 k + beta) t], {n, 1, m}]];
p = Plot3D[T0[x, t, 20], {x, 0, Pi}, {t, 0, .1}, PlotRange → All, AxesLabel → {x, "sec", u}, BaseStyle → 15]

In[»]:= padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Plot[T0[x, t, 20], {x, 0, Pi},
    PlotLabel → Row[{t, " sec,", " U at center=", padIt2[N@T0[Pi/2, t, 100], {3, 2}]}],
    PlotRange → {{0, Pi}, {0, 3.5}}, GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
  ],
  {{t, 0, "time"}, 0, 2, .01, Appearance → "Labeled"}]

In[»]:= r = Table[
      Plot[T0[x, t, 20], {x, 0, Pi},
        PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center = ", padIt2[N@T0[Pi/2, t, 100], {3, 2}]}],
        PlotRange → {{0, Pi}, {0, 3.5}},
        GridLines → Automatic,
        GridLinesStyle → LightGray,
        PlotStyle → Red
      ],
      {t, 0, 2, .01}];

In[»]:= Export["anim.gif", r, "DisplayDurations" → Table[0.1, {Length@r}]]
Out[»]= anim.gif
```

Figure 4.62: Source code

4.1.1.43 [193] Pinchover and Rubinstein 6.23

problem number 193

Added July 2, 2018.

4th example from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.23 .

Solve the heat equation on bar

$$u_t = u_{xx} + g(x,t)$$

Where $g(x,t) = e^{3t}\cos(17\pi x)$ for $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = 3\cos(42\pi x)$.

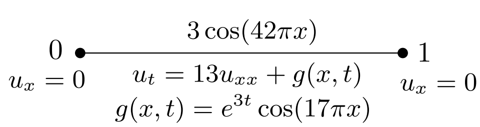


Figure 4.63: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f[x] := 3*Cos[42*Pi*x];
 g[x, t] := Exp[3*t]*Cos[17*x*Pi];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + g[x, t];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\sqrt{2}\cos(\pi xK[1])\left(\int_0^t e^{-\pi^2K[1]^2(t-K[2])}\left(\begin{cases}\frac{e^{3K[2]}}{\sqrt{2}} & K[1]=17\\ 0 & \text{True}\end{cases}\right)dK[2]+e^{-\pi^2tK[1]^2}\left(\right.\right.\right.\right.$$

Maple ✓

```
restart;
f := x->3*cos(42*x*Pi);
g :=(x,t)->exp(3*t)*cos(17*x*Pi);
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + g(x, t);
bc  := (D[1](u))(0, t) = 0, (D[1](u))(1, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t)=\frac{(867\pi^2+9)\cos(42\pi x)\,\mathrm{e}^{-1764\pi^2t}+\left(\mathrm{e}^{3t}-\mathrm{e}^{-289\pi^2t}\right)\cos(17\pi x)}{289\pi^2+3}$$

4.1.1.44 [194] Pinchover and Rubinstein 6.21

problem number 194

added July 2, 2018.

Taken from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.21

Solve the heat equation on bar

$$u_t=u_{xx}+g(x,t)$$

Where $g(x,t) = t\cos(2001x)$ for $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = \pi\cos(2x)$.

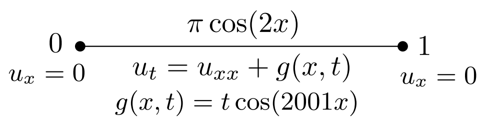


Figure 4.64: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + t*Cos[2001*x];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][Pi, t] == 0};
ic  = u[x, 0] == Pi*Cos[2*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\sqrt{\frac{2}{\pi}}\cos(xK[1])\left(\int_0^t e^{-K[1]^2(t-K[2])}\left(\left\{\begin{array}{cc}\sqrt{\frac{\pi}{2}}K[2] & K[1]=2001\\ 0 & \text{True}\end{array}\right.\right)dK[2]+e^{-tK[1]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + t*cos(2001*x);
bc  := (D[1](u))(0, t) = 0, (D[1](u))(Pi, t) = 0;
ic  := u(x, 0) = Pi*cos(2*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = \pi\cos(2x)\,\mathrm{e}^{-4t} + \frac{(4004001t + \mathrm{e}^{-4004001t} - 1)\cos(2001x)}{16032024008001}$$

4.1.1.45 [195] Diffusion Reaction. Euler-Cauchy Sturm-Liouville

problem number 195

Added April 20, 2019.

Solve the parabolic pde for $u(x,t)$

$$u_t = x^2 u_{xx} + x u_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(1,t) &= 0 \\ h u(b,t) + u_x(b,t) &= 0 \end{aligned}$$

Where $h > 0$. Initial condition is $u(x,0) = \ln x$

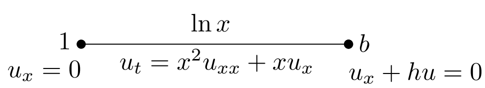


Figure 4.65: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic  = u[x, 0] == Log[x];
bc  = {Derivative[1, 0][u][1, t] == 0, h*u[b, t] + Derivative[1, 0][u][b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t) \to \left\{ \begin{array}{c} \sum_{K[1]=1}^{\infty} \frac{2be^{-tK[2,K[1]]}h\cos\left(\sqrt{K[2,K[1]]}\log(x)\right)\left(\cos\left(\sqrt{K[2,K[1]]}\log(b)\right)+\sqrt{K[2,K[1]]}\log(b)\sin\left(\sqrt{K[2,K[1]]}\log(b)\right)\right.}{K[2,K[1]]\left(\sin^2\left(\sqrt{K[2,K[1]]}\log(b)\right)+bh\log(b)\right)} \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc:=eval( diff(u(x,t),x),x=1)=0,h*u(b,t)+eval( diff(u(x,t),x),x=b)=0;
ic  := u(x,0)=ln(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

$$u(x,t) = \infty \operatorname{signum}\left(c_2 \cos\left(\sqrt{\frac{b^2h^2\left(\cos^2\left(\lambda\_Z65\right)\right)}{\sin\left(\lambda\_Z65\right)^2}} \ln(x)\right) e^{-\frac{b^2h^2t\left(\cos^2\left(\lambda\_Z65\right)\right)}{\sin\left(\lambda\_Z65\right)^2}}\right) \textbf{where} \left\{\lambda\_Z65 - \sqrt{\frac{b^2 h}{\tan(\lambda\_}}\right.$$

Hand solution

Solve

$$u_t = x^2 u_{xx} + x u_x \tag{1}$$

With $1 < x < b, t > 0$. BC

$$u_x(1,t) = 0$$
$$hu(b,t) + u_x(b,t) = 0$$

Where $h > 0$. And initial conditions

$$u(x,0) = \ln x$$

Let $u = X(x)T(x)$. Substituting into (1) gives

$$T'X = x^2X''T + xX'T$$
$$\frac{T'}{T} = x^2\frac{X''}{X} + x\frac{X'}{X} = -\lambda$$

Where λ is the separation constant. From the boundary conditions, we know that λ will be only positive. So we do not need to check for possibility of negative of zero eigenvalue. Letting $\lambda = \alpha^2$, then the above reduces to

$$x^2X'' + xX' + \alpha^2 X = 0 \tag{2}$$
$$T' + \alpha^2 T = 0 \tag{3}$$

Equation (2) is Euler ODE. Assuming $X = x^r$, then $X' = rx^{r-1}, X'' = r(r-1)x^{r-2}$. Substituting back into (2) gives the characteristic equation

$$r(r-1)x^r + rx^r + \alpha^2 x^r = 0$$
$$r^2 + \alpha^2 = 0$$
$$r = \pm i\alpha$$

Hence the solution to (2) is

$$\begin{aligned} X(x) &= Ax^{i\alpha} + Bx^{-i\alpha} \\ &= Ae^{\ln x^{i\alpha}} + Be^{\ln x^{-i\alpha}} \\ &= Ae^{i\alpha \ln x} + Be^{-i\alpha \ln x} \end{aligned}$$

Which using Euler relation can be written as (using new constants, but the name of the constants kept the same for simplicity)

$$X(x) = A\cos(\alpha \ln x) + B\sin(\alpha \ln x)$$

Applying first BC. $X'(1,t) = 0$ gives

$$\begin{aligned} X'(x) &= -\frac{\alpha}{x}A\sin(\alpha \ln x) + B\frac{\alpha}{x}\cos(\alpha \ln x) \\ 0 &= -\alpha A\sin(\alpha \ln 1) + B\alpha\cos(\alpha \ln 1) \\ &= B\alpha \end{aligned}$$

Since $\alpha > 0$ then $B = 0$. Hence the solution becomes

$$X(x) = A\cos(\alpha \ln x)$$

Applying second BC. $hX(b) + X'(b) = 0$ gives

$$\begin{aligned} hA\cos(\alpha \ln b) - A\frac{\alpha}{b}\sin(\alpha \ln b) &= 0 \\ h - \frac{\alpha}{b}\tan(\alpha \ln b) &= 0 \\ \tan(\alpha \ln b) &= \frac{hb}{\alpha} \end{aligned} \tag{4}$$

There is no analytical solution to the above. The eigenvalues α_n are the solutions to the above nonlinear equation. Therefore the eigenfunctions are

$$X_n(x) = \cos(\alpha_n \ln x)$$

With eigenvalues $\alpha_n > 0$ given by solutions to (4). The solution to the time ODE is

$$\begin{aligned} T_n' + \alpha_n^2 T_n &= 0 \\ T_n(t) &= T_n(0)\, e^{-\alpha_n^2 t} \end{aligned}$$

Hence the solution to (1) is

$$u(x,t) = \sum_{n=1}^{\infty} T_n(0)\, e^{-\alpha_n^2 t}\cos(\alpha_n \ln x) \tag{5}$$

At $t = 0$

$$\ln x = \sum_{n=1}^{\infty} T_n(0) \cos(\alpha_n \ln x)$$

Applying orthogonality gives

$$\int_1^b \ln x \cos(\alpha_n \ln x)\, dx = T_n(0) \int_1^b \cos^2(\alpha_n \ln x)\, dx$$

$$T_n(0) = \frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx}$$

Hence the solution (5) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx} \right) e^{-\alpha_n^2 t} \cos(\alpha_n \ln x)$$

4.1.1.46 [196] convection heat loss

problem number 196

Added April 28, 2019

Problem 4, section 74, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are, on the left end $u(0,t) = 0$ and on the right end $u_x(1,t) = -hu(1,t)$ with $h > 0$. Initial conditions $u(x,0) = f(x)$

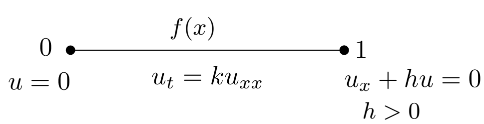


Figure 4.66: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[0, t] == 0, Derivative[1, 0][u][1, t] == -h u[1, t]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty} -\frac{\sqrt{2}e^{-ktK[2,K[1]]}\sqrt{h}\left(\int_0^1 -\frac{2\sqrt{h}f(x)\sin\left(x\sqrt{K[2,K[1]]}\right)}{\sqrt{2h+\cos\left(2\sqrt{K[2,K[1]]}\right)+1}}dx\right)\sin\left(x\sqrt{K[2,K[1]]}\right)}{\sqrt{\cos^2\left(\sqrt{K[2,K[1]]}\right)+h}} \quad h\tan\left(\sqrt{K[2,K[1]]}\right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic  := u(x,0) = f(x);
bc  := u(0,t)=0, eval(diff(u(x,t), x), x = 1)=-h*u(1,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2h\left(\int_0^1 f(x)\sin\left(\frac{hx\sin(\lambda_n)}{\cos(\lambda_n)}\right)dx\right)\mathrm{e}^{-\frac{h^2kt\left(\sin^2(\lambda_n)\right)}{\cos(\lambda_n)^2}}\sin(\lambda_n)\sin\left(\frac{hx\sin(\lambda_n)}{\cos(\lambda_n)}\right)}{-\cos(\lambda_n)\cos\left(\frac{h\,\mathrm{signum}\left(\frac{\sin(\lambda_n)}{\cos(\lambda_n)}\right)\sin(\lambda_n)}{\cos(\lambda_n)}\right)\sin\left(\frac{h\sin(\lambda_n)}{\cos(\lambda_n)}\right)+h\sin(\lambda_n)}\ \textbf{where}\left\{-\sqrt{\tan^2(\lambda_n)}\,h\right.$$

Hand solution

To solve the PDE, we first check the boundary conditions by writing them as

$$a_1u(0,t)+a_2u_x(0,t)=0$$
$$b_1u(1,t)+b_2u_x(1,t)=0$$

Then $a_1=0, a_2=0$. Hence $a_1a_2=0$. And $b_1=1, b_2=h$. Then since it is assumed that $h>0$, then $b_1b_2\geq 0$. And since $q(x)=0$ from the PDE itself, then we know that eigenvalues must be $\lambda\geq 0$.

Let $u = X(x)T(t)$ then the PDE becomes

$$\begin{aligned} T'X &= X''T \\ \frac{T'}{T} &= \frac{X''}{X} = -\lambda \end{aligned}$$

Hence the Sturm Liouville problem is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(1) + hX(1) &= 0 \end{aligned}$$

Where $p(x) = 1$.

Case $\lambda = 0$

Solution is

$$X(x) = Ax + B$$

At $x = 0$

$$0 = B$$

Hence solution becomes

$$X(x) = Ax$$

At $x = 1$ the second boundary conditions gives

$$\begin{aligned} A + hA &= 0 \\ A(1 + h) &= 0 \end{aligned}$$

For non trivial solution $1 + h = 0$ or $h = -1$. But we assumed that $h > 0$. Therefore $\lambda = 0$ is not eigenvalue.

Case $\lambda > 0$

Let $\lambda = \alpha^2, \alpha > 0$. Hence solution is

$$X(x) = A\cos(\alpha x) + B\sin(\alpha x)$$

At $X(0) = 0$

$$0 = A$$

The solution becomes

$$X(x) = B\sin(\alpha x)$$

At $x=1$ the second boundary conditions gives

$$\begin{aligned}
B\alpha\cos(\alpha)+hB\sin(\alpha)&=0\\
\alpha\cos(\alpha)+h\sin(\alpha)&=0\\
\tan(\alpha)&=-\frac{\alpha}{h}
\end{aligned}$$

Therefore the eigenvalues are given by solution to

$$\tan(\alpha_n)=-\frac{\alpha_n}{h}\qquad n=1,2,3,\cdots$$

And eigenfunctions are

$$X_n(x)=\sin(\alpha_n x)$$

The normalized eigenfunctions are

$$\phi_n(x)=\frac{X_n(x)}{\|X_n(x)\|}$$

But

$$\begin{aligned}
\|X_n(x)\|^2&=\int_0^1 p(x)\,X_n^2(x)\,dx\\
&=\int_0^1\sin^2(\alpha_n x)\,dx\\
&=\frac{1}{2}\int_0^1 1-\cos(2\alpha_n x)\,dx\\
&=\frac{1}{2}\left(1-\left[\frac{\sin(2\alpha_n x)}{2\alpha_n}\right]_0^1\right)\\
&=\frac{1}{2}\left(1-\frac{1}{2\alpha_n}\left[\sin(2\alpha_n x)\right]_0^1\right)\\
&=\frac{1}{2}\left(1-\frac{\sin(2\alpha_n)}{2\alpha_n}\right)\\
&=\frac{1}{2}-\frac{\sin(2\alpha_n)}{4\alpha_n}
\end{aligned}$$

But $\sin(2\alpha_n)=2\sin\alpha_n\cos\alpha_n$ and $\alpha_n=-h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}$, therefore the above becomes

$$\begin{aligned}
\|X_n(x)\|^2&=\frac{1}{2}+\frac{2\sin\alpha_n\cos\alpha_n}{4h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}}\\
&=\frac{1}{2}+\frac{\cos^2\alpha_n}{2h}\\
&=\frac{h+\cos^2\alpha_n}{2h}
\end{aligned}$$

Hence

$$\begin{aligned}\phi_n(x) &= \frac{X_n(x)}{\sqrt{\frac{h+\cos^2\alpha_n}{2h}}}\\ &= \sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n x)\end{aligned}$$

Now we use generalized Fourier series to find the solution. Let

$$u(x,t) = \sum_{n=1}^{\infty} B_n(t)\,\phi_n(x) \tag{1}$$

Substituting this back into the PDE gives

$$\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) = k\sum_{n=1}^{\infty} B_n(t)\,\phi_n''(x)$$

But $\phi_n''(x) = -\lambda_n\phi_n(x) = -\alpha_n^2\phi_n(x)$. The above becomes

$$\begin{aligned}\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) &= -k\sum_{n=1}^{\infty} B_n(t)\,\alpha_n^2\phi_n(x)\\ B_n'(t) + k\alpha_n^2 B_n(t) &= 0\end{aligned}$$

The solution is

$$B_n(t) = B_n(0)\,e^{-k\alpha_n^2 t}$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-k\alpha_n^2 t}\phi_n(x)$$

At $t=0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x)$$

Therefore

$$\begin{aligned}B_n(0) &= \langle f(x)\,,\phi_n(x)\rangle\\ &= \int_0^1 p(x)\,f(x)\,\phi_n(x)\,dx\\ &= \sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1 f(x)\sin(\alpha_n x)\,dx\end{aligned}$$

Therefore

$$
\begin{aligned}
B_n(t) &= B_n(0)\, e^{-k\alpha_n^2 t} \\
&= \left(\sqrt{\frac{2h}{h+\cos^2\alpha_n}} \int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t}
\end{aligned}
$$

and solution (1) becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} \sqrt{\frac{2h}{h+\cos^2\alpha_n}} \left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t} \sqrt{\frac{2h}{h+\cos^2\alpha_n}} \sin(\alpha_n x) \\
&= \frac{2h}{h+\cos^2\alpha_n} \sum_{n=1}^{\infty} \left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t} \sin(\alpha_n x)
\end{aligned}
$$

4.1.1.47 [197] Mixed BC

problem number 197

Solve the heat equation

$$
\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}
$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
\frac{\partial u}{\partial x}(0,t) + u(0,t) &= 0 \\
\frac{\partial u}{\partial x}(L,t) + u(L,t) &= 0
\end{aligned}
$$

And initial condition $u(x,0) = f(x)$

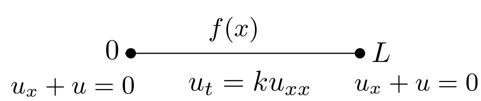


Figure 4.67: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] + u[0, t] == 0, Derivative[1, 0][u][L, t] + u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-ktK[2,K[1]]}\left(\int_0^L \frac{\sqrt{2}f(x)\sqrt{K[2,K[1]]+1}\left(\cos\left(x\sqrt{K[2,K[1]]}\right)\sqrt{K[2,K[1]]}-\sin\left(x\sqrt{K[2,K[1]]}\right)\right)}{\sqrt{(K[2,K[1]]+1)\left(K[2,K[1]]L+L+\cos\left(2L\sqrt{K[2,K[1]]}\right)-1\right)}}\,dx\right)\sqrt{K[2,K[1]]+1}(}{\sqrt{2K[2,K[1]]\cos^2\left(L\sqrt{K[2,K[1]]}\right)+2\cos^2\left(L\sqrt{K[2,K[1]]}\right)+LK[2,K[1]]^2+L+2L}} \right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty} \frac{2\left(L\sin\left(\frac{\pi n x}{L}\right)-\pi n\cos\left(\frac{\pi n x}{L}\right)\right)\left(\int_0^L \left(L\sin\left(\frac{\pi n x}{L}\right)-\pi n\cos\left(\frac{\pi n x}{L}\right)\right)f(x)\,dx\right)\mathrm{e}^{-\frac{\pi^2 k\,n^2 t}{L^2}}}{\left(L^2+\pi^2 n^2\right)L}$$

4.1.1.48 [198] Haberman 8.2.1 (f)

problem number 198

added March 18, 2018.

This is problem 8.2.1, part(f) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \sin\left(\frac{2\pi x}{L}\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

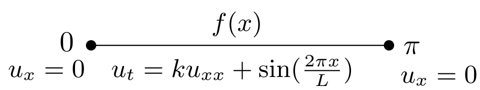


Figure 4.68: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Sin[(2*Pi*x)/L];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(\frac{\left(1-e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\right)\left(\begin{cases} 0 & K[1]=2 \\ \frac{2\sqrt{2}(-1+(-1)^{K[1]})\sqrt{L}}{\pi(K[1]^2-4)} & \text{True} \end{cases}\right)L^2}{k\pi^2 K[1]^2} + e^{-\frac{k\pi^2 t K[1]}{L^2}}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+sin(2*Pi*x/L);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \frac{L^2 \sin\left(\frac{2\pi x}{L}\right)}{4\pi^2 k} - \frac{Lx}{2\pi k} + c_2 + \sum_{n=1}^{\infty} \left(-\frac{\left(\int_0^L \left(L^2 \sin\left(\frac{2\pi x}{L}\right) - 2\pi L x + 4c_2\pi^2 k - 4\pi^2 k f(x)\right) \cos\left(\frac{\pi n x}{L}\right) dx\right)}{2\pi^2 L k} \right.$$

4.1.2 Finite domain (bar), left end homogeneous, right end not

Local contents

4.1.2.1 [199] left end insulated (general case)

problem number 199

Added June 21, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = 0$$
$$u(L,t) = T_0$$

Initial condition is $u(x,0) = f(x)$.

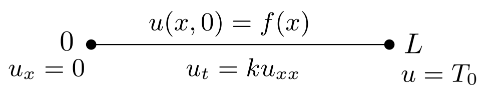


Figure 4.69: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] ;
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
sol =  sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=0}^{\infty} e^{-\frac{k(2\pi n+\pi)^2 t}{4L^2}} \cos\left(\frac{(2n+1)\pi x}{2L}\right) \int_0^L \cos\left(\frac{(2n+1)\pi K[2]}{2L}\right)(f(K[2]) - \mathrm{T0})\, dK[2]}{L} + \mathrm{T0}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = T0+2\left(\sum_{n=0}^{\infty} \frac{\left(\int_0^L (-T0+f(x))\cos\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos\left(\frac{(2n+1)\pi x}{2L}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2 kt}{4L^2}}}{L}\right)$$

Hand solution

Solve $u_t = ku_{xx}$ with $u(x,0) = f(x)$ and $u_x(0,t) = 0, u(L,t) = T_0$. Since the right end is not homogeneous, we need to find a reference function. Let $r(x) = Ax + B$. Then $r'(x) = A$. Since $u_x(0,t) = 0$, then $A = 0$. Hence $r(x) = B$. Since $u(L,0) = T_0$, then $r(L) = T_0$. Hence $B = T_0$. Therefore $r(x) = T_0$. Now let the solution be

$$u(x,t) = v(x,t) + r(x) \tag{1}$$

Where $v(x,t)$ solves the same pde but with homogeneous boundary conditions

$$\begin{aligned} v_t &= kv_{xx} \\ v_x(0,t) &= 0 \\ v(L,0) &= 0 \\ v(x,0) &= u(x,0) - r(x) \\ &= F(x) \end{aligned}$$

The above general PDE was solved in problem 4.1.1.28 on page 478 and the solution is

$$v(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L F(x)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}$$

Since here $F(x) = u(x,0) - r(x) = f(x) - T_0$ the above becomes

$$v(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t} \tag{2}$$

From (1,2) the final solution is

$$u(x,t) = T_0+\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}$$

4.1.2.2 [200] left end insulated (special case)

problem number 200

Added June 21, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$. Using the following values

$$\begin{aligned} L &= 5 \\ T_0 &= 10 \\ k &= \frac{1}{100} \\ f(x) &= 0 \end{aligned}$$

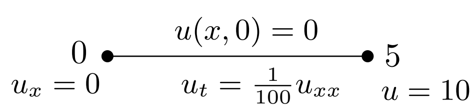


Figure 4.70: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
f=0;
T0=10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] ;
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to \frac{2}{5}\left(\sum_{n=0}^{\infty}-\frac{100e^{-\frac{(2\pi n+\pi)^2 t}{10000}}\cos(n\pi)\cos\left(\frac{1}{10}(2n+1)\pi x\right)}{2\pi n+\pi}+25\right)\right\}\right\}$$

Maple ✓

```
restart;
L:=5;
k:=1/100;
f:=0;
T0:=10;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f;
bc  :=  D[1](u)(0,t)=0, u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) ),out
```

$$u(x,t) = -40\left(\sum_{n=0}^{\infty}\frac{(-1)^n\cos\left(\frac{(2n+1)\pi x}{10}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2 t}{10000}}}{(2n+1)\pi}\right)+10$$

Hand solution

The general solution for this type of PDE is given in problem 4.1.1.28 on page 478 as

$$u(x,t) = T_0+\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}$$

In this problem $u(x,0)=f(x)=0, L=5, k=\frac{1}{100}$ and $T_0=10$, Hence the above becomes

$$u(x,t) = 10 + \frac{2}{5}\sum_{n=0}^{\infty}\left(\int_0^5 -10\cos\left(\frac{(2n+1)\pi}{10}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2 t}$$

But $\int_0^5 -10\cos\left(\frac{(2n+1)\pi}{10}x\right)dx = -\frac{100\cos(\pi n)}{\pi(1+2n)} = \frac{-100(-1)^n}{\pi(1+2n)}$ and the above becomes

$$\begin{aligned} u(x,t) &= 10 + \frac{2}{5}\sum_{n=0}^{\infty}\frac{-100(-1)^n}{\pi(1+2n)}\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2 t} \\ &= 10 - \frac{40}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(1+2n)}\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2 t} \end{aligned}$$

The following is an animation of the solution

Out[ ]=

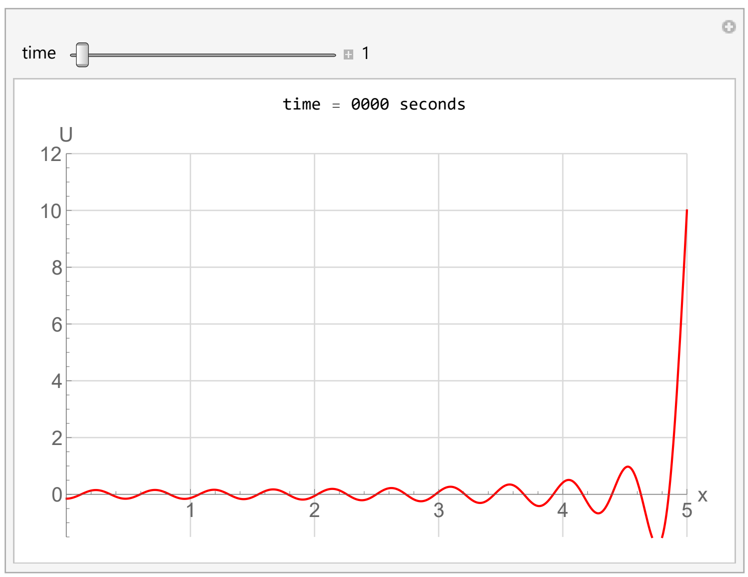


Figure 4.71: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, y, t, n, k]
L = 5;
numberOfTerms = 20;
mySol[x_, t_] = 10 - 40/π Sum[(-1)^n/(1 + 2 n) Exp[-1/100 ((2 n + 1) π/10)^2 t] Cos[(2 n + 1) π/10 x], {n, 0, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 4.72: Source code

```
In[]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-1.5, 12}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 2500, 10}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.73: Code for animation

4.1.2.3 [201] right end nonhomogeneous BC (general case)

problem number 201

Added June 20, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = T_0$$

Initial condition is $u(x,0) = f(x)$

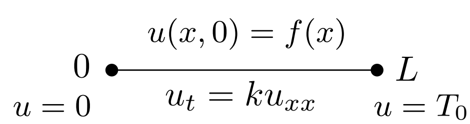


Figure 4.74: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->k>0
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{2e^{-\frac{kn^2\pi^2t}{L^2}}\left(\int_0^L\left(f(x)-\frac{\mathrm{T0}x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx\right)\sin\left(\frac{n\pi x}{L}\right)}{L}+\frac{\mathrm{T0}x}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(L,t) = T0;
ic  := u(x,0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t)=\frac{T0x}{L}+\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L(Lf(x)-T0x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L^2}\right)$$

Hand solution

Since the right side boundary condition is not homogeneous, then we need to first find a reference function. Let $r(x)=Ax+B$. At $x=0,0=B$. Hence $r(x)=Ax$. At $x=L,T_0=AL$, hence $A=\frac{T_0}{L}$. Therefore

$$r(x)=\frac{T_0}{L}x$$

Now let $u(x,t)=v(x,t)+r(x)$ where $v_t=v_{xx}$ but with homogeneous BC $v(0,t)=0,v(L,0)=0$. The basic solution for this type of PDE was already given in problem 4.1.1.1 on page 402 as

$$v(x,t)=\sum_{n=1}^{\infty}B_ne^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Hence

$$\begin{aligned} u(x,t) &= r(x) + v(x,t) \\ &= \frac{T_0}{L}x + \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right) \end{aligned} \tag{1}$$

At $t = 0$

$$f(x) - \frac{T_0}{L}x = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right)$$

Multiplying both sides by $\sin\left(\sqrt{\lambda_{n'}}x\right)$ and integrating

$$\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_{n'}}x\right) dx = \int_0^L \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_{n'}}x\right) \sin\left(\sqrt{\lambda}x\right) dx$$

Moving integration inside summation and by orthogonality of sin function the above reduces to

$$\begin{aligned} \int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx &= B_n \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx \\ \int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx &= \frac{L}{2}B_n \\ B_n &= \frac{2}{L}\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx \end{aligned}$$

Therefore the solution from (1) is

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right)$$

4.1.2.4 [202] right end nonhomogeneous BC (special case)

problem number 202

Added June 20, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$ using these values

$$k = \frac{1}{100}$$
$$L = 100$$
$$T_0 = 100$$
$$f(x) = x$$

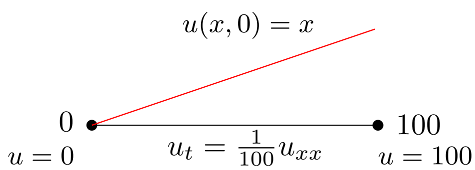


Figure 4.75: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
L=100;
T0=100;
f=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
L:=100;
k:=1/100;
T0:=100;
f:=x;
pde := diff(u(x,t), t) = k*diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(L,t) = T0;
ic  := u(x,0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ),o
```

$$u(x,t) = x$$

Hand solution

The general solution for this type of PDE is given in problem 4.1.2.3 on page 543 as

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

With $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. In this problem

$$\begin{aligned} L &= 100 \\ k &= \frac{1}{100} \\ T_0 &= 100 \\ f(x) &= x \end{aligned}$$

Hence the solution becomes

$$\begin{aligned} u(x,t) &= x + \frac{2}{100}\sum_{n=1}^{\infty}\left(\int_0^{100}(x-x)\sin\left(\frac{n\pi}{100}x\right)dx\right)e^{-\frac{1}{100}\left(\frac{n\pi}{100}\right)^2 t}\sin\left(\frac{n\pi}{100}x\right) \\ &= x + 0 \\ &= x \end{aligned}$$

4.1.2.5 [203] right end nonhomogeneous BC, special case

problem number 203

Added July 2, 2018. Can not find where I found this PDE.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= 1 \end{aligned}$$

Initial condition is $u(x,0) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$

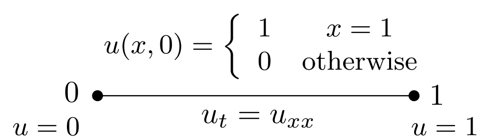


Figure 4.76: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 1};
ic  = u[x, 0] == Piecewise[{{1, x == 1}, {0, True}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{2(-1)^n e^{-n^2\pi^2 t} \sin(n\pi x)}{n\pi} + x\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(1,t) = 1;
ic  := u(x,0) = piecewise(x = 1, 1,  true,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = x + 2\left(\sum_{n=1}^{\infty} \frac{(-1)^n \mathrm{e}^{-\pi^2 n^2 t} \sin(\pi n x)}{\pi n}\right)$$

Hand solution

The general solution for this type of PDE is given in problem 4.1.2.3 on page 543 as

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

With $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$. In this problem

$$L = 1$$
$$k = 1$$
$$T_0 = 1$$
$$f(x) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Hence the solution becomes

$$u(x,t) = x + 2\sum_{n=1}^{\infty}\left(\int_0^1 (f(x) - x)\sin(n\pi x)\,dx\right)e^{-(n\pi)^2 t}\sin(n\pi x) \tag{1}$$

But

$$\begin{aligned}\int_0^1 (f(x) - x)\sin(n\pi x)\,dx &= \int_0^1 f(x)\sin(n\pi x)\,dx - \int_0^1 x\sin(n\pi x)\,dx \\ &= 0 - \int_0^1 x\sin(n\pi x)\,dx\end{aligned}$$

$\int_0^1 x\sin(n\pi x)\,dx = \frac{(-1)^{n+1}}{n\pi}$, hence

$$\begin{aligned}\int_0^1 (f(x) - x)\sin(n\pi x)\,dx &= -\frac{(-1)^{n+1}}{n\pi} \\ &= \frac{(-1)^n}{n\pi}\end{aligned}$$

Therefore (1) becomes

$$u(x,t) = x + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} e^{-(n\pi)^2 t} \sin(n\pi x)$$

This is animation of the solution for 0.3 seconds. (Animation will show only in the HTML version).

Out[ ]=

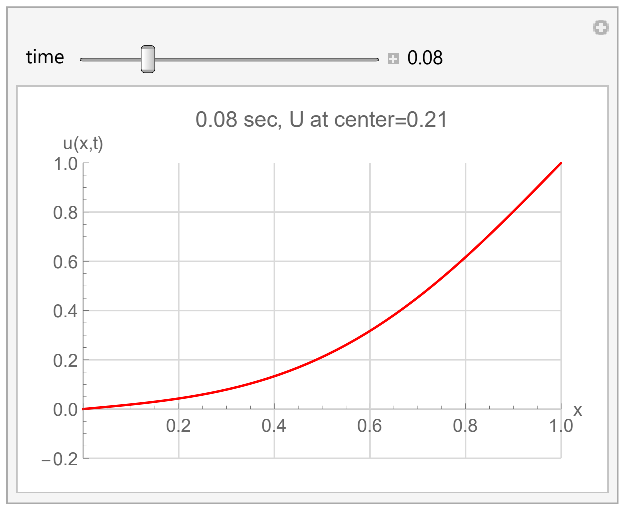


Figure 4.77: Screen shot

Source code used for the above

```
In[•]:= L = 1;
u[x_, t_, m_] := x + 2/Pi Quiet[Sum[(-1)^n/n Sin[n π x] Exp[-(n π)^2 t], {n, 1, m}]];
Plot[u[x, 0, 20], {x, 0, L}, PlotRange → All]

padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
     NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Plot[u[x, t, 20], {x, 0, L},
    PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center=", padIt2[N@u[L/2, t, 20], {3, 2}]}],
    PlotRange → {{0, L}, {-.2, 1}}, GridLines → Automatic,
    GridLinesStyle → LightGray,
    PlotStyle → Red, AxesLabel → {"x", "u(x,t)"},
    BaseStyle → 12
  ],
  {{t, 0, "time"}, 0, 0.4, .01, Appearance → "Labeled"}]

r = Table[
     Plot[u[x, t, 20], {x, 0, L},
       PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center=", padIt2[N@u[L/2, t, 20], {3, 2}]}],
       PlotRange → {{0, L}, {-.2, 1}}, GridLines → Automatic,
       GridLinesStyle → LightGray,
       PlotStyle → Red, AxesLabel → {"x", "u(x,t)"}, BaseStyle → 12
     ],
     {t, 0, 0.3, .01}];

In[•]:= Export["anim.gif", r, "DisplayDurations" → Table[If[i < 10, 0.5, 0.3], {i, Length@r}]]
Out[•]= anim.gif
```

Figure 4.78: Source code

4.1.2.6 [204] convection heat loss

problem number 204

Added April 28, 2019

Problem 2, section 77, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are $u_x(0,t) = hu(0,t)$ and on the right end $u(1,t) = 1$ with $h > 0$. Initial conditions $u(x,0) = 0$

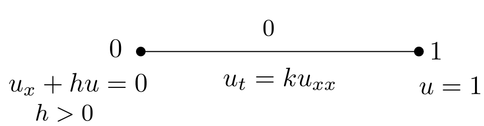


Figure 4.79: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == 0;
bc  = {Derivative[1, 0][u][0, t] == h * u[0,t], u[1, t] == 1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions->h>
```

$$\left\{\left\{u(x,t) \to \left\{ \quad x^2 + \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\sqrt{h}\left(\frac{\sqrt{2}e^{-tK[2,K[1]]}\left(2h+\cos\left(\sqrt{K[2,K[1]]}\right)((h-2)K[2,K[1]]-2h)-\sqrt{K[2,K[1]]}(2h+K[2,K[1]]-2)\sin\left(\sqrt{K[2,K\right.\right.}{K[2,K[1]]^{3/2}\sqrt{h\left(-\cos^2\left(\sqrt{K[2,K[1]]}\right)+h+2\right)+\frac{\left(h-\cos^2\left(\sqrt{K[2,K[1]]}\right)\right)K[2,K[1]]}{h}}}\right.}{\sqrt{h^3-\cos^2\left(\sqrt{K[2,K\right.}}}\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(x,t), t) = (diff(u(x,t), x, x));
ic  := u(x,0) = 0;
bc  := eval(diff(u(x,t), x), x = 0) = h*u(0,t), u(1,t) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

time expired

Hand solution

Solve

$$u_t = u_{xx} \qquad 0 < x < 1, t > 0$$

With boundary conditions

$$\begin{aligned} u_x(0,t) - hu(0,t) &= 0 \\ u(1,t) &= 1 \end{aligned}$$

With $h > 0$. And initial conditions $u(x,0) = f(x)$.

Because the second B.C. is not zero, we need to introduce a reference function $r(x)$ which satisfies the nonhomogeneous boundary conditions.

Let $r(x) = Ax + B$. When $x = 0$ then the first BC gives

$$A - hB = 0$$

And the second BC gives

$$A+B=1$$

From the first equation $A=hB$. Substituting in the second equation give $hB+B=1$ or $B(1+h)=1$ or $B=\frac{1}{1+h}$. Hence $A=\frac{h}{1+h}$. Therefore

$$\begin{aligned}r(x)&=Ax+B\\&=\frac{h}{1+h}x+\frac{1}{1+h}\\&=\frac{hx+1}{1+h}\end{aligned}\tag{1}$$

To verify. $r_x=\frac{h}{1+h}$. When $x=0$ then $r(0)=\frac{1}{1+h}$. Hence $r_x(0)-hr(0)=\frac{h}{1+h}-h\frac{1}{1+h}=0$ as expected. And when $x=1$ then $r(1)=1$ as expected. Now that we found $r(x)$ then we write

$$u(x,t)=v(x,t)+r(x)$$

Where $v(x,t)$ is the solution to the homogenous PDE

$$v_t=v_{xx}\qquad 0<x<1,t>0$$

With boundary conditions

$$\begin{aligned}v_x(0,t)-hv(0,t)&=0\\v(1,t)&=0\end{aligned}$$

We can now solve for $v(x,t)$ using separation of variables since boundary conditions are homogenous. Separation of variables gives

$$\begin{aligned}X''+\lambda X&=0\\X'(0)-hX(0)&=0\\X(1)&=0\end{aligned}$$

The above is known eigenvalue problem which we found before. It has the following eigenfunctions and eigenvalues

$$\begin{aligned}\phi_n(x)&=\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)\qquad n=1,2,\cdots\\\tan\left(\alpha_n\right)&=\frac{-\alpha_n}{h}\end{aligned}$$

With $\alpha_n>0$. Hence the solution $v(x,t)$ using generalized Fourier series is

$$v(x,t)=\sum_{n=1}^{\infty}B_n(t)\,\phi_n(x)\tag{2}$$

Substituting into the PDE $v_t = v_{xx}$ gives

$$\begin{aligned}\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) &= \sum_{n=1}^{\infty} B_n(t)\,\phi_n''(x)\\ &= -\sum_{n=1}^{\infty} B_n(t)\,\alpha_n^2\phi_n(x)\end{aligned}$$

Therefore the ODE is

$$B_n'(t) + \alpha_n^2 B_n(t) = 0$$

The solution is

$$B_n(t) = B_n(0)\,e^{-\alpha_n^2 t}$$

Hence (2) becomes

$$v(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x)$$

And since $u(x,t) = v(x,t) + r(x)$ then

$$u(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x) + \frac{hx+1}{1+h}$$

Now we find $B_n(0)$ from initial conditions. At $t=0$ the above becomes

$$\begin{aligned}0 &= \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x) + \frac{hx+1}{1+h}\\ -\frac{hx+1}{1+h} &= \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x)\end{aligned}$$

Hence

$$\begin{aligned}B_n(0) &= \left\langle -\frac{hx+1}{1+h}, \phi_n(x)\right\rangle\\ &= -\int_0^1 p(x)\,\frac{hx+1}{1+h}\phi_n(x)\,dx\\ &= -\int_0^1 \frac{hx+1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)dx\\ &= -\frac{1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1 (hx+1)\sin\left(\alpha_n(1-x)\right)dx \qquad (3)\end{aligned}$$

But

$$
\begin{aligned}
\int_0^1 (hx+1)\sin(\alpha_n(1-x))\,dx &= \int_0^1 \sin(\alpha_n(1-x))\,dx + h\int_0^1 x\sin(\alpha_n(1-x))\,dx\\
&= \left[\frac{\cos(\alpha_n(1-x))}{\alpha_n}\right]_0^1 + h\left[\frac{\alpha_n x\cos(\alpha_n(1-x))+\sin(\alpha_n(1-x))}{\alpha_n^2}\right]_0^1\\
&= \frac{1-\cos(\alpha_n)}{\alpha_n} + \frac{h}{\alpha_n^2}\left[\alpha_n x\cos(\alpha_n(1-x))+\sin(\alpha_n(1-x))\right]_0^1\\
&= \frac{1-\cos(\alpha_n)}{\alpha_n} + \frac{h}{\alpha_n^2}\left[\alpha_n - \sin\alpha_n\right]\\
&= \frac{\alpha_n - \alpha_n\cos(\alpha_n) + h\alpha_n - h\sin\alpha_n}{\alpha_n^2}
\end{aligned}
$$

But $\frac{\sin(\alpha_n)}{\cos(\alpha_n)} = -\frac{\alpha_n}{h}$ or $h\sin(\alpha_n) = -\alpha_n\cos(\alpha_n)$ or $-h\sin\alpha_n = \alpha_n\cos(\alpha_n)$, hence the above simplifies to

$$
\begin{aligned}
\int_0^1 (hx+1)\sin(\alpha_n(1-x))\,dx &= \frac{\alpha_n + h\alpha_n}{\alpha_n^2}\\
&= \frac{1+h}{\alpha_n}
\end{aligned}
$$

Therefore (3) becomes

$$
\begin{aligned}
B_n(0) &= \frac{-1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\left(\frac{1+h}{\alpha_n}\right)\\
&= -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}
\end{aligned}
$$

Hence final solution becomes

$$
\begin{aligned}
u(x,t) &= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x)\\
&= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n(1-x))\\
&= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n(1-x))\\
&= \frac{hx+1}{1+h} - 2h\sum_{n=1}^{\infty}\frac{\sin(\alpha_n(1-x))}{\alpha_n\left(h+\cos^2\alpha_n\right)}\exp\left(-\alpha_n^2 t\right)
\end{aligned}
$$

4.1.2.7 [205] nonhomogeneous BC

problem number 205

Added July 2, 2018.

Second example from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$.

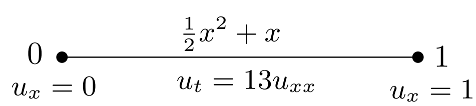


Figure 4.80: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, t], x] == 13*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
ic  = u[x, 0] == (1*x^2)/2 + x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x, x));
bc  := D[1](u)(0,t)=0,D[1](u)(1,t)=1;
ic  := u(x, 0) = 1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t) = \frac{x^2}{2} + 13t + 2\left(\sum_{n=1}^{\infty} \frac{((-1)^n - 1)\cos(\pi n x)\, \mathrm{e}^{-13\pi^2 n^2 t}}{\pi^2 n^2}\right) + \frac{1}{2}$$

4.1.2.8 [206] nonhomogeneous BC

problem number 206

Added March 31, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u_x(\pi,t) = A$$

Initial condition is $u(x,0) = 0$

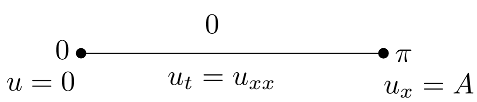


Figure 4.81: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] ;
ic  = u[x, 0] == 0;
bc  = {u[0,t] == 0, Derivative[1, 0][u][Pi, t] == A};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->A>0
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{8(-1)^{K[1]}Ae^{-\frac{1}{4}t(1-2K[1])^2}\sin\left(x\left(K[1]-\frac{1}{2}\right)\right)}{\pi(1-2K[1])^2}+Ax\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic  := u(x, 0) = 0:
bc  := u(0,t)=0,  eval( diff(u(x,t),x),x=Pi)=A:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t)=Ax+\sum_{n=0}^{\infty}\left(-\frac{8A(-1)^n\,\mathrm{e}^{-\frac{(2n+1)^2t}{4}}\sin\left(nx+\frac{1}{2}x\right)}{\pi\left(2n+1\right)^2}\right)$$

4.1.2.9 [207] nonhomogeneous BC

problem number 207

Added April 15, 2019.

Solve the heat equation for $u(x,t)$

$$u_t=ku_{rr}$$

For $0<r<a$ and $t>0$. The boundary conditions are

$$u(0,t)=0$$
$$u(a,t)=a\phi(t)$$

Initial condition is $u(r,0)=rf(r)$

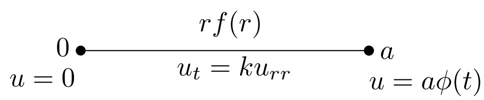


Figure 4.82: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*D[u[r, t], {r, 2}] ;
ic  = u[r, 0] == r*f[r];
bc  = {u[0, t] == 0, u[a, t] == a*phi[t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t)\to \sum_{K[1]=1}^{\infty}\sqrt{2}\sqrt{\frac{1}{a}}\left(e^{-\frac{k\pi^2 tK[1]^2}{a^2}}\int_0^a \sqrt{2}\sqrt{\frac{1}{a}}r(f(r)-\phi(0))\sin\left(\frac{\pi rK[1]}{a}\right)dr+\int_0^t \frac{(-1)^{K[1]}\sqrt{2}e^{-\frac{k\pi}{}}}{\left(\frac{1}{a}\right)^{3}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r, t), t) = k*diff(u(r, t), r$2):
ic  := u(r,0)=r*f(r);
bc  := u(0,t)=0,u(a,t)=a*phi(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(r, t))),
```

$$u(r,t)=r\phi(t)+\int_0^t\left(\sum_{n=1}^{\infty}\frac{2a(-1)^n\left(\frac{d}{d\tau}\phi(\tau)\right)\mathrm{e}^{-\frac{\pi^2(t-\tau)k\,n^2}{a^2}}\sin\left(\frac{\pi n r}{a}\right)}{\pi n}\right)d\tau+\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^a (f(r)-\phi(0))\,r\sin\left(\frac{\pi n}{a}\right.\right.}{a}\right.$$

<u>Hand solution</u>

Solve

$$u_t = ku_{rr} \qquad t>0, 0<r<a \tag{1}$$

With boundary conditions

$$u(0,t)=0$$
$$u(a,t)=a\phi(t)$$

And initial conditions

$$u(r,0) = rf(r)$$

Since the boundary conditions are not homogeneous, the first step is to convert them to homogeneous. This is done using a reference function which needs to only satisfy the boundary conditions. This reference function can be seen to be $v(r,t) = r\phi(t)$. Now we write

$$u(r,t) = w(r,t) + v(r,t)$$

Where $w(r,t)$ satisfies the PDE but with homogeneous B.C. Substituting the above into (1) gives

$$\begin{aligned} w_t(r,t) + r\phi'(t) &= kw_{rr} \\ w_t(r,t) &= kw_{rr} - r\phi'(t) \end{aligned} \tag{2}$$

With boundary conditions

$$\begin{aligned} w(0,t) &= 0 \\ w(a,t) &= 0 \end{aligned}$$

The solution to the homogeneous PDE $w_t(r,t) = kw_{rr}$ with the above boundary conditions is easily found and known. The eigenvalues are $\lambda_n = \left(\frac{n\pi}{a}\right)^2, n = 1,2,\cdots$ and eigenfunctions $\Phi_n(r) = \sin\left(\sqrt{\lambda_n} r\right)$. Let the solution to (2), using eigenfunction expansion be

$$w(r,t) = \sum_{n=1}^{\infty} C_n(t)\,\Phi_n(r) \tag{2A}$$

Substituting the above back into (2) gives

$$\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) = k\sum_{n=1}^{\infty} C_n(t)\,\Phi_n''(r) - \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r) \tag{3}$$

Where $q_n(t)$ are the Fourier coefficients of $r\phi'(t)$ which are found by

$$r\phi'(t) = \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r)$$

Applying orthogonality using $\Phi_n(r)$ gives

$$\begin{aligned} \int_0^a r\phi'(t)\,\Phi_m(r)\,dr &= \int_0^a \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r)\,\Phi_m(r)\,dr \\ &= \sum_{n=1}^{\infty} q_n(t) \int_0^r \Phi_n(r)\,\Phi_m(r)\,dr \end{aligned}$$

But $\int_0^a \Phi_n(r)\,\Phi_m(r)\,dr = \int_0^a \sin\left(\frac{n\pi}{a}r\right)\sin\left(\frac{m\pi}{a}r\right)dr = \frac{a}{2}$ for $n=m$ only, and the above becomes

$$\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(s)\,dr = q_m(t)$$

Substituting the above back into (3) gives

$$\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) = k\sum_{n=1}^{\infty} C_n(t)\,\Phi_n''(r) - \sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)$$

But $\Phi_n''(r) = -\lambda_n\Phi_n(r)$ and above simplifies to

$$\begin{aligned}
\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) + k\sum_{n=1}^{\infty} C_n(t)\,\lambda_n\Phi_n(r) &= -\sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)\\
C_n'(t) + kC_n(t)\,\lambda_n &= -\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\\
&= -\frac{2}{a}\phi'(t)\int_0^a r\sin\left(\frac{n\pi}{a}r\right)dr\\
&= -\frac{2}{a}\phi'(t)\,\frac{(-1)^{n+1}\,a^2}{n\pi}\\
&= -2a\phi'(t)\,\frac{(-1)^{n+1}}{n\pi}
\end{aligned}$$

This is first order ODE in $C(t)$. The solution is

$$C_n(t) = e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau$$

From (2A)

$$w(r,t) = \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right)$$

Hence

$$\begin{aligned}
u(r,t) &= w(r,t) + v(r,t)\\
&= \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right) + r\phi(t)
\end{aligned} \tag{4}$$

At $t=0$ the above becomes

$$\begin{aligned}
rf(r) &= \sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) + r\phi(0)\\
\sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) &= r(f(r) - \phi(0))
\end{aligned}$$

Hence $C_n(0)$ is the Fourier sine coefficients of $r(f(r)-\phi(0))$

$$\frac{a}{2}C_n(0)=\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr$$
$$C_n(0)=\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr$$

Substituting this into (4) gives the final solution as

$$\begin{aligned}u(r,t)&=r\phi(t)+\sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)+2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t\phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin(\\&=r\phi(t)+\sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)+2a\frac{(-1)^{n+1}}{n\pi}\int_0^t\phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\right)\sin(\\&=r\phi(t)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\sin\left(\frac{n\pi}{a}r\right)+\sum_{n=1}^{\infty}2a\frac{(-1)^{n+1}}{n\pi}\int_0^t\phi'(\tau)\,e^{-k\lambda_n(t-\tau}\end{aligned}$$

Or

$$\begin{aligned}u(r,t)&=r\phi(t)\\&\quad+\frac{2}{a}\sum_{n=1}^{\infty}e^{-k\lambda_n t}\sin\left(\frac{n\pi}{a}r\right)\left(\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\\&\quad+\frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi}{a}r\right)\int_0^t\phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\end{aligned}$$

Where $\lambda_n=\left(\frac{n\pi}{a}\right)^2$.

4.1.3 Finite domain (bar), right end homogeneous, left end not

Local contents

4.1.3.1 [208] left end BC depends on time (general case)

problem number 208

Added June 21, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = A(t)$$
$$u(L,t) = 0$$

And initial condition is $u(x,0) = 0$

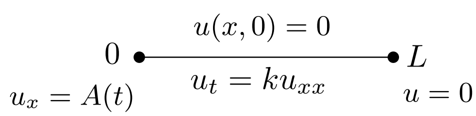


Figure 4.83: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == A[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\frac{4\sqrt{2}e^{-\frac{k\pi^2 t(1-2K[1])^2}{4L^2}}A(0)L^{3/2}}{\pi^2(1-2K[1])^2} + \int_0^t \frac{4\sqrt{2}\exp\left(-\frac{k\pi^2(1-2K[1])^2(t-K[2])}{4L^2}\right)L^{3/2}A'(K}{\pi^2(1-2K[1])^2}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =A(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = (-L+x)\,A(t) + \int_0^t \left(\sum_{n=0}^{\infty} \frac{8L\cos\left(\frac{(2n+1)\pi x}{2L}\right)\left(\frac{d}{d\tau}A(\tau)\right)\mathrm{e}^{-\frac{(t-\tau)\left(n+\frac{1}{2}\right)^2\pi^2 k}{L^2}}}{(2n+1)^2\pi^2} \right) d\tau + 8\left(\sum_{n=0}^{\infty} \frac{A(0)\,L\cos\left(\right.}{(2\right.}$$

<u>Hand solution</u>

This 1D heat PDE has left one end with boundary condition that is time dependent.

$$\begin{aligned} u_t &= k u_{xx} \qquad 0<x<L, t>0 \\ u_x(0,t) &= A(t) \\ u(L,t) &= 0 \\ u(x,0) &= 0 \end{aligned}$$

<u>Solution</u>

Since the boundary condition is not homogeneous, we need to first find a reference function $r(x,t)$. Let

$$r(x,t) = A(t)\,(x-L)$$

This function only needs to satisfy the nonhomogeneous boundary conditions given. i.e. $\frac{\partial r}{\partial x}(0,t) = A(t)\,, r(L,t) = 0$. Now we can write

$$u(x,t) = v(x,t) + r(x,t) \tag{1}$$

Since $r(x,t)$ satisfies the nonhomogeneous B.C's, then $v(x,t)$ satisfies the homogeneous boundary conditions. Substituting the above back into the original PDE gives

$$\begin{aligned} v_t + r_t &= k(v_{xx} + r_{xx}) \\ v_t + A'(t)\,(x-L) &= k v_{xx} \\ v_t &= k v_{xx} + A'(t)\,(L-x) \\ &= k v_{xx} + Q(x,t) \end{aligned} \tag{2}$$

The PDE $v_t = kv_{xx} + Q(x,t)$ has now homogeneous B.C. $v_x(0,t) = 0, v(L,0) = 0$. Where $Q(x,t) = A'(t)(L-x)$. The method of eigenfunction expansion is now used to solve (2). Let

$$v(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x)$$

Substituting this back into (2) gives

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) &= k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + Q(x,t)\\
&= k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)
\end{aligned}$$

Where $Q(x,t) = \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)$. Now, since $\Phi_n''(x) = -\lambda_n\Phi_n(x)$ then the above reduces to

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) + k\sum_{n=1}^{\infty} a_n(t)\,\lambda_n\Phi_n(x) &= \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)\\
a_n'(t) + k\lambda_n a_n(t) &= q_n(t) \qquad (3)
\end{aligned}$$

The eigenfunctions $\Phi_n(x)$ come from solving the eigenvalue problem in $v_t = kv_{xx}$ with homogeneous boundary conditions $v_x(0,t) = 0, v(L,t) = 0$. This was solved before in problem 4.1.1.28 on page 478. The eigenfunctions were found to be $\Phi_n(x) = \cos\left(\sqrt{\lambda_n}x\right), n = 1,3,5,\cdots$ with eigenvalues $\lambda_n = \left(\frac{n\pi}{2L}\right)^2, n = 1,3,5,\cdots$. Before solving the ODE (3), we need to first find $q_n(t)$. Orthogonality is now used to find $q_n(t)$

$$\begin{aligned}
Q(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} q_n(t)\cos\left(\sqrt{\lambda_n}x\right)\\
A'(t)(L-x) &= \sum_{n=1,3,5,\cdots}^{\infty} q_n(t)\cos\left(\sqrt{\lambda_n}x\right)\\
\int_0^L A'(t)(L-x)\cos\left(\sqrt{\lambda_n}x\right)dx &= \frac{L}{2}q_n(t)\\
q_n(t) &= \frac{2A'(t)}{L}\int_0^L (L-x)\cos\left(\frac{n\pi}{2L}x\right)dx\\
&= -\frac{2A'(t)}{L}\left(\frac{4L^2\left(\cos\left(\frac{n\pi}{2}\right)-1\right)}{\pi^2 n^2}\right) \qquad n = 1,3,5,\cdots
\end{aligned}$$

But $\cos\left(\frac{n\pi}{2}\right)-1=-1$ for $n=1,3,5,\cdots$. Hence the above becomes

$$\begin{aligned}
q_n(t) &= \frac{2A'(t)}{L}\left(\frac{4L^2}{\pi^2 n^2}\right) \qquad n=1,3,5,\cdots\\
&= \frac{8A'(t)\,L}{\pi^2 n^2}
\end{aligned}$$

Hence (3) becomes

$$\begin{aligned}
a_n'(t)+k\lambda_n a_n(t) &= q_n(t) \qquad n=1,2,3,\cdots\\
a_n'(t)+k\left(\frac{n\pi}{2L}\right)^2 a_n(t) &= \frac{8A'(t)\,L}{\pi^2 n^2}\\
a_n'(t)+k\lambda_n a_n(t) &= \frac{8A'(t)\,L}{\pi^2 n^2}
\end{aligned}$$

Integrating factor is $e^{k\lambda_n t}$. Hence the above becomes

$$\frac{d}{dt}\left(a(t)\,e^{k\lambda_n t}\right)=\frac{8A'(t)\,L}{\pi^2 n^2}e^{k\lambda_n t}$$

Integrating gives

$$\begin{aligned}
a_n(t)\,e^{k\lambda_n t} &= \int_0^t \frac{8A'(\tau)\,L}{\pi^2 n^2}e^{k\lambda_n \tau}d\tau + a_n(0)\\
a_n(t) &= a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t \frac{8A'(\tau)\,L}{\pi^2 n^2}e^{k\lambda_n \tau}d\tau
\end{aligned}$$

Hence

$$\begin{aligned}
v(x,t) &= \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x)\\
&= \sum_{n=1,3,5,\cdots}^{\infty}\left(a_n(0)\,e^{-k\lambda_n t}+\frac{8L}{\pi^2 n^2}e^{-k\lambda_n t}\int_0^t A'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right)
\end{aligned}$$

Since $u(x,t)=v(x,t)+r(x,t)$ then

$$u(x,t)=A(t)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty}\left(a_n(0)\,e^{-k\lambda_n t}+\frac{8L}{\pi^2 n^2}e^{-k\lambda_n t}\int_0^t A'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right) \tag{4}$$

At $t=0, u(x,0)=0$ and the above becomes

$$\begin{aligned}
0 &= A(0)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty} a_n(0)\cos\left(\sqrt{\lambda_n}x\right)\\
A(0)\,(L-x) &= \sum_{n=1,3,5,\cdots}^{\infty} a_n(0)\cos\left(\sqrt{\lambda_n}x\right)
\end{aligned}$$

Applying orthogonality

$$\int_0^L (L-x)\cos\left(\sqrt{\lambda_n}x\right)dx = a_n(0)\,\frac{L}{2}$$

$$\begin{aligned} a_n(0) &= \frac{2A(0)}{L}\int_0^L (L-x)\cos\left(\sqrt{\lambda_n}x\right)dx \\ &= \frac{2A(0)}{L}\left(-\frac{\cos\left(L\sqrt{\lambda_n}\right)-1}{\lambda_n}\right) \end{aligned}$$

But $\cos\left(L\sqrt{\lambda_n}\right)-1=\cos\left(L\frac{n\pi}{2L}\right)-1=\cos\left(\frac{n\pi}{2}\right)-1=-1$ for $n=1,3,5,\cdots$, and the above becomes

$$a_n(0)=\frac{2A(0)}{L\lambda_n}$$

Therefore the solution (4) is

$$u(x,t)=A(t)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{2A(0)}{L\lambda_n}+\frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\,e^{k\lambda_n\tau}\right)\cos\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{2L}\right)^2$. Hence

$$u(x,t)=A(t)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2A(0)}{L\left(\frac{n\pi}{2L}\right)^2}+\frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\,e^{k\left(\frac{n\pi}{2L}\right)^2\tau}d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

4.1.3.2 [209] left end BC depends on time (special case)

problem number 209

Added June 22, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0<x<L$ and $t>0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= A(t) \\ u(L,t) &= 0 \end{aligned}$$

And initial condition is $u(x,0)=0$. Using the following values

$$\begin{aligned} L &= 5 \\ k &= \frac{1}{100} \\ A(t) &= e^t \end{aligned}$$

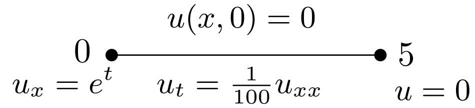


Figure 4.84: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == Exp[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{40e^{-\frac{\pi^2 t(1-2K[1])^2}{10000}}\cos\left(\frac{1}{10}\pi x(2K[1]-1)\right)\left(\pi^2(1-2K[1])^2+10000e^{\frac{\pi^2 t(1-2K[1])^2}{10000}+t}\right)}{\left(\pi^2(1-2K[1])^2+10000\right)\left(\pi-2\pi K[1]\right)^2}+e^t(x\right.\right.$$

Maple ✓

```
restart;
k:=1/100;
L:=5;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =exp(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t)=\left(\sum_{n=0}^{\infty}\frac{10\left(2500\,\mathrm{e}^{t}+\left(n+\frac{1}{2}\right)^2\pi^2\mathrm{e}^{-\frac{\pi^2(2n+1)^2 t}{10000}}\right)\cos\left(\frac{(2n+1)\pi x}{10}\right)}{\left(2500+\left(n+\frac{1}{2}\right)^2\pi^2\right)\left(n+\frac{1}{2}\right)^2\pi^2}\right)+(x-5)\,\mathrm{e}^{t}$$

Hand solution

This PDE general solution was obtained in problem 4.1.3.1 on page 563 as

$$u(x,t) = A(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2A(0)}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\, e^{k\left(\frac{n\pi}{2L}\right)^2 \tau} d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

When $A(t) = e^t$, the above becomes

$$u(x,t) = e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\int_0^t e(\tau)\, e^{k\left(\frac{n\pi}{2L}\right)^2 \tau} d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

But

$$\int_0^t e(\tau)\, e^{k\left(\frac{n\pi}{2L}\right)^2 \tau} d\tau = \frac{e^{k\left(\frac{n\pi}{2L}\right)^2 t + t} - 1}{k\left(\frac{n\pi}{2L}\right)^2 + 1}$$

And the general solution becomes

$$\begin{aligned}
u(x,t) &= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\frac{e^{k\left(\frac{n\pi}{2L}\right)^2 t + t} - 1}{k\left(\frac{n\pi}{2L}\right)^2 + 1}\right)\cos\left(\frac{n\pi}{2L}x\right) \\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{2e^{-k\left(\frac{n\pi}{2L}\right)^2 t}}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\frac{e^{t} - e^{-k\left(\frac{n\pi}{2L}\right)^2 t}}{k\left(\frac{n\pi}{2L}\right)^2 + 1}\right)\cos\left(\frac{n\pi}{2L}x\right) \\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} - \frac{8L}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\right) + \frac{8Le^t}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\cos\left(\frac{n\pi}{2L}\right. \\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{8L}{n^2\pi^2} - \frac{8L}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\right)e^{-k\left(\frac{n\pi}{2L}\right)^2 t} + \frac{8Le^t}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\cos\left(\frac{n\pi}{2L}x\right)
\end{aligned}$$

In this problem $L = 5, k = \frac{1}{100}$, hence the above becomes

$$u(x,t) = e^t(x-5) + \sum_{n=1,3,5,\cdots}^{\infty} \left(\left(\frac{40}{n^2\pi^2} - \frac{40}{\left(k\left(\frac{n\pi}{10}\right)^2 + 1\right)\pi^2 n^2}\right)e^{-\frac{n^2\pi^2}{10000}t} + \frac{40e^t}{\left(k\left(\frac{n\pi}{10}\right)^2 + 1\right)\pi^2 n^2}\right)\cos\left(\frac{n\pi}{10}x\right.$$

The following is an animation of the solution

Out[ ]=

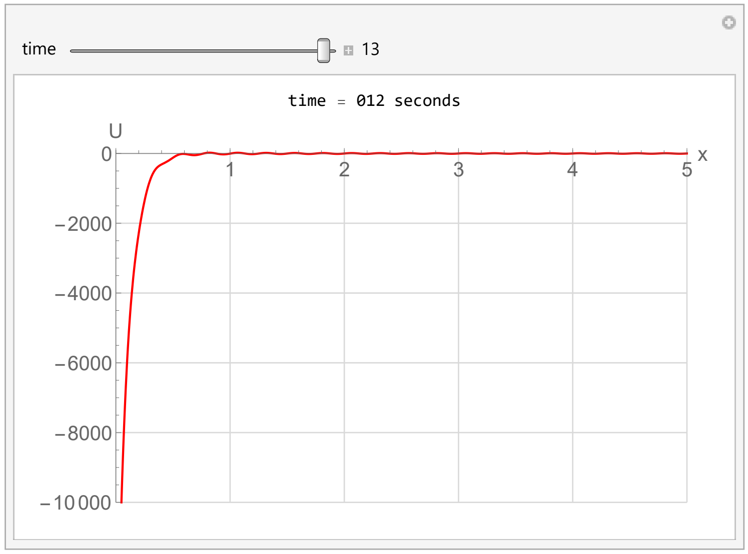


Figure 4.85: Initial state

Source code used for the above

In[ ]:=
```
ClearAll[x, y, t, n, k]
L = 5;
k = 1/100;
numberOfTerms = 80;
mySol[x_, t_] =
   Exp[t] (x - L) + Sum[((8 L/(n^2 π^2) - 8 L/((k (n π/(2 L))^2 + 1) n^2 π^2)) Exp[-k (n π/(2 L))^2 t] + 8 L Exp[t]/((k (n π/(2 L))^2 + 1) n^2 π^2)) Cos[n π/(2 L) x],
     {n, 1, numberOfTerms, 2}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 4.86: Source code

```
In[ ]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-1.5, 12}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 2500, 10}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.87: Code for animation

4.1.3.3 [210] left end BC depends on time (special case)

problem number 210

Added June 22, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = A(t)$$
$$u(L,t) = 0$$

And initial condition is $u(x,0) = 0$. Using the following values

$$L = 5$$
$$k = \frac{1}{100}$$
$$A(t) = \sin(t)$$

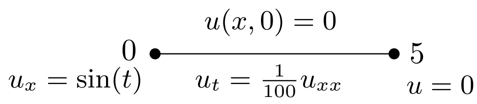


Figure 4.88: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == Sin[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{400000\cos\left(\frac{1}{10}\pi x(2K[1]-1)\right)\left(\pi^2\cos(t)(1-2K[1])^2-e^{-\frac{\pi^2t(1-2K[1])^2}{10000}}\pi^2(1-2K[1])^2+\right.}{\left(\pi^4(1-2K[1])^4+100000000\right)(\pi-2\pi K[1])^2}\right.\right.$$

Maple ✓

```
restart;
k:=1/100;
L:=5;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =sin(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t)=400000\left(\sum_{n=0}^{\infty}\frac{\left(\pi^2\left(n+\frac{1}{2}\right)^2\cos(t)-\left(n+\frac{1}{2}\right)^2\pi^2\mathrm{e}^{-\frac{\pi^2(2n+1)^2t}{10000}}+2500\sin(t)\right)\cos\left(\frac{(2n+1)\pi x}{10}\right)}{16\pi^2\left(6250000+\left(n+\frac{1}{2}\right)^4\pi^4\right)\left(n+\frac{1}{2}\right)^2}\right)+(x-$$

Hand solution

This PDE general solution was obtained in problem 4.1.3.1 on page 563 as

$$u(x,t) = A(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{2A(0)}{L\lambda_n} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{2L}\right)^2, n = 1,3,5,\cdots$. When $A(t) = \sin(t)$, the above becomes

$$u(x,t) = \sin(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{8L}{\pi^2 n^2}\int_0^t \cos(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right)$$

But

$$\int_0^t \cos(\tau)\,e^{k\lambda_n \tau}d\tau = \frac{k\lambda_n e^{k\lambda_n t}\cos(t) + e^{k\lambda_n t}\sin(t) - k\lambda_n}{k^2\lambda_n^2+1}$$

And the general solution becomes

$$\begin{aligned} u(x,t) &= (x-L)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{8L}{\pi^2 n^2}\left(\frac{k\lambda_n e^{k\lambda_n t}\cos(t) + e^{k\lambda_n t}\sin(t) - k\lambda_n}{k^2\lambda_n^2+1}\right)\right)\cos\left(\sqrt{\lambda_n}x\right) \\ &= (x-L)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{8L}{\pi^2 n^2}\left(\frac{k\lambda_n\cos(t) + \sin(t) - k\lambda_n e^{-k\lambda_n t}}{k^2\lambda_n^2+1}\right)\cos\left(\sqrt{\lambda_n}x\right) \end{aligned}$$

In this problem $L=5, k=\frac{1}{100}$, hence the above becomes

$$u(x,t) = (x-5)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{40}{\pi^2 n^2}\left(\frac{\frac{1}{100}\left(\frac{n\pi}{10}\right)^2\cos(t) + \sin(t) - \frac{1}{100}\left(\frac{n\pi}{10}\right)^2 e^{-\frac{1}{100}\left(\frac{n\pi}{10}\right)^2 t}}{\left(\frac{1}{100}\left(\frac{n\pi}{10}\right)^2\right)^2+1}\right)\cos\left(\frac{n\pi}{10}x\right)$$

The following is an animation of the solution

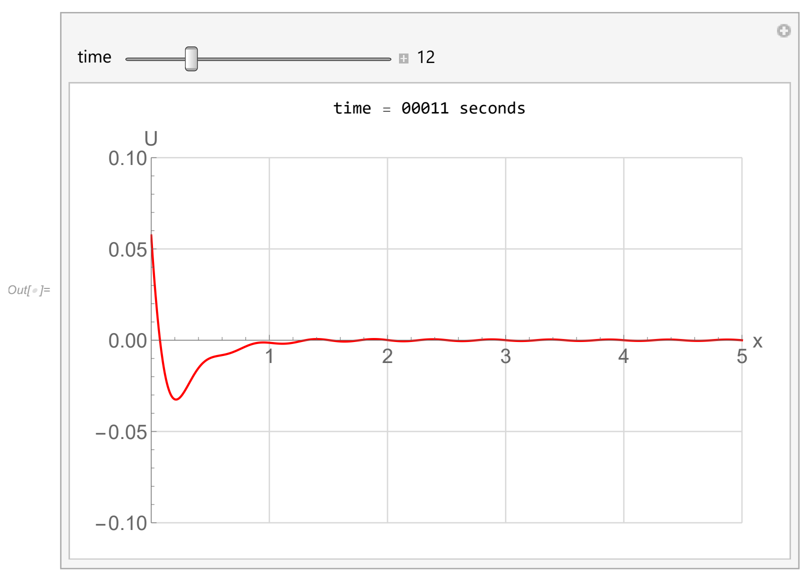


Figure 4.89: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, y, t, n, k]
L = 5;
k = 1 / 100;
λ = (n π / (2 L))^2;
numberOfTerms = 40;
mySol[x_, t_] =
  N[Sin[t] (x - L) + Sum[(8 L)/(n^2 π^2) ((k λ Cos[t] + Sin[t] - k λ Exp[-k λ t])/(k^2 λ^2 + 1)) Cos[Sqrt[λ] x], {n, 1, numberOfTerms, 2}]];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 4.90: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {5, 1}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-0.1, 0.1}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 50, 1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.1]
```

Figure 4.91: Code for animation

4.1.3.4 [211] Haberman 8.3.6 (special case)

problem number 211

Added Nov 25, 2018.

Problem 8.3.6 from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + \sin(5x)e^{-2t}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 1 \\ u(\pi,t) &= 0 \end{aligned}$$

Initial condition is $u(x,0) = 0$

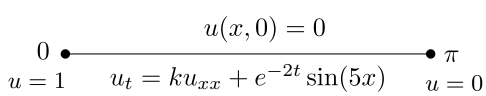


Figure 4.92: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + Sin[5*x]*Exp[-2*t];
bc  = {u[0, t] == 1, u[Pi, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\left(\sqrt{\frac{2}{\pi}}\int_0^t e^{-K[1]^2(t-K[2])}\left(\begin{cases} e^{-2K[2]}\sqrt{\frac{\pi}{2}} & K[1]=5 \\ 0 & \text{True}\end{cases}\right) dK[2] - \frac{2e^{-tK[1]^2}}{\pi K[1]}\right)\sin(xK[$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2)+ sin(5*x)*exp(-2*t);
ic  := u(x,0)=0;
bc  := u(0,t) =1,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t) = \frac{-23x - 46\pi\left(\sum_{n=1}^{\infty} \frac{e^{-n^2 t}\sin(nx)}{\pi n}\right) + \pi(-e^{-25t} + e^{-2t})\sin(5x) + 23\pi}{23\pi}$$

<u>Hand solution</u>

This problem has nonhomogeneous B.C. and non-homogenous in the PDE itself (source present). First step is to use reference function to remove the nonhomogeneous B.C. then use the method of eigenfunction expansion on the resulting problem. Let

$$r(x) = c_1 x + c_2$$

At $x = 0, r(x) = 1$, hence $1 = c_2$ and at $x = \pi, r(x) = 0$, hence $0 = c_1\pi + 1$ or $c_1 = -\frac{1}{\pi}$, therefore

$$r(x) = 1 - \frac{x}{\pi}$$

Therefore

$$u(x,t) = v(x,t) + r(x)$$

Where $v(x,t)$ solution for the given PDE but with homogeneous B.C., therefore

$$\begin{aligned} v_t &= v_{xx} + e^{-2t}\sin 5x \qquad (1)\\ v(0,t) &= 0\\ v(\pi,t) &= 0\\ v(x,0) &= f(x)\\ &= u(x,0) - r(x)\\ &= 0 - \left(1 - \frac{x}{\pi}\right)\\ &= \frac{x}{\pi} - 1 \end{aligned}$$

We now solve (1). This is a PDE with homogeneous B.C. of the form $v_t = v_{xx} + Q(x,t)$. The general solution to above PDE was solved in 4.1.6.4 on page 665 and the solution is

$$v(x,t)=\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau \tag{2}$$

Where

$$\Phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right) \tag{3}$$
$$\lambda_n=\left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots$$

Replacing $L=\pi, f(x)=\frac{x}{\pi}-1, Q(x,t)=e^{-2t}\sin(5x)$ into (3,2) gives

$$\Phi_n(x)=\sin(nx) \tag{3A}$$
$$\lambda_n=n^2 \qquad n=1,2,3,\cdots$$

And

$$v(x,t)=\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\frac{2}{\pi}\int_0^\pi\left(\frac{s}{\pi}-1\right)\sin(ns)\,ds\right)+\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\int_0^t\frac{2}{\pi}e^{kn^2\tau}e^{-2\tau}\left(\int_0^\pi \sin(5s \tag{2A}$$

But $\int_0^\pi\left(\frac{s}{\pi}-1\right)\sin(ns)\,ds=\frac{-1}{n}$ since n is integer. And $\int_0^\pi \sin 5s\sin(ns)\,ds=0$ when $n\neq 5$ and for $n=5$ it becomes $\frac{\pi}{2}$. Using these values in the above gives

$$v(x,t)=\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\frac{-2}{\pi n}\right)+e^{-k(5)^2t}\sin(5x)\left(\int_0^t\frac{2}{\pi}e^{k(5)^2\tau}e^{-2\tau}\left(\frac{\pi}{2}\right)d\tau\right) \tag{2C}$$
$$=-\frac{2}{\pi}\sum_{n=1}^{\infty}e^{-kn^2t}\frac{\sin(nx)}{n}+e^{-25kt}\sin(5x)\left(\int_0^t e^{25k\tau}e^{-2\tau}d\tau\right) \tag{4.1}$$

But $\int_0^t e^{25k\tau}e^{-2\tau}d\tau=\frac{-1+e^{25kt-2t}}{25k-2}$ and the above becomes

$$v(x,t)=-\frac{2}{\pi}\sum_{n=1}^{\infty}e^{-kn^2t}\frac{\sin(nx)}{n}+e^{-25kt}\sin(5x)\left(\frac{-1+e^{25kt-2t}}{25k-2}\right)$$
$$=-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-kn^2t}\sin(nx)+\sin(5x)\left(\frac{-e^{-25kt}+e^{-2t}}{25k-2}\right)$$

Since $u(x,t) = v(x,t) + r(x)$ then the final solution is

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-kn^2 t}\sin(nx)\right) + \sin(5x)\left(\frac{-e^{-25kt} + e^{-2t}}{25k - 2}\right)$$

Animation is below using $k = 1$, the solution becomes

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-n^2 t}\sin(nx)\right) + \sin(5x)\left(\frac{e^{-2t} - e^{-25t}}{23}\right)$$

Out[ ]=

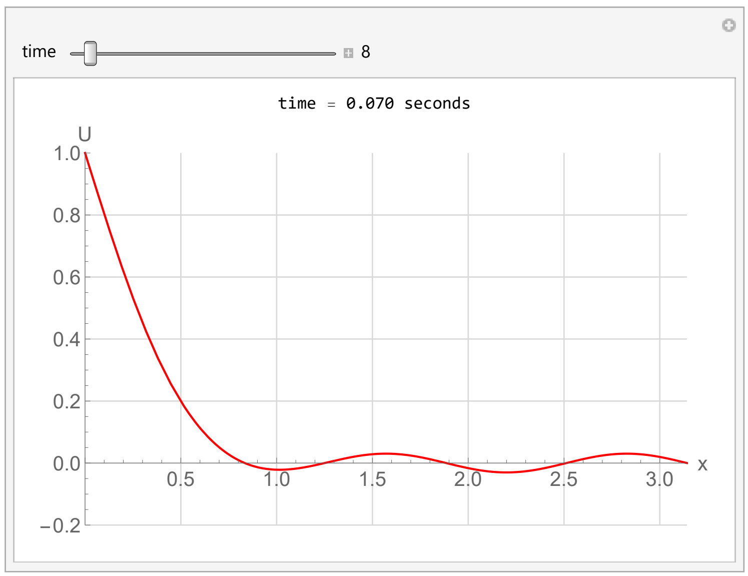


Figure 4.93: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = π;
k = 1;
f[x_] := 0;
Q[x_, t_] := Exp[-2 t] Sin[5 x];
ϕ[x_, n_] := Sin[n x];
λ = n^2;
numberOfTerms = 35;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = N[(1 - x/π) - 2/π Sum[1/n Exp[-k λ t] ϕ[x, n], {n, 1, numberOfTerms}]] + Sin[5 x] ((Exp[-2 t] - Exp[-25 k t])/(25 k - 2));
```

Figure 4.94: Source code

```
In[]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, L}, {-.2, 1}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 2, .01}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.95: Code for animation

4.1.3.5 [212] BC depends on time (special case)

problem number 212

added March 8, 2018. Exam problem

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = t$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

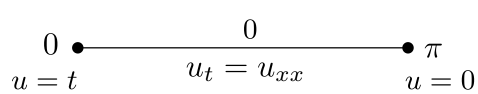


Figure 4.96: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == t, u[Pi, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{\left(2-2e^{-n^2t}\right)\sin(nx)}{n^3\pi}-\frac{tx}{\pi}+t\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=diff(u(x,t),x$2);
bc  := u(0,t)=t,u(Pi,t)=0;
ic  := u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))),outp
```

$$u(x,t)=\frac{12\pi\left(\sum_{n=1}^{\infty}\frac{e^{-n^2t}\sin(nx)}{\pi\, n^3}\right)-6\left(\frac{1}{6}x^2+t-\frac{1}{3}\pi x\right)(x-\pi)}{6\pi}$$

4.1.4 Finite domain (bar), Periodic BC

Local contents

4.1.4.1 [213] Periodic BC (general case). IC given

problem number 213

Solve the heat equation

$$u_t = k u_{xx}$$

For $-L < x < L$ and $t > 0$. The boundary conditions are periodic

$$\begin{aligned} u(-L,t) &= u(L,t) \\ \frac{\partial u}{\partial x}(-L,t) &= \frac{\partial u}{\partial x}(L,t) \end{aligned}$$

And initial conditions $u(x,0) = f(x)$

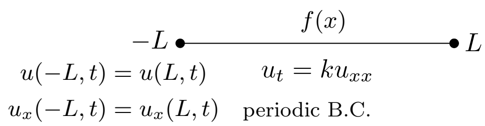


Figure 4.97: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[-L, t] == u[L, t], Derivative[1, 0][u][-L, t] == Derivative[1, 0][u][L, t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\left(\cos\left(\frac{\pi x K[1]}{L}\right)\int_{-L}^{L}\frac{\cos\left(\frac{\pi x K[1]}{L}\right)f(x)}{\sqrt{L}}\,dx+\left(\int_{-L}^{L}\frac{f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{\pi x K[1]}{L}\right)\right)}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(-L,t)=u(L,t),eval(diff(u(r,t),r),r=-L)=eval(diff(u(r,t),r),r=L);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\frac{2L\left(\sum_{n=1}^{\infty}\frac{\left(\left(\int_{-L}^{L}\cos\left(\frac{\pi n x}{L}\right)f(x)dx\right)\cos\left(\frac{\pi n x}{L}\right)+\left(\int_{-L}^{L}f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\sin\left(\frac{\pi n x}{L}\right)\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}}{L}\right)+\int_{-L}^{L}f(x)\,dx}{2L}$$

4.1.4.2 [214] Periodic BC (general case). No IC given

problem number 214

Added Sept 21, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $-\pi < x < \pi$ and $t > 0$. The boundary conditions are periodic

$$u(-\pi,t) = u(\pi,t)$$
$$\frac{\partial u}{\partial x}(-\pi,t) = \frac{\partial u}{\partial x}(\pi,t)$$

No initial conditions give.

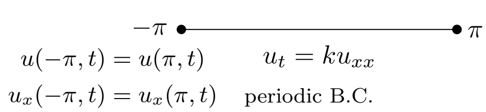


Figure 4.98: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[-Pi, t] == u[Pi, t], Derivative[1, 0][u][-Pi, t] == Derivative[1, 0][u][Pi, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t)\to c_1\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(-Pi,t)=u(Pi,t),D[1](u)(-Pi,0)=D[1](u)(Pi,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) ),output=
```

$$u(x,t)=c_3\left(c_1\mathrm{e}^{ix}+c_2\mathrm{e}^{-ix}\right)\mathrm{e}^{-kt}$$
$$u(x,t)=c_3(c_1+c_2)$$

Hand solution

Solve the heat equation $u_t = ku_{xx}$ with periodic boundary conditions $u(t,-\pi) = u(t,\pi)\,,u_x(t,-\pi) = u_x(t,\pi)$

Solution

Using separation of variables, Let $u(x,t)=T(t)\,X(x)$. Substituting this into $u_t=ku_{xx}$ gives $T'X=TX''$. Dividing by $XT\neq 0$ gives

$$\frac{1}{k}\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

Where λ is the seperation constant. This gives the following ODE's to solve

$$X''(x)+\lambda X(x)=0$$
$$T'(t)+\lambda kT(t)=0$$

Where λ is the separation constant. Eigenfunctions are solutions to the spatial ODE.

$$X(x)=c_1e^{\sqrt{-\lambda}x}+c_2e^{-\sqrt{-\lambda}x} \tag{1}$$

To determine the actual eigenfunctions and eigenvalues, boundary conditions are used. Starting with the spatial ODE above, and transferring the boundary condition to X, it becomes

$$\begin{aligned} X''(x)+\lambda X(x)&=0\\ X(-\pi)&=X(\pi)\\ X'(-\pi)&=X'(\pi)\end{aligned}$$

This is an eigenvalue boundary value problem. The solution is

$$X(x)=c_1e^{\sqrt{-\lambda}x}+c_2e^{-\sqrt{-\lambda}x} \tag{1}$$

case $\lambda<0$

Since $\lambda<0$, then $-\lambda$ is positive. Let $\mu=-\lambda$, where μ is now positive. The solution (1) becomes

$$X(x)=c_1e^{\sqrt{\mu}x}+c_2e^{-\sqrt{\mu}x}$$

The above can be written as

$$X(x)=c_1\cosh\left(\sqrt{\mu}x\right)+c_2\sinh\left(\sqrt{\mu}x\right) \tag{2}$$

Applying first B.C. $X(-\pi)=X(\pi)$ using (2) gives

$$\begin{aligned} c_1\cosh\left(\sqrt{\mu}\pi\right)+c_2\sinh\left(-\sqrt{\mu}\pi\right)&=c_1\cosh\left(\sqrt{\mu}\pi\right)+c_2\sinh\left(\sqrt{\mu}\pi\right)\\ c_2\sinh\left(-\sqrt{\mu}\pi\right)&=c_2\sinh\left(\sqrt{\mu}\pi\right)\end{aligned}$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_2=0$. The solution (2) now reduces to

$$X(x)=c_1\cosh\left(\sqrt{\mu}x\right) \tag{3}$$

Taking derivative gives

$$X'(x)=c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right) \tag{4}$$

Applying the second BC $X'(-\pi)=X'(\pi)$ using (4) gives

$$c_1\sqrt{\mu}\sinh\left(-\sqrt{\mu}\pi\right)=c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right)$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_1=0$. This means a trivial solution. Therefore $\lambda<0$ is not an eigenvalue.

case $\lambda=0$

In this case the solution is $X(x) = c_1 + c_2 x$. Applying first BC $X(-\pi) = X(\pi)$ gives

$$\begin{aligned} c_1 - c_2\pi &= c_1 + c_2\pi \\ -c_2\pi &= c_2\pi \end{aligned}$$

This gives $c_2 = 0$. The solution now becomes $X(x) = c_1$ and $X'(x) = 0$. Applying the second boundary conditions $X'(-\pi) = X'(\pi)$ is not satisfies $(0 = 0)$. Therefore $\underline{\lambda = 0}$ is an eigenvalue with eigenfunction $X_0(0) = 1$ (selected $c_1 = 1$ since an arbitrary constant).

case $\lambda > 0$

The solution in this case is

$$\begin{aligned} X(x) &= c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \\ &= c_1 e^{i\sqrt{\lambda}x} + c_2 e^{-i\sqrt{\lambda}x} \end{aligned}$$

Which can be rewritten as (the constants c_1, c_2 below will be different than the above c_1, c_2, but kept the same name for simplicity).

$$X(x) = c_1 \cos\left(\sqrt{\lambda}x\right) + c_2 \sin\left(\sqrt{\lambda}x\right) \tag{5}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using the above gives

$$\begin{aligned} c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(\sqrt{\lambda}\pi\right) \\ c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_2 \sin\left(\sqrt{\lambda}\pi\right) \end{aligned}$$

There are two choices here. If $\sin\left(-\sqrt{\lambda}\pi\right) \neq \sin\left(\sqrt{\lambda}\pi\right)$, then this implies that $c_2 = 0$. If $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$ then $c_2 \neq 0$. Assuming for now that $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$. This happens when $\sqrt{\lambda}\pi = n\pi, n = 1, 2, 3, \cdots$, or

$$\lambda_n = n^2 \qquad n = 1, 2, 3, \cdots$$

Using this choice, we will now look to see what happens using the second BC. The solution (5) now becomes

$$X(x) = c_1 \cos(nx) + c_2 \sin(nx) \qquad n = 1, 2, 3, \cdots$$

Therefore

$$X'(x) = -c_1 n \sin(nx) + c_2 n \cos(nx)$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using the above gives

$$\begin{aligned} c_1 n \sin(n\pi) + c_2 n \cos(n\pi) &= -c_1 n \sin(n\pi) + c_2 n \cos(n\pi) \\ c_1 n \sin(n\pi) &= -c_1 n \sin(n\pi) \\ 0 &= 0 \end{aligned}$$

Since n is integer. Therefore this means that using $\lambda_n = n^2$ will satisfy both boundary conditions with $c_2 \neq 0, c_1 \neq 0$. This means the solution (5) becomes

$$X_n(x) = A_n \cos(nx) + B_n \sin(nx) \qquad n = 1, 2, 3, \cdots$$

The above says that there are two eigenfunctions in this case. They are

$$X_n(x) = \begin{cases} \cos(nx) \\ \sin(nx) \end{cases}$$

Since there is also zero eigenvalue, then the complete set of eigenfunctions become

$$X_n(x) = \begin{cases} 1 \\ \cos(nx) \\ \sin(nx) \end{cases}$$

Now that the eigenvalues are found, the solution to the time ODE can be found. Recalling that the time ODE from above was found to be

$$T'(t) + \lambda k T(t) = 0$$

For the zero eigenvalue case, the above reduces to $T'(t) = 0$ which has the solution $T_0(t) = C_0$. For non zero eigenvalues $\lambda_n = n^2$, the ODE becomes $T'(t) + n^2 T(t) = 0$, whose solution is $T_0(t) = C_n e^{-kn^2 t}$.

Putting all the above together, gives the fundamental solution as

$$u_n(x,t) = \begin{cases} C_0 & \\ C_n \cos(nx) e^{-kn^2 t} & n = 1, 2, 3, \cdots \\ B_n \sin(nx) e^{-kn^2 t} & n = 1, 2, 3, \cdots \end{cases}$$

Therefore the complete solution is the sum of the above solutions

$$u(x,t) = C_0 + \sum_{n=1}^{\infty} e^{-kn^2 t} (C_n \cos(nx) + B_n \sin(nx))$$

The constants C_0, C_n, B_n can be found from initial conditions.

4.1.4.3 [215] Periodic BC (general case). Damped heat PDE. No IC given

problem number 215

Added Sept 21, 2019

Solve the heat equation

$$u_t = k u_{xx} - u(x,t)$$

For $-\pi < x < \pi$ and $t > 0$. The boundary conditions are periodic

$$u(-\pi,t) = u(\pi,t)$$
$$\frac{\partial u}{\partial x}(-\pi,t) = \frac{\partial u}{\partial x}(\pi,t)$$

No initial conditions give.

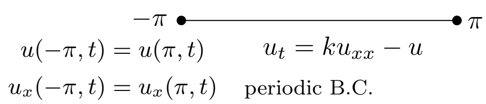


Figure 4.99: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] -u[x,t];
bc  = {u[-Pi, t] == u[Pi, t], Derivative[1, 0][u][-Pi, t] == Derivative[1, 0][u][Pi, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-u(x,t);
bc  := u(-Pi,t)=u(Pi,t),D[1](u)(-Pi,0)=D[1](u)(Pi,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) ),output=
```

$$u(x,t) = c_3\left(c_1 e^{ix} + c_2 e^{-ix}\right) e^{-(k+1)t}$$
$$u(x,t) = c_3(c_1 + c_2) e^{-t}$$

Hand solution

Solve the heat equation $u_t = ku_{xx} - u$ with periodic boundary conditions $u(t,-\pi) = u(t,\pi)\,, u_x(t,-\pi) = u_x(t,\pi)$

Solution

Using separation of variables, Let $u(x,t) = T(t)\,X(x)$. Substituting this into $u_t = ku_{xx}$ gives $T'X = TX''$. Dividing by $XT \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} + XT = \frac{X''}{X} = -\lambda$$

Where λ is the seperation constant. This gives the following ODE's to solve

$$\begin{aligned} X''(x) + \lambda X(x) &= 0 \\ T'(t) + k(1+\lambda)\,T(t) &= 0 \end{aligned}$$

Where λ is the separation constant. Eigenfunctions are solutions to the spatial ODE.

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

To determine the actual eigenfunctions and eigenvalues, boundary conditions are used. Transferring the boundary condition to X, it becomes

$$\begin{aligned} X''(x) + \lambda X(x) &= 0 \\ X(-\pi) &= X(\pi) \\ X'(-\pi) &= X'(\pi) \end{aligned}$$

This is an eigenvalue boundary value problem. The solution is

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

case $\lambda < 0$

Since $\lambda < 0$, then $-\lambda$ is positive. Let $\mu = -\lambda$, where μ is now positive. The solution (1) becomes

$$X(x) = c_1 e^{\sqrt{\mu}x} + c_2 e^{-\sqrt{\mu}x}$$

The above can be written as

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) + c_2 \sinh\left(\sqrt{\mu}x\right) \tag{2}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using (2) gives

$$\begin{aligned} c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(\sqrt{\mu}\pi\right) \\ c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_2 \sinh\left(\sqrt{\mu}\pi\right) \end{aligned}$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_2 = 0$. The solution (2) now reduces to

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) \tag{3}$$

Taking derivative gives

$$X'(x) = c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right) \tag{4}$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using (4) gives

$$c_1\sqrt{\mu}\sinh\left(-\sqrt{\mu}\pi\right) = c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right)$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_1 = 0$. This means a trivial solution. Therefore $\lambda < 0$ is not an eigenvalue.

<u>case $\lambda = 0$</u>

In this case the solution is $X(x) = c_1 + c_2 x$. Applying first BC $X(-\pi) = X(\pi)$ gives

$$\begin{aligned} c_1 - c_2\pi &= c_1 + c_2\pi \\ -c_2\pi &= c_2\pi \end{aligned}$$

This gives $c_2 = 0$. The solution now becomes $X(x) = c_1$ and $X'(x) = 0$. Applying the second boundary conditions $X'(-\pi) = X'(\pi)$ is not satisfies $(0 = 0)$. Therefore <u>$\lambda = 0$</u> is an eigenvalue with eigenfunction $X_0(0) = 1$ (selected $c_1 = 1$ since an arbitrary constant).

<u>case $\lambda > 0$</u>

The solution in this case is

$$\begin{aligned} X(x) &= c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \\ &= c_1 e^{i\sqrt{\lambda}x} + c_2 e^{-i\sqrt{\lambda}x} \end{aligned}$$

Which can be rewritten as (the constants c_1, c_2 below will be different than the above c_1, c_2, but kept the same name for simplicity).

$$X(x) = c_1\cos\left(\sqrt{\lambda}x\right) + c_2\sin\left(\sqrt{\lambda}x\right) \tag{5}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using the above gives

$$\begin{aligned} c_1\cos\left(\sqrt{\lambda}\pi\right) + c_2\sin\left(-\sqrt{\lambda}\pi\right) &= c_1\cos\left(\sqrt{\lambda}\pi\right) + c_2\sin\left(\sqrt{\lambda}\pi\right) \\ c_2\sin\left(-\sqrt{\lambda}\pi\right) &= c_2\sin\left(\sqrt{\lambda}\pi\right) \end{aligned}$$

There are two choices here. If $\sin\left(-\sqrt{\lambda}\pi\right) \neq \sin\left(\sqrt{\lambda}\pi\right)$, then this implies that $c_2 = 0$. If $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$ then $c_2 \neq 0$. Assuming for now that $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$. Then happens when $\sqrt{\lambda}\pi = n\pi, n = 1, 2, 3, \cdots$, or

$$\lambda_n = n^2 \qquad n = 1, 2, 3, \cdots$$

Using this choice, we will now look to see what happens using the second BC. The solution (5) now becomes

$$X(x) = c_1 \cos(nx) + c_2 \sin(nx) \qquad n = 1, 2, 3, \cdots$$

Therefore

$$X'(x) = -c_1 n \sin(nx) + c_2 n \cos(nx)$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using the above gives

$$\begin{aligned} c_1 n \sin(n\pi) + c_2 n \cos(n\pi) &= -c_1 n \sin(n\pi) + c_2 n \cos(n\pi) \\ c_1 n \sin(n\pi) &= -c_1 n \sin(n\pi) \\ 0 &= 0 \end{aligned}$$

Since n is integer. Therefore this means that using $\lambda_n = n^2$ will satisfy both boundary conditions with $c_2 \neq 0, c_1 \neq 0$. This means the solution (5) becomes

$$X_n(x) = A_n \cos(nx) + B_n \sin(nx) \qquad n = 1, 2, 3, \cdots$$

The above says that there are two eigenfunctions in this case. They are

$$X_n(x) = \begin{cases} \cos(nx) \\ \sin(nx) \end{cases}$$

Since there is also zero eigenvalue, then the complete set of eigenfunctions become

$$X_n(x) = \begin{cases} 1 \\ \cos(nx) \\ \sin(nx) \end{cases}$$

Now that the eigenvalues are found, the solution to the time ODE can be found. Recalling that the time ODE from above was found to be

$$T'(t) + k(\lambda + 1) T(t) = 0$$

For the zero eigenvalue case, the above reduces to $T'(t) + kT(t) = 0$ which has the solution $T_0(t) = C_0 e^{-kt}$. For non zero eigenvalues $\lambda_n = n^2$, the ODE becomes $T'(t) + k(n^2 + 1) T(t) = 0$, whose solution is $T_0(t) = C_n e^{-k(n^2+1)t}$.

Putting all the above together, gives the fundamental solution as

$$u_n(t,x) = \begin{cases} C_0 e^{-kt} & \\ C_n \cos(nx)\, e^{-k(n^2+1)t} & n = 1,2,3,\cdots \\ B_n \sin(nx)\, e^{-k(n^2+1)t} & n = 1,2,3,\cdots \end{cases}$$

Therefore the complete solution is the sum of the above solutions

$$u(t,x) = C_0 e^{-kt} + \sum_{n=1}^{\infty} e^{-k(n^2+1)t}(C_n \cos(nx) + B_n \sin(nx))$$

The constants C_0, C_n, B_n can be found from initial conditions.

4.1.5 Finite domain (bar), Both ends nonhomogeneous BC

Local contents

4.1.5.1 [216] both ends non-homogeneous BC (general case)

problem number 216

Added June 22, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

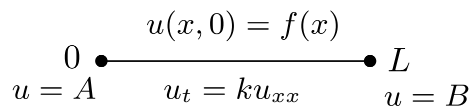


Figure 4.100: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to \sum_{n=1}^{\infty}\frac{2e^{-\frac{kn^2\pi^2t}{L^2}}\left(\int_0^L\left(-A+\frac{(A-B)x}{L}+f(x)\right)\sin\left(\frac{n\pi x}{L}\right)dx\right)\sin\left(\frac{n\pi x}{L}\right)}{L}+\frac{x(B-A)}{L}+A\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{AL - 2L\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L (Bx - Lf(x) + (L-x)A)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}\sin\left(\frac{\pi n x}{L}\right)}{L^2}\right) - (A-B)x}{L}$$

Hand solution

Solve

$$u_t = k u_{xx} \qquad t > 0, 0 < x < L \tag{1}$$

BC are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial conditions

$$u(x,0) = f(x)$$

Solution

Since boundary conditions are nonhomogeneous, then the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables (separation of variables can only be done on a PDE with homogeneous B.C.)

This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies homogeneous version of the B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$0 = kr''(x)$$
$$r(0) = A$$
$$r(L) = B$$

This has the solution $r(x) = c_1 x + c_2$. Using BC at $x=0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1 x + A$. Using BC at $x = L$ gives $B = c_1 L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right) x + A$$

Therefore (1) becomes

$$u(x,t) = v(x,t) + \left(\frac{B-A}{L}\right) x + A$$

Substituting the above back in the original PDE $u_t = k u_{xx}$ gives

$$\begin{aligned} v_t &= k v_{xx} \\ v(0) &= 0 \\ v(L) &= 0 \\ v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\frac{B-A}{L} x + A\right) \end{aligned} \tag{3}$$

The above PDE was solved in problem 4.1.1.1 on page 402 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L F(s) \sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots \end{aligned} \tag{4}$$

Substituting (3) into (4) gives

$$v(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right)$$

From (2)

$$\begin{aligned} u(x,t) &= v(x,t) + r(x) \\ &= A + \left(\frac{B-A}{L}\right) x + \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \\ &= A + \left(\frac{B-A}{L}\right) x + \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\frac{n\pi}{L} s\right) ds\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

4.1.5.2 [217] non-homogeneous BC (special case)

problem number 217

Taken from Maple PDE help pages

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 20$$
$$u(1,t) = 50$$

Initial condition is $u(x,0) = 0$

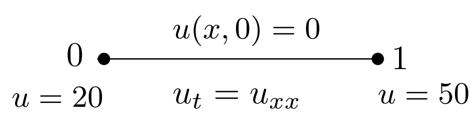


Figure 4.101: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 20, u[1, t] == 50};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to -\frac{2\sum_{n=1}^{\infty}\frac{(20-50(-1)^n)e^{-n^2\pi^2 t}\sin(n\pi x)}{n}}{\pi}+30x+20\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=20, u(1,t)=50;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=30x+20\left(\sum_{n=1}^{\infty}\frac{(5(-1)^n-2)\,\mathrm{e}^{-\pi^2n^2t}\sin(\pi nx)}{\pi n}\right)+20$$

Hand solution

The general solution to

$$u_t=ku_{xx}\qquad t>0,0<x<L \tag{1}$$

BC are

$$u(0,t)=A$$
$$u(L,t)=B$$

Initial conditions

$$u(x,0)=f(x)$$

Is given in problem 4.1.5.1 on page 592 as

$$u(x,t)=A+\left(\frac{B-A}{L}\right)x+\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L\left(f(x)-\left(\frac{B-A}{L}x+A\right)\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$. Substituting $A=20,B=50,L=1,k=1,f(x)=0$ gives the solution as

$$u(x,t)=20+30x+2\sum_{n=1}^{\infty}\left(\int_0^1-(30x+20)\sin(n\pi x)\,dx\right)e^{-kn^2\pi^2t}\sin(n\pi x)$$

But $\int_0^1-(30x+20)\sin(n\pi x)\,dx=\frac{50(-1)^n-20}{n\pi}$ and the above becomes

$$\begin{aligned}u(x,t)&=20+30x+2\sum_{n=1}^{\infty}\left(\frac{50(-1)^n-20}{n\pi}\right)e^{-kn^2\pi^2t}\sin(n\pi x)\\&=20+30x+\sum_{n=1}^{\infty}\left(\frac{100(-1)^n-40}{n\pi}\right)e^{-kn^2\pi^2t}\sin(n\pi x)\end{aligned}$$

4.1.5.3 [218] Articolo 8.4.1 (special case)

problem number 218

Added December 20, 2018.

Example 8.4.1 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 10$$
$$u(1,t) = 20$$

Initial condition is $u(x,0) = 60x - 50x^2 + 10$ and $k = \frac{1}{20}$

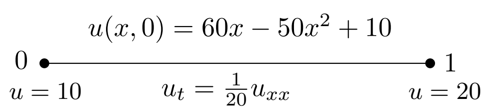


Figure 4.102: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/20;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 10, u[1, t] == 20};
ic  = u[x, 0] == 60*x - 50*x^2 + 10;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} -\frac{200(-1+(-1)^n)\, e^{-\frac{1}{20}n^2\pi^2 t} \sin(n\pi x)}{n^3\pi^3} + 10(x+1)\right\}\right\}$$

Maple ✓

```
restart;
k := 1/20;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = 10, u(1, t) = 20;
ic  := u(x, 0) = 60*x - 50*x^2 + 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t) = 10x - 200\left(\sum_{n=1}^{\infty} \frac{\left((-1)^n - 1\right) e^{-\frac{\pi^2 n^2 t}{20}} \sin(\pi n x)}{\pi^3 n^3}\right) + 10$$

Hand solution

The general solution to

$$u_t = k u_{xx} \qquad t > 0, 0 < x < L \tag{1}$$

BC are

$$u(0,t) = A$$
$$u(L,t) = B$$

with Initial conditions

$$u(x,0) = f(x)$$

Is given in problem 4.1.5.1 on page 592 as

$$u(x,t) = A + \left(\frac{B-A}{L}\right) x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \left(\frac{B-A}{L}x + A\right)\right)\sin\left(\sqrt{\lambda_n}x\right) dx\right) e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. Substituting $A = 10, B = 20, L = 1, k = \frac{1}{20}, f(x) = 60x - 50x^2 + 10$ gives the solution as

$$u(x,t) = 10 + 10x + 2\sum_{n=1}^{\infty}\left(\int_0^1 \left(60x - 50x^2 + 10 - (10x + 10)\right)\sin(n\pi x x)\, dx\right) e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x)$$
$$= 10 + 10x + 2\sum_{n=1}^{\infty}\left(\int_0^1 \left(50x - 50x^2\right)\sin(n\pi x x)\, dx\right) e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x)$$

But $\int_0^1 (50x - 50x^2)\sin(n\pi x x)\, dx = \frac{100-100(-1)^n}{n^3\pi^3}$ and the above becomes

$$u(x,t) = 10 + 10x + 2\sum_{n=1}^{\infty} 100\left(\frac{1-(-1)^n}{n^3\pi^3}\right) e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x)$$
$$= 10 + 10x - \sum_{n=1}^{\infty} 200\left(\frac{(-1)^n - 1}{n^3\pi^3}\right) e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x)$$

4.1.5.4 [219] With source that depends on space only (general case)

problem number 219

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(1,t) = B$$

Initial condition is $u(x,0) = f(x)$

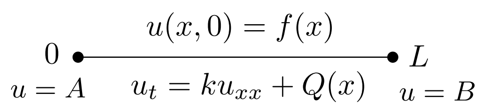


Figure 4.103: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(\frac{\left(1-e^{-\frac{k\pi^2tK[1]^2}{L^2}}\right)\left(\int_0^L\frac{\sqrt{2}Q(x)\sin\left(\frac{\pi xK[1]}{L}\right)}{\sqrt{L}}dx\right)L^2}{k\pi^2K[1]^2}+e^{-\frac{k\pi^2tK[1]^2}{L^2}}\int_0^L\frac{\sqrt{2}(-Bx+A(x-L)+Lf(x))\sin\left(\cdots\right.}{L^{3/2}}\cdots\right.}{\sqrt{L}}\cdots\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q(x);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \frac{-2Lk\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L -\left(L\left(\int_0^{\_a}\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)-\_a\left(\int_0^L\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)+(-AL+Lf(\_a)+(A-B)\_a)k\right)\sin\left(\frac{\pi\_}{L}\right.}{L^2 k}\right.}{}$$

<u>Hand solution</u>

Solving

$$u_t = k u_{xx} + Q(x) \tag{1}$$

With initial conditions $u(x,0) = f(x)$ and boundary conditions $u(0,t) = A, u(L,t) = B$ with $0 < x < L, t > 0$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$\begin{aligned} 0 &= kr''(x) \\ r(0) &= A \\ r(L) &= B \end{aligned}$$

This has the solution $r(x) = c_1 x + c_2$. Using BC at $x = 0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1 x + A$. Using BC at $x = L$ gives $B = c_1 L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right) x + A \tag{3}$$

Substituting (2) back in the original PDE $u_t = ku_{xx} + Q(x)$ gives

$$\begin{aligned} v_t &= kv_{xx} + Q(x) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \end{aligned} \tag{4}$$

The initial conditions are

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\left(\frac{B-A}{L} \right) x + A \right) \end{aligned}$$

The general solution to (4) was solved in 4.1.1.11 on page 431 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t} \Phi_n(x) \left(\frac{2}{L} \int_0^L F(s)\, \Phi_n(s)\, ds \right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t} \Phi_n(x) \left(\int_0^t \frac{2}{L} e^{k\lambda_n \tau} \left(\int_0^L Q(s)\, \Phi_n(s)\, ds \right) d\tau \right) \\ \Phi_n(x) &= \sin\left(\sqrt{\lambda_n} x \right) \\ \lambda_n &= \left(\frac{n\pi}{L} \right)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

But $F(x) = f(x) - \left(\left(\frac{B-A}{L} \right) x + A \right)$ in this case, hence the solution becomes

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x \right) \left(\frac{2}{L} \int_0^L \left(f(s) - \left(\left(\frac{B-A}{L} \right) s + A \right) \right) \sin\left(\frac{n\pi}{L} s \right) ds \right) \\ &\quad + \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x \right) \left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau} \left(\int_0^L Q(s) \sin\left(\frac{n\pi}{L} s \right) ds \right) d\tau \right) \end{aligned}$$

Since $u(x,t) = v(x,t) + r(x)$, then the final solution is

$$\begin{aligned} u(x,t) &= \left(\frac{B-A}{L} \right) x + A + \\ &\quad \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x \right) \left(\frac{2}{L} \int_0^L \left(f(s) - \left(\left(\frac{B-A}{L} \right) s + A \right) \right) \sin\left(\frac{n\pi}{L} s \right) ds \right) \\ &\quad + \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x \right) \left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau} \left(\int_0^L Q(s) \sin\left(\frac{n\pi}{L} s \right) ds \right) d\tau \right) \end{aligned}$$

4.1.5.5 [220] With source that depends on space only (special case)

problem number 220

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(1,t) = B$$

Initial condition is $u(x,0) = f(x)$, Using the following values

$$A = 20$$
$$B = 50$$
$$f(x) = 60 - 2x$$
$$L = 30$$
$$k = \frac{1}{10}$$
$$Q(x) = \frac{x}{10}$$

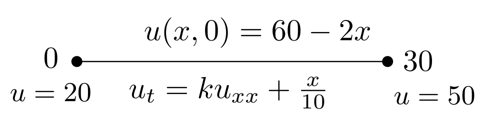


Figure 4.104: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
A=20; B=50; f=60-2*x; L=30; k=1/10; Q=x/10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}-\frac{20e^{-\frac{\pi^2tK[1]^2}{9000}}\left(-\left(\left(4+5(-1)^{K[1]}\right)\pi^2K[1]^2\right)-2700(-1)^{K[1]}+2700(-1)^{K[1]}e^{\frac{\pi^2tK[1]^2}{9000}}\right)}{\pi^3K[1]^3}\right.\right.$$

Maple ✓

```
restart;
A:=20;
B:=50;
f:=60-2*x;
L:=30;
k:=1/10;
Q:=x/10;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q;
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t)=-\frac{x^3}{6}+151x+20\left(\sum_{n=1}^{\infty}\frac{\left(5\pi^2n^2(-1)^n+4\pi^2n^2+2700(-1)^n\right)\mathrm{e}^{-\frac{\pi^2n^2t}{9000}}\sin\left(\frac{\pi nx}{30}\right)}{\pi^3n^3}\right)+20$$

Hand solution

Solving

$$\begin{aligned}u_t&=kv_{xx}+Q(x)\qquad(1)\\u(0,t)&=A\\u(L,t)&=B\\u(x,0)&=f(x)\end{aligned}$$

Where $A = 20, B = 50, f(x) = 60 - 2x, Q(x) = \frac{x}{10}, k = \frac{1}{10}, L = 30$.

The general solution to above PDE was solved in 4.1.5.4 on page 599 and the solution is

$$
\begin{aligned}
u(x,t) = &\left(\frac{B-A}{L}\right)x + A + \\
&\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right)s + A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{L}e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L Q(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Replacing the specific values, the solution becomes

$$
\begin{aligned}
u(x,t) = x + 20 + &\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{30}\int_0^{30} (60 - 2s - (s+20))\sin\left(\frac{n\pi}{30}s\right)ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{30}e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^{30} \frac{s}{10}\sin\left(\frac{n\pi}{30}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) = x + 20 + &\frac{1}{15}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^{30} (40 - 3s)\sin\left(\frac{n\pi}{30}s\right)ds\right) \\
&+ \frac{1}{150}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^{30} s\sin\left(\frac{n\pi}{30}s\right)ds\right)d\tau\right)
\end{aligned}
$$

But $\int_0^{30}(40-3s)\sin\left(\frac{n\pi}{30}s\right)ds = \frac{1500(-1)^n + 1200}{n\pi}$ and $\int_0^{30} s\sin\left(\frac{n\pi}{30}s\right)ds = \frac{-900(-1)^n}{n\pi}$, hence the above becomes

$$
\begin{aligned}
u(x,t) = x + 20 + &\frac{1}{15}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\frac{1500(-1)^n + 1200}{n\pi}\right) \\
&+ \frac{1}{150}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\frac{-900(-1)^n}{n\pi}\right)d\tau\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) = x + 20 + &\frac{300}{15}\sum_{n=1}^{\infty}\left(\frac{5(-1)^n + 4}{n\pi}\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \\
&- \frac{900}{150}\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau} d\tau\right)
\end{aligned}
$$

But $\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau} d\tau = \left[\frac{e^{k\left(\frac{n\pi}{L}\right)^2\tau}}{k\left(\frac{n\pi}{L}\right)^2}\right]_0^t = \frac{e^{k\left(\frac{n\pi}{L}\right)^2 t}-1}{k\left(\frac{n\pi}{L}\right)^2}$, therefore the above becomes

$$\begin{aligned} u(x,t) &= x + 20 + \frac{300}{15}\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right) \\ &\quad - \frac{900}{150}\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{e^{k\left(\frac{n\pi}{L}\right)^2 t}-1}{k\left(\frac{n\pi}{L}\right)^2}\right) \end{aligned}$$

Or

$$\begin{aligned} u(x,t) &= x + 20 + 20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right) \\ &\quad - \frac{900}{150}\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi k\left(\frac{n\pi}{L}\right)^2}\sin\left(\frac{n\pi}{L}x\right)\left(1-e^{-k\left(\frac{n\pi}{L}\right)^2 t}\right) \end{aligned}$$

Finally replacing the remaining variables in the above for L,k gives

$$\begin{aligned} u(x,t) &= x + 20 + 20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\sin\left(\frac{n\pi}{30}x\right) \\ &\quad - \frac{900}{150}\sum_{n=1}^{\infty}\frac{9000(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\right) \end{aligned}$$

Or

$$\begin{aligned} u(x,t) &= x + 20 + 20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\sin\left(\frac{n\pi}{30}x\right) \\ &\quad - 54000\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\right) \end{aligned}$$

Or

$$u(x,t) = (x+20) + 20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{\pi^2 n^2}{9000}t}\sin\left(\frac{n\pi}{30}x\right) - 54000\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{\pi^2 n^2}{9000}t}\right)$$

Animation is below

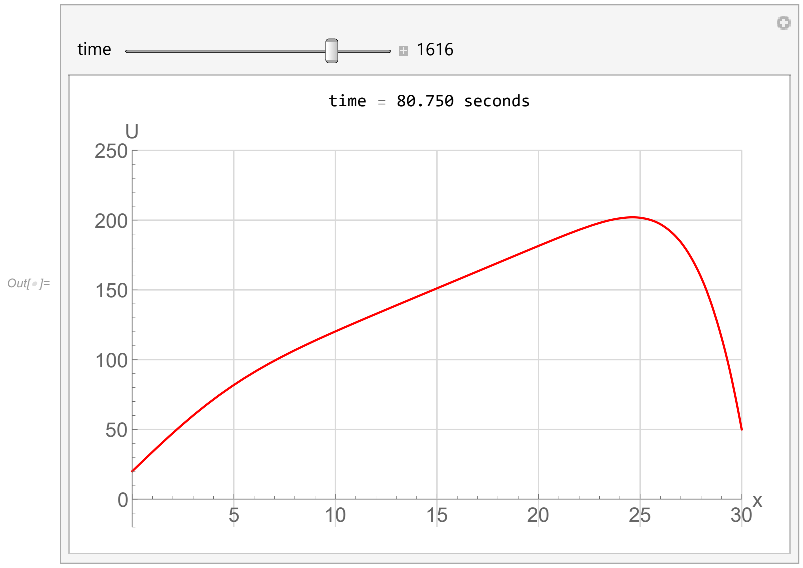


Figure 4.105: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 30;
k = 1 / 10;
f[x_] := 60 - 2 x;
Q[x_] := x / 10;
λ = (n π / L)^2;
ϕ[x_, n_] := Sin[Sqrt[λ] x];
numberOfTerms = 50;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = x + 20 + 20 Chop@N@Sum[(5 (-1)^n + 4)/(n π) Exp[-k λ t] ϕ[x, n] , {n, 1, numberOfTerms}] - 54000 Chop@N@Sum[(-1)^n/(n^3 π^3) (1 - Exp[-k λ t]) ϕ[x, n], {n, 1, numberOfTerms}];
```

Figure 4.106: Source code

```
In[ ]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-20, 250}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 100, .1}];

In[ ]:= Manipulate[tab⟦i⟧, {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.107: Code for animation

4.1.5.6 [221] With source that depends on space and time only (general case)

problem number 221

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= A \\ u(1,t) &= B \end{aligned}$$

Initial condition is $u(x,0) = f(x)$

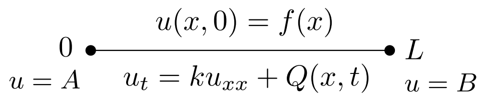


Figure 4.108: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\text{Integrate}\left[\frac{\sqrt{2}Q(x,K[2])\sin\left(\frac{\pi xK[1]}{L}\right)}{\sqrt{L}},\{x,0,L\},\text{Assumptions}\to k>\right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q(x,t);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{AL+L\left(\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi nx}{L}\right)dx\right)e^{-\frac{\pi^2(t-\tau)kn^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L}\right)d\tau\right)-2L\left(\sum_{n=1}^{\infty}\frac{\left(\int_0^L(AL-Lf(\tau)+(-A+B)\tau\right.}{}\right.}{L}$$

Hand solution

Solving

$$u_t = ku_{xx} + Q(x,t) \tag{1}$$

With initial conditions $u(x,0) = f(x)$ and boundary conditions $u(0,t) = A, u(L,t) = B$ with $0 < x < L, t > 0$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$\begin{aligned} 0 &= kr''(x) \\ r(0) &= A \\ r(L) &= B \end{aligned}$$

This has the solution $r(x) = c_1 x + c_2$. Using BC at $x = 0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1 x + A$. Using BC at $x = L$ gives $B = c_1 L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right) x + A \tag{3}$$

Substituting (2) back in the original PDE $u_t = k u_{xx} + Q(x,t)$ gives

$$\begin{aligned} v_t &= k v_{xx} + Q(x,t) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \end{aligned} \tag{4}$$

The initial conditions are

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\left(\frac{B-A}{L}\right) x + A\right) \end{aligned}$$

The general solution to (4) was solved in 4.1.6.4 on page 665 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t} \Phi_n(x) \left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x) \left(\int_0^t \frac{2}{L} e^{k\lambda_n \tau} \left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right) d \right. \\ \Phi_n(x) &= \sin\left(\sqrt{\lambda_n} x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

But $F(x) = f(x) - \left(\left(\frac{B-A}{L}\right) x + A\right)$ in this case, hence the solution becomes

$$
\begin{aligned}
v(x,t) = {} & \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right) s + A\right)\right) \sin\left(\frac{n\pi}{L}s\right) ds\right) \\
& + \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau} \left(\int_0^L Q(s,\tau) \sin\left(\frac{n\pi}{L}s\right) ds\right) d\tau\right)
\end{aligned}
$$

Since $u(x,t) = v(x,t) + r(x)$, then the final solution is

$$
\begin{aligned}
u(x,t) = {} & \left(\frac{B-A}{L}\right) x + A + \\
& \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right) s + A\right)\right) \sin\left(\frac{n\pi}{L}s\right) ds\right) \\
& + \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau} \left(\int_0^L Q(s,\tau) \sin\left(\frac{n\pi}{L}s\right) ds\right) d\tau\right)
\end{aligned}
$$

4.1.5.7 [222] Both ends depend on time (general case)

problem number 222

Added June 23, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= A(t) \\
u(L,t) &= B(t)
\end{aligned}
$$

Initial condition is $u(x,0) = f(x)$

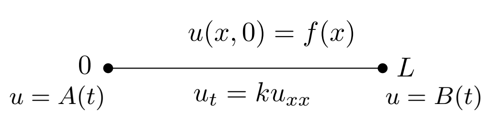


Figure 4.109: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\int_0^L \frac{\sqrt{2}(-LA(0)+xA(0)-xB(0)+Lf(x))\sin\left(\frac{\pi xK[1]}{L}\right)}{L^{3/2}}\,dx+\int_0^t \frac{\sqrt{2}e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\sqrt{L}((\cdot}{\pi K}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = A(t), u(L, t) = B(t);
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{L\left(\int_0^t\left(\sum_{n=1}^{\infty}\left(-\frac{2\left(-(-1)^n\left(\frac{d}{d\tau}B(\tau)\right)+\frac{d}{d\tau}A(\tau)\right)e^{-\frac{\pi^2(t-\tau)k\,n^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{\pi n}\right)\right)d\tau\right)+L\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L\frac{(Lf(x)-B(0)x+A(0)\cdots}{L}\right.}{}\right.}{L}$$

Hand solution

$$\begin{aligned} u_t &= ku_{xx} \qquad (1)\\ u(0,t) &= A(t)\\ u(L,t) &= B(t)\\ u(x,0) &= f(x) \end{aligned}$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. We see that

$$\begin{aligned} r(x,t) &= A(t) + \frac{B(t) - A(t)}{L} x \\ &= A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L} x \end{aligned} \tag{3}$$

Satisfies the nonhomogeneous. Substituting (2) back into the original PDE (1) gives

$$\begin{aligned} \frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) \\ v_t(x,t) + r_t(x,t) &= kv_{xx}(x,t) + kr_{xx}(x,t) \end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t)\left(\frac{L-x}{L}\right) + \frac{B'(t)}{L}x$ and the above PDE becomes

$$v_t(x,t) = kv_{xx}(x,t) - r_t(x,t)$$

Let

$$\begin{aligned} Q(x,t) &= -r_t(x,t) \\ &= -\left(A'(t)\left(\frac{L-x}{L}\right) + \frac{B'(t)}{L}x\right) \end{aligned}$$

Therefore the problem has been transformed to

$$\begin{aligned} v_t &= kv_{xx} + Q(x,t) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \\ v(0,x) &= F(x) \\ &= u(x,0) - r(x,0) \\ &= f(x) - \left(A(0)\left(\frac{L-x}{L}\right) + \frac{B(0)}{L}x\right) \end{aligned}$$

The above problem was solved in 4.1.6.4 on page 665. The solution is

$$v(x,t) = \sum_{n=1}^{\infty}\left[\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right)e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,dx\right)d\tau\right]\Phi_n(x) \tag{4}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots$$

Hence (4) becomes

$$\begin{aligned} v(x,t) &= \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L F(s)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)dx\right)d\tau \end{aligned}$$

Since $u(x,t) = r(x,t) + v(x,t)$ then the final solution becomes

$$\begin{aligned} u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L F(s)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau \end{aligned}$$

Or

$$\begin{aligned} u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0)\left(\frac{L-s}{L}\right) + \frac{B(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right) \end{aligned}$$

Or

$$
\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s)\,L-\left(A(0)\,(L-s)+B(0)\,s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Or, since here $Q(x,t)=-\left(A'(t)\left(\frac{L-x}{L}\right)+\frac{B'(t)}{L}x\right)$ the above becomes

$$
\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s)\,L-\left(A(0)\,(L-s)+B(0)\,s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L \left(\frac{B'(\tau)}{L}s-A'(\tau)\left(\frac{L-s}{L}\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\Bigg(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)-A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)-B(0)\,s\sin\Big(\frac{n\pi}{L}s\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\Bigg(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\Bigg(\int_0^L \frac{B'(\tau)}{L}s\sin\left(\frac{n\pi}{L}s\right)-A'(\tau)\left(\frac{L-s}{L}\right)\sin\left(\frac{n\pi}{L}s\right)ds
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right) \qquad (5)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\Bigg(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)-A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)-B(0)\,s\sin\Big(\frac{n\pi}{L}s\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\Bigg(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\Bigg(\int_0^L B'(\tau)\,s\sin\left(\frac{n\pi}{L}s\right)-A'(\tau)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)ds\Bigg)
\end{aligned}
$$

But

$$\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right) - A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right) - B(0)\,s\sin\left(\frac{n\pi}{L}s\right)ds =$$
$$L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds - B(0)\int_0^L s\sin\left(\frac{n\pi}{L}s\right)ds$$

Since $\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds = \frac{L^2}{n\pi}$ and $\int_0^L s\sin\left(\frac{n\pi}{L}s\right)dx = -\frac{L^2(-1)^n}{n\pi}$, then the above becomes

$$\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right) - A(0)\left(\frac{L-s}{L}\right)\sin\left(\frac{n\pi}{L}s\right) - \frac{B(0)}{L}s\sin\left(\frac{n\pi}{L}s\right)ds =$$
$$L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi} \tag{6}$$

And

$$\int_0^L B'(\tau)\,s\sin\left(\frac{n\pi}{L}s\right) - A'(\tau)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)ds = -A'(\tau)\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds + B'(\tau)\int_0^L s\sin \tag{7}$$
$$= -A'(\tau)\,\frac{L^2}{n\pi} + B'(\tau)\,\frac{L^2(-1)^n}{n\pi}$$

Substituting (6,7) back into (5) gives

$$\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right)\\
&+ \frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi}\right)\\
&- \frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(-B'(\tau)\,\frac{L^2(-1)^n}{n\pi} + A'(\tau)\,\frac{L^2}{n\pi}\right)d\tau\right)
\end{aligned}$$

Or

$$\begin{aligned}
u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right)\\
&+ \frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi}\right)\\
&- 2\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{1}{n\pi}\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(-(-1)^n\,B'(\tau) + A'(\tau)\right)d\tau\right)
\end{aligned}$$

4.1.5.8 [223] Both ends depend on time (special case)

problem number 223

Added June 26, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$ using the following values

$$L = 2$$
$$k = \frac{1}{10}$$
$$f(x) = x$$
$$A(t) = \sin(t)$$
$$B(t) = 2\cos(t)$$

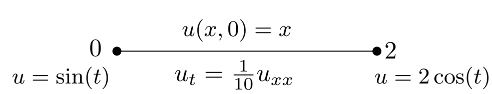


Figure 4.110: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f=x;
L=2;
k=1/10;
A=Sin[t];
B=2*Cos[t];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{80\left(e^{-\frac{1}{40}\pi^2tK[1]^2}\pi^2K[1]^2-80(-1)^{K[1]}e^{-\frac{1}{40}\pi^2tK[1]^2}+\cos(t)\left(80(-1)^{K[1]}-\pi^2K[1]^2\right)-2(\right.}{\pi K[1]\left(\pi^4K[1]^4+1600\right)}\right.\right.$$

Maple ✓

```
restart;
f:=x;
L:=2;
k:=1/10;
A:=sin(t);
B:=2*cos(t);
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) )
```

$$u(x,t)=x\cos(t)-\frac{x\sin(t)}{2}-80\left(\sum_{n=1}^{\infty}\frac{\left(\pi^2n^2\cos(t)+\left(2\pi^2n^2\sin(t)-80\cos(t)\right)(-1)^n+\left(-\pi^2n^2+80(-1\right.\right.}{\pi\left(\pi^4n^4+1600\right)n}\right.$$

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 4.1.5.7 on page 610 as

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right) \\
&+\frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi}+B(0)\,\frac{L^2(-1)^n}{n\pi}\right) \\
&-2\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{1}{n\pi}\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(-(-1)^n B'(\tau)+A'(\tau)\right)d\tau\right)
\end{aligned}
$$

In this problem we have

$$
\begin{aligned}
L &= 2 \\
k &= \frac{1}{10} \\
f(x) &= x \\
A(t) &= \sin(t) \\
B(t) &= 2\cos(t)
\end{aligned}
$$

This is animation of the above solution using these specific values for 20 seconds. (Animation will only show in the HTML version)

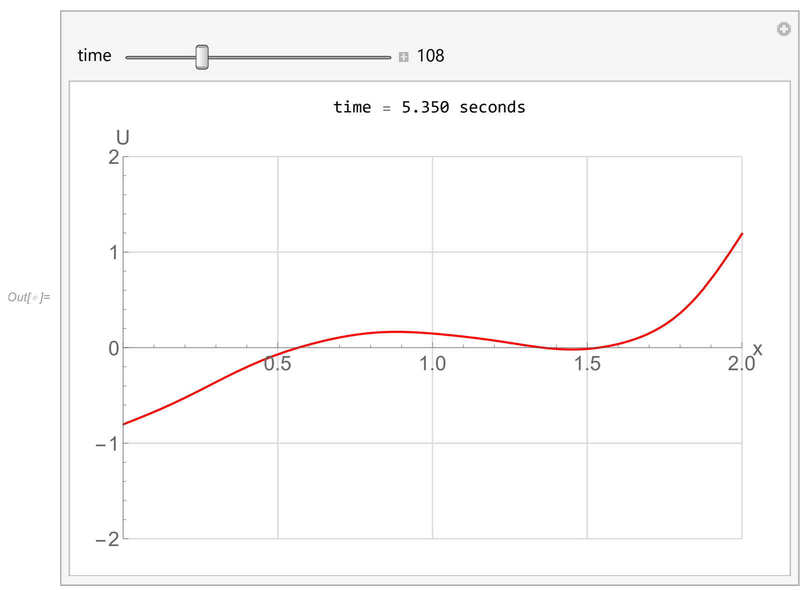


Figure 4.111: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1/10;
f[x_] := x;
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
firstIntegral = Assuming[Element[n, Integers], L Integrate[f[x] Sin[(n Pi)/L x], {x, 0, L}]];
secondIntegral = 1/(n Pi) Integrate[Exp[(k n^2 Pi^2 s)/L^2] (A'[s] - (-1)^n B'[s]), {s, 0, t}];
mySol[x_, t_] =
   N@Simplify[(A[t] ((L - x)/L) + B[t]/L x) + 2.0/L^2 Sum[Exp[-k ((n Pi)/L)^2 t] Sin[(n Pi)/L x] (firstIntegral - (A[0] L^2)/(n Pi) + ((-1)^n B[0] L^2)/(n Pi)), {n, 1, numberOfTerms}] -
      2.0 Sum[Exp[-k ((n Pi)/L)^2 t] Sin[(n Pi)/L x] (secondIntegral), {n, 1, numberOfTerms}]];
```

Figure 4.112: Source code

```
In[ ]:= tab = Table[
         Grid[{
            {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
            {
              Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, 2}, {-2, 2}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

            }}],
         {t, 0, 20, .05}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.113: Code used for animation

4.1.5.9 [224] both ends nonhomogeneous

problem number 224

Added January 18, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = 1$$
$$u_x(1,t) = -1$$

Initial condition is $u(x,0) = \sin(x)$

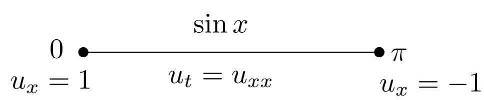


Figure 4.114: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + (x - (1*x^2)/Pi);
ic  = u[x, 0] == Sin[x];
bc  = {Derivative[1, 0][u][0, t] == 1, Derivative[1, 0][u][Pi, t] == -1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{e^{-tK[1]^2}\cos(xK[1])\left(\sqrt{2\pi}K[1]^4\left(\begin{cases}0 & K[1]=1\\ -\frac{(1+(-1)^{K[1]})\sqrt{\frac{2}{\pi}}}{K[1]^2(K[1]^2-1)} & \text{True}\end{cases}\right)-2(1+(-1)^{K}\right.}{\pi K[1]^4}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic  := u(x, 0) = sin(x):
bc  := eval(diff(u(x,t),x),x=0)=1,  eval( diff(u(x,t),x),x=Pi)=-1:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

$$u(x,t) = -\frac{x^2}{\pi} - \frac{2t}{\pi} + x + \sum_{n=2}^{\infty}\left(-\frac{2((-1)^n+1)\cos(nx)\,\mathrm{e}^{-n^2t}}{(n^2-1)\,\pi\,n^2}\right) + \frac{2-\frac{\pi^2}{6}}{\pi}$$

Hand solution

Since the boundary conditions are not homogeneous, we can't use separation of variables. Let the solution be

$$u = v(x,t) + r(x)$$

Where $v(x,t)$ is the solution to $v_t = v_{xx}$ and homogenous B.C. $v_x(0,t) = 0, v_x(\pi,t) = 0$ and $r(x)$ is any reference solution which only needs to satisfy the nonhomogeneous boundary conditions: $r'(0) = 1, r'(\pi) = -1$. By guessing, let $r(x) = Ax + Bx^2$. Let see if this satisfies the boundary conditions. $r' = A + 2Bx$. At $x = 0$ this implies $1 = A$. Hence $r = x + Bx^2$. Now $r' = 1 + 2Bx$. At $x = \pi$ this gives $-1 = 1 + 2B\pi$ or $B = -\frac{1}{\pi}$. Therefore

$$r(x) = x - \frac{1}{\pi}x^2$$

Substituting $u = v(x,t) + r(x)$ into the PDE $u_t = u_{xx}$ and noting that $r''(x) = -\frac{2}{\pi}$ gives

$$v_t = v_{xx} - \frac{2}{\pi} \tag{1}$$

PDE (1) is now solved using eigenfunction expansion. We need to find eigenfunctions and eigenvalues of $v_t = v_{xx}$ with $v_x(0,t) = 0, v_x(\pi,t) = 0$. This is known PDE and have eigenfunctions and eigenvalues as follows. For zero eigenvalue, the eigenfunction is an arbitrary constant. Say β. let $\beta = 1$ since scale is not important.

$$\Phi_0(x) = 1$$

And for $n = 1,2,3,\cdots$

$$\begin{aligned}\Phi_n(x) &= \cos\left(\sqrt{\lambda_n}x\right)\\ &= \cos(nx)\end{aligned}$$

with eigenvalues $\lambda_n = n^2$ for $n = 1,2,3,\cdots$. Now we can eigenfunction expansion and assume the solution to (1) is

$$v(x,t) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n(x) \tag{2}$$

Plugging this into the PDE (1) gives

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n''(x) - \frac{2}{\pi}$$

But $\Phi_n''(x) = -\lambda_n \Phi_n(x)$ and the above simplifies to

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \frac{2}{\pi}$$

Since eigenfunctions are complete, we can expand $\frac{2}{\pi}$ using them and the above becomes

$$\begin{aligned}
\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) &= -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \sum_{n=0}^{\infty} C_n \Phi_n(x) \\
A_n'(t)\,\Phi_n(x) + A_n(t)\,\lambda_n \Phi_n(x) &= -C_n \Phi_n(x) \\
A_n'(t) + A_n(t)\,\lambda_n &= -C_n \qquad (3)
\end{aligned}$$

To find C_n

$$\sum_{n=0}^{\infty} C_n \Phi_n(x) = \frac{2}{\pi}$$

For $n = 0$

$$C_0 \Phi_0(x) = \frac{2}{\pi}$$

But $\Phi_0(x) = 1$, hence

$$C_0 = \frac{2}{\pi}$$

All other C_m for $m > 0$ are zero. Hence (3) becomes, for $n = 0$ (since $\lambda_0 = 0$)

$$\begin{aligned}
A_0'(t) &= -\frac{2}{\pi} \\
A_0(t) &= -\frac{2}{\pi} t + B_0
\end{aligned}$$

Where B_0 is integration constant. For $n > 0$ (3) becomes

$$A_n'(t) + A_n(t)\,n^2 = 0$$

This has the solution

$$A_n(t) = B_n e^{-n^2 t}$$

Where B_n is constant of integration. Hence from (2)

$$\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} A_n(t)\,\Phi_n(x) \\
&= A_0(t) + \sum_{n=1}^{\infty} A_n(t)\,\Phi_n(x) \\
&= -\frac{2}{\pi} t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t} \cos(nx)
\end{aligned}$$

Since $u = v(x,t) + r(x)$ then the solution becomes

$$u(x,t) = \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t}\cos(nx) \tag{4}$$

At $t = 0$

$$\sin(x) = \left(x - \frac{1}{\pi}x^2\right) + B_0 + \sum_{n=1}^{\infty} B_n \cos(nx) \tag{5}$$

case $n = 0$

$$\int_0^\pi \sin(x)\cos\left(\sqrt{\lambda_0}x\right)dx = \int_0^\pi \left(x - \frac{1}{\pi}x^2\right)\cos\left(\sqrt{\lambda_0}x\right)dx + \int_0^\pi B_0\cos\left(\sqrt{\lambda_0}x\right)dx$$

But $\lambda_0 = 0$ hence

$$\begin{aligned}\int_0^\pi \sin(x)\,dx &= \int_0^\pi \left(x - \frac{1}{\pi}x^2\right)dx + \int_0^\pi B_0 dx\\ 2 &= \frac{\pi^2}{6} + B_0\pi\\ B_0 &= \frac{2}{\pi} - \frac{\pi}{6}\end{aligned}$$

For $n > 0$, Multiplying both sides of (5) by $\cos(mx)$ and integrating

$$\int_0^\pi \sin(x)\cos(mx)\,dx = \int_0^\pi \left(x - \frac{1}{\pi}x^2\right)\cos(mx)\,dx + \sum_{n=1}^{\infty} B_n \int_0^\pi \cos(nx)\cos(mx)\,dx$$

For $m = 1$

$$\begin{aligned}0 &= 0 + B_1\frac{\pi}{2}\\ B_1 &= 0\end{aligned}$$

For $m > 1$

$$\begin{aligned}-\frac{1+(-1)^m}{m^2(-1+m^2)} &= \frac{\pi}{2}B_m\\ B_m &= \frac{-2}{\pi}\left(\frac{1}{m^2}\frac{(-1)^m+1}{m^2-1}\right)\end{aligned}$$

Hence solution (4) becomes

$$\begin{aligned}u(x,t) &= \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=1}^{\infty} B_n e^{-n^2 t}\cos(nx)\\ u(x,t) &= \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=2}^{\infty} \frac{-2}{\pi}\left(\frac{1}{n^2}\frac{(-1)^n+1}{n^2-1}\right)e^{-n^2 t}\cos(nx)\end{aligned}$$

4.1.5.10 [225] Haberman 8.2.1 (a) (general case)

problem number 225

Added Nov 27, 2018

This is problem 8.2.1 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= A \\ \frac{\partial u}{\partial x}(L,t) &= B \end{aligned}$$

Initial condition is $u(x,0) = f(x)$

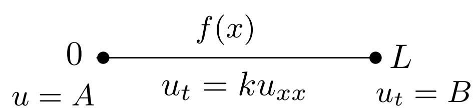


Figure 4.115: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, Derivative[1, 0][u][L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-\frac{k\pi^2 t(1-2K[1])^2}{4L^2}}\left(\int_0^L -\frac{\sqrt{2}(A+Bx-f(x))\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}} + A + Bx\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  := u(0,t)=A, eval( diff(u(x,t),x),x=L)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = Bx+A-2\left(\sum_{n=0}^{\infty} \frac{\left(\int_0^L (Bx+A-f(x)) \sin\left(\frac{(2n+1)\pi x}{2L}\right) dx\right) e^{-\frac{\pi^2 (2n+1)^2 kt}{4L^2}} \sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}\right)$$

Hand solution

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

We can look for $u_E(x)$ which is the steady state solution that satisfies the non-homogenous boundary conditions. In (1) $v(x,t)$ satisfies the PDE itself but with homogenous boundary conditions. The first step is to find $u_E(x)$. We use the equilibrium solution in this case. At equilibrium $\frac{\partial u_E(x,t)}{\partial t} = 0$ and hence the solution is given $\frac{d^2 u_E}{\partial x^2} = 0$ or

$$u_E(x) = c_1 x + c_2$$

At $x = 0, u_E(x) = A$, Hence

$$c_2 = A$$

And solution becomes $u_E(x) = c_1 x + A$. at $x = L$, $\frac{\partial u_E(x)}{\partial x} = c_1 = B$, Therefore

$$u_E(x) = Bx + A$$

Now we plug-in (1) into the original PDE, this gives

$$\frac{\partial v(x,t)}{\partial t} = k\left(\frac{\partial^2 v(x,t)}{\partial x} + \frac{\partial^2 u_E(x)}{\partial x}\right)$$

But $\frac{\partial^2 u_E(x)}{\partial x} = 0$, hence we need to solve

$$\frac{\partial v(x,t)}{\partial t} = k\frac{\partial^2 v(x,t)}{\partial x}$$

for $v(x,t)=u(x,t)-u_E(x)$ with homogenous boundary conditions $v(0,t)=0$, $\frac{\partial v(L,t)}{\partial t}=0$ and initial conditions

$$\begin{aligned} v(x,0) &= u(x,0)-u_E(x) \\ &= f(x)-(Bx+A) \end{aligned}$$

This PDE we already solved before and we know that it has the following solution

$$\begin{aligned} v(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n}x\right) e^{-k\lambda_n t} \\ \lambda_n &= \left(\frac{n\pi}{2L}\right)^2 \qquad n=1,3,5,\cdots \end{aligned} \tag{2}$$

With b_n found from orthogonality using initial conditions $v(x,0)=f(x)-(Bx+A)$

$$v(x,0)=\sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n}x\right)$$

$$\int_0^L (f(x)-(Bx+A))\sin\left(\sqrt{\lambda_m}x\right)dx = \int_0^L \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n}x\right)\sin\left(\sqrt{\lambda_m}x\right)dx$$

$$\int_0^L (f(x)-(Bx+A))\sin\left(\sqrt{\lambda_m}x\right)dx = b_m\frac{L}{2}$$

Hence

$$b_n=\frac{2}{L}\int_0^L (f(x)-(Bx+A))\sin\left(\sqrt{\lambda_n}x\right)dx \qquad n=1,3,5,\cdots \tag{3}$$

Therefore, from (1) the solution is

$$\begin{aligned} u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n}x\right) e^{-k\lambda_n t} + \overbrace{Bx+A}^{u_E(x)} \\ &= Bx+A+\sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L (f(x)-(Bx+A))\sin\left(\sqrt{\frac{n\pi}{L}}x\right)dx\right)\sin\left(\sqrt{\frac{n\pi}{L}}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned}$$

Or

$$u(x,t)=Bx+A+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L (f(x)-(Bx+A))\sin\left(\sqrt{\frac{(2n+1)\pi}{2L}}x\right)dx\right)\sin\left(\sqrt{\frac{(2n+1)\pi}{2L}}x\right)e^{-k\left(\frac{(2n}{}\right.}$$

4.1.5.11 [226] Haberman 8.2.1 (d) (general solution)

problem number 226

This is problem 8.2.1 part(d) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + k$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

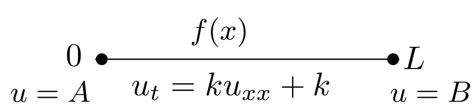


Figure 4.116: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + k;
bc  = {u[0, t] == A0, u[L0, t] == B0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} - \frac{e^{-\frac{k\pi^2 tK[1]^2}{\mathrm{L0}^2}}\left(2\left(-1+(-1)^{K[1]}\right)\left(-1+e^{\frac{k\pi^2 tK[1]^2}{\mathrm{L0}^2}}\right)\mathrm{L0}^2 - \sqrt{2}\sqrt{\frac{1}{\mathrm{L0}}}\pi^3\left(\int_0^{\mathrm{L0}}\sqrt{2}\left(\frac{1}{\mathrm{L0}}\right)^{3/2}(\right.\right.}{\pi^3 K[1]^3}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = -\frac{x^2}{2}+A+\sum_{n=1}^{\infty}\left(-\frac{\left(\int_0^L 2\left(\frac{L^2 x}{2}-Lf(x)+\left(-\frac{x^2}{2}+A\right)L-(A-B)\,x\right)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2 k\, n^2 t}{L^2}}\sin\left(\right.}{L^2}\right.$$

<u>Hand solution</u>

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

Where $u_E(x)$ is the equilibrium solution which needs to satisfy only the nonhomogeneous B.C. And $v(x,t)$ is transient solution to heat PDE with homogeneous B.C.

At equilibrium, $u_t = ku_{xx} + Q(x)$ becomes

$$\begin{aligned}0 &= ku_E'' + Q(x)\\ &= ku_E'' + k\\ &= k(u_E'' + 1)\end{aligned}$$

Hence

$$u_E'' = -1$$

The solution to this ODE is

$$u_E = c_1 x + c_2 - \frac{1}{2}x^2$$

At $x = 0$, the above gives

$$A = c_2$$

And at $x = L$

$$\begin{aligned}B &= c_1 L + A - \frac{1}{2}L^2\\ c_1 &= \frac{B - A + \frac{1}{2}L^2}{L}\\ &= \frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\end{aligned}$$

Hence

$$u_E = \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2$$

Hence from (1)

$$\begin{aligned} u(x,t) &= v(x,t) + u_E \qquad (1A)\\ &= v(x,t) + \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2 \end{aligned}$$

Substituting this in $u_t = k u_{xx} + k$ gives

$$\begin{aligned} v_t &= k(v_{xx} - 1) + k\\ &= k v_{xx} \qquad (2) \end{aligned}$$

We need to solve the above for $v(x,t)$, but with homogeneous B.C. $v(0,t) = 0, v(L,t) = 0$. The eigenvalues for the homogeneous PDE $v_t = k v_{xx}$ with these boundary conditions is known to be $\lambda_n = \left(\frac{n\pi}{L}\right)^2$, for $n = 1, 2, \cdots$ and the corresponding eigenfunctions are $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. Now, using eigenfunction expansion, let

$$v(x,t) = \sum_{n=1}^{\infty} b_n(t)\, X_n(x) \qquad (3)$$

Substituting (3) into (2) gives

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) = k \sum_{n=1}^{\infty} b_n(t)\, X_n''(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, therefore the above becomes

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) + k \sum_{n=1}^{\infty} \lambda_n b_n(t)\, X_n(x) = 0$$

Since the above is true for each n and since eigenfunctions can not be zero, the above simplifies to

$$b_n'(t) + k\lambda_n b_n(t) = 0 \qquad (4)$$

This is linear in $b(t)$. The solution using integrating factor is

$$b_n(t) = b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

Therefore (3) becomes

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} b_n(t)\, X_n(x)\\ &= \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

And from (1)

$$\begin{aligned} u(x,t) &= v(x,t) + u_E(x) \\ &= \overbrace{\left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2}^{u_E} + \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned} \tag{5}$$

At $t = 0$ the above becomes

$$f(x) = \frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2 + \sum_{n=1}^{\infty} b_0(0) \sin\left(\frac{n\pi}{L}x\right)$$

For $n > 0$, and applying orthogonality

$$\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx = \int_0^L \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \sin\left(\frac{n\pi}{L}x\right) dx + \int_0^L b_0(0) \sin^2\left(\frac{n\pi}{L}x\right) dx$$

Hence

$$\int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx = \frac{L}{2} b_0(0)$$

Therefore

$$b_0(0) = \frac{2}{L}\int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx$$

Substituting the above in (5) gives

$$\begin{aligned} u(x,t) &= \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \\ &+ \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx\right) e^{-k\frac{n\pi}{L}t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

4.1.5.12 [227] Both ends depend on time with source that depends on space only (general solution)

problem number 227

Added July 3, 2019

Solve the heat equation

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

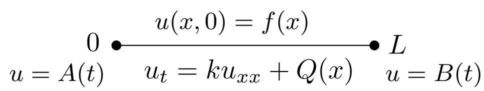


Figure 4.117: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{L>
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2 K[1]^2(t-K[2])}{L^2}}\,\text{Integrate}\left[\frac{\sqrt{2}\sin\left(\frac{\pi x K[1]}{L}\right)(LQ(x)+(x-L)A'(K[2])-xB'(K[2]))}{L^{3/2}},\{x,0,L\},\text{A}\right.\right.}{}\right.\right.$$

Maple ✗

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q(x);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A(t), u(L,t)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

time expired Possible bug. Maple can solve with $Q(x,t)$ source but not with $Q(x)$

Hand solution

Solve

$$\begin{aligned}
u_t &= k u_{xx} + Q(x) \\
u(0,t) &= A(t) \\
u(L,0) &= B(t) \\
u(x,0) &= f(x)
\end{aligned}$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. One can easily see that the reference function is

$$r(x,t) = A(t) + \frac{B(t) - A(t)}{L} x \tag{3}$$

Substituting (1) back into the original PDE gives

$$\begin{aligned}
\frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) + Q(x) \\
v_t(x,t) + r_t(x,t) &= k v_{xx}(x,t) + k r_{xx}(x,t) + Q(x)
\end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t) + \frac{B'(t)-A'(t)}{L}x$ and PDE becomes

$$v_t(x,t) = k v_{xx}(x,t) - r_t(x,t) + Q(x)$$

Let

$$\begin{aligned}
\tilde{Q}(x) &= Q(x) - r_t(x,t) \\
&= Q(x) - \left(A'(t) + \frac{B'(t) - A'(t)}{L} x \right)
\end{aligned} \tag{4}$$

Therefore the problem has been transformed to

$$\begin{aligned}
v_t &= k v_{xx} + \tilde{Q}(x) \\
v(0,t) &= 0 \\
v(L,t) &= 0 \\
v(0,x) &= F(x) \\
&= u(x,0) - r(x,0) \\
&= f(x) - \left(A(0) + \frac{B(0) - A(0)}{L} x \right)
\end{aligned}$$

The basic solution for this type of PDE was already given in problem 4.1.1.11 on page 431 and the solution is

$$
\begin{aligned}
v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right)\\
&\quad+\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}
$$

Where

$$
\begin{aligned}
\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\
\lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots
\end{aligned}
$$

Hence, using our $\tilde{Q}(x)$ and $F(x)$ found above into this solution gives

$$
\begin{aligned}
v(x,t) &= \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)-\left(A(0)+\frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&\quad+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(Q(s)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Since $u(x,t)=v(x,t)+r(x,t)$ then the final solution is

$$
\begin{aligned}
u(x,t) &= A(t)+\frac{B(t)-A(t)}{L}x\\
&\quad+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)-\left(A(0)+\frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&\quad+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(Q(s)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) d
\end{aligned}
$$

4.1.5.13 [228] Both ends depend on time with source that depends on space only (special case)

problem number 228

Added July 4, 2019

Solve the heat equation

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$ using these values

$$L = 2$$
$$k = \frac{1}{10}$$
$$A(t) = \sin(t)$$
$$B(t) = 2\cos(t)$$
$$Q(x) = x$$
$$f(x) = x$$

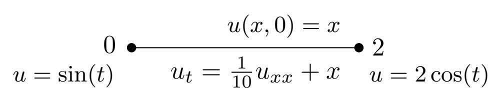


Figure 4.118: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
k=1/10;
A=Sin[t];
B=Cos[t];
f=x;
Q=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\left(-\frac{80\left(\pi^4\cos(t)K[1]^4+40\pi^2\sin(t)K[1]^2-e^{-\frac{1}{40}\pi^2tK[1]^2}\left(\pi^4K[1]^4+2(-1)^{K[1]}\left(\pi^4K[1]^4\right.\right.\right.}{\pi^3K[1]^3\left(\pi^4\right.}\right.\right.\right.$$

Maple ✓

```
restart;
L:=2;
k:=1/10;
A:=sin(t);
B:=cos(t);
f:=x;
Q:=x;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q;
ic  :=  u(x,0)=f;
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t)=-\frac{5x^3}{3}-\frac{(-3\cos(t)+3\sin(t)-40)\,x}{6}+\sum_{n=1}^{\infty}\left(-\frac{2\left(40\pi^2(\pi^2n^2\cos(t)+(\pi^2n^2\sin(t)-40\cos(t))\right.\left(\right.}{}\right.$$

Hand solution

Solve

$$
\begin{aligned}
u_t &= k u_{xx} + Q(x) \\
u(0,t) &= A(t) \\
u(L,0) &= B(t) \\
u(x,0) &= f(x)
\end{aligned}
$$

With $k = \frac{1}{10}, L = 2, f(x) = x, Q(x) = x, A(t) = \sin(t), B(t) = 2\cos(t)$.

The general problem above was solved in 4.1.5.12 on page 630 and the solution is

$$
\begin{aligned}
u(x,t) &= A(t) + \frac{B(t) - A(t)}{L} x \\
&+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) ds\right) \\
&+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L \left(Q(s) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) ds\right) d\tau
\end{aligned}
$$

Substituting the specific values given above into this solution gives

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}x\right)\left(\int_0^2 \left(s - \left(A(0) + \frac{B(0)-A(0)}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right) ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau}\left(\int_0^2 \left(s - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right) ds\right) d\tau
\end{aligned}
$$

But $A(0) = 0, B(0) = 2, A'(t) = \cos(t), B'(t) = -2\sin(t)$ and the above becomes

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}x\right)\left(\int_0^2 \left(s - \left(0 + \frac{2-0}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right) ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau}\left(\int_0^2 \left(s - \left(\cos(\tau) + \left(\frac{-2\sin(\tau) - \cos(\tau)}{2}\right)s\right)\right)\sin\left(\frac{n\pi}{2}s\right) ds
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau}\left(\int_0^2 \left(s - \left(\cos(\tau) - \left(\frac{2\sin(\tau) + \cos(\tau)}{2}\right)s\right)\right)\sin\left(\frac{n\pi}{2}s\right) ds\right)
\end{aligned}
$$

Animation is below

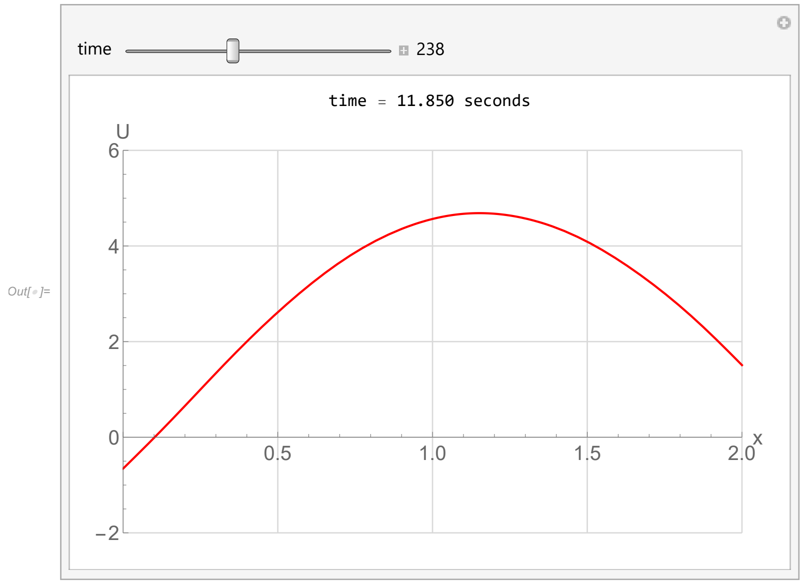


Figure 4.119: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1 / 10;
f[x_] := x;
Q[x_, t_] := x;
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[∘]:= sol = Integrate[(s - (Cos[tau] - ((2 Sin[tau] + Cos[tau])/L) * s)) Sin[(n Pi s)/L], {s, 0, L}];
sol2 = Assuming[Element[n, Integers] && n > 0, Simplify[sol]];
sol3 = N@Integrate[Exp[k ((n Pi)/L)^2 tau] * sol2, {tau, 0, t}];
mySol[x_, t_] = Chop@N@Simplify[(Sin[t] + ((2 Cos[t] - Sin[t])/L) x) + Sum[Exp[-k ((n Pi)/L)^2 t] Sin[(n Pi)/L x] (sol3), {n, 1, numberOfTerms}]];
```

Figure 4.120: Source code

```
In[∘]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, 2}, {-2, 6}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                   ]

              }}],
          {t, 0, 30, .05}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[∘]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.121: Code for animation

4.1.5.14 [229] Both ends depend on time with source that depends on time and space (general solution)

problem number 229

Added June 27, 2019

Solve the heat equation

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

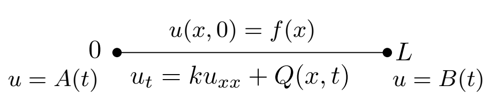


Figure 4.122: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{L>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\text{Integrate}\left[\frac{\sqrt{2}\sin\left(\frac{\pi xK[1]}{L}\right)(LQ(x,K[2])+(x-L)A'(K[2])-xB'(K[2]))}{L^{3/2}},\{x,0,L\right.\right.}{}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q(x,t);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A(t), u(L,t)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\frac{L\left(\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L\left(-\frac{x\left(\frac{d}{d\tau}B(\tau)\right)}{L}+Q(x,\tau)-\left(-\frac{x}{L}+1\right)\left(\frac{d}{d\tau}A(\tau)\right)\right)\sin\left(\frac{\pi nx}{L}\right)dx\right)e^{-\frac{\pi^2(t-\tau)kn^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L}\right)d\tau\right)+L\left(\sum_{n=1}^{\infty}\right.}{L}$$

<u>Hand solution</u>

Solve

$$
\begin{aligned}
u_t &= ku_{xx}+Q(x,t)\\
u(0,t) &= A(t)\\
u(L,0) &= B(t)\\
u(x,0) &= f(x)
\end{aligned}
$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. One can easily see that the reference function is

$$r(x,t) = A(t) + \frac{B(t) - A(t)}{L}x \tag{3}$$

Substituting (1) back into the original PDE gives

$$\begin{aligned}
\frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) + Q(x,t) \\
v_t(x,t) + r_t(x,t) &= kv_{xx}(x,t) + kr_{xx}(x,t) + Q(x,t)
\end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t) + \frac{B'(t)-A'(t)}{L}x$ and PDE becomes

$$v_t(x,t) = kv_{xx}(x,t) - r_t(x,t) + Q(x,t)$$

Let

$$\begin{aligned}
\tilde{Q}(x,t) &= Q(x,t) - r_t(x,t) \\
&= Q(x,t) - \left(A'(t) + \frac{B'(t) - A'(t)}{L}x\right) \qquad (4)
\end{aligned}$$

Therefore the problem has been transformed to

$$\begin{aligned}
v_t &= kv_{xx} + \tilde{Q}(x,t) \\
v(0,t) &= 0 \\
v(L,t) &= 0 \\
v(0,x) &= F(x) \\
&= u(x,0) - r(x,0) \\
&= f(x) - \left(A(0) + \frac{B(0) - A(0)}{L}x\right)
\end{aligned}$$

The basic solution for this type of PDE was already given in problem 4.1.6.4 on page 665 and the solution is

$$\begin{aligned}
v(x,t) = &\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L \tilde{Q}(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence, using our $\tilde{Q}(x,\tau)$ and $F(x)$ found above into this solution gives

$$v(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) ds\right)$$
$$+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L \left(Q(s,\tau) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) ds\right) d$$

Since $u(x,t) = v(x,t) + r(x,t)$ then the final solution is

$$u(x,t) = A(t) + \frac{B(t)-A(t)}{L}x$$
$$+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right) ds\right)$$
$$+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L \left(Q(s,\tau) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s$$

4.1.5.15 [230] Both ends depend on time with source present (special case)

problem number 230

Added June 27, 2019

Solve the heat equation

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$ using these values

$$
\begin{aligned}
L &= 2 \\
k &= \frac{1}{10} \\
A(t) &= \sin(t) \\
B(t) &= 2\cos(t) \\
Q(x,t) &= xte^{-t}\cos(t) \\
f(x) &= x
\end{aligned}
$$

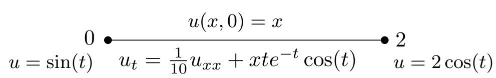


Figure 4.123: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
k=1/10;
A=Sin[t];
B=Cos[t];
f=x;
Q=x*t*Exp[-t]*Cos[t];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty} \frac{2e^{-\frac{1}{40}\pi^2 tK[1]^2}\left(\frac{40\left(-80(-1)^{K[1]}\pi^2\left(\pi^2K[1]^2-80\right)\left(\pi^4K[1]^4+1600\right)K[1]^2-\left(40(-1)^{K[1]}-\pi^2K[1]^2\right)\left(\pi^4K[1]^4-80\pi\right.\right.}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
L:=2;
k:=1/10;
A:=sin(t);
B:=cos(t);
f:=x;
Q:=x*t*exp(-t)*cos(t);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q;
ic  :=  u(x,0)=f;
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t)=\frac{x\cos(t)}{2}+\sum_{n=1}^{\infty}\left(-\frac{2\left(80\left(\left(\pi^6n^6t-120\left(t+\frac{1}{3}\right)\pi^4n^4+6400\left(t+\frac{1}{2}\right)\pi^2n^2-128000t\right)\cos(t)+40(\pi\right.\right.}{}\right.$$

Hand solution

Solve

$$\begin{aligned} u_t &= ku_{xx}+Q(x,t)\\ u(0,t) &= A(t)\\ u(L,0) &= B(t)\\ u(x,0) &= f(x) \end{aligned}$$

With $k=\frac{1}{10}, L=2, f(x)=x, Q(x,t)=xte^{-t}\cos(t), A(t)=\sin(t), B(t)=2\cos(t)$.

The general problem above was solved in 4.1.5.14 on page 638 and the solution is

$$\begin{aligned} u(x,t) &= A(t)+\frac{B(t)-A(t)}{L}x\\ &\quad+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)-\left(A(0)+\frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\ &\quad+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(Q(s,\tau)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{L}x\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau \end{aligned}$$

Substituting the specific values given above into this solution gives

$$u(x,t) = \sin(t) + \frac{2\cos(t)-\sin(t)}{2}x$$
$$+\sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}x\right)\left(\int_0^2 \left(s-\left(A(0)+\frac{B(0)-A(0)}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right)ds\right)$$
$$+\sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2\tau}\left(\int_0^2 \left(s\tau e^{-\tau}\cos(\tau)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right)d$$

But $A(0)=0, B(0)=2, A'(t)=\cos(t), B'(t)=-2\sin(t)$ and the above becomes

$$u(x,t) = \sin(t) + \frac{2\cos(t)-\sin(t)}{2}x$$
$$+\sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}x\right)\left(\int_0^2 \left(s-\left(0+\frac{2-0}{2}s\right)\right)\sin\left(\frac{n\pi}{2}s\right)ds\right)$$
$$+\sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2\tau}\left(\int_0^2 \left(s\tau e^{-\tau}\cos(\tau)-\left(\cos(\tau)+\left(\frac{-2\sin(\tau)-\cos(\tau)}{2}\right)s\right)\right)\sin$$

Or

$$u(x,t) = \sin(t) + \frac{2\cos(t)-\sin(t)}{2}x$$
$$+\sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}x\right)\int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2\tau}\left(\int_0^2 \left(s\tau e^{-\tau}\cos(\tau)-\left(\cos(\tau)-\left(\frac{2\sin(\tau)+\cos(\tau)}{2}\right)s\right)\right)\sin\,($$

Animation is below

Out[ ]=

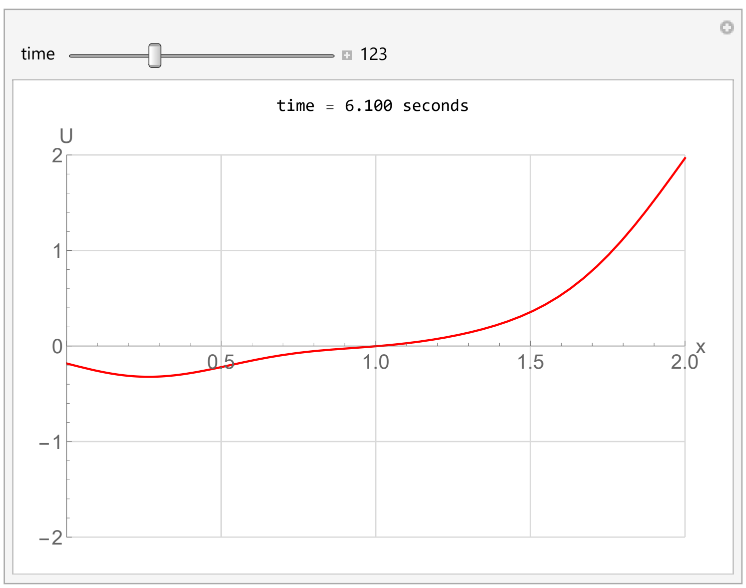


Figure 4.124: Initial state

Source code used for the above

```
In[]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1 / 10;
f[x_] := x;
Q[x_, t_] := x t Exp[-t] Cos[t];
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[]:= sol = Integrate[(s * tau * Exp[-tau] Cos[tau] - (Cos[tau] - ((2 Sin[tau] + Cos[tau])/L) * s)) Sin[(n Pi s)/L], {s, 0, L}];
sol2 = Assuming[Element[n, Integers] && n > 0, Simplify[sol]];
sol3 = N@Integrate[Exp[k ((n Pi)/L)^2 tau] * sol2, {tau, 0, t}];
mySol[x_, t_] = Chop@N@Simplify[(Sin[t] + ((2 Cos[t] - Sin[t])/L) x) + Sum[Exp[-k ((n Pi)/L)^2 t] Sin[(n Pi)/L x] (sol3), {n, 1, numberOfTerms}]];
```

Figure 4.125: Source code

```
In[]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, 2}, {-2, 2}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 20, .05}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.126: Code for animation

4.1.5.16 [231] Pinchover and Rubinstein 6.17

problem number 231

Added July 2, 2018.

Pinchover and Rubinstein's exercise 6.17. Taken from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$\frac{\partial}{\partial t}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 1 + x\cos(t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = \sin(t)$$
$$\frac{\partial u}{\partial x}(1,t) = \sin(t)$$

Initial condition is $u(x,0) = 1 + \cos(2\pi x)$.

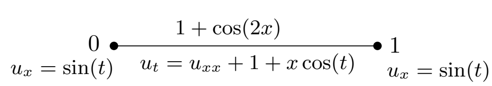


Figure 4.127: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, t], x] == D[u[x, t], {x, 2}] + 1 + x*Cos[t];
bc  = {Derivative[1, 0][u][0, t] == Sin[t], Derivative[1, 0][u][1, t] == Sin[t]};
ic  = u[x, 0] == 1 + Cos[2*Pi*x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic, bc}, u[x, t], x, t]], 60*10]
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + 1+x*cos(t);
bc  := (D[1](u))(0, t) = sin(t), (D[1](u))(1, t) = sin(t);
ic  := u(x, 0) = 1+cos(2*Pi*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = x\sin(t) + \cos(2\pi x)\,\mathrm{e}^{-4\pi^2 t} + t + 1$$

4.1.5.17 [232] nonhomogeneous BC

problem number 232

Added March 28, 2018. A problem from my PDE animation page.

Solve the heat equation

$$u_t = k u_{xx} + x$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = \frac{t\sin t}{5}$$
$$u(\pi,t) = \frac{t\cos t}{10}$$

Initial condition is $u(x,0) = 60 - 20x$.

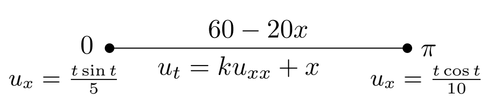


Figure 4.128: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + x;
bc  = {u[0, t] == (t*Sin[t])/5, u[Pi, t] == (t*Cos[t])/10};
ic  = u[x, 0] == 60 - 2*x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {t
```

$$\left\{\left\{u(x,t) \to \frac{10\pi \sum_{K[1]=1}^{\infty} \frac{e^{-tK[1]^2}\left(20(30+(-1)^{K[1]}(-30+\pi))K[1]^2+\frac{(-1)^{K[1]+1}K[1]^8+10(-1)^{K[1]}\pi K[1]^8-10(-1)^{K[1]}e^{tK[1]^2}\pi K[1]^8-4K[1]^6 \cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+x;
ic  :=  u(x,0)=(60-2*x);
bc  :=  u(0,t)=t/5*sin(t), u(Pi,t)=t/10*cos(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \frac{(x\cos(t)+(-2x+2\pi)\sin(t))\,t+2\pi\left(\sum_{n=1}^{\infty}\frac{20\left(-\frac{\left(\left(n^4t-2n^2+t\right)n^2\cos(t)+\left(n^6+n^4t-n^2+t\right)\sin(t)\right)n^2}{10}+\left(-\frac{\left(n^4t-2n^2+t \cdots}{20}\right.\right.}{\cdots}\right.}{\cdots}$$

4.1.5.18 [233] Haberman 8.2.2. (a)

problem number 233

Added Nov 27, 2018.

Problem 8.2.2 part(a) from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = A(t)$$
$$\frac{\partial u}{\partial x}(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

For hand solution see my HW9, Math 322, UW Madison. The text does not actually asks to solve this PDE but only to reduce the problem to one with homogeneous B.C.

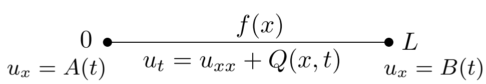


Figure 4.129: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {Derivative[1, 0][u][0, t] == A[t], Derivative[1, 0][u][L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(e^{-\frac{k\pi^2 t K[1]^2}{L^2}} \int_0^L \frac{\cos\left(\frac{\pi x K[1]}{L}\right)(x(x(A(0)-B(0))-2LA(0))+2Lf(x))}{\sqrt{2}L^{3/2}}\,dx + \int_0^t e^{-\frac{k\pi^2 K[1]}{}} \cdots\right.}{\cdots}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+Q(x,t);
ic  :=  u(x,0)=f(x);
bc  := eval( diff(u(x,t),x),x=0)=A(t), eval( diff(u(x,t),x),x=L)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{x^2B(t)}{2L}+\left(-\frac{x^2}{2L}+x\right)A(t)+\int_0^t\left(\left(\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L-\frac{\left(\frac{x^2\left(\frac{d}{d\tau}B(\tau)\right)}{2}+LQ(x,\tau)+\left(L-\frac{x}{2}\right)x\left(\frac{d}{d\tau}A(\tau)\right)-(A(\tau)-B(\tau))k\right)}{L}\right.}{L}\right.\right.\right.$$

<u>Hand solution</u>

Solve

$$\begin{aligned}u_t&=ku_{xx}+Q(x,t)\\u_x(0,t)&=A(t)\\u_x(L,0)&=B(t)\\u(x,0)&=f(x)\end{aligned}$$

Let

$$u(x,t)=v(x,t)+r(x,t)\tag{1}$$

Since the problem has time dependent source function $Q(x,t)$ then $r(x,t)$ is now a reference function that only needs to satisfy the non-homogenous boundary conditions which in this problem are at both ends and $v(x,t)$ has homogenous boundary conditions. The first step is to find $r(x,t)$. Let

$$r(x,t)=c_1(t)\,x+c_2(t)\,x^2$$

Then

$$\frac{\partial r(x,t)}{\partial x}=c_1(t)+2c_2(t)\,x$$

At $x=0$

$$A(t)=c_1(t)$$

And at $x = L$

$$
\begin{aligned}
B(t) &= c_1(t) + 2c_2(t)\, L \\
c_2(t) &= \frac{B(t) - c_1(t)}{2L}
\end{aligned}
$$

Solving for c_1, c_2 gives

$$
r(x,t) = A(t)\, x + \left(\frac{B(t) - A(t)}{2L} \right) x^2 \tag{2}
$$

Replacing (1) into the original PDE $u_t = k u_{xx} + Q(x,t)$ gives

$$
\begin{aligned}
\frac{\partial}{\partial t}(v(x,t) - r(x,t)) &= k \frac{\partial^2}{\partial x}(v(x,t) - r(x,t)) + Q(x,t) \\
\frac{\partial v}{\partial t} - \frac{\partial r}{\partial t} &= k \frac{\partial^2 v}{\partial x^2} - k \frac{\partial^2 r}{\partial x^2} + Q(x,t)
\end{aligned}
$$

But $r_{xx} = \frac{B(t) - A(t)}{L}$, hence the above reduces to

$$
v_t = k v_{xx} + Q(x,t) - k \frac{B(t) - A(t)}{L} + r_t \tag{3}
$$

Let

$$
\tilde{Q}(x,t) = Q(x,t) + r_t - k \frac{B(t) - A(t)}{L}
$$

Then (3) becomes

$$
\begin{aligned}
v_t &= k v_{xx} + \tilde{Q}(x,t) \qquad (4) \\
v_t(0,t) &= 0 \\
v_t(L,t) &= 0
\end{aligned}
$$

And initial condition is

$$
\begin{aligned}
v(x,0) &= F(x) \\
&= u(x,0) - r(x,0) \\
&= f(x) - \left(A(0)\, x + \left(\frac{B(0) - A(0)}{2L} \right) x^2 \right)
\end{aligned}
$$

PDE (4) with its homogenous boundary conditions is standard one, its corresponding eigenvalue boundary value ODE $X'' + \lambda X = 0$ has $\lambda = 0$ as eigenvalue with corresponding eigenfunction $\Phi_0(x) = 1$ and $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1, 2, 3, \cdots$ with corresponding eigenfunctions $\Phi_n(x) = \cos\left(\sqrt{\lambda_n} x\right)$. Using these, we can write the solution to (4) using eigenfunction expansion as

$$v(x,t) = \sum_{n=0}^{\infty} c_n(t)\, \Phi_n(x) \tag{4A}$$

Hence $v_t(x,t) = \sum_{n=0}^{\infty} c_n'(t)\, \Phi_n(x)$ and $v_{xx}(x,t) = \sum_{n=0}^{\infty} c_n(t)\, \Phi_n''(x)$. Substituting these into (4) gives

$$\sum_{n=0}^{\infty} c_n'(t)\, \Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\, \Phi_n''(x) + \tilde{Q}(x,t)$$

Expanding $\tilde{Q}(x,t)$ using same eigenfunctions since they are complete, the above becomes

$$\sum_{n=0}^{\infty} c_n'(t)\, \Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\, \Phi_n''(x) + \sum_{n=0}^{\infty} b_n(t)\, \Phi_n(x)$$

But $\Phi_n''(x) = -\lambda_n \Phi_n(x)$ and the above becomes

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n'(t)\, \Phi_n(x) &= -\sum_{n=0}^{\infty} c_n(t)\, \lambda_n \Phi_n(x) + \sum_{n=0}^{\infty} b_n(t)\, \Phi_n(x) \\
c_n'(t)\, \Phi_n(x) + c_n(t)\, \lambda_n \Phi_n(x) &= b_n(t)\, \Phi_n(x) \\
c_n'(t) + c_n(t)\, \lambda_n &= b_n(t) \\
c_n'(t) + c_n(t)\, \frac{n^2\pi^2}{L^2} &= b_n(t) \qquad (5)
\end{aligned}$$

To find $b_n(t)$, since $\tilde{Q}(x,t) = Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L}$ then

$$Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L} = \sum_{n=0}^{\infty} b_n(t)\, \Phi_n(x)$$

Multiplying both sides by $\Phi_m(x)$ and integrating gives

$$\begin{aligned}
\int_0^L \left(Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L} \right) \Phi_m(x)\, dx &= \int_0^L \sum_{n=0}^{\infty} b_n(t)\, \Phi_n(x)\, \Phi_m(x)\, dx \\
&= \sum_{n=0}^{\infty} b_n(t) \left(\int_0^L \Phi_n(x)\, \Phi_m(x)\, dx \right)
\end{aligned}$$

By orthogonality

$$\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L} \right) \Phi_m(x)\, dx = b_m(t) \int_0^L \Phi_m^2(x)\, dx$$

When $m=0, \Phi_0(x)=1$ and the above gives

$$\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx = b_0(t)\int_0^L dx$$

$$b_0(t)=\frac{1}{L}\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx$$

When $m=1,2,3,\cdots$

$$\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = b_m(t)\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

$$\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = b_m(t)\frac{L}{2}$$

$$b_m(t)=\frac{2}{L}\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right) d$$

Therefore (5) is now solved. When $n=0$ (5) becomes

$$c_0'(t)+c_0(t)\frac{n^2\pi^2}{L^2}=b_0(t)$$

$$c_0'(t)=b_0(t)$$

$$c_0'(t)=\frac{1}{L}\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx$$

Hence

$$c_0(t)=\int_0^t\left(\frac{1}{L}\int_0^L Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}dx\right)dt+C_0$$

For $n=1,2,3,\cdots$ (5) becomes

$$c_n'(t)+c_n(t)\frac{n^2\pi^2}{L^2}=b_n(t)$$

$$=\frac{2}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

Integrating factor is $I=e^{\int\frac{n^2\pi^2}{L^2}dt}=e^{\frac{n^2\pi^2}{L^2}t}$ and the solution to the above becomes

$$\frac{d}{dt}\left(c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t}\right)=\frac{2e^{\frac{n^2\pi^2}{L^2}t}}{L}\int_0^L\left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t}=\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt+C_n$$

$$c_n(t)=e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt+C_ne^{-\frac{n^2\pi^2}{L^2}t}$$

Now that we found $c_n(t)$ for $n = 0, 1, 2, 3, \cdots$ the solution for $v(x,t)$ is found from 4A.

$$
\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} c_n(t)\,\Phi_n(x)\\
&= \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0 + \sum_{n=1}^{\infty} c_n(t)\,\Phi_n(x)\\
&= \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0\\
&+ \sum_{n=1}^{\infty}\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)\cos
\end{aligned}
$$

But

$$
u(x,t) = v(x,t) + r(x,t)
$$

Hence

$$
\begin{aligned}
u(x,t) = A(t)\,x + \left(\frac{B(t)-A(t)}{2L}\right)x^2 + \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0\\
+ \sum_{n=1}^{\infty}\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)\cos
\end{aligned}
$$

But

$$
\begin{aligned}
r_\tau &= \frac{d}{dt}\left(A(\tau)\,x + \left(\frac{B(\tau)-A(\tau)}{2L}\right)x^2\right)\\
&= \frac{2LA'(\tau)\,x + \left(B'(\tau)-A'(\tau)\right)x^2}{2L}
\end{aligned}
$$

Hence

$$
\begin{aligned}
u(x,t) = C_0 + A(t)\,x + \left(\frac{B(t)-A(t)}{2L}\right)x^2\\
+\frac{1}{2L^2}\int_0^t\left(\int_0^L 2LQ(x,\tau) + 2LA'(\tau)\,x + \left(B'(\tau)-A'(\tau)\right)x^2 - 2k(B(\tau)-A(\tau))\,dx\right)dt+\\
\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}x\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{e^{\frac{n^2\pi^2}{L^2}\tau}}{L^2}\int_0^L\left(2LQ(x,\tau)+2LA'(\tau)\,x + \left(B'(\tau)-A'(\tau)\right)x^2 - 2k(B(\tau)-A(\tau))\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)
\end{aligned}
$$

The constants C_0, C_n are found from initial conditions $u(x,0) = f(x)$.

4.1.5.19 [234] Articolo 8.4.3

problem number 234

Added December 20, 2018.

Example 8.4.3 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + t$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 5 \\ u(1,t) + \frac{\partial u}{\partial x}(1,t) &= 10 \end{aligned}$$

Initial condition is $u(x,0) = \frac{-40x^2}{3} + \frac{45x}{2} + 5$ and $k = \frac{1}{20}$

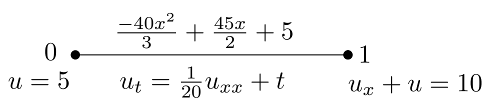


Figure 4.130: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/20;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + t;
bc  = {u[0, t] == 5, u[1, t] + Derivative[1, 0][u][1, t] == 10};
ic  = u[x, 0] == (-40*x^2)/3 + (45*x)/2 + 5;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to \left\{ \frac{5x}{2}+\sum_{K[1]=1}^{\infty}-\frac{\sqrt{2}\left(\frac{40e^{-\frac{1}{20}tK[2,K[1]]}\left(\cos\left(\sqrt{K[2,K[1]]}\right)-1\right)\left(e^{\frac{1}{20}tK[2,K[1]]}(tK[2,K[1]]-20)+20\right)}{\sqrt{\cos\left(2\sqrt{K[2,K[1]]}\right)+3}K[2,K[1]]^{5/2}}+\frac{40e^{-\frac{1}{20}tK[2,K[1]]}\left(\cos\cdots\right.}{\cdots}\right.}{\sqrt{\cos^2\left(\sqrt{K[2,K[1]]}\right)\cdots}}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t) = (1/20)*(diff(u(x, t), x$2))+t;
bc  := u(0, t) = 5, (u(1, t)+ eval( diff(u(x,t),x),x=1)) = 10;
ic  := u(x, 0) = -40*x^2/3+45*x/2+5;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t))),c
```

$$u(x,t)=\frac{5x}{2}+\int_0^t\left(\sum_{n1=0}^{\infty}\left\{\begin{array}{cc}0 & \lambda_n=0\\ \frac{4(\cos(\lambda_n)-1)\tau\,\mathrm{e}^{-\frac{(t-\tau)\lambda_n^2}{20}}\sin(x\lambda_n)}{-2\lambda_n+\sin(2\lambda_n)} & \mathit{otherwise}\end{array}\right.\right)d\tau+\left(\sum_{n=0}^{\infty}\frac{80(\lambda_n^2\cos(\lambda_n)+\lambda_n\sin(\lambda\cdots}{3(-2\lambda\cdots}\right.$$

4.1.6 Semi-infinite domain

Local contents

4.1.6.1 [235] left end constant (general case)

problem number 235

Added July 6, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = A$ and initial conditions $u(x,0) = 0$

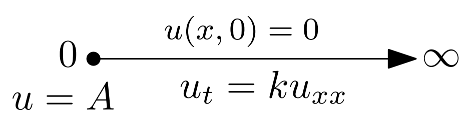


Figure 4.131: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == A;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{Ae^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to \text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=A;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -A\,\mathrm{erf}\left(\frac{x}{2\sqrt{k}\,\sqrt{t}}\right) + A$$

<u>Hand solution</u>

Solving

$$\begin{aligned} u_t &= k u_{xx} \qquad t>0, x>0 \\ u(0,t) &= A \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

And $u(x,t) < \infty$ as $x \to \infty$. This means $u(x,t)$ is bounded. This conditions is always needed to solve these problems.

Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Defined as

$$\mathcal{L}(u,t) = \int_0^\infty e^{-st} u(x,t)\,dt$$

Applying Laplace transform to the original PDE (1) gives

$$sU(x,s) - u(x,0) = kU_{xx}(x,s)$$

But $u(x,0) = 0$, therefore the above becomes

$$U_{xx} - \frac{s}{k}U = 0$$

The solution to this differential equation is

$$U(x,s) = c_1 e^{\sqrt{\frac{s}{k}}x} + c_2 e^{-\sqrt{\frac{s}{k}}x}$$

Since $u(x,t)$ is bounded in the limit as $x \to \infty$ and $k > 0$, therefore it must be that $c_1 = 0$ to keep the solution bounded. The above simplifies to

$$U(x,s) = c_2 e^{-\sqrt{\frac{s}{k}}x} \tag{2}$$

At $x = 0$, $u(0,t) = A$. Therefore $U(0,s) = \mathcal{L}(u(0,t)) = \mathcal{L}(A) = \frac{1}{s}A$. Hence at $x = 0$ the above gives

$$\frac{1}{s}A = c_2$$

Therefore (2) becomes

$$U(x,s) = \frac{A}{s}e^{-\sqrt{\frac{s}{k}}x} \tag{3}$$

From tables, the inverse Laplace transform of the above is (since $x > 0, k > 0$)

$$\begin{aligned} u(x,t) &= A\operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right) \\ &= A\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{kt}}\right)\right) \end{aligned}$$

4.1.6.2 [236] left end constant (special case)

problem number 236

Added July 6, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = ku_{xx}$$

The boundary conditions are $u(0,t) = A$ and initial conditions $u(x,0) = 0$, using

$$\begin{aligned} A &= 60 \\ k &= \frac{1}{10} \end{aligned}$$

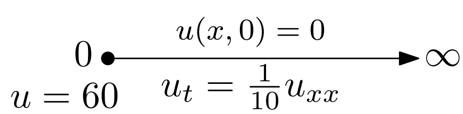


Figure 4.132: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/10; A=60;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == A;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \rightarrow \boxed{\text{Indeterminate if } x \leq 0}\}\}$$

It fail if assumption $x > 0$ is given. A bug

Maple ✓

```
restart;
k:=1/10;
A:=60;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=A;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -60\,\mathrm{erf}\left(\frac{\sqrt{10}\,x}{2\sqrt{t}}\right) + 60$$

Hand solution

Solving on semi-infinite domain

$$\begin{aligned} u_t &= k u_{xx} \qquad t > 0, x > 0 \\ u(0,t) &= A \\ u(x,0) &= 0 \end{aligned}$$

With $A = 60, k = \frac{1}{10}$

The general problem above was solved in 4.1.6.1 on page 657 and the solution is

$$u(x,t) = A\left(1 - \mathrm{erf}\left(\frac{x}{2\sqrt{kt}}\right)\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = 60\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\frac{t}{10}}}\right)\right)$$

Animation is below

Out[]=

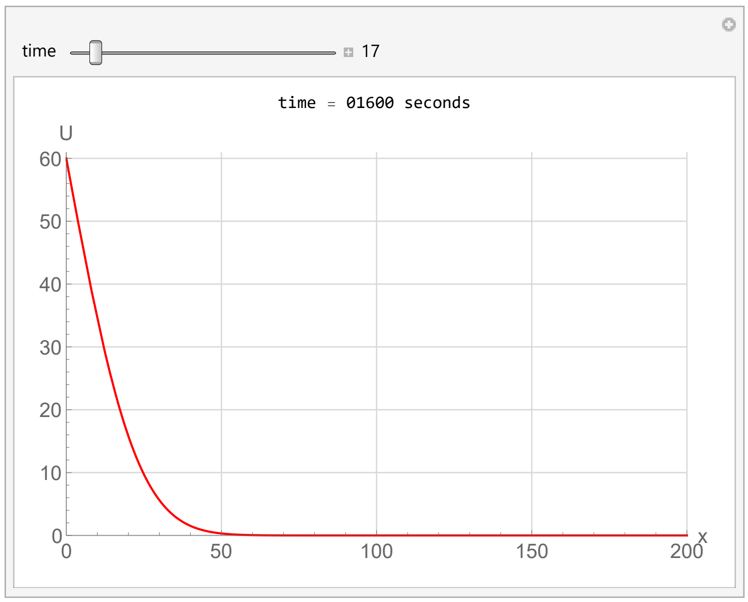


Figure 4.133: Initial state

Source code used for the above

```
In[]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
A = 60;
L = 200;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = A (1 - Erf[x / (2 Sqrt[k t])]);
```

Figure 4.134: Source code

```
In[*]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {5, 0}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {0, 61}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 30000, 100}];

In[*]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[*]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.135: Code for animation

4.1.6.3 [237] Logan p. 76. Left end general function of time (general case)

problem number 237

This is problem at page 76 from David J Logan text book.

Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = f(t)$ and initial conditions $u(x,0) = 0$

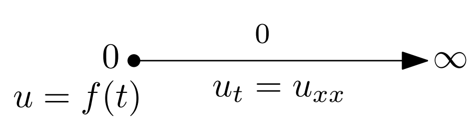


Figure 4.136: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == f[t];
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[2] -> z}
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{f(z)e^{-\frac{x^2}{4kt-4kz}}}{(t-z)^{3/2}},\{z,0,t\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=f(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{x\left(\int_0^t \frac{e^{\frac{x^2}{4(-t+\zeta)k}}f(\zeta)}{(t-\zeta)^{\frac{3}{2}}}d\zeta\right)}{2\sqrt{\pi}\,\sqrt{k}}$$

Hand solution

Solving on semi-infinite domain

$$\begin{aligned} u_t &= ku_{xx} \qquad t>0, x>0 \\ u(0,t) &= f(t) \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

With $k>0$ and $u(x,t)<\infty$ as $x\to\infty$. This means $u(x,t)$ is bounded. This conditions is always needed to solve these problems.

Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Defined as

$$\mathcal{L}(u,t)=\int_0^\infty e^{-st}u(x,t)\,dt$$

Applying Laplace transform to the original PDE (1) gives

$$sU(x,s) - u(x,0) = kU_{xx}(x,s)$$

But $u(x,0) = 0$, therefore the above becomes

$$U_{xx} - \frac{s}{k}U = 0$$

The solution to this differential equation is

$$U(x,s) = c_1 e^{\sqrt{\frac{s}{k}}x} + c_2 e^{-\sqrt{\frac{s}{k}}x}$$

Since $u(x,t)$ is bounded in the limit as $x \to \infty$ and $k > 0$, therefore it must be that $c_1 = 0$ to keep the solution bounded. The above simplifies to

$$U(x,s) = c_2 e^{-\sqrt{\frac{s}{k}}x} \tag{2}$$

At $x = 0$, $u(0,t) = f(t)$. Therefore $U(0,s) = \mathcal{L}(f(t)) = F(s)$. Hence at $x = 0$ the above gives

$$F(s) = c_2$$

Therefore (2) becomes

$$U(x,s) = F(s)\, e^{-\sqrt{\frac{s}{k}}x} \tag{3}$$

By convolution, the above becomes

$$u(x,t) = f(t) \circledast G(x,t) \tag{4}$$

Where $G(x,t)$ is the inverse transform of $e^{-\sqrt{\frac{s}{k}}x}$ which is $\frac{xe^{\frac{-x^2}{4kt}}}{2\sqrt{k\pi}t^{\frac{3}{2}}}$. Hence (4) becomes

$$\begin{aligned} u(x,t) &= f(t) \circledast \frac{xe^{\frac{-x^2}{4kt}}}{2\sqrt{k\pi}t^{\frac{3}{2}}} \\ &= \frac{x}{2\sqrt{k\pi}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4k(t-\tau)}} d\tau \end{aligned}$$

For $k = 1$

$$u(x,t) = \frac{x}{2\sqrt{\pi}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4(t-\tau)}} d\tau$$

4.1.6.4 [238] Left end function of time (special case)

problem number 238

Added July 7, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = \sin(t)$ and initial conditions $u(x,0) = 0$ using $k = \frac{1}{10}$

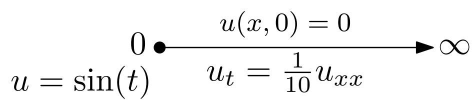


Figure 4.137: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == Sin[t];
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \sqrt{\frac{5}{2\pi}}x\text{Integrate}\left[\frac{\sin(K[2])e^{-\frac{5x^2}{2(t-K[2])}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to\text{True}\right]\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
k:=1/10;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=sin(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -\frac{\sqrt{10}\, x \left(\int_0^t -\frac{e^{-\frac{5x^2}{2\zeta}} \sin(t-\zeta)}{\zeta^{\frac{3}{2}}} d\zeta\right)}{2\sqrt{\pi}}$$

Hand solution

Solving

$$\begin{aligned} u_t &= k u_{xx} \qquad t>0, x>0 \\ u(0,t) &= f(t) \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

Using $k=\frac{1}{10}$ and $f(t)=\sin(t)$.

The general solution was solved in problem 4.1.6.3 on page 662 and the solution was found to be

$$u(x,t) = \frac{x}{2\sqrt{k\pi}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4k(t-\tau)}} d\tau$$

Replacing the given values above, the solution becomes

$$u(x,t) = \sqrt{\frac{5}{2}} \frac{x}{\sqrt{\pi}} \int_0^t \frac{\sin(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-5x^2}{2(t-\tau)}} d\tau$$

We could also use the following form of the solution

$$u(x,t) = \sqrt{\frac{5}{2}} \frac{x}{\sqrt{\pi}} \int_0^t \frac{\sin(t-\tau)}{\tau^{\frac{3}{2}}} e^{\frac{-5x^2}{2\tau}} d\tau$$

Animation is below

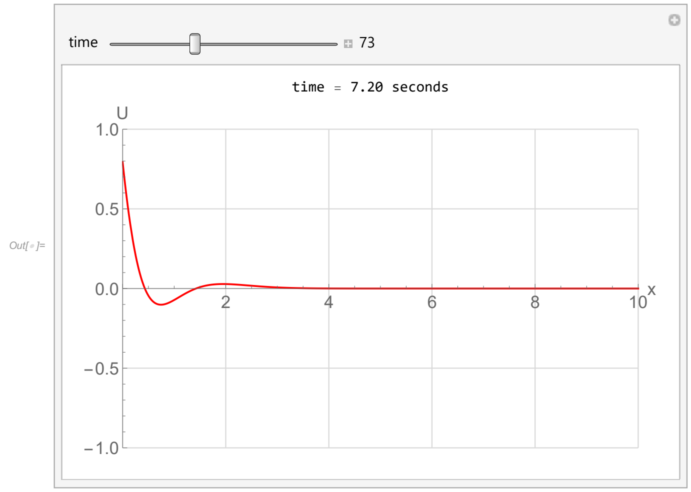


Figure 4.138: Initial state

Source code used for the above

```
In[◦]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_?NumericQ, t_?NumericQ] := x/(2 Sqrt[Pi k]) NIntegrate[Exp[-x^2/(4 k s)]/s^(3/2) Sin[t - s], {s, 0, t}, Method → {"GlobalAdaptive", "SymbolicProcessing" → 0}]
```

Figure 4.139: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
          {
            Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, 10},
                BaseStyle → 15,
                ImageMargins → 3,
                PerformanceGoal → "Quality",
                PlotRange → {{0, 10}, {-1, 1}},
                ImageSize → 500,
                AxesLabel → {"x", "U"},
                GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotStyle → Red
              ]
          }}],
      {t, 0.001, 20, 0.1}];

In[◦]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[◦]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 4.140: Code for animation

4.1.6.5 [239] nonhomogeneous BC

problem number 239

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = 1$ and And initial condition $u(x,0) = 0$

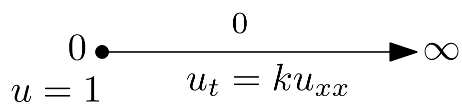


Figure 4.141: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == 1;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0:
bc  := u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) = \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

4.1.6.6 [240] I.C. not zero

problem number 240

Added December 20, 2018.

From https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,t_0) = 10;$$

And boundary conditions

$$u(-x_0,t) = 0$$

For $x > |x_0|$ and $t > |t_0|$.

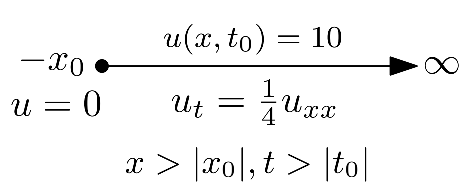


Figure 4.142: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1/4)*D[u[x, t], {x, 2}];
bc  = u[-x0, t] == 0;
ic  = u[x, t0] == 10;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t, Assumptions -> {t
```

$$\left\{\left\{u(x,t)\to\left\{\begin{array}{cc} 10\text{erf}\left(\frac{x+\text{x0}}{\sqrt{t-\text{t0}}}\right) & x+\text{x0}>0 \\ \text{Indeterminate} & \text{True} \end{array}\right.\right\}\right\}$$

due to IC/BC not zero

Maple ✓

```
restart;
pde := diff(u(x, t), t) = (1/4)*(diff(u(x, t), x$2));
bc  := u(-x0, t) = 0;
ic  := u(x, t0) = 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t)) assu
```

$$u(x,t)=10\,\text{erf}\left(\frac{x+x0}{\sqrt{t-t0}}\right)$$

4.1.6.7 [241] nonhomogeneous BC

problem number 241

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

For $x>0$ and $t>0$. The boundary conditions is $u(0,t)=\mu$ and And initial condition $u(x,0)=\lambda$

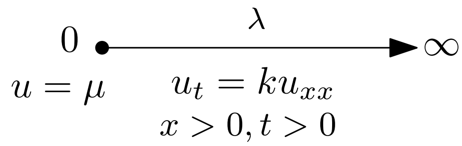


Figure 4.143: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == lambda;
ic  = u[x, 0] == mu;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\sqrt{kt}\text{Integrate}\left[\frac{\lambda e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to\text{True}\right]+\sqrt{k}\text{Integrate}\left[\mu\left(e^{-\frac{(x-K[}{4kt}}\right.\right.}{2\sqrt{\pi}k\sqrt{t}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=mu:
bc  := u(0,t)=lambda;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t)=\mu+(\lambda-\mu)\operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

4.1.6.8 [242] nonhomogeneous BC

problem number 242

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x > 0$ and $t > 0$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0) = \cos x$$

And boundary conditions

$$u(0,t) = 1$$

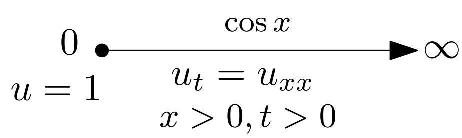


Figure 4.144: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == Cos[x];
bc  = u[0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{pde, ic, bc}, u[x, t], {x, t}]],
```

$$\left\{\left\{u(x,t)\to \left\{ \begin{array}{l} \frac{\text{Integrate}\left[\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)\cos(K[1]),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]+\sqrt{t}x\,\text{Integrate}\left[\frac{e^{-\frac{x^2}{4(t-K[2])}}}{(t-K[2])^{3/2}},\{\right.}{2\sqrt{\pi}\sqrt{t}} \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=cos(x);
bc  := u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = -\frac{\operatorname{erf}\left(\frac{2it-x}{2\sqrt{t}}\right)e^{-ix-t}}{2} + \frac{\operatorname{erf}\left(\frac{2it+x}{2\sqrt{t}}\right)e^{ix-t}}{2} - \operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right) + 1$$

4.1.6.9 [243] nonhomogeneous B.C.

problem number 243

Solve the heat equation for $u(x,t)$ on half the line $x > 0$ and $t > 0$

$$u_t = ku_{xx}$$

With initial condition

$$u(x,0) = 0$$

And boundary conditions $u(0,t) = t$. Solution is bounded at infinity.

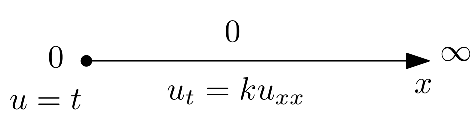


Figure 4.145: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == 0;
bc  = u[0, t] == t;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{K[2]e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to \text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)=k*diff(u(x, t), x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=t;
assume(x>0);
assume(t>0);
assume(k>0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=-\frac{2\sqrt{k}\,tx\,\mathrm{e}^{-\frac{x^2}{4kt}}+\sqrt{\pi}\left(\mathrm{erf}\left(\frac{x}{2\sqrt{k}\sqrt{t}}\right)-1\right)\left(2k\,t^{\frac{3}{2}}+\sqrt{t}\,x^2\right)}{2\sqrt{\pi}\,k\sqrt{t}}$$

4.1.6.10 [244] Unit triangle I.C.

problem number 244

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$u_t=u_{xx}$$

With initial condition

$$u(x,0)=\text{UnitTriagle[x-3]}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0$$

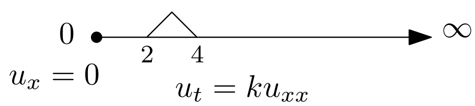


Figure 4.146: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitTriangle[x - 3];
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \Lambda(3-x) & t=0 \\ \frac{2\int_0^\infty \frac{4e^{-tK[1]^2}\cos(3K[1])\cos(xK[1])\sin^2\left(\frac{K[1]}{2}\right)}{K[1]^2}dK[1]}{\pi} & t>0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=piecewise( x>2 and x<3,-2+x, x>3 and x<4, 4-x, 0);
bc:=(D[1](u))(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{t\,e^{-\frac{(x-4)^2}{4t}} - 2t\,e^{-\frac{(x-3)^2}{4t}} + t\,e^{-\frac{(x-2)^2}{4t}} + t\,e^{-\frac{(x+2)^2}{4t}} - 2t\,e^{-\frac{(x+3)^2}{4t}} + t\,e^{-\frac{(x+4)^2}{4t}} + \frac{\left((x-4)\,\mathrm{erf}\left(\frac{x-4}{2\sqrt{t}}\right)+(-2x+6)\,\mathrm{er}\ldots}{\sqrt{\pi}\,\sqrt{t}}}{\ldots}$$

4.1.6.11 [245] I.C. not at $t = 0$

problem number 245

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ for $t > 0, x > 0$

$$u_t = \frac{1}{4} u_{xx}$$

With initial condition

$$u(x,t_0) = 10e^{-x^2}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(x_0,t) = 0$$

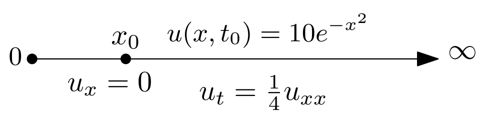


Figure 4.147: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1*D[u[x, t], {x, 2}])/4;
ic  = u[x, t0] == 10*Exp[-x^2];
bc  = Derivative[1, 0][u][x0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} 10e^{-x^2} & t - \\ \frac{2\int_0^\infty \frac{5}{2} e^{-\frac{1}{4}K[1](4ix0+(t-t0+1)K[1])}\sqrt{\pi}\cos((x-x0)K[1])\left(\text{Erfc}\left(x0-\frac{1}{2}iK[1]\right)+e^{2ix0K[1]}\,\text{Erfc}\left(x0+\frac{1}{2}iK[1]\right)\right)dK[1]}{\pi} & t - \\ \text{Indeterminate} & \text{T} \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 1/4*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=x0)=0;
ic  := u(x,t0)=10*exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = -\frac{5\left(-\operatorname{erf}\left(\frac{x+(-t+t0-1)x0}{\sqrt{t-t0}\,\sqrt{t-t0+1}}\right)\mathrm{e}^{\frac{x^2}{-t+t0-1}} - \mathrm{e}^{\frac{x^2}{-t+t0-1}} + \left(\operatorname{erf}\left(\frac{-t0\,x0+x+(t-1)x0}{\sqrt{t-t0}\,\sqrt{t-t0+1}}\right)-1\right)\mathrm{e}^{-\frac{(x-2x0)^2}{t-t0+1}}\right)}{\sqrt{t-t0+1}}$$

4.1.6.12 [246] Diffusion with advection

problem number 246

Added April 5, 2019.

Solve for $u(x,t)$ in

$$u_t = u_{xx} - u_x$$

For $t > 0, x > 0$. With boundary conditions $u(0,t) = 0$ and intitial conditions $u(x,0) = f(x)$

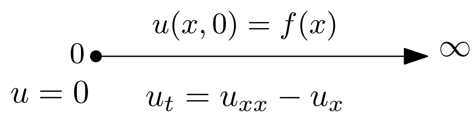


Figure 4.148: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - D[u[x, t], x];
ic  = u[x, 0] == f[x];
bc  = u[0, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t)\to e^{\frac{x}{2}-\frac{t}{4}}\left(\begin{cases} \frac{\text{Integrate}\left[e^{-\frac{K[1]}{2}}\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)f(K[1]),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{t}} & x>0 \\ \text{Indeterminate} & \text{True} \end{cases} \right)\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=diff(u(x,t),x$2)- diff(u(x,t),x);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))as
```

$$u(x,t)=\left(\mathcal{L}^{-1}\left(\frac{\int \frac{\sqrt{e^{\sqrt{4s+1}\,x}}\,f(x)}{\sqrt{e^{x}}}dx}{\sqrt{4s+1}\,\sqrt{e^{\sqrt{4s+1}\,x}}},s,t\right)-\mathcal{L}^{-1}\left(\frac{\left(\int \frac{f(x)}{\sqrt{e^{x}}\,\sqrt{e^{\sqrt{4s+1}\,x}}}dx\right)\sqrt{e^{\sqrt{4s+1}\,x}}}{\sqrt{4s+1}},s,t\right)+\mathcal{L}^{-1}\left(\frac{\int^{0} \frac{f}{\sqrt{e^{-a}}\,\sqrt{}}}{\sqrt{4s+}}\right.\right.$$

4.1.6.13 [247] Practice exam problem

problem number 247

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 13.

Solve for $u(x,t)$ with IC $u(x,0)=x^2+1$ and BC $u_t(0,t)=1$ for $x>0,t>0$

$$u_t = u_{xx}$$

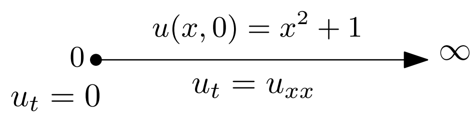


Figure 4.149: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x,t],{x,2}];
ic  = u[x,0]==x^2+1;
bc  = u[0,t]==1;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic,bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \left\{ \frac{\text{Integrate}\left[\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)(K[1]^2+1),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]+\sqrt{t}x\text{Integrate}\left[\frac{e^{-\frac{x^2}{4(t-K[2])}}}{(t-K[2])^{3/2}},\right.}{2\sqrt{\pi}\sqrt{t}} \right.\right.\right.$$
$$\text{Indeterminate}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2);
ic  := u(x,0)=x^2+1;
bc :=u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=x^2+2t-2\mathcal{L}^{-1}\left(\frac{\mathrm{e}^{-\sqrt{s}\,x}}{s^2},s,t\right)+1$$

4.1.7 Infinite domain

Local contents

4.1.7.1 [248] Inverse exponential I.C.

problem number 248

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t = u_{xx}$$

With initial condition

$$u(x,0) = e^{-x^2}$$

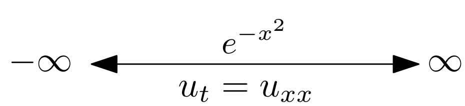


Figure 4.150: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == E^(-x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

On $-\infty < x < \infty, t > 0$ with $u(x,0) = f(x) = e^{-x^2}$. The first step is to find Green function for the above PDE. Taking Fourier transform of both sides w.r.t. x, using $\hat{u}(k,t)$ as the Fourier transform of $u(x,t)$ gives

$$\begin{aligned}\frac{d}{dt}\hat{u}(k,t) &= (ik)^2\,\hat{u}(k,t)\\ &= -k^2\hat{u}(k,t)\end{aligned}$$

$$\frac{d}{dt}\hat{u}(k,t) + k^2\hat{u}(k,t) = 0$$

The solution to the above is

$$\hat{u}(k,t) = Ce^{-k^2t} \tag{1}$$

At $t = 0$,

$$\hat{u}(k,0) = \mathcal{F}(h(x))$$

Therefore

$$C = \mathcal{F}(h(x))$$

And (1) becomes

$$\hat{u}(k,t) = \mathcal{F}(h(x))\,e^{-k^2t}$$

To find Green function, we replace $h(x)$ by $\delta(x-\xi)$ where ξ is the location of the pulse. But $\mathcal{F}(\delta(x-\xi)\,;k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta(x-\xi)\,e^{-ikx}dx = \frac{1}{\sqrt{2\pi}}e^{-i\xi k}$. Therefore the above becomes

$$\hat{G}(k,t) = \frac{1}{\sqrt{2\pi}}e^{-i\xi k}e^{-k^2t}$$

The above is the Fourier transform of the Green function. Now we invert it

$$\begin{aligned} G(x,t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi}} e^{-i\xi k} e^{-k^2 t} \right) e^{ikx} dk \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi k - k^2 t + ikx} dk \end{aligned} \tag{2}$$

We would like to use Gaussian as the integrand, hence we want to change $-i\xi k - k^2 t + ikx$ to $-\left(k\sqrt{t} - A\right)^2$. We do this by completing the square.

$$\begin{aligned} -i\xi k - k^2 t + ikx &= -\left(k\sqrt{t} - A\right)^2 \\ &= -\left(k^2 t + A^2 - 2Ak\sqrt{t}\right) \\ &= -k^2 t - A^2 + 2Ak\sqrt{t} \end{aligned}$$

Comparing sides then $2Ak\sqrt{t} = k(-i\xi + ix)$ or $A = \frac{-i\xi + ix}{2\sqrt{t}}$. Therefore

$$\begin{aligned} -i\xi k - k^2 t + ikx &= -\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2 + A^2 \\ &= -\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2 + \left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2 \end{aligned}$$

Hence

$$\begin{aligned} e^{-i\xi k - k^2 t + ikx} &= e^{-\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2 + \left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \\ &= e^{-\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} e^{\left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \end{aligned}$$

Substituting the above into (2) gives

$$\begin{aligned} G(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} e^{\left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} dk \\ &= \frac{1}{2\pi} e^{\left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} dk \end{aligned}$$

To evaluate $\int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} dk$, let $u = k\sqrt{t}$, then $du = \sqrt{t} dk$. The above becomes

$$\begin{aligned} G(x,t) &= \frac{1}{2\pi} e^{\left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(u - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \frac{du}{\sqrt{t}} \\ &= \frac{1}{2\pi\sqrt{t}} e^{\left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(u - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2} du \end{aligned}$$

Now the integral is Gaussian. $\int_{-\infty}^{\infty} e^{-\left(u-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} du = \sqrt{\pi}$ and the above becomes

$$
\begin{aligned}
G(x,t) &= \frac{\sqrt{\pi}}{2\pi\sqrt{t}} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \\
&= \frac{1}{2\sqrt{\pi t}} e^{\left(i\left(\frac{-\xi+x}{2\sqrt{t}}\right)\right)^2} \\
&= \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}}
\end{aligned}
$$

Now that we found the Green function for the PDE, we can find the solution as

$$
\begin{aligned}
u(x,t) &= \int_{-\infty}^{\infty} G(\xi,t)\, h(\xi)\, d\xi \\
&= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}} h(\xi)\, d\xi \\
&= \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi
\end{aligned}
$$

But $\int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi = \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}}$, hence the above becomes

$$
\begin{aligned}
u(x,t) &= \frac{1}{\sqrt{4\pi t}} \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}} \\
&= \frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}}
\end{aligned}
$$

4.1.7.2 [249] Advection term

problem number 249

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = 12u_{xx} + u_x \sin t$$

With initial condition

$$u(x,0) = x$$

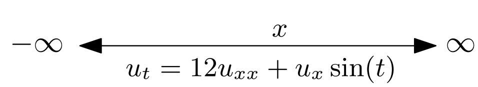


Figure 4.151: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 12*D[u[x, t], {x, 2}] + Sin[t]*D[u[x, t], x];
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t)\to t\sin(t)+x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= 12* diff(u(x,t),x$2)+sin(t)*diff(u(x,t),x);
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t))),output=
```

$$u(x,t)=x-\cos(t)+1$$

4.1.7.3 [250] UnitBox I.C.

problem number 250

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t=u_{xx}$$

With initial condition

$$u(x,0)=\text{UnitBox[x]}$$

Where UnitBox is equal to 1 if $|x|\leq\frac{1}{2}$ and zero otherwise.

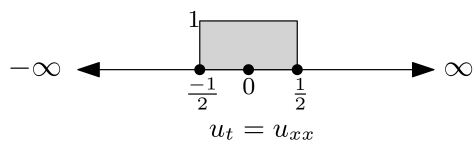


Figure 4.152: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{\int_{-\infty}^{\infty} e^{K[1](ix-tK[1])}\operatorname{sinc}\left(\frac{K[1]}{2}\right)dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = -\frac{\operatorname{erf}\left(\frac{2x-1}{4\sqrt{t}}\right)}{2} + \frac{\operatorname{erf}\left(\frac{2x+1}{4\sqrt{t}}\right)}{2}$$

4.1.7.4 [251] No source

problem number 251

Solve the heat equation

$$u_t = ku_{xx}$$

For $-\infty < x < \infty$ and $t > 0$, and initial condition is $u(x,0) = f(x)$

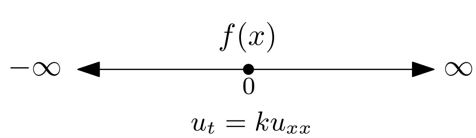


Figure 4.153: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
 sol[[2]] = sol[[2]] /. K[1] -> s;
```

$$\left\{\left\{u(x,t)\to\int_{-\infty}^{\infty}\frac{f(s)e^{-\frac{(s-x)^2}{4kt}}}{2\sqrt{\pi}\sqrt{kt}}\,ds\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t)=\frac{\int_{-\infty}^{\infty}\frac{2\pi^{\frac{3}{2}}\mathrm{e}^{-\frac{(x+\zeta)^2}{4kt}}f(-\zeta)}{\sqrt{k}\,\sqrt{t}}d\zeta}{4\pi^2}$$

4.1.7.5 [252] constant as source

problem number 252

Solve the heat equation

$$u_t = ku_{xx} + m$$

For $-\infty < x < \infty$ and $t > 0$. Initial condition is $u(x,0)=\sin(x)$

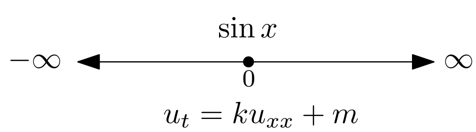


Figure 4.154: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + m;
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to e^{-kt}\sin(x) + mt\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = mt + e^{-kt}\sin(x)$$

4.1.7.6 [253] No intial conditions

problem number 253

Solve the heat equation for $u(x,t)$

$$u_t = u_x x$$

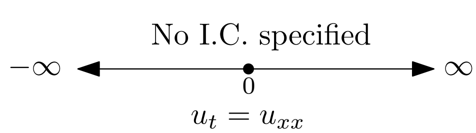


Figure 4.155: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to \cosh(c_2(x+c_2t)+c_1)+\sinh(c_2(x+c_2t)+c_1)+1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t) = c_3\left(c_1 \mathrm{e}^{2x\sqrt{-c_1}} + c_2\right) \mathrm{e}^{t\_c_1} \mathrm{e}^{-x\sqrt{-c_1}}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $t>0, -\infty<x<\infty$. Let $u = X(x)T(t)$ then we obtain

$$T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

(Only positive eigenvalues are possible). The two ODE's are

$$T' + \lambda T = 0 \tag{1}$$

$$X'' + \lambda X = 0 \tag{2}$$

Solution for (2) is $X(x) = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}$ and solution for (1) is $T(t) = C_3 e^{-\lambda t}$. Hence

$$\begin{aligned}
u(x,t) &= C_3 e^{-\lambda t}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right)\\
&= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + C_3 e^{-\lambda t} C_2 e^{-\sqrt{\lambda}x}\\
&= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + \frac{C_3 e^{-\lambda t} C_2}{e^{\sqrt{\lambda}x}}
\end{aligned}$$

4.1.7.7 [254] piecewise I.C.

problem number 254

Added December 20, 2018.

From https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t = \mu u_{xx} - 1$$

With initial condition

$$u(x,1) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$$

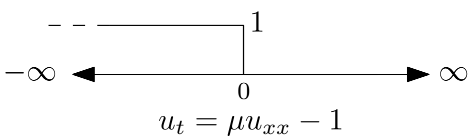


Figure 4.156: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 1 == mu*D[u[x, t], {x, 2}];
ic  = u[x, 1] == Piecewise[{{1, x <= 0}, {0, x > 0}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], x, t, Assumptions -> mu > 0
```

$$\left\{\left\{u(x,t)\to \frac{\int_{-\infty}^{\infty} \frac{i e^{K[1](ix-\mu(t-1)K[1])}}{K[1]} dK[1]}{2\pi}\right\}\right\}$$

due to i.c. not at zero

Maple ✓

```
restart;
pde := diff(u(x, t), t)+1 = mu* diff(u(x, t), x$2);
ic  := u(x, 1) = piecewise(0 <= x, 0, x < 0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t) = -t - \frac{\operatorname{erf}\left(\frac{x}{2\sqrt{t-1}\,\sqrt{\mu}}\right)}{2} + \frac{3}{2}$$

Hand solution

Solve

$$u_t = \mu u_{xx} - 1$$

for $t > 0, -\infty < x < \infty$ with initial conditions $u(x,0) = f(x) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$

Let $v = u + t$. Hence $u = v - t$ and $u_t = v_t - 1$ and $u_x = v_x$ and $u_{xx} = v_{xx}$. The above PDE becomes

$$\begin{aligned} v_t - 1 &= \mu v_{xx} - 1 \\ v_t &= \mu v_{xx} \end{aligned} \tag{1}$$

Initial conditions do not change. They are $v(x,0) = u(x,0) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$. Using Green function for 1D heat PDE on the real line, (also called heat Kernel)

$$G(x,t) = \frac{1}{\sqrt{4\pi\mu t}} e^{\frac{-x^2}{4\mu t}}$$

Then the solution to (1) is

$$\begin{aligned} v(x,t) &= \int_{-\infty}^{\infty} f(x')\, G(x-x',t)\, dx' \\ &= \int_{-\infty}^{0} \frac{1}{\sqrt{4\pi\mu t}} e^{\frac{-(x-x')^2}{4\mu t}} dx' \\ v(x,t) &= \frac{-1}{\sqrt{4\pi\mu t}} \int_{0}^{\infty} e^{\frac{-(x-x')^2}{4\mu t}} dx' \end{aligned}$$

But $\int_0^\infty e^{\frac{-(x-x')^2}{4\mu t}} dx' = \sqrt{\pi\mu t}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$, hence

$$v(x,t) = \frac{-1}{2}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$$

Since $u = v - t$ then

$$\begin{aligned} u(x,t) &= \frac{-1}{2}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right) - t \\ &= -\frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right) - t \end{aligned}$$

4.1.7.8 [255] Practice exam problem

problem number 255

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 14.

Solve for $u(x,t)$ with IC $u(x,0) = x$ for $-\infty < x < \infty, t > 0$

$$u_t = u_{xx}$$

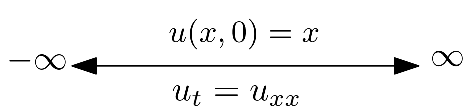


Figure 4.157: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x,t],{x,2}];
ic  = u[x,0]==x;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2);
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x$$

4.2 Diffusion in 2D

Local contents

4.2.1 Cartesian coordinates (Rectangle, Square)

Local contents

4.2.1.1 [256] No source

problem number 256

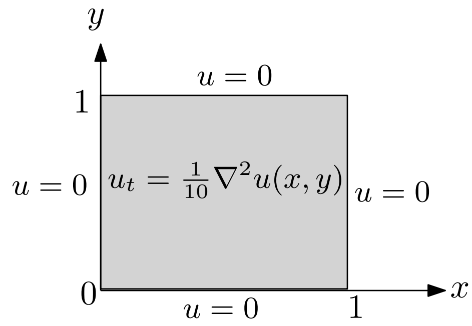
Taken from Maple help pages on PDE. Solve the heat equation for $u(x, y, t)$

$$u_t = \frac{1}{10}\nabla^2 u(x, y)$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0, y, t) &= 0 \\ u(1, y, t) &= 0 \\ u(x, 0, t) &= 0 \\ u(x, 1, t) &= 0 \end{aligned}$$

Initial condition is $u(x, y, 0) = x(1 - x)(1 - y)y$.



At $t = 0, u = x(1-x)(1-y)y$

Figure 4.158: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10;
ic  = u[x, y, 0] == x*(1 - x)*(1 - y)*y;
bc  = {u[0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, u[x, 1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \left\{\ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{16\left(-1+(-1)^{K[1]}\right)\left(-1+(-1)^{K[3]}\right)e^{\frac{1}{10}t\left(-\pi^2K[1]^2-\pi^2K[3]^2\right)}\sin(\pi xK[1])\sin(\pi yK[3])}{\pi^6K[1]^3K[3]^3}\quad (K[1]|K[\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc  := u(0, y, t) = 0, u(1, y, t) = 0, u(x, 0, t) = 0, u(x, 1, t) = 0;
ic  := u(x, y, 0) = x*(1-x)*(1-y)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,y,t))),out
```

$$u(x,y,t)=\sum_{n1=1}^{\infty}\sum_{n=1}^{\infty}\left(-\frac{16\left((-1)^{n}+(-1)^{n1}-(-1)^{n+n1}-1\right)\mathrm{e}^{-\frac{\pi^2\left(n^2+n1^2\right)t}{10}}\sin\left(\pi nx\right)\sin\left(\pi n1y\right)}{\pi^6n^3n1^3}\right)$$

4.2.1.2 [257] Internal source term

problem number 257

Taken from Maple help pages on PDE

Solve the heat equation for $u(x,y,t)$

$$\frac{\partial u}{\partial t} = 1/10\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{5}u(x,y,t);$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned}
\frac{\partial u}{\partial x}u(0,y,t) &= 0 \\
u(1,y,t) &= 0 \\
u(x,0,t) &= 0 \\
\frac{\partial u}{\partial y}u(x,1,t) &= 0
\end{aligned}$$

Initial condition is $u(x,y,0) = (1-x^2)(1-\frac{1}{2}y)y$.

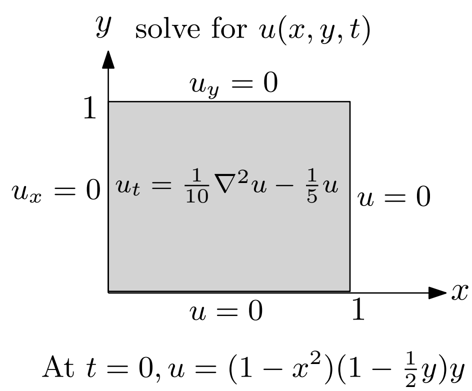


Figure 4.159: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10 - (1*u[x,
ic  = u[x, y, 0] == (-x^2 + 1)*(1 - (1/2)*y)*y;
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \left\{\ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}-\frac{512(-1)^{K[1]}\exp\left(t\left(\frac{1}{10}\left(-\frac{1}{4}\pi^2(2K[1]-1)^2-\frac{1}{4}\pi^2(2K[3]-1)^2\right)-\frac{1}{5}\right)\right)\cos\left(\frac{1}{2}\pi x(2K[1]-1)\right)\sin\left(\frac{1}{2}\pi y(\right.}{\pi^6(2K[1]-1)^3(2K[3]-1)^3}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2)) - 1/5 * u(x
ic  := u(x, y, 0) = (-x^2+1)*(1-(1/2)*y)*y;
bc  := (D[1](u))(0, y, t) = 0,
       u(1, y, t) = 0,
       u(x, 0, t) = 0,
       (D[2](u))(x, 1, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
```

$$u(x,y,t)=\sum_{n1=0}^{\infty}\sum_{n=0}^{\infty}\frac{512(-1)^n\cos\left(\frac{(2n+1)\pi x}{2}\right)\mathrm{e}^{-\frac{\left(\pi^2n^2+\pi^2n1^2+\pi^2n+\pi^2n1+2+\frac{1}{2}\pi^2\right)t}{10}}\sin\left(\frac{(2n1+1)\pi y}{2}\right)}{\pi^6\left(2n+1\right)^3\left(2n1+1\right)^3}$$

4.2.1.3 [258] Articolo 6.6.3

problem number 258

Added December 20, 2018.

Example 6.6.3 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin rectangular plate over the finite two-dimensional domain $D=(x,y)$s.t.$0<x<1,0<y<1$. The lateral surfaces of the plate are insulated. The boundaries $y=0$ and $y=1$ are fixed at temperature 0, the boundary $x=0$ is insulated, and the boundary $x=1$ is losing heat by convection into a surrounding medium at temperature 0. The initial temperature distribution f(x, y) is

$$u(x,y,0)=\left(1-\frac{x^2}{3}\right)y(1-y)$$

The thermal diffusivity is $k=\frac{1}{50}$. Solve for $u(x,y,t)$ the heat PDE

$$\frac{\partial u}{\partial t}=k\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

With $0<x<1, 0<y<1$ and $t>0$. Boundary conditions are

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y,t)&=0\\ \frac{\partial u}{\partial x}(1,y,t)+u(1,y,t)&=0\\ u(x,0,t)&=0\\ u(x,1,t)&=0\end{aligned}$$

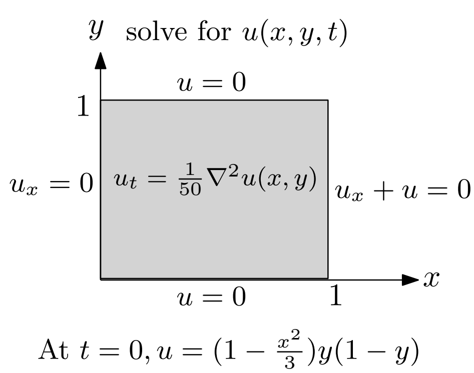


Figure 4.160: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/50;
pde =  D[u[x, y, t], t] == k*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]);
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] + u[1, y, t] ==
ic  = u[x, y, 0] == (1 - (1/3)*x^2)*y*(1 - y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\left\{\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{8\left(-1+(-1)^{K[3]}\right)e^{\frac{1}{50}t\left(-\pi^2K[3]^2-K[2,K[1]]\right)}\cos\left(x\sqrt{K[2,K[1]]}\right)\sin(\pi yK[3])\left(\frac{2\cos\left(\sqrt{K[2,K[1]]}\right)}{K[2,K[1]]}-\frac{2(K\cdots}{\cdots}\right.}{3\pi^3K[3]^3\left(\sin^2\left(\sqrt{K[2,K[1]]}\right)+1\right)}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
k:=1/50;
pde := diff(u(x, y, t), t) = k*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc_left_edge:=eval( diff(u(x,y,t),x),x=0)=0;
bc_right_edge:= eval( diff(u(x,y,t),x),x=1)+u(1,y,t)=0;
bc_bottom_edge:=u(x,0,t)=0;
bc_top_edge:=u(x,1,t)=0;
bc:=bc_left_edge,bc_right_edge,bc_bottom_edge,bc_top_edge;
ic  := u(x, y, 0) = (1-(1/3)*x^2)*y*(1-y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x, y, t))
```

$$u(x,y,t)=\sum_{n1=1}^{\infty}\sum_{n=0}^{\infty}\left(-\frac{32\left((-1)^{n1}-1\right)\left(\lambda_n^2\sin\left(\lambda_n\right)-\lambda_n\cos\left(\lambda_n\right)+\sin\left(\lambda_n\right)\right)\cos\left(x\lambda_n\right)\mathrm{e}^{-\frac{\left(\pi^2 n1^2+\lambda_n^2\right)t}{50}}\sin\left(\pi\cdots\right)}{3\pi^3\left(2\lambda_n+\sin\left(2\lambda_n\right)\right)n1^3\lambda_n^2}\right.$$

4.2.2 Polar coordinates (disk, sector, annulus)

Local contents

4.2.2.1 [259] no θ dependency, insulated (General solution)

problem number 259

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr} + \frac{1}{r}u_r)$$

For $0 < r < L$ and $t > 0$. The boundary conditions are such it is insulated

$$u_r(L,t) = 0$$

Initial condition is $u(r,0) = f(r)$.

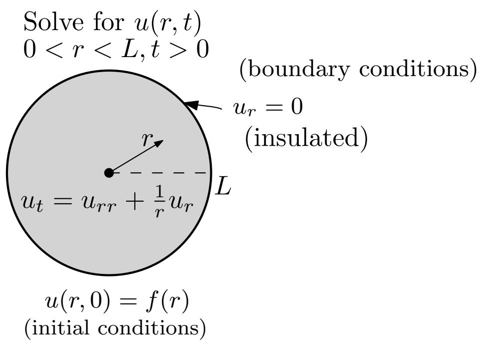


Figure 4.161: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == f[r];
bc  = Derivative[1,0][u][L, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions->{r
```

$$\left\{\left\{u(r,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}e^{-\frac{kt\left(j_{1,K[1]}\right)^2}{L^2}}J_0\left(\frac{rj_{1,K[1]}}{L}\right)\int_0^L\frac{\sqrt{2}rJ_0\left(\frac{rj_{1,K[1]}}{L}\right)f(r)}{LJ_0\left(j_{1,K[1]}\right)}dr}{LJ_0\left(j_{1,K[1]}\right)}+\frac{\sqrt{2}\int_0^L\frac{\sqrt{2}rf(r)}{L}dr}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)=f(r);
bc  := D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t)=\sqrt{k}\left(\int_{-L+r}^{0}\mathcal{L}^{-1}\left(\frac{\mathrm{BesselJ}\left(0,\frac{\sqrt{-s}\,(r-\tau)}{\sqrt{k}}\right)\mathcal{L}\left(\frac{-D(f)(L+\tau)+(-L-\tau)D^{(2)}(f)(L+\tau)}{L+\tau},t,s\right)}{\sqrt{-s}\ \mathrm{BesselJ}\left(1,\frac{\sqrt{-s}\,L}{\sqrt{k}}\right)},s,t\right)d\tau\right)-\sqrt{k}\,\mathcal{L}$$

Cant get series solution

Hand solution

Solve for $u(r,t)$

$$
\begin{aligned}
u_t &= k\left(u_{rr} + \frac{1}{r}u_r\right)\\
|u(0,t)| &< \infty\\
u_r(L,t) &= 0\\
u(r,0) &= f(r)
\end{aligned}
$$

for $0<r<L$. Let $u=RT$. Substituting in the PDE gives

$$
\begin{aligned}
\frac{T'R}{k} &= R''T + \frac{1}{r}R'T\\
\frac{T'}{kT} &= \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\lambda
\end{aligned}
$$

The above gives the following ODE's to solve

$$
T' + \lambda k T = 0
$$

And

$$
\begin{aligned}
R'' + \frac{1}{r}R' + \lambda R = 0 \qquad (1)\\
R'(L) = 0\\
R(0) < \infty
\end{aligned}
$$

Case $\lambda < 0$

Let $\lambda = -n^2$ for positive n. (1) becomes

$$
\begin{aligned}
R'' + \frac{1}{r}R' - n^2 R &= 0\\
r^2R'' + rR' - n^2r^2R &= 0
\end{aligned}
$$

This is a Bessel ODE whose solution is $R(r) = C_1 \operatorname{BesselI}(0,nr) + C_2 \operatorname{Besselk}(0,nr)$. Since $R(r)$ is bounded at $r=0$ then $C_2=0$ since Besselk blows up at $r=0$. The solution becomes $R(r) = C_1 \operatorname{BesselI}(0,nr)$. Then $R'(r) = C_1 \operatorname{BesselI}(1,nr)$. Hence we need to solve for n in the following $R'(L) = 0 = \operatorname{BesselI}(1,nL)$. But BesselI has zero only at zero. Therefore $\lambda<0$ is not possible eigenvalue.

Case $\lambda = 0$

Equation (1) becomes $R'' + \frac{1}{r}R' = 0$. The solution is $R(r) = C_1 \ln(r) + C_2$. Since bounded at $r=0$ then $C_1=0$. The solution becomes $R(r) = C_2$ and $R'(r) = 0$. Which satisfies the boundary condition for any constant C_2. Therefore $\lambda = 0$ is an eigenvalue with eigenfunction $R_0(r) = 1$ as the choice of the constant.

Case $\lambda > 0$

Equation (1) becomes

$$r^2 R'' + rR' + \lambda r^2 R = 0$$

This is a Bessel ODE whose solution is $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$. Since $R(r)$ is bounded at $r = 0$ then $C_2 = 0$ since BesselY blows up at $r = 0$. The solution becomes $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$. Then $R'(r) = -C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} r\right)$. Absorbing the minus sign into the constant, hence $R'(r) = C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} r\right)$. At $r = L$ we want

$$0 = C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} L\right)$$

For non-trivial solution, $\sqrt{\lambda} L$ are the zeros of the $\operatorname{BesselJ}(1, x)$. Let these zeros be Λ_n where $n = 1, 2, \cdots$. Hence

$$\sqrt{\lambda_n} L = \Lambda_n \qquad n = 1, 2, \cdots$$

$$\lambda_n = \left(\frac{\Lambda_n}{L}\right)^2$$

The corresponding eigenfunctions are

$$R_n(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right) \qquad n = 1, 2, \cdots$$

Now that we found the eigenvalues and eigenfunctions, we can solve the time ODE. For the zero eigenvalue $\lambda = 0$ the ODE $T' + \lambda k T = 0$ becomes $T' = 0$, hence $T_0(t)$ is a constant. For $\lambda > 0$ the time ODE has solution $T_n(t) = e^{-k\lambda_n t} = e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t}$. Therefore the complete solution is

$$u(r,t) = C_0 + \sum_{n=1}^{\infty} C_n e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t} \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right)$$

Now we find the constants from the initial conditions. At $t = 0$

$$f(r) = C_0 + \sum_{n=1}^{\infty} C_n \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right)$$

For $n=0$, applying orthogonality gives

$$\begin{aligned}\int_0^L f(r)\, r dr &= \int_0^L C_0 r dr\\ &= C_0 \frac{L^2}{2}\\ C_0 &= \frac{2}{L^2}\int_0^L f(r)\, r dr\end{aligned}$$

For $n>0$

$$\begin{aligned}\int_0^L f(r) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right) r dr &= \int_0^L C_n \operatorname{BesselJ}^2\left(0, \frac{\Lambda_n}{L} r\right) r dr\\ C_n &= \frac{\int_0^L f(r) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right) r dr}{\int_0^L \operatorname{BesselJ}^2\left(0, \frac{\Lambda_n}{L} r\right) r dr}\end{aligned}$$

Hence the final solution is

$$u(r,t) = \frac{2}{L^2}\int_0^L f(r)\, r dr + \sum_{n=1}^{\infty}\left(\frac{\int_0^L f(r) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right) r dr}{\int_0^L \operatorname{BesselJ}^2\left(0, \frac{\Lambda_n}{L} r\right) r dr}\right) e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t} \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right)$$

Where Λ_n are the zeros of the $\operatorname{BesselJ}(1,x)$.

4.2.2.2 [260] no θ dependency, insulated (Specific solution)

problem number 260

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr} + \frac{1}{r} u_r)$$

For $0<r<L$ and $t>0$ and $k=1, L=1$. The boundary conditions are such it is insulated

$$u_r(L,t) = 0$$

Initial condition is $u(r,0) = 2Lr - r^2$.

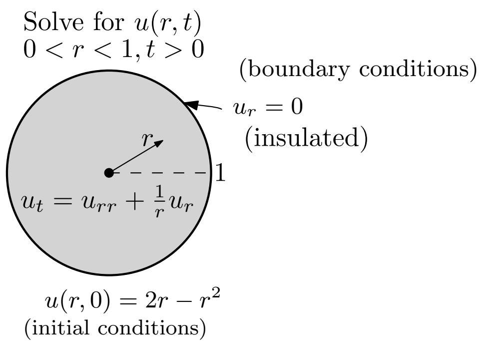


Figure 4.162: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
 L=1;
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == 2*L*r-r^2;
bc  = Derivative[1,0][u][L, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} -\frac{2e^{-t(j_{1,K[1]})^2} J_0\left(r j_{1,K[1]}\right) \left(\frac{2J_2\left(j_{1,K[1]}\right)}{\left(j_{1,K[1]}\right)^2} - \frac{2}{3}\,{}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}\left(j_{1,K[1]}\right)^2\right) - \frac{J_3\left(j_{1,K[1]}\right)}{j_{1,K[1]}}\right)}{J_0\left(j_{1,K[1]}\right)^2} + \frac{5}{6}\right\}\right\}$$

Maple ✓

```
restart;
k:=1;
L:=1;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)= 2*L*r-r^2;
bc  := D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t))),outpu
```

$$u(r,t) = -r^2 + 2r - \left(\int_0^{r-1} \mathcal{L}^{-1}\left(\frac{\operatorname{BesselJ}\left(0, \sqrt{-s}\,(r-\tau)\right)\mathcal{L}\left(\frac{4\tau+2}{\tau+1}, t, s\right)}{\sqrt{-s}\,\operatorname{BesselJ}\left(1, \sqrt{-s}\right)}, s, t\right) d\tau\right)$$

Do not understand Maple solution
Hand solution

Solve for $u(r,t)$

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r\right)$$
$$|u(0,t)| < \infty$$
$$u_r(1,t) = 0$$
$$u(r,0) = 2Lr - r^2$$

for $0 < r < L$.

The basic solution for this type of PDE was already given in problem 4.2.2.1 on page 700 as

$$u(r,t) = \frac{2}{L^2}\int_0^L f(r)\, r dr + \sum_{n=1}^{\infty}\left(\frac{\int_0^L f(r)\operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L}r\right) r dr}{\int_0^L \operatorname{BesselJ}^2\left(0, \frac{\Lambda_n}{L}r\right) r dr}\right) e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t}\operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L}r\right)$$

Where Λ_n are the zeros of the $\operatorname{BesselJ}(1,x)$. In this problem $L=1$ and $k=1, f(r) = 2Lr - r^2$. This is animation of the above solution using these specific values for for 0.2 seconds. (Animation will only show in the HTML version)

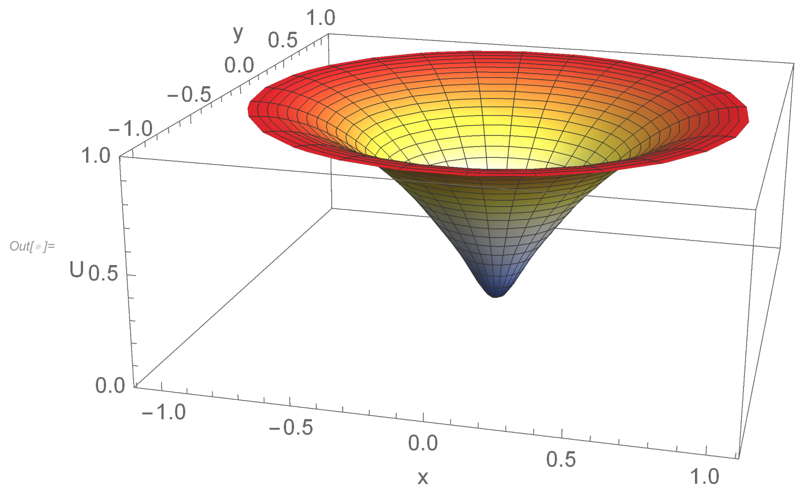


Figure 4.163: Initial state

Source code used for the above

```
In[∘]:= ClearAll[r, u, t, lam, mySol]
L = 1;
k = 1;
f = 2 L r - r^2;
maxTime = 30;
numberOfTerms = 10;
z = N@Table[ BesselJZero[1, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ1*)
An = Table[Integrate[f BesselJ[0, z[[n]]/L r] r, {r, 0, L}] / Integrate[ BesselJ[0, z[[n]]/L r]^2 r, {r, 0, L}], {n, 1, numberOfTerms}];
mySol[r_, t_] = 2/L^2 Integrate[r f, {r, 0, L}] + Sum[An[[n]] Exp[- k (z[[n]]/L)^2 t] BesselJ[0, z[[n]]/L r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
(*display initial conditions*)
```

Figure 4.164: Source code

```
In[ ]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (2 L r-r^2)"}]},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, L}, {θ, 0, 2 Pi},
                 BaseStyle → 15,
                 ImageMargins → 3,
                 Mesh → 25,
                 PerformanceGoal → "Quality",
                 BoxRatios → {1, 1, 0.4},
                 PlotRange → {Automatic, Automatic, {0, (f /. r → L)}},
                 ImageSize → 500,
                 ColorFunctionScaling → False,
                 ColorFunction → ColorData[{"TemperatureMap", {0, (f /. r → L)}}],
                 AxesLabel → {"x", "y", "U"},
                 ViewPoint → {0.623, -2.678, 0.896},
                 Boxed → True, Axes → True
               ]
              }}],
          {t, 0, 0.18, .001}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → Table[.05, {Length[tab]}]]
```

Figure 4.165: Code used for animation

4.2.2.3 [261] no θ dependency

problem number 261

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr} + \frac{1}{r}u_r)$$

For $0 < r < L$ and $t > 0$. The boundary conditions are Newton Law of cooling, where $h > 0$

$$Lu(L,t) = -hu(L,t)$$

Initial condition is $u(r,0) = f(r)$.

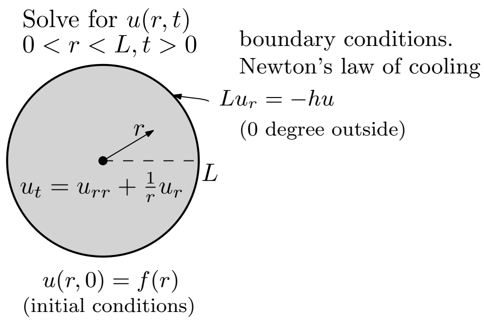


Figure 4.166: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == f[r];
bc  = L*Derivative[1,0][u][L, t] == -h*u[L,t];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions->{r
sol = sol/. K[1]->n;
```

$$\left\{\left\{u(r,t)\to \left\{ \begin{array}{cc} \sum_{n=1}^{\infty} \frac{\sqrt{2}e^{-ktK[2,n]^2}\,\text{BesselJ}(0,rK[2,n])\left(\int_0^L \frac{\sqrt{2}r\,\text{BesselJ}(0,rK[2,n])f(r)K[2,n]}{\text{BesselJ}(0,LK[2,n])\sqrt{h^2+L^2K[2,n]^2}}\,dr\right)K[2,n]}{L\,\text{BesselJ}(0,LK[2,n])\sqrt{\frac{h^2}{L^2}+K[2,n]^2}} & h\,\text{BesselJ}(0,LK[2,n \\ \text{Indeterminate} & \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)=f(r);
bc  := L*D[1](u)(L,t)=-h*u(L,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t)=hf(L)\ln(L)-hf(L)\ln(r)-2\left(\sum_{n=0}^{\infty}\frac{\left(-\cosh\left(\frac{kt\lambda_n^2}{L^2}\right)+\sinh\left(\frac{kt\lambda_n^2}{L^2}\right)\right)\text{BesselJ}\left(0,\frac{r\lambda_n}{L}\right)\left(\int_0^L\left(-hf(L\right.\right.}{\left(\text{BesselJ}\left(0,\lambda_n\right)\right)^2+\text{B}}\right.$$

4.2.2.4 [262] no θ dependency

problem number 262

Taken from Mathematica DSolve help pages

Solve the heat equation in polar coordinates for $u(r,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

For $0 < r < 1$ and $t > 0$. The boundary conditions are

$$u(1,t) = 0$$

Initial condition is $u(r,0) = 1 - r$.

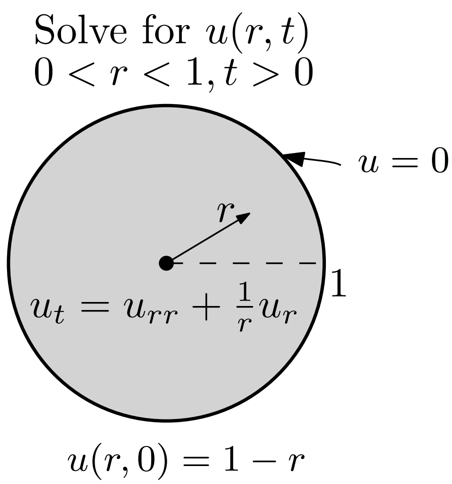


Figure 4.167: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r;
ic  = u[r, 0] == 1 - r;
bc  = u[1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{e^{-t(j_{0,n})^2}\pi J_0(r j_{0,n})\left(J_1(j_{0,n})\boldsymbol{H}_0(j_{0,n}) - J_0(j_{0,n})\boldsymbol{H}_1(j_{0,n})\right)}{J_1(j_{0,n})^2 (j_{0,n})^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r);
ic  := u(r,0)=1-r;
bc  := u(1,t) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t) = \sum_{n=1}^{\infty} \frac{\pi(-\operatorname{BesselJ}(0,\lambda_n)\operatorname{StruveH}(1,\lambda_n)+\operatorname{BesselJ}(1,\lambda_n)\operatorname{StruveH}(0,\lambda_n))\left(\cosh\left(t\lambda_n^2\right)-\sinh\left(t\lambda_n^2\right.\right.}{\left(\operatorname{BesselJ}(0,\lambda_n)^2+\operatorname{BesselJ}(1,\lambda_n)^2\right)\lambda_n^2}$$

4.2.2.5 [263] Haberman 8.3.5 (General solution)

problem number 263

Added Nov 24, 2018.

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$.

One of the problems here, is how to tell CAS the implicit condition when solving this which is that $u(0,t) < \infty$.

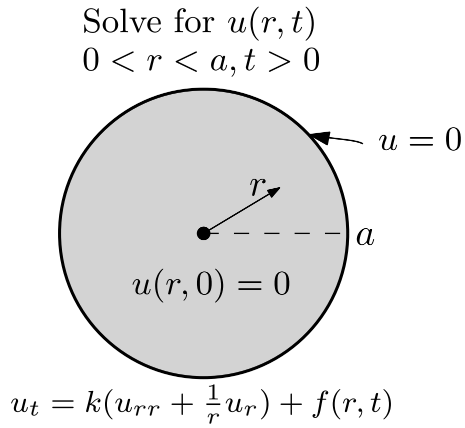


Figure 4.168: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f[r, t];
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2} J_0\left(\frac{r j_{0,K[1]}}{a}\right) \int_0^t e^{-\frac{k\left(j_{0,K[1]}\right)^2 (t-K[2])}{a^2}} \text{Integrate}\left[\frac{\sqrt{2} r J_0\left(\frac{r j_{0,K[1]}}{a}\right) f(r,K[2])}{a J_1\left(j_{0,K[1]}\right)}, \{r,0,a\}, \text{Assumpti}\right.}{a J_1\left(j_{0,K[1]}\right)}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f(r,t);
ic  := u(r,0)=0;
bc  := u(a,t) =0;
#do not use HINT=boundedseries below, Maple will  not solve it then
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t) = -\left(\int_{-a+r}^{0} \mathcal{L}^{-1}\left(\frac{\operatorname{BesselJ}\left(0, \sqrt{-\frac{s}{k}}\,(r-\tau)\right) \mathcal{L}\left(-\frac{f(a+\tau,t)}{k}, t, s\right)}{\operatorname{BesselJ}\left(0, \sqrt{-\frac{s}{k}}\,a\right)}, s, t\right) d\tau\right)$$

Hand solution

Since this problem has homogeneous B.C. but has time dependent source (i.e. non-homogenous in the PDE itself), then we will use the method of eigenfunction expansion. In this method, we first find the eigenfunctions $\phi_n(x)$ of the associated homogenous PDE without the source being present. Then use these $\phi_n(x)$ to expand the source $f(x,t)$ as generalized Fourier series. We now switch to the associated homogenous PDE in order to find the eigenfunctions. $u \equiv u(r,t)$. There is no θ. Hence

$$\begin{aligned} \frac{\partial u(r,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \qquad (1)\\ u(a,t) &= 0\\ |u(0,t)| &< \infty\\ u(r,0) &= 0 \end{aligned}$$

We need to solve the above in order to find the eigenfunctions $\phi_n(r)$. Let $u = R(r)T(t)$. Substituting this back into (1) gives

$$T'R = k\left(R''T + \frac{1}{r}R'T\right)$$

Dividing by RT

$$\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\lambda$$

Where λ is the separation constant. The above gives

$$T' + k\lambda T = 0$$

And

$$rR'' + R' + \lambda r R = 0$$

This is a singular Sturm-Liouville ODE. Standard form is

$$(rR')' = -\lambda r R$$

Hence

$$\begin{aligned} p &= r \\ q &= 0 \\ \sigma &= r \end{aligned}$$

The ODE $rR'' + R' + \lambda r R = 0$ is Bessel ODE whose solution is

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$$

Since $\operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$ blows up at $r = 0$, then $C_2 = 0$ and the solution becomes

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$$

At $r = a$ the above becomes $0 = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} a\right)$. Non trivial solution requires that $\sqrt{\lambda} a$ are the zeros of $\operatorname{BesselJ}(0, x)$. Let the zeros be called $\Lambda_n, n = 1, 2, 3, \cdots$. Therefore $\sqrt{\lambda_n} a = \Lambda_n$ or

$$\lambda_n = \left(\frac{\Lambda_n}{a}\right)^2 \qquad n = 1, 2, 3, \cdots$$

The corresponding eigenfunctions are $R_n(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$. Now that the eigenfunctions for the homogeneous PDE are found, eigenfunction expansion is used to find the general solution. Let

$$u(r,t) = \sum_{n=1}^{\infty} a_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \tag{2}$$

Where $a_n(t)$ is function of time since it includes the time solution in it. Substituting the above back into the original nonhomogeneous PDE

$$\begin{aligned} u_t &= k\nabla^2 u + f(r,t) \\ &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + f(r,t) \end{aligned} \tag{3}$$

Where $\nabla^2 u = -\lambda r u$. Substituting (2) into (3), and using $f(r,t) = \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$ gives

$$\sum_{n=1}^{\infty} a_n'(t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)=ka_n(t)\left(\sum_{n=1}^{\infty}\operatorname{BesselJ}''\left(0,\frac{\Lambda_n}{a}r\right)+\frac{1}{r}\operatorname{BesselJ}'\left(0,\frac{\Lambda_n}{a}r\right)\right)+\sum_{n=1}^{\infty} b_n(t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}\right.$$

But $\sum_{n=1}^{\infty}\operatorname{BesselJ}''\left(0,\frac{\Lambda_n}{a}r\right)+\frac{1}{r}\operatorname{BesselJ}'\left(0,\frac{\Lambda_n}{a}r\right)=-\lambda_n\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$. The above becomes

$$\sum_{n=1}^{\infty} a_n'(t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)=-ka_n(t)\sum_{n=1}^{\infty}\left(\frac{\Lambda_n}{a}\right)^2\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)+\sum_{n=1}^{\infty} b_n(t)\operatorname{BesselJ}\left(0\right.$$

$$\sum_{n=1}^{\infty}\left(a_n'(t)+k\left(\frac{\Lambda_n}{a}\right)^2 a_n(t)\right)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)=\sum_{n=1}^{\infty} b_n(t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

The above simplifies to

$$a_n'(t)+k\left(\frac{\Lambda_n}{a}\right)^2 a_n(t)=b_n(t)$$

The solution is

$$a_n(t)=e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau+a_n(0)\,e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}$$

Hence the solution (2) becomes

$$u(r,t)=\sum_{n=1}^{\infty}\left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)+a_n(0)\,e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\right)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

To find $a_n(0)$, putting $t=0$ in the above gives

$$0=\sum_{n=1}^{\infty} a_n(0)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

Hence $a_n(0)=0$. Therefore $a_n(t)$ becomes.

$$a_n(t)=e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau$$

Hence the solution from (2) now becomes

$$u(r,t)=\sum_{n=1}^{\infty}\left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

And finally, to find $b_n(t)$, which is the generalized Fourier coefficient of the expansion of the source in (3) above, orthogonality is used as follows

$$\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) r dr = b_n(t)\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right) r dr$$
$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) r dr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right) r dr}$$

Summary of solution

$$u(r,t) = \sum_{n=1}^{\infty} a_n(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$
$$= \sum_{n=1}^{\infty}\left(\int_0^t b_n(\tau)\, e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau} d\tau\right) e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

Where

$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) r dr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right) r dr}$$

4.2.2.6 [264] Specific example of the above

problem number 264

Added June 16,2019

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition, but added specific values for parameters in order to do animations

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$. Let $k = \frac{1}{100}, a = 2$. Let $f(r,t) = sin(t)$

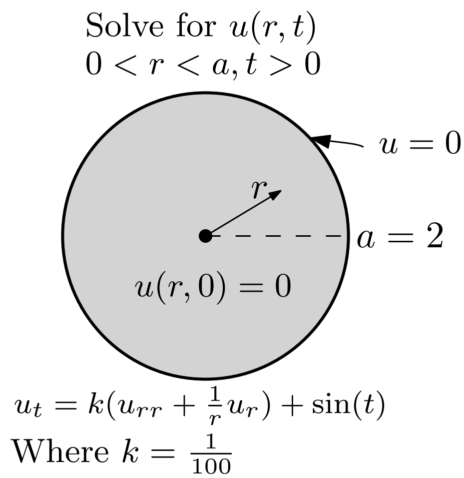


Figure 4.169: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=2;
k=1/100;
f=Sin[t];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f;
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{800\,\text{BesselJ}\left(0, \frac{1}{2}r\,\text{BesselJZero}(0,n)\right)\left(\sin(t)\,\text{BesselJZero}(0,n)^2 + 400\left(e^{-\frac{1}{400}t\,\text{BesselJZero}(0,n}\right.\right.}{\text{BesselJ}(1, \text{BesselJZero}(0,n))\,\text{BesselJZero}(0,n)\left(\text{BesselJZero}(0,n)^4 + 16000\right.}\right.\right.$$

Maple ✓

```
restart;
a:=2;
k:=1/100;
f:=sin(t);
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f;
ic  := u(r,0)=0;
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t)=\mathcal{L}^{-1}\left(\frac{J_0\left(10\sqrt{-s}\,r\right)s}{J_0\left(20\sqrt{-s}\right)\left(s^2+1\right)},s,t\right)-\mathcal{L}^{-1}\left(\frac{J_0\left(10\sqrt{-s}\,r\right)}{J_0\left(20\sqrt{-s}\right)s},s,t\right)-\cos\left(t\right)+1$$

Hand solution

The basic solution for this type of PDE was already given in problem 4.2.2.5 on page 711 as

$$\begin{aligned}u(r,t)&=\sum_{n=1}^{\infty}a_n(t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)\\&=\sum_{n=1}^{\infty}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)e^{-k\left(\frac{\Lambda_n}{a}\right)^2t}\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)\end{aligned}$$

Where

$$b_n(t)=\frac{\int_0^a f(r,t)\operatorname{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \operatorname{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}$$

Where Λ_n are the n^{th} zeros of BesselJ $(0,x)$. In this problem

$$\begin{aligned}a&=2\\k&=\frac{1}{100}\\f(r,t)&=\sin\left(t\right)\end{aligned}$$

This is animation of the above solution using these specific values for for 50 seconds. (Animation will only show in the HTML version)

Out[ ]=

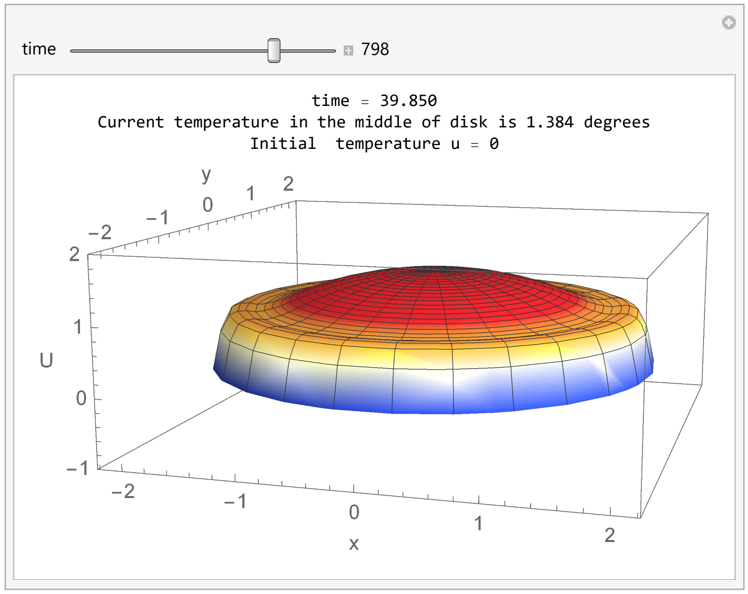


Figure 4.170: Initial state

Source code used for the above

```
In[ ]:= ClearAll[r, u, t, lam, mySol]
a = 2;
k = 1 / 100;
f = Sin[t];
maxTime = 30;
numberOfTerms = 20;
z = N@Table[ BesselJZero[0, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ0*)
bn = Table[ Integrate[f BesselJ[0, z[[n]]/a r] r, {r, 0, a}] / Integrate[ BesselJ[0, z[[n]]/a r]^2 r, {r, 0, a}], {n, 1, numberOfTerms}];
mySol[r_, t_] = Chop@Sum[Integrate[ (bn[[n]] /. t -> s) Exp[ k (z[[n]]/a)^2 s], {s, 0, t}] Exp[- k (z[[n]]/a)^2 t] BesselJ[0, z[[n]]/a r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns -> {"", ""}, NumberPadding -> {"0", "0"}, SignPadding -> True];
```

Figure 4.171: Source code

```
In[◦]:= tab = Table[
        Grid[{
          {Row[{"time = ", padIt2[t, {4, 3}]}]},
          {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]},
          {Row[{"Initial  temperature u = 0"}]},
          {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, a}, {θ, -Pi, Pi},
            BaseStyle → 15,
            ImageMargins → 3,
            Mesh → 25,
            PerformanceGoal → "Speed",
            BoxRatios → {1, 1, 0.4},
            PlotRange → {Automatic, Automatic, {-1, 2}},
            ImageSize → 500,
            ColorFunctionScaling → False,
            ColorFunction → ColorData[{"TemperatureMap", {0, 1}}],
            AxesLabel → {"x", "y", "U"},
            ViewPoint → {0.796 , -2.725 , 0.5471},
            Boxed → True, Axes → True
           ]
          }}],
       {t, 0, 50, .05}];

In[◦]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[◦]:= Export["anim.gif", tab, "DisplayDurations" → Table[.05, {Length[tab]}]]
```

Figure 4.172: Code used for animation

4.2.2.7 [265] Specific example of the above

problem number 265

Added June 16,2019

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition, but added specific values for the parameters in order to do animations

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$. Let $k = \frac{1}{100}, a = 2$. Let $f(r,t) = rte^{-t}$

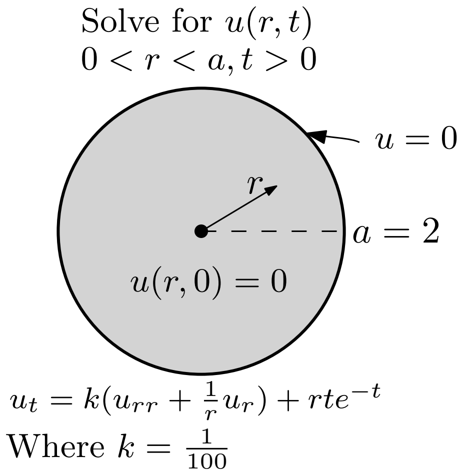


Figure 4.173: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=2;
 k=1/100;
 f=r*t*Exp[-t];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f;
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{1600e^{-\frac{1}{400}t\,\text{BesselJZero}(0,n)^2}\,\text{BesselJ}\left(0, \frac{1}{2}r\,\text{BesselJZero}(0,n)\right)\left(e^{\frac{1}{400}t\,\text{BesselJZero}(0,n)^2-t}(t\,\text{BesselJ}\right.}{3\,\text{BesselJ}(1,\text{BesselJZero}(0,}\right.\right.$$

Maple ✗

```
restart;
a:=2;
k:=1/100;
f:=r*t*exp(-t);
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f;
ic  := u(r,0)=0;
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 4.2.2.5 on page 711 as

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} a_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \\ &= \sum_{n=1}^{\infty} \left(\int_0^t b_n(\tau)\, e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau\right) e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \end{aligned}$$

Where

$$b_n(t) = \frac{\int_0^a f(r,t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) r dr}{\int_0^a \operatorname{BesselJ}^2\left(0, \frac{\Lambda_n}{a} r\right) r dr}$$

Where Λ_n are the n^{th} zeros of $\operatorname{BesselJ}(0, x)$. In this problem

$$\begin{aligned} a &= 2 \\ k &= \frac{1}{100} \\ f(r,t) &= rte^{-t} \end{aligned}$$

This is animation of the above solution using these specific values for for 100 seconds. (Animation will only show in the HTML version)

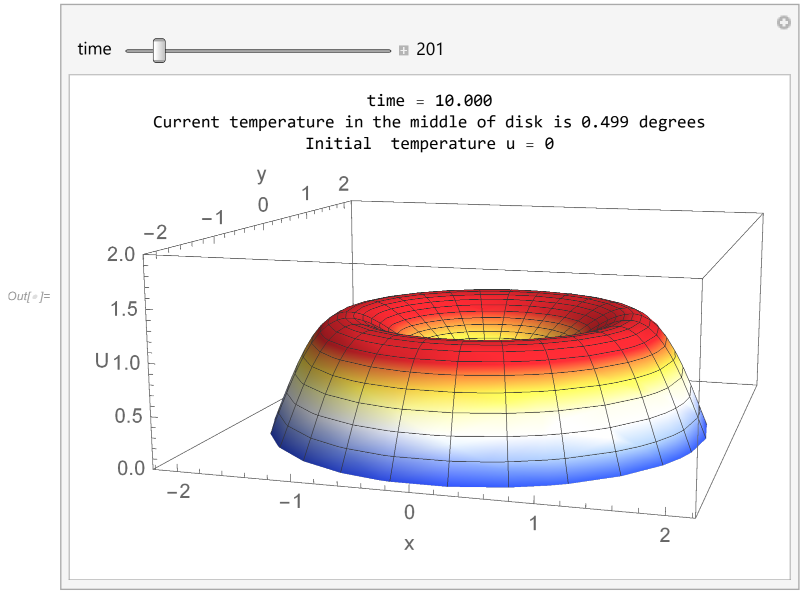


Figure 4.174: Initial state

Source code used for the above

```
In[∘]:= ClearAll[r, u, t, lam, mySol]
a = 2;
k = 1 / 100;
f = r * t * Exp[-t];
maxTime = 30;
numberOfTerms = 10;
z = N@Table[ BesselJZero[0, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ0*)
bn = Table[Integrate[f BesselJ[0, z[[n]]/a r] r, {r, 0, a}] / Integrate[ BesselJ[0, z[[n]]/a r]^2 r, {r, 0, a}], {n, 1, numberOfTerms}];
mySol[r_, t_] = Chop@Sum[Integrate[(bn[[n]] /. t → s) Exp[k (z[[n]]/a)^2 s], {s, 0, t}] Exp[- k (z[[n]]/a)^2 t] BesselJ[0, z[[n]]/a r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 4.175: Source code

```
In[ ]:= tab = Table[
        Grid[{
            {Row[{"time = ", padIt2[t, {4, 3}]}]},
            {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]},
            {Row[{"Initial  temperature u = 0"}]},
            {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, a}, {θ, -Pi, Pi},
                BaseStyle → 15,
                ImageMargins → 3,
                Mesh → 25,
                PerformanceGoal → "Speed",
                BoxRatios → {1, 1, 0.4},
                PlotRange → {Automatic, Automatic, {0, 2}},
                ImageSize → 500,
                ColorFunctionScaling → False,
                ColorFunction → ColorData[{"TemperatureMap", {0, 1}}],
                AxesLabel → {"x", "y", "U"},
                ViewPoint → {0.796 , -2.725 , 0.5471},
                Boxed → True, Axes → True
              ]
            }}],
        {t, 0, 110, .05}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

      Export["anim.gif", tab, "DisplayDurations" → Table[.01, {Length[tab]}]]
```

Figure 4.176: Code used for animation

4.2.2.8 [266] Inside ring

problem number 266

Added May 2,2021

Taken from post at https://www.mapleprimes.com/questions/232084-How-Do-I-Solve-The-Heat-Equation-In

Solve for $u(r,t)$

$$\frac{\partial}{\partial t}u(r,t) = \frac{2r\left(\frac{\partial}{\partial r}u(r,t)\right) + r^2\left(\frac{\partial^2}{\partial r^2}u(r,t)\right)}{r^2}$$

Inside the circle ring where $1 < r < 2$ with boundary condtions $u(1,t) = 0, u(2,t) = 0$ and intitial conditions $u(r,0) = -\sin(\pi r)$.

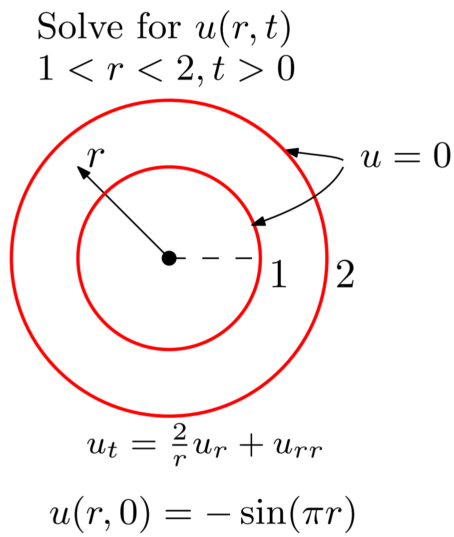


Figure 4.177: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[r, t], t] == D[r^2*D[w[r, t], r], r]/r^2;
bc = {w[1, t] == 0, w[2, t] == 0};
ic = w[r, 0] == -Sin[Pi r];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, w[r, t], {r, t}], 60*10]];
sol = sol /. K[1] -> n;
```

$$\left\{\left\{w(r,t) \to \frac{\sum_{n=1}^{\infty} -\frac{4(1+(-1)^n)e^{-n^2\pi^2 t} n \sin(n\pi(r-1))}{(n^2-1)^2\pi^2}}{r}\right\}\right\}$$

$n = 1$ causs division by zero

Maple ✗

```
restart;
pde := diff(u(r, t), t) = diff(r^2*diff(u(r, t), r), r)/r^2;
bc := u(1, t) = 0,u(2, t) = 0;
ic := u(r, 0) = -sin(Pi*r);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t))),outpu
```

time expired

4.2.2.9 [267] Articolo 6.9.1

problem number 267

Added December 20, 2018.

Example 6.9.1 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution $u(r,\theta,t)$ in a thin circular plate over the two-dimensional domain $D = \{(r,\theta) | 0 < r < 1, 0 < \theta < \frac{\pi}{2}\}$.

The lateral surfaces of the plate are insulated. The edges $r = 1$ and $\theta = 0$ are at a fixed temperature of 0, and the edge $\theta = \frac{\pi}{2}$ is insulated. The initial temperature distribution $u(r,\theta,0) = f(r,\theta)$ is $u(r,\theta,0) = (r - r^3)\sin(\theta)$.

The thermal diffusivity is $k = \frac{1}{50}$.

Solve for $u(r,\theta,t)$ the heat PDE

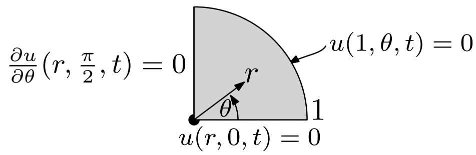
$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$\begin{aligned}|u(0,\theta,t)| &< \infty \\ u(1,\theta,t) &= 0 \\ u(r,0,t) &= 0 \\ \frac{\partial u}{\partial \theta}(1,\frac{\pi}{2},t) &= 0\end{aligned}$$

Solve for $u(r,\theta,t)$

$0 < r < 1, 0 < \theta < \frac{\pi}{2}, t > 0$



I.C. $u(r,\theta,0) = (r - r^3)\sin\theta$

$u_t = \frac{1}{50}(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$

Figure 4.178: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/50;
pde =  D[u[r, theta, t], t] == k*Laplacian[u[r, theta, t], {r, theta}, "Polar"];
bcOnR = u[1, theta, t] == 0;
bcOnTheta = {u[r, 0, t] == 0, Derivative[0, 1, 0][u][r, Pi/2, t] == 0};
ic  = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r
sol = sol /. K[3] -> n;
sol = sol /. K[1] -> m;
```

$$\left\{\left\{u(r,\theta,t) \to \left\{ \quad \sum_{n=1}^{\infty} \frac{4e^{-\frac{1}{50}t\,\mathrm{BesselJZero}(1,n)^2}\,\mathrm{BesselJ}(1,r\,\mathrm{BesselJZero}(1,n))(\mathrm{BesselJ}(3,\mathrm{BesselJZero}(1,n))+\mathrm{BesselJ}(2,\mathrm{BesselJZero}(1,n}{3\,\mathrm{BesselJ}(0,\mathrm{BesselJZero}(1,n))^2\,\mathrm{BesselJZero}(1,n)^2}\right.\right.\right.$$

Indeterminate

Maple ✗

```
restart;
k:=1/50;
pde := diff(u(r, theta, t), t) = k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bc_on_r:= u(1,theta,t)=0;
bc_on_theta:= u(r,0,t)=0, eval(diff(u(r,theta,t),theta),theta=Pi/2)=0;
ic  := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

sol=()

4.2.2.10 [268] Articolo 6.9.2

problem number 268

Added December 20, 2018.

Example 6.9.2 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin circular plate over the two-dimensional domain $D = \{(r,\theta)|0 < r < 1, 0 < \theta < \pi\}$. The lateral surfaces of the plate are insulated. The sides $\theta = 0$ and $\theta = \pi$ are at a fixed temperature of 0, and the edge $r = 1$ is insulated. The initial temperature distribution is $u(r,\theta,0) = \left(r - \frac{r^3}{3}\right)\sin\theta$.

The thermal diffusivity is $k = \frac{1}{25}$. Solve for $u(r,\theta,t)$ the heat PDE

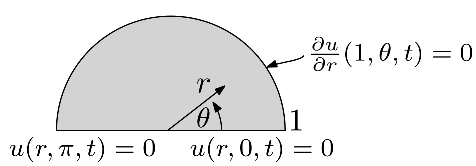
$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$\begin{aligned}|u(0,\theta,t) &< \infty\\ u(1,\theta,t) &= 0\\ u(r,0,t) &= 0\\ u(r,\pi,t)&0\end{aligned}$$

Solve for $u(r,\theta,t)$

$0 < r < 1, 0 < \theta < \pi, t > 0$



I.C. $u(r,\theta,0) = (r - \frac{r^3}{3})\sin\theta$

$u_t = \frac{1}{25}(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$

Figure 4.179: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/25;
pde =  D[u[r, theta, t], t] == k*Laplacian[u[r, theta, t], {r, theta}, "Polar"];
bcOnR = Derivative[1, 0, 0][u][1, theta, t] == 0;
bcOnTheta = {u[r, 0, t] == 0, u[r, Pi, t] == 0};
ic  = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r
sol = sol /. K[3] -> n;
```

$$\left\{\left\{u(r,\theta,t)\to\ \left\{\ \sum_{n=1}^{\infty}\frac{4e^{-\frac{1}{25}tK[2,1,n]^2}\,\text{BesselJ}(1,rK[2,1,n])(\text{BesselJ}(3,K[2,1,n])+\text{BesselJ}(2,K[2,1,n])K[2,1,n])\sin(\theta)}{3K[2,1,n]((\text{BesselJ}(0,K[2,1,n])^2+\text{BesselJ}(1,K[2,1,n])^2)K[2,1,n]-2\,\text{BesselJ}(0,K[2,1,n])\,\text{BesselJ}(1,K[2,1,n]))}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
k:=1/25;
pde := diff(u(r, theta, t), t) = k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bc_on_r:= eval(diff(u(r,theta,t),r),r=1)=0;
bc_on_theta:= u(r,0,t)=0, u(r,Pi,t)=0;
ic  := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\left(-\frac{4(\lambda_n^3\,\text{BesselJ}\left(0,\lambda_n\right)-\lambda_n^2\,\text{BesselJ}\left(1,\lambda_n\right)+4\lambda_n\,\text{BesselJ}\left(0,\lambda_n\right)-8\,\text{BesselJ}\left(1,\lambda_n\right))\,\text{Bess}}{3\left(\lambda_n\,\text{BesselJ}\left(0,\lambda_n\right)^2+\lambda_n\,\text{BesselJ}\left(1,\lambda_n\right)^2-2\,\text{BesselJ}\left(0,\lambda_n\right)\text{BesselJ}\left(1\right.\right.}\right.$$

4.2.2.11 [269] Haberman 8.2.5 with θ dependency (General case)

problem number 269

Added Feb 24, 2019.

Problem 8.2.5 from from Richard Haberman applied partial differential equations book, 5th edition.

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius a) $u_t = k\nabla^2 u$ with time-independent boundary conditions:

$$u(a,\theta,t) = g(\theta)$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

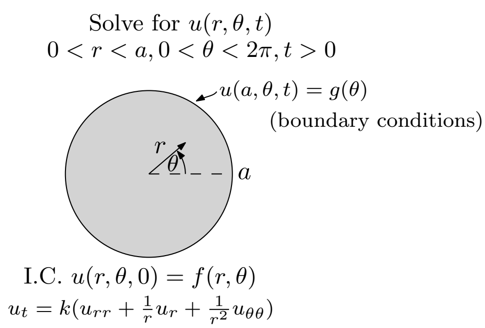


Figure 4.180: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta, t], t] == k*Laplacian[u[r, theta, t], {r, theta}, "Polar"];
bcOnR = u[a, theta, t] == g[theta];
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == f[r, theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
sol = sol /. K[1] -> n;
sol = sol /. K[3] -> m;
```

$$\left\{\left\{u(r,\theta,t)\to\left(\left\{\sum_{m=1}^{\infty}\frac{e^{-\frac{kt\,\text{BesselJZero}(0,m)^2}{a^2}}\text{BesselJ}\left(0,\frac{r\,\text{BesselJZero}(0,m)}{a}\right)\int_0^a\int_{-\pi}^{\pi}r\,\text{BesselJ}\left(0,\frac{r\,\text{BesselJZero}(0,m)}{a}\right)\left(2\pi f(r,\theta)-\text{Inte}\right.}{}\right.\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r,theta,t),t)=k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bcOnR:= u(a,theta,t)=g(theta);
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=f(r,theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$
\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= g(\theta)\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

With initial conditions $u(r,\theta,0)=f(r,\theta)$.

Since the boundary conditions are not homogenous, and since there are no time dependent sources, then in this case we look for $u_E(r,\theta)$ which is solution at steady state which needs to satisfy the nonhomogeneous B.C., where $u(r,\theta,t) = \overbrace{v(r,\theta,t)}^{\text{transient}} + \overbrace{u_E(r,\theta)}^{\text{steady state}}$ and $v(r,\theta,t)$ solves the PDE but with homogenous B.C. Therefore, we need to find equilibrium (steady state) solution for Laplace PDE on disk, that only needs to satisfy the nonhomogeneous B.C.

$$
\begin{aligned}
\nabla^2 u_E &= 0\\
\frac{\partial^2 u_E}{\partial r^2} + \frac{1}{r}\frac{\partial u_E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_E}{\partial \theta^2} &= 0
\end{aligned}
$$

With boundary condition

$$
\begin{aligned}
|u_E(0,\theta)| &< \theta\\
u_E(a,\theta) &= g(\theta)\\
u_E(r,-\pi) &= u_E(r,\pi)\\
\frac{\partial u_E}{\partial \theta}(r,-\pi) &= \frac{\partial u_E}{\partial \theta}(r,\pi)
\end{aligned}
$$

Let

$$
u_E(r,\theta) = R(r)\,\Theta(\theta)
$$

Where $R(r)$ is the solution in radial dimension and $\Theta(\theta)$ is solution in angular dimension. Substituting $u_E(r,\theta)$ in the PDE gives

$$
R''\Theta + \frac{1}{r}R'\Theta + \frac{1}{r^2}\Theta'' R = 0
$$

Dividing by $R(r)\,\Phi(\theta)$

$$
\begin{aligned}
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= 0\\
r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta}
\end{aligned}
$$

Hence each side is equal to constant, say λ and we obtain

$$
\begin{aligned}
r^2\frac{R''}{R} + r\frac{R'}{R} &= \lambda\\
-\frac{\Theta''}{\Theta} &= \lambda
\end{aligned}
$$

Or

$$
r^2 R'' + rR' - \lambda R = 0 \tag{1}
$$

$$
\Theta'' + \lambda\Theta = 0 \tag{2}
$$

We start with Φ ODE. The boundary conditions on (3) are

$$\Theta(-\pi) = \Theta(\pi)$$
$$\frac{\partial \Theta}{\partial \theta}(-\pi) = \frac{\partial \Theta}{\partial \theta}(\pi)$$

case $\lambda = 0$ The solution is $\Phi = c_1\theta + c_2$. Hence we obtain, from first initial conditions

$$-\pi c_1 + c_2 = \pi c_1 + c_2$$
$$c_1 = 0$$

Second boundary conditions just says that $c_2 = c_2$, so any constant will do. Hence $\lambda = 0$ is an eigenvalue with constant being eigenfunction.

case $\lambda > 0$ The solution is

$$\Theta(\theta) = c_1 \cos \sqrt{\lambda}\theta + c_2 \sin \sqrt{\lambda}\theta$$

The first boundary conditions gives

$$c_1 \cos\left(-\sqrt{\lambda}\pi\right) + c_2 \sin\left(-\sqrt{\lambda}\pi\right) = c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(\sqrt{\lambda}\pi\right)$$
$$c_1 \cos\left(\sqrt{\lambda}\pi\right) - c_2 \sin\left(\sqrt{\lambda}\pi\right) = c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(\sqrt{\lambda}\pi\right)$$
$$2c_2 \sin\left(\sqrt{\lambda}\pi\right) = 0 \tag{3}$$

From second boundary conditions we obtain

$$\Theta'(\theta) = -\sqrt{\lambda}c_1 \sin \sqrt{\lambda}\theta + c_2\sqrt{\lambda} \cos \sqrt{\lambda}\theta$$

Therefore

$$-\sqrt{\lambda}c_1 \sin\left(-\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda} \cos\left(-\sqrt{\lambda}\pi\right) = -\sqrt{\lambda}c_1 \sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda} \cos\left(\sqrt{\lambda}\pi\right)$$
$$\sqrt{\lambda}c_1 \sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda} \cos\left(\sqrt{\lambda}\pi\right) = -\sqrt{\lambda}c_1 \sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda} \cos\left(\sqrt{\lambda}\pi\right)$$
$$\sqrt{\lambda}c_1 \sin\left(\sqrt{\lambda}\pi\right) = -\sqrt{\lambda}c_1 \sin\left(\sqrt{\lambda}\pi\right)$$
$$2c_1 \sin\left(\sqrt{\lambda}\pi\right) = 0 \tag{4}$$

Both (3) and (4) are satisfied if

$$\sqrt{\lambda}\pi = n\pi \qquad n = 1, 2, 3, \cdots$$
$$\lambda = n^2 \qquad n = 1, 2, 3, \cdots$$

Therefore

$$\Theta_n(\theta) = \overbrace{A_0}^{\lambda=0} + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

Now we go back to the R ODE (1) given by $r^2R'' + rR' - \lambda_n R = 0$ and solve it. This is Euler ODE whose solution is found by substituting $R(r) = r^\alpha$. The solution comes out to be

$$R_n(r) = c_0 + \sum_{n=1}^{\infty} c_n r^n \tag{6}$$

Combining (5,6) we now find u_E as

$$\begin{aligned} u_{E_n}(r,\theta) &= R_n(r)\,\Theta_n(\theta) \\ u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta)) \end{aligned} \tag{7}$$

Where c_0 was combined with A_0. Now the above equilibrium solution needs to satisfy the non-homogenous B.C. $u_E(a,\theta) = g(\theta)$. Using orthogonality on (7) to find A_n, B_n gives

$$g(\theta) = A_0 + \sum_{n=1}^{\infty} a^n (A_n \cos(n\theta) + B_n \sin(n\theta))$$

For $n = 0$

$$\begin{aligned} \int_0^{2\pi} g(\theta)\,d\theta &= A_0 \int_0^{2\pi} d\theta \\ A_0 &= \frac{1}{2\pi} \int_0^{2\pi} g(\theta)\,d\theta \end{aligned}$$

For $n > 0$, applying orthogonality using cosine to find A_n gives

$$\begin{aligned} \int_0^{2\pi} g(\theta) \cos(n\theta)\,d\theta &= A_n \int_0^{2\pi} \cos^2(n\theta)\, a^n d\theta \\ A_n &= \frac{1}{\pi} \int_0^{2\pi} g(\theta) \cos(n\theta)\,d\theta \end{aligned}$$

Similarly, applying orthogonality using sin to find B_n gives

$$B_n = \frac{1}{\pi} \int_0^{2\pi} g(\theta) \sin(n\theta)\,d\theta$$

Therefore, we have found $u_E(r,\theta)$ completely now. It is given by (7)

$$u_E(r,\theta) = A_0 + \sum_{n=1}^{\infty} r^n \left(A_n \cos(n\theta) + B_n \sin(n\theta)\right) \tag{7A}$$
$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\,d\theta$$
$$A_n = \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\,d\theta$$
$$B_n = \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\,d\theta$$

Now, since $u(r,\theta,t) = v(r,\theta,t) + u_E(r,\theta)$, then we need to solve now for $v(r,\theta,t)$ with homogeneous boundary conditions

$$v_t(r,\theta,t) = k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}\right) \tag{8}$$
$$|v(0,\theta,t)| < \theta$$
$$v(a,\theta,t) = 0$$
$$v(r,-\pi,t) = v(r,\pi,t)$$
$$\frac{\partial v}{\partial \theta}(r,-\pi,t) = \frac{\partial v}{\partial \theta}(r,\pi,t)$$

Let $v(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$. Substituting into (8) gives

$$T'R\Theta = k\left(R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT\right)$$

Dividing by $R(r)\,\Theta(\theta)\,T(t) \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}$$

Let first separation constant be $-\lambda$, hence the above becomes

$$\frac{1}{k}\frac{T'}{T} = -\lambda$$
$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} = -\lambda$$

Or

$$T' + \lambda k T = 0$$
$$r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda = -\frac{\Theta''}{\Theta}$$

We now separate the second equation above using μ giving

$$r^2\frac{R''}{R}+r\frac{R'}{R}+r^2\lambda=\mu$$
$$-\frac{\Theta''}{\Theta}=\mu$$

Or

$$R''+\frac{1}{r}R'+R\left(\lambda-\frac{\mu}{r^2}\right)=0 \tag{9}$$
$$\Theta''+\mu\Theta=0 \tag{10}$$

Equation (9) is Sturm-Liouville ODE with boundary conditions $R(a)=0$ and bounded at $r=0$ and (10) has periodic boundary conditions as was solved above. The solution to (10) is given in (5) above, no change for this part.

$$\Theta_n(\theta)=\overbrace{\alpha_0}^{\lambda=0}+\sum_{n=1}^{\infty}\alpha_n\cos(n\theta)+\beta_n\sin(n\theta) \tag{11}$$

Therefore (9) becomes $R''+\frac{1}{r}R'+R\left(\lambda-\frac{n^2}{r^2}\right)=0$ with $n=0,1,2,\cdots$. We found the solution to this Sturm-Liouville before, it is given by

$$R_{nm}(r)=J_n\left(\sqrt{\lambda_{nm}}r\right)\qquad n=0,1,2,\cdots,m=1,2,3,\cdots \tag{12}$$

Where $\sqrt{\lambda_{nm}}=\frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n. This is found numerically. We now just need to find the time solution from $T'+\lambda_{nm}kT=0$. For This has solution

$$T_{nm}(t)=e^{-k\lambda_{nm}t} \tag{13}$$

Now we combine (11,12,13) to find solution for $v(r,\theta,t)$, and combining constants gives

$$v_{nm}(r,\theta,t)=\Theta_n(\theta)\,R_{nm}(r)\,T_{nm}(t)$$
$$v(r,\theta,t)=\alpha_{0,1}J_0\left(\sqrt{\lambda_{0,1}}r\right)+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta))$$
$$=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta)) \tag{14}$$

We now need to find $\alpha_0,\alpha_{nn},\beta_{nm}$, which are found from initial conditions on $v(r,\theta,0)$ which is given by

$$v(r,\theta,0)=u(r,\theta,0)-u_E(r,\theta)$$
$$=f(r,\theta)-u_E(r,\theta)$$

Hence from (14), at $t=0$

$$f(r,\theta)-u_E(r,\theta)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)\left(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta)\right) \tag{15}$$

For each n, inside the m sum, $\cos(n\theta)$ and $\sin(n\theta)$ will be constant. So we need to apply orthogonality twice in order to remove both sums. Multiplying (15) by $\cos(n'\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos(n'\theta)\,d\theta &= \int_{-\pi}^{\pi}\sum_{n=0}^{\infty}\left(\sum_{m=1}^{\infty}\alpha_{nn}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\cos(n\theta)\cos(n'\theta)\,d\theta\\ &\quad+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\beta_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\sin(n\theta)\cos(n'\theta)\end{aligned}$$

The second sum in the RHS above goes to zero due to $\int_{-\pi}^{\pi}\sin(n\theta)\cos(n'\theta)\,d\theta$ and we end up with

$$\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos(n\theta)\,d\theta=\alpha_{nn}\int_{-\pi}^{\pi}\cos^2(n\theta)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)d\theta$$

We now apply orthogonality again, but on Bessel functions and remembering to add the weight r, the above becomes

$$\begin{aligned}\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm'}}r\right)rd\theta dr &= \alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)J_n\left(\sqrt{\lambda_{nm'}}r\right)r\\ &=\alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm'}}r\right)rd\theta dr\end{aligned}$$

Therefore

$$\alpha_{nn}=\frac{\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

We will repeat the same thing to find β_{nm}. The only difference now is to use $\sin n\theta$. repeating these steps gives

$$\beta_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

This complete the solution.

Summary of solution

$$\begin{aligned}u(r,\theta,t)&=v(r,\theta,t)+u_E(r,\theta)\\ &=u_E(r,\theta)+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}\left(\alpha_{nn}\cos(n\theta)+\beta_{nm}\sin(n\theta)\right)\end{aligned}$$

Where

$$
\begin{aligned}
u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta)) \\
A_0 &= \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\, d\theta \\
A_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\, d\theta \\
B_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\, d\theta
\end{aligned}
$$

And

$$
\alpha_{nn} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta))\cos(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots
$$

And

$$
\beta_{nm} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta))\sin(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots
$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n.

4.2.2.12 [270] With θ dependency (Specific example)

problem number 270

Added June 12, 2019

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius $a = 1$) $u_t = k\nabla^2 u$ with $k = 1$ with time-independent boundary conditions:

$$
u(1,\theta,t) = 0
$$

And initial conditions $u(r,\theta,0) = 1 - r^2$. There is an implied periodic boundary conditions on θ

$$
\begin{aligned}
u(r,-\pi,t) &= u(r,\pi,t) \\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

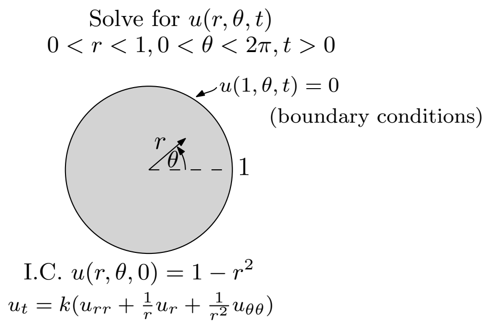


Figure 4.181: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
a=1;
pde = D[u[r, theta, t], t] == k*((Laplacian[u[r, theta, t], {r, theta}, "Polar"]));
bcOnR = u[a, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == 1-r^2;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
sol = sol /. K[1] -> n;
sol = sol /. K[3] -> m
```

$$\left\{\left\{u(r,\theta,t) \to \begin{cases} \sum_{m=1}^{\infty} \frac{4e^{-t\,\text{BesselJZero}(0,m)^2}\,\text{BesselJ}(0,r\,\text{BesselJZero}(0,m))\,\text{BesselJ}(2,\text{BesselJZero}(0,m))}{\text{BesselJ}(1,\text{BesselJZero}(0,m))^2\,\text{BesselJZero}(0,m)^2} & (n|m) \in \mathbb{Z} \land n \geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right.$$

Maple ✗

```
restart;
a:=1;
k:=1;
pde := diff(u(r,theta,t),t)=k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bcOnR:= u(a,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=1-r^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

<u>Hand solution</u>

Solve

$$
\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) \\
|u(0,\theta,t)| &< \infty \\
u(a,\theta,t) &= 0 \\
u(r,-\pi,t) &= u(r,\pi,t) \\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \\
u(r,\theta,0) &= 1-r^2
\end{aligned}
$$

With $t>0, 0<r<a$ where $a=1, k=1$.

This problem was solved in problem 4.2.2.11 on page 729 (since B.C. is zero, we just need to use $v(r,\theta,t)$ in the solution given in the above problem). Hence

$$
u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right) e^{-k\lambda_{nm}t}\left(A_{nm}\cos(n\theta) + B_{nm}\sin(n\theta)\right)
$$

Where

$$
A_{nm} = \frac{\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\cos(n\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}}r\right) r d\theta dr} \qquad n=0,1,2,\cdots, m=1,2,3,\cdots
$$

And

$$
B_{nm} = \frac{\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\sin(n\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}}r\right) r d\theta dr} \qquad n=0,1,2,\cdots, m=1,2,3,\cdots
$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ and z_{nm} is the m^{th} zero of the Bessel function of order n. where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n. Since in this problem $k = 1, a = 1, f(r,\theta) = 1 - r^2$ then

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}} r\right) e^{-\frac{t}{z_{nm}^2}} \left(A_{nm}\cos(n\theta) + B_{nm}\sin(n\theta)\right)$$

$$A_{nm} = \frac{\int_0^1 \int_{-\pi}^{\pi} (1-r^2)\cos(n\theta)\, J_n\left(\frac{r}{z_{nm}}\right) r d\theta dr}{\int_0^1 \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\frac{r}{z_{nm}}\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots$$

$$B_{nm} = \frac{\int_0^1 \int_{-\pi}^{\pi} (1-r^2)\sin(n\theta)\, J_n\left(\frac{r}{z_{nm}}\right) r d\theta dr}{\int_0^1 \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\frac{r}{z_{nm}}\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n.

This is animation of the solution for 0.4 seconds. (Animation will only show in the HTML version)

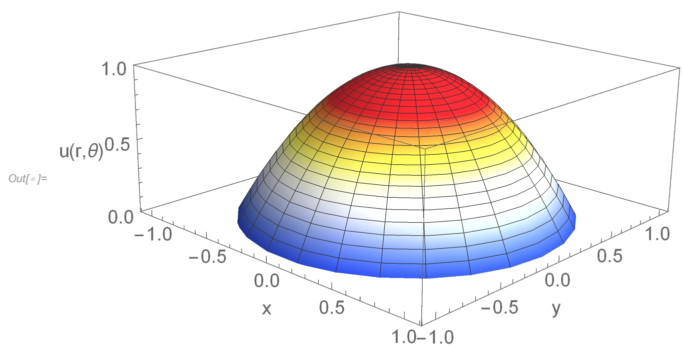


Figure 4.182: Initial state

Source code used for the above

```
ClearAll[a, c, n, m, r, theta, f, g, θ, t, f, n, lam0, A0, A, B, B0, lam];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
a = 1;
k = 1;  (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (a - r^2);
A0[n_, m_] := A[[n + 1, m]];
B0[n_, m_] := B[[n + 1, m]];
lam0[n_, m_] := lam[[n + 1, m]];
getEigenvalue[n_, m_] := Module[{x}, x = BesselJZero[n, m]; N[(x / a)^2]];
lam = Table[getEigenvalue[n, m], {n, 0, maxN - 1}, {m, 1, maxM}];
A = Table[Integrate[f Cos[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Cos[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}], {n, 0, maxN - 1}, {m, 1, maxM}];
B = Table[If[n == 0, 0, Integrate[f Sin[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Sin[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}]], {n, 0, maxN - 1}, {m, 1, maxM}];
u[r_, θ_, t_] := Sum[Sum[BesselJ[n, r Sqrt[lam0[n, m]]] Exp[-k lam0[n, m] t] (A0[n, m] Cos[n θ] + B0[n, m] Sin[n θ]), {m, 1, maxM}], {n, 0, maxN - 1}]
```

Figure 4.183: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {4, 3}]}]},
          {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
          {Row[{"Initial  temperature u =  (1-r^2)"}]},
          {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
              BaseStyle → 15,
              ImageMargins → 5,
              Mesh → 25,
              PerformanceGoal → "Speed",
              BoxRatios → {1, 1, 0.4},
              PlotRange → {Automatic, Automatic, {0, 1}},
              ImageSize → 500,
              ColorFunctionScaling → False,
              ColorFunction → ColorData[{"TemperatureMap", {0, .8}}],
              AxesLabel → {"x", "y", "U(r,θ)"},
              ViewPoint → {2.17, -2.4, 1}
            ]
          }}],
      {t, 0, 0.4, .01}];

Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

Export["anim.gif", tab, "DisplayDurations" → Table[.35, {Length[tab]}]]
```

Figure 4.184: Animation part of code

4.2.2.13 [271] With θ dependency (specific example)

problem number 271

Added June 12, 2019

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius $a = 1$) $u_t = k\nabla^2 u$ with $k = 1$ with time-independent boundary conditions:

$$u(1, \theta, t) = 0$$

And initial conditions $u(r,\theta,0)=(r-r^3)\sin\theta$. There is an implied periodic boundary conditions on θ

$$u(r,-\pi,t)=u(r,\pi,t)$$
$$\frac{\partial u}{\partial\theta}(r,-\pi,t)=\frac{\partial u}{\partial\theta}(r,\pi,t)$$

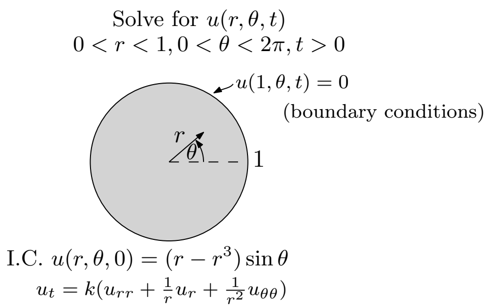


Figure 4.185: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
a=1;
pde = D[u[r, theta, t], t] == k*(Laplacian[u[r, theta, t], {r, theta}, "Polar"]);
bcOnR = u[a, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (r-r^3)*Sin[theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
sol = sol /. K[1] -> n;
sol = sol /. K[3] -> m;
```

$$\left\{\left\{u(r,\theta,t)\to\begin{cases}\sum_{m=1}^{\infty}\frac{4e^{-t\,\text{BesselJZero}(1,m)^2}\,\text{BesselJ}(1,r\,\text{BesselJZero}(1,m))\,\text{BesselJ}(3,\text{BesselJZero}(1,m))\sin(\theta)}{\text{BesselJ}(0,\text{BesselJZero}(1,m))^2\,\text{BesselJZero}(1,m)^2} & (n|m)\in\mathbb{Z}\land n \\ \text{Indeterminate} & \text{Tr}\end{cases}\right.\right.$$

Maple ✗

```
restart;
a:=1;
k:=1;
a:=1;
pde := diff(u(r,theta,t),t)=k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bcOnR:= u(a,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=(r-r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$
\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= 0\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)\\
u(r,\theta,0) &= \left(r-r^3\right)\sin\theta
\end{aligned}
$$

for $0<r<a$ where $a=1,k=1$.

This problem was solved in problem 4.2.2.11 on page 729 (since B.C. is zero, we just need to use $v(r,\theta,t)$ in the solution given in the above problem). Hence

$$
u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}\left(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta)\right)
$$

Where

$$
A_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots
$$

And

$$
B_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots
$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ and z_{nm} is the m^{th} zero of the Bessel function of order n and a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n. Since in this problem $k = 1, a = 1, f(r, \theta) = (r - r^3)\sin\theta$ then the above solution becomes

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_n \cos(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) e^{-\frac{t}{z_{nm}^2}} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_n \sin(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) e^{-\frac{t}{z_{nm}^2}}$$

$$A_n = \frac{\int_0^1 \int_{-\pi}^{\pi} (r - r^3)\sin\theta\cos(n\theta)\, J_n\left(\frac{r}{z_{nm}}\right) r d\theta dr}{\int_0^1 \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\frac{r}{z_{nm}}\right) r d\theta dr}$$

$$B_n = \frac{\int_0^1 \int_{-\pi}^{\pi} (r - r^3)\sin\theta\sin(n\theta)\, J_n\left(\frac{r}{z_{nm}}\right) r d\theta dr}{\int_0^1 \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\frac{r}{z_{nm}}\right) r d\theta dr}$$

This is animation of the solution for 0.11 seconds. (Animation will only show in the HTML version)

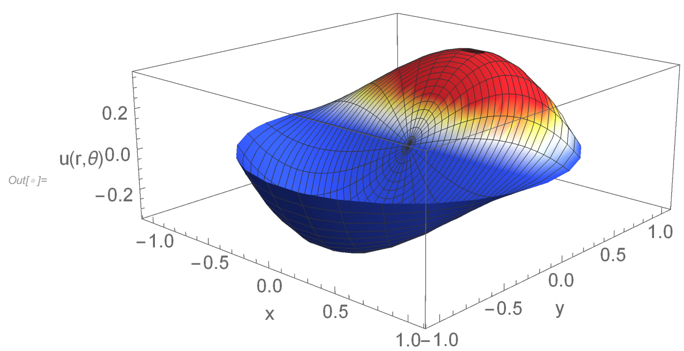


Figure 4.186: Initial state

Source code used for the above

```
In[•]:= ClearAll[a, c, n, m, r, theta, f, g, θ, t, f, n, lam0, A0, A, B, B0, lam];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
a = 1;
k = 1;  (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (r - r^3) Sin[θ];
A0[n_, m_] := A[[n + 1, m]];
B0[n_, m_] := B[[n + 1, m]];
lam0[n_, m_] := lam[[n + 1, m]];
getEigenvalue[n_, m_] := Module[{x}, x = BesselJZero[n, m]; N[(x / a) ^2]];
lam = Table[getEigenvalue[n, m], {n, 0, maxN - 1}, {m, 1, maxM}];
A = Table[Integrate[f Cos[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Cos[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}], {n, 0, maxN - 1}, {m, 1, maxM}];
B = Table[If[n == 0, 0, Integrate[f Sin[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Sin[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}]], {n, 0, maxN - 1}, {m, 1, maxM}];
u[r_, θ_, t_] := Sum[Sum[BesselJ[n, r Sqrt[lam0[n, m]]] Exp[-k lam0[n, m] t] (A0[n, m] Cos[n θ] + B0[n, m] Sin[n θ]), {m, 1, maxM}], {n, 0, maxN - 1}]
```

Figure 4.187: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (1-r^2)"}]},
              {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
                  BaseStyle → 15,
                  ImageMargins → 5,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {-0.35, 0.38}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, .2}}],
                  AxesLabel → {"x", "y", "U(r,0)"},
                  ViewPoint → {2.17, -2.4, 1}
                ]
              }}],
          {t, 0, 0.11, .005}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.5, {Length[tab]}]]
```

Figure 4.188: Animation part of code

4.2.2.14 [272] Insulated with θ dependency (General solution)

problem number 272

Added June 13, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

with time-independent boundary conditions:

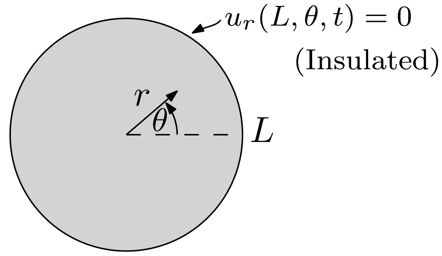
$$u_r(a,\theta,t) = 0$$

And initial conditions $u(r,\theta,0)=f(r,\theta)$. There is an implied periodic boundary conditions on θ

$$u(r,-\pi,t)=u(r,\pi,t)$$
$$\frac{\partial u}{\partial \theta}(r,-\pi,t)=\frac{\partial u}{\partial \theta}(r,\pi,t)$$

The solution is bounded at $r=0$.

Solve for $u(r,\theta,t)$
$0<r<L,-\pi<\theta<\pi,t>0$



I.C. $u(r,\theta,0)=f(r,\theta)$
$u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$

Figure 4.189: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta, t], t] == k*Laplacian[u[r, theta, t], {r, theta}, "Polar"];
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == f[r,theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
sol = sol /. K[3] -> n;
sol = sol /. K[1] -> m;
```

$$\left\{\left\{u(r,\theta,t)\to\ \left\{\ \frac{\int_0^L\int_{-\pi}^{\pi} rf(r,\theta)d\theta dr}{L^2\pi}+\sum_{n=1}^{\infty}\frac{e^{-ktK[2,0,n]^2}\,\text{BesselJ}(0,rK[2,0,n])\left(\int_0^L\int_{-\pi}^{\pi} r\,\text{BesselJ}(0,rK[2,0,n])f(r,\theta)d\theta d}{L^2\pi\sqrt{\text{BesselJ}(0,LK[2,0,n])^2+\text{BesselJ}(1,LK[2,0,n])^2}\sqrt{(\text{BesselJ}(0,LK[2,0,n])^2+\text{Bessel}}}\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r,theta,t),t)=k*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta]);
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=f(r,theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()
Hand solution

Solve

$$\frac{\partial u(r,\theta,t)}{\partial t} = k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u_t(L,\theta,t) &= g(\theta) \\
u(r,-\pi,t) &= u(r,\pi,t) \\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. Let $y(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$. Substituting into (1) gives

$$T'R\Theta = k\left(R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT\right)$$

Dividing by $R(r)\,\Theta(\theta)\,T(t) \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}$$

Let first separation constant be $-\lambda$, hence the above becomes

$$\begin{aligned}
\frac{1}{k}\frac{T'}{T} &= -\lambda \\
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= -\lambda
\end{aligned}$$

Or

$$\begin{aligned}
T' + \lambda k T &= 0 \\
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda = -\frac{\Theta''}{\Theta} &= \mu
\end{aligned}$$

Hence

$$r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda = \mu$$
$$-\frac{\Theta''}{\Theta} = \mu$$

Or

$$r^2R''(r) + rR'(r) + R(r)\left(r^2\lambda - \mu\right) = 0 \tag{2}$$
$$\Theta''(\theta) + \mu\Theta(\theta) = 0 \tag{3}$$

Equation (2) is Sturm-Liouville ODE with boundary conditions $R'(a) = 0$ and bounded at $r = 0$ and (3) has periodic boundary conditions.

We start with Θ ODE. The boundary conditions on (3) are

$$\Theta(-\pi) = \Theta(\pi)$$
$$\frac{\partial\Theta}{\partial\theta}(-\pi) = \frac{\partial\Theta}{\partial\theta}(\pi)$$

Negative μ is not possible, since this gives exponential and the boundary conditions on $\Theta(\theta)$ are periodic.

case $\mu = 0$ The solution is $\Phi = c_1\theta + c_2$. Hence we obtain, from first initial conditions

$$-\pi c_1 + c_2 = \pi c_1 + c_2$$
$$c_1 = 0$$

Second boundary conditions just says that $c_2 = c_2$, so any constant will do. Hence $\mu = 0$ is an eigenvalue with constant being eigenfunction $\Theta_0(\theta) = 1$.

case $\mu > 0$ The solution is

$$\Theta(\theta) = c_1\cos\sqrt{\mu}\theta + c_2\sin\sqrt{\mu}\theta$$

The first boundary conditions gives

$$c_1\cos\left(-\sqrt{\mu}\pi\right) + c_2\sin\left(-\sqrt{\mu}\pi\right) = c_1\cos\left(\sqrt{\mu}\pi\right) + c_2\sin\left(\sqrt{\mu}\pi\right)$$
$$c_1\cos\left(\sqrt{\mu}\pi\right) - c_2\sin\left(\sqrt{\mu}\pi\right) = c_1\cos\left(\sqrt{\mu}\pi\right) + c_2\sin\left(\sqrt{\mu}\pi\right)$$
$$2c_2\sin\left(\sqrt{\mu}\pi\right) = 0 \tag{4}$$

From second boundary conditions we obtain

$$\Theta'(\theta) = -\sqrt{\mu}c_1\sin\sqrt{\mu}\theta + c_2\sqrt{\mu}\cos\sqrt{\mu}\theta$$

Therefore

$$
\begin{aligned}
-\sqrt{\mu} c_1 \sin(-\sqrt{\mu}\pi) + c_2\sqrt{\mu}\cos(-\sqrt{\mu}\pi) &= -\sqrt{\mu}c_1 \sin(\sqrt{\mu}\pi) + c_2\sqrt{\lambda}\cos(\sqrt{\mu}\pi)\\
\sqrt{\mu} c_1 \sin(\sqrt{\mu}\pi) + c_2\sqrt{\mu}\cos(\sqrt{\mu}\pi) &= -\sqrt{\mu}c_1 \sin(\sqrt{\mu}\pi) + c_2\sqrt{\mu}\cos(\sqrt{\mu}\pi)\\
\sqrt{\mu} c_1 \sin(\sqrt{\mu}\pi) &= -\sqrt{\mu}c_1 \sin(\sqrt{\mu}\pi)\\
2c_1 \sin(\sqrt{\mu}\pi) &= 0 \qquad (5)
\end{aligned}
$$

Both (4) and (5) are satisfied if

$$
\begin{aligned}
\sqrt{\mu}\pi &= n\pi \qquad n = 1,2,3,\cdots \qquad (5A)\\
\mu &= n^2 \qquad n = 1,2,3,\cdots
\end{aligned}
$$

Therefore $\mu_n = 1,4,9,16,\cdots$. Or $\sqrt{\mu_n} = n = 1,2,3,\cdots$

$$
\begin{aligned}
\Theta_n(\theta) &= \overbrace{A_0}^{n=0} + \sum_{n=1}^{\infty} A_n \cos(\sqrt{\mu_n}\theta) + B_n \sin(\sqrt{\mu_n}\theta) \qquad (6)\\
&= A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \qquad (4.2)
\end{aligned}
$$

Now we go back to the $R(r)$ ODE (2) given by $r^2R'' + rR' + R(r^2\lambda - \mu) = 0$ or since $\mu_n = n^2$, we write it as $r^2R'' + rR' + R(r^2\lambda - n^2) = 0$ for $n = 0,1,2,\cdots$.

<u>Case $n = 0$</u>

The $R(r)$ ODE becomes $r^2R''+rR'+\lambda r^2 R = 0$. This is a Bessel ODE. If $\lambda < 0$, this leads to solution (contains Besselk and BesselI) that can not satisfy the boundary conditions. If $\lambda = 0$ then the ODE becomes $r^2R''+rR' = 0$ whose solution is $R(r) = C_1 \ln(r) + C_2$. Since bounded at $r = 0$. Then $R(r) = C_2$. Hence $R'(r) = 0$. Which satisfies the boundary condition at $r = L$ for any constant C_2. Therefore <u>$\lambda = 0$ is possible eigenvalue</u> with constant as its eigenfunction $R_0(r) = 1$.

If $\lambda > 0$ the ODE is a Bessel ODE whose solution is

$$
R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)
$$

We set $C_2 = 0$ since $\operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$ blows up at $r = 0$. Hence $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$ and $R'(r) = -C_1\sqrt{\lambda}\operatorname{BesselJ}\left(1, \sqrt{\lambda} r\right)$. At $r = L$ we obtain

$R'(L) = -C_1\sqrt{\lambda}\operatorname{BesselJ}\left(1, \sqrt{\lambda} L\right) = 0$. Therefore for non-trivial solution we want $\sqrt{\lambda}L$ to be the zeros of $\operatorname{BesselJ}(1, x)$. Let these zeros be Λ_m, $m = 1,2,3,\cdots$. Hence

$$
\begin{aligned}
\sqrt{\lambda_{0,m}} L &= \Lambda_m\\
\lambda_{0,m} &= \left(\frac{\Lambda_m}{L}\right)^2 \qquad m = 1,2,3,\cdots
\end{aligned}
$$

Where we added zero as subscript to $\lambda_{0,m}$ to indicate that this is associated with Case $n=0$. Hence the eigenfunctions for $\mu=0$ are

$$R_{0,m}(r)=\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right) \qquad n=0,m=1,2,3,\cdots$$

And $\Theta_0(\theta)=1$.

Case $n>0$

The $R(r)$ ODE becomes $r^2R''+rR'+R(r^2\lambda-n^2)=0$.

This is a Bessel ODE. If $\lambda<0$, this leads to solution that can not satisfy the boundary conditions (contains Besselk and BesselI). If $\lambda=0$ then the ODE becomes $r^2R''+rR'-n^2R=0$. This is Euler ODE whose solution is $R(r)=C_1r^n+C_2r^{-n}$. Since bounded at $r=0$. Then $C_2=0$ and the solution becomes $R(r)=C_1r^n$. Hence $R'(r)=C_1nr^{n-1}$. At $r=L$ this becomes $C_1nL^{n-1}=0$. Since $L\neq0$, then this gives $C_1=0$, trivial solution. Hence $\lambda=0$ is not possible eigenvalue when $\mu>0$.

If $\lambda>0$ the ODE becomes $r^2R''+rR'+R(r^2\lambda-n^2)=0$ with now λ positive. This is a Bessel ODE whose solution is

$$R_n(r)=C_1\,\text{BesselJ}\left(n,\sqrt{\lambda}r\right)+C_2\,\text{BesselY}\left(n,\sqrt{\lambda}r\right) \qquad n=1,2,3,\cdots$$

Since $R(r)$ is bounded at $r=0$ then $C_2=0$ and the above becomes

$$R_n(r)=C_1\,\text{BesselJ}\left(n,\sqrt{\lambda}r\right) \qquad n=1,2,3,\cdots$$

Therefore

$$R_n'(r)=C_1\frac{d}{dr}\,\text{BesselJ}\left(n,\sqrt{\lambda}r\right)$$

At $r=L$

$$R_n'(L)=C_1\frac{d}{dr}\,\text{BesselJ}\left(n,\sqrt{\lambda}L\right)$$

For non-trivial solution we want

$$\frac{d}{dr}\,\text{BesselJ}\left(n,\sqrt{\lambda}L\right)=0 \tag{7}$$

The eigenvalues λ are solved for numerically by finding all zeros of $\frac{d}{dx}\,\text{BesselJ}(n,x)$. Let these zeros be called Γ_{nm} for $n=1,2,3,\cdots,m=1,2,3,\cdots$. Hence

$$\begin{aligned}\sqrt{\lambda_{nm}}L&=\Gamma_{nm}\\ \lambda_{nm}&=\left(\frac{\Gamma_{nm}}{L}\right)^2 \qquad n=1,2,3,\cdots,m=1,2,3,\cdots\end{aligned}$$

With associated eigenfunctions $R_{n,m}(r) = \text{BesselJ}\left(n, \frac{\Gamma_{nm}}{L}r\right)$. Now that we solved for $R_{n,m}(r)$ and $\Theta_{n,m}(\theta)$, what is left is to solve $\frac{1}{k}\frac{T'}{T} = -\lambda_{nm}$.

For $n=0$

For $\lambda = 0$ the solution is constant. Say $T_{00}(t) = 1$. For $\lambda > 0$ the solution is $T_{0m}(t) = e^{-k\lambda_{0m}t}$. Where $\lambda_{0,m} = \left(\frac{\Lambda_m}{L}\right)^2, m = 1,2,3,\cdots$ and Λ_m are the zeros of $\text{BesselJ}(1,x)$.

For $n>0$ In this case $\lambda > 0$ only. The solution is $T_{nm}(t) = e^{-k\lambda_{nm}t}$. Where $\lambda_{n,m}$ are now the solutions of (7). Hence $T_{nm}(t) = e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}$

The solution now becomes

$$u(r,\theta,t) = \overbrace{R_{0,0}(r)\,\Theta_{0,0}(\theta)\,T_{0,0}(t)}^{\text{arbitrary constant}} + \overbrace{R_{0,m}(r)\,\Theta_{0,m}(\theta)\,T_{0.m}(t)}^{n=0,m>0} + \overbrace{R_{n,m}(r)\,\Theta_{n,m}(\theta)\,T_{n.m}(t)}^{n>0,m>0}$$

Or

$$\begin{aligned} u(r,\theta,t) &= \beta_0 \\ &\quad + \sum_{m=1}^{\infty} \alpha_{0,m}\,\text{BesselJ}\left(0, \frac{\Lambda_m}{L}r\right) e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} \\ &\quad + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\,\text{BesselJ}\left(n, \frac{\Gamma_{nm}}{L}r\right)\left(A_{nm}\cos(n\theta) + B_{nm}\sin(n\theta)\right) \end{aligned}$$

Where Γ_{nm} are the m^{th} zeros $\frac{d}{dx}\text{BesselJ}(n,x)$ and Λ_m are the m^{th} zeros of $\text{BesselJ}(1,x)$. These have to be found numerically.

Now we find the constants $\beta_0, \alpha_m, A_{nm}, B_{nm}$. For $n=0, m=0$, and at initial conditions at $t=0$, and applying orthogonality gives (all eigenfunctions are constants, assumed 1, in this case)

$$\begin{aligned} \int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta &= \beta_0 \int_{-\pi}^{\pi}\int_0^L rdrd\theta \\ \beta_0 &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta}{\pi L^2} \end{aligned}$$

For $n=0, m>0$

$$\begin{aligned} \int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right) rdrd\theta &= \alpha_{0,m}\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right) rdrd\theta \\ &= 2\pi\alpha_{0,m}\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right) rdr \\ \alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right) rdrd\theta}{2\pi\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right) rdr} \end{aligned}$$

For $n>0,m>0$, applying orthogonality on $\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)$ gives

$$\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)rdr=\sum_{n=1}^{\infty}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)rdr(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta))$$

applying orthogonality on $\cos(n\theta)$ the above becomes

$$\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta=A_{nm}\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta$$

$$A_{nm}=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta}$$

Similarly

$$B_{nm}=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,rdrd\theta}$$

This complete the solution. In summary, the solution is

$$\begin{aligned}u(r,\theta,t)&=\beta_0\\&+\sum_{m=1}^{\infty}\alpha_{0,m}\operatorname{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)e^{-k\left(\frac{\Lambda_m}{L}\right)^2t}\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2t}\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta))\end{aligned}$$

Where

$$\begin{aligned}\beta_0&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta}{\pi L^2}\\\alpha_{0,m}&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta}{2\pi\int_0^L \operatorname{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdr}\\A_{nm}&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta}\\B_{nm}&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,rdrd\theta}\end{aligned}$$

And Γ_{nm} are the m^{th} zeros of $\frac{d}{dx}\operatorname{BesselJ}(n,x)$ and Λ_m are the m^{th} zeros of $\operatorname{BesselJ}(1,x)$. These have to be found numerically.

4.2.2.15 [273] Insulated with θ dependency (Specific example)

problem number 273

Added June 15, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

For $t>0$ and $0<r<L$ with $L=1,k=1$ and time-independent boundary conditions:

$$u_r(L,\theta,t)=0$$

And initial conditions $u(r,\theta,0)=(2Lr-r^2)\cos\theta\sin\theta$. There is an implied periodic boundary conditions on θ

$$u(r,-\pi,t)=u(r,\pi,t)$$
$$\frac{\partial u}{\partial \theta}(r,-\pi,t)=\frac{\partial u}{\partial \theta}(r,\pi,t)$$

The solution is bounded at $r=0$.

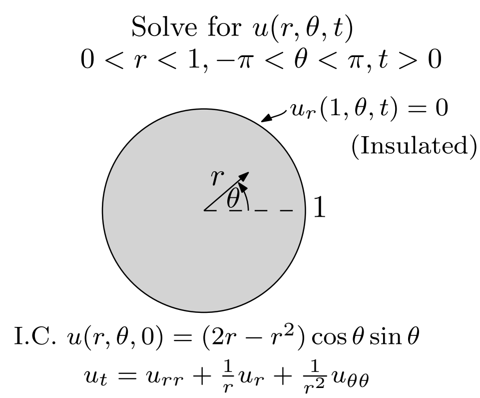


Figure 4.190: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=1;
k=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (2*L*r-r^2)*Cos[theta]*Sin[theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t)\to\ \{\quad \sum_{K[3]=1}^{\infty}\frac{e^{-tK[2,2,K[3]]^2}J_2(rK[2,2,K[3]])\left(2\,{}_1F_2\left(\frac{5}{2};3,\frac{7}{2};-\frac{1}{4}K[2,2,K[3]]^2\right)-5\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}K[2,2,K[3]]^2\right)\right)K[2,2,K[3]]^3\sin}{40((J_1(K[2,2,K[3]]))^2+J_2(K[2,2,K[3]])^2)K[2,2,K[3]]-4J_1(K[2,2,K[3]])J_2(K[2,2,K[3]]))}\right.\right.$$

Indeterminate

Maple ✗

```
restart;
k:=1;
L:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)= (2*L*r-r^2)*cos(theta)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve for $u(r,\theta,t)$

$$u_t = k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}|u(0,\theta,t)| &< \infty\\ u_t(L,\theta,t) &= 0\\ u(r,-\pi,t) &= u(r,\pi,t)\\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)\end{aligned}$$

And initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) = \left(2rL - r^2\right)\cos\theta\sin\theta \\ L &= 1 \\ k &= 1 \end{aligned}$$

The basic solution for this type of PDE was already given in problem 4.2.2.14 on page 746 as

$$\begin{aligned} u(r,\theta,t) = {} & \beta_0 \\ & + \sum_{m=1}^{\infty} \alpha_{0,m} \operatorname{BesselJ}\left(0, \frac{\Lambda_m}{L} r\right) e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} \\ & + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t} \operatorname{BesselJ}\left(n, \frac{\Gamma_{nm}}{L} r\right)\left(A_{nm}\cos(n\theta) + B_{nm}\sin(n\theta)\right) \end{aligned}$$

Where

$$\begin{aligned} \beta_0 &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\, r dr d\theta}{\pi L^2} \\ \alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(0, \frac{\Lambda_m}{L} r\right) r dr d\theta}{2\pi \int_0^L \operatorname{BesselJ}^2\left(0, \frac{\Lambda_m}{L} r\right) r dr} \\ A_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n, \frac{\Gamma_{nm}}{L} r\right)\cos(n\theta)\, r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n, \frac{\Gamma_{nm}}{L} r\right)\cos^2(n\theta)\, r dr d\theta} \\ B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n, \frac{\Gamma_{nm}}{L} r\right)\sin(n\theta)\, r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n, \frac{\Gamma_{nm}}{L} r\right)\sin^2(n\theta)\, r dr d\theta} \end{aligned}$$

And Γ_{nm} are the m^{th} zeros of $\frac{d}{dx}\operatorname{BesselJ}(n,x)$ and Λ_m are the m^{th} zeros of $\operatorname{BesselJ}(1,x)$. These have to be found numerically. This is animation of the solution for 0.2 seconds. (Animation will only show in the HTML version)

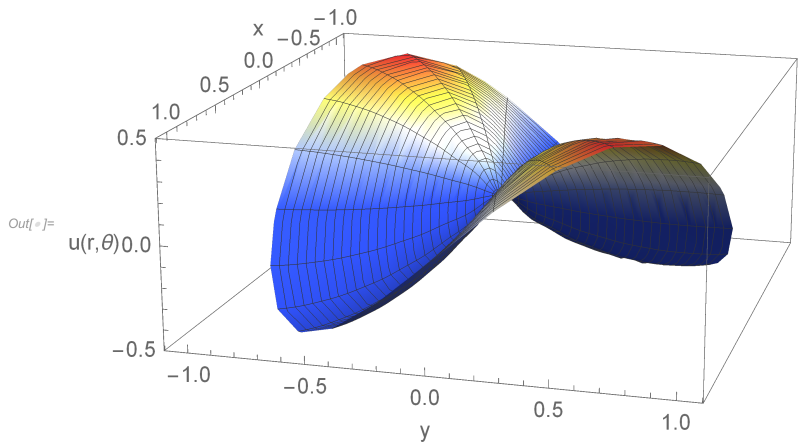


Figure 4.191: Initial state

Source code used for the above

```
Get["G:\\nabbasi\\data\\mathematica_stuff\\besselZeros_old_package\\BesselZeros.m"];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
L = 1;
k = 1; (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (2 * L * r - r^2)  Cos[θ] Sin[θ] ;
besselJ1Zero = N@Table[BesselJZero[1, m], {m, 1, maxM}];
besselJnPrimeZero = N@Table[BesselJPrimeZeros[n, m][[m]], {n, 1, maxN}, {m, 1, maxM}];
beta0 = 1/(π L^2) Integrate[f * r, {θ, -Pi, Pi}, {r, 0, L}];
alpha0M = Table[Integrate[f * r BesselJ[0, besselJ1Zero[[m]]/L r], {θ, -Pi, Pi}, {r, 0, L}] /
    (2 π Integrate[r BesselJ[0, besselJ1Zero[[m]]/L r]^2, {r, 0, L}]), {m, 1, maxM}];
Anm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Cos[n θ], {θ, -Pi, Pi}, {r, 0, L}] /
    Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Cos[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
Bnm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Sin[n θ], {θ, -Pi, Pi}, {r, 0, L}] /
    Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Sin[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
u[r_, θ_, t_] := beta0 +
    Sum[ alpha0M[[m]] BesselJ[0, besselJ1Zero[[m]]/L r] Exp[-k (besselJ1Zero[[m]]/L)^2 t] , {m, 1, maxM}] +
    Sum[Sum[Exp[-k (besselJnPrimeZero[[n, m]]/L)^2 t] BesselJ[n, besselJnPrimeZero[[n, m]]/L r]
      (Anm[[n, m]] Cos[n θ] + Bnm[[n, m]] Sin[n θ]), {n, 1, maxN}], {m, 1, maxM}];
```

Figure 4.192: Source code

```
In[ ]:= tab = Table[
         Grid[{
             {Row[{"time = ", padIt2[t, {4, 3}]}]},
             {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
             {Row[{"Initial  temperature u = (2 L r-r^2) Cos[θ] Sin[θ]"}]},
             {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
                 BaseStyle → 15,
                 ImageMargins → 5,
                 Mesh → 25,
                 PerformanceGoal → "Speed",
                 BoxRatios → {1, 1, 0.4},
                 PlotRange → {Automatic, Automatic, {-0.5, 0.5}},
                 ImageSize → 500,
                 ColorFunctionScaling → False,
                 ColorFunction → ColorData[{"TemperatureMap", {0, .4}}],
                 AxesLabel → {"x", "y", "U(r,θ)"},
                 ViewPoint → {4.03, 1.133, 1.15}
               ]
             }}],
         {t, 0, 0.3, .005}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → Table[.25, {Length[tab]}]]
```

Figure 4.193: Code for animation

4.2.2.16 [274] Insulated with θ dependency (Specific example)

problem number 274

Added June 15, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

For $t>0$ and $0<r<L$ with $L=1, k=1$ and time-independent boundary conditions:

$$u_r(L,\theta,t) = 0$$

And initial conditions $u(r,\theta,0) = (2Lr - r^2)\theta \sin\theta e^{\cos\theta}$. There is an implied periodic boundary conditions on θ

$$u(r,-\pi,t) = u(r,\pi,t)$$
$$\frac{\partial u}{\partial \theta}(r,-\pi,t) = \frac{\partial u}{\partial \theta}(r,\pi,t)$$

The solution is bounded at $r=0$.

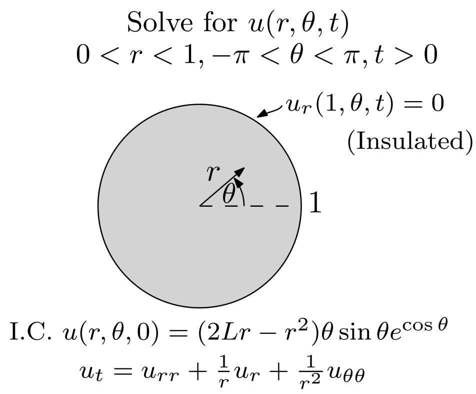


Figure 4.194: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
L=1;
k=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (2*L*r - r^2)*theta*Sin[theta]*Exp[Cos[theta]];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$Aborted

Maple ✗

```
restart;
k:=1;
L:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)= (2*L*r -r^2)*theta*sin(theta)*exp(cos(theta));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()
Hand solution

Solve for $u(r,\theta,t)$

$$u_t = k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u_t(L,\theta,t) &= 0 \\
u(r,-\pi,t) &= u(r,\pi,t) \\
u_\theta(r,-\pi,t) &= u_\theta(r,\pi,t)
\end{aligned}$$

And initial conditions

$$\begin{aligned}
u(r,\theta,0) = f(r,\theta) &= \left(2rL - r^2\right)\theta\sin\theta e^{\cos\theta} \\
L &= 1 \\
k &= 1
\end{aligned}$$

The basic solution for this type of PDE was already given in problem 4.2.2.14 on page 746 as

$$u(r,\theta,t) = \beta_0 + \sum_{m=1}^{\infty}\alpha_{0,m}\operatorname{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\left(A_{nm}\cos(n\right.$$

Where

$$\begin{aligned}
\beta_0 &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\, r dr d\theta}{\pi L^2} \\
\alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right) r dr d\theta}{2\pi\int_0^L \operatorname{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right) r dr} \\
A_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\, r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\, r dr d\theta} \\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\operatorname{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\, r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \operatorname{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\, r dr d\theta}
\end{aligned}$$

And Γ_{nm} are the m^{th} zeros of $\frac{d}{dx}\operatorname{BesselJ}(n,x)$ and Λ_m are the m^{th} zeros of $\operatorname{BesselJ}(1,x)$. These have to be found numerically. This is animation of the solution for0.18 seconds. (Animation will only show in the HTML version)

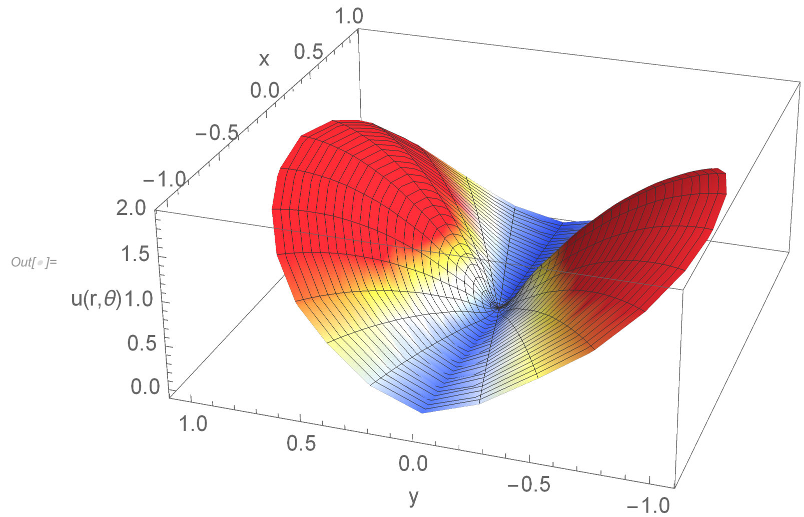


Figure 4.195: Initial state

Source code used for the above

```
In[ ]:= Get["G:\\nabbasi\\data\\mathematica_stuff\\besselZeros_old_package\\BesselZeros.m"];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
L = 1;
k = 1; (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (2 * L * r - r^2) θ Sin[θ] Exp[Cos[θ]] ;
besselJ1Zero = N@Table[BesselJZero[1, m], {m, 1, maxM}];
besselJnPrimeZero = N@Table[BesselJPrimeZeros[n, m][[m]], {n, 1, maxN}, {m, 1, maxM}];
beta0 = 1/(π L^2) Integrate[f * r, {θ, -Pi, Pi}, {r, 0, L}];
alpha0M = Table[Integrate[f * r BesselJ[0, besselJ1Zero[[m]]/L r], {θ, -Pi, Pi}, {r, 0, L}] /
    (2 π Integrate[r BesselJ[0, besselJ1Zero[[m]]/L r]^2, {r, 0, L}]), {m, 1, maxM}];
Anm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Cos[n θ], {θ, -Pi, Pi}, {r, 0, L}] /
    Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Cos[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
Bnm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Sin[n θ], {θ, -Pi, Pi}, {r, 0, L}] /
    Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Sin[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
u[r_, θ_, t_] := beta0 +
    Sum[ alpha0M[[m]] BesselJ[0, besselJ1Zero[[m]]/L r] Exp[-k (besselJ1Zero[[m]]/L)^2 t] , {m, 1, maxM}] +
    Sum[Sum[Exp[-k (besselJnPrimeZero[[n, m]]/L)^2 t] BesselJ[n, besselJnPrimeZero[[n, m]]/L r] (Anm[[n, m]] Cos[n θ] + Bnm[[n, m]] Sin[n θ]), {n, 1, maxN}], {m, 1, maxM}];
```

Figure 4.196: Source code

```
In[ ]:= tab = Table[
           Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (2*L*r-r^2)   θ Sin[θ] Exp[Cos[θ]] "}]},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], u[r, θ, t]}, {r, 0, 1}, {θ, -Pi, Pi},
                 BaseStyle → 15,
                 ImageMargins → 3,
                 Mesh → 25,
                 PerformanceGoal → "Speed",
                 BoxRatios → {1, 1, 0.4},
                 PlotRange → {Automatic, Automatic, {-.1, 2}},
                 ImageSize → 500,
                 ColorFunctionScaling → False,
                 ColorFunction → ColorData[{"TemperatureMap", {0, .8}}],
                 AxesLabel → {"x", "y", "u(r,θ)"},
                 ViewPoint → {-3.70, -1.245, 1.9}
               ]
              }}],
           {t, 0, 0.18, .001}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → Table[.12, {Length[tab]}]]

Out[ ]= anim.gif
```

Figure 4.197: Code for animation

4.3 Diffusion in 3D

Local contents

4.3.1 Spherical coordinates

Local contents

4.3.1.1 [275] No angle dependencies

problem number 275

Added March 28, 2019.

Problem 1, section 41, Fourier series and boundary value problems 8th edition by Brown and Churchill.

Solve $u_t = \nabla u$ where $\nabla u = \frac{1}{r}(ru)_{rr}$ in Spherical coordinates with initial conditions $u(r,0) = 0$ and boundary conditions $u(1,t) = t$

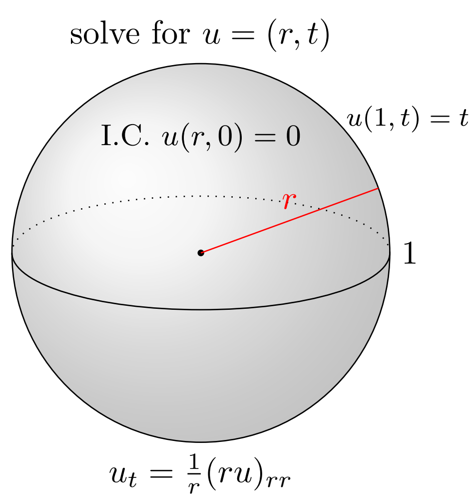


Figure 4.198: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == (k*D[r*u[r, t], {r, 2}])/r;
ic  = u[r, 0] == 0;
bc  = u[1, t] == t;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

$$\left\{\left\{u(r,t)\to \sum_{K[1]=1}^{\infty}\frac{2(-1)^{K[1]}\left(1-e^{-k\pi^2 tK[1]^2}\right)\sin(\pi rK[1])}{k\pi^3 rK[1]^3}+t\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k/r*diff(r*u(r,t),r$2);
ic  := u(r,0)=0;
bc  := u(1,t) =t;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t)=\frac{\mathcal{L}^{-1}\left(\frac{\sinh\left(\frac{r\sqrt{s}}{\sqrt{k}}\right)}{s^2\sinh\left(\frac{\sqrt{s}}{\sqrt{k}}\right)},s,t\right)}{r}$$

Has unresolved Laplace integrals

4.3.2 Cylinderical coordinates

Local contents

4.3.2.1 [276] Haberman 7.9.4 (a)

problem number 276

Added May 26, 2019.

Problem 7.9.4 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r,z)$ independent of θ if the boundary conditions are $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u(a,\theta,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of the full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

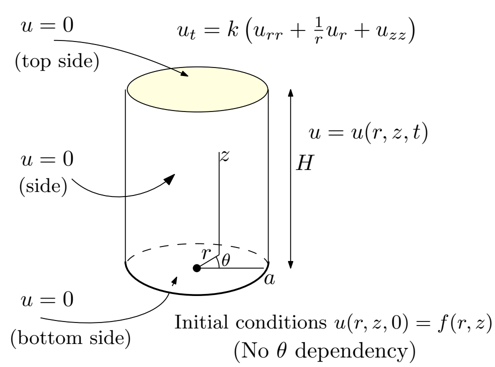


Figure 4.199: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r,  z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, 0, t] == 0, u[r, H, t] == 0, u[a, z,t] == 0};
ic  = u[r, z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r, z,t], {r, z
```

$$\left\{\left\{u(r,z,t)\to \begin{cases} \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\exp\left(-kt\left(\frac{(j_{0,K[3]})^2}{a^2}+\frac{\pi^2K[1]^2}{H^2}\right)\right)J_0\left(\frac{rj_{0,K[3]}}{a}\right)\left(\int_0^a\int_0^H rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,z)\sin\left(\frac{\pi zK[1]}{H}\right)dzdr\right)}{a^2HJ_1(j_{0,K[3]})^2} \\ \text{Indeterminate} \end{cases}\right.\right.$$

Maple ✓

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,0,t)=0,u(r,H,t)=0, u(a,z,t)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

$$u(r,z,t)=\sum_{n1=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\,\mathrm{BesselJ}\left(0,\frac{r\lambda_{n1}}{a}\right)\left(\left(\int_0^a r\,\mathrm{BesselJ}\left(0,\frac{r\lambda_{n1}}{a}\right)()\,dr\right)_{AllSolutions}\right)\mathrm{e}^{-\frac{\left(H^2\lambda_{n1}^2+\pi^2a^2n^2\right)kt}{H^2a^2}}\sin\left(\frac{\pi nz}{H}\right)}{H\,a^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2}\right],[1,2],-\lambda_{n1}^2\right)}$$

Hand solution

Solve $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0)=f(r,z)$ independent of θ if the boundary conditions are $u(r,0,t)=0, u(r,H,t)=0, u(a,z,t)=0$

Let $u(r,z,t)=T(t)\,\Phi(r,z)$. Substituting into the PDE gives

$$T'(t)\,\Phi(r,z)=k\big(T(t)\,\nabla^2\Phi\big)$$
$$\frac{T'}{kT}=\frac{\nabla^2\Phi}{\Phi\,(r,z)}=-\lambda$$

Where λ is the separation constant assumed positive. This gives $T'+\lambda kT=0$ with solution $T(t)=Ce^{-k\lambda t}$. And

$$\nabla^2\Phi+\lambda\Phi(r,z)=0 \tag{1}$$

Where $\nabla^2\Phi(r,z) = \Phi_{rr} + \frac{1}{r}\Phi_r + \Phi_{zz}$ in this case, the Laplacian in cylindrical with no θ dependency. Let $\Phi(r,z) = R(r)\,Z(z)$. Substituting in (1) gives

$$R''Z + \frac{1}{r}R' + Z'' + \lambda RZ = 0$$
$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = -\frac{Z''}{Z} = \upsilon \tag{2}$$

Where υ is the second separation constant assumed positive. This gives

$$Z'' + \upsilon Z = 0$$
$$Z(z) = A\cos\left(\sqrt{\upsilon}z\right) + B\sin\left(\sqrt{\upsilon}z\right)$$

At $z = 0, Z = 0$, hence $A = 0$ and the solution becomes $Z(z) = B\sin\left(\sqrt{\upsilon}z\right)$. At $z = H, Z = 0$, hence for non-trivial solution we want $\sqrt{\upsilon}H = n\pi, n = 1,2,3,\cdots$. Therefore

$$\upsilon_n = \left(\frac{n\pi}{H}\right)^2 \qquad n = 1,2,3,\cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \sin\left(\frac{n\pi}{H}z\right)$$

From (2) the radial equation becomes

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = \left(\frac{n\pi}{H}\right)^2$$
$$R'' + \frac{1}{r}R' + \left(\lambda - \left(\frac{n\pi}{H}\right)^2\right)R = 0$$

This is Bessel ODE. The solution is

$$R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right) + C_2 Y_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right)$$

Since Y_0 blows up at $r = 0$, it is discarded leaving $R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right)$. At $r = a, R_n(a) = 0$. For non-trivial solution we want $J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,a\right) = 0$. Therefore $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,a$ are the zeros of Bessel function $J_0(x)$. This allows us to determine all possible values of $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}$ (since a is given constant). Let the m^{th} zero of $J_0(x)$ be called Λ_m. Hence

$$
\begin{aligned}
\sqrt{\lambda-\left(\frac{n\pi}{H}\right)^2}\,a &= \Lambda_m \qquad m=1,2,3,\cdots\\
\sqrt{\lambda-\left(\frac{n\pi}{H}\right)^2} &= \frac{\Lambda_m}{a}\\
\lambda_{nm} &= \frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2 \qquad n=1,2,3,\cdots,m=1,2,3,\cdots
\end{aligned}
$$

For each n, there are m values of λ_{nm}. The corresponding radial eigenfunction is

$$
R_m(r)=J_0\left(\frac{\Lambda_m}{a}r\right) \qquad m=1,2,3,\cdots
$$

Therefore the complete solution is

$$
\begin{aligned}
u(r,z,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}e^{-k\lambda_{nm}t}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)\\
&= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}e^{-k\left(\frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2\right)t}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)
\end{aligned}
$$

What is left is to determine A_{nm}. This is done using initial conditions by using orthogonality. At $t=0$

$$
f(r,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)
$$

Multiplying both sides by $rJ_0\left(\frac{\Lambda_{m'}}{a}r\right)$ and integrating

$$
\begin{aligned}
\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr &= \int_0^a\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr\\
\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr &= \sum_{n=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{H}z\right)\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdr
\end{aligned}
$$

Multiplying both sides by $\sin\left(\frac{n'\pi}{H}z\right)$ and integrating

$$
\begin{aligned}
\int_0^H\left(\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr\right)\sin\left(\frac{n'\pi}{H}z\right)dz &= \int_0^H\sum_{n=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{H}z\right)\sin\left(\frac{n'\pi}{H}z\right)\left(\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)r\right.\\
\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\sin\left(\frac{n\pi}{H}z\right)rdrdz &= A_{nm}\int_0^H\int_0^a \sin^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz\\
A_{nm} &= \frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\sin\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a \sin^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}
\end{aligned}
$$

Hence the final solution is

$$u(r,z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\sin\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a \sin^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}e^{-k\left(\frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2\right)t}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

Where Λ_m is the m^{th} zero of $J_0(x)$. To verify the solution, it is compared to numerical solution, using the following values $a=1, H=3, k=\frac{1}{100}, f(r,z)=(a-r)\sin\left(\frac{z}{H}\pi\right)$. The summation was taken up to $n=10, m=10$. This animation only looks at cross section of the cylinder in the middle. The height indicates the amount of heat. As time passes the cylinder cools down. It runs for 15 seconds. (The animation will only show on the HTML version, not the PDE version of this report).

The following is the source code used to generate the above

```
ClearAll[t, r, m, n, f];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
      NumberPadding → {"0", "0"}, SignPadding → True];
nTerms = 10;
a = 1;
H = 3;
k = 1 / 100;
f[r_, z_, theta_] := (a - r) Sin[Pi / H * z];
(*f[r_,z_,theta_]:=Sin[r Pi]Sin[z/H *Pi];*)
lam = Table[ BesselJZero[0, m], {m, 1, nTerms}] // N;
A = Table[Integrate[Integrate[f[r, z, 0] * BesselJ[0, lam[[m]] / a * r ] * r, {r, 0, a}] * Sin[n * Pi / H * z], {z, 0, H}] /
       Integrate[Integrate[BesselJ[0, lam[[m]] / a * r ]^2 * r, {r, 0, a}] Sin[n * Pi / H * z]^2, {z, 0, H}],
     {n, 1, nTerms}, {m, 1, nTerms}];
mySol[r_, z_, t_] := Sum[Sum[ A[[n, m]] * Exp[-k * (lam[[m]]^2 / a^2 + (n * Pi / H)^2) t] * Sin[n Pi / H z] BesselJ[0, lam[[m]] * r / a],
      {n, 1, nTerms}], {m, 1, nTerms}];
```

Figure 4.200: Code to generate solution

```
In[ ]:= framesMySolVer2 = Table[
          Print["t=", t];
          Grid[{
             {Row[{"current time = ", padIt2[t, {3, 2}]}]},
             {Row[{"Tempreature in middle of cylinder = ", padIt2[mySol[0, 1.5, t], {5, 4}]}]},
             {Plot3D[mySol[Sqrt[x^2 + y^2], 1.5, t],
                Element[{x, y}, Disk[{0, 0}, 1]],
                PerformanceGoal → "Speed",
                PlotRange → {All, All, {0, 1}}, Mesh → Full,
                ImageSize → 400
              ]
             }}],
          {t, 0, 15, 0.1}];

In[ ]:= Manipulate[
          framesMySolVer2[[i]],
          {{i, 1, "time"}, 1, Length@framesMySolVer2, 1, Appearance → "Labeled"}
        ]

In[ ]:= Export["anim_plot3d_3.gif", framesMySolVer2,
          "DisplayDurations" → Table[0.05, {Length[framesMySolVer2]}]]
```

Figure 4.201: Code to make animation and export it

4.3.2.2 [277] Haberman 7.9.4 (b)

problem number 277

Added May 26, 2019.

Problem 7.9.4 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ independent of θ, subject to boundary conditions $u_z(r,0,t) = 0$, $u_z(r,H,t) = 0$, $u_r(a,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of the full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

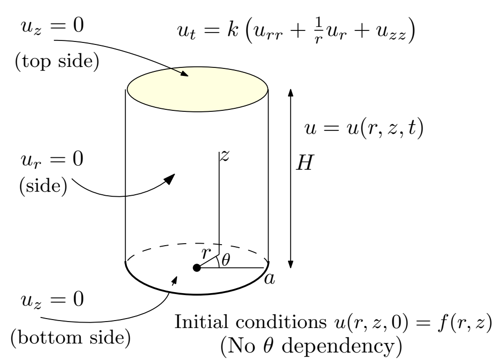


Figure 4.202: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, z, t], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,1,0][u][r,0, t] == 0, Derivative[0,1,0][u][r,  H, t] == 0, Derivative[1,
ic  = u[r,z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r,  z,t], {r,
```

$$\left\{\left\{u(r,z,t)\to \left\{ \frac{2\int_0^a\int_0^H rf(r,z)dzdr}{a^2H} + \sum_{K[1]=1}^{\infty} \frac{4e^{-\frac{k\pi^2 tK[1]^2}{H^2}}\cos\left(\frac{\pi z K[1]}{H}\right)\int_0^a\int_0^H r\cos\left(\frac{\pi z K[1]}{H}\right) f(r,z)dzdr}{a^2H} + \sum_{K[3]=1}^{\infty} \frac{2e^{-}}{a^2H\sqrt{J_0(}}\right.\right.\right.$$

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc:=eval(diff(u(r,z,t),z),z=0)=0,eval(diff(u(r,z,t),z),z=H)=0, eval(diff(u(r,z,t),r),r=a)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

sol=()

<u>Hand solution</u>

Solve $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ independent of θ if the boundary conditions are $u_z(r,0,t) = 0, u_z(r,H,t) = 0, u_r(a,z,t) = 0$

Let $u(r,z,t) = T(t)\,\Phi(r,z)$. Substituting into the PDE gives

$$\begin{aligned} T'(t)\,\Phi(r,z) &= k\big(T(t)\,\nabla^2\Phi\big)\\ \frac{T'}{kT} &= \frac{\nabla^2\Phi}{\Phi\,(r,z)} = -\lambda \end{aligned}$$

Where λ is the separation constant assumed positive. This gives $T' + \lambda k T = 0$ with solution $T(t) = Ce^{-k\lambda t}$. And

$$\nabla^2\Phi + \lambda\Phi(r,z) = 0 \tag{1}$$

Where $\nabla^2\Phi(r,z) = \Phi_{rr} + \frac{1}{r}\Phi_r + \Phi_{zz}$ in this case, the Laplacian in cylindrical with no θ

dependency. Let $\Phi(r,z) = R(r)\,Z(z)$. Substituting in (1) gives

$$R''Z + \frac{1}{r}R' + Z'' + \lambda RZ = 0$$

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = -\frac{Z''}{Z} = \upsilon \tag{2}$$

Where υ is the second separation. Then $Z'' + \upsilon Z = 0$

case $\upsilon = 0$

This gives

$$Z(z) = Az + B$$
$$Z' = A$$

At $z = 0, Z' = 0$, hence $A = 0$. The solution becomes $Z(z) = B$ and $Z'(z) = 0$. Which satisfies the boundary conditions at $z = H$. Hence $\upsilon = 0$ is eigenvalue with $Z_0 = 1$ as eigenfunction.

case $\upsilon > 0$

$$Z'' + \upsilon Z = 0$$
$$Z(z) = A\cos\left(\sqrt{\upsilon}z\right) + B\sin\left(\sqrt{\upsilon}z\right)$$
$$Z'(z) = -A\sqrt{\upsilon}\sin\left(\sqrt{\upsilon}z\right) + B\sqrt{\upsilon}\cos\left(\sqrt{\upsilon}z\right)$$

At $z = 0, Z' = 0$, hence $B = 0$ and the solution becomes $Z(z) = A\cos\left(\sqrt{\upsilon}z\right)$ and $Z'(z) = -A\sqrt{\upsilon}\sin\left(\sqrt{\upsilon}z\right)$. At $z = H, Z_t = 0$, hence for non-trivial solution we want $\sqrt{\upsilon}H = n\pi, n = 1, 2, 3, \cdots$. Therefore

$$\upsilon_n = \left(\frac{n\pi}{H}\right)^2 \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \cos\left(\frac{n\pi}{H}z\right)$$

From (2) the radial equation becomes

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = \upsilon$$

case $\upsilon = 0$

The above becomes

$$R_0'' + \frac{1}{r}R_0' + R_0\lambda_0 = 0$$

This is Bessel ODE whose solution is

$$R_0(r) = c_1 J_0\left(\sqrt{\lambda_0} r\right) + c_2 Y_0\left(\sqrt{\lambda_0} r\right)$$

Since Y_0 blows at $r=0$ the solution becomes

$$\begin{aligned} R_0(r) &= c_1 J_0\left(\sqrt{\lambda_0} r\right) \\ R_0'(r) &= c_1 J_0'\left(\sqrt{\lambda_0} r\right) \\ &= -c_1\sqrt{\lambda_0} J_1\left(\sqrt{\lambda_0} r\right) \end{aligned}$$

At $r=a$ then $R_0'(a)=0$. For nontrivial solution we want $J_1\left(\sqrt{\lambda} a\right)=0$. Hence $\sqrt{\lambda} a$ are zeros of $J_1(x)$. This determines λ. Let Λ_m be the zeros of $J_1(x)$, therefore

$$\begin{aligned} \sqrt{\lambda_0} a &= \Lambda_m \qquad n=0, m=1,2,3,\cdots \\ \lambda_{0,m} &= \left(\frac{\Lambda_m}{a}\right)^2 \qquad m=1,2,3,\cdots \end{aligned}$$

Hence the radial eigenfunction for $\upsilon = 0$ is given by

$$R_{0,m}(r) = c_1 J_0\left(\frac{\Lambda_m}{a} r\right) \qquad m=1,2,3,\cdots$$

case $\upsilon > 0$

$$\begin{aligned} \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda &= \left(\frac{n\pi}{H}\right)^2 \\ R'' + \frac{1}{r}R' + \left(\lambda - \left(\frac{n\pi}{H}\right)^2\right) R &= 0 \end{aligned}$$

This is Bessel ODE. The solution is

$$R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right) + C_2 Y_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$$

Since Y_0 blows up at $r=0$, it is discarded leaving $R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$. Hence $R_n'(r) = C_1 J_0'\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right) = -C_1\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} J_1\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$

At $r=a$, $R_n'(a)=0$. For non-trivial solution we want $J_1\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} a\right) = 0$. Therefore $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} a$ are the zeros of Bessel function $J_1(x)$. This allows us to determine all possible values of $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}$ (since a is given constant). Let the m^{th} zero of $J_1(x)$ be called Λ_m. Hence

$$\begin{aligned}
\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\, a &= \Lambda_m \qquad m = 1,2,3,\cdots \\
\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} &= \frac{\Lambda_m}{a} \\
\lambda_{nm} &= \frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2 \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots
\end{aligned}$$

For each n, there are m values of λ_{nm}. The corresponding radial eigenfunction is

$$R_m(r) = J_0\left(\frac{\Lambda_m}{a} r\right) \qquad m = 1,2,3,\cdots$$

Now we look at the time solution again. $T(t) = Ce^{-k\lambda t}$. For $n = 0$ this gives $T_{0,m}(t) = C_{0,m} e^{-k\lambda_{0,m} t} = C_{0,m} e^{-k\left(\frac{\Lambda_m}{a}\right)^2 t}$. For $n > 0$ the solution becomes

$$T_{n,m}(t) = C_{n,m} e^{-k\lambda_{n,m} t} = Ce^{-k\left(\frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2\right)t}$$

Therefore the complete solution is

$$\begin{aligned}
u(r,z,t) &= \sum_{m=1}^{\infty} B_m R_{0,m}(r)\, T_{0,m}(t) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} e^{-k\lambda_{nm} t} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) \\
&= \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) e^{-k\left(\frac{\Lambda_m}{a}\right)^2 t} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} e^{-k\left(\frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2\right)t} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right)
\end{aligned}$$

What is left is to determine A_{nm} and B_m. This is done using initial conditions by using orthogonality. At $t = 0$

$$f(r,z) = \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right)$$

For $n = 0$

$$\begin{aligned}
f(r,z) &= \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) \\
\int_0^a f(r,z)\, J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr &= \int_0^a \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr \\
\int_0^a f(r,0)\, J_0\left(\frac{\Lambda_m}{a} r\right) r dr &= \int_0^a B_m J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr \\
B_m &= \frac{\int_0^a f(r,0)\, J_0\left(\frac{\Lambda_m}{a} r\right) r dr}{\int_0^a J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr} \qquad m = 1,2,3,\cdots
\end{aligned}$$

Integrating again both sides over z gives (we can think of $\cos\left(\frac{n\pi}{H}z\right)=1$ with $n=0$ as the eigenfunction in this case).

$$\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz = B_m\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz$$

$$B_m = \frac{\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz}{\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

For $n>0$

$$f(r,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

Multiplying both sides by $rJ_0\left(\frac{\Lambda_{m'}}{a}r\right)$ and integrating

$$\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr = \int_0^a\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr$$

$$\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr = \sum_{n=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdr$$

Multiplying both sides by $\cos\left(\frac{n'\pi}{H}z\right)$ and integrating

$$\int_0^H\left(\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr\right)\cos\left(\frac{n'\pi}{H}z\right)dz = \int_0^H\sum_{n=1}^{\infty}A_{nm}\sin\left(\frac{n\pi}{H}z\right)\cos\left(\frac{n'\pi}{H}z\right)\left(\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)$$

$$\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz = A_{nm}\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz$$

$$A_{nm}=\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

Hence the final solution is

$$u(r,z,t)=\sum_{m=1}^{\infty}B_mJ_0\left(\left(\frac{\Lambda_m}{a}\right)^2r\right)e^{-k\left(\frac{\Lambda_m}{a}\right)^2t}+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}e^{-k\left(\frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2\right)t}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

With

$$A_{nm}=\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

$$B_m=\frac{\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz}{\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

Where Λ_m is the m^{th} zero of $J_1(x)$.

To verify the solution, it is compared to numerical solution, using the following values $a = 1, H = 3, k = \frac{1}{100}, f(r,z) = (a-r)\sin\left(\frac{z}{H}\pi\right)$. The summation was taken up to $n = 10, m = 10$. This animation only looks at cross section of the cylinder in the middle. The height indicates the amount of heat. As time passes the initial temperature averages inside (cylinder is insulated). It runs for 15 seconds. (The animation will only show on the HTML version, not the PDE version of this report).

The following is the source code used to generate the above

```
In[]:= ClearAll[t, r, m, n, f, A, B];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
      NumberPadding → {"0", "0"}, SignPadding → True];
nTerms = 10;
a = 1;
H = 3;
k = 1 / 100;
f[r_, z_, theta_] := (a - r) Sin[Pi / H * z];
lam = Table[ BesselJZero[1, m], {m, 1, nTerms}] // N;

B = Table[ Integrate[Integrate[f[r, 0, 0] * BesselJ[0, lam[[m]]/a r] r, {r, 0, a}], {z, 0, H}] /
      Integrate[Integrate[BesselJ[0, lam[[m]]/a * r]^2 * r, {r, 0, a}], {z, 0, H}], {m, 1, nTerms}];

A = Table[ Integrate[Integrate[f[r, z, 0] * BesselJ[0, lam[[m]]/a r] r, {r, 0, a}] Cos[n Pi/H * z], {z, 0, H}] /
      Integrate[Integrate[BesselJ[0, lam[[m]]/a r]^2 r, {r, 0, a}] Cos[n Pi/H * z]^2, {z, 0, H}], {n, 1, nTerms}, {m, 1, nTerms}];

mySol[r_, z_, t_] := Sum[B[[m]] Exp[-k (lam[[m]]^2/a^2) t] BesselJ[0, (lam[[m]]/a)^2 r], {m, 1, nTerms}] +
    Sum[Sum[ A[[n, m]] * Exp[-k (lam[[m]]^2/a^2 + (n Pi/H)^2) t] Cos[n Pi/H * z] BesselJ[0, lam[[m]]/a r], {n, 1, nTerms}], {m, 1, nTerms}];
```

Figure 4.203: Code to generate solution

```
In[]:= framesMySolVer2 = Table[
         Print["t=", t];
         Grid[{
            {Row[{"current time = ", padIt2[t, {3, 2}]}]},
            {Row[{"Tempreature in middle of cylinder = ", padIt2[mySol[0, 1.5, t], {5, 4}]}]},
            {Plot3D[mySol[Sqrt[x^2 + y^2], 1.5, t],
              Element[{x, y}, Disk[{0, 0}, 1]],
              PerformanceGoal → "Speed",
              PlotRange → {All, All, {-0.125, .22}}, Mesh → Full,
              ImageSize → 400
             ]
            }}],
         {t, 0, 15, 0.1}];

In[]:= Manipulate[
       framesMySolVer2[[i]],
       {{i, 1, "time"}, 1, Length@framesMySolVer2, 1, Appearance → "Labeled"}
      ]

In[]:= Export["anim.gif", framesMySolVer2,
       "DisplayDurations" → Table[0.1, {Length[framesMySolVer2]}]]

Out[]= anim.gif
```

Figure 4.204: Code to make animation and export it

4.3.2.3 [278] Haberman 7.9.4 (c)

problem number 278

Added May 26, 2019.

Problem 7.9.4 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ subject to boundary conditions $u(r,0,t) = 0$, $u(r,H,t) = 0$, $u_r(a,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

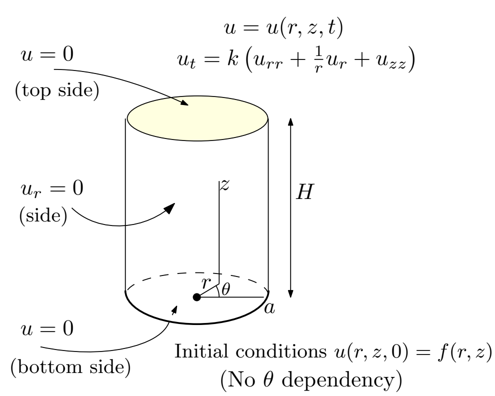


Figure 4.205: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, 0, t] == 0, u[r,  H, t] == 0, Derivative[1,0,0][u][a, z,t] == 0};
ic  = u[r, z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r, z,t], {r, z
```

$$\left\{\left\{u(r,z,t)\to \left\{\sum_{K[1]=1}^{\infty}\frac{4e^{-\frac{k\pi^2 tK[1]^2}{H^2}}\left(\int_0^a\int_0^H rf(r,z)\sin\left(\frac{\pi zK[1]}{H}\right)dzdr\right)\sin\left(\frac{\pi zK[1]}{H}\right)}{a^2H}+\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\exp\left(-kt\left(\frac{\pi^2K[1]^2}{H^2}+K[2\right.\right.}{a^2H\sqrt{J_0}}\right.\right.\right.$$

Indetermi

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,0,t)=0,u(r,H,t)=0, eval(diff(u(r,z,t),r),r=a)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

sol=()

4.3.2.4 [279] Haberman 7.9.3 (a)

problem number 279

Added May 26, 2019.

Problem 7.9.3 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u(r,0,z,t) = 0$, $u(r,\frac{\pi}{2},z,t) = 0$, $u(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

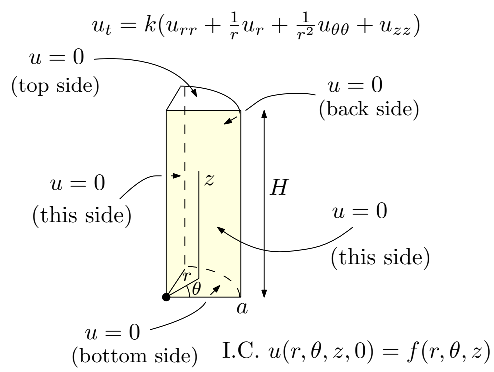


Figure 4.206: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0, t] == 0, u[r, theta, H, t] == 0, u[r, 0, z,t] == 0,u[r,Pi/2,z,t]==0,u[
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc, ic}, u[r, theta
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,theta,0,t)=0,u(r,theta,H,t)=0, u(r,0,z,t)=0, u(r,Pi/2,z,t)=0,u(a,Pi,z,t)=0;
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

4.3.2.5 [280] Haberman 7.9.3 (b)

problem number 280

Added May 26, 2019.

Problem 7.9.3 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u_z(r,\theta,0,t) = 0$, $u_z(r,\theta,H,t) = 0$, $u_\theta(r,0,z,t) = 0$, $u_\theta(r,\frac{\pi}{2},z,t) = 0$, $u_r(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

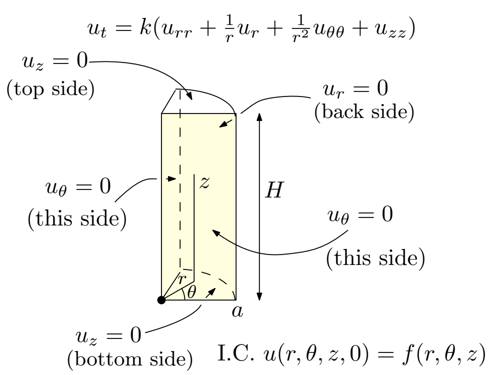


Figure 4.207: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1,0][u][r, theta, 0, t] == 0, Derivative[0,0,1,0][u][r, theta, H, t] =
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc}, u[r, theta, z,
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc:=eval(diff(u(r,theta,z,t),z),z=0)=0,eval(diff(u(r,theta,z,t),z),z=H)=0,eval(diff(u(r,thet
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

4.3.2.6 [281] Haberman 7.9.3 (c)

problem number 281

Added May 26, 2019.

Problem 7.9.3 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u_\theta(r,0,z,t) = 0$, $u(r,\frac{\pi}{2},z,t) = 0$, $u_r(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

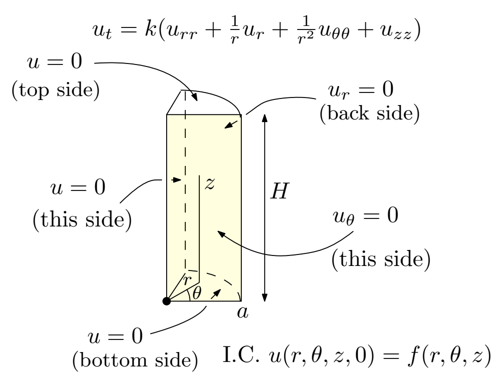


Figure 4.208: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0, t] == 0, u[r, theta, H, t] == 0, u[r, 0, z,t] == 0,u[r,Pi/2,z,t]==0,u[
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc}, u[r, theta, z,
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,theta,0,t)=0,u(r,theta,H,t)=0, eval(diff(u(r,theta,z,t),theta),theta=0)=0, u(r,Pi
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

CHAPTER 5

Elliptic PDE's (Laplace, Poisson, Helmholtz)

Local contents

5.1 Laplace in 2D

Local contents

5.1.1 Cartesian coordinates

Local contents

5.1.1.1 [282] Rectangle, 3 edges zero, buttom edge not

problem number 282

Added Nov 20, 2019

Solve Laplace PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} u(x,0) &= f(x) \\ u(x,H) &= 0 \\ u(0,y) &= 0 \\ u(L,y) &= 0 \end{aligned}$$

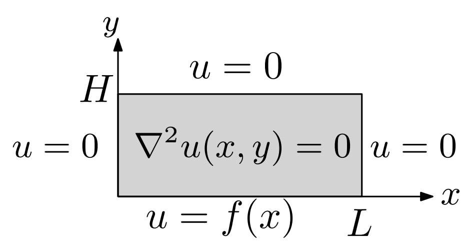


Figure 5.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[x,0]==f[x], u[x, H] == 0, u[0, y] == 0,u[L,y]==0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} \operatorname{csch}\left(\frac{Hn\pi}{L}\right) \text{FourierSinCoefficient}\left[f(x), x, n, \text{FourierParameters} \to \left\{1, \frac{\pi}{L}\right\}\right] \sin\left(\frac{n\pi x}{L}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(x,0)=f(x), u(x, H) = 0, u(0,y) = 0,u(L,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{2\left(-e^{\frac{\pi n y}{L}}+e^{-\frac{\pi(-2H+y)n}{L}}\right)\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\sin\left(\frac{\pi n x}{L}\right)}{\left(e^{\frac{2\pi H n}{L}}-1\right)L}$$

Hand solution

Solve

$$\begin{aligned}\nabla^2 u&=0 \qquad \text{on a rectangle} \qquad R=\{0<x<a,0<y<b\}\\ u(x,0)&=f(x)\\ u(x,H)&=0\\ u(0,y)&=0\\ u(L,y)&=0\end{aligned}$$

Solution

Let $u(x,y)=X(x)Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above.

$$\frac{X''}{X}=-\frac{Y''}{Y}=-\lambda$$

Two ODE's are obtained

$$X''+\lambda X=0 \tag{1}$$

With the boundary conditions

$$X(0) = 0$$
$$X(L) = 0$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$Y(0) = f(x)$$
$$Y(H) = 0$$

<u>Case $\lambda < 0$</u>

The solution to (1) is

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = L$ this gives $X = B\sinh\left(\sqrt{|\lambda|}L\right)$. But $\sinh\left(\sqrt{|\lambda|}L\right) = 0$ only at 0 and $\sqrt{|\lambda|}L \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

<u>Case $\lambda = 0$</u>

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = L$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

<u>Case $\lambda > 0$</u>

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = L$

$$0 = B\sin\left(\sqrt{\lambda}L\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$Y_n = C_n \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \tag{4}$$

Applying B.C. at $y = H$ gives

$$\begin{aligned}
0 &= C_n \cosh\left(\frac{n\pi}{L}H\right) + D_n \sinh\left(\frac{n\pi}{L}H\right) \\
C_n &= -D_n \frac{\sinh\left(\frac{n\pi}{L}H\right)}{\cosh\left(\frac{n\pi}{L}H\right)} \\
&= -D_n \tanh\left(\frac{n\pi}{L}H\right)
\end{aligned}$$

Hence (4) becomes

$$\begin{aligned}
Y_n &= -D_n \tanh\left(\frac{n\pi}{L}H\right) \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\
&= D_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right) \cosh\left(\frac{n\pi}{L}y\right)\right)
\end{aligned}$$

Now the complete solution is produced

$$\begin{aligned}
u_n(x,y) &= Y_n X_n \\
&= D_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right) \cosh\left(\frac{n\pi}{L}y\right)\right) B_n \sin\left(\frac{n\pi}{L}x\right)
\end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x,y) = B_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right) \cosh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right) \cosh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right) \tag{5}$$

The nonhomogeneous boundary condition is now resolved. At $y = 0$

$$u(x,0) = f(x)$$

Therefore (5) becomes

$$f(x) = \sum_{n=1}^{\infty} -B_n \tanh\left(\frac{n\pi}{a}b\right) \sin\left(\frac{n\pi}{a}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}
\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx &= -\int_0^a \sin\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} B_n \tanh\left(\frac{n\pi}{L}H\right)\sin\left(\frac{n\pi}{L}x\right)dx\\
&= -\sum_{n=1}^{\infty} B_n \tanh\left(\frac{n\pi}{L}b\right)\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\
&= -B_n \tanh\left(\frac{m\pi}{L}H\right)\left(\frac{L}{2}\right)
\end{aligned}$$

Hence

$$B_n = -\frac{2}{L}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\tanh\left(\frac{n\pi}{L}H\right)}$$

The solution (5) becomes

$$\begin{aligned}
u(x,y) &= -\frac{2}{L}\sum_{n=1}^{\infty}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\tanh\left(\frac{n\pi}{L}H\right)}\left(\sinh\left(\frac{n\pi}{L}y\right)-\tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)\\
&= -\frac{2}{L}\sum_{n=1}^{\infty}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\left(\frac{\sinh\left(\frac{n\pi}{L}y\right)}{\tanh\left(\frac{n\pi}{L}H\right)}-\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)
\end{aligned}$$

5.1.1.2 [283] Rectangle, 3 edges zero, right edge not

problem number 283

Added January 12, 2020

Solve Laplace PDE inside square $\nabla^2 u(x,y) = 0$ with $0 \leq x \leq 1, 0 \leq y \leq 1$, with following boundary conditions

$$\begin{aligned}
u(x,0) &= 0\\
u(x,1) &= 0\\
u(0,y) &= 0\\
u(1,y) &= y(1-y)
\end{aligned}$$

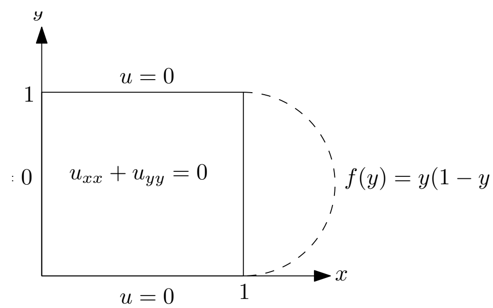


Figure 5.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1;b=1;
pde = Laplacian[u[x,y],{x,y}] == 0;
bc  = {u[x,0]==0, u[x, b] == 0, u[0, y] == 0,u[a,y]==y*(1-y)};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\rightarrow\sum_{n=1}^{\infty}-\frac{4(-1+(-1)^n)\operatorname{csch}(n\pi)\sin(n\pi y)\sinh(n\pi x)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
a:=1;
b:=1;
pde := VectorCalculus:-Laplacian(u(x,y),'cartesian'[x,y])=0;
bc:=u(x,0)=0, u(x, b) = 0, u(0,y) = 0,u(a,y)=y*(1-y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) ),output
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-\frac{4((-1)^n-1)\left(e^{\pi nx}-e^{-\pi nx}\right)\sin(\pi ny)}{\pi^3\left(e^{\pi n}-e^{-\pi n}\right)n^3}\right)$$

Hand solution

a is used for the length of the x dimension and b for the length of the y dimension.

Solution

Let $u(x,y) = X(x)Y(x)$. Substituting this into the PDE gives

$$X''Y + Y''X = 0$$

Dividing throughout by $XY \neq 0$ and simplifying gives

$$\frac{X''}{X} = -\frac{Y''}{Y} = \lambda$$

This gives the eigenvalue ODE

$$\begin{aligned} Y'' + \lambda Y &= 0 \\ Y(0) &= 0 \\ Y(b) &= 0 \end{aligned} \tag{1}$$

The solution to (1) gives the eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1, 2, 3\cdots$ and since $L = b$, this becomes

$$\lambda_n = \left(\frac{n\pi}{b}\right)^2 \qquad n = 1, 2, \cdots$$

And the corresponding eigenfunction

$$\begin{aligned} Y_n(y) &= c_n \sin\left(\sqrt{\lambda_n} y\right) \\ &= c_n \sin\left(\frac{n\pi}{b} y\right) \end{aligned}$$

Therefore the corresponding nonhomogeneous $X(x)$ ODE

$$\begin{aligned} X_n'' - \lambda_n X_n &= 0 \\ X_n(0) &= 0 \\ X_n(a) &= y - y^2 \end{aligned} \tag{2}$$

The solution to (2), since λ_n is positive is

$$\begin{aligned} X_n(x) &= A_n \cosh\left(\sqrt{\lambda_n} x\right) + B_n \sinh\left(\sqrt{\lambda_n} x\right) \\ &= A_n \cosh\left(\frac{n\pi}{b} x\right) + B_n \sinh\left(\frac{n\pi}{b} x\right) \end{aligned}$$

Boundary conditions $X(0) = 0$ gives

$$0 = A_n$$

The solution (3) now simplifies to

$$X_n(x) = B_n \sinh\left(\frac{n\pi}{b} x\right)$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,y) &= X_n Y_n \\ &= c_n \sinh\left(\frac{n\pi}{b}x\right)\sin\left(\frac{n\pi}{b}y\right) \end{aligned}$$

Where the constants B_n is merged with c_n. The solution is

$$u(x,y) = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}x\right)\sin\left(\frac{n\pi}{b}y\right) \tag{3}$$

c_n is now found by applying the boundary condition at $x = a$. The above becomes

$$y - y^2 = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}a\right)\sin\left(\frac{n\pi}{b}y\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{b}y\right)$ and integrating gives

$$\int_0^b (y-y^2)\sin\left(\frac{m\pi}{b}y\right)dy = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}a\right)\left(\int_0^b \sin\left(\frac{m\pi}{b}y\right)\sin\left(\frac{n\pi}{b}y\right)dy\right)$$

By orthogonality the above reduces to

$$\begin{aligned} \int_0^b (y-y^2)\sin\left(\frac{m\pi}{b}y\right)dy &= c_n \sinh\left(\frac{m\pi}{b}a\right)\int_0^b \sin^2\left(\frac{m\pi}{b}y\right)dy \\ &= \frac{b}{2}c_m \sinh\left(\frac{m\pi}{b}a\right) \end{aligned}$$

Therefore

$$c_n = \frac{2}{b}\frac{1}{\sinh\left(\frac{m\pi}{b}a\right)}\int_0^b (y-y^2)\sin\left(\frac{n\pi}{b}y\right)dy$$

Now replacing $\underline{a=1, b=1}$, the above becomes

$$\begin{aligned} c_n &= \frac{2}{\sinh(n\pi)}\int_0^1 (y-y^2)\sin(n\pi y)\,dy \\ &= \frac{2}{\sinh(n\pi)}\left(\frac{-2(-1+(-1)^n)}{n^3\pi^3}\right) \\ &= \frac{-4}{\sinh(n\pi)}\frac{(-1+(-1)^n)}{n^3\pi^3} \end{aligned}$$

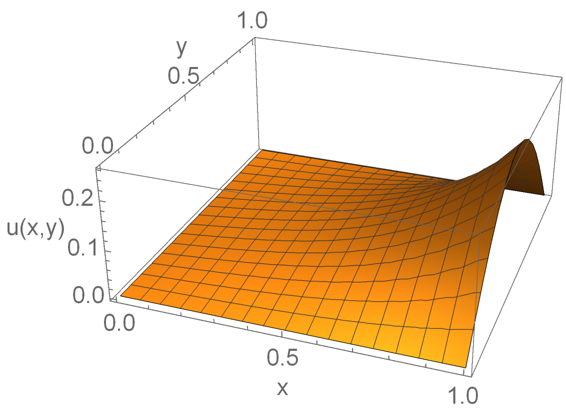
Hence the solution (3) becomes

$$u(x,y) = \frac{-4}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1+(-1)^n)}{n^3}\frac{\sinh(n\pi x)}{\sinh(n\pi)}\sin(n\pi y)$$

This is a 3D plot of the solution.

In[40]:=
```
mySol[x_, y_] := -4 / Pi^3 Sum[ ((-1 + (-1)^n) / n^3) (Sinh[n Pi x] / Sinh[n Pi]) Sin[n Pi y] , {n, 1, 2}]
Plot3D[mySol[x, y], {x, 0, 1}, {y, 0, 1}, AxesLabel → {"x", "y", "u(x,y)"}, BaseStyle → 14]
```

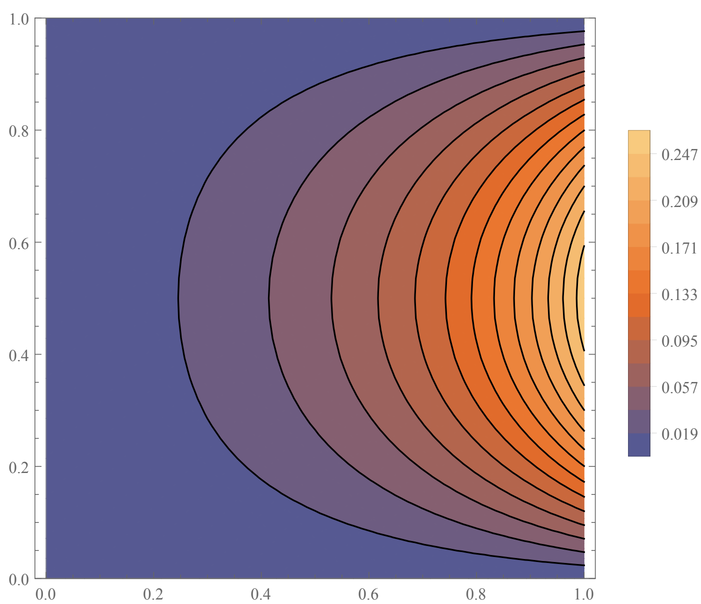
Out[41]=



This is a contour plot

```
ContourPlot[Evaluate[mySol[x, y]], {x, 0, 1}, {y, 0, 1}, AxesLabel → {x, y},
  PlotRange → {-1, 1}, Contours → 100, PlotTheme -> "Scientific", PlotLegends → Automatic]
```

Out[16]=



5.1.1.3 [284] Rectangle, 3 edges zero, buttom edge has impulse

problem number 284

Added Jan. 8, 2020.

This is Problem 6.3.10 from Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve

$$
\begin{aligned}
\nabla^2 u &= 0 \qquad \text{on a rectangle} \qquad R = \{0 < x < a, 0 < y < b\} \\
u(x,0) &= f(x) \\
u(x,b) &= 0 \\
u(0,y) &= 0 \\
u(a,y) &= 0
\end{aligned}
$$

When the boundary data $f(x) = \delta(x - \xi)$ is a delta function at a point $0 < \xi < a$.

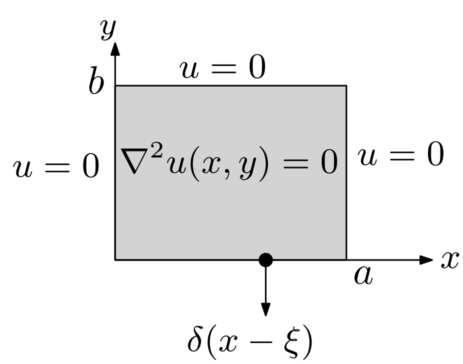


Figure 5.3: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = Laplacian[u[x, y], {x, y}] == 0;
f[x_] := DiracDelta[x - zeta];
bc = {u[x, 0] == f[x], u[x, b] == 0, u[0, y] == 0, u[a, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to \sum_{n=1}^{\infty}\frac{2\operatorname{csch}\left(\frac{bn\pi}{a}\right)\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi\zeta}{a}\right)\sinh\left(\frac{n\pi(b-y)}{a}\right)}{a}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x->Dirac(x-zeta);
bc:=u(x,0)=f(x), u(x, b) = 0, u(0,y) = 0,u(a,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-\frac{2\left(-\mathrm{e}^{\frac{\pi(b-y)n}{a}}+\mathrm{e}^{-\frac{\pi(b-y)n}{a}}\right)\sin\left(\frac{\pi nx}{a}\right)\sin\left(\frac{\pi n\zeta}{a}\right)}{\left(\mathrm{e}^{\frac{\pi bn}{a}}-\mathrm{e}^{-\frac{\pi bn}{a}}\right)a}\right)$$

Hand solution

Let $u(x,y)=X(x)\,Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above.

$$\frac{X''}{X}=-\frac{Y''}{Y}=-\lambda$$

Two ODE's are obtained

$$X'' + \lambda X = 0 \tag{1}$$

With the boundary conditions

$$\begin{aligned} X(0) &= 0 \\ X(a) &= 0 \end{aligned}$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$\begin{aligned} Y(0) &= f(x) \\ Y(b) &= 0 \end{aligned}$$

In all these cases λ will turn out to be positive. This is shown below.

<u>Case $\lambda < 0$</u>

The solution to (1) is

$$X = A \cosh\left(\sqrt{|\lambda|}x\right) + B \sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B \sinh\left(\sqrt{|\lambda|}x\right)$. At $x = a$ this gives $X = B \sinh\left(\sqrt{|\lambda|}a\right)$. But $\sinh\left(\sqrt{|\lambda|}a\right) = 0$ only at 0 and $\sqrt{|\lambda|}a \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

<u>Case $\lambda = 0$</u>

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = a$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

<u>Case $\lambda > 0$</u>

Solution is

$$X = A \cos\left(\sqrt{\lambda}x\right) + B \sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B \sin\left(\sqrt{\lambda}x\right)$. At $x = a$

$$0 = B \sin\left(\sqrt{\lambda}a\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}a\right) = 0$ or $\sqrt{\lambda}a = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{a}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{a}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$Y_n = C_n \cosh\left(\frac{n\pi}{a}y\right) + D_n \sinh\left(\frac{n\pi}{a}y\right) \tag{4}$$

Applying B.C. at $y = b$ gives

$$\begin{aligned} 0 &= C_n \cosh\left(\frac{n\pi}{a}b\right) + D_n \sinh\left(\frac{n\pi}{a}b\right) \\ C_n &= -D_n \frac{\sinh\left(\frac{n\pi}{a}b\right)}{\cosh\left(\frac{n\pi}{a}b\right)} \\ &= -D_n \tanh\left(\frac{n\pi}{a}b\right) \end{aligned}$$

Hence (4) becomes

$$\begin{aligned} Y_n &= -D_n \tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right) + D_n \sinh\left(\frac{n\pi}{a}y\right) \\ &= D_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x,y) &= Y_n X_n \\ &= D_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right) B_n \sin\left(\frac{n\pi}{a}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x,y) = B_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right)\sin\left(\frac{n\pi}{a}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right)\sin\left(\frac{n\pi}{a}x\right) \tag{5}$$

The nonhomogeneous boundary condition is now resolved. At $y = 0$

$$u(x,0) = f(x) = \delta(x - \xi)$$

Therefore (5) becomes

$$\delta(x-\xi)=\sum_{n=1}^{\infty}-B_n\tanh\left(\frac{n\pi}{a}b\right)\sin\left(\frac{n\pi}{a}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{a}x\right)$ and integrating gives

$$\begin{aligned}\int_0^a\delta(x-\xi)\sin\left(\frac{m\pi}{a}x\right)dx&=-\int_0^a\sin\left(\frac{m\pi}{a}x\right)\sum_{n=1}^{\infty}B_n\tanh\left(\frac{n\pi}{a}b\right)\sin\left(\frac{n\pi}{a}x\right)dx\\&=-\sum_{n=1}^{\infty}B_n\tanh\left(\frac{n\pi}{a}b\right)\int_0^a\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}x\right)dx\\&=-B_n\tanh\left(\frac{m\pi}{a}b\right)\left(\frac{a}{2}\right)\end{aligned}$$

Hence

$$B_n=-\frac{2}{a}\frac{\int_0^a\delta(x-\xi)\sin\left(\frac{n\pi}{a}x\right)dx}{\tanh\left(\frac{n\pi}{a}b\right)}$$

But $\int_0^a\delta(x-\xi)\sin\left(\frac{m\pi}{L}x\right)dx=\sin\left(\frac{m\pi}{L}\xi\right)$ by the property delta function. Therefore

$$B_n=-\frac{2}{a}\frac{\sin\left(\frac{n\pi}{a}\xi\right)}{\tanh\left(\frac{n\pi}{a}b\right)}$$

This completes the solution. (4) becomes

$$\begin{aligned}u(x,y)&=-\frac{2}{a}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{a}\xi\right)}{\tanh\left(\frac{n\pi}{a}b\right)}\left(\sinh\left(\frac{n\pi}{a}y\right)-\tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right)\sin\left(\frac{n\pi}{a}x\right)\\&=-\frac{2}{a}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{a}\xi\right)\sin\left(\frac{n\pi}{a}x\right)\left(\frac{\sinh\left(\frac{n\pi}{a}y\right)}{\tanh\left(\frac{n\pi}{a}b\right)}-\cosh\left(\frac{n\pi}{a}y\right)\right)\end{aligned}$$

Looking at the solution above, it is composed of functions that are all differentiable. Hence the solution is infinitely differentiable inside the rectangle.

Here is a plot of the above solution using $a=\pi,b=\frac{1}{2},\xi=1$.

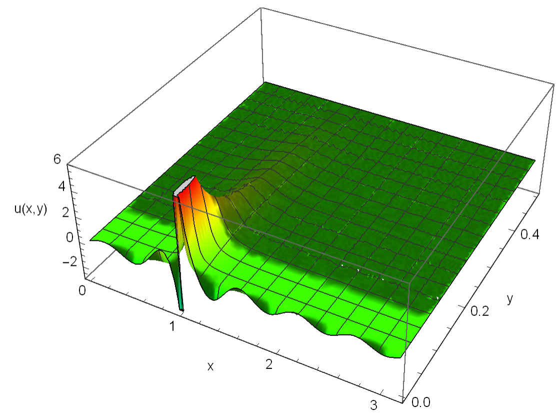


Figure 5.4: Plot of $u(x, y)$

```
u[x_, y_, ξ_] := -2/a Sum[Sin[n π/a ξ] Sin[n π/a x] (Sinh[n π/a y]/Tanh[n π/a b] - Cosh[n π/a y]), {n, 1, 300}];
a = Pi; b = 1/2; ξ = 1;
p = Plot3D[u[x, y, ξ], {x, 0, a}, {y, 0, b}, PlotRange → {Automatic, Automatic, {-3, 7}},
        PlotPoints → 40, AxesLabel → {"x", "y", "u(x,y)"},
        ColorFunction → Function[{x, y, z}, Hue[.45 (1 - z)]]];
```

Figure 5.5: Code used for the above plot

5.1.1.4 [285] Haberman 2.5.1 (a)

problem number 285

This is problem 2.5.1 part (a) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y) &= 0\\ \frac{\partial u}{\partial x}(L,y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= f(x)\end{aligned}$$

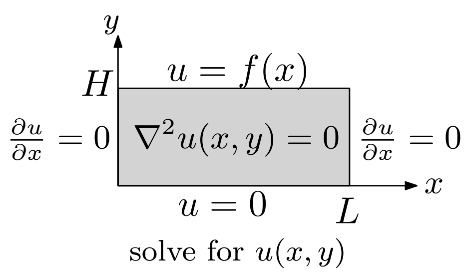


Figure 5.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == 0, Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi x}{L}\right)\operatorname{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L\cos\left(\frac{n\pi x}{L}\right)f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L}+\frac{y\int_0^L f(x)\,dx}{HL}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
#these simplifications below to convert answer to one that match standard;
sol:=convert(sol,trigh);
sol:=simplify(expand(sol));
```

$$u(x,y) = \frac{2H\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L \cos\left(\frac{\pi n x}{L}\right) f(x) dx\right) \cos\left(\frac{\pi n x}{L}\right) \sinh\left(\frac{\pi n y}{L}\right)}{\sinh\left(\frac{\pi H n}{L}\right)}\right) + y\left(\int_0^L f(x)\, dx\right)}{HL}$$

5.1.1.5 [286] Haberman 2.5.1 (b)

problem number 286

This is problem 2.5.1 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial x}(0,y) &= g(y) \\ \frac{\partial u}{\partial x}(L,y) &= 0 \\ u(x,0) &= 0 \\ u(x,H) &= 0 \end{aligned}$$

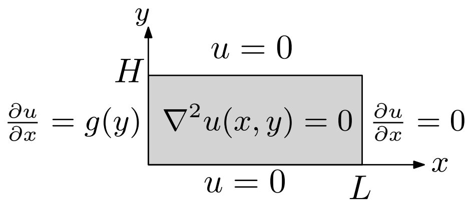


Figure 5.7: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == g[y], Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-\frac{2\left(\cosh\left(\frac{\pi n x}{H}\right)+\cosh\left(\frac{\pi(-2L+x)n}{H}\right)+\sinh\left(\frac{\pi n x}{H}\right)-\sinh\left(\frac{\pi(-2L+x)n}{H}\right)\right)\left(\int_0^H g(y)\sin\left(\frac{\pi n y}{H}\right)\right.}{\pi\left(\cosh\left(\frac{2\pi L n}{H}\right)+\sinh\left(\frac{2\pi L n}{H}\right)-1\right)n}\right.$$

5.1.1.6 [287] Haberman 2.5.1 (c)

problem number 287

This is problem 2.5.1 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial x}(0, y) &= 0 \\ u(L, y) &= g(y) \\ u(x, 0) &= 0 \\ u(x, H) &= 0 \end{aligned}$$

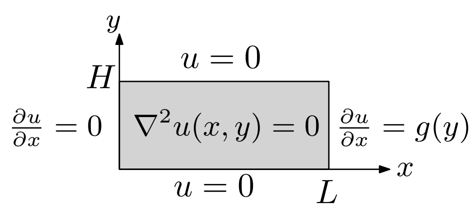


Figure 5.8: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == 0, u[L, y] == g[y], u[x, 0] == 0, u[x, H] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

$$\left\{\left\{u(x,y) \to \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{H}} \cosh\left(\frac{\pi x K[1]}{H}\right) \left(\int_0^H \frac{\sqrt{2}g(y)\sin\left(\frac{\pi y K[1]}{H}\right)}{\sqrt{H}}\, dy\right) \operatorname{sech}\left(\frac{L\pi K[1]}{H}\right) \sin\left(\frac{\pi y K[1]}{H}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^H g(y)\sin\left(\frac{\pi n y}{H}\right)dy\right)\cosh\left(\frac{\pi n x}{H}\right)\sin\left(\frac{\pi n y}{H}\right)}{H\cosh\left(\frac{\pi L n}{H}\right)}$$

5.1.1.7 [288] Haberman 2.5.1 (d)

problem number 288

This is problem 2.5.1 part (d) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\leq x\leq L, 0\leq y\leq H$, with following boundary conditions

$$\begin{aligned}u(0,y)&=g(y)\\u(L,y)&=0\\\frac{\partial u}{\partial y}u(x,0)&=0\\u(x,H)&=0\end{aligned}$$

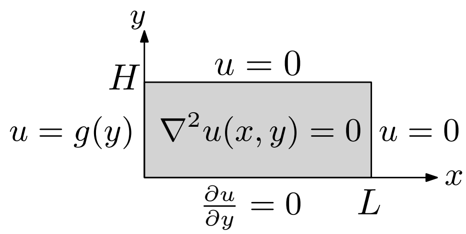


Figure 5.9: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[0, y] == g[y], u[L, y] == 0, Derivative[0, 1][u][x, 0] == 0, u[x, H] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{(2n-1)\pi y}{2H}\right)\operatorname{csch}\left(\frac{L(2n-1)\pi}{2H}\right)\left(\int_0^H\frac{\sqrt{2}\cos\left(\frac{(2n-1)\pi y}{2H}\right)g(y)}{\sqrt{H}}\,dy\right)\sinh\left(\frac{(2n-1)\pi(L-x)}{2H}\right)}{\sqrt{H}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc  := u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=0}^{\infty}\frac{2\left(\cosh\left(\frac{(-2L+x)\pi\left(n+\frac{1}{2}\right)}{H}\right)-\cosh\left(\frac{(2n+1)\pi x}{2H}\right)+\sinh\left(\frac{(-2L+x)\pi\left(n+\frac{1}{2}\right)}{H}\right)+\sinh\left(\frac{(2n+1)\pi x}{2H}\right)\right)\left(\int_0^H\right.}{\left(\cosh\left(\frac{(2n+1)\pi L}{H}\right)-\sinh\left(\frac{(2n+1)\pi L}{H}\right)-1\right)H}$$

5.1.1.8 [289] Haberman 2.5.1 (e)

problem number 289

This is problem 2.5.1 part (e) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(L,y) &= 0 \\ u(x,0) - \frac{\partial u}{\partial y}u(x,0) &= 0 \\ u(x,H) &= f(x) \end{aligned}$$

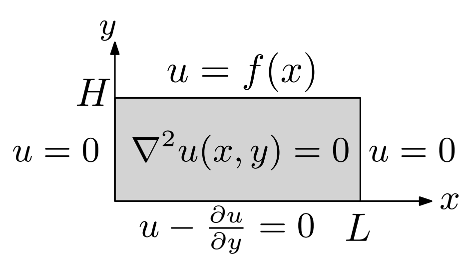


Figure 5.10: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[0, y] == 0, u[L, y] == 0, u[x, 0] - Derivative[0, 1][u][x, 0] == 0, u[x, H] == f[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,y)\rightarrow\sum_{n=1}^{\infty}\frac{\sqrt{2}\sqrt{\frac{1}{L}}\left(\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{n\pi x}{L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{n\pi x}{L}\right)\left(n\pi\cosh\left(\frac{n\pi y}{L}\right)+L\sinh\left(\frac{n\pi y}{L}\right)\right)}{n\pi\cosh\left(\frac{Hn\pi}{L}\right)+L\sinh\left(\frac{Hn\pi}{L}\right)}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc  := u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{2\left(L\sinh\left(\frac{\pi ny}{L}\right)+\pi n\cosh\left(\frac{\pi ny}{L}\right)\right)\left(\int_0^L f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi nx}{L}\right)}{\left(L\sinh\left(\frac{\pi Hn}{L}\right)+\pi n\cosh\left(\frac{\pi Hn}{L}\right)\right)L}$$

Hand solution

Let $u(x,y)=X(x)Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above. Two ODE's (1,2) are obtained as follows

$$X''+\lambda X=0 \tag{1}$$

With the boundary conditions

$$\begin{aligned} X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$\begin{aligned} Y(0) &= Y'(0) \\ Y(H) &= f(x) \end{aligned}$$

In all these cases λ will turn out to be positive. This is shown for this problem only and not be repeated again.

Case $\lambda < 0$

The solution to (1) us

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = L$ this gives $X = B\sinh\left(\sqrt{|\lambda|}L\right)$. But $\sinh\left(\sqrt{|\lambda|}L\right) = 0$ only at 0 and $\sqrt{|\lambda|}L \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = L$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = L$

$$0 = B\sin\left(\sqrt{\lambda}L\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$\begin{aligned} Y_n &= C_n \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ Y_n' &= C_n \frac{n\pi}{L} \sinh\left(\frac{n\pi}{L}y\right) + D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) \end{aligned}$$

Applying B.C. at $y = 0$ gives

$$\begin{aligned} Y(0) &= Y'(0) \\ C_n \cosh(0) &= D_n \frac{n\pi}{L} \cosh(0) \\ C_n &= D_n \frac{n\pi}{L} \end{aligned}$$

The eigenfunctions Y_n are

$$\begin{aligned} Y_n &= D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ &= D_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x,y) &= Y_n X_n \\ &= D_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) B_n \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x,y) = B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x,y) = \sum_{n=1}^{\infty} B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

The nonhomogeneous boundary condition is now resolved. At $y = H$

$$u(x,H) = f(x)$$

Therefore

$$f(x) = \sum_{n=1}^{\infty} B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}H\right) + \sinh\left(\frac{n\pi}{L}H\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sin\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\sin\left(\frac{n\pi}{L}x\right)dx\\ &= \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= B_m\left(\frac{m\pi}{L}\cosh\left(\frac{m\pi}{L}H\right)+\sinh\left(\frac{m\pi}{L}H\right)\right)\frac{L}{2}\end{aligned}$$

Hence

$$B_n = \frac{2}{L}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)} \tag{4}$$

This completes the solution. In summary

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}y\right)+\sinh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)$$

With B_n given by (4).

5.1.1.9 [290] Unit triangle B.C.

problem number 290

Taken from Mathematica DSolve help pages.

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\leq x\leq 1, 0\leq y\leq 2$, with following boundary conditions

$$\begin{aligned}u(0,y)&=0\\ u(1,y)&=0\\ u(x,0)&=\text{UnitTriagle(2 x-1)}\\ u(x,2)&=\text{UnitTriagle(2 x-1)}\end{aligned}$$

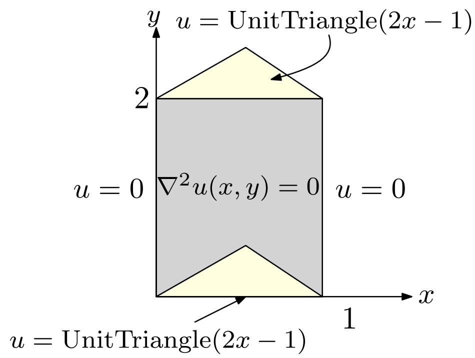


Figure 5.11: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
 L0 = 1;
 H0 = 2;
bc  = DirichletCondition[u[x, y] == Piecewise[{{UnitTriangle[2*x - L0], y == 0 || y == H0}},
 domain = Rectangle[{0, 0}, {L0, H0}];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], Element[{x, y}, do
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{8\operatorname{csch}(2n\pi)\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)(\sinh(n\pi(2-y))+\sinh(n\pi y))}{n^2\pi^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x-> piecewise(x>0 and x<1/2, 2*x, x>1/2 and x<1, 2-2*x);
bc  := u(0,y)=0,u(1,y)=0,u(x,0)=f(x),u(x,2)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-\frac{8\left(e^{\pi(y-2)n}-e^{\pi n y}-e^{-\pi(y-2)n}+e^{-\pi n y}\right)\sin(\pi n x)\sin\left(\frac{\pi n}{2}\right)}{\pi^2\left(-e^{-2\pi n}+e^{2\pi n}\right)n^2}\right)$$

5.1.1.10 [291] Top edge at infinity

problem number 291

Added December 20, 2018.

Example 21, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq x \leq L, 0 \leq y \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0,y) &= A \\ u(L,y) &= 0 \\ u(x,0) &= 0 \end{aligned}$$

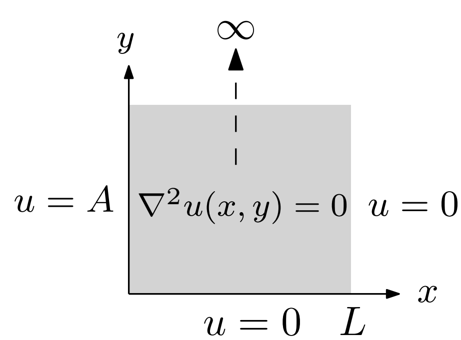


Figure 5.12: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == A, u[L, y] == 0, u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y)\to \frac{A\left(-iL\log\left(1-e^{\frac{i\pi(x+iy)}{L}}\right)+iL\log\left(1-e^{-\frac{\pi(y+ix)}{L}}\right)+\pi(L-x)\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = A;
bc_right_edge:= u(L, y) = 0;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y)=-\frac{Ax}{L}+A+\sum_{n=1}^{\infty}\left(-\frac{2A\,\mathrm{e}^{-\frac{\pi n y}{L}}\sin\left(\frac{\pi n x}{L}\right)}{\pi n}\right)$$

Hand solution

Let

$$u=U+v \tag{1}$$

Where U satisfies $\nabla^2 U=0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0)=A, v(L)=0$. This implies

$$v(x)=A\left(1-\frac{x}{L}\right)$$

Hence $u=U+A\left(1-\frac{x}{L}\right)$. Substituting this back in $\nabla^2 u=0$ gives

$$\nabla^2 U=0$$

But with boundary condition on right edge being zero now. Let $U=X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X}+\frac{Y''}{Y}=0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$\begin{aligned} U(x, y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \end{aligned}$$

Using the above in (1) gives the solution

$$u(x, y) = A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}$$

At $y = 0$ the above gives

$$\begin{aligned} 0 &= A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\ A\left(\frac{x}{L} - 1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $\frac{A}{L} x$

$$\begin{aligned} B_n &= \frac{2}{L} \int_0^L A\left(\frac{x}{L} - 1\right) \sin\left(\frac{n\pi}{L} x\right) dx \\ &= \frac{2A}{L} \int_0^L \left(\frac{x}{L} - 1\right) \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L} \frac{L}{n\pi} \\ &= -\frac{2A}{n\pi} \end{aligned}$$

Hence the solution (2) becomes

$$u(x,y)=A\Big(1-\frac{x}{L}\Big)-2\frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{L}x\right)e^{-\frac{n\pi}{L}y}$$

5.1.1.11 [292] Top edge at infinity

problem number 292

Added March 19, 2019

Solve Laplace equation

$$u_{xx}+u_{yy}=0$$

Inside a rectangle $0\leq x\leq L, 0\leq y\leq\infty$, with following boundary conditions

$$\begin{aligned}u(0,y)&=0\\u(L,y)&=A\\u(x,0)&=0\end{aligned}$$

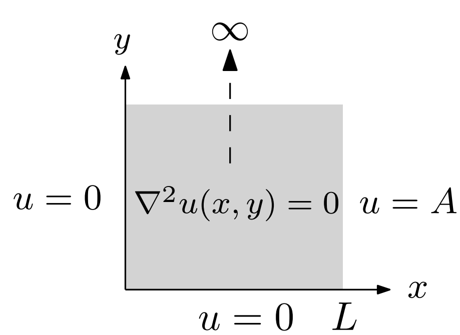


Figure 5.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[L, y] == A, u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y)\to\frac{A\left(iL\log\left(1+e^{\frac{i\pi(x+iy)}{L}}\right)-iL\log\left(1+e^{-\frac{\pi(y+ix)}{L}}\right)+\pi x\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_right_edge:= u(L, y) = A;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y)=\frac{Ax}{L}+\left(\sum_{n=1}^{\infty}\frac{2A(-1)^{n}\,\mathrm{e}^{-\frac{\pi n y}{L}}\sin\left(\frac{\pi n x}{L}\right)}{\pi n}\right)$$

Hand solution

Let

$$u=U+v \tag{1}$$

Where U satisfies $\nabla^2 U=0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0)=0A, v(L)=A$. This implies

$$v(x)=A\frac{x}{L}$$

Hence $u=U+\frac{A}{L}x$. Substituting this back in $\nabla^2 u=0$ gives

$$\nabla^2 U=0$$

But with boundary condition on right edge being zero now. Let $U=X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X}+\frac{Y''}{Y}=0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$\begin{aligned} U(x, y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \end{aligned}$$

Using the above in (1) gives the solution

$$u(x, y) = \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}$$

At $y = 0$ the above gives

$$\begin{aligned} 0 &= \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\ -\frac{A}{L} x &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $-\frac{A}{L} x$

$$\begin{aligned} B_n &= -\frac{2}{L} \int_0^L \frac{A}{L} x \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L^2} \int_0^L x \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L^2} \frac{(-1)^{n+1} L^2}{n\pi} \\ &= \frac{2A}{n\pi} (-1)^n \end{aligned}$$

Hence the solution (2) becomes

$$u(x,y) = \frac{A}{L}x + \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi}{L}x\right)e^{-\frac{n\pi}{L}y}$$

5.1.1.12 [293] Right edge at infinity

problem number 293

Added March 19, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq L, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(x,0) &= A \\ u(x,L) &= 0 \end{aligned}$$

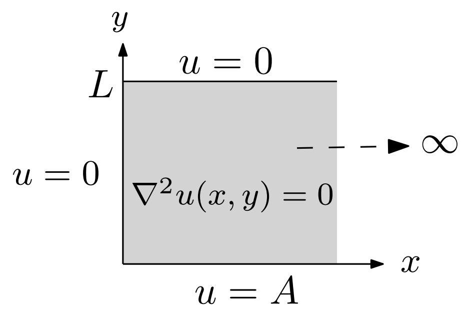


Figure 5.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, 0] == A, u[x, L] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \frac{A\left(-iL\log\left(1-e^{-\frac{\pi(x-iy)}{L}}\right) + iL\log\left(1-e^{-\frac{\pi(x+iy)}{L}}\right) + \pi(L-y)\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = 0;
bc_bottom_edge:= u(x, 0) = A;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y) = -\frac{Ay}{L} + A + \sum_{n=1}^{\infty}\left(-\frac{2A\,\mathrm{e}^{-\frac{\pi n x}{L}}\sin\left(\frac{\pi n y}{L}\right)}{\pi n}\right)$$

Hand solution

Let

$$u(x,y) = U(x,y) + v(y) \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with bottom edge boundary conditions zero, and $v(y)$ satisfies the nonhomogeneous boundary conditions $v(0) = A, v(L) = 0$. This implies

$$v(y) = A\left(1 - \frac{y}{L}\right)$$

Substituting (1) back in $\nabla^2 u = 0$ results in

$$\nabla^2 U = 0$$

But with boundary condition on bottom edge as $U = 0$. Now we can use separation of variables. Let $U = X(x)\,Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the Y direction. Hence

$$\frac{Y''}{Y} = -\frac{X''}{X} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} Y'' + \lambda Y &= 0 \\ Y(0) &= 0 \\ Y(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $Y_n(x) = \sin\left(\sqrt{\lambda_n} y\right)$. The X ode is

$$
\begin{aligned}
X_n'' - \lambda_n X_n &= 0 \\
X_n(0) &= 0
\end{aligned}
$$

Since $\lambda_n > 0$ then the solution is $X_n(y) = c_{1_n} e^{\sqrt{\lambda_n} x} + c_{2_n} e^{-\sqrt{\lambda_n} x}$. Since $X_n(x)$ is bounded, then $c_{1_n} = 0$ and the $X_n(x) = c_{2_n} e^{-\sqrt{\lambda_n} x}$. Hence by superposition the solution is

$$
\begin{aligned}
U(x, y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\
&= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} y\right) e^{-\sqrt{\lambda_n} x} \\
&= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x}
\end{aligned}
$$

Substituting the above in (1) gives

$$
u(x, y) = A\left(1 - \frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x} \tag{2}
$$

At $x = 0$ the above gives

$$
\begin{aligned}
0 &= A\left(1 - \frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) \\
A\left(\frac{y}{L} - 1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right)
\end{aligned}
$$

Therefore B_n are the Fourier sine coefficients of $A\left(\frac{y}{L} - 1\right)$

$$
\begin{aligned}
B_n &= \frac{2}{L} \int_0^L A\left(\frac{y}{L} - 1\right) \sin\left(\frac{n\pi}{L} y\right) dy \\
&= \frac{2A}{L} \int_0^L \left(\frac{y}{L} - 1\right) \sin\left(\frac{n\pi}{L} y\right) dy \\
&= -\frac{2A}{L} \frac{L}{n\pi} \\
&= -\frac{2A}{n\pi}
\end{aligned}
$$

Hence the solution (2) becomes

$$
u(x, y) = A\left(1 - \frac{y}{L}\right) - \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x}
$$

5.1.1.13 [294] Right edge at infinity

problem number 294

Added March 20, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq L, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(x,L) &= e^{-x} \\ u(x,0) &= 0 \end{aligned}$$

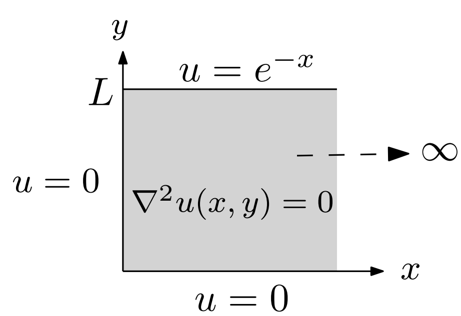


Figure 5.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, L] == Exp[-x], u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y)\to \begin{cases} \int_0^\infty \frac{2\operatorname{csch}(LK[1])K[1]\sin(xK[1])\sinh(yK[1])}{\pi K[1]^2+\pi}dK[1] & x\geq 0\land y\geq 0\land L>0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = exp(-x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
#I need to find out how Maple obtained the above solution. It seems to have unknown constant
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
```

$$u(x,y) = \frac{y\,\mathrm{e}^{-x}}{L} + \sum_{n=1}^{\infty}\left(-\frac{\left(\left(L^2-\pi^2 n^2\right)\_F1(n)\,\mathrm{e}^{-\frac{\pi n x}{L}} - 2\left(-(-1)^n\,\mathrm{e}^{-x} + (L+\pi n)\_F1(n)\right)L + \left((-4L-2\right.\right.}{\pi\,(L+\pi n)\,n}\right.$$

Hand solution

Let $u = X(x)\,Y(x)$. Substituting this in $\nabla^2 u = 0$

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\frac{X''}{X} = -\frac{Y''}{Y} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ |X(x)| &< \infty \end{aligned}$$

case $\lambda < 0$

Solution is $X(x) = c_1\cosh\left(\sqrt{-\lambda}x\right) + c_2\sinh\left(\sqrt{-\lambda}x\right)$. Since $X(0) = 0$ then $c_1 = 0$. Solution becomes $X(x) = c_2\sinh\left(\sqrt{-\lambda}x\right)$. Since sinh is not bounded on $x > 0$ as $x \to \infty$ then $c_2 = 0$. Therefore $\lambda < 0$ is not eigenvalue.

case $\lambda = 0$

Solution is $X(x) = c_1 x + c_2$. At $x = 0$ this gives $c_2 = 0$. Hence solution is $X(x) = c_1 x$. This is bounded as $x \to \infty$ only when $c_1 = 0$. Therefore $\lambda = 0$ is not eigenvalue.

case $\lambda > 0$

Let $\lambda = \alpha^2, \alpha > 0$. Then solution is $X(x) = c_1 \cos(\alpha x) + c_2 \sin(\alpha x)$. At $x = 0$ this results in $0 = c_1$. Hence the eigenvalues are $\lambda = \alpha^2$ for all real positive real numbers and eigenfunctions are

$$X_\alpha(x) = \sin(\alpha x)$$

For the Y ode,

$$\begin{aligned} Y'' - \alpha^2 Y &= 0 \\ Y(0) &= 0 \end{aligned}$$

The solution is $Y_\alpha(y) = c_1 e^{\alpha y} + c_2 e^{-\alpha y}$. Since $Y(0) = 0$ then $c_2 = -c_1$ and the solution becomes $Y_\alpha(y) = c_1(e^{\alpha y} - e^{-\alpha y}) = c_1 \sinh(\alpha y)$. Hence the solution is generalized linear combination of $Y(y)\,X(x)$ given by Fourier integral (since eigenvalues are continuous now and not discrete)

$$\begin{aligned} u(x,y) &= \int_0^\infty A(\alpha)\,Y_\alpha(y)\,X_\alpha(x)\,d\alpha \\ &= \int_0^\infty A(\alpha)\sinh(\alpha y)\sin(\alpha x)\,d\alpha \end{aligned} \tag{1}$$

When $y = L$, then above becomes

$$e^{-x} = \int_0^\infty (A(\alpha)\sinh(\alpha L))\sin(\alpha x)\,d\alpha$$

Hence the coefficient $A(\alpha)\sinh(\alpha L)$ is given by

$$\begin{aligned} A(\alpha)\sinh(\alpha L) &= \frac{2}{\pi}\int_0^\infty e^{-x}\sin(\alpha x)\,dx \\ &= \frac{2}{\pi}\frac{\alpha}{1+\alpha^2} \end{aligned}$$

Therefore $A(\alpha) = \frac{2}{\pi \sinh(\alpha L)}\frac{\alpha}{1+\alpha}$. The solution (1) becomes

$$u(x,y) = \frac{2}{\pi}\int_0^\infty \frac{\alpha \sinh(\alpha y)\sin(\alpha x)}{(1+\alpha^2)\sinh(\alpha L)}d\alpha$$

5.1.1.14 [295] Right edge at infinity

problem number 295

Added April 4, 2019.

Second midterm exam problem, Math 4567, UMN. Spring 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq 1, 0 \leq x \leq \infty$, with following boundary conditions

$$u(0, y) = 0$$
$$u(x, 1) = f(x)$$
$$u(x, 0) = 0$$

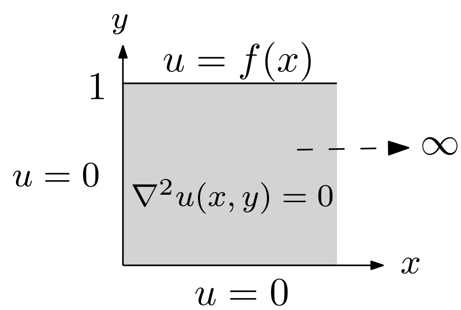


Figure 5.16: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, 1] == f[x], u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \begin{cases} \int_0^\infty \frac{2\operatorname{csch}(K[1])\left(\int_0^\infty f(x)\sin(xK[1])\,dx\right)\sin(xK[1])\sinh(yK[1])}{\pi} dK[1] & x \geq 0 \land y \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, 1) = f(x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
#Maple can not solve it when using boundedseries(x = infinity)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
```

$$u(x,y) = yf(x) + \int_0^x \left(\sum_{n=1}^{\infty} \left(\frac{2(-1)^n \left(\frac{d^2}{d\tau^2} f(\tau)\right) e^{\pi(-\tau+x)n}}{\pi n} - \_F1(n) e^{\pi(-\tau+x)n} + \_F1(n) e^{-\pi(-\tau+x)n} \right) \sin (\right.$$

5.1.1.15 [296] Laplace PDE in 2D Cartesian with boundary condition as Dirac function

problem number 296

Added December 20, 2018

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary condition

$$u(x,0) = \delta(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[x, 0] == DiracDelta[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], x, y]], 60*10]];
```

$$\left\{\left\{u(x,y)\to\left\{\begin{array}{cc}\frac{y}{\pi(x^2+y^2)} & y\geq 0\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(x, 0) = Dirac(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y),method=Fo
sol:=convert(sol,Int);
```

$$u(x,y)=\frac{\int_{-\infty}^{\infty}\mathrm{e}^{(ix-y)s}ds}{2\pi}$$

5.1.1.16 [297] One side homogeneous

problem number 297

Added December 20, 2018

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation for $u(x,y)$

$$u_{xx}+u_{yy}=0$$

With boundary condition

$$\begin{aligned}u(0,y)&=0\\ u(\pi,y)&=\sinh(\pi)\cos(y)\\ u(x,0)&=\sin(x)\\ u(x,\pi)&=-\sinh(x)\end{aligned}$$

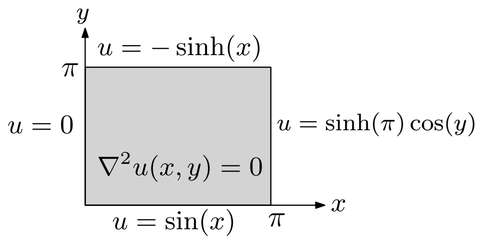


Figure 5.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[Pi, y] == Sinh[Pi]*Cos[y], u[x, 0] == Sin[x], u[x, Pi] == -Sinh[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

$$\left\{\left\{u(x,y)\to\sum_{K[1]=1}^{\infty}\operatorname{csch}(\pi K[1])\left(\delta(K[1]-1)\sin(xK[1])\sinh((\pi-y)K[1])+\frac{2K[1]\sinh(\pi)\left((1+(-1)^{K\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_side   :=  u(0,y) = 0;
bc_right_side  := u(Pi,y) = sinh(Pi)*cos(y);
bc_bottom_side := u(x,0)  = sin(x);
bc_top_side    := u(x,Pi) = -sinh(x);
bc  := bc_left_side,bc_right_side,bc_bottom_side,bc_top_side;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\frac{(e^{2\pi}-1)\left(\sum_{n=1}^{\infty}\frac{(e^{2\pi}-1)(e^{2ny}-1)n(-1)^{n}e^{-ny+(n-1)\pi}\sin(nx)}{\pi(n^{2}+1)(e^{2\pi n}-1)}\right)+(2e^{2\pi}-2)\left(\sum_{n=2}^{\infty}\frac{\sinh(\pi)((-1)^{n}+1)(e^{2nx}-1)n\,e^{(-}}{\pi(e^{2\pi n}-1)(n^{2}-1)}\right.}{e^{2\pi}-1}$$

5.1.1.17 [298] In right half plane

problem number 298

PDE example 18 from Maple help page

see `march_20_2019_11_pm.tex` for start of solution. Not completed yet

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

With boundary conditions

$$u(0, y) = \frac{\sin y}{y}$$

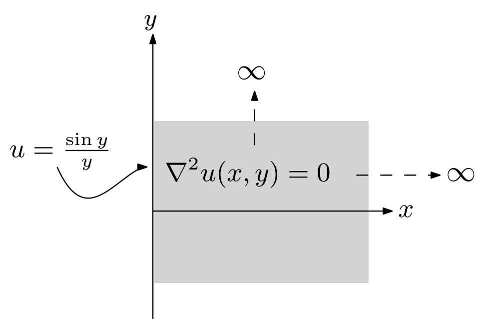


Figure 5.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = u[0, y] == Sin[y]/y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> 0 <
```

$$\left\{\left\{u(x, y) \to \frac{-2y \sinh(x) \sin(y) - 2x \cosh(x) \cos(y) + x}{x^2 + y^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(0,y)=sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y) = \_F2(ix+y) + \frac{(-ix+y)\_F2(-ix+y) + \sin(ix-y)}{ix-y}$$

5.1.1.18 [299] Right edge at infinity

problem number 299

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

With boundary conditions

$$\begin{aligned} u(0,y) &= \sin y \\ u(x,0) &= 0 \\ u(x,a) &= 0 \\ u(\infty,y) &= 0 \end{aligned}$$

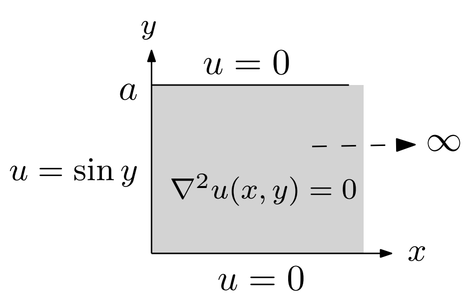


Figure 5.19: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(x,0)=0, u(x,a)=0, u(0,y)=sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({pde, bc}, u(x,y)) assumin
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{\left(-2\pi n\,\mathrm{e}^{\frac{\pi n x}{a}}\left(\left\{\begin{array}{cc}-1 & a=\pi n\\ -\frac{(-1)^{n}\sin(a)}{a-\pi n} & \mathit{otherwise}\end{array}\right.\right)+(a+\pi n)\left(-\mathrm{e}^{\frac{\pi n x}{a}}+\mathrm{e}^{-\frac{\pi n x}{a}}\right)\_\mathit{F1}(n)\right)\sin\left(\frac{\pi n y}{a}\right)}{a+\pi n}$$

5.1.1.19 [300] Dirichlet problem Upper half

problem number 300

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx}+y_{yy}=0$$

Boundary conditions $u(x,0)=1$ for $-\frac{1}{2}\leq x\leq\frac{1}{2}$ and $x=0$ otherwise. This is called UnitBox in Mathematica.

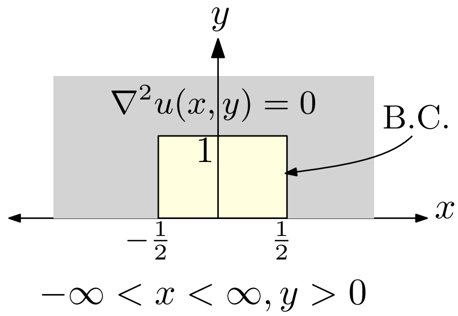


Figure 5.20: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {y >
```

$$\left\{\left\{u(x,y) \to \frac{\tan^{-1}\left(\frac{\frac{1}{2}-x}{y}\right) + \tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right)}{\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x,0) =piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y) = -\_F2(ix - y) + \_F2(ix + y) + \left(\begin{cases} 0 & iy + x < -\frac{1}{2} \\ 1 & iy + x \le \frac{1}{2} \\ 0 & \frac{1}{2} < iy + x \end{cases}\right)$$

5.1.1.20 [301] Right half-plane

problem number 301

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions $u(0,y) = \text{sinc}(y)$.

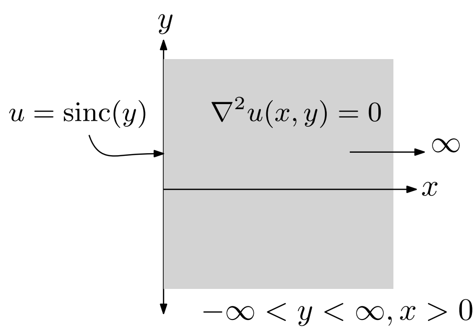


Figure 5.21: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[0, y] == Sinc[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {x >
```

$$\left\{\left\{u(x,y) \to \frac{-2y\sinh(x)\sin(y) - 2x\cosh(x)\cos(y) + x}{x^2+y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(0,y) =sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming x
```

$$u(x,y) = \_F2(ix+y) + \frac{(-ix+y)\_F2(-ix+y)+\sin(ix-y)}{ix-y}$$

5.1.1.21 [302] First quadrant

problem number 302

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions

$$u(x,0) = -\frac{1}{(x-2)^2+3}$$
$$u(0,y) = \frac{1}{(y-3)^2+1}$$

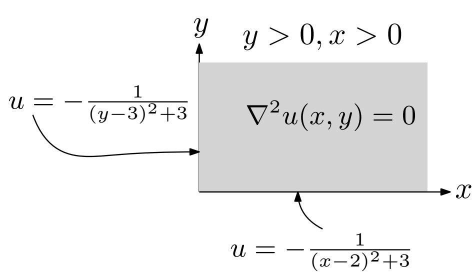


Figure 5.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == -((x - 2)^2 + 3)^(-1), u[0, y] == 1/((y - 3)^2 + 1)};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {x >
```

$$\left\{\left\{u(x,y)\to \frac{2\left(\frac{3(y(3\pi(x+1)(2x^2(y^2+12)+x^4-4x^3-4x(y^2+10)+y^4-16y^2+100)+x(2x^2(y^2-10)+x^4+y^4+20y^2-260)\log(x^2+y^2)+x}{}\right.}{}\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x, 0) = (-1/((x - 2)^2 + 3)), u(0, y) = 1/((y - 3)^2 + 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming x
```

sol=()

5.1.1.22 [303] Neumann problem upper half-plane

problem number 303

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$\nabla^2 u(x,y) = 0$$

Boundary conditions $\frac{\partial u}{\partial y}(x,0) = \text{UnitBox[x]}$ where UnitBox[x] is 1 for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and 0 otherwise. This is called UnitBox in Mathematica.

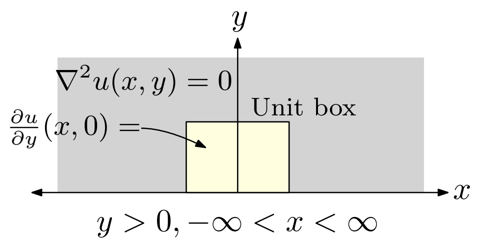


Figure 5.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = Derivative[0, 1][u][x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> y>0],
```

$$\left\{\left\{u(x,y) \to \frac{-2x\log\left((1-2x)^2+4y^2\right)+\log\left((1-2x)^2+4y^2\right)+2x\log\left((2x+1)^2+4y^2\right)+\log\left((2x+1\right.}{4\pi}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=eval(diff(u(x,y),y),y=0)= piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming y
sol:=convert(sol,Int);
```

$$u(x,y) = \frac{i\left(-\left(\int_{-\infty}^{\infty}\frac{e^{\frac{(2ix-2y-i)s}{2}}}{s^2}ds\right)+\int_{-\infty}^{\infty}\frac{e^{\frac{(2ix-2y+i)s}{2}}}{s^2}ds\right)}{2\pi}$$

used convert(sol,Int).

5.1.1.23 [304] Dirichlet problem in a rectangle

problem number 304

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(x,0) = x^2(1-x), u(x,2) = 0, u(0,y) = 0, u(1,y) = 0.$

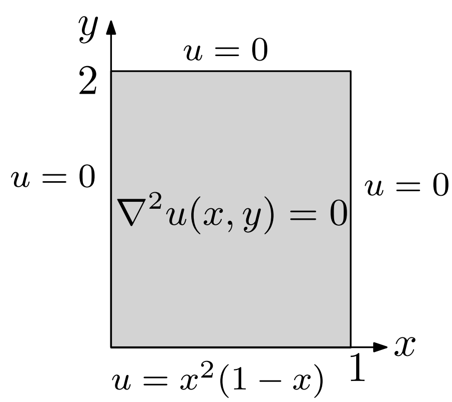


Figure 5.24: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == x^2*(1 - x), u[x, 2] == 0, u[0, y] == 0, u[1, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} -\frac{4(1+2(-1)^n)\operatorname{csch}(2n\pi)\sin(n\pi x)\sinh(n\pi(2-y))}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x, 0) = x^2*(1 - x),u(x, 2) = 0, u(0, y) = 0, u(1, y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{8\left(-\mathrm{e}^{\pi(y-2)n}+\mathrm{e}^{-\pi(y-2)n}\right)\left((-1)^{n}+\frac{1}{2}\right)\sin\left(\pi n x\right)}{\left(\mathrm{e}^{-2\pi n}-\mathrm{e}^{2\pi n}\right)\pi^{3}n^{3}}$$

5.1.1.24 [305] Strip in upper half

problem number 305

Added December 20, 2018.

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions

$$u(x,0)=0$$
$$u(x,b)=h(x)$$

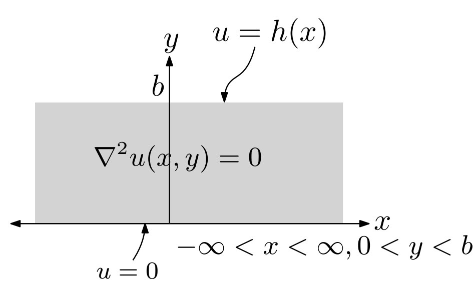


Figure 5.25: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == 0, u[x, b] == h[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

$$\left\{\left\{u(x,y)\to \frac{\int_{-\infty}^{\infty} e^{ixK[1]}\operatorname{csch}(bK[1])\sinh(yK[1])\int_{-\infty}^{\infty} e^{-ixK[1]}h(x)dxdK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2)=0;
bc  := u(x,0)=0,u(x,b)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
sol:=convert(sol,Int);
```

$$u(x,y)=\frac{-b\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h(x)dx\right)\mathrm{e}^{(ix+b-y)s}}{\mathrm{e}^{2bs}-1}ds\right)+b\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h(x)dx\right)\mathrm{e}^{(ix+b+y)s}}{\mathrm{e}^{2bs}-1}ds\right)-y\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h\right.}{\mathrm{e}^{2b}}\right.}{2\pi b}$$

5.1.1.25 [306] in Rectangle, right edge at infinity

problem number 306

Added December 20, 2018.

Example 23, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions

$$
\begin{aligned}
u(x,0) &= 0 \\
u(x,a) &= 0 \\
u(0,y) &= \sin(y) \\
u(\infty,y) &= 0
\end{aligned}
$$

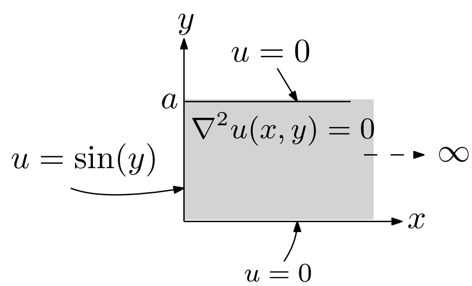


Figure 5.26: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge:=u(0, y) = sin(y);
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,a)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

$$u(x,y) = \sum_{n=1}^{\infty} \frac{\left(-2\pi n\, e^{\frac{\pi n x}{a}} \left(\left\{\begin{array}{cc} -1 & a = \pi n \\ -\frac{(-1)^n \sin(a)}{a-\pi n} & \textit{otherwise} \end{array}\right.\right) + (a+\pi n)\left(-e^{\frac{\pi n x}{a}} + e^{-\frac{\pi n x}{a}}\right) \_F1(n)\right) \sin\left(\frac{\pi n y}{a}\right)}{a+\pi n}$$

5.1.2 Polar coordinates

Local contents

5.1.2.1 [307] Laplace PDE inside quarter disk, Neumann BC at edge

problem number 307

Added December 20, 2018.

Example 20, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation in polar coordinates inside quarter disk with $0 < r < 1$ and $0 < \theta < \frac{\pi}{2}$

Solve for $u(r,\theta)$

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

Boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\frac{\pi}{2}) &= 0 \\ u_r(1,\theta) &= f(\theta) \end{aligned}$$

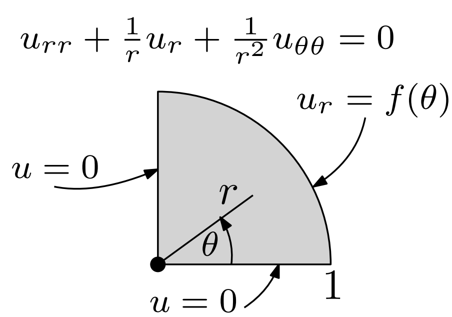


Figure 5.27: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bcOnR = {Derivative[1, 0][u][1, theta] == f[theta]};
 bcOnTheta = {u[r, 0] == 0, u[r, Pi/2] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta}, u[r, theta], {r, theta
```

$$\left\{\left\{u(r,\theta)\to\sum_{K[1]=1}^{\infty}\frac{r^{2K[1]}\left(\int_0^{\frac{\pi}{2}}\frac{2f(\theta)\sin(2\theta K[1])}{\sqrt{\pi}}\,d\theta\right)\sin(2\theta K[1])}{\sqrt{\pi}K[1]}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2)+1/r* diff(u(r, theta), r)+1/r^2* diff(u(r, theta), theta$2)= 0
bc_on_theta:=u(r, 0) = 0, u(r,Pi/2) = 0;
bc_on_r:= eval( diff(u(r,theta),r),r=1)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_theta,bc_on_r]
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{2r^{2n}\left(\int_0^{\frac{\pi}{2}}f(\theta)\sin(2n\theta)\,d\theta\right)\sin(2n\theta)}{\pi n}$$

5.1.2.2 [308] $r = 4$ and $u = x^4$ at boundary of disk

problem number 308

Added Nov 10, 2019.

Problem 4.3.25 part c. Peter J. Olver, Introduction to Partial Differential Equations, 2014 edition.

Solve Laplace equation in polar coordinates inside a disk

Solve $\nabla^2 u = 0$ with $x^2 + y^2 < 4$ and boundary conditions $u = x^4, x^2 + y^2 = 4$.

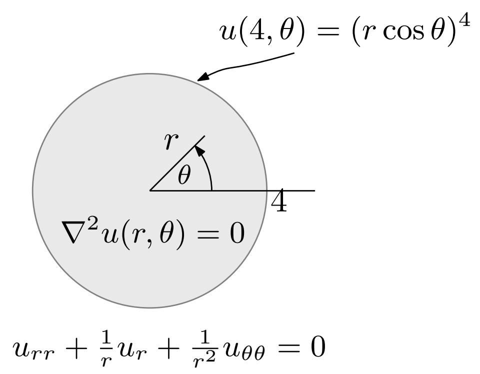


Figure 5.28: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (D[u[r, theta], r])/r + (D[u[r, theta], {theta, 2}])/r^2 ==
bc  = u[4, theta] == (4*Cos[theta])^4;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,\theta) \to \frac{1}{8}r^4\cos(4\theta) + 8r^2\cos(2\theta) + 96\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2) + diff(u(r, theta), r)/r + diff(u(r, theta), theta$2)/r^2 = 0;
bc  := u(4, theta) = (4*cos(theta))^4, u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta))),c
```

$$u(r,\theta) = \frac{r^4\cos(4\theta)}{8} + 8r^2\cos(2\theta) + 96$$

Hand solution

Solve the following boundary value problems $\nabla^2 u = 0, x^2 + y^2 < 4, u = x^4, x^2 + y^2 = 4$

Solution

In polar coordinates, where $x = r\cos\theta, y = r\sin\theta$, we need to solve for $u(r,\theta)$ inside disk of radius $r_0 = 4$. The Laplace PDE in polar coordinates is

$$\begin{aligned} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} &= 0 \qquad 0 < r < r_0, -\pi < \theta < \pi \\ u(r_0,\theta) &= f(\theta) = (r_0 \cos\theta)^4 \\ u(-\pi) &= u(\pi) \\ u_\theta(-\pi) &= u_\theta(\pi) \end{aligned}$$

Let the solution be

$$u(r,\theta) = R(r)\,\Theta(\theta)$$

Substituting this assumed solution back into the (A) gives

$$r^2 R''\Theta + rR'\Theta + R\Theta'' = 0$$

Dividing the above by $R\Theta$ gives

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + \frac{\Theta''}{\Theta} &= 0 \\ r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to the same constant. say λ.

$$r^2\frac{R''}{R} + r\frac{R'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

This results in the following two ODE's. The boundaries conditions in original PDE are transferred to each ODE which results in

$$\begin{aligned} \Theta'' + \lambda\Theta &= 0 \qquad (1) \\ \Theta(-\pi) &= \Theta(\pi) \\ \Theta'(-\pi) &= \Theta'(\pi) \end{aligned}$$

And

$$r^2 R'' + rR' - \lambda R = 0 \qquad (2)$$

Starting with ODE (1) with periodic boundary conditions.

Case $\lambda < 0$ The solution is

$$\Theta(\theta) = A\cosh\left(\sqrt{|\lambda|}\theta\right) + B\sinh\left(\sqrt{|\lambda|}\theta\right)$$

First B.C. gives

$$
\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
A\cosh\left(-\sqrt{|\lambda|}\pi\right) + B\sinh\left(-\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\
A\cosh\left(\sqrt{|\lambda|}\pi\right) - B\sinh\left(\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\
2B\sinh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}
$$

But $\sinh = 0$ only at zero and $\lambda \neq 0$, hence $B = 0$ and the solution becomes

$$
\begin{aligned}
\Theta(\theta) &= A\cosh\left(\sqrt{|\lambda|}\theta\right) \\
\Theta'(\theta) &= A\sqrt{\lambda}\cosh\left(\sqrt{|\lambda|}\theta\right)
\end{aligned}
$$

Applying the second B.C. gives

$$
\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi) \\
A\sqrt{|\lambda|}\cosh\left(-\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) \\
A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) \\
2A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}
$$

But cosh is never zero, hence $A = 0$. Therefore trivial solution and $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$ The solution is $\Theta = A\theta + B$. Applying the first B.C. gives

$$
\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
-A\pi + B &= \pi A + B \\
2\pi A &= 0 \\
A &= 0
\end{aligned}
$$

And the solution becomes $\Theta = B_0$. A constant. Hence $\lambda = 0$ is an eigenvalue.

Case $\lambda > 0$

The solution becomes

$$
\begin{aligned}
\Theta &= A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right) \\
\Theta' &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\theta\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\theta\right)
\end{aligned}
$$

Applying first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
A\cos\left(-\sqrt{\lambda}\pi\right) + B\sin\left(-\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
A\cos\left(\sqrt{\lambda}\pi\right) - B\sin\left(\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
2B\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)
\end{aligned}$$

Applying second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi) \\
-A\sqrt{\lambda}\sin\left(-\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) \\
2A\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)
\end{aligned}$$

Equations (3,4) can be both zero only if $A = B = 0$ which gives trivial solution, or when $\sin\left(\sqrt{\lambda}\pi\right) = 0$. Therefore taking $\sin\left(\sqrt{\lambda}\pi\right) = 0$ gives a non-trivial solution. Hence

$$\begin{aligned}
\sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\
\lambda_n &= n^2 \qquad n = 1, 2, 3, \cdots
\end{aligned}$$

Hence the eigenfunctions are

$$\{1, \cos(n\theta), \sin(n\theta)\} \qquad n = 1, 2, 3, \cdots \qquad (5)$$

Now the R equation is solved

The case for $\lambda = 0$ gives from (2)

$$\begin{aligned}
r^2 R'' + rR' &= 0 \\
R'' + \frac{1}{r}R' &= 0 \qquad r \neq 0
\end{aligned}$$

The solution to this is

$$R_0(r) = A\ln r + C$$

Since u is bounded at $r = 0$ we want $A = 0$. Hence $R_0(r)$ is just a constant.

Case $\lambda > 0$ The ODE (2) becomes

$$r^2R'' + rR' - n^2R = 0 \qquad n = 1,2,3,\cdots$$

Let $R = r^p$, the above becomes

$$\begin{aligned} r^2p(p-1)\,r^{p-2} + rpr^{p-1} - n^2r^p &= 0 \\ p(p-1)\,r^p + pr^p - n^2r^p &= 0 \\ p(p-1) + p - n^2 &= 0 \\ p^2 &= n^2 \\ p &= \pm n \end{aligned}$$

Hence the solution is

$$R_n(r) = C_nr^n + D_n\frac{1}{r^n} \qquad n = 1,2,3,\cdots$$

Since u is bounded at $r = 0$ we want $D = 0$. Hence $R_n(r) = C_nr^n$.

The complete solution for $R(r)$ is

$$R(r) = C_0 + \sum_{n=1}^{\infty} C_nr^n \tag{6}$$

Using (5),(6) gives

$$\begin{aligned} u_n(r,\theta) &= R_n\Theta_n \\ u(r,\theta) &= \left(C_0 + \sum_{n=1}^{\infty} C_nr^n\right)\left(A_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta) + B_n\sin(n\theta)\right) \end{aligned}$$

Combining constants to simplify things gives

$$u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r^n(A_n\cos(n\theta) + B_n\sin(n\theta)) \tag{7}$$

When $r = r_0$ the above becomes

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r_0^n(A_n\cos(n\theta) + B_n\sin(n\theta))$$

Hence

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta \\ r_0^nA_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta \\ A_n &= \frac{1}{\pi r_0^n}\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta \end{aligned}$$

And

$$\begin{aligned} r_0^n B_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta \\ B_n &= \frac{1}{\pi r_0^n}\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta \end{aligned}$$

Hence (7) becomes

$$u(r,\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta + \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{r}{r_0}\right)^n \left(\cos(n\theta)\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta + \sin(n\theta)\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta\right) \tag{8}$$

In this problem $f(\theta) = (r_0\cos\theta)^4$ where $r_0 = 4$, hence

$$\begin{aligned} A_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} 256\cos^4\theta d\theta \\ &= \frac{256}{\pi}\int_{-\pi}^{\pi}\cos^4\theta d\theta \\ &= \frac{256}{\pi}\left(\frac{3\theta}{8} + \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)\right)_{-\pi}^{\pi} \\ &= \frac{256}{\pi}\left(\frac{3\pi}{8} + \frac{3\pi}{8}\right) \\ &= \frac{256}{\pi}\left(\frac{3\pi}{4}\right) \\ &= 192 \end{aligned}$$

And

$$\begin{aligned} A_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} 256\cos^4(\theta)\cos(n\theta)\,d\theta \\ &= \frac{256}{\pi}\int_{-\pi}^{\pi}\cos^4(\theta)\cos(n\theta)\,d\theta \end{aligned}$$

To evaluate the above integral, we will start by using the identity

$$\cos^4(\theta) = \frac{3}{8} + \frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta)$$

Therefore the integral now becomes

$$\begin{aligned} A_n &= \frac{256}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{8} + \frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta)\right)\cos(n\theta)\,d\theta \\ &= \frac{256}{\pi}\left[\frac{3}{8}\int_{-\pi}^{\pi}\cos(n\theta)\,d\theta + \frac{1}{8}\int_{-\pi}^{\pi}\cos(4\theta)\cos(n\theta)\,d\theta + \frac{1}{2}\int_{-\pi}^{\pi}\cos(2\theta)\cos(n\theta)\,d\theta\right] \end{aligned} \tag{1}$$

But $\int_{-\pi}^{\pi}\cos(n\theta)\,d\theta = 0$ and $\int_{-\pi}^{\pi}\cos(4\theta)\cos(n\theta)\,d\theta$ is not zero, only for $n = 4$ by orthogonality of cosine functions. Hence

$$\begin{aligned}\int_{-\pi}^{\pi}\cos(4\theta)\cos(n\theta)\,d\theta &= \int_{-\pi}^{\pi}\cos^2(4\theta)\,d\theta\\ &= \pi\end{aligned}$$

And similarly, $\int_{-\pi}^{\pi}\cos(2\theta)\cos(n\theta)\,d\theta$ is not zero, only for $n = 2$ by orthogonality of cosine functions. Hence

$$\begin{aligned}\int_{-\pi}^{\pi}\cos(2\theta)\cos(n\theta)\,d\theta &= \int_{-\pi}^{\pi}\cos^2(2\theta)\,d\theta\\ &= \pi\end{aligned}$$

Using these results in (1) gives, for $n = 2$

$$\begin{aligned}A_2 &= \frac{256}{\pi}\left[\frac{1}{2}\int_{-\pi}^{\pi}\cos^2(2\theta)\,d\theta\right]\\ &= \frac{256}{\pi}\left(\frac{\pi}{2}\right)\\ &= 128\end{aligned}$$

And for $n = 4$

$$\begin{aligned}A_4 &= \frac{256}{\pi}\left[\frac{1}{8}\int_{-\pi}^{\pi}\cos^2(4\theta)\,d\theta\right]\\ &= \frac{256}{\pi}\left(\frac{\pi}{8}\right)\\ &= 32\end{aligned}$$

And all other A_n are zero. Now that we found all A_n, and since $B_n = 0$ for all n (because $f(\theta)$ is even function) then the solution (8) becomes

$$\begin{aligned}u(r,\theta) &= \frac{192}{2} + a_2\left(\frac{r}{4}\right)^2\cos(2\theta) + a_4\left(\frac{r}{4}\right)^4\cos(4\theta)\\ &= 96 + 128\left(\frac{r^2}{16}\right)\cos(2\theta) + 32\frac{r^4}{256}\cos(4\theta)\end{aligned}$$

Therefore

$$u(r,\theta) = 96 + 8r^2\cos(2\theta) + \frac{1}{8}r^4\cos(4\theta)$$

Here is plot of the above solution.

```
sol = 96 + 8 r^2 Cos[2 θ] + 1/8 r^4 Cos[4 θ];
ParametricPlot3D[{r Cos[θ], r Sin[θ], sol}, {r, 0, 4}, {θ, 0, 2 Pi},
  AxesLabel → {x, y, "u(x,y"}, ImageSize → 400, BoxRatios → {1,  1,  1}, BaseStyle → 14]
```

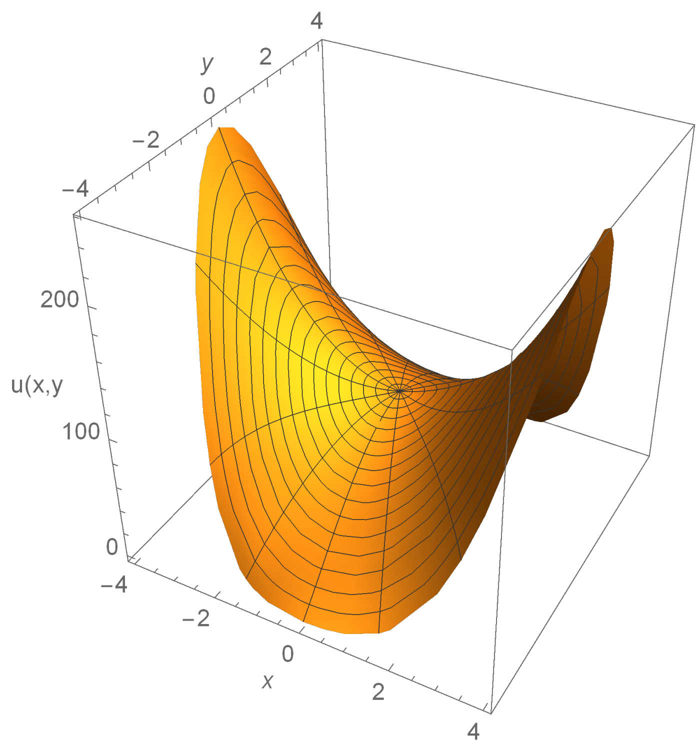


Figure 5.29: Solution plot to the above problem with code used

5.1.2.3 [309] $r = 1$ and $u_r = x$ at boundary of disk

problem number 309

Added January 8, 2020.

Problem 4.3.25 part d, Peter J. Olver, Introduction to Partial Differential Equations, 2014 edition.

Solve Laplace equation in polar coordinates inside a disk of radius 1.

Solve $\nabla^2 u = 0$ and boundary conditions $\frac{\partial u}{\partial n} = x$.

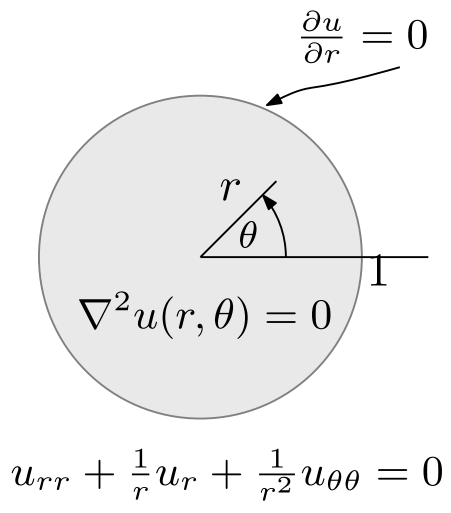


Figure 5.30: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
bc = {Derivative[1, 0][u][1, theta] == Cos[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde:=VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta])=0;
bc  := D[1](u)(1, theta) = cos(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta))),c
```

sol=()

Hand solution

In polar coordinates, where $x = r\cos\theta, y = r\sin\theta$, we need to solve for $u(r,\theta)$ inside disk of radius $r_0 = 1$. The Laplace PDE in polar coordinates is

$$
\begin{aligned}
u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} &= 0 \qquad 0 < r < 1, -\pi < \theta < \pi \\
u_r(1,\theta) &= f(\theta) = \cos\theta \\
u(-\pi) &= u(\pi) \\
u_\theta(-\pi) &= u_\theta(\pi)
\end{aligned}
$$

Using separation of variables, let $u(r,\theta) = R(r)\,\Theta(\theta)$ the solution is given by

$$
u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n r^n \cos(n\theta) + b_n r^n \sin(n\theta) \tag{1}
$$

At $r = r_0 = 1$ we have that $\frac{\partial u(r,\theta)}{\partial r} = \cos\theta$ (since $x = r\cos\theta$ but $r = 1$ at boundary). The above becomes

$$
\cos\theta = \sum_{n=1}^{\infty} n a_n r^{n-1} \cos(n\theta) + n b_n r^{n-1} \sin(n\theta)
$$

Therefore $n = 1$ is only term that survives in the sum. Hence $a_1 = 1$ and all others are zero. The solution (1) becomes

$$
u(r,\theta) = \frac{a_0}{2} + r\cos(\theta)
$$

The solution is not unique as there is a_0 arbitrary constant.

5.1.2.4 [310] Laplace inside disk. General solution

problem number 310

Solve Laplace equation in polar coordinates inside a disk

Solve for $u(r,\theta)$

$$
u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0
$$

With $0 \leq r \leq a, 0 < \theta \leq 2\pi$ Boundary conditions

$$
\begin{aligned}
u(a,\theta) &= f(\theta) \\
|u(0,\theta)| &< \infty \\
u(r,0) &= u(r,2\pi) \\
\frac{\partial u}{\partial \theta}(r,0) &= \frac{\partial u}{\partial \theta}(r,2\pi)
\end{aligned}
$$

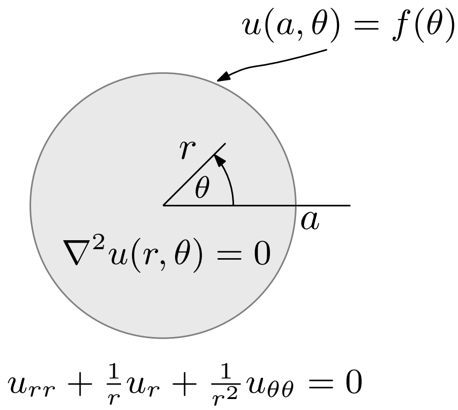


Figure 5.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
bc  = u[a, theta] == f[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,\theta) \to \sum_{n=1}^{\infty} \frac{\left(\frac{r}{a}\right)^n \left(\cos(n\theta) \int_{-\pi}^{\pi} \cos(n\theta) f(\theta)\, d\theta + \left(\int_{-\pi}^{\pi} f(\theta) \sin(n\theta)\, d\theta\right) \sin(n\theta)\right)}{\pi} + \frac{\int_{-\pi}^{\pi} f(\theta)\, d\theta}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r +(diff(u(r, theta), theta, theta))/r^2 = 0;
bc  := u(a, theta) = f(theta),
      u(r, -Pi) = u(r, Pi),
      (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta)=\frac{\int_{-\pi}^{\pi} f(\theta)\,d\theta+2\pi\left(\sum_{n=1}^{\infty}\frac{\left(\left(\int_{-\pi}^{\pi}\cos(n\theta)f(\theta)d\theta\right)\cos(n\theta)+\left(\int_{-\pi}^{\pi}f(\theta)\sin(n\theta)d\theta\right)\sin(n\theta)\right)\left(\frac{a}{r}\right)^{-n}}{\pi}\right)}{2\pi}$$

5.1.2.5 [311] Laplace inside disk. Specific boundary conditions

problem number 311

Added January 12, 2020

Solve $u_{xx}+u_{yy}=0$ on disk $x^2+y^2<1$ with boundary condition xy^2 when $x^2+y^2=a$. Where $a=1$ in this problem. Express solution in x,y

The first step is to convert the boundary condition to polar coordinates. Since $x=r\cos\theta, y=r\sin\theta$, then at the boundary $u(r,\theta)=r\cos\theta(r\sin\theta)^2$. But $r=1$ (the radius). Hence at the boundary, $u(1,\theta)=f(\theta)$ where

$$\begin{aligned} f(\theta)&=\cos\theta\sin^2\theta\\ &=\cos\theta\left(1-\cos^2\theta\right)\\ &=\cos\theta-\cos^3\theta \end{aligned}$$

But $\cos^3\theta=\frac{3}{4}\cos\theta+\frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta)&=\cos\theta-\left(\frac{3}{4}\cos\theta+\frac{1}{4}\cos 3\theta\right)\\ &=\frac{1}{4}\cos\theta-\frac{1}{4}\cos 3\theta \end{aligned}\tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

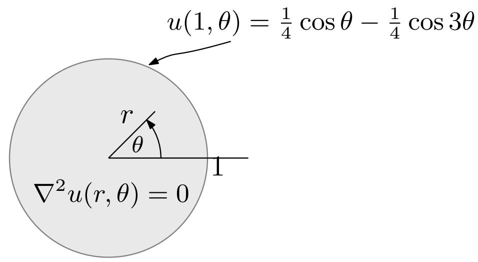


Figure 5.32: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a = 1;
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
f[theta_] := 1/4*(Cos[theta] - Cos[3*theta]);
bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Derivat
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta)\to\frac{1}{4}\left(r\cos(\theta)-r^3\cos(3\theta)\right)\right\}\right\}$$

Maple ✓

```
restart;
f:=theta-> 1/4*(cos(theta) - cos(3*theta));
a:=1;
pde := VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta]);
bc := u(a, theta) = f(theta),u(r, -Pi) = u(r, Pi),(D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
sol:=simplify(subs(cos(theta)^3=trigsubs(cos(theta)^3)[2],sol),size);
```

$$u(r,\theta)=-\frac{\left(r^2\cos\left(3\theta\right)-\cos\left(\theta\right)\right)r}{4}$$

<u>Hand solution</u>

Solve $u_{xx}+u_{yy}=0$ on disk $x^2+y^2<1$ with boundary condition xy^2 when $x^2+y^2=a$. Where $a=1$ in this problem. Express solution in x,y

The first step is to convert the boundary condition to polar coordinates. Since $x =$

$r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$
\begin{aligned}
f(\theta) &= \cos\theta\sin^2\theta \\
&= \cos\theta(1-\cos^2\theta) \\
&= \cos\theta - \cos^3\theta
\end{aligned}
$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$
\begin{aligned}
f(\theta) &= \cos\theta - \left(\frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta\right) \\
&= \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta \qquad (1)
\end{aligned}
$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

The solution is known to be

$$u(r,\theta) = \frac{c_0}{2} + \sum_{n=1}^{\infty} r^n(c_n\cos(n\theta) + k_n\sin(n\theta)) \qquad (2)$$

Since the above solution is the same as $f(\theta)$ when $r = 1$, then equating (2) when $r = 1$ to (1) gives

$$\frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta = \frac{c_0}{2} + \sum_{n=1}^{\infty}(c_n\cos(n\theta) + k_n\sin(n\theta))$$

By comparing terms on both sides, this shows by inspection that

$$
\begin{aligned}
c_0 &= 0 \\
c_1 &= \frac{1}{4} \\
c_3 &= \frac{-1}{4}
\end{aligned}
$$

And all other c_n, k_n are zero. Using the above result back in (2) gives the solution as

$$\boxed{u(r,\theta) = \frac{r}{4}\cos\theta - \frac{r^3}{4}\cos 3\theta} \qquad (3)$$

This solution is now converted to xy using the formula

$$
\begin{aligned}
r^n\cos n\theta &= \sum_{\substack{k=0\\ \text{even}}}^{n} \binom{n}{k} x^{n-k}(-1)^{\frac{k}{2}} y^k \\
&= \sum_{\substack{k=0\\ \text{even}}}^{n} \frac{n!}{k!\,(n-k)!} x^{n-k}(-1)^{\frac{k}{2}} y^k
\end{aligned}
$$

For $n=1$ the above gives

$$\begin{aligned} r\cos\theta &= \frac{1!}{0!\,(1-0)!}x^{1-0}(-1)^0\,y^0 \\ &= x \end{aligned} \tag{4}$$

And for $n=3$

$$\begin{aligned} r^3\cos 3\theta &= \frac{3!}{0!\,(3-0)!}x^{3-0}(-1)^0\,y^0 + \frac{3!}{2!\,(3-2)!}x^{3-2}(-1)^1\,y^2 \\ &= x^3 - 3xy^2 \end{aligned} \tag{5}$$

Using (4,5) in (3) gives the solution in x, y

$$\boxed{u(x,y) = \tfrac{1}{4}x - \tfrac{1}{4}(x^3 - 3xy^2)} \tag{6}$$

This is now verified that is satisfies the PDE $u_{xx} + u_{yy} = 0$.

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{1}{4} - \frac{1}{4}(3x^2 - 3y^2) \\ \frac{\partial^2 u}{\partial x^2} &= -\frac{6}{4}x \end{aligned}$$

And

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{6}{4}xy \\ \frac{\partial^2 u}{\partial y^2} &= \frac{6}{4}x \end{aligned}$$

Therefore $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Now the boundary conditions $u(x,y) = xy^2$ are also verified. This condition applies when $x^2 + y^2 = 1$ or $y^2 = 1 - x^2$. Substituting this into (6) gives

$$u(x,y)_{@D} = \frac{1}{4}x - \frac{1}{4}\left(x^3 - 3x\overbrace{(1-x^2)}^{y2}\right)$$

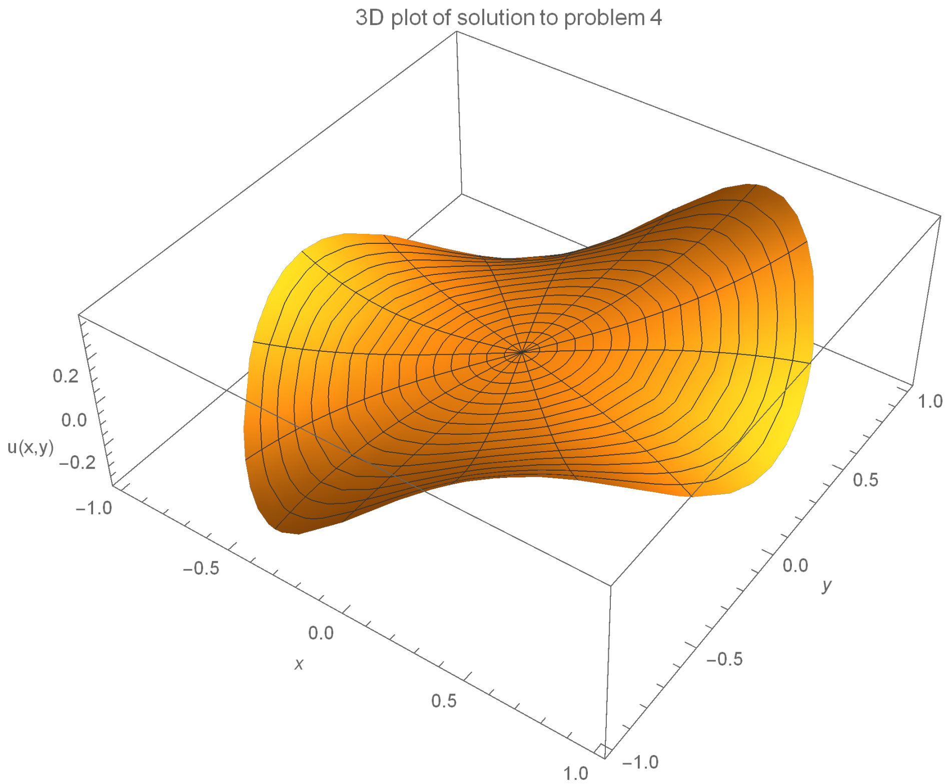
Simplifying gives

$$\begin{aligned} u(x,y)_{@D} &= \frac{1}{4}x - \frac{1}{4}\left(x^3 - \left(3x - 3x^3\right)\right) \\ &= \frac{1}{4}x - \frac{1}{4}x^3 + \frac{1}{4}\left(3x - 3x^3\right) \\ &= \frac{1}{4}x - \frac{1}{4}x^3 + \frac{3}{4}x - \frac{3}{4}x^3 \\ &= x - x^3 \\ &= x\left(1 - x^2\right) \\ &= xy^2 \end{aligned}$$

Verified. This is 3D plot of the solution

```
In[76]:= ParametricPlot3D[{r Cos[t], r Sin[t], r/4 Cos[t] - r^3/4 Cos[3 t]},
   {r, 0, 1}, {t, 0, 2 Pi}, AxesLabel → {x, y, "u(x,y)"},
   PlotLabel → "3D plot of solution to problem 4", ImageSize → 500]
```

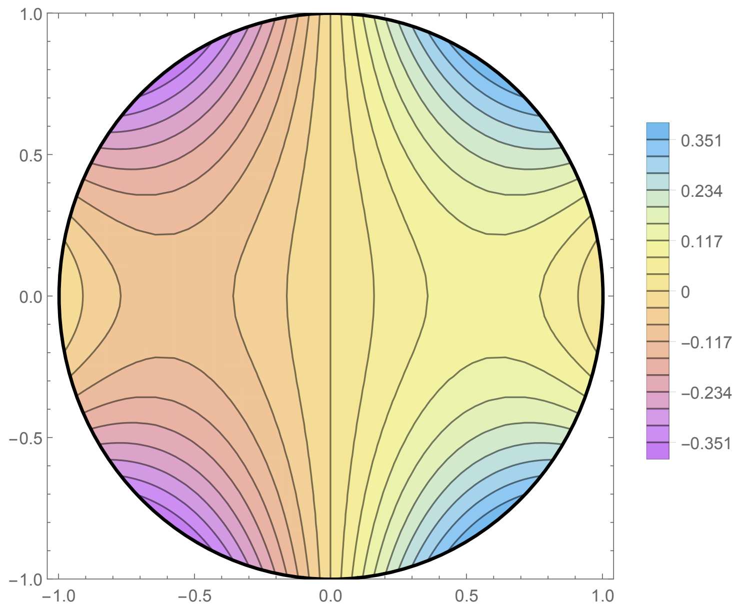
Out[76]=



This is a contour plot

```
In[96]:= ContourPlot[1 / 4 x - 1 / 4 (x^3 - 3 x y^2), {x, -1, 1}, {y, -1, 1}, AxesLabel → {x, y},
    Contours → 50, PlotLegends → Automatic, ColorFunction → "Pastel",
    Epilog → {Thick, Circle[]},
    PlotRange → {-1, 1},
    RegionFunction → Function[{x, y, z}, Norm[{x, y}] < 1.]]
```

Out[96]=



5.1.2.6 [312] Haberman 2.5.5 (c)

problem number 312

This is problem 2.5.5 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

Inside quarter circle of radius 1 with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 1$, with following boundary conditions

$$u(r,0) = 0$$
$$u(r,\frac{\pi}{2}) = 0$$
$$\frac{\partial u}{\partial r}(1,\theta) = f(\theta)$$

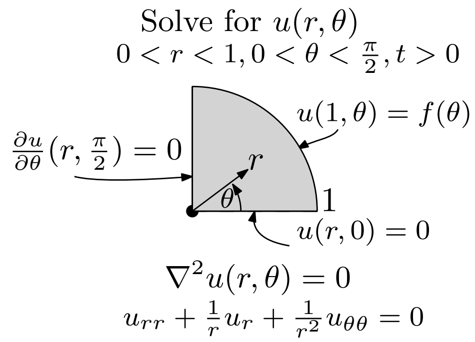


Figure 5.33: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) =
bc  = {Derivative[1, 0][u][1, theta] == f[theta], u[r, Pi/2] == 0, u[r, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r,theta),r$2)+ 1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta),HINT=b
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{2r^{2n}\left(\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta\right)\sin(2n\theta)}{\pi n}$$

Hand solution

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0 \tag{A}$$

With boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u\left(r,\frac{\pi}{2}\right) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta) = R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2 R''\Theta + rR'\Theta + R\Theta'' = 0$$

Dividing the above by $R\Theta \neq 0$ gives

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + \frac{\Theta''}{\Theta} &= 0 \\ r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R} + r\frac{R'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This gives

$$\begin{aligned} \Theta'' + \lambda\Theta &= 0 \\ \Theta(0) &= 0 \\ \Theta\left(\frac{\pi}{2}\right) &= 0 \end{aligned} \tag{1}$$

And

$$\begin{aligned} r^2R'' + rR' - \lambda R &= 0 \\ |R(0)| &< \infty \end{aligned} \tag{2}$$

Starting with (1). Consider the Case $\lambda < 0$. The solution in this case will be

$$\Theta = A\cosh\left(\sqrt{\lambda}\theta\right) + B\sinh\left(\sqrt{\lambda}\theta\right)$$

Applying first B.C. gives $A=0$. The solution becomes $\Theta = B\sinh\left(\sqrt{\lambda}\theta\right)$. Applying second B.C. gives

$$0 = B\sinh\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

But sinh is zero only when $\sqrt{\lambda}\frac{\pi}{2} = 0$ which is not the case here. Therefore $B=0$ and hence trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$ The ODE becomes $\Theta'' = 0$ with solution $\Theta = A\theta + B$. First B.C. gives $0 = B$. The solution becomes $\Theta = A\theta$. Second B.C. gives $0 = A\frac{\pi}{2}$, hence $A = 0$ and trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$ The ODE becomes $\Theta'' + \lambda\Theta = 0$ with solution

$$\Theta = A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)$$

The first B.C. gives $0 = A$. The solution becomes

$$\Theta = B\sin\left(\sqrt{\lambda}\theta\right)$$

And the second B.C. gives

$$0 = B\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right) = 0$ or $\sqrt{\lambda}\frac{\pi}{2} = n\pi$ for $n = 1,2,3,\cdots$. Hence the eigenvalues are

$$\begin{aligned}\sqrt{\lambda_n} &= 2n\\ \lambda_n &= 4n^2 \qquad n = 1,2,3,\cdots\end{aligned}$$

And the eigenfunctions are

$$\Theta_n(\theta) = B_n \sin(2n\theta) \qquad n = 1,2,3,\cdots \tag{3}$$

Now the R ODE is solved. There is one case to consider, which is $\lambda > 0$ based on the above. The ODE is

$$\begin{aligned}r^2R'' + rR' - \lambda_n R &= 0\\ r^2R'' + rR' - 4n^2R &= 0 \qquad n = 1,2,3,\cdots\end{aligned}$$

This is Euler ODE. Let $R(r) = r^p$. Then $R' = pr^{p-1}$ and $R'' = p(p-1)\,r^{p-2}$. This gives

$$\begin{aligned}r^2\left(p(p-1)\,r^{p-2}\right) + r\left(pr^{p-1}\right) - 4n^2r^p &= 0\\ \left(\left(p^2-p\right)r^p\right) + pr^p - 4n^2r^p &= 0\\ r^pp^2 - pr^p + pr^p - 4n^2r^p &= 0\\ p^2 - 4n^2 &= 0\\ p &= \pm 2n\end{aligned}$$

Hence the solution is

$$R(r) = Cr^{2n} + D\frac{1}{r^{2n}}$$

Applying the condition that $|R(0)| < \infty$ implies $D = 0$, and the solution becomes

$$R_n(r) = C_n r^{2n} \qquad n = 1, 2, 3, \cdots \tag{4}$$

Using (3,4) the solution $u_n(r, \theta)$ is

$$\begin{aligned} u_n(r, \theta) &= R_n \Theta_n \\ &= C_n r^{2n} B_n \sin(2n\theta) \\ &= B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Where $C_n B_n$ was combined into one constant B_n. (No need to introduce new symbol). The final solution is

$$\begin{aligned} u(r, \theta) &= \sum_{n=1}^{\infty} u_n(r, \theta) \\ &= \sum_{n=1}^{\infty} B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Now the nonhomogeneous condition is applied to find B_n.

$$\frac{\partial}{\partial r} u(r, \theta) = \sum_{n=1}^{\infty} B_n (2n)\, r^{2n-1} \sin(2n\theta)$$

Hence $\frac{\partial}{\partial r} u(1, \theta) = f(\theta)$ becomes

$$f(\theta) = \sum_{n=1}^{\infty} 2B_n n \sin(2n\theta)$$

Multiplying by $\sin(2m\theta)$ and integrating gives

$$\begin{aligned} \int_0^{\frac{\pi}{2}} f(\theta) \sin(2m\theta)\, d\theta &= \int_0^{\frac{\pi}{2}} \sin(2m\theta) \sum_{n=1}^{\infty} 2B_n n \sin(2n\theta)\, d\theta \\ &= \sum_{n=1}^{\infty} 2nB_n \int_0^{\frac{\pi}{2}} \sin(2m\theta) \sin(2n\theta)\, d\theta \end{aligned} \tag{5}$$

When $n=m$ then

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}} \sin^2(2n\theta)\,d\theta\\ &= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2}-\frac{1}{2}\cos 4n\theta\right)d\theta\\ &= \frac{1}{2}[\theta]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin 4n\theta}{4n}\right]_0^{\frac{\pi}{2}}\\ &= \frac{\pi}{4} - \left(\frac{1}{8n}\left(\sin\frac{4n}{2}\pi\right) - \sin(0)\right)\end{aligned}$$

And since n is integer, then $\sin\frac{4n}{2}\pi = \sin 2n\pi = 0$ and the above becomes $\frac{\pi}{4}$.

Now for the case when $n\neq m$ using $\sin A\sin B = \frac{1}{2}(\cos(A-B)-\cos(A+B))$ then

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos(2m\theta-2n\theta)-\cos(2m\theta+2n\theta))\,d\theta\\ &= \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(2m\theta-2n\theta)\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(2m\theta+2n\theta)\,d\theta\\ &= \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos((2m-2n)\theta)\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos((2m+2n)\theta)\,d\theta\\ &= \frac{1}{2}\left[\frac{\sin((2m-2n)\theta)}{(2m-2n)}\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin((2m+2n)\theta)}{(2m+2n)}\right]_0^{\frac{\pi}{2}}\\ &= \frac{1}{4(m-n)}[\sin((2m-2n)\theta)]_0^{\frac{\pi}{2}} - \frac{1}{4(m+n)}[\sin((2m+2n)\theta)]_0^{\frac{\pi}{2}}\\ &= \frac{1}{4(m-n)}\left[\sin\left((2m-2n)\frac{\pi}{2}\right)-0\right] - \frac{1}{4(m+n)}\left[\sin\left((2m+2n)\frac{\pi}{2}\right)-0\right]\end{aligned}$$

Since $2m-2n\frac{\pi}{2} = \pi(m-n)$ which is integer multiple of π and also $(2m+2n)\frac{\pi}{2}$ is integer multiple of π then the whole term above becomes zero. Therefore (5) becomes

$$\int_0^{\frac{\pi}{2}} f(\theta)\sin(2m\theta)\,d\theta = 2mB_m\frac{\pi}{4}$$

Hence

$$B_n = \frac{2}{\pi n}\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta$$

Summary: the final solution is

$$u(r,\theta) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta\right]\left(r^{2n}\sin(2n\theta)\right)$$

5.1.2.7 [313] semi-circle

problem number 313

Solve Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

Inside semi-circle of radius 1 with $0 \leq \theta \leq \pi$ and $0 \leq r \leq 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned}$$

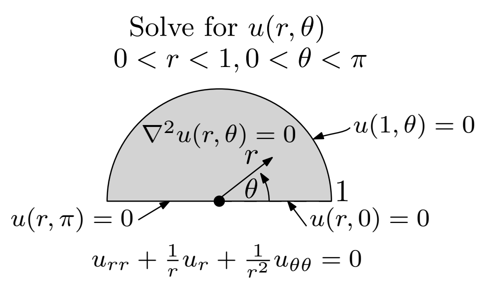


Figure 5.34: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta], {r, 2}] + (1/r) D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] ==
bc  = {u[r, 0] == 0, u[r, Pi] == 0, u[1, theta] == f[theta]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta) \to \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} r^{K[1]} \left(\int_0^{\pi} \sqrt{\frac{2}{\pi}} f(\theta) \sin(\theta K[1])\, d\theta\right) \sin(\theta K[1])\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(r,0)=0,u(r,Pi)=0,u(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta),HINT=b
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{2r^n\left(\int_0^\pi f(\theta)\sin(n\theta)\,d\theta\right)\sin(n\theta)}{\pi}$$

5.1.2.8 [314] Haberman 2.5.8 (b)

problem number 314

This is problem 2.5.8 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation $\nabla^2 u(r,\theta)=0$ or

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

Inside circular annulus $a<r<b$ subject to the following boundary conditions

$$\frac{\partial u}{\partial r}(a,\theta)=0$$
$$u(b,\theta)=g(\theta)$$

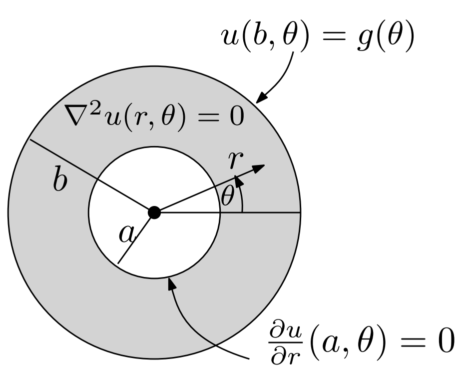


Figure 5.35: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta], {r, 2}] + 1/r*D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] == 0
bc  = {Derivative[1, 0][u][a, theta] == 0, u[b, theta] == g[theta]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta) \to \left\{ \frac{\int_0^{2\pi} g(K[2])\,dK[2]}{2\pi} + \sum_{K[1]=1}^{\infty}\left(\cos(\theta K[1])\left(\frac{a^{2K[1]}b^{K[1]}\left(\int_0^{2\pi}\frac{\cos(K[1]K[2])g(K[2])}{\pi}\,dK[2]\right)r^{-K[1]}}{a^{2K[1]}+b^{2K[1]}} + \frac{b^{K[1]}\left(\int_0^{2\pi}\right.}{}\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta)) assum
```

$$u(r,\theta) = g(\theta) - \mathcal{F}^{-1}\left(\frac{a^s b^{-s}\mathcal{F}(g(\theta),\theta,s)}{a^s b^{-s}+a^{-s}b^s}, s, \theta\right) + \mathcal{F}^{-1}\left(\frac{a^s r^{-s}\mathcal{F}(g(\theta),\theta,s)}{a^s b^{-s}+a^{-s}b^s}, s, \theta\right) - \mathcal{F}^{-1}\left(\frac{a^{-s} b^{s}\mathcal{F}(g(\theta),\theta,s)}{a^s b^{-s}+a^{-s}b^s}\right.$$

Hand solution

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0 \tag{A}$$

With

$$\begin{aligned} \frac{\partial u}{\partial r}(a,\theta) &= 0 \\ u(b,\theta) &= g(\theta) \end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta) = R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2 R''\Theta + rR'\Theta + R\Theta'' = 0$$

Dividing the above by $R\Theta$ gives

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + \frac{\Theta''}{\Theta} &= 0 \\ r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R} + r\frac{R'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This results in

$$\begin{aligned} \Theta'' + \lambda\Theta &= 0 \\ \Theta(-\pi) &= \Theta(\pi) \\ \Theta'(-\pi) &= \Theta'(\pi) \end{aligned} \tag{1}$$

And

$$\begin{aligned} r^2R'' + rR' - \lambda R &= 0 \\ R'(a) &= 0 \end{aligned} \tag{2}$$

Starting with (1)

Case $\lambda < 0$ The solution is

$$\Theta(\theta) = A\cosh\left(\sqrt{|\lambda|}\theta\right) + B\sinh\left(\sqrt{|\lambda|}\theta\right)$$

First B.C. gives

$$\begin{aligned} \Theta(-\pi) &= \Theta(\pi) \\ A\cosh\left(-\sqrt{|\lambda|}\pi\right) + B\sinh\left(-\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\ A\cosh\left(\sqrt{|\lambda|}\pi\right) - B\sinh\left(\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\ 2B\sinh\left(\sqrt{|\lambda|}\pi\right) &= 0 \end{aligned}$$

But $\sinh = 0$ only at zero and $\lambda \neq 0$, hence $B = 0$ and the solution becomes

$$\begin{aligned} \Theta(\theta) &= A\cosh\left(\sqrt{|\lambda|}\theta\right) \\ \Theta'(\theta) &= A\sqrt{\lambda}\cosh\left(\sqrt{|\lambda|}\theta\right) \end{aligned}$$

Applying the second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi)\\
A\sqrt{|\lambda|}\cosh\left(-\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
2A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}$$

But cosh is never zero, hence $A = 0$. Therefore trivial solution and $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$ The solution is $\Theta = A\theta + B$. Applying the first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
-A\pi + B &= \pi A + B\\
2\pi A &= 0\\
A &= 0
\end{aligned}$$

And the solution becomes $\Theta = B_0$. A constant. Hence $\lambda = 0$ is an eigenvalue.

Case $\lambda > 0$

The solution becomes

$$\begin{aligned}
\Theta &= A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)\\
\Theta' &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\theta\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\theta\right)
\end{aligned}$$

Applying first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
A\cos\left(-\sqrt{\lambda}\pi\right) + B\sin\left(-\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right)\\
A\cos\left(\sqrt{\lambda}\pi\right) - B\sin\left(\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right)\\
2B\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)
\end{aligned}$$

Applying second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi)\\
-A\sqrt{\lambda}\sin\left(-\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right)\\
2A\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)
\end{aligned}$$

Equations (3,4) can be both zero only if $A = B = 0$ which gives trivial solution, or when $\sin\left(\sqrt{\lambda}\pi\right) = 0$. Therefore taking $\sin\left(\sqrt{\lambda}\pi\right) = 0$ gives a non-trivial solution. Hence

$$\begin{aligned}\sqrt{\lambda}\pi &= n\pi \qquad n = 1,2,3,\cdots \\ \lambda_n &= n^2 \qquad n = 1,2,3,\cdots\end{aligned}$$

Hence the solution for Θ is

$$\Theta = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

Now the R equation is solved

The case for $\lambda = 0$ gives

$$\begin{aligned}r^2 R'' + rR' &= 0 \\ R'' + \frac{1}{r} R' &= 0 \qquad r \neq 0\end{aligned}$$

As was done in last problem, the solution to this is

$$R(r) = A \ln|r| + C$$

Since $r > 0$ no need to keep worrying about $|r|$ and is removed for simplicity. Applying the B.C. gives

$$R' = A\frac{1}{r}$$

Evaluating at $r = a$ gives

$$0 = A\frac{1}{a}$$

Hence $A = 0$, and the solution becomes

$$R(r) = C_0$$

Which is a constant.

Case $\lambda > 0$ The ODE in this case is

$$r^2 R'' + rR' - n^2 R = 0 \qquad n = 1,2,3,\cdots$$

Let $R = r^p$, the above becomes

$$\begin{aligned}r^2 p(p-1)\, r^{p-2} + rpr^{p-1} - n^2 r^p &= 0 \\ p(p-1)\, r^p + pr^p - n^2 r^p &= 0 \\ p(p-1) + p - n^2 &= 0 \\ p^2 &= n^2 \\ p &= \pm n\end{aligned}$$

Hence the solution is

$$R_n(r) = Cr^n + D\frac{1}{r^n} \qquad n = 1, 2, 3, \cdots$$

Applying the boundary condition $R'(a) = 0$ gives

$$\begin{aligned}
R_n'(r) &= nC_n r^{n-1} - nD_n \frac{1}{r^{n+1}} \\
0 &= R_n'(a) \\
&= nC_n a^{n-1} - nD_n \frac{1}{a^{n+1}} \\
&= nC_n a^{2n} - nD_n \\
&= C_n a^{2n} - D_n \\
D_n &= C_n a^{2n}
\end{aligned}$$

The solution becomes

$$\begin{aligned}
R_n(r) &= C_n r^n + C_n a^{2n}\frac{1}{r^n} \qquad n = 1, 2, 3, \cdots \\
&= C_n\left(r^n + \frac{a^{2n}}{r^n}\right)
\end{aligned}$$

Hence the complete solution for $R(r)$ is

$$R(r) = C_0 + \sum_{n=1}^{\infty} C_n\left(r^n + \frac{a^{2n}}{r^n}\right) \tag{6}$$

Using (5),(6) gives

$$\begin{aligned}
u_n(r,\theta) &= R_n\Theta_n \\
u(r,\theta) &= \left[C_0 + \sum_{n=1}^{\infty} C_n\left(r^n + \frac{a^{2n}}{r^n}\right)\right]\left[A_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta) + B_n\sin(n\theta)\right] \\
&= D_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta)\, C_n\left(r^n + \frac{a^{2n}}{r^n}\right) + \sum_{n=1}^{\infty} B_n\sin(n\theta)\, C_n\left(r^n + \frac{a^{2n}}{r^n}\right)
\end{aligned}$$

Where $D_0 = C_0A_0$. To simplify more, A_nC_n is combined to A_n and B_nC_n is combined to B_n. The full solution is

$$u(r,\theta) = D_0 + \sum_{n=1}^{\infty} A_n\left(r^n + \frac{a^{2n}}{r^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n\left(r^n + \frac{a^{2n}}{r^n}\right)\sin(n\theta)$$

The final nonhomogeneous B.C. is applied.

$$\begin{aligned}
u(b,\theta) &= g(\theta) \\
g(\theta) &= D_0 + \sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\sin(n\theta)
\end{aligned}$$

For $n=0$, integrating both sides give

$$\int_{-\pi}^{\pi} g(\theta)\,d\theta = \int_{-\pi}^{\pi} D_0 d\theta$$
$$D_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\theta)\,d\theta$$

For $n>0$, multiplying both sides by $\cos(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta\\ &+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\cos(m\theta)\cos(n\theta)\,d\theta\\ &+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\cos(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta\\ &+\sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta\\ &+\sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta\end{aligned}\tag{7}$$

But

$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = \pi \qquad n=m\neq 0$$
$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = 0 \qquad n\neq m$$

And

$$\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta = 0$$

Then (7) becomes

$$\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta = \pi A_n\left(b^n+\frac{a^{2n}}{b^n}\right)$$
$$A_n = \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta}{b^n+\frac{a^{2n}}{b^n}}\tag{8}$$

Again, multiplying both sides by $\sin(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0 \sin(m\theta)\,d\theta \\ &\quad + \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\sin(m\theta)\cos(n\theta)\,d\theta \\ &\quad + \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\sin(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0 \sin(m\theta)\,d\theta \\ &\quad + \sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta \\ &\quad + \sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta \end{aligned} \tag{9}$$

But

$$\begin{aligned}\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= \pi \qquad n = m \neq 0 \\ \int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= 0 \qquad n \neq m\end{aligned}$$

And

$$\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\sin(m\theta)\,d\theta = 0$$

Then (9) becomes

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta &= \pi B_n\left(b^n + \frac{a^{2n}}{b^n}\right) \\ B_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}}\end{aligned}$$

This complete the solution. Summary

$$
\begin{aligned}
u(r,\theta) &= D_0 + \sum_{n=1}^{\infty} A_n \left(r^n + \frac{a^{2n}}{r^n} \right) \cos(n\theta) + \sum_{n=1}^{\infty} B_n \left(r^n + \frac{a^{2n}}{r^n} \right) \sin(n\theta) \\
D_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} g(\theta)\, d\theta \\
A_n &= \frac{1}{\pi} \frac{\int_{-\pi}^{\pi} g(\theta) \cos(n\theta)\, d\theta}{b^n + \frac{a^{2n}}{b^n}} \\
B_n &= \frac{1}{\pi} \frac{\int_{-\pi}^{\pi} g(\theta) \sin(n\theta)\, d\theta}{b^n + \frac{a^{2n}}{b^n}}
\end{aligned}
$$

5.1.2.9 [315] Circular annulus

problem number 315

Solve Laplace equation

$$
u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0
$$

Inside circular annulus $1 < r < 2$ subject to the following boundary conditions

$$
\begin{aligned}
u(1,\theta) &= 0 \\
u(2,\theta) &= \sin\theta
\end{aligned}
$$

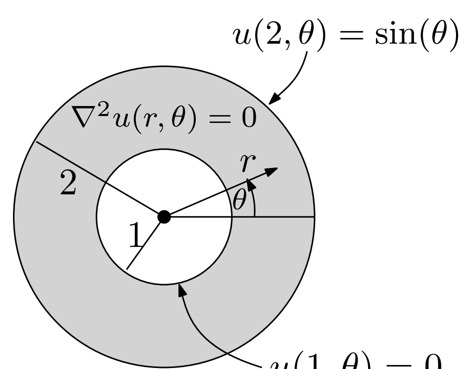


Figure 5.36: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
bc  = {u[1, theta] == 0, u[2, theta] == Sin[theta]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta)\to\left\{\begin{array}{cc}\frac{2(r^2-1)\sin(\theta)}{3r} & 1\leq r\leq 2\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(1,theta)=0,u(2,theta)=sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta))),outp
```

$$u(r,\theta)=\frac{2(r^2-1)\sin(\theta)}{3r}$$

5.1.2.10 [316] Outside a disk

problem number 316

Solve Laplace equation in polar coordinates outside a disk

Solve for $u(r,\theta)$

$$\begin{aligned}u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}&=0\\ a&\leq r\\ 0<\theta&\leq 2\pi\end{aligned}$$

Boundary conditions

$$\begin{aligned}u(a,\theta)&=f(\theta)\\ |u(0,\theta)|&<\infty\\ u(r,0)&=u(r,2\pi)\\ \frac{\partial u}{\partial\theta}(r,0)&=\frac{\partial u}{\partial\theta}(r,2\pi)\end{aligned}$$

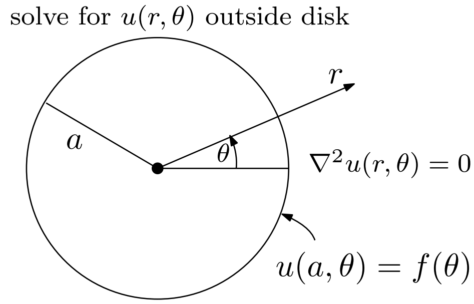


Figure 5.37: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + 1/r*D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] ==
bc  = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Deriva
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta) \to \sum_{K[1]=1}^{\infty} \frac{a^{-K[1]} r^{K[1]} \left(\cos(\theta K[1]) \int_{-\pi}^{\pi} \frac{\cos(\theta K[1]) f(\theta)}{\sqrt{\pi}}\, d\theta + \left(\int_{-\pi}^{\pi} \frac{f(\theta)\sin(\theta K[1])}{\sqrt{\pi}}\, d\theta\right) \sin(\theta K[1])\right)}{\sqrt{\pi}} + \frac{\int_{-\pi}^{\pi}}{\sqrt{}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r, theta), r$2) + 1/r* diff(u(r,theta),r) + 1/r^2* diff(u(r, theta), theta$2)
bc  := u(a, theta) = f(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta) = \frac{\int_{-\pi}^{\pi} f(\theta)\, d\theta + 2\pi \left(\sum_{n=1}^{\infty} \frac{\left(\left(\int_{-\pi}^{\pi} \cos(n\theta) f(\theta) d\theta\right) \cos(n\theta) + \left(\int_{-\pi}^{\pi} f(\theta) \sin(n\theta) d\theta\right) \sin(n\theta)\right) \left(\frac{r}{a}\right)^{-n}}{\pi} \right)}{2\pi}$$

5.1.2.11 [317] Outside a disk

problem number 317

Added January 13, 2020

Laplace PDE in polar coordinates outside a disk

Solve $u_{xx} + u_{yy} = 0$ outside disk $x^2 + y^2 > 1$ with boundary condition xy^2 when $x^2 + y^2 = a$. Where $a = 1$ in this problem. Express solution in x, y

The first step is to convert the boundary condition to polar coordinates. Since $x = r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned} f(\theta) &= \cos\theta\sin^2\theta \\ &= \cos\theta\left(1-\cos^2\theta\right) \\ &= \cos\theta - \cos^3\theta \end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta) &= \cos\theta - \left(\frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta\right) \\ &= \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta \end{aligned} \tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

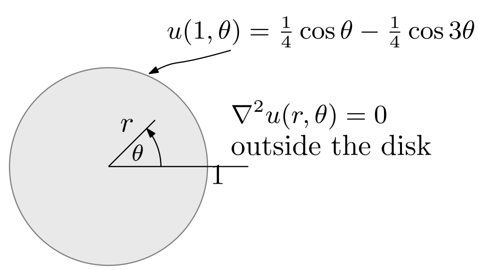


Figure 5.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1;
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
f[theta_] := 1/4*(Cos[theta] - Cos[3*theta]);
bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Derivat
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta)\to\frac{1}{4}\left(r\cos(\theta)-r^3\cos(3\theta)\right)\right\}\right\}$$

Maple ✓

```
restart;
f:=theta-> 1/4*(cos(theta) - cos(3*theta));
a:=1;
pde := VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta]);
bc := u(a, theta) = f(theta),u(r, -Pi) = u(r, Pi),(D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
sol:=simplify(subs(cos(theta)^3=trigsubs(cos(theta)^3)[2],expand(sol)),size);
```

$$u(r,\theta)=\frac{r^2\cos(\theta)-\cos(3\theta)}{4r^3}$$

Hand solution

The first step is to convert the boundary condition to polar coordinates. Since $x = r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned}f(\theta)&=\cos\theta\sin^2\theta\\&=\cos\theta\left(1-\cos^2\theta\right)\\&=\cos\theta-\cos^3\theta\end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned}f(\theta)&=\cos\theta-\left(\frac{3}{4}\cos\theta+\frac{1}{4}\cos 3\theta\right)\\&=\frac{1}{4}\cos\theta-\frac{1}{4}\cos 3\theta\end{aligned}\tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

The solution is known to be

$$u(r,\theta)=\frac{c_0}{2}+\sum_{n=1}^{\infty}r^{-n}(c_n\cos(n\theta)+k_n\sin(n\theta)) \tag{2}$$

Since the above solution is the same as $f(\theta)$ when $r=1$, then equating (2) when $r=1$ to (1) gives

$$\frac{1}{4}\cos\theta-\frac{1}{4}\cos 3\theta=\frac{c_0}{2}+\sum_{n=1}^{\infty}(c_n\cos(n\theta)+k_n\sin(n\theta))$$

By comparing terms on both sides, this shows by inspection that

$$\begin{aligned}c_0&=0\\ c_1&=\frac{1}{4}\\ c_3&=\frac{-1}{4}\end{aligned}$$

And all other c_n, k_n are zero. Using the above result back in (2) gives the solution as

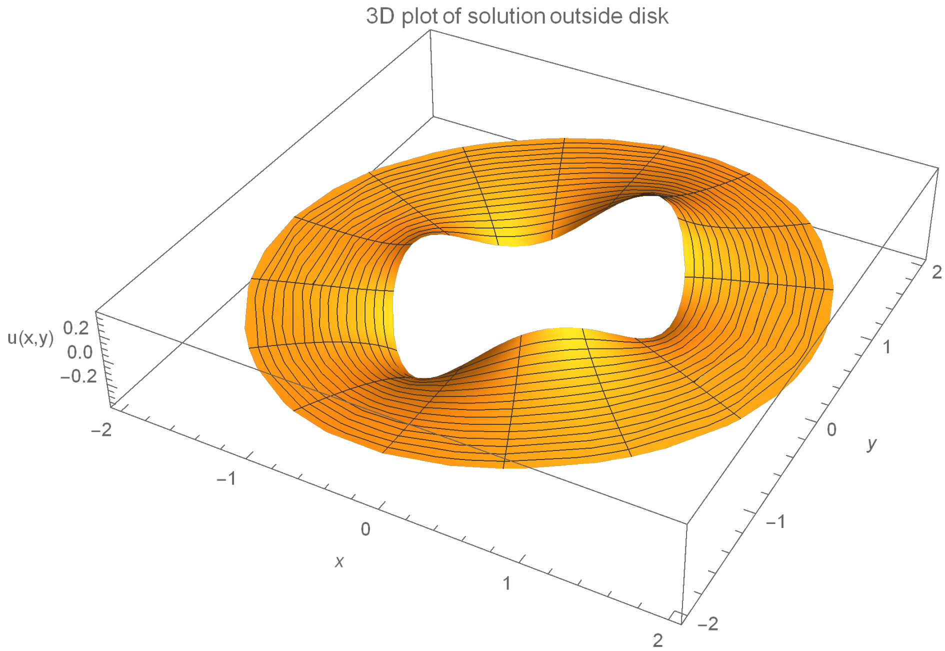
$$\begin{aligned}u(r,\theta)&=\frac{r^{-1}}{4}\cos\theta-\frac{r^{-3}}{4}\cos 3\theta\\ &=\frac{r^2\cos(\theta)-\cos(3\theta)}{4r^3}\end{aligned} \tag{3}$$

This is 3D plot of the solution

```
polarSolution[r_, phi_] := (r^-1) / 4 Cos[phi] - (r^-3) / 4 Cos[3 phi];
```

```
In[49]:= ParametricPlot3D[{r Cos[t], r Sin[t], polarSolution[r, t]},
  {r, 1, 2}, {t, 0, 2 Pi}, AxesLabel → {x, y, "u(x,y)"},
  PlotLabel → "3D plot of solution outside disk", ImageSize → 500]
```

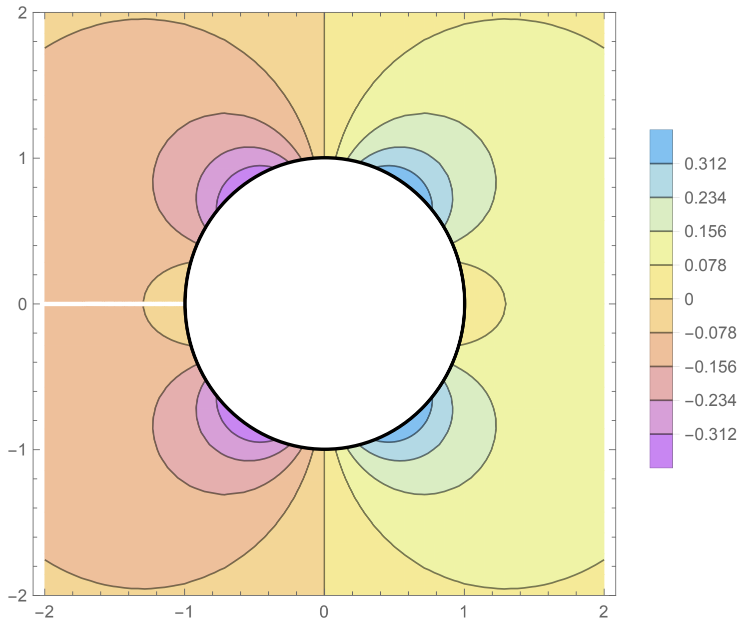
Out[49]=



This is a contour plot

In[52]:=
```
ContourPlot[polarSolution[Sqrt[x^2 + y^2], ArcTan[x, y]], {x, -2, 2}, {y, -2, 2},
  AxesLabel → {x, y},
  Contours → 50, PlotLegends → Automatic, ColorFunction → "Pastel",
  Epilog → {Thick, Circle[]},
  PlotRange → {-2, 2},
  RegionFunction → Function[{x, y, z}, Norm[{x, y}] > 1.]]
```

Out[52]=



5.2 Laplace in 3D

Local contents

5.2.1 Spherical coordinates

Local contents

5.2.1.1 [318] In a sphere

problem number 318

Taken from Maple pdsolve help pages

Solve for $u(r,\theta,\phi)$. Where θ is the polar angle and ϕ is the azimuthal angle. Hence $0<\theta<\pi$ and $-\pi<\phi<\pi$.

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2}=0$$

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[f[r, theta, phi], {r, theta, phi}, "Spherical"];
sol = AbsoluteTiming[TimeConstrained[DSolve[lap == 0, f[r, theta, phi], {r, theta, phi}], 60
```

$$\left\{\left\{f(r,\theta,\phi)\to\ \{\quad \sqrt{2}r^{-\frac{1}{2}\sqrt{4c_7+1}-\frac{1}{2}}\left(c_1r^{\sqrt{4c_7+1}}+c_2\right)\left(c_4\,{}_2F_1\left(\tfrac{1}{4}\left(-\sqrt{4c_7+1}+2\sqrt{c_8}+1\right),\tfrac{1}{4}\left(\sqrt{4c_7+1}+\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
PDE := diff(r^2*diff(F(r,theta,phi),r),r)+ 1/sin(theta)*diff(sin(theta)*diff(F(r,theta,phi),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(PDE,F(r,theta,phi),'build'
```

$$F(r,\theta,\phi) = \frac{\left(c_5 \sin\left(\phi\sqrt{\_c_2}\right) + c_6 \cos\left(\phi\sqrt{\_c_2}\right)\right)\left(c_1 r^{\frac{\sqrt{4\_c_1+1}}{2}} + c_2 r^{-\frac{\sqrt{4\_c_1+1}}{2}}\right)\left(c_4\sqrt{2}\,\text{hypergeom}\left(\left[\frac{\sqrt{\_c_2}}{2} + \right.\right.\right.}{}$$

5.2.2 Cylinderical coordinates

Local contents

5.2.2.1 [319] Haberman 7.9.1 (a)

problem number 319

Added May 25, 2019.

Problem 7.9.1 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0) = f(r,\theta)$, $u(r,\theta,H) = 0$, $u(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

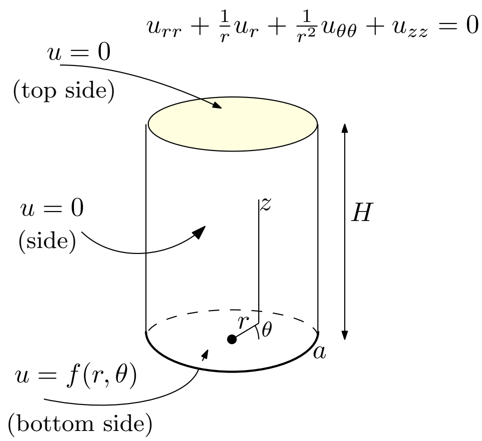


Figure 5.39: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == f[r, theta], u[r, theta, H] == 0, u[a, theta, z] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = f(r, theta), u(r, theta, H) = 0, u(a, theta, z) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.2 [320] Haberman 7.9.1 (b)

problem number 320

Added May 25, 2019.

Problem 7.9.1 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0) = f(r)\sin(7\theta)$, $u(r,\theta,H) = 0$, $u(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

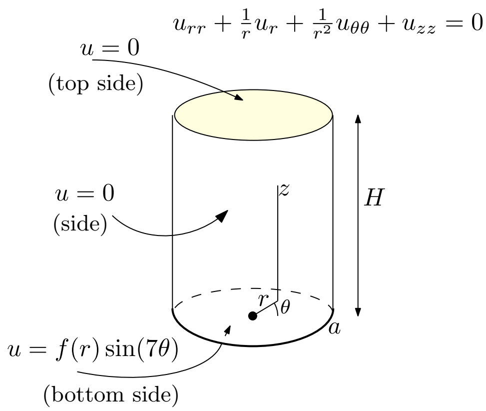


Figure 5.40: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == f[r]*Sin[7*theta], u[r, theta, H] == 0, u[a, theta, z] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = f(r)*sin(7*theta), u(r, theta, H) = 0, u(a, theta, z) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.3 [321] Haberman 7.9.1 (c)

problem number 321

Added May 25, 2019.

Problem 7.9.1 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0)=0$, $u(r,\theta,H)=f(r)\cos(3\theta)$, $u_r(a,\theta,z)=0$.

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}+u_{zz}=0$$

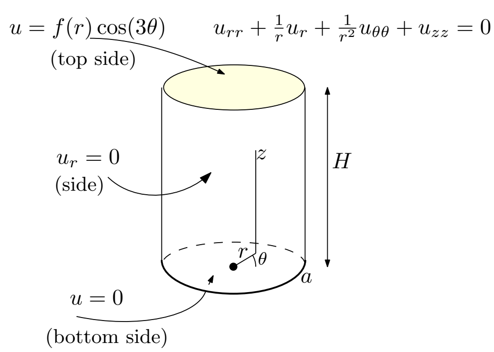


Figure 5.41: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, theta, H] == f[r]*Cos[3*theta], Derivative[1,0,0][u][a, the
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = 0, u(r, theta, H) = f(r)*cos(3*theta), eval(diff(u(r,theta,z),r),r=a
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.4 [322] Haberman 7.9.1 (d)

problem number 322

Added May 25, 2019.

Problem 7.9.1 (d) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u_z(r,\theta,0) = f(r)\sin(3\theta)$, $u_z(r,\theta,H) = 0$, $u_r(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

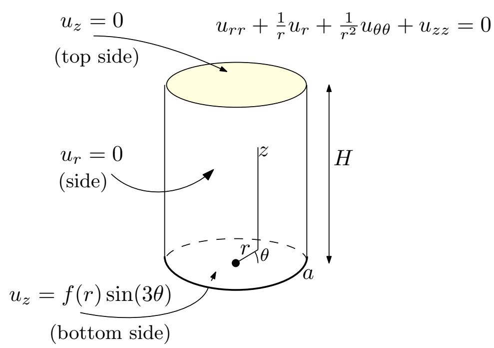


Figure 5.42: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == f[r]*Sin[3*theta], Derivative[0,0,1][u][r, theta
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=f(r)*sin(3*theta), eval(diff(u(r,theta,z),z),z=H)= 0, eva
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.5 [323] Haberman 7.9.1 (e)

problem number 323

Added May 25, 2019.

Problem 7.9.1 (e) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u_z(r,\theta,0) = f(r,\theta)$, $u_z(r,\theta,H) = 0$, $u_r(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

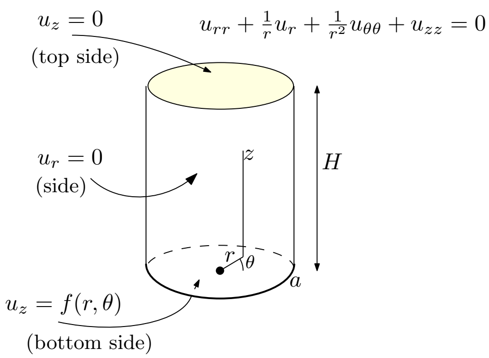


Figure 5.43: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == f[r,theta], Derivative[0,0,1][u][r, theta, H] ==
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=f(r,theta), eval(diff(u(r,theta,z),z),z=H)= 0, eval(diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.6 [324] Haberman 7.9.2 (a)

problem number 324

Added May 25, 2019.

Problem 7.9.2 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r,\theta,0) = 0$, $u(r,\theta,H) = f(r,\theta)$, $u(r,0,z) = 0$, $u(r,\pi,z) = 0$, $u(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

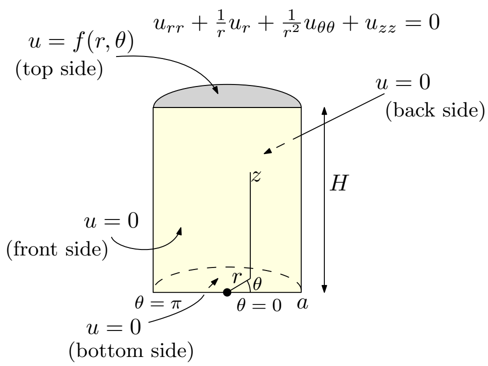


Figure 5.44: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, theta, H] == f[r,theta], u[r, 0, z] == 0,u[r,Pi,z]==0,u[a,t
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0, u(r,theta,H)= f(r,theta), u(r,0,z)=0, u(r,Pi,z)=0,u(a,theta,z)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.7 [325] Haberman 7.9.2 (b)

problem number 325

Added May 25, 2019.

Problem 7.9.2 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r,\theta,0) = 0$, $u_z(r,\theta,H) = 0$, $u(r,0,z) = 0$, $u(r,\pi,z) = 0$, $u(a,\theta,z) = g(\theta,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

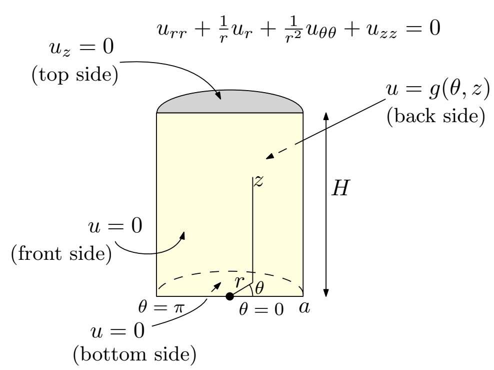


Figure 5.45: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, Derivative[0,0,1][u][r, theta, H] == 0, u[r, 0, z] == 0,u[r,Pi,z
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0, eval(diff(u(r,theta,z),z),z=H)=0, u(r,0,z)=0, u(r,Pi,z)=0,u(a,theta,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.8 [326] Haberman 7.9.2 (c)

problem number 326

Added May 25, 2019.

Problem 7.9.2 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u_z(r,\theta,0) = 0$, $u_z(r,\theta,H) = 0$, $u_\theta(r,0,z) = 0$, $u_\theta(r,\pi,z) = 0$, $u_r(a,\theta,z) = g(\theta,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

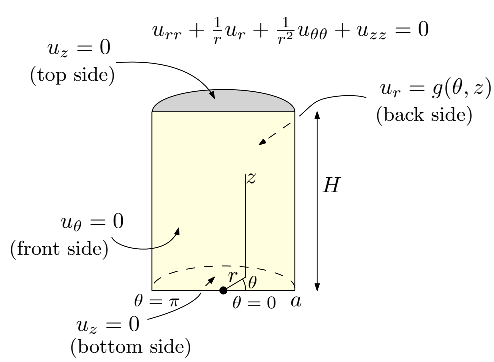


Figure 5.46: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == 0, Derivative[0,0,1][u][r, theta, H] == 0, Deriv
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=0, eval(diff(u(r,theta,z),z),z=H)=0, eval(diff(u(r,theta,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.2.2.9 [327] Haberman 7.9.2 (d)

problem number 327

Added May 26, 2019.

Problem 7.9.2 (d) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r,\theta,0) = 0$, $u(r,0,z) = 0$, $u(a,\theta,z) = 0$, $u(r,\theta,H) = 0$, $u_\theta(r,\pi,z) = f(r,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

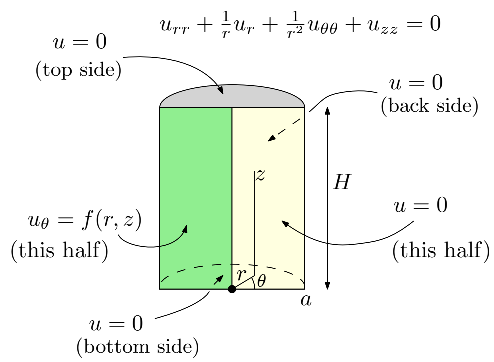


Figure 5.47: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, 0, z] == 0, u[a, theta, z] == 0,u[r,theta,H]==0,Derivative[
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0,u(r,0,z)=0, u(a,theta,z)=0, u(r,theta,H)=0,eval(diff(u(r,theta,z),thet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

5.3 Poisson in 2D

Local contents

5.3.1 Cartesian coordinates

Local contents

5.3.1.1 [328] All boundaries at zero

problem number 328

Added March 13, 2019.

Solve for $u(x,y)$

$$\frac{u_{xx}}{A}+\frac{u_{xx}}{B}=-2\theta$$

Where A,B,θ are constants, and the boundary conditions are

$$\begin{aligned} u(x,-b)&=0\\ u(x,b)&=0\\ u(-a,y)&=0\\ u(a,y)&=0 \end{aligned}$$

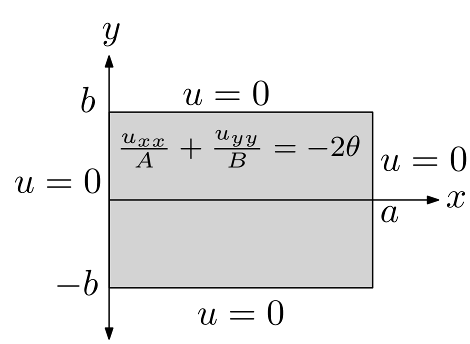


Figure 5.48: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}]/A + D[u[x, y], {y, 2}]/B == -2*theta;
bc  = {u[x, -b] == 0, u[x, b] == 0, u[-a, y] == 0, u[a, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)/A+diff(u(x,y),y$2)/B  = -2*theta;
bc  := u(x,-b)=0, u(x,b)=0, u(-a,y)=0, u(a,y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

sol=()

Hand solution

solve

$$\begin{aligned}\frac{u_{xx}}{A}+\frac{u_{yy}}{B}&=-2\theta\\ Bu_{xx}+Au_{yy}&=-2\theta AB\\ &=C\end{aligned}$$

Where $C=-2\theta AB$ is a new constant. With boundary conditions

$$\begin{aligned}u(x,-b)&=0\\ u(x,b)&=0\\ u(-a,y)&=0\\ u(a,y)&=0\end{aligned}$$

To simplify solution, shift the rectangle so its lower left corner on the origin. Let $\tilde{x}=x+a$, and $\tilde{y}=y+b$. The boundary conditions becomes

$$\begin{aligned}u(\tilde{x},0)&=0\\ u(\tilde{x},2b)&=0\\ u(0,\tilde{y})&=0\\ u(2a,\tilde{y})&=0\end{aligned}$$

And the pde becomes $Bu_{\tilde{x}\tilde{x}}+Au_{\tilde{y}\tilde{y}}=C$. Instead of keep writing $\tilde{x},\tilde{y}$, will use x,y, but remember that these are shifted version. At the end, we shift back.

Hence the PDE to solve is $Bu_{xx} + Au_{yy} = C$ with BC

$$\begin{aligned} u(x,0) &= 0 \\ u(x,2b) &= 0 \\ u(0,y) &= 0 \\ u(2a,y) &= 0 \end{aligned}$$

Using eigenfunction expansion method. Let

$$u(x,y) = \sum_{n=1}^{\infty} b_n(y)\, X_n(x) \tag{1}$$

Where $X_n(x)$ is eigenfunctions for $X''(x) + \lambda_n X(x) = 0$ with boundary conditions $X(0) = X(2a) = 0$. This has eigenfunctions as $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$ with eigenvalues $\lambda_n = \left(\frac{n\pi}{2a}\right)^2$ for $n = 1, 2, \cdots$.

Substituting (1) into the PDE $Bu_{xx} + Au_{yy} = C$ gives

$$B \sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A \sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = C$$

Expanding C (a constant) as Fourier sine series the above becomes

$$B \sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A \sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = \sum_{n=1}^{\infty} q_n X_n(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, hence the above becomes

$$-B \sum_{n=1}^{\infty} \lambda_n b_n(y)\, X_n(x) + A \sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = \sum_{n=1}^{\infty} q_n X_n(x)$$

$$A b_n''(y) - B\lambda_n b_n(y) = q_n \tag{1A}$$

But

$$\begin{aligned} C &= \sum_{n=1}^{\infty} q_n X_n(x) \\ \int_0^{2a} C X_n(x)\, dx &= q_n \int_0^{2a} X_n^2(x)\, dx \\ \int_0^{2a} C \sin\left(\sqrt{\lambda_n} x\right) dx &= q_n \int_0^{2a} \sin^2\left(\sqrt{\lambda_n} x\right) dx \\ \frac{-C}{\sqrt{\lambda_n}}\left((-1)^n - 1\right) &= q_n a \\ q_n &= \frac{-C}{a\sqrt{\lambda_n}}\left((-1)^n - 1\right) \end{aligned}$$

Hence (1A) becomes

$$Ab_n''(y) - B\lambda_n b_n(y) = \frac{-C}{a\sqrt{\lambda_n}}((-1)^n - 1)$$

This is standard second order linear ODE. The solution is

$$b_n(y) = D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1)$$

Using the above in (1) gives the solution

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1)\right)X_n(x) \tag{1A}$$

We now need to find D_n, E_n.

<u>Case n even</u>

When n is even $((-1)^n - 1) = 0$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y}\right)X_n(x)$$

At $y = 0$ the above gives

$$0 = \sum_{n=1}^{\infty}(D_n + E_n)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n + E_n = 0 \tag{2}$$

And at $y = 2b$

$$0 = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b} = 0 \tag{3}$$

From (2,3) we see that $D_n = E_n = 0$, Hence $u(x,y) = 0$ when n even.

<u>Case n odd</u>

When n is odd $((-1)^n - 1) = -2$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} - \frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)X_n(x)$$

At $y=0$ the above gives

$$0=\sum_{n=1}^{\infty}\left(D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{4}$$

And at $y=2b$

$$0=\sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{5}$$

Solving (4,5) for D_n, E_n gives

$$D_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$
$$E_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$

Therefore the final solution from (1A) becomes

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)X_n(x)$$
$$=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}y}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda}\right.$$

Where $\lambda_n=\left(\frac{n\pi}{2a}\right)^2$. Switching back to original coordinates using $\tilde{x}=x+a$, and $\tilde{y}=y+b$, then the above is

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\left(-\sqrt{\frac{B}{A}\lambda_n}y+b\right)}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)$$

Where $C=-2\theta AB$, hence

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\right.$$
$$=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta A}{a\lambda_n^{\frac{3}{2}}}\right)\sin$$

5.3.1.2 [329] Dirichlet problem in a rectangle

problem number 329

Taken from Mathematica DSolve help pages.

Solve for $u(x,y)$

$$u_{xx} + u_{yy} = 6x - 6y$$

Boundary conditions

$$\begin{aligned} u(x,0) &= 1 + 11x + x^3 \\ u(x,2) &= -7 + 11x + x^3 \\ u(0,y) &= 1 - y^3 \\ u(4,y) &= 109 - y^3 \end{aligned}$$

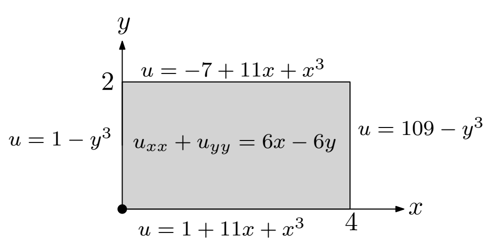


Figure 5.49: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 6*x - 6*y;
bc  = {u[x, 0] == 1 + 11*x + x^3, u[x, 2] == -7 + 11*x + x^3, u[0, y] == 1 - y^3, u[4, y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to x^3 + 11x - y^3 + 1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=6*x-6*y;
bc  := u(x,0)=1+11*x+x^3,
    u(x,2)=-7+11*x+x^3,
    u(0,y)=1-y^3,
    u(4,y)=109-y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = x^3 - y^3 + 11x + 1$$

5.3.1.3 [330] Poisson PDE in whole 2D plane

problem number 330

Added January 13, 2020

Solve Poisson PDE

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 6y$$

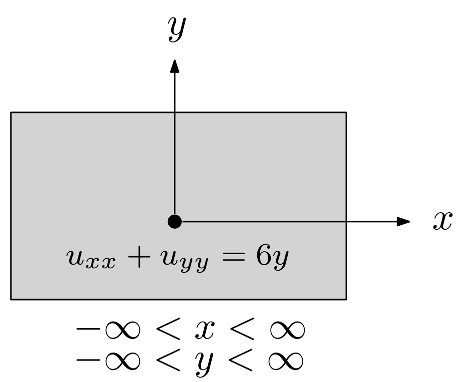


Figure 5.50: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 6*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1(x-iy)+c_2(x+iy)+3x^2y\right\}\right\}$$

Maple ✓

```
restart;
pde:=VectorCalculus:-Laplacian(u(x,y),'cartesian'[x,y])=6*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=3x^2y+\_F1(-ix+y)+\_F2(ix+y)$$

Hand solution

Solve Poisson PDE

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=6y$$

The solution is

$$f=f_h+f_p \tag{1}$$

Where f_h is the homogenous solution to Laplace $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$ and f_p is a particular solution. The homogeneous solution is easily found as follows. Let

$$f_h=F(mx+y)$$

Then $\frac{\partial f_h}{\partial x}=mF'$ and $\frac{\partial^2 f_h}{\partial x^2}=m^2F''$ and $\frac{\partial f_h}{\partial y}=F'$ and $\frac{\partial^2 f_h}{\partial y^2}=F''$. Substituting these into $\frac{\partial^2 f_h}{\partial x^2}+\frac{\partial^2 f_h}{\partial y^2}=0$ gives

$$\begin{aligned}m^2F''+F''&=0\\ m^2+1&=0\\ m&=\pm i\end{aligned}$$

Since we assumed $f_h=F(mx+y)$, then the homogeneous solution is sum of two arbitrary functions (one for each root of m)

$$f_h=F_1(ix+y)+F_2(-ix+y)$$

To find particular solution, let

$$f_p = Ax^n y^m \tag{2}$$

Hence $\frac{\partial f_p}{\partial x} = Anx^{n-1}y^m$ and $\frac{\partial^2 f_p}{\partial x^2} = An(n-1)\,x^{n-2}y^m$ and $\frac{\partial f_p}{\partial y} = Ax^n m y^{m-1}$ and $\frac{\partial^2 f_p}{\partial y^2} = Ax^n m(m-1)\,y^{m-2}$. Substituting these into $\frac{\partial^2 f_p}{\partial x^2} + \frac{\partial^2 f_p}{\partial y^2} = 6y$ gives

$$\begin{aligned} An(n-1)\,x^{n-2}y^m + Ax^n m(m-1)\,y^{m-2} &= 6y \\ y^m\left(An(n-1)\,x^{n-2} + Ax^n m(m-1)\,y^{-2}\right) &= 6y \end{aligned}$$

Comparing terms, then $m = 1$ and

$$An(n-1)\,x^{n-2} + Ax^n m(m-1)\,y^{-2} = 6$$

Since $m = 1$ then the above simplifies to

$$An(n-1)\,x^{n-2} = 6$$

Since there is no x in RHS, then $n - 2 = 0$ or $n = 2$ and the above becomes

$$\begin{aligned} 2A(2-1) &= 6 \\ A &= 3 \end{aligned}$$

Hence (2) becomes

$$f_p = Ax^n y^m = 3x^2 y$$

And the complete solution (1) is

$$\begin{aligned} f(x,y) &= f_h + f_p \\ &= F_1(ix+y) + F_2(-ix+y) + 3x^2 y \end{aligned}$$

5.4 Helmholtz in 2D

Local contents

5.4.0.1 [331] In rectangle

problem number 331

Taken from Mathematica DSolve help pages.

Solve for $u(x,y)$

$$u_{xx} + u_{yy} + 5u(x,y) = 0$$

Boundary conditions

$$\begin{aligned} u(x,0) &= \text{UnitTriangle[x-2]} \\ u(x,2) &= 0 \\ u(0,y) &= 0 \\ u(4,y) &= 0 \end{aligned}$$

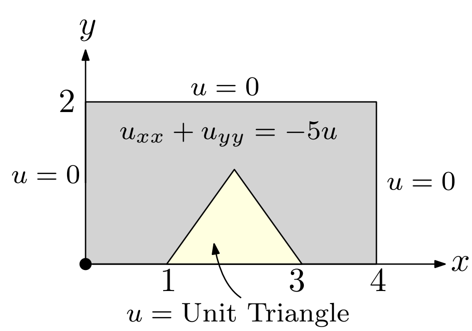


Figure 5.51: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
bc  = {u[x, 0] == Piecewise[{{-1 + x, x > 1 && x < 2}, {3 - x, x > 2 && x < 3}}], u[x, 2] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{64\left(\cos\left(\frac{n\pi}{8}\right)+\cos\left(\frac{3n\pi}{8}\right)\right)\operatorname{csch}\left(\frac{1}{2}\sqrt{n^2\pi^2-80}\right)\sin^3\left(\frac{n\pi}{8}\right)\sin\left(\frac{n\pi x}{4}\right)\sinh\left(\frac{1}{4}\sqrt{n^2\pi^2-80}(2-\right.}{n^2\pi^2}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
bc  := u(x,0)=piecewise( x>1 and x<2, -1+x,x>2 and x<3 ,3-x),
       u(x,2)=0,
       u(0,y)=0,
       u(4,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{32\left(\left(\cos\left(\frac{\pi n}{4}\right)-1\right)\cos\left(\frac{\sqrt{-\pi^2n^2+80}}{2}\right)\cos\left(\frac{\pi n}{4}\right)\sin\left(\frac{\pi n}{4}\right)\sin\left(\frac{\sqrt{-\pi^2n^2+80}\,y}{4}\right)+\frac{\left(\sin\left(\frac{\pi n}{2}\right)-\frac{\sin\left(\frac{\pi n}{4}\right)}{2}-\cdots\right.}{\cdots}\right.}{\pi^2n^2\sin\left(\frac{\sqrt{-\pi^2n^2+80}}{2}\right)}$$

5.4.0.2 [332] On whole plane

problem number 332

Added December 27, 2018.

Solve for $u(x,y)$

$$u_{xx}+u_{yy}+5u(x,y)=0$$

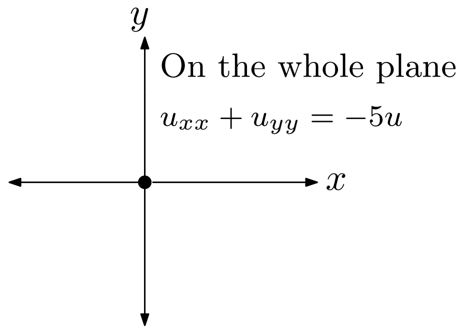


Figure 5.52: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to e^{\sqrt{c_5}(-x)}\left(c_1e^{2\sqrt{c_5}x}+c_2\right)\left(c_4\cos\left(\sqrt{5+c_5}y\right)+c_3\sin\left(\sqrt{5+c_5}y\right)\right)\right\}\right\}$$

why? It solved earlier with BC?

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y)=\left(c_1\mathrm{e}^{2x\sqrt{\_c_1}}+c_2\right)\left(c_3\sin\left(\sqrt{\_c_1+5}\,y\right)+c_4\cos\left(\sqrt{\_c_1+5}\,y\right)\right)\mathrm{e}^{-x\sqrt{\_c_1}}$$

5.4.0.3 [333] Reduced Helmholtz Inside square

problem number 333

Added December 20, 2018.

Example 24, taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - ku(x,y) = 0$$

With $k > 0$. It is called reduced Helmholtz, because of the minus sign above. Otherwise, standard Helmholtz has a positive sign.

Boundary conditions

$$\begin{aligned} u(x,0) &= 0 \\ u(x,\pi) &= 0 \\ u(0,y) &= 1 \\ u(\pi,y) &= 0 \end{aligned}$$

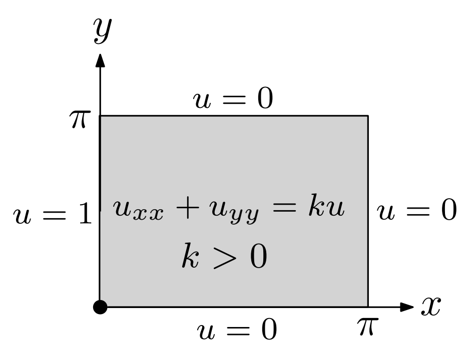


Figure 5.53: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] - k*u[x, y] == 0;
bc  = {u[x, 0] == 0, u[x, Pi] == 0, u[0, y] == 1, u[Pi, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> k >
```

$$\left\{\left\{u(x,y)\to\sum_{K[1]=1}^{\infty}-\frac{2\left(-1+(-1)^{K[1]}\right)\operatorname{csch}\left(\pi\sqrt{K[1]^2+k}\right)\sin(yK[1])\sinh\left((\pi-x)\sqrt{K[1]^2+k}\right)}{\pi K[1]}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2)-k*u(x, y) = 0;
bc_left_edge:=u(0, y) = 1;
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,Pi)=0;
bc_right_edge:=u(Pi,y)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge,bc_right_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{2((-1)^n-1)\left(e^{\sqrt{n^2+k}\,x}-e^{-(x-2\pi)\sqrt{n^2+k}}\right)\sin(ny)}{\left(e^{2\sqrt{n^2+k}\,\pi}-1\right)\pi n}$$

5.5 Helmholtz in 3D

Local contents

5.5.1 Spherical coordinates

Local contents

5.5.1.1 [334] Chain reaction PDE

problem number 334

Added May 7, 2019.

Assume ϕ independence. Solve for $u(r,\theta,t)$

$$u_t = k(\lambda u + \nabla^2(u))$$

Where $\nabla^2(u) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)$ with $k > 0$.

Boundary conditions $u(R,\theta,t) = 0$.

Mathematica ✗

```
ClearAll["Global`*"];
U = u[r, theta, t];
pde = D[U, t] == k*(lambda*U + (1/r^2)*D[r^2*D[U, r], r] + (1/(r^2*Sin[theta]))*D[Sin[theta]
bc  = u[R, theta, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, U, {r, theta, t}, Assumptions -> {k >
```

Failed

Maple ✗

```
restart;
U   := u(r,theta,t);
pde := diff(U,t) = k*(lambda*U +1/r^2* diff(r^2*diff(U,r),r)+ 1/(r^2*sin(theta))*diff(sin(th
bc  := u(R,theta,t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc], U) assuming k>0,
```

$$u(r, theta, t) = 0$$

trivial solution

Hand solution

Solve for $u(r,\theta,t)$ in spherical coordinates (assuming ϕ independence) the chain reaction equation $\frac{1}{k}u_t = \lambda u + \nabla^2 u$ with boundary conditions $u(R,\theta,t) = 0$.

$$\begin{aligned}
\frac{1}{k}u_t &= \lambda u + \nabla^2 u \\
&= \lambda u + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)\right) \\
&= \lambda u + \frac{1}{r^2}(2ru_r + r^2u_{rr}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)
\end{aligned}$$

Let $u = R(r)\,\Theta(\theta)\,T(t)$. Substituting into the above gives

$$\begin{aligned}
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{1}{r^2}(2rR'\Theta T + r^2R''\Theta T) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta(\Theta' RT)) \\
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{2}{r}R'\Theta T + R''\Theta T + \frac{RT}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

Dividing by $TR\Theta$ gives

$$\frac{1}{k}\frac{T'}{T} = \lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)$$

The left side depends on t only and the right depends on r,θ only. Let the separation variable be $-n$. This gives the following 2 equations

$$\frac{1}{k}\frac{T'}{T} = -n \tag{1}$$

$$\lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = -n \tag{2}$$

Now we consider (2). Multiplying both sides of (2) by r^2 gives

$$\begin{aligned}
\lambda r^2 + 2r\frac{R'}{R} + r^2\frac{R''}{R} + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) &= -nr^2 \\
2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda + n) &= -\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

The left side depends on r and the right side depends on θ. Let the separation variable be $l(l+1)$ where l is integer. Hence we obtain the following two equations

$$-\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = l(l+1) \tag{4}$$
$$2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda+n) = l(l+1) \tag{5}$$

Starting with (4)

$$\frac{\partial}{\partial\theta}(\Theta'\sin\theta) + l(l+1)\,\Theta\sin\theta = 0$$
$$\Theta''\sin\theta + \Theta'\cos\theta + l(l+1)\,\Theta\sin\theta = 0$$

Using the substitution $z=\cos\theta$ the above becomes

$$\left(1-z^2\right)\Theta'' - 2z\Theta' + l(l+1)\,\Theta = 0$$

This Legendre ODE. Solution is $P_l(\theta)$. The other solution to the above ODE is ignored as not bounded. Now back to solving (5). Writing it as

$$2rR' + r^2R'' + r^2(\lambda+n)\,R = l(l+1)\,R$$
$$r^2R'' + 2rR' + \left(r^2(\lambda+n) - l(l+1)\right)R = 0 \tag{6}$$

This can be converted to Bessel ODE using substitution. First let $v = r\sqrt{(\lambda+n)}$. Then $R'(r) = \sqrt{(\lambda+n)}R'(v)\,, R''(r) = (\lambda+n)\,R''(v)$ and (6) becomes

$$(\lambda+n)\,r^2R''(v) + 2r\sqrt{(\lambda+n)}R'(v) + \left(r^2(\lambda+n) - l(l+1)\right)R = 0$$
$$v^2R''(v) + 2vR'(v) + \left(v^2 - l(l+1)\right)R = 0 \tag{7}$$

Now, we apply second transformation $R(v) = \frac{Z(v)}{\sqrt{v}}$ Then

$$R'(v) = \frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}}$$
$$R''(v) = \frac{Z''(v)}{\sqrt{v}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}\left(-\frac{3}{2}\right)Z(v)\,\frac{1}{v^{\frac{5}{2}}}$$
$$= \frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}}$$

Hence (7) becomes

$$v^2\left(\frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}}\right) + 2v\left(\frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}}\right) + \left(v^2 - l(l+1)\right)\frac{Z(v)}{\sqrt{v}} = 0$$

Multiplying by $\sqrt{v}$ gives

$$v^2\left(Z''(v)-Z'(v)\frac{1}{v}+\frac{3}{4}Z(v)\frac{1}{v^2}\right)+2v\left(Z'(v)-\frac{1}{2}Z(v)\frac{1}{v}\right)+\left(v^2-l(l+1)\right)Z(v)=0$$

$$\left(v^2Z''(v)-vZ'(v)+\frac{3}{4}Z(v)\right)+\left(2vZ'(v)-Z(v)\right)+\left(v^2-l(l+1)\right)Z(v)=0$$

$$v^2Z''(v)+vZ'(v)+\frac{1}{4}Z(v)+\left(v^2-l(l+1)\right)Z(v)=0$$

$$v^2Z''(v)+vZ'(v)+\left(v^2-l(l+1)+\frac{1}{4}\right)Z(v)=0$$

$$v^2Z''(v)+vZ'(v)+\left(v^2-l^2+l+\frac{1}{4}\right)Z(v)=0$$

$$v^2Z''(v)+vZ'(v)+\left(v^2-\left(l+\frac{1}{2}\right)^2\right)Z(v)=0$$

This is now in standard Bessel ODE form. Comparing it to $v^2Z''(v)+vZ'(v)+\left(v^2-d^2\right)Z(v)=0$ shows the order is $d=l+\frac{1}{2}$. The solutions are

$$Z(v)=c_1J_{l+\frac{1}{2}}(v)+c_2Y_{l+\frac{1}{2}}(v)$$

But $R(v)=\frac{Z(v)}{\sqrt{v}}$, hence

$$R(v)=c_1\frac{J_{l+\frac{1}{2}}(v)}{\sqrt{v}}+c_2\frac{Y_{l+\frac{1}{2}}(v)}{\sqrt{v}}$$

But $v=r\sqrt{\lambda+n}$ then above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}+c_2\frac{Y_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

The solution assumed bounded at $r=0$ hence $c_2=0$ and the above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

Let $m^2=(\lambda+n)$, hence

$$\begin{aligned}R(r)&=c_1\frac{J_{l+\frac{1}{2}}(mr)}{\sqrt{mr}}\\&=j_l(mr)\end{aligned}$$

where $j_l(mr)$ are the spherical Bessel functions. Boundary conditions at $r = R$ gives

$$j_l(mR) = 0$$

Hence mR or $\sqrt{\lambda + n}R$ are the zeros of spherical Bessel functions $j_l(mR)$. There are infinite zeros for each l. Let the v^{th} zero of j_l be called $Z_{l,v}$. Then $mR_{l,v} = Z_{l,v}$ or $\sqrt{\lambda + n}_{l,v} = \frac{Z_{l,v}}{R}$. or

$$n = \left(\frac{Z_{l,v}}{R}\right)^2 - \lambda$$

The solution to the time ODE is therefore

$$\begin{aligned} T_{l,v} &= A_{l,v} e^{-nkt} \\ &= A_{l,v} e^{-\left(\left(\frac{Z_{l,v}}{R}\right)^2 - \lambda\right) kt} \end{aligned}$$

Hence the complete solution is

$$\begin{aligned} u(r,\theta,t) &= e^{-i\alpha kt} P_l(\theta)\, j_l(mr) \\ &= \sum_{l=1}^{\infty} \sum_{v=0}^{\infty} A_{l,v} e^{-nkt} P_l(\theta)\, j_l\left(\frac{Z_{l,v}}{R} r\right) \end{aligned}$$

$A_{l.v}$ constants still need to be found from initial conditions. For each l, we have infinite sum over all $v's$ zeros of j_l.

CHAPTER 6

Hyperbolic PDE's (Wave)

Local contents

6.1 Wave PDE in 1D

Local contents

6.1.1 Finite length string

Local contents

6.1.1.1 [335] General solution for both ends fixed. Domain is $0 \dots L$

problem number 335

Added July 7,2019

Solve for $u(x,t)$ for $t>0$ and $0<x<L$

$$u_{tt} = c^2 u_{xx}$$

With boundary condition both ends fixed

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

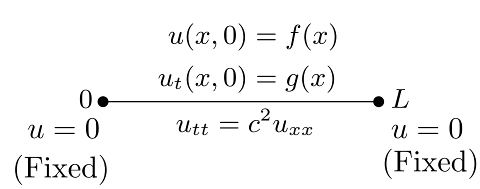


Figure 6.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions->L>
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty}\sqrt{2}\sqrt{\frac{1}{L}}\sin\left(\frac{\pi x K[1]}{L}\right)\left(\cos\left(\pi t\sqrt{\frac{c^2K[1]^2}{L^2}}\right)\int_0^L \frac{\sqrt{2}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx + \frac{L\left(\int_0^L \frac{\sqrt{2}g(x)\sin\left(\frac{\pi x}{}\right.}{\sqrt{L}}\right.}{}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\pi cn\left(\int_0^L f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\cos\left(\frac{\pi cnt}{L}\right)+L\left(\int_0^L g(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi cnt}{L}\right)\right)\sin\left(\frac{\pi nx}{L}\right)}{\pi Lcn}$$

Hand solution

Solving for $t>0, 0<x<L$

$$u_{tt}=c^2u_{xx}$$

With BC

$$u(0,t)=0$$
$$u(L,t)=0$$

And initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=g(x)$$

Let $u = X(x)T(t)$. The PDE becomes

$$\frac{T''X}{c^2} = X''T$$
$$\frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} = -\lambda$$

Where λ is separation constant. Hence the eigenvalue ODE is

$$X'' + \lambda X = 0$$
$$X(0) = 0$$
$$X(L) = 0$$

From the boundary conditions, we see that $\lambda > 0$ is the only possible value. Therefore the solution to the above ODE is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then $A = 0$ and the solution becomes $X(x) = B\sin\left(\sqrt{\lambda}x\right)$. Since $X(L) = 0$ then for non trivial solution we want $\sqrt{\lambda}L = n\pi$ or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE now becomes

$$T'' + c^2\left(\frac{n\pi}{L}\right)^2 T = 0$$

Which has the solution

$$T(t) = D_n\cos\left(c\frac{n\pi}{L}t\right) + E_n\sin\left(c\frac{n\pi}{L}t\right)$$

Therefore the complete solution becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(D_n\cos\left(c\frac{n\pi}{L}t\right) + E_n\sin\left(c\frac{n\pi}{L}t\right)\right)\Phi_n(x) \tag{1}$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} D_n\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx &= D_n \int_0^L \Phi_n^2(x)\,dx \\ &= \frac{L}{2} D_n\end{aligned}$$

Hence

$$D_n = \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t) = \sum_{n=1}^{\infty}\left(-c\frac{n\pi}{L}D_n \sin\left(c\frac{n\pi}{L}t\right) + E_n c\frac{n\pi}{L}\cos\left(c\frac{n\pi}{L}t\right)\right)\Phi_n(x)$$

At $t=0$ the above becomes

$$g(x) = \sum_{n=1}^{\infty} E_n c\frac{n\pi}{L}\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L g(x)\,\Phi_n(x)\,dx &= E_n c\frac{n\pi}{L}\int_0^L \Phi_n^2(x)\,dx \\ &= \frac{L}{2}E_n c\frac{n\pi}{L} \\ &= \frac{1}{2}E_n c n\pi\end{aligned}$$

Hence

$$E_n = \frac{2}{cn\pi}\int_0^L g(x)\,\Phi_n(x)\,dx \tag{3}$$

Using (2,3) in (1) gives the final solution as

$$\begin{aligned}u(x,t) = {}&\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right) \\ &+ \frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

6.1.1.2 [336] both ends fixed, inital position zero (special case)

problem number 336

Added July 8,2019

Solve for $u(x,t)$ for $t>0$ and $0<x<L$

$$u_{tt} = c^2 u_{xx}$$

With boundary condition both ends fixed

$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= f(x) \\ u_t(x,0) &= g(x) \end{aligned}$$

Using the following values

$$\begin{aligned} L &= 10 \\ c &= 2 \\ f(x) &= 0 \\ g(x) &= \frac{8x(L-x)^2}{L^3} \end{aligned}$$

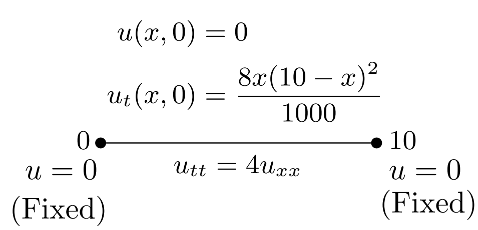


Figure 6.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=10; c=2; f=0; g=(8*x*(L-x)^2)/L^3;
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f, Derivative[0, 1][u][x, 0] == g};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{160(2+(-1)^n)\sin\left(\frac{n\pi t}{5}\right)\sin\left(\frac{n\pi x}{10}\right)}{n^4\pi^4}\right\}\right\}$$

Maple ✓

```
restart;
L:=10;
c:=2;
f:=0;
g:=(8*x*(L-x)^2)/L^3;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,eval(diff(u(x,t),t),t=0)=g;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{160((-1)^n+2)\sin\left(\frac{\pi n x}{10}\right)\sin\left(\frac{\pi n t}{5}\right)}{\pi^4 n^4}$$

Hand solution

Solving the wave PDE on string with both ends fixed

$$u_{tt}=c^2u_{xx}\qquad t>0,x>0$$

With BC

$$u(0,t)=0$$
$$u(L,t)=0$$

And initial conditions

$$u(x,0) = f(x) = 0$$
$$u_t(x,0) = g(x) = \frac{8x(L-x)^2}{L^3}$$

Using $c = 2, L = 10$.

The general problem PDE was solved in 6.1.1.1 on page 916 and the solution is

$$u(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)$$
$$+\frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx\right)\sin\left(2\frac{n\pi}{10}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

But $\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx = \frac{160(2+(-1)^n)}{n^3\pi^3}$, hence the solution becomes

$$u(x,t) = \frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{160(2+(-1)^n)}{n^4}\sin\left(\frac{n\pi}{5}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

Animation is below

Out[ ]=

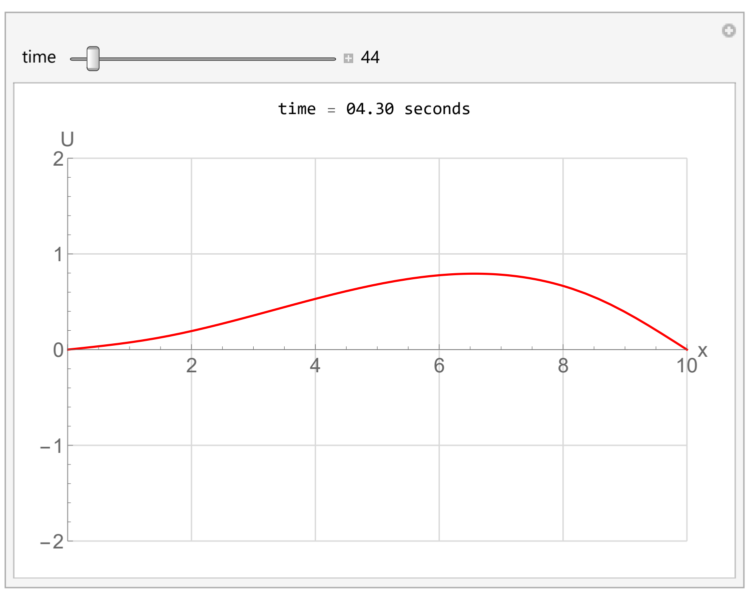


Figure 6.3: Initial state

Source code used for the above

```
In[*]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 2;
L = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 15;
mySol[x_, t_] = 1/π^4 Sum[(160 (2 + (-1)^n))/n^4 Sin[n π/5 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 6.4: Source code

```
In[*]:= tab = Table[
        Grid[{
           {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
           {
             Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                 BaseStyle → 15,
                 ImageMargins → 3,
                 PerformanceGoal → "Quality",
                 PlotRange → {{0, L}, {-2, 2}},
                 ImageSize → 500,
                 AxesLabel → {"x", "U"},
                 GridLines → Automatic,
                 GridLinesStyle → LightGray,
                 PlotStyle → Red
               ]

           }}],
        {t, 0, 100, 0.1}];

In[*]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[*]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.5: Code for animation

6.1.1.3 [337] both ends fixed, inital velocity zero (special case)

problem number 337

Added July 8,2019

Solve for $u(x,t)$ for $t>0$ and $0<x<L$

$$u_{tt}=c^2u_{xx}$$

With boundary condition both ends fixed

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

Using the following values

$$L = 10$$
$$c = 2$$
$$f(x) = \frac{8x(L-x)^2}{L^3}$$
$$g(x) = 0$$

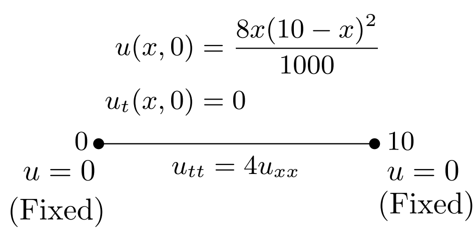


Figure 6.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=10; c=2; g=0; f=(8*x*(L-x)^2)/L^3;
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f, Derivative[0, 1][u][x, 0] == g};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{32(2+(-1)^n)\cos\left(\frac{n\pi t}{5}\right)\sin\left(\frac{n\pi x}{10}\right)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
L:=10;
c:=2;
g:=0;
f:=(8*x*(L-x)^2)/L^3;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,eval(diff(u(x,t),t),t=0)=g;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{32\left(\left(-1\right)^{n}+2\right)\cos\left(\frac{\pi n t}{5}\right)\sin\left(\frac{\pi n x}{10}\right)}{\pi^{3}n^{3}}$$

Hand solution

Solving the wave PDE on string with both ends fixed

$$u_{tt}=c^2u_{xx}\qquad t>0,x>0$$

With BC

$$\begin{aligned}u(0,t)&=0\\ u(L,t)&=0\end{aligned}$$

And initial conditions

$$\begin{aligned}u(x,0)&=f(x)=\frac{8x(L-x)^2}{L^3}\\ u_t(x,0)&=g(x)=0\end{aligned}$$

Using $c=2,L=10$.

The general problem PDE was solved in 6.1.1.1 on page 916 and the solution is

$$\begin{aligned}u(x,t)=&\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\\ &+\frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

Substituting the specific values given above into this solution gives

$$u(x,t)=\frac{1}{5}\sum_{n=1}^{\infty}\left(\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx\right)\cos\left(\frac{n\pi}{5}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

But $\int_0^{10} \frac{8x(10-x)^2}{10^3} \sin\left(\frac{n\pi}{10}x\right) dx = \frac{160(2+(-1)^n)}{n^3\pi^3}$, hence the solution becomes

$$u(x,t) = 32 \sum_{n=1}^{\infty} \frac{(2+(-1)^n)}{n^3\pi^3} \cos\left(\frac{n\pi}{5}t\right) \sin\left(\frac{n\pi}{10}x\right)$$

Animation is below

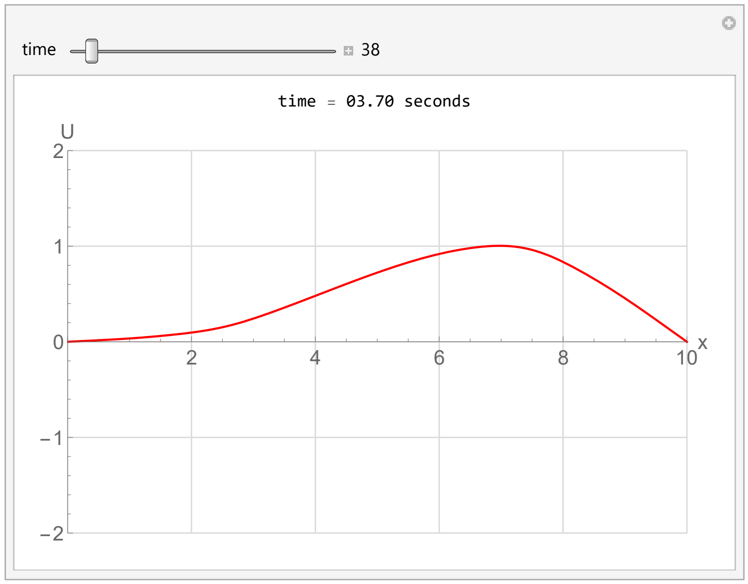


Figure 6.7: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 2;
L = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 15;
mySol[x_, t_] = 32 Sum[(2 + (-1)^n)/(n^3 π^3) Sin[n π/5 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 6.8: Source code

```
In[]:= tab = Table[
         Grid[{
             {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
             {
               Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-2, 2}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

             }}],
         {t, 0, 100, 0.1}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.9: Code for animation

6.1.1.4 [338] both ends fixed but domain is $-\pi \ldots \pi$. zero intial position, non zero initial velocity

problem number 338

Added sept 23 ,2019

Solve for $u(x,t)$ for $t>0$ and $-\pi<x<\pi$

$$u_{tt}=c^2u_{xx}$$

With boundary condition

$$\begin{aligned} u(-\pi,t)&=0\\ u(\pi,t)&=0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0)&=0\\ u_t(x,0)&=\sin(x)^2 \end{aligned}$$

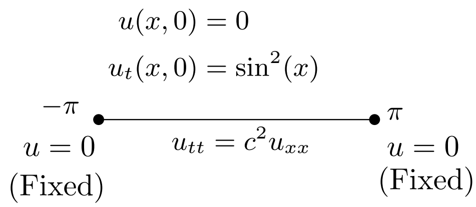


Figure 6.10: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == Sin[x]^2};
bc  = {u[-Pi, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{32\left(-1+(-1)^{K[1]}\right)\sin\left(\frac{1}{2}\sqrt{c^2}tK[1]\right)\sin\left(\frac{1}{2}(x+\pi)K[1]\right)}{\sqrt{c^2}\pi K[1]^2\left(K[1]^2-16\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(-Pi,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=sin(x)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=-\frac{32\left(-315\pi c\left(\sum_{n=5}^{\infty}\frac{\left((-1)^n-1\right)\sin\left(\frac{(x+\pi)n}{2}\right)\sin\left(\frac{cnt}{2}\right)}{\pi(n^2-16)c\,n^2}\right)-42\cos\left(\frac{x}{2}\right)\sin\left(\frac{ct}{2}\right)+10\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3ct}{2}\right)\right)}{315\pi c}$$

Hand solution

Solve

$$u_{tt}=c^2u_{xx}$$

With BC

$$
\begin{aligned}
u(-\pi, t) &= 0 \\
u(\pi, t) &= 0
\end{aligned}
$$

And initial conditions

$$
\begin{aligned}
u(x, 0) &= 0 \\
u_t(x, 0) &= \sin^2(x)
\end{aligned}
$$

Let $\xi = x + \pi$. When $x = -\pi, \xi = 0$ and when $x = \pi, \xi = 2\pi$. In terms of ξ, the new pde in $U(\xi, t)$ becomes

$$
U_{tt} = c^2 U_{\xi\xi}
$$

With BC

$$
\begin{aligned}
U(0, t) &= 0 \\
U(2\pi, t) &= 0
\end{aligned}
$$

And initial conditions

$$
\begin{aligned}
U(\xi, 0) &= 0 \\
U_t(\xi, 0) &= \sin^2(\xi)
\end{aligned}
$$

Let $U = X(\xi)T(t)$. The PDE becomes

$$
\begin{aligned}
\frac{T''X}{c^2} &= X''T \\
\frac{1}{c^2}\frac{T''}{T} &= \frac{X''}{X} = -\lambda
\end{aligned}
$$

Where λ is separation constant. Hence the eigenvalue ODE is

$$
\begin{aligned}
X'' + \lambda X &= 0 \\
X(0) &= 0 \\
X(2\pi) &= 0
\end{aligned}
$$

From the boundary conditions, we see that $\lambda > 0$ is the only possible value. Therefore the solution to the above ODE is

$$
X(x) = A\cos\left(\sqrt{\lambda}\xi\right) + B\sin\left(\sqrt{\lambda}\xi\right)
$$

Since $X(0) = 0$ then $A = 0$ and the solution becomes $X(\xi) = B\sin\left(\sqrt{\lambda}\xi\right)$. Since $X(2\pi) = 0$ then for non trivial solution we want $\sqrt{\lambda}2\pi = n\pi$ or

$$\lambda_n = \left(\frac{n}{2}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence the eigenfunctions are

$$X_n(\xi) = \sin\left(\frac{n}{2}\xi\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE now becomes

$$\begin{aligned} T_n'' + c^2\lambda_n T_n &= 0 \\ T_n'' + c^2\left(\frac{n}{2}\right)^2 T_n &= 0 \\ T_n'' + \frac{c^2n^2}{4} T_n &= 0 \end{aligned}$$

Which has the solution

$$T_n(t) = D_n \cos\left(\frac{cn}{2}t\right) + E_n \sin\left(\frac{cn}{2}t\right)$$

Therefore the complete solution becomes

$$U(\xi, t) = \sum_{n=1}^{\infty} \left(D_n \cos\left(\frac{cn}{2}t\right) + E_n \sin\left(\frac{cn}{2}t\right)\right) \sin\left(\frac{n}{2}\xi\right) \tag{1}$$

Switching back to x the above becomes

$$u(x, t) = \sum_{n=1}^{\infty} \left(D_n \cos\left(\frac{cn}{2}t\right) + E_n \sin\left(\frac{cn}{2}t\right)\right) \sin\left(\frac{n}{2}(x + \pi)\right) \tag{1A}$$

At $t = 0$ the above becomes

$$\begin{aligned} 0 &= \sum_{n=1}^{\infty} D_n \sin\left(\frac{n}{2}(x + \pi)\right) \\ D_n &= 0 \end{aligned}$$

The solution (1A) simplifies to

$$u(x, t) = \sum_{n=1}^{\infty} E_n \sin\left(\frac{cn}{2}t\right) \sin\left(\frac{n}{2}(x + \pi)\right) \tag{2}$$

Taking time derivative of (2) gives

$$u_t(x,t) = \sum_{n=1}^{\infty}\left(E_n \frac{cn}{2}\cos\left(\frac{cn}{2}t\right)\right)\sin\left(\frac{n}{2}(x+\pi)\right)$$

At $t=0$ the above becomes

$$\sin^2(x) = \sum_{n=1}^{\infty} E_n \frac{cn}{2}\sin\left(\frac{n}{2}(x+\pi)\right)$$

Applying orthogonality gives

$$\begin{aligned}\int_{-\pi}^{\pi}\sin^2(x)\sin\left(\frac{n}{2}(x+\pi)\right)dx &= E_n\frac{cn}{2}\int_{-\pi}^{\pi}\sin^2\left(\frac{n}{2}(x+\pi)\right)dx\\ &= E_n\frac{\pi cn}{2}\end{aligned}$$

For the LHS, for n even $\int_{-\pi}^{\pi}\sin^2(x)\sin\left(\frac{n}{2}(x+\pi)\right)dx = 0$. Hence $E_n = 0$ for all n even. For n odd

$$\begin{aligned}\int_{-\pi}^{\pi}\sin^2(x)\sin\left(\frac{n}{2}(x+\pi)\right)dx &= \frac{16(\cos(n\pi)-1)}{(n^2-16)\,n}\\ &= \frac{16((-1)^n-1)}{(n^2-16)\,n}\end{aligned}$$

But n is odd, hence the above simplifies more to

$$\int_{-\pi}^{\pi}\sin^2(x)\sin\left(\frac{n}{2}(x+\pi)\right)dx = \frac{-32}{(n^2-16)\,n}$$

Therefore

$$\begin{aligned}E_n &= \frac{2}{n\pi c}\frac{-32}{(n^2-16)\,n}\\ &= \frac{-64}{n^2\pi c\,(n^2-16)}\end{aligned}$$

Therefore the final solution (2) now becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty}\frac{-64}{n^2\pi c\,(n^2-16)}\sin\left(\frac{cn}{2}t\right)\sin\left(\frac{n}{2}(x+\pi)\right)$$

The following is an animation of the solution for $c=2$. Using $c=2$ then the solution above becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{-32}{n^2\pi\left(n^2-16\right)} \sin\left(nt\right) \sin\left(\frac{n}{2}(x+\pi)\right)$$

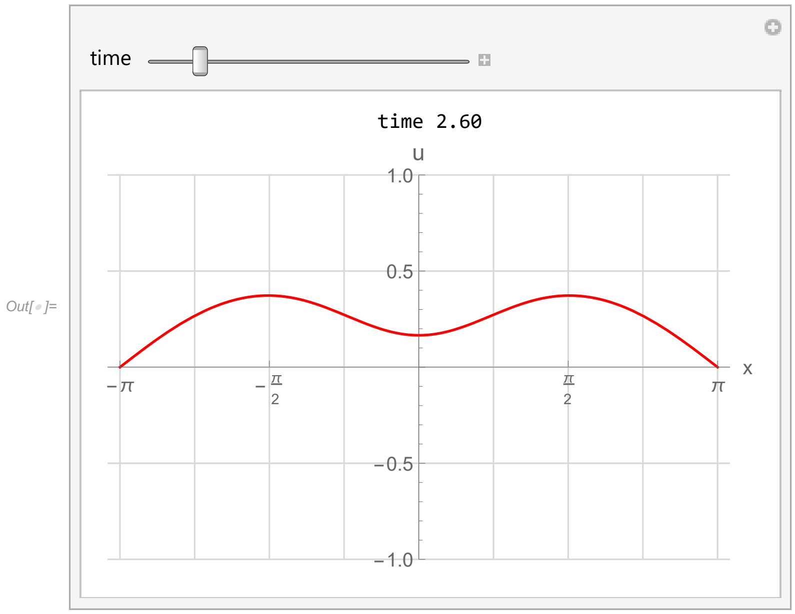


Figure 6.11: snap shot

Source code used for the above

```
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_, max_] := Sum[ -32/(n^2 π (n^2 - 16)) Sin[n t] Sin[n/2 (x + π)], {n, 1, max, 2}]
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
        {
          Quiet@Plot[u[x, time, 7], {x, -Pi, Pi},
                PlotRange → {All, {-1, 1}},
                AxesLabel → {Style["x", 12], Style["u", 14]},
                BaseStyle → 12,
                ImageSize → 400, PlotStyle → Red,
                GridLines → {Range[-Pi, Pi, Pi / 4], Automatic},
                GridLinesStyle → LightGray,
                Ticks → {Range[-Pi, Pi, Pi / 2], Automatic}
              ]
        }}
  ]
  ,
  {{time, 0, "time"}, 0, 20, .1},
  TrackedSymbols :→ {time}
]
```

Figure 6.12: Source code 2D

6.1.1.5 [339] both ends fixed but domain is $-1 \ldots 1$. intial position is an impulse, zero initial velocity

problem number 339

Added January 8 ,2020

Problem 6.3.31 Introduction to Partial Dfferential Equations by Peter Olver, ISBN 9783319020983.

Solve

$$
\begin{aligned}
u_{tt} &= u_{xx} \\
u(-1,t) &= 0 \\
u(1,t) &= 0 \\
u(x,0) &= \delta(x) \\
\frac{\partial u(x,0)}{\partial t} &= 0
\end{aligned}
$$

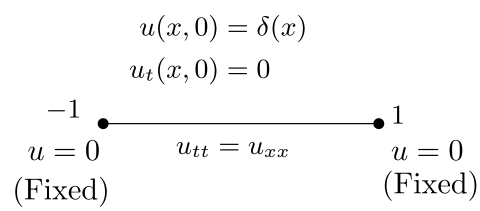


Figure 6.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
bc = {u[-1, t] == 0, u[1, t] == 0};
ic = {u[x, 0] == DiracDelta[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \cos\left(\frac{1}{2}\pi t K[1]\right) \sin\left(\frac{1}{2}\pi K[1]\right) \sin\left(\frac{1}{2}\pi(x+1)K[1]\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=diff(u(x,t),x$2);
bc:=u(-1,t)=0,u(1,t)=0;
ic  := u(x,0)=Dirac(x), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\cos\left(\frac{\pi n t}{2}\right)\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi(x+1)\,n}{2}\right)$$

Hand solution

Since the boundary conditions are at $x=-1$ and at $x=1$, it is a little easier to solve this by first shifting the boundaries to $x=0$ and $x=2$. This is done by transformation. Let

$$z=x+1$$

When $x=-1$ then $z=0$ and when $x=1$ then $z=2$. The PDE in terms of z remains the same but the B.C. are shifted. Hence we want to solve for $v(z,t)$ in

$$\begin{aligned} v_{tt}&=v_{zz}\\ v(0,t)&=0\\ v(2,t)&=0 \end{aligned}$$

No need to worry about initial conditions now, since we will transform back to x before applying initial conditions and therefore will use the original initial conditions. This PDE is now solved by separation. Let $v=Z(z)T(t)$. Substituting into the PDE gives

$$\begin{aligned} T''Z&=Z''T\\ \frac{T''}{T}&=\frac{Z''}{Z}=-\lambda \end{aligned}$$

This gives the boundary value ODE

$$\begin{aligned} Z''+\lambda Z&=0 \qquad (1)\\ Z(0)&=0\\ Z(2)&=0 \end{aligned}$$

And the time ODE

$$T''+\lambda T=0 \qquad (2)$$

Solving (1). From the boundary conditions we know only $\lambda > 0$ is an eigenvalue. Hence for $\lambda > 0$ the solution is

$$Z(z) = A\cos\left(\sqrt{\lambda}z\right) + B\sin\left(\sqrt{\lambda}z\right)$$

At $z = 0$ this gives $A = 0$. Hence the solution now becomes $Z(z) = B\sin\left(\sqrt{\lambda}z\right)$. At $z = 2$ the above gives $0 = B\sin\left(2\sqrt{\lambda}\right)$. For non-trivial solution we want $\sin\left(2\sqrt{\lambda}\right) = 0$ which implies $2\sqrt{\lambda} = n\pi$ or

$$\lambda_n = \left(\frac{n\pi}{2}\right)^2 \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \sin\left(\frac{n\pi}{2}z\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE (2) now becomes

$$T'' + \left(\frac{n\pi}{2}\right)^2 T = 0$$

Which has solution

$$T_n(t) = A_n \cos\left(\frac{n\pi}{2}t\right) + B_n \sin\left(\frac{n\pi}{2}t\right)$$

Hence the complete solution is

$$v(z,t) = \sum_{n=1}^{\infty} \left(A_n \cos\left(\frac{n\pi}{2}t\right) + B_n \sin\left(\frac{n\pi}{2}t\right)\right) \sin\left(\frac{n\pi}{2}z\right)$$

We are now ready to switch back from z to x. Since $z = x + 1$ then the above becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos\left(\frac{n\pi}{2}t\right) + B_n \sin\left(\frac{n\pi}{2}t\right)\right) \sin\left(\frac{n\pi}{2}(x+1)\right) \tag{3}$$

Now we apply initial conditions to find A_n, B_n. At $t = 0, u(x,0) = \delta(x)$. Hence the above gives

$$\delta(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{2}(x+1)\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2}(x+1)\right)$ and Integrating gives

$$\int_{-1}^{1} \delta(x) \sin\left(\frac{m\pi}{2}(x+1)\right) dx = \sum_{n=1}^{\infty} A_n \int_{-1}^{1} \sin\left(\frac{n\pi}{2}(x+1)\right) \sin\left(\frac{m\pi}{2}(x+1)\right) dx$$

By orthogonality of sin functions only term survives and the above simplifies to

$$\begin{aligned}\int_{-1}^{1} \delta(x) \sin\left(\frac{m\pi}{2}(x+1)\right) dx &= A_m \overbrace{\int_{-1}^{1} \sin^2\left(\frac{m\pi}{2}(x+1)\right) dx}^{1} \\ &= A_m\end{aligned}$$

But $\int_{-1}^{1} \delta(x) \sin\left(\frac{m\pi}{2}(x+1)\right) dx = \sin\left(\frac{m\pi}{2}\right)$ since that is where $x=0$. The above reduces to

$$A_n = \sin\left(\frac{n\pi}{2}\right) \qquad n = 1,2,3,\cdots$$

The solution (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2}t\right) + B_n \sin\left(\frac{n\pi}{2}t\right)\right) \sin\left(\frac{n\pi}{2}(x+1)\right) \tag{4}$$

Taking time derivatives

$$\frac{\partial}{\partial t}u(x,t) = \sum_{n=1}^{\infty} \left(-\frac{n\pi}{2}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}t\right) + \frac{n\pi}{2}B_n \cos\left(\frac{n\pi}{2}t\right)\right) \sin\left(\frac{n\pi}{2}(x+1)\right)$$

At $t=0$ the above becomes

$$0 = \sum_{n=1}^{\infty} \frac{n\pi}{2} B_n \sin\left(\frac{n\pi}{2}(x+1)\right)$$

Therefore $B_n = 0$. Hence the solution (4) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2}t\right)\sin\left(\frac{n\pi}{2}(x+1)\right) \tag{5}$$

Notice that $\sin\left(\frac{n\pi}{2}\right)$ is zero when n is even.

6.1.1.6 [340] Logan book, page 28. Both ends fixed

problem number 340

This is problem at page 28, David J Logan textbook, applied PDE textbook. No initial conditions given

$$u_{tt} = c^2 u_{xx}$$

With boundary condition

$$\begin{aligned}u(0,t) &= 0\\ u(L,t) &= 0\end{aligned}$$

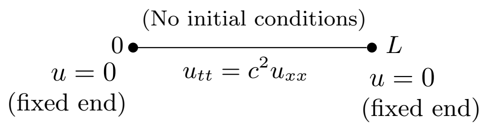


Figure 6.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}, Assumptions -> {L >
```

$$\left\{\left\{u(x,t) \to \left\{ \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}} \sin\left(\frac{\pi x K[1]}{L}\right) \left(\cos\left(\frac{\pi t|c|K[1]}{L}\right) \int_0^L \sqrt{2}\sqrt{\frac{1}{L}} \sin\left(\frac{\pi x K[1]}{L}\right) u(x,0)\, dx + \frac{L\left(\int_0^L \frac{\sqrt{2}\,\mathrm{s}}{}\right.}{} \right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) assuming
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\_F1(n) \sin\left(\frac{\pi c n t}{L}\right) + \_F2(n) \cos\left(\frac{\pi c n t}{L}\right)\right) \sin\left(\frac{\pi n x}{L}\right)$$

6.1.1.7 [341] non-zero initial velocity. Both ends fixed

problem number 341

Added Feb 25, 2019. Exam 1 problem, MATH 4567 Applied Fourier Analysis, University of Minnesota, Twin Cities.

Solve for $u(x,t)$

$$u_{tt} = u_{xx} - u$$

With boundary condition

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 1$$

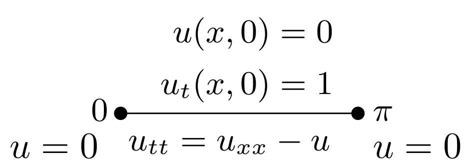


Figure 6.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] - u[x, t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 1};
bc  = {u[0, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \sum_{K[1]=1}^{\infty} \frac{2\left(1+(-1)^{K[1]+1}\right)\sin(xK[1])\sin\left(t\sqrt{K[1]^2+1}\right)}{\sqrt{\pi}K[1]\sqrt{\pi K[1]^2+\pi}} & K[1] \in \mathbb{Z} \land K[1] \geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=diff(u(x,t),x$2)-u(x,t);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(-\frac{2((-1)^n - 1) \sin(nx) \sin\left(\sqrt{n^2+1}\, t\right)}{\pi \sqrt{n^2+1}\, n} \right)$$

6.1.1.8 [342] Logan book page 149)

problem number 342

This is problem at page 149, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + p(x,t)$$

With boundary conditions

$$u(\pi, 0) = 0$$
$$u(0, t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x, 0) = 0$$
$$u(x, 0) = 0$$

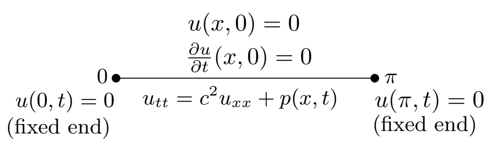


Figure 6.16: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + p[x, t];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\left\{\begin{array}{cc}\sum_{K[1]=1}^{\infty}\sqrt{\frac{2}{\pi}}\left(\int_0^t\frac{\left(\int_0^\pi\sqrt{\frac{2}{\pi}}p(x,K[2])\sin(xK[1])\,dx\right)\sin\left(\sqrt{c^2K[1]^2}(t-K[2])\right)}{\sqrt{c^2K[1]^2}}dK[2]\right)\sin(xK[1]) & K[1]\in \\ \text{Indeterminate} & \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^\pi p(x,\tau)\sin(nx)\,dx\right)\sin(nx)\sin((t-\tau)\,cn)}{\pi cn}\right)d\tau$$

6.1.1.9 [343] Haberman 8.5.2 (a)

problem number 343

Added Nov 25, 2018.

This is problem 8.5.2 (a), Richard Haberman applied partial differential equations book, 5th edition

Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space.

Consider a vibrating string with time-dependent forcing:

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+Q(x,t)$$

With boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

my hand solution in in the file **feb_24_2019_4_24_pm.tex**, but I need to go over my solution again to make sure it is correct.

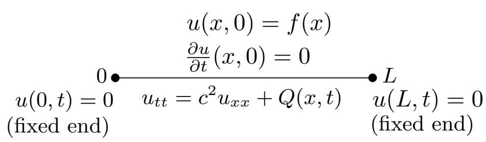


Figure 6.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}} \left(\cos\left(\pi t \sqrt{\frac{c^2 K[1]^2}{L^2}}\right) \int_0^L \sqrt{2}\sqrt{\frac{1}{L}} f(x) \sin\left(\frac{\pi x K[1]}{L}\right) dx + \int_0^t \frac{\left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}} Q(x,K[2]) \right.}{} \right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+Q(x,t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x), eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \int_0^t \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi n x}{L}\right) dx\right)\sin\left(\frac{\pi n x}{L}\right)\sin\left(\frac{\pi(t-\tau)cn}{L}\right)}{\pi c n} \right) d\tau + \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right) dx \right.}{L} \right.$$

6.1.1.10 [344] Haberman 8.5.2 (b)

problem number 344

Added Nov 25, 2018.

This is problem 8.5.2 (b), Richard Haberman applied partial differential equations book, 5th edition.

Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space.

Consider a vibrating string with time-dependent forcing:

$$u_{tt} = c^2 u_{xx} + g(x)\cos(\omega t)$$

With boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u_t(x,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

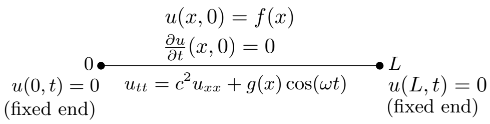


Figure 6.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + g[x]*Cos[w*t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}}\left(\cos\left(\pi t\sqrt{\frac{c^2K[1]^2}{L^2}}\right)\int_0^L \sqrt{2}\sqrt{\frac{1}{L}}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)dx + \int_0^t \frac{\left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}}\cos(wK[2\right.}{}\right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+ g(x)*cos(w*t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \frac{-2\left(\sum_{n=1}^{\infty} \frac{\pi L c\left(\int_0^L g(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos(tw)\sin\left(\frac{\pi n x}{L}\right)}{L^2w^2-\pi^2c^2n^2}\right) + 2\left(\sum_{n=1}^{\infty} \frac{\pi L c\left(\int_0^L g(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos\left(\frac{\pi c n t}{L}\right)\sin\left(\frac{\pi n x}{L}\right)}{L^2w^2-\pi^2c^2n^2}\right)}{\pi c}$$

Hand solution

Let

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$$

Where we used $=$ instead of $\sim$ above, since the PDE given has homogeneous B.C. We know that $\phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$ for $n = 1, 2, 3, \cdots$ where $\lambda_n = \left(\frac{n\pi}{L}\right)^2$. Substituting the above in the given PDE gives

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = c^2 \sum_{n=1}^{\infty} A_n(t)\,\frac{d^2\phi_n(x)}{dx^2} + Q(x,t)$$

But $Q(x,t) = \sum_{n=1}^{\infty} q_n(t)\,\phi_n(x)$, hence the above becomes

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = c^2 \sum_{n=1}^{\infty} A_n(t)\,\frac{d^2\phi_n(x)}{dx^2} + \sum_{n=1}^{\infty} g_n(t)\,\phi_n(x)$$

But $\frac{d^2\phi_n(x)}{dx^2} = -\lambda_n\phi_n(x)$, hence

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = -c^2 \sum_{n=1}^{\infty} \lambda_n A_n(t)\,\phi_n(x) + \sum_{n=1}^{\infty} g_n(t)\,\phi_n(x)$$

Multiplying both sides by $\phi_m(x)$ and integrating gives

$$\int_0^L \sum_{n=1}^{\infty} A_n''(t)\,\phi_m(x)\,\phi_n(x)\,dx = -c^2 \int_0^L \sum_{n=1}^{\infty} \lambda_n A_n(t)\,\phi_m(x)\,\phi_n(x)\,dx + \int_0^L \sum_{n=1}^{\infty} g_n(t)\,\phi_m(x)\,\phi_n(x)\,dx$$

$$A_n''(t)\int_0^L \phi_n^2(x)\,dx = -c^2\lambda_n A_n(t)\int_0^L \phi_n^2(x)\,dx + g_n(t)\int_0^L \phi_n^2(x)\,dx$$

Hence

$$A_n''(t) + c^2\lambda_n A_n(t) = g_n(t)$$

Now we solve the above ODE. Let solution be

$$A_n(t) = A_n^h(t) + A_n^p(t)$$

Which is the sum of the homogenous and particular solutions. The homogenous solution is

$$A_n^h(t) = c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right)$$

And the particular solution depends on $q_n(t)$. Once we find $q_n(t)$, we plug-in everything back into $u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$ and then use initial conditions to find c_{1_n}, c_{2_n}, the two constant of integrations. Now we are given that $Q(x,t) = g(x)\cos(\omega t)$. Hence

$$g_n(t) = \frac{\int_0^L Q(x,t)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \frac{\cos(\omega t)\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \cos(\omega t)\,\gamma_n$$

Where

$$\gamma_n = \frac{\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx}$$

is constant that depends on n. Now we use the above in result found in part (a)

$$A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n \cos(\omega t) \tag{1}$$

We know the homogenous solution from part (a).

$$A_n^h(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n} t\right)$$

We now need to find the particular solution. Will solve using method of undetermined coefficients.

Case 1 $\omega \neq c\sqrt{\lambda_n}$ (no resonance)

We can now guess

$$A_n^p(t) = z_1 \cos(\omega t) + z_2 \sin(\omega t)$$

Plugging this back into (1) gives

$$\begin{aligned}
(z_1 \cos(\omega t) + z_2 \sin(\omega t))'' + c^2\lambda_n (z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t) \\
(-\omega z_1 \sin(\omega t) + \omega z_2 \cos(\omega t))' + c^2\lambda_n (z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t) \\
-\omega^2 z_1 \cos(\omega t) - \omega^2 z_2 \sin(\omega t) + c^2\lambda_n (z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t)
\end{aligned}$$

Collecting terms

$$\cos(\omega t)\left(-\omega^2 z_1 + c^2\lambda_n z_1\right) + \sin(\omega t)\left(-\omega^2 z_2 + c^2\lambda_n z_2\right) = \gamma_n \cos(\omega t)$$

Therefore we obtain two equations in two unknowns

$$\begin{aligned}
-\omega^2 z_1 + c^2\lambda_n z_1 &= \gamma_n \\
-\omega^2 z_2 + c^2\lambda_n z_2 &= 0
\end{aligned}$$

From the second equation, $z_2 = 0$ and from the first equation

$$\begin{aligned}
z_1\left(c^2\lambda_n - \omega^2\right) &= \gamma_n \\
z_1 &= \frac{\gamma_n}{c^2\lambda_n - \omega^2}
\end{aligned}$$

Hence

$$\begin{aligned}
A_n^p(t) &= z_1 \cos(\omega t) + z_2 \sin(\omega t) \\
&= \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Therefore

$$\begin{aligned}
A_n(t) &= A_n^h(t) + A_n^p(t) \\
&= c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Now we need to find c_{1_n}, c_{2_n}. Since

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x) \\
&= \sum_{n=1}^{\infty} \left(c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t) \right) \sin\left(\frac{n\pi}{L}x\right)
\end{aligned}
$$

At $t=0$ the above becomes

$$
\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} \left(c_{1_n} + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \right) \sin\left(\frac{n\pi}{L}x\right) \\
&= \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L}x\right) + \sum_{n=1}^{\infty} \frac{\gamma_n}{c^2\lambda_n - \omega^2} \sin\left(\frac{n\pi}{L}x\right)
\end{aligned}
$$

Applying orthogonality

$$
\begin{aligned}
\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx &= \int_0^L \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right) dx + \int_0^L \sum_{n=1}^{\infty} \frac{\gamma_n}{c^2\lambda_n - \omega^2} \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right) \\
\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx &= c_{1_n} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx &= c_{1_n} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx \\
c_{1_n} &= \frac{\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx}{\int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx} - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \\
&= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2}
\end{aligned}
$$

We now need to find c_{2_n}. For this we need to differentiate the solution once.

$$
\frac{\partial u(x,t)}{\partial t} = \sum_{n=1}^{\infty} \left(-c\sqrt{\lambda_n} c_{1_n} \sin\left(c\sqrt{\lambda_n}t\right) + c\sqrt{\lambda_n} c_{2_n} \cos\left(c\sqrt{\lambda_n}t\right) - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \omega \sin(\omega t) \right) \sin\left(\frac{n\pi}{L}x\right)
$$

Applying initial conditions $\frac{\partial u(x,0)}{\partial t} = 0$ gives

$$
0 = \sum_{n=1}^{\infty} c\sqrt{\lambda_n} c_{2_n} \sin\left(\frac{n\pi}{L}x\right)
$$

Hence

$$
c_{2_n} = 0
$$

Therefore the final solution is

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2}\cos(\omega t)$$

And

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\sin\left(\frac{n\pi}{L}x\right)$$

Where

$$c_{1_n} = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2}$$

Case 2 $\omega = c\sqrt{\lambda_n}$ Resonance case. Now we can't guess $A_n^p(t) = z_1\cos(\omega t) + z_2\sin(\omega t)$ so we have to use

$$A_n^p(t) = z_1 t\cos(\omega t) + z_2 t\sin(\omega t)$$

Substituting this in $A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n\cos(\omega t)$ gives

$$(z_1 t\cos(\omega t) + z_2 t\sin(\omega t))'' + c^2\lambda_n(z_1 t\cos(\omega t) + z_2 t\sin(\omega t)) = \gamma_n\cos(\omega t) \tag{2}$$

But

$$\begin{aligned}
(z_1 t\cos(\omega t) + z_2 t\sin(\omega t))'' &= (z_1\cos(\omega t) - z_1\omega t\sin(\omega t) + z_2\sin(\omega t) + z_2\omega t\cos(\omega t))' \\
&= -z_1\omega\sin(\omega t) - \left(z_1\omega\sin(\omega t) + z_1\omega^2 t\cos(\omega t)\right) \\
&+ z_2\omega\cos(\omega t) + \left(z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t)\right) \\
&= -2z_1\omega\sin(\omega t) - z_1\omega^2 t\cos(\omega t) + 2z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t)
\end{aligned}$$

Hence (2) becomes

$$-2z_1\omega\sin(\omega t) - z_1\omega^2 t\cos(\omega t) + 2z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t) + c^2\lambda_n(z_1 t\cos(\omega t) + z_2 t\sin(\omega t)) = \gamma_n\cos(\omega t$$

Comparing coefficients we see that $2z_2\omega = \gamma_n$ or

$$z_2 = \frac{\gamma_n}{2\omega}$$

And $z_1 = 0$. Therefore

$$A_n^p(t) = \frac{\gamma_n}{2\omega}t\sin(\omega t)$$

Therefore

$$\begin{aligned}
A_n(t) &= A_n^h(t) + A_n^p(t) \\
&= c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}}t\sin(\omega t)
\end{aligned}$$

We now can find c_{1_n}, c_{2_n} from initial conditions.

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\, \phi_n(x) \\
&= \sum_{n=1}^{\infty} \left(c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t) \right) \sin\left(\frac{n\pi}{L} x\right) \quad (4)
\end{aligned}
$$

At $t = 0$

$$
\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L} x\right) \\
c_{1n} &= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx
\end{aligned}
$$

Taking time derivative of (4) and setting it to zero will give c_{2n}. Since initial speed is zero then $c_{2_n} = 0$. Hence

$$
A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)
$$

This completes the solution.

Summary of solution

The solution is given by

$$
u(x,t) = \sum_{n=1}^{\infty} A_n(t)\, \phi_n(x)
$$

Case $\omega \neq c\sqrt{\lambda_n}$

$$
A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{c^2 \lambda_n - \omega^2} \cos(\omega t)
$$

And

$$
c_{1_n} = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{m\pi}{L} x\right) dx - \frac{\gamma_n}{c^2 \lambda_n - \omega^2}
$$

And

$$
\gamma_n = \frac{\int_0^L g(x)\, \phi_n(x)\, dx}{\int_0^L \phi_n^2(x)\, dx}
$$

And $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3,$

Case $\omega = c\sqrt{\lambda_n}$ (resonance)

$$
A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)
$$

And

$$
c_{1_n} = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx
$$

6.1.1.11 [345] Both I.C. not zero

problem number 345

Added July 2, 2018.

Taken from Maple 2018.1 improvements to PDE's document. Solve

$$v_{tt} = v_{xx}$$

For $t > 0$ and $0 < x < 1$. With boundary conditions

$$v(0,t) = 0$$
$$v(1,0) = 0$$

With initial conditions

$$v(x,0) = f(x)$$
$$\frac{\partial v}{\partial t}(x,0) = g(x)$$

Where $f(x) = -\frac{e^{2}x - e^{x+1} - x + e^{1-x}}{e^{2}-1}$ and $g(x) = 1 + \frac{e^{2}x - e^{x+1} - x + e^{1-x}}{e^{2}-1}$

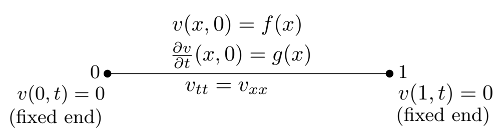


Figure 6.19: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[v[x, t], {t, 2}] == D[v[x, t], {x, 2}];
bc  = {v[0, t] == 0, v[1, t] == 0};
ic  = {v[x, 0] == -((Exp[2]*x - Exp[x + 1] - x + Exp[1 - x])/(Exp[2] - 1)), Derivative[0, 1]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, v[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{v(x,t) \to \sum_{n=1}^{\infty} -\frac{2\left((-1)^{n+1} n\pi \cos(n\pi t) + \left(\left(-1+(-1)^{n}\right)\pi^{2}n^{2} + 2(-1)^{n} - 1\right)\sin(n\pi t)\right)\sin(n\pi x)}{\pi^{4}n^{4} + \pi^{2}n^{2}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(v(x, t), t, t)=(diff(v(x, t), x, x));
bc  := v(0, t) = 0, v(1, t) = 0;
ic  := v(x, 0) = -(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1),
                (D[2](v))(x, 0) = 1+(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],v(x,t))),outp
```

$$v(x,t) = \sum_{n=1}^{\infty}\left(-\frac{2(-\pi n(-1)^{n}\cos(\pi n t) + (\pi^{2}n^{2}(-1)^{n} - \pi^{2}n^{2} + 2(-1)^{n} - 1)\sin(\pi n t))\sin(\pi n x)}{\pi^{2}(\pi^{2}n^{2}+1)\,n^{2}}\right)$$

6.1.1.12 [346] With constant source

problem number 346

Added July 2, 2018.

Third example, from Maple 2018.1 improvements to PDE's document. What_is_New_after_Maple_2018.pdf

Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} + 1$$

For $t > 0$ and $0 < x < L$. With boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

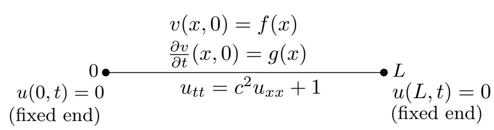


Figure 6.20: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + 1;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\left\{\sum_{K[1]=1}^{\infty}\boxed{\sqrt{2}\sqrt{\frac{1}{L}}\sin\left(\frac{\pi xK[1]}{L}\right)\left(\frac{\sqrt{2}\left(1+(-1)^{K[1]+1}\right)\left(L-L\cos\left(\frac{c\pi tK[1]}{L}\right)\right)L^{3/2}}{c^2\pi^3K[1]^3}+\frac{\left(\int_0^L\frac{\sqrt{2}g(x)\sin\left(\frac{\pi xK[1]}{L}\right)}{\sqrt{L}}dx\right.}{\pi|c||K}\right.}\right.\right.\right.$$

Indeter

Maple ✓

```
restart;
interface(showassumed=0);
f='f';
pde :=diff(u(x, t), t, t) = c^2* diff(u(x, t), x, x) + 1;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x), (D[2](u))(x, 0) = g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t)=\left(\sum_{n=1}^{\infty}\frac{\left(2Lc\left(\int_0^L g(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi cnt}{L}\right)-\pi n\left(\int_0^L\left(-2c^2f(x)+Lx-x^2\right)\sin\left(\frac{\pi nx}{L}\right)dx\right)\cos\left(\right.\right.}{\pi L c^2 n}\right.$$

6.1.1.13 [347] Logan page 213

problem number 347

This is problem at page 213, David J Logan textbook, applied PDE textbook. Both ends fixed end, with source.

$$u_{tt} = c^2 u_{xx} + Ax$$

With boundary conditions

$$u(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

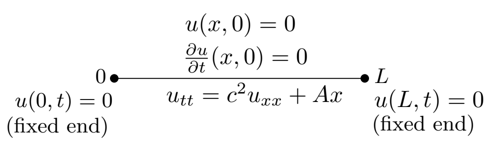


Figure 6.21: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + A*x;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \quad -\frac{iAL^3\left(\text{PolyLog}\left(3,-e^{-\frac{i\pi(ct-x)}{L}}\right)-\text{PolyLog}\left(3,-e^{\frac{i\pi(ct-x)}{L}}\right)+2\,\text{PolyLog}\left(3,-e^{-\frac{i\pi x}{L}}\right)-2\,\text{PolyLog}\left(3,-e^{\frac{i\pi x}{L}}\right)-\text{Po}\right.}{2c^2\pi^3} \right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))  assu
```

$$u(x,t) = \frac{A\,L^2 x}{6c^2} - \frac{A\,x^3}{6c^2} + \left(\sum_{n=1}^{\infty} \frac{2A\,L^3(-1)^n \cos\left(\frac{\pi c n t}{L}\right) \sin\left(\frac{\pi n x}{L}\right)}{\pi^3 c^2 n^3}\right)$$

6.1.1.14 [348] Telegraphy PDE

problem number 348

Both ends fixed with damping Solve

$$u_{tt} + 2u_t = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

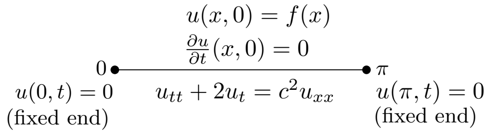


Figure 6.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] + 2*D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \sum_{K[1]=1}^{\infty} e^{-t}\sqrt{\frac{2}{\pi}}\sin(xK[1])\left(\cos\left(\frac{1}{2}t\sqrt{4K[1]^2-4}\right)\int_0^{\pi}\sqrt{\frac{2}{\pi}}f(x)\sin(xK[1])\,dx + \frac{\sin\left(\frac{1}{2}t\sqrt{4K\right.}}{} \right. \right. \right. \right. \quad \text{Indeterminate}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+2*diff(u(x,t),t)=diff(u(x,t),x$2);
ic  :=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=f(x);
bc  := u(0,t)=0,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \sum_{n=1}^{\infty} \begin{cases} \frac{2(t+1)e^{-t}\left(\int_0^{\pi} f(x)\sin(x)dx\right)\sin(x)}{\pi} & n=1 \\ -\frac{\left(\left(i-\sqrt{n^2-1}\right)e^{\left(i\sqrt{n^2-1}-1\right)t}+\left(-i-\sqrt{n^2-1}\right)e^{-\left(i\sqrt{n^2-1}+1\right)t}\right)\left(\int_0^{\pi} f(x)\sin(nx)dx\right)\sin(nx)}{\sqrt{n^2-1}\,\pi} & \text{otherwise} \end{cases}$$

6.1.1.15 [349] Dispersion term present (general case)

problem number 349

Added July 12, 2019.

Solve

$$u_{tt} + \gamma^2 u(x,t) = c^2 u_{xx}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time.

With $0 < x < L$ and $t > 0$ and with boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

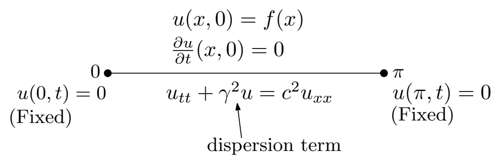


Figure 6.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2 D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t},Assumptions->L>0
```

$$\left\{\left\{u(x,t) \to \left\{ \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}} \cos\left(t\sqrt{c^2\left(\frac{\gamma^2}{c^2} + \frac{\pi^2 K[1]^2}{L^2}\right)}\right)\left(\int_0^L \frac{\sqrt{2} f(x) \sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\, dx\right) \sin\left(\frac{\pi x K[1]}{L}\right) \quad K[1] \right.\right.\right.$$

Indeterminate

Due to adding dispersion term

Maple ✓

```
restart;
interface(showassumed=0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_{0}^{L}f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos\left(\frac{\sqrt{\pi^{2}c^{2}n^{2}+\gamma^{2}L^{2}}\,t}{L}\right)\sin\left(\frac{\pi n x}{L}\right)}{L}$$

Hand solution

Solving for $t>0, 0<x<L$

$$\frac{\partial^2 u}{\partial t^2}+\gamma^2 u=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<L, t>0$$

With BC

$$u(0,t)=0$$
$$u(L,t)=0$$

And initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=0$$

Dispersion term $\gamma^2 u$ causes the shape of the original wave to distort with time. Using separation of variables, Let $u=X(x)T(t)$. Substituting this back in the PDE gives

$$T''X+\gamma^2 XT=c^2X''T$$
$$\frac{1}{c^2}\left(\frac{T''}{T}+\gamma^2\right)=\frac{X''}{X}=-\lambda$$

The eigenvalue ODE is

$$X''+\lambda X=0$$
$$X(0)=0$$
$$X(L)=0$$

The eigenvalues are $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and the eigenfunctions are $X_n(x) = c_n \sin\left(\frac{n\pi}{L}x\right)$. The time ODE becomes

$$T'' + \left(\gamma^2 + c^2\lambda_n\right) T = 0$$

The solution is

$$T_n(t) = A_n \cos\left(\sqrt{\gamma^2 + c^2\lambda_n} t\right) + B_n \sin\left(\sqrt{\gamma^2 + c^2\lambda_n} t\right)$$

Taking time derivatives gives

$$T_n'(t) = -\sqrt{\gamma^2 + c^2\lambda_n} A_n \sin\left(\sqrt{\gamma^2 + c^2\lambda_n} t\right) + B_n \sqrt{\gamma^2 + c^2\lambda_n} \cos\left(\sqrt{\gamma^2 + c^2\lambda_n} t\right)$$

At time $t = 0$, the above is zero (initial velocity is zero), which gives

$$0 = B_n \sqrt{\gamma^2 + c^2\lambda_n}$$

Hence $B_n = 0$ and the time ODE solution becomes

$$T_n(t) = A_n \cos\left(\sqrt{\gamma^2 + c^2\lambda_n} t\right)$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,t) &= T_n X_n \\ &= c_n \cos\left(\sqrt{\gamma^2 + c^2\left(\frac{n\pi}{L}\right)^2} t\right) \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Therefore the solution is

$$u(x,t) = \sum_{n=1}^{\infty} c_n \cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2 + c^2 n^2 \pi^2)} t\right) \sin\left(\frac{n\pi}{L}x\right)$$

c_n is found from initial position. At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{L}x\right)$$

Applying orthogonality gives

$$\begin{aligned} \int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx &= c_n \frac{L}{2} \\ c_n &= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx \end{aligned}$$

Hence solution is

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L f(s) \sin\left(\frac{n\pi}{L}s\right) ds\right) \cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2 + c^2 n^2 \pi^2)} t\right) \sin\left(\frac{n\pi}{L}x\right)$$

6.1.1.16 [350] Dispersion term present

problem number 350

Solve

$$u_{tt} + \gamma^2 u(x,t) = c^2 u_{xx}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time. With $0 < x < \pi$ and $t > 0$ and with boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

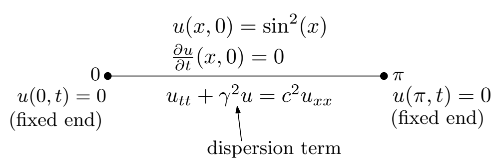


Figure 6.24: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} \cos\left(t\sqrt{c^2\left(\frac{\gamma^2}{c^2}+K[1]^2\right)}\right) \left(\begin{cases} 0 & K[1]=2 \\ \frac{2\left(-1+(-1)^{K[1]}\right)\sqrt{\frac{2}{\pi}}}{K[1]^3-4K[1]} & \text{True} \end{cases} \right) \sin(xK[1]) \right.\right.\right.$$

Indeterminate

Due to adding dispersion term

Maple ✓

```
restart;
interface(showassumed=0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=sin(x)^2,(D[2](u))(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = \frac{8\cos\left(\sqrt{c^2+\gamma^2}\, t\right)\sin(x)}{3\pi} + \left(\sum_{n=3}^{\infty} \frac{4((-1)^n-1)\cos\left(\sqrt{c^2n^2+\gamma^2}\, t\right)\sin(nx)}{(n^2-4)\,\pi n}\right)$$

Hand solution

Solving for $t > 0, 0 < x < L$

$$\frac{\partial^2 u}{\partial t^2} + \gamma^2 u = \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < L, t > 0 \tag{1}$$

With BC

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Where now $L = \pi, f(x) = \sin^2(x)$.

The general solution for (1) was found in problem 6.1.1.15 on page 956 as

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right) ds\right)\cos\left(\frac{t}{L}\sqrt{L^2\gamma^2+c^2n^2\pi^2}\right)\sin\left(\frac{n\pi}{L}x\right)$$

Using the above specific values for this problem, the solution becomes

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} \left(\frac{2}{\pi}\int_0^\pi f(s)\sin\left(\frac{n\pi}{\pi}s\right) ds\right)\cos\left(\frac{t}{\pi}\sqrt{\pi^2\gamma^2+c^2n^2\pi^2}\right)\sin\left(\frac{n\pi}{\pi}x\right) \\ &= \sum_{n=1}^{\infty} \left(\frac{2}{\pi}\int_0^\pi f(s)\sin(ns)\, ds\right)\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx) \end{aligned}$$

But

$$
\begin{aligned}
\int_0^\pi \sin^2(s)\sin(ns)\,ds &= \int_0^\pi \left(\frac{1}{2}-\frac{1}{2}\cos(2s)\right)\sin(ns)\,ds\\
&= \int_0^\pi \frac{1}{2}\sin(ns)\,ds - \frac{1}{2}\int_0^\pi \cos(2s)\sin(ns)\,ds\\
&= \frac{1-(-1)^n}{2n} - \frac{1}{2}\int_0^\pi \cos(2s)\sin(ns)\,ds
\end{aligned}
$$

$\int_0^\pi \cos(2s)\sin(ns)\,ds = -\frac{2}{3}$for $n=1$ and $\int_0^\pi \cos(2s)\sin(ns)\,ds = 0$ For $n=2$ and $\int_0^\pi \cos(2s)\sin(ns)\,ds = \frac{(1-(-1)^n)n}{n^2-4}$ for $n>2$. Hence

$$
\int_0^\pi \sin^2(s)\sin(ns)\,ds = \begin{cases} \frac{1-(-1)}{2}+\frac{1}{3}=\frac{4}{3} & n=1\\ \frac{1-(-1)^n}{2n}=0 & n=2\\ \frac{1-(-1)^n}{2n}-\frac{1}{2}\frac{(1-(-1)^n)n}{n^2-4}=\frac{2}{n}\frac{(-1)^n-1}{n^2-4} & n>2 \end{cases}
$$

The solution becomes

$$
\begin{aligned}
u(x,t) &= \left(\frac{2}{\pi}\right)\frac{4}{3}\cos\left(\sqrt{\gamma^2+c^2}t\right)\sin(x) + \sum_{n=3}^{\infty}\left(\frac{2}{\pi}\frac{2}{n}\frac{(-1)^n-1}{n^2-4}\right)\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx)\\
&= \frac{8}{3\pi}\cos\left(\sqrt{\gamma^2+c^2}t\right)\sin(x) + \frac{4}{\pi}\sum_{n=3}^{\infty}\frac{(-1)^n-1}{n(n^2-4)}\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx)
\end{aligned}
$$

6.1.1.17 [351] Dispersion term present (specific case)

problem number 351

Added July 12, 2019 Solve

$$
u_{tt}+\gamma^2 u(x,t) = c^2 u_{xx}
$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time. With $0<x<L$ and $t>0$ and with boundary conditions

$$
\begin{aligned}
u(0,t) &= 0\\
u(\pi,0) &= 0
\end{aligned}
$$

With initial conditions

$$
\begin{aligned}
\frac{\partial u}{\partial t}(x,0) &= 0\\
u(x,0) &= f(x)
\end{aligned}
$$

Using the following values

$$
\begin{aligned}
L &= 10 \\
\gamma &= \frac{1}{8} \\
f(x) &= \begin{cases} x-4 & 4 \leq x \leq 5 \\ 6-x & 5 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases} \\
c &= 1
\end{aligned}
$$

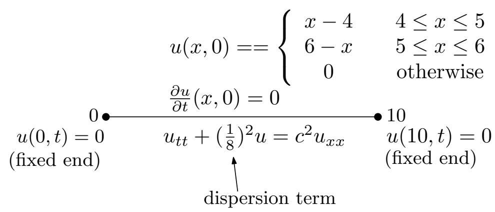


Figure 6.25: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
gamma=1/8; c=1; L=10;
f=Piecewise[{{x-4,4<=x<=5},{6-x,5<x<=6},{0,True}}];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \left\{ \begin{array}{ll} \sum_{K[1]=1}^{\infty} -\frac{20\cos\left(t\sqrt{\frac{1}{100}\pi^2 K[1]^2+\frac{1}{64}}\right)\left(\sin\left(\frac{2}{5}\pi K[1]\right)-2\sin\left(\frac{1}{2}\pi K[1]\right)+\sin\left(\frac{3}{5}\pi K[1]\right)\right)\sin\left(\frac{1}{10}\pi x K[1]\right)}{\pi^2 K[1]^2} & K[1] \in \mathbb{Z} \\ \text{Indeterminate} & \text{Tr} \end{array}\right.\right.\right.
$$

Due to adding dispersion term

Maple ✓

```
restart;
local gamma;
gamma:=1/8;
c:=1;
L:=10;
f:=piecewise(4<=x and x<=5, x-4,5<x and x<=6,6-x,true,0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,(D[2](u))(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(-\frac{40\left(\cos\left(\frac{\pi n}{2}\right) - 2\cos\left(\frac{\pi n}{5}\right) + 2\cos\left(\frac{\pi n}{10}\right) - 2\cos\left(\frac{2\pi n}{5}\right) + 2\cos\left(\frac{3\pi n}{10}\right) - 1\right)\cos\left(\frac{\sqrt{16\pi^2 n^2+25}\,t}{40}\right)}{\pi^2 n^2}\right.$$

Hand solution

Solve for $0 < x < L, t > 0$

$$\frac{\partial^2 u}{\partial t^2} + \gamma^2 u = c^2 \frac{\partial^2 u}{\partial x^2}$$

Boundary conditions, $t > 0$ (both ends fixed)

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial conditions, $t = 0$

$$\frac{\partial u(x,0)}{\partial t} = 0$$
$$u(x,0) = f(x)$$

Using $L = 10, \gamma = 1/8, c = 1$ and initial position

$$f(x) = \begin{cases} x-4 & 4 \leq x \leq 5 \\ 6-x & 5 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

The general solution to the above PDE was given in problem 6.1.1.15 on page 956 as

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2+c^2n^2\pi^2)}t\right)\sin\left(\frac{n\pi}{L}x\right)$$

Replacing given values in the above solution results in

$$\begin{aligned}
u(x,t)&=\sum_{n=1}^{\infty}\left(\frac{2}{10}\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{10}\sqrt{\left(100\left(\frac{1}{8}\right)^2+n^2\pi^2\right)t}\right)\sin\left(\frac{n\pi}{10}x\right)\\
&=\frac{2}{10}\sum_{n=1}^{\infty}\left(\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{10}\sqrt{\frac{1}{16}\left(\frac{100}{4}+16n^2\pi^2\right)t}\right)\sin\left(\frac{n\pi}{10}x\right) \qquad (1)\\
&=\frac{2}{10}\sum_{n=1}^{\infty}\left(\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2t}\right)\sin\left(\frac{n\pi}{10}x\right) \qquad (6.1)
\end{aligned}$$

But

$$\int_0^{10} f(x)\sin\left(\frac{n\pi}{10}x\right)ds=\frac{100\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)}{n^2\pi^2}$$

Hence the solution (1) becomes

$$\begin{aligned}
u(x,t)&=\frac{2}{10}\sum_{n=1}^{\infty}\frac{100\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)}{n^2\pi^2}\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2t}\right)\sin\left(\frac{n\pi}{10}x\right)\\
&=\frac{20}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2t}\right)\sin\left(\frac{n\pi}{10}x\right)
\end{aligned}$$

Animation is below. The left one uses $\gamma=\frac{1}{8}$ and the right one uses larger value of $\gamma=\frac{5}{8}$ in order to show the effect of larger dispersion.

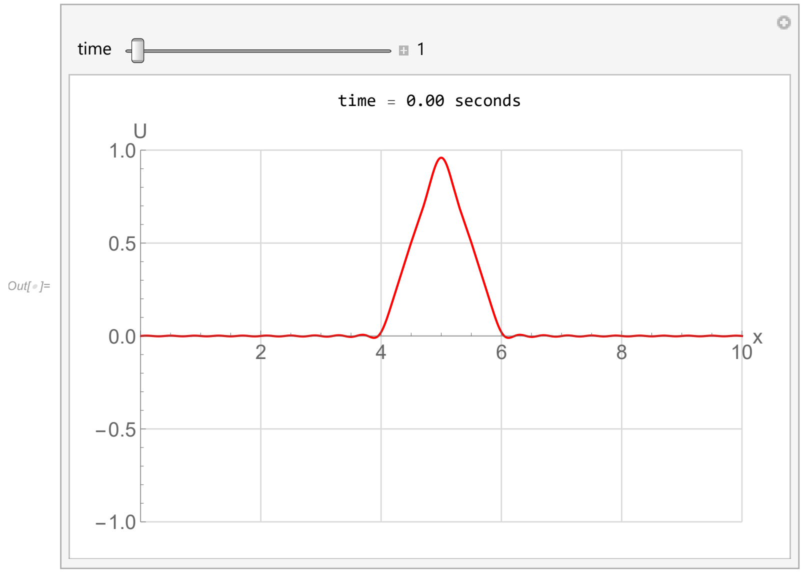


Figure 6.26: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
gamma = 1 / 8; c = 1; L = 10;
f = Piecewise[{{x - 4, 4 ≤ x ≤ 5}, {6 - x, 5 < x ≤ 6}, {0, True}}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 50;
mySol[x_, t_] = 20/π^2 Sum[1/n^2 (2 Sin[n π/2] - Sin[2 n π/5] - Sin[3 n π/5]) (Cos[1/10 Sqrt[50/32 + n^2 π^2] t]) Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 6.27: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
          {
            Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                BaseStyle → 15,
                ImageMargins → 3,
                PerformanceGoal → "Quality",
                PlotRange → {{0, L}, {-1, 1}},
                ImageSize → 500,
                AxesLabel → {"x", "U"},
                GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotStyle → Red
              ]

          }}],
      {t, 0, 80, 0.1}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.28: Code for animation

6.1.1.18 [352] non-zero initial position

problem number 352

Added March 9, 2018. Solve

$$u_{tt} = 4u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

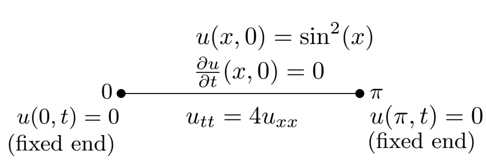


Figure 6.29: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}];
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
bc  = {u[0, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4(\cos(n\pi)-1)\cos(2nt)\sin(nx)}{(n^3-4n)\,\pi}\right\}\right\}$$

But sum should not include $n=2$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)= 4*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=sin(x)^2,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\frac{8\cos(2t)\sin(x)}{3\pi}+\left(\sum_{n=3}^{\infty}\frac{4((-1)^n-1)\cos(2nt)\sin(nx)}{\pi(n^2-4)\,n}\right)$$

Handled $n=2$ case correctly

6.1.1.19 [353] With source

problem number 353

Added December 20, 2018.

Example 18, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $0<x<1$ and $t>0$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+xe^{-t}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(1,0) = 0$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

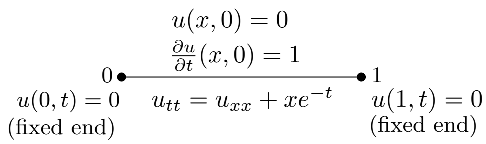


Figure 6.30: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + x*Exp[-t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[0, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \left\{ \frac{e^{-i((-i+\pi)t+\pi x)}\left(ie^{2i\pi t+t}\pi^2(i+\pi)\,{}_2F_1\left(1,\frac{-i+\pi}{\pi};2-\frac{i}{\pi};-e^{i\pi(t-x)}\right)+e^{i\pi t}(1-i\pi)\pi^2\,{}_2F_1\left(1,\frac{-i+\pi}{\pi};2-\frac{i}{\pi};-e^{-i\pi x}\right)+ie^{i\pi(t+\cdots}}{\cdots}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2)+x*exp(-t);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(-\frac{2(\pi(-\cos(\pi n t) + e^{-t})\, n(-1)^{n} + (\pi^{2} n^{2}(-1)^{n} - \pi^{2} n^{2} + 2(-1)^{n} - 1)\sin(\pi n t))\sin(\pi n x)}{\pi^{2}(\pi^{2} n^{2} + 1)\, n^{2}}\right)$$

6.1.1.20 [354] Right end free (general case)

problem number 354

Added July 8, 2019

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

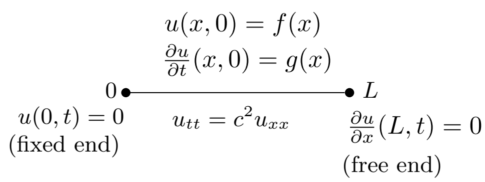


Figure 6.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == g[x], u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \left\{\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\cos\left(\frac{1}{2}\pi t\sqrt{\frac{c^2(2K[1]-1)^2}{L^2}}\right)\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx+\frac{2L\left(\int_0^L\frac{\sqrt{2}g(x)\sin\left(\frac{\pi x(2K[1]}{2L}\right.}{\sqrt{L}}\right.}{\pi|c}\right.}{\sqrt{L}}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=g(x),u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{4\left(L\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\sin\left(\frac{(2n+1)\pi ct}{2L}\right)+\pi\left(n+\frac{1}{2}\right)c\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos\right.}{\pi\left(2n+1\right)Lc}$$

Hand solution

Solving for $0<x<L$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<L,t>0$$

Boundary conditions, $t>0$

$$u(0,t)=0$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t = 0$

$$
\begin{aligned}
u_t(x,0) &= g(x) \\
u(x,0) &= f(x)
\end{aligned}
$$

Separation of variables gives the eigenvalue ODE

$$
\begin{aligned}
X'' + \lambda X &= 0 \\
X(0) &= 0 \\
X'(L) &= 0
\end{aligned}
$$

Only $\lambda > 0$ gives non-trivial solution from the nature of the boundary conditions. Hence solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then the above gives $0 = A$ and the solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivatives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Since $X'(L) = 0$ the above becomes

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

Which implies $\sqrt{\lambda}L = \frac{n\pi}{2}$ for $n = 1,3,5,\cdots$ or

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right) \qquad n = 1,3,5,\cdots$$

The time ODE now becomes

$$T'' + c^2\left(\frac{n\pi}{2L}\right)^2 T = 0$$

Which has the solution

$$T(t) = D_n\cos\left(c\frac{n\pi}{2L}t\right) + E_n\sin\left(c\frac{n\pi}{2L}t\right)$$

Therefore the complete solution becomes

$$u(x,t)=\sum_{n=1,3,5,\cdots}^{\infty}\left(D_n\cos\left(c\frac{n\pi}{2L}t\right)+E_n\sin\left(c\frac{n\pi}{2L}t\right)\right)\Phi_n(x) \tag{1}$$

At $t=0$ the above becomes

$$f(x)=\sum_{n=1,3,5,\cdots}^{\infty}D_n\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx &= D_n\int_0^L \Phi_n^2(x)\,dx\\ &= \frac{L}{2}D_n\end{aligned}$$

Hence

$$D_n=\frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t)=\sum_{n=1,3,5,\cdots}^{\infty}\left(-c\frac{n\pi}{2L}D_n\sin\left(c\frac{n\pi}{2L}t\right)+E_nc\frac{n\pi}{L}\cos\left(c\frac{n\pi}{2L}t\right)\right)\Phi_n(x)$$

At $t=0$ the above becomes

$$g(x)=\sum_{n=1,3,5,\cdots}^{\infty}E_nc\frac{n\pi}{2L}\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L g(x)\,\Phi_n(x)\,dx &= E_nc\frac{n\pi}{L}\int_0^L \Phi_n^2(x)\,dx\\ &= \frac{L}{2}E_nc\frac{n\pi}{2L}\\ &= \frac{1}{4}E_ncn\pi\end{aligned}$$

Hence

$$E_n=\frac{4}{cn\pi}\int_0^L g(x)\,\Phi_n(x)\,dx \tag{3}$$

Using (2,3) in (1) gives the final solution as

$$
\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\cos\left(c\frac{n\pi}{2L}t\right)\sin\left(\frac{n\pi}{2L}x\right)\\
&+\frac{4}{c\pi}\sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\sin\left(c\frac{n\pi}{2L}t\right)\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)\\
&+\frac{4}{c\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

6.1.1.21 [355] Right end free, zero initial velocity (general case)

problem number 355

Added July 8, 2019

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions

$$
\begin{aligned}
u(0,t) &= 0\\
u_x(L,t) &= 0
\end{aligned}
$$

With initial conditions

$$
\begin{aligned}
u(x,0) &= f(x)\\
u_t(x,0) &= 0
\end{aligned}
$$

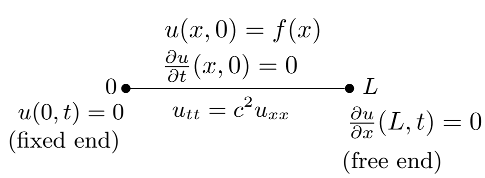


Figure 6.32: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \left\{ \begin{array}{cc} \sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{1}{2}\pi t\sqrt{\frac{c^2(2K[1]-1)^2}{L^2}}\right)\left(\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}} & K[1]\in\mathbb{Z}\land((L < \\ \text{Indeterminate} & \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos\left(\frac{(2n+1)\pi ct}{2L}\right)\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}$$

Hand solution

Solving for $0 < x < L$

$$u_{tt}=c^2u_{xx}\qquad 0<x<L,t>0$$

Boundary conditions, $t>0$

$$u(0,t)=0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t=0$

$$u_t(x,0)=g(x)=0$$
$$u(x,0)=f(x)$$

The general PDE was solved in 6.1.1.20 on page 969 and the solution is

$$u(x,t)=\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$
$$+\frac{4}{c\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

But here $g(x)=0$, hence the above reduces to

$$u(x,t)=\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

6.1.1.22 [356] Right end free, zero initial velocity (special case)

problem number 356

Added July 8, 2019

$$u_{tt}=c^2u_{xx}$$

With boundary conditions

$$\begin{aligned}u(0,t)&=0\\u_x(L,t)&=0\end{aligned}$$

With initial conditions

$$\begin{aligned}u(x,0)&=f(x)\\u_t(x,0)&=0\end{aligned}$$

Using the following values

$$\begin{aligned}c&=4\\L&=3\\h&=\frac{1}{10}\\f(x)&=\begin{cases}\frac{3h}{L}x & 0<x<\frac{L}{3}\\ h & \frac{L}{3}<x<L\end{cases}\end{aligned}$$

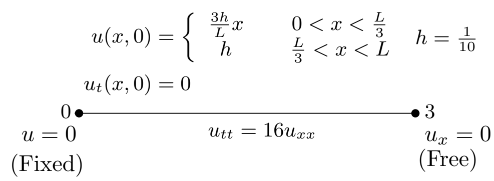


Figure 6.33: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \sum_{K[1]=1}^{\infty} -\frac{12\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{2}{3}\pi t\sqrt{(2K[1]-1)^2}\right)\sin\left(\frac{1}{6}\pi x(2K[1]-1)\right)}{5\pi^2(1-2K[1])^2} & K[1]\in\mathbb{Z}\land K[1]\geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{12\cos\left(\frac{(4n+2)\pi t}{3}\right)\sin\left(\frac{(2n+1)\pi x}{6}\right)\sin\left(\frac{1}{3}\pi n+\frac{1}{6}\pi\right)}{5\pi^2\left(2n+1\right)^2}$$

Hand solution

Solving the wave PDE on string

$$u_{tt} = c^2 u_{xx} \qquad t > 0, x > 0$$

Boundary conditions, $t > 0$

$$u(0,t) = 0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

Initial conditions, $t = 0$

$$u(x,0) = f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$
$$u_t(x,0) = 0$$

Using $c = 4, L = 3, h = \frac{1}{10}$. Hence $f(x) = \begin{cases} \frac{1}{10}x & 0 < x < 1 \\ \frac{1}{10} & 1 < x < 3 \end{cases}$

The general problem PDE was solved in 6.1.1.21 on page 973 and the solution is

$$u(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)\cos\left(\frac{(2n+1)\pi}{2L}ct\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = \frac{2}{3}\sum_{n=0}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)\cos\left(4\frac{(2n+1)\pi}{6}t\right)\sin\left(\frac{(2n+1)\pi}{6}x\right)$$

But

$$\begin{aligned}
\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx &= \int_0^{\frac{L}{3}} f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx + \int_{\frac{L}{3}}^L f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx \\
&= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx + \frac{1}{10}\int_1^3 \sin\left(\frac{(2n+1)\pi}{6}x\right)dx \\
&= \frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)
\end{aligned}$$

Hence the solution becomes

$$\begin{aligned}
u(x,t) &= \frac{2}{3}\sum_{n=0}^{\infty}\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\cos\left(\frac{2}{3}(2n+1)\pi t\right)\sin\left(\frac{1}{6}(2n+1)\pi x\right) \\
&= \frac{36}{15\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\cos\left(\frac{2}{3}(2n+1)\pi t\right)\sin\left(\frac{1}{6}(2n+1)\pi x\right)
\end{aligned}$$

Animation is below

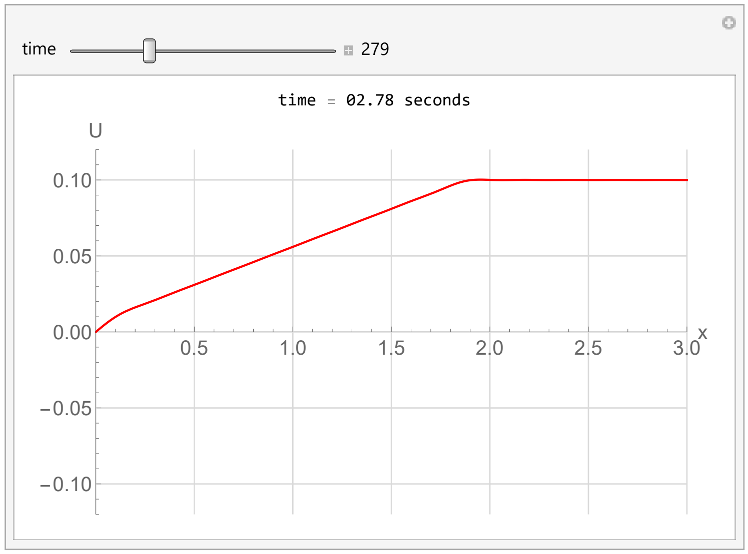


Figure 6.34: Initial state

Source code used for the above

```
In[=]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 4;
L = 3;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 25;
mySol[x_, t_] = 36/(15 π^2) Sum[1/(2 n + 1)^2 Sin[((2 n + 1) π)/6] Cos[2/3 (2 n + 1) π t] Sin[1/6 (2 n + 1) π x], {n, 0, numberOfTerms}];
```

Figure 6.35: Source code

```
In[]:= tab = Table[
         Grid[{
            {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
            {
              Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-0.12, 0.12}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

            }}],
         {t, 0, 10, 0.01}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.36: Code for animation

6.1.1.23 [357] Right end free, zero initial velocity, damping present (general case)

problem number 357

Added July 9, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

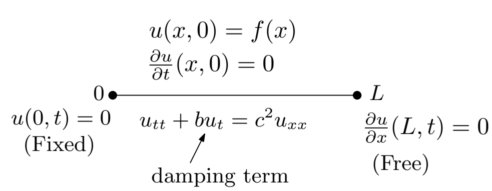


Figure 6.37: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions->{b
```

$$\left\{\left\{u(x,t)\to \left\{ \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-\frac{bt}{2}}\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\cos\left(\frac{1}{2}t\sqrt{\frac{c^2\pi^2(2K[1]-1)^2}{L^2}-b^2}\right)\int_0^L \frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx+\frac{2\left(\int_0^L \frac{bf(x)\sin\left(\frac{\pi}{}\right.}{\sqrt{}}\right.}{}\right.}{\sqrt{L}} \right. \right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{\left(\left(Lb+\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi^2c^2}\right)\mathrm{e}^{-\frac{\left(Lb-\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi^2c^2}\right)t}{2L}}-\left(Lb-\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi\right.\right.}{\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi}}$$

Hand solution

Solving for $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L, t>0$$

Boundary conditions

$$u(0,t) = 0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

Initial conditions, $t = 0$

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

Separation of variables gives

$$T''X + bT'X = c^2 X''T$$
$$\frac{1}{c^2}\left(\frac{T''}{T} + b\frac{T'}{T}\right) = \frac{X''}{X} = -\lambda$$

The eigenvalue ODE is

$$X'' + \lambda X = 0$$
$$X(0) = 0$$
$$X'(L) = 0$$

Only $\lambda > 0$ gives non-trivial solution from the nature of the boundary conditions. Hence solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then the above gives $0 = A$ and the solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivatives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Since $X'(L) = 0$ the above becomes

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

Which implies $\sqrt{\lambda}L = \frac{n\pi}{2}$ for $n = 1,3,5,\cdots$ or

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right) \qquad n = 1,3,5,\cdots$$

The time ODE now becomes

$$\begin{aligned}\frac{1}{c^2}\left(\frac{T''}{T} + b\frac{T'}{T}\right) &= -\lambda_n \\ \frac{T''}{T} + b\frac{T'}{T} &= -c^2\lambda_n \\ T'' + bT' + c^2\lambda_n T &= 0\end{aligned}$$

The characteristic equation $r^2 + br + c^2\lambda_n = 0$ has the roots $r = \frac{-B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC} \rightarrow r = \frac{-b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4c^2\lambda_n}$ or

$$r = \frac{-b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4c^2\lambda_n}$$

Case $b^2 < 4c^2\lambda_n$ for all n. This is called the underdamped case, which generates damped oscillations. The roots becomes

$$r = \frac{-b}{2} \pm \frac{1}{2}i\sqrt{4c^2\lambda_n - b^2}$$

Let $\beta_n = \frac{1}{2}\sqrt{4c^2\lambda_n - b^2}$, then

$$r = \frac{-b}{2} \pm i\beta_n$$

Hence the solution is

$$T_n(t) = e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))$$

Therefore the complete solution becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))\,\Phi_n(x) \tag{1}$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1,3,5,\cdots}^{\infty} D_n\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx &= D_n\int_0^L \Phi_n^2(x)\,dx \\ &= \frac{L}{2}D_n\end{aligned}$$

Hence

$$D_n = \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{-b}{2} e^{\frac{-b}{2}t}\left(D_n \cos(\beta_n t) + E_n \sin(\beta_n t)\right)\Phi_n(x) + e^{\frac{-b}{2}t}\left(-\beta_n D_n \sin(\beta_n t) + E_n\beta_n \cos(\beta_n t)\right)\Phi_n$$

At $t=0$ since $g(x)=0$ then the above becomes

$$\begin{aligned}
0 &= \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{-b}{2}D_n + E_n\beta_n\right)\Phi_n(x)\\
0 &= \frac{-b}{2}D_n + E_n\beta_n\\
E_n &= \frac{bD_n}{2\beta_n}
\end{aligned} \tag{3}$$

Using (2,3), the solution (1) now becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}\left(D_n \cos(\beta_n t) + \frac{bD_n}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)\\
&= \sum_{n=1,3,5,\cdots}^{\infty} D_n e^{\frac{-b}{2}t}\left(\cos(\beta_n t) + \frac{b}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)
\end{aligned}$$

Or

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) e^{\frac{-b}{2}t}\left(\cos(\beta_n t) + \frac{b}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)$$

But $\beta_n = \frac{1}{2}\sqrt{4c^2\lambda_n - b^2}$,$\lambda_n = \left(\frac{n\pi}{2L}\right)^2$ and $\Phi_n(x) = \sin\left(\frac{n\pi}{2L}x\right)$, hence the above becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right) e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2 t}\right)\sin\left(\frac{n\pi}{2L}x\right)\\
&+ \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right) e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2 t}\right)}{\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2}}\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}$$

Or

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)\sin\left(\frac{(2n+1)}{2L}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

Case $b^2=4c^2\left(\frac{n\pi}{2L}\right)^2$. We see that for $n=1$ it becomes critical, because then $b=\frac{\pi c}{L}$ and now and the discriminant is zero in this case. This is called the critical damped case. For $n>1$, it becomes underdamped, which is the above case. So we only need to find solution for $n=1$. In this case, the solution to $T''+bT'+c^2\lambda_1 T=0$ is

$$T_1(t)=D_1e^{\frac{-b}{2}t}+E_1te^{\frac{-b}{2}t}$$

Therefore the complete solution becomes

$$u(x,t)=\left(D_1e^{\frac{-b}{2}t}+E_1te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)+\sum_{n=3,5,\cdots}^{\infty}\left(D_ne^{\frac{-b}{2}t}+E_nte^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)\quad(4)$$

For $n=1$, At $t=0$, from intial conditions (4) becomes

$$f(x)=D_1\sin\left(\frac{\pi}{2L}x\right)$$

By orthognailty the above gives

$$D_1=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\tag{5}$$

Taking time derivative of (4) for $n=1$, gives

$$u(x,t)=\left(\frac{-b}{2}D_1e^{\frac{-b}{2}t}+E_1\left(e^{\frac{-b}{2}t}-\frac{b}{2}te^{\frac{-b}{2}t}\right)\right)\sin\left(\frac{\pi}{2L}x\right)$$

At $t=0$ and since $g(x)=0$, then the above becomes

$$0=\left(\frac{-b}{2}D_1+E_1\right)\sin\left(\frac{\pi}{2L}x\right)$$

$$\frac{-b}{2}D_1+E_1=0$$

$$E_1=\frac{b}{2}D_1\tag{6}$$

Using (5,6) then (4) becomes

$$
\begin{aligned}
u(x,t) &= D_1\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)+\sum_{n=3,5,\cdots}^{\infty}\left(D_n e^{\frac{-b}{2}t}+E_n te^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)\\
&= \left(\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\right)\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)\\
&\quad+\sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\sqrt{\lambda_n}s\right)ds\right)\left(e^{\frac{-b}{2}t}+\frac{bt}{2}e^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)
\end{aligned}
$$

For the case of $n>1$, the solution was found in the above underdamped case. Putting all these together, gives the solution as

$$
\begin{aligned}
u(x,t) &= \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{\pi}{2L}s\right)ds\right)\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)\\
&\quad+\sum_{n=2}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)\sin\left(\frac{(2n+1)\pi}{2L}\right.\\
&\quad+\sum_{n=2}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

<u>Case</u> $b^2>4c^2\left(\frac{n\pi}{2L}\right)^2$. Will consider only the case when this is true for $n=1$ only. If this is true for larger n, then same solution needs to be summed for each mode. But for simplicity, will consider $n=1$ here. In this case, the roots are

$$
r=\frac{-b}{2}\pm\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}
$$

Where now $b^2-4c^2\lambda_1$ is positive. Hence we get $r_1=\frac{-b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}, r_2=\frac{-b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}$ or

$$
\begin{aligned}
r_1 &= \frac{-b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}\\
r_1 &= \frac{-b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}
\end{aligned}
$$

And the solution to $T_1''+bT_1'+c^2\lambda_1T=0$ is

$$
T_1(t)=e^{\frac{-b}{2}t}\left(D_1e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}+E_1e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)
$$

For the rest of the modes, the solution is from above

$$T_n(t) = e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t)) \qquad n = 3, 5, 7, \cdots$$

Hence the complete solution becomes

$$\begin{aligned} u(x,t) &= e^{\frac{-b}{2}t}\left(D_1 e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t} + E_1 e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)\sin\left(\frac{\pi}{2L}x\right) \\ &+ \sum_{n=3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))\sin\left(\frac{n\pi}{2L}x\right) \end{aligned} \tag{7}$$

At $t = 0$ and for $n = 1$, the above becomes

$$f(x) = (D_1 + E_1)\sin\left(\frac{\pi}{2L}x\right)$$

Hence

$$(D_1 + E_1) = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx \tag{8}$$

Taking time derivative of (7) and for $n = 1$ at $t = 0$ it gives

$$0 = \left(\frac{-b}{2}(D_1 + E_1) + \left(\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}D_1 - E_1\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right)\right)\sin\left(\frac{\pi}{2L}x\right)$$

Hence

$$\begin{aligned} \frac{-b}{2}(D_1 + E_1) + \left(\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}D_1 - E_1\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right) &= 0 \\ -E_1\left(\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right) &= \left(\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right)D_1 \\ E_1 &= \frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}D_1 \end{aligned} \tag{9}$$

From (8,9)

$$D_1 = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx - E_1$$

$$D_1 - \frac{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}} D_1 = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx$$

$$D_1 = \frac{\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx}{1 - \frac{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}}$$

$$= \frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}} \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx$$

And therefore

$$E_1 = \frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}} \left(\frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}\right) \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx$$

$$= \frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}} \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx$$

For $n > 1$ the solution is the same as the underdamped case above. Hence the complete solution becomes from (7)

$$u(x,t) = e^{\frac{-b}{2}t}\left(D_1 e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t} + E_1 e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)\sin\left(\frac{\pi}{2L}x\right)$$
$$+ \sum_{n=3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}\left(D_n\cos\left(\beta_n t\right) + E_n\sin\left(\beta_n t\right)\right)\sin\left(\frac{n\pi}{2L}x\right)$$

Or

$$
\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+\sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2t}\right)\sin\left(\frac{n\pi}{2L}x\right)\\
&+\sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2t}\right)}{\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2}}\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

6.1.1.24 [358] Right end free, zero initial velocity, damping present (special case, underdamped)

problem number 358

Added July 9, 2019

$$u_{tt}+bu_t=c^2u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Using the following values

$$L = 3$$
$$c = 4$$
$$f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$
$$h = \frac{1}{10}$$
$$b = \frac{1}{2}\frac{\pi c}{L}$$

Hence $b = \frac{2\pi}{3}$

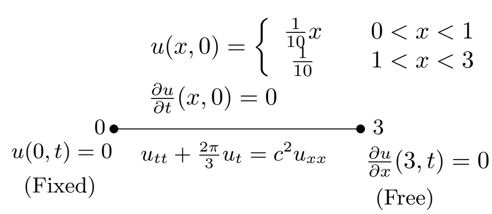


Figure 6.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=1/2*(Pi*c/L);
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\left\{\sum_{K[1]=1}^{\infty}\sqrt{\frac{2}{3}}e^{-\frac{\pi t}{3}}\sin\left(\frac{1}{6}\pi x(2K[1]-1)\right)\left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2-\frac{4\pi^2}{9}}\right)}{5\pi^2(1-2K[1])^2}-\cdots\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=1/2*(Pi*c/L);
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}\left(-\frac{6i\left(i\sqrt{16n^2+16n+3}\,\mathrm{e}^{\frac{\pi\left(i\sqrt{16n^2+16n+3}-1\right)t}{3}}+\mathrm{e}^{\frac{\pi\left(i\sqrt{16n^2+16n+3}-1\right)t}{3}}+i\sqrt{16n^2+16n+3}\,\mathrm{e}^{-\frac{\pi\left(i\sqrt{1\cdots}\right.}{}}\cdots\right.}{5\sqrt{16n^2+16n+3}\,\pi^2\,(2n+\cdots}\right.$$

Hand solution

Solving the wave PDE on string underdamped case $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L, t>0$$

Boundary conditions

$$
\begin{aligned}
u(0,t) &= 0 \\
\left.\frac{\partial u}{\partial x}\right|_{x=L} &= 0
\end{aligned}
$$

Initial conditions, $t=0$

$$
\begin{aligned}
u_t(x,0) &= 0 \\
u(x,0) &= f(x)
\end{aligned}
$$

Using

$$
\begin{aligned}
f(x) &= \begin{cases} \frac{3h}{L}x & 0<x<\frac{L}{3} \\ h & \frac{L}{3}<x<L \end{cases} \\
b &= \frac{2\pi}{3} \\
c &= 4 \\
L &= 3
\end{aligned}
$$

Hence the PDE becomes $u_{tt}+\frac{2\pi}{3}u_t=16u_{xx}$. The general solution to the above PDE was given in problem 6.1.1.23 on page 979. The eigenvalues are given as

$$
\lambda_n = \left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n=0,1,2,\cdots
$$

And the discriminant is $b^2-4c^2\lambda_n = b^2-4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 = b^2-(4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n=0$ this gives $b^2-\frac{16}{9}\pi^2$. But $b=\frac{2\pi}{3}$. Hence discriminant is $\left(\frac{2\pi}{3}\right)^2-\frac{16}{9}\pi^2=-\frac{4}{3}\pi^2$. Since discriminant is negative, then this is underdamped wave with damped oscillations as the solution given from the above problem as

$$
\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right) \\
&+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2t}\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

Replacing given values in the above solution results in

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-\pi}{3}t}\left(\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}\,t\right)\right)\sin\left(\frac{(2n+1)}{6}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-\pi}{3}t}\left(\frac{2\pi}{3}\frac{\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}}\right)\sin\left(\frac{(2n+1)\pi}{6}x\right. \tag{1}$$

But

$$\begin{aligned}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)&=\frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx+\frac{1}{10}\int_1^3\sin\left(\frac{(2n+1)\pi}{6}x\right)dx\\&=\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\end{aligned}$$

Hence the solution (1) becomes

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{3}\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\right)e^{\frac{-\pi}{3}t}\left(\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}\,t\right)\right)\sin\left(\frac{(2n+1}{6}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{3}\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\right)e^{\frac{-\pi}{3}t}\left(\frac{2\pi}{3}\frac{\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{2\pi}{3}\right)^2}}\right)\sin\left(\frac{(2n+1)\pi}{6}\right. \tag{2}$$

Animation is below

Out[=]=

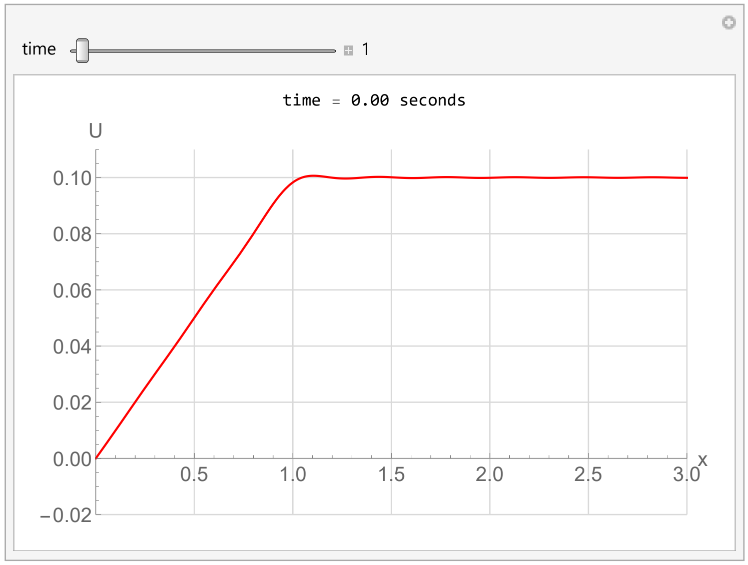


Figure 6.39: Initial state

Source code used for the above

```
ClearAll[x, t, n, f, A, B, s, mySol]
c = 4;
L = 3;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 16;
z = 2/3 18/(5 π^2 (2 n + 1)^2) Sin[((2 n + 1) π)/6];
mySol[x_, t_] = Sum[z Exp[(-π t)/3] Sin[((2 n + 1) π)/6 x] (Cos[1/2 Sqrt[64 (((2 n + 1) π)/6)^2 - ((2 π)/3)^2] t]), {n, 0, numberOfTerms}]
 + Sum[z Exp[(-π t)/3] Sin[((2 n + 1) π)/6 x] ((2 π)/3 Sin[1/2 Sqrt[64 (((2 n + 1) π)/6)^2 - ((2 π)/3)^2] t]/Sqrt[64 (((2 n + 1) π)/6)^2 - ((2 π)/3)^2]), {n, 0, numberOfTerms}];
```

Figure 6.40: Source code

```
In[ ]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, L}, {-0.02, 0.11}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 4, 0.01}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.41: Code for animation

6.1.1.25 [359] Right end free, zero initial velocity, damping present (special case, critical damped)

problem number 359

Added July 10, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Using the following values

$$
\begin{aligned}
L &= 3\\
c &= 4\\
f(x) &= \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}\\
h &= \frac{1}{10}\\
b &= \frac{\pi c}{L}
\end{aligned}
$$

Hence $b = \frac{4\pi}{3}$

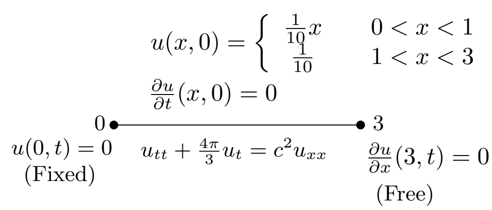


Figure 6.42: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=Pi*c/L;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \quad \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{3}} e^{-\frac{2\pi t}{3}} \sin\left(\frac{1}{6}\pi x(2K[1]-1)\right) \left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2 - \frac{16\pi^2}{9}}\right)}{5\pi^2(1-2K[1])^2} - \right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=Pi*c/L;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}\left\{\begin{array}{c}\frac{4\left(\pi t+\frac{3}{2}\right)\mathrm{e}^{-\frac{2\pi t}{3}}\sin\left(\frac{\pi x}{6}\right)}{5\pi^{2}}\\ \frac{3\left(\cos\left(\frac{\pi n}{3}\right)+\sqrt{3}\,\sin\left(\frac{\pi n}{3}\right)\right)\left(\left(2\sqrt{n+1}\,\sqrt{n}+i\right)\mathrm{e}^{\frac{2i\pi\left(-2\sqrt{n+1}\,\sqrt{n}+i\right)t}{3}}+\left(2\sqrt{n+1}\,\sqrt{n}-i\right)\mathrm{e}^{\frac{2i\pi\left(2\sqrt{n+1}\,\sqrt{n}+i\right)t}{3}}\right)\sin\left(\frac{(2n+1)\pi x}{6}\right)}{10\sqrt{n+1}\,\pi^{2}(2n+1)^{2}\sqrt{n}}\end{array}\right.$$

Hand solution

Solving the wave PDE on string underdamped case $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L, t>0$$

Boundary conditions

$$u(0,t)=0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t=0$

$$u_t(x,0)=0$$
$$u(x,0)=f(x)$$

Using

$$
\begin{aligned}
f(x) &= \begin{cases} \frac{3h}{L}x & 0<x<\frac{L}{3} \\ h & \frac{L}{3}<x<L \end{cases} \\
b &= \frac{4\pi}{3} \\
c &= 4 \\
L &= 3 \\
h &= \frac{1}{10}
\end{aligned}
$$

Hence the PDE becomes $u_{tt}+\frac{4\pi}{3}u_t=16u_{xx}$. The general solution to the above PDE was given in problem 6.1.1.23 on page 979. The eigenvalues are given as

$$
\lambda_n=\left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n=0,1,2,\cdots
$$

And the discriminant is $b^2-4c^2\lambda_n=b^2-4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2=b^2-(4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n=0$ this gives $b^2-\left(\frac{4}{3}\pi\right)^2$. But $b=\frac{4\pi}{3}$. Hence discriminant is zero for $n=0$. This means this is critically damped in first mode. Using the the solution for this case from the above general solution as

$$
\begin{aligned}
u(x,t) &= \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{\pi}{2L}s\right)ds\right)\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right) \\
&+\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right) \\
&+\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

Replacing given values in the above solution results in

$$
\begin{aligned}
u(x,t) &= \left(\frac{2}{3}\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx\right)\left(e^{\frac{-2\pi t}{3}}+\frac{b}{2}te^{\frac{-2\pi t}{3}}\right)\sin\left(\frac{\pi}{6}x\right) \qquad (1)\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-2\pi t}{3}}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)\sin\left(\frac{(2n}{2}\right.\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-b}{2}t}\frac{\left(\frac{4\pi}{3}\right)\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}}\sin\left(\frac{(2n+1}{6}\right.
\end{aligned}
$$

But

$$
\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx=\frac{9}{5\pi^2}
$$

And

$$
\begin{aligned}
\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right) &= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx+\frac{1}{10}\int_1^3\sin\left(\frac{(2n+1)\pi}{6}x\right)dx\\
&= \frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)
\end{aligned}
$$

Hence the solution (1) becomes

$$
\begin{aligned}
u(x,t) &= \frac{6}{5\pi^2}\left(e^{\frac{-2\pi t}{3}}+\frac{2}{3}\pi te^{\frac{-2\pi t}{3}}\right)\sin\left(\frac{\pi}{6}x\right) \qquad (1)\\
&+\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{\frac{-2\pi t}{3}}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)\sin\left(\frac{(2n+1}{6}\right.\\
&+\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{\frac{-2\pi t}{3}}\frac{\left(\frac{4\pi}{3}\right)\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}}\sin\left(\frac{(2n+1)\pi}{6}\right.
\end{aligned}
$$

Animation is below

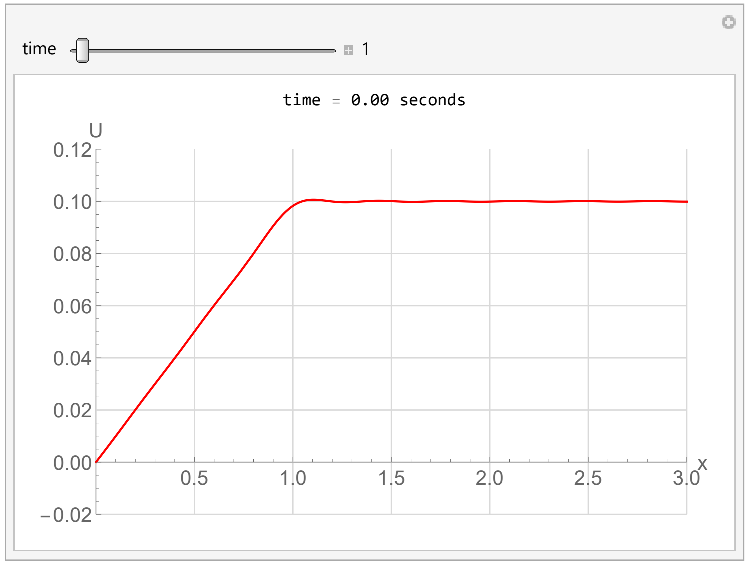


Figure 6.43: Initial state

Source code used for the above

```
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 50;
z = 12/(5 π^2 (2 n + 1)^2) Sin[((2 n + 1) π)/6];
mySol[x_, t_] = 6/(5 π^2) (Exp[(-2 π t)/3] + 2/3 π t Exp[(-2 π t)/3]) Sin[π/6 x] + Sum[z Exp[(-2 π t)/3] Sin[((2 n + 1) π)/6 x] (Cos[1/2 Sqrt[64 (((2 n + 1) π)/6)^2 - ((4 π)/3)^2] t]), {n, 1, numberOfTerms}] +
    Sum[z Exp[(-2 π t)/3] Sin[((2 n + 1) π)/6 x] ((4 π)/3 Sin[1/2 Sqrt[64 (((2 n + 1) π)/6)^2 - ((4 π)/3)^2] t]/Sqrt[64 (((2 n + 1) π)/6)^2 - ((4 π)/3)^2]), {n, 1, numberOfTerms}];
```

Figure 6.44: Source code

```
In[∘]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, L}, {-0.01, 0.11}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 2.9, 0.01}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[∘]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.45: Code for animation

6.1.1.26 [360] Right end free, zero initial velocity, damping present (special case, over damped)

problem number 360

Added July 11, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Using the following values

$$L = 3$$
$$c = 4$$
$$f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$
$$h = \frac{1}{10}$$
$$b = \frac{3}{2}\frac{\pi c}{L}$$

Hence $b = 2\pi$

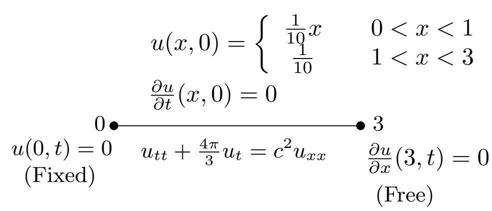


Figure 6.46: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=3/2*Pi*c/L;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \quad \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{3}} e^{-\pi t} \sin\left(\frac{1}{6}\pi x(2K[1]-1)\right) \left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2-4\pi^2}\right)}{5\pi^2(1-2K[1])^2} - \frac{18\sqrt{}}{} \right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=3/2*Pi*c/L;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{6\left(\left(\sqrt{-16n^2-16n+5}+3\right)e^{\frac{\pi\left(\sqrt{-16n^2-16n+5}-3\right)t}{3}}+\left(\sqrt{-16n^2-16n+5}-3\right)e^{-\frac{\pi\left(\sqrt{-16n^2-16n+5+\cdots}\right.}{3}}\cdots\right.}{5\sqrt{-16n^2-16n+5}\,\pi^2\left(2n+1\right)^2}$$

<u>Hand solution</u>

Solving the wave PDE on string underdamped case $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L,t>0$$

Boundary conditions

$$u(0,t)=0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t=0$

$$u_t(x,0)=0$$
$$u(x,0)=f(x)$$

Using

$$f(x)=\begin{cases}\frac{3h}{L}x & 0<x<\frac{L}{3}\\ h & \frac{L}{3}<x<L\end{cases}$$
$$b=2\pi$$
$$c=4$$
$$L=3$$
$$h=\frac{1}{10}$$

Hence the PDE becomes $u_{tt} + 2\pi u_t = 16u_{xx}$. The general solution to the above PDE was given in problem 6.1.1.23 on page 979. The eigenvalues are given as

$$\lambda_n = \left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n = 0,1,2,\cdots$$

And the discriminant is $b^2 - 4c^2\lambda_n = b^2 - 4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 = b^2 - (4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n=0$ this gives $b^2 - \left(\frac{4}{3}\pi\right)^2$. But $b = 2\pi$. Hence discriminant is positive for $n=0$. This means this is critically damped in first mode. Using the the solution for this case from the above general solution as

$$\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right.\\
&+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}$$

Replacing given values in the above solution results in

$$u(x,t) = e^{-\pi t}\left(\frac{\pi + \frac{1}{2}\sqrt{4\pi^2 - 64\left(\frac{\pi}{6}\right)^2}}{\sqrt{4\pi^2 - 64\left(\frac{\pi}{6}\right)^2}}\right)\frac{2}{3}\left(\int_0^3 f(s)\sin\left(\sqrt{\frac{\pi}{6}}s\right)ds\right)e^{\frac{1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right) \tag{1}$$

$$+ e^{-\pi t}\left(\frac{-\pi + \frac{1}{2}\sqrt{4\pi^2 - 64\left(\frac{\pi}{6}\right)^2}}{\sqrt{4\pi^2 - 64\left(\frac{\pi}{6}\right)^2}}\right)\frac{2}{3}\left(\int_0^3 f(s)\sin\left(\sqrt{\frac{\pi}{6}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right)$$

$$+ \sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{-\pi t}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2 - 4\pi^2 t}\right)\sin\left(\frac{(2n+1)}{6}\right.$$

$$+ \sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{-\pi t}\frac{2\pi\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2 - 4\pi^2 t}\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2 - 4\pi^2}}\sin\left(\frac{(2n+1)\pi}{6}x\right.$$

But

$$\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx = \frac{9}{5\pi^2}$$

And

$$\begin{aligned}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right) &= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx + \frac{1}{10}\int_1^3 \sin\left(\frac{(2n+1)\pi}{6}x\right)dx \\ &= \frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\end{aligned}$$

Hence the solution (1) becomes

$$
\begin{aligned}
u(x,t) &= e^{-\pi t}\left(\frac{\pi+\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}{2\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}\right)\left(\frac{6}{5\pi^2}\right)e^{\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right) \qquad (2)\\
&+e^{-\pi t}\left(\frac{-\pi+\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}{2\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}\right)\left(\frac{6}{5\pi^2}\right)e^{-\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right)\\
&+\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{-\pi t}\cos\left(\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2}t\right)\sin\left(\frac{(2n+1)\pi}{6}x\right)\\
&+\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{-\pi t}\frac{\pi\sin\left(\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2}t\right)}{\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2}}\sin\left(\frac{(2n+1)\pi}{6}x\right)
\end{aligned}
$$

Animation is below

Out[ ]=

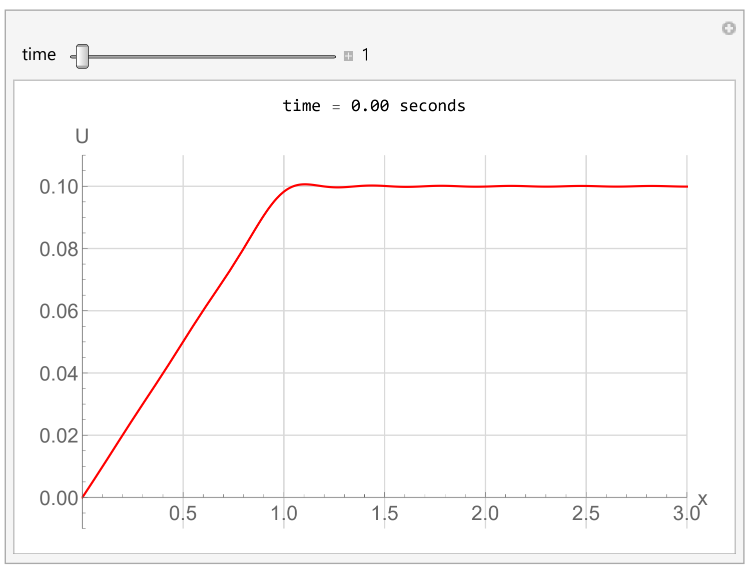


Figure 6.47: Initial state

Source code used for the above

```
In[•]:= padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 16;
z = 12/(5 π^2 (2 n + 1)^2) Sin[((2 n + 1) π)/6];
mySol[x_, t_] = Exp[-π t] ((π + Sqrt[π^2 - 16 (π/6)^2])/(2 Sqrt[π^2 - 16 (π/6)^2])) 6/(5 π^2) Exp[Sqrt[π^2 - 16 (π/6)^2] t] Sin[π/6 x] + Exp[-π t] ((-π + Sqrt[π^2 - 16 (π/6)^2])/(2 Sqrt[π^2 - 16 (π/6)^2])) 6/(5 π^2) Exp[-Sqrt[π^2 - 16 (π/6)^2] t] Sin[π/6 x] +
    Sum[z Exp[- π t] Cos[Sqrt[16 (((2 n + 1) π)/6)^2 - π^2] t] Sin[((2 n + 1) π)/6 x], {n, 1, numberOfTerms}] +
    Sum[z Exp[- π t] (π Sin[Sqrt[16 (((2 n+1) π)/6)^2 - π^2] t])/Sqrt[16 (((2 n+1) π)/6)^2 - π^2] Sin[((2 n + 1) π)/6 x], {n, 1, numberOfTerms}];
```

Figure 6.48: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-0.01, 0.11}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 4, 0.01}];

In[•]:= Manipulate[tab〚i〛, {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.49: Code for animation

6.1.1.27 [361] I.C. at different times, right end free, with source

problem number 361

Added July 2, 2018. This is Example 2 (pde 10) taken from Maple document What_is_New_after_Maple_2018.pdf

Solve

$$-u_{tt} + u(x,t) = u_{xx} + 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)$$

With boundary condition

$$u(0,t) = 0$$
$$\frac{\partial u(1,t)}{\partial x} = 0$$

And initial conditions

$$u(x,0) = x^2 - 2x$$
$$u(x,1) = u(x,\frac{1}{2}) + e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}}$$

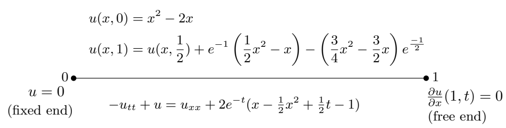


Figure 6.50: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  -D[u[x, t], {t, 2}] + u[x, t] == D[u[x, t], {x, 2}] + 2*Exp[-t]*(x - (1/2)*x^2 + (1/2
bc  = {u[0, t] == 0, Derivative[1, 0][u][1, t] == 0};
ic  = {u[x, 0] == x^2 - 2*x, u[x, 1] == u[x, 1/2] + ((1/2)*x^2 - x)*Exp[-1] - ((3*x^2)/4 - (
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := -diff(u(x, t), t, t) + u(x, t) = diff(u(x, t), x, x)+ 2*exp(-t)*(x-(1/2)*x^2+(1/2)*t-
ic  := u(x, 0) = x^2-2*x,
       u(x, 1) = u(x, 1/2)+((1/2)*x^2-x)*exp(-1)-(3/4*(x^2)-3/2*x)*exp(-1/2);
bc  := u(0, t) = 0, eval(diff(u(x, t), x), {x = 1}) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = -\frac{(x-2)\,(t-2)\,x\,\mathrm{e}^{-t}}{2}$$

Hand solution

Solve

$$-u_{tt} + u = u_{xx} + 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right) \qquad t > 0, 0 < x < 1 \tag{1}$$

Boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ \left.\frac{\partial u}{\partial x}\right|_{x=1} &= 0 \end{aligned}$$

Initial conditions, $t = 0$

$$\begin{aligned} u(x,0) &= x^2 - 2x \\ u(x,1) &= u\left(x, \frac{1}{2}\right) + e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right) e^{\frac{-1}{2}} \end{aligned}$$

Since boundary conditions are homogeneous, we can directly use eigenfunction expansion method. Let the solution be

$$u(x,t) = \sum_{n=1}^{\infty} c_n(t)\, \Phi_n(x) \tag{2}$$

Where $\Phi_n(x)$ are the eigenfunctions of the corresponding homogeneous PDE $-u_{tt} + u = u_{xx}$. Using separation of variables, Let $u = X(x)\,T(t)$. Substituting this back in $-u_{tt} + u = u_{xx}$ gives

$$\begin{aligned} -T''X + XT &= X''T \\ -\frac{T''}{T} + 1 &= \frac{X''}{X} = -\lambda \end{aligned}$$

The eigenvalue ODE is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(1) &= 0 \end{aligned}$$

This is known to have the eigenvalues are $\lambda_n = \left(\frac{n\pi}{2L}\right)^2 = \left(\frac{n\pi}{2}\right)^2$, since $L = 1$. This is for $n = 1, 3, 5, \cdots$ and the eigenfunctions are $\Phi_n(x) = \sin\left(\sqrt{\lambda_n} x\right) = \sin\left(\frac{n\pi}{2}x\right)$. Therefore the solution (2) is

$$\begin{aligned} u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} c_n(t) \sin\left(\frac{n\pi}{2}x\right) \\ &= \sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\, \Phi_n(x) \end{aligned}$$

Substituting this back into (1) gives

$$-\sum_{n=1,3,5,\cdots}^{\infty} c_n''(t)\,\Phi_n(x) + \sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n(x) = \sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n''(x) + \sum_{n=1,3,5,\cdots}^{\infty} b_n(t)\,\Phi_n(x) \tag{3}$$

Where $\sum_{n=1,3,5,\cdots}^{\infty} b_n(t)\,\Phi(x) = 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)$. By orthogonality this becomes

$$\int_0^1 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)\Phi_n(x)\,dx = b_n(t)\int_0^1 \Phi_n^2(x)\,dx$$

$$2e^{-t}\int_0^1 \left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)\sin\left(\frac{n\pi}{2}x\right)dx = \frac{1}{2}b_n(t)$$

But $2e^{-t}\int_0^1 \left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)\sin\left(\frac{n\pi}{2}x\right)dx = \frac{2e^{-t}(8+(t-2)n^2\pi^2)}{n^3\pi^3}$. Hence the above gives

$$b_n(t) = \frac{4e^{-t}(8 + (t-2)\,n^2\pi^2)}{n^3\pi^3}$$

Substituting the above into (3) gives

$$-\sum_{n=1,3,5,\cdots}^{\infty} c_n''(t)\,\Phi_n(x) + \sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n(x) = \sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n''(x) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{4e^{-t}(8 + (t-2)\,n^2\pi^2)}{n^3\pi^3}\Phi_n(x)$$

But $\Phi_n''(x) = -\lambda_n\Phi_n(x)$, hence the above simplifies to

$$-c_n''(t) + c_n(t) = -\lambda_n c_n(t) + \frac{4e^{-t}(8 + (t-2)\,n^2\pi^2)}{n^3\pi^3}$$

$$-c_n''(t) + (1+\lambda_n)\,c_n(t) = \frac{4e^{-t}(8 + (t-2)\,n^2\pi^2)}{n^3\pi^3}$$

$$c_n''(t) - \left(1 + \frac{n^2\pi^2}{4}\right)c_n(t) = -\frac{4e^{-t}(8 + (t-2)\,n^2\pi^2)}{n^3\pi^3}$$

The solution to this second order ODE can be found to be

$$c_n(t) = A_n e^{\frac{-\sqrt{n^2\pi^2+4t}}{2}} + B_n e^{\frac{\sqrt{n^2\pi^2+4t}}{2}} + \frac{16(t-2)\,e^{-t}}{n^3\pi^3}$$

Hence (2) becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty}\left(A_n e^{\frac{-\sqrt{n^2\pi^2+4t}}{2}} + B_n e^{\frac{\sqrt{n^2\pi^2+4t}}{2}} + \frac{16(t-2)\,e^{-t}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right) \tag{4}$$

At $t=0$ the above becomes

$$x^2 - 2x = \sum_{n=1,3,5,\cdots}^{\infty}\left(A_n + B_n - \frac{32}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

Applying orthogonality gives

$$\int_0^1 \left(x^2-2x\right)\sin\left(\frac{n\pi}{2}x\right)dx = \left(A_n + B_n - \frac{32}{n^3\pi^3}\right)\frac{1}{2}$$

But $\int_0^1 \left(x^2-2x\right)\sin\left(\frac{n\pi}{2}x\right)dx = -\frac{16}{n^3\pi^3}$, hence the above gives

$$\begin{aligned} -\frac{32}{n^3\pi^3} &= A_n + B_n - \frac{32}{n^3\pi^3} \\ A_n &= -B_n \end{aligned} \tag{5}$$

Therefore the solution (4) now becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}t}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}t}{2}} + \frac{16(t-2)\,e^{-t}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right) \tag{6}$$

The second initial conditions is $u(x,1) = u\left(x,\frac{1}{2}\right) + e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}}$. At $t=1$ the above gives

$$u(x,1) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}}{2}} - \frac{16e^{-1}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

At $t=\frac{1}{2}$ Eq (6) gives

$$u\left(x,\frac{1}{2}\right) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{4}} - e^{\frac{\sqrt{n^2\pi^2+4}}{4}} - \frac{24e^{-\frac{1}{2}}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

Hence the second initial conditions implies

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}}{2}} - \frac{16}{n^3\pi^3}e^{-1}\right)\sin\left(\frac{n\pi}{2}x\right) \\ &\quad - \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{4}} - e^{\frac{\sqrt{n^2\pi^2+4}}{4}} - \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Or

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right) - \frac{16}{n^3\pi^3}e^{-1}\right)\sin\left(\frac{n\pi}{2}x\right) \\ &\quad - \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right) - \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Simplifying gives

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right) - 2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right) - \frac{16}{n^3\pi^3}e^{-1} + \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = \\ &\qquad e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Applying orthogonality gives

$$A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)-\frac{16}{n^3\pi^3}e^{-1}+\frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\frac{1}{2}=\int_0^1\left(e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right)dx$$

But $\int_0^1\left(e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right)dx=-\frac{8e^{-\frac{1}{2}}-12e^{-1}}{n^3\pi^3}$, hence the above becomes

$$A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)-\frac{16}{n^3\pi^3}e^{-1}+\frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)=-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}$$
$$2A_n\left(\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)\right)+A_n\left(-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}\right)=-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}$$

Since this is true for all $n=1,3,5,\cdots$ then

$$A_n\left(\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)\right)=0$$
$$A_n=1$$

Which implies $\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)=0$ but this is not possible for $n=1,3,5,\cdots$. Something went wrong. I need to look at this again.

6.1.1.28 [362] Right end oscillates

problem number 362

Added December 20, 2018.

Left end fixed, right end oscillates, initially at rest. With source that depends on time and space.

Example 19, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $0<x<\pi$ and $t>0$

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}+(1+t)x$$

With boundary conditions

$$u(0,t)=0$$
$$u(\pi,0)=\sin(t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

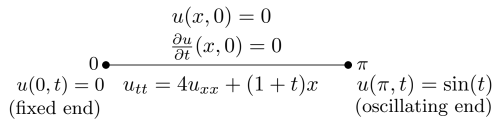


Figure 6.51: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}] + (1 + t)*x;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[0, t] == 0, u[Pi, t] == Sin[t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \frac{x\sin(t)}{\pi} + \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} \sin(xK[1]) \left(\frac{(-1)^{K[1]}(-8\sin(t)K[1]^3 + 2\pi(-t+\cos(2tK[1])-1)(4K[1]^2-1)K[1] + (4(1+}{4\sqrt{2\pi}K[1]^4(4K[1]^2-1)} \right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = 4*diff(u(x, t), x$2)+(1+t)*x;
bc  := u(0,t)=0,u(Pi,t)=sin(t);
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \frac{x\sin(t)}{\pi} + \left(\sum_{n=1}^{\infty} \frac{4\left(-\frac{\left(n^2\sin(t) + \left(-\pi n^2 + \frac{1}{4}\pi\right)\cos(2nt) + \left(n-\frac{1}{2}\right)\left(n+\frac{1}{2}\right)(t+1)\pi\right)n}{2} + \left(n^4 + \frac{1}{4}\pi n^2 - \frac{1}{16}\pi\right)\sin(2nt)\right)}{\pi\left(4n^2-1\right)n^4} \right.$$

6.1.1.29 [363] Perioidic B.C.

problem number 363

Added May 26, 2019.

Taken from midterm 2 sample exam. UMN Math 5587, Fall 2016. Problem 8

Solve for $u(x,t)$ with $-\pi < x < \pi$ and $t > 0$

$$u_{tt} = u_{xx}$$

With boundary conditions

$$\begin{aligned} u(-\pi,t) &= u(\pi,t) \\ u_x(-\pi,0) &= u_x(\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,0) &= x \\ u_t(x,0) &= 0 \end{aligned}$$

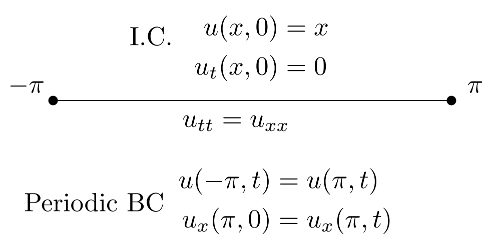


Figure 6.52: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] ;
ic  = {u[x, 0] == x, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[-Pi, t] == u[Pi,t], Derivative[1, 0][u][-Pi, t]  ==Derivative[1, 0][u][Pi, t] };
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
bc  := u(-Pi,t)=u(Pi,t),eval(diff(u(x,t),x),x=-Pi)=eval(diff(u(x,t),x),x=Pi);
ic  := u(x,0)=x,eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

sol=()

6.1.1.30 [364] Mixed B.C.

problem number 364

Added May 26, 2019.

Taken from midterm 2 sample exam. UMN Math 5587, Fall 2016. Problem 10

Solve for $u(x,t)$ with $0 < x < \pi$ and $t > 0$

$$u_{tt} = u_{xx}$$

With boundary conditions $u(0,t) = u_t(\pi,t)$ and initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 1$$

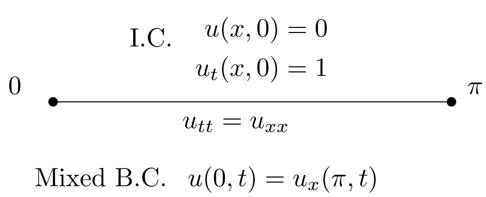


Figure 6.53: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] ;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 1};
bc  = u[0, t] == Derivative[1, 0][u][Pi, t] ;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
bc  := u(0,t)=eval(diff(u(x,t),x),x=Pi);
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

sol=()

6.1.1.31 [365] Left end fixed, right end non-homogeneous Neumann BC. Zero initial conditions

problem number 365

Added January 12, 2020.

Solve for $u(x,t)$ with $0 < x < L$ and $t > 0$

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions $u(0,t) = 0, u_x(L,t) = C$ and zero initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 0$$

For animations use $L = 10, c = 1, C = 5$

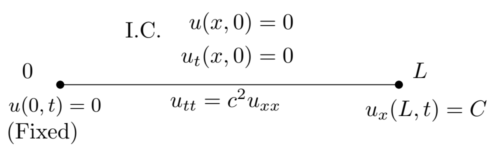


Figure 6.54: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == C0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t},Assumptions->L>0
```

$$\left\{\left\{u(x,t)\to\begin{cases} \text{C0}x+\sum_{K[1]=1}^{\infty}\frac{8(-1)^{K[1]}\text{C0}L\cos\left(\frac{\pi t\sqrt{c^2(2K[1]-1)^2}}{2L}\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\pi^2(1-2K[1])^2} & K[1]\in\mathbb{Z}\land((c<0\land K[1]\geq \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*diff(u(x, t), x$2);
bc  := u(0,t)=0, D[1](u)(L,t)=C0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=C0x-8\left(\sum_{n=0}^{\infty}\frac{C0L(-1)^n\cos\left(\frac{(2n+1)\pi ct}{2L}\right)\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{\pi^2(2n+1)^2}\right)$$

<u>Hand solution</u>

Let

$$u(x,t)=v(x,t)+u_E(x) \tag{2}$$

$u_E(x)$ is the steady state solution which only needs to satisfy the non-homogeneous boundary conditions. At equilibrium $\frac{\partial^2 u(x,t)}{\partial t^2}=0$ and the PDE becomes $\frac{\partial^2 u_E(x,t)}{\partial t^2}=0$ or the ODE $\frac{d^2u_E(x,t)}{dx^2}=0$ with B.C. $u_E(0)=0,u'_E(L)=C$. The solution to this ODE is

$$u_E(x)=c_1x+c_2$$

At first B.C.

$$0=c_2$$

Solution becomes $u_E(x)=c_1x$. At second B.C. $u'_E(x)=c_1=C$. Therefore solution is

$$u_E(x)=Cx$$

Hence

$$u(x,t) = v(x,t) + Cx$$

$v(x,t)$ is the solution to the PDE but with homogeneous B.C. Plugging (2) into (1) gives

$$\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial^2 u_E(x,t)}{\partial t^2} = c\left(\frac{\partial^2 v(x,t)}{\partial x^2} + \frac{\partial^2 u_E(x,t)}{\partial x^2}\right)$$

But $\frac{\partial^2 u_E(x,t)}{\partial x^2} = 0$ and also $\frac{\partial^2 u_E(x,t)}{\partial t^2} = 0$, hence above becomes

$$\frac{\partial^2 v(x,t)}{\partial t^2} = c\frac{\partial^2 v(x,t)}{\partial x^2}$$

With $v(x,t)$ having now homogeneous B.C.

$$\begin{aligned} v(0,t) &= 0 \\ \frac{\partial v(L,t)}{\partial x} &= 0 \end{aligned}$$

And initial conditions given by

$$\begin{aligned} v(x,0) &= u(x,0) - u_E(x) \\ &= 0 - Cx \\ &= -Cx \end{aligned}$$

And

$$\begin{aligned} \frac{\partial v(x,0)}{\partial t} &= \frac{\partial u(x,0)}{\partial t} - \frac{\partial u_E(x)}{\partial t} \\ &= 0 \end{aligned}$$

In summary, the PDE to solve for $v(x,t)$ is

$$\begin{aligned} \frac{\partial^2 v(x,t)}{\partial t^2} &= c\frac{\partial^2 v(x,t)}{\partial x^2} \\ v(0,t) &= 0 \\ \frac{\partial v(L,t)}{\partial x} &= 0 \\ v(x,0) &= -Cx \\ \frac{\partial v(x,0)}{\partial t} &= 0 \end{aligned} \tag{3}$$

Now we solve for PDE (3) for $v(x,t)$ using separation of variables since the boundary conditions in space are now homogeneous. Let $v(x,t) = X(x)T(t)$ and the PDE becomes

$$\frac{1}{c}T''X = X''T$$

Dividing by $XT \neq 0$ gives

$$\frac{1}{c}\frac{T''}{T} = \frac{X''}{X} = -\lambda \tag{4}$$

Where λ is some real positive constant. The space ODE becomes

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Case $\lambda < 0$: Let The solution is

$$X(x) = c_1 \cosh\left(\sqrt{\lambda} x\right) + c_2 \sinh\left(\sqrt{\lambda} x\right)$$

At $x = 0$

$$0 = c_1$$

Hence solution becomes

$$X(x) = c_2 \sinh\left(\sqrt{\lambda} x\right)$$

Taking derivative

$$X'(x) = \sqrt{\lambda} c_2 \cosh\left(\sqrt{\lambda} x\right)$$

Using second boundary conditions gives

$$0 = \sqrt{\lambda} c_2 \cosh\left(\sqrt{\lambda} L\right)$$

Since cosh is zero only when its argument is zero. But we assumed $\sqrt{\lambda}$ not zero here, then $c_2 = 0$ in only other choice. Hence this gives trivial solution. Therefore $\lambda < 0$ is not possible.

Case $\lambda = 0$

$$\begin{aligned} X'' &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Solution is $X(x) = c_1 x + c_2$. First B.C. gives $0 = c_2$. Solution becomes $X(x) = c_1 x$. Second B.C. gives $c_1 = 0$. This gives trivial solution again. Hence $\lambda = 0$ is not possible eigenvalue.

Case $\lambda > 0$: The solution becomes

$$X(x) = B_1 \cos\left(\sqrt{\lambda} x\right) + B_2 \sin\left(\sqrt{\lambda} x\right)$$

AT first B.C.

$$0 = B_1$$

Hence solution becomes

$$X(x) = B_2 \sin\left(\sqrt{\lambda}x\right)$$

Taking derivative

$$X'(x) = \sqrt{\lambda}B_2 \cos\left(\sqrt{\lambda}x\right)$$

At second B.C.

$$0 = \sqrt{\lambda}B_2 \cos\left(\sqrt{\lambda}L\right)$$

To avoid trivial solution, take $\cos\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \cdots$ or

$$\begin{aligned}
\sqrt{\lambda} &= \frac{\pi}{2L}, \frac{3\pi}{2L}, \frac{5\pi}{2L}, \cdots \\
\sqrt{\lambda_n} &= \left(\frac{n\pi}{2L}\right) \qquad n = 1, 3, 5, \cdots \\
&= \frac{(2n+1)\,\pi}{2L} \qquad n = 0, 1, 2, \cdots
\end{aligned} \tag{5}$$

Hence the space solution is

$$X_n(x) = B_n \sin\left(\sqrt{\lambda_n}x\right) \qquad n = 1, 3, 5, \cdots \tag{6}$$

Now we solve the time ODE $T(t)$ from (4), which is

$$T'' + \lambda c T = 0$$

The solution is

$$T_n(t) = D_n \cos\left(\sqrt{\lambda_n c}t\right) + E_n \sin\left(\sqrt{\lambda_n c}t\right)$$

Therefore

$$\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} T_n(t)\, X_n(x) \\
&= \sum_{n=0}^{\infty} \left(D_n \cos\left(\sqrt{\lambda_n c}t\right) + E_n \sin\left(\sqrt{\lambda_n c}t\right)\right) \sin\left(\sqrt{\lambda_n}x\right)
\end{aligned}$$

Where constant B_n merged with the other constants. Now At $t = 0$

$$-Cx = \sum_{n=0}^{\infty} D_n \sin\left(\sqrt{\lambda_n}x\right)$$

Applying orthogonality

$$
\begin{aligned}
-\int_0^L Cx\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx &= D_n\int_0^L \sin^2\left(\frac{(2n+1)\pi}{2L}x\right)dx\\
-C\int_0^L x\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx &= D_n\frac{L}{2}\\
-C\left(\frac{4(-1)^n L^2}{(\pi+2n\pi)^2}\right) &= D_n\frac{L}{2}\\
D_n &= -C\frac{8(-1)^n L}{(\pi+2n\pi)^2}
\end{aligned}
$$

Therefore

$$
v(x,t)=\sum_{n=0}^{\infty}\left(-C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\sqrt{\lambda_n}ct\right)+E_n\sin\left(\sqrt{\lambda_n}ct\right)\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

Taking time derivative

$$
\frac{\partial v(x,t)}{\partial t}=\sum_{n=0}^{\infty}\left(C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\frac{(2n+1)\pi}{2L}\sin\left(\sqrt{\lambda_n}ct\right)+E_n\sqrt{\lambda_n}c\cos\left(\sqrt{\lambda_n}ct\right)\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

At $t=0$

$$
0=\sum_{n=0}^{\infty}E_n\sqrt{\lambda_n}c\sin\left(\sqrt{\lambda_n}x\right)
$$

Hence $E_n=0$. Therefore solution becomes

$$
v(x,t)=\sum_{n=0}^{\infty}-C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\sqrt{\lambda_n}ct\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

Therefore, since $u(x,t)=v(x,t)+u_E(x)$ then

$$
u(x,t)=Cx-C\sum_{n=0}^{\infty}\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\frac{(2n+1)\pi}{2L}\sqrt{c}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right) \tag{7}
$$

This animation runs for 40 seconds for $L=10, c=1, C=5$. The solution becomes

$$
u(x,t)=5x-5\sum_{n=0}^{\infty}\frac{80(-1)^n}{(\pi+2n\pi)^2}\cos\left(\frac{(2n+1)\pi}{20}\sqrt{10}t\right)\sin\left(\frac{(2n+1)\pi}{20}x\right) \tag{7}
$$

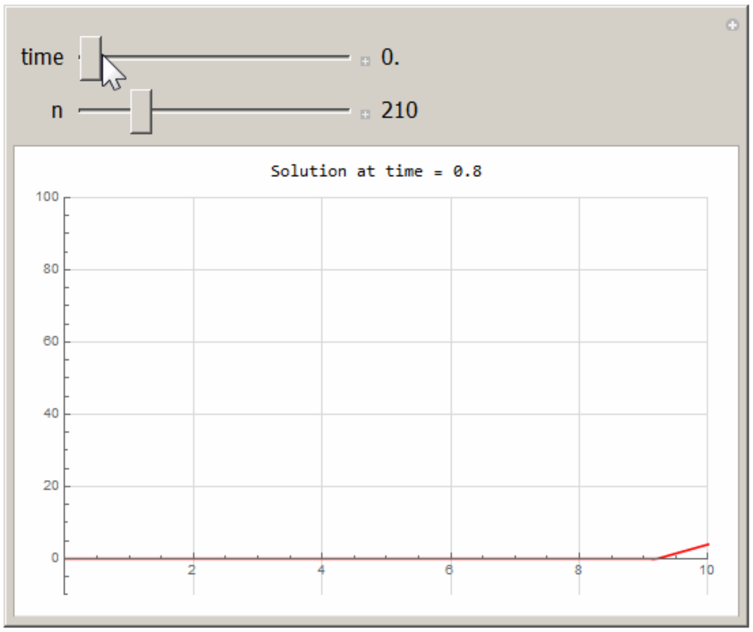


Figure 6.55: snap shot

Code used for the above is

```
L=10;c=1; C0=5;
sqrtLam= (2 n +1) Pi/(2 L);
Manipulate[
mysol=C0 x-C0 Sum[(8.0(-1)^n L)/(Pi+2n \[Pi])^2 Sin[sqrtLam  x] Cos[sqrtLam  Sqrt[c] t],{n,0
];
Grid[{{Row[{"Solution at time = ",i}]},
{Plot[mysol/.t->i,{x,0,10},PlotRange->{{0,10},{-10,100}}%
,ImageSize->500,GridLines->Automatic,GridLinesStyle->LightGray,PlotStyle->Red]}%
}],
{{i,0,"time"},0,100,.1,Appearance->"Labeled"},
{{numberOfTerms,1,"n"},1,1000,1,Appearance->"Labeled"}
]
```

6.1.2 Semi-infinite domain

Local contents

6.1.2.1 [366] Left end fixed, (general case)

problem number 366

Added July 12, 2019 Solve for $u(x,t)$ with $t > 0$ and $x > 0$

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

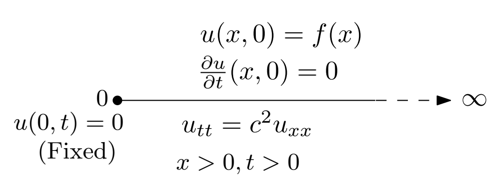


Figure 6.56: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = u[0, t] == 0;
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\left\{\begin{array}{cc}\frac{1}{2}(f(x-ct)+f(ct+x)) & x>ct\\ \frac{1}{2}(f(ct+x)-f(ct-x)) & x\leq ct\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT =
```

sol=()

Hand solution

Solving on semi-infinite domain for $u(x,t)$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<\infty,t>0 \tag{1}$$

With BC

$$u(0,t)=0$$
$$u(\infty,t)<\infty$$

And initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=0$$

Seperation of variables method

Let $u(x,t) = X(x)T(t)$. The PDE in (1) becomes

$$\frac{T''}{c^2T} = \frac{X''}{X} = -\lambda$$

Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(\infty) &< \infty \end{aligned}$$

It is clear that λ can not be negative because it gives a solution that blows up. For $\lambda = 0$, the solution is $X(x) = Ax + B$ and because $X(0) = 0$ this implies $B = 0$. Hence solution is $X(x) = Ax$. And this blows up as x increases unless $A = 0$. Hence $\lambda = 0$ is not valid eigenvalue. Therefore $\lambda > 0$. Let $\lambda = \alpha^2, \alpha > 0$ and the solution becomes

$$X(x) = A_\alpha \cos(\alpha x) + B_\alpha \sin(\alpha x)$$

At $x = 0$ the above gives

$$0 = A_\alpha$$

Therefore the solution becomes

$$X_\alpha(x) = B_\alpha \sin(\alpha x) \qquad \alpha > 0 \tag{1}$$

The time domain ODE becomes

$$\begin{aligned} T'' + c^2\alpha^2 T &= 0 \\ T &= C_\alpha \cos(\alpha ct) + D_\alpha \sin(\alpha ct) \\ T'(t) &= -c\alpha C_\alpha \sin(\alpha ct) + c\alpha D_\alpha \cos(\alpha ct) \end{aligned}$$

And at $t = 0, T'(0) = 0$, hence the above becomes

$$0 = c\alpha D_\alpha$$

Which means $D_\alpha = 0$. Therefore

$$T = C_\alpha \cos(\alpha ct) \qquad \alpha > 0 \tag{2}$$

From (1,2) the complete solution is therefore

$$u(x,t) = \int_0^\infty A_\alpha \cos(\alpha ct) \sin(\alpha x)\, d\alpha \tag{3}$$

Where A_α, C_α are merged into one constant. Now the last initial condition is applied, which is $u(x,0) = f(x)$ to the above which gives

$$f(x) = \int_0^\infty A_\alpha \sin(\alpha x)\, d\alpha$$

Hence

$$A_\alpha = \frac{2}{\pi} \int_0^\infty f(x) \sin(\alpha x)\, dx$$

Using the above in (3) gives the final solution as

$$u(x,t) = \frac{2}{\pi} \int_0^\infty \left(\int_0^\infty f(s) \sin(\alpha s)\, ds \right) \cos(\alpha c t) \sin(\alpha x)\, d\alpha$$

D'Alambert's formula method

For the half line, the D'Alambert's is given by, using $v_0(x) = u_t(x,0)$ as the initial velocity

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct) + f(x-ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} v_0(s)\, ds & x > ct \geq 0 \\ \\ \frac{1}{2}(f(x+ct) - f(ct-x)) + \frac{1}{2c}\int_{ct-x}^{x+ct} v_0(s)\, ds & x \leq ct \end{cases}$$

Hence, since $u_t(x,0) = 0$ in this problem

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct) + f(x-ct)) & x > ct \geq 0 \\ \\ \frac{1}{2}(f(x+ct) - f(ct-x)) & x \leq ct \end{cases}$$

6.1.2.2 [367] Left end fixed with specific initial position

problem number 367

Taken from Mathematica DSolve help pages.

Solve for $u(x,t)$ initial value wave PDE on infinite domain with $t > 0$ and $x > 0$.

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$\begin{aligned} u(x,0) &= \sin^2(x) \qquad \pi < x < 2\pi \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

And boundary conditions $u(0,t)=0$

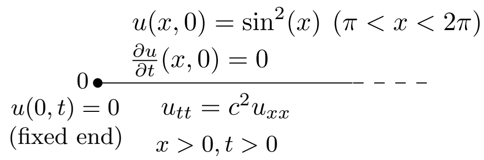


Figure 6.57: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == Piecewise[{{Sin[x]^2, Pi < x < 2*Pi}}], Derivative[0, 1][u][x, 0] == 0};
bc  = u[0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->c>0
```

$$\left\{\left\{u(x,t)\to\ \left\{\begin{array}{l}\frac{1}{2}\left(\left(\left(\begin{cases}\sin^2(ct-x) & \pi<x-ct<2\pi\\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases}\sin^2(ct+x) & \pi<ct+x<2\pi\\ 0 & \text{True}\end{cases}\right.\right.\right.\\ \frac{1}{2}\left(\left(\left(\begin{cases}\sin^2(ct+x) & \pi<ct+x<2\pi\\ 0 & \text{True}\end{cases}\right)-\left(\begin{cases}\sin^2(ct-x) & \pi<ct-x<2\pi\\ 0 & \text{True}\end{cases}\right.\right.\right.\\ \qquad\qquad\qquad\qquad\text{Indeterminate}\end{array}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2);
ic  := u(x,0)= piecewise(Pi<x and x<2*Pi,sin(x)^2),(D[2](u))(x,0)=0;
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{\left(\begin{cases} -\left(\begin{cases} 0 & ct-x\le\pi \\ \sin^2(ct-x) & ct-x<2\pi \\ 0 & 2\pi\le ct-x \end{cases}\right) + \left(\begin{cases} 0 & ct+x\le\pi \\ \sin^2(ct+x) & ct+x<2\pi \\ 0 & 2\pi\le ct+x \end{cases}\right) & x<ct \\ \left(\begin{cases} 0 & -ct+x\le\pi \\ \sin^2(ct-x) & -ct+x<2\pi \\ 0 & 2\pi\le -ct+x \end{cases}\right) + \left(\begin{cases} 0 & ct+x\le\pi \\ \sin^2(ct+x) & ct+x<2\pi \\ 0 & 2\pi\le ct+x \end{cases}\right) & ct<x \end{cases}\right)}{2}$$

Hand solution

Solving on semi-infinite domain

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \qquad t>0, x>0 \\ u(0,t) &= 0 \\ u(x,0) &= f(x) = \sin^2(x) \qquad \pi<x<2\pi \end{aligned} \tag{1}$$

With $k>0$ and $u(x,t)<\infty$ as $x\to\infty$. This means $u(x,t)$ is bounded. The general solution to the above PDE was given in problem 6.1.2.1 on page 1022 as (using the D'Alambert's solution and not the Fourier integral solution)

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct)+f(x-ct)) & x>ct\ge 0 \\ \frac{1}{2}(f(x+ct)-f(ct-x)) & x\le ct \end{cases}$$

But $f(x) = \sin^2(x)$ and the above becomes

$$u(x,t) = \begin{cases} \frac{1}{2}(\sin^2(x+ct) + \sin^2(x-ct)) & x > ct \geq 0 \\ \\ \frac{1}{2}(\sin^2(x+ct) - \sin^2(ct-x)) & x \leq ct \end{cases}$$

But here $f(x)$ is restricted to $\pi < x < 2\pi$. Hence the above solution is modified as follows

$$u(x,t) = \begin{cases} \begin{cases} \frac{1}{2}\sin^2(x+ct) & \pi < x+ct < 2\pi \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{2}\sin^2(x-ct) & \pi < x-ct < 2\pi \\ 0 & \text{otherwise} \end{cases} & x > ct \geq \\ \\ \begin{cases} \frac{1}{2}\sin^2(x+ct) & \pi < x+ct < 2\pi \\ 0 & \text{otherwise} \end{cases} - \begin{cases} \frac{1}{2}\sin^2(ct-x) & \pi < ct-x < 2\pi \\ 0 & \text{otherwise} \end{cases} & 0 < x \leq \end{cases}$$

6.1.2.3 [368] Logan page 115, left end fixed with source

problem number 368

This is problem at page 115, David J Logan textbook, applied PDE textbook.

Falling cable lying on a table that is suddenly removed.

$$u_{tt} = c^2 u_{xx} - g$$

With boundary condition

$$u(0,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

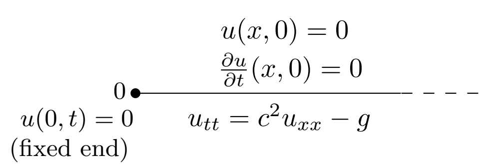


Figure 6.58: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] - g;
bc  = u[0, t] == 0;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \begin{cases} -\frac{gt^2}{2} & ct\leq x \\ \frac{gx(x-2ct)}{2c^2} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic  :=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t),HINT = bou
```

$$u(x,t)=\frac{\left(-c^2t^2+(ct-x)^2\,\theta\left(t-\frac{x}{c}\right)\right)g}{2c^2}$$

6.1.2.4 [369] Left moving boundary condition

problem number 369

Solve for $u(x,t)$ with $t>0$ and $x>0$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t)=g(t)$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=0$$

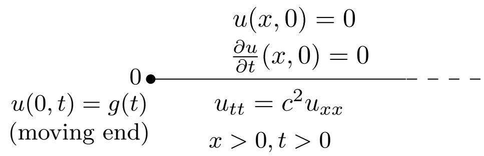


Figure 6.59: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = u[0, t] == g[t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \left\{ \begin{array}{cc} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\leq ct \\ \text{Indeterminate} & \text{True} \end{array} \right.\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=0,D[2](u)(x,0)=0;
bc  := u(0,t)=g(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) =$$

6.1.2.5 [370] moving Left end

problem number 370

Taken from Mathematica DSolve help pages. Initial value problem with a Neumann condition on the half-line.

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = \sin^3(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 1 - e^{-\frac{x}{10}}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

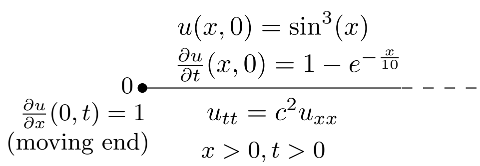


Figure 6.60: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == Sin[x]^3, Derivative[0, 1][u][x, 0] == 1 - E^(-(x/10))};
bc  = Derivative[1, 0][u][0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t},Assumptions
```

$$\left\{\left\{u(x,t) \to \{ \begin{array}{cc} 0 & x > ct \\ g\left(t - \frac{x}{c}\right) & x \leq ct \\ \text{Indeterminate} & \text{True} \end{array} \right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2);
ic  := u(x,0)= sin(x)^3,(D[2](u))(x,0)=1-exp(-x/10);
bc:=(D[1](u))(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{\begin{cases} -c(\sin^3(ct-x)) + c(\sin^3(ct+x)) + 2ct + \\ 10\,\mathrm{e}^{-\frac{ct}{10}-\frac{x}{10}} - 10\,\mathrm{e}^{\frac{ct}{10}-\frac{x}{10}} & ct < x \\ c(\sin^3(ct-x)) + c(\sin^3(ct+x)) - 2c^2t + (2t+2x)\,c + \\ 10\,\mathrm{e}^{-\frac{ct}{10}-\frac{x}{10}} + 10\,\mathrm{e}^{-\frac{ct}{10}+\frac{x}{10}} - 20 & x < ct \end{cases}}{2c}$$

6.1.2.6 [371] I.C. at $t = 1$

problem number 371

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018` Solve

$$u_{tt} = u_{xx}$$

With initial conditions

$$u(x,1) = e^{-(x-6)^2} + e^{-(x+6)^2}$$
$$\frac{\partial u}{\partial t}(x,1) = \frac{1}{2}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

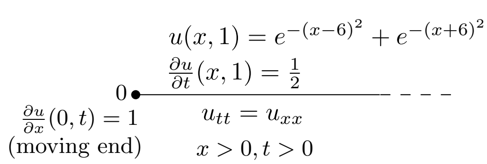


Figure 6.61: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 1] == Sin[x]^3, Derivative[0, 1][u][x, 1] == 1 - E^(-(x/10))};
bc  = Derivative[1, 0][u][0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t}, Assumption
```

$$\left\{\left\{u(x,t)\to \left\{ \begin{array}{cc} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\leq ct \\ \text{Indeterminate} & \text{True} \end{array} \right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
ic  := u(x, 1) = exp(-(x-6)^2)+exp(-(x+6)^2), eval(diff(u(x,t),t),t=1)= 1/2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t)=\frac{t}{2}+\frac{\mathrm{e}^{-(t-x-7)^2}}{2}+\frac{\mathrm{e}^{-(t-x+5)^2}}{2}+\frac{\mathrm{e}^{-(t+x-7)^2}}{2}+\frac{\mathrm{e}^{-(t+x+5)^2}}{2}-\frac{1}{2}$$

6.1.2.7 [372] B.C. at $x = 1$

problem number 372

Added December 20, 2018.

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0)=e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0)=0$$

And Boundary conditions $\frac{\partial u}{\partial x}(1,t)=0$

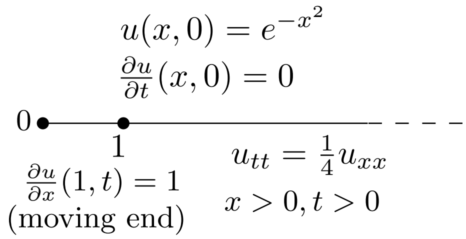


Figure 6.62: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == (1*D[u[x, t], {x, 2}])/4;
ic  = {u[x, 0] == Exp[-x^2], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \left\{ \begin{array}{cc} e^{-x^2} & x-1 \\ \frac{2\int_0^\infty \frac{1}{4}e^{-\frac{1}{4}K[1](K[1]+4i)}\sqrt{\pi}\cos\left(\frac{1}{2}tK[1]\right)\cos((x-1)K[1])\left(\text{Erfc}\left(1-\frac{1}{2}iK[1]\right)+e^{2iK[1]}\,\text{Erfc}\left(\frac{1}{2}iK[1]+1\right)\right)dK[1]}{\pi} & x-1 \\ \text{Indeterminate} & \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2)=(1/4)*(diff(u(x, t), x$2));
bc  := eval(diff(u(x,t),x),x=1)=0;
ic  :=  u(x, 0) = exp(-x^2), eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],u(x,t)) assum
```

$$u(x,t) = \frac{\left(\begin{cases} e^{-\frac{(t-2x)^2}{4}} + e^{-\frac{(t+2x)^2}{4}} & \frac{t}{2} < x-1 \\ e^{-\frac{(t+2x)^2}{4}} + e^{-\frac{(t-2x+4)^2}{4}} & x-1 < \frac{t}{2} \end{cases}\right)}{2}$$

6.1.2.8 [373] Left end free. zero initial velocity (general solution)

problem number 373

Added July 13, 2019.

Solve for $u(x,t)$ with $x>0, t>0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

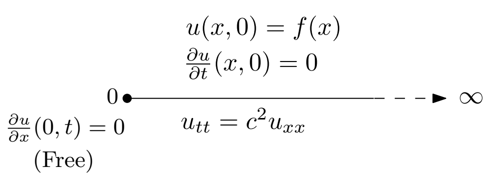


Figure 6.63: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} f(x) & x \geq 0 \land t = 0 \\ \frac{2\int_0^\infty \cos(ctK[1])\cos(xK[1])\int_0^\infty \cos(xK[1])f(x)dxdK[1]}{\pi} & x \geq 0 \land t > 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\left(\begin{cases} f(-ct+x)+f(ct+x) & ct<x \\ f(ct-x)+f(ct+x) & x<ct \end{cases}\right)}{2}$$

6.1.2.9 [374] Left end free. zero initial velocity (Special solution)

problem number 374

Added July 14, 2019.

Solve for $u(x,t)$ with $x>0,t>0$

$$u_{tt}=c^2u_{xx}$$

With initial conditions

$$u(x,0)=f(x)$$
$$\frac{\partial u}{\partial t}(x,0)=0$$

And boundary condition $\frac{\partial u}{\partial x}(0,t)=0$ using

$$c=3$$
$$f(x)=\begin{cases} 1 & 4\leq x\leq 5 \\ 0 & \text{otherwise} \end{cases}$$

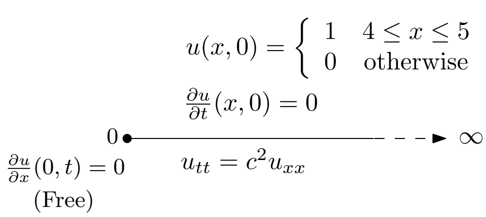


Figure 6.64: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=3;
f[x_]:=Piecewise[{{1,4<x<5},{0,True}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\ \left\{\begin{array}{cc} \frac{1}{2}\left(\left(\left\{\begin{array}{cc} 1 & 4<x-3t<5 \\ 0 & \text{True} \end{array}\right.\right)+\left(\left\{\begin{array}{cc} 1 & 4<3t+x<5 \\ 0 & \text{True} \end{array}\right.\right)\right) & x>3t \\ \frac{1}{2}\left(\left(\left\{\begin{array}{cc} 1 & 4<3t-x<5 \\ 0 & \text{True} \end{array}\right.\right)+\left(\left\{\begin{array}{cc} 1 & 4<3t+x<5 \\ 0 & \text{True} \end{array}\right.\right)\right) & x\leq 3t \end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
c:=3;
f:=piecewise(4<x and x<5,1,true,0);
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\left(\begin{cases}\left(\begin{cases}1 & 4<6t+x<5\\ 0 & \mathit{otherwise}\end{cases}\right)(3t+x)+\left(\begin{cases}1 & 4<x-6t<5\\ 0 & \mathit{otherwise}\end{cases}\right)(-3t+x) & 3t<x\\ \left(\begin{cases}1 & 4<6t+x<5\\ 0 & \mathit{otherwise}\end{cases}\right)(3t+x)+\left(\begin{cases}1 & 4<x<5\\ 0 & \mathit{otherwise}\end{cases}\right)(3t-x) & x<3t\end{cases}\right)}{2}$$

<u>Hand solution</u>

Solve $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ for $x\geq 0, t\geq 0$ with initial conditions $u(x,0)=f(x)=\begin{cases}1 & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}$ and $\frac{\partial u(x,0)}{\partial t}=0$ and boundary condition $\frac{\partial u(0,t)}{\partial x}=0$ (Free end)

The general solution by D'Almbert's is given by

$$u(x,t)=\frac{1}{2}\begin{cases}f(x+ct)+f(x-ct) & x>ct\geq 0\\ f(x+ct)+f(ct-x) & x\leq ct\end{cases}$$

But $f(x)$ is defined for $4\leq x\leq 5$ only, hence the solution becomes

$$u(x,t)=\begin{cases}\begin{cases}\frac{1}{2}f(x+ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}+\begin{cases}\frac{1}{2}f(x-ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases} & x>ct\geq 0\\ \\ \begin{cases}\frac{1}{2}f(x+ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}+\begin{cases}\frac{1}{2}f(ct-x) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases} & 0<x\leq ct\end{cases}$$

Animation is below

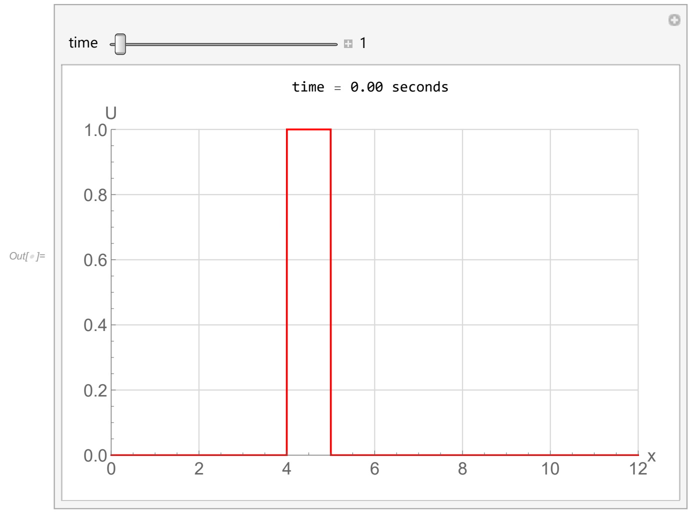


Figure 6.65: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
c = 3;
f[x_] := Piecewise[{{1, 4 < x < 5}, {0, True}}];
mySol[x_, t_] := Piecewise[{
      {(f[x + c t] + f[x - c t]) / 2, x > c t},
      {(f[x + c t] + f[c t - x]) / 2, 0 ≤ x ≤ c t},
      {0, True}}]
```

Figure 6.66: Source code

```
In[]:= tab = Table[
         Grid[{
             {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
             {
               Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, 12},
                    Exclusions → None,
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 12}, {0, 1}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]
             }}],
         {t, 0, 5.5, 0.01}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.67: Code for animation

6.1.2.10 [375] Left end fixed. zero initial velocity (Special solution)

problem number 375

Added January 9, 2020.

Solve for $u(x,t)$ with $x>0,t>0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$\begin{aligned} u(x,0) &= f(x) \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

And boundary condition $u(0,t)=0$ using

$$c = 3$$
$$f(x) = \begin{cases} 1 & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

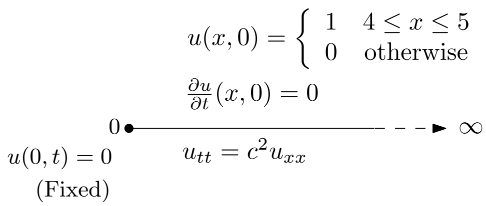


Figure 6.68: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=3;
f[x_]:=Piecewise[{{1,4<x<5},{0,True}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = u[0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(\left(\begin{cases} 1 & 4 < x - 3t < 5 \\ 0 & \text{True} \end{cases}\right) + \left(\begin{cases} 1 & 4 < 3t + x < 5 \\ 0 & \text{True} \end{cases}\right)\right) & x > 3t \\ \frac{1}{2}\left(\left(\begin{cases} 1 & 4 < 3t + x < 5 \\ 0 & \text{True} \end{cases}\right) - \left(\begin{cases} 1 & 4 < 3t - x < 5 \\ 0 & \text{True} \end{cases}\right)\right) & x \le 3t \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
c:=3;
f:=piecewise(4<x and x<5,1,true,0);
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := u(0,t)=0;
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\left(\begin{cases}\left(\begin{cases}1 & 4<6t+x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t+x)-\left(\begin{cases}1 & 4<x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t-x) & x<3t\\ \left(\begin{cases}1 & 4<6t+x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t+x)+\left(\begin{cases}1 & 4<x-6t<5\\ 0 & \text{otherwise}\end{cases}\right)(-3t+x) & 3t<x\end{cases}\right)}{2}$$

Hand solution

Solve $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ for $x\geq 0, t\geq 0$ with initial conditions $u(x,0)=f(x)=\begin{cases}1 & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}$ and $\frac{\partial u(x,0)}{\partial t}=0$ and boundary condition $u(0,t)=0$ (Fixed end)

The general solution by D'Almbert's is given by

$$u(x,t)=\frac{1}{2}\begin{cases}f(x+ct)+f(x-ct) & x>ct\geq 0\\ f(x+ct)-f(ct-x) & x\leq ct\end{cases}$$

But $f(x)$ is defined for $4\leq x\leq 5$ only, hence the solution becomes

$$u(x,t)=\begin{cases}\begin{cases}\frac{1}{2}f(x+ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}+\begin{cases}\frac{1}{2}f(x-ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases} & x>ct\geq 0\\ \begin{cases}\frac{1}{2}f(x+ct) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases}+\begin{cases}-\frac{1}{2}f(ct-x) & 4\leq x\leq 5\\ 0 & \text{otherwise}\end{cases} & 0<x\leq ct\end{cases}$$

Animation is below

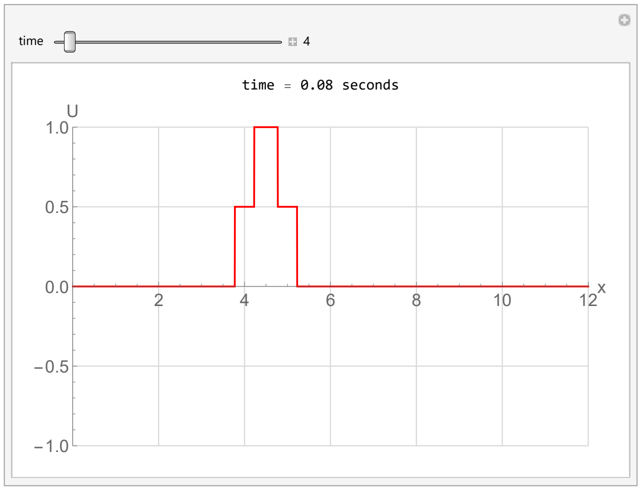


Figure 6.69: Initial state

Source code used for the above

```
In[]:= ClearAll[x, t, n, f, A, B, s, u]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
c = 3;
f[x_] := Piecewise[{{1, 4 < x < 5}, {0, True}}];
g[x_] := 0
u[x_, t_] := Piecewise[{
        {(f[x + c t] + f[x - c t]) / 2 + 1 / (2 c) Integrate[g[s], {s, -x - c t, x + c t}], t ≤ x / c},
        {(-f[c t - x] + f[x + c t]) / 2 + 1 / (2 c) Integrate[g[s], {s, c t - x, x + c t}], t > x / c}, {0, True}}
   ]

In[]:= tab = Table[
        Grid[{
             {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
             {
               Quiet@Plot[Evaluate[u[x, t]], {x, 0, 12},
                    Exclusions → None,
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 12}, {-1, 1}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]
             }}],
        {t, 0, 3, 0.025}];

In[]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 6.70: Source code

6.1.2.11 [376] Left end free. zero initial position (general solution)

problem number 376

Added July 14, 2019.

Solve for $u(x,t)$ with $x>0,t>0$

$$u_{tt}=c^2u_{xx}$$

With initial conditions

$$\begin{aligned}u(x,0)&=0\\ \frac{\partial u}{\partial t}(x,0)&=g(x)\end{aligned}$$

And boundary condition $\frac{\partial u}{\partial x}(0,t)=0$.

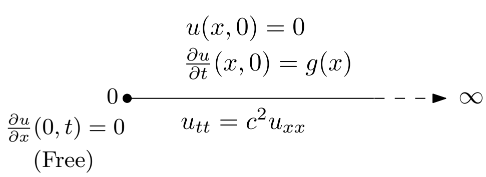


Figure 6.71: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == g[x]};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\ \left\{\begin{array}{cc}0 & x\geq 0\land t=0\\ \frac{2\int_0^\infty\frac{\cos(xK[1])\sin(ctK[1])\int_0^\infty\cos(xK[1])g(x)dx}{cK[1]}dK[1]}{\pi} & x\geq 0\land t>0\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \frac{\begin{cases} \int_{-ct+x}^{ct+x} g(\zeta)\,d\zeta & ct < x \\ \int_0^{ct-x} g(\zeta)\,d\zeta + \int_0^{ct+x} g(\zeta)\,d\zeta & x < ct \end{cases}}{2c}$$

6.1.2.12 [377] Left end free. Non zero initial position and velocity (general solution)

problem number 377

Added July 14, 2019.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

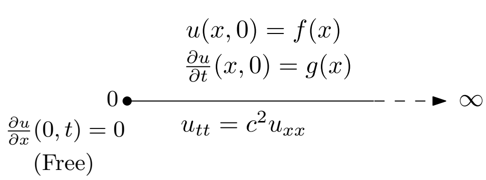


Figure 6.72: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \begin{cases} f(x) & x\geq 0\land t \\ \frac{2\int_0^\infty \frac{\sqrt{\frac{\pi}{2}}\cos(xK[1])\left(c\sqrt{\frac{2}{\pi}}\cos(ctK[1])K[1]\int_0^\infty \cos(xK[1])f(x)dx+\sqrt{\frac{2}{\pi}}\sin(ctK[1])\int_0^\infty \cos(xK[1])g(x)dx\right)}{cK[1]}dK[1]}{\pi} & x\geq 0\land t \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\left(\begin{cases} f(-ct+x)+f(ct+x)+\frac{\int_{-ct+x}^{ct+x} g(\zeta)d\zeta}{c} & ct<x \\ \frac{cf(ct-x)+cf(ct+x)+\int_0^{ct-x} g(\zeta)d\zeta+\int_0^{ct+x} g(\zeta)d\zeta}{c} & x<ct \end{cases}\right)}{2}$$

6.1.2.13 [378] Left end free with source

problem number 378

Added December 20, 2018.

Example 17, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Left end free with initial position and velocity given.

Solve for $u(x,t)$ with $x>0, t>0$

$$\frac{\partial^2 u}{\partial t^2}=9\frac{\partial^2 u}{\partial x^2}+f(x,t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = x^3$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

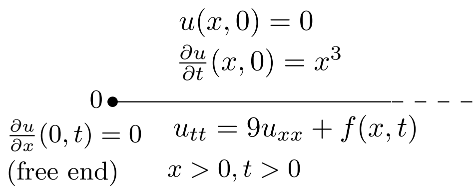


Figure 6.73: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 9*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^3};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \left\{ \begin{array}{cc} tx(9t^2+x^2) & x > 3t \\ \frac{1}{12}(81t^4+54x^2t^2+x^4) & x \leq 3t \end{array} \right\}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = 9*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=x^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \begin{cases} 9t^3x + t\,x^3 & 3t < x \\ \frac{27}{4}t^4 + \frac{9}{2}t^2x^2 + \frac{1}{12}x^4 & x < 3t \end{cases}$$

6.1.3 Infinite domain

Local contents

6.1.3.1 [379] General case. $u_{tt} = u_{xx}$ with $u(x,0) = f(x), u_t(x,0) = g(x)$

problem number 379

Added May 26, 2019.

Solve for $u(x,t)$ for all x and $t > 0$ with $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$

$$u_{tt} = u_{xx}$$

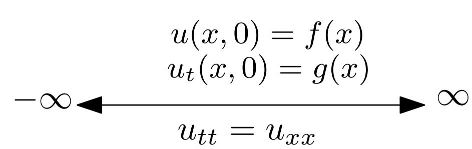


Figure 6.74: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==f[x], Derivative[0,1][u][x,0]==g[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}(f(x-t)+f(t+x))+\frac{1}{2}\int_{x-t}^{t+x} g(K[1])dK[1] & t \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=f(x), eval( diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{\left(\int_{-t+x}^{t+x} g(x1)\, dx1\right)}{2} + \frac{f(-t+x)}{2} + \frac{f(t+x)}{2}$$

6.1.3.2 [380] General case. No IC given. $u_{tt} + u_{xt} = c^2 u_{xx}$

problem number 380

From Mathematica DSolve help pages (slightly modified)

Solve for $u(x,t)$ with $t > 0$ on real line

$$u_{tt} + u_{xt} = c^2 u_{xx}$$

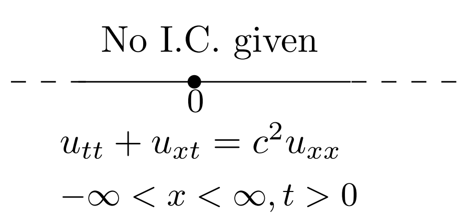


Figure 6.75: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] + D[u[x, t], x, t] == c^2*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t},Assumptions->c>0], 60*10]]
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{\left(\sqrt{4c^2+1}-1\right)x}{2c^2}\right)+c_2\left(\frac{2c^2t+\sqrt{4c^2+1}x+x}{2c^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+diff(u(x,t),t,x)=c^2*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))) assuming t>0,x
```

$$u(x,t)=\_F1\left(\frac{2c^2t+\sqrt{4c^2+1}\,x+x}{2c^2}\right)+\_F2\left(\frac{2c^2t-\sqrt{4c^2+1}\,x+x}{2c^2}\right)$$

6.1.3.3 [381] $u_{tt}=c^2u_{xx}+f(x,t)$, IC at $t=1$,$u(x,1)=g(x),u_t(x,1)=h(x)$

problem number 381

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$u_{tt}=c^2u_{xx}+f(x,t)$$

With initial conditions not at zero

$$u(x,1)=g(x)$$
$$\frac{\partial u}{\partial t}(x,1)=h(x)$$

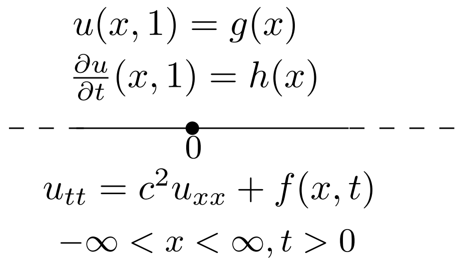


Figure 6.76: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 1] == g[x], Derivative[0, 1][u][x, 1] == h[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(g\left(x-\sqrt{c^2}(t-1)\right)+g\left(\sqrt{c^2}(t-1)+x\right)\right)+\frac{\int_{x-\sqrt{c^2}(t-1)}^{\sqrt{c^2}(t-1)+x} h(K[1])dK[1]}{2\sqrt{c^2}} & t-1\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2)) + f(x, t);
ic  := u(x, 1) = g(x),eval(diff(u(x,t),t),t=1)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t)) assumi
```

$$u(x,t) = \frac{2cg(x) + (2t-2)\,ch(x) + \int_0^{t-1}\int_{(-t+\tau+1)c+x}^{(t-\tau-1)c+x}\left(c^2\tau\left(\frac{d^2}{d\zeta^2}h(\zeta)\right) + c^2\left(\frac{d^2}{d\zeta^2}g(\zeta)\right) + f(\zeta,\tau+1)\right)d\zeta d\tau}{2c}$$

6.1.3.4 [382] No source. $u_{tt} = u_{xx}$, with $u(x,0) = e^{-x^2}, u_t(x,0) = 1$

problem number 382

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$u_{tt} = u_{xx}$$

With initial conditions

$$u(x,0) = e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

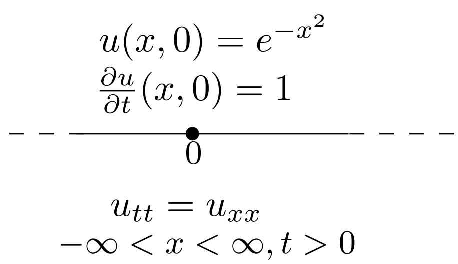


Figure 6.77: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
ic  = {u[x, 0] == E^(-x^2), Derivative[0, 1][u][x, 0] == 1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(e^{-(x-t)^2}+e^{-(t+x)^2}\right)+t & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t$2) = diff(u(x,t), x$2);
ic  := u(x, 0) = exp(-x^2), (D[2](u))(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = t + \frac{e^{-(t-x)^2}}{2} + \frac{e^{-(t+x)^2}}{2}$$

6.1.3.5 [383] With source term. $u_{tt} = u_{xx} + m$

problem number 383

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$u_{tt} = u_{xx} + m$$

With initial conditions

$$u(x,0) = \sin x - \frac{\cos 3x}{e^{\frac{abs(x)}{6}}}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

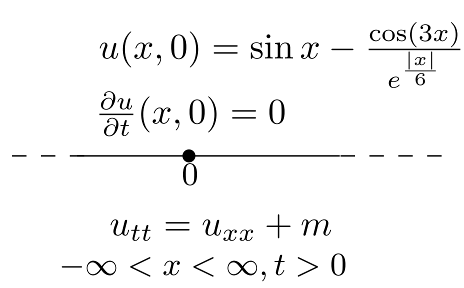


Figure 6.78: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + m};
ic  = {u[x, 0] == Sin[x] - Cos[3*x]/E^(Abs[x]/6), Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(mt^2 - e^{-\frac{|t-x|}{6}}\cos(3t-3x) - e^{-\frac{|t+x|}{6}}\cos(3(t+x)) + 2\cos(t)\sin(x)\right) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2) + m;
ic  := u(x, 0) = sin(x) - cos(3*x)/exp(abs(x)/6), (D[2](u))(x, 0) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = \frac{\left(-\cos(3t-3x)\,\mathrm{e}^{\frac{|t+x|}{6}} + \left(-\cos(3t+3x) + (m\,t^2 - \sin(t-x) + \sin(t+x))\,\mathrm{e}^{\frac{|t+x|}{6}}\right)\mathrm{e}^{\frac{|t-x|}{6}}\right)\mathrm{e}^{-\frac{|t-}{6}}}{2}$$

6.1.3.6 [384] non-linear (Solitons) $u_t + 6u(x,t)u_x + u_{xxx} = 0$

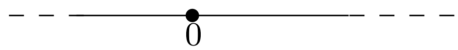
problem number 384

This was first solved analytically by (Krvskal, Zabrsky 1965).

Solve

$$u_t + 6u(x,t)u_x + u_{xxx} = 0$$

No I.C. given



$$u_t + 6u + u_{xxx} = 0$$
$$-\infty < x < \infty, t > 0$$

Figure 6.79: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 6*u[x, t]*D[u[x, t], x] + D[u[x, t], {x, 3}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{12{c_1}^3\tanh^2(c_2t+c_1x+c_3)-8{c_1}^3+c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)+6*u(x,t)*diff(u(x,t),x)+diff(u(x,t),x$3)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t)) assuming t>0,x
```

$$u(x,t) = -2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)+\frac{8c_2^3-c_3}{6c_2}$$

Hand solution

Solve

$$u_t + 6u\ u_x + u_{xxx} = 0$$

Assuming special solution $u = f(\xi)$ where $\xi = x - ct$, this PDE is transformed to non-linear first order ODE

$$-c\frac{f^2}{2} + f^3 + \frac{1}{2}\left(\frac{df}{d\xi}\right)^2 = 0$$

The above is solved analytically (Krvskal, Zabrsky 1965) and the solution is

$$f(x,c,t) = \left(\frac{1}{2}c\right)\operatorname{sech}^2\left(\frac{\sqrt{c}}{2}(x-ct)\right)$$

Tall waves move fast but have smaller period, while short wave move slow. Tall wave pass through short wave and leave in same shape they entered. Here are two animations and the above solution. This first animation has one tall wave passing though short wave

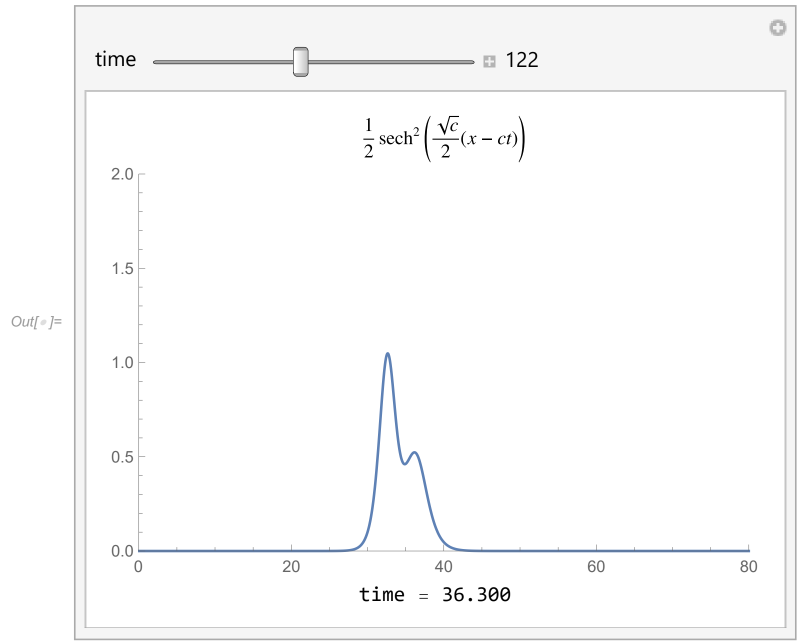


Figure 6.80: Screen shot

Source code used for the above

```
f[x_, c_, t_] := 1/2 c Sech[Sqrt[c]/2 (x - c t)]^2;
tab = Table[Grid[{
          {
            Plot[
              If[t > 20, f[x, 2, t - 20] + f[x, 1, t], f[x, 1, t]], {x, 0, 80},
              PlotRange → {{0, 80}, {0, 2}},
              ImageSize → 400,
              Frame → False,
              PlotLabel → title]
          },
          {Row[{"time = ", NumberForm[t, {5, 3}]}]}
        }
      ], {t, 0, 80, .3}
    ];
```

Figure 6.81: Source code

This animation shows three waves interacting

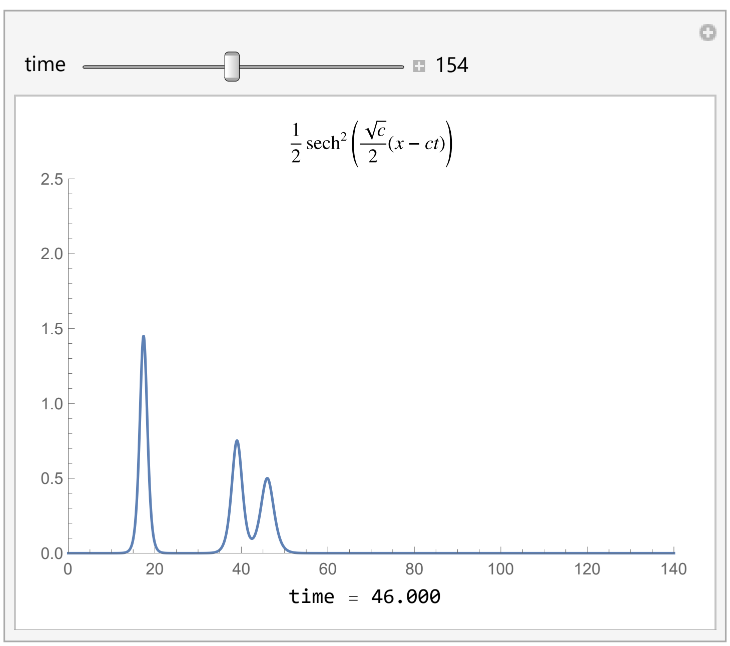


Figure 6.82: Screen shot

Source code used for the above

```
In[ ]:= tab = Table[Grid[{
            {
              Plot[
                If[t > 40, f[x, 2.9, t - 40] + f[x, 1.5, t - 20] + f[x, 1, t],
                  If[t > 20, f[x, 1.5, t - 20] + f[x, 1, t],
                    f[x, 1, t]
                  ]
                ], {x, 0, 140},
                PlotRange → {{0, 140}, {0, 2.5}},
                ImageSize → 400,
                Frame → False,
                PlotLabel → title]
            },
            {Row[{"time = ", NumberForm[t, {5, 3}]}]}
          }
        ],
        {t, 0.1, 100, .3}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[ ]:= Export["anim_2.gif", tab, "DisplayDurations" → 0.06]
```

Figure 6.83: Code for animation

6.1.3.7 [385] Inhomogeneous PDE $3u_{xx} - u_{tt} + u_{xt} = 1$

problem number 385

From Mathematica DSolve help pages. Inhomogeneous hyperbolic PDE with constant coefficients.

Solve for $u(x,t)$

$$3u_{xx} - u_{tt} + u_{xt} = 1$$

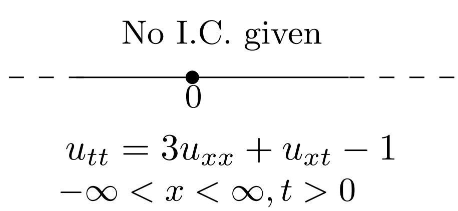


Figure 6.84: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ode = 3*D[u[x, t], {x, 2}] - D[u[x, t], {t, 2}] + D[u[x, t], x, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{1}{6}\left(1+\sqrt{13}\right)x\right)+c_2\left(t+\frac{1}{6}\left(\sqrt{13}-1\right)x\right)+\frac{x^2}{6}\right\}\right\}$$

Maple ✓

```
restart;
pde := 3*diff(u(x, t), x$2) - diff(u(x, t),t$2)+diff(u(x, t), x,t) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, t))),output='rea
```

$$u(x,t)=-\frac{3t^2}{13}+\frac{tx}{13}+\frac{x^2}{13}+\_F1\left(\frac{x}{2}+\frac{(-6t+x)\sqrt{13}}{26}\right)+\_F2\left(t+\frac{(-1+\sqrt{13})\,x}{6}\right)$$

6.1.3.8 [386] Practice exam problem Math 5587

problem number 386

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 10.

Solve for $u(x,t)$ with $u(x,0)=\sin(x)$ and $u_t(x,0)=\cos(x)$

$$u_{tt}=u_{xx}$$

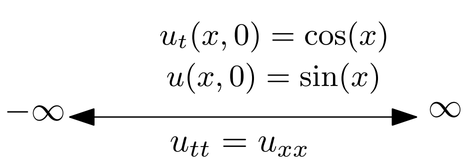


Figure 6.85: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==Sin[x], Derivative[0,1][u][x,0]==Cos[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\left\{\begin{array}{cc}\sin(t+x) & t\geq 0\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=sin(x), eval( diff(u(x,t),t),t=0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\sin(t+x)$$

6.1.3.9 [387] Practice exam problem Math 5587

problem number 387

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 11.

Solve for $u(x,t)$ with $u(x,0)=x^2$ and $u_t(x,0)=x$

$$u_{tt}=u_{xx}$$

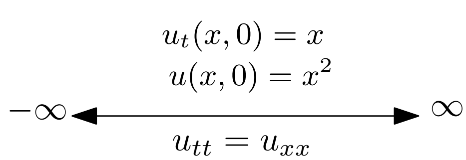


Figure 6.86: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==x^2, Derivative[0,1][u][x,0]==x};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\left\{\begin{array}{cc} t^2+xt+x^2 & t\geq 0 \\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=x^2, eval( diff(u(x,t),t),t=0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=t^2+tx+x^2$$

6.1.3.10 [388] Practice exam problem Math 5587

problem number 388

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 12.

Solve for $u(x,t)$ with $u(x,0)=0$ and $u_t(x,0)=\frac{4x}{x^2+1}$

$$u_{tt}=u_{xx}$$

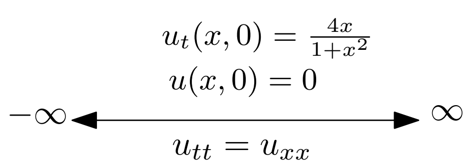


Figure 6.87: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==0, Derivative[0,1][u][x,0]==4*x/(x^2+1)};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \log\left((t+x)^2+1\right)-\log\left((t-x)^2+1\right) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=4*x/(x^2+1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = -\ln\left(t^2-2tx+x^2+1\right)+\ln\left(t^2+2tx+x^2+1\right)$$

6.1.3.11 [389] zero initial velocity

problem number 389

Added January 9, 2020

Solve for $u(x,t)$ with $u(x,0)=\frac{1}{x^2+1}$ and $u_t(x,0)=0$

$$u_{tt}=u_{xx}$$

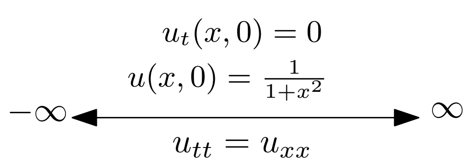


Figure 6.88: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==1/(x^2+1), Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\begin{cases}\frac{t^2+x^2+1}{t^4-2(x^2-1)t^2+(x^2+1)^2} & t\geq 0\\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=1/(1+x^2), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\frac{1}{2\left(-t+x\right)^2+2}+\frac{1}{2\left(t+x\right)^2+2}$$

Hand solution

Solve the wave equation $u_{tt}=u_{xx}$ for infinite domain $-\infty<x<\infty$ with initial position $u(x,0)=\frac{1}{1+x^2}$ and zero initial velocity $g(x)=0$.

The solution for wave PDE $u_{tt}=a^2u_{xx}$ on infinite domain can be given as either series solution, or using D'Alembert solution. Using D'Alembert, the solution is

$$u(x,t)=\frac{1}{2}(f(x-at)+f(x+at))+\frac{1}{2a}\int_{x-at}^{x+at}g(s)\,ds$$

But here $c=1$ and $g(x)$ is zero. Therefore the above simplifies to

$$u(x,t)=\frac{1}{2}(f(x-t)+f(x+t))$$

Since $f(x)=\frac{1}{1+x^2}$, the above becomes

$$u(x,t)=\frac{1}{2}\left(\frac{1}{1+\left(x-t\right)^2}+\frac{1}{1+\left(x+t\right)^2}\right)$$

Animation is below

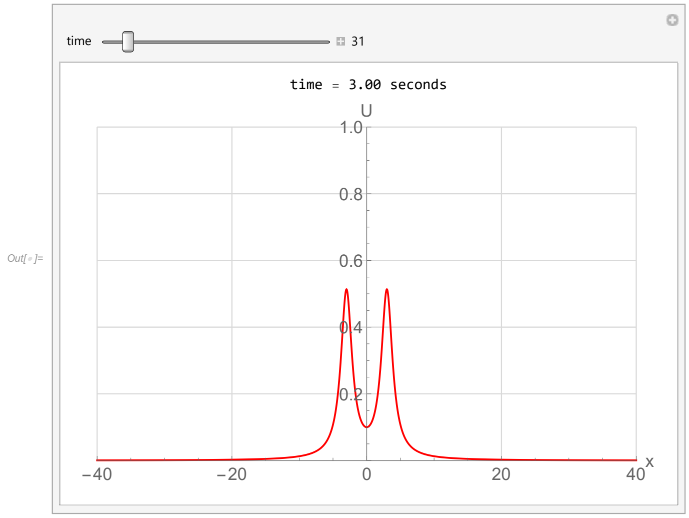


Figure 6.89: Initial state

Source code used for the above

```
In[=]:= ClearAll[x, t, u]
        padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
        fLeft[x_, t_] := 1 / (1 + (x + t) ^2)
        fRight[x_, t_] := 1 / (1 + (x - t) ^2)
        u[x_, t_] := 1 / 2 (fRight[x, t] + fLeft[x, t]);

In[=]:= tab = Table[
           Grid[{
               {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
               {
                 Quiet@Plot[Evaluate[u[x, t]], {x, -40, 40},
                        Exclusions → None,
                       BaseStyle → 15,
                       ImageMargins → 3,
                       PerformanceGoal → "Quality",
                       PlotRange → {{-40, 40}, {0, 1}},
                       ImageSize → 500,
                       AxesLabel → {"x", "U"},
                       GridLines → Automatic,
                       GridLinesStyle → LightGray,
                       PlotStyle → Red
                    ]
               }}],
           {t, 0, 40, 0.1}];

In[=]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 6.90: Source code

6.1.3.12 [390] zero initial velocity

problem number 390

Added January 10, 2020

Solve for $u(x,t)$ with $u(x,0)=\sin(x)$ from $-\pi<x<\pi$ and zero everywhere else and $u_t(x,0)=0$

$$u_{tt}=u_{xx}$$

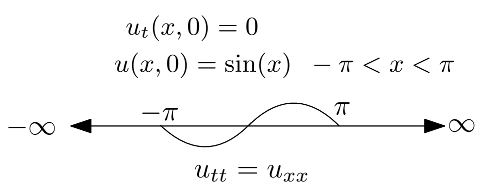


Figure 6.91: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
ic = {u[x, 0] == Piecewise[{{Sin[x], -Pi < x < Pi}, {0, True}}], Derivative[0, 1][u][x, 0] =
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \left\{ \quad \frac{1}{2}\left(\left(\left\{ \begin{array}{cc} -\sin(t-x) & t<x+\pi\land x<t+\pi \\ 0 & \text{True} \end{array}\right.\right)+\left(\left\{ \begin{array}{cc} \sin(t+x) & -\pi<t+x< \\ 0 & \text{True} \end{array}\right.\right.\right.\right.\right.$$

$$\text{Indeterminate}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)= piecewise(-Pi<x and x<Pi,sin(x), true,0) , D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = -\frac{\left(\begin{cases} 0 & -t+x \leq -\pi \\ \sin(t-x) & -t+x < \pi \\ 0 & \pi \leq -t+x \end{cases}\right)}{2} + \frac{\left(\begin{cases} 0 & t+x \leq -\pi \\ \sin(t+x) & t+x < \pi \\ 0 & \pi \leq t+x \end{cases}\right)}{2}$$

Hand solution

The solution for wave PDE $u_{tt} = u_{xx}$ on infinite domain using D'Alembert solution with zero initial velocity is

$$\begin{aligned} u(x,t) &= \frac{1}{2}(f(x-t) + f(x+t)) \\ &= \frac{1}{2}(\sin(x-t) + \sin(x+t)) \end{aligned}$$

The following is an animation Here is animation for 10 seconds.

Animation is below

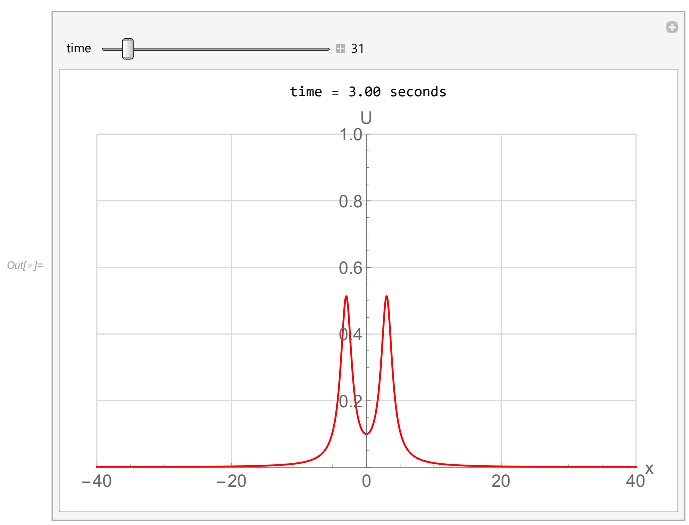


Figure 6.92: Initial state

Source code used for the above

```
In[*]:= ClearAll[x, t, u]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
fLeft[x_, t_] := 1 / (1 + (x + t) ^2)
fRight[x_, t_] := 1 / (1 + (x - t) ^2)
u[x_, t_] := 1 / 2 (fRight[x, t] + fLeft[x, t]);

In[*]:= tab = Table[
        Grid[{
            {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
            {
              Quiet@Plot[Evaluate[u[x, t]], {x, -40, 40},
                    Exclusions → None,
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{-40, 40}, {0, 1}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]
            }}],
        {t, 0, 40, 0.1}];

In[*]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 6.93: Source code

6.1.3.13 [391] General case $u_{tt} = u_{xx}$ with $u(x,0) = \sin x, u_t(x,0) = -2xe^{-x^2}$

problem number 391

Added May 26, 2019.

Taken from Final exam, Math 5587 UMN, Fall 2016.

Solve for $u(x,t)$ for all x and $t > 0$ with $u(x,0) = \sin(x)$ and $u_t(x,0) = -2xe^{-x^2}$

$$u_{tt} = u_{xx}$$

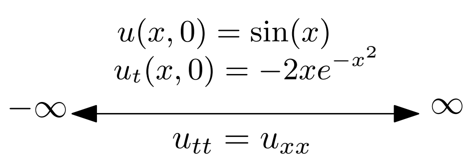


Figure 6.94: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==Sin[x], Derivative[0,1][u][x,0]==-2*x*Exp[-x^2]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(2\cos(t)\sin(x)-e^{-(t-x)^2}+e^{-(t+x)^2}\right) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=sin(x), eval( diff(u(x,t),t),t=0)=-2*x*exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=-\frac{e^{-(t-x)^2}}{2}+\frac{e^{-(t+x)^2}}{2}-\frac{\sin(t-x)}{2}+\frac{\sin(t+x)}{2}$$

6.1.3.14 [392] General case. $u_{tt}=u_{xx}$ dAlembert solution, box function as initial position

problem number 392

Added Sept, 15, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. Problem 2.4.2

Solve for $u(x,t)$ for all x and $t>0$ with $u(x,0)=1$ for $1<x<2$ and zero otherwise. $u_t(x,0)=0$

$$u_{tt}=u_{xx}$$

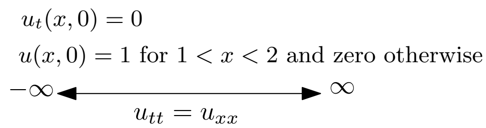


Figure 6.95: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0] == Piecewise[{{1, 1 < x < 2}, {0, True}}], Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(\left(\begin{cases} 1 & 1<x-t<2 \\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases} 1 & 1<t+x<2 \\ 0 & \text{True}\end{cases}\right)\right) & t\geq 0 \\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic:=u(x,0)=piecewise(1<x and x<2,1,true,0),eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\left(\begin{cases} 0 & -t+x\leq 1 \\ \frac{1}{2} & -t+x<2 \\ 0 & 2\leq -t+x\end{cases}\right)+\left(\begin{cases} 0 & t+x\leq 1 \\ \frac{1}{2} & t+x<2 \\ 0 & 2\leq t+x\end{cases}\right)$$

<u>Hand solution</u>

Solve the wave equation $u_{tt}=u_{xx}$ when the initial displacement is the box function $u(0,x)=\begin{cases} 1 & 1<x<2 \\ 0 & \text{otherwise}\end{cases}$, while the initial velocity is zero.

<u>Solution</u>

d'Alembert solution of the wave equation is

$$u(t,x) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds$$

Where c is the wave speed which is $c = 1$ in this problem and $f(x) = u(0,x)$ and $g(x) = u_t(0,x) = 0$ in this problem. Hence the above simpifies to

$$\begin{aligned} u(t,x) &= \frac{1}{2}(f(x-t) + f(x+t)) \\ &= \frac{1}{2}\left(\begin{cases} 1 & 1 < x-t < 2 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 1 & 1 < x+t < 2 \\ 0 & \text{otherwise} \end{cases}\right) \end{aligned}$$

The following is an animation of the solution

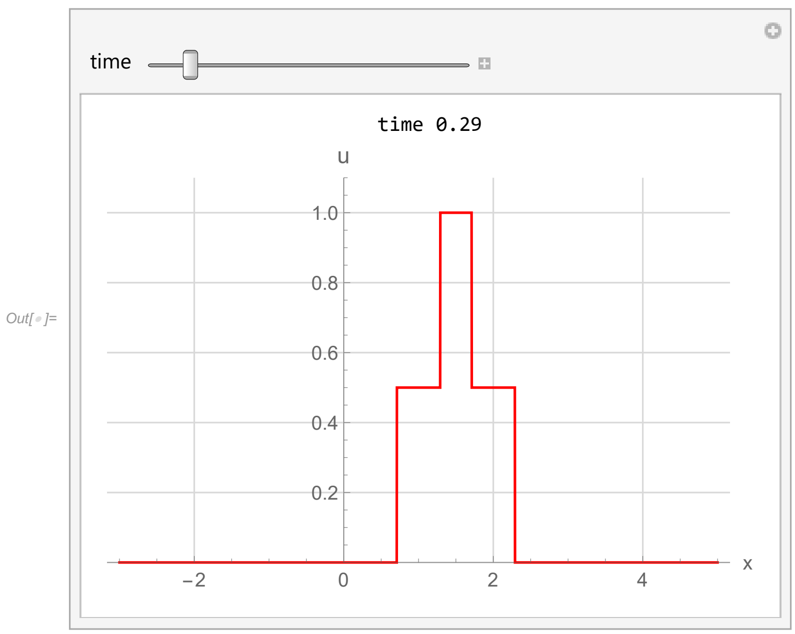


Figure 6.96: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1/2 (Piecewise[{{1, 1 < x - t < 2}, {0, True}}] + Piecewise[{{1, 1 < x + t < 2}, {0, True}}]);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
        {
          Quiet@Plot[u[x, time], {x, -3, 5},
                PlotRange → {All, {0, 1.1}},
                AxesLabel → {Style["x", 12], Style["u", 14]},
                BaseStyle → 12,
                ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                GridLinesStyle → LightGray,
                Exclusions → None
              ]
        }}
   ]
   ,
   {{time, 0, "time"}, 0, 3, .01},
   TrackedSymbols :→ {time}
]
```

Figure 6.97: Source code 2D

6.1.3.15 [393] $u_{tt} = 4u_{xx} + \cos(t)$ dAlembert solution with $u(x,0) = \sin x, u_t(x,0) = \cos x$

problem number 393

Added Oct 8, 2019

Exam 1 problem, math 5587, fall 2019, UMN.

Solve for $u(x,t)$

$$u_{tt} = 4u_{xx} + \cos(t)$$

With initial conditions $u(x,0) = \sin x, u_t(x,0) = \cos x$

$$u_t(x,0) = \sin x$$
$$u(x,0) = \cos x$$
$$-\infty \longleftrightarrow \infty$$
$$u_{tt} = 4u_{xx} + \cos t$$

Figure 6.98: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == 4* D[u[x,t],{x,2}]+ Cos[t];
ic  = {u[x,0]== Sin[x], Derivative[0,1][u][x,0]==Cos[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \sin(x)\cos^2(t)+(\cos(x)\sin(t)-1)\cos(t)-\sin^2(t)\sin(x)+1 & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= 4*diff(u(x,t),x$2)+cos(t);
ic:=u(x,0)=sin(x),D[2](u)(x,0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=-\cos\left(t\right)-\frac{\sin\left(2t-x\right)}{4}+\frac{3\sin\left(2t+x\right)}{4}+1$$

Hand solution

Solve the wave equation $u_{tt}=4u_{xx}+\cos t$ when initial conditions $u(x,0)=\sin x$, $u_t(x,0)=\cos x$

Solution

d'Alembert solution of the wave equation is

$$u(t,x)=\frac{1}{2}(f(x-ct)+f(x+ct))+\frac{1}{2c}\int_{x-ct}^{x+ct}g(s)\,ds+\frac{1}{2c}\int_0^t\int_{x-c(t-s)}^{x+c(t-s)}F(s)\,dyds$$

Where c is the wave speed which is $c=2$ in this problem and $f(x)=u(0,x)=\sin x$ and $g(x)=u_t(0,x)=\cos x$ and the force $F(t)=\cos t$ in this problem. Hence the above simpifies to

$$u(t,x)=\frac{1}{2}(\sin\left(x-2t\right)+\sin\left(x+2t\right))+\frac{1}{4}\int_{x-2t}^{x+2t}\cos\left(s\right)ds+\frac{1}{4}\int_0^t\int_{x-2(t-s)}^{x+2(t-s)}\cos\left(s\right)dyds$$

But $\frac{1}{4}\int_{x-2t}^{x+2t}\cos\left(s\right)ds=\frac{1}{4}[\sin\left(s\right)]_{x-2t}^{x+2t}=\frac{1}{4}(\sin\left(x+2t\right)-\sin\left(x-2t\right))$. Hence the above becomes

$$
\begin{aligned}
u(t,x) &= \frac{1}{2}(\sin(x-2t)+\sin(x+2t)) + \frac{1}{4}(\sin(x+2t)-\sin(x-2t)) + \frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds \\
&= \frac{1}{4}\sin(x-2t) + \frac{3}{4}\sin(x+2t) + \frac{1}{4}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} \cos(s)\,dyds \qquad (1A)
\end{aligned}
$$

But

$$
\begin{aligned}
\frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds &= \frac{1}{4}\int_0^t \cos(s)\int_{x-2(t-s)}^{x+2(t-s)} dyds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(x+2(t-s)-x+2(t-s))\,ds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(2(t-s)+2(t-s))\,ds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(2t-2s+2t-2s)\,ds \\
&= \int_0^t \cos(s)\,(t-s)\,ds \\
&= \int_0^t t\cos(s)\,ds - \int_0^t s\cos(s)\,ds \\
&= t[\sin(s)]_0^t - \int_0^t s\cos(s)\,ds \\
&= t\sin t - \int_0^t s\cos(s)\,ds \qquad (1)
\end{aligned}
$$

Integration by parts. $udv = uv - \int vdu$. Let $u = s, dv = \cos s$, then $du = 1, v = \sin(s)$, then

$$
\begin{aligned}
\int_0^t s\cos(s)\,ds &= [s\sin(s)]_0^t - \int_0^t \sin s ds \\
&= t\sin t - [-\cos s]_0^t \\
&= t\sin t + (\cos t - 1) \qquad (2)
\end{aligned}
$$

Using (2) in (1) gives

$$
\begin{aligned}
\frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds &= t\sin t - (t\sin t + (\cos t - 1)) \\
&= 1 - \cos t
\end{aligned}
$$

Substituting the above in (1A) gives

$$
u(x,t) = \frac{1}{4}\sin(x-2t) + \frac{3}{4}\sin(x+2t) + 1 - \cos t
$$

The following is an animation of the solution

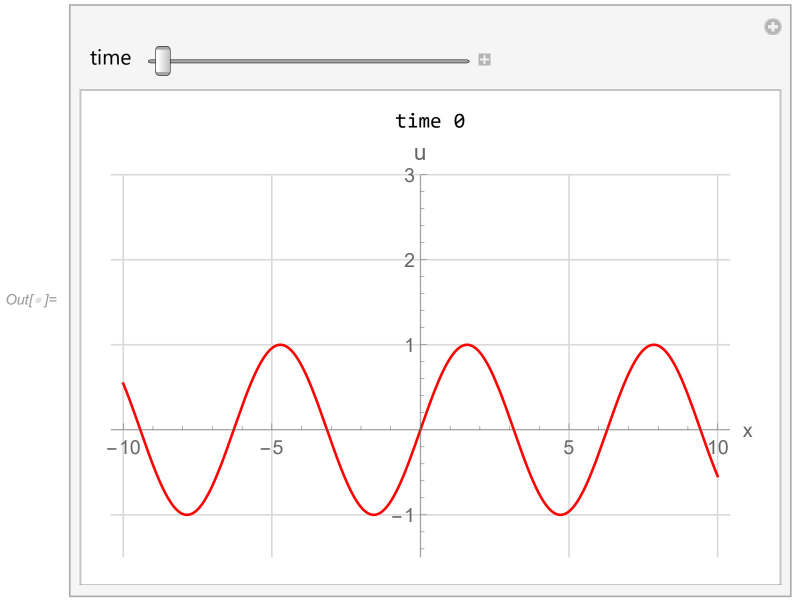


Figure 6.99: snap shot

Source code used for the above

In[ ]:=
```
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1 / 4 * Sin[x - 2 t] + 3 / 4 * Sin[x + 2 t] + 1 - Cos[t];
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
       {
         Plot[u[x, time], {x, -10, 10},
           PlotRange → {All, {-1.5, 3}},
           AxesLabel → {Style["x", 12], Style["u", 14]},
           BaseStyle → 12,
           ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
           GridLinesStyle → LightGray,
           Exclusions → None
         ]
       }}
  ]
  ,
  {{time, 0, "time"}, 0, 30, .01},
  TrackedSymbols :→ {time}
]
```

Figure 6.100: Source code 2D

6.1.3.16 [394] $u_{tt} = c^2 u_{xx}$ dAlembert solution with $u(x,0) = \delta(x-a), u_t(x,0) = 0$

problem number 394

Added January 8, 2020

Problem 6.3.27 Introduction to Partial Differential Equations by Peter Olver, ISBN 9783319020983.

Consider the wave equation $u_{tt} = c^2 u_{xx}$ on the line $-\infty < x < \infty$. Use the d'Alembert formula to solve the initial value problem $u(x,0) = \delta(x-a)\,, u_t(x,0) = 0$.

$$u(x,t) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds \tag{2.82}$$

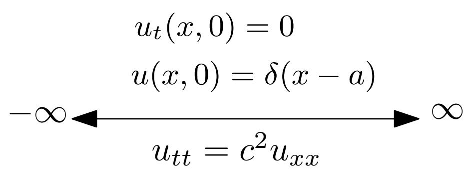


Figure 6.101: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic = {u[x, 0] == DiracDelta[x - a], Derivative[0, 1][u][x, 0] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> a
```

$$\left\{\left\{u(x,t) \to \begin{cases} \delta(x-a) & x \geq 0 \land t = 0 \\ \frac{2\int_0^\infty \cos(aK[1]) \cos\left(\sqrt{c^2} t K[1]\right) \cos(xK[1]) dK[1]}{\pi} & x \geq 0 \land t > 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=Dirac(x-a), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \frac{\delta(-ct+a-x)}{2} + \frac{\delta(ct+a-x)}{2}$$

Hand solution

$$u(x,t) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\, ds \tag{2.82}$$

In (2.82), the function f is the initial conditions and the function g is the initial velocity. Hence the above becomes

$$u(x,t) = \frac{1}{2}(\delta((x-a)-ct) + \delta((x-a)+ct))$$

But $\delta((x-a)-ct)=\delta(x-a-ct)=\delta(x-(a+ct))$ and $\delta((x-a)+ct)=\delta(x-a+ct)=\delta(x-(a-ct))$. Hence the above becomes

$$u(x,t)=\frac{1}{2}\delta(x-(a+ct))+\frac{1}{2}\delta(x-(a-ct)) \tag{1}$$

The above is two half strength delta pulses, one traveling to the left and one traveling to the right from the starting position.

6.1.3.17 [395] system of 2 inhomogeneous linear hyperbolic system with constant coefficients

problem number 395

From Mathematica DSolve help pages

Solve for $u(x,t), v(x,t)$

$$\begin{aligned}\frac{\partial u}{\partial t}&=\frac{\partial v}{\partial x}+1\\ \frac{\partial v}{\partial t}&=-\frac{\partial u}{\partial x}-1\end{aligned}$$

With initial conditions

$$\begin{aligned}u(x,0)&=\cos^2 x\\ v(x,0)&=\sin x\end{aligned}$$

Mathematica ✓

```
ClearAll["Global`*"];
eqns = {D[u[x, t], t] == D[v[x, t], x] + 1, D[v[x, t], t] == -D[u[x, t], x] - 1};
ic  = {u[x, 0] == Cos[x]^2, v[x, 0] == Sin[x]};
sol =  AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{eqns, ic}, {u[x, t], v[x, t]}, {x
```

$$\left\{\left\{u(x,t)\to \sinh(t)\cos(x)+\frac{1}{2}\cosh(2t)\cos(2x)+t+\frac{1}{2}, v(x,t)\to \cosh(t)\sin(x)(2\sinh(t)\cos(x)+1)-\right.\right.$$

Maple ✗

```
restart;
pde1 := diff(u(x, t), t) = diff(v(x, t), x) + 1;
pde2 := diff(v(x, t), t) = -diff(u(x, t), x) - 1;
ic  := u(x, 0) = cos(x)^2, v(x, 0) = sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2, ic], {u(x, t),
```

sol=()

6.2 Wave PDE in 2D

Local contents

6.2.1 Cartesian coordinates

Local contents

6.2.1.1 [396] Rectangular membrane. Fixed on all edges, General solution

problem number 396

Added January 10, 2020.

Solve

$$u_{tt} = c^4(u_{xx} + u_{yy})$$

$$0 < x < L$$
$$0 < y < H$$

Boundary conditions on x

$$u(0, y, t) = 0$$
$$u(L, y, t) = 0$$

And boundary conditions on y

$$u(x,0,t) = 0$$
$$u(x,H,t) = 0$$

Initial conditions

$$u(x,y,0) = f(x,y)$$
$$\frac{\partial u}{\partial t}(x,y,0) = g(x,y)$$

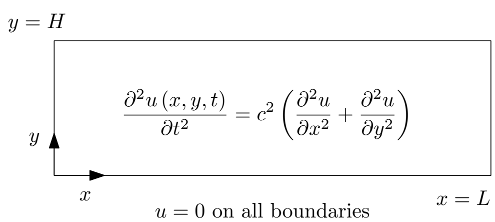


Figure 6.102: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}];
ic = {Derivative[0, 0, 1][u][x, y, 0] == g[x, y], u[x, y, 0] == f[x, y]};
bc = {u[0, y, t] == 0, u[0, H, t] == 0, u[x, 0, t] == 0, u[x, L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t},Assumption
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) = c^2*VectorCalculus:-Laplacian(u(x,y,t),[x,y]);
bc  := u(0,y,t)=0,
       u(L,y,t)=0,
       u(x, 0, t) = 0,
       u(x, H, t) = 0;
ic  := u(x, y, 0) = f(x,y),(D[3](u))(x, y, 0) = g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))assum
sol := subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{4\left(HL\left(\left(\int_0^H ()\sin\left(\frac{\pi m y}{H}\right)dy\right)_{AllSolutions}\right)\sin\left(\frac{\pi\sqrt{H^2n^2+L^2m^2}\,ct}{HL}\right)+\pi\sqrt{H^2n^2+L^2m^2}\,c\left(\left(\cdots\right.\right.\right.}{\sqrt{H^2n^2+L^2m^2}\,\pi \cdots}$$

<u>Hand solution</u>

Assuming $u = X(x)\,Y(y)\,T(t)$ and substituting into the PDE gives

$$\frac{1}{c^2}T''XY = X''YT + Y''XT$$
$$\frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} + \frac{Y''}{Y}$$

Therefore

$$\frac{1}{c^2}\frac{T''}{T} = -\lambda$$
$$\frac{X''}{X} + \frac{Y''}{Y} = -\lambda$$

The time ODE becomes

$$T'' + c^2\lambda T = 0$$

And the space ODE is

$$\frac{X''}{X} + \frac{Y''}{Y} = -\lambda$$

Separating this again gives

$$\frac{X''}{X} = -\lambda - \frac{Y''}{Y}$$

Let the second separation variable be μ. This gives two new ODE's to solve

$$\frac{X''}{X} = -\mu$$
$$-\lambda - \frac{Y''}{Y} = -\mu$$

Or

$$
\begin{aligned}
X'' + \mu X &= 0 \\
Y'' + Y(\lambda - \mu) &= 0
\end{aligned}
$$

Solving for $X(x)$ ODE first, and knowing that only $\mu > 0$ will give non trivial solutions (from the nature of the boundary conditions), gives the solution as

$$X(x) = A\cos\left(\sqrt{\mu}x\right) + B\sin\left(\sqrt{\mu}x\right)$$

Applying B.C. at $x = 0$ results in

$$0 = A$$

Therefore $X(x) = B\sin\left(\sqrt{\mu}x\right)$. Applying the B.C. at $x = L$ gives

$$0 = B\sin\left(\sqrt{\mu}L\right)$$

For non trivial solution

$$
\begin{aligned}
\sqrt{\mu}L &= n\pi \\
\mu &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots
\end{aligned}
$$

Therefore the $X_n(x)$ eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Now, solving the $Y(y)$ ODE above

$$Y'' + Y\left(\lambda - \left(\frac{n\pi}{L}\right)^2\right) = 0$$

The solution is

$$Y_n(y) = A\cos\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\, y\right) + B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\, y\right)$$

Applying first B.C. gives

$$0 = A$$

Hence

$$Y_n(y) = B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\, y\right)$$

Applying the second B.C. gives

$$0 = B \sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2} H\right)$$

For non trivial solution

$$\begin{aligned}
\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2} H &= m\pi \qquad m = 1,2,3,\cdots \\
\lambda_{nm} - \left(\frac{n\pi}{L}\right)^2 &= \left(\frac{m\pi}{H}\right)^2 \\
\lambda_{nm} &= \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots
\end{aligned}$$

Hence the $Y_{nm}(y)$ solution is

$$Y_{nm} = B_{nm} \sin\left(\frac{m\pi}{H} y\right) \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots$$

The time ode $T(t)$ is now solved

$$\begin{aligned}
T''_{nm} + c^2 \lambda_{nm} T_{nm} &= 0 \\
T_{nm}(t) &= A_{nm} \cos\left(c\sqrt{\lambda_{nm}} t\right) + B_{nm} \sin\left(c\sqrt{\lambda_{nm}} t\right)
\end{aligned}$$

Combining all solutions, and merging all constants into two results in

$$\begin{aligned}
u_{nm}(x,y,t) &= X_n(x)\, Y_{nm}(y)\, T_{nm}(t) \\
u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_m(x)\, Y_{mn}(y)\, T_{mn}(t) \\
&= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \cos\left(c\sqrt{\lambda_{nm}} t\right) \\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \sin\left(c\sqrt{\lambda_{nm}} t\right) \qquad (1)
\end{aligned}$$

Initial conditions are now used to find A_{nm}, B_{nm}. At $t = 0$

$$\begin{aligned}
u(x,y,0) &= f(x,y) \\
\frac{\partial u}{\partial t}(x,y,0) &= g(x,y)
\end{aligned}$$

Applying first initial condition to (1) gives

$$f(x,y) = \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{m\pi}{H} y\right)\right) \sin\left(\left(\frac{n\pi}{L}\right) x\right)$$

Applying 2D orthogonality gives

$$
\begin{aligned}
\int_0^L \int_0^H f(x,y) \sin\left(\left(\frac{n\pi}{L}\right) x\right) \sin\left(\frac{m\pi}{H} y\right) \ dxdy &= A_{nm} \left(\frac{L}{2}\right)\left(\frac{H}{2}\right) \\
A_{nm} &= \frac{4}{LH} \int_0^L \int_0^H f(x,y) \sin\left(\left(\frac{n\pi}{L}\right) x\right) \sin\left(\frac{m\pi}{H} y\right) \ dxdy
\end{aligned}
$$

Taking time derivative of (1) gives

$$
\begin{aligned}
\frac{\partial u}{\partial t}(x,y,t) = &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}} A_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \sin\left(c\sqrt{\lambda_{nm}} t\right) \\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}} B_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \cos\left(c\sqrt{\lambda_{nm}} t\right)
\end{aligned}
$$

At $t = 0$ the above becomes

$$
g(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}} B_{nm} \sin\left(\left(\frac{n\pi}{L}\right) x\right) \sin\left(\frac{m\pi}{H} y\right)
$$

Applying 2D orthogonality gives

$$
\begin{aligned}
\int_0^L \int_0^H g(x,y) \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \ dxdy &= B_{nm} \left(\frac{L}{2}\right)\left(\frac{H}{2}\right) \\
B_{nm} &= \frac{4}{LH} \int_0^L \int_0^H g(x,y) \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{H} y\right) \ dxdy
\end{aligned}
$$

Summary of solution

$$
\begin{aligned}
u(x,y,t) = \sum_{n=1}^{\infty} \left(\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{m\pi}{H} y\right) \cos\left(c\sqrt{\lambda_{nm}} t\right)\right) \sin\left(\frac{n\pi}{L} x\right) + \sum_{n=1}^{\infty} \left(\sum_{m=1}^{\infty} B_{nm} \sin\left(\frac{m\pi}{H} y\right) \sin\left(c\sqrt{\lambda_{n}} \right.\right. \\
A_{nm} &= \frac{4}{LH} \int_0^L \int_0^H f(x,y) \sin\left(\left(\frac{n\pi}{L}\right) x\right) \mathrm{si} \\
B_{nm} &= \frac{4}{LH} \int_0^L \int_0^H g(x,y) \sin\left(\left(\frac{n\pi}{L}\right) x\right) \mathrm{si} \\
\lambda_{nm} &= \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2
\end{aligned}
$$

6.2.1.2 [397] Rectangular membrane. Fixed on all edges, zero velocity. Specific example

problem number 397

Added January 10, 2020.

Solve

$$u_{tt} = c^4(u_{xx} + u_{yy})$$

$$\begin{aligned} 0 &< x < L \\ 0 &< y < H \end{aligned}$$

Boundary conditions on x

$$\begin{aligned} u(0, y, t) &= 0 \\ u(L, y, t) &= 0 \end{aligned}$$

And boundary conditions on y

$$\begin{aligned} u(x, 0, t) &= 0 \\ u(x, H, t) &= 0 \end{aligned}$$

Initial conditions

$$\begin{aligned} u(x, y, 0) &= f(x, y) \\ \frac{\partial u}{\partial t}(x, y, 0) &= g(x, y) \end{aligned}$$

Using $L = 1, H = 2, c = \frac{1}{10}, f(x, y) = x\cos(y), g(x, y) = 0$.

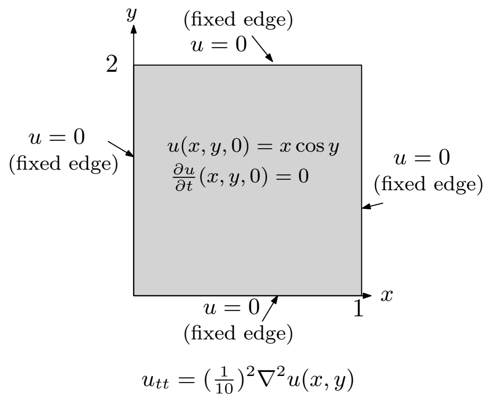


Figure 6.103: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
L=1;H=2;c=1/10;
f[x_,y_]:=x*Cos[y];
g[x_,y_]:=0;
pde = D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}];
ic = {Derivative[0, 0, 1][u][x, y, 0] == g[x, y], u[x, y, 0] == f[x, y]};
bc = {u[0, y, t] == 0, u[0, H, t] == 0, u[x, 0, t] == 0, u[x, L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
L:=1;
H:=2;
c:=1/10;
f:=(x,y)->x*cos(y);
g:=(x,y)->0;
pde := diff(u(x, y, t), t$2) = c^2*VectorCalculus:-Laplacian(u(x,y,t),[x,y]);
bc  := u(0,y,t)=0,
       u(L,y,t)=0,
       u(x, 0, t) = 0,
       u(x, H, t) = 0;
ic  := u(x, y, 0) = f(x,y),(D[3](u))(x, y, 0) = g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))),out
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\left(-(-1)^{n}+\cos(2)\,(-1)^{m+n}\right)m\cos\left(\frac{\pi\sqrt{m^{2}+4n^{2}}\,t}{20}\right)\sin(\pi n x)\sin\left(\frac{\pi m y}{2}\right)}{\left(\pi^{2}m^{2}-4\right)n}$$

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 6.2.1.1 on page 1080 as

$$u(x,y,t)=\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}A_{nm}\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\lambda_{nm}}t\right)\right)\sin\left(\frac{n\pi}{L}x\right)+\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}B_{nm}\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\lambda_{n}}\right.\right.$$

$$A_{nm}=\frac{4}{LH}\int_{0}^{L}\int_{0}^{H}f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\right.$$

$$B_{nm}=\frac{4}{LH}\int_{0}^{L}\int_{0}^{H}g(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\right.$$

$$\lambda_{nm}=\left(\frac{m\pi}{H}\right)^{2}+\left(\frac{n\pi}{L}\right)^{2}\qquad n=1,2,3,\cdots$$

In this problem

$$\begin{aligned} L &= 1 \\ H &= 2 \\ c &= \frac{1}{10} \\ f(x,y) &= x\cos y \\ g(x,y) &= 0 \end{aligned}$$

Hence the solution becomes

$$u(x,y,t) = \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{m\pi}{2}y\right)\cos\left(\frac{1}{10}\sqrt{\lambda_{nm}}t\right)\right)\sin(n\pi x) + \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{m\pi}{2}y\right)\sin\left(\frac{1}{10}\sqrt{\right.$$

$$\begin{aligned} A_{nm} &= 2\int_0^1\int_0^2 x\cos y\sin(n\pi x)\sin\left(\frac{m\pi}{2}y\right. \\ B_{nm} &= 0 \\ \lambda_{nm} &= \left(\frac{m\pi}{2}\right)^2 + (n\pi)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

But

$$\begin{aligned} A_{nm} &= 2\int_0^1\int_0^2 x\cos y\sin(n\pi x)\sin\left(\frac{m\pi}{2}y\right)\,dxdy \\ &= \frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)} \end{aligned}$$

Hence the solution simplifies to

$$\begin{aligned} u(x,y,t) &= \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)}\sin\left(\frac{m\pi}{2}y\right)\cos\left(\frac{1}{10}\sqrt{\left(\frac{m\pi}{2}\right)^2+(n\pi)^2}t\right)\right)\sin(n\pi x \\ &= \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)}\cos\left(\pi\frac{\sqrt{m^2+4n^2}}{20}t\right)\sin\left(\frac{m\pi}{2}y\right)\right)\sin(n\pi x) \end{aligned}$$

Animation is below

Out[∘]=

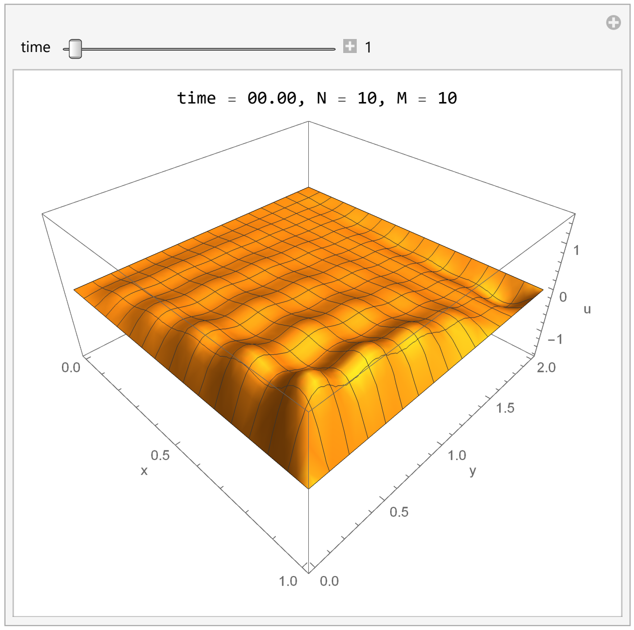


Figure 6.104: Initial state

Source code used for the above

```
ClearAll[x, t, u]
max = 10;
L0 = 1;
H0 = 2;

u[x_, y_, t_] := Sum[Sum[(4 (-1)^n m (-1 + (-1)^m Cos[2]))/(n (m^2 π^2 - 4)) Cos[π Sqrt[m^2 + 4 n^2]/20 t] Sin[m π/2 y] * Sin[n π x], {m, 1, max}],
    {n, 1, max}]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 6.105: Source code

```
tab = Table[
       Grid[{
            {Row[{"time = ", padIt2[t, {4, 2}], ", N = ", 10, ", M = ", 10}]},
            {Plot3D[Evaluate[u[x, y, t]], {x, 0, L0}, {y, 0, H0},
                 PlotRange → {Automatic, Automatic, {-1.65, 1.6}},
                 AxesLabel → {"x", "y", "u"}, BaseStyle → 10,
                 ImageMargins → 5, ViewPoint → {1, -1, 1},
                 BoxRatios → {1, 1, .5}, ImageSize → 400,
                 PerformanceGoal → "Quality"]}}
       ],
       {t, 0, 15, .1}
     ];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[∘]:= Export["anim.gif", tab, "DisplayDurations" → 0.15] (*,"AnimationRepetitions"→Infinity]*)
```

Figure 6.106: Source code

6.2.1.3 [398] All 4 edges fixed, zero initial velocity, Specific example

problem number 398

Added June 17, 2019

Solve for $u(x,y,t)$ with $0<x<L$ and $0<y<H$ and $t>0$.

Solve

$$u_{tt}=c^2\nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned} u(x,0,t)&=0\\ u(0,y,t)&=0\\ u(L,y,t)&=0\\ u(x,H,t)&=0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0)&=3f_1(x)f_2(y)\\ u_t(x,y,0)&=0 \end{aligned}$$

And

$$f_1(x)=\begin{cases} x & 0<x<\frac{L}{2}\\ L-x & \frac{L}{2}<x<L \end{cases}$$

Where

$$f_2(y)=\begin{cases} y & 0<y<\frac{H}{2}\\ H-y & \frac{H}{2}<y<H \end{cases}$$

And $L=2, H=3$ and $c=\frac{1}{3}$.

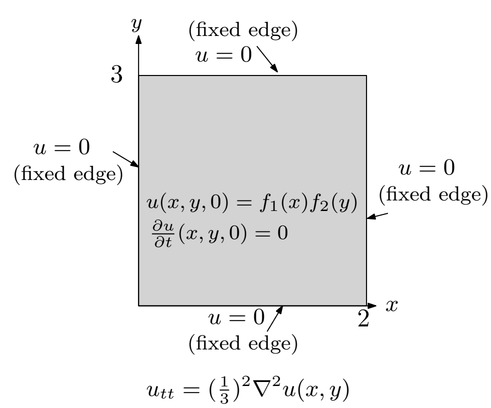


Figure 6.107: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
H=3;
c=1/3;
f1[x_] :=Piecewise[{{x, x < L/2}, {L - x, x > L/2}}];
f2[y_] := Piecewise[{{y, y < H/2}, {H - y, y > H/2}}];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == 3*f1[x]*f2[y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to\left\{\begin{array}{cc}\sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{288\cos\left(\frac{1}{18}\pi t\sqrt{9n^2+4K[3]^2}\right)\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{2}\right)\sin\left(\frac{1}{2}\pi K[3]\right)\sin\left(\frac{1}{3}\pi yK[3]\right)}{n^2\pi^4K[3]^2} & (n|K[3])\in\mathbb{Z}\land n \\ \text{Indeterminate} & \text{Tru}\end{array}\right.\right.\right.$$

Maple ✓

```
restart;
L   := 2;
H   := 3;
c   := 1/3;
f1  := x-> piecewise(x < L/2,x, x > L/2,L - x);
f2  := y-> piecewise(y < H/2,y, y > H/2,H - y);
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=3*f1(x)*f2(y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{288\cos\left(\frac{\pi\sqrt{4m^2+9n^2}\,t}{18}\right)\sin\left(\frac{\pi m}{2}\right)\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi my}{3}\right)\sin\left(\frac{\pi nx}{2}\right)}{\pi^4m^2n^2}$$

Hand solution

The basic solution for this type of PDE was already given in problem 6.2.1.8 on page

1106 as

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right)$$
$$A_{nm}=\frac{4}{LH}\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\,dxdy$$

In this problem

$$L=2$$
$$H=3$$
$$c=\frac{1}{3}$$
$$f(x,y)=3f_1(x)\,f_2(y)$$

And

$$f_1(x)=\begin{cases}x & 0<x<\frac{L}{2}\\ L-x & \frac{L}{2}<x<L\end{cases}$$

And

$$f_2(y)=\begin{cases}y & 0<y<\frac{H}{2}\\ H-y & \frac{H}{2}<y<H\end{cases}$$

This is animation of the above solution using these specific values for for 40 seconds. (Animation will only show in the HTML version)

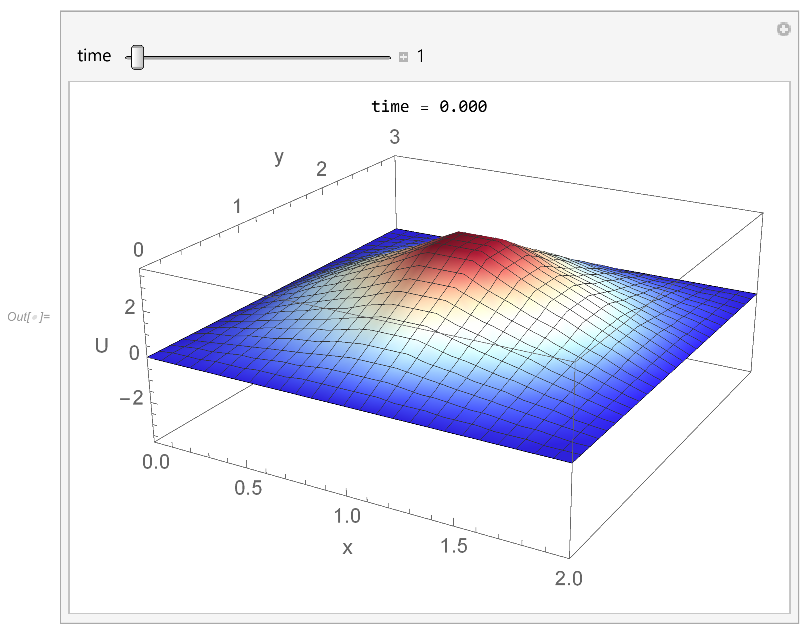


Figure 6.108: Initial state

Source code used for the above

```
ClearAll[x, y, t, mySol]
L = 2;
H = 3;
c = 0.3;
f1[x_] := Piecewise[{{x, 0 ≤ x ≤ L / 2}, {L - x, L / 2 < x ≤ L}}];
f2[y_] := Piecewise[{{y, 0 ≤ y ≤ H / 2}, {H - y, H / 2 < y ≤ H}}];
numberOfTerms = 10;
Anm = N[4/(L H) * Table[Print["n=", n, ", m=", m]; Integrate[3 f1[x] * f2[y] Sin[(n π)/L x] Sin[(m π)/H y], {x, 0, L}, {y, 0, H}],
        {n, 1, numberOfTerms}, {m, 1, numberOfTerms}]];
mySol[x_, y_, t_] = Chop@Sum[Anm[[n, m]] Sin[(n π)/L x] Sin[(m π)/H y] Cos[c t Sqrt[((m π)/H)^2 + ((n π)/L)^2]],
       {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 6.109: Source code

```
In[◦]:= tab = Table[
          Grid[{
             {Row[{"time = ", padIt2[t, {4, 3}]}]},
             {Plot3D[Evaluate[mySol[x, y, t]], {x, 0, L}, {y, 0, H},
                BaseStyle → 15,
                ImageMargins → 3,
                Mesh → 25,
                PerformanceGoal → "Speed",
                BoxRatios → {1, 1, 0.4},
                PlotRange → {{0, L}, {0, H}, {-3.8, 3.8}},
                ImageSize → 500,
                ColorFunctionScaling → True,
                (*ColorFunction→ColorData[{"TemperatureMap",{0,.5}}],*)
                AxesLabel → {"x", "y", "U"},
                ColorFunction → Function[{x, y, z}, ColorData["ThermometerColors"][z]],
                ViewPoint → {1.542, -2.736, 1.258}
              ]
             }}],
          {t, 0, 40, .3}];

In[◦]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[◦]:= Export["anim.gif", tab, "DisplayDurations" → Table[.3, {Length[tab]}]]
```

Figure 6.110: Code used for animation

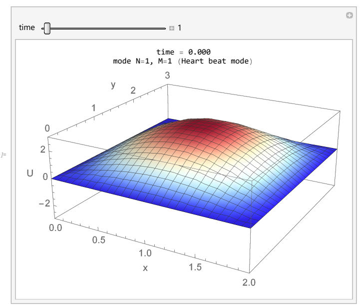
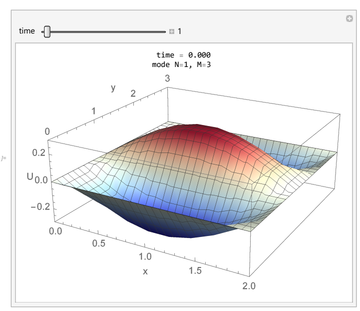
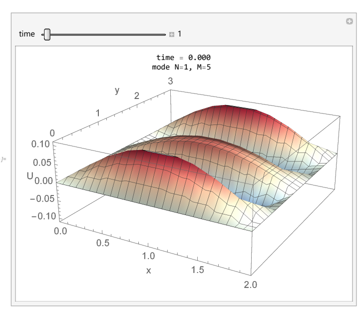
The following shows selected modes. For example, for $n = 1, m = 1$ the solution becomes

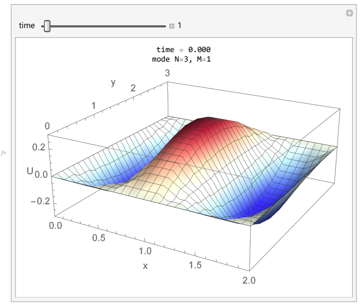
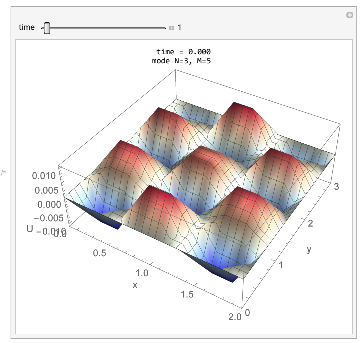
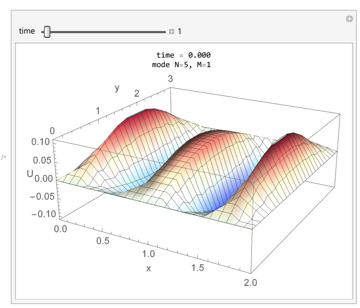
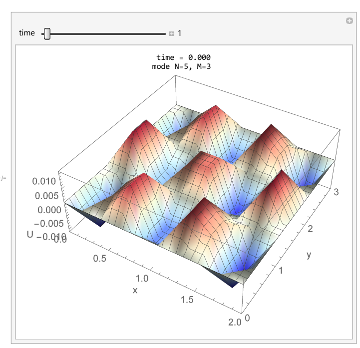
$$u(x,y,t) = A_{1,1}\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{\pi}{H}\right)^2+\left(\frac{1\pi}{L}\right)^2}\,t\right)$$

And for $n=1, m=5$ then the solution becomes

$$u(x,y,t) = A_{1,5}\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{5\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{5\pi}{H}\right)^2+\left(\frac{\pi}{L}\right)^2}t\right)$$

And so on.

n	m	animation
1	1	
1	3	
1	5	

3	1	 time = 0.000 mode N=3, M=1
3	5	 time = 0.000 mode N=3, M=5
5	1	 time = 0.000 mode N=5, M=1
5	3	 time = 0.000 mode N=5, M=3

6.2.1.4 [399] All 4 edges fixed, zero initial velocity, Specific example, delta in center

problem number 399

Added June 18, 2019

Solve for $u(x,y,t)$ with $0<x<L$ and $0<y<H$ and $t>0$.

Solve

$$u_{tt}=c^2\nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned}u(x,0,t)&=0\\u(0,y,t)&=0\\u(L,y,t)&=0\\u(x,H,t)&=0\end{aligned}$$

With initial conditions

$$\begin{aligned}u(x,y,0)&=f(x,y)\\u_t(x,y,0)&=0\end{aligned}$$

And

$$\begin{aligned}L&=20\\H&=30\\c&=\frac{1}{3}\\f(x,y)&=f_1(x)\,f_2(y)\end{aligned}$$

Where $f(x,y)$ is an approximation of delta in the middle of the membrane

$$f_1(x)=\begin{cases}1 & \frac{45}{100}L<x<\frac{55}{100}L\\ 0 & \text{otherwise}\end{cases}$$

And

$$f_2(y)=\begin{cases}1 & \frac{45}{100}H<y<\frac{55}{100}H\\ 0 & \text{otherwise}\end{cases}$$

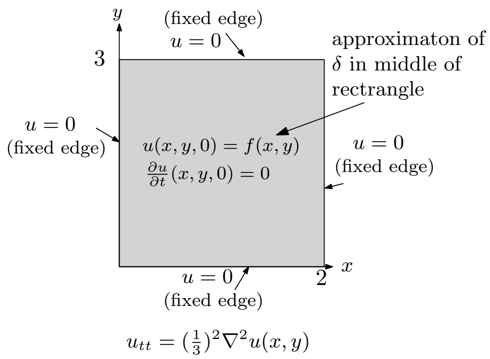


Figure 6.111: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L  = 20;
H  = 30;
c  = 1/3;
f1[x] := Piecewise[{{1,45/100*L <= x<= 55/100*L},{0,True}}];
f2[y] := Piecewise[{{1,45/100*H<= y<= 55/100*H},{0,True}}];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == f1[x]*f2[y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to \left\{ \begin{array}{c} \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\left(\cos\left(\frac{9n\pi}{20}\right)-\cos\left(\frac{11n\pi}{20}\right)\right)\left(\cos\left(\frac{9}{20}\pi K[3]\right)-\cos\left(\frac{11}{20}\pi K[3]\right)\right)\cos\left(\frac{1}{180}\pi t\sqrt{9n^2+4K[3]^2}\right)\sin\left(\frac{n\pi x}{20}\right)\sin\left(\frac{1}{30}\pi y\right.}{n\pi^2 K[3]} \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
L   := 20;
H   := 30;
c   := 1/3;
f1  := x-> piecewise(x>45/100*L and x< 55/100*L,1, true,0);
f2  := y-> piecewise(y>45/100*H and y< 55/100*H,1, true,0);
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=f1(x)*f2(y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{16\left(2\cos\left(\frac{\pi n}{5}\right)-2\cos\left(\frac{\pi n}{10}\right)+1\right)\left(-2\cos\left(\frac{\pi n}{10}\right)+2\cos\left(\frac{\pi n}{20}\right)-1\right)\left(2\cos\left(\frac{\pi n}{10}\right)+2\cos\left(\frac{\pi n}{20}\right.\right.}{}$$

Hand solution

The basic solution for this type of PDE was already given in problem 6.2.1.8 on page 1106 as

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right)$$

$$A_{nm}=\frac{4}{LH}\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\,dxdy$$

In this problem

$$L=20$$
$$H=30$$
$$c=\frac{1}{3}$$
$$f(x,y)=f_1(x)\,f_2(y)$$

Where $f(x,y)$ is an approximation of delta in the middle of the membrane

$$f_1(x)=\begin{cases}1 & \frac{45}{100}L<x<\frac{55}{100}L\\ 0 & \text{otherwise}\end{cases}$$

And

$$f_2(y)=\begin{cases}1 & \frac{45}{100}H<y<\frac{55}{100}H\\ 0 & \text{otherwise}\end{cases}$$

This is animation of the above solution using these specific values for for 40 seconds. (Animation will only show in the HTML version)

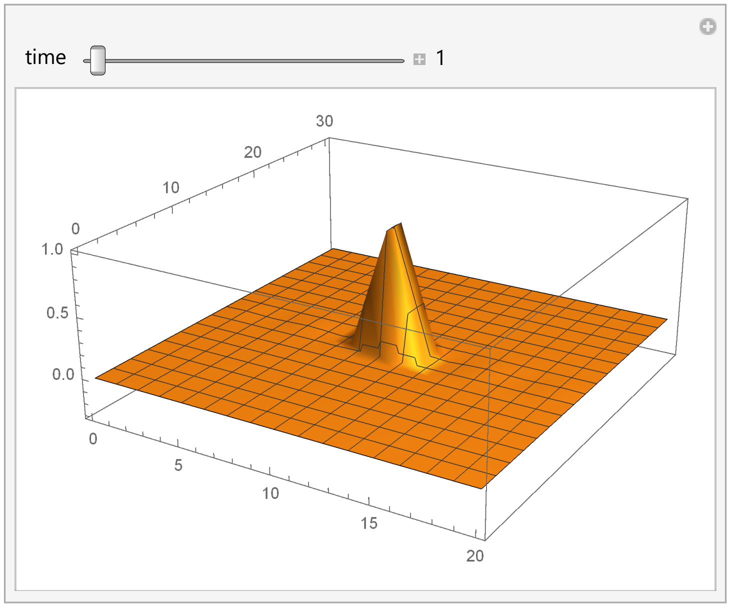


Figure 6.112: Initial state

Source code used for the above

```
ClearAll[x, y, t, n, m]
L = 20;
H = 30;
c = 3 / 10;
f1[x_] := Piecewise[{{1, 45 / 100 L ≤ x ≤ 55 / 100 L}, {0, True}}];
f2[y_] := Piecewise[{{1, 45 / 100 H ≤ y ≤ 55 / 100 H}, {0, True}}];
numberOfTerms = 150;
res[n_, m_] = N@Integrate[f1[x] * f2[y] * Sin[ (n Pi) / L x] * Sin[ (m Pi) / H y], {x, 0, L}, {y, 0, H}]

Anm = Table[ If[Mod[m, 20] == 0, Print["n=", n, ", m=", m]; Pause[0.05]];
      Evaluate[res[n, m]], {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
mySol[x_, y_, t_] = 4/(L H) * Sum[Anm[[n, m]] Sin[(n π)/L x] Sin[(m π)/H y] Cos[c t Sqrt[((m π)/H)^2 + ((n π)/L)^2]], {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 6.113: Source code

```
In[•]:=
    data = Reap@Do[
            Print["n=", n]; Pause[0.05];
            t = n;
            Sow[Table[Evaluate[{x, y, Chop[N[mySol[x, y, t]]]}], {x, 0, L, 1}, {y, 0, H, 1}]]
            ,
            {n, 0, 100, 0.1}
          ];
    data = data[[2, 1]];

In[•]:= Manipulate[
      f = Flatten[data[[n, All]], 1];
      ListPlot3D[f, PlotRange → {{0, L}, {0, H}, {-0.3, .7}}, PerformanceGoal → "Quality", ImageSize → 400, ViewPoint → {1.604, -2.6836, 1.2937}],
      {{n, 1, "time"}, 1, Length@data, 1, Appearance → "Labeled"},
      TrackedSymbols :→ {n}
    ]

In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[(n - 1) * 0.1, {3, 2}], " seconds"}]},
              {
                f = Flatten[data[[n, All]], 1];
                ListPlot3D[f, PlotRange → {{0, L}, {0, H}, {-0.3, .7}}, PerformanceGoal → "Quality", ImageSize → 400, ViewPoint → {1.604, -2.6836, 1.2937}
                ]
              }}],
          {n, 1, Length@data, 2}
        ];

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[0.06, {Length[tab]}]]
```

Figure 6.114: Code used for animation

6.2.1.5 [400] All 4 edges fixed

problem number 400

Taken from Mathematica helps pages on DSolve

Solve for $u(x,y,t)$ with $0<x<1$ and $0<y<2$ and $t>0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

With boundary conditions

$$\begin{aligned} u(x,0,t) &= 0 \\ u(0,y,t) &= 0 \\ u(1,y,t) &= 0 \\ u(x,2,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= \frac{1}{10}(x-x^2)(2y-y^2) \\ \frac{\partial u}{\partial t}(x,y,0) &= 0 \end{aligned}$$

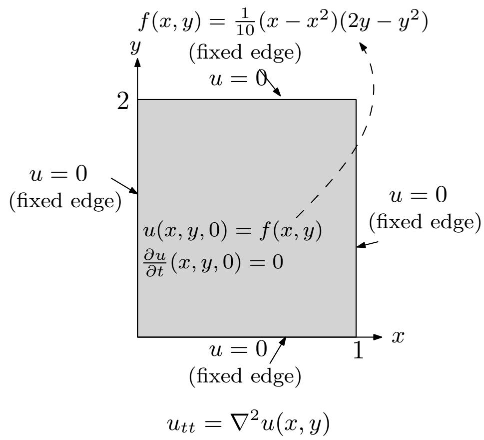


Figure 6.115: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == (1/10)*(x - x^2)*(2*y - y^2), Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[1, y, t] == 0, u[x, 2, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
sol =  Assuming[Element[{n, m}, Integers], FullSimplify[sol]];
```

$$\left\{\left\{u(x,y,t)\to \begin{cases} \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{32(-1+(-1)^n)\left(-1+(-1)^{K[3]}\right)\cos\left(\frac{1}{2}\pi t\sqrt{4n^2+K[3]^2}\right)\sin(n\pi x)\sin\left(\frac{1}{2}\pi yK[3]\right)}{5n^3\pi^6K[3]^3} & K[3]\in\mathbb{Z}\land n\geq \\ \text{Indeterminate} & \text{Tru} \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) =  VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y]);
ic  := u(x,y,0)=(1/10)*(x-x^2)*(2*y-y^2),(D[3](u))(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(1,y,t)=0,u(x,2,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(-\frac{32\left((-1)^{m}+(-1)^{n}-(-1)^{m+n}-1\right)\cos\left(\frac{\pi\sqrt{m^{2}+4n^{2}}\,t}{2}\right)\sin\left(\pi n x\right)\sin\left(\frac{\pi m y}{2}\right)}{5\pi^{6}m^{3}n^{3}}\right)$$

6.2.1.6 [401] All edges fixed (Haberman 8.5.5 (a)

problem number 401

Added Nov 27, 2018.

This is problem 8.5.5 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the initial value problem for membrane with time-dependent forcing and fixed boundaries $u=0$.

$$u_{tt}=c^{2}\nabla^{2}u+Q(x,y,t)$$

If the memberane is rectangle $(0<x<L,0<y<H)$. With initial conditions

$$u(x,y,0)=f(x,y)$$
$$\frac{\partial u}{\partial t}(x,y,0)=0$$

See my HW9, Math 322, UW Madison.

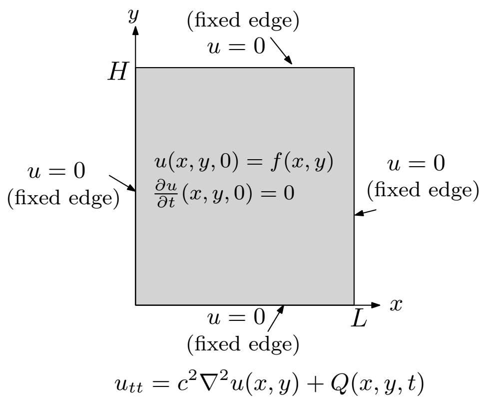


Figure 6.116: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}] + Q[x, y, t];
ic  = {u[x, y, 0] == f[x, y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[0, y, t] == 0, u[L, y, t] == 0, u[x, 0, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}, Assumptio
```

$$\left\{\left\{u(x,y,t)\to\ \{\quad \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}2\sqrt{\frac{1}{H}}\sqrt{\frac{1}{L}}\left(\int_0^t\frac{2\left(\int_0^L\int_0^H Q(x,y,K[4])\sin\left(\frac{\pi xK[1]}{L}\right)\sin\left(\frac{\pi yK[3]}{H}\right)dydx\right)\sin\left(c\pi\sqrt{\frac{K[1]^2}{L^2}+\frac{K[3]^2}{H^2}}(t\right.}{c\pi\sqrt{\frac{HK[1]^2}{L}+\frac{LK[3]^2}{H}}}\right.\right.\right.$$

I

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x,y,t),t$2)=c^2*(diff(u(x,y,t),x$2)+diff(u(x,y,t),y$2))+Q(x,y,t);
bc  := u(0,y,t)=0,u(L,y,t)=0,u(x,0,t)=0,u(x,H,t)=0;
ic  := u(x,y,0)=f(x,y), eval( diff(u(x,y,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t)) assu
```

sol=()

6.2.1.7 [402] 2 edgs fixed, 2 free, zero initial velocity

problem number 402

Taken from Maple PDE help pages. This wave PDE inside square with free to move on left edge and right edge, and top and bottom edges are fixed. It has zero initial velocity, but given a non-zero initial position. Where $0 < x < \pi$ and $0 < y < \pi$ and $t > 0$.

Solve

$$u_{tt} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}u(0, y, t) &= 0\\ \frac{\partial u}{\partial x}u(\pi, y, t) &= 0\\ u(x, 0, t) &= 0\\ u(x, \pi, 0) &= 0\end{aligned}$$

With initial conditions

$$\begin{aligned}\frac{\partial u}{\partial t}(x, y, 0) &= 0\\ u(x, 0) &= xy(\pi - y)\end{aligned}$$

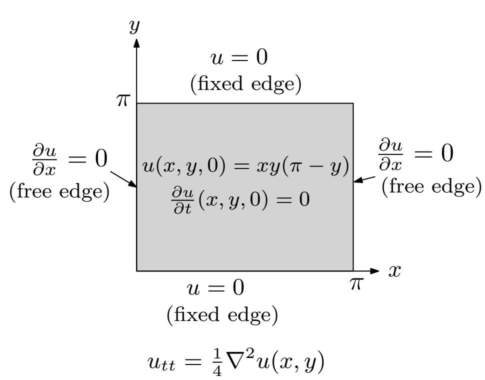


Figure 6.117: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4;
ic  = {Derivative[0, 0, 1][u][x, y, 0] == 0, u[x, y, 0] == x*y*(Pi - y)};
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][Pi, y, t] == 0, u[x, 0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\left\{\sum_{K[3]=1}^{\infty}-\frac{2\left(-1+(-1)^{K[3]}\right)\cos\left(\frac{1}{2}tK[3]\right)\sin(yK[3])}{K[3]^3}+\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}-\frac{8\left(-1+(-1)^{K[1]}\right)\left(-1+(-1)^{K[3]}\right)\cos(x}{\pi^2K[}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, y, t), t, t) = (1/4)*(diff(u(x, y, t), x, x))+(1/4)*(diff(u(x, y, t), y, y)
bc  := (D[1](u))(0, y, t) = 0,
       (D[1](u))(Pi, y, t) = 0,
       u(x, 0, t) = 0,
       u(x, Pi, t) = 0;
ic  := u(x, y, 0) = x*y*(Pi-y),(D[3](u))(x, y, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))),out
sol := subs(n1=m,sol);
```

$$u(x,y,t)=-2\left(\sum_{n=1}^{\infty}\frac{((-1)^n-1)\cos\left(\frac{nt}{2}\right)\sin(ny)}{n^3}\right)+\left(\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{8\left((-1)^m+(-1)^n-(-1)^{m+n}-1\right)\cos(mx}{\pi^2m^2n^3}\right.$$

6.2.1.8 [403] All 4 edges fixed, zero initial velocity, general solution

problem number 403

Added June 16, 2019

Solve for $u(x,y,t)$ with $0<x<L$ and $0<y<H$ and $t>0$.

Solve

$$u_{tt}=c^2\nabla^2u(x,y)$$

With boundary conditions

$$u(x,0,t)=0$$
$$u(0,y,t)=0$$
$$u(L,y,t)=0$$
$$u(x,H,t)=0$$

With initial conditions

$$u(x,y,0)=f(x,y)$$
$$u_t(x,y,0)=0$$

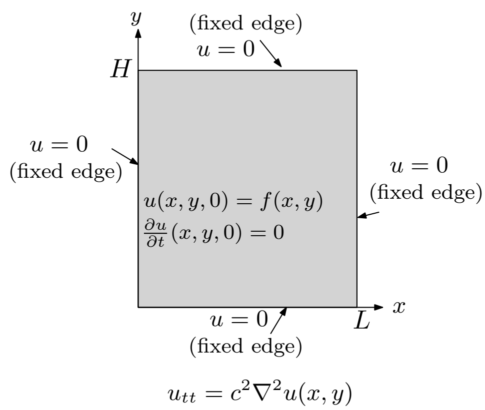


Figure 6.118: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == f[x,y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to \left\{\quad \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\cos\left(\pi t\sqrt{c^2\left(\frac{n^2}{L^2}+\frac{K[3]^2}{H^2}\right)}\right)\left(\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\pi yK[3]}{H}\right)dydx\right)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\pi yK[3]}{H}\right)}{HL}\quad (n\right.\right.\right.$$

Indeterminate

Maple ✗

```
restart;
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=f(x,y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

time expired
Hand solution

Solve for $u(r,\theta,t)$

$$u_{tt} = c^2\nabla^2 u(x,y)$$

With boundary conditions such that all edges are fixed, and initial conditions $u(x,y,0) = f(x,y)$ and initial velocity $g(x,y) = 0$.

Let $u = X(x)\,Y(y)\,T(t)$. Substituting into the above PDE gives

$$\begin{aligned}\frac{1}{c^2}T''XY &= X''YT + Y''XT\\ \frac{1}{c^2}\frac{T''}{T} &= \frac{X''}{X} + \frac{Y''}{Y}\end{aligned}$$

Hence

$$\begin{aligned}\frac{1}{c^2}\frac{T''}{T} &= -\lambda\\ \frac{X''}{X} + \frac{Y''}{Y} &= -\lambda\end{aligned}$$

The time ODE becomes

$$T'' + c^2\lambda T = 0$$

And the space ODE is

$$\begin{aligned}\frac{X''}{X} + \frac{Y''}{Y} &= -\lambda\\ \frac{X''}{X} &= -\lambda - \frac{Y''}{Y}\end{aligned}$$

Using a new separation variable μ gives the following two ODE's

$$\begin{aligned}\frac{X''}{X} &= -\mu\\ -\lambda - \frac{Y''}{Y} &= -\mu\end{aligned}$$

Or

$$
\begin{aligned}
X'' + \mu X &= 0 \\
X(0) &= 0 \\
X(L) &= 0
\end{aligned}
$$

And

$$
\begin{aligned}
Y'' + Y(\lambda - \mu) &= 0 \\
Y(0) &= 0 \\
Y(H) &= 0
\end{aligned}
$$

Solving first for the $X(x)$ ODE, and knowing that μ must be positive only here from the nature of the boundary conditions gives

$$
X = A\cos\left(\sqrt{\mu}x\right) + B\sin\left(\sqrt{\mu}x\right)
$$

Applying B.C. at $x = 0$

$$
0 = A
$$

Hence solution becomes $X(x) = B\sin\left(\sqrt{\mu}x\right)$. Applying the B.C. at $x = L$ gives

$$
0 = B\sin\left(\sqrt{\mu}L\right)
$$

Non trivial solution requires that

$$
\begin{aligned}
\sqrt{\mu}L &= n\pi \qquad n = 1,2,3,\cdots \\
\mu_n &= \left(\frac{n\pi}{L}\right)^2
\end{aligned}
$$

Therefore the eigenfunctions $X_n(x)$ are

$$
X_n(x) = \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1,2,3,\cdots
$$

Solving the $Y(y)$ ODE

$$
Y_n'' + \left(\lambda - \left(\frac{n\pi}{L}\right)^2\right)Y_n = 0 \qquad n = 1,2,3,\cdots
$$

The nature of the boundary conditions on $Y(y)$ suggests that $\left(\lambda - \left(\frac{n\pi}{L}\right)^2\right)$ must be positive (if $\lambda - \left(\frac{n\pi}{L}\right)^2 = 0$ or $\lambda - \left(\frac{n\pi}{L}\right)^2 < 0$, trivial solutions result).

Hence the solution for $Y_n(y)$ becomes

$$Y_n(y) = A\cos\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}y\right) + B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}y\right)$$

Applying first B.C. $Y(0) = 0$ gives

$$0 = A$$

The solution becomes

$$Y_n(y) = B_n\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}y\right)$$

Applying second B.C. $Y(H) = 0$ gives

$$0 = B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}H\right)$$

Non trivial solution requires that

$$\begin{aligned}
\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}H &= m\pi \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots \\
\lambda_{nm} - \left(\frac{n\pi}{L}\right)^2 &= \left(\frac{m\pi}{H}\right)^2 \\
\lambda_{nm} &= \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2
\end{aligned}$$

Hence the $Y_{nm}(y)$ eigenfunctions are

$$Y_{nm}(y) = \sin\left(\frac{m\pi}{H}y\right) \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots$$

Now the time $T(t)$ ode is solved, and since λ_{nm} is positive, then

$$T''_{nm} + c^2\lambda_{nm}T_{nm} = 0$$

$$\begin{aligned}
T_{nm}(t) &= A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) \\
&= A_{nm}\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}t\right) + B_{nm}\sin\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}t\right)
\end{aligned}$$

Combining all solution , and merging all constants into two results in

$$\begin{aligned}
u_{nm}(x,y,t) &= X_n(x)Y_{nm}(y)T_{nm}(t) \\
u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_n(x)Y_{nm}(y)T_{nm}(t) \\
&= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}t\right) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin
\end{aligned} \tag{1}$$

Initial conditions are now used to find A_{nm}, B_{nm}. At $t=0$

$$
\begin{aligned}
u(x,y,0) &= f(x,y) \\
\frac{\partial u}{\partial t}(x,y,0) &= 0
\end{aligned}
$$

Applying first initial condition to (1) gives

$$
f(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)
$$

Applying 2D orthogonality gives

$$
\begin{aligned}
\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\, dxdy &= A_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right) \\
A_{nm} &= \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\, dxdy
\end{aligned}
$$

Taking time derivative of (1) gives

$$
\begin{aligned}
\frac{\partial u}{\partial t}(x,y,t) = &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right)
\end{aligned}
$$

AT $t=0$ the above becomes

$$
\int_0^H g(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)
$$

Applying 2D orthogonality gives

$$
\int_0^L \int_0^H g(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\sin\left(\frac{m\pi}{H}y\right)\, dxdy = B_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right)
$$

But the initial velocity $g(x,y)=0$. Hence $B_{nm}=0$ for all n,m.

<u>Summary of solution</u>

$$
\begin{aligned}
u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right) \\
A_{nm} &= \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\, dxdy
\end{aligned}
$$

6.2.1.9 [404] With damping

problem number 404

Taken from Maple PDE help pages. This wave PDE inside square with damping present.

Membrane is free to move on the right edge and also on top edge. But fixed at left edge and bottom edge.

It has zero initial position, but given a non-zero initial velocity. Where $0 < x < 1$ and $0 < y < 1$ and $t > 0$. Solve

$$u_{tt} + \frac{1}{10}u_t = \frac{1}{4}\nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned} u(0,y,t) &= 0 \\ \frac{\partial u}{\partial x}u(1,y,t) &= 0 \\ u(x,0,t) &= 0 \\ \frac{\partial u}{\partial y}u(x,1,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= 0 \\ \frac{\partial u}{\partial t}(x,y,0) &= x(1-\frac{1}{2}x)(1-\frac{1}{2}y)y \end{aligned}$$

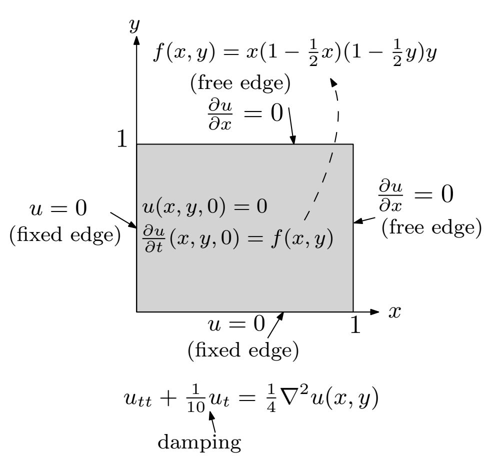


Figure 6.119: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4 - (1*D
ic  = {u[x, y, 0] == 0, Derivative[0, 0, 1][u][x, y, 0] == x*(1 - (1/2)*x)*(1 - (1/2)*y)*y};
bc  = {u[0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to \left\{ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{5120e^{-t/20}\sin\left(\frac{1}{2}\pi x(2K[1]-1)\right)\sin\left(\frac{1}{2}\pi y(2K[3]-1)\right)\sin\left(\frac{1}{20}t\sqrt{50\pi^2(2K[1]^2-2K[1]+2K[3]^2-2K[3]\cdots}\right.}{\pi^6(2K[1]-1)^3(2K[3]-1)^3\sqrt{50\pi^2(2K[1]^2-2K[1]+2K[3]^2-2K[3]+1)-1}}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) = 1/4*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2))-(1/10)*(dif
bc  := u(0, y, t) = 0,
      (D[1](u))(1, y, t) = 0,
      u(x, 0, t) = 0,
      (D[2](u))(x, 1, t) = 0;
ic  := u(x, y, 0) = 0, (D[3](u))(x, y, 0) = x*(1-(1/2)*x)*(1-(1/2)*y)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{5120\,\mathrm{e}^{-\frac{t}{20}}\sin\left(\frac{\sqrt{-1+(100m^2+100n^2+100m+100n+50)\pi^2}\,t}{20}\right)\sin\left(\frac{(2m+1)\pi y}{2}\right)\sin\left(\frac{(2n+1)\pi x}{2}\right)}{\sqrt{-1+(100m^2+100n^2+100m+100n+50)\,\pi^2}\,\pi^6\,(2m+1)^3\,(2n+1)^3}$$

6.2.1.10 [405] On the whole plane

problem number 405

From Mathematica DSolve help pages.

Hyperbolic partial differential equation with non-rational coefficients.

Solve for $u(x,y)$

$$u_{xx}-2\sin x u_{xy}-\cos^2 x u_{yy}-\cos x u_y=0$$

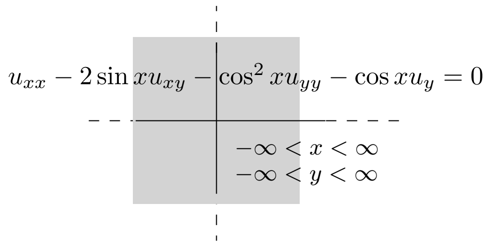


Figure 6.120: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ode = D[u[x, y], {x, 2}] - 2*Sin[x]*D[u[x, y], x, y] - Cos[x]^2*D[u[x, y], {y, 2}] - Cos[x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(x - \cos(x) + y) + c_2(-x - \cos(x) + y)\}\}$$

Maple ✗

```
restart;
interface(showassumed=0);
ode := diff(u(x, y), x$2) - 2*sin(x)*diff(u(x, y),x,y)-cos(x)^2*diff(u(x, y), y$2) - cos(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(ode, u(x, y))),output='rea
```

sol=()

6.2.2 Polar coordinates

Local contents

6.2.2.1 [406] no θ dependency, fixed boundary, general case

problem number 406

Added January 12, 2020

Circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0<r<a$ and $t>0$.

$$u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r\right)$$

With boundary conditions

$$u(a,t) = 0$$

With initial conditions

$$\begin{aligned} u(r,0) &= f(r) \\ \frac{\partial u}{\partial t}(r,0) &= g(r) \end{aligned}$$

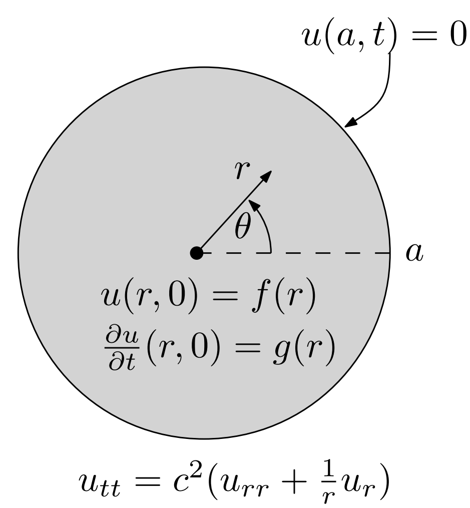


Figure 6.121: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == f[r], Derivative[0, 1][u][r, 0] == g[r]};
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t},Assumptions->{t>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t)\to\sum_{n=1}^{\infty}\boxed{\frac{2J_0\left(\frac{rj_{0,n}}{a}\right)\left(\sqrt{c^2}j_{0,n}\cos\left(\frac{\sqrt{c^2}tj_{0,n}}{a}\right)\int_0^a rJ_0\left(\frac{rj_{0,n}}{a}\right)f(r)\,dr+a\left(\int_0^a rJ_0\left(\frac{rj_{0,n}}{a}\right)g(r)\,dr\right)\sin\left(\frac{\sqrt{c^2}tj_{0,n}}{a}\right)\right)}{a^2\sqrt{c^2}(J_0(j_{0,n})^2+J_1(j_{0,n})^2)j_{0,n}}\ \text{if } j_{0,n}\geq}$$

Maple ✓

```
restart;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=f(r), D[2](u)(r,0)=g(r);
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t),HINT
```

$$u(r,t)=\frac{-\mathcal{L}^{-1}\left(\text{BesselK}\left(0,\frac{rs}{c}\right)\left(\int(sf(a)+g(a))\,a\,\text{BesselI}\left(0,\frac{as}{c}\right)da\right),s,t\right)+\mathcal{L}^{-1}\left(\text{BesselK}\left(0,\frac{rs}{c}\right)\left(\int(sf(\right.\right.}{}$$

Has unresolved Invlaplace calls

Hand solution

Assuming $u=T(t)\,R(r)$. Substituting in the PDE gives

$$\frac{1}{c^2}T''R=R''T+\frac{1}{r}R'T$$

Dividing by RT

$$\frac{1}{c^2}\frac{T''}{T}=\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}$$

Hence

$$\frac{1}{c^2}\frac{T''}{T}=-\lambda$$
$$\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}=-\lambda$$

The time ODE is

$$T'' + c^2\lambda T = 0$$

And the r ODE is (Sturm-Liouville)

$$rR'' + R' + \lambda r R = 0$$

Where $p = r, q = 0, \sigma = r$. This is singular SL. The solution turns out to be

$$R_n(r) = A_n J_0\left(\sqrt{\lambda_n} r\right) \qquad n = 1, 2, 3, \cdots$$

Where λ_n is found from roots of $0 = J_n\left(\sqrt{\lambda_n} a\right)$ giving the eigenvalues. Now the time ODE is solved

$$\begin{aligned} T_n'' + c^2\lambda_n T_n &= 0 \\ T_n &= B_n \cos\left(c\sqrt{\lambda_n} t\right) + C_n \sin\left(c\sqrt{\lambda_n} t\right) \qquad n = 1, 2, 3, \ldots, \end{aligned}$$

Hence the solution is

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} T_n R_n \\ &= \sum_{n=1}^{\infty} A_n \cos\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) + B_n \sin\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) \qquad (1) \end{aligned}$$

Now initial conditions $u(r,0) = f(r)$ is used to find A_n using orthogonality. At $t = 0$ the solution simplifies to

$$u(r,0) = \sum_{n=1}^{\infty} A_n J_0\left(\sqrt{\lambda_n} r\right)$$

Hence

$$\begin{aligned} f(r) &= \sum_{n=1}^{\infty} A_n J_0\left(\sqrt{\lambda_n} r\right) \\ \int_0^a f(r) J_0\left(\sqrt{\lambda_n} r\right) r dr &= A_n \int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr \\ A_n &= \frac{\int_0^a f(r) J_0\left(\sqrt{\lambda_n} r\right) r dr}{\int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr} \end{aligned}$$

Now we will look at the second initial conditions $\frac{\partial u}{\partial t}(r,0) = g(r)$. Taking derivative w.r.t. time t of the solution in (1) gives

$$\frac{\partial u}{\partial t}(r,t) = \sum_{n=1}^{\infty} -c\sqrt{\lambda_n} A_n \sin\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) + B_n c\sqrt{\lambda_n} \cos\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right)$$

At time $t = 0$ the above becomes

$$g(r) = \sum_{n=1}^{\infty} B_n c\sqrt{\lambda_n} J_0\left(\sqrt{\lambda_n} r\right)$$

Now orthogonality is used. The above becomes

$$B_n = \frac{\int_0^a g(r)\, J_0\left(\sqrt{\lambda_n} r\right) r dr}{c\sqrt{\lambda_n} \int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr}$$

Summary of solution

$$u(r,t) = \sum_{n=1}^{\infty} A_n \cos\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) + B_n \sin\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right)$$

$$A_n = \frac{\int_0^a f(r)\, J_0\left(\sqrt{\lambda_n} r\right) r dr}{\int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr}$$

$$B_n = \frac{\int_0^a g(r)\, J_0\left(\sqrt{\lambda_n} r\right) r dr}{c\sqrt{\lambda_n} \int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr}$$

With λ_n being the solutions for $0 = J_0\left(\sqrt{\lambda_n} a\right)$. We have infinite number of zeros. This generates all the needed λ_n. Hence $\sqrt{\lambda_n} a = BesselJZero(0,n)$, therefore $\sqrt{\lambda_n} = \frac{a}{BesselJZero(0,n)}$

6.2.2.2 [407] no θ dependency. Specific example. Both initial conditions not zero

problem number 407

Taken from Mathematica helps pages on DSolve

In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0 < r < 1$ and $t > 0$.

$$u_{tt} = c^2\left(u_{rr} + \frac{1}{r} u_r\right)$$

With boundary conditions

$$u(1,t) = 0$$

With initial conditions

$$u(r,0) = 1$$
$$\frac{\partial u}{\partial t}(r,0) = \frac{r}{3}$$

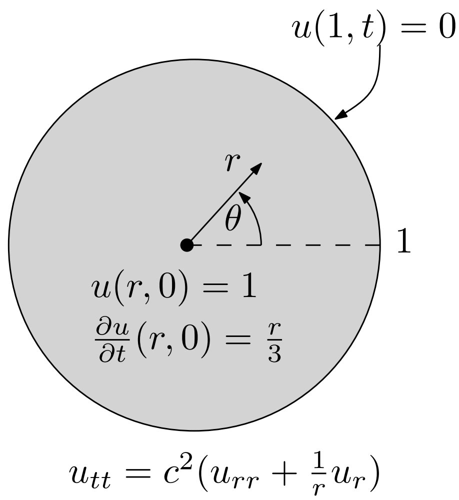


Figure 6.122: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == 1, Derivative[0, 1][u][r, 0] == r/3};
bc  = u[1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
sol =  sol /. K[1] -> n;
sol =  FullSimplify[sol];
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2J_0(rj_{0,n})\left(9\sqrt{c^2}J_1(j_{0,n})\cos\left(ctj_{0,n}\right) + {}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}(j_{0,n})^2\right)\sin\left(\sqrt{c^2}tj_{0,n}\right)\right)}{9\sqrt{c^2}\left(J_0\left(j_{0,n}\right)^2 + J_1\left(j_{0,n}\right)^2\right)j_{0,n}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=1, eval( diff(u(r,t),t),t=0)=r/3;
bc  := u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t)) ass
```

$$u(r,t) = \frac{rt}{3} + \frac{\pi\left(-\mathcal{L}^{-1}\left(\frac{\mathrm{StruveL}\left(0,\frac{rs}{c}\right)}{s^3}, s, t\right) + \mathcal{L}^{-1}\left(\frac{\mathrm{BesselI}\left(0,\frac{rs}{c}\right)\mathrm{StruveL}\left(0,\frac{s}{c}\right)}{s^3\,\mathrm{BesselI}\left(0,\frac{s}{c}\right)}, s, t\right)\right)c}{6} - \frac{\mathcal{L}^{-1}\left(\frac{\mathrm{BesselI}\left(0,\frac{rs}{c}\right)}{s^2\,\mathrm{BesselI}\left(0,\frac{s}{c}\right)}, s, t\right)}{3} - \mathcal{L}$$

Has unresolved Invlaplace calls

6.2.2.3 [408] no θ dependency. Specific example. Both initial conditions not zero

problem number 408

Added January 12, 2020.

In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0 < r < a$ and $t > 0$.

$$u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r\right)$$

With boundary conditions

$$u(a,t) = 0$$

With initial conditions

$$u(r,0) = f(r)$$
$$\frac{\partial u}{\partial t}(r,0) = g(r)$$

Using $a = 1, c = \frac{2}{10}, g(r) = 0, f(r) = r$.

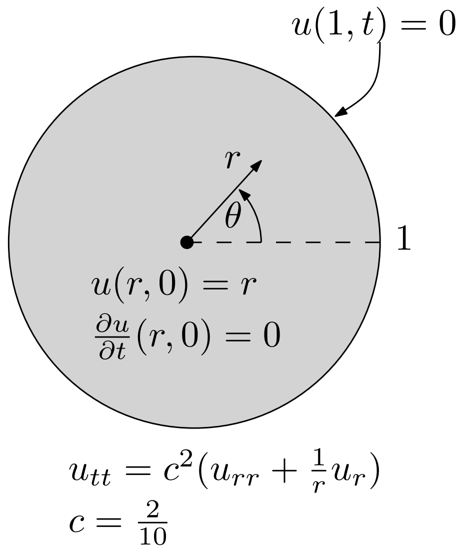


Figure 6.123: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=2/10; a=1;
g[r_]:=0;
f[r_]:=r;
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == f[r], Derivative[0, 1][u][r, 0] == g[r]};
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t},Assumptions->{t>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t)\to\sum_{n=1}^{\infty}\frac{2J_0(rj_{0,n})\cos\left(\frac{tj_{0,n}}{5}\right)\,{}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}(j_{0,n})^2\right)}{3\left(J_0\left(j_{0,n}\right)^2+J_1\left(j_{0,n}\right)^2\right)}\right\}\right\}$$

Maple ✓

```
restart;
c:=2/10;
a:=1;
g:=r->0;
f:=r->r;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=f(r), D[2](u)(r,0)=g(r);
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t)) ass
```

$$u(r,t) = r+\int_0^{r-1} \mathcal{L}^{-1}\left(\frac{\operatorname{BesselI}(0,5(r-\tau)\,s)\,\mathcal{L}(-\frac{1}{\tau+1},t,s)}{\operatorname{BesselI}(0,5s)},s,t\right)d\tau+\mathcal{L}^{-1}\left(\frac{\operatorname{BesselI}(0,5rs)\,\mathcal{L}(-1,t,s)}{\operatorname{BesselI}(0,5s)},s,\right.$$

Has unresolved Invlaplace calls. How to get series solution?
Hand solution

The basic solution for this type of PDE was already given in problem 6.2.2.1 on page 1114 as

$$u(r,t) = \sum_{n=1}^{\infty} A_n \cos\left(c\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right) + B_n \sin\left(c\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right)$$

$$A_n = \frac{\int_0^a f(r)\,J_0\left(\sqrt{\lambda_n}r\right) r dr}{\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right) r dr}$$

$$B_n = \frac{\int_0^a g(r)\,J_0\left(\sqrt{\lambda_n}r\right) r dr}{c\sqrt{\lambda_n}\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right) r dr}$$

With λ_n being the solutions for $0 = J_0\left(\sqrt{\lambda_n}a\right)$. We have infinite number of zeros. This generates all the needed λ_n. Hence $\sqrt{\lambda_n}a = BesselJZero(0,n)$, therefore $\sqrt{\lambda_n} = \frac{a}{BesselJZero(0,n)}$.

In this problem $c = \frac{2}{10}, a = 1, g(r) = 0$ and $f(r) = r$, hence the solution becomes

$$u(r,t) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{2}{10}\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right)$$

Where $\sqrt{\lambda_n} = \frac{1}{BesselJZero(0,n)}$.

This animation runs for 40 seconds.

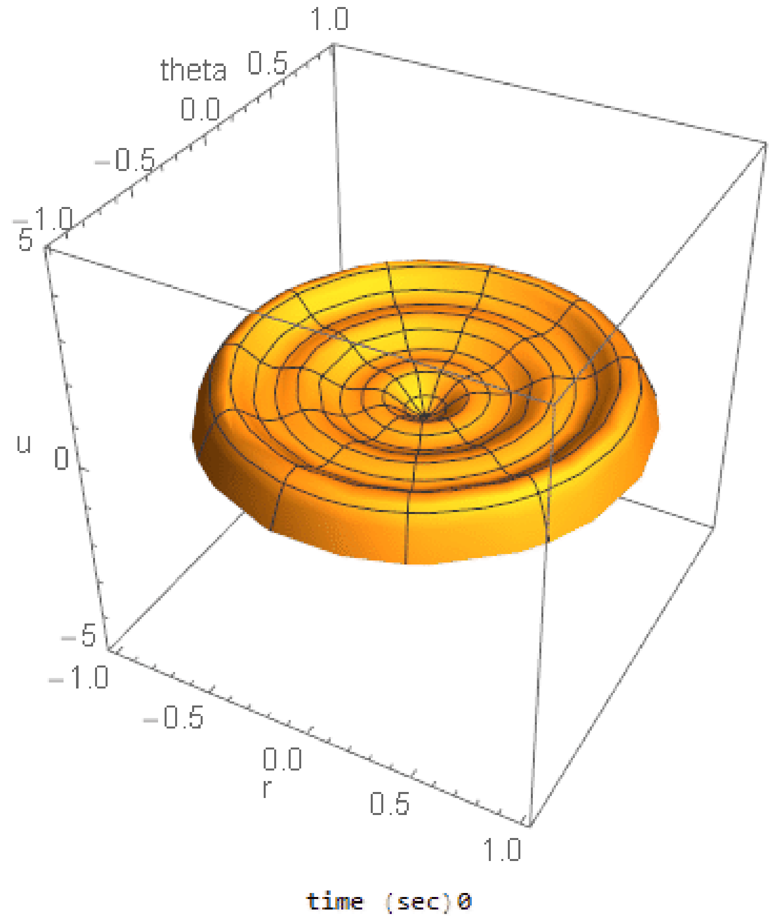


Figure 6.124: snap shot

Source code for all the above animation

```
(*By Nasser M Abbasi*)
SetDirectory[NotebookDirectory[]]

(*Axis symmetric*)
(*definitions*)
ClearAll[a,c,n,m,r,theta,f,g,u]
A0[n_,a_,lam0_]:= Module[{num,den,theta,r,f},
   f=r;
   num=N[(BesselJ[1,lam0] (2 lam0-Pi StruveH[0,lam0])+
        Pi BesselJ[0,lam0] StruveH[1,lam0])/(2 lam0^2)];
```

```
    den=0.5 (BesselJ[0,lam0]^2+BesselJ[1,lam0]^2);
    num/den
];

u[r_,theta_,t_]:=Sum[A0tbl[[n]]*Cos[c lamtbl[[n]] t] *BesselJ[0,lamtbl[[n]]*r],{n,1,maxN}];

maxN=6;
a=1;
c=.2;

lam[n_,a_]:=Module[{x},
   x=BesselJZero[0,n];
   N[(x/a)]
];


lamtbl=Table[lam[n,a],{n,1,maxN}];
A0tbl=Table[A0[n,a,lamtbl[[n]]],{n,1,maxN}];


t=.1
ParametricPlot3D[{r Cos[theta],r Sin[theta],Evaluate[u[r,theta,t]]},{r,0,1},
      {theta,0,2 Pi},AxesLabel->{"r","theta","u"},ImageMargins->5,
      PerformanceGoal->"Quality",BoxRatios->{1,1,1},
      PlotRange->{Automatic,Automatic,{-5,5}},Mesh->10,MaxRecursion->1]

Animate[ParametricPlot3D[{r Cos[theta],r Sin[theta],
        Evaluate[u[r,theta,t]]},{r,0,1},{theta,0,2 Pi},
        AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
        PerformanceGoal->"Speed",BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{-5,5}},
        Mesh->10,MaxRecursion->1],{t,0,50,.01}]

r=Table[
      Labeled[ParametricPlot3D[{r Cos[theta],r Sin[theta],
           Evaluate[u[r,theta,t]]},{r,0,1},{theta,0,2 Pi},AxesLabel->{"r","theta","u"},
           BaseStyle->15,ImageMargins->5,PerformanceGoal->"Speed",BoxRatios->{1,1,1},
           PlotRange->{Automatic,Automatic,{-5,5}},Mesh->10,MaxRecursion->1],
           Row[{"time (sec)",Round@t}]],{t,0,40,.1}];

Export["anim_axis.gif",r,"DisplayDurations"->Table[.05,{Length[r]}]]
```

6.2.2.4 [409] no θ dependency. Using integral transforms. Source present. Specific example

problem number 409

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$\frac{\partial^2}{\partial r^2}u(r,t) + \frac{\frac{\partial}{\partial r}u(r,t)}{r} + \frac{\partial^2}{\partial t^2}u(r,t) = -Q_0\, q(r)$$

With initial conditions

$$u(r,0) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, t], {r, 2}] + D[u[r, t], r]/r + D[u[r, t], {t, 2}] == -Q0*q[r];
ic = u[r, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t) \to \mathrm{Q0}\left(\int_0^\infty \frac{J_0(rK[1]) \int_0^\infty rq(r)J_0(rK[1])\, dr}{K[1]}\, dK[1] - \int_0^\infty \frac{e^{-tK[1]}J_0(rK[1]) \int_0^\infty rq(r)J_0(rK[1])}{K[1]}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r, t), r$2) + diff(u(r, t), r)/r + diff(u(r, t), t$2) = -Q__0*q(r);
iv := u(r, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(r,t))),output='
sol:=convert(sol,Int,only = hankel);
```

$$u(r,t) = \left(\int_0^\infty \frac{\mathrm{BesselJ}\,(0,rs)\,\left(\int_0^\infty r\,\mathrm{BesselJ}\,(0,rs)\,q(r)\,dr\right)}{s} ds - \left(\int_0^\infty \frac{\mathrm{BesselJ}\,(0,rs)\,\left(\int_0^\infty r\,\mathrm{BesselJ}\,(0,rs\right.}{s}\right.\right.$$

6.2.2.5 [410] no θ dependency. Using integral transforms. Source present. Specific example

problem number 410

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$c^2\left(\frac{\partial^2}{\partial r^2}u(r,t)+\frac{\frac{\partial}{\partial r}u(r,t)}{r}\right)=\frac{\partial^2}{\partial t^2}u(r,t)$$

With initial conditions

$$u(r,0)=\frac{Aa}{\sqrt{a^2+r^2}}$$
$$\frac{\partial u(r,0)}{\partial t}=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = c^2*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) == D[u[r, t], {t, 2}];
ic = {u[r, 0] == A*a*(a^2 + r^2)^(-1/2), Derivative[0, 1][u][r, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[r, t], {r, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := c^2*(diff(u(r, t), r, r) + diff(u(r, t), r)/r) = diff(u(r, t), t, t);
iv := u(r, 0) = A*a*(a^2 + r^2)^(-1/2), D[2](u)(r, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(r,t),method = H
```

$$u(r,t)=\frac{\left(\sqrt{2iact-c^2t^2+a^2+r^2}+\sqrt{-2iact-c^2t^2+a^2+r^2}\right)Aa}{2\sqrt{-2iact-c^2t^2+a^2+r^2}\,\sqrt{2iact-c^2t^2+a^2+r^2}}$$

6.2.2.6 [411] θ dependency, fixed on edges, general solution

problem number 411

Added January 11, 2020

Solve for $u(r,\theta,t)$ with $0<r<a$ and $t>0$ and $-\pi<\theta<\pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) \\ \frac{\partial u}{\partial t}(r,\theta,0) &= g(r,\theta) \end{aligned}$$

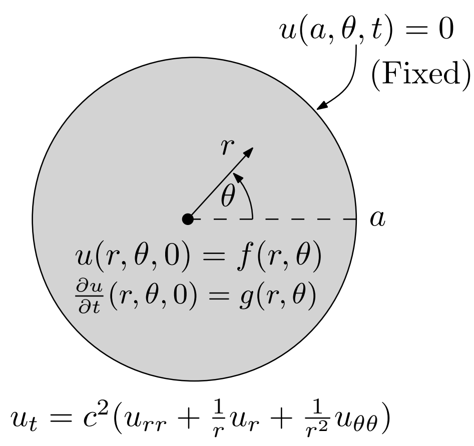


Figure 6.125: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == g[r,theta]};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to \left\{ \sum_{K[3]=1}^{\infty} -\frac{\sqrt{\frac{2}{\pi}}J_0\left(\frac{rj_{0,K[3]}}{a}\right)\left(-\frac{\sqrt{\frac{2}{\pi}}\cos\left(\frac{\sqrt{c^2}tj_{0,K[3]}}{a}\right)\int_0^a\int_{-\pi}^{\pi} rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,\theta)d\theta dr}{aJ_1\left(j_{0,K[3]}\right)} - \frac{\sqrt{\frac{2}{\pi}}\left(\int_0^a\int_{-\pi}^{\pi} rJ_0\left(\frac{rj_{0,K[3]}}{a}\right.\right.}{|c|J_1\left(j_{0\ldots}\right.}\right.}{aJ_1\left(j_{0,K[3]}\right)}\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = g(r,theta);
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

<u>Hand solution</u>

Assuming $u = T(t)\,R(r)\,\Theta(\theta)$ and substituting in the PDE gives

$$\frac{1}{c^2}T''R\Theta = R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT$$

Dividing by $RT\Theta$

$$\frac{1}{c^2}\frac{T''}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}$$

Hence

$$\frac{1}{c^2}\frac{T''}{T} = -\lambda$$

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} = -\lambda$$

The time ODE is

$$T'' + c^2 \lambda T = 0$$

Now we separate again the space ODE's (remember to move the λ with the R and not the Θ)

$$\begin{aligned} \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda &= -\frac{1}{r^2}\frac{\Theta''}{\Theta} \\ r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Let the new separation constant be μ, therefore

$$\begin{aligned} -\frac{\Theta''}{\Theta} &= \mu \\ \Theta'' + \mu\Theta &= 0 \end{aligned}$$

With periodic boundary conditions and

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= \mu \\ r^2 R'' + rR' + \lambda r^2 R - \mu R &= 0 \\ rR'' + R' - \frac{\mu}{r}R &= -\lambda r R \end{aligned}$$

Now it is in Sturm Liouville form, where $p = r, q = -\frac{\mu}{r}, \sigma = r$. This is singular SL. Can be written as

$$R'' + \frac{1}{r}R' + \left(\lambda - \frac{\mu}{r^2}\right) R = 0$$

Before we solve the above R ODE, we solve the $\Theta'' + \mu\Theta = 0$ to find μ Eigenvalues. The solution is

$$\Theta = A\cos\left(\sqrt{\mu}\theta\right) + B\sin\left(\sqrt{\mu}\theta\right)$$

With B.C $\Theta(-\pi) = \Theta(\pi)$ and $\Theta'(-\pi) = \Theta'(\pi)$. From first B.C. we obtain

$$\begin{aligned} A\cos\left(\sqrt{\mu}\pi\right) - B\sin\left(\sqrt{\mu}\pi\right) &= A\cos\left(\sqrt{\mu}\pi\right) + B\sin\left(\sqrt{\mu}\pi\right) \\ 2B\sin\left(\sqrt{\mu}\pi\right) &= 0 \end{aligned} \tag{1}$$

Looking at second B.C. $\Theta'(-\pi) = \Theta'(\pi)$

$$\Theta'(\theta) = -A\sqrt{\mu}\sin\left(\sqrt{\mu}\theta\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\theta\right)$$

Hence

$$\begin{aligned} A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\pi\right) &= -A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\pi\right) \\ A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) &= -A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) \\ 2A\sin\left(\sqrt{\mu}\pi\right) &= 0 \end{aligned} \tag{2}$$

From (1,2), we see that both are satisfied if

$$\begin{aligned}\sqrt{\mu}\pi &= n\pi \qquad n = 1,2,3,\dots\\ \mu &= n^2\end{aligned}$$

Hence

$$\Theta_n = A_n \cos(n\theta) + B_n \sin(n\theta)$$

There is another solution for $\mu = 0$ which is constant (that is why one of the sums below starts from $n = 0$). We can combine the zero eigenvalue with the above and write

$$\Theta_n = A_n \cos(n\theta) + B_n \sin(n\theta) \qquad n = 0,1,2,3,\dots$$

Since at $n = 0$ the above reduces to constant A_0.

Now that we know $\mu_n = n^2$, from solving the θ part, we go and solve the r ODE. For each n, the solution to the r (Bessel) ode

$$R'' + \frac{1}{r}R' + \left(\lambda - \frac{n^2}{r^2}\right)R = 0$$

The solution turns out to be

$$R_{nm}(r) = J_n\left(\sqrt{\lambda_{nm}}r\right) \qquad m = 1,2,3,\cdots$$

Where λ_{nm} is found from roots of $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$ giving the eigenvalues. Now the time ODE is solved

$$\begin{aligned}&T''_{nm} + c^2\lambda_{nm}T_{nm} = 0\\ &T_{nm} = C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) \qquad n = 0,1,2,3,\dots,m = 1,2,3,\cdots\end{aligned}$$

Hence the solution is

$$\begin{aligned}u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} T_{nm}R_{nm}\Theta_n\\ &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\left(A_n\cos(n\theta) + B_n\sin(n\theta)\right)\end{aligned}$$

We now break this sum as follows

$$\begin{aligned}u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right)J_n\left(\sqrt{\lambda_{nm}}r\right)A_n\cos(n\theta)\\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right)J_n\left(\sqrt{\lambda_{nm}}r\right)B_n\sin(n\theta)\end{aligned}$$

Or

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) A_n\cos\left(n\theta\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) A_n\cos \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) B_n\sin\left(n\theta\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) B_n\sin
\end{aligned}
$$

Then we break the above into 4 sums

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) A_n\cos\left(n\theta\right) \\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) A_n\cos\left(n\theta\right) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) B_n\sin\left(n\theta\right) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) B_n\sin\left(n\theta\right)
\end{aligned}
$$

Finally, we merge constants in the above as follows

$$
\begin{aligned}
A_n C_{nm} &\equiv A_{nm} \\
A_n D_{nm} &\equiv B_{nm} \\
B_n C_{nm} &\equiv C_{nm} \\
B_n D_{nm} &\equiv D_{nm}
\end{aligned}
$$

Hence the final solution now becomes

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) \cos\left(n\theta\right) \\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) \cos\left(n\theta\right) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) \sin\left(n\theta\right) \\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right) \sin\left(n\theta\right) \qquad (3)
\end{aligned}
$$

Now initial conditions $u(r,\theta,0) = f(r,\theta)$ is used to find A_{nm}, C_{nm} using orthogonality.

At $t=0$ the solution simplifies to (all terms with $\sin\left(c\sqrt{\lambda_{nm}}t\right)$ vanish giving

$$
\begin{aligned}
u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&\quad+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}
$$

Hence

$$
f(r,\theta) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta) \tag{4}
$$

When iterating over m index, the terms $\cos(n\theta)$ and $\sin(n\theta)$ will be constant. So for each n, we have $\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)$ and $\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)$. So orthogonality is carried out on the m index on the Bessel functions. Multiplying (4) by $J_n\left(\sqrt{\lambda_{nk}}r\right)$ and integrating

$$
\begin{aligned}
\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nk}}r\right) r dr &= \sum_{n=0}^{\infty}\left(\int_0^a \sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right) J_n\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\cos(n\theta)\\
&\quad+\sum_{n=1}^{\infty}\left(\int_0^a \sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right) J_n\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\sin(n\theta)
\end{aligned}
$$

Or

$$
\begin{aligned}
\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nk}}r\right) r dr &= \sum_{n=0}^{\infty} A_{nk}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\cos(n\theta)\\
&\quad+\sum_{n=1}^{\infty} C_{nk}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\sin(n\theta)
\end{aligned}
$$

Replacing k back with m, the above becomes

$$
\begin{aligned}
\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r dr &= \sum_{n=0}^{\infty} A_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\cos(n\theta)\\
&\quad+\sum_{n=1}^{\infty} C_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\sin(n\theta) \qquad (5)
\end{aligned}
$$

We now apply orthogonality on n using the $\cos(n\theta)$ results in

$$
\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta = A_{nm}\int_{-\pi}^{\pi}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\cos^2(n\theta)\, d\theta
$$

But $\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr$ does not depend on θ, therefore the above becomes

$$\begin{aligned}\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\,d\theta &= A_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr\right)\int_{-\pi}^{\pi}\cos^2(n\theta)\,d\theta\\ &= A_{nm}\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr\end{aligned}$$

Therefore

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

Similarly for $\sin(n\theta)$, which gives

$$C_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

Now we will look at the second initial conditions $\frac{\partial u}{\partial t}(r,\theta,0) = g(r,\theta)$. Taking derivative w.r.t. time t of the solution in (3) gives

$$\begin{aligned}\frac{\partial u}{\partial t}(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}}A_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}}C_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

At time $t=0$ the above becomes (all terms with $\sin\left(c\sqrt{\lambda_{nm}}t\right)$ vanish).

$$\begin{aligned}g(r,\theta) =&\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

Now orthogonality is used. At $t=0$ the above becomes

$$\begin{aligned}g(r,\theta) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

Similarly to the above we now find B_{nm} and D_{nm}. The only difference, is that now we have extra $c\sqrt{\lambda_{nm}}$ terms that show up. The final result will be

$$B_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}$$

And

$$D_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}$$

Summary of solution

$$\begin{aligned}
u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}$$

$$\begin{aligned}
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}
\end{aligned}$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$

6.2.2.7 [412] θ dependency, fixed on edges, zero initial velocity, general solution

problem number 412

Solve for $u(r,\theta,t)$ with $0<r<a$ and $t>0$ and $-\pi<\theta<\pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) \\ \frac{\partial u}{\partial t}(r,\theta,0) &= 0 \end{aligned}$$

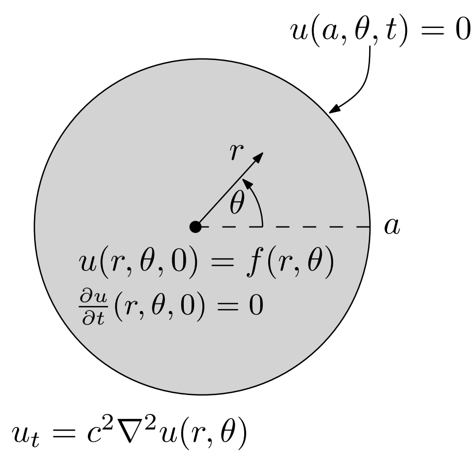


Figure 6.126: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to\left\{\sum_{K[3]=1}^{\infty}\frac{2J_0\left(\frac{rj_{0,K[3]}}{a}\right)\cos\left(\frac{\sqrt{c^2}tj_{0,K[3]}}{a}\right)\int_0^a\int_{-\pi}^{\pi}rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,\theta)d\theta dr}{a^2\pi J_1\left(j_{0,K[3]}\right)^2}+\sum_{K[3]=1}^{\infty}\left(\sum_{K[1]=1}^{\infty}\left(\frac{2J_{K[1]}\left(\frac{rj_{K[1}}{}\right.}{}\right.\right.\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) =c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 6.2.2.6 on page 1126 as

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos\left(n\theta\right)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos\left(n\theta\right)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin\left(n\theta\right)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin\left(n\theta\right)
\end{aligned}
$$

$$
\begin{aligned}
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}
\end{aligned}
$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$. Since $g(r,\theta) = 0$ in this case, then $B_{nm} = 0, D_{nm} = 0$ and the solution simplifies to

$$
u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin
$$

6.2.2.8 [413] θ dependency, fixed on edges, zero initial velocity, specific example

problem number 413

Added January 11 2020.

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$
\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)
$$

With boundary conditions

$$
\begin{aligned}
u(a,\theta,t) &= 0\\
|u(0,\theta,t)| &< \infty\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

With initial conditions

$$u(r,\theta,0) = f(r,\theta)$$
$$\frac{\partial u}{\partial t}(r,\theta,0) = 0$$

Using $a = 1, c = 0.2, f(r,\theta) = r\theta$.

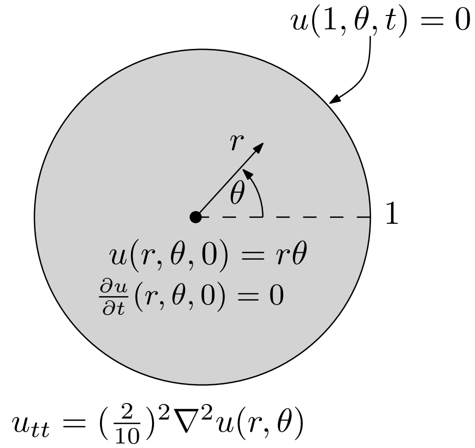


Figure 6.127: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f[r_,theta_]:=r*theta;
c=2/10; a=1;
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t) \to \left\{ \sum_{K[3]=1}^{\infty} \left(\sum_{K[1]=1}^{\infty} \frac{(-1)^{K[1]+1} J_{K[1]}(r j_{K[1],K[3]}) j_{K[1],K[3]} \cos\left(\frac{1}{5} t j_{K[1],K[3]}\right) \text{Gamma}\left(\frac{1}{2}(K[1]+3)\right) {}_1\tilde{F}_2\left(\frac{1}{2}(K[1]+3); \frac{1}{2}\right.}{J_{K[1]-1}(j_{K[1],K[3]})\, {}_0\tilde{F}_1\left(;K[1];-\frac{1}{4}(j_{K[1],K[3]})^2\right) K[1} \right.\right.\right.\right.$$

Indeterminate

Maple ✗

```
restart;
f:=(r,theta)->r*theta;
c:=2/10;
a:=1;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 6.2.2.6 on page 1126 as

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}
$$

$$
\begin{aligned}
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,dr\right)\cos(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right)r dr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,dr\right)\sin(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right)r dr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\,r\,d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\,r\,d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,dr}
\end{aligned}
$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1},\lambda_{n,2},\lambda_{n,3},\cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$.

In this problem $g(r,\theta) = 0, f(r,\theta) = r\theta, a = 1, c = \frac{2}{10}$, then $B_{nm} = 0, D_{nm} = 0$ and the solution simplifies to

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(\frac{2}{10}\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(\frac{2}{10}\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}\right.$$

Where

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a r\theta J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$
$$C_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a r\theta J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

The following animations run for 80 seconds. They are for different n, m modes. (This only show in the HTML version)

6.2.2.8.1 Cases for $n = 0$

6.2.2.8.2 Cases for $n = 1$

6.2.2.8.3 Cases for $n = 2$

6.2.2.8.4 Cases for $n = 3$

Source code for all the above animations

```
(*By Nasser M. Abbasi*)
SetDirectory[NotebookDirectory[]]
 X:\data\public_html\my_notes\PDE_animations\problems\4

(*definitions*)
ClearAll[a,c,n,m,r,theta,f,g,u,maxM,maxN,t]
maxN=4;
maxM=4;
a=1;
c=.2;
minZ=-8;
```

```
maxZ=10;
A0[n_,m_]:= Module[{num,den,r,theta,f},
f=r*theta;
num=Integrate[f* BesselJ[n,lamtbl[[n+1,m]]  r] Cos[n theta] r,{r,0,a},{theta,0,2Pi}];
den=Integrate[(BesselJ[n,lamtbl[[n+1,m]] r])^2  (Cos[n theta])^2 r,{r,0,a},{theta,0,2Pi}];
num/den
];

C0[n_,m_]:= Module[{num,den,f,r,theta},
   f=r*theta;
   num=Integrate[f* BesselJ[n,lamtbl[[n+1,m]]  r] Sin[n theta] r,{r,0,a},{theta,0,2Pi}];
   den=Integrate[(BesselJ[n,lamtbl[[n+1,m]] r])^2  (Sin[n theta])^2 r,{r,0,a},{theta,0,2Pi}]
   num/den
   ];

lam[n_,m_]:=Module[{x},
    x=BesselJZero[n,m];
    N[(x/a)]
   ];

u[r_,theta_,t_,maxN_,maxM_]:=Module[{tmp,n,m},
     tmp=Sum[A0tbl[[n+1,m]]*Cos[c lamtbl[[n+1,m]] t] *BesselJ[n,lamtbl[[n+1,m]]*r]*
         Cos[n*theta],{n,0,maxN},{m,1,maxM}];

     tmp=tmp+Sum[C0tbl[[n+1,m]]*Cos[c lamtbl[[n+1,m]] t] *BesselJ[n,lamtbl[[n+1,m]]*r]*
        Sin[n*theta],{n,1,maxN},{m,1,maxM}]
   ];

lamtbl=Table[lam[n,m],{n,0,maxN},{m,1,maxN}];

A0tbl=Table[A0[n,m],{n,0,maxN},{m,1,maxM}]
C0tbl=Table[C0[n,m],{n,1,maxN},{m,1,maxM}]

(*n=0,m=1*)
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,12,0,1]]},
   {r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
   PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
   PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
     Evaluate[u[r0,theta0,t,0,1]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
```

```
     BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
     MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
     Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_0_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]



(*n=0,m=2*)
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,0,0,2]]},{r0,0,1},
   {theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
   PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
   PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

(*to speed it up, make Z in {t,0,100,Z} larger, and then make Z in Table[Z,{Length[r]}] smal
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
     Evaluate[u[r0,theta0,t,0,2]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
     BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
     MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
     Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_0_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=0,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
          Evaluate[u[r0,theta0,t,0,3]]},{r0,0,1},{theta0,0,2 Pi},
          AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
          PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
          PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",3}]],{t,0,80,.5}];

Export["anim_n_0_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]



(*n=0,m=4*)
t=1;
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,t,0,4]]},
        {r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,
        ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
        BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
```

```
          Evaluate[u[r0,theta0,t,0,4]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"
          BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
          BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_0_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=1*)
t=1;
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
       Evaluate[u[r0,theta0,t,1,1]]},{r0,0,1},{theta0,0,2 Pi},
       AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
       PerformanceGoal->"Speed",Mesh->10, BoxRatios->{1,1,1},
       PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
            Evaluate[u[r0,theta0,t,1,1]]},{r0,0,1},{theta0,0,2 Pi},
            AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
            PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
            BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
            Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_1_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=2*)

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,1,2]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
         BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_1_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
    Evaluate[u[r0,theta0,t,1,3]]},{r0,0,1},{theta0,0,2 Pi},
    AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
    PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
    PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
```

```
    Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",3}]],{t,0,80,.5}];

Export["anim_n_1_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
           Evaluate[u[r0,theta0,t,1,4]]},{r0,0,1},{theta0,0,2 Pi},
           AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
           PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
           BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
           Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_1_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=1*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
          Evaluate[u[r0,theta0,t,2,1]]},{r0,0,1},{theta0,0,2 Pi},
          AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
          PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
          PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",1}]]
          ,{t,0,80,.5}];

Export["anim_n_2_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=2*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,2,2]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
         PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_2_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
       Evaluate[u[r0,theta0,t,2,3]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
       BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
       MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
       Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",3}]],{t,0,80,.5}];
```

```
Export["anim_n_2_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,2,4]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
         PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_2_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=1*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
      Evaluate[u[r0,theta0,t,3,1]]},{r0,0,1},{theta0,0,2 Pi},
      AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
      PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
      PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
      Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_3_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=2*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
            Evaluate[u[r0,theta0,t,3,2]]},{r0,0,1},{theta0,0,2 Pi},
            AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
            PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
            BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
            Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_3_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
           Evaluate[u[r0,theta0,t,3,3]]},{r0,0,1},{theta0,0,2 Pi},
           AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
           PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
           PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
           Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",3}]],{t,0,80,.5}];
Export["anim_n_3_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]
```

```
(*n=3,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
          Evaluate[u[r0,theta0,t,3,4]]},{r0,0,1},{theta0,0,2 Pi},
          AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
          PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
          PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_3_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]
```

6.2.2.9 [414] θ dependency, fixed on edges, zero initial position, specific example

problem number 414

Added January 11, 2020

Math 322 UW exam problem. 2018.

Solve for $u(r,\theta,t)$ with $0<r<a$ and $t>0$ and $-\pi<\theta<\pi$

$$\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}u(a,\theta,t)&=0\\|u(0,\theta,t)|&<\infty\\u(r,-\pi,t)&=u(r,\pi,t)\\\frac{\partial u}{\partial \theta}(r,-\pi,t)&=\frac{\partial u}{\partial \theta}(r,\pi,t)\end{aligned}$$

With initial conditions

$$\begin{aligned}u(r,\theta,0)&=0\\u_t(r,\theta,0)&=\begin{cases}\frac{1}{\pi\epsilon^2} & \text{if } r\le\epsilon\\0 & \text{otherwise}\end{cases}\end{aligned}$$

Where $0<\epsilon<1$

Using $a=1,c=1,\epsilon=\frac{1}{2}$.

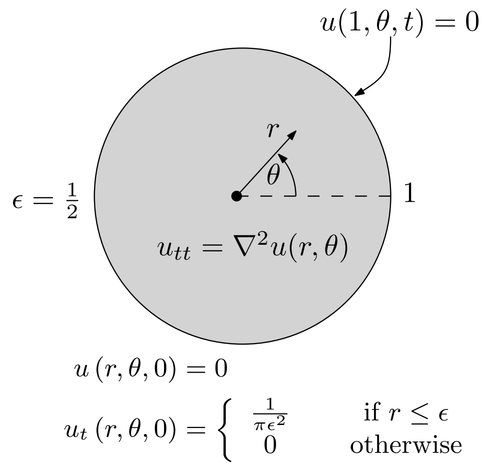


Figure 6.128: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1; c=1; epsilon=1/2;
f[r_,theta_]:=0;
g[r_,theta_]:=Piecewise[{{1/(Pi*epsilon^2),r<epsilon},{0,True}}];
c=1; a=1;
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == g[r,theta]};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to \begin{cases} \sum_{K[3]=1}^{\infty} \frac{2J_0\left(rj_{0,K[3]}\right)\,{}_0\tilde{F}_1\left(;2;-\frac{1}{16}\left(j_{0,K[3]}\right)^2\right)\sin\left(tj_{0,K[3]}\right)}{\pi J_1\left(j_{0,K[3]}\right)^2 j_{0,K[3]}} & (K[1]|K[3])\in\mathbb{Z}\land K[1]\geq 1\land K[3]\geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right.$$

Maple ✗

```
restart;
c:=1;
a:=1;
epsilon:=1/2;
f:=(r,theta)->r*theta;
g:=(r,theta)->piecewise(r<epsilon,1/(Pi*epsilon^2),true,0);
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = g(r,theta);
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 6.2.2.6 on page 1126 as

$$
\begin{aligned}
u(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta) J_n\left(\sqrt{\lambda_{nm}}r\right) r\, dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta) J_n\left(\sqrt{\lambda_{nm}}r\right) r\, dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\, d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\, dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\, d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\, dr}
\end{aligned}
$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$.

In this problem $f(r,\theta) = 0, a = 1, c = 1$, then $A_{nm} = 0, C_{nm} = 0$ and the solution simplifies to

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin$$

Taking time derivative gives

$$u_t(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\cos(n\theta)\,\lambda_{nm}\cos(\lambda_{nm}t)\,J_n(\lambda_{nm}r) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin(n\theta)\,\lambda_{nm}\cos(\lambda_{nm}t)\,J_n(\lambda_{nm}r)$$

Applying the second initial condition at $t = 0$ gives

$$\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\cos(n\theta)\,\lambda_{nm}J_n(\lambda_{nm}r) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin(n\theta)\,\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} \frac{1}{\pi\epsilon^2} & \text{if } r \le \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Case $n = 0$ (9) becomes

$$\sum_{m=1}^{\infty} B_{0m}\lambda_{0m}J_0(\lambda_{0m}r) = \begin{cases} \frac{1}{\pi\epsilon^2} & \text{if } r \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Applying orthogonality on $J_0(\lambda_{0m}r)$ results in

$$\begin{aligned} B_{0m}\lambda_{0m}\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr &= \frac{1}{\pi\epsilon^2}\int_0^\epsilon rJ_0(\lambda_{0m}r)\,dr \\ B_{0m} &= \frac{1}{\pi\epsilon^2\lambda_{0m}}\frac{\int_0^\epsilon rJ_0(\lambda_{0m}r)\,dr}{\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr} \end{aligned} \tag{9A}$$

Case $n > 1$ Applying orthogonality on $\cos(n\theta)$, equation (9) becomes

$$\begin{aligned} \sum_{m=1}^{\infty} B_{nm}\left(\int_{-\pi}^{\pi}\cos^2(n\theta)\,d\theta\right)\lambda_{nm}J_n(\lambda_{nm}r) &= \begin{cases} \frac{1}{\pi\epsilon^2}\int_{-\pi}^{\pi}\cos(n\theta)\,d\theta & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases} \\ \sum_{m=1}^{\infty}\pi B_{nm}\lambda_{nm}J_n(\lambda_{nm}r) &= \begin{cases} 0 & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Hence $B_{nm} = 0$ for all $n > 0$.

The same is now done to find $\bar{D}_{nm}$. Applying orthogonality on $\sin(n\theta)$, equation (9) becomes

$$\sum_{m=1}^{\infty} D_{nm} \left(\int_{-\pi}^{\pi} \sin^2(n\theta)\, d\theta \right) \lambda_{nm} J_n(\lambda_{nm} r) = \begin{cases} \frac{1}{\pi\epsilon^2} \int_{-\pi}^{\pi} \sin(n\theta)\, d\theta & \text{if } r^2 \leq \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{m=1}^{\infty} D_{nm} \left(\int_{-\pi}^{\pi} \sin^2(n\theta)\, d\theta \right) \lambda_{nm} J_n(\lambda_{nm} r) = \begin{cases} 0 & \text{if } r^2 \leq \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Hence all $\underline{D_{nm} = 0 \text{ for all } n > 0}$.

Therefore the solution (8) reduces to only using $n = 0, m = 1, 2, 3, \cdots$. The solution can now be written as

$$u(r,\theta,t) = \sum_{m=1}^{\infty} B_{0m} \sin(\lambda_{0m} t)\, J_0(\lambda_{0m} r) \tag{10}$$

Where $B_{0m} = \frac{1}{\pi\epsilon^2 \lambda_{0m}} \frac{\int_0^{\epsilon} r J_0(\lambda_{0m} r) dr}{\int_0^1 r J_0^2(\lambda_{0m} r) dr}$ And λ_{0m} are all the positive zeros of $J_0(z)$, $m = 1, 2, 3, \cdots$.

B_{0m} is now simplified more. Considering first the numerator of B_{0m} which is $\int_0^{\epsilon} r J_0(\lambda_{0m} r)\, dr$. The hint given says that

$$\frac{d}{dr}(r J_1(r)) = r J_0(r)$$

This is the same as saying

$$r J_1(r) = \int r J_0(r)\, dr \tag{10A}$$

However the integral in B_{0m} is $\int r J_0(\lambda_{0m} r)\, dr$ and not $\int r J_0(r)\, dr$. To transform it so that the hint can be used, let $\lambda_{0m} r = \bar{r}$, then $\frac{dr}{d\bar{r}} = \frac{1}{\lambda_{0m}}$ or $dr = \frac{d\bar{r}}{\lambda_{0m}}$. Now $\int r J_0(\lambda_{0m} r)\, dr$ becomes $\int \frac{\bar{r}}{\lambda_{0m}} J_0(\bar{r}) \frac{d\bar{r}}{\lambda_{0m}}$ or $\frac{1}{\lambda_{0m}^2} \int \bar{r} J_0(\bar{r})\, d\bar{r}$ and now the hint (10A) can be used on this integral giving

$$\frac{1}{\lambda_{0m}^2} \left(\int \bar{r} J_0(\bar{r})\, d\bar{r} \right) = \frac{1}{\lambda_{0m}^2} (\bar{r} J_1(\bar{r}))$$

Replacing $\bar{r}$ back by $\lambda_{0m} r$, gives the result needed

$$\begin{aligned} \frac{1}{\lambda_{0m}^2} (\bar{r} J_1(\bar{r})) &= \frac{1}{\lambda_{0m}^2} (\lambda_{0m} r J_1(\lambda_{0m} r)) \\ &= \frac{1}{\lambda_{0m}} r J_1(\lambda_{0m} r) \end{aligned}$$

Now the limits are applied, using the fundamental theory of calculus

$$\begin{aligned} \int_0^{\epsilon} r J_0(\lambda_{0m} r)\, dr &= \frac{1}{\lambda_{0m}} [r J_1(\lambda_{0m} r)]_0^{\epsilon} \\ &= \frac{\epsilon}{\lambda_{0m}} J_1(\lambda_{0m} \epsilon) \end{aligned} \tag{10B}$$

This completes finding the numerator integral in B_{0m}. The denominator integral in B_{0m} is $\int_0^1 r J_0^2(\lambda_{0m} r)\, dr$. This was found before which is

$$\int_0^1 r J_0^2(\lambda_{0m} r)\, dr = \frac{1}{2}[J_0'(\lambda_{0m})]^2$$

But $J_0'(\lambda_{0m}) = -J_1(\lambda_{0m})$, hence the above becomes

$$\int_0^1 r J_0^2(\lambda_{0m} r)\, dr = \frac{1}{2} J_1^2(\lambda_{0m}) \tag{10C}$$

Applying (10B) and (10C), B_{0m} simplifies to the following expression

$$\begin{aligned} B_{0m} &= \frac{1}{\pi\epsilon^2 \lambda_{0m}} \frac{\frac{\epsilon}{\lambda_{0m}} J_1(\lambda_{0m}\epsilon)}{\frac{1}{2} J_1^2(\lambda_{0m})} \\ &= \frac{2}{\pi\epsilon\lambda_{0m}^2} \frac{J_1(\lambda_{0m}\epsilon)}{J_1^2(\lambda_{0m})} \end{aligned}$$

Therefore the final solution becomes

$$u(r,\theta,t) = \sum_{m=1}^{\infty} B_{0m} \sin(\lambda_{0m} t)\, J_0(\lambda_{0m} r)$$

$$u(r,\theta,t) = \frac{2}{\pi\epsilon} \sum_{m=1}^{\infty} \frac{1}{\lambda_{0m}^2} \frac{J_1(\lambda_{0m}\epsilon)}{J_1^2(\lambda_{0m})} J_0(\lambda_{0m} r) \sin(\lambda_{0m} t) \tag{11}$$

When $\epsilon = \frac{1}{2}$, the above solution (11) becomes

$$u(r,\theta,t) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{1}{\lambda_{0m}^2} \frac{J_1\left(\frac{1}{2}\lambda_{0m}\right)}{J_1^2(\lambda_{0m})} J_0(\lambda_{0m} r) \sin(\lambda_{0m} t) \tag{11A}$$

Here is animation for 5 seconds made in Mathematica

Mathematica Source code for all the above animations

```
(*By Nasser M. Abbasi. Animation of problem 4 solution*)

ClearAll[t,r,m];
padIt2[v_,f_List]:=AccountingForm[v,f,NumberSigns->{"",""},
                          NumberPadding->{"0","0"},SignPadding->True];
nTerms=40;
lam = Table[ BesselJZero[0,m],{m,1,nTerms}]//N;
c   = Table[1/lam[[m]]^2 BesselJ[1,lam[[m]]/2]/BesselJ[1,lam[[m]] ]^2,{m,1,nTerms}];
mySol[r_,t_]:=4/Pi   Sum[c[[m]]BesselJ[0,lam[[m]] r] Sin[lam[[m]] t],{m,1,nTerms}];
```

```
frames=Table[
  Print["t=",t];
  Grid[{
   {Row[{"time = ",padIt2[t,{3,2}]}]},
   {ParametricPlot3D[{r Cos[theta],r Sin[theta],mySol[r,t]},{r,0,1},{theta,0,2 Pi},
    PlotRange->{Automatic,Automatic,{-0.6,0.6}},
    PerformanceGoal->"Speed",Boxed->True,
    Axes->True,Mesh->20,
    ViewPoint->{2.17,-2.4,1},
    ImageSize->400,
    BoxRatios->{1, 1, 1}]
  }}],
    {t,0,5,0.05}
];

Manipulate[
  frames[[i]],
  {{i,1,"time"},1,Length@frames,1,Appearance->"Labeled"}
]

Export["anim.gif",frames,"DisplayDurations"->Table[.2,{Length@frames}]]
```

Here is the same animation made in Maple 2018

Maple source code for all the above animations

```
#by Nasser M. Abbasi, May 23,2018

restart;
currentdir("X:/data/public_html/my_notes/PDE_animations/problems/wave_disk_exam_problem_4");
nTerms := 20:
lam    := evalf([BesselJZeros(0,1..nTerms)]):
c      := seq(1/lam[n]^2*BesselJ(1,lam[n]/2)/BesselJ(1,lam[n])^2,n=1..nTerms):
mySol  := proc(r,t)
  local n;
  4/Pi*sum(c[n]*BesselJ(0,lam[n]*r)*sin(lam[n]*t),n=1..nTerms);
end proc:

maxTime := 5: (*seconds*)
delay   := 0.05:
nFrames := round(maxTime/delay):
```

```
frames  := [seq( plot3d([ r, theta, mySol(r,(i*delay)) ],
                  r      = 0..1,
                  theta  = 0..2*Pi,
                  coords = cylindrical,
                  axes   = none,
                  title  = sprintf("%s %3.2f %s","time ",(i*delay),"seconds")
               ),
            i=0..nFrames-1)
          ]:
plots:-display(convert(frames,list),insequence=true);
```

Here is the same animation made in Matlab 2016a

Matlab source code for all the above animations

```
function nma_HW4_math_322
%By Nasser M. Abbasi, May 23, 2018

close all;

GENERATE_GIF=true; %turn to false to not generate animated gif

lam = zeros(80,1); %eigenvalues
for i = 1:80
  lam(i) = fzero(@(x)besselj(0,x),i);
end;

lam    = uniquetol(lam); %must use  uniquetol
nTerms = 20;
c      = zeros(nTerms,1);

for i = 1:nTerms
  c(i) = 1/lam(i)^2*besselj(1,lam(i)/2)/besselj(1,lam(i))^2;
end

  %------------- inner function --------
  function tot = mySol(r,t)
    tot = 0;
    for ii =1:nTerms
        tot = tot + (c(ii)*besselj(0,lam(ii).*r).*sin(lam(ii)*t));
    end;
    tot = 4/pi*tot;
```

```
  end
  %-------------------

maxTime = 5; %seconds
delay   = 0.05;
nFrames = round(maxTime/delay);

r   = 0:.05:1;
phi = 0:pi/20:2*pi;
[R,PHI] = meshgrid(r,phi);

fig_handle   = figure();
set(fig_handle,'Name',....
   'Math 322, Final exam problem 4 animations by Nasser M. Abbasi');

for i=1:nFrames
  Z = mySol(R,((i-1)*delay));
  surf(R.*cos(PHI), R.*sin(PHI), Z);
  set(gca,'nextplot','replacechildren','visible','on');
  colormap cool ;
  title(sprintf('time = %3.2f', (i-1)*delay));
  zlim([-0.6 0.6]);
  drawnow;
  pause(.01);
  if GENERATE_GIF
      frame = getframe(gcf);
      im = frame2im(frame);
      [imind,cm] = rgb2ind(im,256);
      if i == 1
          imwrite(imind,cm,'matlab_animations.gif','gif', ...
                                   'DelayTime',0.1,'LoopCount',0);
      else
          imwrite(imind,cm,'matlab_animations.gif','gif',...
                 'WriteMode','append','DelayTime',0.1,'LoopCount',0);
      end
  end
end

end
```

6.2.2.10 [415] θ dependency, fixed on edges, zero initial position with internal source (Haberman 8.5.5. (b)

problem number 415

Added January 15, 2020

Problem 8.5.5. (b) Richard Haberman applied partial di fb00erential equations book, 5th edition

Solve wave PDE inside circular membrane for $u(r,\theta,t)$ with $0<r<a$ and $t>0$ and $-\pi<\theta<\pi$

$$u_{tt} = c^2\nabla^2 u(r,\theta) + Q(r,\theta,t)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0\\ |u(0,\theta,t)| &< \infty\\ u(r,-\pi,t) &= u(r,\pi,t)\\ \frac{\partial u}{\partial\theta}(r,-\pi,t) &= \frac{\partial u}{\partial\theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta)\\ u_t(r,\theta,0) &= 0 \end{aligned}$$

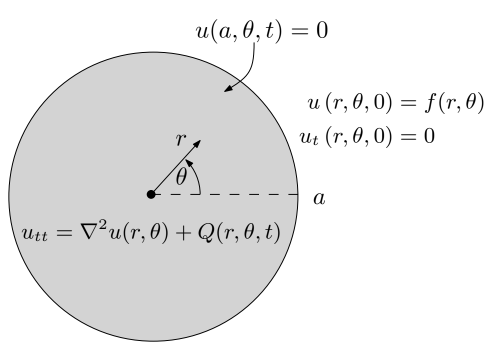


Figure 6.129: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"]+Q[r,theta,
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

Failed

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The solution to the corresponding homogeneous wave PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2$$

Is known to be

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} a_n(t)\, J_n\left(\sqrt{\lambda_{nm}} r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_n(t)\, J_n\left(\sqrt{\lambda_{nm}} r\right)\sin(n\theta)$$

Where λ_{nm} are found by solving roots of $J_n\left(\sqrt{\lambda_{nm}} a\right) = 0$. To make things simpler, we will write

$$u(r,\theta,t) = \sum_i a_i(t)\,\Phi_i(r,\theta)$$

Where the above means the double sum of all eigenvalues λ_i. So $\Phi_i(r,\theta)$ represents $J_n\left(\sqrt{\lambda_{nm}} r\right)\{\cos(n\theta), \sin(\theta)\}$ combined. So double sum is implied everywhere. Given this, we now expand the source term

$$Q(r,\theta,t) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

And the original PDE becomes

$$\sum_i a_i''(t)\,\Phi(\lambda_i) = c^2 \sum_i a_i(t)\,\nabla^2(\Phi_i(r,\theta)) + \sum_i q_i(t)\,\Phi_i(r,\theta) \tag{1}$$

But

$$\nabla^2(\Phi_i(r,\theta)) = -\lambda_i \Phi_i(r,\theta)$$

Hence (1) becomes

$$\sum_i a_i''(t)\,\Phi_i(r,\theta) + c^2\lambda_i a_i(t)\,\Phi_i(r,\theta) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

$$\sum_i \left(a_i''(t) + c^2\lambda_i a_i(t)\right)\Phi_i(r,\theta) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

Applying orthogonality gives

$$a_i''(t) + c^2\lambda_i a_i(t) = q_i(t)$$

Where

$$q_i(t) = \frac{\int_0^a \int_{-\pi}^{\pi} Q(r,\theta,t)\,\Phi_i(r,\theta)\,r dr d\theta}{\int_0^a \int_{-\pi}^{\pi} \Phi_i^2(r,\theta)\,r dr d\theta}$$

The solution to the homogenous ODE is

$$a_i^h(t) = A_i \cos\left(c\sqrt{\lambda_i} t\right) + B_i \sin\left(c\sqrt{\lambda_i} t\right)$$

And the particular solution is found if we know what $Q(r,\theta,t)$ and hence $q_i(t)$. For now, lets call the particular solution as $a_i^p(t)$. Hence the solution for $a_i(t)$ is

$$a_i(t) = A_i \cos\left(c\sqrt{\lambda_i} t\right) + B_i \sin\left(c\sqrt{\lambda_i} t\right) + a_i^p(t)$$

Plugging the above into the $u(r,\theta,t) = \sum_i a_i(t)\,\Phi_i(r,\theta)$, gives

$$u(r,\theta,t) = \sum_i \left(A_i \cos\left(c\sqrt{\lambda_i} t\right) + B_i \sin\left(c\sqrt{\lambda_i} t\right) + a_i^p(t)\right)\Phi_i(r,\theta) \tag{2}$$

We now find A_i, B_i from initial conditions. At $t = 0$

$$f(r,\theta) = \sum_i \left(A_i + a_i^p(0)\right)\Phi_i(r,\theta)$$

Applying orthogonality

$$\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta = \int_0^a \int_{-\pi}^{\pi} \sum_i \left(A_i + a_i^p(0)\right)\Phi_i(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta$$

$$\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta = \left(A_j + a_j^p(0)\right)\int_0^a \int_{-\pi}^{\pi} \Phi_j^2(r,\theta)\,r dr d\theta$$

$$\left(A_i + a_i^p(0)\right) = \frac{\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_i(r,\theta)\,r dr d\theta}{\int_0^a \int_{-\pi}^{\pi} \Phi_i^2(r,\theta)\,r dr d\theta}$$

Taking time derivative of (2)

$$\frac{\partial u(r,\theta,t)}{\partial t}=\sum_i\left(-A_ic\sqrt{\lambda_i}\sin\left(c\sqrt{\lambda_i}t\right)+c\sqrt{\lambda_i}B_i\cos\left(c\sqrt{\lambda_i}t\right)+\frac{da_i^p(t)}{dt}\right)\Phi_i(r,\theta)$$

At $t=0$

$$0=\sum_i\left(c\sqrt{\lambda_i}B_i+\frac{da_i^p(0)}{dt}\right)\Phi_i(r,\theta)$$

Hence $B_i=0$. Therefore the final solution is

$$u(r,\theta,t)=\sum_i\left(A_i\cos\left(c\sqrt{\lambda_i}t\right)+a_i^p(t)\right)\Phi_i(r,\theta)$$

Where

$$\left(A_i+a_i^p(0)\right)=\frac{\int_0^a\int_{-\pi}^{\pi}f(r,\theta)\,\Phi_i(r,\theta)\,rdrd\theta}{\int_0^a\int_{-\pi}^{\pi}\Phi_i^2\,(r,\theta)\,rdrd\theta}$$

This complete the solution.

6.3 Wave PDE in 3D

Local contents

6.3.1 Spherical coordinates

Local contents

6.3.1.1 [416] No I.C. no B.C.

problem number 416

Added Jan 10, 2019.

Solve for $u(r,\theta,\phi,t)$ the wave PDE in 3D

$$u_{tt} = c^2\nabla^2 u$$

Using the Physics convention for Spherical coordinates system.

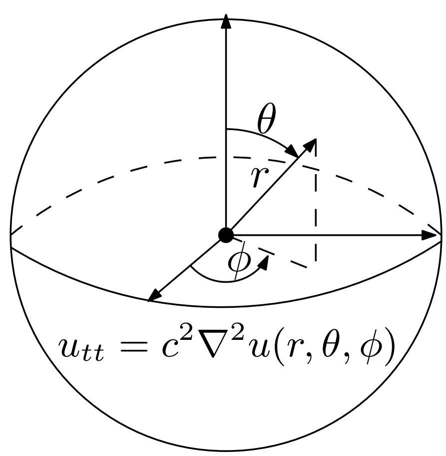


Figure 6.130: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, phi, t], {r, theta, phi}, "Spherical"];
pde =  D[u[r, theta, phi, t], {t, 2}] == c^2*lap;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, theta, phi, t], {r, theta, phi, t}, A
```

$$\left\{\left\{u(r,\theta,\phi,t)\to \left\{ \frac{\sqrt{2}e^{-\frac{1}{2}\sqrt{c_{10}}(2\phi+\pi)-t\sqrt{c_{11}}}\left(J_{\frac{1}{2}\sqrt{\frac{4c_9}{c^2}+1}}\left(\frac{r\sqrt{-c_{11}}}{\sqrt{c^2}}\right)c_1+Y_{\frac{1}{2}\sqrt{\frac{4c_9}{c^2}+1}}\left(\frac{r\sqrt{-c_{11}}}{\sqrt{c^2}}\right)c_2\right)\left(e^{2\phi\sqrt{c_{10}}}c_5+c_6\right)\left(e^{2t\sqrt{c_{11}}}c_7+c\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
lap:=VectorCalculus:-Laplacian( u(r,theta,phi,t), 'spherical'[r,theta,phi] );
pde := diff(u(r,theta,phi,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,theta,phi,t),'buil
sol := simplify(sol);
```

$$u(r,\theta,\phi,t)=\frac{\left(c_7\mathrm{e}^{2t\sqrt{-\_c_4}}+c_8\right)\left(c_5\mathrm{e}^{2\phi\sqrt{-\_c_3}}+c_6\right)\left(c_1\,\mathrm{BesselJ}\left(\frac{\sqrt{\frac{c^2-4\_c_1}{c^2}}}{2},\frac{\sqrt{-\_c_4}\,r}{c}\right)+c_2\,\mathrm{BesselY}\left(\frac{\sqrt{\frac{c^2-4\_c_1}{c^2}}}{2},\right.\right.}{}$$

6.3.2 Cylindrical coordinates

Local contents

6.3.2.1 [417] No I.C. no B.C.

problem number 417

Added Jan 10, 2019.

Solve for $u(r,\phi,z,t)$ the wave PDE in 3D

$$u_{tt} = c^2\nabla^2 u$$

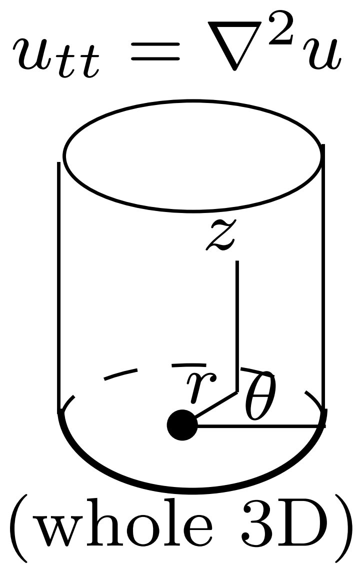


Figure 6.131: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, phi, z, t], {r, phi, z}, "Cylindrical"];
pde =  D[u[r, phi, z, t], {t, 2}] == c^2*lap;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, phi, z, t], {r, phi, z, t}], 60*10]];
```

$$\left\{\left\{u(r,\phi,z,t)\to \left\{ \begin{array}{l} e^{-\sqrt{c_9}\phi-z\sqrt{c_{10}}-t\sqrt{c_{11}}}\left(J_{\sqrt{-c_9}}\left(\frac{r\sqrt{c^2c_{10}-c_{11}}}{\sqrt{c^2}}\right)c_1+Y_{\sqrt{-c_9}}\left(\frac{r\sqrt{c^2c_{10}-c_{11}}}{\sqrt{c^2}}\right)c_2\right)\left(e^{2\phi\sqrt{c_9}}c_3+c_4\right. \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
restart;
lap :=VectorCalculus:-Laplacian( u(r,phi,z,t), 'cylindrical'[r,phi,z] );
pde := diff(u(r,phi,z,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,phi,z,t),'build'))
```

$$u(r,\phi,z,t) = \left(c_3 \mathrm{e}^{2\phi\sqrt{-\_c_2}} + c_4\right)\left(c_1 \operatorname{BesselJ}\left(\sqrt{-\_c_2}, \frac{\sqrt{\_c_3 c^2 - \_c_4}\, r}{c}\right) + c_2 \operatorname{BesselY}\left(\sqrt{-\_c_2}, \frac{\sqrt{\_c_3 c^2 -}}{c}\right.\right.$$

CHAPTER 7

Handbook of first order partial differential equations

Local contents

7.1 chapter 1

Local contents

7.1.1 problem number 1

problem number 418

Added January 2, 2019.

Problem 1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1],y)dK[1]+c_1(y)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \int f(x,y)\,dx + \_F1(y)$$

7.1.2 problem number 2

problem number 419

Added January 2, 2019.

Problem 1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^y f(x,K[1])dK[1]+c_1(x)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),y)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int f(x,y)\,dy+\_F1(x)$$

7.1.3 problem number 3

problem number 420

Added January 2, 2019.

Problem 1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x = wf(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == w[x, y]*f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y)\exp\left(\int_1^x f(K[1],y)dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(y)\,\mathrm{e}^{\int f(x,y)dx}$$

Hand solution

$$\begin{aligned}\frac{\partial w}{\partial x} &= wf(x,y)\\ \frac{1}{w}\frac{\partial w}{\partial x} &= f(x,y)\end{aligned}$$

Integrating both sides w.r.t. x gives

$$\begin{aligned}\ln(w) &= \int_0^x f(s,y)\,ds + G(y)\\ w &= e^{\int_0^x f(s,y)ds+G(y)}\\ &= F(y)\,e^{\int_0^x f(s,y)ds}\end{aligned}$$

Where $F(y)=e^{G(y)}$

7.1.4 problem number 4

problem number 421

Added January 2, 2019.

Problem 1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_y = wf(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == w[x, y]*f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \rightarrow c_1(x) \exp\left(\int_1^y f(x, K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1(y)\, e^{\int f(x,y)dx}$$

7.1.5 problem number 5

problem number 422

Added January 2, 2019.

Problem 1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x = wf(x, y) + g(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == w[x, y]*f[x, y] + g[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x f(K[1],y)dK[1]\right)\left(\int_1^x \exp\left(-\int_1^{K[2]} f(K[1],y)dK[1]\right)g(K[2],y)dK[2]+c_1(y)\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \mathrm{e}^{-\left(\int f(x,y)dx\right)}g(x,y)\,dx+\_F1(y)\right)\mathrm{e}^{\int f(x,y)dx}$$

Hand solution

$$\begin{aligned}\frac{\partial w}{\partial x} &= wf(x,y)+g(x,y)\\ \frac{\partial w}{\partial x}-wf(x,y) &= g(x,y)\end{aligned}$$

We can treat this similar to linear ODE and use an integrating factor $I=e^{-\int f(x,y)dx}$ hence the above becomes

$$\begin{aligned}\frac{\partial}{\partial x}\left(we^{-\int f(x,y)dx}\right) &= e^{-\int wdx}g(x,y)\\ we^{-\int f(x,y)dx} &= \int e^{-\int f(x,y)dx}g(x,y)\,dx+G(y)\\ w &= \left(\int e^{-\int f(x,y)dx}g(x,y)\,dx+G(y)\right)e^{\int f(x,y)dx}\end{aligned}$$

7.1.6 problem number 6

problem number 423

Added January 2, 2019.

Problem 1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = wf(x,y) + g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == w[x, y]*f[x, y] + g[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^y f(x,K[1])dK[1]\right)\left(\int_1^y \exp\left(-\int_1^{K[2]} f(x,K[1])dK[1]\right)g(x,K[2])dK[2]+c_1(x)\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),y)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int e^{-\left(\int f(x,y)dy\right)} g(x,y)\,dy + \_F1(x)\right) e^{\int f(x,y)dy}$$

7.2 chapter 2

Local contents

7.2.1 2.1

Local contents

7.2.1.1 [424] problem number 1

problem number 424

Added January 2, 2019.

Problem 2.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)$$

Hand solution

$$aw_x + bw_y = 0$$

The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dw}{0}$$

The first pair of equations results in $bdx = ady$ or $bx = ay + C_1$. Hence

$$C_1 = bx - ay$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$ where F is arbitrary function, therefore the solution is

$$w(x, y) = F(bx - ay)$$

7.2.1.2 [425] problem number 2

problem number 425

Added January 2, 2019.

Problem 2.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + (bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + (b*x + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{-2ay+bx^2+2cx}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+(b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-b\,x^2+2ya-2cx}{2a}\right)$$

<u>Hand solution</u>

Solve $aw_x + (bx+c)\,w_y = 0$. The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{(bx+c)} = \frac{dw}{0}$$

The first pair of equations gives $\frac{(bx+c)}{a}dx = dy$. Integrating results in

$$\frac{1}{a}\left(\frac{bx^2}{2}+cx\right) = y + C_1$$
$$C_1 = \frac{1}{a}\left(\frac{bx^2}{2}+cx\right) - y$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Where F is arbitray function. Therefore

$$w(x,y) = F\left(\frac{bx^2}{2a}+\frac{c}{a}x-y\right)$$

7.2.1.3 [426] problem number 3

problem number 426

Added January 2, 2019.

Problem 2.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax+by+c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x + b*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-bx}(abx+a+b(by+c))}{b^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(b^2y+a+(ax+c)\,b)\,\mathrm{e}^{-bx}}{b^2}\right)$$

Hand solution

Solve

$$w_x+(ax+by+c)\,w_y=0 \tag{1}$$

The Lagrange-charpit equations are

$$dx=\frac{dy}{(ax+by+c)}=\frac{dw}{0}$$

The first pair of equations gives

$$\begin{aligned}\frac{dy}{dx}&=ax+by+c\\ \frac{dy}{dx}-by&=ax+c\end{aligned}$$

This is linear. Integrating factor is $I = e^{-bx}$. Hence the above becomes

$$
\begin{aligned}
\frac{d}{dx}\left(ye^{-bx}\right) &= (ax+c)\,e^{-bx} \\
ye^{-bx} &= a\int xe^{-bx} + c\int e^{-bx} + C_1 \\
ye^{-bx} &= a\left(-\frac{(1+bx)\,e^{-bx}}{b^2}\right) - c\frac{e^{-bx}}{b} + C_1 \\
y &= -a\frac{(1+bx)}{b^2} - \frac{c}{b} - C_1 e^{bx} \\
C_1 &= -\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}
\end{aligned}
$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$
\begin{aligned}
w(x,y) &= F\left(-\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}\right) \\
&= F\left(\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}\right)
\end{aligned}
$$

7.2.1.4 [427] problem number 4

problem number 427

Added January 2, 2019.

Problem 2.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$
axw_x + byw_y = 0
$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$
\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{b}{a}}\right)$$

Hand solution

Solve

$$axw_x + byw_y = 0 \tag{1}$$

The Lagrange-charpit equations are

$$\frac{dx}{ax} = \frac{dy}{by} = \frac{dw}{0}$$

The first pair of equations gives

$$\begin{aligned}\frac{b}{a}\frac{dx}{x} &= \frac{dy}{y}\\ \frac{b}{a}\ln x &= \ln y + C_1\\ x^{\frac{b}{a}} &= C_1 y\\ C_1 &= \frac{x^{\frac{b}{a}}}{y}\end{aligned}$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$w(x,y) = F\left(\frac{x^{\frac{b}{a}}}{y}\right)$$

7.2.1.5 [428] problem number 5

problem number 428

Added January 2, 2019.

Problem 2.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y^2a-b\,x^2}{a}\right)$$

Hand solution

$$ayw_x+bxw_y=0$$

Using method of charaterstics the lagrange-Charpit equations are

$$\frac{dx}{ay}=\frac{dy}{bx}=\frac{du}{0}$$

The first two equations give $aydy=bxdx$. Hence

$$\begin{aligned}a\frac{y^2}{2}&=\frac{bx^2}{2}+C_1\\C_1&=a\frac{y^2}{2}-\frac{bx^2}{2}\end{aligned}$$

And $du=0$. This gives $u=C_2$. Now let $C_2=F(C_1)$. This gives

$$u=F\left(\frac{ay^2-bx^2}{2}\right)$$

7.2.1.6 [429] problem number 6

problem number 429

Added January 2, 2019.

Problem 2.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + (y+a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + (y + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\left(-\log\left(e^{-\frac{a+y}{a}}(a+y)\right)\right)-a-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(w(x,y),x)+(y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-a\ln(a+y)-x+y)$$

Hand solution

$$yw_x + (y+a)\,w_y = 0$$

Using method of charaterstics the lagrange-Charpit equations are

$$\frac{dx}{y} = \frac{dy}{y+a} = \frac{du}{0}$$

The first two equations give $dx = \frac{y}{y+a}dy$. Hence by integrating

$$x = y - a\ln(y+a) + C_1$$
$$C_1 = x - y + a\ln(y+a)$$

And $du = 0$. This gives $u = C_2$. Now let $C_2 = F(C_1)$. This gives

$$u = F(x - y + a\ln(y+a))$$

7.2.1.7 [430] problem number 7

problem number 430

Added January 2, 2019.

Problem 2.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay+bx+c)w_x-(by+kx+s)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y + b*x + c)*D[w[x, y], x] - (b*y + k*x + s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y(ay+2bx+2c)+kx^2+2sx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=(a*y+b*x+c)*diff(w(x,y),x)-(b*y+k*x+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{}{\sqrt{a^3k^2y^2-2a^2b^2k\,y^2+2a^2b\,k^2xy+a^2k^3x^2+a\,b^4y^2-4a\,b^3kxy-2a\,b^2k^2x^2+2b^5xy+b^4}}\right.$$

Hand solution

Solve

$$(ay+bx+c)\,w_x-(by+kx+s)\,w_y=0$$

The lagrange-Charpit equations are

$$\frac{dx}{ay+bx+c}=\frac{dy}{-(by+kx+s)}=\frac{dw}{0}$$

The first two equations give

$$\frac{dy}{dx}=\frac{-(by+kx+s)}{ay+bx+c}$$

Need to solve the above, then solve for C_1 to finish the solution.

7.2.1.8 [431] problem number 8

problem number 431

Added January 2, 2019.

Problem 2.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x + b_1y + c_1)w_x + (a_2x + b_2y + c_2)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a1*x + b1*y + c1)*D[w[x, y], x] + (a1*x + b2*y + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a1*x+b1*y+c1)*diff(w(x,y),x)+(a2*x+b2*y+c2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2(a1 + b2)\arctan\left(\frac{-2a2\,b1^2y+(-2a2\,c1+(-a2x+c2)a1+(2a1y+a2x+c2)b2)b1+(a1-b2)(a1x+c1)b2}{\sqrt{-a1^2+2a1\,b2-4a2\,b1-b2^2}\,((-a2x-c2)b1+(a1x+c1)b2)}\right) -$$

7.2.2 2.2

Local contents

7.2.2.1 [432] problem number 1

problem number 432

Added January 2, 2019.

Problem 2.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^2 + bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2 + b*x + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{ax^3}{3}-\frac{bx^2}{2}-cx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x^2+b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{3}a\,x^3-\frac{1}{2}b\,x^2-cx+y\right)$$

7.2.2.2 [433] problem number 2

problem number 433

Added January 2, 2019.

Problem 2.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay^2+by+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^2 + b*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2ay+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*y^2+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sqrt{4ca-b^2}\,x - 2\arctan\left(\frac{2ya+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$$

7.2.2.3 [434] problem number 3

problem number 434

Added January 2, 2019.

Problem 2.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^2 + cx)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^2 + c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-ax}(b(a^2x^2+2ax+2)+a(a^2y+acx+c))}{a^3}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*y+b*x^2+c*x)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a^3y + (bx+c)\,a^2x + (2bx+c)\,a + 2b)\,e^{-ax}}{a^3}\right)$$

7.2.2.4 [435] problem number 4

problem number 435

Added January 2, 2019.

Problem 2.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (axy + bx^2 + cx + ky + s)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x*y + b*x^2 + c*x + k*y + s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{1}{2}x(ax+2k)}\left(2\sqrt{a}(a^2y+a(bx+c)-bk)-\sqrt{2\pi}e^{\frac{(ax+k)^2}{2a}}\operatorname{erf}\left(\frac{ax+k}{\sqrt{2}\sqrt{a}}\right)(a^2s+a(b-ck)+bk\right.}{2a^{5/2}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x*y+b*x^2+c*x+k*y+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(\sqrt{\pi}\,\sqrt{2}\,(a^2s-ack+(k^2+a)\,b)\operatorname{erf}\left(\frac{\sqrt{2}\,(ax+k)}{2\sqrt{a}}\right)\mathrm{e}^{\frac{2a^2x^2+4kxa+k^2}{2a}}+2\left(-a^{\frac{5}{2}}y+\sqrt{a}\,bk+(-\right.\right.}{2a^{\frac{5}{2}}}\right.$$

7.2.2.5 [436] problem number 5

problem number 436

Added January 2, 2019.

Problem 2.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 - a^2x^2 + 3a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2*x^2 + 3*a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(ax-y)\,\text{ParabolicCylinderD}\left(-2,i\sqrt{2}\sqrt{a}x\right)+i\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(-1,i\sqrt{2}\sqrt{\right.}\right.}{(ax+y)\,\text{ParabolicCylinderD}\left(1,\sqrt{2}\sqrt{a}x\right)-\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(2,\sqrt{2}\sqrt{a}x\right)}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2*x^2+3*a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\,x^2+xy+1}{-\sqrt{\pi}\,\left((-a)^{\frac{3}{2}}\,x^2+\sqrt{-a}\,xy+\sqrt{-a}\right)\text{erf}\left(\sqrt{-a}\,x\right)+(ax-y)\,\mathrm{e}^{a\,x^2}}\right)$$

7.2.2.6 [437] problem number 6

problem number 437

Added January 2, 2019.

Problem 2.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 - a^2x^2 + a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2*x^2 + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{\pi}(y-ax)\operatorname{Erfi}\left(\sqrt{a}x\right)+2\sqrt{a}e^{ax^2}}{2\sqrt{a}(ax-y)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2*x^2+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(ax-y)\sqrt{\pi}}{\sqrt{\pi}\ (ax-y)\operatorname{erf}\left(\sqrt{-a}\,x\right)-2\sqrt{-a}\,\mathrm{e}^{a\,x^2}}\right)$$

7.2.2.7 [438] problem number 7

problem number 438

Added January 2, 2019.

Problem 2.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + axy + a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*y + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)-\frac{ye^{\frac{ax^2}{2}}}{\sqrt{2}\sqrt{a}(xy+1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+a*x*y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(xy+1)\,a\,\text{erf}\left(\frac{\sqrt{-2a}\,x}{2}\right)-\sqrt{-\frac{2a}{\pi}}\,y\,e^{\frac{a\,x^2}{2}}}{\sqrt{-\frac{2a}{\pi}}\,(xy+1)}\right)$$

7.2.2.8 [439] problem number 8

problem number 439

Added January 2, 2019.

Problem 2.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(y^2+axy-abx-b^2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*y - a*b*x - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{2b^2}{a}}\left(\sqrt{2\pi}(y-b)\operatorname{Erfi}\left(\frac{ax+2b}{\sqrt{2}\sqrt{a}}\right)+2\sqrt{a}e^{\frac{(ax+2b)^2}{2a}}\right)}{2\sqrt{a}(b-y)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+a*x*y-a*b*x-b^2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(\sqrt{\pi}\,(b-y)\operatorname{erf}\left(\frac{(ax+2b)\sqrt{2}}{2\sqrt{-a}}\right)+\sqrt{2}\,\sqrt{-a}\,\mathrm{e}^{\frac{(ax+2b)^2}{2a}}\right)\sqrt{2}\,\mathrm{e}^{-\frac{2b^2}{a}}}{\sqrt{-a}\,(2b-2y)}\right)$$

7.2.2.9 [440] problem number 9

problem number 440

Added January 2, 2019.

Problem 2.2.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+k(ax+by+c)^2w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*(a*x + a*y + c)^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-2ia\sqrt{k}x}\left(ia\sqrt{k}(x+y)+ic\sqrt{k}+1\right)}{2a\sqrt{k}\left(a\sqrt{k}(x+y)+c\sqrt{k}+i\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+k*(a*x+a*y+c)^2*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\sqrt{k}\,x-\arctan\left(\left((x+y)\,a+c\right)\sqrt{k}\right)}{a\sqrt{k}}\right)$$

7.2.2.10 [441] problem number 10

problem number 441

Added January 2, 2019.

Problem 2.2.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+(ay^2+cx^2+y)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + c*x^2 + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tan^{-1}\left(\frac{\sqrt{a}y}{\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+(a*y^2+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\sqrt{ac}\,x+\arctan\left(\frac{ay}{\sqrt{ac}\,x}\right)}{\sqrt{ac}}\right)$$

7.2.2.11 [442] problem number 11

problem number 442

Added January 2, 2019.

Problem 2.2.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+(ay^2+bxy+cx^2+y)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + b*x*y + c*x^2 + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2ay+bx}{x\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+(a*y^2+b*x*y+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\sqrt{4ac-b^2}\,x-2\arctan\left(\frac{2ya+bx}{\sqrt{4ac-b^2}\,x}\right)}{\sqrt{4ac-b^2}}\right)$$

7.2.2.12 [443] problem number 12

problem number 443

Added January 2, 2019.

Problem 2.2.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+c)w_x+\left(\alpha(ay+bx)^2+\beta(ay+bx)-bx+\gamma\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + c)*D[w[x, y], x] + (alpha*(a*y + b*x)^2 + beta*(a*y + b*x) - b*x + gamma)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(2\tan^{-1}\left(\frac{2\alpha(ay+bx)+\beta}{a\alpha\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}}\right)-a\alpha\log(ax+c)\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x + c)*diff(w(x,y),x)+(alpha*(a*y+b*x)^2+beta*(a*y+b*x)-b*x+g)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-2a^2\arctan\left(\frac{(2a\alpha y+2\alpha bx+\beta)a^2}{\sqrt{4a^4\alpha g-a^4\beta^2+4a^3\alpha bc}}\right)+\sqrt{4a^3\alpha bc+(4g\alpha-\beta^2)\,a^4}\,\ln(ax+c)}{\sqrt{4a^3\alpha bc+(4g\alpha-\beta^2)\,a^4}}\right)$$

7.2.2.13 [444] problem number 13

problem number 444

Added January 2, 2019.

Problem 2.2.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x+by^2w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{b}{ax}-\frac{1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ax-by}{axy}\right)$$

7.2.2.14 [445] problem number 14

problem number 445

Added January 2, 2019.

Problem 2.2.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2+b)w_x-\left(y^2-2xy+(1-a)x^2-b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^2 + b)*D[w[x, y], x] - (y^2 - 2*x*y + (1 - a)*x^2 - b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\frac{(y-x)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}-1}{x-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x^2+b)*diff(w(x,y),x)-(y^2-2*x*y+(1-a)*x^2-b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(x-y)\arctan\left(\frac{ax}{\sqrt{ab}}\right)+\sqrt{ab}}{\sqrt{ab}\,(x-y)}\right)$$

7.2.2.15 [446] problem number 15

problem number 446

Added January 2, 2019.

Problem 2.2.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2+b_1x+x_1)w_x+(a_2y^2+b_2y+c_2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x^2 + b1*x + c1)*D[w[x, y], x] + (a2*y^2 + b2*y + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2\text{a2}y+\text{b2}}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}\right)}{\sqrt{4\text{a2}\text{c2}-\text{b2}^2}}-\frac{2\tan^{-1}\left(\frac{2\text{a1}x+\text{b1}}{\sqrt{4\text{a1}\text{c1}-\text{b1}^2}}\right)}{\sqrt{4\text{a1}\text{c1}-\text{b1}^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a1*x^2+b1*x+c1)*diff(w(x,y),x)+ (a2*y^2+b2*y+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(\sqrt{4c2\,a2-b2^2}\,\arctan\left(\frac{2a1x+b1}{\sqrt{4c1\,a1-b1^2}}\right)-\sqrt{4c1\,a1-b1^2}\,\arctan\left(\frac{2a2y+b2}{\sqrt{4c2\,a2-b2^2}}\right)\right)}{\sqrt{4c1\,a1-b1^2}\,\sqrt{4c2\,a2-b2^2}}\right)$$

7.2.2.16 [447] problem number 16

problem number 447

Added January 2, 2019.

Problem 2.2.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)(x-b)w_x - \left(y^2+k(y+x-a)(y+x-b)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*(x - b)*D[w[x, y], x] - (y^2 + k*(y + x - a)*(y + x - b))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-a)-\log(x-b))}{2(a-b)}-\tan^{-1}\left(\frac{ak+bk-2(k(x+y)+y)}{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde := (x-a)*(x-b)*diff(w(x,y),x)- (y^2+k*(y+x-a)*(y+x-b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{((b-x-y)k-y)(a-x)^{-k}(b-x)^k}{(a-x-y)k-y}\right)$$

7.2.2.17 [448] problem number 17

problem number 448

Added January 2, 2019.

Problem 2.2.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1y^2+b_1y+c_1)w_x+(a_2x^2+b_2x+c_2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*y^2 + b1*y + c1)*D[w[x, y], x] + (a2*x^2 + b2*x + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{6}(2\text{a1}y^3-2\text{a2}x^3+3\text{b1}y^2-3\text{b2}x^2+6\text{c1}y-6\text{c2}x)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a1*y^2+b1*y+c1)*diff(w(x,y),x)+ (a2*x^2+b2*x+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{3}a1\,y^3-\frac{1}{3}a2\,x^3+\frac{1}{2}b1\,y^2-\frac{1}{2}b2\,x^2+c1y-c2x\right)$$

7.2.2.18 [449] problem number 18

problem number 449

Added January 2, 2019.

Problem 2.2.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$y(ax + b)w_x + (ay^2 - cx)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*(a*x + b)*D[w[x, y], x] + (a*y^2 - c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a(ay^2-2cx)-bc}{a^2(ax+b)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  y*(a*x+b)*diff(w(x,y),x)+ (a*y^2-c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a^2y^2-2acx-bc}{(ax+b)^2\,a^2}\right)$$

7.2.2.19 [450] problem number 19

problem number 450

Added January 2, 2019.

Problem 2.2.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ay^2 + bx)w_x - (cx^2 + by)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x)*D[w[x, y], x] - (x*x^2 + b*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^3}{3}+bxy+\frac{x^4}{4}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+b*x)*diff(w(x,y),x)- (x*x^2+b*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{3}a\,y^3-\frac{1}{4}x^4-bxy\right)$$

7.2.2.20 [451] problem number 20

problem number 451

Added January 2, 2019.

Problem 2.2.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx^2)w_x+2bxw_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b\,x^2 + (y^2+2y+2)\,a)\,\mathrm{e}^{-y}}{b}\right)$$

7.2.2.21 [452] problem number 21

problem number 452

Added January 2, 2019.

Problem 2.2.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2 + bx^2)w_x + 2bxyw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\log\left(\frac{bx^2}{y}-ay\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{y^2a - b\,x^2}\right)$$

7.2.2.22 [453] problem number 22

problem number 453

Added January 2, 2019.

Problem 2.2.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+x^2)w_x+(bx^2+c-2xy)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + x^2)*D[w[x, y], x] + (b*x^2 + c - 2*x*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{3}\left(ay^3-bx^3-3cx+3x^2y\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+x^2)*diff(w(x,y),x)+(b*x^2+c-2*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{3}a\,y^3+\frac{1}{3}b\,x^3-x^2y+cx\right)$$

7.2.2.23 [454] problem number 23

problem number 454

Added January 2, 2019.

Problem 2.2.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2+Bx^2-a^2B)w_x+(Cy^2+2Bxy)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x^2 - a^2*B)*D[w[x, y], x] + (C0*y^2 + 2*B*x*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a^2(-B)+y(\mathrm{C0}x-Ay)+Bx^2}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (A*y^2+B*x^2-a^2*B)*diff(w(x,y),x)+(C*y^2+2*B*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-A\,y^2-a^2B+B\,x^2+Cxy}{y}\right)$$

7.2.2.24 [455] problem number 24

problem number 455

Added January 2, 2019.

Problem 2.2.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx^2+cy)w_x+2bxw_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2 + c*y)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2+c*y)*diff(w(x,y),x)+2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a\,y^2+b\,x^2+2a+c+(2a+c)\,y)\,\mathrm{e}^{-y}}{b}\right)$$

7.2.2.25 [456] problem number 25

problem number 456

Added January 2, 2019.

Problem 2.2.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Axy+Bx^2+kx)w_x+(Dy^2+Exy+Fx^2+ky)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x*y + B*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F*x^2 + k*y)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  (A*x*y+B*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.2.26 [457] problem number 26

problem number 457

Added January 2, 2019.

Problem 2.2.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Axy + Aky + Bx^2 + Bkx)w_x + (Cy^2 + Dxy + k(D - B)y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x*y + A*k*y + B*x^2 + B*k*x)*D[w[x, y], x] + (C0*y^2 + D0*x*y + k*(D0 - B)*y)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*x*y+A*k*y+B*x^2+B*k*x)*diff(w(x,y),x)+(C0*y^2+D0*x*y+k*(D0-B)*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.2.27 [458] problem number 27

problem number 458

Added January 2, 2019.

Problem 2.2.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Ay^2 + Bxy + Cx^2 + kx)w_x + (Dy^2 + Exy + Fx^2 + ky)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x*y + C0*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2 + k*y)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*y^2+B*x*y+C0*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.2.28 [459] problem number 28

problem number 459

Added January 2, 2019.

Problem 2.2.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + Bxy + Cx^2)w_x + (Dy^2 + Exy + Fx^2)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x*y + C0*x^2)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (A*y^2+B*x*y+C0*x^2)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\ln(x) - \frac{\left(A\,\mathrm{RootOf}\left(A\_Z^3 + (B - D0)\_Z^2 - F0 + (C0 - E0)\_Z\right)^2 + B\,\mathrm{RootOf}\left(A\right.\right.}{3A\,\mathrm{RootOf}\left(A\_Z^3 + (B - D0)\_Z^2 - F0 + (C0 - E0)\_Z\right)}\right.$$

solution contains RootOf

7.2.2.29 [460] problem number 29

problem number 460

Added January 2, 2019.

Problem 2.2.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + 2Bxy + Dx^2 + a)w_x - (Dy^2 + 2Dxy - Ex^2 - b)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + 2*B*x*y + D0*x^2 + a)*D[w[x, y], x] - (D0*y^2 + 2*D0*x*y - E0*x^2 - b)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*y^2+2*B*x*y+D0*x^2+a)*diff(w(x,y),x)-(D0*y^2+2*D0*x*y-E0*x^2-b)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.2.30 [461] problem number 30

problem number 461

Added January 2, 2019.

Problem 2.2.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(y^2 - 2xy + x^2 + ay)w_x + ayw_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y^2 - 2*x*y + x^2 + a*y)*D[w[x, y], x] + a*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (y^2-2*x*y+x^2+a*y)*diff(w(x,y),x)+a*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{a + (x - y)\ln(y)}{x - y}\right)$$

7.2.2.31 [462] problem number 31, Hesse's equation

problem number 462

Added January 2, 2019.

Problem 2.2.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke (1965).

Solve for $w(x, y)$

$$(xf_1 - f_2)w_x + (yf_1 - f_3)w_y = 0$$

Where $f_n = a_n + b_n x + c_n y$.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (x*(a1 + b1*x + c1*y) - (a2 + b2*x + c2*y))*D[w[x, y], x] + (y*(a1 + b1*x + c1*y) - (
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (x *(a1+b1*x+c1*y)-(a2+b2*x+c2*y))*diff(w(x,y),x)+(y*(a1+b1*x+c1*y)-(a3+b3*x+c3*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

7.2.3 2.3

Local contents

7.2.3.1 [463] problem number 1

problem number 463

Added January 2, 2019.

Problem 2.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + bx^2y - a^2 - abx^2)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + b*x^2*y - a^2 - a*b*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{e^{\frac{bx^3}{3}}\left((bx^2+y)\,\text{HeunT}[a^2,-((a-2)b),0,0,b,x]+\text{HeunTPrime}[a^2,-((a-2)b),0,0,b\right.}{y\text{HeunT}\left[a^2,-ab,0,0,-b,x\right]+\text{HeunTPrime}\left[a^2,-ab,0,0,-b,x\right]}\right.\right.\right.$$

But it can't solve it when assuming $b>0$ which is strange.

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+b*x^2*y-a^2-a*b*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{3\left((b\,x^2\mathrm{csgn}(b)-b\,x^2-2y)\,HT\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)^{\frac{1}{3}}},-\frac{3(a-1)\sqrt{b^2}}{b},0,\frac{3^{\frac{2}{3}}(b^2)}{3}\right.\right.}{-3\left((b\,x^2\mathrm{csgn}(b)-b\,x^2-2y)\,HT\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)^{\frac{1}{3}}},-\frac{3(a-1)\sqrt{b^2}}{b},0,\frac{3^{\frac{2}{3}}(b^2)^{\frac{1}{6}}x}{3}\right)-\frac{2\,3^{\frac{2}{3}}(b^2)^{\frac{1}{6}}HT'\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)}\right.}{}\right.}\right.$$

7.2.3.2 [464] problem number 2

problem number 464

Added January 2, 2019.

Problem 2.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ax^2y+bx^3+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2*y + b*x^3 + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt[3]{3}b\,\text{Gamma}\left(\frac{4}{3},\frac{ax^3}{3}\right)}{a^{4/3}}+\frac{c\,\text{Gamma}\left(\frac{1}{3},\frac{ax^3}{3}\right)}{3^{2/3}\sqrt[3]{a}}+ye^{-\frac{ax^3}{3}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*x^3+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{3\left(3^{\frac{1}{6}}(ac+b)\,x\,\text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},\frac{a\,x^3}{3}\right)\mathrm{e}^{\frac{a\,x^3}{6}}+\frac{4(a\,x^3)^{\frac{1}{6}}(cx-y)a}{3}\right)\mathrm{e}^{-\frac{a\,x^3}{3}}}{4\,(a\,x^3)^{\frac{1}{6}}\,a}\right)$$

7.2.3.3 [465] problem number 3

problem number 465

Added January 2, 2019.

Problem 2.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ax^2y+by^3)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2*y + b*y^3)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{2ax^3}{3}}}{y^2}+\frac{i(\sqrt{3}+i)\,b\,\mathrm{Gamma}\left(\frac{1}{3},-\frac{2ax^3}{3}\right)}{\sqrt[3]{2}3^{2/3}\sqrt[3]{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*y^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2\,2^{\frac{2}{3}}3^{\frac{5}{6}}Obx\,y^2-3\,2^{\frac{2}{3}}3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)bx\,y^2\Gamma\left(\frac{1}{3},-\frac{2a\,x^3}{3}\right)+9\Gamma\left(\frac{2}{3}\right)O\,\mathrm{e}^{\frac{2a\,x^3}{3}}}{9\Gamma\left(\frac{2}{3}\right)O\,y^2}\right)$$

7.2.3.4 [466] problem number 4

problem number 466

Added January 2, 2019.

Problem 2.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(axy+b)y^2w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x*y + b)*y^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x*y+b)*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\sqrt{b^2-4a}\,b\,\text{arctanh}\left(\frac{\sqrt{b^2-4a}\,(2axy+b)}{-b^2+4a}\right)-8\left(-\frac{b^2}{4}+a\right)\left(-\frac{\ln((a\,x^2y^2+bxy+1)\,x^2)}{2}+\ln(xy)\right)}{-2b^2+8a}\right)$$

7.2.3.5 [467] problem number 5

problem number 467

Added January 2, 2019.

Problem 2.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + A(ax+by+c)^3y^2w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + A*(a*x + b*y + c)^3*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  A*(a*x+b*y+c)^3*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \frac{\ln\left(\frac{ax+(-\operatorname{RootOf}(A\,b^4\_Z^3+3A\,b^3c\_Z^2+3A\,b^2c^2\_Z+Ab\,c^3+a)+y)b}{b}\right)}{3Ab\left(\operatorname{RootOf}\left(A\,b^4\_Z^3+3A\,b^3c\_Z^2+3A\,b^2c^2\_Z+Ab\,c^3+a\right)b+c\right)^2}\right)$$

Answer contains RootOf

7.2.3.6 [468] problem number 6

problem number 468

Added January 2, 2019.

Problem 2.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ax^4y^3 + (bx^2-1)y + cx)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^4*y^3 + (b*x^2 - 1)*y + c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+  (a*x^4*y^3+(b*x^2-1)*y+c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-bxy-\mathrm{RootOf}(c^2a\_Z^3+b^3\_Z-b^3)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2a\_Z^3+b^3\_Z-b^3\right)^2 a\,c^2+b^3} - \frac{b\,x^2}{2}\right)$$

Answer contains RootOf

7.2.3.7 [469] problem number 7

problem number 469

Added January 2, 2019.

Problem 2.2.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + (ax^2y^2 + bxy + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + (a*x^2*y^2 + b*x*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{\sqrt{-4ac+b^2+2b+1}}\left(\sqrt{-4ac+b^2+2b+1}+2axy+b+1\right)}{\sqrt{-4ac+b^2+2b+1}-2axy-b-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+  (a*x^2*y^2+b*x*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2axy+b+1}{\sqrt{4ac-b^2-2b-1}}\right)+\sqrt{4ac-b^2-2b-1}\,\ln(x)}{\sqrt{4ac-b^2-2b-1}}\right)$$

7.2.3.8 [470] problem number 8

problem number 470

Added January 2, 2019.

Problem 2.2.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2y+b)w_x-(axy^2+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^2*y + b)*D[w[x, y], x] - (a*x*y^2 + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ax^2y^2+2by+2cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x^2*y+b)*diff(w(x,y),x)- (a*x*y^2+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{2}a\,x^2y^2-by-cx\right)$$

7.2.3.9 [471] problem number 9

problem number 471

Added January 2, 2019.

Problem 2.2.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+by^3)w_x-(cx^3+ay)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x + b*y^3)*D[w[x, y], x] - (c*x^3 + a*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  (a*x+b*y^3)*diff(w(x,y),x)- (c*x^3+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{4}b\,y^4-\frac{1}{4}c\,x^4-axy\right)$$

7.2.4 2.4

Local contents

7.2.4.1 [472] problem number 1

problem number 472

Added January 2, 2019.

Problem 2.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a\sqrt{x}y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\frac{2}{3}ax^{3/2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{2a\,x^{\frac{3}{2}}}{3}}\right)$$

7.2.4.2 [473] problem number 2

problem number 473

Added January 2, 2019.

Problem 2.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a\sqrt{x}y + b\sqrt{y})w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y + b*Sqrt[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{b\,\text{Gamma}\left(\frac{2}{3},\frac{1}{3}ax^{3/2}\right)}{\sqrt[3]{3}a^{2/3}}-\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(\frac{b\,\text{Gamma}\left(\frac{2}{3},\frac{1}{3}ax^{3/2}\right)}{\sqrt[3]{3}a^{2/3}}+\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(3\,3^{\frac{1}{3}}bx\,\text{WhittakerM}\left(\frac{1}{3},\frac{5}{6},\frac{a\,x^{\frac{3}{2}}}{3}\right)\mathrm{e}^{\frac{a\,x^{\frac{3}{2}}}{6}}+5\left(a\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}bx-10\left(a\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}\sqrt{y}\right)\mathrm{e}^{-\frac{a\,x^{\frac{3}{2}}}{3}}}{10\left(a\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}}\right)$$

7.2.4.3 [474] problem number 3

problem number 474

Added January 2, 2019.

Problem 2.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(a\sqrt{x}y+bx\sqrt{y})w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y + b*x*Sqrt[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt[3]{3}b\,\text{Gamma}\left(\frac{4}{3},\frac{1}{3}ax^{3/2}\right)}{a^{4/3}}+\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*x*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(3\,3^{\frac{1}{6}}b\sqrt{x}\ \text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},\frac{a\,x^{\frac{3}{2}}}{3}\right)\mathrm{e}^{\frac{a\,x^{\frac{3}{2}}}{6}}-4\left(a\,x^{\frac{3}{2}}\right)^{\frac{1}{6}}a\sqrt{y}\right)\mathrm{e}^{-\frac{a\,x^{\frac{3}{2}}}{3}}}{4\left(a\,x^{\frac{3}{2}}\right)^{\frac{1}{6}}a}\right)$$

7.2.4.4 [475] problem number 4

problem number 475

Added January 2, 2019.

Problem 2.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+A\sqrt{ax+by+c}w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + A*Sqrt[a*x + b*y + c]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(x-\frac{a\log\left(\frac{e^{\frac{2\sqrt{A^2b^2(ax+by+c)}}{a}}}{\left(\sqrt{A^2b^2(ax+by+c)}+a\right)^2}\right)}{A^2b^2}\right)\right\},\left\{w(x,y)\to c_1\left(x-\frac{a\log\left(\frac{e^{-\frac{2\sqrt{A^2b^2(ax+by+c)}}{a}}}{\left(a-\sqrt{A^2b^2(ax+by+c)}\right)^2}\right)}{A^2b^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ A*sqrt(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{A^2b^2x-2\sqrt{ax+by+c}\,Ab-a\ln\left(\sqrt{ax+by+c}\,Ab-a\right)+a\ln\left(\sqrt{ax+by+c}\,Ab+a\right)+}{A^2b^2}\right.$$

7.2.4.5 [476] problem number 5

problem number 476

Added January 2, 2019.

Problem 2.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ay+b\sqrt{y^2+cx^2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y + b*Sqrt[y^2 + c*x^2])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*diff(w(x,y),x)+ ( a*y + b *sqrt(y^2+c*x^2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.4.6 [477] problem number 6

problem number 477

Added January 2, 2019.

Problem 2.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax + b\sqrt{y})\, w_x - (c\sqrt{x} + ay)\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b*Sqrt[y])*D[w[x, y], x] - (c*Sqrt[x] + a*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{3a^3x^3}{8b^2}+\frac{2}{3}cx^{3/2}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(axy-\frac{2}{3}by^{3/2}+\frac{2}{3}cx^{3/2}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(axy+\frac{2}{3}by^{3/2}+\frac{2}{3}cx^{3/2}\right)\right\}$$

Maple ✓

```
restart;
pde := (a*x+b*sqrt(y))* diff(w(x,y),x)- (c*sqrt(x)+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\mathrm{RootOf}\left(3a^4y^4+8ab\,c^2y^{\frac{5}{2}}-2\left(-a^3y^3-4b\,c^2y^{\frac{3}{2}}-6\_Zc^2+2\sqrt{2a^3b\,y^{\frac{9}{2}}+3\_Za^3y^3+4b\right.\right.\right.$$

7.2.4.7 [478] problem number 7

problem number 478

Added January 2, 2019.

Problem 2.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sqrt{f(x)}w_x+\sqrt{f(y)}w_y=0$$

Where $f(t)=\sum_{n=0}^{4}a_nt^n$

Mathematica ✗

```
ClearAll["Global`*"];
f[t_] := Sum[a[n]*t^n, {n, 1, 4}];
pde =  Sqrt[f[x]]*D[w[x, y], x] + Sqrt[f[y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
f:=t->sum(a[n]*t^n,n=1..4);
pde := sqrt(f(x))* diff(w(x,y),x)+ sqrt(f(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

7.2.5 2.5

Local contents

7.2.5.1 [479] problem number 1

problem number 479

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^k)\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ba^{-k-1}\,\text{Gamma}(k+1,ax)+ye^{-ax}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y+b*x^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-b\,x^k(ax)^{-\frac{k}{2}}\,\text{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}}+(k+1)\,ay\right)\mathrm{e}^{-ax}}{(k+1)\,a}\right)$$

7.2.5.2 [480] problem number 2

problem number 480

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^k y + bx^n)\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^k*y + b*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(b(k+1)^{\frac{n-k}{k+1}}a^{-\frac{n+1}{k+1}}\,\text{Gamma}\left(\frac{n+1}{k+1},\frac{ax^{k+1}}{k+1}\right)+ye^{-\frac{ax^{k+1}}{k+1}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^k*y+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-(k+1)^2\left(a\,x^{n+1}+(k+n+2)\,x^{-k+n}\right)b\left(\frac{a\,x^{k+1}}{k+1}\right)^{\frac{-k-n-2}{2k+2}}\text{WhittakerM}\left(\frac{-k+n}{2k+2},\frac{2k+n+3}{2k+2},\frac{a\,x^{k+1}}{k+1}\right.}{}\right.$$

7.2.5.3 [481] problem number 3

problem number 481

Added January 2, 2019.

Problem 2.2.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ay^2+bx^n\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^2 + b*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol=Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-2axy\,\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\,\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{}}{}\right.}{(2axy+1)\,\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)+\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\left(\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-\text{B}\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y^2+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{axy\,\text{BesselY}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,x\,x^{\frac{n}{2}}}{n+2}\right)-\sqrt{ab}\,x\,x^{\frac{n}{2}}\,\text{BesselY}\left(\frac{n+3}{n+2},\frac{2\sqrt{ab}\,x\,x^{\frac{n}{2}}}{n+2}\right)+\text{BesselY}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,x}{n+}\right.}{-axy\,\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,x\,x^{\frac{n}{2}}}{n+2}\right)+\sqrt{ab}\,x\,x^{\frac{n}{2}}\,\text{BesselJ}\left(\frac{n+3}{n+2},\frac{2\sqrt{ab}\,x\,x^{\frac{n}{2}}}{n+2}\right)-\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,x}{n+}\right.}\right.$$

7.2.5.4 [482] problem number 4

problem number 482

Added January 2, 2019.

Problem 2.2.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+anx^{n-1}-a^2x^{2n}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*n*x^(n - 1) - a^2*x^(2*n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*n*x^(n-1)-a^2*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{}{-3\left(\left(n+\frac{4}{3}\right)a\,x^{n+1}+\frac{(n+2)(-xy+n+1)}{3}\right)(n+2)\,\mathrm{WhittakerM}\left(\frac{n+2}{2n+2},\frac{2n+3}{2n+2},-\frac{2a\,x^{n+1}}{n+1}\right)\mathrm{e}^{-\frac{a\,x^{n}}{n+}}\right.$$

7.2.5.5 [483] problem number 5

problem number 483

Added January 2, 2019.

Problem 2.2.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y + ax^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x^n*y + a*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left((-1)^{\frac{1}{n+1}}(n+1)^{-\frac{n+2}{n+1}}a^{\frac{1}{n+1}}\,\mathrm{Gamma}\left(-\frac{1}{n+1},-\frac{ax^{n+1}}{n+1}\right)-\frac{e^{\frac{ax^{n+1}}{n+1}}}{x^2y+x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*x^n*y+a*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(n+1)^2\left(axy - ny\,x^{-n} - n\,x^{-n-1} + a\right)\left(-\frac{a\,x^{n+1}}{n+1}\right)^{-\frac{n}{2n+2}} \text{WhittakerM}\left(\frac{-n-2}{2n+2}, \frac{2n+1}{2n+2}, -\frac{a\,x^{n}}{n+}\right.}{(xy}\right.$$

7.2.5.6 [484] problem number 6

problem number 484

Added January 2, 2019.

Problem 2.2.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y - abx^n - b^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x^n*y - a*b*x^n - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*x^n*y-a*b*x^n-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-b+y)\left(\int e^{\frac{(a\,x^n+2(n+1)b)x}{n+1}}dx\right)+e^{\frac{(a\,x^n+2(n+1)b)x}{n+1}}}{b-y}\right)$$

7.2.5.7 [485] problem number 7

problem number 485

Added January 2, 2019.

Problem 2.2.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^(-n - 2))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{x^{\sqrt{(n+1)^2-4ab}}\left(\sqrt{(n+1)^2-4ab}+2ayx^{n+1}+n+1\right)}{\sqrt{(n+1)^2-4ab}-2ayx^{n+1}-n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2axy\,x^n+n+1}{\sqrt{4ab-n^2-2n-1}}\right)+\sqrt{4ab-n^2-2n-1}\,\ln(x)}{\sqrt{4ab-n^2-2n-1}}\right)$$

7.2.5.8 [486] problem number 8

problem number 486

Added January 2, 2019.

Problem 2.2.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bmx^{m-1} - ab^2x^{n+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(n + 2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*m*x^(m-1) -a*b^2*x^(n+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{-3\left(\left(m+\frac{4n}{3}+\frac{4}{3}\right)ab\,x^{m+n+1}-\frac{(m+2n+2)(ay\,x^{n+1}-m-n-1)}{3}\right)(m+2n+2)\,\mathrm{WhittakerM}\left(\frac{m+}{2m-}\right.}\right.$$

7.2.5.9 [487] problem number 9

problem number 487

Added January 2, 2019.

Problem 2.2.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((n+1)x^n y^2 - a x^{n+m+1} y + a x^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((n + 1)*x^n*y^2 - a*x^(n + m + 1)*y + a*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((n+1)*x^n*y^2 - a*x^(n+m+1)* y + a*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(m+2n+3)\left((m-n)\,ax\,x^m \operatorname{hypergeom}\left(\left[\frac{2m+2}{m+n+2}\right], \left[\frac{2m+n+3}{m+n+2}\right], \frac{a\,x^2 x^m x^n}{m+n+2}\right) - (\cdots}{\left((m+1)\,a\,x^2 x^m x^n \operatorname{hypergeom}\left(\left[\frac{2m+n+3}{m+n+2}\right], \left[\frac{2m+3n+5}{m+n+2}\right], \frac{a\,x^2 x^m x^n}{m+n+2}\right) - (m+2n+3)\left((ax\,x^{\cdots}\right.\right.}\right.$$

7.2.5.10 [488] problem number 10

problem number 488

Added January 2, 2019.

Problem 2.2.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a x^n y^2 + b x^m y + b c x^m - a c^2 x^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + b*c*x^m - a*c^2*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*x^m*y+ b*c*x^m -a*c^2*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-c-y)\left(\int a\,x^n \mathrm{e}^{\frac{-2(m+1)ac\,x^{n+1}+(n+1)b\,x^{m+1}}{(m+1)(n+1)}}dx\right) - \mathrm{e}^{\frac{-2(m+1)ac\,x^{n+1}+(n+1)b\,x^{m+1}}{(m+1)(n+1)}}}{c+y}\right)$$

7.2.5.11 [489] problem number 11

problem number 489

Added January 2, 2019.

Problem 2.2.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - ax^n(bx^m + c)y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*x^m + c)*y + b*m*x^(m - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2-a*x^n*(b*x^m +c)*y+ b*m*x^(m-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.12 [490] problem number 12

problem number 490

Added January 2, 2019.

Problem 2.2.5.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(anx^{n-1}y^2 - cx^m(ax^n + b) + cx^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (a*n*x^(n - 1)*y^2 - c*x^m*(a*x^n + b) + c*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)- (a*n*x^(n-1)*y^2 - c*x^m*(a*x^n+b) + c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.13 [491] problem number 13

problem number 491

Added January 2, 2019.

Problem 2.2.5.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bx^m y + ckx^{k-1} - bcx^{m+k} - ac^2 x^{n+2k}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + c*k*x^(k - 1) - b*c*x^(m + k) - a*c^2*x^(n + 2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^m*y+ c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.14 [492] problem number 14

problem number 492

Added January 2, 2019.

Problem 2.2.5.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^{2n+1}y^3 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + b*x^(-n - 2))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^(2*n+1)*y^3 + b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\ln(x) - \frac{\ln\left(xy\,x^n - \mathrm{RootOf}\left(\_Z^3a + (n+1)\_Z + b\right)\right)}{3\,\mathrm{RootOf}\left(\_Z^3a + (n+1)\_Z + b\right)^2 a + n + 1}\right)$$

Solution contains RootOf

7.2.5.15 [493] problem number 15

problem number 493

Added January 2, 2019.

Problem 2.2.5.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^3 + 3abx^{n+m}y^2 - bmx^{m-1} - 2ab^3x^{n+3m}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 - b*m*x^(m - 1) - 2*a*b^3*x^(n + 3*m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{6^{-\frac{n+1}{2m+n+1}}(2m+n+1)^{-\frac{2m}{2m+n+1}}b^{-\frac{2(n+1)}{2m+n+1}}e^{-\frac{6ab^2x^{2m+n+1}}{2m+n+1}}\left(6^{\frac{n+1}{2m+n+1}}(2m+n+1)^{\frac{2m}{2m+n+1}}b^{\frac{2(n+}{2m+n}}\right.}{(bx^m+y)^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2 - b*m*x^(m-1) - 2*a*b^3*x^(n+3*m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left((n+1)\left(n^2-n+1\right)\left(y^2x^{-2m}+b^2\right)2^{\frac{m}{2m+n+1}} + \left(4\left(m^3+\left(\frac{5n}{2}+\frac{5}{2}\right)m^2+\frac{3n^2}{2}+2(n+1)^2\right.\right.\right.}{}\right.$$

7.2.5.16 [494] problem number 16

problem number 494

Added January 2, 2019.

Problem 2.2.5.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^3 + 3abx^{n+m}y^2 + cx^ky - 2ab^3x^{n+3m} + bcx^{m+l} - bmx^{m-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 + c*x^k*y - 2*a*b^3*x^(n + 3*m) + b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2+ c*x^k*y-2*a*b^3*x^(n+3*m) + b*c*x^(m+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.17 [495] problem number 17

problem number 495

Added January 2, 2019.

Problem 2.2.5.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay^n + bx^{\frac{n}{1-n}}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^n + b*x^(n/(1 - n)))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y^n+b*x^(n/(1-n)))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-n\left(\int_{\_b}^{y} \frac{x^{\frac{n}{n-1}}}{(n-1)\,bx + ((n-1)\,ax\_a^n + \_a)\,x^{\frac{n}{n-1}}} d\_a\right) + \int_{\_b}^{y} \frac{x^{\frac{n}{n-1}}}{(n-1)\,bx + ((n-1)\,ax\_$$

7.2.5.18 [496] problem number 18

problem number 496

Added January 2, 2019.

Problem 2.2.5.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^{m-n-(mn)}y^n + bx^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^(m - n - m*n)*y^n + b*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^(m-n-(m*n))*y^n + b*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} -\frac{x^n x^{mn}}{ax\_a^n x^m + (bx\,x^m - (m+1)\_a)\,x^n x^{mn}} d\_a + \ln(x)\right)$$

7.2.5.19 [497] problem number 19

problem number 497

Added January 2, 2019.

Problem 2.2.5.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^k + bx^m y\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^k + b*x^m*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(a(-1)^{\frac{m-n}{m+1}}(m+1)^{\frac{n-m}{m+1}} b^{-\frac{n+1}{m+1}} (k-1)^{\frac{m-n}{m+1}} \operatorname{Gamma}\left(\frac{n+1}{m+1}, -\frac{b(k-1)x^{m+1}}{m+1}\right) + y^{1-k} e^{\frac{b(k-}{m}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^k + b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(m+1)^2\left((k-1)\,b\,x^{n+1} + (-m-n-2)\,x^{-m+n}\right)a\left(-\frac{(k-1)b\,x^{m+1}}{m+1}\right)^{\frac{-m-n-2}{2m+2}}\text{WhittakerM}\left(\right.}{}\right.$$

7.2.5.20 [498] problem number 20

problem number 498

Added January 2, 2019.

Problem 2.2.5.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^2 + by + cx^{2b}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + b*y + c*x^(2*b))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{a}y\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)+\sqrt{c}x^b\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{c}x^b\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)-\sqrt{a}y\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^2 + b*y+ c*x^(2*b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\sqrt{a}\,\sqrt{c}\,x^{b} - b\arctan\left(\frac{\sqrt{a}\,y\,x^{-b}}{\sqrt{c}}\right)}{b}\right)$$

7.2.5.21 [499] problem number 21

problem number 499

Added January 2, 2019.

Problem 2.2.5.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^2 + (n + bx^n)y + cx^{2n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + (n + b*x^n)*y + c*x^(2*n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}\left(x^n\sqrt{b^2-4ac} + 2ay + bx^n\right)}{x^n\sqrt{b^2-4ac} - 2ay - bx^n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^2+(n+b*x^n)*y + c*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2bn\arctan\left(\frac{2aby\,x^{-n}+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\sqrt{4ac\,b^2-b^4}\,x^n\right)b}{\sqrt{4ac\,b^2-b^4}\,n}\right)$$

7.2.5.22 [500] problem number 22

problem number 500

Added January 2, 2019.

Problem 2.2.5.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + by + cx^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + b*y + c/x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{x^{\sqrt{-4ac+b^2+2bn+n^2}}\left(\sqrt{-4ac+b^2+2bn+n^2}+2ayx^n+b+n\right)}{-\sqrt{-4ac+b^2+2bn+n^2}+2ayx^n+b+n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^n*y^2+b*y+c*x^(-n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2ay\,x^n+b+n}{\sqrt{4ac-b^2-2bn-n^2}}\right)+\sqrt{4ac-b^2-2bn-n^2}\,\ln(x)}{\sqrt{4ac-b^2-2bn-n^2}}\right)$$

7.2.5.23 [501] problem number 23

problem number 501

Added January 2, 2019.

Problem 2.2.5.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + my - ab^2 x^{x+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + m*y - a*b^2*x^(x + 2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+  (a*x^n*y^2+ m*y- a*b^2*x^(x+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Failed to convert to latex

7.2.5.24 [502] problem number 24

problem number 502

Added January 2, 2019.

Problem 2.2.5.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x^{2n}y^2 + (m-n)y + x^{2m}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x^(2*n)*y^2 + (m - n)*y + x^(2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(yx^{n-m}\right) - \frac{x^{m+n}}{m+n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+  (x^(2*n)*y^2+(m-n)*y+ x^(2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{m+n} + (-m-n)\arctan\left(y\,x^{-m+n}\right)}{m+n}\right)$$

7.2.5.25 [503] problem number 25

problem number 503

Added January 2, 2019.

Problem 2.2.5.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n}y^2 + (bx^n - n)y + c\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*y^2 + (b*x^n - n)*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}\left(\sqrt{b^2-4ac}+2ayx^n+b\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n)*y^2+ (b*x^n -n)*y + c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(-2bn\arctan\left(\frac{2aby\,x^n+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\sqrt{4ac\,b^2-b^4}\,x^n\right)b}{\sqrt{4ac\,b^2-b^4}\,n}\right)$$

7.2.5.26 [504] problem number 26

problem number 504

Added January 2, 2019.

Problem 2.2.5.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n+m}y^2+(bx^{n+m}-n)y+cx^m\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n + m)*y^2 + (b*x^(n + m) - n)*y + c*x^m)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{\sqrt{b^2-4ac}x^{m+n}}{m+n}}\left(\sqrt{b^2-4ac}+2ayx^n+b\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n + m)*y^2 +(b*x^(n+m)-n)*y+ c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(-2(m+n)\,b\arctan\left(\frac{2aby\,x^n+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\sqrt{4ac\,b^2-b^4}\,x^m x^n\right)b}{\sqrt{4ac\,b^2-b^4}\,(m+n)}\right)$$

7.2.5.27 [505] problem number 27

problem number 505

Added January 2, 2019.

Problem 2.2.5.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ay^3+3abx^ny^2-bnx^n-2ab^3x^{3n}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^3 + 3*a*b*x^n*y^2 - b*n*x^n - 2*a*b^3*x^(3*n))*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{3ab^2x^{2n}}{n}}\left(ae^{\frac{3ab^2x^{2n}}{n}}(bx^n+y)^2\operatorname{Ei}\left(-\frac{3ab^2x^{2n}}{n}\right)+n\right)}{n\left(bx^n+y\right)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^3+3*a*b*x^n*y^2 - b*n*x^n -2*a*b^3*x^(3*n) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-(b^2x^{2n}+2by\,x^n+y^2)\,a\operatorname{expIntegral}\left(1,\frac{3a\,b^2x^{2n}}{n}\right)+n\,\mathrm{e}^{-\frac{3a\,b^2x^{2n}}{n}}}{(b^2x^{2n}+2by\,x^n+y^2)\,n}\right)$$

7.2.5.28 [506] problem number 28

problem number 506

Added January 2, 2019.

Problem 2.2.5.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n+1}y^3+(bx-n)y+cx^{1-n}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + (b*x - n)*y + c*x^(1 - n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n +1)*y^3 + (b*x-n)*y + c*x^(1-n) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-by\,x^n - \mathrm{RootOf}\left(c^2a\_Z^3+\_Zb^3-b^3\right)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2a\_Z^3+\_Zb^3-b^3\right)^2 a\,c^2+b^3} - bx\right)$$

Solution contains RootOf

7.2.5.29 [507] problem number 29

problem number 507

Added January 2, 2019.

Problem 2.2.5.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{n+2}y^3 + (bx^n - 1)y + cx^{n-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(n + 2)*y^3 + (b*x^n - 1)*y + c*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(n+2)*y^3+ (b*x^n-1)*y + c*x^(n-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\frac{b^2 n \ln\left(\frac{-bxy-\mathrm{RootOf}\left(c^2 a \_Z^3+\_Z b^3-b^3\right)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2 a \_Z^3+\_Z b^3-b^3\right)^2 a c^2+b^3}-x^n\right)b}{n}\right)$$

Solution contains RootOf

7.2.5.30 [508] problem number 30

problem number 508

Added January 2, 2019.

Problem 2.2.5.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(y + ax^{n-m}y^m + bx^{n-k}y^k\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (y + a*x^(n - m)*y^m + b*x^(n - k)*y^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ ( y+a*x^(n - m)*y^m+b*x^(n-k)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)\,x\left(\int^{y}_{\_b} -\frac{x^k x^m}{(a\_a^m x^k + b\_a^k x^m)\,x} d\_a\right) + x^n}{(n-1)\,x}\right)$$

7.2.5.31 [509] problem number 31

problem number 509

Added January 2, 2019.

Problem 2.2.5.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + \left(x^{n-1}((1+2n)x+an)y - nx^{2n}(x+a)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + (x^(n - 1)*((1 + 2*n)*x + a*n)*y - n*x^(2*n)*(x + a))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := y*diff(w(x,y),x)+ ( x^(n-1)*((1+2*n)*x+a*n)*y-n*x^(2*n)*(x+a) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(a+\frac{x}{2}\right) n \, \mathrm{e}^{\frac{2\arctan\left(\frac{\left(-2a\,x^{n}+y-x^{n+1}\right)n}{\sqrt{-n^2}\left(y-x^{n+1}\right)}\right)}{\sqrt{-n^2}}} - \sqrt{-n^2}\, x \left(\int^{-\frac{2\arctan\left(\frac{\left(-2a\,x^{n}+y-x^{n+1}\right)n}{\sqrt{-n^2}\left(y-x^{n+1}\right)}\right)}{\sqrt{-n^2}}} \mathrm{e}^{-\_a} \tan\left(\frac{\sqrt{-n^2}}{2}\right.\right.}{2x}\right.$$

7.2.5.32 [510] problem number 32

problem number 510

Added January 2, 2019.

Problem 2.2.5.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + \left((a(2n+k)x^k + b)x^{n-1}y - (a^2nx^{2k} + abx^k - c)x^{2n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + ((a*(2*n + k)*x^k + b)*x^(n - 1)*y - (a^2*n*x^(2*k) + a*b*x^k - c)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := y*diff(w(x,y),x)+ ( (a*(2*n+k)*x^k+b)*x^(n-1)*y -(a^2*n*x^(2*k)+ a*b*x^k-c)*x^(2*n-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.33 [511] problem number 33

problem number 511

Added January 2, 2019.

Problem 2.2.5.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2axy+b)w_x-\left(a(m+3)xy^2+b(m+2)y-cx^m\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(2*a*x*y + b)*D[w[x, y], x] - (a*(m + 3)*x*y^2 + b*(m + 2)*y - c*x^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{m+2}(2(m+1)y(axy+b)-cx^m)}{2a(m+1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*(2*a*x*y+b)*diff(w(x,y),x)- ( a*(m+3)*x*y^2+b*(m+2)*y-c*x^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{2\left(-\frac{cx^m}{2}+(m+1)\left(axy+b\right)y\right)x^2x^m}{2m+2}\right)$$

7.2.5.34 [512] problem number 34

problem number 512

Added January 2, 2019.

Problem 2.2.5.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2(2axy+b)w_x - \left(4ax^2y^2+3bxy-cx^2-k\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*(2*a*x*y + b)*D[w[x, y], x] - (4*a*x^2*y^2 + 3*b*x*y - c*x^2 - k)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^2(x(4y(axy+b)-cx)-2k)}{4a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*(2*a*x*y+b)*diff(w(x,y),x)- ( 4*a*x^2*y^2 + 3*b*x*y-c*x^2 - k )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-a\,x^4y^2 - b\,x^3y + \frac{1}{4}c\,x^4 + \frac{1}{2}k\,x^2\right)$$

7.2.5.35 [513] problem number 35

problem number 513

Added January 2, 2019.

Problem 2.2.5.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^mw_x + by^nw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^m*D[w[x, y], x] + b*y^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-m}}{a(m-1)}-\frac{y^{1-n}}{n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^m*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(m-1)\,a\,y^{-n+1}-(n-1)\,b\,x^{-m+1}}{(m-1)\,a}\right)$$

7.2.5.36 [514] problem number 36

problem number 514

Added January 2, 2019.

Problem 2.2.5.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x+(by+cx^m)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*y + c*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{\frac{bx^{1-n}}{a(n-1)}}-\frac{c(a-an)^{\frac{-m+n-1}{n-1}}b^{\frac{m-n+1}{n-1}}\text{Gamma}\left(\frac{-m+n-1}{n-1},\frac{bx^{1-n}}{a-an}\right)}{a(n-1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+ (b*y+c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)\,(m-2n+2)^2\,ac\,x^m\left(-\frac{b\,x^{-n+1}}{(n-1)a}\right)^{\frac{m-2n+2}{2n-2}}\text{WhittakerM}\left(\frac{-m+2n-2}{2n-2},\frac{-m+3n-3}{2n-2},-\frac{b\,x^{-n+1}}{(n-1)a}\right.}{}\right.$$

7.2.5.37 [515] problem number 37

problem number 515

Added January 2, 2019.

Problem 2.2.5.37 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + (y^n + bx^m y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + (y^n + b*x^m*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> {n != 1}],
```

$$\left\{\left\{w(x,y)\to c_1\left((k-m-1)^{\frac{m}{k-m-1}}a^{\frac{m}{k-m-1}}b^{\frac{1-k}{k-m-1}}(n-1)^{\frac{m}{-k+m+1}}\,\text{Gamma}\left(\frac{k-1}{k-m-1},\frac{b(n-1)x^{-k+m+1}}{a(k-m-1)}\right)\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^k*diff(w(x,y),x)+ (y^n+b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n<>1)
```

$$w(x,y) = \_F1\left(\frac{\left(-4(k-m-1)\left(k-\frac{m}{2}-1\right)^2 a\,x^{-m} y^{\frac{1}{k-m-1}} y^{\frac{m}{k-m-1}} y^{\frac{kn}{k-m-1}} \left(\frac{(n-1)b}{(k-m-1)a}\right)^{\frac{-k+1}{k-m-1}} \left(\frac{(n-1)b}{(k-m-1)a}\right)^{\frac{}{k}}\right.}{}\right.$$

7.2.5.38 [516] problem number 38

problem number 516

Added January 2, 2019.

Problem 2.2.5.38 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^k+b)w_x + \left(\alpha x^n y^2 + (\beta - anx^k)y + \gamma x^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(a*x^k + b)*D[w[x, y], x] + (alpha*x^n*y^2 + (beta - a*n*x^k)*y + gamma/x^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}+2\alpha y x^n+bn+\beta\right)\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}(k\log(x)-\log(ax^k+b))\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}}{bk}\right)}{-\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}+2\alpha y x^n+bn+\beta}\right)\right.\right.$$

Maple ✓

```
restart;
pde :=x*(a*x^k+b)*diff(w(x,y),x)+ (alpha*x^n*y^2+(beta-a*n*x^k)*y+g*x^(-n))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(2(bn+\beta)\,bk\,\operatorname{arctanh}\left(\frac{(bn+\beta)(2\alpha y\,x^n+bn+\beta)}{\sqrt{(bn+\beta)^2(b^2n^2+2b\beta n-4\alpha g+\beta^2)}}\right)+\sqrt{(bn+\beta)^2\left(b^2n^2+2b\beta n-4\alpha g\right.}\right.}{\sqrt{(bn+\beta)^2\left(b^2n^2+2b\beta n-4\alpha g+\beta^2\right)}\,bk}\right.$$

7.2.5.39 [517] problem number 39

problem number 517

Added January 2, 2019.

Problem 2.2.5.39 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y+Ax^n+a)w_x-\left(nAx^{n-1}y+kx^m+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (y + A*x^n + a)*D[w[x, y], x] - (n*A*x^(n - 1)*y + k*x^m + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y(2a+2Ax^n+y)+2bx+\frac{2kx^{m+1}}{m+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=(y+ A*x^n + a)*diff(w(x,y),x)- ( n*A*x^(n-1)*y + k*x^m + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2kx\,x^m - 2(m+1)\left(Ay\,x^n + ay + bx + \frac{y^2}{2}\right)}{2m+2}\right)$$

7.2.5.40 [518] problem number 40

problem number 518

Added January 2, 2019.

Problem 2.2.5.40 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y + ax^{n+1} + bx^n)w_x + \left(anx^n + cx^{n-1}\right)yw_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y + a*x^(n + 1) + b*x^n)*D[w[x, y], x] + (a*n*x^n + c*x^(n - 1))*y*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(y+ a*x^(n+1)+b*x^n)*diff(w(x,y),x)+ ( a*n*x^n + c*x^(n-1))*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.41 [519] problem number 41

problem number 519

Added January 2, 2019.

Problem 2.2.5.41 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2ax^n y + b)w_x - \left(a(3n+m)x^n y^2 + b(2n+m)y - Ax^m - Cx^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(2*a*x^n*y + b)*D[w[x, y], x] - (a*(3*n + m)*x^n*y^2 + b*(2*n + m)*y - A*x^m - C0/x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{x^{m+n}(x^n(2y(m+n)\left(ayx^n+b\right) - Ax^m) - 2\mathrm{C0})}{2a(m+n)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*(2*a*x^n*y+b)*diff(w(x,y),x)- ( a*(3*n+m)*x^n*y^2+b*(2*n+m)*y-A*x^m -C*x^(-n))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2(m+n)\,a\,y^2 x^{m+3n} - 2(m+n)\,by\,x^{m+2n} + A\,x^{2m+2n} + 2C\,x^{m+n}}{2m+2n}\right)$$

7.2.5.42 [520] problem number 42

problem number 520

Added January 2, 2019.

Problem 2.2.5.42 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^2 + xy)w_x + \left(cx^n + bxy + y^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^2 + x*y)*D[w[x, y], x] + (c*x^n + b*x*y + y^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n+b*x^2+ x*y)*diff(w(x,y),x)+ ( c*x^n + b*x*y+ y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\ln\left(\frac{9((n-1)a^2x^n+(cx+(-bx+(bx+y)n-2y)a)x)(n^2-3n+3)}{(2n-3)(b\,x^2+a\,x^n+xy)a}\right)+(-n+1)\ln\left(\frac{9(n^2-3n+3)(a^2x^n+(ab+c)x^2)}{(n-3)(a\,x^n+(bx+y)x)a}\right.\right.}{3\,(n-}\right.$$

7.2.5.43 [521] problem number 43

problem number 521

Added January 2, 2019.

Problem 2.2.5.43 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^n + bx^2 + cxy)w_x + (ky^n + bxy + cy^2)\,w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^n + b*x^2 + c*x*y)*D[w[x, y], x] + (k*y^n + b*x*y + c*y^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*y^n+b*x^2+c*x*y)*diff(w(x,y),x)+ ( k*y^n+ b*x*y+c*y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n^2-3n+3)\left((-n+1)\ln\left(\frac{9(k^2y^n+(ab+ck)y^2)(n^2-3n+3)}{(n-3)(bxy+cy^2+ky^n)k}\right)+\ln\left(\frac{9((n-1)k^2y^n+(aby+((n-2)bx+(n-}{2(bxy+cy^2+ky^n)(n}\right.\right.}{3}\right.$$

7.2.5.44 [522] problem number 44

problem number 522

Added January 2, 2019.

Problem 2.2.5.44 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n+bx^m+c)w_x+\left(cy^2-bx^{m-1}y+ax^{n-2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (c*y^2 - b*x^(m - 1)*y + a*x^(n - 2))*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( c*y^2-b*x^(m-1)*y+ a*x^(n-2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.45 [523] problem number 45

problem number 523

Added January 2, 2019.

Problem 2.2.5.45 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left(ax^{n-2}y^2 + bx^{m-1}y + c\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (a*x^(n - 2)*y^2 + b*x^(m - 1)*y + c)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( a*x^(n-2)*y^2 + b*x^(m-1)*y + c)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.46 [524] problem number 46

problem number 524

Added January 2, 2019.

Problem 2.2.5.46 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left(\alpha x^k y^2 + \beta x^s y - \alpha\lambda^2 x^k + \beta\lambda x^s\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(\lambda+y)\,\alpha\left(\int \frac{x^k \mathrm{e}^{-\left(\int \frac{2\alpha\lambda\,x^k-\beta\,x^s}{a\,x^n+b\,x^m+c}dx\right)}}{a\,x^n+b\,x^m+c}dx\right)-\mathrm{e}^{-\left(\int \frac{2\alpha\lambda\,x^k-\beta\,x^s}{a\,x^n+b\,x^m+c}dx\right)}}{\lambda+y}\right)$$

7.2.5.47 [525] problem number 47

problem number 525

Added January 2, 2019.

Problem 2.2.5.47 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n+bx^m+c)w_x - \left(sx^k y^2 - (ax^n+bx^m+c)y - s\lambda x^{k+2}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(a*x^n + b*x^m + c)*D[w[x, y], x] - (s*x^k*y^2 - (a*x^n + b*x^m + c)*y - s*lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tanh^{-1}\left(\frac{y}{\sqrt{\lambda}x}\right)}{\sqrt{\lambda}}-\int_1^x\frac{sK[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*(a*x^n + b*x^m + c)*diff(w(x,y),x)- (s*x^k*y^2 -(a*x^n + b*x^m+c)*y - s*lambda*x^(k+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\sqrt{\lambda}\,s\left(\int\frac{x^k}{a\,x^n+b\,x^m+c}dx\right)+\operatorname{arctanh}\left(\frac{y}{\sqrt{\lambda}\,x}\right)}{\sqrt{\lambda}\,s}\right)$$

7.2.5.48 [526] problem number 48

problem number 526

Added January 2, 2019.

Problem 2.2.5.48 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n+bx^m+c)w_x+\left((ax^n+bx^m+c)y^2-an(n-1)x^{n-2}-bm(m-1)x^{m-2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + ((a*x^n + b*x^m + c)*y^2 - a*n*(n - 1)*x^(n - 2)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ((a*x^n+b*x^m + c)*y^2-a*n*(n-1)*x^(n-2)-b*m*(m-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a^2n\,x^{2n} + a^2y\,x^{2n+1} + 2aby\,x^{m+n+1} + 2acn\,x^n + 2acy\,x^{n+1} + b^2m\,x^{2m} + b^2y\,x^{2m+1} + 2bc}{}\right.$$

7.2.5.49 [527] problem number 49

problem number 527

Added January 2, 2019.

Problem 2.2.5.49 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + by^n + x)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*y^n + x)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n - m) + y)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n + b*y^n + x)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k) +beta*x^m*y^(n-m) + y )*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.50 [528] problem number 50

problem number 528

Added January 2, 2019.

Problem 2.2.5.50 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + by^n + Ax^2 + Bxy)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + Axy + By^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*y^n + A*x^2 + B*x*y)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n + b*y^n + A*x^2 + B*x*y)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k)+beta*x^m*y^(n-m) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.51 [529] problem number 51

problem number 529

Added January 2, 2019.

Problem 2.2.5.51 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^m + bx^n + s)w_x - \left(\alpha x^k + bnx^{n-1}y + \beta\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^m + b*x^n + s)*D[w[x, y], x] - (alpha*x^k + b*n*x^(n - 1)*y + beta)*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*y^m + b*x^n + s)*diff(w(x,y),x)- (alpha*x^k + b*n*x^(n-1)* y + beta)*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(m+1)\,\alpha x\,x^k - (ay\,y^m + (m+1)\,by\,x^n + (\beta x + sy)\,(m+1))\,(k+1)}{(m+1)\,(k+1)}\right)$$

7.2.5.52 [530] problem number 52

problem number 530

Added January 2, 2019.

Problem 2.2.5.52 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + x)w_x + \left(bx^k y^{n+m-k} + y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + x)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n*y^m +x)*diff(w(x,y),x)+  (b*x^k*y^(n+m-k) + y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.53 [531] problem number 53

problem number 531

Added January 2, 2019.

Problem 2.2.5.53 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n y^m + \alpha)w_x - y(bx^n y^m + \beta)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*x^n*y^m + alpha)*D[w[x, y], x] - y*(b*x^n*y^m + beta)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*(a*x^n*y^m +alpha)*diff(w(x,y),x)-  y*( b*x^n*y^m + beta )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\Big(x^{(an-bm)\beta m}(y^m)^{(an-bm)\alpha}\,(\alpha n - \beta m + (an - bm)\,x^n y^m)^{-(a\beta-\alpha b)m}\Big)$$

7.2.5.54 [532] problem number 54

problem number 532

Added January 2, 2019.

Problem 2.2.5.54 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(anx^ky^{n+k}+s)w_x - y\left(bmx^{m+k}y^k+s\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*n*x^k*y^(n + k) + s)*D[w[x, y], x] - y*(b*m*x^(m + k)*y^k + s)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(a*n*x^k*y^(n+k) + s)*diff(w(x,y),x)- y*( b*m*x^(m+k)*y^k + s )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.55 [533] problem number 55

problem number 533

Added January 2, 2019.

Problem 2.2.5.55 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^ny^m + Ax^2 + Bxy)w_x + \left(bx^ky^{n+m-k} + Axy + By^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + A*x^2 + B*x*y)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + A*x*y + B*y^2)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n*y^m + A*x^2 + B*x*y)*diff(w(x,y),x)+ (b*x^k*y^(n+m-k) + A*x*y+ B*y^2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.5.56 [534] problem number 56

problem number 534

Added January 2, 2019.

Problem 2.2.5.56 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxy^k)w_x + (\alpha y^s + \beta)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + (alpha*y^s + beta)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*y^m + b*x*y^k)*diff(w(x,y),x)+ (alpha*y^s + beta)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \frac{y^m \mathrm{e}^{(n-1)b\left(\int \frac{y^k}{\alpha\, y^s+\beta}dy\right)}}{\alpha\, y^s+\beta}dy\right)+x^{-n+1}\mathrm{e}^{(n-1)b\left(\int \frac{y^k}{\alpha\, y^s+\beta}dy\right)}\right)$$

7.2.6 3.1

Local contents

7.2.6.1 [535] problem number 1

problem number 535

Added January 2, 2019.

Problem 2.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ae^{\lambda x}w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Exp[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*exp(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda}\right)$$

7.2.6.2 [536] problem number 2

problem number 536

Added January 7, 2019.

Problem 2.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{ae^{\lambda x}}{\lambda}-bx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\,\mathrm{e}^{\lambda x} - (bx - y)\,\lambda}{\lambda}\right)$$

7.2.6.3 [537] problem number 3

problem number 537

Added January 7, 2019.

Problem 2.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\frac{e^{\lambda y}}{ae^{\lambda y}+b}\right)}{b\lambda} - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\ln\left(a\,\mathrm{e}^{b\lambda x} + b\,\mathrm{e}^{(bx-y)\lambda}\right)}{b\lambda}\right)$$

7.2.6.4 [538] problem number 4

problem number 538

Added January 7, 2019.

Problem 2.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y+\beta x} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\log\left(a\lambda e^{x(b\lambda+\beta)} + \beta e^{\lambda(bx-y)} + b\lambda e^{\lambda(bx-y)}\right)}{b\lambda+\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(bx-y)\lambda - \ln\left(\frac{1}{a\lambda\, e^{\beta x+\lambda y} + b\lambda + \beta}\right)}{b\lambda+\beta}\right)$$

7.2.6.5 [539] problem number 5

problem number 539

Added January 7, 2019.

Problem 2.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y+\beta x} + be^{\gamma x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b*Exp[gamma*x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b*exp(g*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda\left(\int \mathrm{e}^{\frac{b\lambda\,\mathrm{e}^{gx}}{g}+\beta x}dx\right) - \mathrm{e}^{\frac{(b\,\mathrm{e}^{gx}-gy)\lambda}{g}}}{\lambda}\right)$$

7.2.6.6 [540] problem number 6

problem number 540

Added January 7, 2019.

Problem 2.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a\lambda\,\mathrm{e}^{\lambda x} + b\beta\,\mathrm{e}^{\beta y})\,\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)$$

7.2.6.7 [541] problem number 7

problem number 541

Added January 7, 2019.

Problem 2.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^{\lambda x} + b)\,w_x + (ce^{\beta x} + d)\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\beta(c+d)\log\left(ae^{\lambda x}+b\right) - \lambda e^{\beta x}\,{}_2F_1\left(1,\frac{\beta}{\lambda};\frac{\beta+\lambda}{\lambda};-\frac{ae^{\lambda x}}{b}\right) - \beta\lambda(-by+cx+dx)}{b\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{c+d+\mathrm{e}^{\beta x}}{a\,\mathrm{e}^{\lambda x}+b}dx\right)\right)$$

7.2.6.8 [542] problem number 8

problem number 542

Added January 7, 2019.

Problem 2.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^{\lambda x}+b)\,w_x+(ce^{\beta y}+d)\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*y] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\log\left(\left(e^{\beta y}+c+d\right)e^{\frac{\beta x(c+d)}{b}-\beta y}\left(ae^{\lambda x}+b\right)^{-\frac{\beta(c+d)}{b\lambda}}\right)}{\beta(c+d)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*y)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-b\lambda\,\mathrm{RootOf}\left(\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{\beta d}{b\lambda}}\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{\beta c^2}{(c+d)b\lambda}}\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{\beta cd}{(c+d)b\lambda}}\mathrm{e}^{\frac{dy\beta b-dx\beta c-d^2x\beta+c\_Zb}{(c+d)b}}\mathrm{e}^{\frac{\beta cy}{c+d}}\mathrm{e}^{\frac{\beta d^2}{(c+d}}\right.}{}\right.$$

Has RootOf

7.2.6.9 [543] problem number 9

problem number 543

Added January 7, 2019.

Problem 2.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^{\lambda y}+b)\,w_x+(ce^{\beta x}+d)\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*y] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ae^{\lambda y}}{\lambda}+by-\frac{e^{\beta x}}{\beta}-cx-dx\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*y)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}+\left(\left(by+(-c-d)\,x\right)\beta-\mathrm{e}^{\beta x}\right)\lambda}{\beta\lambda}\right)$$

7.2.6.10 [544] problem number 10

problem number 544

Added January 7, 2019.

Problem 2.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^{\lambda x}+be^{\beta y})\,w_x+a\lambda e^{\lambda x}w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b*Exp[beta*y])*D[w[x, y], x] + a*lambda*Exp[lambda*x]*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),x)+ a*lambda*exp(lambda*x)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda x - y + \ln\left(-b\,\mathrm{e}^{\beta y - \lambda x} + (\beta - 1)\,a\right)}{\beta - 1}\right)$$

7.2.6.11 [545] problem number 11

problem number 545

Added January 7, 2019.

Problem 2.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x+\beta y} + c\mu\right) w_x - \left(be^{\gamma x+muy} + c\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x + beta*y] + c*mu)*D[w[x, y], x] - (b*Exp[gamma*x + mu*y] + c*lambda)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (a*exp(lambda*x+beta*y)+c*mu)*diff(w(x,y),x)- (b*exp(g*x+ mu*y)+c*lambda)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7 3.2

Local contents

7.2.7.1 [546] problem number 1

problem number 546

Added January 7, 2019.

Problem 2.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\text{Ei}\left(\frac{2ae^{\lambda x}}{\lambda}\right)\left(y-ae^{\lambda x}\right)+\lambda e^{\frac{2ae^{\lambda x}}{\lambda}}}{ae^{\lambda x}-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+a*lambda*exp(lambda*x)- a^2*exp(2*lambda *x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\,e^{\lambda x}+y}{\lambda\,e^{\frac{2a\,e^{\lambda x}}{\lambda}}+\left(a\,e^{\lambda x}-y\right)\text{expIntegral}\left(1,-\frac{2a\,e^{\lambda x}}{\lambda}\right)}\right)$$

7.2.7.2 [547] problem number 2

problem number 547

Added January 7, 2019.

Problem 2.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + by + a(\lambda - b)e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + b*y + a*(lambda - b)*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2^{b/\lambda}\lambda^{-\frac{b}{\lambda}}e^{bx}a^{b/\lambda}\left(\left(a\left(-e^{\lambda x}\right)+b+y\right)\text{LaguerreL}\left(-\frac{b}{\lambda},\frac{b}{\lambda},\frac{2ae^{\lambda x}}{\lambda}\right)-2ae^{\lambda x}\,\text{LaguerreL}\left(-\frac{b+}{\lambda}\right.\right.\right.}{ae^{\lambda x}-y}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+b*y+ a*(lambda-b)*exp(lambda*x) - a^2*exp(2*lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-a\,\mathrm{e}^{\lambda x}+y\right)\left(\int \mathrm{e}^{\frac{b\lambda x+2a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)+\mathrm{e}^{\frac{b\lambda x+2a\,\mathrm{e}^{\lambda x}}{\lambda}}}{a\,\mathrm{e}^{\lambda x}-y}\right)$$

7.2.7.3 [548] problem number 3

problem number 548

Added January 7, 2019.

Problem 2.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ae^{\lambda x}y - abe^{\lambda x} - b^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Exp[lambda*x]*y - a*b*Exp[lambda*x] - b^2)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(2b(-1)^{-\frac{b}{\lambda}}\left(-\frac{\text{Gamma}\left(\frac{2b}{\lambda},0,-\frac{ae^{\lambda x}}{\lambda}\right)}{\lambda}+\frac{\lambda^{-\frac{2b}{\lambda}}a^{\frac{2b}{\lambda}}e^{\frac{ae^{\lambda x}+2b\lambda x+2i\pi b}{\lambda}}}{b-y}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+a*exp(lambda*x)*y-a*b*exp(lambda*x)- b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(b-y)\left(\int \mathrm{e}^{\frac{2b\lambda x+a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)-\mathrm{e}^{\frac{2b\lambda x+a\,\mathrm{e}^{\lambda x}}{\lambda}}}{b-y}\right)$$

7.2.7.4 [549] problem number 4

problem number 549

Added January 7, 2019.

Problem 2.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(y^2 - axe^{\lambda x}y + ae^{\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (y^2 - a*x*Exp[lambda*x]*y + a*Exp[lambda*x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{ae^{\lambda x}(\lambda x-1)}{\lambda^2}}}{x(xy-1)}-\int_1^x\frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-  (y^2-a*x*exp(lambda*x)*y + a*exp(lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(x^2y-x)\left(\int\frac{\mathrm{e}^{\frac{(\lambda x-1)a\,\mathrm{e}^{\lambda x}}{\lambda^2}}}{x^2}dx\right)-\mathrm{e}^{\frac{(\lambda x-1)a\,\mathrm{e}^{\lambda x}}{\lambda^2}}}{(xy-1)\,\lambda^2x}\right)$$

7.2.7.5 [550] problem number 5

problem number 550

Added January 7, 2019.

Problem 2.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + be^{-\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{x\left(-\sqrt{\lambda^2-4ab}\right)}\left(\sqrt{\lambda^2-4ab}-2aye^{\lambda x}-\lambda\right)}{a\left(2ye^{\lambda x}\sqrt{\lambda^2-4ab}-4b\right)+\lambda\left(\sqrt{\lambda^2-4ab}+\lambda\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(2\lambda\arctan\left(\frac{2a\lambda y\,e^{\lambda x}+\lambda^2}{\sqrt{4\lambda^2ab-\lambda^4}}\right)-\sqrt{4\lambda^2ab-\lambda^4}\,x\right)\lambda}{\sqrt{4\lambda^2ab-\lambda^4}}\right)$$

7.2.7.6 [551] problem number 6

problem number 551

Added January 7, 2019.

Problem 2.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + b\mu e^{\mu x} - ab^2e^{(\lambda+2\mu)x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*mu*Exp[mu*x] - a*b^2*Exp[(lambda + 2*mu)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*mu*exp(mu*x) - a*b^2*exp((lambda + 2*mu)*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.7 [552] problem number 7

problem number 552

Added January 7, 2019.

Problem 2.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + by + ce^{-\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*y + c*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{x\left(-\sqrt{-4ac+b^2+2b\lambda+\lambda^2}\right)}\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}-2aye^{\lambda x}-b-\right.}{a\left(2ye^{\lambda x}\sqrt{-4ac+b^2+2b\lambda+\lambda^2}-4c\right)+b\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+2\lambda\right)+\lambda\left(\sqrt{-4a\right.}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*y +c*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(-\frac{\sqrt{(b+\lambda)^2(4ac-b^2-2b\lambda-\lambda^2)}\,x}{2} + (b+\lambda)\arctan\left(\frac{(b+\lambda)(2ay\,e^{\lambda x}+b+\lambda)}{\sqrt{(b+\lambda)^2(4ac-b^2-2b\lambda-\lambda^2)}}\right)\right)(b+\lambda)}{\sqrt{(b+\lambda)^2\,(4ac-b^2-2b\lambda-\lambda^2)}}\right)$$

7.2.7.8 [553] problem number 8

problem number 553

Added January 7, 2019.

Problem 2.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + \mu y - ab^2 e^{(\lambda+2\mu)x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + mu*y - a*b^2*Exp[(lambda + 2*mu)*x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + mu*y - a*b^2*exp((lambda+2*mu)*x))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-b\cosh\left(\frac{ab\,e^{(\lambda+\mu)x}}{\lambda+\mu}\right)e^{(\lambda+\mu)x} - y\,e^{\lambda x}\sinh\left(\frac{ab\,e^{(\lambda+\mu)x}}{\lambda+\mu}\right)}{b\,e^{(\lambda+\mu)x}\sinh\left(\frac{ab\,e^{(\lambda+\mu)x}}{\lambda+\mu}\right) + y\cosh\left(\frac{ab\,e^{(\lambda+\mu)x}}{\lambda+\mu}\right)e^{\lambda x}}\right)$$

7.2.7.9 [554] problem number 9

problem number 554

Added January 7, 2019.

Problem 2.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}y^2 + ae^{\mu x}y + a\lambda e^{(\mu - lambda)x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda)*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left((-1)^{\lambda/\mu}\mu^{-\frac{\lambda}{\mu}}a^{\lambda/\mu}\,\text{Gamma}\left(-\frac{\lambda}{\mu}, -\frac{ae^{\mu x}}{\mu}\right) - \frac{\mu e^{\frac{ae^{\mu x}}{\mu}-\lambda x}}{ye^{\lambda x}+\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (exp(lambda*x)*y^2  + a*exp(mu*x)*y+a*lambda*exp((mu-lambda)*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-\lambda+\mu)\left(y\,\mathrm{e}^{\lambda x}+\lambda\right)\mathrm{e}^{\lambda x}}{a\lambda\,\text{hypergeom}\left(\left[\frac{-\lambda+\mu}{\mu}\right],\left[\frac{-\lambda+2\mu}{\mu}\right],\frac{a\,\mathrm{e}^{\mu x}}{\mu}\right)\mathrm{e}^{\mu x} - (-\lambda+\mu)\,y\,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right],\left[\frac{-\lambda+\mu}{\mu}\right],\frac{a}{}\right.}\right.$$

7.2.7.10 [555] problem number 10

problem number 555

Added January 7, 2019.

Problem 2.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(\lambda e^{\lambda x}y^2 - ae^{\mu x}y + ae^{(\mu - lambda)x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (lambda*Exp[lambda*x]*y^2 - a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\mu\left(ae^{\mu x}\,\text{LaguerreL}\left(-\frac{-\lambda^2+\lambda+\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{ae^{\mu x}}{\mu}\right) + \lambda\left(ye^{\lambda x}-1\right)\text{LaguerreL}\left(\frac{(\lambda-}{\mu}\right.\right.}{\lambda\left(a(\lambda-1)e^{\mu x}\,\text{HypergeometricU}\left(\frac{-\lambda^2+\lambda+\mu}{\mu}, \frac{\lambda}{\mu}+2, \frac{ae^{\mu x}}{\mu}\right) + \left(\mu - \mu ye^{\lambda x}\right)\text{Hypergeometr}\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-  (lambda*exp(lambda*x)*y^2  - a*exp(mu*x)*y + a*lambda*exp((mu-lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-\lambda^2-\mu\right)\text{KummerM}\left(\frac{-\lambda^2+\lambda-\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a\,e^{\mu x}}{\mu}\right) - \left(-\lambda y\,e^{\lambda x} + a\,e^{\mu x} - \lambda^2 + \lambda - \mu\right)\text{KummerM}}{-\mu\,\text{KummerU}\left(\frac{-\lambda^2+\lambda-\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a\,e^{\mu x}}{\mu}\right) + \left(-\lambda y\,e^{\lambda x} + a\,e^{\mu x} - \lambda^2 + \lambda - \mu\right)\text{KummerU}\left(-\right.}\right.$$

7.2.7.11 [556] problem number 11

problem number 556

Added January 7, 2019.

Problem 2.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + abe^{(\lambda+\mu)x}y - b\mu e^{\mu x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*b*Exp[(lambda + mu)*x]*y - b*mu*Exp[mu*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2+ a*b*exp((lambda +mu)*x)*y - b*mu*exp(mu*x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{}{2\left(2\lambda+\mu\right)^2\left(\frac{3\lambda}{2}+\mu\right)\mathrm{WhittakerM}\left(\frac{4\lambda+3\mu}{2\lambda+2\mu},\frac{3\lambda+2\mu}{2\lambda+2\mu},\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}}{\lambda+\mu}\right)\mathrm{e}^{\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}-3\left(\frac{2\lambda}{3}+\mu\right)(\lambda+\mu)x}{2\lambda+2\mu}}+2\left(\right.}\right.$$

7.2.7.12 [557] problem number 12

problem number 557

Added January 7, 2019.

Problem 2.3.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{(2\lambda+\mu)x}y^2 + \left(be^{(\lambda+\mu)x} - \lambda\right)y + ce^{\mu x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[(2*lambda + mu)*x]*y^2 + (b*Exp[(lambda + mu)*x] - lambda)*y +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{i\pi e^{-\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}}\left(\sqrt{b^2-4ac}-2aye^{\lambda x}-b\right)}{2\left(\left(2aye^{\lambda x}+b\right)\cosh\left(\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}\right)+\sqrt{b^2-4ac}\sinh\left(\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}\right)\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp((2*lambda +mu)*x)*y^2+ (b*exp((lambda +mu)*x) -lambda)*y + c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2(\lambda+\mu)\,b\arctan\left(\frac{2aby\,\mathrm{e}^{\lambda x}+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\sqrt{4ac\,b^2-b^4}\,\mathrm{e}^{(\lambda+\mu)x}\right)b}{\sqrt{4ac\,b^2-b^4}\,(\lambda+\mu)}\right)$$

7.2.7.13 [558] problem number 13

problem number 558

Added January 7, 2019.

Problem 2.3.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}(y - be^{\mu x})^2 + b\mu e^{\mu x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*(y - b*Exp[mu*x])^2 + b*mu*Exp[mu*x])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{b\left(-e^{x(\lambda+\mu)}\right) + ye^{\lambda x} + \lambda}{\lambda\left(be^{\mu x} - y\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( exp(lambda*x) *(y- b*exp(mu*x))^2 + b*mu*exp(mu*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b\,\mathrm{e}^{(\lambda+\mu)x} - y\,\mathrm{e}^{\lambda x} - \lambda}{(b\,\mathrm{e}^{\mu x} - y)\,\lambda}\right)$$

7.2.7.14 [559] problem number 14

problem number 559

Added January 7, 2019.

Problem 2.3.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + bnx^{n-1} - ab^2e^{\lambda x}x^{2n}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*n*x^(n - 1) - a*b^2*Exp[lambda*x]*x^(2*n))*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^2+ b*n*x^(n-1) - a*b^2*exp(lambda*x)*x^(2*n))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.15 [560] problem number 15

problem number 560

Added January 7, 2019.

Problem 2.3.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}y^2 + ax^n y + a\lambda x^n e^{-\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*x^n*y + a*lambda*x^n*Exp[-(lambda*x)])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( exp(lambda*x)*y^2+ a*x^n*y + a*lambda*x^n*exp(-lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-y\,\mathrm{e}^{\lambda x}-\lambda\right)\left(\int \mathrm{e}^{\frac{(a\,x^n-(n+1)\lambda)x}{n+1}}dx\right)-\mathrm{e}^{\frac{(a\,x^n-(n+1)\lambda)x}{n+1}}}{y\,\mathrm{e}^{\lambda x}+\lambda}\right)$$

7.2.7.16 [561] problem number 16

problem number 561

Added January 7, 2019.

Problem 2.3.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda e^{\lambda x}y^2 + ax^n e^{\lambda x}y - ax^n e^{2\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Exp[lambda*x]*y^2 + a*x^n*Exp[lambda*x]*y - a*x^n*Exp[2*lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+  ( lambda*exp(lambda*x)*y^2+ a*x^n*exp(lambda*x)*y - a*x^n*exp(2*lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.17 [562] problem number 17

problem number 562

Added January 7, 2019.

Problem 2.3.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 - abx^ne^{\lambda x}y + bnx^{n-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 - a*b*x^n*Exp[lambda*x]*y + b*n*x^(n - 1))*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  ( a*exp(lambda*x)*y^2- a*b*x^n*exp(lambda*x)*y + b*n*x^(n-1))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(b\,x^n - y)\,a}{(b\,x^n - y)\,a\left(\int \lambda\,\mathrm{e}^{\frac{(-\Gamma(n)+\Gamma(n,-\lambda x))abn\,x^n(-\lambda)^n(-\lambda)^{-n}(-\lambda x)^{-n}+ab\,x^n(-\lambda)^n(-\lambda)^{-n}\mathrm{e}^{\lambda x}+\lambda^2 x}{\lambda}}dx\right) - \lambda\,\mathrm{e}^{\frac{((-\Gamma(}{}}}\right.$$

7.2.7.18 [563] problem number 18

problem number 563

Added January 7, 2019.

Problem 2.3.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^2 + b\lambda e^{\lambda x} - ab^2x^ne^{2\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*lambda*Exp[lambda*x] - a*b^2*x^n*Exp[2*lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 + b*lambda*exp(lambda*x) - a*b^2*x^n*exp(2*lambda*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.19 [564] problem number 19

problem number 564

Added January 7, 2019.

Problem 2.3.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^2 + \lambda y - ab^2x^ne^{2\lambda x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + lambda*y - a*b^2*x^n*Exp[2*lambda*x])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-i\left(ab(-1)^{-n}\lambda^{-n-1}\,\text{Gamma}(n+1,-\lambda x) + \tanh^{-1}\left(\frac{ye^{-\lambda x}}{b}\right)\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  ( a*x^n*y^2 + lambda*y - a*b^2*x^n*exp(2*lambda*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-i\left(\left(n\Gamma(n,-\lambda x)-\Gamma(n+1)\right)(-\lambda x)^{-n}+\mathrm{e}^{\lambda x}\right)ab\,x^{n}-i\lambda\,\mathrm{arctanh}\left(\frac{y\,\mathrm{e}^{-\lambda x}}{b}\right)}{\lambda}\right)$$

7.2.7.20 [565] problem number 20

problem number 565

Added January 7, 2019.

Problem 2.3.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - abx^n e^{\lambda x} y + b\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*b*x^n*Exp[lambda*x]*y + b*lambda*Exp[lambda*x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*b*x^n*exp(lambda*x)*y + b*lambda*exp(lambda*x) )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.21 [566] problem number 21

problem number 566

Added January 7, 2019.

Problem 2.3.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - ax^n \left(be^{\lambda x} + c\right) y + b\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*Exp[lambda*x] + c)*y + b*lambda*Exp[lambda*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*x^n*(b*exp(lambda*x) + c )*y + b*lambda*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.22 [567] problem number 22

problem number 567

Added January 7, 2019.

Problem 2.3.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n e^{2\lambda x} y^2 + \left(bx^n e^{\lambda x} - \lambda\right) y + cx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*Exp[2*lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - lambda)*y + c*x^n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\sqrt{a}\sqrt{c}\left((-1)^{1-n}\lambda^{-n-1}\operatorname{Gamma}(n+1,-\lambda x)-\frac{2\tan^{-1}\left(\frac{b-2aye^{\lambda x}}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^n*exp(2*lambda*x)*y^2 + (b*x^n*exp(lambda*x) - lambda)*y + c*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2\left(b\lambda\arctan\left(\frac{2aby\,e^{\lambda x}+b^2}{\sqrt{4ac\,b^2-b^4}}\right)-\frac{\left((n\Gamma(n,-\lambda x)-\Gamma(n+1))(-\lambda x)^{-n}+e^{\lambda x}\right)\sqrt{4ac\,b^2-b^4}\,x^n}{2}\right)b}{\sqrt{4ac\,b^2-b^4}\,\lambda}\right)$$

7.2.7.23 [568] problem number 23

problem number 568

Added January 10, 2019.

Problem 2.3.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}(y-bx^n-c)^2+bnx^{n-1}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*(y - b*x^n - c)^2 + b*n*x^(n - 1))*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow c_1\left(\frac{ae^{\lambda x}}{\lambda}-\frac{1}{bx^n+c-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (  a*exp(lambda*x)*(y- b*x^n - c)^2 +b*n*x^(n-1))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(b\,x^n+c-y)\,a\,\mathrm{e}^{\lambda x}-\lambda}{(b\,x^n+c-y)\,\lambda}\right)$$

7.2.7.24 [569] problem number 24

problem number 569

Added January 10, 2019.

Problem 2.3.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+2a\lambda xe^{\lambda x^2}-a^2e^{2\lambda x^2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*Exp[2*lambda*x^2])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ (   y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*exp(2*lambda*x^2))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.7.25 [570] problem number 25

problem number 570

Added January 10, 2019.

Problem 2.3.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{-\lambda x^2}y^2 + \lambda xy + ab^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[-(lambda*x^2)]*y^2 + lambda*x*y + a*b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right)-\frac{\sqrt{\frac{\pi}{2}}ab\,\text{erf}\left(\frac{\sqrt{\lambda}x}{\sqrt{2}}\right)}{\sqrt{\lambda}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (    a*exp(-lambda*x^2)*y^2 + lambda*x*y + a*b^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\sqrt{\pi}\sqrt{2}\,ab\,\text{erf}\left(\frac{\sqrt{2}\sqrt{\lambda}\,x}{2}\right)-2\sqrt{\lambda}\,\arctan\left(\frac{y\,e^{-\frac{\lambda x^2}{2}}}{b}\right)}{2\sqrt{\lambda}}\right)$$

7.2.7.26 [571] problem number 26

problem number 571

Added January 10, 2019.

Problem 2.3.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + \lambda xy + ab^2 x^n e^{\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (a*x^n*y^2 + lambda*x*y + a*b^2*x^n*Exp[lambda*x^2])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right) - iabi^{-n}2^{\frac{n-1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}\,\text{Gamma}\left(\frac{n+1}{2}, -\frac{\lambda x^2}{2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (   a*x^n*y^2 + lambda*x*y + a*b^2*x^n*exp(lambda*x^2) )*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(-\lambda\,x^2\right)^{-\frac{n}{2}-\frac{1}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2}\right) - ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(-\lambda\,x^2\right)^{-\frac{n}{2}-\frac{1}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2}, -\frac{\lambda\,x^2}{2}\right) - a\right.$$

7.2.7.27 [572] problem number 27

problem number 572

Added January 10, 2019.

Problem 2.3.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{2\lambda x}y^3 + be^{\lambda x}y^2 + cy + de^{-\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[2*lambda*x]*y^3 + b*Exp[lambda*x]*y^2 + c*y + d*Exp[-(lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (  a*exp(2*lambda*x)*y^3 + b*exp(lambda*x)*y^2 + c*y+ d*exp(-lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \frac{\ln\left(y\,e^{\lambda x} - \mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\,\_Z + d\right)\right)}{3\,\mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\,\_Z + d\right)^2 a + 2\,\mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\,\_\right.}\right.$$

Solution contains RootOf

7.2.7.28 [573] problem number 28

problem number 573

Added January 10, 2019.

Problem 2.3.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^3 + 3abe^{\lambda x}y^2 + cy - 2ab^3e^{\lambda x} + bc\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^3 + 3*a*b*Exp[lambda*x]*y^2 + c*y - 2*a*b^3*Exp[la
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{6ab^2e^{\lambda x}}{\lambda}}\left(2(b+y)^2e^{\frac{6ab^2e^{\lambda x}}{\lambda}}\int_1^x ae^{(2c+\lambda)K[1]-\frac{6ab^2e^{\lambda K[1]}}{\lambda}}dK[1]+e^{2cx}\right)}{(b+y)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^3 + 3*a*b*exp(lambda*x)*y^2 + c*y- 2*a*b^3*exp(l
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2(b+y)^2a\left(\int \mathrm{e}^{\frac{-6a\,b^2\mathrm{e}^{\lambda x}+2\left(c+\frac{\lambda}{2}\right)\lambda x}{\lambda}}dx\right)+\mathrm{e}^{-\frac{6a\,b^2\mathrm{e}^{\lambda x}}{\lambda}+2cx}}{(b+y)^2}\right)$$

7.2.7.29 [574] problem number 29

problem number 574

Added January 10, 2019.

Problem 2.3.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ae^{\lambda x}y^2+ky+ab^2x^{2k}e^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + k*y + a*b^2*x^(2*k)*Exp[lambda*x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\sqrt{b^2}x^k(-\lambda x)^{-k}\operatorname{Gamma}(k,-\lambda x)+\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ ( a*exp(lambda*x)* y^2 + k*y + a*b^2*x^(2*k)*exp(lambda*x) )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(ab\,x^k(-\lambda x)^{-k}\,\Gamma(k)-ab\,x^k(-\lambda x)^{-k}\,\Gamma(k,-\lambda x)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.7.30 [575] problem number 30

problem number 575

Added January 10, 2019.

Problem 2.3.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n}e^{\lambda x}y^2+(bx^ne^{\lambda x}-n)y+ce^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*Exp[lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - n)*y + c*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(c(-\lambda x)^{-n}\sqrt{\frac{ax^{2n}}{c}}\,\text{Gamma}(n,-\lambda x)-\frac{2\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{\frac{b^2}{ac}}-2y\sqrt{\frac{ax^{2n}}{c}}\right)}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ (  a*x^(2*n)*exp(lambda*x)*y^2 + (b*x^n*exp(lambda*x) - n)*y + c*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2\left(b\arctan\left(\frac{2aby\,x^n+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\frac{\sqrt{4ac\,b^2-b^4}\,(-\Gamma(n)+\Gamma(n,-\lambda x))x^n(-\lambda x)^{-n}}{2}\right)b}{\sqrt{4ac\,b^2-b^4}}\right)$$

7.2.7.31 [576] problem number 31

problem number 576

Added January 10, 2019.

Problem 2.3.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x+e^{\lambda x}\left((2a\lambda x+a+b)y-e^{\lambda x}(a^2\lambda x^2+abx-c)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + Exp[lambda*x]*((2*a*lambda*x + a + b)*y - Exp[lambda*x]*(a^2*lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  y*diff(w(x,y),x)+ exp(lambda*x)* ( (2*a*lambda*x+a + b)*y - exp(lambda*x)*(a^2*lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\sqrt{\frac{-b^2-4\lambda c}{a^2}}\, a \left(\int \frac{2\arctan\left(\frac{2a\lambda x+b-2\lambda y\,\mathrm{e}^{-\lambda x}}{a\sqrt{\frac{-b^2-4\lambda c}{a^2}}}\right)}{\sqrt{\frac{-b^2-4\lambda c}{a^2}}}\, \mathrm{e}^{-\_a} \tan\left(\frac{\sqrt{\frac{-b^2-4\lambda c}{a^2}}\,\_a}{2}\right) d\_a\right) - (2a\lambda x+b)\,\mathrm{e}^{-\frac{2\,\mathrm{arc}}{}}}{2a}\right.$$

7.2.7.32 [577] problem number 32

problem number 577

Added January 10, 2019.

Problem 2.3.2.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^m w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^m*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{y^{1-m}}{m-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*exp(lambda*x)*diff(w(x,y),x)+  b*y^m*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\lambda\, y^{-m+1}-(m-1)\, b\, \mathrm{e}^{-\lambda x}}{a\lambda}\right)$$

7.2.7.33 [578] problem number 33

problem number 578

Added January 10, 2019.

Problem 2.3.2.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^y+bx)w_x+w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[y] + b*x)*D[w[x, y], x] + D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*exp(y)+b*x)*diff(w(x,y),x)+ diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(a\,\mathrm{e}^{by} + (b-1)\,x\,\mathrm{e}^{(b-1)y}\right)\mathrm{e}^{(-2b+1)y}}{b-1}\right)$$

7.2.7.34 [579] problem number 34

problem number 579

Added January 10, 2019.

Problem 2.3.2.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n e^{\lambda y} + bxy^m)w_x + e^{\mu y}w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Exp[lambda*y] + b*x*y^m)*D[w[x, y], x] + Exp[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*exp(lambda*y)+ b*x*y^m)*diff(w(x,y),x)+ exp(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{\frac{1}{m+1}}x^{\frac{m}{m+1}}x^{-\frac{n}{m+1}}x^{-\frac{mn}{m+1}}\mathrm{e}^{\frac{bn\,y^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)\mathrm{e}^{-\frac{\mu y}{2}}}{(m+1)\mu}}\mathrm{e}^{-\frac{b\,y^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)\mathrm{e}^{-}}{(m+1)\mu}}\right.$$

7.2.7.35 [580] problem number 35

problem number 580

Added January 10, 2019.

Problem 2.3.2.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxe^{\lambda y})w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*Exp[lambda*y])*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+ b *x*exp(lambda*y))*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x\,x^{-n}\mathrm{e}^{\frac{bn\,y^{-k}\mathrm{e}^{\lambda y}}{\lambda}}\,\mathrm{e}^{\frac{b\,y^{-k}(-\lambda y)^k\Gamma(-k+1)}{\lambda}}\,\mathrm{e}^{\frac{bk\,y^{-k}(-\lambda y)^k\Gamma(-k,-\lambda y)}{\lambda}}\,\mathrm{e}^{-\frac{b\,y^{-k}\mathrm{e}^{\lambda y}}{\lambda}}\,\mathrm{e}^{-\frac{bn\,y^{-k}(-\lambda y)^k\Gamma(-k+1)}{\lambda}}\,\mathrm{e}^{-\frac{bkn\,y^{-k}(-\lambda y)}{\lambda}}$$

7.2.7.36 [581] problem number 36

problem number 581

Added January 10, 2019.

Problem 2.3.2.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxy^k)w_x + e^{\lambda y}w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + Exp[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+ b *x*y^k)*diff(w(x,y),x)+ exp(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{\frac{1}{k+1}} x^{\frac{k}{k+1}} x^{-\frac{n}{k+1}} x^{-\frac{kn}{k+1}} \mathrm{e}^{\frac{bn\, y^k (\lambda y)^{-\frac{k}{2}} \,\mathrm{WhittakerM}\left(\frac{k}{2}, \frac{k}{2}+\frac{1}{2}, \lambda y\right) \mathrm{e}^{-\frac{\lambda y}{2}}}{(k+1)\lambda}} \mathrm{e}^{-\frac{b\, y^k (\lambda y)^{-\frac{k}{2}} \,\mathrm{WhittakerM}\left(\frac{k}{2}, \frac{k}{2}+\frac{1}{2}, \lambda y\right) \mathrm{e}^{-\frac{\lambda y}{2}}}{(k+1)\lambda}} + \right.$$

7.2.8 4.1

Local contents

7.2.8.1 [582] problem number 1

problem number 582

Added January 10, 2019.

Problem 2.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\cosh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\cosh(\lambda x)+\lambda y}{\lambda}\right)$$

7.2.8.2 [583] problem number 2

problem number 583

Added January 10, 2019.

Problem 2.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu x - 2\,\mathrm{arctanh}\left(\mathrm{e}^{\mu y}\right)}{a\mu}\right)$$

7.2.8.3 [584] problem number 3

problem number 584

Added January 10, 2019.

Problem 2.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\sinh(\lambda x) - a^2\sinh^2(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sinh[lambda*x] - a^2*Sinh[lambda*x]^2)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}} - \int_1^{e^{\lambda x}}\frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2-a^2 + a*lambda*sinh(lambda*x) - a^2*sinh(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(-\left(-\frac{(\sinh^2(\lambda x))}{2} + i\sinh(\lambda x) + \frac{1}{2}\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda}, -\right.\right.}{-(-\sinh(\lambda x) + i)\left(\sinh^2(\lambda x) + 1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda}, \frac{1}{2}, -\frac{1}{2}, \frac{2ia}{\lambda}, -\frac{8ia-3\lambda}{8\lambda}, -\frac{i\sinh(\lambda x)}{2}\right.}\right.$$

7.2.8.4 [585] problem number 4

problem number 585

Added January 10, 2019.

Problem 2.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \lambda\left(\sinh(\lambda x)y^2 - \sinh^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + lambda*(Sinh[lambda*x]*y^2 - Sinh[lambda*x]^3)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ lambda*(sinh(lambda*x)*y^2 - sinh(lambda*x)^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y-\cosh(\lambda x))\sqrt{\pi}}{-\sqrt{\pi}\,y\,\mathrm{erfi}(\cosh(\lambda x)) + \sqrt{\pi}\,\mathrm{erfi}(\cosh(\lambda x))\cosh(\lambda x) - 2\,\mathrm{e}^{\cosh^2(\lambda x)}}\right)$$

7.2.8.5 [586] problem number 5

problem number 586

Added January 10, 2019.

Problem 2.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((a\sinh^2(\lambda x) - \lambda)y^2 - a\sinh^2(\lambda x) + \lambda - a\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((a*Sinh[lambda*x]^2 - lambda)*y^2 - a*Sinh[lambda*x]^2 + lambda - a)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((a*sinh(lambda*x)^2-lambda)*y^2 - a*sinh(lambda*x)^2 + lambda - a)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{\cosh(2\lambda x)+1}\,(\cosh(2\lambda x)}{4\sqrt{\cosh(2\lambda x)-1}\,(a\cosh(2\lambda x)-a-2\lambda)\,\lambda\,\mathrm{e}^{\frac{a\cosh(2\lambda x)}{2\lambda}}\sinh(2\lambda x)+2\sqrt{\cosh(2\lambda x)+1}}\right.$$

7.2.8.6 [587] problem number 6

problem number 587

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\lambda x)w_x + a(\sinh(\mu y))\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[lambda*x]*D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\tanh^{-\frac{a\mu}{\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(lambda*x)*diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2a\mu\operatorname{arctanh}\left(e^{\lambda x}\right)-2\lambda\operatorname{arctanh}\left(e^{\mu y}\right)}{a\lambda\mu}\right)$$

7.2.8.7 [588] problem number 7

problem number 588

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\mu y)w_x+a(\sinh(\lambda x))\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[mu*yx]*D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\cosh(\lambda x)\operatorname{csch}(\mu \mathrm{yx})}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(mu*y)*diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu\cosh(\lambda x) + \lambda\cosh(\mu y)}{a\lambda\mu}\right)$$

7.2.9 4.2

Local contents

7.2.9.1 [589] problem number 1

problem number 589

Added January 10, 2019.

Problem 2.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a(\cosh(\lambda x))\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\sinh(\lambda x)+\lambda y}{\lambda}\right)$$

7.2.9.2 [590] problem number 2

problem number 590

Added January 10, 2019.

Problem 2.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a(\cosh(\lambda x))\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*cosh(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda x + 2\arctan\left(e^{\lambda y}\right)}{a\lambda}\right)$$

7.2.9.3 [591] problem number 3

problem number 591

Added January 10, 2019.

Problem 2.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((a\cosh^2(\lambda x) - \lambda)y^2 - a\cosh^2(\lambda x) + \lambda + a\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((a*Cosh[lambda*x]^2 - lambda)*y^2 - a*Cosh[lambda*x]^2 + lambda + a)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ( (a *cosh(lambda*x)^2-lambda)*y^2 - a*cosh(lambda*x)^2+ lambda + a)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{8\left(\left(a\left(\cosh^2(\lambda x)\right) - \lambda\right) y \left(\cosh^4\right.\right.}{4\sqrt{\cosh(2\lambda x)+1}\,(a\cosh(2\lambda x) + a - 2\lambda)\,\lambda\, e^{\frac{a\cosh(2\lambda x)}{2\lambda}}\sinh(2\lambda x) + 8\left(\left(a\left(\cosh^2(\lambda x)\right)\right.\right.}\right.$$

7.2.9.4 [592] problem number 4

problem number 592

Added January 10, 2019.

Problem 2.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2w_x + \left((a-\lambda+a\cosh(\lambda x))y^2 + a + \lambda - a\cosh(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((a - lambda + a*Cosh[lambda*x])*y^2 + a + lambda - a*Cosh[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+ ( (a - lambda + a*cosh(lambda*x))*y^2 + a+ lambda- a *cosh(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\cosh(\lambda x)+1)^{\frac{3}{2}}\sqrt{\cosh(\lambda x)-1}\,(y\cosh(\lambda x)}{-2(\cosh(\lambda x)+1)\lambda\,\mathrm{e}^{\frac{a\cosh(\lambda x)}{\lambda}}\sinh(\lambda x)+\sqrt{\cosh(\lambda x)-1}\left(-(\cosh(\lambda x)+1)^{\frac{5}{2}}y+(\cosh\right.}\right.$$

7.2.9.5 [593] problem number 5

problem number 593

Added January 10, 2019.

Problem 2.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cosh^m(y))\,w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}e^{(n-1)b\left(\int y^{-k}(\cosh^m(y))dy\right)}dy\right) + x^{-n+1}e^{(n-1)b\left(\int y^{-k}(\cosh^m(y))dy\right)}\right)$$

7.2.9.6 [594] problem number 6

problem number 594

Added January 10, 2019.

Problem 2.4.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cosh^m(y))\,w_x + \cosh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int (\cosh^{-k}(\lambda y))\,\mathrm{e}^{(n-1)b\left(\int (\cosh^m(y))(\cosh^{-k}(\lambda y))dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\cosh^m(y))(\cos\right.}\right.$$

7.2.9.7 [595] problem number 7

problem number 595

Added January 10, 2019.

Problem 2.4.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^ny^m + bx)\,w_x + \cosh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*y^m+ b*x)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m(\cosh^{-k}(\lambda y))\,\mathrm{e}^{(n-1)b\left(\int (\cosh^{-k}(\lambda y))dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\cosh^{-k}(\lambda y))dy\right)}\right)$$

7.2.9.8 [596] problem number 8

problem number 596

Added January 10, 2019.

Problem 2.4.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(\cosh(\mu y))\, w_x + a\cosh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cosh[mu*y]*D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sinh(\mu y)}{\mu}-\frac{a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := cosh(mu*y)*diff(w(x,y),x)+a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu\sinh(\lambda x)+\lambda\sinh(\mu y)}{a\lambda\mu}\right)$$

7.2.10 4.3

Local contents

7.2.10.1 [597] problem number 1

problem number 597

Added January 10, 2019.

Problem 2.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tanh(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tanh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\log(\cosh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\ln(\tanh(\lambda x)-1)+a\ln(\tanh(\lambda x)+1)+2\lambda y}{2\lambda}\right)$$

7.2.10.2 [598] problem number 2

problem number 598

Added January 10, 2019.

Problem 2.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tanh(\lambda y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tanh[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log(\sinh(\lambda y))}{\lambda}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\ln(\tanh(\lambda x)-1)+a\ln(\tanh(\lambda x)+1)+2\lambda y}{2\lambda}\right)$$

7.2.10.3 [599] problem number 3

problem number 599

Added January 10, 2019.

Problem 2.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+a\lambda-a(a+\lambda)\tanh^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Tanh[lambda*x]^2)*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\lambda e^{-2ax}\left({}_2F_1\left(-\frac{2a}{\lambda},-\frac{a}{\lambda};1-\frac{a}{\lambda};-e^{2\lambda x}\right)\left(a\left(e^{2\lambda x}-1\right)-y\left(e^{2\lambda x}+1\right)\right)+2a\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}+1}\right.}{2a\left(a\left(-e^{2\lambda x}\right)+a+ye^{2\lambda x}+y\right)}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2+a*lambda - a*(a+lambda)*tanh(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda \operatorname{LegendreP}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x)\right) - (y + (a+\lambda)\tanh(\lambda x)) \operatorname{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x)\right)}{-\lambda \operatorname{LegendreQ}\left(\frac{a+\lambda}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x)\right) + (y + (a+\lambda)\tanh(\lambda x)) \operatorname{LegendreQ}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x)\right)}\right.$$

7.2.10.4 [600] problem number 4

problem number 600

Added January 10, 2019.

Problem 2.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + 3a\lambda - \lambda^2 - a(a+\lambda)\tanh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Tanh[lambda*x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2+3*a*lambda - lambda^2 -a*(a+lambda)*tanh(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-(\cosh(\lambda x) + \sinh(\lambda x))(a - \lambda + y)\left(\lambda + \frac{\sqrt{-a^2-4a\lambda+\lambda^2}}{2}\right) a \operatorname{hypergeom}\left(\left[\frac{a+\lambda-\sqrt{-a^2-4a\lambda+}}{2\lambda}\right.\right.\right.}{\left(\lambda - \sqrt{-a^2-4a\lambda+\lambda^2}\right)\left(a + \lambda - \sqrt{-a^2-4a\lambda+\lambda^2}\right)\left((\cosh\right.}\right.$$

7.2.10.5 [601] problem number 5

problem number 601

Added January 10, 2019.

Problem 2.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tanh^m(y))\,w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tanh^m(y))dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tanh^m(y))dy\right)}\right)$$

7.2.10.6 [602] problem number 6

problem number 602

Added January 10, 2019.

Problem 2.4.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tanh^m(y))\,w_x + \tanh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\tanh^{-k}(\lambda y)\right) e^{(n-1)b\left(\int (\tanh^m(y))(\tanh^{-k}(\lambda y))dy\right)} dy\right) + x^{-n+1} e^{(n-1)b\left(\int (\tanh^m(y))(\mathrm{ta}\right.}\right.$$

7.2.10.7 [603] problem number 7

problem number 603

Added January 10, 2019.

Problem 2.4.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)\, w_x + \tanh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n*y^m + b*x)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m \left(\tanh^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int \left(\tanh^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int \left(\tanh^{-k}(\lambda y)\right)dy\right)}\right)$$

7.2.10.8 [604] problem number 8

problem number 604

Added January 10, 2019.

Problem 2.4.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \tanh^m y + bx)\, w_x + y^k w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^n*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y K[1]^{-k}\tanh^m(K[1])dK[1] + \frac{x^{1-n}}{a(n-1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := ( a*x^n*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\left(\tanh^m(y)\right)dy\right) + x^{-n+1}\right)$$

7.2.11 4.4

Local contents

7.2.11.1 [605] problem number 1

problem number 605

Added January 10, 2019.

Problem 2.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\coth(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Coth[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\log(\sinh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*coth(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\ln(\coth(\lambda x)-1)+a\ln(\coth(\lambda x)+1)+2\lambda y}{2\lambda}\right)$$

7.2.11.2 [606] problem number 2

problem number 606

Added January 10, 2019.

Problem 2.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\coth(\lambda y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Coth[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log(\cosh(\lambda y))}{\lambda}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*coth(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2a\lambda x+\ln(\coth(\lambda y)-1)+\ln(\coth(\lambda y)+1)-2\ln(\coth(\lambda y))}{2a\lambda}\right)$$

7.2.11.3 [607] problem number 3

problem number 607

Added January 10, 2019.

Problem 2.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda - a(a+\lambda)\coth^2(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\lambda e^{-2ax}\left(\frac{{}_2F_1\left(-\frac{2a}{\lambda},-\frac{a}{\lambda};1-\frac{a}{\lambda};e^{2\lambda x}\right)}{a}-\frac{2\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}+1}}{ae^{2\lambda x}+a-ye^{2\lambda x}+y}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 + a*lambda - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\lambda\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)-(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)}{-\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)+(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)}\right.$$

7.2.11.4 [608] problem number 4

problem number 608

Added January 10, 2019.

Problem 2.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+3a\lambda-\lambda^2-a(a+\lambda)\coth^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda e^{2x(\lambda-a)}\left((a-2\lambda)F_1\left(1-\frac{a}{\lambda};-\frac{2a}{\lambda},2;2-\frac{a}{\lambda};e^{2\lambda x},-e^{2\lambda x}\right)\left(a\left(2e^{2\lambda x}+3e^{4\lambda x}-1\right)-\left(e^{2\lambda x}\right.\right.\right.}{2\left(a^2-3a\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 + a*lambda  -lambda^2 - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{\sqrt{a^2+\lambda^2}}{\lambda},\coth(\lambda x)\right)+\left(-a-\lambda+\sqrt{a^2+\lambda^2}\right)\mathrm{Legen}}{-(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{\sqrt{a^2+\lambda^2}}{\lambda},\coth(\lambda x)\right)+\left(a+\lambda-\sqrt{a^2+\lambda^2}\right)\mathrm{Legen}}\right.$$

7.2.12 4.5

Local contents

7.2.12.1 [609] problem number 1

problem number 609

Added January 10, 2019.

Problem 2.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\lambda x)\cosh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[lambda*x]*Cosh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{2a\cosh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(lambda*x)*cosh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\mu\cosh(\lambda x)+2\lambda\arctan(e^{\mu y})}{a\lambda\mu}\right)$$

7.2.12.2 [610] problem number 2

problem number 610

Added January 10, 2019.

Problem 2.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cosh(\lambda x)\sinh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*x]*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh^2\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{2a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cosh(lambda*x)*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\mu\,\mathrm{e}^{\lambda x}+a\mu\,\mathrm{e}^{-\lambda x}-4\lambda\operatorname{arctanh}\left(\mathrm{e}^{\mu y}\right)}{2a\lambda\mu}\right)$$

7.2.12.3 [611] problem number 3

problem number 611

Added January 10, 2019.

Problem 2.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2-2\lambda^2\tanh^2(\lambda x)-2\lambda^2\coth^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*lambda^2*Tanh[lambda*x]^2 - 2*lambda^2*Coth[lambda*x]^2)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-4\lambda x}\left(16\lambda^2xe^{4\lambda x}\left(e^{4\lambda x}+1\right)+y\left(e^{4\lambda x}+1\right)\left(e^{4\lambda x}-1\right)^2+2\lambda\left(e^{4\lambda x}-1\right)\left(-2e^{4\lambda x}(2xy+3\right.\right.}{2\left(-ye^{4\lambda x}+2\lambda\left(e^{4\lambda x}+1\right)+y\right)}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 -2 *lambda^2*tanh(lambda*x)^2 - 2*lambda^2*coth(lambda*x)^2)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{2\left(-4\sinh\left(\lambda x\right)-3\sinh\left(3\lambda x\right)+\sinh\left(5\lambda x\right)\right)\lambda\coth\left(\lambda x\right)+\left(-\lambda\left(\coth^2\left(\lambda x\right)\right)-y\coth\left(\lambda x\right.\right.}\right.$$

7.2.12.4 [612] problem number 4

problem number 612

Added January 10, 2019.

Problem 2.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) - 2ab - a(a+\lambda)\tanh^2(\lambda x) - b(b+\lambda)\coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*(a + b) - 2*a*b - a*(a + lambda)*Tanh[lambda*x]^2 - b*(
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda e^{-2x(a+b)}\left((a+b-\lambda)F_1\left(-\frac{a+b}{\lambda};-\frac{2b}{\lambda},-\frac{2a}{\lambda};-\frac{a+b-\lambda}{\lambda};e^{2\lambda x},-e^{2\lambda x}\right)\left(a\left(-2e^{2\lambda x}+3e^{4\lambda x}-1\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 +lambda*(a+b)-2*a*b -a*(a+lambda)*tanh(lambda*x)^2 - b*(b+lambda)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(2a+3\lambda)\left(a\left(\cosh^2(\lambda x)\right)+b\left(\cosh^2(\lambda x)\right)\right.}{4\left(b-\frac{\lambda}{2}\right)\lambda\,\mathrm{hypergeom}\left(\left[2,-\frac{2b-3\lambda}{2\lambda}\right],\left[\frac{2a+5\lambda}{2\lambda}\right],\frac{\cosh^2(\lambda x)}{\sinh(\lambda x)^2}\right)\left(\cosh^2(\lambda x)\right)+2\left(a+\frac{3\lambda}{2}\right)(\cosh(}\right.$$

7.2.12.5 [613] problem number 5

problem number 613

Added January 10, 2019.

Problem 2.4.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\lambda y)w_x + a\cosh(\beta x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[lambda*y]*D[w[x, y], x] + a*Cosh[beta*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\cosh(\lambda y)}{\lambda}-\frac{a\sinh(\beta x)}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(lambda*y)*diff(w(x,y),x)+a*cosh(beta*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda\sinh(\beta x)+\beta\cosh(\lambda y)}{a\beta\lambda}\right)$$

7.2.12.6 [614] problem number 6

problem number 614

Added January 10, 2019.

Problem 2.4.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \cosh^m(\lambda y) + bx)\, w_x + \sinh^k(\beta y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Cosh[lambda*y]^m + b*x)*D[w[x, y], x] + Sinh[beta*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*cosh(lambda*y)^m+b*x)*diff(w(x,y),x)+sinh(beta*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\, a \left(\int (\cosh^m(\lambda y)) \left(\sinh^{-k}(\beta y)\right) e^{(n-1)b\left(\int (\sinh^{-k}(\beta y))dy\right)} dy\right) + x^{-n+1} e^{(n-1)b\left(\int (\sinh^{-k}\right.}\right.$$

7.2.13 5.1

Local contents

7.2.13.1 [615] problem number 1

problem number 615

Added January 14, 2019.

Problem 2.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \ln^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{a \log^k(\lambda x)(-\log(\lambda x))^{-k} \operatorname{Gamma}(k+1, -\log(\lambda x))}{\lambda} - bx + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*ln(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a \ln(\lambda x)^k \, dx\right)\right)$$

7.2.13.2 [616] problem number 3

problem number 616

Added January 14, 2019.

Problem 2.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \ln^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\log^k(\lambda K[1])+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*ln(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(x-\left(\int \frac{1}{a\ln\left(\lambda y\right)^k+b}dy\right)\right)$$

7.2.13.3 [617] problem number 4

problem number 617

Added January 14, 2019.

Problem 2.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\ln^k(x+\lambda y)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Log[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*ln(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{a\lambda\ln\left(\_a\lambda\right)^{k}+1}d\_a\right)+x\right)$$

7.2.14 5.2

Local contents

7.2.14.1 [618] problem number 1

problem number 618

Added January 14, 2019.

Problem 2.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ax^n \ln^k(\lambda y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*x^n*Log[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(-\log(\lambda y))^k \log^{-k}(\lambda y)\,\text{Gamma}(1-k,-\log(\lambda y))}{\lambda}-\frac{ax^{n+1}}{n+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*x^n*ln(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\,x^{n+1}+(-n-1)\left(\int \ln(\lambda y)^{-k}\,dy\right)}{a}\right)$$

7.2.14.2 [619] problem number 2

problem number 619

Added January 14, 2019.

Problem 2.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ay^n \ln^k(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y^n*Log[lambda*x]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{a\log^k(\lambda x)(-\log(\lambda x))^{-k}\operatorname{Gamma}(k+1,-\log(\lambda x))}{\lambda}-\frac{y^{1-n}}{n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*y^n*ln(lambda*x)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\left((n-1)\,a\,y^n\left(\int\ln(\lambda x)^k\,dx\right)+y\right)y^{-n}\right)$$

7.2.14.3 [620] problem number 3

problem number 620

Added January 14, 2019.

Problem 2.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+a\ln(\beta x)y-ab\ln(\beta x)-b^2\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Log[beta*x]*y - a*b*Log[beta*x] - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+ a*ln(beta*x)* y - a*b*ln(beta*x) - b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(\beta x)^{ax}\,\mathrm{e}^{-(a-2b)x} + (b-y)\left(\int (\beta x)^{ax}\,\mathrm{e}^{-(a-2b)x}dx\right)}{b-y}\right)$$

7.2.14.4 [621] problem number 4

problem number 621

Added January 14, 2019.

Problem 2.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\ln^m(bx)y + a\ln^m(bx)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Log[b*x]^m*y + a*Log[b*x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\int_1^x \frac{\exp\left(\frac{2^{-m-1}a\,\mathrm{Gamma}(m+1,-2(\log(b)+\log(K[1])))(-\log(b)-\log(K[1]))^{-m}(\log(b)+\log(K[1]))^m}{b^2}\right)}{K[1]^2}dK[1\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+ a*x*ln(b*x)^m * y + a *ln(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{a\,x^2 \ln(bx)^m-2}{x}dx}dx\right) + x\,e^{\int \frac{a\,x^2 \ln(bx)^m-2}{x}dx} + \int e^{\int \frac{a\,x^2 \ln(bx)^m-2}{x}dx}dx}{xy+1}\right)$$

7.2.14.5 [622] problem number 5

problem number 622

Added January 14, 2019.

Problem 2.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - abx^{n+1} y \ln(x) + b\ln(x) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*b*x^(n + 1)*y*Log[x] + b*Log[x] + b)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*x^n*y^2- a*b*x^(n+1)*y*ln(x)  + b*ln(x) + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.6 [623] problem number 6

problem number 623

Added January 14, 2019.

Problem 2.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((n+1)x^n y^2 - a x^{n+1}(\ln x)^m y + a(\ln x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((n + 1)*x^n*y^2 - a*x^(n + 1)*Log[x]^m*y + a*Log[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-((n+1)*x^n*y^2 - a*x^(n+1)*ln(x)^m*y + a*ln(x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{n+1} e^{\int \frac{a x\, x^{n+1} \ln(x)^m - 2n - 2}{x} dx} + \left(y\, x^{n+1} - 1\right)(n+1)\left(\int \frac{x^n e^{a\left(\int x^{n+1} \ln(x)^m dx\right) - 2n \ln(x)}}{x^2} dx\right)}{y\, x^{n+1} - 1}\right)$$

7.2.14.7 [624] problem number 7

problem number 624

Added January 14, 2019.

Problem 2.5.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 + b m x^{m-1} - a b^2 x^{2m} (\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(2*m)*Log[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a *ln(x)^n*y^2 + b*m*x^(m-1) - a*b^2*x^(2*m)* ln(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.8 [625] problem number 8

problem number 625

Added January 14, 2019.

Problem 2.5.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 - abxy(\ln x)^{n+1} + b\ln x + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 - a*b*x*y*Log[x]^(n + 1) + b*Log[x] + b)*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*ln(x)^n*y^2 - a*b*x*y*(ln(x))^(n+1) + b*ln(x)+ b )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.9 [626] problem number 9

problem number 626

Added January 14, 2019.

Problem 2.5.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^k (y - bx^n - c)^3 + bnx^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^k*(y - b*x^n - c)^3 + b*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2(bx^n + c - y)^2 \int_1^x a \log^k(K[1]) dK[1] + 1}{(bx^n + c - y)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*(ln(x))^k*(y - b*x^n-c)^3 + b*n*x^(n-1) ) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(b^2x^{2n} + 2(c-y)\left(b\,x^n + \frac{c}{2} - \frac{y}{2}\right)\right) a\left(\int \ln(x)^k\, dx\right) + 1}{(b\,x^n + c - y)^2}\right)$$

7.2.14.10 [627] problem number 10

problem number 627

Added January 14, 2019.

Problem 2.5.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 + b(\ln x)^m y + bc(\ln x)^m - ac^2(\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 + b*Log[x]^m*y + b*c*Log[x]^m - a*c^2*Log[x]^n)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*(ln(x))^n*y^2 + b*(ln(x))^m *y+ b*c* (ln(x))^m - a*c^2* (ln(x))^n)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(c+y)\,a\left(\int \ln(x)^n \,\mathrm{e}^{-\left(\int (2ac\ln(x)^n - b\ln(x)^m)dx\right)}dx\right) - \mathrm{e}^{-\left(\int (2ac\ln(x)^n - b\ln(x)^m)dx\right)}}{c+y}\right)$$

7.2.14.11 [628] problem number 11

problem number 628

Added January 14, 2019.

Problem 2.5.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ay + b\ln x)^2 w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y + b*Log[x])^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ay+b\log(x)}{a\sqrt{\frac{b}{a^3}}}\right)-a^2\sqrt{\frac{b}{a^3}}\log(x)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*y+ b*ln(x))^2 *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\arctan\left(\frac{(ay+b\ln(x))a}{\sqrt{ab}}\right)-\sqrt{ab}\,\ln(x)}{\sqrt{ab}\,a}\right)$$

7.2.14.12 [629] problem number 12

problem number 629

Added January 14, 2019.

Problem 2.5.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(xy^2-A^2x(\ln\beta x)^2+A\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^2 + A)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^2 + A) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.13 [630] problem number 13

problem number 630

Added January 14, 2019.

Problem 2.5.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(xy^2 - A^2x(\ln(\beta x))^{2k} + kA(\ln(\beta x))^{k-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^(2*k) + k*A*Log[beta*x]^(k - 1))*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^(2*k) + k*A*(ln(beta*x))^(k-1))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.14 [631] problem number 14

problem number 631

Added January 14, 2019.

Problem 2.5.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + b - ab^2 x^n (\ln x)^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + b - a*b^2*x^n*Log[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(a*x^n*y^2 + b - a*b^2*x^n*(ln(x))^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.14.15 [632] problem number 15

problem number 632

Added January 14, 2019.

Problem 2.5.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a(\ln(\lambda x))^m y^2 + ky + ab^2 x^{2k} (\ln(\lambda x))^m\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Log[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Log[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\sqrt{b^2}x^k(\lambda x)^{-k}\log^{m+1}(\lambda x)(-k\log(\lambda x))^{-m-1}\,\mathrm{Gamma}(m+1,-k\log(\lambda x))+\tan^{-1}\left(\frac{yx^{-}}{\sqrt{b}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*(ln(lambda*x))^m*y^2 + k*y+ a*b^2*x^(2*k)* (ln(lambda*x))^m )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(ab\left(\int x^{k-1}\ln\left(\lambda x\right)^{m}dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.14.16 [633] problem number 16

problem number 633

Added January 14, 2019.

Problem 2.5.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^n(y+b\ln x)^2-b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*(y + b*Log[x])^2 - b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{abx^n\log(x)+ayx^n+n}{bn\log(x)+ny}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*x^n*(y + b*ln(x))^2 - b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b\ln(x)+y)\,a\,x^n+n}{(b\ln(x)+y)\,n}\right)$$

7.2.14.17 [634] problem number 17

problem number 634

Added January 14, 2019.

Problem 2.5.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n}\ln(x)y^2 + (bx^n\ln x - n)y + c\ln x\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*Log[x]*y^2 + (b*x^n*Log[x] - n)*y + c*Log[x])*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\left(\sqrt{b^2-4ac}+2ayx^n+b\right)e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*x^(2*n)*ln(x)* y^2 + (b* x^n *ln(x) - n)*y + c *ln(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2b\,n^2 \arctan\left(\frac{2aby\,x^n+b^2}{\sqrt{4ac\,b^2-b^4}}\right)+\sqrt{4ac\,b^2-b^4}\,(n\ln(x)-1)\,x^n\right)b}{\sqrt{4ac\,b^2-b^4}\,n^2}\right)$$

7.2.14.18 [635] problem number 18

problem number 635

Added January 14, 2019.

Problem 2.5.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^k w_x + (ay^n(\ln x)^m + by(\ln x)^s)\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y*Log[x]^s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left((n-1)\int_1^x a\exp\left(-b(k-1)^{-s-1}(n-1)\,\text{Gamma}(s+1,(k-1)\log(K[1]))\right)K[1]^{-k}\log^m\right.\right.\right.$$

Maple ✓

```
restart;
pde := x^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m + b*y*(ln(x))^s )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int x^{-k}\ln(x)^m\,\mathrm{e}^{(n-1)b\left(\int x^{-k}\ln(x)^s dx\right)}dx\right) + y^{-n+1}\mathrm{e}^{(n-1)b\left(\int x^{-k}\ln(x)^s dx\right)}\right)$$

7.2.14.19 [636] problem number 19

problem number 636

Added January 14, 2019.

Problem 2.5.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left(y^2 + c(\ln x)^n y - \lambda^2 + \lambda c(\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Log[x] + b)*D[w[x, y], x] + (y^2 + c*Log[x]^n*y - lambda^2 + lambda*c*Log[x]^n)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*ln(x)+b)*diff(w(x,y),x)+(y^2+ c*(ln(x))^n*y- lambda^2 + lambda*c*(ln(x))^n )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{e^{\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)e^{\int \frac{-c\ln(x)^n+2\lambda}{a\ln(x)+b}dx+\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx} - y\left(\int \frac{e^{\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) - e^{\int \frac{c\ln(x)^n-}{a\ln(x)+}}}{\lambda\,e^{\int \frac{-c\ln(x)^n+2\lambda}{a\ln(x)+b}dx+\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx} + y}\right.$$

7.2.14.20 [637] problem number 20

problem number 637

Added January 14, 2019.

Problem 2.5.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left((\ln x)^n y^2 - cy - \lambda^2(\ln x)^n + c\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Log[x] + b)*D[w[x, y], x] + (Log[x]^n*y^2 - c*y - lambda^2*Log[x]^n + c*lambda)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*ln(x)+b)*diff(w(x,y),x)+((ln(x))^n*y^2- c*y - lambda^2*(ln(x))^n + c*lambda )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{\ln(x)^n e^{\int \frac{2\lambda\ln(x)^n - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) e^{\int \frac{-2\lambda\ln(x)^n + c}{a\ln(x)+b}dx + \int \frac{2\lambda\ln(x)^n - c}{a\ln(x)+b}dx} + y\left(\int \frac{\ln(x)^n e^{\int \frac{2\lambda\ln(x)^n - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) +}{\lambda e^{\int \frac{-2\lambda\ln(x)^n + c}{a\ln(x)+b}dx + \int \frac{2\lambda\ln(x)^n - c}{a\ln(x)+b}dx} - y}\right.$$

7.2.14.21 [638] problem number 21

problem number 638

Added January 14, 2019.

Problem 2.5.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2\ln(ax)w_x - \left(x^2y^2\ln(ax) + 1\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x^2*Log[a*x]*D[w[x, y], x] - (x^2*y^2*Log[a*x] + 1)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x^2*ln(a*x)*diff(w(x,y),x)-(x^2*y^2* ln(a*x)+ 1  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\ln(ax) - 1}{a\,x^2y + xy\,\mathrm{expIntegral}\left(1, -\ln(ax)\right)\ln(ax) - \mathrm{expIntegral}\left(1, -\ln(ax)\right)}\right)$$

7.2.14.22 [639] problem number 22

problem number 639

Added January 14, 2019.

Problem 2.5.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n + by\ln^m x)\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[lambda*x]^k*D[w[x, y], x] + (a*y^n + b*y*Log[x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left((n-1)\int_1^x a\exp\left((n-1)\int_1^{K[2]} b\log^m(K[1])(\log(\lambda)+\log(K[1]))^{-k}dK[1]\right)(\log(\lambda)+\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n+ b*y* (ln(x))^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \ln(\lambda x)^{-k}\,e^{(n-1)b\left(\int \ln(x)^m \ln(\lambda x)^{-k}dx\right)}dx\right) + y^{-n+1}e^{(n-1)b\left(\int \ln(x)^m \ln(\lambda x)^{-k}dx\right)}\right)$$

7.2.14.23 [640] problem number 23

problem number 640

Added January 14, 2019.

Problem 2.5.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n \ln^m x + by)\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[lambda*x]^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left((n-1)\int_1^x a\exp\left(\frac{b(n-1)\,\text{Gamma}(1-k,-\log(\lambda)-\log(K[1]))(-\log(\lambda)-\log(K[1])\right.}{\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m+ b*y )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \ln{(x)}^m \ln{(\lambda x)}^{-k} \mathrm{e}^{(n-1)b\left(\int \ln(\lambda x)^{-k}dx\right)}dx\right) + y^{-n+1}\mathrm{e}^{(n-1)b\left(\int \ln(\lambda x)^{-k}dx\right)}\right)$$

7.2.15 6.1

Local contents

7.2.15.1 [641] problem number 1

problem number 641

Added January 14, 2019.

Problem 2.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \sin^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y},Assumptions -> {Element[k,
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\sin^2(\lambda x)\right)}{k\lambda+\lambda}-bx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming k::in
```

$$w(x,y) = \_F1\left(\frac{a\left(\sum_{i=0}^{\lceil\frac{k}{2}\rceil-1}\frac{\sin^{-2i+k-1}(\lambda x)}{\prod_{j=1}^{i}\frac{2j-k}{2j-k-1}}\right)\cos(\lambda x) - \left(ax\left(\prod_{j=0}^{\lceil\frac{k}{2}\rceil-1}\frac{2j-k+1}{2j-k}\right) + bx - y\right)k\lambda}{k\lambda}\right)$$

7.2.15.2 [642] problem number 2

problem number 642

Added January 14, 2019.

Problem 2.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda y) + b\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a\sin^k(\lambda K[1]) + b}dK[1] - x\right)\right\}\right\}$$

contains unresolved integral

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\left(\sin^k\left(\lambda y\right)\right) + b} dy\right)\right)$$

contains unresolved integral

7.2.15.3 [643] problem number 3

problem number 643

Added January 14, 2019.

Problem 2.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(\lambda y)\sin^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[lambda*x]^k*Sin[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\cos^2(\mu y)}\sec(\mu y)\sin^{1-n}(\mu y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\mu y)\right)}{\mu - \mu n} - \frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(}{k\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sin(lambda*x)^k*sin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \left(\sin^k(\lambda x)\right) dx\right) + \int \frac{\sin^{-n}(\mu y)}{a} dy\right)$$

contains unresolved integral

7.2.15.4 [644] problem number 4

problem number 644

Added January 14, 2019.

Problem 2.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(x+\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sin(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y + x}{\lambda}} \frac{1}{a\lambda\left(\sin^k(\_a\lambda)\right)+1} d\_a\right) + x\right)$$

contains unresolved integral

7.2.15.5 [645] problem number 5

problem number 645

Added January 14, 2019.

Problem 2.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\sin(\lambda x) + a^2\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sin[lambda*x] + a^2*Sin[lambda*x]^2)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2 + a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(\frac{\lambda \operatorname{HeunCPrime}\left(\frac{4a}{\lambda},-\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\sin(\lambda x)}{2}+\frac{1}{2}\right)\cos(\lambda x)\operatorname{csgn}(}{2}\right.}{2\left(\operatorname{csgn}\left(\sin\left(\lambda x\right)\right)+\sin\left(\lambda x\right)\right)\lambda \operatorname{HeunCPrime}\left(\frac{4a}{\lambda},\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\sin(\lambda x)}{2}+\frac{1}{2}\right)\cos\left(\lambda x\right.}\right.$$

7.2.15.6 [646] problem number 6

problem number 646

Added January 14, 2019.

Problem 2.6.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\sin(\beta x)y + ab\sin(\beta x) - b^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Sin[beta*x]*y + a*b*Sin[beta*x] - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2 + a*sin(beta*x)* y + a*b*sin(beta*x)-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b+y)\left(\int \mathrm{e}^{\frac{-2b\beta x-a\cos(\beta x)}{\beta}}dx\right)+\mathrm{e}^{\frac{-2b\beta x-a\cos(\beta x)}{\beta}}}{b+y}\right)$$

contains unresolved integrals

7.2.15.7 [647] problem number 7

problem number 647

Added January 14, 2019.

Problem 2.6.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\sin^m(bx)y + a\sin^m(bx)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Sin[b*x]^m*y + a*Sin[b*x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\int_1^x \frac{\exp\left(\frac{a\sin^{m+1}(bK[1])\left(\frac{2b\cos(bK[1])\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};\sin^2(bK[1])\right)K[1]}{m+1}-2^{-m-1}\sqrt{\pi}\,\text{Gamma}(m+1)\,{}_3\tilde{F}_2\left(1,\frac{m-}{2}\right.\right.}{2b^2}\right.}{K[1]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2 + a*x*sin(b*x)^m*y + a*sin(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{xy\left(\int e^{\int\frac{a\,x^2(\sin^m(bx))-2}{x}dx}dx\right)+x\,e^{\int\frac{a\,x^2(\sin^m(bx))-2}{x}dx}+\int e^{\int\frac{a\,x^2(\sin^m(bx))-2}{x}dx}dx}{xy+1}\right)$$

7.2.15.8 [648] problem number 8

problem number 648

Added January 14, 2019.

Problem 2.6.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(\lambda\sin(\lambda x)y^2+\lambda\sin^3(\lambda x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + lambda*Sin[lambda*x]^3)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(lambda*sin(lambda*x)*y^2 + lambda*sin(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sqrt{\pi}\,(y+\cos(\lambda x))}{\sqrt{\pi}\,y\,\mathrm{erfi}(\cos(\lambda x))+\sqrt{\pi}\,\mathrm{erfi}(\cos(\lambda x))\cos(\lambda x)-2\,\mathrm{e}^{\cos^2(\lambda x)}}\right)$$

7.2.15.9 [649] problem number 9

problem number 649

Added January 14, 2019.

Problem 2.6.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2w_x + \left((\lambda + a - a\sin(\lambda x))y^2 + \lambda - a - a\sin(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((lambda + a - a*Sin[lambda*x])*y^2 + lambda - a - a*Sin[lambda*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+((lambda+a-a*sin(lambda*x))*y^2 +lambda -a -a*sin(lambda*x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{((\cos(\lambda x)+1)(\cos(\lambda}{2(a\sin(\lambda x)-a-\lambda)^2(\sin(\lambda x)-1)(\cos(\lambda x)+1)(-a\sin(\lambda x)+(a+\lambda)\operatorname{csgn}(\sin(\lambda x}\right.$$

7.2.15.10 [650] problem number 10

problem number 650

Added January 14, 2019.

Problem 2.6.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((\lambda + a\sin^2(\lambda x))y^2 + \lambda - a + a\sin^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((lambda + a*Sin[lambda*x]^2)*y^2 + lambda - a + a*Sin[lambda*x]^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+((lambda+a*sin(lambda*x)^2)*y^2 + lambda -a +a*sin(lambda*x)^2)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{\cos(2\lambda x)+1}\,\left(-(a(\sin^2(\lambda x)\right.}{-4\sqrt{\cos(2\lambda x)-1}\,(a\cos(2\lambda x)-a-2\lambda)\,\lambda\,\mathrm{e}^{\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)+2\sqrt{\cos(2\lambda x)+1}\,\left(-\right.}\right.$$

7.2.15.11 [651] problem number 11

problem number 651

Added January 14, 2019.

Problem 2.6.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - a x^{k+1}(\sin x)^m y + a(\sin x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Sin[x]^m*y + a*Sin[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-((k+1)*x^k*y^2 - a*x^(k+1)*(sin(x))^m*y + a*(sin(x))^m)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x\,x^{k}\mathrm{e}^{\int \frac{a\,x^{2}x^{k}(\sin^{m}(x))-2k-2}{x}dx} + \left(-xy\,x^{k}+1\right)\left(\int -(k+1)\,x^{k}\mathrm{e}^{\int \frac{a\,x^{2}x^{k}(\sin^{m}(x))-2k-2}{x}dx}dx\right)}{xy\,x^{k}-1}\right)$$

7.2.15.12 [652] problem number 12

problem number 652

Added January 14, 2019.

Problem 2.6.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*x + mu)^k * (y-b*x^n -c)^2 + y - b*x^n + b*n*x^(n-1) - c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.15.13 [653] problem number 13

problem number 653

Added January 14, 2019.

Problem 2.6.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\sin^m(\lambda x)y^2 + ky + ab^2x^{2k}\sin^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Sin[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Sin[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*sin(lambda*x)^m*y^2 + k*y + a*b^2*x^(2*k)*sin(lambda*x)^m)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab\left(\int x^{k-1}(\sin^m(\lambda x))\,dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.15.14 [654] problem number 14

problem number 654

Added January 14, 2019.

Problem 2.6.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\sin(\lambda x)+b)w_x + \left(y^2 + c\sin(\mu x)y - k^2 + ck\sin(\mu x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Sin[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Sin[mu*x]*y - k^2 + c*k*Sin[mu*x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a *sin(lambda*x) + b)*diff(w(x,y),x)+(y^2+ c*sin(mu*x)* y - k^2 + c*k*sin(mu*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(k+y)\left(\int \frac{e^{\frac{\sqrt{-a^2+b^2}\,c\lambda\left(\int \frac{\sin(\mu x)}{a\sin(\lambda x)+b}dx\right)-4k\arctan\left(\frac{a\cos\left(\frac{\lambda x}{2}\right)+b\sin\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2+b^2}\cos\left(\frac{\lambda x}{2}\right)}\right)}{\sqrt{-a^2+b^2}\,\lambda}}}{a\sin(\lambda x)+b}dx\right)+e^{\frac{\sqrt{-a^2+b^2}\,c\lambda\left(\int \frac{\sin(\mu x)}{a\sin(\lambda x)+b}dx\right.}{\sqrt{}}}}{k+y}\right.$$

7.2.16 6.2

Local contents

7.2.16.1 [655] problem number 1

problem number 655

Added January 14, 2019.

Problem 2.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cos[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cos^2(\lambda x)\right)}{k\lambda+\lambda}-bx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*cos(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx+y-\left(\int a\left(\cos^k(\lambda x)\right)dx\right)\right)$$

Contains unresolved integral

7.2.16.2 [656] problem number 2

problem number 656

Added January 14, 2019.

Problem 2.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cos[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\cos^k(\lambda K[1])+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*cos(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x-\left(\int \frac{1}{a\left(\cos^k\left(\lambda y\right)\right)+b}dy\right)\right)$$

7.2.16.3 [657] problem number 3

problem number 657

Added January 14, 2019.

Problem 2.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cos^k(\lambda x)\cos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cos[lambda*y]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cos^{-k}(\lambda K[1])\cos^{-n}(\mu K[1])dK[1]-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cos(lambda*y)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ax-\left(\int\left(\cos^{-n}(\mu y)\right)\left(\cos^{-k}(\lambda y)\right)dy\right)}{a}\right)$$

7.2.16.4 [658] problem number 4

problem number 658

Added January 14, 2019.

Problem 2.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\cos^k(x+\lambda y)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cos[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cos(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{a\lambda\left(\cos^k\left(\_a\lambda\right)\right)+1} d\_a\right)+x\right)$$

7.2.16.5 [659] problem number 5

problem number 659

Added January 14, 2019.

Problem 2.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\cos(\lambda x) + a^2\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Cos[lambda*x] + a^2*Cos[lambda*x]^2)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2-a^2 + a *lambda*cos(lambda*x) + a^2*cos(lambda*x)^2)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(\frac{\lambda \operatorname{HeunCPrime}\left(\frac{4a}{\lambda},-\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\cos(\lambda x)}{2}+\frac{1}{2}\right)\sin(\lambda x)}{2} + (a\sin(\lambda x) -}{2\left(\cos(\lambda x)+1\right)\lambda \operatorname{HeunCPrime}\left(\frac{4a}{\lambda},\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\cos(\lambda x)}{2}+\frac{1}{2}\right)\sin(\lambda x) + (-4(\cos(}\right.$$

7.2.16.6 [660] problem number 6

problem number 660

Added January 14, 2019.

Problem 2.6.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \cos(\lambda x) y^2 + \lambda \cos^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Cos[lambda*x]*y^2 + lambda*Cos[lambda*x]^3)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(lambda*cos(lambda*x)*y^2 + lambda*cos(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(y-\sin(\lambda x))\,\mathrm{KummerM}\left(1, \frac{3}{2}, -(\sin^2(\lambda x))\right)\sin(\lambda x) - 1}{(y-\sin(\lambda x))\,\mathrm{KummerU}\left(1, \frac{3}{2}, -(\sin^2(\lambda x))\right)\sin(\lambda x) - 2}\right)$$

7.2.16.7 [661] problem number 7

problem number 661

Added January 14, 2019.

Problem 2.6.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2w_x + \left((\lambda + a + a\cos(\lambda x))y^2 + \lambda - a + a\cos(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((lambda + a + a*Cos[lambda*x])*y^2 + lambda - a + a*Cos[lambda*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+ ((lambda+a+a*cos(lambda*x))*y^2 +lambda - a + a *cos(lambda*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y\cos(\lambda x) + y - \sin(\lambda x))\sqrt{\cos(\lambda x) - 1}\,\sqrt{\cos(\lambda x) + 1}\,e^{\frac{a\cos(}{\lambda}}}{\left((y\cos(\lambda x) + y - \sin(\lambda x))\sqrt{\cos(\lambda x) - 1}\,\sqrt{\cos(\lambda x) + 1}\left(\int \frac{(a\cos(\lambda x)+a+\lambda)e^{-\frac{a\cos(\lambda x)}{\lambda}}\sin(\lambda}{(\cos(\lambda x)+1)^{\frac{3}{2}}\sqrt{\cos(\lambda x)-1}}\right.\right.}\right.$$

7.2.16.8 [662] problem number 8

problem number 662

Added January 14, 2019.

Problem 2.6.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((\lambda + a\cos^2(\lambda x))y^2 + \lambda - a + a\cos^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((lambda + a*Cos[lambda*x]^2)*y^2 + lambda - a + a*Cos[lambda*x]^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((lambda+a*cos(lambda*x)^2)*y^2 + lambda - a + a*cos(lambda*x)^2)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(\left(a(\cos^2(\lambda x)) + \lambda\right) y(\cos^4(\lambda x)) - \frac{(\cos(}{}\right.}{\left(4\left(\left(a\left(\cos^2(\lambda x)\right) + \lambda\right) y\left(\cos^4(\lambda x)\right) - \frac{(\cos(2\lambda x)+1)(a\cos(2\lambda x)+a+2\lambda)\sin(2\lambda x)}{8}\right)\sqrt{\cos(2\lambda x) - 1}\right.}\right.$$

7.2.16.9 [663] problem number 9

problem number 663

Added January 14, 2019.

Problem 2.6.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+b*x)*diff(w(x,y),x)+ cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\, a\left(\int y^m\left(\cos^{-k}(\lambda y)\right) e^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1} e^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right)}\right)$$

7.2.16.10 [664] problem number 10

problem number 664

Added January 14, 2019.

Problem 2.6.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cos^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cos^m(y))dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cos^m(y))dy\right)}\right)$$

7.2.16.11 [665] problem number 11

problem number 665

Added January 14, 2019.

Problem 2.6.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cos^m y)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\cos^{-k}(\lambda y)\right) e^{(n-1)b\left(\int (\cos^m(y))\left(\cos^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1} e^{(n-1)b\left(\int (\cos^m(y))\left(\cos^{-k}(\lambda\right.\right.}\right.$$

7.2.16.12 [666] problem number 12

problem number 666

Added January 14, 2019.

Problem 2.6.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \cos^m y + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Cos[y]^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*cos(y)^m+b*x)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int (\cos^m(y))\left(\cos^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right.}\right.$$

7.2.17 6.3

Local contents

7.2.17.1 [667] problem number 1

problem number 667

Added January 14, 2019.

Problem 2.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a + Tan[lambda*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-ax-bx+\frac{\log(\cos(\lambda x))}{\lambda}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a+tan(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{((-2a-2b)\,x+2y)\,\lambda-\ln\left(\tan^2(\lambda x)+1\right)}{2\lambda}\right)$$

7.2.17.2 [668] problem number 2

problem number 668

Added January 14, 2019.

Problem 2.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\tan^k(\lambda y)+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a + Tan[lambda*y] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-x+\frac{-i(a+b-i)\log(-\tan(\lambda y)+i)+i(a+b+i)\log(\tan(\lambda y)+i)+2\log(a+b+\text{t}}{2\lambda(a+b-i)(a+b+i)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a+tan(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(2a^2x + 4abx + 2b^2x + 2x)\lambda + (-2a-2b)\arctan(\tan(\lambda y)) + \ln(\tan^2(\lambda y)+1) - 2\ln(a}{2(a^2+2ab+b^2+1)\lambda}\right.$$

7.2.17.3 [669] problem number 3

problem number 669

Added January 14, 2019.

Problem 2.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tan^k(\lambda x)\tan^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tan[lambda*x]^k*Tan[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\tan^{1-n}(\mu y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(\mu y)\right)}{\mu-\mu n} - \frac{a\tan^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};-\tan^2(\lambda x\right.}{k\lambda+\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a *tan(lambda*x)^k * tan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \left(\tan^k(\lambda x)\right) dx\right) + \int \frac{\tan^{-n}(\mu y)}{a} dy\right)$$

Has unresolved integrals

7.2.17.4 [670] problem number 4

problem number 670

Added January 14, 2019.

Problem 2.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+ a *lambda + a*(lambda -a) *tan(lambda*x)^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2(\cos(2\lambda x)+1)\lambda \operatorname{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right) + \left(-4y(\cos^3(\lambda x)) - a\sin(\lambda x) - a\,\mathrm{s}\right.}{-2(\cos(2\lambda x)+1)\lambda \operatorname{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right) + \left(4y(\cos^3(\lambda x)) + a\sin(\lambda x) + a\,\mathrm{s}\right.}\right.$$

7.2.17.5 [671] problem number 5

problem number 671

Added January 14, 2019.

Problem 2.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda^2 + 3a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  y^2+ lambda^2 +3*a*lambda +a*(lambda-a)*tan(lambda*x)^2)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{4\lambda \operatorname{LegendreP}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right)(\cos^2(\lambda x)) - 2\left(\frac{\lambda(\sin^3(\lambda x))}{2} + \left(a + \frac{3\lambda}{2}\right)(\cos^2(\lambda x)) \mathrm{si}\right.}{-4\lambda \operatorname{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right)(\cos^2(\lambda x)) + 2\left(\frac{\lambda(\sin^3(\lambda x))}{2} + \left(a + \frac{3\lambda}{2}\right)(\cos^2(\lambda x)) \mathrm{s}\right.}\right.$$

7.2.17.6 [672] problem number 6

problem number 672

Added January 14, 2019.

Problem 2.6.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\tan^k(bx)y + a\tan^k(bx)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Tan[b*x]^k*y + a*Tan[b*x]^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\exp\left(-\int_1^x -aK[5]\tan^k(bK[5])dK[5]\right)}{x^2y+x}-\int_1^x\frac{\exp\left(-\int_1^{K[6]} -aK[5]\tan^k(bK[5])dK[5]\right)}{K[6]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  y^2+ a*x  *tan(b*x)^k * y + a*tan(b*x)^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{xy\left(\int \mathrm{e}^{\int \frac{a\,x^2\left(\tan^k(bx)\right)-2}{x}dx}dx\right)+x\,\mathrm{e}^{\int \frac{a\,x^2\left(\tan^k(bx)\right)-2}{x}dx}+\int \mathrm{e}^{\int \frac{a\,x^2\left(\tan^k(bx)\right)-2}{x}dx}dx}{xy+1}\right)$$

7.2.17.7 [673] problem number 7

problem number 673

Added January 14, 2019.

Problem 2.6.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x-\left((k+1)x^ky^2-ax^{k+1}(\tan x)^my+a(\tan x)^m\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Tan[x]^m*y + a*Tan[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2- a*x^(k+1)*tan(x)^m*y + a*tan(x)^m )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}\mathrm{e}^{\int \frac{a x\, x^{k+1}(\tan^m(x))-2k-2}{x}dx} + \left(y\, x^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}\mathrm{e}^{a\left(\int x^{k+1}\left(-\frac{i\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}\right)^m dx\right)}}{x^2}dx\right)}{y\, x^{k+1}-1}\right)$$

7.2.17.8 [674] problem number 8

problem number 674

Added January 20, 2019.

Problem 2.6.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^n(\lambda x)y^2 - ab^2\tan^{n+2}(\lambda x) + b\lambda\tan^2(\lambda x) + b\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Tan[lambda*x]^n*y^2 - a*b^2*Tan[lambda*x]^(n + 2) + b*lambda*Tan[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(a*tan(lambda*x)^n*y^2- a*b^2*tan(lambda*x)^(n+2) + b*lambda*tan(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.17.9 [675] problem number 9

problem number 675

Added January 20, 2019.

Problem 2.6.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Tan[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(a *tan(lambda*x+mu)^k*(y-b*x^n-c)^2 + y- b*x^n + b*n*x^(n-1)-c)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.17.10 [676] problem number 10

problem number 676

Added January 20, 2019.

Problem 2.6.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\tan^m(\lambda x)y^2 + ky + ab^2x^{2k}\tan^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Tan[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Tan[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ (a*tan(lambda*x)^m*y^2 +k*y+ a*b^2*x^(2*k)*tan(lambda*x)^m )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab\left(\int x^{k-1}(\tan^m(\lambda x))\,dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.17.11 [677] problem number 11

problem number 677

Added January 20, 2019.

Problem 2.6.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\tan(\lambda x)+b)w_x + \left(y^2 + c\tan(\mu x)y - k^2 + ck\tan(\mu x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Tan[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Tan[mu*x]*y - k^2 + c*k*Tan[mu*x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*tan(lambda*x)+b)*diff(w(x,y),x)+ (y^2+ c *tan(mu*x)*y - k^2 + c*k*tan(mu*x) )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left((k+y)\left(e^{2i\mu x}+1\right)^{\frac{ia^2c}{(ib+a)\left(a^2+b^2\right)\mu}}\left(e^{2i\mu x}+1\right)^{\frac{ib^2c}{(ib+a)\left(a^2+b^2\right)\mu}}\left(\int\frac{\left(\frac{a\sin(\lambda x)+b\cos(\lambda x)}{\cos(\lambda x)}\right)^{-\frac{2ak}{\left(a^2+b^2\right)\lambda}}\left(\frac{2}{\cos(2\lambda\right.\right.}{}\right.\right.}{}\right.$$

7.2.17.12 [678] problem number 12

problem number 678

Added January 20, 2019.

Problem 2.6.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^ny^m + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*y^m + b*x)*diff(w(x,y),x)+ tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m\left(\tan^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}\right)$$

7.2.17.13 [679] problem number 13

problem number 679

Added January 20, 2019.

Problem 2.6.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tan^m(y))dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tan^m(y))dy\right)}\right)$$

7.2.17.14 [680] problem number 14

problem number 680

Added January 20, 2019.

Problem 2.6.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\tan^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int (\tan^m(y))\left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\tan^m(y))\left(\tan^{-k}(\right.\right.}\right.$$

7.2.17.15 [681] problem number 15

problem number 681

Added January 20, 2019.

Problem 2.6.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \tan^m y + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Tan[y]^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*tan(y)^m+ b*x)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int (\tan^m(y))\left(\tan^{-k}(\lambda y)\right) e^{(n-1)b\left(\int \left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}e^{(n-1)b\left(\int \left(\tan^{-k}(\lambda y)\right)d\right.}\right.$$

7.2.18 6.4

Local contents

7.2.18.1 [682] problem number 1

problem number 682

Added January 20, 2019.

Problem 2.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cot^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cot[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\cot^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};-\cot^2(\lambda x)\right)}{k\lambda+\lambda}-bx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (a*cot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a\left(\cot^k(\lambda x)\right)dx\right)\right)$$

Has unresolved integral

7.2.18.2 [683] problem number 2

problem number 683

Added January 20, 2019.

Problem 2.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cot^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cot[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\cot^k(\lambda K[1])+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (a*cot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x-\left(\int \frac{1}{a\left(\cot^k\left(\lambda y\right)\right)+b}dy\right)\right)$$

7.2.18.3 [684] problem number 3

problem number 684

Added January 20, 2019.

Problem 2.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cot^k(x + \lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cot[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  cot(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{\lambda\left(\cot^k\left(\_a\lambda\right)\right)+1}d\_a\right)+x\right)$$

7.2.18.4 [685] problem number 4

problem number 685

Added January 20, 2019.

Problem 2.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\cot^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ ( y^2+a*lambda + a*(lambda-a)*cot(lambda*x)^2  )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2(\cos(2\lambda x)-1)\lambda \operatorname{LegendreP}\left(\frac{2a+\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)+(-a\cos(\lambda x)+a\cos(3\lambda x)+3y\,\mathrm{s}}{2(\cos(2\lambda x)-1)\lambda \operatorname{LegendreQ}\left(\frac{2a+\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)+(a\cos(\lambda x)-a\cos(3\lambda x)-3y\sin}\right.$$

7.2.18.5 [686] problem number 5

problem number 686

Added January 20, 2019.

Problem 2.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda^2 + 3a\lambda + a(\lambda - a)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (  y^2+lambda^2 + 3*a*lambda +a*(lambda-a)*cot(lambda*x)^2   )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{4\lambda \operatorname{LegendreP}\left(\frac{2a+3\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)(\sin^2(\lambda x)) - 2\left(\frac{\lambda(\cos^3(\lambda x))}{2} + y(\cos^2(\lambda x))\sin(\lambda x)\right.}{-4\lambda \operatorname{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)(\sin^2(\lambda x)) + 2\left(\frac{\lambda(\cos^3(\lambda x))}{2} + y(\cos^2(\lambda x))\sin(\lambda x)\right.}\right.$$

7.2.18.6 [687] problem number 6

problem number 687

Added January 20, 2019.

Problem 2.6.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - 2a\cot(ax)y + b^2 - a^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*a*Cot[a*x]*y + b^2 - a^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{y - a\cot(ax)}{\sqrt{b^2}}\right) - \sqrt{b^2}x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (  y^2-2*a*cot(a*x)*y + b^2-a^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-ia\cot(ax) + b + iy)\,\mathrm{e}^{-2ibx}}{2(-a\cot(ax) + ib + y)\,b}\right)$$

7.2.18.7 [688] problem number 7

problem number 688

Added January 20, 2019.

Problem 2.6.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)w_x + a\cot(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*D[w[x, y], x] + a*Cot[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\cos(\mu y)\cos^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(lambda*x)*diff(w(x,y),x)+ a*cot(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\mu\ln\left(\frac{\cot^2(\lambda x)+1}{\cot(\lambda x)^2}\right)+\lambda\ln\left(\cos^2(\mu y)\right)}{2\lambda\mu}\right)$$

7.2.18.8 [689] problem number 8

problem number 689

Added January 20, 2019.

Problem 2.6.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\mu y)w_x + a\cot(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sin(\mu y)\sin^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\frac{\sqrt{(\tan^2(\mu y)+1)\left(-\frac{2}{\cos(2\lambda x)-1}\right)^{\frac{a\mu}{\lambda}}}\tan(\mu y)}{\tan^2(\mu y)+1}\right)}{a\mu}\right)$$

7.2.18.9 [690] problem number 9

problem number 690

Added January 20, 2019.

Problem 2.6.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\mu y)w_x + a\cot^2(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4\sin(\mu y)e^{\frac{a\mu(\lambda x+\cot(\lambda x))}{\lambda}}}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\lambda\ln\left(\frac{\tan^2(\mu y)}{\tan^2(\mu y)+1}\right)\sin(\lambda x)+2\left(\operatorname{arccot}\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)\sin(\lambda x)+\cos(\lambda x)-\frac{\pi\sin(\lambda x)}{2}\right)a\mu}{2a\lambda\mu\sin(\lambda x)}\right)$$

7.2.18.10 [691] problem number 10

problem number 691

Added January 20, 2019.

Problem 2.6.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(y+a)w_x+c\cot(x+b)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[y + a]*D[w[x, y], x] + c*Cot[x + b]^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(4\sin(a+y)e^{c(\cot(b+x)+b+x)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(y+a)*diff(w(x,y),x)+ c*cot(x+b)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\tan(b)+\tan(x))\ln\left(\frac{1}{\sin(y)^2}\right)\tan(b)-2(\tan(b)+\tan(x))\ln\left(\frac{\cos(y)\tan(a)+\sin(y)}{\sin(y)\tan(a)}\right)\tan(b)}{2(\tan(b)+\tan(x))\,\mathrm{ta}}\right.$$

7.2.18.11 [692] problem number 11

problem number 692

Added January 20, 2019.

Problem 2.6.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x + aw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sin(\mu y)\cos^{\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\frac{\sqrt{(\tan^2(\mu y)+1)\left(\cos^{\frac{2a\mu}{\lambda}}(\lambda x)\right)}\,\tan(\mu y)}{\tan^2(\mu y)+1}\right)}{a\mu}\right)$$

7.2.18.12 [693] problem number 12

problem number 693

Added January 20, 2019.

Problem 2.6.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x + a\cot(vx)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*Cot[v*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*cot(v*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\left(e^{2ivx}-1\right)^{\frac{ia\mu}{v}} \operatorname{csgn}\left(\frac{1}{\cos(\mu y)}\right) e^{a\mu x} e^{\int -\frac{\left(-2\,e^{2ivx}-2\right)a\mu}{\left(e^{2i\lambda x}+1\right)\left(e^{2ivx}-1\right)}dx} \sin(\mu y)\right)}{a\mu}\right)$$

7.2.19 6.5

Local contents

7.2.19.1 [694] problem number 1

problem number 694

Added January 20, 2019.

Problem 2.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(\lambda x)\cos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[lambda*x]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{\sin^2(\mu y)}\csc(\mu y)\cos^{1-n}(\mu y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\mu y)\right)}{\mu(n-1)} - \frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}}{k}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ a*sin(lambda*x)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int\left(\sin^k(\lambda x)\right)dx\right) + \int\frac{\cos^{-n}(\mu y)}{a}dy\right)$$

Has unresolved integrals

7.2.19.2 [695] problem number 2

problem number 695

Added January 20, 2019.

Problem 2.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - y\tan x + a(1-a)\cot^2 x\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - y*Tan[x] + a*(1 - a)*Cot[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\left(-\sin^2(x)\right)^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}}\left(i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}\cos(x)+2y\sin(x)+\cos(x)\right)}{-i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}\cos(x)+2y\sin(x)+\cos(x)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-y *tan(x)+a*(1-a)*cot(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a\cos(x)+y\sin(x))\left(\sin^{2a-1}(x)\right)}{a\cos(x)-y\sin(x)-\cos(x)}\right)$$

7.2.19.3 [696] problem number 3

problem number 696

Added January 20, 2019.

Problem 2.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - my\tan x + b^2\cos^{2m}x\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - m*y*Tan[x] + b^2*Cos[x]^(2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(x)\right)}{m+1}+\tan^{-1}\left(\frac{y\cos^{-m}(x)}{\sqrt{b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-m*y*tan(x)+b^2*cos(x)^(2*m) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{3\left(\text{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\sin^2(x)\right)+\frac{(\cos(x)-1)(\cos(x)+1)(m-1)\,\text{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}+\frac{3}{2}\right],\left[\frac{5}{2}\right]\right.}{3}}{3\left(\text{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\sin^2(x)\right)+\frac{(\cos(x)-1)(\cos(x)+1)(m-1)\,\text{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}+\frac{3}{2}\right],\left[\frac{5}{2}\right]\right.}{3}}\right.$$

Mathematica answer is simpler

7.2.19.4 [697] problem number 4

problem number 697

Added January 20, 2019.

Problem 2.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+my\cot x+b^2\sin^m x\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + m*y*Cot[x] + b^2*Sin[x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   diff(w(x,y),x)+ (y^2+m*y*cot(x)+b^2*sin(x)^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.19.5 [698] problem number 5

problem number 698

Added January 20, 2019.

Problem 2.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - 2\lambda^2\tan^2(\lambda x) - 2\lambda^2\cot^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*lambda^2*Tan[lambda*x]^2 - 2*lambda^2*Cot[lambda*x]^2)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-2*lambda^2*tan(lambda*x)^2-2*lambda^2*cot(lambda*x)^2)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{}{8\sqrt{2\cos(2\lambda x)-2}\,\lambda\cos(2\lambda x)\ln\left(\cos(\lambda x)+\sqrt{\cos^2(\lambda x)-1}\right)-4\sqrt{2\cos(2\lambda x)-2}\,y\,\mathrm{l}\right.$$

7.2.19.6 [699] problem number 6

problem number 699

Added January 20, 2019.

Problem 2.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) + 2ab + a(\lambda-a)\tan^2(\lambda x) + b(\lambda-b)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*(a + b) + 2*a*b + a*(lambda - a)*Tan[lambda*x]^2 + b*(l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ ( y^2+lambda*(a+b)+2*a*b+a*(lambda -a)*tan(lambda*x)^2+ b*(lambda -
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(a-\frac{3\lambda}{2}\right)\left(a(\sin^2(\lambda x)) - b(\cos^2(\lambda x)) -\right.}{-4\left(a+b-\lambda\right)\lambda\,\text{hypergeom}\left(\left[2, \frac{-a-b+2\lambda}{\lambda}\right], \left[-\frac{2a-5\lambda}{2\lambda}\right], \cos^2(\lambda x)\right)\left(\cos^2(\lambda x)\right)\left(\sin^2(\lambda x)\right)+\right.}\right.$$

7.2.19.7 [700] problem number 7

problem number 700

Added January 20, 2019.

Problem 2.6.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x)y^2 + a\cos^n(\lambda x)y - a\cos^{n-1}(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Cos[lambda*x]^n*y - a*Cos[lambda*x]^(n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)* y^2 + a*cos(lambda*x)^n*y-a*cos(lambda*x)^(n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.19.8 [701] problem number 8

problem number 701

Added January 20, 2019.

Problem 2.6.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(y \cos(\lambda x) - 1) \,\mathrm{e}^{\frac{a \cos(\lambda x)}{\lambda}}}{(y \cos(\lambda x) - 1)\, a \,\mathrm{expIntegral}\left(1, \frac{a \cos(\lambda x)}{\lambda}\right) \mathrm{e}^{\frac{a \cos(\lambda x)}{\lambda}} - \lambda y}\right)$$

7.2.19.9 [702] problem number 9

problem number 702

Added January 20, 2019.

Problem 2.6.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(y\cos(\lambda x)-1)\,\mathrm{e}^{\frac{a\cos(\lambda x)}{\lambda}}}{(y\cos(\lambda x)-1)\,a\,\mathrm{expIntegral}\left(1,\frac{a\cos(\lambda x)}{\lambda}\right)\mathrm{e}^{\frac{a\cos(\lambda x)}{\lambda}}-\lambda y}\right)$$

7.2.19.10 [703] problem number 10

problem number 703

Added January 20, 2019.

Problem 2.6.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(Ae^{\lambda x}\cos(ay) + Be^{\mu x}\sin(ay) + Ae^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (A*Exp[lambda*x]*Cos[a*y] + B*Exp[mu*x]*Sin[a*y] + A*Exp[lambda*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (A*exp(lambda*x)*cos(a*y) + B*exp(mu*x)*sin(a*y) + A*exp(lambda*x))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\cos(ay)+1)\,Aa\left(\int \mathrm{e}^{\frac{-Ba\,\mathrm{e}^{\mu x}+\lambda\mu x}{\mu}}dx\right) - \mathrm{e}^{-\frac{Ba\,\mathrm{e}^{\mu x}}{\mu}}\sin(ay)}{2(-\lambda+\mu)\,a\cos\left(\frac{ay}{2}\right)^2}\right)$$

7.2.19.11 [704] problem number 11

problem number 704

Added January 20, 2019.

Problem 2.6.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin^{n+1}(2x)w_x + \left(ay^2\sin^{2n}x + b\cos^{2n}x\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Sin[2*x]^(n + 1)*D[w[x, y], x] + (a*y^2*Sin[x]^(2*n) + b*Cos[x]^(2*n))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=   sin(2*x)^(n+1)*diff(w(x,y),x)+ (a*y^2*sin(x)^(2*n) + b*cos(x)^(2*n) )*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\left((-3\cos(3x)+\cos(5x))\left(\cos^{\sqrt{-ab4^{-n}+n^2}}(x)\right)+2\left(\cos^{\sqrt{-ab4^{-n}+n^2}+1}(x)\right)\right)ay\left(\sin^{2n-\sqrt{-}}\right.}{-2n\left(\sin^{n}(2x)\right)\sin(x)\sin(4x)+(2\cos(x)-3\cos(3x)+\cos(5x))\,a}\right.$$

7.2.20 7.1

Local contents

7.2.20.1 [705] problem number 1

problem number 705

Added January 20, 2019.

Problem 2.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arcsin^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcSin[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{ia \sin^{-1}(\lambda x)^k \left(\sin^{-1}(\lambda x)^2\right)^{-k} \left(\left(i \sin^{-1}(\lambda x)\right)^k \text{Gamma}\left(k+1, -i \sin^{-1}(\lambda x)\right) - \left(-i \sin^{-1}\right.\right.}{2\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-\arcsin\left(\lambda x\right)^{k}\arcsin\left(\lambda x\right)^{\frac{3}{2}} + \mathrm{LommelS1}\left(k+\frac{3}{2},\frac{1}{2},\arcsin\left(\lambda x\right)\right)\arcsin\left(\lambda x\right)\right)\sqrt{-\lambda^{2}x^{2}+}}{}\right.$$

7.2.20.2 [706] problem number 2

problem number 706

Added January 20, 2019.

Problem 2.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\arcsin^k(\lambda y) + b\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcSin[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\sin^{-1}(\lambda K[1])^k+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a \arcsin(\lambda y)^k + b} dy\right)\right)$$

7.2.20.3 [707] problem number 3

problem number 707

Added January 20, 2019.

Problem 2.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k \arcsin^n(ax + by + c) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*Arcsin[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ k*arcsin(a*x + b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk \arcsin(b\_a + c)^n + a} d\_a\right) + x\right)$$

7.2.20.4 [708] problem number 4

problem number 708

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a \arcsin^k(\lambda x) \arcsin^n(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Arcsin[lambda*x]^k*Arcsin[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \mathrm{Arcsin}(\mu K[1])^{-n}dK[1]-\int_1^x a\mathrm{Arcsin}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arcsin(lambda*x)^k*arcsin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(\lambda x-1)(\lambda x+1)\left((n-1)\left(\arcsin(\lambda x)^k-\frac{\mathrm{LommelS1}\left(k+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)\sqrt{-\lambda^2x^2+1}\,a\mu 2^k\right.}{}\right.$$

7.2.20.5 [709] problem number 5

problem number 709

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arcsin x)^n y - a^2 + a\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*Arcsin[x]^n*y - a^2 + a*lambda*Arcsin[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+ lambda*arcsin(x)^n*y -a^2 + a *lambda*arcsin(x)^n)*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int \mathrm{e}^{-\frac{2(x+1)(x-1)\left(\frac{\left(-\arcsin(x)^n+\frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(x)\right)}{\sqrt{\arcsin(x)}}\right)\sqrt{-x^2+1}\,\lambda 2^n 2^{-n}\arcsin(x)}{2}+\left(-\frac{\lambda n 2^n 2^{-n}\,\mathrm{Lor}\cdots}{\cdots}\right.\cdots}{\cdots}}\cdots\right.}{\cdots}\right.$$

7.2.20.6 [710] problem number 6

problem number 710

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arcsin x)^n y + \lambda(\arcsin y)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*Arcsin[x]^n*y + lambda*Arcsin[x]^n)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda \text{Arcsin}(K[5])^n K[5] dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda \text{Arcsin}(K[5])^n K[5] dK\right.}{K[6]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+ lambda*x*arcsin(x)^n*y + lambda*arcsin(x)^n)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int \mathrm{e}^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x} dx} dx\right) + x\,\mathrm{e}^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x} dx} + \int \mathrm{e}^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x} dx} dx}{xy+1}\right)$$

7.2.20.7 [711] problem number 7

problem number 711

Added January 20, 2019.

Problem 2.7.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arcsin x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*Arcsin[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2 - lambda*arcsin(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1} e^{\int \frac{\lambda x\, x^{k+1} \arcsin(x)^n - 2k - 2}{x} dx} + \left(y\, x^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k} e^{\lambda\left(\int x^{k+1} \arcsin(x)^n dx\right)}}{x^2} dx\right)}{y\, x^{k+1} - 1}\right)$$

7.2.20.8 [712] problem number 8

problem number 712

Added January 20, 2019.

Problem 2.7.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + ay + ab - b^2\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 + a*y + a*b - b^2*lambda*Arcsin[x]^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + a*y+ a*b -b^2 * lambda*arcsin(x)^n)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(b+y)\,\lambda\left(\int \arcsin(x)^n \mathrm{e}^{\frac{(x+1)\left(-2\left(\arcsin(x)^n - \frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(x)\right)}{\sqrt{\arcsin(x)}}\right)\sqrt{-x^2+1}\,b\lambda 2^n 2^{-n}\arcsin(x) + \left(-2b\lambda\right.}{}}\right.}{}\right.$$

7.2.20.9 [713] problem number 9

problem number 713

Added January 29, 2019.

Problem 2.7.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 - b\lambda x^m (\arcsin x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 - b*lambda*x^m*ArcSin[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 - b*lambda*x^m*arcsin(x)^n*y+b*m*x^(m-1) )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.20.10 [714] problem number 10

problem number 714

Added January 29, 2019.

Problem 2.7.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m}(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcSin[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + b*m*x^(m-1) - lambda*b^2*x^(2*m)*arcsin(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.20.11 [715] problem number 11

problem number 715

Added January 29, 2019.

Problem 2.7.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arcsin x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcSin[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{1}{2}i\lambda\left(i\sin^{-1}(x)\right)^n \sin^{-1}(x)^n \left(\sin^{-1}(x)^2\right)^{-n} \text{Gamma}\left(n+1, -i\sin^{-1}(x)\right) + \frac{1}{2}i\lambda\left(-i\sin^{-1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*(y - a*x^m -b)^2 + a*m*x^(m-1)  )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{-(a\,x^m + b - y)\,\lambda n x 2^n 2^{-n}\,\text{LommelS1}\left(n+\frac{1}{2}, \frac{3}{2}, \arcsin(x)\right)\arcsin(x) - (a\,x^m + b - y)\right.$$

7.2.20.12 [716] problem number 12

problem number 716

Added January 29, 2019.

Problem 2.7.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arcsin x)^n y^2 + ky + \lambda b^2 x^{2k}(\arcsin x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcSin[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda\sin^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 +k*y+ lambda*b^2*x^(2*k)*arcsin(x)^n  )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(b\lambda\left(\int x^{k-1}\arcsin(x)^n\,dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.21 7.2

Local contents

7.2.21.1 [717] problem number 1

problem number 717

Added January 29, 2019.

Problem 2.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\arccos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcCos[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{a\cos^{-1}(\lambda x)^k\left(-i\cos^{-1}(\lambda x)\right)^{-k}\text{Gamma}\left(k+1,-i\cos^{-1}(\lambda x)\right)+a\left(i\cos^{-1}(\lambda x)\right)^{-k}\cos^-}{2\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arccos(lambda*x)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx+\frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-k}\,\text{LommelS1}\left(k+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(k+2)}+\frac{\sqrt{-\lambda^2x^2+1}\,2^{-k}\arccos(\lambda x)^{k+1}}{\sqrt{\pi}\,(k+2)}-\right.}{\lambda}\right.$$

7.2.21.2 [718] problem number 2

problem number 718

Added January 29, 2019.

Problem 2.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arccos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcCos[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a\cos^{-1}(\lambda K[1])^k + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arccos(lambda*y)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\arccos\left(\lambda y\right)^k + b} dy\right)\right)$$

7.2.21.3 [719] problem number 3

problem number 719

Added January 29, 2019.

Problem 2.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\arccos^n(ax + by + c) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcCos[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k*arccos(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\arccos\left(b\_a+c\right)^{n}+a}d\_a\right)+x\right)$$

7.2.21.4 [720] problem number 4

problem number 720

Added January 29, 2019.

Problem 2.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arccos^k(\lambda x)\arccos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcCos[lambda*x]^k*ArcCos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(\cos^{-1}(\lambda x)^2)^{-k}\left(-a(i\cos^{-1}(\lambda x))^k\cos^{-1}(\lambda x)^k\,\text{Gamma}\left(k+1,-i\cos^{-1}(\lambda x)\right)-a(-i\,\text{co}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arccos(lambda*x)^k*arccos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(n-2)\left(-\frac{2(k+2)2^{-k-1}\,\mathrm{LommelS1}\left(k+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)}{\sqrt{\arccos(\lambda x)}} + \left(\mathrm{LommelS1}\left(k+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\mathrm{a}\right.\right.}{}\right.$$

7.2.21.5 [721] problem number 5

problem number 721

Added January 29, 2019.

Problem 2.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arccos x)^n y - a^2 + a\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcCos[x]^n*y - a^2 + a*lambda*ArcCos[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+lambda*arccos(x)^n*y- a^2 + a*lambda*arccos(x)^n )*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int \frac{\left(-\left(-(n+2)n\,\mathrm{LommelS1}\left(n-\frac{1}{2},\frac{1}{2},\arccos(x)\right)\arccos(x)^3+\arccos(x)^n\arccos(x)^{\frac{7}{2}}-\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\mathrm{a}\right.\right.\right.}{}\right.}{}\right.$$

7.2.21.6 [722] problem number 6

problem number 722

Added January 29, 2019.

Problem 2.7.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arccos x)^n y + \lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*ArcCos[x]^n*y + a*lambda*ArcCos[x]^n)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( y^2+lambda*x*arccos(x)^n*y + a*lambda*arccos(x)^n )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.21.7 [723] problem number 7

problem number 723

Added January 29, 2019.

Problem 2.7.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arccos x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCos[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2 -lambda*arccos(x)^n*(x^(k+1)*y-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}e^{\int \frac{\lambda x\, x^{k+1} \arccos(x)^n - 2k - 2}{x} dx} + \left(y\, x^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k} e^{\lambda\left(\int x^{k+1} \arccos(x)^n dx\right)}}{x^2} dx\right)}{y\, x^{k+1} - 1}\right)$$

7.2.21.8 [724] problem number 8

problem number 724

Added January 29, 2019.

Problem 2.7.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + ay + ab - b^2\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCos[x]^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ a*y+ a*b - b^2*lambda*arccos(x)^n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-b-y)\left(\int \frac{\left(2\left(-(n+2)n\,\mathrm{LommelS1}\left(n-\frac{1}{2},\frac{1}{2},\arccos(x)\right)\arccos(x)^2-\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x)^2+(n+\right.\right.}{}\right.}{}\right.$$

7.2.21.9 [725] problem number 9

problem number 725

Added January 29, 2019.

Problem 2.7.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 - b\lambda x^m (\arccos x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 - b*lambda*x^m*ArcCos[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2- b*lambda*x^m*arccos(x)^n*y + b*m*x^(m-1) )*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.21.10 [726] problem number 10

problem number 726

Added January 29, 2019.

Problem 2.7.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCos[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccos(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.21.11 [727] problem number 11

problem number 727

Added January 29, 2019.

Problem 2.7.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n(y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{1}{2}\left(\lambda\left(i\cos^{-1}(x)\right)^n \cos^{-1}(x)^n \left(\cos^{-1}(x)^2\right)^{-n} \text{Gamma}\left(n+1, -i\cos^{-1}(x)\right) + \lambda\left(-i\cos^{-1}(x\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*(y- a*x^m-b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(\arccos(x)^n \arccos(x)^{\frac{3}{2}} - \mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x) + (n+2)\,\mathrm{Lomm}\right.}{}\right.$$

7.2.21.12 [728] problem number 12

problem number 728

Added January 29, 2019.

Problem 2.7.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arccos x)^n y^2 + ky + \lambda b^2 x^{2k}(\arccos x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCos[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda\cos^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ k*y + lambda*b^2*x^(2*k)*arccos(x)^n )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\arccos(x)^n\,dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.22 7.3

Local contents

7.2.22.1 [729] problem number 1

problem number 729

Added January 29, 2019.

Problem 2.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arctan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcTan[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \left(a \tan^{-1}(\lambda K[1])^k+b\right) dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arctan(lambda*x)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a \arctan(\lambda x)^k \, dx\right)\right)$$

7.2.22.2 [730] problem number 2

problem number 730

Added January 29, 2019.

Problem 2.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arctan^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcTan[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a \tan^{-1}(\lambda K[1])^k + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arctan(lambda*y)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a \arctan(\lambda y)^k + b} dy\right)\right)$$

7.2.22.3 [731] problem number 3

problem number 731

Added January 29, 2019.

Problem 2.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\arctan^n(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcTan[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k *arctan(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\arctan\left(\_ab+c\right)^n + a}d\_a\right) + x\right)$$

7.2.22.4 [732] problem number 4

problem number 732

Added January 29, 2019.

Problem 2.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arctan^k(\lambda x)\arctan^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcTan[lambda*x]^k*ArcTan[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \tan^{-1}(\mu K[1])^{-n}dK[1]-\int_1^x a\tan^{-1}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arctan(lambda*x)^k*arctan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\left(\int \arctan(\lambda x)^k\,dx\right)+\int \frac{\arctan(\mu y)^{-n}}{a}dy\right)$$

7.2.22.5 [733] problem number 5

problem number 733

Added January 29, 2019.

Problem 2.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+\lambda(\arctan x)^n y-a^2+a\lambda(\arctan x)^n\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcTan[x]^n*y - a^2 + a*lambda*ArcTan[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 + lambda*arctan(x)^n*y -a^2 + a *lambda*arctan(x)^n )*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int \mathrm{e}^{-\left(\int(-\lambda\arctan(x)^n+2a)dx\right)}dx\right)-\mathrm{e}^{-\left(\int(-\lambda\arctan(x)^n+2a)dx\right)}}{a+y}\right)$$

7.2.22.6 [734] problem number 6

problem number 734

Added January 29, 2019.

Problem 2.7.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arctan x)^n y + \lambda(\arctan x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*ArcTan[x]^n*y + lambda*ArcTan[x]^n)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda\tan^{-1}(K[5])^n K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda\tan^{-1}(K[5])^n K[5]dK\right.}{K[6]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 + lambda*x*arctan(x)^n*y + lambda*arctan(x)^n )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x}dx}dx\right) + x\,e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x}dx} + \int e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x}dx}dx}{yx+1}\right)$$

7.2.22.7 [735] problem number 7

problem number 735

Added Feb. 1, 2019.

Problem 2.7.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arctan x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcTan[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2 - lambda*arctan(x)^n*(x^(k+1)*y-1)  )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}\mathrm{e}^{\int \frac{\lambda x\, x^{k+1} \arctan(x)^n - 2k - 2}{x} dx} + \left(y\, x^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k} \mathrm{e}^{\lambda\left(\int x^{k+1} \arctan(x)^n dx\right)}}{x^2} dx\right)}{y\, x^{k+1} - 1}\right)$$

7.2.22.8 [736] problem number 8

problem number 736

Added Feb. 1, 2019.

Problem 2.7.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arctan x)^n + ay + ab - b^2\lambda(\arctan x)^n n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n + a*y + a*b - b^2*lambda*ArcTan[x]^n*n)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]}\left(\left(\lambda - b^2\lambda n\right)\tan^{-1}(K[1])^n + ab\right) dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda* arctan(x)^n +a*y+ a*b - b^2*lambda*arctan(x)^n*n  )*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} + \int -\left(ab + \left(-b^2n + 1\right)\lambda\arctan\left(x\right)^n\right)\mathrm{e}^{-ax}dx\right)$$

7.2.22.9 [737] problem number 9

problem number 737

Added Feb. 1, 2019.

Problem 2.7.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(arctanx)^n y^2 - b\lambda x^m(\arctan x)^n y + bmx^{m-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 - b*lambda*x^m*ArcTan[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 - b*lambda*x^m*arctan(x)^n*y+ b*m*x^(m-1))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.22.10 [738] problem number 10

problem number 738

Added Feb. 1, 2019.

Problem 2.7.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(arctanx)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcTan[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 +b*m*x^(m-1) - lambda*b^2*x^(2*m)*arctan(x)^n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.22.11 [739] problem number 11

problem number 739

Added Feb. 1, 2019.

Problem 2.7.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(arctanx)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^x \lambda \tan^{-1}(K[2])^n dK[2]-\frac{1}{ax^m+b-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*(y-a*x^m -b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(a\,x^m+b-y)\left(\int \lambda \arctan(x)^n\,dx\right)-1}{a\,x^m+b-y}\right)$$

7.2.22.12 [740] problem number 12

problem number 740

Added Feb. 1, 2019.

Problem 2.7.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(\lambda(\arctan x)^n y^2+ky+\lambda b^2x^{2k}(\arctan x)^n\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcTan[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda \tan^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+(lambda*arctan(x)^n*y^2+k*y+lambda*b^2*x^(2*k)*arctan(x)^n  )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\arctan(x)^n\,dx\right) - \arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.23 7.4

Local contents

7.2.23.1 [741] problem number 1

problem number 741

Added Feb. 1, 2019.

Problem 2.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\,\mathrm{arccot}^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCot[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \left(\lambda\cot^{-1}(\lambda K[1])^k+b\right)dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-bx+y-\left(\int\lambda\left(-\arctan\left(\lambda x\right)+\frac{\pi}{2}\right)^k dx\right)\right)$$

7.2.23.2 [742] problem number 2

problem number 742

Added Feb. 1, 2019.

Problem 2.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\operatorname{arccot}^k(\lambda y)+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCot[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{\lambda\cot^{-1}(\lambda K[1])^k+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{\lambda \left(-\arctan\left(\lambda y\right) + \frac{\pi}{2}\right)^{k} + b}dy\right)\right)$$

7.2.23.3 [743] problem number 3

problem number 743

Added Feb. 1, 2019.

Problem 2.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\operatorname{arccot}^n(ax + by + c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcCot[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k*arccot(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\left(-\arctan\left(\_ab + c\right) + \frac{\pi}{2}\right)^{n} + a}d\_a\right) + x\right)$$

7.2.23.4 [744] problem number 4

problem number 744

Added Feb. 1, 2019.

Problem 2.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k \operatorname{arccot}^k(\lambda x) \operatorname{arccot}^n(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcCot[lambda*x]^k*ArcCot[lambda*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \cot^{-1}(\lambda K[1])^{-n} dK[1] - \int_1^x a \cot^{-1}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arccot(lambda*x)^k*arccot(lambda*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \left(-\arctan(\lambda x) + \frac{\pi}{2}\right)^k dx\right) + \int \frac{\left(-\arctan(\lambda y) + \frac{\pi}{2}\right)^{-n}}{a} dy\right)$$

7.2.23.5 [745] problem number 5

problem number 745

Added Feb. 1, 2019.

Problem 2.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\operatorname{arccot} x)^n y - a^2 + a\lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcCot[x]^n*y - a^2 + a*lambda*ArcCot[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+lambda*arccot(x)^n*y - a^2 +a*lambda*arccot(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int \mathrm{e}^{-\left(\int\left(-\lambda\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}+2a\right)dx\right)}dx\right)-\mathrm{e}^{-\left(\int\left(-\lambda\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}+2a\right)dx\right)}}{a+y}\right)$$

7.2.23.6 [746] problem number 6

problem number 746

Added Feb. 1, 2019.

Problem 2.7.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\operatorname{arccot} x)^n y + \lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (y^2 + lambda*x*ArcCot[x]^n*y + lambda*ArcCot[x]^n)*D[w[x, y], y] == 0
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda\cot^{-1}(K[5])^n K[5]dK[5]\right)}{x^2y+x}-\int_1^x\frac{\exp\left(-\int_1^{K[6]} -\lambda\cot^{-1}(K[5])^n K[5]dK\right.}{K[6]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+lambda*x*arccot(x)^n*y +lambda*arccot(x)^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} dx\right) + x\, e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} + \int e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} dx}{yx + 1}\right)$$

7.2.23.7 [747] problem number 7

problem number 747

Added Feb. 1, 2019.

Problem 2.7.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\operatorname{arccot} x)^n (x^{k+1} y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCot[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y]
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2- lambda*arccot(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}\mathrm{e}^{\int \frac{\lambda x\, x^{k+1}\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}-2k-2}{x}dx} + \left(y\, x^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}\mathrm{e}^{\lambda\left(\int x^{k+1}\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}dx\right)}}{x^{2}}d\right.}{y\, x^{k+1}-1}\right.$$

7.2.23.8 [748] problem number 8

problem number 748

Added Feb. 1, 2019.

Problem 2.7.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + ay + ab - b^2\lambda(\operatorname{arccot} x)^n n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCot[x]^n*n)*D[w[x,
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2+a*y + a*b -b^2*lambda*arccot(x)^n*n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.23.9 [749] problem number 9

problem number 749

Added Feb. 1, 2019.

Problem 2.7.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 - b\lambda x^m (\operatorname{arccot} x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 - b*lambda*x^m*ArcCot[x]^n*y + b*m*x^(m - 1))*
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2- b*lambda*x^m*arccot(x)^n*y+ b*m*x^(m-1)  )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.23.10 [750] problem number 10

problem number 750

Added Feb. 1, 2019.

Problem 2.7.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m}(\operatorname{arccot} x^n)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCot[x]
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccot(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.2.23.11 [751] problem number 11

problem number 751

Added Feb. 1, 2019.

Problem 2.7.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y] =
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x \lambda \cot^{-1}(K[2])^n dK[2] - \frac{1}{ax^m + b - y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*(y-a*x^m-b)^2+a*m*x^(m-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a\,x^m+b-y)\left(\int \lambda \mathrm{arccot}(x)^n\,dx\right)-1}{a\,x^m+b-y}\right)$$

7.2.23.12 [752] problem number 12

problem number 752

Added Feb. 1, 2019.

Problem 2.7.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + ky + \lambda b^2 x^{2k}(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCot[x]^n)*D[w[
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda\cot^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ k*y+ lambda*b^2*x^(2*k)*arccot(x)^n )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\left(-\arctan(x)+\frac{\pi}{2}\right)^n dx\right)-\arctan\left(\frac{y\,x^{-k}}{b}\right)\right)$$

7.2.24 8.1

Local contents

7.2.24.1 [753] problem number 1

problem number 753

Added Feb. 4, 2019.

Problem 2.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (f(x)y + g(x))\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (f[x]*y + g[x])*D[w[x, y], y] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right)-\int_1^x\exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right)g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y+g(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\left(\int f(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f(x)dx\right)} g(x)\,dx\right)\right)$$

7.2.24.2 [754] problem number 2

problem number 754

Added Feb. 4, 2019.

Problem 2.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y + g(x)y^k\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y + g[x]*y^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2] + y^{1-k}\exp\left((k-1)\int_1^x f\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y+g(x)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f(x)dx\right)} g(x)\,dx\right)\right)$$

7.2.24.3 [755] problem number 3

problem number 755

Added Feb. 4, 2019.

Problem 2.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + f(x)y - a^2 - af(x))\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + f[x]*y - a^2 - a*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+f(x)*y -a^2 -a*f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a-y)\left(\int \mathrm{e}^{2ax+\int f(x)dx}dx\right) - \mathrm{e}^{2ax+\int f(x)dx}}{a-y}\right)$$

7.2.24.4 [756] problem number 4

problem number 756

Added Feb. 4, 2019.

Problem 2.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + xf(x)y + f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + x*f[x]*y + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\exp\left(-\int_1^x -f(K[5])K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -f(K[5])K[5]dK[5]\right)}{K[6]^2}dK[6]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+x*f(x)*y + f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int \mathrm{e}^{\int \frac{x^2f(x)-2}{x}dx}dx\right) + x\,\mathrm{e}^{\int \frac{x^2f(x)-2}{x}dx} + \int \mathrm{e}^{\int \frac{x^2f(x)-2}{x}dx}dx}{yx+1}\right)$$

7.2.24.5 [757] problem number 5

problem number 757

Added Feb. 4, 2019.

Problem 2.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^ky^2 - x^{k+1}f(x)y + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - x^(k + 1)*f[x]*y + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2-x^(k+1)*f(x)*y+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}e^{\int \frac{x\,x^{k+1}f(x)-2k-2}{x}dx} + \left(y\,x^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}e^{\int x^{k+1}f(x)dx}}{x^2}dx\right)}{y\,x^{k+1}-1}\right)$$

7.2.24.6 [758] problem number 6

problem number 758

Added Feb. 4, 2019.

Problem 2.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + ay - ab - b^2 f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*y - a*b - b^2*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2+a*y-a*b- b^2*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b-y)\left(\int e^{ax+2b\left(\int f(x)dx\right)} f(x)\, dx\right) - e^{ax+2b\left(\int f(x)dx\right)}}{b-y}\right)$$

7.2.24.7 [759] problem number 7

problem number 759

Added Feb. 4, 2019.

Problem 2.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f[x]y^2 - ax^n f[x]y + anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x^n*f[x]*y + a*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*f(x)*y+a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.24.8 [760] problem number 8

problem number 760

Added Feb. 4, 2019.

Problem 2.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + anx^{n-1} - a^2x^{2n}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*n*x^(n - 1) - a^2*x^(2*n)*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.24.9 [761] problem number 9

problem number 761

Added Feb. 4, 2019.

Problem 2.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y - a^2f(x) - ag(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y - a^2*f[x] - a*g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)* y-a^2*f(x)-a*g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a-y)\left(\int \mathrm{e}^{2a\left(\int f(x)dx\right)+\int g(x)dx} f(x)\, dx\right) - \mathrm{e}^{2a\left(\int f(x)dx\right)+\int g(x)dx}}{a-y}\right)$$

7.2.24.10 [762] problem number 10

problem number 762

Added Feb. 4, 2019.

Problem 2.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y + anx^{n-1} - ax^n g(x) - a^2x^{2n}f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*n*x^(n - 1) - a*x^n*g[x] - a^2*x^(2*n)*f[x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)*y+a*n*x^(n-1) - a*x^n*g(x)-a^2*x^(2*n))*f(x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.24.11 [763] problem number 11

problem number 763

Added Feb. 4, 2019.

Problem 2.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - ax^n g(x)y + anx^{n-1} + a^2x^{2n}(g(x) - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x^n*g*x*y + a*n*x^(n - 1) + a^2*x^(2*n)*(g*x - f*x))*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*g(x)*y+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.24.12 [764] problem number 12

problem number 764

Added Feb. 4, 2019.

Problem 2.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(f(x)y^2 + ny + ax^{2n}f(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (f[x]*y^2 + n*y + a*x^(2*n)*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-n}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x f(K[1])K[1]^{n-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( f(x)*y^2+n*y+a*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int x^{n-1}f(x)\,dx\right)-\arctan\left(\frac{y\,x^{-n}}{\sqrt{a}}\right)\right)$$

7.2.24.13 [765] problem number 13

problem number 765

Added Feb. 4, 2019.

Problem 2.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + \left(x^{2n} f(x) y^2 + (a x^n f(x) - n) y + b f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x^(2*n)*f[x]*y^2 + (a*x^n*f[x] - n)*y + b*f[x])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\int_1^x \frac{b f(K[5])\sqrt{\frac{K[5]^{2n}}{b}}}{K[5]}dK[5] - \frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\left(\sqrt{\frac{a^2}{b}}-2y\sqrt{\frac{x^{2n}}{b}}\right)}{\sqrt{4b-a^2}}\right)}{\sqrt{4b-a^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x* diff(w(x,y),x)+( x^(2*n)* f(x)*y^2+(a*x^n*f(x)-n)*y+b*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{2\left(a\,\mathrm{arctanh}\left(\frac{(2y\,x^n+a)a}{\sqrt{(a^2-4b)a^2}}\right)+\frac{\sqrt{(a^2-4b)a^2}\left(\int \frac{x^n f(x)}{x}dx\right)}{2}\right)a}{\sqrt{(a^2-4b)\,a^2}}\right)$$

7.2.25 8.2

Local contents

7.2.25.1 [766] problem number 1

problem number 766

Added Feb. 4, 2019.

Problem 2.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + ae^{\lambda x}f(x)y + \lambda f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*Exp[lambda*x]*f[x]*y + lambda*f[x])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\lambda \exp\left(-\int_1^{e^{\lambda x}} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda}dK[5] - \lambda x\right)}{aye^{\lambda x} + \lambda} - \int_1^{e^{\lambda x}} \frac{\exp\left(-\int_1^{K[6]} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda}dK[5]\right)}{K[6]^2} d\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( a*exp(lambda*x)*y^2 + a*exp(lambda*x)*f(x)*y+lambda*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(ay\,e^{\lambda x} + \lambda\right)\lambda\,e^{\lambda x}}{ay\left(\int e^{a\left(\int e^{\lambda x} f(x)dx\right) - \lambda x}dx\right)e^{2\lambda x} + \lambda\left(\int e^{a\left(\int e^{\lambda x} f(x)dx\right) - \lambda x}dx\right)e^{\lambda x} + e^{a\left(\int e^{\lambda x} f(x)dx\right)}}\right)$$

7.2.25.2 [767] problem number 2

problem number 767

Added Feb. 4, 2019.

Problem 2.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x}f(x)y + a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*f[x]*y + a*lambda*Exp[lambda*x])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*exp(lambda*x)*f(x)*y+a*lambda*exp(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

sol=()

7.2.25.3 [768] problem number 3

problem number 768

Added Feb. 4, 2019.

Problem 2.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + a\lambda e^{\lambda x} - a^2e^{2\lambda x}f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x]*f[x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.2.25.4 [769] problem number 4

problem number 769

Added Feb. 4, 2019.

Problem 2.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + \lambda y + ae^{2\lambda x}f(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + lambda*y + a*Exp[2*lambda*x]*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x e^{\lambda K[1]}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+lambda*y+ a*exp(2*lambda*x)* f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int \mathrm{e}^{\lambda x} f(x)\, dx\right) - \arctan\left(\frac{y\,\mathrm{e}^{-\lambda x}}{\sqrt{a}}\right)\right)$$

7.2.25.5 [770] problem number 5

problem number 770

Added Feb. 4, 2019.

Problem 2.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - (ae^{\lambda x} + b)f(x)y + a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - (a*Exp[lambda*x] + b)*f[x]*y + a*lambda*Exp[lambda*x])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2-(a*exp(lambda*x)+b)*f(x)*y+a *lambda*exp(lambda*x))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.25.6 [771] problem number 6

problem number 771

Added Feb. 4, 2019.

Problem 2.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x} f(x) y^2 + (af(x) - \lambda)y + be^{-\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*f[x]*y^2 + (a*f[x] - lambda)*y + b*Exp[-(lambda*x)]*f[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  exp(lambda*x)*f(x)*y^2+(a*f(x)-lambda)*y+b*exp(-lambda*x)*f(x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{2\left(a \operatorname{arctanh}\left(\frac{(2y\,e^{\lambda x}+a)a}{\sqrt{(a^2-4b)a^2}}\right) + \frac{\sqrt{(a^2-4b)a^2}\,\left(\int f(x)dx\right)}{2}\right)a}{\sqrt{(a^2-4b)\,a^2}}\right)$$

7.2.25.7 [772] problem number 7

problem number 772

Added Feb. 4, 2019.

Problem 2.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y + a\lambda e^{\lambda x} - ae^{\lambda x} g(x) - a^2 e^{2\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*lambda*Exp[lambda*x] - a*Exp[lambda*x]*g[x] -
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+ g(x)*y+a*lambda*exp(lambda*x) -a*exp(lambda*x)*g(x)-a^2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.25.8 [773] problem number 8

problem number 773

Added Feb. 7, 2019.

Problem 2.8.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x}g(x)y + a\lambda e^{\lambda x} + a^2e^{2\lambda x}(g(x) - f(x))\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*g[x]*y + a*lambda*Exp[lambda*x] + a^2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2- a*exp(lambda*x)*g(x)*y + a*lambda*exp(lambda*x) +a^2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.25.9 [774] problem number 9

problem number 774

Added Feb. 7, 2019.

Problem 2.8.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + 2a\lambda x e^{\lambda x^2} - a^2 f(x) e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*f[x]*Exp[2*lambda*x^2]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*f(x)*exp(2*lambda*x^2))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.25.10 [775] problem number 10

problem number 775

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + 2\lambda xy + af(x)e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + 2*lambda*x*y + a*f[x]*Exp[2*lambda*x^2])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x^2}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x e^{\lambda K[1]^2}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+2*lambda*x*y+ a*f(x)*exp(2*lambda*x^2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y)=\_F1\left(\sqrt{a}\left(\int e^{\lambda x^2}f(x)\,dx\right)-\arctan\left(\frac{y\,e^{-\lambda x^2}}{\sqrt{a}}\right)\right)$$

7.2.25.11 [776] problem number 11

problem number 776

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f(x)e^{\lambda y}+g(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*Exp[lambda*y] + g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(   f(x)*exp(lambda*y) + g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int g(x)dx\right)} f(x)\, dx\right) - e^{-\left(y-\left(\int g(x)dx\right)\right)\lambda}}{\lambda}\right)$$

7.2.26 8.3

Local contents

7.2.26.1 [777] problem number 1

problem number 777

Added Feb. 7, 2019.

Problem 2.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda \sinh(\lambda x) - a^2 f(x)\sinh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sinh[lambda*x] - a^2*f[x]*Sinh[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a^2*f(x) + a*lambda*sinh(lambda*x) - a^2*f(x)*sinh(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.26.2 [778] problem number 2

problem number 778

Added Feb. 7, 2019.

Problem 2.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a(af(x) + \lambda)\tanh^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Tanh[lambda*x]^2 + a*lambda)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*tanh(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.26.3 [779] problem number 3

problem number 779

Added Feb. 7, 2019.

Problem 2.8.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) + \lambda)\coth^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Coth[lambda*x]^2 + a*lambda)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*coth(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.27 8.4

Local contents

7.2.27.1 [780] problem number 1

problem number 780

Added Feb. 7, 2019.

Problem 2.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(ay^2 \ln x - axy(\ln x - 1)f(x) + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (a*y^2*Log[x] - a*x*y*(Log[x] - 1)*f[x] + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-(   a*y^2*ln(x) -a*x*y* (ln(x)-1)*f(x)+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-(\ln(x)-1)\,x\,\mathrm{e}^{\int \frac{a\,x^2 f(x)\ln(x)^2 + a\,x^2 f(x) + \left(-2a\,x^2 f(x) - 2\right)\ln(x)}{(\ln(x)-1)x}dx} + (axy\ln(x) - axy - 1)\left(\int \frac{\mathrm{e}^{\left(\int \frac{xf}{\ln(x}\right.}}{\cdots}\right.}{(axy\ln(x) - axy - 1)\,a}\right.$$

7.2.27.2 [781] problem number 2

problem number 781

Added Feb. 7, 2019.

Problem 2.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ax(\ln x)f(x)y + a\ln x + a\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x*Log[x]*f[x]*y + a*Log[x] + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)* y^2 -a*x*ln(x)*f(x)*y+a*ln(x)+a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.27.3 [782] problem number 3

problem number 782

Added Feb. 7, 2019.

Problem 2.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(f(x)y^2 + a - a^2(\ln x)^2 f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (f[x]*y^2 + a - a^2*Log[x]^2*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+( f(x)*y^2 +a -a^2* ln(x)^2 *f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

sol=()

7.2.27.4 [783] problem number 4

problem number 783

Added Feb. 7, 2019.

Problem 2.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left((y + a\ln x)^2 f(x) - a\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + ((y + a*Log[x])^2*f[x] - a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x \frac{f(K[2])}{K[2]}dK[2] + \frac{1}{a\log(x) + y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+( (y+a *ln(x))^2*f(x)-a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{(a\ln(x)+y)\left(\int \frac{f(x)}{x}dx\right)+1}{a\ln(x)+y}\right)$$

7.2.28 8.5

Local contents

7.2.28.1 [784] problem number 1

problem number 784

Added Feb. 7, 2019.

Problem 2.8.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda\sin(\lambda x)y^2 + f(x)\cos(\lambda x)y - f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + f[x]*Cos[lambda*x]*y - f[x])*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( lambda*sin(lambda*x)*y^2 + f(x)*cos(lambda*x)*y-f(x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y\,c}{-y\left(\int -\lambda\,\mathrm{e}^{\int \frac{-2\lambda\left(\cos^2(\lambda x)\right)+\sqrt{-\frac{\cos(2\lambda x)}{2}+\frac{1}{2}}\left(\cos^2(\lambda x)\right)f(x)+2\lambda}{\cos(\lambda x)\sin(\lambda x)}dx}\sin(\lambda x)\,dx\right)\cos(\lambda x)+\cos(\lambda x)\,\mathrm{e}^{\int -}}\right.$$

7.2.28.2 [785] problem number 2

problem number 785

Added Feb. 7, 2019.

Problem 2.8.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda\sin(\lambda x) + a^2 f(x)\sin^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sin[lambda*x] + a^2*f[x]*Sin[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.28.3 [786] problem number 3

problem number 786

Added Feb. 7, 2019.

Problem 2.8.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda\cos(\lambda x) + a^2 f(x)\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Cos[lambda*x] + a^2*f[x]*Cos[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.28.4 [787] problem number 4

problem number 787

Added Feb. 7, 2019.

Problem 2.8.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda)\tan^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Tan[lambda*x]^2 + a*lambda)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*tan(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

sol=()

7.2.28.5 [788] problem number 5

problem number 788

Added Feb. 7, 2019.

Problem 2.8.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda)\cot^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Cot[lambda*x]^2 + a*lambda)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*cot(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.29 8.6

Local contents

7.2.29.1 [789] problem number 1

problem number 789

Added Feb. 7, 2019.

Problem 2.8.6.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - f(x)g(x)y + g'(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - f[x]*g[x]*y + Derivative[1][g][x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2 -f(x)*g(x)*y+ diff(g(x),x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.29.2 [790] problem number 2

problem number 790

Added Feb. 7, 2019.

Problem 2.8.6.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(f'(x)y^2 - f(x)g(x)y + g(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (Derivative[1][f][x]*y^2 - f[x]*g[x]*y + g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-( diff(f(x),x)*y^2 -f(x)*g(x)*y+ g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y\left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{\int f(x)g(x)dx}}{f(x)^2}dx\right)f(x) - e^{-\left(\int \frac{-f(x)^2g(x)+2\frac{d}{dx}f(x)}{f(x)}dx\right)}f(x) - \left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{\int f(x)g(x)dx}}{f(x)^2}dx\right.}{yf(x)-1}\right.$$

7.2.29.3 [791] problem number 3

problem number 791

Added Feb. 7, 2019.

Problem 2.8.6.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(g(x)(y-f(x))^2 + f'(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (g[x]*(y - f[x])^2 + Derivative[1][f][x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x g(K[2])dK[2] + \frac{1}{y-f(x)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(g(x)*(y-f(x))^2 + diff(f(x),x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{(y-f(x))\left(\int g(x)\,dx\right)+1}{y-f(x)}\right)$$

7.2.29.4 [792] problem number 4

problem number 792

Added Feb. 7, 2019.

Problem 2.8.6.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\frac{f'(x)}{g(x)}y^2 - \frac{g'(x)}{f(x)}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((Derivative[1][f][x]*y^2)/g[x] - Derivative[1][g][x]/f[x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(f(x),x)/g(x)* y^2 - diff(g(x),x)/f(x)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\left(-yf(x)^2 - f(x)\,g(x)\right)\left(\int \frac{\frac{d}{dx}f(x)}{f(x)^2 g(x)}dx\right)-1}{\left(yf(x)+g(x)\right)f(x)}\right)$$

7.2.29.5 [793] problem number 5

problem number 793

Added Feb. 7, 2019.

Problem 2.8.6.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f^2(x)w_x + \left(f'(x)y^2 - g(x)(y - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]^2*D[w[x, y], x] + (Derivative[1][f][x]*y^2 - g[x]*(y - f[x]))*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f(x)^2*diff(w(x,y),x)+(diff(f(x),x)*y^2 -g(x)*(y-f(x))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

sol=()

7.2.29.6 [794] problem number 6

problem number 794

Added Feb. 7, 2019.

Problem 2.8.6.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - \frac{f''(x)}{f(x)}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - Derivative[2][f][x]/f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 - diff(f(x),x,x)/f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\left(-yf(x)^2 - \left(\frac{d}{dx}f(x)\right)f(x)\right)\left(\int \frac{1}{f(x)^2}dx\right) - 1}{\left(yf\left(x\right) + \frac{d}{dx}f\left(x\right)\right)f\left(x\right)}\right)$$

7.2.29.7 [795] problem number 7

problem number 795

Added Feb. 7, 2019.

Problem 2.8.6.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$g(x)w_x + \left(af(x)g(x)y^3 + (bf(x)g^3(x) + g'(x))y + cf(x)g^4(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  g[x]*D[w[x, y], x] + (a*f[x]*g[x]*y^3 + (b*f[x]*g[x]^3 + Derivative[1][g][x])*y + c*f
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  g(x)*diff(w(x,y),x)+(a*f(x)*g(x)*y^3 + (b*f(x)*g(x)^3 + diff(g(x),x))*y+ c*f(x)*g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-\operatorname{RootOf}(a\,c^2\_Z^3+b^3\_Z-b^3)cg(x)-by}{cg(x)}\right)}{3\operatorname{RootOf}\left(a\,c^2\_Z^3+b^3\_Z-b^3\right)^2 a\,c^2+b^3} - b\left(\int f(x)\,g(x)^2\,dx\right)\right)$$

7.2.29.8 [796] problem number 8

problem number 796

Added Feb. 7, 2019.

Problem 2.8.6.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f(x)y^3+3f(x)h(x)y^2+(g(x)+3f(x)h^2(x))y+f(x)h^3(x)+g(x)h(x)-h'(x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^3 + 3*f[x]*h[x]*y^2 + (g[x] + 3*f[x]*h[x]^2)*y + f[x]*h[x]^3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2(h(x)+y)^2\int_1^x \exp\left(2\int_1^{K[2]} g(K[1])dK[1]\right)f(K[2])dK[2]+\exp\left(2\int_1^x g(K[1])dK[1]\right)}{(h(x)+y)^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(f(x)*y^3+3*f(x)*h(x)*y^2+(g(x)+3*f(x)*h(x)^2)*y+ f(x)*h(x)^3 + g(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{2(y+h(x))^2\left(\int \mathrm{e}^{2(\int g(x)dx)}f(x)\,dx\right)+\mathrm{e}^{2(\int g(x)dx)}}{(y+h(x))^2}\right)$$

7.2.29.9 [797] problem number 9

problem number 797

Added Feb. 7, 2019.

Problem 2.8.6.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(\frac{g'(x)}{f^2(x)(ag(x)+b)^3}y^3+\frac{f'(x)}{f(x)}y+f(x)g'(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((Derivative[1][g][x]*y^3)/(f[x]^2*(a*g[x] + b)^3) + (Derivative[1][f
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(g(x),x)/(f(x)^2 *(a*g(x)+b)^3)*y^3 + diff(f(x),x)/f(x) * y + f(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{a^3 \ln\left(\frac{-(ag(x)+b)\operatorname{RootOf}(-a^3\_Z+\_Z^3+a^3)f(x)+ay}{(ag(x)+b)f(x)}\right)}{a^3-3\operatorname{RootOf}\left(-a^3\_Z+\_Z^3+a^3\right)^2}-\ln\left(ag(x)+b\right)\right)$$

7.2.29.10 [798] problem number 10

problem number 798

Added Feb. 7, 2019.

Problem 2.8.6.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left((y-f(x))(y-g(x))\left(y-\frac{af(x)+bg(x)}{a+b}\right)h(x)+\frac{y-g(x)}{f(x)-g(x)}f'(x)+\frac{y-f(x)}{g(x)-f(x)}g'(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((y - f[x])*(y - g[x])*(y - (a*f[x] + b*g[x])/(a + b))*h[x] + ((y - g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+((y-f(x))*(y-g(x))*(y- (a*f(x)+b*g(x))/(a+b))*h(x)+(y-g(x))/(f(x)-g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\left((a+b)\,b\ln\left(\frac{9(a+b)(a^2+ab+b^2)(y-g(x))}{(f(x)-g(x))(2a+b)}\right)+\left(2\left(-\frac{\left(\int f(x)^2h(x)dx\right)}{2}-\frac{\left(\int g(x)^2h(x)dx\right)}{2}+\int f(x)\,g(x\right.\right.\right.}{}\right.$$

7.2.29.11 [799] problem number 11

problem number 799

Added Feb. 7, 2019.

Problem 2.8.6.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g'(x)y + af(x)e^{2g(x)}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + Derivative[1][g][x]*y + a*f[x]*Exp[2*g[x]])*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-g(x)}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x e^{g(K[1])}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(f(x)*y^2 + diff(g(x),x)*y+ a*f(x)*exp(2*g(x)) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int e^{g(x)} f(x)\,dx\right) - \arctan\left(\frac{y\,e^{-g(x)}}{\sqrt{a}}\right)\right)$$

7.2.29.12 [800] problem number 12

problem number 800

Added Feb. 7, 2019.

Problem 2.8.6.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f'(x)y^2 + ae^{\lambda x} f(x)y + ae^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Derivative[1][f][x]*y^2 + a*Exp[lambda*x]*f[x]*y + a*Exp[lambda*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(f(x),x)*y^2+ a*exp(lambda*x)* f(x)*y+a*exp(lambda*x) )*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))  ),output='rea
```

$$w(x,y) = \_F1\left(\frac{-y\left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{a\left(\int e^{\lambda x}f(x)dx\right)}}{f(x)^2}dx\right)f(x) - e^{-\left(\int \frac{-a\,e^{\lambda x}f(x)^2+2\frac{d}{dx}f(x)}{f(x)}dx\right)}f(x) - \left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{a\left(\int e^{\lambda x}f}}{f(x)^2}\right.}{yf(x)+1}\right.$$

7.2.30 9.1

Local contents

7.2.30.1 [801] problem number 1

problem number 801

Added Feb. 7, 2019.

Problem 2.9.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1]-\int_1^x \frac{1}{f(K[2])}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='rea
```

$$w(x,y)=\_F1\left(-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy\right)$$

7.2.30.2 [802] problem number 2

problem number 802

Added Feb. 7, 2019.

Problem 2.9.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(x)+g(y))w_x+f'(x)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[x] + g[y])*D[w[x, y], x] + Derivative[1][f][x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (f(x)+g(y))*diff(w(x,y),x)+diff(f(x),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(e^{-y}f(x) - \left(\int e^{-y}g(y)\, dy\right)\right)$$

7.2.30.3 [803] problem number 3

problem number 803

Added Feb. 7, 2019.

Problem 2.9.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x^n f(y) + xg(y))w_x + h(y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (x^n*f[y] + x*g[y])*D[w[x, y], x] + h[y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (x^n*f(y) + x*g(y))*diff(w(x,y),x)+h(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(x^{-n+1}e^{(n-1)\left(\int \frac{g(y)}{h(y)}dy\right)} + (n-1)\left(\int \frac{e^{(n-1)\left(\int \frac{g(y)}{h(y)}dy\right)} f(y)}{h\left(y\right)}dy\right)\right)$$

7.2.30.4 [804] problem number 4

problem number 804

Added Feb. 7, 2019.

Problem 2.9.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(y) + amx^n y^{m-1})w_x - (g(x) + anx^{n-1}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[y] + a*m*x^n*y^(m - 1))*D[w[x, y], x] - (g[x] + a*n*x^(n - 1)*y^m)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(y) + a*m*x^n*y^(m-1))*diff(w(x,y),x)-(g(x)+a*n*x^(n-1)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.30.5 [805] problem number 5

problem number 805

Added Feb. 7, 2019.

Problem 2.9.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(e^{\alpha x} f(y) + c\beta)w_x - (e^{\beta y} g(x) + c\alpha)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (Exp[alpha*x]*f[y] + c*beta)*D[w[x, y], x] - (Exp[beta*y]*g[x] + c*alpha)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (exp(alpha*x)* f(y) + c*beta)*diff(w(x,y),x)-(exp(beta*y)*g(x) + c*alpha)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.31 9.2

Local contents

7.2.31.1 [806] problem number 1

problem number 806

Added Feb. 7, 2019.

Problem 2.9.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + f(ax + by + c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + f[a*x + b*y + c]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ f(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bf(\_ab+c)+a} d\_a\right) + x\right)$$

7.2.31.2 [807] problem number 2

problem number 807

Added Feb. 7, 2019.

Problem 2.9.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + f(\frac{y}{x})w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + f[y/x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ f(y/x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int^{\frac{y}{x}} \frac{1}{-\_a + f(\_a)} d\_a - \ln(x)\right)$$

7.2.31.3 [808] problem number 3

problem number 808

Added Feb. 7, 2019.

Problem 2.9.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(y + ax^n + b) - anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*x^n + b] - a*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (f(y+a*x^n+b) - a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-x + \int_{\_b}^{y} \frac{1}{f\left(a\,x^n + \_a + b\right)} d\_a\right)$$

7.2.31.4 [809] problem number 4

problem number 809

Added Feb. 7, 2019.

Problem 2.9.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yf(x^n y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*f[x^n*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*f(x^n*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{m\left(\int_{\_b}^{y} \frac{1}{(mf(\_a^m x^n)+n)\_a} d\_a\right) - \ln\left(x\right)}{m}\right)$$

7.2.31.5 [810] problem number 5

problem number 810

Added Feb. 7, 2019.

Problem 2.9.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^{m-1}w_x + x^{n-1}f(ax^n + by^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y^(m - 1)*D[w[x, y], x] + x^(n - 1)*f[a*x^n + b*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := y^(m-1)*diff(w(x,y),x)+  x^(n-1)*f(a*x^n+b*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.31.6 [811] problem number 6

problem number 811

Added Feb. 7, 2019.

Problem 2.9.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{-\lambda x}f(e^{\lambda x}y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[-(lambda*x)]*f[Exp[lambda*x]*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   exp(-lambda*x)*f(exp(lambda*x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x - \left(\int^{y\,\mathrm{e}^{\lambda x}} \frac{1}{\_a\lambda + f(\_a)} d\_a\right)\right)$$

7.2.31.7 [812] problem number 7

problem number 812

Added Feb. 7, 2019.

Problem 2.9.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(e^{\lambda y} x) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*y]*f[Exp[lambda*y]*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   exp(lambda*y)*f(exp(lambda*y)*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int^{\frac{\lambda y+\ln(x)}{\lambda}}\frac{1}{\lambda\,\mathrm{e}^{\_a\lambda}f(\mathrm{e}^{-\_a\lambda})+1}d\_a\right)+\ln(x)}{\lambda}\right)$$

7.2.31.8 [813] problem number 8

problem number 813

Added Feb. 7, 2019.

Problem 2.9.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + yf(e^{\alpha x}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + y*f[Exp[alpha*x]*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   y*f(exp(alpha*x)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{m\left(\int_{\_b}^{y}\frac{1}{(mf(\_a^m\mathrm{e}^{\alpha x})+\alpha)\_a}d\_a\right)-x}{m}\right)$$

7.2.31.9 [814] problem number 9

problem number 814

Added Feb. 7, 2019.

Problem 2.9.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x w_x + f(x^n e^{\alpha y}) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + f[x^n*Exp[alpha*y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+   f(x^n*exp(alpha*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x, y) = \_F1\left(\frac{\alpha\left(\int_{\_b}^{y} \frac{1}{\alpha f(x^n e^{\_a \alpha}) + n} d\_a\right) - \ln(x)}{\alpha}\right)$$

7.2.31.10 [815] problem number 10

problem number 815

Added Feb. 7, 2019.

Problem 2.9.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + e^{\lambda x - \beta y} f(a e^{\lambda x} + b e^{\beta y}) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*x - beta*y]*f[a*Exp[lambda*x] + b*Exp[beta*y]]*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  exp(lambda*x-beta*y)*f(a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{a\,\lambda^2\left(\int^{\frac{-a\,e^{\lambda x}-b\,e^{\beta y}}{a\lambda}} \frac{1}{b\beta f(-\_a a\lambda)+a\lambda} d\_a\right)+e^{\lambda x}}{\lambda}\right)$$

7.2.31.11 [816] problem number 11

problem number 816

Added Feb. 7, 2019.

Problem 2.9.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f\left(y + ae^{\lambda x} + b\right) - a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*Exp[lambda*x] + b] - a*lambda*Exp[lambda*x])*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (f(y+a*exp(lambda*x)+b)-a * lambda*exp(lambda*x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-x + \int_{\_b}^{y} \frac{1}{f(a\,e^{\lambda x} + \_a + b)} d\_a\right)$$

7.2.31.12 [817] problem number 12

problem number 817

Added Feb. 7, 2019.

Problem 2.9.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\alpha xyw_x + (\alpha f(x^n e^{\alpha y}) - ny)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  alpha*x*y*D[w[x, y], x] + (alpha*f[x^n*Exp[alpha*y]] - n*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  alpha*x*y*diff(w(x,y),x)+  (alpha*f(x^n*exp(alpha*y)) - n*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.31.13 [818] problem number 13

problem number 818

Added Feb. 7, 2019.

Problem 2.9.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mx(\ln y)w_x + (yf(x^n y^m) - ny\ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*x*Log[y]*D[w[x, y], x] + (y*f[x^n*y^m] - n*y*Log[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   m*x*ln(y)*diff(w(x,y),x)+  (y*f(x^n*y^m) - n*y*ln[y])*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

sol=()

7.2.31.14 [819] problem number 14

problem number 819

Added Feb. 7, 2019.

Problem 2.9.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(y + a\tan x) - a\tan^2 x\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*Tan[x]] - a*Tan[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (f(y+a*tan(x)) - a*tan(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-x + \int^{a\tan(x)+y} \frac{1}{a + f(\_a)} d\_a\right)$$

7.2.31.15 [820] problem number 15

problem number 820

Added Feb. 7, 2019.

Problem 2.9.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$e^{\lambda x} w_x + f(\lambda x + \ln y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Exp[lambda*x]*D[w[x, y], x] + f[lambda*x + Log[y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=    exp(lambda*x)*diff(w(x,y),x)+  f(lambda*x+ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(x - \left(\int^{y\,e^{\lambda x}} \frac{1}{\_a\lambda + f(\ln(\_a))} d\_a\right)\right)$$

7.2.31.16 [821] problem number 16

problem number 821

Added Feb. 7, 2019.

Problem 2.9.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(\lambda y + \ln x) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*y]*f[lambda*y + Log[x]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  exp(lambda*y)*f(lambda*y+ln(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int^{\frac{\lambda y+\ln(x)}{\lambda}} \frac{1}{\lambda\,e^{-\_a\lambda} f(\_a\lambda)+1} d\_a\right) + \ln(x)}{\lambda}\right)$$

7.2.32 9.3

Local contents

7.2.32.1 [822] problem number 1

problem number 822

Added Feb. 7, 2019.

Problem 2.9.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x - \left(ny - xy^k f(x) g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] - (n*y - x*y^k*f[x]*g[x^n*y^m])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   m*x*diff(w(x,y),x)- ( n*y -x*y^k*f(x)*g(x^n*y^m)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-\left(\int_{\_b}^{x} \frac{n\_a^{\frac{-kn-m+n}{m}} y^{-k+1} - \_a^{-\frac{(k-1)n}{m}} f(\_a)\, g(\_a^n y^m)}{g(\_a^n y^m)} d\_a\right) - \left(\int \frac{m\, x^{-\frac{(k-1)n}{m}} y^{-k} + n\right.\right.$$

7.2.32.2 [823] problem number 2

problem number 823

Added Feb. 9, 2019.

Problem 2.9.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^n w_x - \left(ax^n + g(x)f(y^{n+1} + ax^{n+1})\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y^n*D[w[x, y], x] - (a*x^n + g[x]*f[y^(n + 1) + a*x^(n + 1)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  y^n*diff(w(x,y),x)- ( a*x^n + g(x)*f(y^(n+1) + a*x^(n+1))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.3 [824] problem number 3

problem number 824

Added Feb. 9, 2019.

Problem 2.9.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(\frac{y}{x}) + x^{\alpha}h(\frac{y}{x})\right) w_x + \left(g(\frac{y}{x}) + yx^{\alpha-1}h(\frac{y}{x})\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[y/x] + x^alpha*h[y/x])*D[w[x, y], x] + (g[y/x] + y*x^(alpha - 1)*h[y/x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(y/x)+x^alpha * h(y/x))*diff(w(x,y),x)+ ( g(y/x)+y*x^(alpha-1)*h(y/x))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.4 [825] problem number 4

problem number 825

Added Feb. 9, 2019.

Problem 2.9.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(ax+by)+bxg(ax+by))\,w_x+(h(ax+by)-axg(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*x*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*x*g[a*x + b*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(a*x+b*y)+b*x*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*x*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.5 [826] problem number 5

problem number 826

Added Feb. 9, 2019.

Problem 2.9.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(ax+by)+byg(ax+by))\,w_x+(h(ax+by)-ayg(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*y*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*y*g[a*x + b*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(a*x+b*y)+b*y*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*y*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.6 [827] problem number 6

problem number 827

Added Feb. 9, 2019.

Problem 2.9.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(f(x^n y^m) + m x^k g(x^n y^m)\right) w_x + y\left(h(x^n y^m) - n x^k g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*x^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*x^k*g[x^n*y^m])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(f(x^n*y^m)+m*x^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*x^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.7 [828] problem number 7

problem number 828

Added Feb. 9, 2019.

Problem 2.9.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(f(x^n y^m) + m y^k g(x^n y^m)\right) w_x + y\left(h(x^n y^m) - n y^k g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*y^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*y^k*g[x^n*y^m])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(f(x^n*y^m)+m*y^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*y^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.8 [829] problem number 8

problem number 829

Added Feb. 9, 2019.

Problem 2.9.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x\left(sf(x^n y^m) - mg(x^k y^s)\right) w_x + y\left(ng(x^k y^s) - kf(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(s*f[x^n*y^m] - m*g[x^k*y^s])*D[w[x, y], x] + y*(n*g[x^k*y^s] - k*f[x^n*y^m])*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(s*f(x^n*y^m)-m*g(x^k*y^s))*diff(w(x,y),x)+ y*(n*g(x^k*y^s)-k*f(x^n*y^m))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.9 [830] problem number 9

problem number 830

Added Feb. 9, 2019.

Problem 2.9.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x, y)$

$$f_y * w_x - f_x w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[f[x, y], y]*D[w[x, y], x] - D[f[x, y], x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to c_1(\text{InverseFunction}[\text{InverseFunction}[f,2,2],2,2][x,y])\}\}$$

Maple ✓

```
restart;
pde :=  diff(f(x,y),y)*diff(w(x,y),x)-diff(f(x,y),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1(-f(x,y))$$

7.2.32.10 [831] problem number 11

problem number 831

Added Feb. 9, 2019.

Problem 2.9.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$xw_x + (xf(x)g(x^n e^y) - n)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*f[x]*g[x^n*Exp[y]] - n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+(x*f(x)*g(x^n*exp(y))-n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} \frac{1}{g\left(x^n \mathrm{e}^{\_a}\right)} d\_a - \left(\int f(x)\, dx\right)\right)$$

7.2.32.11 [832] problem number 12

problem number 832

Added Feb. 9, 2019.

Problem 2.9.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$m w_x + \left(m y^k f(x) g(e^{\alpha x} y^m) - \alpha y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*D[w[x, y], x] + (m*y^k*f[x]*g[Exp[alpha*x]*y^m] - alpha*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  m*diff(w(x,y),x)+(m*y^k*f(x)*g(exp(alpha*x)*y^m)- alpha*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-m\left(\int \frac{\alpha\left(\int_{\_b}^{x} \frac{m\,y^{-k+m}\mathrm{e}^{-\frac{(k-m-1)\_a\alpha}{m}} D(g)(y^m\mathrm{e}^{\_a\alpha})+(k-1)y^{-k}\mathrm{e}^{-\frac{(k-1)\_a\alpha}{m}} g(y^m\mathrm{e}^{\_a\alpha})}{g(y^m\mathrm{e}^{\_a\alpha})^2} d\_a\right) g(y^m\mathrm{e}^{\alpha x})+m\,y^{-k}\mathrm{e}^{-\frac{(k}{}}}{mg(y^m\mathrm{e}^{\alpha x})}\right.}{m}\right.$$

7.2.32.12 [833] problem number 13

problem number 833

Added Feb. 9, 2019.

Problem 2.9.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(ax+by)+be^{\lambda y}g(ax+by)\right)w_x+\left(h(ax+by)-ae^{\lambda y}g(ax+by)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*Exp[lambda*y]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(a*x+b*y)+ b*exp(lambda*y)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)-  a*exp(lambda*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.13 [834] problem number 14

problem number 834

Added Feb. 9, 2019.

Problem 2.9.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(ax+by)+be^{\lambda x}g(ax+by)\right)w_x+\left(h(ax+by)-ae^{\lambda x}g(ax+by)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*Exp[lambda*x]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(a*x+b*y)+ b*exp(lambda*x)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)-  a*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.14 [835] problem number 15

problem number 835

Added Feb. 9, 2019.

Problem 2.9.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^n e^{\alpha y}) + \alpha y g(x^n e^{\alpha y}))\, w_x + (h(x^n e^{\alpha y}) - n y g(x^n e^{\alpha y})))\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*Exp[alpha*y]] + alpha*y*g[x^n*Exp[alpha*y]])*D[w[x, y], x] + (h[x^n*Exp[alph
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*exp(alpha*y))+alpha*y*g(x^n*exp(alpha*y)))*diff(w(x,y),x)+(h(x^n*exp(alpha*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.15 [836] problem number 16

problem number 836

Added Feb. 9, 2019.

Problem 2.9.3.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(e^{\alpha x} y^m) + m x g(e^{\alpha x} y^m))\, w_x + y(h(e^{\alpha x} y^m) - \alpha x g(e^{\alpha x} y^m)))\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[Exp[alpha*x]*y^m] + m*x*g[Exp[alpha*x]*y^m])*D[w[x, y], x] + y*(h[Exp[alpha*x]*y^m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(exp(alpha*x)*y^m)+m*x*g(exp(alpha*x)*y^m))*diff(w(x,y),x)+ y*(h(exp(alpha*x)*y^m)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.16 [837] problem number 17

problem number 837

Added Feb. 9, 2019.

Problem 2.9.3.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (xyf(x)g(x^n \ln y) - ny \ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y*f[x]*g[x^n*Log[y]] - n*y*Log[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+ (x*y*f(x)*g(x^n*ln(y))-n*y*ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.17 [838] problem number 18

problem number 838

Added Feb. 9, 2019.

Problem 2.9.3.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^n y^m) + mg(x^n y^m)\ln y)\, w_x + y(h(x^n y^m) - ng(x^n y^m)\ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[y])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(y))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.18 [839] problem number 19

problem number 839

Added Feb. 9, 2019.

Problem 2.9.3.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^n y^m) + mg(x^n y^m) \ln x)\, w_x + y(h(x^n y^m) - ng(x^n y^m) \ln x)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[x])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(x))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.19 [840] problem number 20

problem number 840

Added Feb. 9, 2019.

Problem 2.9.3.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cos y w_x + (f(x)g(\sin x \sin y) - \cot x \sin y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Cos[y]*D[w[x, y], x] + (f[x]*g[Sin[x]*Sin[y]] - Cot[x]*Sin[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=cos(y)*diff(w(x,y),x)+ (f(x)* g(sin(x)*sin(y)) - cot(x)*sin(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.20 [841] problem number 21

problem number 841

Added Feb. 9, 2019.

Problem 2.9.3.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin 2x w_x + \left(\sin 2x \cos^2 y f(x) g(\tan x \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Sin[2*x]*D[w[x, y], x] + (Sin[2*x]*Cos[y]^2*f[x]*g[Tan[x]*Tan[y]] - Sin[2*y])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=sin(2*x)*diff(w(x,y),x)+ (sin(2*x)*cos(y)^2*f(x)*g(tan(x)*tan(y)) -sin(2*y))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.21 [842] problem number 22

problem number 842

Added Feb. 9, 2019.

Problem 2.9.3.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x\cos^2 y f(x) g(x^{2n}\tan y) - n\sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*Cos[y]^2*f[x]*g[x^(2*n)*Tan[y]] - n*Sin[2*y])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=x*diff(w(x,y),x)+ (x *cos(y)^2* f(x)* g(x^(2*n)*tan(y)) - n*sin(2*y))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.2.32.22 [843] problem number 23

problem number 843

Added Feb. 9, 2019.

Problem 2.9.3.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\cos^2 y f(x) g(e^{2x} \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Cos[y]^2*f[x]*g[Exp[2*x]*Tan[y]] - Sin[2*y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (cos(y)^2* f(x)* g(exp(2*x)*tan(y)) -sin(2*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

7.3 chapter 3

Local contents

7.3.1 Examples

Local contents

7.3.1.1 [844] Example 1

problem number 844

Added Feb. 9, 2019.

Problem Chapter 3, example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log(x)}{a}+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\_F1\left(y\,x^{-\frac{b}{a}}\right)+c\ln\left(x\right)}{a}$$

7.3.1.2 [845] Example 2

problem number 845

Added Feb. 9, 2019.

Problem Chapter 3, example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^x w_x + b w_y = c e^{2x} y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[x]*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[2*x]*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(ae^x y+b(-x)+b)}{a^2}+c_1\left(\frac{be^{-x}}{a}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(x)*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*exp(2*x)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=-\frac{\left(-a\,\mathrm{e}^{-\frac{b\,\mathrm{e}^{-x}}{a}+x}+b\,\mathrm{expIntegral}\left(1,\frac{b\,\mathrm{e}^{-x}}{a}\right)\right)cy\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-x}}{a}}}{a^2}+\_F1\left(y\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-x}}{a}}\right)$$

7.3.1.3 [846] Example 3

problem number 846

Added Feb. 9, 2019.

Problem Chapter 3, example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = b$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to bx + c_1(y - ax)\}\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = bx + \_F1(-ax + y)$$

7.3.2 2.1

Local contents

7.3.2.1 [847] Problem 1

problem number 847

Added Feb. 9, 2019.

Problem Chapter 3.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{cx}{a}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{cx}{a}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.2.2 [848] Problem 2

problem number 848

Added Feb. 9, 2019.

Problem Chapter 3.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{x(a(\alpha x+2\beta y+2\gamma)-b\beta x)}{2a^2}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2a^2\_F1\left(\frac{ay-bx}{a}\right)+(-b\beta x+(\alpha x+2\beta y+2\gamma)\,a)\,x}{2a^2}$$

7.3.2.3 [849] Problem 3

problem number 849

Added Feb. 9, 2019.

Problem Chapter 3.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a\alpha x+2a\log(x)(\beta y+\gamma)-b\beta\log^2(x)}{2a^2}+c_1\left(y-\frac{b\log(x)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{-b\beta\ln(x)^2+2a^2\_F1\left(\frac{ay-b\ln(x)}{a}\right)+2a\alpha x+2(\beta y+\gamma)\,a\ln(x)}{2a^2}$$

7.3.2.4 [850] Problem 4

problem number 850

Added Feb. 9, 2019.

Problem Chapter 3.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+bxw_y=c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log(x)}{a}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{a\_F1\left(\frac{ay-bx}{a}\right) + c\ln(x)}{a}$$

7.3.2.5 [851] Problem 5

problem number 851

Added Feb. 9, 2019.

Problem Chapter 3.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + \frac{\log(ax+b)(-a\beta d + ac\gamma - \alpha bc)}{a^2c} + \frac{\alpha x}{a} + \frac{\beta(cy+d)}{c^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x+b)* diff(w(x,y),x)+(c*y+d)*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='real
```

$$w(x,y) = \frac{a^2c^2\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + (-\alpha bc + (-\beta d + \gamma c)\, a)\, c\ln(ax+b) + (\alpha\, c^2 x + (cy+d)\, a\beta)\, a}{a^2c^2}$$

7.3.2.6 [852] Problem 6

problem number 852

Added Feb. 9, 2019.

Problem Chapter 3.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{y^2}{2} - \frac{bx}{a}\right) - \frac{\alpha(ay^2)^{3/2}}{3\sqrt{a}b^2} + \frac{\sqrt{ay^2}(\alpha x + \gamma)}{\sqrt{a}b} + \frac{\beta x}{a}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{y^2}{2} - \frac{bx}{a}\right) + \frac{\alpha(ay^2)^{3/2}}{3\sqrt{a}b^2} - \frac{\sqrt{ay^2}(\alpha x + \gamma)}{\sqrt{a}b} + \frac{\beta x}{a}\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{6a^2b^2\_F1\left(\frac{y^2a-2bx}{a}\right) + 6a\,b^2\beta x - 3(a\alpha\,y^2 - 2(\alpha x+\gamma)\,b)\,\sqrt{a^2y^2}\,a + (a^2y^2)^{\frac{3}{2}}\,\alpha}{6a^2b^2}$$

7.3.2.7 [853] Problem 7

problem number 853

Added Feb. 9, 2019.

Problem Chapter 3.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to -\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}} + c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$
$$\left\{w(x,y) \to \frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}} + c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\ln\left(\frac{abx}{\sqrt{ab}} + \sqrt{a^2y^2}\right) + \sqrt{ab}\_F1\left(\frac{y^2a-b\,x^2}{a}\right)}{\sqrt{ab}}$$

7.3.2.8 [854] Problem 8

problem number 854

Added Feb. 9, 2019.

Problem Chapter 3.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cx + ky$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x + k*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{ay^2 - bx^2}{2a}\right) - \frac{c\sqrt{ay^2}}{\sqrt{ab}} + \frac{kx}{a}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{ay^2 - bx^2}{2a}\right) + \frac{c\sqrt{ay^2}}{\sqrt{ab}} + \frac{kx}{a}\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x+k*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{kx}{a} + \frac{cy}{b} + \_F1\left(\frac{y^2a - b\,x^2}{a}\right)$$

7.3.3 2.2

Local contents

7.3.3.1 [855] Problem 1

problem number 855

Added Feb. 9, 2019.

Problem Chapter 3.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^2 + dy^2 + kxy + n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{x(a^2(2cx^2+6dy^2+3kxy+6n)-abx(6dy+kx)+2b^2dx^2)}{6a^3}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{6a^3\_F1\left(\frac{ay-bx}{a}\right)+2\left(b^2d\,x^2-3\left(dy+\frac{kx}{6}\right)abx+\left(c\,x^2+3d\,y^2+\frac{3}{2}kxy+3n\right)a^2\right)x}{6a^3}$$

7.3.3.2 [856] Problem 2

problem number 856

Added Feb. 9, 2019.

Problem Chapter 3.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cx^2+dy^2+kxy+n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2ab(a+b)c_1\left(yx^{-\frac{b}{a}}\right)+a^2dy^2+abcx^2+abdy^2+2abkxy+2bn(a+b)\log(x)+b^2cx^2}{2ab(a+b)}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kxy}{a+b}+\frac{c\,x^2}{2a}+\frac{d\,y^2}{2b}+\frac{n\ln(x)}{a}+\_F1\left(y\,x^{-\frac{b}{a}}\right)$$

7.3.3.3 [857] Problem 3

problem number 857

Added Feb. 9, 2019.

Problem Chapter 3.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x+bxw_y=cxy+d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}+\frac{cx^2}{2a}\right\}$$
$$\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}+\frac{cx^2}{2a}\right\}$$

Maple ✓

```
restart;
pde := a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x*y+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ad\ln\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2y^2}\right)+\frac{\sqrt{ab}\,c\,x^2}{2}+\sqrt{ab}\,a\_F1\left(\frac{y^2a-b\,x^2}{a}\right)}{\sqrt{ab}\,a}$$

7.3.3.4 [858] Problem 4

problem number 858

Added Feb. 9, 2019.

Problem Chapter 3.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x+by^2w_y=cx^2+dy^2+kxy+nx+my+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n*x + m*y + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{abx(ax-by)c_1\left(\frac{b}{ax}-\frac{1}{y}\right)-a^2mx^2\log\left(\frac{ax}{y}\right)+abcx^3-abdxy^2+abkx^2y\log\left(\frac{ax}{y}\right)+x\log(x}{abx(ax-by)}\right.\right.$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) =c*x^2+d*y^2+ k*x*y+ n*x+ m*y+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kxy\ln\left(\frac{ax}{y}\right)}{ax-by}-\frac{d\,y^2}{ax-by}+\frac{cx}{a}+\frac{n\ln(x)}{a}+\frac{m\ln(x)}{b}-\frac{m\ln\left(\frac{ax}{y}\right)}{b}+\_F1\left(\frac{ax-by}{axy}\right)-\frac{s}{ax}$$

7.3.3.5 [859] Problem 5

problem number 859

Added Feb. 9, 2019.

Problem Chapter 3.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x+axyw_y=by^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{by^2}{x-2ax}+c_1\left(yx^{-a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*diff(w(x,y),x)+a*x*y*diff(w(x,y),y) =b*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{b\,y^2}{(2a-1)\,x}+\_F1\left(y\,x^{-a}\right)$$

7.3.3.6 [860] Problem 6

problem number 860

Added Feb. 9, 2019.

Problem Chapter 3.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2w_x+bx^2w_y=cx^2+d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + b*x^2*D[w[x, y], y] == c*x^2 + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to \frac{bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3}{\sqrt[3]{a}b\left(ay^3\right)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

$$\left\{w(x,y)\to -\frac{\sqrt[3]{-1}\left(bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3\right)}{\sqrt[3]{a}b\left(ay^3\right)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

$$\left\{w(x,y)\to \frac{(-1)^{2/3}\left(bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3\right)}{\sqrt[3]{a}b\left(ay^3\right)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y^2*diff(w(x,y),x)+b*x^2*diff(w(x,y),y) =c*x^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{(\_a^2c+d)\,a}{\left(\left(\_a^3b+a\,\mathrm{RootOf}\left(ay-(a^2b\,x^3+a^3\_Z)^{\frac{1}{3}}\right)\right)a^2\right)^{\frac{2}{3}}}d\_a+\_F1\Big(\mathrm{RootOf}\left(ay-(a^2b\,x^3+a\right.$$

Contains unresolved integral with RootOf

7.3.3.7 [861] Problem 7

problem number 861

Added Feb. 9, 2019.

Problem Chapter 3.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2w_x + bxyw_y = cx^2 + dy^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + b*x*y*D[w[x, y], y] == c*x^2 + d*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)-\frac{\sqrt{a}c\sqrt{y^2-\frac{bx^2}{a}}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}\sqrt{y^2-\frac{bx^2}{a}}}\right)}{b^{3/2}}+\frac{dx}{a}+\frac{cx}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*y^2*diff(w(x,y),x)+b*x*y*diff(w(x,y),y) =c*x^2+d*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-(y^2a-b\,x^2)\,ac\arctan\left(\frac{bx}{\sqrt{(y^2a-b\,x^2)\,b}}\right)+\left(ab\_F1\left(\frac{y^2a-b\,x^2}{a}\right)+(ac+bd)\,x\right)\sqrt{(y^2a-b\,x^2)\,b}}{\sqrt{(y^2a-b\,x^2)\,b}\,ab}$$

7.3.4 2.3

Local contents

7.3.4.1 [862] Problem 1

problem number 862

Added Feb. 9, 2019.

Problem Chapter 3.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a\sqrt{x^2 + y^2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to a\sqrt{x^2+y^2}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*sqrt(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \sqrt{x^2+y^2}\,a + \_F1\left(\frac{y}{x}\right)$$

7.3.4.2 [863] Problem 2

problem number 863

Added Feb. 9, 2019.

Problem Chapter 3.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x w_x + b y w_y = c x y^2 + d x^2 y + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x*y^2 + d*x^2*y + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a(2a^2+5ab+2b^2)\,c_1\left(yx^{-\frac{b}{a}}\right)+k(2a^2+5ab+2b^2)\log(x)+axy(2acy+adx+bcy+2bdx}{a(2a+b)(a+2b)}\right.\right.$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x*y^2+d*x^2*y+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{cx\,y^2}{a+2b}+\frac{d\,x^2y}{2a+b}+\frac{k\ln(x)}{a}+\_F1\left(y\,x^{-\frac{b}{a}}\right)$$

7.3.4.3 [864] Problem 3

problem number 864

Added Feb. 9, 2019.

Problem Chapter 3.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cxy^2 + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y^2 + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to -\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)+cy^2\sqrt{ay^2}}{3\sqrt{a}b}+c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$
$$\left\{w(x,y)\to \frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)+cy^2\sqrt{ay^2}}{3\sqrt{a}b}+c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y*diff(w(x,y),x) + b*x*diff(w(x,y),y) =c*x*y^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a^2bd\ln\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2y^2}\right)+\left(a^2b\_F1\left(\frac{a\,y^2-b\,x^2}{a}\right)+\left(\left(a\,y^2-b\,x^2\right)ay+\left(-\frac{2a\,y^2}{3}+b\,x^2\right)\sqrt{a^2y^2}\right)c\right)}{\sqrt{ab}\,a^2b}$$

7.3.4.4 [865] Problem 4

problem number 865

Added Feb. 9, 2019.

Problem Chapter 3.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = kx^3 + ny^3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == k*x^3 + n*y^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-\frac{\log(ax+b)\left(a^3d^3n+b^3c^3k\right)}{a^4c^3}+\frac{b^2kx}{a^3}-\frac{bkx^2}{2a^2}+\frac{kx^3}{3a}+\frac{n(-3c^2d\right.\right.$$

Maple ✓

```
restart;
pde := (a*x+b)*diff(w(x,y),x) + (c*y+d)*diff(w(x,y),y) =k*x^3+n*y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{6a^4c^4\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)+2\left(a^2c^4k\,x^3-\frac{3ab\,c^4k\,x^2}{2}+3b^2c^4kx+\left(c^2y^2-\frac{5}{2}cdy+\frac{11}{2}d^2\right)(cy+d)\,c\right.}{6a^4c^4}$$

7.3.4.5 [866] Problem 5

problem number 866

Added Feb. 9, 2019.

Problem Chapter 3.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xyw_y = y^2(ax + by)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{y^2(ax+by)}{2x}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*diff(w(x,y),x) + x*y*diff(w(x,y),y) =y^2*(a*x + b*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\,y^2}{2}+\frac{b\,y^3}{2x}+\_F1\left(\frac{y}{x}\right)$$

7.3.4.6 [867] Problem 6

problem number 867

Added Feb. 9, 2019.

Problem Chapter 3.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x^{3} w_{x}+b y^{3} w_{y}=c x+d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^3*D[w[x, y], x] + b*y^3*D[w[x, y], y] == c*x + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{2cx+d}{2ax^2}+c_1\left(\frac{1}{2}\left(\frac{b}{ax^2}-\frac{1}{y^2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^3*diff(w(x,y),x) + b*y^3*diff(w(x,y),y) =c*x+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\_F1\left(\frac{a\,x^{2}-b\,y^{2}}{a\,x^{2}y^{2}}\right)-\frac{c}{ax}-\frac{d}{2a\,x^{2}}$$

7.3.5 2.4

Local contents

7.3.5.1 [868] Problem 1

problem number 868

Added Feb. 9, 2019.

Problem Chapter 3.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n + dy^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + d*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{cx^{n+1}}{an+a}+\frac{dy^{m+1}}{bm+b}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n+d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{(m+1)\,(n+1)\,ab\_F1\left(\frac{ay-bx}{a}\right)+(n+1)\,ad\,y^{m+1}+(m+1)\,bc\,x^{n+1}}{(m+1)\,(n+1)\,ab}$$

7.3.5.2 [869] Problem 2

problem number 869

Added Feb. 9, 2019.

Problem Chapter 3.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-cx^{n+1}(bx-a(n+2)y)+a^2(n^2+3n+2)\,c_1\left(y-\frac{bx}{a}\right)}{a^2(n+1)(n+2)}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{(n+2)\,(n+1)\,a^2\_F1\left(\frac{ay-bx}{a}\right)+((n+2)\,ay-bx)\,c\,x^{n+1}}{(n+2)\,(n+1)\,a^2}$$

7.3.5.3 [870] Problem 3

problem number 870

Added Feb. 9, 2019.

Problem Chapter 3.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2 + y^2)^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a(x^2+y^2)^k}{2k}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*(x^2+y^2)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a(x^2+y^2)^k+2k\_F1\left(\frac{y}{x}\right)}{2k}$$

7.3.5.4 [871] Problem 4

problem number 871

Added Feb. 9, 2019.

Problem Chapter 3.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cx^ny^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{cy^mx^n}{an+bm}+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c\,x^{n}y^{m}}{na+bm}+\_F1\left(y\,x^{-\frac{b}{a}}\right)$$

7.3.5.5 [872] Problem 5

problem number 872

Added Feb. 9, 2019.

Problem Chapter 3.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cx^n+dy^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n + d*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)+\frac{cx^n}{an}+\frac{dy^m}{bm}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n + d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{c\_a^{n}+d\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^{m}}{\_aa}d\_a+\_F1\left(y\,x^{-\frac{b}{a}}\right)$$

Result has unresolved integral

7.3.5.6 [873] Problem 6

problem number 873

Added Feb. 9, 2019.

Problem Chapter 3.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = (ax^n + by^m)^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{(ax^n+by^m)^k}{kmn}+c_1\left(yx^{-\frac{n}{m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=m*x*diff(w(x,y),x) + n*y*diff(w(x,y),y) =(a*x^n+b*y^m)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{kmn\_F1\left(y\,x^{-\frac{n}{m}}\right)+(a\,x^n+b\,y^m)^k}{kmn}$$

Result has unresolved integral

7.3.5.7 [874] Problem 7

problem number 874

Added Feb. 9, 2019.

Problem Chapter 3.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^m w_y = cx^k + dy^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == c*x^k + d*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-m}}{m-1}\right)+\frac{cx^{k-n+1}}{ak-an+a}-\frac{dy^{1-m}\left(\left(y^{m-1}\right)^{\frac{1}{m-1}}\right)^s}{b(m-s-1)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*y^m*diff(w(x,y),y) =c*x^k+d*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = -\frac{(k-n+1)\,a^2 d\, a^{\frac{s}{m-1}-1} y^{-m+1}\left((n-1)\,a\,y^{-m+1}\right)^{-\frac{s}{m-1}}(n-1)^{\frac{s}{m-1}}\,\mathrm{e}^{-\frac{i\left(\mathrm{csgn}\left(\frac{i}{n-1}\right)\mathrm{csgn}\left(ia\,y^{-m+1}\right)^2-\mathrm{csgn}\left(i\right.}{}}}{}$$

Result has unresolved integral

7.3.5.8 [875] Problem 8

problem number 875

Added Feb. 9, 2019.

Problem Chapter 3.2.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^k y^s + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^k*y^s + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{1-n}\left(\frac{d}{a-an}-\frac{cx^k y^s e^{-\frac{bsx^{m-n+1}}{am-an+a}}\left(-\frac{bsx^{m-n+1}}{am-an+a}\right)^{\frac{-k+n-1}{m-n+1}}\text{Gamma}\left(\frac{k-n+1}{m-n+1},-\frac{bsx^{m-n+1}}{am-an+a}\right)}{a(m-n+1)}\right)+c_1\left(\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =c*x^k*y^s+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c\_a^{k-n}\left(y\,e^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)n}{(m-n+1)a}}\right)^s+d\_a^{-n}}{a}d\_a+\_F1\left(y\,e^{-\frac{n\,x^{m-n+1}}{(m-n+1)a}}\right)$$

Result has unresolved integral

7.3.5.9 [876] Problem 9

problem number 876

Added Feb. 9, 2019.

Problem Chapter 3.2.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k)w_y = sx^p y^q + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == s*x^p*y^q + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{K[1]^{-n}\left(s\left(b^{-\frac{k+1}{m-n+1}}\exp\left(-\frac{b\left(x^{m-n+1}-K[1]^{m-n+1}\right)}{a(m-n+1)}\right)(a(m-n+1))^{-\frac{m}{m-n+1}}\left(b^{\frac{n}{m-n+1}}ce^{\frac{bx^{m-}}{ma-n}}\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{s\_a^{-n+p}\left(y\,\mathrm{e}^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)n}{(m-n+1)a}}\right)^q+d\_a^{-n}}{a}d\_a+\_F1\left(y\,\mathrm{e}^{-\frac{n\,x^{m-n+1}}{(m-n+1)a}}\right)$$

7.3.5.10 [877] Problem 10

problem number 877

Added Feb. 9, 2019.

Problem Chapter 3.2.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y^k + cx^r y)w_y = sx^p y^q + d$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y^k + c*x^r*y)*D[w[x, y], y] == s*x^p*y^q + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) +(b*x^m*y^k + c*x^r*y)*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int^x \frac{s\_a^{-n+p} \left(\left(\left(\frac{-(n-r-1)(m-2n+r+2)^2 ab\_a^{m-r} y^{\frac{1}{n-r-1}} y^{\frac{r}{n-r-1}} y^{\frac{kn}{n-r-1}} \left(\frac{(k-1)c}{(n-r-1)a}\right)^{\frac{-m+n-1}{n-r-1}} \left(\frac{(k-1)c}{(n-r-1)a}\right)^{\frac{m-n+1}{n-r-1}}}{}\right.\right.\right.}{}$$

7.3.5.11 [878] Problem 11

problem number 878

Added Feb. 9, 2019.

Problem Chapter 3.2.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^m w_y = cx^m + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^m*D[w[x, y], y] == c*x^m + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(bdx\left(\frac{a(m+1)y^{k+1}}{a(m+1)y^{k+1}-b(k+1)x^{m+1}}\right)^{\frac{k}{k+1}}\ {}_2F_1\left(\frac{k}{k+1},\frac{1}{m+1};1+\frac{1}{m+1};\frac{b(k+1)x^{m+1}}{b(k+1)x^{m+1}-a(m+\right.\right.}{ab}\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x) +b*x^n*diff(w(x,y),y) =c*x^m+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x \frac{(c\_a^m+d)\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_a^{n+1}-(k+1)b\,x^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}}{a}d\_a+\_F1\left(\frac{(n+1)\,a\,y^{k+1}-(k+}{(n+1)\,a}\right.$$

7.3.6 3.1

Local contents

7.3.6.1 [879] Problem 1

problem number 879

Added Feb. 9, 2019.

Problem Chapter 3.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\lambda x} + de^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x] + d*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{ce^{\lambda x}}{a\lambda}+\frac{de^{\mu y}}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x)+d*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ad\lambda\,\mathrm{e}^{\mu y}+bc\mu\,\mathrm{e}^{\lambda x}}{ab\lambda\mu}$$

7.3.6.2 [880] Problem 2

problem number 880

Added Feb. 9, 2019.

Problem Chapter 3.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ce^{\lambda x+\beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{ce^{\beta y+\lambda x}}{a\lambda+b\beta}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c\,\mathrm{e}^{\beta y+\lambda x}}{a\lambda+b\beta}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.6.3 [881] Problem 3

problem number 881

Added Feb. 9, 2019.

Problem Chapter 3.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{ce^{-\lambda x}}{a\lambda}+c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(-\frac{(a\lambda\,\mathrm{e}^{\lambda x}-b\beta\,\mathrm{e}^{\beta y})\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)-c\,\mathrm{e}^{-\lambda x}}{a\lambda}$$

7.3.6.4 [882] Problem 4

problem number 882

Added Feb. 9, 2019.

Problem Chapter 3.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\lambda y} w_x + b e^{\beta x} w_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\beta x - \log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)\right)}{a\beta e^{\lambda y} - b\lambda e^{\beta x}} + c_1\left(\frac{e^{\lambda y}}{\lambda} - \frac{b e^{\beta x}}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{\left(-\ln\left(\frac{a\beta\,\mathrm{e}^{\lambda y}}{b\lambda}\right) + \ln\left(\mathrm{e}^{\beta x}\right)\right)c + \left(a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}\right)\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)}{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}}$$

7.3.6.5 [883] Problem 5

problem number 883

Added Feb. 9, 2019.

Problem Chapter 3.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\alpha x} w_x + b e^{\beta y} w_y = c e^{\gamma x - \beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2(2\alpha^2-3\alpha\gamma+\gamma^2)\,c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta}\right)+ac(\gamma-2\alpha)e^{-\alpha x-\beta y+\gamma x}+b\beta ce^{x(\gamma-2\alpha)}}{a^2\left(2\alpha^2-3\alpha\gamma+\gamma^2\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}\right)-\frac{\left(-\frac{\mathrm{e}^{(-2\alpha+\gamma)x}}{-2\alpha+\gamma}-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y+(-2\alpha+\gamma)x}}{(-\alpha+\gamma)b\beta}\right)b\beta c}{a^2\alpha}$$

7.3.6.6 [884] Problem 6

problem number 884

Added Feb. 9, 2019.

Problem Chapter 3.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x+be^{\beta y}w_y=ce^{\gamma x-2\beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - 2*beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-2(\alpha x+\beta y)}\left(-ce^{x(\gamma-\alpha)}\left(a^2\left(6\alpha^2-5\alpha\gamma+\gamma^2\right)e^{2\alpha x}-2ab\beta(3\alpha-\gamma)e^{\alpha x+\beta y}+2b^2\beta^2e^{2\beta y}\right)+a^3(-\right.}{a^3(\alpha-\gamma)(2\alpha-\gamma)(3\alpha-\gamma)}\right.\right.$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-2*beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{\left(a\alpha\,e^{\alpha x}-b\beta\,e^{\beta y}\right)e^{-\alpha x-\beta y}}{\alpha b\beta}\right)+\frac{\left(\frac{e^{(-3\alpha+\gamma)x}}{-3\alpha+\gamma}+\frac{2\left(a\alpha\,e^{\alpha x}-b\beta\,e^{\beta y}\right)e^{-\beta y+(-3\alpha+\gamma)x}}{(-2\alpha+\gamma)b\beta}+\frac{\left(a\alpha\,e^{\alpha x}-b\beta\,e^{\beta y}\right)^2e^{-2}}{(-\alpha+\gamma)b^2}\right.}{a^3\alpha^2}$$

7.3.6.7 [885] Problem 7

problem number 885

Added Feb. 9, 2019.

Problem Chapter 3.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x+be^{\beta y}w_y=ce^{\gamma x}+se^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x] + s*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta}\right)-\frac{ce^{\gamma x-\alpha x}}{a\alpha-a\gamma}-\frac{s\left(e^{-\beta y}\right)^{1-\frac{\mu}{\beta}}}{b\beta-b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x) + s*exp(mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{(-\beta+\mu)\left(\left(-\alpha+\gamma\right)a\_F1\left(-\frac{(a\alpha\,e^{\alpha x}-b\beta\,e^{\beta y})e^{-\alpha x-\beta y}}{\alpha b\beta}\right)+c\,e^{(-\alpha+\gamma)x}\right)\alpha b+\left(b\beta\,e^{-\alpha x}+\left(a\alpha\,e^{\alpha x}-b\beta\right.\right.}{(-\alpha+\gamma)\left(-\beta+\mu\right)a\alpha b}$$

7.3.6.8 [886] Problem 8

problem number 886

Added Feb. 9, 2019.

Problem Chapter 3.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x+(be^{\gamma x}+ce^{\lambda y})w_y=se^{\mu x}+ke^{\delta y}+p$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{k\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_\_ba\beta-b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_\_b}}{(\beta-\gamma)a}}d\_\_b\right)+a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a}\right)^{-\frac{\delta}{\lambda}}\mathrm{e}}{a}$$

7.3.6.9 [887] Problem 9

problem number 887

Added Feb. 9, 2019.

Problem Chapter 3.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x+\delta y} + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{s\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_\_ba\beta-b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_\_b}}{(\beta-\gamma)a}}d\_\_b\right)+a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a}\right)^{-\frac{\delta}{\lambda}} \mathrm{e}}{a}$$

7.3.6.10 [888] Problem 10

problem number 888

Added Feb. 9, 2019.

Problem Chapter 3.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = ce^{\mu x+\delta y} + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x + lambda*y]*D[w[x, y], y] == c*Exp[mu*x +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{\gamma x-\beta x}}{a\beta-a\gamma} - \frac{e^{-\lambda y}}{\lambda}\right) + \frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\delta/\lambda} e^{-\gamma x-\lambda y+\mu x}\,{}_2F_1\left(1, \frac{\mu-\gamma}{\beta-\gamma}; \frac{\beta\delta-\gamma\delta-\gamma\lambda+\lambda\mu}{\beta\lambda-\gamma\lambda}; 1 - \frac{ae^{\beta x-\gamma x-}}{b}\right.}{b(\beta(\lambda-\delta)+\delta\gamma-\lambda\mu)}\right.\right.$$

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +b*exp(gamma*x+lambda*y)*diff(w(x,y),y) =c*exp(mu*x+delta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{c\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_a}+(\beta-\gamma)a\,\mathrm{e}^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{(-\beta+\mu)\_a}+k\,\mathrm{e}^{-\_a\beta}}{a}d\_a+\_F1\left(-\frac{\left(-b\lambda\,\mathrm{e}^{\lambda y+(-}\right.}{}\right.$$

7.3.6.11 [889] Problem 11

problem number 889

Added Feb. 9, 2019.

Problem Chapter 3.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma x} + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\lambda y}}{\lambda}-\frac{be^{\beta x}}{a\beta}\right)-\frac{d\gamma\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)-\beta d\gamma x+c\lambda\left(e^{\lambda y}\right)^{\gamma/\lambda}}{a\beta\gamma e^{\lambda y}-b\gamma\lambda e^{\beta x}}-\frac{a\beta c\lambda\left(e^{\lambda y}\right)^{\frac{\gamma+\lambda}{\lambda}}\,{}_2F_1\left(1,\frac{\gamma+\lambda}{\lambda};\frac{\gamma}{\lambda}+\right.}{(\gamma+\lambda)\left(a\beta e^{\lambda y}-b\lambda e\right.}\right.\right.$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c*exp(gamma*y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}\frac{\left(c\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-\left(-\mathrm{e}^{-a\beta}+\mathrm{e}^{\beta x}\right)b\lambda}{a\beta}\right)^{\frac{\gamma}{\lambda}}+d\right)\beta}{a\beta\,\mathrm{e}^{\lambda y}-\left(-\mathrm{e}^{-a\beta}+\mathrm{e}^{\beta x}\right)b\lambda}d\_a+\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)$$

7.3.7 3.2

Local contents

7.3.7.1 [890] Problem 1

problem number 890

Added Feb. 9, 2019.

Problem Chapter 3.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cye^{\lambda x}+kxe^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*y*Exp[lambda*x] + k*x*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{bce^{\lambda x}}{a^2\lambda^2}-\frac{ake^{\mu y}}{b^2\mu^2}+\frac{cye^{\lambda x}}{a\lambda}+\frac{kxe^{\mu y}}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*y*exp(lambda*x)+k*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a^2b^2\lambda^2\mu^2\_F1\left(\frac{ay-bx}{a}\right)-(-b\mu x+a)\,a^2k\,\lambda^2\mathrm{e}^{\mu y}+(a\lambda y-b)\,b^2c\,\mu^2\mathrm{e}^{\lambda x}}{a^2b^2\lambda^2\mu^2}$$

7.3.7.2 [891] Problem 2

problem number 891

Added Feb. 9, 2019.

Problem Chapter 3.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=ax^ke^{\lambda y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == a*x^k*Exp[lambda*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{x^k(-a\lambda x)^{-k}e^{\lambda(y-ax)}\,\mathrm{Gamma}(k+1,-a\lambda x)}{\lambda}+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +a*diff(w(x,y),y) =a*x^k*exp(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{(-\Gamma(k)+\Gamma(k,-a\lambda x))\,k\,x^k(-a\lambda x)^{-k}\,\mathrm{e}^{(-ax+y)\lambda}+\lambda\_F1(-ax+y)+x^k\mathrm{e}^{\lambda y}}{\lambda}$$

7.3.7.3 [892] Problem 3

problem number 892

Added Feb. 9, 2019.

Problem Chapter 3.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay+be^{\lambda x})w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*Exp[lambda*x])*D[w[x, y], y] == c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ce^{\beta x}}{\beta}+c_1\left(e^{-ax}\left(\frac{be^{\lambda x}}{a-\lambda}+y\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*y+b*exp(lambda*x))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{\beta\_F1\left(\frac{(b\,\mathrm{e}^{ax}+(a-\lambda)y\,\mathrm{e}^{(a-\lambda)x})\mathrm{e}^{-(2a-\lambda)x}}{a-\lambda}\right)+c\,\mathrm{e}^{\beta x}}{\beta}$$

7.3.7.4 [893] Problem 4

problem number 893

Added Feb. 9, 2019.

Problem Chapter 3.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (aye^{\lambda x} + be^{\beta x}y^k)w_y = ce^{\mu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y*Exp[lambda*x] + b*Exp[beta*x]*y^k)*D[w[x, y], y] == c*Exp[mu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ce^{\mu x}}{\mu}+c_1\left((k-1)\int_1^x be^{\frac{ae^{\lambda K[1]}(k-1)}{\lambda}+\beta K[1]}dK[1]+y^{1-k}e^{\frac{a(k-1)e^{\lambda x}}{\lambda}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*y*exp(lambda*x)+b*exp(beta*x)*y^k)*diff(w(x,y),y) =c*exp(mu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c\,e^{\mu x}+\mu\_F1\left((k-1)\,b\left(\int e^{\frac{\beta\lambda x+(k-1)a\,e^{\lambda x}}{\lambda}}dx\right)+y^{-k+1}e^{\frac{(k-1)a\,e^{\lambda x}}{\lambda}}\right)}{\mu}$$

7.3.7.5 [894] Problem 5

problem number 894

Added Feb. 9, 2019.

Problem Chapter 3.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^k + bx^n e^{\lambda y})w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^k + b*x^n*Exp[lambda*y])*D[w[x, y], y] == c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ce^{\beta x}}{\beta} + c_1\left(\frac{b\lambda x^{n+1}\left(-\frac{a\lambda x^{k+1}}{k+1}\right)^{-\frac{n+1}{k+1}} \text{Gamma}\left(\frac{n+1}{k+1}, -\frac{a\lambda x^{k+1}}{k+1}\right) - (k+1)e^{-\frac{\lambda\left(-ax^{k+1}+ky+y\right)}{k+1}}}{ab(k+1)\lambda^2(k-n)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*x^k+b*x^n*exp(lambda*y))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \frac{\beta\_F1\left(\frac{-\left(a\lambda x^{n+1}+(-k-n-2)x^{-k+n}\right)(k+1)^2 b\left(-\frac{a\lambda x^{k+1}}{k+1}\right)^{\frac{-k-n-2}{2k+2}} \text{WhittakerM}\left(\frac{-k+n}{2k+2}, \frac{2k+n+3}{2k+2}, -\frac{a\lambda x^{k+1}}{k+1}\right) e^{\frac{a\lambda x^{k+1}}{2k+2}} -2(k-}{(}\right.}{}$$

7.3.7.6 [895] Problem 6

problem number 895

Added Feb. 9, 2019.

Problem Chapter 3.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*x*exp(lambda *x+ mu* y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\,\mathrm{e}^{\lambda x+\mu y}}{\lambda+\frac{\mu y}{x}}+\_F1\left(\frac{y}{x}\right)$$

7.3.7.7 [896] Problem 7

problem number 896

Added Feb. 9, 2019.

Problem Chapter 3.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = aye^{\lambda x} + bxe^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*y*Exp[lambda*x] + b*x*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)+\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*y*exp(lambda*x)  + b*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ay\,\mathrm{e}^{\lambda x}}{\lambda x}+\frac{bx\,\mathrm{e}^{\mu y}}{\mu y}+\_F1\left(\frac{y}{x}\right)$$

7.3.7.8 [897] Problem 8

problem number 897

Added Feb. 9, 2019.

Problem Chapter 3.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + be^{\lambda y} w_y = cx^n + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == c*x^n + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{1-k}\left(\frac{cx^n}{a(-k)+an+a}+\frac{s}{a-ak}\right)+c_1\left(\frac{bx^{1-k}}{a(k-1)}-\frac{e^{-\lambda y}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^k*diff(w(x,y),x) +b*exp(lambda*y)*diff(w(x,y),y) =c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{c\,x^{-k+n+1}}{(-k+n+1)\,a}-\frac{s\,x^{-k+1}}{(k-1)\,a}+\_F1\left(\frac{b\lambda\,x^{-k+1}-(k-1)\,a\,\mathrm{e}^{-\lambda y}}{(k-1)\,b\lambda}\right)$$

7.3.7.9 [898] Problem 9

problem number 898

Added Feb. 9, 2019.

Problem Chapter 3.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + be^{\lambda x} w_y = ce^{\mu x} + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == c*Exp[mu*x] + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\left((k+1)\mu s\,{}_2F_1\left(1,\frac{1}{k+1};\frac{k+2}{k+1};\frac{a\lambda y^{k+1}}{a\lambda y^{k+1}-be^{\lambda x}(k+1)}\right)-c\lambda e^{\mu x}\,{}_2F_1\left(1,\frac{\lambda+k\mu+\mu}{k\lambda+\lambda};\frac{\lambda\text{-}}{}\right.\right.}{\mu\left(b(k+1)e^{\lambda x}-a\lambda y^{k+1}\right)}\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x) +b*exp(lambda*x)*diff(w(x,y),y) =c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}\frac{\left(c\,\mathrm{e}^{-a\mu}+s\right)\left(\left(\frac{a\lambda\,y^{k+1}+(k+1)b\,\mathrm{e}^{-a\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k}}{a}d\_a+\_F1\left(\frac{a\lambda y\,y^{k}-(k+1)\,b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)$$

7.3.7.10 [899] Problem 10

problem number 899

Added Feb. 9, 2019.

Problem Chapter 3.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x+by^k w_y=cx^n+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*x^n + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-cx^n(\lambda x)^{-n}\operatorname{Gamma}(n+1,\lambda x)+a\lambda c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{y^{1-k}}{k-1}\right)-se^{-\lambda x}}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*y^k*diff(w(x,y),y) = c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c\,x^n(\lambda x)^{-\frac{n}{2}}\operatorname{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},\lambda x\right)\mathrm{e}^{-\frac{\lambda x}{2}}+(n+1)\,a\lambda\_F1\left(\frac{a\lambda\,y^{-k+1}-(k-1)b\,\mathrm{e}^{-\lambda x}}{a\lambda}\right)-(n+1)\,(\mathrm{e}}{(n+1)\,a\lambda}$$

7.3.7.11 [900] Problem 11

problem number 900

Added Feb. 9, 2019.

Problem Chapter 3.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+bx^kw_y=cExp[\mu x]+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == c*Exp[mu*x] + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{(k+1)\left(e^{\mu K[1]}c+s\right)}{ae^{\lambda y}(k+1)+b\lambda\left(K[1]^{k+1}-x^{k+1}\right)}dK[1]+c_1\left(\frac{e^{\lambda y}}{\lambda}-\frac{bx^{k+1}}{ak+a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*x^k*diff(w(x,y),y) = c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x\frac{(c\,\mathrm{e}^{-a\mu}+s)\,(k+1)}{b\lambda\_a^{k+1}-b\lambda\,x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}d\_a+\_F1\left(\frac{-b\lambda\,x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}{(k+1)\,b\lambda}\right)$$

7.3.8 4.1

Local contents

7.3.8.1 [901] Problem 1

problem number 901

Added Feb. 9, 2019.

Problem Chapter 3.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\sinh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Sinh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\cosh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x)+k*sinh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ak\lambda\cosh\left(\mu y\right)+bc\mu\cosh\left(\lambda x\right)}{ab\lambda\mu}$$

7.3.8.2 [902] Problem 2

problem number 902

Added Feb. 9, 2019.

Problem Chapter 3.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\cosh(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{c\cosh\left(\lambda x+\mu y\right)}{a\lambda+\mu b}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.8.3 [903] Problem 3

problem number 903

Added Feb. 9, 2019.

Problem Chapter 3.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cx\sinh(\lambda x+\mu y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x*Sinh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*x*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-(a\sinh(\lambda x+\mu y)-(a\lambda+\mu b)\,x\cosh(\lambda x+\mu y))\,c+(a\lambda+\mu b)^2 \_F1\left(\frac{ay-bx}{a}\right)}{(a\lambda+\mu b)^2}$$

7.3.8.4 [904] Problem 4

problem number 904

Added Feb. 9, 2019.

Problem Chapter 3.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda x)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted Kernel Exception

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(lambda*x)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\sinh^m(\_a\mu)) + s\left(\sinh^k\left(\frac{(a\lambda y+b\cosh(\_a\lambda)-b\cosh(\lambda x))\beta}{a\lambda}\right)\right)}{a} d\_a + \_F1\left(\frac{a\lambda y - b\cosh(\lambda x)}{a\lambda}\right)$$

7.3.8.5 [905] Problem 5

problem number 905

Added Feb. 9, 2019.

Problem Chapter 3.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*y]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{s\sinh^k\left(\frac{2\beta\tanh^{-1}\left(e^{\frac{b\lambda(K[1]-x)}{a}}\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda}\right)+c\sinh^m(\mu K[1])}{a}dK[1]+c_1\left(\frac{\log\left(\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(lambda*y)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\sinh^m(\_a\mu))+s\left(\sinh^k\left(\frac{\beta\ln\left(\tanh\left(\frac{a\,\mathrm{arctanh}\left(e^{\lambda y}\right)+\frac{(-\_a+x)b\lambda}{2}}{a}\right)\right)}{\lambda}\right)\right)}{a}d\_a+\_F1\left(\frac{-b\lambda x-2a\,\mathrm{ar}}{b\lambda}\right.$$

7.3.9 4.2

Local contents

7.3.9.1 [906] Problem 1

problem number 906

Added Feb. 9, 2019.

Problem Chapter 3.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x] + k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\sinh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ak\lambda\sinh\left(\mu y\right)+bc\mu\sinh\left(\lambda x\right)}{ab\lambda\mu}$$

7.3.9.2 [907] Problem 2

problem number 907

Added Feb. 9, 2019.

Problem Chapter 3.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\sinh(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{c\sinh\left(\lambda x+\mu y\right)}{a\lambda+\mu b}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.9.3 [908] Problem 3

problem number 908

Added Feb. 9, 2019.

Problem Chapter 3.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a(x(a\lambda+b\mu)\sinh(\lambda x+\mu y)-a\cosh(\lambda x+\mu y))}{(a\lambda+b\mu)^2}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = a*x*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{(-a\cosh(\lambda x+\mu y)+(a\lambda+\mu b)\,x\sinh(\lambda x+\mu y))\,a+(a\lambda+\mu b)^2\_F1\left(\frac{ay-bx}{a}\right)}{(a\lambda+\mu b)^2}$$

7.3.9.4 [909] Problem 4

problem number 909

Added Feb. 9, 2019.

Problem Chapter 3.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda x)w_y = c\cosh^m(\mu x) + s\cosh^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(lambda*x)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{c(\cosh^{m}(\_b\mu))+s\left(\cosh^{k}\left(\frac{\left(b\left(\int(\cosh^{n}(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\cosh^{n}(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a}d\_b+\_F1\Bigg(y-\Bigg($$

7.3.9.5 [910] Problem 5

problem number 910

Added Feb. 9, 2019.

Problem Chapter 3.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cosh^{n}(\lambda y)w_y=c\cosh^{m}(\mu x)+s\cosh^{k}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y\frac{\cosh^{-n}(\lambda K[1])\left(s\cosh^k(\beta K[1])+c\cosh^m\left(\frac{\mu\left(\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(\lambda y)\right)\sinh(\lambda y)\cosh^{1-n}(\lambda y)}{\sqrt{-\sinh^2(\lambda y)}}-b\lambda\right.}{}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(lambda*y)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^y\frac{\left(c\left(\cosh^m\left(\frac{\left(a\left(\int\left(\cosh^{-n}\left(\_b\lambda\right)\right)d\_b\right)-a\left(\int\left(\cosh^{-n}(\lambda y)\right)dy\right)+bx\right)\mu}{b}\right)\right)+s\left(\cosh^k\left(\_b\beta\right)\right)\right)\left(\cosh^{-n}\left(\_b\lambda\right)\right)}{b}$$

7.3.10 4.3

Local contents

7.3.10.1 [911] Problem 1

problem number 911

Added Feb. 9, 2019.

Problem Chapter 3.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x] + k*Tanh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\log(\cosh(\lambda x))}{a\lambda}+\frac{k\log(\cosh(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x)+ k *tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)-bc\mu\ln\left(\tanh\left(\lambda x\right)-1\right)-bc\mu\ln\left(\tanh\left(\lambda x\right)+1\right)-\left(\ln\left(\tanh\left(\mu y\right)-1\right)+\ln\right.}{2ab\lambda\mu}$$

7.3.10.2 [912] Problem 2

problem number 912

Added Feb. 9, 2019.

Problem Chapter 3.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log(\cosh(\lambda x+\mu y))}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{-(\ln(\tanh(\lambda x+\mu y)-1)+\ln(\tanh(\lambda x+\mu y)+1))\,c+(2a\lambda+2\mu b)\_F1\left(\frac{ay-bx}{a}\right)}{2a\lambda+2\mu b}$$

7.3.10.3 [913] Problem 3

problem number 913

Added Feb. 11, 2019.

Problem Chapter 3.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\log(\cosh(\lambda x+\mu y))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-ax\ln\left(\tanh\left(\lambda x+\mu y\right)-1\right)-ax\ln\left(\tanh\left(\lambda x+\mu y\right)+1\right)+2\lambda x\_F1\left(\frac{y}{x}\right)+2\mu y\_F1\left(\frac{y}{x}\right)}{2\lambda x+2\mu y}$$

7.3.10.4 [914] Problem 4

problem number 914

Added Feb. 11, 2019.

Problem Chapter 3.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tanh^n(\lambda x)w_y=c\tanh^m(\mu x)+s\tanh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{s\tanh^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\lambda x)\right)\tanh^{n+1}(\lambda x)+b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\lambda K[1])\right)\tanh^{n+1}(\lambda K[1])+a\lambda(n\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*tanh(lambda*x)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{c(\tanh^m(\_b\mu))+s\left(\frac{\sinh\left(\frac{\left(b\left(\int(\tanh^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\tanh^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}{\cosh\left(\frac{\left(b\left(\int(\tanh^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\tanh^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\right.\right.$$

7.3.10.5 [915] Problem 5

problem number 915

Added Feb. 11, 2019.

Problem Chapter 3.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tanh^n(\lambda y)w_y=c\tanh^m(\mu x)+s\tanh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y\frac{\tanh^{-n}(\lambda K[1])\left(s\tanh^k(\beta K[1])+c\tanh^m\left(\frac{-a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\tanh^2(\lambda y)\right)\tanh^{1-n}(\lambda y)+a\mu\,{}_2F_1}{b\lambda}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*tanh(lambda*y)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^y\frac{\left(c\left(-\frac{\sinh\left(\frac{\left(-a\left(\int\left(\tanh^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}{\cosh\left(\frac{\left(-a\left(\int\left(\tanh^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}\right)^m+s\left(\tanh^k(\_b\beta)\right)\right)\left(\tanh^{-n}(\_b\lambda\right.}{b}$$

7.3.11 4.4

Local contents

7.3.11.1 [916] Problem 1

problem number 916

Added Feb. 11, 2019.

Problem Chapter 3.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x) + k\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x] + k*Coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ak\lambda\log(\tanh(\mu y)) + ak\lambda\log(\cosh(\mu y)) + bc\mu\log(\sinh(\lambda x))}{ab\lambda\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x)+k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{2ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right) - bc\mu\ln\left(\coth\left(\lambda x\right)-1\right) - bc\mu\ln\left(\coth\left(\lambda x\right)+1\right) - \left(\ln\left(\coth\left(\mu y\right)-1\right) + \ln\left(\right.\right.}{2ab\lambda\mu}$$

7.3.11.2 [917] Problem 2

problem number 917

Added Feb. 11, 2019.

Problem Chapter 3.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(\log(\tanh(\lambda x+\mu y))+\log(\cosh(\lambda x+\mu y)))}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{-(\ln(\coth(\lambda x+\mu y)-1)+\ln(\coth(\lambda x+\mu y)+1))\,c+(2a\lambda+2\mu b)\_F1\left(\frac{ay-bx}{a}\right)}{2a\lambda+2\mu b}$$

7.3.11.3 [918] Problem 3

problem number 918

Added Feb. 11, 2019.

Problem Chapter 3.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax(\log(\tanh(\lambda x+\mu y))+\log(\cosh(\lambda x+\mu y)))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-ax\ln(\coth(\lambda x+\mu y)-1) - ax\ln(\coth(\lambda x+\mu y)+1) + 2\lambda x\_F1\left(\frac{y}{x}\right) + 2\mu y\_F1\left(\frac{y}{x}\right)}{2\lambda x + 2\mu y}$$

7.3.11.4 [919] Problem 4

problem number 919

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda x)w_y = c\coth^m(\mu x) + s\coth^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*coth(lambda*x)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{c(\coth^{m}(\_b\mu))+s\left(\frac{\cosh\left(\frac{\left(b(\int(\coth^{n}(\_b\lambda))d\_b)+\left(y-\left(\int\frac{b(\coth^{n}(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}{\sinh\left(\frac{\left(b(\int(\coth^{n}(\_b\lambda))d\_b)+\left(y-\left(\int\frac{b(\coth^{n}(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}\right)^{k}}{a}d\_b+\_F1\left(y-\left(\int \frac{b}{}\right.\right.$$

7.3.11.5 [920] Problem 5

problem number 920

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\coth^{n}(\lambda y)w_y=c\coth^{m}(\mu x)+s\coth^{k}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_{1}^{y}\frac{\left(s\coth^{k}(\beta K[1])+c\coth^{m}\left(\frac{-a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\coth^{2}(\lambda y)\right)\coth^{1-n}(\lambda y)+b\lambda\mu x-b\lambda\mu nx+a\mu\coth^{1-n}(\lambda\right.}{b\lambda-b\lambda n}\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*coth(lambda*y)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{y}\frac{\left(c\left(-\frac{\cosh\left(\frac{\left(-a\left(\int\left(\coth^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}{\sinh\left(\frac{\left(-a\left(\int\left(\coth^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}\right)^{m}+s\left(\coth^{k}(\_b\beta)\right)\right)\left(\coth^{-n}(\_b\lambda)\right.}{b}$$

7.3.12 4.5

Local contents

7.3.12.1 [921] Problem 1

problem number 921

Added Feb. 11, 2019.

Problem Chapter 3.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*sinh(lambda*x)+ k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ak\lambda\sinh(\mu y)+bc\mu\cosh(\lambda x)}{ab\lambda\mu}$$

7.3.12.2 [922] Problem 2

problem number 922

Added Feb. 11, 2019.

Problem Chapter 3.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=\tanh(\lambda x)+k\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Tanh[lambda*x] + k*Coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{\log(\cosh(\lambda x))}{a\lambda}+\frac{k(\log(\tanh(\mu y))+\log(\cosh(\mu y)))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  tanh(lambda*x)+ k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) + \frac{k\ln(\sinh(\mu y))}{b\mu} + \frac{\ln(\cosh(\lambda x))}{a\lambda}$$

7.3.12.3 [923] Problem 3

problem number 923

Added Feb. 11, 2019.

Problem Chapter 3.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sinh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sinh[lambda*x] + k*Tanh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{\cosh(\lambda x)}{a\lambda} + \frac{k\log(\cosh(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  sinh(lambda*x)+ k*tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) + \frac{k\ln(\cosh(\mu y))}{b\mu} + \frac{\cosh(\lambda x)}{a\lambda}$$

7.3.12.4 [924] Problem 4

problem number 924

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh(\mu y)w_y = \sinh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == Sinh[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{\cosh(\lambda x)}{a\lambda}+c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(mu*y)*diff(w(x,y),y) =  sinh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{-b\mu x+2a\arctan(e^{\mu y})}{b\mu}\right)+\cosh(\lambda x)}{a\lambda}$$

7.3.12.5 [925] Problem 5

problem number 925

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh(\mu y)w_y = \cosh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[mu*y]*D[w[x, y], y] == Cosh[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\sinh(\lambda x)}{a\lambda}+c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(mu*y)*diff(w(x,y),y) =  cosh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{-b\mu x-2a\,\mathrm{arctanh}(\mathrm{e}^{\mu y})}{b\mu}\right)+\sinh(\lambda x)}{a\lambda}$$

7.3.13 5.1

Local contents

7.3.13.1 [926] Problem 1

problem number 926

Added Feb. 11, 2019.

Problem Chapter 3.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c(a\beta y-b\beta x)\log(a(\beta y+\lambda x))}{a(a\lambda+b\beta)}+\frac{cx\log(\beta y+\lambda x)}{a}-\frac{cx}{a}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{(\ln(\beta y+\lambda x)-1)(\beta y+\lambda x)c+(a\lambda+b\beta)\_F1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\beta}$$

7.3.13.2 [927] Problem 2

problem number 927

Added Feb. 11, 2019.

Problem Chapter 3.5.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x) + k\ln(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x] + k*Log[beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{abc_1\left(y-\frac{bx}{a}\right)+aky\log(\beta y)+bcx\log(\lambda x)-bcx-bkx}{ab}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x)+k*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{aky\ln(\beta y)+bcx\ln(\lambda x)+ab\_F1\left(\frac{ay-bx}{a}\right)-aky-bcx}{ab}$$

7.3.13.3 [928] Problem 3

problem number 928

Added Feb. 11, 2019.

Problem Chapter 3.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln(\lambda x)\ln(\beta y)w_y = c\ln(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]*Log[beta*y]*D[w[x, y], y] == c*Log[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\beta\lambda c_1\left(\frac{\text{li}(\beta y)}{\beta}-\frac{bx(\log(\lambda x)-1)}{a}\right)+c\left(a\lambda\text{li}(\beta y)-b\beta\left(\log\left(e^{W\left(\frac{\lambda x(\log(\lambda x)-1)}{e}\right)+1}\right)-1\right)\text{Ei}\left(\log\left(e\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)*ln(beta*y)*diff(w(x,y),y) =  c*ln(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{-\left(\text{LambertW}\left((\ln(\lambda x)-1)\,\mathrm{e}^{-1}\lambda x\right)-\ln\left(\frac{(\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda x}{\text{LambertW}((\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda x)}\right)\right)\left(\ln\left(\frac{(\ln(\lambda x)-1)\mathrm{e}^{-1}x}{\text{LambertW}((\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda\right.\right.}{}$$

7.3.13.4 [929] Problem 4

problem number 929

Added Feb. 11, 2019.

Problem Chapter 3.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda x)w_y = c\ln^m(\mu x) + s\ln^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{s\log^k\left(\frac{\beta(-b\,\mathrm{Gamma}(n+1,-\log(\lambda x))\log^n(\lambda x)(-\log(\lambda x))^{-n}+b\,\mathrm{Gamma}(n+1,-\log(\lambda K[1]))(-\log(\lambda K[1]))^{-n}\log^n}{a\lambda}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{c\ln(\_b\mu)^m+s\ln\left(\frac{\left(b\left(\int\ln(\_b\lambda)^n d\_b\right)+\left(y-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)a\right)\beta}{a}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right.\right.$$

7.3.13.5 [930] Problem 5

problem number 930

Added Feb. 11, 2019.

Problem Chapter 3.5.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda y)w_y=c\ln^m(\mu x)+s\ln^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y\frac{\log^{-n}(\lambda K[1])\left(s\log^k(\beta K[1])+c\log^m\left(\frac{\mu(-a\,\mathrm{Gamma}(1-n,-\log(\lambda y))(-\log(\lambda y))^n\log^{-n}(\lambda y)+a\,\mathrm{Gamm}}{b\lambda}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*y)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^y\frac{\left(c\ln\left(-\frac{\left(-a\left(\int\ln(\_b\lambda)^{-n}d\_b\right)+a\left(\int\ln(\lambda y)^{-n}dy\right)-bx\right)\mu}{b}\right)^m+s\ln(\_b\beta)^k\right)\ln(\_b\lambda)^{-n}}{b}d\_b+\_F1\Big(-$$

7.3.13.6 [931] Problem 6

problem number 931

Added Feb. 11, 2019.

Problem Chapter 3.5.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x+b\ln^k(\beta y)w_y=c\ln^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[lambda*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*ln(lambda*x)^n*diff(w(x,y),x) + b*ln(lambda*y)^k*diff(w(x,y),y) =  c*ln(gamma*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{c(\ln(x)+\ln(\gamma))^m \ln(\lambda x)^{-n}}{a} dx + \_F1\left(-\left(\int \ln(\lambda x)^{-n} dx\right) + \int \frac{a \ln(\lambda y)^{-k}}{b} dy\right)$$

7.3.14 5.2

Local contents

7.3.14.1 [932] Problem 1

problem number 932

Added Feb. 11, 2019.

Problem Chapter 3.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = c x^n + s \ln^k(\lambda y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + s*Log[lambda*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{x(-cx^n+ns+s)}{a(n+1)}+\frac{sy\log(\lambda y)}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*x^n+s*ln(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{bc\,x^{n+1}+(n+1)\left(b\_F1\left(\frac{ay-bx}{a}\right)+(\ln(\lambda y)-1)\,sy\right)a}{(n+1)\,ab}$$

7.3.14.2 [933] Problem 2

problem number 933

Added Feb. 11, 2019.

Problem Chapter 3.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=by^2+cx^ny+s\ln^k(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*y^2 + c*x^n*y + s*Log[lambda*x]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{(-\log(\lambda x))^{-k}\left(3(n^2+3n+2)\,s\log^k(\lambda x)\,\text{Gamma}(k+1,-\log(\lambda x))+3\lambda(n^2+3n+2)\,(-\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*y^2+c*x^n*y+s*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\left(ac\_a^{n+1}-(ax-y)\,c\_a^{n}+s\ln(\_a\lambda)^{k}+((-\_a+x)\,a-y)^{2}\,b\right)d\_a+\_F1(-ax+y)$$

Result has unresolved integrals

7.3.14.3 [934] Problem 3

problem number 934

Added Feb. 11, 2019.

Problem Chapter 3.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=bln^k(\lambda x)\ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x b\log^k(\lambda K[1])\log^n(\beta(y+a(K[1]-x)))dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}b\ln\left(\_a\lambda\right)^{k}\ln\left(-\left(\left(-\_a+x\right)a-y\right)\beta\right)^{n}d\_a+\_F1(-ax+y)$$

7.3.15 5.3

Local contents

7.3.15.1 [935] Problem 4

problem number 935

Added Feb. 11, 2019.

Problem Chapter 3.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay+bx^n)w_y=cln^k(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{c\log^k(\lambda x)(-\log(\lambda x))^{-k}\operatorname{Gamma}(k+1,-\log(\lambda x))}{\lambda}+c_1\left(ba^{-n-1}\operatorname{Gamma}(n+1,ax)+ye^{-a\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +  (a*y+b*x^n)*diff(w(x,y),y) =  c*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int c\ln(\lambda x)^k\,dx+\_F1\left(\frac{\left(-b\,x^n(ax)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}}+(n+1)\,ay\right)\mathrm{e}^{-ax}}{(n+1)\,a}\right)$$

Result has unresolved integrals

7.3.15.2 [936] Problem 5

problem number 936

Added Feb. 11, 2019.

Problem Chapter 3.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=x^k(n\ln x+m\ln y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{x^k(akm\log(y)+akn\log(x)-an-bm)}{a^2k^2}+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x) +  b*y*diff(w(x,y),y) =  x^k*(n*ln(x)+m*ln(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2a^2k^2\_F1\left(y\,x^{-\frac{b}{a}}\right) + \left(2akn\ln(x) - 2bm + \left(-\left(-2\ln\left(x^{\frac{b}{a}}\right) - 2\ln\left(y\,x^{-\frac{b}{a}}\right) + \left(i\text{csgn}(iy)^3 - ic\right.\right.\right.\right.}{}$$

7.3.15.3 [937] Problem 6

problem number 937

Added Feb. 11, 2019.

Problem Chapter 3.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^kw_x + by^nw_y = c\ln^m(\lambda x) + s\ln^l(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == c*Log[lambda*x]^m + s*Log[beta*y]^l;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{K[1]^{-k}\left(s\log^l\left(\beta\left(\frac{a(k-1)x^ky^nK[1]^k}{a(k-1)x^kyK[1]^k - b(n-1)y^n(xK[1]^k - x^kK[1])}\right)^{\frac{1}{n-1}}\right) + c\log^m(\lambda K[1])\right)}{a}dK[1]\right.\right.$$

Maple ✓

```
restart;
pde := a*x^k*diff(w(x,y),x) +  b*y^n*diff(w(x,y),y) =  c*ln(lambda*x)+s*ln(beta*y)^l;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}\frac{\left(c\ln\left(\_a\lambda\right)+s\ln\left(\beta\left(\frac{(k-1)a\,y^{-n+1}+(n-1)b\_a^{-k+1}-(n-1)b\,x^{-k+1}}{(k-1)a}\right)^{-\frac{1}{n-1}}\right)^{l}\right)\_a^{-k}}{a}d\_a+\_F1\Bigg(\frac{(k}{}$$

7.3.16 6.1

Local contents

7.3.16.1 [938] Problem 1

problem number 938

Added Feb. 11, 2019.

Problem Chapter 3.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+by^nw_y=c\sin(\lambda x)+k\sin(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x] + k*Sin[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{c\cos(\lambda x)}{a\lambda}-\frac{k\cos(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x)+k*sin(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)-ak\lambda\cos(\mu y)-bc\mu\cos(\lambda x)}{ab\lambda\mu}$$

7.3.16.2 [939] Problem 2

problem number 939

Added Feb. 11, 2019.

Problem Chapter 3.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+by^nw_y=c\sin(\lambda x+\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{c\cos(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = -\frac{c\cos(\lambda x+\mu y)}{a\lambda+\mu b} + \_F1\left(\frac{ay-bx}{a}\right)$$

7.3.16.3 [940] Problem 3

problem number 940

Added Feb. 11, 2019.

Problem Chapter 3.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x+\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{ax\cos(\lambda x+\mu y)}{\lambda x+\mu y} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = -\frac{a\cos(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}} + \_F1\left(\frac{y}{x}\right)$$

7.3.16.4 [941] Problem 4

problem number 941

Added Feb. 11, 2019.

Problem Chapter 3.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\sin^k\left(\frac{\beta\left(-b\sqrt{\cos^2(\lambda x)}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)\sec(\lambda x)\sin^{n+1}(\lambda x)+b\sqrt{\cos^2(\lambda K[1])}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda K[\right.\right.}{a\lambda(n+1)}}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*sin(lambda*x)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\sin^m(\_b\mu))+s\left(\sin^k\left(\frac{\left(b\left(\int(\sin^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\sin^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a}d\_b+\_F1\left(y-\left(\left(\int\frac{b(\right.\right.\right.$$

7.3.16.5 [942] Problem 5

problem number 942

Added Feb. 11, 2019.

Problem Chapter 3.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\sin^{-n}(\lambda K[1])\left(s\sin^k(\beta K[1])+c\sin^m\left(\frac{-a\mu\sqrt{\cos^2(\lambda y)}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\lambda y)\right)\sec(\lambda y)\sin^{1-n}(\lambda y)+\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*sin(lambda*y)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\sin^m\left(\frac{(a(\int(\sin^{-n}(\_b\lambda))d\_b)-a(\int(\sin^{-n}(\lambda y))dy)+bx)\mu}{b}\right)\right)+s\left(\sin^k(\_b\beta)\right)\right)\left(\sin^{-n}(\_b\lambda)\right)}{b}d\_b+\_$$

Result has unresolved integrals

7.3.17 6.2

Local contents

7.3.17.1 [943] Problem 1

problem number 943

Added Feb. 11, 2019.

Problem Chapter 3.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cos(\lambda x) + k\cos(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x] + k*Cos[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\sin(\lambda x)}{a\lambda}+\frac{k\sin(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x)+k*cos(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ak\lambda\sin(\mu y)+bc\mu\sin(\lambda x)}{ab\lambda\mu}$$

7.3.17.2 [944] Problem 2

problem number 944

Added Feb. 11, 2019.

Problem Chapter 3.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\sin(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{c\sin\left(\lambda x+\mu y\right)}{a\lambda+\mu b}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.17.3 [945] Problem 3

problem number 945

Added Feb. 11, 2019.

Problem Chapter 3.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\sin\left(\lambda x+\mu y\right)}{\lambda+\frac{\mu y}{x}}+\_F1\left(\frac{y}{x}\right)$$

7.3.17.4 [946] Problem 4

problem number 946

Added Feb. 11, 2019.

Problem Chapter 3.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda x)w_y = c\cos^m(\mu x) + s\cos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{s\cos^k\left(\frac{\beta\left(b\csc(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(\lambda x)\right)\sqrt{\sin^2(\lambda x)}\cos^{n+1}(\lambda x)+a\lambda(n+1)y-b\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,{}_2F_1\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cos(lambda*x)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{c(\cos^m(\_b\mu))+s\left(\cos^k\left(\frac{\left(b\left(\int(\cos^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a}d\_b+\_F1\left(y-\left(\left(\int\frac{b}{}\right.\right.\right.$$

7.3.17.5 [947] Problem 5

problem number 947

Added Feb. 11, 2019.

Problem Chapter 3.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cos^n(\lambda y)w_y=c\cos^m(\mu x)+s\cos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\cos^{-n}(\lambda K[1])\left(s\cos^k(\beta K[1])+c\cos^m\left(\frac{\mu\left(a\csc(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\lambda y)\right)\sqrt{\sin^2(\lambda y)}\cos^{1-n}(\lambda y)\right.}{}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cos(lambda*y)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\cos^m\left(\frac{\left(a\left(\int\left(\cos^{-n}(\_b\lambda)\right)d\_b\right)-a\left(\int\left(\cos^{-n}(\lambda y)\right)dy\right)+bx\right)\mu}{b}\right)\right)+s\left(\cos^k\left(\_b\beta\right)\right)\right)\left(\cos^{-n}\left(\_b\lambda\right)\right)}{b}d\_b+$$

7.3.18 6.3

Local contents

7.3.18.1 [948] Problem 1

problem number 948

Added Feb. 11, 2019.

Problem Chapter 3.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\tan(\lambda x) + k\tan(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x] + k*Tan[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{c\log(\cos(\lambda x))}{a\lambda}-\frac{k\log(\cos(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x)+k*tan(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ak\lambda\ln\left(\tan^2(\mu y)+1\right)+bc\mu\ln\left(\tan^2(\lambda x)+1\right)}{2ab\lambda\mu}$$

7.3.18.2 [949] Problem 2

problem number 949

Added Feb. 11, 2019.

Problem Chapter 3.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{c\log(\cos(\lambda x+\mu y))}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{c\ln\left(\tan^2(\lambda x+\mu y)+1\right)}{2a\lambda+2\mu b}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.18.3 [950] Problem 3

problem number 950

Added Feb. 11, 2019.

Problem Chapter 3.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{ax\log(\cos(\lambda x+\mu y))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\ln\left(\tan^2\left(\lambda x+\mu y\right)+1\right)}{2\lambda+\frac{2\mu y}{x}}+\_F1\left(\frac{y}{x}\right)$$

7.3.18.4 [951] Problem 4

problem number 951

Added Feb. 11, 2019.

Problem Chapter 3.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda x)w_y=c\tan^m(\mu x)+s\tan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{s\tan^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda x)\right)\tan^{n+1}(\lambda x)+b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda K[1])\right)\tan^{n+1}(\lambda K[1])+a\lambda(n+1\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(lambda*x)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{c(\tan^m(\_b\mu))+s\left(\frac{\tan\left(\left(y-\left(\int\frac{b(\tan^n(\lambda x))}{a}dx\right)\right)\beta\right)+\tan\left(\frac{b\beta\left(\int(\tan^n(\_b\lambda))d\_b\right)}{a}\right)}{-\tan\left(\left(y-\left(\int\frac{b(\tan^n(\lambda x))}{a}dx\right)\right)\beta\right)\tan\left(\frac{b\beta\left(\int(\tan^n(\_b\lambda))d\_b\right)}{a}\right)+1}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\right.\right.$$

7.3.18.5 [952] Problem 5

problem number 952

Added Feb. 11, 2019.

Problem Chapter 3.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda y)w_y=c\tan^m(\mu x)+s\tan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^y \frac{\tan^{-n}(\lambda K[1]) \left(s\tan^k(\beta K[1]) + c\tan^m\left(\frac{-a\mu\, {}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(\lambda y)\right)\tan^{1-n}(\lambda y)+a\mu\, {}_2F_1\left(1,\frac{1}{2}-\right.}{b\lambda-b\lambda n}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(lambda*y)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{\left(c\left(\frac{-\tan\left(\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)-\tan\left(\frac{a\mu\left(\int\left(\tan^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)}{\tan\left(\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\tan\left(\frac{a\mu\left(\int\left(\tan^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)-1}\right)^m + s\left(\tan^k(\_b\beta)\right)\right)\left(\tan^{-n}(\_b\right.}{b}$$

7.3.19 6.4

Local contents

7.3.19.1 [953] Problem 1

problem number 953

Added Feb. 11, 2019.

Problem Chapter 3.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cot(\lambda x) + k\cot(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x] + k*Cot[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ak\lambda\log(\tan(\mu y))+ak\lambda\log(\cos(\mu y))+bc\mu\log(\sin(\lambda x))}{ab\lambda\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x)+k*cot(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)-ak\lambda\ln\left(\cot^2(\mu y)+1\right)-bc\mu\ln\left(\cot^2(\lambda x)+1\right)}{2ab\lambda\mu}$$

7.3.19.2 [954] Problem 2

problem number 954

Added Feb. 11, 2019.

Problem Chapter 3.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b y^n w_y = c\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{c\ln\left(\cot^2\left(\lambda x+\mu y\right)+1\right)}{2a\lambda+2\mu b}+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.19.3 [955] Problem 3

problem number 955

Added Feb. 11, 2019.

Problem Chapter 3.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + y w_y = a x\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=-\frac{a\ln\left(\cot^2\left(\lambda x+\mu y\right)+1\right)}{2\left(\lambda+\frac{\mu y}{x}\right)}+\_F1\left(\frac{y}{x}\right)$$

7.3.19.4 [956] Problem 4

problem number 956

Added Feb. 11, 2019.

Problem Chapter 3.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cot^n(\lambda x)w_y=c\cot^m(\mu x)+s\cot^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{s\cot^k\left(\frac{\beta\left(b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda x)\right)\cot^{n+1}(\lambda x)+a\lambda(n+1)y-b\cot^{n+1}(\lambda K[1])\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda K[1])\right)\right)}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cot(lambda*x)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{c(\cot^m(\_b\mu))+s\left(\frac{\cot\left(\left(y-\left(\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\right)\beta\right)\cot\left(\frac{b\beta\left(\int(\cot^n(\_b\lambda))d\_b\right)}{a}\right)-1}{\cot\left(\left(y-\left(\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\right)\beta\right)+\cot\left(\frac{b\beta\left(\int(\cot^n(\_b\lambda))d\_b\right)}{a}\right)}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\frac{b(}{}\right.\right.$$

7.3.19.5 [957] Problem 5

problem number 957

Added Feb. 11, 2019.

Problem Chapter 3.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cot^n(\lambda y)w_y=c\cot^m(\mu x)+s\cot^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\left(s\cot^k(\beta K[1])+c\cot^m\left(\frac{a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\cot^2(\lambda y)\right)\cot^{1-n}(\lambda y)+b\lambda\mu x-b\lambda\mu nx-a\mu\cot^{1-n}(\lambda K[1])\,{}_2}{b\lambda-b\lambda n}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cot(lambda*y)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\frac{\cot\left(\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\cot\left(\frac{a\mu\left(\int\left(\cot^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)-1}{\cot\left(\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)+\cot\left(\frac{a\mu\left(\int\left(\cot^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)}\right)^m+s\left(\cot^k\left(\_b\beta\right)\right)\right)\left(\cot^{-n}\left(\_b\lambda\right.\right.}{b}$$

7.3.20 6.5

Local contents

7.3.20.1 [958] Problem 1

problem number 958

Added Feb. 11, 2019.

Problem Chapter 3.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sin(\lambda x) + c\cos(\mu y) + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x] + c*Cos[mu*y] + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{kx}{a}-\frac{\cos(\lambda x)}{a\lambda}+\frac{c\sin(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)+c*cos(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{kx}{a}+\frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right)+ac\lambda\sin(\mu y)-b\mu\cos(\lambda x)}{ab\lambda\mu}$$

7.3.20.2 [959] Problem 2

problem number 959

Added Feb. 11, 2019.

Problem Chapter 3.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \tan(\lambda x) + c\sin(\mu y) + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Tan[lambda*x] + c*Sin[mu*y] + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{k\lambda x-\log(\cos(\lambda x))}{a\lambda}-\frac{c\cos(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  tan(lambda*x)+c*sin(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{kx}{a}+\_F1\left(\frac{ay-bx}{a}\right)-\frac{c\cos(\mu y)}{b\mu}-\frac{\ln(\cos(\lambda x))}{a\lambda}$$

7.3.20.3 [960] Problem 3

problem number 960

Added Feb. 11, 2019.

Problem Chapter 3.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sin(\lambda x)\cos(\mu y) + c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x]*Cos[mu*y] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{(a^3\lambda^2-ab^2\mu^2)\,c_1\left(y-\frac{bx}{a}\right)+a^2c\lambda^2x-a^2\lambda\cos(\lambda x)\cos(\mu y)-ab\mu\sin(\lambda x)\sin(\mu y)-b^2c\mu^2x}{a(a\lambda-b\mu)(a\lambda+b\mu)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)*cos(mu*y)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}+\_F1\left(\frac{ay-bx}{a}\right)-\frac{(a\lambda+\mu b)\cos\left(\lambda x-\mu y\right)+(a\lambda-\mu b)\cos\left(\lambda x+\mu y\right)}{2\left(a\lambda-\mu b\right)\left(a\lambda+\mu b\right)}$$

7.3.20.4 [961] Problem 4

problem number 961

Added Feb. 11, 2019.

Problem Chapter 3.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\sin(\mu y)w_y=\cos(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == Cos[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{a\lambda c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)+c\lambda x+\sin(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +  b*sin(mu*y)*diff(w(x,y),y) =  cos(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}\right)\right)\right)}{b\mu}\right)+c\lambda x+\sin(\lambda x)}{a\lambda}$$

7.3.20.5 [962] Problem 5

problem number 962

Added Feb. 11, 2019.

Problem Chapter 3.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan(\mu y)w_y=\sin(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Sin[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\lambda x-\cos(\lambda x)}{a\lambda}+c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  sin(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{-b\mu x+a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)+c\lambda x-\cos(\lambda x)}{a\lambda}$$

7.3.20.6 [963] Problem 6

problem number 963

Added Feb. 11, 2019.

Problem Chapter 3.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan(\mu y)w_y=\cot(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Cot[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)+c\lambda x+\log(\sin(\lambda x))}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  cot(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2a\lambda\_F1\left(\frac{-b\mu x+a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)+2c\lambda x-\ln\left(\cot^2(\lambda x)+1\right)}{2a\lambda}$$

7.3.21 7.1

Local contents

7.3.21.1 [964] Problem 1

problem number 964

Added Feb. 11, 2019.

Problem Chapter 3.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arcsin\frac{x}{\lambda} + k\arcsin\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[x/lambda] + k*ArcSin[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{a^2b\beta c_1\left(y-\frac{bx}{a}\right)-\frac{bkx\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{aky\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}}{a^2b\beta}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arcsin(x/lambda)+k*arcsin(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{bcx\arcsin\left(\frac{x}{\lambda}\right)+ab\_F1\left(\frac{ay-bx}{a}\right)+\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda+\left(y\arcsin\left(\frac{y}{\beta}\right)+\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,\beta\right)ak}{ab}$$

7.3.21.2 [965] Problem 2

problem number 965

Added Feb. 11, 2019.

Problem Chapter 3.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}+(\beta y+\lambda x)\sin^{-1}(\beta y+\lambda x)\right)}{a\lambda+b\beta}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{(\beta y+\lambda x)\,c\arcsin\left(\beta y+\lambda x\right)+\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\,c+(a\lambda+b\beta)\_F1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\beta}$$

7.3.21.3 [966] Problem 3

problem number 966

Added Feb. 11, 2019.

Problem Chapter 3.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax \arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to ax\left(\frac{\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}}{\beta y+\lambda x}+\sin^{-1}(\beta y+\lambda x)\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{\sqrt{-\left(\frac{\beta y}{x}+\lambda\right)^2x^2+1}\,ax+(\beta y+\lambda x)\left(ax\arcsin(\beta y+\lambda x)+\_F1\left(\frac{y}{x}\right)\right)}{\beta y+\lambda x}$$

7.3.21.4 [967] Problem 4

problem number 967

Added Feb. 11, 2019.

Problem Chapter 3.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda x)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\left(\frac{\sin^{-1}\left(\frac{\beta\left(ib\sin^{-1}(\lambda x)^n\left((i\sin^{-1}(\lambda x))^n\operatorname{Gamma}(n+1,-i\sin^{-1}(\lambda x))-(-i\sin^{-1}(\lambda x))^n\operatorname{Gamma}(n+1,i\sin^{-1}(\lambda x)\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*x)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^x\left(\arcsin(\_a\mu)^m+\frac{\arcsin\left(\frac{\left(\sqrt{-\_a^2\lambda^2+1}\,b-\sqrt{-\lambda^2x^2+1}\,b+(\_ab\arcsin(\_a\lambda)-bx\arcsin(\lambda x)+ay)\lambda\right)\beta}{a\lambda}\right)^k}{a}\right)d\_$$

7.3.21.5 [968] Problem 5

problem number 968

Added Feb. 11, 2019.

Problem Chapter 3.7.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda y)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^y \frac{\left(\sin^{-1}(\beta K[1])^k + a\sin^{-1}\left(\frac{\mu(ia((-i\sin^{-1}(\lambda y))^n \operatorname{Gamma}(1-n,-i\sin^{-1}(\lambda y)) - (i\sin^{-1}(\lambda y))^n \operatorname{Gamma}(1-n\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*y)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^y \frac{a\left(-\arcsin\left(\frac{(-b\lambda x - a\operatorname{cosineIntegral}(\arcsin(\_a\lambda)) + a\operatorname{cosineIntegral}(\arcsin(\lambda y)))\mu}{b\lambda}\right)\right)^m + \arcsin(\_a\beta)^k}{b\arcsin(\_a\lambda)} d\_a +$$

7.3.22 7.2

Local contents

7.3.22.1 [969] Problem 1

problem number 969

Added Feb. 11, 2019.

Problem Chapter 3.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \arccos \frac{x}{\lambda} + k \arccos \frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[x/lambda] + k*ArcCos[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{a^2 b\beta c_1\left(y-\frac{bx}{a}\right)+\frac{bkx\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\frac{ay}{\sqrt{a^2\left(\beta^2-y^2\right)}}\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{aky\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(-\right.}{\sqrt{1-\frac{y^2}{\beta^2}}}}{a^2b\beta}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccos(x/lambda)+k*arccos(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{bcx\arccos\left(\frac{x}{\lambda}\right)+ab\_F1\left(\frac{ay-bx}{a}\right)-\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda-\left(-y\arccos\left(\frac{y}{\beta}\right)+\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,\beta\right)ak}{ab}$$

7.3.22.2 [970] Problem 2

problem number 970

Added Feb. 11, 2019.

Problem Chapter 3.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\arccos(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{c\left(\beta(bx-ay)\sin^{-1}(\beta y+\lambda x)+x(a\lambda+b\beta)\cos^{-1}(\beta y+\lambda x)+a\left(-\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x\right.\right.}{a(a\lambda+b\beta)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{(\beta y+\lambda x)\,c\arccos(\beta y+\lambda x)-\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\,c+(a\lambda+b\beta)\_F1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\beta}$$

7.3.22.3 [971] Problem 3

problem number 971

Added Feb. 11, 2019.

Problem Chapter 3.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\arccos(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to ax\left(\cos^{-1}(\beta y+\lambda x)-\frac{\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}}{\beta y+\lambda x}\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{-\sqrt{-\left(\frac{\beta y}{x}+\lambda\right)^2 x^2+1}\, ax + (\beta y+\lambda x)\left(ax \arccos\left(\beta y+\lambda x\right) + \_F1\left(\frac{y}{x}\right)\right)}{\beta y+\lambda x}$$

7.3.22.4 [972] Problem 4

problem number 972

Added Feb. 11, 2019.

Problem Chapter 3.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda x)w_y = c\arccos^m(\mu x) + s\arccos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \left(\frac{\cos^{-1}\left(\frac{\beta\left(\cos^{-1}(\lambda K[1])^2\right)^{-n}\left(\left(\cos^{-1}(\lambda K[1])^2\right)^n\left(-b\left(i\cos^{-1}(\lambda x)\right)^n \operatorname{Gamma}\left(n+1,-i\cos^{-1}(\lambda x)\right)\cos^{-1}(\lambda x)^n - b(\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccos(lambda*x)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\left(\arccos\left(\_a\mu\right)^{m}+\frac{\arccos\left(\frac{\left(-\sqrt{-\_a^{2}\lambda^{2}+1}\,b+\sqrt{-\lambda^{2}x^{2}+1}\,b+\left(\_ab\arccos(\_a\lambda)-bx\arccos(\lambda x)+ay\right)\lambda\right)\beta}{a\lambda}\right)^{k}}{a}\right)d$$

7.3.22.5 [973] Problem 5

problem number 973

Added Feb. 11, 2019.

Problem Chapter 3.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arccos^n(\lambda y)w_y=c\arccos^m(\mu x)+s\arccos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y\frac{\left(\cos^{-1}(\beta K[1])^k+a\cos^{-1}\left(-\frac{a\mu\cos^{-1}(\lambda K[1])^{-n}\left(-\text{Gamma}\left(1-n,-i\cos^{-1}(\lambda K[1])\right)\left(-i\cos^{-1}(\lambda K[1])\right)^{n}\right.}{}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccos(lambda*y)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int^{y} \frac{a\arccos\left(\frac{(b\lambda x - a\,\mathrm{sinIntegral}(\arccos(\_a\lambda)) + a\,\mathrm{sinIntegral}(\arccos(\lambda y)))\mu}{b\lambda}\right)^{m} + \arccos(\_a\beta)^{k}}{b\arccos(\_a\lambda)} d\_a + \_F1\left(\frac{b\lambda x}{\right.$$

7.3.23 7.3

Local contents

7.3.23.1 [974] Problem 1

problem number 974

Added Feb. 11, 2019.

Problem Chapter 3.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arctan\frac{x}{\lambda} + k\arctan\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[x/lambda] + k*ArcTan[y/beta];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{a\beta k\log\left(a^2(\beta^2+y^2)\right)-2aky\tan^{-1}\left(\frac{y}{\beta}\right)+bc\lambda\log\left(\lambda^2+x^2\right)-2bcx\tan^{-1}\left(\frac{x}{\lambda}\right)}{2ab}+c_1\left(y-\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arctan(x/lambda)+k*arctan(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{-a\beta k\ln\left(\frac{\beta^2+y^2}{\beta^2}\right)+2aky\arctan\left(\frac{y}{\beta}\right)-bc\lambda\ln\left(\frac{x^2}{\lambda^2}+1\right)+2bcx\arctan\left(\frac{x}{\lambda}\right)+2ab\_F1\left(\frac{ay-bx}{a}\right)}{2ab}$$

7.3.23.2 [975] Problem 2

problem number 975

Added Feb. 11, 2019.

Problem Chapter 3.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\arctan(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{2(\beta y+\lambda x)\,c\arctan\left(\beta y+\lambda x\right)-c\ln\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)+(2a\lambda+2b\beta)\_F1\left(\frac{ay-bx}{a}\right)}{2a\lambda+2b\beta}$$

7.3.23.3 [976] Problem 3

problem number 976

Added Feb. 11, 2019.

Problem Chapter 3.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\arctan(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)-\frac{ax\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2(\beta y+\lambda x)}+ax\tan^{-1}(\beta y+\lambda x)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-ax\ln\left(\left(\frac{\beta y}{x}+\lambda\right)^2x^2+1\right)+2(\beta y+\lambda x)\left(ax\arctan\left(\beta y+\lambda x\right)+\_F1\left(\frac{y}{x}\right)\right)}{2\beta y+2\lambda x}$$

7.3.23.4 [977] Problem 4

problem number 977

Added Feb. 11, 2019.

Problem Chapter 3.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arctan^n(\lambda x)w_y=c\arctan^m(\mu x)+s\arctan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\left(\frac{\tan^{-1}\left(\beta\left(y-\int_1^x\frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]}\frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1]\right)\right)^k}{a}+\tan^{-1}(\mu K[2])\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arctan(lambda*x)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\arctan(\_a\mu)^{m} + \frac{\arctan\left(\frac{\left(-\frac{b\ln(\_a^2\lambda^2+1)}{2}+\frac{b\ln(\lambda^2x^2+1)}{2}+(\_ab\arctan(\_a\lambda)-bx\arctan(\lambda x)+ay)\lambda\right)\beta}{a\lambda}\right)^{k}}{a} \right.$$

7.3.23.5 [978] Problem 5

problem number 978

Added Feb. 11, 2019.

Problem Chapter 3.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda y)w_y = c\arctan^m(\mu x) + s\arctan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^y \frac{\tan^{-1}(\lambda K[2])^{-n}\left(\tan^{-1}(\beta K[2])^k + a\tan^{-1}\left(\frac{\mu\left(bx-a\int_1^y \tan^{-1}(\lambda K[1])^{-n}dK[1]+a\int_1^{K[2]}\tan^{-1}(\lambda K[1\right.}{b}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arctan(lambda*y)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{y}\frac{a\arctan\left(\frac{a\mu\left(\int\frac{1}{\arctan(\_b\lambda)}d\_b\right)}{b}+\left(x-\left(\int\frac{a}{b\arctan(\lambda y)}dy\right)\right)\mu\right)^{m}+\arctan(\_b\beta)^{k}}{b\arctan(\_b\lambda)}d\_b+\_F1\Big(x$$

7.3.24 7.4

Local contents

7.3.24.1 [979] Problem 1

problem number 979

Added Feb. 11, 2019.

Problem Chapter 3.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\operatorname{arccot}\frac{x}{\lambda}+k\operatorname{arccot}\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[x/lambda] + k*ArcCot[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2abc_1\left(y-\frac{bx}{a}\right)+a\beta k\log\left(a^2(\beta^2+y^2)\right)-2aky\tan^{-1}\left(\frac{y}{\beta}\right)+2bkx\tan^{-1}\left(\frac{y}{\beta}\right)+2bkx\cot^{-1}\left(\right.}{2ab}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccot(x/lambda)+k*arccot(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\beta k\ln\left(\frac{\beta^2+y^2}{\beta^2}\right)-2aky\arctan\left(\frac{y}{\beta}\right)+bc\lambda\ln\left(\frac{x^2}{\lambda^2}+1\right)-2bcx\arctan\left(\frac{x}{\lambda}\right)+2ab\_F1\left(\frac{ay-bx}{a}\right)+\pi(}{2ab}$$

7.3.24.2 [980] Problem 2

problem number 980

Added Feb. 11, 2019.

Problem Chapter 3.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\operatorname{arccot}(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(a\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right)+2\beta(bx-ay)\tan^{-1}(\beta y+\lambda x)+2x(a\lambda+b\beta)\cot^{-1}}{2a(a\lambda+b\beta)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ac\ln\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)-2\left(\left(\beta y+\lambda x\right)a\arctan\left(\beta y+\lambda x\right)-\frac{\pi(a\lambda+b\beta)x}{2}\right)c+\left(2\lambda a^2+2\right.}{2\left(a\lambda+b\beta\right)a}$$

7.3.24.3 [981] Problem 3

problem number 981

Added Feb. 11, 2019.

Problem Chapter 3.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\operatorname{arccot}(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to ax\left(\frac{\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2\beta y+2\lambda x}+\cot^{-1}(\beta y+\lambda x)\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ax \ln\left(\left(\frac{\beta y}{x}+\lambda\right)^2 x^2+1\right)+(\beta y+\lambda x)\left(-2ax \arctan(\beta y+\lambda x)+\pi ax+2\_F1\left(\frac{y}{x}\right)\right)}{2\beta y+2\lambda x}$$

7.3.24.4 [982] Problem 4

problem number 982

Added Feb. 11, 2019.

Problem Chapter 3.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b \operatorname{arccot}^n(\lambda x) w_y = c \operatorname{arccot}^m(\mu x) + s \operatorname{arccot}^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \left(\frac{\cot^{-1}\left(\beta\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]} \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]\right)\right)^k}{a}+\cot^{-1}(\mu K[2])^m\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccot(lambda*x)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\left(-\arctan(\_a\mu) + \frac{\pi}{2}\right)^{m} + \frac{\left(-\arctan\left(\frac{\left(\frac{b\ln(\_a^2\lambda^2+1)}{2} - \frac{b\ln(\lambda^2x^2+1)}{2} + \left(-\_ab\arctan(\_a\lambda)+bx\arctan(\right.\right.}{a\lambda}\right.\right.}{a}\right.$$

7.3.24.5 [983] Problem 5

problem number 983

Added Feb. 11, 2019.

Problem Chapter 3.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \operatorname{arccot}^n(\lambda y) w_y = c \operatorname{arccot}^m(\mu x) + s \operatorname{arccot}^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^y \frac{\cot^{-1}(\lambda K[2])^{-n}\left(\cot^{-1}(\beta K[2])^k + a\cot^{-1}\left(\frac{\mu\left(bx - a\int_1^y \cot^{-1}(\lambda K[1])^{-n}dK[1] + a\int_1^{K[2]}\cot^{-1}(\lambda K[1])\right.}{b}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccot(lambda*y)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a\left(-\arctan\left(\frac{2a\mu\left(\int \frac{1}{-2\arctan(\_b\lambda)+\pi}d\_b\right)}{b} + \left(x - \left(\int \frac{2a}{(-2\arctan(\lambda y)+\pi)b}dy\right)\right)\mu\right) + \frac{\pi}{2}\right)^{m} + \left(-\mathrm{arcta}\right.}{b\,\mathrm{arccot}\left(\_b\lambda\right)}$$

7.3.25 8.1

Local contents

7.3.25.1 [984] Problem 1

problem number 984

Added Feb. 11, 2019.

Problem Chapter 3.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{f(K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int \frac{f(x)}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.25.2 [985] Problem 2

problem number 985

Added Feb. 11, 2019.

Problem Chapter 3.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = yf(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1])(y+a(K[1]-x))dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +  a*diff(w(x,y),y) =  y*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x -((-\_a+x)\,a-y)\,f(\_a)\,d\_a+\_F1(-ax+y)$$

7.3.25.3 [986] Problem 3

problem number 986

Added Feb. 11, 2019.

Problem Chapter 3.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=y^2f(x)+yg(x)+h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y^2*f[x] + y*g[x] + h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \left(f(K[1])(y+a(K[1]-x))^2+g(K[1])(y+a(K[1]-x))+h(K[1])\right)dK[1]+c_1(y-ax$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^2*f(x)+y*g(x)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}\left(\left(\left(-\_a+x\right)a-y\right)^{2}f(\_a)+\left(\left(\_a-x\right)a+y\right)g(\_a)+h(\_a)\right)d\_a+\_F1(-ax+y)$$

7.3.25.4 [987] Problem 4

problem number 987

Added Feb. 11, 2019.

Problem Chapter 3.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=y^kf(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y^k*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x f(K[1])(y+a(K[1]-x))^k dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^k*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x}\left(\left(\_a-x\right)a+y\right)^{k}f(\_a)\,d\_a+\_F1(-ax+y)$$

7.3.25.5 [988] Problem 5

problem number 988

Added Feb. 11, 2019.

Problem Chapter 3.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = e^{\lambda y} f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == Exp[lambda*y]*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x e^{\lambda(y+a(K[1]-x))} f(K[1])dK[1] + c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  exp(lambda*y)*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='real
```

$$w(x,y) = \int^x \mathrm{e}^{-((-\_a+x)a-y)\lambda} f(\_a)\, d\_a + \_F1(-ax+y)$$

7.3.25.6 [989] Problem 6

problem number 989

Added Feb. 11, 2019.

Problem Chapter 3.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x g(K[2])dK[2]+c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int g(x)\,dx+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right)\right)$$

7.3.25.7 [990] Problem 7

problem number 990

Added Feb. 11, 2019.

Problem Chapter 3.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay+f(x))w_y=y^k g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == y^k*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x g(K[2])\left(e^{aK[2]}\left(e^{-ax}y-\int_1^x e^{-aK[1]}f(K[1])dK[1]+\int_1^{K[2]} e^{-aK[1]}f(K[1])dK[1]\right)\right)^k d$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  y^k*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\left(\left(y\,\mathrm{e}^{-ax}+\int \mathrm{e}^{-\_ba}f(\_b)\,d\_b-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right)\right)\mathrm{e}^{-ba}\right)^{k}g(\_b)\,d\_b+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\right.\right.$$

7.3.25.8 [991] Problem 8

problem number 991

Added Feb. 11, 2019.

Problem Chapter 3.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+y^k w_y=g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{g(K[2])}{f(K[2])}dK[2]+c_1\left(-\int_1^x\frac{1}{f(K[1])}dK[1]-\frac{y^{1-k}}{k-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + y^k*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int g(x)\,dx+\_F1\left(\left((k-1)\,x\,y^k+y\right)y^{-k}\right)$$

7.3.25.9 [992] Problem 9

problem number 992

Added Feb. 11, 2019.

Problem Chapter 3.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+a)w_y = by + c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == b*y + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{c + b\exp\left(\int_1^{K[3]} \frac{1}{f(K[1])} dK[1]\right)\left(\exp\left(-\int_1^x \frac{1}{f(K[1])} dK[1]\right) y - \int_1^x \frac{a\exp\left(-\int_1^{K[2]} \frac{1}{f(K[1])} dK[1]\right)}{f(K[2])} d\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y+a)*diff(w(x,y),y) =  b*y+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = cx + ((-x+1)\,a + y)\,b + \_F1\left((a+y)\,\mathrm{e}^{-x}\right)$$

7.3.25.10 [993] Problem 10

problem number 993

Added Feb. 11, 2019.

Problem Chapter 3.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+ax)w_y = g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{g(K[3])}{f(K[3])}dK[3]+c_1\left(y\exp\left(-\int_1^x\frac{1}{f(K[1])}dK[1]\right)-\int_1^x\frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK[1]\right)}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y+a*x)*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int g(x)\,dx+\_F1\left((ax+a+y)\,\mathrm{e}^{-x}\right)$$

7.3.25.11 [994] Problem 11

problem number 994

Added Feb. 11, 2019.

Problem Chapter 3.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(yg_1(x)+g_0(x))w_y=y^2h_2(x)+yh_1(x)+h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y*g1[x] + g0[x])*D[w[x, y], y] == y^2*h2[x] + y*h1[x] + h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{\exp\left(2\int_1^{K[3]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\mathrm{h2}(K[3])\left(\exp\left(-\int_1^x\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)y-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\mathrm{g1}(K[1])}{f(K[1])}\right.}{f(K[2}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y*g1(x)+g0(x))*diff(w(x,y),y) =   y^2*h2(x)+y*h1(x)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\left(y^2\mathrm{e}^{2\left(\int g1(\_f)d\_f\right)}\mathrm{e}^{\int -2g1(x)dx}h2(\_f)+y\,\mathrm{e}^{-\left(\int g1(x)dx\right)}\mathrm{e}^{\int g1(\_f)d\_f}h1(\_f)+\left(\int \mathrm{e}^{-\left(\int g1(\_f)d\_f\right)}g0(\right.\right.$$

7.3.25.12 [995] Problem 12

problem number 995

Added Feb. 11, 2019.

Problem Chapter 3.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(yg_1(x)+y^kg_2(x))w_y=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y*g1[x] + y^k*g2[x])*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{h(K[3])}{f(K[3])}dK[3]+c_1\left((k-1)\int_1^x\frac{\exp\left((k-1)\int_1^{K[2]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\text{g2}(K[2])}{f(K[2])}dK[2]+y^{1-}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y*g1(x)+y^k*g2(x))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int h(x)\,dx+\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int g1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int g1(x)dx\right)}g2(x)\,dx\right)\right)$$

7.3.25.13 [996] Problem 13

problem number 996

Added Feb. 11, 2019.

Problem Chapter 3.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)+e^{\lambda y}g_2(x))w_y=h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x] + Exp[lambda*y])*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := f(x)*diff(w(x,y),x) +(g1(x)+exp(lambda*y))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{h(x)}{f(x)}dx + \_F1\left(\frac{-\lambda\left(\int \frac{e^{\lambda\left(\int \frac{g1(x)}{f(x)}dx\right)}}{f(x)}dx\right) - e^{-\left(y-\left(\int \frac{g1(x)}{f(x)}dx\right)\right)\lambda}}{\lambda}\right)$$

7.3.25.14 [997] Problem 14

problem number 997

Added Feb. 11, 2019.

Problem Chapter 3.8.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^k f(x)w_x + g(x)w_y = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^k*f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{h(K[2])\left(\left(y^{k+1} - (k+1)\int_1^x \frac{g(K[1])}{f(K[1])}dK[1] + (k+1)\int_1^{K[2]} \frac{g(K[1])}{f(K[1])}dK[1]\right)^{\frac{1}{k+1}}\right)^{-k}}{f(K[2])}dK[2] +$$

Maple ✓

```
restart;
pde := y^k*f(x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{\left(\left(y^{k+1}+\int\frac{(-k-1)g(x)}{f(x)}dx+\int\frac{(k+1)g(\_b)}{f(\_b)}d\_b\right)^{\frac{1}{k+1}}\right)^{-k}h(\_b)}{f(\_b)}d\_b+\_F1\left(y\,y^{k}+(-k-1)\left(\int\right.\right.$$

7.3.25.15 [998] Problem 15

problem number 998

Added Feb. 11, 2019.

Problem Chapter 3.8.1.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^{k}f(x)w_{x}+(y^{k+1}g_{1}(x)+g_{0}(x))w_{y}=y^{3k+1}h_{2}(x)+y^{2k+1}h_{1}(x)+y^{k}h_{0}(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^k*f[x]*D[w[x, y], x] + (y^(k + 1)*g1[x] + g0[x])*D[w[x, y], y] == y^(3*k + 1)*h2[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_{1}^{x}\frac{\mathrm{h1}(K[3])\left(\left(\exp\left(-\left((k+1)\left(\int_{1}^{x}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]-\int_{1}^{K[3]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\right)\right)\left(y^{k+1}-\exp\left(\right.\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := y^k*f(x)*diff(w(x,y),x) +(y^(k+1)* g1(x) + g0(x))*diff(w(x,y),y) =   y^(3*k +1)*h2(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\left(\left(\left(y^{k+1} \mathrm{e}^{-(k+1)\left(\int \frac{g1(x)}{f(x)} dx\right)} + (k+1)\left(\int \frac{\mathrm{e}^{-(k+1)\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)} g0(\_f)}{f(\_f)} d\_f\right) + (-k-1)\left(\int \frac{\mathrm{e}^{-(k+1)\left(\right.}}{}\right.\right.\right.\right.\right.}{}$$

7.3.25.16 [999] Problem 16

problem number 999

Added Feb. 11, 2019.

Problem Chapter 3.8.1.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda x}w_x + g(x)w_y = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*Exp[lambda*x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{e^{-\lambda K[2]}h(K[2])}{f(K[2])}dK[2] + c_1\left(y - \int_1^x \frac{e^{-\lambda K[1]}g(K[1])}{f(K[1])}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := f(x)*exp(lambda*x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int \frac{e^{-\lambda x} h(x)}{f(x)}dx+\_F1\left(y-\left(\int \frac{e^{-\lambda x} g(x)}{f(x)}dx\right)\right)$$

7.3.26 8.2

Local contents

7.3.26.1 [1000] Problem 1

problem number 1000

Added Feb. 11, 2019.

Problem Chapter 3.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = f(x) + g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x] + g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +b*diff(w(x,y),y) =   f(x)+g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{f(\_a)+g\left(\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.26.2 [1001] Problem 2

problem number 1001

Added Feb. 11, 2019.

Problem Chapter 3.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x f(K[1])g(-ax+y+aK[1])dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =   f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x} f(\_a)\,g((\_a-x)\,a+y)\,d\_a+\_F1(-ax+y)$$

7.3.26.3 [1002] Problem 3

problem number 1002

Added Feb. 11, 2019.

Problem Chapter 3.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay+f(x))w_y=g(x)h(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x g(K[2])h\left(e^{aK[2]}\left(e^{-ax}y-\int_1^x e^{-aK[1]}f(K[1])dK[1]+\int_1^{K[2]} e^{-aK[1]}f(K[1])dK[1]\right)\right)dK$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +(a*y+f(x) )*diff(w(x,y),y) =   g(x)*h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^{x} g(\_b)\,h\left(\left(y\,\mathrm{e}^{-ax}+\int \mathrm{e}^{-\_ba}f(\_b)\,d\_b-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right)\right)\mathrm{e}^{ba}\right)d\_b+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\right.\right.$$

7.3.26.4 [1003] Problem 4

problem number 1003

Added Feb. 11, 2019.

Problem Chapter 3.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = h_1(x) + h_2(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == h1[x] + h2[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +g(y)*diff(w(x,y),y) =   h1(x)+h2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h1(\_f) + h2\left(\mathrm{RootOf}\left(\int \frac{1}{f(\_f)}d\_f - \left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy - \left(\int^{\_Z} \frac{1}{g(\_a)}d\_a\right)\right)\right)}{f(\_f)} d\_f + \_F1$$

7.3.26.5 [1004] Problem 5

problem number 1004

Added Feb. 11, 2019.

Problem Chapter 3.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x + (f_2(x)y + y^k f_3(x))w_y = g(x)h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y], x] + (y*f2[x] + y^k*f3[x])*D[w[x, y], y] == g[x]*h[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{g(K[3])h(K[3])}{\text{f1}(K[3])}dK[3]+c_1\left((k-1)\int_1^x\frac{\exp\left((k-1)\int_1^{K[2]}\frac{\text{f2}(K[1])}{\text{f1}(K[1])}dK[1]\right)\text{f3}(K[2])}{\text{f1}(K[2])}dK[2\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y),x) +(y*f2(x)+y^k*f3(x))*diff(w(x,y),y) =  g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int\frac{g(x)\,h(x)}{f1\,(x)}dx+\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int\frac{f2(x)}{f1(x)}dx\right)}+(k-1)\left(\int\frac{\mathrm{e}^{(k-1)\left(\int\frac{f2(x)}{f1(x)}dx\right)}f3(x)}{f1\,(x)}dx\right)\right)$$

7.3.26.6 [1005] Problem 6

problem number 1005

Added Feb. 11, 2019.

Problem Chapter 3.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(x)w_x+f_2(x)g_2(x)w_y=h_1(x)h_2(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*g1[x]*D[w[x, y], x] + f2[x]*g2[x]*D[w[x, y], y] == h1[x]*h2[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{\text{h1}(K[2])\text{h2}(K[2])}{\text{f1}(K[2])\text{g1}(K[2])}dK[2]+c_1\left(y-\int_1^x\frac{\text{f2}(K[1])\text{g2}(K[1])}{\text{f1}(K[1])\text{g1}(K[1])}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f1(x)*g1(x)*diff(w(x,y),x) +f2(x)*g2(x)*diff(w(x,y),y) =  h1(x)*h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int\frac{h1(x)\,h2(x)}{f1\,(x)\,g1\,(x)}dx+\_F1\left(y-\left(\int\frac{f2(x)\,g2(x)}{f1\,(x)\,g1\,(x)}dx\right)\right)$$

7.3.26.7 [1006] Problem 7

problem number 1006

Added Feb. 11, 2019.

Problem Chapter 3.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x+f_2(x)g_2(y)w_y=h_1(x)+h_2(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x] + h2[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h1(x)+h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{h1(\_f)+h2(\_f)}{f1(\_f)\,g1\left(\mathrm{RootOf}\left(\int\frac{f2(\_f)}{f1(\_f)}d\_f-\left(\int\frac{f2(x)}{f1(x)}dx\right)+\int\frac{g1(y)}{g2(y)}dy-\left(\int^{\_Z}\frac{g1(\_a)}{g2(\_a)}d\_a\right)\right)\right)}d\_f+\_F$$

7.3.27 8.3

Local contents

7.3.27.1 [1007] Problem 1

problem number 1007

Added Feb. 11, 2019.

Problem Chapter 3.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=f(\alpha x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{f\left(\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) =  f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{f\left(\frac{-(-\_a+x)b\beta+(\_a\alpha+\beta y)a}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)$$

7.3.27.2 [1008] Problem 2

problem number 1008

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=xf(\frac{y}{x})$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to xf\left(\frac{y}{x}\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = xf\left(\frac{y}{x}\right) + \_F1\left(\frac{y}{x}\right)$$

7.3.27.3 [1009] Problem 3

problem number 1009

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = f(x^2 + y^2)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{f\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]} dK[1] + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  f(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{f\left(\frac{\_a^2 y^2}{x^2} + \_a^2\right)}{\_a} d\_a + \_F1\left(\frac{y}{x}\right)$$

7.3.27.4 [1010] Problem 4

problem number 1010

Added Feb. 11, 2019.

Problem Chapter 3.8.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x}) + g(x^2+y^2)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x] + g[x^2 + y^2];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\left(f\left(\frac{y}{x}\right)+\frac{g\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]}\right)dK[1]+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x)+g(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x\frac{\_af\left(\frac{y}{x}\right)+g\left(\frac{\_a^2y^2}{x^2}+\_a^2\right)}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)$$

7.3.27.5 [1011] Problem 5

problem number 1011

Added Feb. 11, 2019.

Problem Chapter 3.8.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^kf(x^ny^m)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*x^m];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{f\left(K[1]^{m+n}\right)K[1]^{k-1}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  x^k*f(x^n*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x\frac{\_a^{k-1}f\left(\_a^n\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m\right)}{a}d\_a+\_F1\left(y\,x^{-\frac{b}{a}}\right)$$

7.3.27.6 [1012] Problem 6

problem number 1012

Added Feb. 11, 2019.

Problem Chapter 3.8.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x+nyw_y=f(ax^n+by^m)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*x^m];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{f(bK[1]^m+aK[1]^n)}{mK[1]}dK[1]+c_1\left(yx^{-\frac{n}{m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  m*x*diff(w(x,y),x) +n*y*diff(w(x,y),y) =  f(a*x^n+b*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{f\left(\_a\_a^{n} + b\left(y\_a^{\frac{n}{m}} x^{-\frac{n}{m}}\right)^{m}\right)}{\_am} d\_a + \_F1\left(y\, x^{-\frac{n}{m}}\right)$$

7.3.27.7 [1013] Problem 7

problem number 1013

Added Feb. 17, 2019.

Problem Chapter 3.8.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xyw_y = y^k f(\alpha x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{f\left(\left(\alpha + \frac{\beta y}{x}\right) K[1]\right) \left(\frac{yK[1]}{x}\right)^k}{K[1]^2} dK[1] + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x) +x*y*diff(w(x,y),y) =  y^k*f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{\left(\frac{\_ay}{x}\right)^{k} f\left(\left(\alpha+\frac{\beta y}{x}\right)\_a\right)}{\_a^{2}}d\_a+\_F1\left(\frac{y}{x}\right)$$

7.3.27.8 [1014] Problem 8

problem number 1014

Added Feb. 17, 2019.

Problem Chapter 3.8.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)}w_x+\frac{g(y)}{g'(y)}w_y=h(f(x)+g(y))$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[y]*D[w[x, y], y])/Derivative[1][g][y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow\int_{1}^{x}\frac{h\left(f(K[1])+g\left(\text{InverseFunction}\left[\text{InverseFunction}\left[g^{(-1)},1,1\right],1,1\right]\left[\frac{f(K[1])\text{InverseFunction}}{f(x)}\right.\right.\right.}{f(K[1])}\right.\right.$$

Maple ✓

```
restart;
pde := f(x)/diff(f(x),x)*diff(w(x,y),x) +g(y)/diff(g(y),y)*diff(w(x,y),y) =  h(f(x)+g(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\frac{d}{d\_a} f(\_a)\right) h\left(\frac{(f(x)+g(y))f(\_a)}{f(x)}\right)}{f(\_a)} d\_a + \_F1\left(\ln\left(\frac{g(y)}{f(x)}\right)\right)$$

7.3.28 8.4

Local contents

7.3.28.1 [1015] Problem 1

problem number 1015

Added Feb. 17, 2019.

Problem Chapter 3.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x f(K[1], -ax + y + aK[1])dK[1] + c_1(y - ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} f(\_a, (\_a - x)\, a + y)\, d\_a + \_F1(-ax + y)$$

7.3.28.2 [1016] Problem 2

problem number 1016

Added Feb. 17, 2019.

Problem Chapter 3.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{f\left(K[1], x^{-\frac{b}{a}} y K[1]^{\frac{b}{a}}\right)}{aK[1]} dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{f\left(\_a, y\, \_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_aa} d\_a + \_F1\left(y\, x^{-\frac{b}{a}}\right)$$

7.3.28.3 [1017] Problem 3

problem number 1017

Added Feb. 17, 2019.

Problem Chapter 3.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{h\left(K[2],\exp\left(\int_1^{K[2]}\frac{g(K[1])}{f(K[1])}dK[1]-\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]\right)y\right)}{f(K[2])}dK[2]+c_1\left(y\exp\left(-\int_1^x\frac{g(K[1}{f(K[1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +g(x)*y*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x\frac{h\left(\_b,y\,\mathrm{e}^{\int\frac{g(\_b)}{f(\_b)}d\_b-\left(\int\frac{g(x)}{f(x)}dx\right)}\right)}{f(\_b)}d\_b+\_F1\left(y\,\mathrm{e}^{-\left(\int\frac{g(x)}{f(x)}dx\right)}\right)$$

7.3.28.4 [1018] Problem 4

problem number 1018

Added Feb. 17, 2019.

Problem Chapter 3.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{h\left(K[3],\exp\left(\int_1^{K[3]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\left(\exp\left(-\int_1^x\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)y-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\text{g1}(K[1])}{f(K[1])}dK\right.}{f(K[2])}\right.\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{h\left(\_f,\left(y\,\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}+\int\frac{\mathrm{e}^{-\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}g0(\_f)}{f(\_f)}d\_f-\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int\frac{g1(\_f)}{f(\_f)}d\_f}\right)}{f\left(\_f\right)}d\_$$

7.3.28.5 [1019] Problem 5

problem number 1019

Added Feb. 17, 2019.

Problem Chapter 3.8.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{h\left(K[3],\left(\exp\left(-\int_1^x \frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]-(k-1)\int_1^{K[3]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)y^{-k}\left(\exp\left(\int_1^x \frac{\mathrm{g1}(K[1])}{f(K[1])}d\right.\right.\right.\right.}{}$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x \frac{h\left(\_f,\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int\frac{g1(x)}{f(x)}dx\right)}+(-k+1)\left(\int\frac{\mathrm{e}^{(k-1)\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}g0(\_f)}{f(\_f)}d\_f\right)+(k-1)\left(\int\frac{\mathrm{e}^{(k-1)}}{}\right.\right.\right.}{f(\_f)}$$

7.3.28.6 [1020] Problem 6

problem number 1020

Added Feb. 17, 2019.

Problem Chapter 3.8.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x] + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int^x \frac{h\left(\_f, \frac{\lambda\left(\int \frac{g1(\_f)}{f(\_f)}d\_f\right)+\ln\left(\frac{1}{-\lambda\left(\int \frac{e^{\lambda\left(\int \frac{g1(\_f)}{f(\_f)}d\_f\right)}g0(\_f)}{f(\_f)}d\_f\right)+\lambda\left(\int \frac{e^{\lambda\left(\int \frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)+e^{-\left(y-\left(\int \frac{g1(x)}{f(x)}dx\right)\right)\lambda}}\right)}{\lambda}\right)}{f(\_f)} d$$

7.3.28.7 [1021] Problem 7

problem number 1021

Added Feb. 17, 2019.

Problem Chapter 3.8.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int^{x} \frac{h\left(\_f, \mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_a\right)\right)\right)}{f1(\_f)\, g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_a\right)\right)\right)} d\_f + \_F$$

Contains RootOf

7.4 chapter 4

Local contents

7.4.1 1.1

Local contents

7.4.1.1 [1022] Example 1

problem number 1022

Added Feb. 17, 2019.

Chapter 4.1.1 example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ayw_y = by^2w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y*D[w[x, y], y] == b*y^2*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{by^2}{2a}}c_1\left(ye^{-ax}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) =  b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax}\right)\mathrm{e}^{\frac{b\,y^2}{2a}}$$

7.4.1.2 [1023] Example 2

problem number 1023

Added Feb. 17, 2019.

Chapter 4.1.1 example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ayw_y = be^{\lambda x}yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y*D[w[x, y], y] == b*Exp[lambda*x]*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-ax}\right)e^{\frac{bye^{\lambda x}}{a+\lambda}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) = b*exp(lambda*x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax}\right)\mathrm{e}^{\frac{by\,\mathrm{e}^{\lambda x}}{a+\lambda}}$$

7.4.1.3 [1024] Example 3

problem number 1024

Added Feb. 17, 2019.

Chapter 4.1.1 example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = bw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{bx}c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1(-ax+y)\,\mathrm{e}^{bx}$$

7.4.2 2.1

Local contents

7.4.2.1 [1025] Problem 1

problem number 1025

Added Feb. 17, 2019.

Problem Chapter 4.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)e^{\frac{cx}{a}}$$

7.4.2.2 [1026] Problem 2

problem number 1026

Added Feb. 17, 2019.

Problem Chapter 4.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+yw_y=bw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{bx}{a}}c_1\left(ye^{-\frac{x}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +y*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{x}{a}}\right)\mathrm{e}^{\frac{bx}{a}}$$

7.4.2.3 [1027] Problem 3

problem number 1027

Added Feb. 17, 2019.

Problem Chapter 4.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = aw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^a c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = x^a \_F1\left(\frac{y}{x}\right)$$

7.4.2.4 [1028] Problem 4

problem number 1028

Added Feb. 17, 2019.

Problem Chapter 4.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(aw_x - bw_y) = cyw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(D[w[x, y], x] - b*D[w[x, y], y]) == c*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-bcx}x^{c(bx+y)}c_1(bx+y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*(diff(w(x,y),x) -b*diff(w(x,y),y)) = c*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^{(bx+y)c}\_F1(bx+y)\,\mathrm{e}^{-bcx}$$

7.4.2.5 [1029] Problem 5

problem number 1029

Added Feb. 17, 2019.

Problem Chapter 4.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{ax}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*x*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{ax}$$

7.4.2.6 [1030] Problem 6

problem number 1030

Added Feb. 17, 2019.

Problem Chapter 4.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)w_x+(y-b)w_y=w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\left((a-x)c_1\left(\frac{b-y}{a-x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x-a)*diff(w(x,y),x) +(y-b)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = (a-x)\_F1\left(\frac{-b+y}{a-x}\right)$$

7.4.2.7 [1031] Problem 7

problem number 1031

Added Feb. 17, 2019.

Problem Chapter 4.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y+ax)w_x + (y-ax)w_y = bw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y + a*x)*D[w[x, y], x] + (y - a*x)*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (y+a*x)*diff(w(x,y),x) +(y-a*x)*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

time expired

7.4.3 2.2

Local contents

7.4.3.1 [1032] Problem 1

problem number 1032

Added Feb. 17, 2019.

Problem Chapter 4.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (x^2 - y^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (x^2 - y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{x(a^2(x^2-3y^2)+3abxy-b^2x^2)}{3a^3}\right)c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = (x^2-y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{(a^2x^2-3a^2y^2+3axyb-b^2x^2)x}{3a^3}}$$

7.4.3.2 [1033] Problem 2

problem number 1033

Added Feb. 17, 2019.

Problem Chapter 4.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axyw_y = by^2 w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{by^2}{x-2ax}}c_1\left(yx^{-a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x) +a*x*y*diff(w(x,y),y) = b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\,x^{-a}\right)\mathrm{e}^{\frac{b\,y^2}{(2a-1)x}}$$

7.4.3.3 [1034] Problem 3

problem number 1034

Added Feb. 17, 2019.

Problem Chapter 4.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2 w_x + by^2 w_y = (x+cy)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == (x + c*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{1}{a}+\frac{c}{b}}\left(\frac{ax}{y}\right)^{-\frac{c}{b}}c_1\left(\frac{b}{ax}-\frac{1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x) +b*y^2*diff(w(x,y),y) = (x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=x^{\frac{c}{b}+\frac{1}{a}}\left(\frac{ax}{y}\right)^{-\frac{c}{b}}\_F1\left(\frac{ax-by}{axy}\right)$$

7.4.3.4 [1035] Problem 4

problem number 1035

Added Feb. 17, 2019.

Problem Chapter 4.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x+ay^2w_y=(bx^2+cxy+dy^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x^2 + c*x*y + d*y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\left(\frac{x}{y}\right)^{\frac{cxy}{x-ay}}c_1\left(\frac{a}{x}-\frac{1}{y}\right)e^{\frac{bx^2-dy^2}{x-ay}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) = (b*x^2+c*x*y+d*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \left(\frac{x}{y}\right)^{-\frac{cxy}{ay-x}} \_F1\left(\frac{-ay+x}{xy}\right) \mathrm{e}^{\frac{d\,y^2+(ay-x)bx}{ay-x}}$$

7.4.3.5 [1036] Problem 5

problem number 1036

Added Feb. 17, 2019.

Problem Chapter 4.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^2 w_x + a x^2 w_y = (b x^2 + c y^2) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[w[x, y], x] + a*x^2*D[w[x, y], y] == (b*x^2 + c*y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{1}{3}\left(y^3 - a x^3\right)\right) e^{\frac{b\sqrt[3]{y^3}}{a}+cx}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{1}{3}\left(y^3 - a x^3\right)\right) e^{cx-\frac{\sqrt[3]{-1}b\sqrt[3]{y^3}}{a}}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{1}{3}\left(y^3 - a x^3\right)\right) e^{\frac{(-1)^{2/3}b\sqrt[3]{y^3}}{a}+cx}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(w(x,y),x) +a*x^2*diff(w(x,y),y) =(b*x^2+c*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-a\,x^3 + y^3\right) \mathrm{e}^{\frac{cax+by}{a}}$$

7.4.3.6 [1037] Problem 6

problem number 1037

Added Feb. 17, 2019.

Problem Chapter 4.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xyw_x + ay^2w_y = (bx + cy + d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*y*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x + c*y + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^c c_1\left(yx^{-a}\right) e^{-\frac{\frac{bx}{a-1}+\frac{d}{a}}{y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*y*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) =(b*x+c*y+d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^c\_F1\left(y\,x^{-a}\right) \mathrm{e}^{\frac{(-bx-d)a+d}{(a-1)ay}}$$

7.4.3.7 [1038] Problem 7

problem number 1038

Added Feb. 17, 2019.

Problem Chapter 4.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ay+b)w_x + (ay^2 - bx)w_y = ayw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*y + b)*D[w[x, y], x] + (a*y^2 - b*x)*D[w[x, y], y] == a*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(a*y+b)*diff(w(x,y),x) +(a*y^2-b*x)*diff(w(x,y),y) =a*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{ay + (x+y)\,a\ln\left(\frac{-9ax+9b}{2ay+2b}\right) - (x+y)\,a\ln\left(-\frac{9(x+y)(ax-b)a}{(ay+b)x}\right) + b}{3\,(x+y)\,a}\right) \mathrm{e}^{\frac{\left(\int^x \frac{9\_aa+2b\,\mathrm{e}^{\mathrm{RootOf}\left(2\_Zax\,\mathrm{e}\cdots\right.}}{\cdots}\right.}{\cdots}}$$

7.4.3.8 [1039] Problem 8

problem number 1039

Added Feb. 17, 2019.

Problem Chapter 4.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ky - x + a)w_x - y(kx - y + a)w_y = b(y - x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(k*y - x + a)*D[w[x, y], x] - y*(k*x - y + a)*D[w[x, y], y] == b*(y - x)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(k*y-x+a)*diff(w(x,y),x)-y*(k*x-y+a)*diff(w(x,y),y) = b*(y-x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(-\frac{(k^2+k+1)\left(-k\ln\left(-\frac{(k+1)(k^2+k+1)(a-x)}{(k+2)(ky+a-x)}\right)+k\ln(-a+x)-\ln(x)-\ln\left(\frac{(k+1)(k^2+k+1)k}{(2k+1)(ky+a-x}\right.\right.}{3(k+1)k}\right.$$

7.4.4 2.3

Local contents

7.4.4.1 [1040] Problem 1

problem number 1040

Added Feb. 17, 2019.

Problem Chapter 4.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (cx^3 + dy^3)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^3 + d*y^3)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{1}{4}\left(\frac{cx^4}{a}+\frac{dy^4}{b}\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^3+d*y^3)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left(c\,x^3a^3+4a^3d\,y^3-6a^2bdx\,y^2+4a\,b^2d\,x^2y-b^3d\,x^3\right)x}{4a^4}}$$

7.4.4.2 [1041] Problem 2

problem number 1041

Added Feb. 17, 2019.

Problem Chapter 4.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = a\sqrt{x^2 + y^2}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{a\sqrt{x^2+y^2}}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{\sqrt{x^2+y^2}\,a}$$

7.4.4.3 [1042] Problem 3

problem number 1042

Added Feb. 17, 2019.

Problem Chapter 4.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x + xyw_y = y^2(ax+by)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\,y^2}{2}+\frac{b\,y^3}{2x}}$$

7.4.4.4 [1043] Problem 4

problem number 1043

Added Feb. 17, 2019.

Problem Chapter 4.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2yw_x+axy^2w_y=(bxy+cx+dy+k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*y*D[w[x, y], x] + a*x*y^2*D[w[x, y], y] == (b*x*y + c*x + d*y + k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^bc_1\left(yx^{-a}\right)\exp\left(-\frac{a^2dy+acx+ady+ak+cx}{a^2xy+axy}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*y*diff(w(x,y),x)+a*x*y^2*diff(w(x,y),y) =(b*x*y +c*x+ d*y + k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=x^b\_F1\left(y\,x^{-a}\right)\mathrm{e}^{\frac{-a^2dy-cx+(-cx-dy-k)a}{(a+1)axy}}$$

7.4.4.5 [1044] Problem 5

problem number 1044

Added Feb. 17, 2019.

Problem Chapter 4.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axy^2w_x + bx^2yw_y = (any^2 + bmx^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*y^2*D[w[x, y], x] + b*x^2*y*D[w[x, y], y] == (a*n*y^2 + b*m*x^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^n\left(ay^2\right)^{m/2}c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*y^2*diff(w(x,y),x)+b*x^2*y*diff(w(x,y),y) = (a*n*y^2+ b*m*x^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=x^n\left(a\,y^2\right)^{\frac{m}{2}}\_F1\left(\frac{a\,y^2-b\,x^2}{a}\right)$$

7.4.4.6 [1045] Problem 6

problem number 1045

Added Feb. 17, 2019.

Problem Chapter 4.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^3w_x + ay^3w_y = x^2(bx + cy)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^3*D[w[x, y], x] + a*y^3*D[w[x, y], y] == x^2*(b*x + c*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(\frac{a}{x^2}-\frac{1}{y^2}\right)\right)\exp\left(bx-\frac{c\tan^{-1}\left(\frac{x\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}{\sqrt{\frac{x^2}{y^2}}}\right)}{\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(\frac{a}{x^2}-\frac{1}{y^2}\right)\right)\exp\left(\frac{c\tan^{-1}\left(\frac{x\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}{\sqrt{\frac{x^2}{y^2}}}\right)}{\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}+bx\right)\right\}$$

Maple ✓

```
restart;
pde :=x^3*diff(w(x,y),x)+a*y^3*diff(w(x,y),y) =  x^2*(b*x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\left(\sqrt{\frac{-a\,y^2+x^2}{x^2y^2}}\,x+\sqrt{\frac{x^2}{y^2}}\right)^{\frac{c}{\sqrt{\frac{-a\,y^2+x^2}{x^2y^2}}}}\_F1\left(\frac{-a\,y^2+x^2}{x^2y^2}\right)\mathrm{e}^{bx}$$

7.4.5 2.4

Local contents

7.4.5.1 [1046] Problem 1

problem number 1046

Added Feb. 17, 2019.

Problem Chapter 4.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + dy^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + d*y^m)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{cx^{n+1}}{an+a}+\frac{dy^{m+1}}{bm+b}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n + d*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{(n+1)ad\,y^{m+1}+(m+1)bc\,x^{n+1}}{(n+1)(m+1)ab}}$$

7.4.5.2 [1047] Problem 2 case $n \neq -1, n \neq -2$

problem number 1047

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $neq - 1, neq - 2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol[[2]] = Assuming[{n != -1, n != -2}, Simplify[sol[[2]]]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{cx^{n+1}(a(n+2)y - bx)}{a^2(n+1)(n+2)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol)  assuming n<>-1, n<>-2;
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right) e^{\frac{\left((n+2)ay\,x^{n+1} - b\,x^{n+2}\right)c}{(n+2)(n+1)a^2}}$$

7.4.5.3 [1048] Problem 2 case $n = -1$

problem number 1048

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n = -1$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -1],
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c(\log(x)(ay-bx)+bx)}{a^2}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-1)
```

$$w(x,y)=x^{\frac{(ay-bx)c}{a^2}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{bcx}{a^2}}$$

7.4.5.4 [1049] Problem 2 case $n = -2$

problem number 1049

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n = -2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -2],
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c(-ay+bx+bx\log(x))}{a^2x}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-2)
```

$$w(x,y) = x^{\frac{bc}{a^2}}\_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{-\frac{(ay-bx)c}{a^2x}}$$

7.4.5.5 [1050] Problem 3

problem number 1050

Added Feb. 17, 2019.

Problem Chapter 4.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2+y^2)^k w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{a\left(x^2+y^2\right)^k}{2k}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*(x^2+y^2)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) \mathrm{e}^{\frac{a\left(x^2+y^2\right)^k}{2k}}$$

7.4.5.6 [1051] Problem 4

problem number 1051

Added Feb. 17, 2019.

Problem Chapter 4.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cy^m x^n}{an+bm}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =   c*x^n*y^m*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{b}{a}}\right)\mathrm{e}^{\frac{c\,x^n y^m}{na+bm}}$$

7.4.5.7 [1052] Problem 5

problem number 1052

Added Feb. 17, 2019.

Problem Chapter 4.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = (cx^n + ky^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == (c*x^n + k*y^m)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cx^n}{an}+\frac{ky^m}{bm}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =  (c*x^n + k*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y\,x^{-\frac{b}{a}}\right)\mathrm{e}^{\frac{akn\,y^m+bcm\,x^n}{abmn}}$$

7.4.5.8 [1053] Problem 6

problem number 1053

Added Feb. 17, 2019.

Problem Chapter 4.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x+nyw_y=(ax^n+by^m)^k w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{n}{m}}\right)e^{\frac{(ax^n+by^m)^k}{kmn}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) =  (a*x^n + b*y^m)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{n}{m}}\right) \mathrm{e}^{\frac{\left(a\,x^{n}+b\,y^{m}\right)^{k}}{k m n}}$$

7.4.5.9 [1054] Problem 7

problem number 1054

Added Feb. 17, 2019.

Problem Chapter 4.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^m w_y = (cx^k + dy^s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == (c*x^k + d*y^s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-m}}{m-1}\right)\exp\left(\frac{\frac{cx^{k-n+1}}{a}+\frac{d(-k+n-1)y^{1-m}\left(\left(y^{m-1}\right)^{\frac{1}{m-1}}\right)^{s}}{b(m-s-1)}}{k-n+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*y^m*diff(w(x,y),y) =  (c*x^k + d*y^s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)\,a\,y^{-m+1}-(m-1)\,b\,x^{-n+1}}{(n-1)\,a}\right)\mathrm{e}^{\frac{-(k-n+1)a^2 d\, a^{\frac{s}{m-1}-1} y^{-m+1}\left((n-1)a\,y^{-m+1}\right)^{-\frac{s}{m-1}}(n-1)^{\frac{s}{m-1}}\mathrm{e}^{-\frac{i\left(cs\right.}{}}}{}}$$

7.4.5.10 [1055] Problem 8

problem number 1055

Added Feb. 17, 2019.

Problem Chapter 4.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = (cx^k y^s + d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == (c*x^k*y^s + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\frac{bx^{m-n+1}}{am-an+a}}\right)\exp\left(\frac{x^{1-n}\left(\frac{d}{1-n}-\frac{cx^k y^s e^{-\frac{bsx^{m-n+1}}{am-an+a}}\left(-\frac{bsx^{m-n+1}}{am-an+a}\right)^{\frac{-k+n-1}{m-n+1}}\text{Gamma}\left(\frac{k-n+1}{m-n+1},-\frac{bsx^{m-n+}}{am-an+a}\right.}{m-n+1}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y*diff(w(x,y),y) =  (c*x^k*y^s + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{b\,x^{m-n+1}}{(m-n+1)a}}\right)\mathrm{e}^{\int^{x}\frac{\left(c\_a^{k}\left(y\,\mathrm{e}^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)b}{(m-n+1)a}}\right)^{s}+d\right)\_a^{-n}}{a}d\_a}$$

7.4.5.11 [1056] Problem 9

problem number 1056

Added Feb. 17, 2019.

Problem Chapter 4.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^{n}w_{x} + (bx^{m}y + cx^{k})w_{y} = (sx^{p}y^{q} + d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == (s*x^p*y^q + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(c(a(m-n+1))^{\frac{k-m}{m-n+1}}b^{\frac{-k+n-1}{m-n+1}}\,\text{Gamma}\left(\frac{k-n+1}{m-n+1},\frac{bx^{m-n+1}}{am-an+a}\right)+ye^{-\frac{bx^{m-n+1}}{am-an+a}}\right)\text{exp}\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+(b*x^m*y+c*x^k)*diff(w(x,y),y) =   (s*x^p*y^q + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(m-n+1)\,(k+m-2n+2)^2\,ac\,x^{k-m}\left(\frac{b\,x^{m-n+1}}{(m-n+1)a}\right)^{\frac{-k-m+2n-2}{2m-2n+2}}\mathrm{WhittakerM}\left(\frac{k+m-2n+2}{2m-2n+2},\right.}{}\right.$$

7.4.5.12 [1057] Problem 10

problem number 1057

Added Feb. 17, 2019.

Problem Chapter 4.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y^k w_y = (cx^p y^q + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == (c*x^p*y^q + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\frac{y^{1-k}(-m+n-1)}{k-1}-\frac{bx^{m-n+1}}{a}}{m-n+1}\right)\exp\left(\frac{x^{1-n}\left(-\frac{cx^p\left(\left(y^{k-1}\right)^{\frac{1}{k-1}}\right)^q\left(\frac{ay(m-n+1)x^n}{ay(m-n+1)x^n+b(k-1)y^kx^{m+1}}\right)^{\frac{q}{k-1}}{}_2F_1\left(\right.}{n-p-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y^k*diff(w(x,y),y) =    (c*x^p*y^q + s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(m-n+1)\,a\,y^{-k+1}+(k-1)\,b\,x^{m-n+1}}{(m-n+1)\,a}\right)\mathrm{e}^{\int^x \frac{\left(c\_a^p\left(\left(\frac{(m-n+1)a\,y^{-k+1}-(k-1)b\_a^{m-n+1}+(k-1)b\,x^{m-n-}}{(m-n+1)a}\right.\right.\right.}{a}}$$

7.4.5.13 [1058] Problem 11

problem number 1058

Added Feb. 17, 2019.

Problem Chapter 4.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = (cx^m + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == (c*x^m + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\frac{bx^{n+1}}{an+a}\right)\exp\left(\frac{x\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\left(cx^m\,{}_2F_1\left(\frac{k}{k+1},\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x)+b*x^n*diff(w(x,y),y) =   (c*x^m+ s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(n+1)\,a\,y^{k+1}-(k+1)\,b\,x^{n+1}}{(n+1)\,a}\right)\mathrm{e}^{\int^{x}\frac{(c\_a^{m}+s)\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_a^{n+1}-(k+1)b\,x^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}}{a}d\_a}$$

7.4.5.14 [1059] Problem 12

problem number 1059

Added Feb. 17, 2019.

Problem Chapter 4.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(x^n+(an-1)y^n)w_x+y(y^n+(an-1)x^n)w_y=kn(x^n+y^n)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(x^n + (a*n - 1)*y^n)*D[w[x, y], x] + y*(y^n + (a*n - 1)*x^n)*D[w[x, y], y] == k*n*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*(x^n + (a*n -1)*y^n)*diff(w(x,y),x)+y*(y^n + (a*n -1)*x^n)*diff(w(x,y),y) =   k*n*(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\left(-x^{-\frac{1}{a}}y^{n}+x^{\frac{na-1}{a}}\right)(y^{n})^{-\frac{1}{an}}\right)\mathrm{e}^{\int^{x}\frac{\left(\_a^{n}+\mathrm{RootOf}\left(-\_a^{\frac{1}{a}}x^{-\frac{1}{a}}y^{n}(\_Z^{n})\right.\right.}{\left(an\,\mathrm{RootOf}\left(-\_a^{\frac{1}{a}}x^{-\frac{1}{a}}y^{n}(\_Z^{n})^{\frac{1}{an}}(y^{n})^{-\frac{1}{an}}+\_a^{\frac{1}{a}}x^{\frac{na-1}{a}}(\_Z^{n})^{\frac{1}{an}}(y^{n})^{-\frac{1}{an}}+\_\right.\right.}}$$

7.4.5.15 [1060] Problem 13

problem number 1060

Added Feb. 17, 2019.

Problem Chapter 4.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x((n-2)y^n-2x^n)w_x+y(2y^n-(n-2)x^n)w_y = ((a(n-2)+2b)y^n-(2a+b(n-2))x^n)\,w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = x*((n - 2)*y^n - 2*x^n)*D[w[x, y], x] + y*(2*y^n - (n - 2)*x^n)*D[w[x, y], y] == ((a*(
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*((n-2)*y^n -2*x^n )*diff(w(x,y),x)+y*(2*y^n - (n-2)*x^n)*diff(w(x,y),y) = ((a*(n-2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(x^n+y^n)\,(y^n)^{-\frac{2}{n}}}{x^2}\right) \mathrm{e}^{\int^x \frac{2\left(\frac{1}{2}bn+a-b\right)\_a^n+(-na+2a-2b)\operatorname{RootOf}\left(\_a^2x^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}\right.}{\left(-n\operatorname{RootOf}\left(\_a^2x^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}+\_a^2y^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}-x^2\_Z^n-x^2\_a^n\right)^n+2\_a^n+2\operatorname{Root}\right.}}$$

7.4.6 3.1

Local contents

7.4.6.1 [1061] Problem 1

problem number 1061

Added Feb. 23, 2019.

Problem Chapter 4.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\alpha x+\beta y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x + beta*y]*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{ce^{\alpha x+\beta y}}{a\alpha+b\beta}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*exp(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{c\,e^{\alpha x+\beta y}}{a\alpha+b\beta}}$$

7.4.6.2 [1062] Problem 2

problem number 1062

Added Feb. 23, 2019.

Problem Chapter 4.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (ce^{\lambda x} + ke^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x] + k*Exp[mu*y])*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{ce^{\lambda x}}{a\lambda}+\frac{ke^{\mu y}}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*exp(lambda*x)+k*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{ak\lambda\,e^{\mu y}+bc\mu\,e^{\lambda x}}{ab\lambda\mu}}$$

7.4.6.3 [1063] Problem 3

problem number 1063

Added Feb. 23, 2019.

Problem Chapter 4.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ce^{-\lambda x}}{a\lambda}}c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{(a\lambda\,\mathrm{e}^{\lambda x}-b\beta\,\mathrm{e}^{\beta y})\,\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)\mathrm{e}^{-\frac{c\,\mathrm{e}^{-\lambda x}}{a\lambda}}$$

7.4.6.4 [1064] Problem 4

problem number 1064

Added Feb. 23, 2019.

Problem Chapter 4.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+be^{\beta x}w_y=cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\lambda y}}{\lambda}-\frac{be^{\beta x}}{a\beta}\right)\exp\left(\frac{c\left(\beta x-\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)\right)}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\frac{a\beta\,\mathrm{e}^{\lambda y}}{b\lambda}\right)^{-\frac{c}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}}\left(\mathrm{e}^{\beta x}\right)^{\frac{c}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}}\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)$$

7.4.6.5 [1065] Problem 5

problem number 1065

Added Feb. 23, 2019.

Problem Chapter 4.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x+be^{\beta x}w_y=ce^{\gamma y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)}+y\right)\exp\left(\int_1^x\frac{c\exp\left(y\gamma-\frac{b\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K[1]}\right)\gamma}{a(\beta-\lambda)}-\lambda K[1]\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*exp(gamma*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\beta-\lambda)\,ay - b\,\mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\,a}\right)\mathrm{e}^{\int^{x}\frac{c\,\mathrm{e}^{\frac{\gamma b\,\mathrm{e}^{(\beta-\lambda)\_a}-\gamma b\,\mathrm{e}^{(\beta-\lambda)x}+(\beta-\lambda)(-\_a\lambda+\gamma y)a}{(\beta-\lambda)a}}}{a}d\_a}$$

7.4.6.6 [1066] Problem 6

problem number 1066

Added Feb. 23, 2019.

Problem Chapter 4.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = (ce^{\gamma y} + se^{\delta y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == (c*Exp[gamma*y] + s*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{e^{-\beta y}}{\beta}\right)\exp\left(-\frac{e^{-\lambda x}\left(e^{-\beta y}\right)^{-\frac{\delta+\gamma}{\beta}}\left((\beta-\delta)\left(c\gamma\left(e^{-\beta y}\right)^{\frac{\delta}{\beta}}\left(\frac{a\lambda e^{\lambda x-\beta y}}{b\beta}\right)^{\frac{\gamma}{\beta}}{}_2F_1\left(\frac{\beta+\gamma}{\beta},\frac{\gamma}{\beta}\right.\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  (c*exp(gamma*y)+s*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(a\lambda\,\mathrm{e}^{\lambda x} - b\beta\,\mathrm{e}^{\beta y})\,\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)\mathrm{e}^{-\frac{\left((\beta-\delta)c\left(\frac{a\lambda}{b\beta\,\mathrm{e}^{-\lambda x}+(a\lambda\,\mathrm{e}^{\lambda x}-b\beta\,\mathrm{e}^{\beta y})\mathrm{e}^{-\beta y-\lambda x}}\right)^{\frac{\gamma}{\beta}}+(\beta-\gamma)s\left(\frac{a\lambda}{b\beta\,\mathrm{e}^{-\lambda x}+(a\lambda\,\mathrm{e}^{\lambda x}-b\beta}\right.\right.}{(\beta-\gamma)(\beta-\delta)ab\lambda}}$$

7.4.6.7 [1067] Problem 7

problem number 1067

Added Feb. 23, 2019.

Problem Chapter 4.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = (se^{\mu x} + ke^{\delta y} + p)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{\lambda}\right)\mathrm{e}^{\int^{x}\frac{k\left(a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}d\right.\right.}{}}$$

7.4.6.8 [1068] Problem 8

problem number 1068

Added Feb. 23, 2019.

Problem Chapter 4.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = (se^{\mu x+\delta y} + k)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{\lambda}\right)\mathrm{e}^{\int^{x}\frac{s\left(a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}d\right.\right.}{}}$$

7.4.6.9 [1069] Problem 9

problem number 1069

Added Feb. 23, 2019.

Problem Chapter 4.3.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = (ce^{\mu x+\delta y} + k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x + lambda*y])*D[w[x, y], y] == (c*Exp[mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{\gamma x-\beta x}}{a\beta-a\gamma}-\frac{e^{-\lambda y}}{\lambda}\right)\exp\left(\frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\delta/\lambda}e^{-\gamma x-\lambda y+\mu x}\,{}_2F_1\left(1,\frac{\mu-\gamma}{\beta-\gamma};\frac{\beta\delta-\gamma\delta-\gamma\lambda+\lambda\mu}{\beta\lambda-\gamma\lambda};1-\frac{ae^{\beta x}}{}\right.}{b(\beta(\lambda-\delta)+\delta\gamma-\lambda\mu)}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x+lambda*y))*diff(w(x,y),y) =  (c*exp(mu*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-b\lambda\, \mathrm{e}^{\lambda y+(-\beta+\gamma)x} + (\beta-\gamma)\, a\right) \mathrm{e}^{-\lambda y}}{(\beta-\gamma)\, b\lambda}\right) \mathrm{e}^{\int^x \frac{\left(c\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_a}+(\beta-\gamma)a\,\mathrm{e}^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{-a\mu}}{a}}$$

7.4.6.10 [1070] Problem 10

problem number 1070

Added Feb. 23, 2019.

Problem Chapter 4.3.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = (ce^{\mu x} + k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == (c*Exp[mu*x] + k)*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{\frac{ce^{x(\mu-\lambda)}}{\lambda-\mu}+\frac{ke^{-\lambda x}}{\lambda}}{a}} c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  (c*exp(mu*x) + k)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\beta-\lambda)\, ay - b\, \mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\, a}\right) \mathrm{e}^{\frac{c\lambda\, \mathrm{e}^{(-\lambda+\mu)x} - (-\lambda+\mu)k\, \mathrm{e}^{-\lambda x}}{(-\lambda+\mu)a\lambda}}$$

7.4.7 3.2

Local contents

7.4.7.1 [1071] Problem 1

problem number 1071

Added Feb. 23, 2019.

Problem Chapter 4.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cye^{\lambda x} + kxe^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y*Exp[lambda*x] + k*x*Exp[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(-\frac{bce^{\lambda x}}{a^2\lambda^2}-\frac{ake^{\mu y}}{b^2\mu^2}+\frac{cye^{\lambda x}}{a\lambda}+\frac{kxe^{\mu y}}{b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*y*exp(lambda*x) + k*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) e^{\frac{-(-b\mu x+a)a^2k\,\lambda^2 e^{\mu y}+(a\lambda y-b)b^2c\,\mu^2 e^{\lambda x}}{a^2b^2\lambda^2\mu^2}}$$

7.4.7.2 [1072] Problem 2

problem number 1072

Added Feb. 23, 2019.

Problem Chapter 4.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*exp(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\frac{a\,e^{\lambda x+\mu y}}{\lambda+\frac{\mu y}{x}}}$$

7.4.7.3 [1073] Problem 3

problem number 1073

Added Feb. 23, 2019.

Problem Chapter 4.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = (aye^{\lambda x} + bxe^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x] + b*x*Exp[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =    (a*y*exp(lambda*x)+ b*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{\left(\frac{a\mu\, y^2\mathrm{e}^{\lambda x}}{x^2}+b\lambda\,\mathrm{e}^{\mu y}\right)x}{\lambda\mu y}}$$

7.4.7.4 [1074] Problem 4

problem number 1074

Added Feb. 23, 2019.

Problem Chapter 4.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + be^{\lambda y}w_y = (cx^n + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == (c*x^n + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x^{1-k}\left(\frac{cx^n}{-k+n+1}+\frac{s}{1-k}\right)}{a}}c_1\left(\frac{bx^{1-k}}{a(k-1)}-\frac{e^{-\lambda y}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^k*diff(w(x,y),x)+b*exp(lambda*y)*diff(w(x,y),y) =    (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{b\lambda\,x^{-k+1}-(k-1)\,a\,\mathrm{e}^{-\lambda y}}{(k-1)\,b\lambda}\right)\mathrm{e}^{-\frac{((k-1)c\,x^n+(k-n-1)s)x^{-k+1}}{(k-1)(k-n-1)a}}$$

7.4.7.5 [1075] Problem 5

problem number 1075

Added Feb. 23, 2019.

Problem Chapter 4.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + be^{\lambda x}w_y = (ce^{\mu x}+s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\frac{be^{\lambda x}}{a\lambda}\right)\exp\left(-\frac{y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\left(c\lambda e^{\mu x}\,{}_2F_1\left(1,\frac{\lambda+k\mu+\mu}{k\lambda+\lambda};\frac{\lambda+\mu}{\lambda};\frac{be^{\lambda x}(k+1)}{be^{\lambda x}(k+1)-a\lambda y^{k+1}}\right.\right.}{\mu\left(b(k+1)e^{\lambda x}-a\lambda\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*y^k*diff(w(x,y),x)+b*exp(lambda*x)*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\lambda y\, y^{k} - (k+1)\, b\, e^{\lambda x}}{a\lambda}\right) e^{\int^{x} \frac{(c\, e^{-a\mu}+s)\left(\left(\frac{a\lambda\, y^{k+1}+(k+1) b\, e^{-a\lambda}-(k+1) b\, e^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k}}{a} d\_a}$$

7.4.7.6 [1076] Problem 6

problem number 1076

Added Feb. 23, 2019.

Problem Chapter 4.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^k w_y = (cx^n + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == (c*x^n + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{y^{1-k}}{k-1}\right) \exp\left(-\frac{cx^n(\lambda x)^{-n}\,\text{Gamma}(n+1,\lambda x) + se^{-\lambda x}}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*y^k*diff(w(x,y),y) =   (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\lambda\, y^{-k+1} - (k-1)\, b\, \mathrm{e}^{-\lambda x}}{a\lambda}\right) \mathrm{e}^{\frac{c\, x^{n} (\lambda x)^{-\frac{n}{2}}\ \mathrm{WhittakerM}\left(\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, \lambda x\right) \mathrm{e}^{-\frac{\lambda x}{2}} - \left(\mathrm{e}^{-\lambda x}-1\right)(n+1)s}{(n+1)a\lambda}}$$

7.4.7.7 [1077] Problem 7

problem number 1077

Added Feb. 23, 2019.

Problem Chapter 4.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y} w_x + bx^k w_y = (ce^{\mu x} + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\lambda y}}{\lambda} - \frac{bx^{k+1}}{ak+a}\right) \exp\left(\int_1^x \frac{(k+1)\left(e^{\mu K[1]}c + s\right)}{ae^{\lambda y}(k+1) + b\lambda\left(K[1]^{k+1} - x^{k+1}\right)} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*x^k*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-b\lambda\, x^{k+1} + (k+1)\, a\, \mathrm{e}^{\lambda y}}{(k+1)\, b\lambda}\right) \mathrm{e}^{\int^{x} \frac{\left(c\, \mathrm{e}^{\_a\mu} + s\right)(k+1)}{b\lambda \_a^{k+1} - b\lambda\, x^{k+1} + (k+1) a\, \mathrm{e}^{\lambda y}} d\_a}$$

7.4.8 4.1

Local contents

7.4.8.1 [1078] Problem 1

problem number 1078

Added Feb. 23, 2019.

Problem Chapter 4.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\sinh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Sinh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\cosh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*sinh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\cosh(\mu y)+bc\mu\cosh(\lambda x)}{ab\lambda\mu}}$$

7.4.8.2 [1079] Problem 2

problem number 1079

Added Feb. 23, 2019.

Problem Chapter 4.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{c\cosh(\lambda x+\mu y)}{a\lambda+\mu b}}$$

7.4.8.3 [1080] Problem 3

problem number 1080

Added Feb. 23, 2019.

Problem Chapter 4.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sinh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ax\cosh(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\cosh(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

7.4.8.4 [1081] Problem 4

problem number 1081

Added Feb. 23, 2019.

Problem Chapter 4.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\sinh^n(\lambda x)w_y=(c\sinh^m(\mu x)+s\sinh^k(\beta y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*sinh(lambda*x)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\sinh^n(\lambda x))}{a} dx\right)\right) \mathrm{e}^{\int^x \frac{c(\sinh^m(\_b\mu))+s\left(-\sinh\left(\left(-y-\left(\int \frac{b(\sinh^n(\_b\lambda))}{a} d\_b\right)+\int \frac{b(\sinh^n(\lambda x))}{a} dx\right)\beta\right)\right)^k}{a} d\_}$$

7.4.8.5 [1082] Problem 5

problem number 1082

Added Feb. 23, 2019.

Problem Chapter 4.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = (c\sinh^m(\mu x) + s\sinh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*y]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\cosh^2(\lambda y)}\operatorname{sech}(\lambda y)\sinh^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; -\sinh^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\sinh}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*sinh(lambda*y)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\left(\int \left(\sinh^{-n}(\lambda y)\right) dy\right) + bx}{b}\right) e^{\int^y \frac{\left(c\left(-\sinh\left(\frac{\left(a\left(\int\left(\sinh^{-n}(\lambda y)\right)dy\right) - bx - b\left(\int \frac{a\left(\sinh^{-n}(\_b\lambda)\right)}{b} d\_b\right)\right)\mu}{b}\right)\right)\right)^m +}{b}}$$

7.4.9 4.2

Local contents

7.4.9.1 [1083] Problem 1

problem number 1083

Added Feb. 23, 2019.

Problem Chapter 4.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cosh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cosh[lambda*x] + k*Cosh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sinh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*cosh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\sinh(\mu y)+bc\mu\sinh(\lambda x)}{ab\lambda\mu}}$$

7.4.9.2 [1084] Problem 2

problem number 1084

Added Feb. 23, 2019.

Problem Chapter 4.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\cosh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sinh(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) e^{\frac{c\sinh(\lambda x+\mu y)}{a\lambda+\mu b}}$$

7.4.9.3 [1085] Problem 3

problem number 1085

Added Feb. 23, 2019.

Problem Chapter 4.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cosh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ax\sinh(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\frac{a\sinh(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

7.4.9.4 [1086] Problem 4

problem number 1086

Added Feb. 23, 2019.

Problem Chapter 4.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda x)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*cosh(lambda*x)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\cosh^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x \frac{c(\cosh^m(\_b\mu)) + s\left(\cosh^k\left(\left(-y - \left(\int \frac{b(\cosh^n(\_b\lambda))}{a}d\_b\right) + \int \frac{b(\cosh^n(\lambda x))}{a}dx\right)\beta\right)\right)}{a} d\_$$

7.4.9.5 [1087] Problem 5

problem number 1087

Added Feb. 23, 2019.

Problem Chapter 4.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda y)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{-\sinh^2(\lambda y)}\operatorname{csch}(\lambda y)\cosh^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(\lambda y)\right)}{\lambda-\lambda n}-\frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cosh}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*cosh(lambda*y)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\left(\int\left(\cosh^{-n}(\lambda y)\right)dy\right)+bx}{b}\right)\mathrm{e}^{\int^y \frac{\left(c\left(\cosh^m\left(\frac{\left(a\left(\int\left(\cosh^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\cosh^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)+s}{b}}$$

7.4.10 4.3

Local contents

7.4.10.1 [1088] Problem 1

problem number 1088

Added Feb. 23, 2019.

Problem Chapter 4.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\tanh(\lambda x) + k\tanh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tanh[lambda*x] + k*Tanh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cosh^{\frac{c}{a\lambda}}(\lambda x)\cosh^{\frac{k}{b\mu}}(\mu y)c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*tanh(lambda*x) + k*tanh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x)-1)^{-\frac{c}{2a\lambda}}(\tanh(\lambda x)+1)^{-\frac{c}{2a\lambda}}(\tanh(\mu y)-1)^{-\frac{k}{2b\mu}}(\tanh(\mu y)+1)^{-\frac{k}{2b\mu}}\_F1\left(\frac{ay-bx}{a}\right.$$

7.4.10.2 [1089] Problem 2

problem number 1089

Added Feb. 23, 2019.

Problem Chapter 4.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cosh^{\frac{c}{a\lambda+b\mu}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=(\tanh(\lambda x+\mu y)-1)^{-\frac{c}{2a\lambda+2\mu b}}(\tanh(\lambda x+\mu y)+1)^{-\frac{c}{2a\lambda+2\mu b}}\_F1\left(\frac{ay-bx}{a}\right)$$

7.4.10.3 [1090] Problem 3

problem number 1090

Added Feb. 23, 2019.

Problem Chapter 4.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\tanh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cosh^{\frac{ax}{\lambda x+\mu y}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x+\mu y)-1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}} (\tanh(\lambda x+\mu y)+1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}} \_F1\left(\frac{y}{x}\right)$$

7.4.10.4 [1091] Problem 4

problem number 1091

Added Feb. 23, 2019.

Problem Chapter 4.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda x)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\tanh^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x \frac{s\tanh^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tan\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*tanh(lambda*x)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\tanh^n(\lambda x))}{a} dx\right)\right) e^{\int^x \frac{c(\tanh^m(\_b\mu))+s\left(-\tanh\left(\left(-y-\left(\int \frac{b(\tanh^n(\_b\lambda))}{a} d\_b\right)+\int \frac{b(\tanh^n(\lambda x))}{a} dx\right)\beta\right)\right)^k}{a}}$$

7.4.10.5 [1092] Problem 5

problem number 1092

Added Feb. 23, 2019.

Problem Chapter 4.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda y)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\tanh^{1-n}(\lambda y)\,{}_2F_1\left(1, \frac{1}{2} - \frac{n}{2}; \frac{3}{2} - \frac{n}{2}; \tanh^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right) \exp\left(\int_1^y \frac{\tanh^{-n}(\lambda K[1]) \left(s\tanh^{k}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*tanh(lambda*y)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int \left(\tanh^{-n}(\lambda y)\right) dy\right)}{b} + x\right) \mathrm{e}^{\int^{y} \frac{\left(c\left(-\tanh\left(-\mu\left(\int \frac{a\left(\tanh^{-n}(\_b\lambda)\right)}{b} d\_b\right) - \left(-\frac{a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)}{b}}$$

7.4.11 4.4

Local contents

7.4.11.1 [1093] Problem 1

problem number 1093

Added Feb. 23, 2019.

Problem Chapter 4.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\coth(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Coth[lambda*x] + k*Coth[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \sinh^{\frac{c}{a\lambda}}(\lambda x) c_1\left(y - \frac{bx}{a}\right) e^{\frac{k(\log(\tanh(\mu y)) + \log(\cosh(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*coth(lambda*x) + k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\coth(\lambda x)-1)^{-\frac{c}{2a\lambda}} (\coth(\lambda x)+1)^{-\frac{c}{2a\lambda}} (\coth(\mu y)-1)^{-\frac{k}{2b\mu}} (\coth(\mu y)+1)^{-\frac{k}{2b\mu}} \_F1\left(\frac{ay-bx}{a}\right)$$

7.4.11.2 [1094] Problem 2

problem number 1094

Added Feb. 23, 2019.

Problem Chapter 4.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{c(\log(\tanh(\lambda x+\mu y)) + \log(\cosh(\lambda x+\mu y)))}{a\lambda + b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\coth(\lambda x+\mu y)-1)^{-\frac{c}{2a\lambda+2\mu b}} (\coth(\lambda x+\mu y)+1)^{-\frac{c}{2a\lambda+2\mu b}} \_F1\left(\frac{ay-bx}{a}\right)$$

7.4.11.3 [1095] Problem 3

problem number 1095

Added Feb. 23, 2019.

Problem Chapter 4.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\frac{ax(\log(\tanh(\lambda x+\mu y))+\log(\cosh(\lambda x+\mu y)))}{\lambda x+\mu y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=(\coth(\lambda x+\mu y)-1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}}(\coth(\lambda x+\mu y)+1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}}\_F1\left(\frac{y}{x}\right)$$

7.4.11.4 [1096] Problem 4

problem number 1096

Added Feb. 23, 2019.

Problem Chapter 4.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda x)w_y = (c\coth^m(\mu x) + s\coth^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\coth^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x\frac{s\coth^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*coth(lambda*x)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b(\coth^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c(\coth^m(\_b\mu))+s\left(-\coth\left(\left(-y-\left(\int\frac{b(\coth^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\coth^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}}$$

7.4.11.5 [1097] Problem 5

problem number 1097

Added Feb. 23, 2019.

Problem Chapter 4.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\coth^n(\lambda y)w_y=(c\coth^m(\mu x)+s\coth^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\coth^{1-n}(\lambda y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\coth^2(\lambda y)\right)}{\lambda-\lambda n}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\left(s\coth^k(\beta K[1])+c\,\mathrm{cot}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*coth(lambda*y)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mathrm{e}^{\int^y\frac{\left(c\left(-\coth\left(-\mu\left(\int\frac{a\left(\coth^{-n}(\_b\lambda)\right)}{b}d\_b\right)-\left(-\frac{a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)^m\right.}{b}}$$

7.4.12 4.5

Local contents

7.4.12.1 [1098] Problem 1

problem number 1098

Added Feb. 23, 2019.

Problem Chapter 4.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Cosh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{ak\lambda\sinh(\mu y)+bc\mu\cosh(\lambda x)}{ab\lambda\mu}}$$

7.4.12.2 [1099] Problem 2

problem number 1099

Added Feb. 23, 2019.

Problem Chapter 4.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (\tanh(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (Tanh[lambda*x] + k*Coth[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\cosh(\lambda x)}c_1\left(y-\frac{bx}{a}\right)e^{\frac{k(\log(\tanh(\mu y))+\log(\cosh(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (tanh(lambda*x)+k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\cosh^{\frac{1}{a\lambda}}(\lambda x)\right)\left(\sinh^{\frac{k}{b\mu}}(\mu y)\right)\_F1\left(\frac{ay-bx}{a}\right)$$

7.4.12.3 [1100] Problem 3

problem number 1100

Added Feb. 23, 2019.

Problem Chapter 4.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\sinh(\mu y)w_y=b\cosh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Cosh[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sinh(\lambda x)}{\lambda}}c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*cosh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu x - 2\operatorname{arctanh}\left(e^{\mu y}\right)}{a\mu}\right) e^{\frac{b\sinh(\lambda x)}{\lambda}}$$

7.4.12.4 [1101] Problem 4

problem number 1101

Added Feb. 23, 2019.

Problem Chapter 4.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = b\tanh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Tanh[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cosh^{\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu} - ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*tanh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x) - 1)^{-\frac{b}{2\lambda}} (\tanh(\lambda x) + 1)^{-\frac{b}{2\lambda}} \_F1\left(\frac{-a\mu x - 2\operatorname{arctanh}\left(e^{\mu y}\right)}{a\mu}\right)$$

7.4.12.5 [1102] Problem 5

problem number 1102

Added Feb. 23, 2019.

Problem Chapter 4.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh(\lambda x)w_x + b\cosh(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\tanh\left(\frac{\lambda x}{2}\right)}c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{b\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*sinh(lambda*x)*diff(w(x,y),x)+b*cosh(mu*y)^n*diff(w(x,y),y) =  w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda\left(\int\left(\cosh^{-n}(\mu y)\right)dy\right)-2b\operatorname{arctanh}\left(e^{\lambda x}\right)}{a\lambda}\right)e^{\int\frac{\cosh^{-n}(\mu y)}{b}dy}$$

7.4.12.6 [1103] Problem 6

problem number 1103

Added Feb. 23, 2019.

Problem Chapter 4.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh(\lambda x)w_x + b\coth(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y], x] + b*Coth[mu*y]*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\sinh(\lambda x)}c_1\left(-\frac{2a\cosh(\mu y)\sinh^{-\frac{b\mu}{a\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*tanh(lambda*x)*diff(w(x,y),x)+b*coth(mu*y)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y),'build')),outpu
```

$$w(x,y)=(\tanh(\lambda x)-1)^{-\frac{1}{2a\lambda}}(\tanh(\lambda x)+1)^{-\frac{1}{2a\lambda}}\left(\tanh^{\frac{1}{a\lambda}}(\lambda x)\right)\_F1\left(\frac{a\lambda\ln\left(\frac{\sqrt{(\coth^2(\mu y)-1)\left(-\frac{1}{(e^{2\lambda x}-1)^2}\right)}}{\coth^2(\mu y)-1}\right.}{b\lambda\mu}\right.$$

7.4.13 5.1

Local contents

7.4.13.1 [1104] Problem 1

problem number 1104

Added Feb. 25, 2019.

Problem Chapter 4.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\frac{(a\beta y-b\beta x)\log(a(\beta y+\lambda x))}{a\lambda+b\beta}+x\log(\beta y+\lambda x)-x\right)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*ln(lambda*x + beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\beta y+\lambda x)^{\frac{(\beta y+\lambda x)c}{a\lambda+b\beta}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{-\frac{(\beta y+\lambda x)c}{a\lambda+b\beta}}$$

7.4.13.2 [1105] Problem 2

problem number 1105

Added Feb. 25, 2019.

Problem Chapter 4.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\ln(\lambda x) + k\ln(\beta y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Log[lambda*x] + k*Log[beta*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{x(c+k)}{a}}(\lambda x)^{\frac{cx}{a}}(\beta y)^{\frac{ky}{b}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*ln(lambda*x)+k*ln(beta*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\beta y)^{\frac{ky}{b}}\,(\lambda x)^{\frac{cx}{a}}\,\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{-kay-bcx}{ab}}$$

7.4.13.3 [1106] Problem 3

problem number 1106

Added Feb. 25, 2019.

Problem Chapter 4.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda x)w_y = \left(c\ln^m(\mu x) + s\ln^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b(-\log(\lambda x))^{-n}\log^n(\lambda x)\,\text{Gamma}(n+1,-\log(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x\frac{s\log^k\left(\frac{\beta(-b\,\text{Gamma}(}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*ln(lambda*x)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)e^{\int^x\frac{c\ln(\_b\lambda)^m+s\ln\left(\left(y+\int\frac{b\ln(\_b\lambda)^n}{a}d\_b-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)\beta\right)^k}{a}d\_b}$$

7.4.13.4 [1107] Problem 4

problem number 1107

Added Feb. 25, 2019.

Problem Chapter 4.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda y)w_y=\left(c\ln^m(\mu x)+s\ln^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(-\log(\lambda y))^n\log^{-n}(\lambda y)\,\text{Gamma}(1-n,-\log(\lambda y))}{\lambda}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\log^{-n}(\lambda K[1])\left(s\log\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*ln(lambda*y)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\ln(\lambda y)^{-n}\,dy\right)}{b}+x\right)\mathrm{e}^{\int^y\frac{\left(c\ln\left(\left(\left(-\frac{a\left(\int\ln(\lambda y)^{-n}dy\right)}{b}+x+\int\frac{a\ln(\_b\lambda)^{-n}}{b}d\_b\right)\lambda\right)^m+s\ln(\_b\beta)^k\right)\ln(\_b\lambda)^{-n}}{b}d\_b}$$

7.4.13.5 [1108] Problem 5

problem number 1108

Added Feb. 25, 2019.

Problem Chapter 4.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln(\beta y)w_x+a\ln(\lambda x)w_y=bw\ln(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[beta*y]*D[w[x, y], x] + a*Log[lambda*x]*D[w[x, y], y] == b*w[x, y]*Log[beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{bx}c_1\left(y\left(\log\left(\beta y e^{\frac{ax}{y}}x^{-\frac{ax}{y}}\lambda^{-\frac{ax}{y}}\right)-1\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := ln(beta*y)*diff(w(x,y),x)+a*ln(lambda*x)*diff(w(x,y),y) =  b*w(x,y)*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-ax\ln(\lambda x)+ax+y\ln(\beta y)-y}{a}\right)e^{bx}$$

7.4.13.6 [1109] Problem 6

problem number 1109

Added Feb. 25, 2019.

Problem Chapter 4.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln(\lambda x)^n w_x + b\ln(\beta y)^k w_y = c\ln(\gamma x)^m w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[beta*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*ln(lambda*x)^n*diff(w(x,y),x)+b*ln(beta*y)^k*diff(w(x,y),y) = c*log(gamma*x)^m*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \ln(\lambda x)^{-n}\, dx\right) + \int \frac{a \ln(\beta y)^{-k}}{b} dy\right) e^{\int \frac{c(\ln(x)+\ln(\gamma))^m \ln(\lambda x)^{-n}}{a} dx}$$

7.4.14 5.2

Local contents

7.4.14.1 [1110] Problem 1

problem number 1110

Added Feb. 25, 2019.

Problem Chapter 4.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + s\ln^k(\lambda y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + s*Log[gamma*y]^k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{s\log^k(\gamma y)(-\log(\gamma y))^{-k}\operatorname{Gamma}(k+1,-\log(\gamma y))}{b\gamma}+\frac{cx^{n+1}}{an+a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n+s*ln(gamma*y)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\int^x \frac{c\_a^n+s\left(\ln\left(\frac{ay-(-\_a+x)b}{a}\right)+\ln(\gamma)\right)^k}{a}d\_a}$$

7.4.14.2 [1111] Problem 2

problem number 1111

Added Feb. 25, 2019.

Problem Chapter 4.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=(by^2+cx^ny+s\ln^k(\lambda x))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*y^2 + c*x^n*y + s*Log[lambda*x]^k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\frac{s\log^k(\lambda x)(-\log(\lambda x))^{-k}\operatorname{Gamma}(k+1,-\log(\lambda x))}{\lambda}+\frac{1}{3}a^2bx^3+abx^2(y-a\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = (b*y^2+c*x^n*y+ s*ln(lambda*x)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(-ax+y)\,\mathrm{e}^{\int^x\left(ac\_a^{n+1}-(ax-y)c\_a^n+s\ln(\_a\lambda)^k+((-\_a+x)a-y)^2b\right)d\_a}$$

7.4.14.3 [1112] Problem 3

problem number 1112

Added March 9, 2019.

Problem Chapter 4.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=b\ln^k(\lambda x)\ln^n(\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x b\log^k(\lambda K[1])\log^n(\beta(y+a(K[1]-x)))dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b*ln(lambda*x)^k*ln(beta*y)^n*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x b\ln(\_a\lambda)^k \ln(-((-\_a+x)a-y)\beta)^n d\_a}$$

7.4.14.4 [1113] Problem 4

problem number 1113

Added March 9, 2019.

Problem Chapter 4.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^n)w_y = c\ln^k(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ba^{-n-1}\,\mathrm{Gamma}(n+1,ax) + ye^{-ax}\right)\exp\left(\frac{c(-\log(\lambda x))^{-k}\log^k(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\right.\right.\right.}{\lambda}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a*y+b*x^n)*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-b\,x^{n}(ax)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}}+(n+1)\,ay\right)\mathrm{e}^{-ax}}{(n+1)\,a}\right)\mathrm{e}^{\int c\ln(\lambda x)^{k}dx}$$

7.4.14.5 [1114] Problem 5

problem number 1114

Added March 9, 2019.

Problem Chapter 4.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k(n\ln x + m\ln y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\frac{x^k(akm\log(y)+akn\log(x)-an-bm)}{a^2k^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = x^k*(n*ln(x)+m*ln(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = x^{\frac{n\,x^k}{ak}} \left(x^{\frac{b}{a}}\right)^{\frac{m\,x^k}{ak}} \left(y\,x^{-\frac{b}{a}}\right)^{\frac{m\,x^k}{ak}} \_F1\left(y\,x^{-\frac{b}{a}}\right) \mathrm{e}^{-\frac{\left(i\pi akm\mathrm{csgn}(iy)^3-i\pi akm\mathrm{csgn}(iy)^2\mathrm{csgn}\left(ix^{\frac{b}{a}}\right)-i\left(\mathrm{csgn}(iy)-\mathrm{csgn}\left(ix^{\frac{b}{a}}\right)\right)\pi ak}{2a^2k^2}}$$

7.4.14.6 [1115] Problem 6

problem number 1115

Added March 9, 2019.

Problem Chapter 4.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + by^n w_y = (c\ln^m(\lambda x) + s\ln^t(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*y]^t)*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-k}}{a(k-1)}-\frac{y^{1-n}}{n-1}\right)\exp\left(\int_1^x \frac{K[1]^{-k}\left(c\log^m(\lambda K[1])+s\log^t\left(\beta\left(\frac{a(k-1)x^ky^n}{a(k-1)x^kyK[1]^k-b(n-1)y}\right.\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^k*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = (c*ln(lambda*x)^m+s*ln(beta*y)^t)*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(k-1)\,a\,y^{-n+1}-(n-1)\,b\,x^{-k+1}}{(k-1)\,a}\right) e^{\int^{x} \frac{\left(c\ln(\_a\lambda)^{m}+s\ln\left(\beta\left(\frac{(k-1)a\,y^{-n+1}+(n-1)b\_a^{-k+1}-(n-1)b\,x^{-k+1}}{(k-1)a}\right)^{-}\right.\right.}{a}}$$

7.4.15 6.1

Local contents

7.4.15.1 [1116] Problem 1

problem number 1116

Added March 9, 2019.

Problem Chapter 4.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sin(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{-\frac{c\cos(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{-\frac{c\cos(\lambda x+\mu y)}{a\lambda+\mu b}}$$

7.4.15.2 [1117] Problem 2

problem number 1117

Added March 9, 2019.

Problem Chapter 4.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c\sin(\lambda x)+k\sin(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sin[lambda*x] + k*Sin[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{-\frac{c\cos(\lambda x)}{a\lambda}-\frac{k\cos(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*sin(lambda*x)+k*sin(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) e^{\frac{-ak\lambda\cos(\mu y)-bc\mu\cos(\lambda x)}{ab\lambda\mu}}$$

7.4.15.3 [1118] Problem 3

problem number 1118

Added March 9, 2019.

Problem Chapter 4.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{-\frac{ax\cos(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{-\frac{a\cos(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

7.4.15.4 [1119] Problem 4

problem number 1119

Added March 9, 2019.

Problem Chapter 4.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x\frac{s\sin^k\left(\frac{\beta\left(-b\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*x)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b(\sin^n(\lambda x))}{a}dx\right)\right)e^{\int^x\frac{c(\sin^m(\_b\mu))+s\left(-\sin\left(\left(-y-\left(\int\frac{b(\sin^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\sin^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}d\_b}$$

7.4.15.5 [1120] Problem 5

problem number 1120

Added March 9, 2019.

Problem Chapter 4.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{\cos^2(\lambda y)}\sec(\lambda y)\sin^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\lambda y)\right)}{\lambda-\lambda n}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\sin^{-n}(\lambda K[1]\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*y)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\left(\int\left(\sin^{-n}(\lambda y)\right)dy\right)+bx}{b}\right)e^{\int^y\frac{\left(c\left(-\sin\left(\frac{\left(a\left(\int\left(\sin^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\sin^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)^m+s\left(\sin^k\right.\right.}{b}}$$

7.4.16 6.2

Local contents

7.4.16.1 [1121] Problem 1

problem number 1121

Added March 9, 2019.

Problem Chapter 4.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cos(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sin(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{c\sin(\lambda x+\mu y)}{a\lambda+\mu b}}$$

7.4.16.2 [1122] Problem 2

problem number 1122

Added March 9, 2019.

Problem Chapter 4.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cos(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cos[lambda*x] + k*Cos[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sin(\lambda x)}{a\lambda}+\frac{k\sin(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cos(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\sin(\mu y)+bc\mu\sin(\lambda x)}{ab\lambda\mu}}$$

7.4.16.3 [1123] Problem 3

problem number 1123

Added March 9, 2019.

Problem Chapter 4.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + y w_y = a x \cos(\lambda x + \mu y) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow c_1\left(\frac{y}{x}\right)e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\sin(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

7.4.16.4 [1124] Problem 4

problem number 1124

Added March 9, 2019.

Problem Chapter 4.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\cos^n(\lambda x) w_y = (c\cos^m(\mu x) + s\cos^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{b\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(\lambda x)\right)}{a\lambda n+a\lambda}+y\right)\exp\left(\int_1^x\frac{s\cos^k\left(\frac{\beta\left(b\cos\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*x)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c(\cos^m(\_b\mu))+s\left(\cos^k\left(\left(-y-\left(\int\frac{b(\cos^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\beta\right)\right)}{a}d\_b}$$

7.4.16.5 [1125] Problem 5

problem number 1125

Added March 9, 2019.

Problem Chapter 4.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cos^n(\lambda y)w_y=(c\cos^m(\mu x)+s\cos^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{\sin^2(\lambda y)}\csc(\lambda y)\cos^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\lambda y)\right)}{\lambda(n-1)}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\cos^{-n}(\lambda K[1\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*y)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\left(\int\left(\cos^{-n}(\lambda y)\right)dy\right)+bx}{b}\right)\mathrm{e}^{\int^y\frac{\left(c\left(\cos^m\left(\frac{\left(a\left(\int\left(\cos^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\cos^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)+s\left(\cos^k\left(\right.\right.}{b}}$$

7.4.17 6.3

Local contents

7.4.17.1 [1126] Problem 1

problem number 1126

Added March 9, 2019.

Problem Chapter 4.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cos^{-\frac{c}{a\lambda+b\mu}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\tan^2(\lambda x+\mu y)+1\right)^{\frac{c}{2a\lambda+2\mu b}} \_F1\left(\frac{ay-bx}{a}\right)$$

7.4.17.2 [1127] Problem 2

problem number 1127

Added March 9, 2019.

Problem Chapter 4.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\tan(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tan[lambda*x] + k*Tan[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{c}{a\lambda}}(\lambda x)\cos^{-\frac{k}{b\mu}}(\mu y)c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*tan(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\tan^2(\lambda x)+1\right)^{\frac{c}{2a\lambda}}\left(\tan^2(\mu y)+1\right)^{\frac{k}{2b\mu}}\_F1\left(\frac{ay-bx}{a}\right)$$

7.4.17.3 [1128] Problem 3

problem number 1128

Added March 9, 2019.

Problem Chapter 4.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cos^{-\frac{ax}{\lambda x+\mu y}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\tan^2(\lambda x+\mu y)+1\right)^{\frac{a}{2\lambda+\frac{2\mu y}{x}}}\_F1\left(\frac{y}{x}\right)$$

7.4.17.4 [1129] Problem 4

problem number 1129

Added March 9, 2019.

Problem Chapter 4.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda x)w_y=(c\tan^m(\mu x)+s\tan^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\tan^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x\frac{s\tan^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*x)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\tan^n(\lambda x))}{a} dx\right)\right) \mathrm{e}^{\int^x \frac{c(\tan^m(\_b\mu))+s\left(-\tan\left(\left(-y-\left(\int \frac{b(\tan^n(\_b\lambda))}{a} d\_b\right)+\int \frac{b(\tan^n(\lambda x))}{a} dx\right)\beta\right)\right)^k}{a} d\_b}$$

7.4.17.5 [1130] Problem 5

problem number 1130

Added March 9, 2019.

Problem Chapter 4.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\tan^n(\lambda y) w_y = (c\tan^m(\mu x) + s\tan^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\tan^{1-n}(\lambda y)\,{}_2F_1\left(1, \frac{1}{2} - \frac{n}{2}; \frac{3}{2} - \frac{n}{2}; -\tan^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right) \exp\left(\int_1^y \frac{\tan^{-n}(\lambda K[1])\left(s\tan^k(\beta\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*y)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int \left(\tan^{-n}(\lambda y)\right) dy\right)}{b} + x\right) e^{\int^y \frac{\left(c\left(-\tan\left(-\mu\left(\int \frac{a\left(\tan^{-n}(\_b\lambda)\right)}{b} d\_b\right) - \left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right) dy\right)}{b} + x\right)\mu\right)\right)^m + s(t}{b}}$$

7.4.18 6.4

Local contents

7.4.18.1 [1131] Problem 1

problem number 1131

Added March 9, 2019.

Problem Chapter 4.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cot(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\cot^2(\lambda x+\mu y)+1\right)^{-\frac{c}{2a\lambda+2\mu b}}\_F1\left(\frac{ay-bx}{a}\right)$$

7.4.18.2 [1132] Problem 2

problem number 1132

Added March 9, 2019.

Problem Chapter 4.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c\cot(\lambda x)+k\cot(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cot[lambda*x] + k*Cot[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\sin^{\frac{c}{a\lambda}}(\lambda x)c_1\left(y-\frac{bx}{a}\right)e^{\frac{k(\log(\tan(\mu y))+\log(\cos(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cot(lambda*x)+k*cot(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cot^2(\lambda x) + 1\right)^{-\frac{c}{2a\lambda}} \left(\cot^2(\mu y) + 1\right)^{-\frac{k}{2b\mu}} \_F1\left(\frac{ay - bx}{a}\right)$$

7.4.18.3 [1133] Problem 3

problem number 1133

Added March 9, 2019.

Problem Chapter 4.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cot(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) \exp\left(\frac{ax(\log(\tan(\lambda x + \mu y)) + \log(\cos(\lambda x + \mu y)))}{\lambda x + \mu y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cot^2(\lambda x + \mu y) + 1\right)^{-\frac{a}{2\left(\lambda + \frac{\mu y}{x}\right)}} \_F1\left(\frac{y}{x}\right)$$

7.4.18.4 [1134] Problem 4

problem number 1134

Added March 9, 2019.

Problem Chapter 4.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot^n(\lambda x)w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{b\cot^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda x)\right)}{a\lambda n+a\lambda}+y\right)\exp\left(\int_1^x \frac{s\cot^k\left(\frac{\beta\left(b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*x)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y-\left(\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\right)e^{\int^x \frac{c(\cot^m(\_b\mu))+s\left(-\cot\left(\left(-y-\left(\int\frac{b(\cot^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}d\_b}$$

7.4.18.5 [1135] Problem 5

problem number 1135

Added March 9, 2019.

Problem Chapter 4.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot^n(\lambda y)w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\cot^{1-n}(\lambda y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\cot^2(\lambda y)\right)}{\lambda(n-1)}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\left(s\cot^k(\beta K[1])+c\cot^m\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*y)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)e^{\int^y\frac{\left(c\left(-\cot\left(-\mu\left(\int\frac{a\left(\cot^{-n}(\_b\lambda)\right)}{b}d\_b\right)-\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)^m+s\left(\cot\right.\right.}{b}}$$

7.4.19 6.5

Local contents

7.4.19.1 [1136] Problem 1

problem number 1136

Added March 9, 2019.

Problem Chapter 4.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = (b\sin(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Cos[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)e^{\frac{k\sin(\mu y)}{a\mu}-\frac{b\cos(\lambda x)}{\lambda}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\frac{-ab\mu\cos(\lambda x)+k\lambda\sin(\mu y)}{a\lambda\mu}}$$

7.4.19.2 [1137] Problem 2

problem number 1137

Added March 9, 2019.

Problem Chapter 4.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin(\mu y)w_y = (b\sin(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Tan[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{b\cos(\lambda x)}{\lambda}}c_1(y-ax)\cos^{-\frac{k}{a\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cos^{-\frac{k}{a\mu}}(\mu y)\right)\_F1(-ax+y)\,\mathrm{e}^{-\frac{b\cos(\lambda x)}{\lambda}}$$

7.4.19.3 [1138] Problem 3

problem number 1138

Added March 9, 2019.

Problem Chapter 4.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin(\mu y)w_y = b\tan(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[mu*y]*D[w[x, y], y] == b*Tan[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*sin(mu*y)*diff(w(x,y),y) = b*tan(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\tan^2(\lambda x)+1\right)^{\frac{b}{2\lambda}}\_F1\left(\frac{\ln\left(\operatorname{RootOf}\left(\mu y-\arctan\left(\frac{2\_Z\mathrm{e}^{a\mu x}}{\_Z^2\mathrm{e}^{2a\mu x}+1},-\frac{\_Z^2\mathrm{e}^{2a\mu x}-1}{\_Z^2\mathrm{e}^{2a\mu x}+1}\right)\right)\right)}{a\mu}\right)$$

7.4.19.4 [1139] Problem 4

problem number 1139

Added March 9, 2019.

Problem Chapter 4.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\tan(\mu y)w_y=b\sin(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tan[mu*y]*D[w[x, y], y] == b*Sin[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{b\cos(\lambda x)}{\lambda}}c_1\left(\frac{\log(\sin(\mu y))}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*tan(mu*y)*diff(w(x,y),y) = b*sin(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\ln\left(\operatorname{csgn}\left(\frac{1}{\cos(\mu y)}\right)\mathrm{e}^{-a\mu x}\sin(\mu y)\right)}{a\mu}\right)\mathrm{e}^{-\frac{b\cos(\lambda x)}{\lambda}}$$

7.4.19.5 [1140] Problem 5

problem number 1140

Added March 9, 2019.

Problem Chapter 4.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin(\lambda x)w_x+aw_y=b\cos(\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sin[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Cos[mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sin(\mu y)}{a\mu}}c_1\left(\frac{-a\log\left(\sin\left(\frac{\lambda x}{2}\right)\right)+a\log\left(\cos\left(\frac{\lambda x}{2}\right)\right)+\lambda y}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  sin(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*cos(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\ln\left(-\cot\left(\lambda x\right)+\csc\left(\lambda x\right)\right)+\lambda y}{\lambda}\right)\mathrm{e}^{\frac{b\sin(\mu y)}{a\mu}}$$

7.4.19.6 [1141] Problem 6

problem number 1141

Added March 9, 2019.

Problem Chapter 4.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)w_x + aw_y = b\tan(\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Tan[mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\cos^{-\frac{b}{a\mu}}(\mu y)c_1\left(\frac{a\log(\cos(\lambda x))}{\lambda}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  cot(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*tan(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cos^{-\frac{b}{a\mu}}(\mu y)\right) \_F1\left(\frac{-a\ln\left(\cot^2(\lambda x)+1\right)+2a\ln\left(\cot(\lambda x)\right)+2\lambda y}{2\lambda}\right)$$

7.4.20 7.1

Local contents

7.4.20.1 [1142] Problem 1

problem number 1142

Added March 9, 2019.

Problem Chapter 4.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arcsin(\frac{x}{\lambda} + k\arcsin(\frac{y}{\beta})\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcSin[x/lambda] + k*ArcSin[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{\frac{k\left(\sqrt{a^2(\beta^2-y^2)}(ay-bx)\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)+a^2(\beta^2-y^2)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}}+akx\sin^{-1}\left(\frac{y}{\beta}\right)+ac\lambda\right.}{a^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arcsin(x/lambda)+k*arcsin(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)e^{\frac{aky\arcsin\left(\frac{y}{\beta}\right)+bcx\arcsin\left(\frac{x}{\lambda}\right)+\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,a\beta k+\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda}{ab}}$$

7.4.20.2 [1143] Problem 2

problem number 1143

Added March 9, 2019.

Problem Chapter 4.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\arcsin(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}+(\beta y+\lambda x)\sin^{-1}(\beta y+\lambda x)\right)}{a\lambda+b\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left((\beta y+\lambda x)\arcsin(\beta y+\lambda x)+\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)c}{a\lambda+b\beta}}$$

7.4.20.3 [1144] Problem 3

problem number 1144

Added March 9, 2019.

Problem Chapter 4.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ax\arcsin(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{a\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}(-3a\beta y+a\lambda x+4b\beta x)+\sin^{-1}(\beta y+\lambda\right.}{4(a\lambda+b\beta)^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left(\left(2(\beta y+\lambda x)b\beta x+\left(-\beta^2y^2+\lambda^2x^2-\frac{1}{2}\right)a\right)\arcsin(\beta y+\lambda x)+\left(2b\beta x+\left(-\frac{3\beta y}{2}+\frac{\lambda x}{2}\right)a\right)\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)a}{2(a\lambda+b\beta)^2}}$$

7.4.20.4 [1145] Problem 4

problem number 1145

Added March 9, 2019.

Problem Chapter 4.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^n(\lambda x)w_y=\left(c\arcsin^m(\mu x)+s\arcsin^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x\frac{b\sin^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x\frac{s\sin^{-1}\left(\beta\left(y-\int_1^x\frac{b\sin^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*x)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\left(-\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)\arcsin(\lambda x)+\arcsin(\lambda x)^{n+\frac{3}{2}}\right)\sqrt{-\lambda^2x^2+1}\,b+\left(\right.}{}\right.$$

7.4.20.5 [1146] Problem 5

problem number 1146

Added March 9, 2019.

Problem Chapter 4.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^n(\lambda y)w_y=\left(c\arcsin^m(\mu x)+s\arcsin^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{bx}{a}-\frac{i\sin^{-1}(\lambda y)^{-n}\left(\left(-i\sin^{-1}(\lambda y)\right)^n\text{Gamma}\left(1-n,-i\sin^{-1}(\lambda y)\right)-\left(i\sin^{-1}(\lambda y)\right)^n\text{G}\right.}{2\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*y)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\left(-\text{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin\left(\lambda y\right)\right)\arcsin\left(\lambda y\right)+\arcsin\left(\lambda y\right)^{-n+\frac{3}{2}}\right)\sqrt{-\lambda^2y^2+1}\,a-\left(\right.}{}\right.$$

7.4.21 7.2

Local contents

7.4.21.1 [1147] Problem 1

problem number 1147

Added March 9, 2019.

Problem Chapter 4.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arccos(\frac{x}{\lambda} + k\arccos(\frac{y}{\beta})\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCos[x/lambda] + k*ArcCos[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{-\frac{k\left(\sqrt{a^2(\beta^2-y^2)}(ay-bx)\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)+a^2(\beta^2-y^2)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}}+akx\cos^{-1}\left(\frac{y}{\beta}\right)-ac}{a^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccos(x/lambda)+k*arccos(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\frac{aky\arccos\left(\frac{y}{\beta}\right)+bcx\arccos\left(\frac{x}{\lambda}\right)-\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,a\beta k-\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda}{ab}}$$

7.4.21.2 [1148] Problem 2

problem number 1148

Added March 9, 2019.

Problem Chapter 4.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arccos(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\beta(bx-ay)\sin^{-1}(\beta y+\lambda x)+x(a\lambda+b\beta)\cos^{-1}(\beta y+\lambda x)+a\left(-\sqrt{-\beta^2\right.\right.}{a(a\lambda+b\beta)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left((\beta y+\lambda x)\arccos(\beta y+\lambda x)-\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)c}{a\lambda+b\beta}}$$

7.4.21.3 [1149] Problem 3

problem number 1149

Added March 9, 2019.

Problem Chapter 4.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = ax \arccos(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{(a^2 + 2\beta^2(bx - ay)^2)\sin^{-1}(\beta y + \lambda x) - a\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}(\cdot}{4(a\lambda + b\beta)^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right) e^{\frac{\left(\frac{a\arcsin(\beta y+\lambda x)}{2} + (2b\beta x + (-\beta y + \lambda x)a)(\beta y + \lambda x)\arccos(\beta y + \lambda x) + \left(-2b\beta x + \left(\frac{3\beta y}{2} - \frac{\lambda x}{2}\right)a\right)\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}\right)}{2(a\lambda + b\beta)^2}}$$

7.4.21.4 [1150] Problem 4

problem number 1150

Added March 9, 2019.

Problem Chapter 4.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda x)w_y = \left(c\arccos^m(\mu x) + s\arccos^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{b\cos^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x \frac{s\cos^{-1}\left(\beta\left(y-\int_1^x \frac{b\cos^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*x)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}+\arccos(\lambda x)^{n+1}+\frac{(n+2)\operatorname{LommelS1}\left(n+\frac{1}{2}\right.}{\sqrt{\arccos(\lambda}}\right.}{}\right.\right.$$

7.4.21.5 [1151] Problem 5

problem number 1151

Added March 9, 2019.

Problem Chapter 4.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda y)w_y = \left(c\arccos^m(\mu x) + s\arccos^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \cos^{-1}(\lambda K[1])^{-n}dK[1] - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cos^{-1}(\lambda K[2])^{-n}\left(s\cos^{-1}(\beta K[2])^k + c\cos^{-1}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*y)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\text{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)} - \arccos(\lambda y)^{-n+1} + \frac{(n-2)\,\text{LommelS1}(-n}{\sqrt{\arccos}}\right.}{}\right.$$

7.4.22 7.3

Local contents

7.4.22.1 [1152] Problem 1

problem number 1152

Added March 9, 2019.

Problem Chapter 4.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arctan(\frac{x}{\lambda} + k\arctan(\frac{y}{\beta})\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcTan[x/lambda] + k*ArcTan[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to (\lambda^2+x^2)^{-\frac{c\lambda}{2a}}c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{k\left(2y\tan^{-1}\left(\frac{y}{\beta}\right)-\beta\log\left(a^2(\beta^2+y^2)\right)\right)}{2b}+\frac{cx\tan^{-1}\left(\frac{x}{\lambda}\right)}{a}\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arctan(x/lambda)+k*arctan(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{\beta^2+y^2}{\beta^2}\right)^{-\frac{\beta k}{2b}}\left(\frac{x^2}{\lambda^2}+1\right)^{-\frac{c\lambda}{2a}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{aky\arctan\left(\frac{y}{\beta}\right)+bcx\arctan\left(\frac{x}{\lambda}\right)}{ab}}$$

7.4.22.2 [1153] Problem 2

problem number 1153

Added March 9, 2019.

Problem Chapter 4.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arctan(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\beta^2 y^2 + 2\beta\lambda xy + \lambda^2 x^2 + 1\right)^{-\frac{c}{2a\lambda+2b\beta}} \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{(\beta y+\lambda x)c \arctan(\beta y+\lambda x)}{a\lambda+b\beta}}$$

7.4.22.3 [1154] Problem 3

problem number 1154

Added March 9, 2019.

Problem Chapter 4.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax \arctan(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{1}{4}\left(2x^2 \tan^{-1}(\beta y + \lambda x) + \frac{i(a+i\beta(bx-ay))^2 \log(a(\beta y + \lambda x + i)) + i(b\beta\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\beta^2 y^2 + 2\beta\lambda xy + \lambda^2 x^2 + 1\right)^{\frac{(ay-bx)a\beta}{2(a\lambda+b\beta)^2}} \_F1\left(\frac{ay-bx}{a}\right) e^{\frac{\left(-(\beta y+\lambda x)a+\left(2(\beta y+\lambda x)b\beta x+\left(-\beta^2y^2+\lambda^2x^2+1\right)a\right)\arctan(\beta y}{2(a\lambda+b\beta)^2}}$$

7.4.22.4 [1155] Problem 4

problem number 1155

Added March 9, 2019.

Problem Chapter 4.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda x)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x \frac{s\tan^{-1}\left(\beta\left(y - \int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1] + \int_1^K\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*x)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b\arctan(\lambda x)^n}{a}dx\right)\right) \mathrm{e}^{\int^x \frac{c\arctan(\_b\mu)^m + s\arctan\left(\left(y+\int \frac{b\arctan(\_b\lambda)^n}{a}d\_b - \left(\int \frac{b\arctan(\lambda x)^n}{a}dx\right)\right)\beta\right)^k}{a} d\_}$$

7.4.22.5 [1156] Problem 5

problem number 1156

Added March 9, 2019.

Problem Chapter 4.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda y)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \tan^{-1}(\lambda K[1])^{-n}dK[1] - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\tan^{-1}(\lambda K[2])^{-n}\left(s\tan^{-1}(\beta K[2])^k + c\tan^{-}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*y)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int \arctan(\lambda y)^{-n} dy\right)}{b} + x\right) e^{\int^y \frac{\left(c \arctan\left(\left(\left(-\frac{a\left(\int \arctan(\lambda y)^{-n} dy\right)}{b} + x + \int \frac{a \arctan(\_b\lambda)^{-n}}{b} d\_b\right)\mu\right)^m + s \arctan(\right.}{b}}$$

7.4.23 7.4

Local contents

7.4.23.1 [1157] Problem 1

problem number 1157

Added March 9, 2019.

Problem Chapter 4.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c \operatorname{arccot}(\frac{x}{\lambda} + k \operatorname{arccot}(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCot[x/lambda] + k*ArcCot[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\left(\lambda^2+x^2\right)^{\frac{c\lambda}{2a}}c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{k\left(a\beta\log\left(a^2(\beta^2+y^2)\right)+2\tan^{-1}\left(\frac{y}{\beta}\right)(bx-ay)+2bx\cot^{-}\right.}{2ab}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccot(x/lambda)+k*arccot(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\frac{\beta^2+y^2}{\beta^2}\right)^{\frac{\beta k}{2b}}\left(\frac{x^2}{\lambda^2}+1\right)^{\frac{c\lambda}{2a}}\_F1\left(\frac{ay-bx}{a}\right)e^{\frac{-2aky\arctan\left(\frac{y}{\beta}\right)-2\left(c\arctan\left(\frac{x}{\lambda}\right)-\frac{\pi(c+k)}{2}\right)bx}{2ab}}$$

7.4.23.2 [1158] Problem 2

problem number 1158

Added March 9, 2019.

Problem Chapter 4.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\operatorname{arccot}(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(a\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right)+2\beta(bx-ay)\tan^{-1}(\beta y+\lambda x)+}{2a(a\lambda+b\beta)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)^{\frac{c}{2a\lambda+2b\beta}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{(-2(\beta y+\lambda x)a\arctan(\beta y+\lambda x)+\pi(a\lambda+b\beta)x)c}{2(a\lambda+b\beta)a}}$$

7.4.23.3 [1159] Problem 3

problem number 1159

Added March 9, 2019.

Problem Chapter 4.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ax\operatorname{arccot}(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{1}{4}\left(2x^2\cot^{-1}(\beta y+\lambda x)+\frac{i(ia\beta y+a-ib\beta x)^2\log(-a(\beta y+\lambda x-i))+i(\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)^{-\frac{(ay-bx)a\beta}{2(a\lambda+b\beta)^2}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{2\pi ab\beta\lambda x^2+\pi b^2\beta^2x^2+\left(\pi\lambda^2x^2+2\beta y+2\lambda x\right)a^2-2\left(2(\beta y+\lambda x)b\right.}{4(a\lambda+b\beta)^2}}$$

7.4.23.4 [1160] Problem 4

problem number 1160

Added March 9, 2019.

Problem Chapter 4.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\operatorname{arccot}^n(\lambda x)w_y=\left(c\operatorname{arccot}^m(\mu x)+s\operatorname{arccot}^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x\frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x\frac{s\cot^{-1}\left(\beta\left(y-\int_1^x\frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*x)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c\left(-\arctan(\_b\mu)+\frac{\pi}{2}\right)^m+s\left(-\arctan\left(\left(y+\int\frac{b\left(-\arctan(\_b\lambda)+\frac{\pi}{2}\right)^n}{a}d\_\right.\right.\right.}{a}}$$

7.4.23.5 [1161] Problem 5

problem number 1161

Added March 9, 2019.

Problem Chapter 4.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\operatorname{arccot}^n(\lambda y)w_y=\left(c\operatorname{arccot}^m(\mu x)+s\operatorname{arccot}^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cot^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cot^{-1}(\lambda K[2])^{-n}\left(s\cot^{-1}(\beta K[2])^k+c\cot^{-1}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*y)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x\right)\mathrm{e}^{\int^y \frac{\left(c\left(-\arctan\left(\left(-\frac{a\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x+\int\frac{a\left(-\arctan\right.}{}\right.\right.\right.\right.}{}}$$

7.4.24 8.1

Local contents

7.4.24.1 [1162] Problem 1

problem number 1162

Added March 10, 2019.

Problem Chapter 4.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =f(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)e^{\int \frac{f(x)}{a}dx}$$

7.4.24.2 [1163] Problem 2

problem number 1163

Added March 10, 2019.

Problem Chapter 4.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x)yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])(y+a(K[1]-x))dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x -((-\_a+x)a-y)f(\_a)d\_a}$$

7.4.24.3 [1164] Problem 3

problem number 1164

Added March 10, 2019.

Problem Chapter 4.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = (f(x)y^2 + g(x)y + h(x))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (f[x]*y^2 + g[x]*y + h[x])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x\left(f(K[1])(y+a(K[1]-x))^2+g(K[1])(y+a(K[1]-x))+h(K[1])\right)dK\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =(f(x)*y^2+g(x)*y+h(x))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(-ax+y)\,\mathrm{e}^{\int^x\left(((-\_a+x)a-y)^2f(\_a)+((\_a-x)a+y)g(\_a)+h(\_a)\right)d\_a}$$

7.4.24.4 [1165] Problem 4

problem number 1165

Added March 10, 2019.

Problem Chapter 4.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)y^kw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])(y+a(K[1]-x))^k dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^{x}((\_a-x)a+y)^{k}f(\_a)d\_a}$$

7.4.24.5 [1166] Problem 5

problem number 1166

Added March 10, 2019.

Problem Chapter 4.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x)e^{\lambda y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*Exp[lambda*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x e^{\lambda(y+a(K[1]-x))}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*exp(lambda*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^{x}\mathrm{e}^{-((-\_a+x)a-y)\lambda}f(\_a)d\_a}$$

7.4.24.6 [1167] Problem 6

problem number 1167

Added March 10, 2019.

Problem Chapter 4.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x g(K[2])dK[2]\right)c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right)\right)\mathrm{e}^{\int g(x)dx}$$

7.4.24.7 [1168] Problem 7

problem number 1168

Added March 10, 2019.

Problem Chapter 4.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)y^k w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*y^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\exp\left(\int_1^x g(K[2])\left(e^{aK[2]}\left(e^{-ax}y - \int_1^x e^{-aK[1]}f(K[\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,e^{-ax} - \left(\int e^{-ax} f(x)\,dx\right)\right) e^{\int^x \left(\left(y\,e^{-ax} + \int e^{-\_ba} f(\_b) d\_b - \left(\int e^{-ax} f(x) dx\right)\right) e^{-\_ba}\right)^k g(\_b) d\_b}$$

7.4.24.8 [1169] Problem 8

problem number 1169

Added March 10, 2019.

Problem Chapter 4.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + y^k w_y = g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g(K[2])}{f(K[2])}dK[2]\right)c_1\left(-\int_1^x \frac{1}{f(K[1])}dK[1]-\frac{y^{1-k}}{k-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\left((k-1)\,y^k\left(\int \frac{1}{f(x)}dx\right)+y\right)y^{-k}\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}$$

7.4.24.9 [1170] Problem 9

problem number 1170

Added March 10, 2019.

Problem Chapter 4.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(y+a)w_y=(by+c)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == (b*y + c)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x\frac{1}{f(K[1])}dK[1]\right)-\int_1^x\frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK[1]\right)}{f(K[2])}dK[2]\right)\exp\left(\int_1^x\frac{c+\cdots}{\cdots}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (y+a)*diff(w(x,y),y) =(b*y+c)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left((a+y)\,\mathrm{e}^{-\left(\int\frac{1}{f(x)}dx\right)}\right)\mathrm{e}^{(a+y)b+(-ab+c)\left(\int\frac{1}{f(x)}dx\right)}$$

7.4.24.10 [1171] Problem 10

problem number 1171

Added March 10, 2019.

Problem Chapter 4.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(y+ax)w_y=g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g(K[3])}{f(K[3])}dK[3]\right)c_1\left(y\exp\left(-\int_1^x \frac{1}{f(K[1])}dK[1]\right)-\int_1^x \frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (y+a*x)*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-a\left(\int \frac{x\,\mathrm{e}^{-\left(\int \frac{1}{f(x)}dx\right)}}{f\left(x\right)}dx\right)+y\,\mathrm{e}^{-\left(\int \frac{1}{f(x)}dx\right)}\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}$$

7.4.24.11 [1172] Problem 11

problem number 1172

Added March 10, 2019.

Problem Chapter 4.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_0(x))w_y=\left(h_2(x)y^2+h_1(x)y+h_0(x)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == (h2[x]*y^2 + h1[x]*y + h0[x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\mathrm{g0}(K[2])}{f(K[2])}dK[2]\right)\exp\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) =(h2(x)*y^2+h1(x)*y+h0(x))*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y\,\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}-\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int^x\frac{\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}g0(\_f)}{f(\_f)}d\_f\right)^2\mathrm{e}^{2\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}h2(\_f}{}}$$

7.4.24.12 [1173] Problem 12

problem number 1173

Added March 10, 2019.

Problem Chapter 4.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_2(x)y^k)w_y=h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*y^k)*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h(K[3])}{f(K[3])}dK[3]\right)c_1\left((k-1)\int_1^x \frac{\exp\left((k-1)\int_1^{K[2]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\text{g2}(K[2])}{f(K[2])}dK[2]\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*y^k)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol := simplify(sol);
```

$$w(x,y) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (k-1)\left(\int \frac{\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} g2(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int \frac{h(x)}{f(x)}dx}$$

7.4.24.13 [1174] Problem 13

problem number 1174

Added March 10, 2019.

Problem Chapter 4.8.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_2(x)e^{\lambda y})w_y = h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*Exp[lambda*y])*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*exp(lambda*y))*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.4.24.14 [1175] Problem 14

problem number 1175

Added March 10, 2019.

Problem Chapter 4.8.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)y^k w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*y^k*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]\right)\exp\left(\int_1^x\frac{h(K[2])\left(\left(y^{k+1}-(k+1)\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]+(k\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*y^k*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,y^{k} + (-k-1)\left(\int \frac{g(x)}{f(x)}dx\right)\right) \mathrm{e}^{\int^{x} \frac{\left(\left(y^{k+1}+\int \frac{(-k-1)g(x)}{f(x)}dx+\int \frac{(k+1)g(\_b)}{f(\_b)}d\_b\right)^{\frac{1}{k+1}}\right)^{-k} h(\_b)}{f(\_b)}d\_b}$$

7.4.24.15 [1176] Problem 15

problem number 1176

Added March 10, 2019.

Problem Chapter 4.8.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda y}w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*Exp[lambda*y]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\lambda y}}{\lambda} - \int_1^x \frac{g(K[1])}{f(K[1])}dK[1]\right) \exp\left(\int_1^x \frac{h(K[2])}{f(K[2])\left(-\lambda \int_1^x \frac{g(K[1])}{f(K[1])}dK[1] + e^{\lambda y} + \lambda \int_1^{K[2]} \frac{g(K[1]}{f(K[1]}\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*exp(lambda*y)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{g(x)}{f(x)}dx\right) + e^{\lambda y}}{\lambda}\right) e^{\int^x \frac{h(\_b)}{\left(\lambda\left(\int \frac{g(\_b)}{f(\_b)}d\_b\right) - \lambda\left(\int \frac{g(x)}{f(x)}dx\right) + e^{\lambda y}\right) f(\_b)}d\_b}$$

7.4.25 8.2

Local contents

7.4.25.1 [1177] Problem 1

problem number 1177

Added March 10, 2019.

Problem Chapter 4.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (f(x) + g(y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x] + g[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) =(f(x)+g(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)e^{\int^x\frac{f(\_a)+g\left(\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a}$$

7.4.25.2 [1178] Problem 2

problem number 1178

Added March 10, 2019.

Problem Chapter 4.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)g(y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])g(-ax+y+aK[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x)*g(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a)g((\_a-x)a+y)d\_a}$$

7.4.25.3 [1179] Problem 3

problem number 1179

Added March 10, 2019.

Problem Chapter 4.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)h(y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\exp\left(\int_1^x g(K[2])h\left(e^{aK[2]}\left(e^{-ax}y-\int_1^x e^{-aK[1]}f(K\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a*y+f(x))*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right)\right) \mathrm{e}^{\int^x g(\_b)h\left(\left(y\,\mathrm{e}^{-ax}+\int \mathrm{e}^{-\_ba} f(\_b)d\_b-\left(\int \mathrm{e}^{-ax} f(x)dx\right)\right)\mathrm{e}^{\_ba}\right)d\_b}$$

7.4.25.4 [1180] Problem 4

problem number 1180

Added March 10, 2019.

Problem Chapter 4.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy\right) \mathrm{e}^{\int^x \frac{h1(\_f)+h2\left(\mathrm{RootOf}\left(\int \frac{1}{f(\_f)}d\_f-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy-\left(\int^{\_Z} \frac{1}{g(\_a)}d\_a\right)\right)\right)}{f(\_f)}d}$$

contains RootOf

7.4.25.5 [1181] Problem 5

problem number 1181

Added March 10, 2019.

Problem Chapter 4.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x + (f_2(x) + f_3(x)y^k)w_y = g(x)h(y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x, y], x] + (f2[x] + f3[x]*y^k)*D[w[x, y], y] == g[x]*h[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f1(x)*diff(w(x,y),x)+(f2(x)+f3(x)*y^k)*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.4.25.6 [1182] Problem 6

problem number 1182

Added March 10, 2019.

Problem Chapter 4.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h_1(x)h_2(y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x]*h2[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h1(x)*h2(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1\,(x)}dx\right) + \int \frac{g1\,(y)}{g2\,(y)}dy\right) \mathrm{e}^{\int^x \frac{h1\,(\_f) h2\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1\,(\_f)}d\_f - \left(\int \frac{f2(x)}{f1\,(x)}dx\right) + \int \frac{g1\,(y)}{g2\,(y)}dy - \left(\int^{\_Z} \frac{g1\,(\_a)}{g2\,(\_a)}d\_a\right.\right.\right.}{f1\,(\_f) g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1\,(\_f)}d\_f - \left(\int \frac{f2(x)}{f1\,(x)}dx\right) + \int \frac{g1\,(y)}{g2\,(y)}dy - \left(\int^{\_Z} \frac{g1\,(\_a)}{g2\,(\_a)}d\_a\right.\right.\right.}}$$

has RootOf

7.4.25.7 [1183] Problem 7

problem number 1183

Added March 10, 2019.

Problem Chapter 4.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1\,(x)}dx\right) + \int \frac{g1\,(y)}{g2\,(y)}dy\right) \mathrm{e}^{\int^x \frac{h1(\_f)+h2\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right)+\int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_\right.\right.\right.}{f1(\_f)g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right)+\int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_a\right.\right.\right.}}$$

has RootOf

7.4.26 8.3

Local contents

7.4.26.1 [1184] Problem 1

problem number 1184

Added March 10, 2019.

Problem Chapter 4.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = f(\alpha x + \beta y) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{f\left(\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\int^x\frac{f\left(\frac{-(-\_a+x)b\beta+(\_a\alpha+\beta y)a}{a}\right)}{a}d\_a}$$

7.4.26.2 [1185] Problem 2

problem number 1185

Added March 10, 2019.

Problem Chapter 4.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=xf(\frac{y}{x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{xf\left(\frac{y}{x}\right)}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = x*f(y/x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{xf\left(\frac{y}{x}\right)}$$

7.4.26.3 [1186] Problem 3

problem number 1186

Added March 10, 2019.

Problem Chapter 4.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=f(x^2+y^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\int_1^x\frac{f\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = f(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\int^{x} \frac{f\left(\frac{\_a^2 y^2}{x^2}+\_a^2\right)}{\_a} d\_a}$$

7.4.26.4 [1187] Problem 4

problem number 1187

Added March 10, 2019.

Problem Chapter 4.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k f(x^n * y^m) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*y^m]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right) \exp\left(\int_1^x \frac{f\left(K[1]^n \left(x^{-\frac{b}{a}} y K[1]^{\frac{b}{a}}\right)^m\right) K[1]^{k-1}}{a} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = x^k*f(x^n+y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{b}{a}}\right)\mathrm{e}^{\int^{x}\frac{\_a^{k-1}f\left(\_a^{n}+\left(y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^{m}\right)}{a}d\_a}$$

7.4.26.5 [1188] Problem 5

problem number 1188

Added March 10, 2019.

Problem Chapter 4.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = f(ax^n + by^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*y^m]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{n}{m}}\right)\exp\left(\int_1^x\frac{f\left(b\left(x^{-\frac{n}{m}}yK[1]^{\frac{n}{m}}\right)^m+aK[1]^n\right)}{mK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) = f(a*x^n+b*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{n}{m}}\right)\mathrm{e}^{\int^{x}\frac{f\left(a\,\_a^{n}+b\left(y\,\_a^{\frac{n}{m}}x^{-\frac{n}{m}}\right)^{m}\right)}{\_a m}d\_a}$$

7.4.26.6 [1189] Problem 6

problem number 1189

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x^2 w_x + xy w_y = y^k f(\alpha x^n + \beta y^m) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\int_1^x \frac{f\left(\left(\alpha+\frac{\beta y}{x}\right)K[1]\right)\left(\frac{yK[1]}{x}\right)^k}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^k*f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{\int^x \frac{\left(\frac{\_a y}{x}\right)^k f\left(\left(\alpha+\frac{\beta y}{x}\right)\_a\right)}{\_a^2}d\_a}$$

7.4.26.7 [1190] Problem 7

problem number 1190

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)}w_x + \frac{g(x)}{g'(x)}w_y = h(f(x)+g(y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[x]*D[w[x, y], y])/Derivative[1][g][x] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])}dK[1]\right)\exp\left(\int_1^x \frac{h\left(f(K[2])+g\left(y-\int_1^x \frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])}dK[1]+\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)/diff(f(x),x)*diff(w(x,y),x)+g(x)/diff(g(x),x)*diff(w(x,y),y) = h(f(x)+g(y))*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y-\left(\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{\left(\frac{d}{dx}g(x)\right)f(x)}dx\right)\right)\mathrm{e}^{\int^x \frac{\left(\frac{d}{d\_b}f(\_b)\right)h\left(f(\_b)+g\left(y+\int \frac{\left(\frac{d}{d\_b}f(\_b)\right)g(\_b)}{\left(\frac{d}{d\_b}g(\_b)\right)f(\_b)}d\_b-\left(\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{\left(\frac{d}{dx}g(x)\right)f(x)}dx\right)\right)\right)}{f(\_b)}d}$$

7.4.27 8.4

Local contents

7.4.27.1 [1191] Problem 1

problem number 1191

Added March 10, 2019.

Problem Chapter 4.8.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1],-ax+y+aK[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a,(\_a-x)a+y)d\_a}$$

7.4.27.2 [1192] Problem 2

problem number 1192

Added March 10, 2019.

Problem Chapter 4.8.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\int_1^x \frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,x^{-\frac{b}{a}}\right)\mathrm{e}^{\int^x \frac{f\left(\_a,y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_aa}d\_a}$$

7.4.27.3 [1193] Problem 3

problem number 1193

Added March 10, 2019.

Problem Chapter 4.8.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[2],\exp\left(\int_1^{K[2]}\frac{g(K[1])}{f(K[1])}dK[1]-\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]\right)y\right)}{f(K[2])}dK[2]\right)c_1\left(y\exp\left(-\int_1^x\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+g(x)*y*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\left(\int \frac{g(x)}{f(x)}dx\right)}\right)\mathrm{e}^{\int^x \frac{h\left(\_b,\,y\,\mathrm{e}^{\int \frac{g(\_b)}{f(\_b)}d\_b-\left(\int \frac{g(x)}{f(x)}dx\right)}\right)}{f(\_b)}d\_b}$$

7.4.27.4 [1194] Problem 4

problem number 1194

Added March 10, 2019.

Problem Chapter 4.8.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[3], \exp\left(\int_1^{K[3]} \frac{\text{g1}(K[1])}{f(K[1])} dK[1]\right)\left(\exp\left(-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])} dK[1]\right) y - \int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{\text{g1}}{f(}\right.}{f}\right.\right.}{f(K[3])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} - \left(\int \frac{e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} g0(x)}{f(x)} dx\right)\right) e^{\int^x \frac{h\left(\_f, \left(y\,e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} + \int \frac{e^{-\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)} g0(\_f)}{f(\_f)} d\_f\right.\right.}{f(\_f)}}$$

7.4.27.5 [1195] Problem 5

problem number 1195

Added March 10, 2019.

Problem Chapter 4.8.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[3], \left(\exp\left(-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}dK[1]-(k-1)\int_1^{K[3]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)y^{-k}\left(\exp\left(\int_1^x \frac{\text{g1}}{f}\right.\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (k-1)\left(\int \frac{\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)}g0(x)}{f\left(x\right)}dx\right)\right)\mathrm{e}^{\int^x \frac{h\left(\_f, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}\right.}\right.\right.}{}}$$

7.4.27.6 [1196] Problem 6

problem number 1196

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x,y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y]*w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.4.27.7 [1197] Problem 7

problem number 1197

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x,y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy\right) \mathrm{e}^{\int^{x} \frac{h\left(\_f, \mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_a\right)\right)}{f1(\_f)\,g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)}d\_a\right)}}$$

has RootOf

7.5 chapter 5

Local contents

7.5.1 2.1

Local contents

7.5.1.1 [1198] Problem 1

problem number 1198

Added March 10, 2019.

Problem Chapter 5.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{d}{c}+e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = c*w(x,y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{c\_F1\left(\frac{ay-bx}{a}\right)e^{\frac{cx}{a}}-d}{c}$$

7.5.1.2 [1199] Problem 2

problem number 1199

Added March 10, 2019.

Problem Chapter 5.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)w_x + (y-b)w_y = w - c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y] - c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c+(x-a)c_1\left(\frac{b-y}{a-x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x-a)*diff(w(x,y),x)+(y-b)*diff(w(x,y),y) = w(x,y)-c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = c + (a-x)\_F1\left(\frac{-b+y}{a-x}\right)$$

7.5.1.3 [1200] Problem 3

problem number 1200

Added March 10, 2019.

Problem Chapter 5.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cx+d)w_y = \alpha w + \beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*x + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(bc-ad)\log(ax+b)+a(ay-cx)}{a^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{a^2y-acx-ad\ln(ax+b)+bc\ln(ax+b)}{a^2}\right)-\beta}{\alpha}$$

7.5.1.4 [1201] Problem 4

problem number 1201

Added March 10, 2019.

Problem Chapter 5.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha w + \beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-\beta}{\alpha}$$

7.5.1.5 [1202] Problem 5

problem number 1202

Added March 10, 2019.

Problem Chapter 5.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x+(cy+d)w_y=\alpha w+\beta y+\gamma x$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta*y + gamma*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\alpha(a-\alpha)(\alpha-c)(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-a\beta(\alpha y+d)+\alpha^2\beta y+\alpha^2\gamma x+\alpha b\gamma+\alpha\beta d}{\alpha(\alpha-a)(\alpha-c)}\right.\right.$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta*y+gamma*x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{a\beta d+(-\alpha+c)\,(a-\alpha)\,\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)+(-\beta y-\gamma x)\,\alpha^2+\gamma bc+(a\beta y-\beta d}{(a-\alpha)\,(-\alpha+c)\,\alpha}$$

7.5.1.6 [1203] Problem 6

problem number 1203

Added March 10, 2019.

Problem Chapter 5.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x+(cx+dy)w_y=\alpha w+\beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*x + d*y)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(ax+b)^{-\frac{d}{a}}(-d(-ay+cx+dy)-bc)}{d(a-d)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d*y)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(dya-cxd-d^2y-bc)(ax+b)^{-\frac{d}{a}}}{(a-d)d}\right)-\beta}{\alpha}$$

7.5.1.7 [1204] Problem 7

problem number 1204

Added March 10, 2019.

Problem Chapter 5.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x+a_0)w_x+(b_2y+b_1x+b_0)w_y=(c_2y+c_1x+c_0)w+k_2y+k_1x+k_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (a1*x + a0)*D[w[x, y], x] + (b2*y + b1*x + b0)*D[w[x, y], y] == (c2*y + c1*x + c0)*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to(\text{a0}+\text{a1}x)^{\frac{\text{a0b1c2}-\text{a0b2c1}-\text{a1b0c2}+\text{a1b2c0}}{\text{a1}^2\text{b2}}}\exp\left(\frac{-\text{a1}(\text{a0b1c2}+\text{b2}(\text{b0c2}+\text{b1c2}x-\text{b2c1}x+\text{b2c2}y))}{\text{a1b2}^2(\text{a1}-\text{b}}\right.\right.\right.$$

Maple ✗

```
restart;
pde :=  (a1*x+a0)*diff(w(x,y),x)+ (b2*y+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,y)+k2*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

7.5.1.8 [1205] Problem 8

problem number 1205

Added March 10, 2019.

Problem Chapter 5.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + (b_1x + b_0)w_y = (c_1x + c_0)w + s_1x + s0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*y*D[w[x, y], x] + (b1*x + b0)*D[w[x, y], y] == (c1*x + c0)*w[x, y] + s1*x + s0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to \exp\left(\frac{(\mathrm{b0c1}-\mathrm{b1c0})\tanh^{-1}\left(\frac{\mathrm{b0}+\mathrm{b1}x}{\sqrt{\mathrm{b1}}\sqrt{ay^2}}\right)-\sqrt{\mathrm{b1}}\mathrm{c1}\sqrt{ay^2}}{\sqrt{a}\mathrm{b1}^{3/2}}\right)\left(\int_1^x -\frac{\exp\left(\frac{\sqrt{-\mathrm{b1}x^2-2\mathrm{b0}x+ay^2+\mathrm{b1}K[1]^2+}}{\mathrm{b1}}\right.}{\sqrt{a}}\right.\right.$$

$$\left\{w(x,y)\to \exp\left(\frac{(\mathrm{b1c0}-\mathrm{b0c1})\tanh^{-1}\left(\frac{\mathrm{b0}+\mathrm{b1}x}{\sqrt{\mathrm{b1}}\sqrt{ay^2}}\right)+\sqrt{\mathrm{b1}}\mathrm{c1}\sqrt{ay^2}}{\sqrt{a}\mathrm{b1}^{3/2}}\right)\left(\int_1^x \frac{\exp\left(-\frac{\sqrt{-\mathrm{b1}x^2-2\mathrm{b0}x+ay^2+\mathrm{b1}K[2]^2+}}{\mathrm{b1}}\right.}{\sqrt{a}}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b1*x+b0)*diff(w(x,y),y) =  (c1*x+c0)*w(x,y)+s1*x+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{(\_as1+s0)\left(\frac{(\_ab1+b0)a+\sqrt{(a\,y^2+(2\_a-2x)b0+(\_a^2-x^2)b1)a}\sqrt{ab1}}{\sqrt{ab1}}\right)^{\frac{b0\,c1-b1\,c0}{\sqrt{ab1}\,b1}}\mathrm{e}^{-\frac{\sqrt{\left(a\,y^2+(2\_a-2x)b0+\left(\_a^2\right.\right.}}{ab1}}}{\sqrt{(a\,y^2+(2\_a-2x)\,b0+(\_a^2-x^2)\,b1)\,a}}\right.$$

7.5.2 2.2

Local contents

7.5.2.1 [1206] Problem 1

problem number 1206

Added March 10, 2019.

Problem Chapter 5.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \beta xy + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + beta*x*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{a\beta(2b+cy)+c(b\beta x+\beta cxy+c\gamma)}{c^3}+e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c^3\_F1\left(\frac{ay-bx}{a}\right) e^{\frac{cx}{a}} - 2ab\beta - (ay+bx)\beta c + (-\beta xy - \gamma)c^2}{c^3}$$

7.5.2.2 [1207] Problem 2

problem number 1207

Added March 10, 2019.

Problem Chapter 5.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{c^3\left(-e^{\frac{cx}{a}}\right)c_1\left(y-\frac{bx}{a}\right) + 2a^2\beta + a(2b\gamma + 2\beta cx + c\gamma y) + c(b\gamma x + c(\beta x^2 + \delta + \gamma xy))}{c^3}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{c^{3}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{cx}{a}}-2a^{2}\beta-2\gamma ab+\left(-\beta\,x^{2}-\gamma xy-\delta\right)c^{2}+\left(-2a\beta x+\left(-ay-bx\right)\gamma\right)c}{c^{3}}$$

7.5.2.3 [1208] Problem 3

problem number 1208

Added March 10, 2019.

Problem Chapter 5.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=w+ax^2+by^2+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == w[x, y] + a*x^2 + b*y^2 + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to xc_1\left(\frac{y}{x}\right)+ax^2+by^2-c\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =  w(x,y)+a*x^2+b*y^2+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=a\,x^{2}+b\,y^{2}+x\_F1\left(\frac{y}{x}\right)-c$$

7.5.2.4 [1209] Problem 4

problem number 1209

Added March 10, 2019.

Problem Chapter 5.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax w_x + by w_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(2a-c)(a+b-c)x^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right)-2a^2\delta-2ab\delta+ac(x(\beta x+2\gamma y)+3\delta)+bc(\beta x^2+\delta)-}{c(c-2a)(-a-b+c)}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\beta x^2}{2a-c}+\frac{\gamma y\, x^{-\frac{b}{a}+\frac{a+b}{a}}}{a+b-c}+x^{\frac{c}{a}}\_F1\left(y\, x^{-\frac{b}{a}}\right)-\frac{\delta}{c}$$

7.5.2.5 [1210] Problem 5

problem number 1210

Added March 10, 2019.

Problem Chapter 5.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + (b_2x^2 + b_1x + b_0)w_y = (c_2x^2 + c_1x + c_0)w + s_2x^2 + s_1x + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + (b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*x^2 + c1*x + c0)*w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*x^2+c1*x+c0)*w(x,y)+s2*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

7.5.2.6 [1211] Problem 6

problem number 1211

Added March 10, 2019.

Problem Chapter 5.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2w_x + (b_1x^2 + b_0)w_y = (c_1x^2 + c_0)w + s_1x^2 + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + (b1*x^2 + b0)*D[w[x, y], y] == (c1*x^2 + c0)*w[x, y] + s1*x^2 +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b1*x^2+b0)*diff(w(x,y),y) =  (c1*x^2+c0)*w(x,y)+s1*x^2+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

7.5.2.7 [1212] Problem 7

problem number 1212

Added March 10, 2019.

Problem Chapter 5.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(a_1x^2 + a_0)w_x + (y + b_2x^2 + b_1x + b_0)w_y = (c_2y + c_1x + c_0)w + k_{22}y^2 + k12xy + k_{11}x^2 + k_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (y + b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*y + c1
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (y+b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(y\, e^{-\frac{\arctan\left(\frac{a1\,x}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}} + \int \frac{(\_f^2 b2 + \_f b1 + b0)\, e^{-\frac{\arctan\left(\frac{\_f a1}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}}}{\_f^2 a1 + a0} d\_f - \left(\int \frac{(b2\,x^2 + b1 x + b0)\, e^{-\frac{\arctan\left(\frac{a1\,x}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}}}{a1\,x^2 + a0} d \right.}{}\right.$$

7.5.2.8 [1213] Problem 8

problem number 1213

Added March 10, 2019.

Problem Chapter 5.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2 + a_0)w_x + (b_2y^2 + b_1xy)w_y = (c_2y^2 + c_1x^2)w + s_{22}y^2 + s_{12}xy + s_{11}x^2 + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (b2*y^2 + b1*x^2)*D[w[x, y], y] == (c2*y^2 + c1*x^2)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (b2*y^2+b1*x^2)*diff(w(x,y),y) =  (c2*y^2+c1*x^2)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

7.5.3 2.3

Local contents

7.5.3.1 [1214] Problem 1

problem number 1214

Added March 12, 2019.

Problem Chapter 5.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = \alpha yw + \beta\sqrt{xy} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == alpha*y*w[x, y] + beta*Sqrt[x*y] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{\frac{\alpha y}{b}}\left(-\beta\sqrt{xy}\left(\frac{\alpha y}{b}\right)^{-\frac{a+b}{2b}}\text{Gamma}\left(\frac{a+b}{2b},\frac{\alpha y}{b}\right)+bc_1\left(yx^{-\frac{b}{a}}\right)+\gamma\text{Ei}\left(-\frac{\alpha y}{b}\right)\right)}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = alpha*y*w(x,y)+ beta*sqrt(x*y)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(-4\sqrt{xy}\,(2\alpha y+a+3b)\,a\,b^3\beta\,x^{-\frac{a+b}{2a}}x^{\frac{b}{2a}+\frac{1}{2}}\left(\frac{\alpha y}{b}\right)^{-\frac{a+3b}{4b}}\left(\frac{\alpha y\,x^{-\frac{b}{a}}}{b}\right)^{-\frac{a}{2b}-\frac{1}{2}}\left(\frac{\alpha y\,x^{-\frac{b}{a}}}{b}\right)^{\frac{a}{2b}+\frac{1}{2}}\text{Whittake}}{}$$

7.5.3.2 [1215] Problem 2

problem number 1215

Added March 12, 2019.

Problem Chapter 5.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = \lambda\sqrt{xy}w + \beta xy + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == lambda*Sqrt[x*y]*w[x, y] + beta*x*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{2\lambda\sqrt{xy}}{a+b}}\left(\int_1^x \frac{e^{-\frac{2\lambda\sqrt{x^{-\frac{b}{a}}yK[1]^{\frac{a+b}{a}}}}{a+b}}\left(\beta yK[1]^{\frac{a+b}{a}}x^{-\frac{b}{a}}+\gamma\right)}{aK[1]}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = lambda*sqrt(x*y)*w(x,y)+ beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(\left(a+b\right)\left(a+b+2\sqrt{xy}\,\lambda\right)\beta\,\mathrm{e}^{-\frac{2\sqrt{xy}\,\lambda}{a+b}}-2\left(-2\gamma\,\mathrm{expIntegral}\left(1,\frac{2\sqrt{xy}\,\lambda}{a+b}\right)+(a+b)\_F1\left(y\,x^{-\frac{b}{a}}\right)\right.\right.}{2\left(a+b\right)\lambda^2}$$

7.5.3.3 [1216] Problem 3

problem number 1216

Added March 12, 2019.

Problem Chapter 5.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to e^{-\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}}\left(\int_1^x -\frac{\exp\left(\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}K[1]}{\sqrt{-bx^2+ay^2+bK[1]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[1]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[1]^2-x^2\right)}}dK[1]+c_1\left(\frac{ay^2-b\right.\right.\right.$$

$$\left\{w(x,y)\to e^{\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}}\left(\int_1^x \frac{\exp\left(-\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}K[2]}{\sqrt{-bx^2+ay^2+bK[2]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[2]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[2]^2-x^2\right)}}dK[2]+c_1\left(\frac{ay^2-bx}{2a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\sqrt{\_a}\,\beta+\gamma\right)\left(\frac{\_aab+\sqrt{(a\,y^2+(\_a^2-x^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}}}{\sqrt{(a\,y^2+(\_a^2-x^2)\,b)\,a}}d\_a+\_F1\left(\frac{a\,y^2-b\,x^2}{a}\right)\right)\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2\right.$$

7.5.3.4 [1217] Problem 4

problem number 1217

Added March 12, 2019.

Problem Chapter 5.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to e^{-\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}}\left(\int_1^x -\frac{\exp\left(\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}K[1]}{\sqrt{-bx^2+ay^2+bK[1]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[1]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[1]^2-x^2\right)}}dK[1]+c_1\left(\frac{ay^2-b\right.\right.\right.$$

$$\left\{w(x,y)\to e^{\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}}\left(\int_1^x \frac{\exp\left(-\frac{\alpha\tanh^{-1}\left(\frac{\sqrt{b}K[2]}{\sqrt{-bx^2+ay^2+bK[2]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[2]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[2]^2-x^2\right)}}dK[2]+c_1\left(\frac{ay^2-bx}{2a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(\sqrt{\_a}\,\beta+\gamma\right)\left(\frac{\_aab+\sqrt{(a\,y^2+(\_a^2-x^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}}}{\sqrt{(a\,y^2+(\_a^2-x^2)\,b)\,a}}d\_a+\_F1\left(\frac{a\,y^2-b\,x^2}{a}\right)\right)\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2\right.$$

7.5.3.5 [1218] Problem 5

problem number 1218

Added March 12, 2019.

Problem Chapter 5.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x+b\sqrt{y}w_y=\alpha w+\beta x+\gamma y+\delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*x + gamma*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to-\frac{-2\alpha^3e^{\frac{2\alpha\sqrt{x}}{a}}c_1\left(2\sqrt{y}-\frac{2b\sqrt{x}}{a}\right)+a^2\beta+2a\alpha\beta\sqrt{x}+2\alpha^2\beta x+2\alpha^2\delta+2\alpha^2\gamma y+2\alpha b\gamma\sqrt{y}+b^2\cdots}{2\alpha^3}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*x+gamma*y+de
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(-2\alpha^3\_F1\left(\frac{-a\sqrt{y}+b\sqrt{x}}{b}\right) + \left(2a\alpha\beta\sqrt{x} + a^2\beta + 2\gamma\alpha b\sqrt{y} + (2\beta x + 2\delta + 2\gamma y)\,\alpha^2 + \gamma\, b^2\right) e^{-\frac{2\alpha\sqrt{y}}{b}}\right)}{2\alpha^3}$$

7.5.3.6 [1219] Problem 6

problem number 1219

Added March 12, 2019.

Problem Chapter 5.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x + b\sqrt{y}w_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{a\beta + 2\alpha\left(\beta\sqrt{x}+\gamma\right)}{2\alpha^2} + e^{\frac{2\alpha\sqrt{x}}{a}}c_1\left(2\sqrt{y} - \frac{2b\sqrt{x}}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{y}\frac{\left(\gamma\_a+\sqrt{\frac{\left(\sqrt{\_a}\,a-a\sqrt{y}+b\sqrt{x}\right)^{2}}{b^{2}}}\,\beta+\delta\right)e^{-\frac{2\sqrt{\_a}\,\alpha}{b}}}{\sqrt{\_a}\,b}d\_a+\_F1\left(\frac{-a\sqrt{y}+b\sqrt{x}}{b}\right)\right)e^{\frac{2\alpha\sqrt{y}}{b}}$$

7.5.3.7 [1220] Problem 7

problem number 1220

Added March 12, 2019.

Problem Chapter 5.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{y}w_x+b\sqrt{x}w_y=\alpha w+\beta\sqrt{x}+\gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[y]*D[w[x, y], x] + b*Sqrt[x]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\frac{\alpha x\sqrt[3]{\frac{ay^{3/2}}{ay^{3/2}-bx^{3/2}}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{bx^{3/2}}{bx^{3/2}-ay^{3/2}}\right)}{a\sqrt[3]{y^{3/2}}}\right)\left(\int_1^x\frac{\exp\left(-\frac{\alpha\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{bK[1]^{3/2}}{bx^{3/2}-ay^{3/2}}\right)K[1]\sqrt[3]{1-}}{a\sqrt[3]{y^{3/2}+\frac{b\left(K[1]^{3/2}-x^{3/2}\right.}{a}}}\right.}{a\sqrt[3]{y^{3/2}+\frac{b(K[1]}{}}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sqrt(y)*diff(w(x,y),x)+ b*sqrt(x)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{y}\frac{\left(\gamma\_b+\sqrt{\frac{\left(\left(\_b^{\frac{3}{2}}a+b\operatorname{RootOf}\left(b^{2}x-\left(a\,b^{2}y^{\frac{3}{2}}+\_Z b^{3}\right)^{\frac{2}{3}}\right)\right)b^{2}\right)^{\frac{2}{3}}}{b^{2}}}\,\beta+\delta\right)\mathrm{e}^{-\frac{\alpha\left(\int\frac{1}{\sqrt{\frac{\left(\left(\_b^{\frac{3}{2}}a+b\operatorname{RootOf}\left(b^{2}x-\left(a\right.\right.\right.\right.}{b^{2}}}}\right.}{b}}}{\sqrt{\frac{\left(\left(\_b^{\frac{3}{2}}a+b\operatorname{RootOf}\left(b^{2}x-\left(a\,b^{2}y^{\frac{3}{2}}+\_Z b^{3}\right)^{\frac{2}{3}}\right)\right)b^{2}\right)^{\frac{2}{3}}}{b^{2}}}\,b}\right.$$

contains RootOf

7.5.4 2.4

Local contents

7.5.4.1 [1221] Problem 1

problem number 1221

Added March 12, 2019.

Problem Chapter 5.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + kx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + k*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}kK[1]^n\left(y+\frac{b(K[1]-x)}{a}\right)^m}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\_a^n\left(\frac{ay-(-\_a+x)b}{a}\right)^m \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.4.2 [1222] Problem 2

problem number 1222

Added March 12, 2019.

Problem Chapter 5.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + yw_y = bw + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x(b-m)}{a}}\left(-\frac{cy^m x^n \left(\frac{x(b-m)}{a}\right)^{-n}\text{Gamma}\left(n+1,\frac{x(b-m)}{a}\right)}{b-m}+e^{\frac{mx}{a}}c_1\left(ye^{-\frac{x}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ y*diff(w(x,y),y) = b*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{c\_a^n\left(y\,\mathrm{e}^{-\frac{\_a-x}{a}}\right)^m \mathrm{e}^{-\frac{\_ab}{a}}}{a}d\_a+\_F1\left(y\,\mathrm{e}^{-\frac{x}{a}}\right)\right)\mathrm{e}^{\frac{bx}{a}}$$

7.5.4.3 [1223] Problem 3

problem number 1223

Added April 1, 2019.

Problem Chapter 5.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = axw + bx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y] + b*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{ax}\left(-by^m x^n(ax)^{-m-n}\,\text{Gamma}(m+n,ax)+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{b\,x^n y^m (ax)^{-\frac{m}{2}-\frac{n}{2}}\,\text{WhittakerM}\left(\frac{m}{2}+\frac{n}{2},\frac{m}{2}+\frac{n}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}}}{(m+n+1)\,(m+n)}+\frac{b\,x^{n-1}y^m(ax)^{-\frac{m}{2}-\frac{n}{2}}\,\text{WhittakerM}\left(\frac{}{}\right.}{(m+n}$$

7.5.4.4 [1224] Problem 4

problem number 1224

Added April 1, 2019.

Problem Chapter 5.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}w + bx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2+y^2]*w[x,y] + b*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{a\sqrt{x^2+y^2}}\left(\int_1^x be^{-a\sqrt{\left(\frac{y^2}{x^2}+1\right)K[1]^2}}K[1]^{n-1}\left(\frac{yK[1]}{x}\right)^m dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left((m+n+1)\,b\,x^{-m}x^{m+n}y^m\left(\sqrt{x^2+y^2}\,a\right)^{-\frac{m}{2}-\frac{n}{2}}\left(\frac{\sqrt{x^2+y^2}\,a}{x}\right)^{-m-n}\left(\frac{\sqrt{x^2+y^2}\,a}{x}\right)^{m+n}\mathrm{WhittakerM}\left(\right.\right.}{}$$

7.5.4.5 [1225] Problem 5

problem number 1225

Added April 1, 2019.

Problem Chapter 5.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m w + px^k y^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x,y] + p*x^k*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cy^m x^n}{an+bm}}\left(\int_1^x \frac{\exp\left(-\frac{cK[1]^n\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^m}{bm+an}\right)pK[1]^{k-1}\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^s}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*x^n*y^m*w(x,y)+ p*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left((an+bm)^2\left(cx^ny^m+(k+n)a+(m+s)b\right)px^{\frac{bs+(k-n)a}{a}}x^{-\frac{bs}{a}}y^sy^{-m}\left(\frac{cx^ny^m}{an+bm}\right)^{\frac{(-k-n)a-(m+s)b}{2an+2bm}}\left(\frac{cx^-}{an}\right.\right.}{}$$

7.5.4.6 [1226] Problem 6

problem number 1226

Added April 1, 2019.

Problem Chapter 5.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = (cx^n + py^m)w + qx^ky^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*(x^n+p*y^m)*w[x,y] + q*x^k*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^n}{an}+\frac{cpy^m}{bm}}\left(\int_1^x \frac{\exp\left(-\frac{c\left(\frac{ap\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^m}{bm}+\frac{K[1]^n}{n}\right)}{a}\right)qK[1]^{k-1}\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^s}{a}dK[1]+c_1\left(yx^{\cdots}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*(x^n+y^m)*w(x,y)+ q*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{q\_b^{k-1}\left(y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^s \mathrm{e}^{-\frac{c\left(\int\frac{\_b^n+\left(y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m}{\_b}d\_b\right)}{a}}}{a}d\_b+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int^x \frac{\left(\_a^n+\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^{m}\cdots\right.}{\_aa}}$$

7.5.4.7 [1227] Problem 7

problem number 1227

Added April 1, 2019.

Problem Chapter 5.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axyw_y = by^2 w + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x,y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{by^2}{x-2ax}}\left(-\frac{cy^m x^{n-1}\left(-\frac{by^2}{x-2ax}\right)^{-\frac{am+n-1}{2a-1}}\text{Gamma}\left(\frac{am+n-1}{2a-1},-\frac{by^2}{x-2ax}\right)}{2a-1}+c_1\left(yx^{-a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+ a*x*y*diff(w(x,y),y) = b*y^2*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{4\left(\frac{((m+2)a+n-2)^2\left(a-\frac{1}{2}\right)cx\,x^{-am}x^{am+n}y^m\left(\frac{b\,y^2x-2a}{2a-1}\right)^{\frac{-am-n+1}{2a-1}}\left(\frac{b\,y^2x-2a}{2a-1}\right)^{\frac{am+n-1}{2a-1}}\left(\frac{b\,y^2}{(2a-1)x}\right)^{\frac{(-m-2)a-n+2}{4a-2}}\text{Whittaker}}{2}\right.}{}$$

7.5.4.8 [1228] Problem 8

problem number 1228

Added April 1, 2019.

Problem Chapter 5.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xy w_y = y^2(ax+by)w + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x+b*y)*w[x,y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -c2^{\frac{1}{2}(m+n-3)}y^m x^{n-1}e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}\left(y^2\left(a+\frac{by}{x}\right)\right)^{\frac{1}{2}(-m-n+1)}\text{Gamma}\left(\frac{1}{2}(m+n-1),\frac{1}{2}y^2\left(a+\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+ x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left((m+n+3)\left(b\,y^3+\left(a\,y^2+m+n+1\right)x\right)cx2^{-\frac{m}{4}-\frac{n}{4}+\frac{3}{4}}2^{\frac{m}{2}+\frac{n}{2}+\frac{1}{2}}x^{-m-2}x^{m+n-1}y^{m+2}\left(\left(a+\frac{by}{x}\right)y^2\right.\right.}{}$$

7.5.4.9 [1229] Problem 9

problem number 1229

Added April 1, 2019.

Problem Chapter 5.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^p y^q w + sx^\gamma y^\delta + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^p*y^q*w[x,y] + s*x^gamma*y^delta+d
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(-\frac{cy^q x^{-n+p+1} e^{-\frac{bqx^{m-n+1}}{am-an+a}} \left(-\frac{bqx^{m-n+1}}{am-an+a}\right)^{\frac{n-p-1}{m-n+1}} \text{Gamma}\left(\frac{-n+p+1}{m-n+1}, -\frac{bqx^{m-n+1}}{am-an+a}\right)}{a(m-n+1)}\right) \left(\int_1^x - \right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y*diff(w(x,y),y) = c*x^p*y^q*w(x,y)+ s*x^gamma*y^delta+d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s\_b^{-n+\gamma}\left(y\,\mathrm{e}^{-\frac{\left(-\_b^{m-n+1}+x^{m-n+1}\right)b}{(m-n+1)a}}\right)^{\delta} + d\_b^{-n}\right)\mathrm{e}^{-\frac{c\left(\int \_b^{-n+p}\left(y\,\mathrm{e}^{-\frac{\left(-\_b^{m-n+1}+x^{m-n+1}\right)b}{(m-n+1)a}}\right)^{q} d\_b\right.}{a}}}{a}\right.$$

7.5.4.10 [1230] Problem 10

problem number 1230

Added April 1, 2019.

Problem Chapter 5.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k)w_y = sx^p y^q w + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y+x*x^k)*D[w[x, y], y] == s*x^p*y^q*w[x,y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{s\left(b^{-\frac{k+2}{m-n+1}}\exp\left(-\frac{b\left(x^{m-n+1}-K[1]^{m-n+1}\right)}{a(m-n+1)}\right)(a(m-n+1))^{-\frac{m}{m-n+1}}\left(b^{\frac{n}{m-n+1}}e^{\frac{bx^{m-n+1}}{ma-na+a}}\left(\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ (b*x^m*y+c*x^k)*diff(w(x,y),y) = s*x^p*y^q*w(x,y)+ d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{d\_\_b^{-n}\mathrm{e}^{-\frac{s\left(\int \_\_b^{-n+p}\left(\frac{-(m-n+1)(k+m-2n+2)^2 ac\, x^{k-m}\left(\frac{b\, x^{m-n+1}}{(m-n+1)a}\right)^{\frac{-k-m+2n-2}{2m-2n+2}}\mathrm{WhittakerM}\left(\frac{k+m-2n+2}{2m-2n+2},\frac{k+2m-3n+}{2m-2n+2}\right.}{}\right.\right.}{}}}{}\right.$$

7.5.4.11 [1231] Problem 11

problem number 1231

Added April 1, 2019.

Problem Chapter 5.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y^k w_y = cw + sx^p y^q + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == c*w[x,y] + s*x^p*y^q+d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-n}}{a(n-1)}}K[1]^{-n}\left(s\left(\left(-\frac{a(m-n+1)x^n y^k K[1]^n}{a(-m+n-1)x^n y K[1]^n-b(k-1)y^k(x^{m+1}K[1]^n-x^n K[1]^{m+1})}\right)^{\frac{1}{k-1}}\right)^q K\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y^k*diff(w(x,y),y) = c*w(x,y)+ s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(s\_a^{-n+p}\left(\left(\frac{(m-n+1)a\,y^{-k+1}-(k-1)b\_a^{m-n+1}+(k-1)b\,x^{m-n+1}}{(m-n+1)a}\right)^{-\frac{1}{k-1}}\right)^q+d\_a^{-n}\right)e^{\frac{c\_a^{-n+1}}{(n-1)a}}}{a}d\_a\right.$$

7.5.4.12 [1232] Problem 12

problem number 1232

Added April 1, 2019.

Problem Chapter 5.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y] + s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\ {}_2F_1\left(\frac{k}{k+1},\frac{1}{n+1};1+\frac{1}{n+1};\frac{b(k+1)x^{n+}}{b(k+1)x^{n+1}-a(n-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{s\_b^{m}\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_\_b^{n+1}-(k+1)b\,x^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}\mathrm{e}^{-\frac{c\left(\int\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_\_b^{n+1}-(k+1)b\,x^{n+1}}{(n+1)a}\right)^{\frac{1}{k}}\right.\right.}{a}}}{a}\right.$$

7.5.5 3.1

Local contents

7.5.5.1 [1233] Problem 1

problem number 1233

Added April 1, 2019.

Problem Chapter 5.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (ce^{\lambda x} + se^{\mu y})w + ke^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x]+s*Exp[mu*y])*w[x,y] + k*Exp[nu*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\lambda x}}{a\lambda}+\frac{se^{\mu y}}{b\mu}}\left(\int_1^x\frac{\exp\left(-\frac{e^{\lambda K[1]}c}{a\lambda}-\frac{e^{\mu\left(y+\frac{b(K[1]-x)}{a}\right)}s}{b\mu}+\nu K[1]\right)k}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*exp(lambda*x)+s*exp(mu*y))*w(x,y)+ k*exp(nu*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{k\,\mathrm{e}^{\frac{-a\lambda s\,\mathrm{e}^{\frac{(ay-(-\_a+x)b)\mu}{a}}+\left(\_a a\lambda\nu-c\,\mathrm{e}^{\_a\lambda}\right)b\mu}{ab\lambda\mu}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{a\lambda s\,\mathrm{e}^{\mu y}+bc\mu\,\mathrm{e}^{\lambda x}}{ab\lambda\mu}}$$

7.5.5.2 [1234] Problem 2

problem number 1234

Added April 1, 2019.

Problem Chapter 5.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\alpha x+\beta y}w + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x+beta*y]*w[x,y] + k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\alpha x+\beta y}}{a\alpha+b\beta}}\left(\int_1^x \frac{\exp\left(\gamma K[1]-\frac{ce^{\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}}}{a\alpha+b\beta}\right)k}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*exp(alpha*x+beta*y)*w(x,y)+ k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\,\mathrm{e}^{\frac{-c\,\mathrm{e}^{\frac{-(-\_a+x)b\beta+(\_a\alpha+\beta y)a}{a}}}{a\alpha+b\beta}+\gamma(a\alpha+b\beta)\_a}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\alpha x+\beta y}}{a\alpha+b\beta}}$$

7.5.5.3 [1235] Problem 3

problem number 1235

Added April 1, 2019.

Problem Chapter 5.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma y}w + se^{\mu x+\delta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\exp\left(y\gamma-\frac{b\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K[1]}\right)\gamma}{a(\beta-\lambda)}-\lambda K[1]\right)}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\frac{b\delta\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K}\right.}{a(\beta-\lambda)}\right.}{\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*exp(gamma*y)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{s\,\mathrm{e}^{\frac{b\delta\,\mathrm{e}^{(\beta-\lambda)x}-b\delta\,\mathrm{e}^{(\beta-\lambda)\_b}-(-\beta+\lambda)c\left(\int \mathrm{e}^{\frac{-\gamma b\,\mathrm{e}^{(\beta-\lambda)x}+\gamma b\,\mathrm{e}^{(\beta-\lambda)\_b}+(\beta-\lambda)(-\_b\lambda+\gamma y)a}{(\beta-\lambda)a}}d\_b\right)-(-\beta+\lambda)(\_b\lambda-\_b\mu-\delta y)a}{(-\beta+\lambda)a}}}{a}d\_b+\right.$$

7.5.5.4 [1236] Problem 4

problem number 1236

Added April 1, 2019.

Problem Chapter 5.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\beta x} w_x + (b e^{\gamma x} + c e^{\lambda y}) w_y = s w + k e^{\mu x + \delta y}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k \left(\frac{a\lambda \left(\int \frac{c\, e^{\frac{-(\beta-\gamma)a\beta x - b\lambda\, e^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a} dx \right) - c\lambda \left(\int e^{\frac{-(\beta-\gamma)\_b a\beta - b\lambda\, e^{-(\beta-\gamma)\_b}}{(\beta-\gamma)a}} d\_b \right) + a\, e^{-\frac{\left((\beta-\gamma)ay + b\, e^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a} \right)^{-}}{a} \right.$$

7.5.5.5 [1237] Problem 5

problem number 1237

Added April 1, 2019.

Problem Chapter 5.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x+\delta y}w + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\,e^{\frac{-\_\_f a\beta - s\left(\int \left(\frac{a\lambda\left(\int \frac{c\,e^{\frac{-(\beta-\gamma)a\beta x - b\lambda e^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right) - c\lambda\left(\int e^{\frac{-(\beta-\gamma)\_\_f a\beta - b\lambda e^{-(\beta-\gamma)\_\_f}}{(\beta-\gamma)a}}d\_\_f\right) + a\,e^{-\frac{\left((\beta-\gamma)ay + b\,e^{(-\beta+\gamma)x}\right)\cdots}{(\beta-\gamma)a}}\cdots}{a}\cdots\right.\right.}{a}}}{a}\cdots\right.$$

7.5.5.6 [1238] Problem 6

problem number 1238

Added April 1, 2019.

Problem Chapter 5.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = ce^{\sigma y}w + ke^{\mu x+deltay} + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x+lambda*y]*D[w[x, y], y] == c*Exp[sigma*y]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\frac{\sigma}{\lambda}}e^{-\gamma x-\lambda y}\,{}_2F_1\left(1,-\frac{\gamma}{\beta-\gamma};\frac{\beta\sigma-\gamma(\lambda+\sigma)}{(\beta-\gamma)\lambda};1-\frac{ae^{\beta x-\gamma x-\lambda y}(\beta-\gamma)}{b\lambda}\right)}{b(\beta(\lambda-\sigma)+\gamma\sigma)}\right)\left(\int_1^x \frac{\exp\left(\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ b*exp(gamma*x+lambda*y)*diff(w(x,y),y) = c*exp(sigma*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\left(\frac{(\beta-\gamma)a}{-b\lambda\,e^{-\lambda y}e^{\lambda y+(-\beta+\gamma)x}+b\lambda\,e^{-(\beta-\gamma)\_b}+(\beta-\gamma)a\,e^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}e^{\frac{(-\beta+\mu)\_ba-c\left(\int\left(\frac{(\beta-\gamma)a}{-b\lambda\,e^{-\lambda y}e^{\lambda y+(-\beta+\gamma)x}+b\lambda\,e^{-(\beta-\gamma)\_b}+(\beta\right.\right.}{a}}}{a}\right.$$

7.5.5.7 [1239] Problem 7

problem number 1239

Added April 1, 2019.

Problem Chapter 5.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = cw + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x,y]+s*Exp[gamma*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ce^{-\lambda x}}{a\lambda}}\left(\int_1^x \frac{e^{\frac{e^{-\lambda K[1]}c}{a\lambda}+(\gamma-\lambda)K[1]}s}{a}dK[1]+c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)}+y\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*w(x,y)+s*exp(gamma*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s\,\mathrm{e}^{\frac{(-\lambda+\gamma)a\lambda x+c\,\mathrm{e}^{-\lambda x}}{a\lambda}}}{a}dx+\_F1\left(\frac{(\beta-\lambda)\,ay-b\,\mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\,a}\right)\right)\mathrm{e}^{-\frac{c\,\mathrm{e}^{-\lambda x}}{a\lambda}}$$

7.5.5.8 [1240] Problem 8

problem number 1240

Added April 1, 2019.

Problem Chapter 5.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^{\beta x}w_y = ce^{\gamma x}w + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*x^(beta*x)*D[w[x, y], y] == c*Exp[gamma*x]*w[x,y]+s
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{x(\gamma-\lambda)}}{a(\gamma-\lambda)}}\left(\int_1^x \frac{\exp\left(-\frac{e^{(\gamma-\lambda)K[2]}c}{a(\gamma-\lambda)}-\lambda K[2]\right)s}{a}dK[2]+c_1\left(y-\int_1^x \frac{be^{-\lambda K[1]}K[1]^{\beta K[1]}}{a}dK[1]\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*x^(beta*x)*diff(w(x,y),y) = c*exp(gamma*x)*w(x,y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s\,\mathrm{e}^{\frac{-(-\lambda+\gamma)a\lambda x-c\,\mathrm{e}^{(-\lambda+\gamma)x}}{(-\lambda+\gamma)a}}}{a}dx + \_F1\left(\frac{ay-b\left(\int x^{\beta x}\mathrm{e}^{-\lambda x}dx\right)}{a}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{(-\lambda+\gamma)x}}{(-\lambda+\gamma)a}}$$

7.5.6 3.2

Local contents

7.5.6.1 [1241] Problem 1

problem number 1241

Added April 2, 2019.

Problem Chapter 5.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + bx^n)w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*x^n)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}}-\int_1^x be^{-\frac{ae^{\lambda K[1]}}{\lambda}}K[1]^n dK[1]\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*x^n)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\frac{k\,\mathrm{e}^{(-c+\gamma)x}}{-c+\gamma}+\_F1\left(-b\left(\int x^n\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)+y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)\mathrm{e}^{cx}$$

7.5.6.2 [1242] Problem 2

problem number 1242

Added April 2, 2019.

Problem Chapter 5.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ae^{\lambda x}y+be^{\beta x})w_y=cw+ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*Exp[gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}}-\int_1^x be^{\beta K[1]-\frac{ae^{\lambda K[1]}}{\lambda}}dK[1]\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*exp(ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(k\,\mathrm{e}^{(-c+\gamma)x} + (-c+\gamma)\_F1\left(-b\left(\int \mathrm{e}^{\frac{\beta\lambda x - a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right) + y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)\mathrm{e}^{cx}}{c-\gamma}$$

7.5.6.3 [1243] Problem 3

problem number 1243

Added April 2, 2019.

Problem Chapter 5.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + be^{\beta x})w_y = cw + kx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{cx}\left(-\frac{kx^n(cx)^{-n}\,\mathrm{Gamma}(n+1,cx)}{c} + c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}} - \int_1^x be^{\beta K[1]-\frac{ae^{\lambda K[1]}}{\lambda}}dK[1]\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left(k\,x^{n}(cx)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},cx\right)\mathrm{e}^{-\frac{cx}{2}} + (n+1)\,c\_F1\left(-b\left(\int \mathrm{e}^{\frac{\beta\lambda x-a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)+y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right.}{(n+1)\,c}$$

7.5.6.4 [1244] Problem 4

problem number 1244

Added April 2, 2019.

Problem Chapter 5.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda y} + bx^k)w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y]+b*x^k)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(\frac{a\lambda x\left(-\frac{b\lambda x^{k+1}}{k+1}\right)^{-\frac{1}{k+1}}\mathrm{Gamma}\left(\frac{1}{k+1},-\frac{b\lambda x^{k+1}}{k+1}\right)-(k+1)e^{-\frac{\lambda\left(-bx^{k+1}+ky+y\right)}{k+1}}}{abk(k+1)\lambda^2}\right)\right)}{c-\gamma}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*y)+b*x^k)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(k\,\mathrm{e}^{(-c+\gamma)x} + (-c+\gamma)\_F1\left(\frac{(k+1)(k+2)^2 a\,x^{-k}\left(-\frac{b\lambda\,x^{k+1}}{k+1}\right)^{\frac{-k-2}{2k+2}} \mathrm{WhittakerM}\left(\frac{k+2}{2k+2},\frac{2k+3}{2k+2},-\frac{b\lambda\,x^{k+1}}{k+1}\right)\mathrm{e}^{\frac{b\lambda\,x^{k+1}}{2k+2}} -}{}\right.\right.}{}$$

7.5.6.5 [1245] Problem 5

problem number 1245

Added April 2, 2019.

Problem Chapter 5.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w + be^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x+mu*y]*w[x,y]+b*Exp[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}}\left(\int_1^x \frac{b\exp\left(\nu K[1] - \frac{ae^{\left(\lambda+\frac{\mu y}{x}\right)K[1]}x}{\lambda x+\mu y}\right)}{K[1]}dK[1] + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) =  a*x*exp(lambda*x+mu*y)*w(x,y)+k*exp(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{k\,\mathrm{e}^{-\frac{ax\,\mathrm{e}^{-a\lambda+\frac{a\mu y}{x}}-(\lambda x+\mu y)\_a\nu}{\lambda x+\mu y}}}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{a\,\mathrm{e}^{\lambda x+\mu y}}{\lambda+\frac{\mu y}{x}}}$$

7.5.6.6 [1246] Problem 6

problem number 1246

Added April 2, 2019.

Problem Chapter 5.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=(aye^{\lambda x}+bxe^{\mu y})w+ce^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x]+b*x*Exp[mu*y])*w[x,y]+c*Exp[n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\left(\int_{1}^{x}\frac{c\exp\left(-\frac{be^{\frac{\mu yK[1]}{x}}x}{\mu y}+\nu K[1]-\frac{ae^{\lambda K[1]}y}{\lambda x}\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) = (a*y*exp(lambda*x)+b*x*exp(mu*y))*w(x,y)+c*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{c\,\mathrm{e}^{-\frac{\left(-\frac{\_a\lambda\mu\nu y}{x}+\frac{a\mu\,y^{2}\mathrm{e}^{\_a\lambda}}{x^{2}}+b\lambda\,\mathrm{e}^{\frac{\_a\mu y}{x}}\right)x}{\lambda\mu y}}}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{\left(\frac{a\mu\,y^{2}\mathrm{e}^{\lambda x}}{x^{2}}+b\lambda\,\mathrm{e}^{\mu y}\right)x}{\lambda\mu y}}$$

7.5.6.7 [1247] Problem 7

problem number 1247

Added April 2, 2019.

Problem Chapter 5.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^{k}w_{x}+be^{\lambda x}w_{y}=w+ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(-\frac{(k+1)y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}{}_{2}F_{1}\left(1,\frac{1}{k+1};\frac{k+2}{k+1};\frac{a\lambda y^{k+1}}{a\lambda y^{k+1}-be^{\lambda x}(k+1)}\right)}{a\lambda y^{k+1}-b(k+1)e^{\lambda x}}\right)\left(\int_{1}^{x}\frac{c\exp\left((k+1)(\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k* diff(w(x,y),x)+ b*exp(lambda*x)*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{c\left(\left(\frac{a\lambda\, y^{k+1}+(k+1)b\,\mathrm{e}^{-b\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k} \mathrm{e}^{\frac{-\_ba\beta-\left(\int\left(\left(\frac{a\lambda\, y^{k+1}+(k+1)b\,\mathrm{e}^{-b\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k} d\_b\right)}{a}}}{a} d\_b$$

7.5.6.8 [1248] Problem 8

problem number 1248

Added April 2, 2019.

Problem Chapter 5.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + byw_y = w + ce^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y*D[w[x, y], y] == w[x,y]+c*Exp[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{e^{-\frac{e^{-\lambda x}}{a\lambda}}\left(-c\mathrm{Ei}\left(\frac{e^{-\lambda x}}{a\lambda}\right)+a\lambda c_1\left(ye^{\frac{be^{-\lambda x}}{a\lambda}}\right)\right)}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)* diff(w(x,y),x)+ b*y*diff(w(x,y),y) = w(x,y)+c*exp(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left(a\lambda\_F1\left(y\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-\lambda x}}{a\lambda}}\right)+c\,\mathrm{expIntegral}\left(1,-\frac{\mathrm{e}^{-\lambda x}}{a\lambda}\right)\right)\mathrm{e}^{-\frac{\mathrm{e}^{-\lambda x}}{a\lambda}}}{a\lambda}$$

7.5.6.9 [1249] Problem 9

problem number 1249

Added April 2, 2019.

Problem Chapter 5.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+bx^kw_y=w+ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\frac{(k+1)x\,{}_2F_1\left(1,\frac{1}{k+1};1+\frac{1}{k+1};\frac{b\lambda x^{k+1}}{b\lambda x^{k+1}-ae^{\lambda y}(k+1)}\right)}{a(k+1)e^{\lambda y}-b\lambda x^{k+1}}\right)\left(\int_1^x\frac{c\exp\left(\left(\beta-\frac{(k+1)\,{}_2F_1\left(1,\frac{1}{k+1};1+\right.}{ae^{\lambda y}(k\right.}\right.}{ae^{\lambda y}(k+1)+}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*x^k*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{(k+1)\,c\,\mathrm{e}^{\frac{\_bb\beta\lambda+(-k-1)\left(\int \frac{b\lambda}{b\lambda\_b^{k+1}-b\lambda\,x^{k+1}+(k+1)a\,\mathrm{e}^{\lambda y}}d\_b\right)}{b\lambda}}}{b\lambda\_b^{k+1}-b\lambda\,x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}d\_b+\_F1\left(\frac{-b\lambda\,x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}{(k+1)\,b\lambda}\right)\right.$$

7.5.6.10 [1250] Problem 10

problem number 1250

Added April 2, 2019.

Problem Chapter 5.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = w + cx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == w[x,y]+c*x^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{\beta x-\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right)\left(\int_1^x \frac{\beta c\exp\left(\frac{\log\left(\frac{ae^{\lambda y}\beta}{\lambda}+b\left(-e^{\beta x}+e^{\beta K[1]}\right)\right)-\beta K[1]}{a\beta e^{\lambda y}-be^{\beta x}\lambda}\right)K[1]^k}{ae^{\lambda y}\beta+b\left(-e^{\beta x}+e^{\beta K[1]}\right)\lambda}dK[1]+c_1\left(\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = w(x,y)+c*x^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\beta c\_a^{k}\left(\mathrm{e}^{-a\beta} + \frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\lambda}\right)^{\frac{-a\beta\,\mathrm{e}^{\lambda y}+b\lambda\,\mathrm{e}^{\beta x}+1}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}}\left(\mathrm{e}^{-a\beta}\right)^{-\frac{1}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}}}{b\lambda} d\_a + \_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right.\right.$$

7.5.7 4.1

Local contents

7.5.7.1 [1251] Problem 1

problem number 1251

Added April 3, 2019.

Problem Chapter 5.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sinh^k(\lambda x)\sinh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sinh[lambda*x]^k*Sinh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sinh^k(\lambda K[1])\sinh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*sinh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\sinh^k(\_a\lambda)\right)\left(\sinh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)e^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

7.5.7.2 [1252] Problem 2

problem number 1252

Added April 3, 2019.

Problem Chapter 5.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\sinh^k(\lambda x)w+s\sinh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x]^k*w[x,y]+ s*Sinh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\sqrt{\cosh^2(\lambda x)}\operatorname{sech}(\lambda x)\sinh^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};-\sinh^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(-\frac{c\sqrt{c}}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sinh(lambda*x)^k*w(x,y)+s*sinh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s(\sinh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\sinh^k(\lambda x)\right)dx\right)}{a}}}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int\frac{c\left(\sinh^k(\lambda x)\right)}{a}dx}$$

7.5.7.3 [1253] Problem 3

problem number 1253

Added April 3, 2019.

Problem Chapter 5.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c_1\sinh^{n_1}(\lambda_1 x)+c_2\sinh^{n_2}(\lambda_2 y))\,w+s_1\sinh^{k_1}(\beta_1 x)+s_2\sinh^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Sinh[lambda1*x]^n1 + c2*Sinh[lambda2*y]^n2)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{\text{c1}\sqrt{\cosh^2(\text{lambda1}x)}\text{sech}(\text{lambda1}x)\sinh^{\text{n1}+1}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{\text{n1}+1}{2};\frac{\text{n1}+3}{2};-\sinh^2\right.}{a\text{lambda1n1}+a\text{lambda1}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*sinh(lambda1*x)^n1 + c2*sinh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{\left(s1\left(\sinh^{k1}\left(\_b\beta 1\right)\right)+s2\left(\sinh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right)e^{-\frac{\int\left(c1\left(\sinh^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\sinh^{n2}\left(\frac{(ay-(-\_b+x)}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.7.4 [1254] Problem 4

problem number 1254

Added April 3, 2019.

Problem Chapter 5.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^n(\lambda x)w_x+b\sinh^m(\mu x)w_y=c\sinh^k(\nu x)w+p\sinh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c2^{n-1}e^{-\nu x}\left(e^{\lambda x}-e^{-\lambda x}\right)^{-n}\left(1-e^{2\lambda x}\right)^{n}\left((\lambda n+\nu)\,{}_2F_1\left(n,\frac{\lambda n-\nu}{2\lambda};\frac{1}{2}\left(n-\frac{\nu}{\lambda}+2\right);e^{2\lambda x}\right)+e\right.}{a(\nu-\lambda n)(\lambda n+\nu)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^{x}\frac{\left(cw_{x,y}\sinh\left(\_b\nu\right)+p\left(\sinh^{s}\left(\frac{(ay+b(\int(\sinh^{-n}(\_b\lambda))(\sinh^{m}(\_b\mu))d\_b)-b(\int(\sinh^{-n}(\lambda x))(\sinh^{m}(\mu x))dx))\beta}{a}\right)\right.\right.}{a}$$

7.5.7.5 [1255] Problem 5

problem number 1255

Added April 3, 2019.

Problem Chapter 5.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^{n}(\lambda x)w_{x}+b\sinh^{m}(\mu x)w_{y}=c\sinh^{k}(\nu y)w+p\sinh^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sinh^{-n}(\lambda K[2])\sinh\left(\nu\left(y-\int_1^x \frac{b\sinh^{-n}(\lambda K[1])\sinh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\sinh^{-n}(\lambda K[1}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*y)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(cw_{x,y}\sinh\left(\frac{(ay+b(\int(\sinh^{-n}(\_b\lambda))(\sinh^m(\_b\mu))d\_b)-b(\int(\sinh^{-n}(\lambda x))(\sinh^m(\mu x))dx))\nu}{a}\right)+p(\sinh^s(\_b\beta))\right.}{a}$$

7.5.8 4.2

Local contents

7.5.8.1 [1256] Problem 1

problem number 1256

Added April 3, 2019.

Problem Chapter 5.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cosh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Cosh[lambda*x]^k*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cosh^k(\lambda K[1])\cosh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+cosh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\cosh^k(\_a\lambda)\right)\left(\cosh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)e^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

7.5.8.2 [1257] Problem 2

problem number 1257

Added April 3, 2019.

Problem Chapter 5.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh^k(\lambda x)w + s\cosh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x]^k*w[x,y]+ s*Cosh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\sqrt{-\sinh^2(\lambda x)}\operatorname{csch}(\lambda x)\cosh^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cosh^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(\frac{c\cosh^k}{\right.}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cosh(lambda*x)^k*w(x,y)+s*cosh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s(\cosh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\cosh^k(\lambda x)\right)dx\right)}{a}}}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int\frac{c\left(\cosh^k(\lambda x)\right)}{a}dx}$$

7.5.8.3 [1258] Problem 3

problem number 1258

Added April 3, 2019.

Problem Chapter 5.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \cosh^{n_1}(\lambda_1 x) + c_2 \cosh^{n_2}(\lambda_2 y))\, w + s_1 \cosh^{k_1}(\beta_1 x) + s_2 \cosh^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Cosh[lambda1*x]^n1 + c2*Cosh[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\text{c1}\sqrt{-\sinh^2(\text{lambda1}x)}\text{csch}(\text{lambda1}x)\cosh^{\text{n1}+1}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{\text{n1}+1}{2};\frac{\text{n1}+3}{2};\cosh^2\right.}{a\text{lambda1n1}+a\text{lambda1}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*cosh(lambda1*x)^n1 + c2*cosh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1\left(\cosh^{k1}(\_b\beta 1)\right) + s2\left(\cosh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right)e^{-\frac{\int\left(c1\left(\cosh^{n1}(\_b\lambda 1)\right)+c2\left(\cosh^{n2}\left(\frac{(ay-(-\_b+}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.8.4 [1259] Problem 4

problem number 1259

Added April 3, 2019.

Problem Chapter 5.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cosh(\lambda x + \mu y)w + b\cosh(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x+my*y]+b*Cosh[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)+\frac{ax\sinh(\lambda x+\text{my}y)}{\lambda x+\text{my}y}+b\text{Chi}(\nu x)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cosh(lambda*x+my*y)+b*cosh(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{a\sinh\left(\lambda x+myy\right)}{\lambda+\frac{myy}{x}}+b\,\text{hyperbolicCosineIntegral}\left(\nu x\right)+\_F1\left(\frac{y}{x}\right)$$

7.5.8.5 [1260] Problem 5

problem number 1260

Added April 3, 2019.

Problem Chapter 5.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cosh^n(\lambda x)w_x + b\cosh^m(\mu x)w_y = c\cosh^k(\nu x)w + p\cosh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k+p*C
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{\cosh^{-n}(\lambda K[2])\left(c\cosh^k(\nu K[2])+p\cosh^s\left(\beta\left(y-\int_1^x\frac{b\cosh^{-n}(\lambda K[1])\cosh^m(\mu K[1])}{a}dK[1]+\int\right.\right.\right.\right)}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x\frac{\left(c\left(\cosh^k\left(\_b\nu\right)\right)+p\left(\cosh^s\left(\frac{\left(ay+b\left(\int\left(\cosh^{-n}\left(\_b\lambda\right)\right)\left(\cosh^m\left(\_b\mu\right)\right)d\_b\right)-b\left(\int\left(\cosh^{-n}(\lambda x)\right)\left(\cosh^m(\mu x)\right)dx\right)\right)\beta}{a}\right)\right.\right.}{a}$$

7.5.8.6 [1261] Problem 6

problem number 1261

Added April 3, 2019.

Problem Chapter 5.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cosh^n(\lambda x)w_x+b\cosh^m(\mu x)w_y=c\cosh^k(\nu y)w+p\cosh^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*y]^k+p*C
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{\cosh^{-n}(\lambda K[2])\left(c\cosh^k\left(\nu\left(y-\int_1^x \frac{b\cosh^{-n}(\lambda K[1])\cosh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]}\frac{b\cosh^{-n}(\lambda K[1])\,\mathrm{c}}{a}\right.\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*y)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(c\left(\cosh^k\left(\frac{(ay+b(\int(\cosh^{-n}(\_b\lambda))(\cosh^m(\_b\mu))d\_b)-b(\int(\cosh^{-n}(\lambda x))(\cosh^m(\mu x))dx))\nu}{a}\right)\right)+p(\cosh^s(\_b\beta)\right.}{a}$$

7.5.9 4.3

Local contents

7.5.9.1 [1262] Problem 1

problem number 1262

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \tanh^k(\lambda x)\tanh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Tanh[lambda*x]^k*Tanh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tanh^k(\lambda K[1])\tanh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+tanh(lambda*x)^k*tanh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\tanh^k\left(\_a\lambda\right)\right)\left(\tanh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.9.2 [1263] Problem 2

problem number 1263

Added April 3, 2019.

Problem Chapter 5.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh^k(\lambda x)w + s\tanh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x]^k*w[x,y]+ s*Tanh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\tanh^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\tanh^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(-\frac{c\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\tanh^2(\lambda K[1])\right)\text{ta}}{a\lambda+ak\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tanh(lambda*x)^k*w(x,y)+s*tanh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s(\tanh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\tanh^k(\lambda x)\right)dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int\frac{c\left(\tanh^k(\lambda x)\right)}{a}dx}$$

7.5.9.3 [1264] Problem 3

problem number 1264

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \tanh^{n_1}(\lambda_1 x) + c_2 \tanh^{n_2}(\lambda_2 y))\, w + s_1 \tanh^{k_1}(\beta_1 x) + s_2 \tanh^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Tanh[lambda1*x]^n1 + c2*Tanh[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*tanh(lambda1*x)^n1 + c2*tanh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1\left(\tanh^{k1}\left(\_b\beta 1\right)\right) + s2\left(\tanh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right) e^{-\frac{\int \left(c1\left(\tanh^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\tanh^{n2}\left(\frac{(ay-(-\_b+}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.9.4 [1265] Problem 4

problem number 1265

Added April 3, 2019.

Problem Chapter 5.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a \tanh^n(\lambda x) w_x + b \tanh^m(\mu x) w_y = c \tanh^k(\nu x) w + p \tanh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tanh^{-n}(\lambda K[2])\tanh(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\tanh^{-n}(\lambda K[2])\tanh(\nu K[2])}{a}d\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(cw_{x,y}\tanh\left(\_b\nu\right)+p\left(\frac{\sinh\left(\frac{\left(ay-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\frac{\sinh(\mu x)}{\cosh(\mu x)}\right)^m dx\right)+b\left(\int\left(\tanh^{-n}\left(\_b\lambda\right)\right)\left(\tanh^m\left(\_b\mu\right)\right)d\_b\right)\right)\beta}{a}\right)}{\cosh\left(\frac{\left(ay-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\frac{\sinh(\mu x)}{\cosh(\mu x)}\right)^m dx\right)+b\left(\int\left(\tanh^{-n}\left(\_b\lambda\right)\right)\left(\tanh^m\left(\_b\mu\right)\right)d\_b\right)\right)\beta}{a}\right)}\right)\right.}{a}$$

7.5.9.5 [1266] Problem 5

problem number 1266

Added April 3, 2019.

Problem Chapter 5.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^n(\lambda x)w_x + b\tanh^m(\mu x)w_y = c\tanh^k(\nu y)w + p\tanh^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tanh^{-n}(\lambda K[2])\tanh\left(\nu\left(y-\int_1^x \frac{b\tanh^{-n}(\lambda K[1])\tanh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]}\frac{b\tanh^{-n}(\lambda K}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}\left(\frac{\sinh(\_f\beta)}{\cosh(\_f\beta)}\right)^{s}\mathrm{e}^{-\frac{c\left(\int\frac{\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\sinh(\_f\mu)}{\cosh(\_f\mu)}\right)^m\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sinh(\lambda x}{\cosh(\lambda x}\right.\right.}{a}\right)}{\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\sinh(\_f\mu)}{\cosh(\_f\mu)}\right)^m\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\right.\right.}{a}\right)}\right)}{a}}}{a}\right.$$

7.5.10 4.4

Local contents

7.5.10.1 [1267] Problem 1

problem number 1267

Added April 3, 2019.

Problem Chapter 5.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \coth^k(\lambda x)\coth^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Coth[lambda*x]^k*Coth[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\coth^k(\lambda K[1])\coth^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+coth(lambda*x)^k*coth(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{(\coth^k(\_a\lambda))\left(\coth^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.10.2 [1268] Problem 2

problem number 1268

Added April 3, 2019.

Problem Chapter 5.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth^k(\lambda x)w + s\coth^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x]^k*w[x,y]+ s*Coth[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\coth^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\coth^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\left(\int_1^x \frac{\exp\left(-\frac{c\coth^{k+1}(\lambda K[1])\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\mathrm{co}\right.}{a\lambda+ak\lambda}\right.}{a}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*coth(lambda*x)^k*w(x,y)+s*coth(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s(\coth^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\coth^k(\lambda x)\right)dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{c\left(\coth^k(\lambda x)\right)}{a}dx}$$

7.5.10.3 [1269] Problem 3

problem number 1269

Added April 3, 2019.

Problem Chapter 5.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \coth^{n_1}(\lambda_1 x) + c_2 \coth^{n_2}(\lambda_2 y))\, w + s_1 \coth^{k_1}(\beta_1 x) + s_2 \coth^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Coth[lambda1*x]^n1 + c2*Coth[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*coth(lambda1*x)^n1 + c2*coth(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1\left(\coth^{k1}(\_b\beta 1)\right) + s2\left(\coth^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right) e^{-\frac{\int \left(c1\left(\coth^{n1}(\_b\lambda 1)\right)+c2\left(\coth^{n2}\left(\frac{(ay-(-\_b+}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.10.4 [1270] Problem 4

problem number 1270

Added April 3, 2019.

Problem Chapter 5.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a \coth^n(\lambda x) w_x + b \coth^m(\mu x) w_y = c \coth^k(\nu x) w + p \coth^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\coth^{-n}(\lambda K[2])\coth(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\coth^{-n}(\lambda K[2])\coth(\nu K[2])}{a}dK\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*x)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}\left(\frac{\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)}{\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)}\right)^s \mathrm{e}^{-}}{a}\right.$$

7.5.10.5 [1271] Problem 5

problem number 1271

Added April 3, 2019.

Problem Chapter 5.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\coth^n(\lambda x)w_x+b\coth^m(\mu x)w_y=c\coth^k(\nu y)w+p\coth^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\coth^{-n}(\lambda K[2])\coth\left(\nu\left(y-\int_1^x \frac{b\coth^{-n}(\lambda K[1])\coth^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]}\frac{b\coth^{-n}(\lambda K\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cosh(\_f\beta)}{\sinh(\_f\beta)}\right)^s\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}e^{-\frac{c\left(\int\frac{\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x}{\sinh(\lambda x}\right.\right.}{a}\right.}{\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\left(\right.\right.}{a}\right.}\right.}{a}}}{a}\right.$$

7.5.11 4.5

Local contents

7.5.11.1 [1272] Problem 1

problem number 1272

Added April 4, 2019.

Problem Chapter 5.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1\sinh^k(\lambda x) + c_2\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sinh[lambda*x]^k+c2*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}c_1\left(y-\frac{bx}{a}\right)+\frac{\text{c1}\left(e^{2\lambda x}-1\right)\sinh^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k-\frac{1}{a\lambda}+2\right);\frac{1}{2}\left(-k-\frac{1}{a\lambda}+2\right);e^{2\lambda x}\right)}{ak\lambda+1}-\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sinh(lambda*x)^k+c2*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(c1\left(\sinh^{k}\left(\_a\lambda\right)\right)+c2\left(\cosh^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.11.2 [1273] Problem 2

problem number 1273

Added April 4, 2019.

Problem Chapter 5.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\sinh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sinh[lambda*x]^k*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_{1}^{x}\frac{e^{-\frac{cK[1]}{a}}\cosh^{n}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\sinh^{k}(\lambda K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(\cosh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\left(\sinh^k(\_a\lambda)\right)e^{-\frac{\_ac}{a}}}{a}d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

7.5.11.3 [1274] Problem 3

problem number 1274

Added April 4, 2019.

Problem Chapter 5.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tanh(\lambda x) + s\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Tanh[lambda*x]+s*coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\left(s\coth\left(\mu\left(y+\frac{b(K[1]-x)}{a}\right)\right)+k\tanh(\lambda K[1])\right)}{a}dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*tanh(lambda*x)+s*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x -\frac{\left((k-s)\cosh\left(\frac{-(-\_a+x)b\mu+(-\_a\lambda+\mu y)a}{a}\right)-(k+s)\cosh\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{\left(\sinh\left(\frac{-(-\_a+x)b\mu+(-\_a\lambda+\mu y)a}{a}\right)+\sinh\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\right)a} d\_$$

7.5.11.4 [1275] Problem 4

problem number 1275

Added April 4, 2019.

Problem Chapter 5.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh(\lambda x)w_y = cw + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*w[x,y]+ k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}k\cosh\left(\frac{\mu(a\lambda y-b\cosh(\lambda x)+b\cosh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\cosh(\lambda x)}{a\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sinh(lambda*x)*diff(w(x,y),y) = c*w(x,y)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{k\cosh\left(\frac{(a\lambda y+b\cosh(\_a\lambda)-b\cosh(\lambda x))\mu}{a\lambda}\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{a\lambda y-b\cosh(\lambda x)}{a\lambda}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.11.5 [1276] Problem 5

problem number 1276

Added April 4, 2019.

Problem Chapter 5.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^{n}(\lambda x)w_{x}+b\cosh^{m}(\mu x)w_{y}=c\cosh^{k}(\nu x)w+p\sinh^{s}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\int_{1}^{x}\frac{c\cosh^{k}(\nu K[2])\sinh^{-n}(\lambda K[2])}{a}dK[2]\right)\left(\int_{1}^{x}\frac{\exp\left(-\int_{1}^{K[3]}\frac{c\cosh^{k}(\nu K[2])\sinh^{-n}(\lambda K[2])}{a}d\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p\left(\sinh^{-n}\left(\_f\lambda\right)\right)\left(\sinh^{s}\left(\frac{\left(ay+b\left(\int\left(\cosh^{m}\left(\_f\mu\right)\right)\left(\sinh^{-n}\left(\_f\lambda\right)\right)d\_f\right)-b\left(\int\left(\cosh^{m}\left(\mu x\right)\right)\left(\sinh^{-n}\left(\lambda x\right)\right)dx\right)\right)\beta}{a}\right)\right) \mathrm{e}^{-}}{a}\right.$$

7.5.11.6 [1277] Problem 6

problem number 1277

Added April 4, 2019.

Problem Chapter 5.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^{n}(\lambda x)w_{x} + b\coth^{m}(\mu x)w_{y} = c\tanh^{k}(\nu y)w + p\coth^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]^k*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \rightarrow \exp\left(\int_{1}^{x} \frac{c\tanh^{-n}(\lambda K[2])\tanh^{k}\left(\nu\left(y - \int_{1}^{x} \frac{b\coth^{m}(\mu K[1])\tanh^{-n}(\lambda K[1])}{a}dK[1] + \int_{1}^{K[2]} \frac{b\coth^{m}(\mu K}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\left(\frac{\cosh(\_f\beta)}{\sinh(\_f\beta)}\right)^s \left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} e^{-\frac{c \int \left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} \left(\frac{\sinh\left(\frac{\left(ay+b\left(\int \left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m \left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int \left(\frac{\cosh(\mu}{\sinh(\mu}\right.\right.}{a}\right)}{\cosh\left(\frac{\left(ay+b\left(\int \left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m \left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int \left(\frac{\cosh(\mu}{\sinh(\mu}\right.\right.}{a}\right)}\right.}{a}}}{a} \right.$$

7.5.12 5.1

Local contents

7.5.12.1 [1278] Problem 1

problem number 1278

Added April 5, 2019.

Problem Chapter 5.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \ln^k(\lambda x)\ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Log[lambda*x]^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\log^k(\lambda K[1])\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\ln(\_a\lambda)^k \ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.12.2 [1279] Problem 2

problem number 1279

Added April 5, 2019.

Problem Chapter 5.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln^k(\lambda x)w + s\ln^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x]^k*w[x,y]+s*Log[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^k(\lambda K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{c\log^k(\lambda K[1])}{a}dK[1]\right)s\log^n(\beta K[2])}{a}dK[2]+\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y)+s*ln(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s\ln(\beta x)^n\,\mathrm{e}^{-\frac{c\left(\int \ln(\lambda x)^k dx\right)}{a}}}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{c\ln(\lambda x)^k}{a}dx}$$

7.5.12.3 [1280] Problem 3

problem number 1280

Added April 5, 2019.

Problem Chapter 5.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \ln^{n_1}(\lambda_1 x) + c_2 \ln^{n_2}(\lambda_2 y))\, w + s_1 \ln^{k_1}(\beta_1 x) + s_2 \ln^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*Log[lambda1*x]^n1 +c2*Log[lambda2*y]^n2)*w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\text{c1} \log^{\text{n1}}(\text{lambda1}x)(-\log(\text{lambda1}x))^{-\text{n1}}\,\text{Gamma}(\text{n1}+1, -\log(\text{lambda1}x))}{a\text{lambda1}} + \frac{\text{c2}(-\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*ln(lambda1*x)^n1 +c2*ln(lambda2*y)^n2)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(k2\, s2 \ln\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right) + s1 \ln(\_b\beta 1)^{k1}\right) e^{-\frac{\int\left(c1 \ln(\_b\lambda 1)^{n1} + c2 \ln\left(\frac{(ay-(-\_b+x)b)\lambda 2}{a}\right)^{n2}\right) d\_b}{a}}}{a} d\_b\right.$$

7.5.12.4 [1281] Problem 4

problem number 1281

Added April 5, 2019.

Problem Chapter 5.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln(\lambda x)w_x + b\ln(\mu y)w_y = cw + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]*D[w[x, y], x] + b*Log[mu*y]*D[w[x, y], y] == c*w[x,y]+k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)*diff(w(x,y),x)+ b*ln(mu*y)*diff(w(x,y),y) =c*w(x,y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c\_F1\left(\frac{-a\lambda\,\mathrm{expIntegral}(1,-\ln(\mu y))+b\mu\,\mathrm{expIntegral}(1,-\ln(\lambda x))}{b\lambda\mu}\right)\mathrm{e}^{-\frac{c\,\mathrm{expIntegral}(1,-\ln(\lambda x))}{a\lambda}} - k}{c}$$

7.5.12.5 [1282] Problem 5

problem number 1282

Added April 5, 2019.

Problem Chapter 5.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x + b\ln^m(\mu x)w_y = c\ln^k(\nu x)w + p\ln^s(\beta y) + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^{-n}(\lambda K[2])\log^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\log^{-n}(\lambda K[2])\log^k(\nu K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*x)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(p\ln\left(\frac{\left(ay+b\left(\int \ln(\_f\lambda)^{-n}\ln(\_f\mu)^m d\_f\right)-b\left(\int \ln(\lambda x)^{-n}\ln(\mu x)^m dx\right)\right)\beta}{a}\right)^s+q\right)\ln(\_f\lambda)^{-n}\mathrm{e}^{-\frac{c\left(\int \ln(\_f\lambda)^{-n}\right.}{a}}}{a}\right.$$

7.5.12.6 [1283] Problem 6

problem number 1283

Added April 5, 2019.

Problem Chapter 5.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x + b\ln^m(\mu x)w_y = c\ln^k(\nu y)w + p\ln^s(\beta x) + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^{-n}(\lambda K[2])\log^k\left(\nu\left(y-\int_1^x \frac{b\log^{-n}(\lambda K[1])\log^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\log^{-n}(\lambda K[1])\log}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*y)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{(p\ln(\_f\beta)^s+q)\ln(\_f\lambda)^{-n}\,\mathrm{e}^{-\frac{c\left(\int \ln(\_f\lambda)^{-n}\ln\left(\frac{\left(ay+b\left(\int \ln(\_f\lambda)^{-n}\ln(\_f\mu)^m d\_f\right)-b\left(\int \ln(\lambda x)^{-n}\ln(\mu x)^m dx\right)\right)\nu}{a}\right)^k d\right.}{a}}}{a}\right.$$

7.5.13 5.2

Local contents

7.5.13.1 [1284] Problem 1

problem number 1284

Added April 8, 2019.

Problem Chapter 5.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 x^k + c_2 \ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*x^k+c2*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}K[1]^k+\text{c2}\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*x^k+c2*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\_a^k+c2\ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.13.2 [1285] Problem 2

problem number 1285

Added April 8, 2019.

Problem Chapter 5.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + x^k \ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}K[1]^k \log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\_a^k \ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.13.3 [1286] Problem 3

problem number 1286

Added April 8, 2019.

Problem Chapter 5.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-k}}{a-ak}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-k}}{a(k-1)}}sK[1]^{-k}\log^m(\beta K[1])}{a}dK[1]+c_1\left(y-\frac{bx^{-k+n+1}}{a(-k)+an+a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s\,x^{-k}\ln(\beta x)^m\,\mathrm{e}^{\frac{c\,x^{-k+1}}{(k-1)a}}}{a}dx+\_F1\left(\frac{(k-n-1)\,ay+b\,x^{-k+n+1}}{(k-n-1)\,a}\right)\right)\mathrm{e}^{-\frac{c\,x^{-k+1}}{(k-1)a}}$$

7.5.13.4 [1287] Problem 4

problem number 1287

Added April 8, 2019.

Problem Chapter 5.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^k w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-n}}{a(n-1)}}sK[1]^{-n}\log^m(\beta K[1])}{a}dK[1]+c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-k}}{k-1}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*y^k*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s\,x^{-n}\ln(\beta x)^m\,\mathrm{e}^{\frac{c\,x^{-n+1}}{(n-1)a}}}{a}dx+\_F1\left(\frac{(n-1)\,a\,y^{-k+1}-(k-1)\,b\,x^{-n+1}}{(n-1)\,a}\right)\right)\mathrm{e}^{-\frac{c\,x^{-n+1}}{(n-1)a}}$$

7.5.13.5 [1288] Problem 5

problem number 1288

Added April 8, 2019.

Problem Chapter 5.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + b\ln^n(\lambda x)w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\frac{sx^{m-n+1}\left(\frac{cx^{1-n}}{a-an}\right)^{\frac{m-n+1}{n-1}}\text{Gamma}\left(\frac{-m+n-1}{n-1},\frac{cx^{1-n}}{a-an}\right)}{a(n-1)}+c_1\left(\frac{(n-1)^{-n-1}\left(b\lambda^n\text{Gamma}(\right.}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left(-(n-1)\left(m-2n+2\right)^2 as\,x^m\left(-\frac{c}{(n-1)a}\right)^{\frac{-m+n-1}{n-1}}\left(-\frac{c}{(n-1)a}\right)^{\frac{m-n+1}{n-1}}\left(-\frac{c\,x^{-n+1}}{(n-1)a}\right)^{\frac{m-2n+2}{2n-2}}\text{Whittaker}\right.}{}$$

7.5.13.6 [1289] Problem 6

problem number 1289

Added April 8, 2019.

Problem Chapter 5.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}{}_2F_1\left(\frac{k}{k+1},\frac{1}{n+1};1+\frac{1}{n+1};\frac{b(k+1)x^{n+}}{b(k+1)x^{n+1}-a(n-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{s\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_b^{n+1}-(k+1)b\,x^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}\ln\left(\_b\beta\right)^m \mathrm{e}^{-\frac{c\left(\int\left(\left(\frac{(n+1)a\,y^{k+1}+(k+1)b\_b^{n+1}-(k+1)b}{(n+1)a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.13.7 [1290] Problem 7

problem number 1290

Added April 8, 2019.

Problem Chapter 5.5.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + b\ln^n(\lambda x) w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\left(\left(y^{k+1}-(k+1)\int_1^x \frac{b\log^n(\lambda K[1])}{a}dK[1]+(k+1)\int_1^{K[2]}\frac{b\log^n(\lambda K[1])}{a}dK[1]\right)^{\frac{1}{k+1}}\right)^{-k}}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{s\_f^m\left(\left(\frac{a\,y^{k+1}+\int(k+1)b\ln(\_f\lambda)^n d\_f+\int(-k-1)b\ln(\lambda x)^n dx}{a}\right)^{\frac{1}{k+1}}\right)^{-k} e^{-\frac{\left((k+1)\_fb+\left(ay\,y^k-(k+1)b\left(\int\ln(\lambda x)^n dx\right)\right)\ln}{}}}{a}\right.$$

7.5.14 6.1

Local contents

7.5.14.1 [1291] Problem 1

problem number 1291

Added April 8, 2019.

Problem Chapter 5.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{k((a\lambda+b\mu)\cos(\lambda x+\mu y)+c\sin(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2}+e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left(-(c\sin(\lambda x+\mu y)+(a\lambda+\mu b)\cos(\lambda x+\mu y))\,k\,e^{-\frac{cx}{a}}+(\lambda^2a^2+2ab\lambda\mu+b^2\mu^2+c^2)\_F1\left(\frac{ay-bx}{a}\right.\right.}{\lambda^2a^2+2ab\lambda\mu+b^2\mu^2+c^2}$$

7.5.14.2 [1292] Problem 2

problem number 1292

Added April 8, 2019.

Problem Chapter 5.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \sin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sin[lambda*x]^k+c2*Sin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} c_1\left(y - \frac{bx}{a}\right) - i\left(\frac{\text{c1}\left(-1 + e^{2i\lambda x}\right) \sin^k(\lambda x)\, {}_2F_1\left(1, \frac{1}{2}\left(k + \frac{i}{a\lambda} + 2\right); \frac{1}{2}\left(-k + \frac{i}{a\lambda} + 2\right); e^{2i\lambda x}\right)}{ak\lambda - i}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sin(lambda*x)^k+c2*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(c1\left(\sin^k\left(\_a\lambda\right)\right) + c2\left(\sin^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right) \mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay - bx}{a}\right)\right) \mathrm{e}^{\frac{x}{a}}$$

7.5.14.3 [1293] Problem 3

problem number 1293

Added April 8, 2019.

Problem Chapter 5.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sin^k(\lambda x)\sin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sin[lambda*x]^k*Sin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^k(\lambda K[1])\sin^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sin(lambda*x)^k*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\sin^k(\_a\lambda)\right)\left(\sin^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.14.4 [1294] Problem 4

problem number 1294

Added April 8, 2019.

Problem Chapter 5.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\sin\left(\beta yK[1]^{\frac{b}{a}}x^{-\frac{b}{a}}+\lambda K[1]\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\_a^{\frac{-a-c}{a}}\sin\left(\beta y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}+\_a\lambda\right)}{a}d\_a + \_F1\left(y\,x^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

7.5.14.5 [1295] Problem 5

problem number 1295

Added April 8, 2019.

Problem Chapter 5.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)w + b\sin(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x+beta*y]*w[x,y]+ b*Sin[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ax\cos(\beta y+\lambda x)}{\beta y+\lambda x}}\left(\int_1^x \frac{be^{\frac{ax\cos\left(\left(\lambda+\frac{\beta y}{x}\right)K[1]\right)}{\lambda x+\beta y}}\sin(\nu K[1])}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*sin(lambda*x+beta*y)*w(x,y)+ b*sin(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{b\,\mathrm{e}^{-a\left(\int \sin(b\beta\ln(\_b)+\_b\lambda+(-b\ln(x)+y)\beta)d\_b\right)}\sin\left(\_b\nu\right)}{\_b}d\_b + \_F1(-b\ln\left(x\right)+y)\right)\mathrm{e}^{\int^x a\sin(b\beta\ln(\_a$$

7.5.14.6 [1296] Problem 6

problem number 1296

Added April 8, 2019.

Problem Chapter 5.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sin^{-n}(\lambda K[2])\sin^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\sin^{-n}(\lambda K[2])\sin^k(\nu K[2])}{a}dK[2]\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*x)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p(\sin^{-n}(\_f\lambda))\left(\sin^s\left(\frac{(ay+b(\int(\sin^m(\_f\mu))(\sin^{-n}(\_f\lambda))d\_f)-b(\int(\sin^{-n}(\lambda x))(\sin^m(\mu x))dx))\beta}{a}\right)\right)e^{-\frac{c(\int(\sin^-}{}}}{a}\right.$$

7.5.14.7 [1297] Problem 7

problem number 1297

Added April 8, 2019.

Problem Chapter 5.6.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu y)w + p\sin^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sin^{-n}(\lambda K[2])\sin^k\left(\nu\left(y-\int_1^x \frac{b\sin^{-n}(\lambda K[1])\sin^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]}\frac{b\sin^{-n}(\lambda K[1])\sin^n}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*y)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p(\sin^s(\_f\beta))(\sin^{-n}(\_f\lambda))\,e^{-\frac{c\left(\int(\sin^{-n}(\_f\lambda))\left(\sin^k\left(\frac{\left(ay+b\left(\int(\sin^m(\_f\mu))(\sin^{-n}(\_f\lambda))d\_f\right)-b\left(\int(\sin^{-n}(\lambda x))(\sin^m\right.\right.}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.15 6.2

Local contents

7.5.15.1 [1298] Problem 1

problem number 1298

Added April 11, 2019.

Problem Chapter 5.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{k((a\lambda+b\mu)\sin(\lambda x+\mu y)-c\cos(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2}+e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left(\left(-c\cos\left(\lambda x+\mu y\right)+\left(a\lambda+\mu b\right)\sin\left(\lambda x+\mu y\right)\right)k\,\mathrm{e}^{-\frac{cx}{a}}+\left(\lambda^2a^2+2ab\lambda\mu+b^2\mu^2+c^2\right)\_F1\left(\frac{ay-bx}{a}\right.}{\lambda^2a^2+2ab\lambda\mu+b^2\mu^2+c^2}$$

7.5.15.2 [1299] Problem 2

problem number 1299

Added April 11, 2019.

Problem Chapter 5.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1\cos^k(\lambda x) + c_2\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cos[lambda*x]^k + c2*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{e^{\frac{x}{a}}(ak\lambda-i)(b\beta n-i)c_1\left(y-\frac{bx}{a}\right)+\text{c1}\left(1+e^{2i\lambda x}\right)(1+ib\beta n)\cos^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k+\frac{i}{a\lambda}+2\right);\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cos(lambda*x)^k + c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{\left(c1\left(\cos^k\left(\_a\lambda\right)\right)+c2\left(\cos^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.15.3 [1300] Problem 3

problem number 1300

Added April 11, 2019.

Problem Chapter 5.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cos^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cos[lambda*x]^k * Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^k(\lambda K[1])\cos^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cos(lambda*x)^k *cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\cos^k\left(\_a\lambda\right)\right)\left(\cos^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.15.4 [1301] Problem 4

problem number 1301

Added April 11, 2019.

Problem Chapter 5.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{k\cos\left(\mu yK[1]^{\frac{b}{a}}x^{-\frac{b}{a}}+\lambda K[1]\right)K[1]^{-\frac{a+c}{a}}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\_a^{\frac{-a-c}{a}}\cos\left(\mu y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}+\_a\lambda\right)}{a}d\_a+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

7.5.15.5 [1302] Problem 5

problem number 1302

Added April 11, 2019.

Problem Chapter 5.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)w + b\cos(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x+mu*y]*w[x,y]+b*Cos[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\left(\int_1^x \frac{b\exp\left(-\frac{ax\sin\left(\left(\lambda+\frac{\mu y}{x}\right)K[1]\right)}{\lambda x+\mu y}\right)\cos(\nu K[1])}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =a*x*cos(lambda*x+mu*y)*w(x,y)+b*cos(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{b\cos(\_a\nu)\,\mathrm{e}^{-\frac{a\sin\left(\_a\lambda+\frac{\_a\mu y}{x}\right)}{\lambda+\frac{\mu y}{x}}}}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{a\sin(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

7.5.15.6 [1303] Problem 6

problem number 1303

Added April 11, 2019.

Problem Chapter 5.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cos^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\cos^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cos[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cos^{-n}(\lambda K[2])\cos^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\cos^{-n}(\lambda K[2])\cos^k(\nu K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cos(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =c*cos(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p(\cos^{-n}(\_f\lambda))\left(\cos^s\left(\frac{(ay+b(\int(\cos^m(\_f\mu))(\cos^{-n}(\_f\lambda))d\_f)-b(\int(\cos^{-n}(\lambda x))(\cos^m(\mu x))dx))\beta}{a}\right)\right)e^{-\frac{c(\int(\cos}{}}}{a}\right.$$

7.5.16 6.3

Local contents

7.5.16.1 [1304] Problem 1

problem number 1304

Added April 11, 2019.

Problem Chapter 5.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}} c_1\left(y - \frac{bx}{a}\right) - \frac{ik\left(-2\,{}_2F_1\left(1, -\frac{ic}{2a\lambda+2b\mu}; \frac{-ic+2a\lambda+2b\mu}{2(a\lambda+b\mu)}; -e^{-2i(\lambda x+\mu y)}\right) + 2e^{\frac{2i\mu(ay-bx)}{a}}\,{}_2F_1\left(1, \ldots\right.\right.}{c\left(1+e^{\frac{2i\mu(ay-bx)}{a}}\right)}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{k\,\mathrm{e}^{-\frac{\_ac}{a}}\tan\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.16.2 [1305] Problem 2

problem number 1305

Added April 11, 2019.

Problem Chapter 5.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=w+c_1\tan^k(\lambda x)+c_2\tan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Tan[lambda*x]^k + c2*Tan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x\frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\tan^k(\lambda K[1])+\text{c2}\tan^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*tan(lambda*x)^k + c2*tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(c1\left(\tan^{k}\left(\_a\lambda\right)\right)+c2\left(\tan^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)e^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{x}{a}}$$

7.5.16.3 [1306] Problem 3

problem number 1306

Added April 11, 2019.

Problem Chapter 5.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\tan^k(\lambda x)\tan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Tan[lambda*x]^k * Tan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_{1}^{x}\frac{e^{-\frac{cK[1]}{a}}\tan^{k}(\lambda K[1])\tan^{n}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ tan(lambda*x)^k *tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(\tan^{k}\left(\_a\lambda\right)\right)\left(\tan^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.16.4 [1307] Problem 4

problem number 1307

Added April 11, 2019.

Problem Chapter 5.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan(\mu y)w_y=c\tan(\lambda x)w+k\tan(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Tan[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\cos^{-\frac{c}{a\lambda}}(\lambda x)\left(\int_{1}^{x}\frac{k\cos^{\frac{c}{a\lambda}}(\lambda K[1])\tan(\nu K[1])}{a}dK[1]+c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) = c*tan(lambda*x)*w(x,y)+ k*tan(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{k\left(\frac{2}{\cos(2\lambda x)+1}\right)^{-\frac{c}{2a\lambda}}\sin(\nu x)}{a\cos(\nu x)}dx+\_F1\left(\frac{-b\mu x+a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)\right)\left(\tan^2(\lambda x)+1\right)^{\frac{c}{2a\lambda}}$$

7.5.16.5 [1308] Problem 5

problem number 1308

Added April 11, 2019.

Problem Chapter 5.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cw+k\tan(\lambda x+\nu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+nu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\tan\left(\nu yK[1]^{\frac{b}{a}}x^{-\frac{b}{a}}+\lambda K[1]\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\_a^{\frac{-a-c}{a}} \tan\left(\nu y\_a^{\frac{b}{a}} x^{-\frac{b}{a}} + \_a\lambda\right)}{a} d\_a + \_F1\left(y\,x^{-\frac{b}{a}}\right)\right) x^{\frac{c}{a}}$$

7.5.16.6 [1309] Problem 6

problem number 1309

Added April 11, 2019.

Problem Chapter 5.6.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^{n}(\lambda x)w_x + b\tan^{m}(\mu x)w_y = c\tan^{k}(\nu x)w + p\tan^{s}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_{1}^{x} \frac{c\tan^{-n}(\lambda K[2])\tan^{k}(\nu K[2])}{a} dK[2]\right)\left(\int_{1}^{x} \frac{\exp\left(-\int_{1}^{K[3]} \frac{c\tan^{-n}(\lambda K[2])\tan^{k}(\nu K[2])}{a} dK[\right.}{\ldots}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n} \left(\frac{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\left(\frac{\sin(\mu x)}{\cos(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)}{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\left(\frac{\sin(\mu x)}{\cos(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)} \right)^s e^{-\frac{c\left(\int\left(\frac{\mathrm{s}}{\mathrm{c}}\right.\right.}{}}}{a}\right.$$

7.5.16.7 [1310] Problem 7

problem number 1310

Added April 11, 2019.

Problem Chapter 5.6.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\tan^m(\mu x)w_y = c\tan^k(\nu y)w + p\tan^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-n}(\lambda K[2])\tan^k\left(\nu\left(y-\int_1^x \frac{b\tan^{-n}(\lambda K[1])\tan^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]}\frac{b\tan^{-n}(\lambda K[1])\,t}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\sin(\_f\beta)}{\cos(\_f\beta)}\right)^s\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}\mathrm{e}^{-\frac{c\int\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}\left(\frac{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\right.\right.}{a}\right)}{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\right.\right.}{a}\right)}\right.}{a}}}{a}\right.$$

7.5.17 6.4

Local contents

7.5.17.1 [1311] Problem 1

problem number 1311

Added April 11, 2019.

Problem Chapter 5.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{c(c-2i(a\lambda+b\mu))e^{\frac{x(c-2ib\mu)}{a}}\left(e^{\frac{2ib\mu x}{a}}-e^{2i\mu y}\right)c_1\left(y-\frac{bx}{a}\right)+k\left(-2(2a\lambda+2b\mu+ic)e^{\frac{2i\mu(ay-bx)}{a}}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{k\cot\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.17.2 [1312] Problem 2

problem number 1312

Added April 11, 2019.

Problem Chapter 5.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=w+c_1\cot^k(\lambda x)+c_2\cot^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cot[lambda*x]^k + c2*Cot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x\frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\cot^k(\lambda K[1])+\text{c2}\cot^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cot(lambda*x)^k + c2*cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\cot^{k}\left(\_a\lambda\right)\right)+c2\left(\cot^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.17.3 [1313] Problem 3

problem number 1313

Added April 11, 2019.

Problem Chapter 5.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cot^k(\lambda x)\cot^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cot[lambda*x]^k * Cot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cot^k(\lambda K[1])\cot^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cot(lambda*x)^k *cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\cot^{k}(\_a\lambda)\right)\left(\cot^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.17.4 [1314] Problem 4

problem number 1314

Added April 11, 2019.

Problem Chapter 5.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot(\mu y)w_y = c\cot(\lambda x)w + k\cot(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[mu*y]*D[w[x, y], y] == c*Cot[lambda*x]*w[x,y]+k*Cot[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to\sin^{\frac{c}{a\lambda}}(\lambda x)\left(\int_{1}^{x}\frac{k\cot(\nu K[1])\sin^{-\frac{c}{a\lambda}}(\lambda K[1])}{a}dK[1]+c_1\left(\frac{\log(\sec(\mu y))}{\mu}-\frac{bx}{a}\right)\right)\right\}$$
$$\left\{w(x,y)\to\sin^{\frac{c}{a\lambda}}(\lambda x)\left(\int_{1}^{x}\frac{k\cot(\nu K[2])\sin^{-\frac{c}{a\lambda}}(\lambda K[2])}{a}dK[2]+c_1\left(\frac{\log(\sec(\mu y))}{\mu}-\frac{bx}{a}\right)\right)\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(mu*y)*diff(w(x,y),y) = c*cot(lambda*x)*w(x,y)+ k*cot(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(-\left(\int^{y} \frac{\left(\sin\left(\frac{2a\nu\ln\left(\frac{\cos(\_a\mu)}{\sin(\_a\mu)}\right)-a\nu\ln\left(-\frac{2}{\cos(2\_a\mu)-1}\right)+a\nu\ln(\cot^2(\mu y)+1)-2a\nu\ln(\cot(\mu y))-2(-\_a\mu+\nu x)b\mu}{2b\mu}\right) - \sin\right.}{\left(\sin\left(\frac{2a\nu\ln\left(\frac{\cos(\_a\mu)}{\sin(\_a\mu)}\right)-a\nu\ln\left(-\frac{2}{\cos(2\_a\mu)-1}\right)+a\nu\ln(\cot^2(\right.\right.}{2b\mu}} \right.\right.$$

7.5.17.5 [1315] Problem 5

problem number 1315

Added April 11, 2019.

Problem Chapter 5.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\cot(\lambda x + \nu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+nu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{k\cot\left(\nu y K[1]^{\frac{b}{a}}x^{-\frac{b}{a}}+\lambda K[1]\right)K[1]^{-\frac{a+c}{a}}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{k\_a^{\frac{-a-c}{a}}\cot\left(\nu y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}+\_a\lambda\right)}{a}d\_a+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

7.5.17.6 [1316] Problem 6

problem number 1316

Added April 11, 2019.

Problem Chapter 5.6.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^{n}(\lambda x)w_{x}+b\cot^{m}(\mu x)w_{y}=c\cot^{k}(\nu x)w+p\cot^{s}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\int_{1}^{x}\frac{c\cot^{-n}(\lambda K[2])\cot^{k}(\nu K[2])}{a}dK[2]\right)\left(\int_{1}^{x}\frac{\exp\left(-\int_{1}^{K[3]}\frac{c\cot^{-n}(\lambda K[2])\cot^{k}(\nu K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{p\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}\left(\frac{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^{m}\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m}dx\right)\right)\beta}{a}\right)}{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^{m}\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m}dx\right)\right)\beta}{a}\right)}\right)^{s}\mathrm{e}^{-\frac{c\left(\int\left(\frac{c}{s}\right.\right.}{}}}{a}\right.$$

7.5.17.7 [1317] Problem 7

problem number 1317

Added April 11, 2019.

Problem Chapter 5.6.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^{n}(\lambda x)w_{x}+b\cot^{m}(\mu x)w_{y}=c\cot^{k}(\nu y)w+p\cot^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cot^{-n}(\lambda K[2])\cot^k\left(\nu\left(y-\int_1^x \frac{b\cot^{-n}(\lambda K[1])\cot^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\cot^{-n}(\lambda K[1])\cot}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cos(\_f\beta)}{\sin(\_f\beta)}\right)^s\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}\mathrm{e}^{-\frac{c\left(\int\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}\left(\frac{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^m\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\right.\right.}{a}\right)}{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^m\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\right.\right.}{a}\right)}\right.\right.}{a}}}{a}\right.$$

7.5.18 6.5

Local contents

7.5.18.1 [1318] Problem 1

problem number 1318

Added April 11, 2019.

Problem Chapter 5.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*Sin[lambda*x]^k+c2*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}c_1\left(y-\frac{bx}{a}\right)-\frac{ic1\left(-1+e^{2i\lambda x}\right)\sin^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k+\frac{i}{a\lambda}+2\right);\frac{1}{2}\left(-k+\frac{i}{a\lambda}+2\right);e^{2i\lambda x}\right)}{ak\lambda-i}+\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*sin(lambda*x)^k+c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\sin^{k}\left(\_a\lambda\right)\right)+c2\left(\cos^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.18.2 [1319] Problem 2

problem number 1319

Added April 11, 2019.

Problem Chapter 5.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sin^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sin[lambda*x]^k*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\sin^k(\lambda K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+sin(lambda*x)^k*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(\cos^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\left(\sin^{k}\left(\_a\lambda\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.18.3 [1320] Problem 3

problem number 1320

Added April 11, 2019.

Problem Chapter 5.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\sin(\mu y)w_y=c\sin(\lambda x)w+k\cos(\nu x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Cos[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_{1}^{x}\frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(s+k\cos(\nu K[1]))}{a}dK[1]+c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*cos(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\cos(\nu x)+s)\,\mathrm{e}^{\frac{c\cos(\lambda x)}{a\lambda}}}{a}dx + \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y - \arctan\left(\frac{2\_Z\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}, -\frac{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}\right.\right.\right.}{b\mu}\right.\right.$$

7.5.18.4 [1321] Problem 4

problem number 1321

Added April 11, 2019.

Problem Chapter 5.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = c\sin(\lambda x)w + k\tan(\nu x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Tan[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_1^x \frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(s+k\tan(\nu K[1]))}{a}dK[1] + c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*tan(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\sin(\nu x) + s\cos(\nu x))\, \mathrm{e}^{\frac{c\cos(\lambda x)}{a\lambda}}}{a\cos(\nu x)} dx + \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y - \arctan\left(\frac{2\_Z\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}, -\frac{\_Z}{\_Z}\right.\right.\right.}{b\mu}\right.\right.$$

7.5.18.5 [1322] Problem 5

problem number 1322

Added April 11, 2019.

Problem Chapter 5.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = c\tan(\lambda x)w + k\cot(\nu x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Cot[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cos^{-\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{\cos^{\frac{c}{a\lambda}}(\lambda K[1])(s + k\cot(\nu K[1]))}{a} dK[1] + c_1\left(\frac{\log(\sin(\mu y))}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) =  c*tan(lambda*x)*w(x,y)+k*cot(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\cos(\nu x)+s\sin(\nu x))\left(\cos^{\frac{c}{a\lambda}}(\lambda x)\right)}{a\sin(\nu x)}dx + \_F1\left(\frac{-b\mu x + a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)\right)\left(\cos^{-\frac{c}{a\lambda}}(\lambda$$

7.5.18.6 [1323] Problem 6

problem number 1323

Added April 11, 2019.

Problem Chapter 5.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cos^k(\nu K[2])\sin^{-n}(\lambda K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\cos^k(\nu K[2])\sin^{-n}(\lambda K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =  c*cos(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p(\sin^{-n}(\_f\lambda)) \left(\sin^{s}\left(\frac{(ay+b(\int(\cos^{m}(\_f\mu))(\sin^{-n}(\_f\lambda))d\_f)-b(\int(\cos^{m}(\mu x))(\sin^{-n}(\lambda x))dx))\beta}{a}\right)\right) e^{-\frac{c(\int(\cos^{k}}{}}}{a}\right.$$

7.5.18.7 [1324] Problem 7

problem number 1324

Added April 11, 2019.

Problem Chapter 5.6.5.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^{n}(\lambda x)w_x + b\cot^{m}(\mu x)w_y = c\tan^{k}(\nu x)w + p\cot^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_{1}^{x} \frac{c\tan^{-n}(\lambda K[2])\tan^{k}(\nu K[2])}{a} dK[2]\right) \left(\int_{1}^{x} \frac{\exp\left(-\int_{1}^{K[3]} \frac{c\tan^{-n}(\lambda K[2])\tan^{k}(\nu K[2])}{a} dK[\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =  c*tan(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{p\left(\frac{\cos(\beta x)}{\sin(\beta x)}\right)^{s} \left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n} \mathrm{e}^{-\frac{c\left(\int \left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n} \left(\frac{\sin(\nu x)}{\cos(\nu x)}\right)^{k} dx\right)}{a}}}{a} dx + \_F1\left(\frac{ay - b\left(\int \left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m} \left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right.\right.}{a}\right.\right.$$

7.5.19 7.1

Local contents

7.5.19.1 [1325] Problem 1

problem number 1325

Added April 13, 2019.

Problem Chapter 5.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arcsin^k(\lambda x) + c_2 \arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcSin[lambda*x]^k+c2*ArcSin[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\mathrm{c1}\sin^{-1}(\lambda K[1])^k+\mathrm{c2}\sin^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arcsin(lambda*x)^k+c2*arcsin(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\arcsin(\_a\lambda)^k+c2\arcsin\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.19.2 [1326] Problem 2

problem number 1326

Added April 13, 2019.

Problem Chapter 5.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\arcsin^k(\lambda x)\arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcSin[lambda*x]^k*ArcSin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^{-1}(\lambda K[1])^k \sin^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arcsin(lambda*x)^k*arcsin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\arcsin\left(\_a\lambda\right)^k \arcsin\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.19.3 [1327] Problem 3

problem number 1327

Added April 13, 2019.

Problem Chapter 5.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=\left(c_1\arcsin(\lambda_1 x)+c_2\arcsin(\lambda_2 y)\right)w+s_1\arcsin^n(\beta_1 x)+s_2\arcsin^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcSin[lambda1*x] + c2*ArcSin[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{\text{c1}\sqrt{1-\text{lambda1}^2x^2}}{a\text{lambda1}}+\frac{\text{c1}x\sin^{-1}(\text{lambda1}x)}{a}+\frac{\text{c2}\sqrt{1-\text{lambda2}^2y^2}}{b\text{lambda2}}+\frac{\text{c2}y\sin^{-1}(\text{la}}{b}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arcsin(lambda1*x) + c2*arcsin(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(s1\arcsin\left(\_a\beta 1\right)^n+s2\arcsin\left(\frac{(ay-(-\_a+x)b)\beta 2}{a}\right)^k\right)\mathrm{e}^{-\sqrt{-\frac{(-(-\_a+x)b\lambda 2+(\lambda 2y-1)a)(-(-\_a+x)b\lambda 2+(\lambda 2}{a^2}}}}{a}\right.$$

7.5.19.4 [1328] Problem 4

problem number 1328

Added April 13, 2019.

Problem Chapter 5.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^m(\mu x)w_y=c\arcsin^k(\nu x)w+p\arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*x]^k*w[x,y]+p*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\frac{ic\sin^{-1}(\nu x)^k\left(\sin^{-1}(\nu x)^2\right)^{-k}\left(\left(-i\sin^{-1}(\nu x)\right)^k\text{Gamma}\left(k+1,i\sin^{-1}(\nu x)\right)-\left(i\sin^{-1}\right.\right.}{2a\nu}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*x)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{p\left(-\arcsin\left(\frac{(\_f\mu+1)(\_f\mu-1)\left(-\left(\arcsin(\_f\mu)^m-\frac{\text{LommelS1}\left(m+\frac{3}{2},\frac{1}{2},\arcsin(\_f\mu)\right)}{\sqrt{\arcsin(\_f\mu)}}\right)\sqrt{-\_f^2\mu^2+1}\,b2^m2^{-m}\arcsin(\_f\right.}{}\right.\right.}{}\right.$$

7.5.19.5 [1329] Problem 5

problem number 1329

Added April 13, 2019.

Problem Chapter 5.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^m(\mu x)w_y=c\arcsin^k(\nu y)w+p\arcsin^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*y]^k*w[x,y]+p*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\int_1^x\frac{c\sin^{-1}\left(\nu\left(y-\int_1^x\frac{b\sin^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]}\frac{b\sin^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\left(\int\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*y)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{p\arcsin(\_f\beta)^n\,\mathrm{e}^{-\frac{c\left(\int\left(-\arcsin\left(\frac{(\_f\mu+1)(\_f\mu-1)\left(-\left(\arcsin(\_f\mu)^m-\frac{\mathrm{LommelS1}\left(m+\frac{3}{2},\frac{1}{2},\arcsin(\_f\mu)\right)}{\sqrt{\arcsin(\_f\mu)}}\right)\sqrt{-\_f^2\mu^2+1}\,b2^m2^{-}\right.}{}\right.\right.\right.}{}}}{}\right.$$

7.5.20 7.2

Local contents

7.5.20.1 [1330] Problem 1

problem number 1330

Added April 13, 2019.

Problem Chapter 5.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = w + c_1 \arccos^k(\lambda x) + c_2 \arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCos[lambda*x]^k+c2*ArcCos[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\cos^{-1}(\lambda K[1])^k+\text{c2}\cos^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccos(lambda*x)^k+c2*arccos(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\arccos(\_a\lambda)^k+c2\arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

7.5.20.2 [1331] Problem 2

problem number 1331

Added April 13, 2019.

Problem Chapter 5.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \arccos^k(\lambda x)\arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCos[lambda*x]^k*ArcCos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^{-1}(\lambda K[1])^k \cos^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccos(lambda*x)^k*arccos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\arccos(\_a\lambda)^k \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.20.3 [1332] Problem 3

problem number 1332

Added April 13, 2019.

Problem Chapter 5.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \arccos(\lambda_1 x) + c_2 \arccos(\lambda_2 y))\, w + s_1 \arccos^n(\beta_1 x) + s_2 \arccos^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCos[lambda1*x] + c2*ArcCos[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(-\frac{\text{c1}\sqrt{1-\text{lambda1}^2x^2}}{a\text{lambda1}}+\frac{\text{c1}x\cos^{-1}(\text{lambda1}x)}{a}+\frac{\text{c2}x\sin^{-1}(\text{lambda2}y)}{a}+\frac{\text{c2}x\cos^{-1}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccos(lambda1*x) + c2*arccos(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1 \arccos(\_a\beta1)^n + s2 \arccos\left(\frac{(ay-(-\_a+x)b)\beta2}{a}\right)^k\right) e^{\sqrt{-\frac{(-(-\_a+x)b\lambda2+(\lambda2y-1)a)(-(-\_a+x)b\lambda2+(\lambda2y}{a^2}}}}{a}\right.$$

7.5.20.4 [1333] Problem 4

problem number 1333

Added April 13, 2019.

Problem Chapter 5.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^m(\mu x)w_y = c\arccos^k(\nu x)w + p\arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*x]^k*w[x,y]+p*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\cos^{-1}(\nu x)^k\left(\cos^{-1}(\nu x)^2\right)^{-k}\left(\left(-i\cos^{-1}(\nu x)\right)^k \text{Gamma}\left(k+1, i\cos^{-1}(\nu x)\right)+\left(i\cos^{-1}\right.\right.}{2a\nu}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*x)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\arccos\left(\left(\frac{\left(-\text{LommelS1}\left(m+\frac{3}{2},\frac{3}{2},\arccos(\mu x)\right)\arccos(\mu x)+\arccos(\mu x)^{m+\frac{3}{2}}+(m+2)\,\text{LommelS1}\left(m+\frac{1}{2},\frac{1}{2},\arccos(\mu x)\right)\right)\sqrt{-\mu^2x^2+}}{\sqrt{\arccos(\mu x)}}\right.\right.}{}\right.$$

7.5.20.5 [1334] Problem 5

problem number 1334

Added April 13, 2019.

Problem Chapter 5.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \arccos^m(\mu x) w_y = c \arccos^k(\nu y) w + p \arccos^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*y]^k*w[x,y]+p*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cos^{-1}\left(\nu\left(y-\int_1^x \frac{b\cos^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]} \frac{b\cos^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\left(\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*y)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p \arccos(\_b\beta)^{n} e^{-\frac{c \left(\int \arccos \left(\frac{\left(\frac{\left(-\operatorname{LommelS1}\left(m+\frac{3}{2}, \frac{3}{2}, \arccos(\mu x)\right) \arccos(\mu x) + \arccos(\mu x)^{m+\frac{3}{2}} + (m+2) \operatorname{LommelS1}\left(m+\frac{1}{2}, \frac{1}{2}\right.}{\sqrt{\arccos(\mu x)}}}{}}{}\right.\right.}{}}}{}\right.$$

7.5.21 7.3

Local contents

7.5.21.1 [1335] Problem 1

problem number 1335

Added April 13, 2019.

Problem Chapter 5.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = w + c_1 \arctan^k(\lambda x) + c_2 \arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcTan[lambda*x]^k+c2*ArcTan[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x\frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\tan^{-1}(\lambda K[1])^k+\text{c2}\tan^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arctan(lambda*x)^k+c2*arctan(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{\left(c1\arctan(\_a\lambda)^k+c2\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)e^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{x}{a}}$$

7.5.21.2 [1336] Problem 2

problem number 1336

Added April 13, 2019.

Problem Chapter 5.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\arctan^k(\lambda x)\arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcTan[lambda*x]^k*ArcTan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tan^{-1}(\lambda K[1])^k\tan^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arctan(lambda*x)^k*arctan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\arctan(\_a\lambda)^k\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n e^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

7.5.21.3 [1337] Problem 3

problem number 1337

Added April 13, 2019.

Problem Chapter 5.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c_1\arctan(\lambda_1x)+c_2\arctan(\lambda_2y))\,w+s_1\arctan^n(\beta_1x)+s_2\arctan^k(\beta_2y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcTan[lambda1*x] + c2*ArcTan[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\left(\text{lambda1}^2x^2+1\right)^{-\frac{\text{c1}}{2a\text{lambda1}}}\exp\left(\frac{\text{c2}\left(2\text{lambda2}y\tan^{-1}(\text{lambda2}y)-\log\left(a^2\left(\text{lambda2}^2y^2+\right.\right.\right.}{2b\text{lambda2}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arctan(lambda1*x) + c2*arctan(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(s1\arctan\left(\_a\beta1\right)^{n}+s2\arctan\left(\frac{(ay-(-\_a+x)b)\beta2}{a}\right)^{k}\right)\left(\frac{a^2+(ay-(-\_a+x)b)^2\lambda2^2}{a^2}\right)^{\frac{c2}{2b\lambda2}}\left(\_a^2\lambda1^2\right.}{a}\right.$$

7.5.21.4 [1338] Problem 4

problem number 1338

Added April 13, 2019.

Problem Chapter 5.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arctan^m(\mu x)w_y=c\arctan^k(\nu x)w+p\arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*x]^k*w[x,y]+p*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-1}(\nu K[2])^k}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\tan^{-1}(\nu K[2])^k}{a}dK[2]\right)p\tan^{-1}\left(\beta\left(y-\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*x)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\arctan\left(\frac{\left(b\left(\int \arctan(\_f\mu)^m d\_f\right)+\left(y-\left(\int \frac{b\arctan(\mu x)^m}{a}dx\right)\right)a\right)\beta}{a}\right)^n \mathrm{e}^{-\frac{c\left(\int \arctan(\_f\nu)^k d\_f\right)}{a}}}{a}d\_f+\_F1\left(y-\right.\right.$$

7.5.21.5 [1339] Problem 5

problem number 1339

Added April 13, 2019.

Problem Chapter 5.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^m(\mu x)w_y = c\arctan^k(\nu y)w + p\arctan^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*y]^k*w[x,y]+p*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\exp\left(\int_1^x\frac{c\tan^{-1}\left(\nu\left(y-\int_1^x\frac{b\tan^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]}\frac{b\tan^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\left(\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*y)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{p\arctan(\_f\beta)^n\,\mathrm{e}^{-\frac{c\left(\int\arctan\left(\frac{\left(b\left(\int\arctan(\_f\mu)^m d\_f\right)+\left(y-\left(\int\frac{b\arctan(\mu x)^m}{a}dx\right)\right)a\right)\nu}{a}\right)^k d\_f\right)}{a}}}{a}d\_f+\_F1\left(y-\left(\right.\right.\right.$$

7.5.22 7.4

Local contents

7.5.22.1 [1340] Problem 1

problem number 1340

Added April 13, 2019.

Problem Chapter 5.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \operatorname{arccot}^k(\lambda x) + c_2 \operatorname{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCot[lambda*x]^k+c2*ArcCot[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\cot^{-1}(\lambda K[1])^k+\text{c2}\cot^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccot(lambda*x)^k+c2*arccot(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\left(\int^x \frac{\left(c1\left(-\arctan\left(\_a\lambda\right)+\frac{\pi}{2}\right)^k+c2\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)+\frac{\pi}{2}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay\right.\right.\right.$$

7.5.22.2 [1341] Problem 2

problem number 1341

Added April 13, 2019.

Problem Chapter 5.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \operatorname{arccot}^k(\lambda x) \operatorname{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCot[lambda*x]^k*ArcCot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cot^{-1}(\lambda K[1])^k \cot^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccot(lambda*x)^k*arccot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(-\arctan\left(\_a\lambda\right)+\frac{\pi}{2}\right)^k\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)+\frac{\pi}{2}\right)^n \mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)$$

7.5.22.3 [1342] Problem 3

problem number 1342

Added April 13, 2019.

Problem Chapter 5.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \operatorname{arccot}(\lambda_1 x) + c_2 \operatorname{arccot}(\lambda_2 y))\, w + s_1 \operatorname{arccot}^n(\beta_1 x) + s_2 \operatorname{arccot}^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCot[lambda1*x] + c2*ArcCot[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \left(\text{lambda1}^2x^2+1\right)^{\frac{\text{c1}}{2a\text{lambda1}}}\exp\left(\frac{\frac{\text{c2}\left(a\log\left(a^2\left(\text{lambda2}^2y^2+1\right)\right)+2\text{lambda2}\tan^{-1}(\text{lambda2}y)(bx-ay)+2b\text{lambda}\right.}{b\text{lambda2}}}{2a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccot(lambda1*x) + c2*arccot(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{\left(s1\left(-\arctan\left(\_a\beta 1\right)+\frac{\pi}{2}\right)^n+s2\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta 2}{a}\right)+\frac{\pi}{2}\right)^k\right)\left(\frac{a^2+(ay-(-\_a+x)b)^2}{a^2}\right.}{a}\right.$$

7.5.22.4 [1343] Problem 4

problem number 1343

Added April 13, 2019.

Problem Chapter 5.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\,\mathrm{arccot}^m(\mu x)w_y = c\,\mathrm{arccot}^k(\nu x)w + p\,\mathrm{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*x]^k*w[x,y]+p*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cot^{-1}(\nu K[2])^k}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\cot^{-1}(\nu K[2])^k}{a}dK[2]\right)p\cot^{-1}\left(\beta\left(y-\int\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*x)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\left(-\arctan\left(\frac{\left(b\left(\int\left(-\arctan(\_f\mu)+\frac{\pi}{2}\right)^m d\_f\right)+\left(y-\left(\int\frac{b\left(-\arctan(\mu x)+\frac{\pi}{2}\right)^m}{a}dx\right)\right)a\right)\beta}{a}\right)+\frac{\pi}{2}\right)^n \mathrm{e}^{-\frac{c\left(\int\left(-\mathrm{arc}\right.\right.}{}}}{a}\right.$$

7.5.22.5 [1344] Problem 5

problem number 1344

Added April 13, 2019.

Problem Chapter 5.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \operatorname{arccot}^m(\mu x) w_y = c \operatorname{arccot}^k(\nu y) w + p \operatorname{arccot}^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*y]^k*w[x,y]+p*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c \cot^{-1}\left(\nu\left(y - \int_1^x \frac{b\cot^{-1}(\mu K[1])^m}{a} dK[1] + \int_1^{K[2]} \frac{b\cot^{-1}(\mu K[1])^m}{a} dK[1]\right)\right)^k}{a} dK[2]\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*y)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\left(-\arctan\left(\_f\beta\right) + \frac{\pi}{2}\right)^n \mathrm{e}^{-\frac{c\left(\int\left(-\arctan\left(\frac{\left(b\left(\int\left(-\arctan(\_f\mu)+\frac{\pi}{2}\right)^m d\_f\right)+\left(y-\left(\int\frac{b\left(-\arctan(\mu x)+\frac{\pi}{2}\right)^m}{a}dx\right)\right)a\right)\nu}{a}\right.\right.\right.}{a}}}{a}\right.$$

7.5.23 8.1

Local contents

7.5.23.1 [1345] Problem 1

problem number 1345

Added April 13, 2019.

Problem Chapter 5.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)w + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])}{a}dK[1]\right)g(K[2])}{a}dK[2]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{\mathrm{e}^{-\frac{\int f(x)dx}{a}} g(x)}{a} dx + \_F1\left(\frac{ay-bx}{a}\right)\right) \mathrm{e}^{\int \frac{f(x)}{a} dx}$$

7.5.23.2 [1346] Problem 2

problem number 1346

Added April 13, 2019.

Problem Chapter 5.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cy+k)w + f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y+k)*w[x,y]+f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x(2a(cy+k)-bcx)}{2a^2}} \left(\int_1^x \frac{\exp\left(-\frac{K[1](2a(k+cy)+bc(K[1]-2x))}{2a^2}\right) f(K[1])}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*y+k)*w(x,y)+f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\frac{\left(-\left(-\frac{\_a}{2}+x\right)bc+(cy+k)a\right)\_a}{a^2}} f(\_a)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\frac{\left(-\frac{bcx}{2}+(cy+k)a\right)x}{a^2}}$$

7.5.23.3 [1347] Problem 3

problem number 1347

Added April 13, 2019.

Problem Chapter 5.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)yw + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*y*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{f(K[1])(ay+b(K[1]-x))}{a^2} dK[1]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{f(K[1])(ay+b(K[1]-x))}{a^2} dK[1]\right)}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*y*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\frac{\int (ay+(\_b-x)b)f(\_b)d\_b}{a^2}} g(\_b)}{a} d\_b + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\int^{x} \frac{(ay-(-\_a+x)b)f(\_a)}{a^2} d\_a}$$

7.5.23.4 [1348] Problem 4

problem number 1348

Added April 13, 2019.

Problem Chapter 5.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x)w + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x]*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{f(K[1])}{aK[1]} dK[1]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{f(K[1])}{aK[1]} dK[1]\right) g(K[2])}{aK[2]} dK[2] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{\mathrm{e}^{-\frac{\int \frac{f(x)}{x}dx}{a}} g(x)}{ax}dx+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int \frac{f(x)}{ax}dx}$$

7.5.23.5 [1349] Problem 5

problem number 1349

Added April 13, 2019.

Problem Chapter 5.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(ay+b)w_y=cw+g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (a+y+b)*D[w[x, y], y] == c*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c}{f(K[3])}dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]}\frac{c}{f(K[3])}dK[3]\right)g(K[4])}{f(K[4])}dK[4]+c_1\left(y\exp\left(-\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (a*y+b)*diff(w(x,y),y) = c*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-c\left(\int \frac{1}{f(x)}dx\right)} g(x)}{f(x)} dx + \_F1\left(\frac{(ay+b)\, e^{-a\left(\int \frac{1}{f(x)}dx\right)}}{a}\right)\right) e^{\int \frac{c}{f(x)}dx}$$

7.5.23.6 [1350] Problem 6

problem number 1350

Added April 13, 2019.

Problem Chapter 5.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)w_y = h(x)w + p(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x,y]+p[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{h(K[2])}{f(K[2])} dK[2]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{h(K[2])}{f(K[2])} dK[2]\right) p(K[3])}{f(K[3])} dK[3] + c_1\left(y - \int_1^x \frac{g(}{f(}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x)*w(x,y)+p(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\left(\int \frac{h(x)}{f(x)}dx\right)} p(x)}{f(x)} dx + \_F1\left(y - \left(\int \frac{g(x)}{f(x)} dx\right)\right)\right) e^{\int \frac{h(x)}{f(x)} dx}$$

7.5.23.7 [1351] Problem 7

problem number 1351

Added April 13, 2019.

Problem Chapter 5.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_1(x)w + h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h1[x]*w[x,y]+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\text{h1}(K[3])}{f(K[3])} dK[3]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\text{h1}(K[3])}{f(K[3])} dK[3]\right) \text{h0}(K[4])}{f(K[4])} dK[4] + c_1\left(y \exp\left(\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h1(x)*w(x,y)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\left(\int \frac{h1(x)}{f(x)}dx\right)} h0(x)}{f(x)}dx + \_F1\left(y\,e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} - \left(\int \frac{e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} g0(x)}{f(x)}dx\right)\right)\right) e^{\int \frac{h1(x)}{f(x)}dx}$$

7.5.23.8 [1352] Problem 8

problem number 1352

Added April 13, 2019.

Problem Chapter 5.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_2(x)w + h_1(x)y + h0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\text{h2}(K[3])}{f(K[3])}dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\text{h2}(K[3])}{f(K[3])}dK[3]\right)\left(\text{h0}(K[4]) + \exp\left(\int_1^{K[4]} \frac{\text{g1}(K[}{f(K[}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{e^{-\left(\int \frac{h2(\_g)}{f(\_g)} d\_g\right)} h0(\_g) + \left(y\, e^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} + \int \frac{e^{-\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right)} g0(\_g)}{f(\_g)} d\_g - \left(\int \frac{e^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)} \cdots \right.\right.}{f(\_g)} \cdots \right.$$

7.5.23.9 [1353] Problem 9

problem number 1353

Added April 13, 2019.

Problem Chapter 5.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h_2(x)w + h_1(x)y^n + h0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y^n+h0[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\text{h2}(K[3])}{f(K[3])} dK[3]\right) \left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\text{h2}(K[3])}{f(K[3])} dK[3]\right) \left(\text{h1}(K[4]) \left(\left(\exp\left(-\int_1^x \frac{\text{g1}(K \cdots}{f(K \cdots}\right.\right.\right.\right.}{\cdots}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y^n+h0(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(\left(\left(y^{-k+1} e^{(k-1)\left(\int \frac{g1(x)}{f(x)} dx\right)} + (k-1)\left(\int \frac{e^{(k-1)\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)} dx\right) + (-k+1)\left(\int \frac{e^{(k-1)\left(\int \frac{g1}{f(}\right.}}{f}\right.\right.\right.\right.}{f(\_g)}\right.$$

7.5.23.10 [1354] Problem 10

problem number 1354

Added April 13, 2019.

Problem Chapter 5.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h_2(x)w + h_1(x)e^{\beta y} + h0(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]+g0[x]*Exp[lambda*y])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\frac{1}{\int \frac{\lambda\, e^{\int \frac{\lambda g1(x)}{f(x)} dx} g0(x)}{f(x)} dx + \int -\frac{\lambda\, e^{\int \frac{\lambda g1(\_g)}{f(\_g)} d\_g} g0(\_g)}{f(\_g)} d\_g + e^{-\left(y+\int -\frac{g1(x)}{f(x)} dx\right)\lambda}} \right)^{\frac{\beta}{\lambda}} e^{\beta\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right) - \left(\int \frac{h2(\_g)}{f(\_g)} d\_g\right)}}{f(\_g)} \right.$$

7.5.23.11 [1355] Problem 11

problem number 1355

Added April 13, 2019.

Problem Chapter 5.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x) y^k w_x + f_2(x) w_y = g(x) w + h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f1[x]*y^k*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{g(K[2]) \left(\left(y^{k+1} - (k+1) \int_1^x \frac{\mathrm{f2}(K[1])}{\mathrm{f1}(K[1])} dK[1] + (k+1) \int_1^{K[2]} \frac{\mathrm{f2}(K[1])}{\mathrm{f1}(K[1])} dK[1]\right)^{\frac{1}{k+1}}\right)^{-k}}{\mathrm{f1}(K[2])} d\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f1(x)*y^k*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\left(y^{k+1} + \int \frac{(-k-1)f2(x)}{f1(x)}dx + \int \frac{(k+1)f2(\_f)}{f1(\_f)}d\_f \right)^{\frac{1}{k+1}} \right)^{-k} e^{-\left(\int \frac{\left(\left(y^{k+1} + \int \frac{(-k-1)f2(x)}{f1(x)}dx + \int \frac{(k+1)f2(\_f)}{f1(\_f)}d\_ \right.\right.}{f1(\_f)} \right.}}{f1(\_f)} \right.$$

7.5.23.12 [1356] Problem 12

problem number 1356

Added April 13, 2019.

Problem Chapter 5.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)e^{\lambda y}w_x + f_2(x)w_y = g(x)w + h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f1[x]*Exp[lambda*y]*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{g(K[2])}{\text{f1}(K[2])\left(-\lambda \int_1^x \frac{\text{f2}(K[1])}{\text{f1}(K[1])}dK[1] + e^{\lambda y} + \lambda \int_1^{K[2]} \frac{\text{f2}(K[1])}{\text{f1}(K[1])}dK[1]\right)}dK[2]\right)\left(\int_1^x \frac{\exp\left(\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f1(x)*exp(lambda*y)*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\mathrm{e}^{-\frac{\int\frac{g(\_f)}{\left(\int\frac{f2(\_f)}{f1(\_f)}d\_f+\frac{-\lambda\left(\int\frac{f2(x)}{f1(x)}dx\right)+\mathrm{e}^{\lambda y}}{\lambda}\right)f1(\_f)}d\_f}{\lambda}}h(\_f)}{\left(\lambda\left(\int\frac{f2(\_f)}{f1(\_f)}d\_f\right)-\lambda\left(\int\frac{f2(x)}{f1(x)}dx\right)+\mathrm{e}^{\lambda y}\right)f1(\_f)}d\_f+\_F1\left(\frac{-\lambda\left(\int\frac{f2(x)}{f1(x)}dx\right)+\mathrm{e}^{\lambda y}}{\lambda}\right)\right)\mathrm{e}$$

7.5.24 8.2

Local contents

7.5.24.1 [1357] Problem 1

problem number 1357

Added April 13, 2019.

Problem Chapter 5.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+f(x)g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+f[x]*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}f(K[1])g(K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+f(x)*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{\mathrm{e}^{-\frac{cx}{a}}f(x)\,g(x)}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.24.2 [1358] Problem 2

problem number 1358

Added April 13, 2019.

Problem Chapter 5.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+xf(x)+yg(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x*f[x]+y*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}(af(K[1])K[1]+g(K[1])(-bx+ay+bK[1]))}{a^2}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x*f(x)+y*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{(\_aaf(\_a)+(ay-(-\_a+x)b)g(\_a))\,\mathrm{e}^{-\frac{\_ac}{a}}}{a^2}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

7.5.24.3 [1359] Problem 3

problem number 1359

Added April 13, 2019.

Problem Chapter 5.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=f(x)w+g(x)h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x]*h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])}{a}dK[1]\right)g(K[2])h(K[2])}{a}dK[2]+c_1\left(y-\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{\mathrm{e}^{-\frac{\int f(x)dx}{a}}g(x)\,h(x)}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{f(x)}{a}dx}$$

7.5.24.4 [1360] Problem 4

problem number 1360

Added April 13, 2019.

Problem Chapter 5.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(f(x)+g(y))w+p(x)+q(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x]+g[y])*w[x,y]+p[x]+q[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (f(x)+g(y))*w(x,y)+p(x)+q(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(p(\_b)+q\left(\frac{ay-(-\_b+x)b}{a}\right)\right)\mathrm{e}^{-\frac{\int\left(f(\_b)+g\left(\frac{ay-(-\_b+x)b}{a}\right)\right)d\_b}{a}}}{a}d\_b+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int^x \frac{f(\_a)+g\left(\right.}{}}$$

7.5.24.5 [1361] Problem 5

problem number 1361

Added April 13, 2019.

Problem Chapter 5.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cw+f(x)g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+f[x]*g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{f(K[1])g\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)K[1]^{-\frac{a+c}{a}}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\_a^{\frac{-a-c}{a}}f(\_a)\,g\left(y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{a}d\_a+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

7.5.24.6 [1362] Problem 6

problem number 1362

Added April 13, 2019.

Problem Chapter 5.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x+f_2(y)w_y=aw+g_1(x)+g_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x, y], x] + f2[y]*D[w[x, y], y] == a*w[x,y]+g1[x]+g2[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y),x)+ f2(y)*diff(w(x,y),y) = a*w(x,y)+g1(x)+g2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{\left(g1\left(\_f\right)+g2\left(\mathrm{RootOf}\left(\int\frac{1}{f1\left(\_f\right)}d\_f-\left(\int\frac{1}{f1\left(x\right)}dx\right)+\int\frac{1}{f2\left(y\right)}dy-\left(\int^{\_Z}\frac{1}{f2\left(\_a\right)}d\_a\right)\right)\right)\right)\mathrm{e}^{-}}{f1\left(\_f\right)}\right.$$

7.5.25 8.3

Local contents

7.5.25.1 [1363] Problem 1

problem number 1363

Added April 13, 2019.

Problem Chapter 5.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=xf(\frac{y}{x})w+g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x,y]+g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{xf\left(\frac{y}{x}\right)}\left(\int_1^x \frac{e^{-f\left(\frac{y}{x}\right)K[1]}g\left(K[1],\frac{yK[1]}{x}\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = x*f(y/x)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\mathrm{e}^{-\_af\left(\frac{y}{x}\right)}g\left(\_a,\frac{\_ay}{x}\right)}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{xf\left(\frac{y}{x}\right)}$$

7.5.25.2 [1364] Problem 2

problem number 1364

Added April 13, 2019.

Problem Chapter 5.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=f(x,y)w+g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x,y]*w[x,y]+g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)g\left(K[2\right.}{aK[2]}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x,y)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\mathrm{e}^{-\int \frac{f\left(\_b,y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_b a}d\_b}g\left(\_b,y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_ba}d\_b+\_F1\left(y\,x^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int^x \frac{f\left(\_a,y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_aa}d\_a}$$

7.5.25.3 [1365] Problem 3

problem number 1365

Added April 13, 2019.

Problem Chapter 5.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+g(x)w_y=h(x,y)w+F(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[2],y-\int_1^x \frac{g(K[1])}{f(K[1])}dK[1]+\int_1^{K[2]} \frac{g(K[1])}{f(K[1])}dK[1]\right)}{f(K[2])}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{h\left(\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{F\left(\_f,y+\int \frac{g(\_f)}{f(\_f)}d\_f-\left(\int \frac{g(x)}{f(x)}dx\right)\right)\mathrm{e}^{-\left(\int \frac{h\left(\_f,y+\int \frac{g(\_f)}{f(\_f)}d\_f-\left(\int \frac{g(x)}{f(x)}dx\right)\right)}{f(\_f)}d\_f\right)}}{f(\_f)}d\_f+\_F1\left(y-\left(\right.\right.\right.$$

7.5.25.4 [1366] Problem 4

problem number 1366

Added April 13, 2019.

Problem Chapter 5.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_0(x))w_y=h(x,y)w+F(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[3],\exp\left(\int_1^{K[3]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\left(\exp\left(-\int_1^x\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)y-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\text{g1}}{f(}\right.}{f}\right.\right.}{f(K[3])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{F\left(\_g,\left(y\,\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}+\int\frac{\mathrm{e}^{-\left(\int\frac{g1(\_g)}{f(\_g)}d\_g\right)}g0(\_g)}{f(\_g)}d\_g-\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int\frac{g1(\_g)}{f(\_g)}d\_\ldots}\right.}{f(\_g}\right.$$

7.5.25.5 [1367] Problem 5

problem number 1367

Added April 13, 2019.

Problem Chapter 5.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{F\left(\_g, \left(y^{-k+1} e^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (k-1)\left(\int \frac{e^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} g0(x)}{f(x)}dx\right) + (-k+1)\left(\int \frac{e^{(k-1)(}}{}\right.\right.\right.}{}\right.$$

7.5.25.6 [1368] Problem 6

problem number 1368

Added April 13, 2019.

Problem Chapter 5.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)e^{\lambda y})w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*Exp[lambda*y])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

7.6 chapter 6

Local contents

7.6.1 2.1

Local contents

7.6.1.1 [1369] Problem 1

problem number 1369

Added April 13, 2019.

Problem Chapter 6.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y) + c*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{za-cx}{a}\right)$$

<u>Hand solution</u>

Solve

$$aw_x + bw_y + cw_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c} = \frac{dw}{0}$$

From first two pair of equations, integrating gives $\frac{b}{a}x - y = C_1$ and from $\frac{dx}{a} = \frac{dz}{c}$ by Integrating gives $\frac{c}{a}x - z = C_2$. Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(\frac{b}{a}x - y, \frac{c}{a}x - z\right)$$

7.6.1.2 [1370] Problem 2

problem number 1370

Added April 13, 2019.

Problem Chapter 6.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + axw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ax^2}{2},\frac{1}{3}abx^3-bxy+z\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*x*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{a\,x^2}{2}+y,\frac{(a\,x^2-3y)\,bx}{3}+z\right)$$

Hand solution

Solve

$$w_x+axw_y+byw_z=0 \tag{1}$$

Using Lagrange-charpit

$$dx=\frac{dy}{ax}=\frac{dz}{by}=\frac{dw}{0}$$

From first two pair of equations, integrating gives

$$a\frac{x^2}{2}-y=C_1 \tag{1}$$

And from $dx=\frac{dz}{by}$ we obtain

$$bydx=dz$$

But from (1) $y=\frac{ax^2}{2}-C_1$ and the above becomes

$$b\left(\frac{ax^2}{2}-C_1\right)dx=dz$$

Now we can integrate and the result is

$$b\left(\frac{ax^3}{6}-C_1x\right)-z=C_2$$

Using (1) again the above becomes

$$
\begin{aligned}
b\left(\frac{ax^3}{6} - \left(a\frac{x^2}{2} - y\right)x\right) - z &= C_2 \\
b\left(\frac{ax^3}{6} - a\frac{x^3}{2} + yx\right) - z &= C_2 \\
-\frac{b}{3}ax^3 + byx - z &= C_2
\end{aligned}
$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$
\begin{aligned}
u(x,y,z) &= F\left(a\frac{x^2}{2} - y, -\frac{b}{3}ax^3 + byx - z\right) \\
&= F\left(y - a\frac{x^2}{2}, \frac{1}{3}abx^3 - byx + z\right)
\end{aligned}
$$

7.6.1.3 [1371] Problem 3

problem number 1371

Added April 13, 2019.

Problem Chapter 6.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$
aw_x + byw_y + czw_z = 0
$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$
\left\{\left\{w(x,y,z) \to c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y) + c*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

Hand solution

Solve

$$aw_x + byw_y + czw_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{by} = \frac{dz}{cz} = \frac{dw}{0}$$

From first two pair of equations, integrating gives

$$\begin{aligned} abx + C_1 &= \ln y \\ y &= C_1 e^{abx} \\ C_1 &= ye^{-abx} \end{aligned} \tag{1}$$

And from $\frac{dx}{a} = \frac{dz}{cz}$ we obtain

$$\begin{aligned} \frac{c}{a}x + C_2 &= \ln z \\ z &= C_2 e^{\frac{c}{a}x} \\ C_2 &= ze^{-\frac{c}{a}x} \end{aligned}$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(ye^{-abx}, ze^{-\frac{c}{a}x}\right)$$

7.6.1.4 [1372] Problem 4

problem number 1372

Added April 13, 2019.

Problem Chapter 6.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{z^2a-b\,y^2}{a},-\frac{-\sqrt{ab}\,x+\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)$$

Hand solution

Solve

$$w_x + azw_y + byw_z = 0$$

Using Lagrange-charpit

$$dx = \frac{dy}{az} = \frac{dz}{by} = \frac{dw}{0}$$

Starting with $\frac{dy}{az}=\frac{dz}{by}$ or $\frac{b}{a}ydy=zdz$ and integrating gives

$$\begin{aligned}\frac{b}{a}\frac{y^2}{2}&=\frac{z^2}{2}+C_1\\ \frac{b}{a}y^2&=z^2+C_1\\ C_1&=\frac{b}{a}y^2-z^2 \qquad (1)\end{aligned}$$

Now we can either consider $dx=\frac{dz}{by}$ or $dx=\frac{dy}{az}$. Both will give valid solutions. Lets try both to see.

case $dx=\frac{dz}{by}$

From (1) solving for y in terms of z gives

$$\sqrt{\frac{a}{b}C_1+\frac{a}{b}z^2}=y$$

Hence $dx=\frac{dz}{by}$ becomes $bdx=\frac{dz}{\sqrt{\frac{a}{b}C_1+\frac{a}{b}z^2}}$. Integrating gives

$$bx=\sqrt{\frac{b}{a}}\ln\left(z+\sqrt{z^2+C_1}\right)+C_2$$

Using (1) in the above gives

$$\begin{aligned}bx&=\sqrt{\frac{b}{a}}\ln\left(z+\sqrt{z^2+\left(\frac{b}{a}y^2-z^2\right)}\right)+C_2\\ &=\sqrt{\frac{b}{a}}\ln\left(z+\sqrt{\frac{b}{a}y^2}\right)+C_2\\ C_2&=bx-\sqrt{\frac{b}{a}}\ln\left(z+\sqrt{\frac{b}{a}y^2}\right)\end{aligned}$$

Since $dw=0$ then $w=C_3$. Where C_1,C_2,C_3 are constants. But $C_3=F(C_1,C_2)$ where F is arbitrary function. Hence

$$u(x,y,z)=F\left(\frac{b}{a}y^2-z^2,bx-\sqrt{\frac{b}{a}}\ln\left(\sqrt{\frac{b}{a}}y+z\right)\right) \qquad (2)$$

case $dx=\frac{dy}{az}$

From (1) we solve for z in terms of y. This gives $z^2=\frac{b}{a}y^2-C_1$ or $z=\sqrt{\frac{b}{a}y^2-C_1}$, taking the positive root only. Hence $dx=\frac{dy}{az}$ becomes $adx=\frac{dy}{\sqrt{\frac{b}{a}y^2-C_1}}$ or $adx=\sqrt{\frac{a}{b}}\frac{dy}{\sqrt{y^2-\frac{a}{b}C_1}}$

Integrating give

$$ax = \sqrt{\frac{a}{b}} \ln\left(y + \sqrt{y^2 - \frac{a}{b}C_1}\right) + C_2$$

Using (1) in the above gives

$$ax = \sqrt{\frac{a}{b}} \ln\left(y + \sqrt{y^2 - \frac{a}{b}\left(\frac{b}{a}y^2 - z^2\right)}\right) + C_2$$

$$ax = \sqrt{\frac{a}{b}} \ln\left(y + \frac{a}{b}z\right) + C_2$$

Hence

$$C_2 = ax - \sqrt{\frac{a}{b}} \ln\left(y + \sqrt{\frac{a}{b}}z\right)$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(\frac{b}{a}y^2 - z^2, ax - \sqrt{\frac{a}{b}} \ln\left(y + \frac{a}{b}z\right)\right) \tag{3}$$

Both (2,3) are valid solutions.

7.6.1.5 [1373] Problem 5

problem number 1373

Added April 13, 2019.

Problem Chapter 6.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + ayw_y + bzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*y*D[w[x, y,z], y] +b*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-a}, zx^{-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ a*y*diff(w(x,y,z),y) + b*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,x^{-a}, z\,x^{-b}\right)$$

7.6.1.6 [1374] Problem 6

problem number 1374

Added April 13, 2019.

Problem Chapter 6.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{\sqrt{a}z\,s}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{z^2a - b\,y^2}{a}, x\left(az + \sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)$$

7.6.1.7 [1375] Problem 7

problem number 1375

Added April 13, 2019.

Problem Chapter 6.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + (ax + by)w_y + (\alpha x + \beta y + \gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(alpha*x+beta*y+gamma*z)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{x^{-b}(ax + (b-1)y)}{b-1}, \frac{x^{-\gamma}(-a\beta x + \alpha x(b-\gamma) - (\gamma-1)(-bz + \beta y + \gamma z))}{(\gamma-1)(b-\gamma)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (alpha*x+beta*y+gamma*z)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(ax+(b-1)\,y)\,x^{-b}}{b-1}, \frac{-(-b^2z+(\beta y+z)\,b+(ax-y)\,\beta+(b-1)\,\gamma z)\,(-1+\gamma)\,x^{-\gamma}-(}{(-1+\gamma)\,(b-1)\,(b-\gamma)}\right.$$

7.6.1.8 [1376] Problem 8

problem number 1376

Added April 13, 2019.

Problem Chapter 6.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + (ay+bz)(bw_y - aw_z) = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + (a*y+b*z)*(b*D[w[x, y,z], y] -a*D[w[x,y,z],z])==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*b*x*diff(w(x,y,z),x)+ (a*y+b*z)*(b*diff(w(x,y,z),y) - a*diff(w(x,y,z),z))= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay+bz}{b}, x\,\mathrm{e}^{-\frac{ay}{ay+bz}}\right)$$

7.6.1.9 [1377] Problem 9

problem number 1377

Added April 13, 2019.

Problem Chapter 6.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + b(ay + bz)w_y + a(ay - bz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*b*x*diff(w(x,y,z),x)+ b*(a*y+b*z)*diff(w(x,y,z),y) + a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{\sqrt{-a^2y^2+2abyz+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2abyz+b^2z^2} + \left(\frac{ay}{\sqrt{-a^2y^2+2abyz+b^2z^2}} + \frac{bz}{\sqrt{-a^2y^2+2abyz+b^2z^2}}\right.}{\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}\right.\right.$$

7.6.1.10 [1378] Problem 10

problem number 1378

Added April 13, 2019.

Problem Chapter 6.2.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b^2cyw_x + a^2cxw_y - ab(ax + by)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b^2*c*y*D[w[x, y,z], x] + a^2*c*x*D[w[x, y,z], y] -a*b*(a*x+b*y)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{1}{2}\left(y^2-\frac{a^2x^2}{b^2}\right),\frac{ax+by+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  b^2*c*y*diff(w(x,y,z),x)+ a^2*c*x*diff(w(x,y,z),y) - a*b*(a*x+b*y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+b^2y^2}{b^2},\frac{ax+by+cz}{c}\right)$$

7.6.1.11 [1379] Problem 11

problem number 1379

Added April 13, 2019.

Problem Chapter 6.2.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$czw_x + (ax + by)w_y + (ax + by + cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  c*z*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(a*x+b*y+c*z)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  c*z*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (a*x+b*y+c*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.12 [1380] Problem 12

problem number 1380

Added April 13, 2019.

Problem Chapter 6.2.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b^2czw_x - a^2cxw_y + ab^2yw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b^2*c*z*D[w[x, y,z], x] - a^2*c*x*D[w[x, y,z], y] +a*b^2*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  b^2*c*z*diff(w(x,y,z),x)-a^2*c*x*diff(w(x,y,z),y) + a*b^2*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.13 [1381] Problem 13

problem number 1381

Added April 13, 2019.

Problem Chapter 6.2.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(x+a)w_x + (y+b)xw_y + (z+c)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x+a)*D[w[x, y,z], x] + (y+b)*D[w[x, y,z], y] +(z+c)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b+y}{a+x},\frac{c+z}{a+x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x+a)*diff(w(x,y,z),x)+(y+b)*diff(w(x,y,z),y) + (z+c)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{b+y}{a+x},\frac{c+z}{a+x}\right)$$

7.6.1.14 [1382] Problem 14

problem number 1382

Added April 13, 2019.

Problem Chapter 6.2.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$2bc(ax-by)w_x - ac(ax-by-cz)w_y - ab(ax-by-3cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*b*c*(a*x-b*y)*D[w[x, y,z], x] -a*c*(a*x-b*y-c*z)*D[w[x, y,z], y] - a*b*(a*x -b*y-3*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b*c*(a*x-b*y)*diff(w(x,y,z),x)-a*c*(a*x-b*y-c*z)*diff(w(x,y,z),y)- a*b*(a*x -b*y-3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.15 [1383] Problem 15

problem number 1383

Added April 13, 2019.

Problem Chapter 6.2.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(y-z)w_x + ac(z-x)w_y + ab(x-y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*(y-z)*D[w[x, y,z], x] +a*c*(z-x)*D[w[x, y,z], y] + a*b*(x -y)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  b*c*(y-z)*diff(w(x,y,z),x)+a*c*(z-x)*diff(w(x,y,z),y)+ a*b*(x -y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3c_4c_5\mathrm{e}^{c_1x}\mathrm{e}^{\frac{c_1by}{a}}\mathrm{e}^{\frac{c_1cz}{a}}\mathrm{e}^{\frac{c_2x^2}{2}}\mathrm{e}^{\frac{c_2b\,y^2}{2a}}\mathrm{e}^{\frac{c_2c\,z^2}{2a}}$$

7.6.1.16 [1384] Problem 16

problem number 1384

Added April 13, 2019.

Problem Chapter 6.2.1.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(by-2cz)w_x + ac(3cz-ax)w_y + ab(2ax-3by)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*(b*y-2*c*z)*D[w[x, y,z], x] +a*c*(3*c*z-a*x)*D[w[x, y,z], y] + a*b*(2*a*x -3*b*y)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  b*c*(b*y-2*c*z)*diff(w(x,y,z),x)+a*c*(3*c*z-a*x)*diff(w(x,y,z),y)+ a*b*(2*a*x-3*b*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3c_4c_5\mathrm{e}^{c_1x}\mathrm{e}^{\frac{c_2x^2}{2}}\mathrm{e}^{\frac{c_2b^2y^2}{2a^2}}\mathrm{e}^{\frac{2c_1by}{3a}}\mathrm{e}^{\frac{c_1cz}{3a}}\mathrm{e}^{\frac{c_2c^2z^2}{2a^2}}$$

7.6.1.17 [1385] Problem 17

problem number 1385

Added April 13, 2019.

Problem Chapter 6.2.1.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2bc(by - cz)w_x - ac(4ax - 3by - cz)w_y + 3ab(4ax - by - 3cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*b*c*(b*y-c*z)*D[w[x, y,z], x] -a*c*(4*a*x-3*b*y-c*z)*D[w[x, y,z], y] + 3*a*b*(4*a*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b*c*(b*y-c*z)*diff(w(x,y,z),x)-a*c*(4*a*x-3*b*y-c*z)*diff(w(x,y,z),y)+ 3*a*b*(4*a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.18 [1386] Problem 18

problem number 1386

Added April 13, 2019.

Problem Chapter 6.2.1.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(ax + y - z)w_x - (x + ay - z)w_y + (a - 1)(y - x)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x+y-z)*D[w[x, y,z], x] -(x+a*y-z)*D[w[x, y,z], y] + (a-1)*(y-x)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  (a*x+y-z)*diff(w(x,y,z),x)-(x+a*y-z)*diff(w(x,y,z),y)+ (a-1)*(y-x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.19 [1387] Problem 19

problem number 1387

Added April 13, 2019.

Problem Chapter 6.2.1.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2bc(3ax - 2by + cz)w_x - 2ac(2ax - 5by + 3cz)w_y + ab(2ax - 6by + 11cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   2*b*c*(3*a*x-2*b*y+c*z)*D[w[x, y,z], x] -2*a*c(2*a*x-5*b*y+3*c*z)*D[w[x, y,z], y] +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   2*b*c*(3*a*x-2*b*y+c*z)*diff(w(x,y,z),x)-2*a*c*(2*a*x-5*b*y+3*c*z)*diff(w(x,y,z),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.20 [1388] Problem 20

problem number 1388

Added April 13, 2019.

Problem Chapter 6.2.1.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(Ax + cy + bz)w_x + (cx + By + az)w_y + (bx + ay + Cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x+c*y+b*z)*D[w[x, y,z], x] +(c*x+B*y+a*z)*D[w[x, y,z], y] +(b*x+a*y+C1*z)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   (A*x+c*y+b*z)*diff(w(x,y,z),x)+(c*x+B*y+a*z)*diff(w(x,y,z),y)+ (b*x+a*y+C1*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.1.21 [1389] Problem 21

problem number 1389

Added April 13, 2019.

Problem Chapter 6.2.1.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(a_1x + b_1y + c_1z + d_1)w_x + (a_2x + b_2y + c_2z + d_2)w_y + (a_3x + b_3y + c_3z + d_3)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   (a1*x+b1*y+c1*z+d1)*D[w[x, y,z], x] +(a2*x+b2*y+c2*z+d2)*D[w[x, y,z], y] +(a3*x+b3*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   (a1*x+b1*y+c1*z+d1)*diff(w(x,y,z),x)+(a2*x+b2*y+c2*z+d2)*diff(w(x,y,z),y)+ (a3*x+b3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.2 2.2

Local contents

7.6.2.1 [1390] Problem 1

problem number 1390

Added April 14, 2019.

Problem Chapter 6.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1xy + b_1x^2 + c_1x)w_y + (a_2xy + b_2x^2 + c_2x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*y+b2*x^2+c2*x)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-2\text{a1b2}x^3-3\text{a1c2}x^2+6\text{a1}z+2\text{a2b1}x^3+3\text{a2c1}x^2-6\text{a2}y}{6\text{a1}},\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*y+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-\frac{\pi\sqrt{2}\,b1\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}+\sqrt{\pi}\left(a1^{\frac{3}{2}}y+(b1x+c1)\sqrt{a1}\right)\mathrm{e}^{-\frac{a1\,x^2}{2}}}{\sqrt{\pi}\,a1^{\frac{3}{2}}},-\frac{-\frac{3\sqrt{2}\left(\frac{\sqrt{\pi}\sqrt{\frac{a1}{\pi}}}{\sqrt{a1}}-1\right)a2\,b1\,\mathrm{erf}}{2}}{}\right.$$

7.6.2.2 [1391] Problem 2

problem number 1391

Added April 14, 2019.

Problem Chapter 6.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+b_1x^2+c_1x)w_y+(a_2xz+b_2x^2+c_2x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*z+b2*x^2+c2*x)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}-\frac{\sqrt{\frac{\pi}{2}}\text{b1}\text{erf}\left(\frac{\sqrt{\text{a1}}x}{\sqrt{2}}\right)}{\text{a1}^{3/2}},\frac{e^{-\frac{\text{a2}x^2}{2}}(\text{a2}z+\text{b2}x+\text{c2})}{\text{a2}}-\frac{\sqrt{\frac{\pi}{2}}\text{b2}\text{erf}\left(\frac{\sqrt{\right.}}{\right.}\right.}{\text{a2}^{3/2}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*z+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-\frac{\pi\sqrt{2}\,b1\,\text{erf}\left(\frac{\sqrt{2}\,\sqrt{a1}\,x}{2}\right)}{2}+\sqrt{\pi}\left(a1^{\frac{3}{2}}y+(b1x+c1)\sqrt{a1}\right)\mathrm{e}^{-\frac{a1\,x^2}{2}}}{\sqrt{\pi}\,a1^{\frac{3}{2}}},\frac{-\frac{\pi\sqrt{2}\,b2\,\text{erf}\left(\frac{\sqrt{2}\,\sqrt{a2}\,x}{2}\right)}{2}+\sqrt{\pi}\right.\right.$$

7.6.2.3 [1392] Problem 3

problem number 1392

Added April 14, 2019.

Problem Chapter 6.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+b_1x^2+c_1x)w_y+(a_2yz+b_2y^2+c_2y)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*y*z+b2*y^2+c2*y)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*y*z+b2*y^2+c2*y)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\frac{\pi\sqrt{2}\, b1\, \mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a1}\, x}{2}\right)}{2} + \sqrt{\pi}\left(a1^{\frac{3}{2}} y + (b1 x + c1)\sqrt{a1}\right) \mathrm{e}^{-\frac{a1\, x^2}{2}}}{\sqrt{\pi}\, a1^{\frac{3}{2}}}, z\, \mathrm{e}^{-\frac{\int^x \frac{2\left(-\frac{\sqrt{2}\,\pi\left(-\mathrm{erf}\left(\frac{\sqrt{2}\, \_b\sqrt{a1}}{2}\right.\right.}{}\right.}{}}{}}\right.$$

7.6.2.4 [1393] Problem 4

problem number 1393

Added April 14, 2019.

Problem Chapter 6.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1xy + b_1y^2)w_y + (a_2xz + b_2z^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x+b1*y^2)*D[w[x, y,z], y] +(a2*x*z+b2*z^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{2\left(\text{b1}xyJ_{\frac{1}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}J_{-\frac{2}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/}\right.\right.}{(2\text{b1}xy+1)J_{-\frac{1}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}J_{-\frac{4}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)-\sqrt{\text{a1}}\sqrt{\text{b1}}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x+b1*y^2)*diff(w(x,y,z),y)+ (a2*x*z+b2*z^2)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{b1\,y\,\mathrm{AiryBi}\left(-(a1\,b1)^{\frac{1}{3}}\,x\right)-(a1\,b1)^{\frac{1}{3}}\,\mathrm{AiryBi}\left(1,-(a1\,b1)^{\frac{1}{3}}\,x\right)}{-b1\,y\,\mathrm{AiryAi}\left(-(a1\,b1)^{\frac{1}{3}}\,x\right)+(a1\,b1)^{\frac{1}{3}}\,\mathrm{AiryAi}\left(1,-(a1\,b1)^{\frac{1}{3}}\,x\right)},\frac{\sqrt{\pi}\,b2\,z\,\mathrm{erf}\left(\frac{\sqrt{-2a2}}{2}\right.}{\sqrt{-2}}\right.$$

7.6.2.5 [1394] Problem 5

problem number 1394

Added April 14, 2019.

Problem Chapter 6.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + xzw_y - xyw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*D[w[x, y,z], x] +x*z*D[w[x, y,z], y] -x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\sin\left(\frac{x^2}{2a}\right)+z\cos\left(\frac{x^2}{2a}\right),y\cos\left(\frac{x^2}{2a}\right)-z\sin\left(\frac{x^2}{2a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*diff(w(x,y,z),x)+x*z*diff(w(x,y,z),y)- x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y^2+z^2,-2a\arctan\left(\frac{y}{z}\right)+x^2\right)$$

7.6.2.6 [1395] Problem 6

problem number 1395

Added April 14, 2019.

Problem Chapter 6.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxw_x+cyw_y+(ax^2+by^2)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*D[w[x, y,z], x] +c*y*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},-\frac{ax^2+by^2-2cz}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   c*x*diff(w(x,y,z),x)+c*y*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-a\,x^2 - b\,y^2 + 2cz}{2c}\right)$$

7.6.2.7 [1396] Problem 7

problem number 1396

Added April 14, 2019.

Problem Chapter 6.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$czw_x - a(2ax-b)yw_y + a(2ax-b)zw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*z*D[w[x, y,z], x] -a*(2*a*x-b)*y*D[w[x, y,z], y] +a*(2*a*x-b)*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(cyz, \frac{-a^2x^2 + abx + cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   c*z*diff(w(x,y,z),x)-a*(2*a*x-b)*diff(w(x,y,z),y)+a*(2*a*x-b)*z*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2 + axb + cz}{c}, y + \ln(cz)\right)$$

7.6.2.8 [1397] Problem 8

problem number 1397

Added April 14, 2019.

Problem Chapter 6.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$acx^2w_x - acxyw_y - b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*c*x^2*D[w[x, y,z], x] -a*c*x*y*D[w[x, y,z], y] -b^2*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(xy, z-\frac{b^2y^2}{3acx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*c*x^2*diff(w(x,y,z),x) -a*c*x*y*diff(w(x,y,z),y)-b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(xy, \frac{3zac\,x^3 - b^2x^2y^2}{3ac\,x^3}\right)$$

7.6.2.9 [1398] Problem 9

problem number 1398

Added April 14, 2019.

Problem Chapter 6.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2w_x + by^2w_y + cz^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*x^2*D[w[x, y,z], x] +b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b}{ax} - \frac{1}{y}, \frac{c}{ax} - \frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*x^2*diff(w(x,y,z),x) +b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax - by}{axy}, \frac{ax - cz}{axz}\right)$$

7.6.2.10 [1399] Problem 10

problem number 1399

Added April 14, 2019.

Problem Chapter 6.2.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abx^2w_x + cz^2w_y + 2abxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*b*x^2*D[w[x, y,z], x] +c*z^2*D[w[x, y,z], y] +2*a*b*x*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{z}{x^2},y-\frac{cz^2}{3abx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*b*x^2*diff(w(x,y,z),x) +c*z^2*diff(w(x,y,z),y)+2*a*b*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{z}{x^2},\frac{3axyb-c\,z^2}{3abx}\right)$$

7.6.2.11 [1400] Problem 11

problem number 1400

Added April 14, 2019.

Problem Chapter 6.2.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxyw_x+a^2cx^2w_y-by(2ax+cz)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*x*y*D[w[x, y,z], x] +a^2*c*x^2*D[w[x, y,z], y] -b*y*(2*a*x+c*z)*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-a^2x^2}{2b},\frac{x(ax+cz)}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=     b*c*x*y*diff(w(x,y,z),x) +a^2*c*x^2*diff(w(x,y,z),y)-b*y*(2*a*x+c*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+b\,y^2}{b}, \frac{(ax+cz)\,x}{c}\right)$$

7.6.2.12 [1401] Problem 12

problem number 1401

Added April 14, 2019.

Problem Chapter 6.2.2.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxyw_x + c^2yzw_y + b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*x*y*D[w[x, y,z], x] +c^2*y*z*D[w[x, y,z], y] +b^2*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow c_1\left(\frac{by+cz}{2bx}, \frac{x(by-cz)}{2b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y*diff(w(x,y,z),x) +c^2*y*z*diff(w(x,y,z),y)+b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b^2y^2+c^2z^2}{c^2}, x(by\,\mathrm{csgn}(b)+cz)^{-\mathrm{csgn}(b)}\right)$$

7.6.2.13 [1402] Problem 13

problem number 1402

Added April 14, 2019.

Problem Chapter 6.2.2.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xyw_x + y(y-a)w_y + z(y-a)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*y*D[w[x, y,z], x] +y*(y-a)*D[w[x, y,z], y] +z*(y-a)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y-a}{x},\frac{z}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*y*diff(w(x,y,z),x) +y*(y-a)*diff(w(x,y,z),y)+z*(y-a)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a+y}{x},\frac{z}{y}\right)$$

7.6.2.14 [1403] Problem 14

problem number 1403

Added April 14, 2019.

Problem Chapter 6.2.2.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$by^2w_x - axyw_y + cxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*y^2*D[w[x, y,z], x] -a*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ax^2+by^2}{2b},z\left(-by^2\right)^{\frac{c}{2a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*y^2*diff(w(x,y,z),x) -a*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\,x^2+b\,y^2}{b},z\left(-b\,y^2\right)^{\frac{c}{2a}}\right)$$

7.6.2.15 [1404] Problem 15

problem number 1404

Added April 14, 2019.

Problem Chapter 6.2.2.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x + 2axyw_y - (2ax+cz)zw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] +2*a*x*y*D[w[x, y,z], y] -(2*a*x+c*z)*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-cxyz,x\left(\frac{ax}{c}+z\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x) +2*a*x*y*diff(w(x,y,z),y)-(2*a*x+c*z)*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(ax+cz)\,x}{c},-cxyz\right)$$

7.6.2.16 [1405] Problem 16

problem number 1405

Added April 14, 2019.

Problem Chapter 6.2.2.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x+cyzw_y+abxyw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] +c*y*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{cz^2-abxy}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x) +c*y*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-axyb + c\,z^2}{c}\right)$$

7.6.2.17 [1406] Problem 17

problem number 1406

Added April 14, 2019.

Problem Chapter 6.2.2.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x - cyzw_y + (by^2 - ax)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(b*y^2-a*x)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(xy, \frac{2ax + by^2 + cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(b*y^2-a*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(xy, \frac{b\,x^2y^2 + z^2c\,x^2 + 2a\,x^3}{c\,x^2}\right)$$

7.6.2.18 [1407] Problem 18

problem number 1407

Added April 14, 2019.

Problem Chapter 6.2.2.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x - cyzw_y + (ax^2 + by^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(xy,\frac{-ax^2+by^2+cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(xy,\frac{-a\,x^2+b\,y^2+c\,z^2}{c}\right)$$

7.6.2.19 [1408] Problem 19

problem number 1408

Added April 14, 2019.

Problem Chapter 6.2.2.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xzw_x + yzw_y + (ax^2 + ay^2 + bz^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*z*D[w[x, y,z], x] +y*z*D[w[x, y,z], y] +(a*x^2+a*y^2+b*z^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{x^{-2b}(a(x^2+y^2)+(b-1)z^2)}{b-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*z*diff(w(x,y,z),x)+y*z*diff(w(x,y,z),y)+(a*x^2+a*y^2+b*z^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},\frac{((b-1)\,z^2+(x^2+y^2)\,a)\,x^{-2b}}{b-1}\right)$$

7.6.2.20 [1409] Problem 20

problem number 1409

Added April 14, 2019.

Problem Chapter 6.2.2.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$2cxzw_x + 2cyzw_y + (cz^2 - ax^2 - by^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   2*c*x*z*D[w[x, y,z], x] +2*c*y*z*D[w[x, y,z], y] +(c*z^2-a*x^2-b*y^2)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{ax^2+by^2+cz^2}{cx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := 2*c*x*z*diff(w(x,y,z),x)+2*c*y*z*diff(w(x,y,z),y)+(c*z^2-a*x^2-b*y^2)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{a\,x^2+b\,y^2+c\,z^2}{cx}\right)$$

7.6.2.21 [1410] Problem 21

problem number 1410

Added April 14, 2019.

Problem Chapter 6.2.2.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcyzw_x+acxzw_y+abxyw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*y*z*D[w[x, y,z], x] +a*c*x*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-ax^2}{2b},\frac{cz^2-ax^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*y*z*diff(w(x,y,z),x)+a*c*x*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,x^{2}+b\,y^{2}}{b}, \frac{-a\,x^{2}+c\,z^{2}}{c}\right)$$

7.6.2.22 [1411] Problem 22

problem number 1411

Added April 14, 2019.

Problem Chapter 6.2.2.22, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(x^2-a^2)w_x + c(bxy+acz)w_y + b(cxz+aby)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*(x^2-a^2)*D[w[x, y,z], x] +c*(b*x*y+a*c*z)*D[w[x, y,z], y] +b*(c*x*z + a*b*y)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{acz+bxy}{a^2b-bx^2}, \frac{a(aby+cxz)}{b\,(a^2-x^2)}\right)\right\}\right\}$$

Maple ✗

```
restart;
pde :=  b*c*(x^2-a^2)*diff(w(x,y,z),x)+c*(b*x*y+a*c*z)*diff(w(x,y,z),y)+b*(c*x*z + a*b*y)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.2.23 [1412] Problem 23

problem number 1412

Added April 14, 2019.

Problem Chapter 6.2.2.23, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bx(by+c)w_x + (b^2y^2 - acx)w_y + b^2yzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   b*x*(b*y+c)*D[w[x, y,z], x] + (b^2*y^2-a*c*x )*D[w[x, y,z], y] + b^2*y*z*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  b*x*(b*y+c)*diff(w(x,y,z),x)+(b^2*y^2-a*c*x )*diff(w(x,y,z),y)+b^2*y*z*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{by + c + (ax+by)\ln\left(\frac{-9ax+9c}{2by+2c}\right) + (-ax-by)\ln\left(-\frac{9(ax+by)(ax-c)}{(by+c)x}\right)}{3ax+3by}, z\,e^{-\frac{\left(\int^x \frac{9\_aa+2c\,e^{\mathrm{RootO}}}{\cdots}\right.}{\cdots}}\right.$$

7.6.2.24 [1413] Problem 24

problem number 1413

Added April 14, 2019.

Problem Chapter 6.2.2.24, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(by-cz)w_x + y(cz-ax)w_y + z(ax-by)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*(b*y -c*z)*D[w[x, y,z], x] + y*(c*z-a*x)*D[w[x, y,z], y] + z*(a*x - b*y)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cxyz}{b},\frac{ax+by+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   x*(b*y -c*z)*diff(w(x,y,z),x)+ y*(c*z-a*x)*diff(w(x,y,z),y)+z*(a*x - b*y)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z)=\frac{c_4c_5x^{c_2}y^{c_2}z^{c_2}\mathrm{e}^{c_2}\mathrm{e}^{-c_1x}\mathrm{e}^{-\frac{c_1by}{a}}\mathrm{e}^{-\frac{c_1cz}{a}}}{c_3}$$

7.6.2.25 [1414] Problem 25

problem number 1414

Added April 14, 2019.

Problem Chapter 6.2.2.25, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a(y+\beta)(z+\gamma)w_x - b(x+\alpha)(z+\gamma)w_y - c(x+\alpha)(y+\beta)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*(y+beta)*(z+gamma)*D[w[x, y,z], x] -b*(x+alpha)*(z+gamma)*D[w[x, y,z], y] - c*(x+a
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ay(2\beta+y)+2\alpha bx+bx^2}{2a},\frac{az(2\gamma+z)+2\alpha cx+cx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*(y+beta)*(z+gamma)*diff(w(x,y,z),x)-b*(x+alpha)*(z+gamma)*diff(w(x,y,z),y)- c*(x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z)=c_2c_4c_5\mathrm{e}^{c_1\beta y}\mathrm{e}^{c_3\alpha x}\mathrm{e}^{\frac{c_3\gamma az}{c}}\mathrm{e}^{\frac{c_1y^2}{2}}\mathrm{e}^{\frac{c_3x^2}{2}}\mathrm{e}^{-\frac{c_1bz^2}{2c}}\mathrm{e}^{\frac{c_3az^2}{2c}}\mathrm{e}^{-\frac{c_1\gamma bz}{c}}$$

7.6.2.26 [1415] Problem 26

problem number 1415

Added April 14, 2019.

Problem Chapter 6.2.2.26, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(acxz+b^2y^2)w_x + ac(bcyz-2a^2x^2)w_y - ab(2abxy+c^2z^2)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   b*c*(a*c*x*z + b^2*y^2)*D[w[x, y,z], x] +a*c*(b*c*y*z-2*a^2*x^2)*D[w[x, y,z], y] - a
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=   b*c*(a*c*x*z + b^2*y^2)*diff(w(x,y,z),x)+a*c*(b*c*y*z-2*a^2*x^2)*diff(w(x,y,z),y)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.2.27 [1416] Problem 27

problem number 1416

Added April 14, 2019.

Problem Chapter 6.2.2.27, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a(y^2 + z^2)w_x + x(bz - ay)w_y - x(by + az)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   a*(y^2+z^2)*D[w[x, y,z], x] +x*(b*z-a*y)*D[w[x, y,z], y] -x*(b*y + a*z)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*(y^2+z^2)*diff(w(x,y,z),x)+x*(b*z-a*y)*diff(w(x,y,z),y)-x*(b*y + a*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1\_F5\left(-\frac{-2a\arctan\left(\frac{z}{y}\right) + b\ln\left(y^2+z^2\right)}{2b}\right) e^{\frac{x^2\_c_1}{2}} e^{-a\_c_1\left(\int^y -\frac{}{a\cos\left(2\,\mathrm{RootOf}\left(2\_Za - 2a\arctan\left(\frac{z}{y}\right) - b\ln\left(\frac{}{\cos}\right.\right.\right.}\right.}$$

7.6.2.28 [1417] Problem 28

problem number 1417

Added April 14, 2019.

Problem Chapter 6.2.2.28, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b(by + cz)^2 w_x - ax(by + 2cz)w_y + abxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b*(b*y + c*z)^2*D[w[x, y,z], x] - a*x*(b*y + 2*c*z)*D[w[x, y,z], y] +a*b*x*z*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(\frac{2(ax^2+c^2z^2)}{b},\log(z(by+cz))\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{2(ax^2+c^2z^2)}{b},\log(z(cz-by))\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{2(ax^2+(by-cz)^2)}{b},\log(z(cz-by))\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{2(ax^2+(by+cz)^2)}{b},\log(z(by+cz))\right)\right\}$$

Maple ✓

```
restart;
pde :=  b*(b*y + c*z)^2*diff(w(x,y,z),x)- a*x*(b*y + 2*c*z)*diff(w(x,y,z),y)+a*b*x*z*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1\_F5\left(\frac{(by+cz)\,z}{b}\right)\mathrm{e}^{\frac{x^2\_c_1}{2}}\,\mathrm{e}^{\frac{b^2y^2\_c_1}{2a}}\,\mathrm{e}^{\frac{bcyz\_c_1}{2a}}$$

7.6.2.29 [1418] Problem 29

problem number 1418

Added April 14, 2019.

Problem Chapter 6.2.2.29, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(f_0 x - f_1)w_x + (f_0 y - f_2)w_y + (f_0 z - f_3)w_z = 0$$

Where

$$f_n = a_n + b_n x + c_n y + d_n z$$

Mathematica ✗

```
ClearAll["Global`*"];
f[n_]:= a[n] + b[n]*x + c[n]*y+ d[n]*z;
pde = (f[0]*x - f[1])*D[w[x, y,z], x] +(f[0]*y-f[2])*D[w[x, y,z], y] +(f[0]*z -f[3])*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
f:= n ->  a[n] + b[n]*x + c[n]*y+ d[n]*z;
pde :=  (f(0)*x - f(1))*diff(w(x,y,z),x)+(f(0)*y-f(2))*diff(w(x,y,z),y)+(f(0)*z -f(3))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.3 2.3

Local contents

7.6.3.1 [1419] Problem 1

problem number 1419

Added April 15, 2019.

Problem Chapter 6.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2b^2xzw_x + by(b^2z^2 + 1)w_y + axy(bz + 1)^2w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = 2*b^2*x*z*D[w[x, y,z], x] + b*y*(b^2*z^2 +1)*D[w[x, y,z], y] + a*x*y*(b*z +1)^2*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b^2*x*z*diff(w(x,y,z),x)+b*y*(b^2*z^2 +1)*diff(w(x,y,z),y)+a*x*y*(b*z +1)^2*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.3.2 [1420] Problem 2

problem number 1420

Added April 15, 2019.

Problem Chapter 6.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$bcxy^2 w_x + 2bcy^3 w_y + 2(cyz - ax^2)^2 w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = b*c*x*y^2*D[w[x, y,z], x] + 2*b*c*y^3*D[w[x, y,z], y] + 2*(c*y*z-a*x^2)^2*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x^2},\frac{x^4(\log(x)\left(2cyz-2ax^2\right)+by)}{bcy^2\left(ax^2-cyz\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y^2*diff(w(x,y,z),x)+2*b*c*y^3*diff(w(x,y,z),y)+2*(c*y*z-a*x^2)^2*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x^2},\frac{by+\left(-2a\,x^2+2cyz\right)\ln\left(x\right)}{2a\,x^2-2cyz}\right)$$

7.6.3.3 [1421] Problem 3

problem number 1421

Added April 15, 2019.

Problem Chapter 6.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$bc^2y^2zw_x + ac^2xz^2w_y - abxy^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b*c*x*y^2*D[w[x, y,z], x] + a*c^2*x*z^2*D[w[x, y,z], y] - a*b*x*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ax}{c}+z,\frac{by^3+c^2z^3}{3b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y^2*diff(w(x,y,z),x)+a*c^2*x*z^2*diff(w(x,y,z),y)- a*b*x*y^2*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax+cz}{c}, \frac{-a^3x^3-3a^2c\,x^2z-3a\,c^2x\,z^2+bc\,y^3}{bc}\right)$$

7.6.3.4 [1422] Problem 4

problem number 1422

Added April 15, 2019.

Problem Chapter 6.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(by^2-cz^2)w_x + y(cz^2-ax^2)w_y + z(ax^2-by^2)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(b*y^2-c*z^2)*D[w[x, y,z], x] + y*(c*z^2-a*x^2)*D[w[x, y,z], y] + z*(a*x^2-b*y^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(b*y^2-c*z^2)*diff(w(x,y,z),x)+ y*(c*z^2-a*x^2)*diff(w(x,y,z),y) + z*(a*x^2-b*y^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3c_4c_5x^{\frac{c_2}{2}}y^{\frac{c_2}{2}}z^{\frac{c_2}{2}}\mathrm{e}^{\frac{c_2}{4}}\mathrm{e}^{-\frac{c_1x^2}{4}}\mathrm{e}^{-\frac{c_1by^2}{4a}}\mathrm{e}^{-\frac{c_1cz^2}{4a}}$$

7.6.3.5 [1423] Problem 5

problem number 1423

Added April 15, 2019.

Problem Chapter 6.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$by(3ax^2+by^2+cz^2)w_x - 2ax(ax^2+cz^2)w_y + 2abxyzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*y*(3*a*x^2+ b*y^2+c*z^2)*D[w[x, y,z], x] - 2*a*x*(a*x^2+c*z^2)*D[w[x, y,z], y] + 2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := b*y*(3*a*x^2+ b*y^2+c*z^2)*diff(w(x,y,z),x)- 2*a*x*(a*x^2+c*z^2)*diff(w(x,y,z),y) + 2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.3.6 [1424] Problem 6

problem number 1424

Added April 15, 2019.

Problem Chapter 6.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b(a(a^2x^2 + b^2y^2 - 1)x + by)w_x + a(b(a^2x^2 + b^2y^2 - 1)y - ax)w_y + 2abzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*D[w[x, y,z], x] +a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*diff(w(x,y,z),x)+a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{(a^2x^2 + b^2y^2)\, e^{-2\arctan\left(\frac{by}{ax}\right)}}{a^2x^2 + b^2y^2 - 1}, z\, e^{-2\left(\int^x \frac{}{\left(\_a^2a^2 - \left(\cos^2\left(\text{RootOf}\left(2\_Z - \ln\left(-\frac{\left(a^2x^2+b^2y^2-1\right)\_a^2a^2 e^{2\arctan\left(\frac{by}{ax}\right)}}{\left(a^2x^2+b^2y^2\right)\left(-\_a^2a^2+\cos^2(\_Z)\right)}\right.\right.\right.\right.\right.}\right.}\right.$$

7.6.3.7 [1425] Problem 7

problem number 1425

Added April 15, 2019.

Problem Chapter 6.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$x(b^3y^3 - 2a^3x^3)w_x + y(2b^3y^3 - a^3x^3)w_y + 9z(a^3x^3 - b^3y^3)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(b^3*y^3 - 2*a^3*x^3)*D[w[x, y,z], x] +y*(2*b^3*y^3 -a^3*x^3)*D[w[x, y,z], y] +9*z*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := x*(b^3*y^3 - 2*a^3*x^3)*diff(w(x,y,z),x)+y*(2*b^3*y^3 -a^3*x^3)*diff(w(x,y,z),y) + 9*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.6.3.8 [1426] Problem 8

problem number 1426

Added April 15, 2019.

Problem Chapter 6.2.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2(abxy - c^2z^2)w_x + axy(abxy - c^2z^2)w_y + byz(bcyz + 2a^2x^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*(a*b*x*y-c^2*z^2)*D[w[x, y,z], x] +a*x*y*(a*b*x*y-c^2*z^2)*D[w[x, y,z], y] +b*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{\log\left(\frac{xz}{a^2bx^2y+ac^2xz^2+b^2cy^2z}\right)}{a}\right)\right\}\right\}$$

Maple ✗

```
restart;
pde := a*x^2*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),x)+a*x*y*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),y) + b
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.3.9 [1427] Problem 9

problem number 1427

Added April 15, 2019.

Problem Chapter 6.2.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(cz^4-by^4)w_x+y(ax^4-2cz^4)w_y+z(2by^4-ax^4)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(c*z^4 - b*y^4)*D[w[x, y,z], x] +y*(a*x^4-2*c*z^4)*D[w[x, y,z], y] +z*(2*b*y^4-a*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*(c*z^4 - b*y^4)*diff(w(x,y,z),x)+y*(a*x^4-2*c*z^4)*diff(w(x,y,z),y) + z*(2*b*y^4-a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3c_4c_5x^{\frac{c_2}{4}}y^{\frac{c_2}{8}}z^{\frac{c_2}{8}}\mathrm{e}^{\frac{c_2}{16}}\mathrm{e}^{-\frac{c_1x^4}{16}}\mathrm{e}^{-\frac{c_1by^4}{16a}}\mathrm{e}^{-\frac{c_1cz^4}{16a}}$$

7.6.3.10 [1428] Problem 10

problem number 1428

Added April 15, 2019.

Problem Chapter 6.2.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + a\sqrt{x^2+y^2}w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},z-a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + a*sqrt(x^2+y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},-\sqrt{x^2+y^2}\,a+z\right)$$

7.6.3.11 [1429] Problem 11

problem number 1429

Added April 15, 2019.

Problem Chapter 6.2.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + (z - a\sqrt{x^2+y^2+z^2})w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{1}{2}\log\left(\frac{x^{2a}(y^2+2z^2)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}\right)\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)+2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}$$
$$\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{1}{2}\log\left(\frac{x^{2a}(y^2+2z^2)+2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}\right)\right)\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + (z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)$$

7.6.3.12 [1430] Problem 12

problem number 1430

Added April 15, 2019.

Problem Chapter 6.2.3.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$z\sqrt{y^2+z^2}w_x + az\sqrt{x^2+z^2}w_y - (x\sqrt{y^2+z^2} + ay\sqrt{x^2+z^2})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  z*Sqrt[y^2+z^2]*D[w[x, y,z], x] +a*z*Sqrt[x^2+z^2]*D[w[x, y,z], y] -(x*Sqrt[y^2+z^2]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := z*sqrt(y^2+z^2)*diff(w(x,y,z),x)+a*z*sqrt(x^2+z^2)*diff(w(x,y,z),y) -(x*sqrt(y^2+z^2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.3.13 [1431] Problem 13

problem number 1431

Added April 15, 2019.

Problem Chapter 6.2.3.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(y-z)\sqrt{f(x)}w_x + (z-x)\sqrt{f(y)}w_y + (x-y)\sqrt{f(z)}w_z = 0$$

Where

$$f(t) = a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0$$

Mathematica ✗

```
ClearAll["Global`*"];
f[t_]:= a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
pde =  (y-z)*Sqrt[f[x]]*D[w[x, y,z], x] +(z-x)*Sqrt[f[y]]*D[w[x, y,z], y] +(x-y)*Sqrt[f[z]]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
f := t-> a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
pde :=  (y-z)*sqrt(f(x))*diff(w(x,y,z),x)+(z-x)*sqrt(f(y))*diff(w(x,y,z),y)+(x-y)*sqrt(f(z))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.6.4 2.4

Local contents

7.6.4.1 [1432] Problem 1

problem number 1432

Added May 18, 2019.

Problem Chapter 6.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +a*x^n*y^m*D[w[x, y,z], y] +b*x^nu *y^mu* z^lambda *D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{ax^{n+1}}{n+1}-(m-1)^{\frac{1}{m-1}}y\left(\frac{(m-1)^{\frac{1}{1-m}}}{y}\right)^m,-\frac{b(m-1)^{\frac{\mu}{1-m}}x^{\nu+1}\left((m-1)^{\frac{1}{m-1}}y\left(\frac{(m-1)}{y}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(m-1)\,a\,x^{n+1}+(n+1)\,y^{-m+1}}{n+1},(\lambda-1)\,b\left(\int^{x}\_a^{\nu}\left(\left(\frac{(-\_a^{n+1}+x^{n+1})\,(m-1)\,a+(}{n+1}\right.\right.\right.\right.$$

7.6.4.2 [1433] Problem 2

problem number 1433

Added May 18, 2019.

Problem Chapter 6.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}y+b_2x^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m1)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},-\int_1^x\frac{\left((-1)^{-\frac{\mathrm{n2}}{\mathrm{n1}}}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-\left(a1\,x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)(n1+1)^2\,b1\left(\frac{a1\,x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\right.}{}\right.$$

7.6.4.3 [1434] Problem 3

problem number 1434

Added May 18, 2019.

Problem Chapter 6.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2x^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m1)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},\mathrm{b2}(\mathrm{n2}+1)^{\frac{\mathrm{m1}-\mathrm{n2}}{\mathrm{n2}+1}}\mathrm{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(a1\,x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)b1\left(\frac{a1\,x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\right.}{}\right.$$

7.6.4.4 [1435] Problem 4

problem number 1435

Added May 18, 2019.

Problem Chapter 6.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2y^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*y^m1)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\text{b1}(\text{n1}+1)^{\frac{\text{m1}-\text{n1}}{\text{n1}+1}}\text{a1}^{-\frac{\text{m1}+1}{\text{n1}+1}}\text{Gamma}\left(\frac{\text{m1}+1}{\text{n1}+1},\frac{\text{a1}x^{\text{n1}+1}}{\text{n1}+1}\right)+ye^{-\frac{\text{a1}x^{\text{n1}+1}}{\text{n1}+1}},ze^{-\frac{\text{a2}x^{\text{n2}+1}}{\text{n2}+1}}-\int_1^x\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*y^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(a1\,x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)b1\left(\frac{a1\,x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\text{WhittakerM}\left(\right.}{}\right.$$

7.6.4.5 [1436] Problem 5

problem number 1436

Added May 18, 2019.

Problem Chapter 6.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1}y^k1)w_y+(a_2x^{n_2}z+b_2x^{m_2}z^{k_2})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1*y^k1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2*z^k1)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\text{b1}(-1)^{\frac{\text{n1}-\text{m1}}{\text{n1}+1}}(\text{n1}+1)^{\frac{\text{m1}-\text{n1}}{\text{n1}+1}}\text{a1}^{-\frac{\text{m1}+1}{\text{n1}+1}}(\text{k1}-1)^{\frac{\text{n1}-\text{m1}}{\text{n1}+1}}\text{Gamma}\left(\frac{\text{m1}+1}{\text{n1}+1},-\frac{\text{a1}(\text{k1}-1)x^{\text{n1}+1}}{\text{n1}+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1*y^k1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2*z^k1)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(-(k1-1)\,a1\,x^{m1+1}+(m1+n1+2)\,x^{m1-n1})\,(n1+1)^2\,b1\left(-\frac{(k1-1)a1\,x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1}{2n1+2}}}{}\right.$$

7.6.4.6 [1437] Problem 6

problem number 1437

Added May 18, 2019.

Problem Chapter 6.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^n w_x + by^m w_y + cz^l w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x^n*D[w[x, y,z], x] +b*y^m*D[w[x, y,z], y] +c*z^L*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{\left(\frac{1}{y}\right)^{m-1}}{m-1},\frac{cx^{1-n}}{a(n-1)}-\frac{\left(\frac{1}{z}\right)^{L-1}}{L-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y,z),x)+b*y^m*diff(w(x,y,z),y)+c*z^L*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(n-1)\,a\,y^{-m+1}-(m-1)\,b\,x^{-n+1}}{(n-1)\,a},\frac{(n-1)\,a\,z^{-L+1}-(L-1)\,c\,x^{-n+1}}{(n-1)\,a}\right)$$

7.6.4.7 [1438] Problem 7

problem number 1438

Added May 18, 2019.

Problem Chapter 6.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ay^m w_x + bx^n w_y + cz^l w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y^m*D[w[x, y,z], x] +b*x^n*D[w[x, y,z], y] +c*z^L*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y^m*diff(w(x,y,z),x)+b*x^n*diff(w(x,y,z),y)+c*z^L*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(n+1)\,a\,y^{m+1}-(m+1)\,b\,x^{n+1}}{(n+1)\,a}, \frac{a\,z^{-L+1}+(L-1)\,c\left(\int^{x}\left(\left(\frac{(n+1)a\,y^{m+1}+(m+1)b\_a^{n+1}-(n}{(n+1)a}\right.\right.\right.}{a}\right.$$

7.6.4.8 [1439] Problem 8

problem number 1439

Added May 18, 2019.

Problem Chapter 6.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(y^n-z^n)w_x+y(z^n-x^n)w_y+z(x^n-y^n)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(y^n-z^n)*D[w[x, y,z], x] +y*(z^n-x^n)*D[w[x, y,z], y] +z*(x^n-y^n)*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*(y^n-z^n)*diff(w(x,y,z),x)+y*(z^n-x^n)*diff(w(x,y,z),y)+z*(x^n-y^n)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_4c_5x^{\frac{c_1}{n}}\,(y^n)^{\frac{c_1}{n^2}}\,(z^n)^{\frac{c_1}{n^2}}\,\mathrm{e}^{\frac{c_1}{n^2}}\,\mathrm{e}^{-\frac{c_2}{n}}\,\mathrm{e}^{-\frac{c_3x^n}{n^2}}\,\mathrm{e}^{-\frac{c_3y^n}{n^2}}\,\mathrm{e}^{-\frac{c_3z^n}{n^2}}$$

7.6.5 3.1

Local contents

7.6.5.1 [1440] Problem 1

problem number 1440

Added May 18, 2019.

Problem Chapter 6.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + be^{\alpha x}w_y + ce^{\beta y}w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[beta*y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{be^{\alpha x}}{a\alpha},z-\frac{c\text{Ei}\left(\frac{b\beta e^{\alpha x}}{a\alpha}\right)e^{\beta\left(y-\frac{be^{\alpha x}}{a\alpha}\right)}}{a\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(beta*y)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha y - b\,\mathrm{e}^{\alpha x}}{a\alpha}, \frac{a\alpha z + c\,\mathrm{expIntegral}\left(1, -\frac{b\beta\,\mathrm{e}^{\alpha x}}{a\alpha}\right)\mathrm{e}^{\frac{(a\alpha y - b\,\mathrm{e}^{\alpha x})\beta}{a\alpha}}}{a\alpha}\right)$$

7.6.5.2 [1441] Problem 2

problem number 1441

Added May 18, 2019.

Problem Chapter 6.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} - \frac{e^{-\gamma z}}{\gamma}, y - \frac{be^{\alpha x}}{a\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha y - b\,\mathrm{e}^{\alpha x}}{a\alpha}, \frac{-a\,\mathrm{e}^{-\gamma z} - \gamma c x}{\gamma c}\right)$$

7.6.5.3 [1442] Problem 3

problem number 1442

Added May 18, 2019.

Problem Chapter 6.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + be^{\beta y}w_y + ce^{\gamma z}w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{bx}{a}-\frac{e^{-\beta y}}{\beta},-\frac{cx}{a}-\frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x - a\,\mathrm{e}^{-\beta y}}{b\beta}, \frac{-a\,\mathrm{e}^{-\gamma z} - \gamma c x}{\gamma c}\right)$$

7.6.5.4 [1443] Problem 4

problem number 1443

Added May 18, 2019.

Problem Chapter 6.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (A_1 e^{\alpha_1 x} + B_1 e^{\nu_1 x + \lambda y}) w_y + (A_2 e^{\alpha_2 x} + B_2 e^{\nu_2 x + \beta y}) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(A1*Exp[alpha1*x]+B1*Exp[nu1*x+lambda*y])*D[w[x, y,z], y] +(A2*Exp[al
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(A1*exp(alpha1*x)+B1*exp(nu1*x+lambda*y))*diff(w(x,y,z),y)+(A2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-B1\,\lambda\left(\int \mathrm{e}^{\frac{A1\,\lambda\,\mathrm{e}^{\alpha 1 x}}{\alpha 1}+\nu 1 x}dx\right) - \mathrm{e}^{\frac{\left(A1\,\mathrm{e}^{\alpha 1 x}-\alpha 1 y\right)\lambda}{\alpha 1}}}{\lambda}, z - \left(\int^{x} \left(B2\left(-B1\,\lambda\left(\int \mathrm{e}^{\frac{A1\,\lambda\,\mathrm{e}^{-b\alpha 1}+\_b\alpha 1\nu 1}{\alpha 1}}\right.\right.\right.\right.\right.$$

7.6.5.5 [1444] Problem 5

problem number 1444

Added May 18, 2019.

Problem Chapter 6.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a e^{\alpha x} w_x + b e^{\beta y} w_y + c e^{\gamma z} w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] +b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta},\frac{ce^{-\alpha x}}{a\alpha}-\frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta},\frac{\left(-a\alpha\,\mathrm{e}^{\alpha x}+\gamma c\,\mathrm{e}^{\gamma z}\right)\mathrm{e}^{-\alpha x-\gamma z}}{\gamma\alpha c}\right)$$

7.6.5.6 [1445] Problem 6

problem number 1445

Added May 18, 2019.

Problem Chapter 6.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\beta y}w_x+be^{\alpha x}w_y+ce^{\gamma z}w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*y]*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{\beta y}}{\beta}-\frac{be^{\alpha x}}{a\alpha},-\frac{c\gamma\log\left(\frac{a\alpha e^{\beta y}}{\beta}\right)-a\alpha e^{\beta y-\gamma z}+b\beta e^{\alpha x-\gamma z}-\alpha c\gamma x}{b\beta\gamma e^{\alpha x}-a\alpha\gamma e^{\beta y}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(beta*y)*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\alpha\,\mathrm{e}^{\beta y}-b\beta\,\mathrm{e}^{\alpha x}}{\alpha b\beta},-\frac{\left(a\alpha\,\mathrm{e}^{\beta y}\mathrm{e}^{-\gamma z}-b\beta\,\mathrm{e}^{\alpha x}\mathrm{e}^{-\gamma z}+\gamma\alpha cx-\gamma c\ln\left(\frac{a\alpha\,\mathrm{e}^{\beta y}}{b\beta}\right)\right)b\beta}{\gamma\left(a\alpha\,\mathrm{e}^{\beta y}-b\beta\,\mathrm{e}^{\alpha x}\right)\alpha c}\right)$$

7.6.5.7 [1446] Problem 7

problem number 1446

Added May 18, 2019.

Problem Chapter 6.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1+a_2e^{\alpha x})w_x+(b_1+b_2e^{\beta y}w_y+(c_1+c_2e^{\gamma z})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] +(b1+b2*Exp[beta*y])*D[w[x, y,z], y] +(c1+c2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\frac{e^{\beta y}(\mathrm{a1}+\mathrm{a2}e^{\alpha x})^{\frac{\mathrm{b1}\beta}{\mathrm{a1}\alpha}}}{\mathrm{b1}+\mathrm{b2}e^{\beta y}}\right)}{\mathrm{b1}\beta}-\frac{x}{\mathrm{a1}},\frac{\log\left(\frac{e^{\gamma z}(\mathrm{a1}+\mathrm{a2}e^{\alpha x})^{\frac{\mathrm{c1}\gamma}{\mathrm{a1}\alpha}}}{\mathrm{c1}+\mathrm{c2}e^{\gamma z}}\right)}{\mathrm{c1}\gamma}-\frac{x}{\mathrm{a1}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+(b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b1\,\beta x+a1\,\ln\left(\frac{\left(-b1+\mathrm{RootOf}\left(a1\,\alpha\beta y-a1\,\alpha\ln\left(\frac{(\_Z-b1)(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1\,\beta}{a1\,\alpha}}}{b2}\right)+b1\,\beta\ln(a2\,\mathrm{e}^{\alpha x}+a1)\right)\right)(a2\,\mathrm{e}^{\alpha}}{b2\,\mathrm{RootOf}\left(a1\,\alpha\beta y-a1\,\alpha\ln\left(\frac{(\_Z-b1)(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1\,\beta}{a1\,\alpha}}}{b2}\right)+b1\,\beta\ln(a2\,\mathrm{e}^{\alpha x}+a1)\right)}\right.}{a1\,b1\,\beta}\right.$$

7.6.5.8 [1447] Problem 8

problem number 1447

Added May 18, 2019.

Problem Chapter 6.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$e^{\beta y}(a_1+a_2e^{\alpha x})w_x+e^{\alpha x}(b_1+b_2e^{\beta y}w_y+ce^{\beta y+\gamma z}w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] +Exp[alpha*x]*(b1+b2*Exp[beta*y])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\log\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)}{\mathrm{a1}\alpha}-\frac{cx}{\mathrm{a1}}-\frac{e^{-\gamma z}}{\gamma},\frac{\log\left(\mathrm{b1}+\mathrm{b2}e^{\beta y}\right)}{\mathrm{b2}\beta}-\frac{\log\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)}{\mathrm{a2}\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+exp(alpha*x)*(b1+b2*exp(beta*y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}\operatorname{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{\_Z-b2}\right)-b2\beta\ln\left(a2\,e^{\alpha x}+a1\right)\right)}{-b2+\operatorname{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{\_Z-b2}\right)-b2\beta\ln\left(a2\,e^{\alpha x}+a1\right)\right)}\right)}{b2\beta},\frac{-a1\,\alpha\,e^{-}}{}\right.$$

7.6.6 3.2

Local contents

7.6.6.1 [1448] Problem 1

problem number 1448

Added May 18, 2019.

Problem Chapter 6.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a y e^{\alpha x} w_x + b e^{\beta y} w_y + c e^{\gamma z} w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left(\left(\beta y+1\right) a\alpha\, \mathrm{e}^{\alpha x-\beta y}-b\,\beta^{2}\right) \mathrm{e}^{-\alpha x}}{\alpha b\,\beta^{2}}, \frac{\left(\gamma\alpha b\,\beta^{2} c\left(\int^{x} -\frac{}{\left(\left(\beta y+1\right) a\alpha\,\mathrm{e}^{-\left(-\_a+x\right)\alpha}\mathrm{e}^{\alpha x-\beta y}-\left(\mathrm{e}^{-\left(-\_a+x\right)}\right.\right.}\right.\right.}{}\right.$$

7.6.6.2 [1449] Problem 2

problem number 1449

Added May 18, 2019.

Problem Chapter 6.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aye^{\alpha x}w_x + be^{\beta y}w_y + cze^{\gamma z}w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*z*Exp[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*z*exp(gamma*z)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{((\beta y+1)\,a\alpha\,\mathrm{e}^{\alpha x-\beta y}-b\,\beta^2)\,\mathrm{e}^{-\alpha x}}{\alpha b\,\beta^2}, \frac{b\beta\,\mathrm{expIntegral}\,(1,\gamma z)\,\mathrm{LambertW}\,(-(\beta y+1)\,\mathrm{e}^{-\alpha x-}}{\alpha b\,\beta^2 c\,\mathrm{LambertW}\,(-\,(\beta y+1)\,\mathrm{e}}\right.$$

7.6.6.3 [1450] Problem 3

problem number 1450

Added May 18, 2019.

Problem Chapter 6.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left[y^2 + ae^{\alpha x}(\alpha - ae^{\alpha x})\right] w_y + \left[z^2 + bz + ce^{\beta x}(\beta - b - ce^{\beta x})\right] w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ a*Exp[alpha*x]*(alpha-a*Exp[alpha*x]))*D[w[x, y,z], y] +(z^2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\mathrm{Ei}\left(\frac{2ae^{\alpha x}}{\alpha}\right)(y-ae^{\alpha x})+\alpha e^{\frac{2ae^{\alpha x}}{\alpha}}}{ae^{\alpha x}-y}, \frac{2^{b/\beta}\beta^{-\frac{b}{\beta}}e^{bx}c^{b/\beta}\left(\left(b-ce^{\beta x}+z\right)\mathrm{LaguerreL}\left(-\frac{b}{\beta},\frac{b}{\beta},\frac{2c}{}\right.\right.}{ce^{\beta x}-z}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ a*exp(alpha*x)*(alpha-a*exp(alpha*x)))*diff(w(x,y,z),y)+(z^2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,\mathrm{e}^{\alpha x}+y}{\alpha\,\mathrm{e}^{\frac{2a\,\mathrm{e}^{\alpha x}}{\alpha}}+(a\,\mathrm{e}^{\alpha x}-y)\,\mathrm{expIntegral}\left(1,-\frac{2a\,\mathrm{e}^{\alpha x}}{\alpha}\right)}, \frac{(-c\,\mathrm{e}^{\beta x}+z)\left(\int \mathrm{e}^{\frac{b\beta x+2c\,\mathrm{e}^{\beta x}}{\beta}}dx\right)+\mathrm{e}^{\frac{b\beta x+2c}{\beta}}}{c\,\mathrm{e}^{\beta x}-z}\right.$$

7.6.6.4 [1451] Problem 4

problem number 1451

Added May 18, 2019.

Problem Chapter 6.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left[y^2+by+ae^{\alpha x}(\alpha-b-ae^{\alpha x})\right]w_y+\left[z^2+ce^{\beta x}(z-k)-k^2\right]w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ b*y + a*Exp[alpha*x]*(alpha-b-a*Exp[alpha*x]))*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(2k(-1)^{-\frac{k}{\beta}}\left(-\frac{\mathrm{Gamma}\left(\frac{2k}{\beta},0,-\frac{ce^{\beta x}}{\beta}\right)}{\beta}+\frac{\beta^{-\frac{2k}{\beta}}c^{\frac{2k}{\beta}}e^{\frac{ce^{\beta x}+2\beta kx+2i\pi k}{\beta}}}{k-z}\right),\frac{2^{\frac{b}{\alpha}}\alpha^{-\frac{b}{\alpha}}e^{bx}a^{\frac{b}{\alpha}}((a(-\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ b*y+a*exp(alpha*x)*(alpha-b-a*exp(alpha*x)))*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(-a\,\mathrm{e}^{\alpha x}+y)\left(\int \mathrm{e}^{\frac{\alpha bx+2a\,\mathrm{e}^{\alpha x}}{\alpha}}dx\right)+\mathrm{e}^{\frac{\alpha bx+2a\,\mathrm{e}^{\alpha x}}{\alpha}}}{a\,\mathrm{e}^{\alpha x}-y}, \frac{(k-z)\left(\int \mathrm{e}^{\frac{2\beta kx+c\,\mathrm{e}^{\beta x}}{\beta}}dx\right)-\mathrm{e}^{\frac{2\beta kx+c\,\mathrm{e}^{\beta x}}{\beta}}}{k-z}\right)$$

7.6.6.5 [1452] Problem 5

problem number 1452

Added May 18, 2019.

Problem Chapter 6.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left[y^2 + by + ae^{\alpha x}(y-b) - b^2\right] w_y + \left[z^2 + c(xz-1)e^{\beta x}\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ b*y + a*Exp[alpha*x]*(y-b)-b^2)*D[w[x, y,z], y] +(z^2 +c*(x*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ b*y+a*exp(alpha*x)*(y-b)-b^2)*diff(w(x,y,z),y)+(z^2 +c*(x*z-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.6.6 [1453] Problem 6

problem number 1453

Added May 18, 2019.

Problem Chapter 6.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left[y^2 - ae^{\alpha x}(xy-1)\right] w_y + \left(ce^{\beta x}z^2 + be^{-\beta x}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2- a*Exp[alpha*x]*(x*y-1))*D[w[x, y,z], y] +(c*Exp[beta*x]*z^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2- a*exp(alpha*x)*(x*y-1))*diff(w(x,y,z),y)+(c*exp(beta*x)*z^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.6.7 [1454] Problem 7

problem number 1454

Added May 18, 2019.

Problem Chapter 6.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(ay^2e^{\alpha x} + be^{-\alpha x}\right) w_y + \left[de^{\beta x}z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x})\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a*y^2- a*Exp[alpha*x]+ b * Exp[-alpha*x])*D[w[x, y,z], y] +(d*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( a*y^2- a*exp(alpha*x)+ b * exp(-alpha*x))*diff(w(x,y,z),y)+(d*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.6.8 [1455] Problem 8

problem number 1455

Added May 18, 2019.

Problem Chapter 6.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[be^{\alpha x}y^2 + ae^{\beta x}(\beta - abe^{(\alpha+\beta)x})\right] w_y + \left(cz^2e^{\gamma x} + dz + ke^{-\gamma x}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (b*Exp[alpha*x]*y^2 + a*Exp[beta*x]*(beta- a*b*Exp[(alpha+beta)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (b*exp(alpha*x)*y^2 + a*exp(beta*x)*(beta- a*b*exp((alpha+beta)*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.6.9 [1456] Problem 9

problem number 1456

Added May 18, 2019.

Problem Chapter 6.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ae^{\alpha x}y^2 + by + ce^{\alpha x})\, w_y + (e^{\beta x}z^2 + de^{\gamma x}(z + \beta e^{-\beta x}))\, w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[alpha*x]*y^2 + b*y +  c*Exp[alpha*x])*D[w[x, y,z], y] +(Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*exp(alpha*x)*y^2 + b*y +  c*exp(alpha*x))*diff(w(x,y,z),y)+(exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\sqrt{a}\, y \operatorname{BesselJ}\left(-\frac{\alpha+b}{2\alpha}, \frac{\sqrt{a}\sqrt{c}\, e^{\alpha x}}{\alpha}\right) + \sqrt{c} \operatorname{BesselJ}\left(\frac{\alpha-b}{2\alpha}, \frac{\sqrt{a}\sqrt{c}\, e^{\alpha x}}{\alpha}\right)}{\sqrt{a}\, y \operatorname{BesselY}\left(-\frac{\alpha+b}{2\alpha}, \frac{\sqrt{a}\sqrt{c}\, e^{\alpha x}}{\alpha}\right) - \sqrt{c} \operatorname{BesselY}\left(\frac{\alpha-b}{2\alpha}, \frac{\sqrt{a}\sqrt{c}\, e^{\alpha x}}{\alpha}\right)}, -\frac{}{\beta d \operatorname{hypergeom}\left(\left[\frac{-\beta}{}\right.\right.}\right.$$

7.6.6.10 [1457] Problem 10

problem number 1457

Added May 18, 2019.

Problem Chapter 6.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[e^{\alpha x}y^2 + aye^{\beta x} + a\alpha e^{(\beta-\alpha)x}\right] w_y + \left[\gamma e^{\gamma x}z^2 + be^{\delta x}(z + e^{-\gamma x})\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( Exp[alpha*x]*y^2 + a*y*Exp[beta*x] + a*alpha*Exp[(beta-alpha)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( exp(alpha*x)*y^2 + a*y*exp(beta*x) + a*alpha*exp((beta-alpha)*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{(-\alpha+\beta)\left(y\,\mathrm{e}^{\alpha x}+\alpha\right)\mathrm{e}^{\alpha x}}{a\alpha\,\mathrm{hypergeom}\left(\left[\frac{-\alpha+\beta}{\beta}\right], \left[\frac{-\alpha+2\beta}{\beta}\right], \frac{a\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{\beta x} - (-\alpha+\beta)\,y\,\mathrm{hypergeom}\left(\left[-\frac{\alpha}{\beta}\right], \left[\frac{-\alpha+\beta}{\beta}\right]\right.}\right.$$

7.6.6.11 [1458] Problem 11

problem number 1458

Added May 18, 2019.

Problem Chapter 6.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[\alpha e^{\alpha x} y^2 + a e^{\beta x}(y + e^{-\alpha x})\right] w_y + \left[e^{\gamma x}(z - b e^{\delta x})^2 + b\delta e^{\delta x}\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( alpha*Exp[alpha*x]*y^2 + a*Exp[beta*x]*(y+Exp[-alpha*x]))*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( alpha*exp(alpha*x)*y^2 + a*exp(beta*x)*(y+exp(-alpha*x)))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(-\alpha+\beta)\,(y\,e^{\alpha x}+1)\,e^{\alpha x}}{a\,\mathrm{hypergeom}\left(\left[\frac{-\alpha+\beta}{\beta}\right],\left[\frac{-\alpha+2\beta}{\beta}\right],\frac{a\,e^{\beta x}}{\beta}\right)e^{\beta x} - (-\alpha+\beta)\,y\,\mathrm{hypergeom}\left(\left[-\frac{\alpha}{\beta}\right],\left[\frac{-\alpha+\beta}{\beta}\right],\right.}\right.$$

7.6.6.12 [1459] Problem 12

problem number 1459

Added May 18, 2019.

Problem Chapter 6.3.2.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x w_x + \left(a_1 e^{\alpha x} y^2 + \beta y + a_1 b_2^2 x^{2\beta} e^{\alpha x}\right) w_y + \left[a_2 x^{2n} z^2 e^{\lambda x} + (b_2 x^n e^{\lambda x} - n) z + c e^{\lambda x}\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + ( a1*Exp[alpha*x]*y^2 + beta*y+ a1*b2^2*x^(2*beta)*Exp[alpha*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*diff(w(x,y,z),x)+ ( a1*exp(alpha*x)*y^2 + beta*y+ a1*b2^2*x^(2*beta)*exp(alpha*x))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.6.6.13 [1460] Problem 13

problem number 1460

Added May 18, 2019.

Problem Chapter 6.3.2.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k\right) w_y + \left(a_2 e^{\lambda_2 x} z + b_2 e^{\beta_1 x} z^m\right) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*Exp[lambda1*x]*y + b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +(a2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left((k-1)\int_1^x \mathrm{b1}e^{\frac{\mathrm{a1}e^{\mathrm{lambda1}K[1]}(k-1)}{\mathrm{lambda1}}+\mathrm{beta1}K[1]}dK[1]+y^{1-k}e^{\frac{\mathrm{a1}(k-1)e^{\mathrm{lambda1}x}}{\mathrm{lambda1}}},(m-1)\int_1^x \mathrm{b2}e^{\frac{\mathrm{a2}}{}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a1*exp(lambda1*x)*y + b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+(a2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((k-1)\,b1\left(\int \mathrm{e}^{\frac{\beta1\lambda1x+(k-1)a1\,\mathrm{e}^{\lambda1x}}{\lambda1}}dx\right)+y^{-k+1}\mathrm{e}^{\frac{(k-1)a1\,\mathrm{e}^{\lambda1x}}{\lambda1}},(m-1)\,b2\left(\int \mathrm{e}^{\frac{\beta1lamba2x+(m-1)a2\,\mathrm{e}}{lamba2}}\right.\right.$$

7.6.6.14 [1461] Problem 14

problem number 1461

Added May 18, 2019.

Problem Chapter 6.3.2.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(a_1 e^{\beta_1 x} y + b_1 e^{\gamma_1 x} y^k\right) w_y + \left(a_2 e^{\beta_2 x} + b_2 e^{\gamma_1 x + \lambda z}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*Exp[beta1*x]*y + b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +(a2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a1*exp(beta1*x)*y + b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+(a2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((k-1)\,b1\left(\int \mathrm{e}^{\frac{\beta 1 \gamma 1 x+(k-1) a1\, \mathrm{e}^{\beta 1 x}}{\beta 1}}dx\right) + y^{-k+1}\mathrm{e}^{\frac{(k-1) a1\, \mathrm{e}^{\beta 1 x}}{\beta 1}}, \frac{-b2\lambda\left(\int \mathrm{e}^{\frac{a2\lambda\, \mathrm{e}^{\beta 2 x}}{\beta 2}+\gamma 1 x}dx\right) - \mathrm{e}^{-}}{\lambda}\right.$$

7.6.6.15 [1462] Problem 15

problem number 1462

Added May 18, 2019.

Problem Chapter 6.3.2.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^n + b_1x^m e^{\lambda y})\, w_y + (a_2x^k + b_2x^s e^{\beta z})\, w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*x^n+b1*x^m*Exp[lambda*y])*D[w[x, y,z], y] +(a2*x^k+b2*x^2*Exp[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\text{b2}\beta x^3\left(-\frac{\text{a2}\beta x^{k+1}}{k+1}\right)^{-\frac{3}{k+1}}\text{Gamma}\left(\frac{3}{k+1},-\frac{\text{a2}\beta x^{k+1}}{k+1}\right)-(k+1)e^{-\frac{\beta\left(-\text{a2}x^{k+1}+kz+z\right)}{k+1}}}{\text{a2b2}\beta^2\left(k^2-k-2\right)},\frac{(n+1)e\cdots}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( a1*x^n+b1*x^m*exp(lambda*y))*diff(w(x,y,z),y)+(a2*x^k+b2*x^2*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{\left(-a1\,\lambda\,x^{m+1}+(m+n+2)\,x^{m-n}\right)(n+1)^2\,b1\left(-\frac{a1\,\lambda\,x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}}\text{WhittakerM}\left(\frac{m-n}{2n+2},\cdots\right.}{}\right.$$

7.6.6.16 [1463] Problem 16

problem number 1463

Added May 18, 2019.

Problem Chapter 6.3.2.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(ax^n e^{\lambda y} + bxy^m)w_x + e^{\mu y}w_y + (cy^l z^k + dy^p z)\, w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Exp[lambda*y] + b*x*y^m)*D[w[x, y,z], x] +Exp[mu*y]*D[w[x, y,z], y] +(c*y^L*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*exp(lambda*y) + b*x*y^m)*diff(w(x,y,z),x)+ exp(mu*y)*diff(w(x,y,z),y)+(c*y^L*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x^{\frac{1}{m+1}} x^{\frac{m}{m+1}} x^{-\frac{n}{m+1}} x^{-\frac{mn}{m+1}} \mathrm{e}^{\frac{bn\, y^m (\mu y)^{-\frac{m}{2}}\ \mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right) \mathrm{e}^{-\frac{\mu y}{2}}}{(m+1)\mu}} \mathrm{e}^{-\frac{b\, y^m (\mu y)^{-\frac{m}{2}}\ \mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)}{(m+1)\mu}}\right.$$

7.6.6.17 [1464] Problem 17

problem number 1464

Added May 18, 2019.

Problem Chapter 6.3.2.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 + 2a\alpha e^{\alpha x^2} - a^2 e^{2\alpha x^2}) w_y + \left(ce^{-2\beta x^2} z^2 + 2\beta xz + b^2 c\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2 + 2*a*alpha*Exp[alpha*x^2]-a^2*Exp[2*alpha*x^2] )*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (y^2 + 2*a*alpha*exp(alpha*x^2)-a^2*exp(2*alpha*x^2) )*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.6.18 [1465] Problem 18

problem number 1465

Added May 18, 2019.

Problem Chapter 6.3.2.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ae^{2\alpha x^2}y^2 + 2\alpha xy + ab^2)w_y + \left(cx^\beta z^2 + 2\gamma xz + cd^2x^\beta e^{2\gamma x^2}\right)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[2*alpha*x^2]*y^2 + 2*alpha*x*y + a*b^2 )*D[w[x, y,z], y] +(
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*exp(2*alpha*x^2)*y^2 + 2*alpha*x*y + a*b^2 )*diff(w(x,y,z),y)+(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.7 4.1

Local contents

7.6.7.1 [1466] Problem 1

problem number 1466

Added May 19, 2019.

Problem Chapter 6.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Sinh[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\cosh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*sinh(lambda*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c\cosh(\lambda x)}{a\lambda}\right)$$

7.6.7.2 [1467] Problem 2

problem number 1467

Added May 19, 2019.

Problem Chapter 6.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{c\cosh(\lambda x)}{a\lambda},\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x-2a\,\mathrm{arctanh}\left(\mathrm{e}^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\cosh\left(\lambda x\right)}{a\lambda}\right)$$

7.6.7.3 [1468] Problem 3

problem number 1468

Added May 19, 2019.

Problem Chapter 6.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x-2a\operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta},\frac{-2a\operatorname{arctanh}\left(e^{\gamma z}\right)-\gamma cx}{\gamma c}\right)$$

7.6.7.4 [1469] Problem 4

problem number 1469

Added May 19, 2019.

Problem Chapter 6.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta},\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-2a\lambda\operatorname{arctanh}\left(e^{\beta y}\right)+2b\beta\operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{-2a\lambda\operatorname{arctanh}\left(e^{\gamma z}\right)+2\gamma c\operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

7.6.7.5 [1470] Problem 5

problem number 1470

Added May 19, 2019.

Problem Chapter 6.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x+b\sinh(\lambda x)w_y+c\sinh(\gamma z)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh\left(\beta y\right)-b\beta\cosh\left(\lambda x\right)}{b\beta\lambda}, \frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{\gamma z}\right)-\gamma c\left(\int^{x}\frac{}{\sqrt{\frac{b\beta\cosh(\_a\lambda)-b\beta\cosh(\lambda x)+(\cosh(\beta y)-1)}{a\lambda}}}\right.}{\gamma c}\right.$$

7.6.7.6 [1471] Problem 6

problem number 1471

Added May 19, 2019.

Problem Chapter 6.4.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x + b\sinh(\lambda x)w_y + c\sinh(\lambda x)\sinh(\beta y)\sinh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[lambda*x]*S
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(lambda*x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh(\beta y) - b\beta\cosh(\lambda x)}{b\beta\lambda}, -\frac{4a\lambda\operatorname{arctanh}(e^{\gamma z}) + \gamma c\,e^{\lambda x} + \gamma c\,e^{-\lambda x}}{2\gamma c\lambda}\right)$$

7.6.8 4.2

Local contents

7.6.8.1 [1472] Problem 1

problem number 1472

Added May 19, 2019.

Problem Chapter 6.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cosh(\beta x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cosh[beta*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sinh(\beta x)}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cosh(beta*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\beta z-c\sinh(\beta x)}{a\beta}\right)$$

7.6.8.2 [1473] Problem 2

problem number 1473

Added May 19, 2019.

Problem Chapter 6.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta x)w_y+c\cosh(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*x]*D[w[x, y,z], y] +c*Cosh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{b\sinh(\beta x)}{a\beta},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*x)*diff(w(x,y,z),y)+c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\beta y - b\sinh(\beta x)}{a\beta}, \frac{a\lambda z - c\sinh(\lambda x)}{a\lambda}\right)$$

7.6.8.3 [1474] Problem 3

problem number 1474

Added May 19, 2019.

Problem Chapter 6.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cosh(\beta y)w_y + c\cosh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{bx}{a}, z - \frac{c\sinh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + 2a\arctan\left(e^{\beta y}\right)}{b\beta}, \frac{a\lambda z - c\sinh(\lambda x)}{a\lambda}\right)$$

7.6.8.4 [1475] Problem 4

problem number 1475

Added May 19, 2019.

Problem Chapter 6.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cosh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{2a\arctan\left(e^{\gamma z}\right)-\gamma cx}{\gamma c}\right)$$

7.6.8.5 [1476] Problem 5

problem number 1476

Added May 19, 2019.

Problem Chapter 6.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a\cosh(\lambda x)w_x + b\cosh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{2b\tan^{-1}\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda},\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{2c\tan^{-1}\left(\tanh\left(\right.\right.}{a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2a\lambda\arctan\left(e^{\beta y}\right)-2b\beta\arctan\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{2a\lambda\arctan\left(e^{\gamma z}\right)-2\gamma c\arctan\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

7.6.8.6 [1477] Problem 6

problem number 1477

Added May 19, 2019.

Problem Chapter 6.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a\cosh(\beta y)w_x + b\cosh(\lambda x)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Cosh[beta*y]*D[w[x, y,z], x] + b*Cosh[lambda*x]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*cosh(beta*y)*diff(w(x,y,z),x)+ b*cosh(lambda*x)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\sinh(\beta y) - b\beta\sinh(\lambda x)}{b\beta\lambda}, -\frac{4\left(-\frac{\left((a\lambda-b\beta)\sqrt{-(\sinh^2(\beta y))-1}+\sqrt{\sinh^2(\beta y)+1}\,(a\lambda\sinh(\beta y)-b\beta\sinh(\right.}{}\right.}{}\right.$$

7.6.9 4.3

Local contents

7.6.9.1 [1478] Problem 1

problem number 1478

Added May 19, 2019.

Problem Chapter 6.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log(\sinh(\gamma z))}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*tanh(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \mathrm{RootOf}\left(\gamma z + \operatorname{arctanh}\left(\frac{\sqrt{\left(\mathrm{e}^{\frac{2\gamma(\_Z+x)c}{a}}-1\right)\mathrm{e}^{-\frac{2\gamma(\_Z+x)c}{a}}}\,\mathrm{e}^{\frac{2\gamma(\_Z+x)c}{a}}}{\mathrm{e}^{\frac{2\gamma(\_Z+x)c}{a}}-1}\right)\right)\right)$$

7.6.9.2 [1479] Problem 2

problem number 1479

Added May 19, 2019.

Problem Chapter 6.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta x)w_y + c\tanh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*x]*D[w[x, y,z], y] +c*Tanh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{b\log(\cosh(\beta x))}{a\beta}, z - \frac{c\log(\cosh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*x)*diff(w(x,y,z),y)+c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\beta y + b\ln(\tanh(\beta x) - 1) + b\ln(\tanh(\beta x) + 1)}{2a\beta}, \frac{2a\lambda z + c\ln(\tanh(\lambda x) - 1) + c\ln(t}{2a\lambda}\right.$$

7.6.9.3 [1480] Problem 3

problem number 1480

Added May 19, 2019.

Problem Chapter 6.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(z - \frac{c\log(\cosh(\lambda x))}{a\lambda}, \frac{\log(\sinh(\beta y))}{\beta} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta}, \frac{a\lambda z - c\ln(\cosh}{a\lambda}\right.$$

7.6.9.4 [1481] Problem 4

problem number 1481

Added May 19, 2019.

Problem Chapter 6.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\sinh(\beta y))}{\beta}-\frac{bx}{a},\frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma}-\frac{2c\log(\sinh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta}, \frac{b\beta\ln\left(\sqrt{-\left(-\right.}\right.}{}\right.$$

7.6.9.5 [1482] Problem 5

problem number 1482

Added May 19, 2019.

Problem Chapter 6.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\tanh(\lambda x)w_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda \ln\left(\sqrt{-\left(-\frac{1}{\left(e^{2\lambda x}-1\right)^2}\right)^{\frac{b\beta}{a\lambda}}}\sinh\left(\beta y\right)\right)+\left(\lambda x+\ln\left(2\right)\right)b\beta}{b\beta\lambda}, \frac{a\lambda \ln\left(\sqrt{-\left(-\frac{1}{\left(e^{2\lambda x}-1\right)^2}\right)^{\frac{\gamma c}{a\lambda}}}\right.}{}\right.$$

7.6.9.6 [1483] Problem 6

problem number 1483

Added May 19, 2019.

Problem Chapter 6.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\tanh(\beta y)w_x + b\tanh(\lambda x)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*tanh(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1(\tanh(\gamma z) - 1)^{-\frac{\_c_3}{2\gamma}} (\tanh(\gamma z) + 1)^{-\frac{\_c_3}{2\gamma}} \left(\tanh^{\frac{\_c_3}{\gamma}}(\gamma z)\right) \_F5\left(\frac{a \ln\left(\mathrm{RootOf}\left(\beta y - \mathrm{arccosh}\right.\right.}{}\right.$$

7.6.10 4.4

Local contents

7.6.10.1 [1484] Problem 1

problem number 1484

Added May 19, 2019.

Problem Chapter 6.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c \coth(\gamma z) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Coth[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log(\cosh(\gamma z))}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*coth(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\frac{-a\ln\left(\frac{(\mathrm{RootOf}(\gamma z-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\gamma z-\mathrm{arccoth}(\_Z-1))-2}\right)+a\ln\left(\mathrm{RootOf}\left(\gamma z-\mathrm{arccoth}(\_Z-1)\right)\right)}{2\gamma c}\right.$$

7.6.10.2 [1485] Problem 2

problem number 1485

Added May 19, 2019.

Problem Chapter 6.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta x)w_y+c\coth(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*x]*D[w[x, y,z], y] +c*Coth[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{b\log(\sinh(\beta x))}{a\beta},z-\frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*x)*diff(w(x,y,z),y)+c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2a\beta y+b\ln\left(\coth\left(\beta x\right)-1\right)+b\ln\left(\coth\left(\beta x\right)+1\right)}{2a\beta},\frac{2a\lambda z+c\ln\left(\coth\left(\lambda x\right)-1\right)+c\ln\left(c\right.}{2a\lambda}\right.$$

7.6.10.3 [1486] Problem 3

problem number 1486

Added May 19, 2019.

Problem Chapter 6.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta y)w_y+c\coth(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\cosh(\beta y))}{\beta}-\frac{bx}{a},z-\frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\text{RootOf}(\beta y-\text{arccoth}(\_Z-1))-1)^2}{\text{RootOf}(\beta y-\text{arccoth}(\_Z-1))-2}\right) - a\ln\left(\text{RootOf}\left(\beta y - \text{arccoth}(\_Z-1)\right)\right)}{2b\beta},\right.$$

7.6.10.4 [1487] Problem 4

problem number 1487

Added May 19, 2019.

Problem Chapter 6.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\cosh(\beta y))}{\beta} - \frac{bx}{a}, \frac{b\log\left(\cosh^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\cosh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-2}\right) - a\ln\left(\mathrm{RootOf}\left(\beta y-\mathrm{arccoth}(\_Z-1)\right)\right)}{2b\beta}, -$$

7.6.10.5 [1488] Problem 5

problem number 1488

Added May 19, 2019.

Problem Chapter 6.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\coth(\lambda x)w_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\ln\left(\frac{\coth(\beta y)}{\sqrt{\coth^2(\beta y)-1}}\right)+\left(\lambda x+\ln\left(\frac{2}{e^{2\lambda x}+1}\right)\right)b\beta}{b\beta\lambda}, \frac{a\lambda\ln\left(\frac{\coth(\gamma z)}{\sqrt{\coth^2(\gamma z)-1}}\right)+\gamma\left(\lambda x+\ln\left(\frac{2}{e^{2\lambda x}\right.\right.}{\gamma c\lambda}\right.$$

7.6.10.6 [1489] Problem 6

problem number 1489

Added May 19, 2019.

Problem Chapter 6.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\coth(\beta y)w_x + b\coth(\lambda x)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Coth[lambda*x]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*coth(lambda*x)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1(\coth(\gamma z)-1)^{-\frac{-c_3}{2\gamma}} (\coth(\gamma z)+1)^{-\frac{-c_3}{2\gamma}} \left(\coth^{\frac{-c_3}{\gamma}}(\gamma z)\right) \_F5\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\operatorname{arcsinh}\right.\right.}{}\right.$$

7.6.11 4.5

Local contents

7.6.11.1 [1490] Problem 1

problem number 1490

Added May 19, 2019.

Problem Chapter 6.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\sinh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{c\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda},\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-2a\lambda\operatorname{arctanh}\left(e^{\beta y}\right)+2b\beta\operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{2a\lambda\arctan\left(e^{\gamma z}\right)+2\gamma c\operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

7.6.11.2 [1491] Problem 2

problem number 1491

Added May 19, 2019.

Problem Chapter 6.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x+b\cosh(\beta y)w_y+c\cosh(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{b\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda},\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{c\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}\right)\right\}\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2a\lambda\arctan\left(e^{\beta y}\right)+2b\beta\operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{2a\lambda\arctan\left(e^{\gamma z}\right)+2\gamma c\operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

7.6.11.3 [1492] Problem 3

problem number 1492

Added May 19, 2019.

Problem Chapter 6.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x+b\sinh(\lambda x)w_y+c\sinh(\lambda x)\sinh(\beta y)\cosh(\gamma z)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[lambda*x]*S
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(lambda*x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh(\beta y) - b\beta\cosh(\lambda x)}{b\beta\lambda}, \frac{2a\lambda\arctan(e^{\gamma z}) - \gamma c\cosh(\lambda x)}{\gamma c\lambda}\right)$$

7.6.11.4 [1493] Problem 4

problem number 1493

Added May 19, 2019.

Problem Chapter 6.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\cosh(\beta y)w_x + b\tanh(\lambda x)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Cosh[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*cosh(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\lambda \sinh(\beta y) + b\beta \ln(\tanh(\lambda x) - 1) + b\beta \ln(\tanh(\lambda x) + 1)}{2b\beta\lambda}, \frac{a \arctan(\mathrm{e}^{\gamma z}) - \gamma c\left(\int^{x}\right.}{}\right.$$

7.6.11.5 [1494] Problem 5

problem number 1494

Added May 19, 2019.

Problem Chapter 6.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\coth(\beta y)w_x + b\tanh(\lambda x)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1(\tanh(\gamma z)-1)^{-\frac{\_c_3}{2\gamma}}(\tanh(\gamma z)+1)^{-\frac{\_c_3}{2\gamma}}\left(\tanh^{\frac{\_c_3}{\gamma}}(\gamma z)\right)\_F5\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\mathrm{arcsinh}\right.\right.}{}\right.$$

7.6.11.6 [1495] Problem 6

problem number 1495

Added May 19, 2019.

Problem Chapter 6.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\coth(\beta y)w_x + b\tanh(\lambda x)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1(\coth(\gamma z)-1)^{-\frac{=c3}{2\gamma}}(\coth(\gamma z)+1)^{-\frac{=c3}{2\gamma}}\left(\coth^{\frac{=c3}{\gamma}}(\gamma z)\right)\_F5\left(\frac{a\ln\left(\text{RootOf}\left(\beta y-\text{arcsinh}\right.\right.}{}\right.$$

7.6.12 5.1

Local contents

7.6.12.1 [1496] Problem 1

problem number 1496

Added May 26, 2019.

Problem Chapter 6.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\ln(\beta y)\ln(\lambda z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Log[beta*y]*Log[lambda*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\text{LogIntegral}(\lambda z)}{\lambda}+\frac{cx}{a}-\frac{cy\log(\beta y)}{b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*ln(beta*y)*ln(lambda*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-ay+bx}{b},\frac{-(\ln(\beta y)-1)\,c\lambda y-b\,\text{expIntegral}\,(1,-\ln(\lambda z))}{c\lambda}\right)$$

7.6.12.2 [1497] Problem 2

problem number 1497

Added May 26, 2019.

Problem Chapter 6.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\ln(\beta x)w_y+c\ln(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Log[beta*x]*D[w[x, y,z], y] +c*Log[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ay-bx\log(\beta x)+bx}{a},\frac{az-cx\log(\lambda x)+cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*ln(beta*x)*diff(w(x,y,z),y)+c*ln(lambda*x)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-bx\ln(\beta x)+ay+bx}{a}, \frac{-cx\ln(\lambda x)+az+cx}{a}\right)$$

7.6.12.3 [1498] Problem 3

problem number 1498

Added May 26, 2019.

Problem Chapter 6.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\ln(\beta x)\ln(\lambda y)w_y + c\ln(\mu x)\ln(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Log[beta*x]*Log[lambda*y]*D[w[x, y,z], y] +c*Log[mu*x]*Log[gamm
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\text{LogIntegral}(\lambda y)}{\lambda}-\frac{bx(\log(\beta x)-1)}{a},\frac{\text{LogIntegral}(\gamma z)}{\gamma}-\frac{cx(\log(\mu x)-1)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*ln(beta*x)*ln(lambda*y)*diff(w(x,y,z),y)+c*ln(mu*x)*ln(gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(\ln(\beta x)-1)\,b\lambda x + a\,\mathrm{expIntegral}\left(1,-\ln(\lambda y)\right)}{a\lambda}, -\frac{\left(-\gamma\left(-\ln\left(\frac{(\ln(\beta x)-1)\beta x}{\mathrm{LambertW}((\ln(\beta x)-1)\mathrm{e}^{-1}\beta x)}\right)\right.\right.}{}\right.$$

7.6.12.4 [1499] Problem 4

problem number 1499

Added May 26, 2019.

Problem Chapter 6.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln(\beta x)w_x + b\ln(\lambda y)w_y + c\ln(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[beta*x]*D[w[x, y,z], x] + b*Log[lambda*y]*D[w[x, y,z], y] +c*Log[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\mathrm{LogIntegral}(\lambda y)}{\lambda} - \frac{b\,\mathrm{LogIntegral}(\beta x)}{a\beta}, \frac{\mathrm{LogIntegral}(\gamma z)}{\gamma} - \frac{c\,\mathrm{LogIntegral}(\beta x)}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*ln(beta*x)*diff(w(x,y,z),x)+ b*ln(lambda*y)*diff(w(x,y,z),y)+c*ln(gamma*z)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\beta\,\mathrm{expIntegral}\left(1,-\ln\left(\lambda y\right)\right)+b\lambda\,\mathrm{expIntegral}\left(1,-\ln\left(\beta x\right)\right)}{b\beta\lambda}, \frac{-a\beta\,\mathrm{expIntegral}\left(1,-\ln\right.}{}\right.$$

7.6.13 5.2

Local contents

7.6.13.1 [1500] Problem 1

problem number 1500

Added May 26, 2019.

Problem Chapter 6.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + b\ln^k(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +b*Log[lambda*x]^k*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1},z-\frac{b(-\log(\lambda x))^{-k}\log^k(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+b*ln(lambda*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-ax\,x^n+(n+1)\,y}{n+1},-bx\ln(\lambda x)+bx+z\right)$$

7.6.13.2 [1501] Problem 2

problem number 1501

Added May 26, 2019.

Problem Chapter 6.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+c\ln^k(\lambda x))w_y+(bz+s\ln^n(\lambda x))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+c*Log[lambda*x]^k)*D[w[x, y,z], y] +(b*z+s*Log[lambda*x]^n)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-ax}-\int_1^x ce^{-aK[1]}(\log(\lambda)+\log(K[1]))^k dK[1], ze^{-bx}-\int_1^x e^{-bK[2]}s(\log(\lambda)+\log(K\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*y+c*ln(lambda*x)^k)*diff(w(x,y,z),y)+(b*z+s*log(lambda*x)^n)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-c\left(\int \ln(\lambda x)^k e^{-ax}dx\right)+y\,e^{-ax},-s\left(\int \ln(\lambda x)^n e^{-bx}dx\right)+z\,e^{-bx}\right)$$

7.6.13.3 [1502] Problem 3

problem number 1502

Added May 26, 2019.

Problem Chapter 6.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+(c\ln^n(\lambda x)+s\ln^k(\beta y))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +(c*Log[lambda*x]^n+s*Log[beta*y]^k)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},-\frac{c\log^{n+1}(\lambda x)}{an+a}-\frac{s\log^{k+1}(\beta y)}{bk+b}+z\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+(c*ln(lambda*x)^n+s*ln(beta*y)^k)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,x^{-\frac{b}{a}},\frac{\left(i\pi\,\mathrm{csgn}(i\beta)\,\mathrm{csgn}(iy)\,\mathrm{csgn}(i\beta y)+i\pi\mathrm{csgn}(iy)^3-i\pi\mathrm{csgn}(iy)^2\,\mathrm{csgn}\left(ix^{\frac{b}{a}}\right)+i\pi\mathrm{csg}\right.}{}\right.$$

7.6.13.4 [1503] Problem 4

problem number 1503

Added May 26, 2019.

Problem Chapter 6.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln(\lambda x)w_x+by\ln(\beta y)w_y+cz\ln(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[lambda*x]*D[w[x, y,z], x] + b*y*Log[beta*y]*D[w[x, y,z], y] +c*Log[gamma*z]*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\log\left((\log(\beta)+\log(y))(\log(\lambda)+\log(x))^{-\frac{b}{a}}\right),\frac{\text{li}(\gamma z)}{\gamma}-\frac{c\log(\log(\lambda x))}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*ln(lambda*x)*diff(w(x,y,z),x)+ b*y*ln(beta*y)*diff(w(x,y,z),y)+c*ln(gamma*z)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\ln\left(\ln\left(\beta y\right)\right)+b\ln\left(\ln\left(\lambda x\right)\right)}{a},\frac{-b\,\text{expIntegral}\left(1,-\ln\left(z\right)-\ln\left(\gamma\right)\right)-\gamma c\ln\left(\ln\left(\beta y\right)\right)}{\gamma c}\right)$$

7.6.13.5 [1504] Problem 5

problem number 1504

Added May 26, 2019.

Problem Chapter 6.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln(\lambda x)w_x+by\ln(\beta y)w_y+cz\ln(\gamma x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[lambda*x]*D[w[x, y,z], x] + b*y*Log[beta*y]*D[w[x, y,z], y] +c*Log[gamma*x]*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{az-c\log(\gamma x)\log(\log(\lambda x))+c\log(\lambda x)\log(\log(\lambda x))-c\log(x)}{a},\log\left((\log(\beta)+\log(y)\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*ln(lambda*x)*diff(w(x,y,z),x)+ b*y*ln(beta*y)*diff(w(x,y,z),y)+c*ln(gamma*x)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln(\ln(\beta y))-b\ln(\ln(\lambda x))}{b},\frac{2az-2c\ln(x)-\left(i\pi\operatorname{csgn}(i\lambda)\operatorname{csgn}(ix)\operatorname{csgn}(i\lambda x)+i\pi\operatorname{csg}\right.}{}\right.$$

7.6.13.6 [1505] Problem 6

problem number 1505

Added May 26, 2019.

Problem Chapter 6.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln^n(x)w_x+by\ln^m(y)w_y+cz\ln^k(z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +c*z*Log[z]^k*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\log^{1-n}(x)}{a(n-1)}-(m-1)^{\frac{1}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m,\frac{c\log^{1-n}(x)}{a(n-1)}-(k-1)^{\frac{1}{k-1}}\log(z)\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+ b*y*ln(y)^m*diff(w(x,y,z),y)+c*z*ln(z)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n-1)\,a\ln\left(y\right)^{-m+1}+(m-1)\,b\ln\left(x\right)^{-n+1}}{(n-1)\,(m-1)\,b},\frac{-(n-1)\,a\ln\left(z\right)^{-k+1}+(k-1)\,c\ln\left(x\right)^{-}}{(n-1)\,(k-1)\,c}\right.$$

7.6.14 6.1

Local contents

7.6.14.1 [1506] Problem 1

problem number 1506

Added May 26, 2019.

Problem Chapter 6.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sin(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Sin[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*sin(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\arctan\left(\frac{2\_Z\mathrm{e}^{\frac{\gamma cx}{a}}}{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}+1}\right)\right)\right)}{\gamma c}\right)$$

7.6.14.2 [1507] Problem 2

problem number 1507

Added May 26, 2019.

Problem Chapter 6.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sin(\beta y)w_y + c\sin(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Sin[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\cos(\lambda x)}{a\lambda}+z,\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*sin(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\lambda z+c\cos(\lambda x)}{a\lambda}\right)$$

7.6.14.3 [1508] Problem 3

problem number 1508

Added May 26, 2019.

Problem Chapter 6.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\beta y)w_y+c\sin(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Sin[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*sin(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\mathrm{arcta}\right.\right.}{}\right.$$

7.6.14.4 [1509] Problem 4

problem number 1509

Added May 26, 2019.

Problem Chapter 6.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\lambda x)\sin(\beta y)w_y+cw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[lambda*x]*Sin[beta*y]*D[w[x, y,z], y] +c*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{cx}{a},\frac{a\log\left(\tan^2\left(\frac{\beta y}{2}\right)\right)}{\beta}+\frac{2b\cos(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(lambda*x)*sin(beta*y)*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\lambda\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z}{\_Z^2+1},\frac{\_Z^2-1}{\_Z^2+1}\right)\right)\right)+b\beta\cos(\lambda x)}{b\beta\lambda},\frac{az-cx}{a}\right)$$

7.6.14.5 [1510] Problem 5

problem number 1510

Added May 26, 2019.

Problem Chapter 6.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin^n(\lambda x)\sin^m(\beta y)w_y+c\sin^k(\mu x)\sin^r(\gamma*z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[lambda*x]^n*Sin[beta*y]^m*D[w[x, y,z], y] +c*Sin[mu*x]^k*Si
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\sqrt{\cos^2(\beta y)}\sec(\beta y)\sin^{1-m}(\beta y)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\sin^2(\beta y)\right)}{\beta-\beta m}-\frac{b\sqrt{\cos\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*sin(lambda*x)^n*sin(beta*y)^m*diff(w(x,y,z),y)+c*sin(mu*x)^k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\left(\int(\sin^n(\lambda x))\,dx\right)+\int\frac{a(\sin^{-m}(\beta y))}{b}dy,-\left(\int(\sin^k(\mu x))\,dx\right)+\int\frac{a(\sin^{-r}(\gamma z))}{c}\right.$$

7.6.15 6.2

Local contents

7.6.15.1 [1511] Problem 1

problem number 1511

Added May 26, 2019.

Problem Chapter 6.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\frac{\cosh^{-1}\left(-\frac{\sec(\gamma z)\left(2\left(2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}+\sinh^3\left(\cdots\right.\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\cdots\right.}\right.}{\gamma}\cdots\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\cosh^{-1}\left(-\frac{\sec(\gamma z)\left(2\left(2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}\right)+\sinh^3\left(\frac{c\gamma}{a}\cdots\right.\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)-\cdots}\right.}{\gamma}\cdots\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\frac{\cosh^{-1}\left(\frac{\sec(\gamma z)\left(-2\left(-2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}\right)+\sinh^3\cdots\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)\cdots}\right.}{\gamma}\cdots\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\cosh^{-1}\left(\frac{\sec(\gamma z)\left(-2\left(-2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}\right)+\sinh^3\left(\frac{c}{}\cdots\right.\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)-\cdots}\right.}{\gamma}\cdots\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\arctan\left(\frac{\_Z^2 \mathrm{e}^{\frac{2\gamma c x}{a}}-1}{\_Z^2 \mathrm{e}^{\frac{2\gamma c x}{a}}+1}, \frac{2\_Z \mathrm{e}^{\frac{\gamma c x}{a}}}{\_Z^2 \mathrm{e}^{\frac{2\gamma c x}{a}}+1}\right)\right)\right)}{\gamma c}\right)$$

7.6.15.2 [1512] Problem 2

problem number 1512

Added May 26, 2019.

Problem Chapter 6.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b\cos(\beta y) w_y + c\cos(\lambda x) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sinh\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\left(\right.}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(\frac{\sec(\beta y)\left(-2\left(-2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+s\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\frac{\cosh^{-1}\left(\frac{\sec(\beta y)\left(-2\left(-2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sinh\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\left(2\right.}\right.}{\beta}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\lambda z-c\sin(\lambda x)}{a\lambda}\right)$$

7.6.15.3 [1513] Problem 3

problem number 1513

Added May 26, 2019.

Problem Chapter 6.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},\frac{2\_Z\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\arctan\right.\right.}{}\right.$$

7.6.15.4 [1514] Problem 4

problem number 1514

Added May 26, 2019.

Problem Chapter 6.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\arctan\right.\right.}{}\right.$$

7.6.15.5 [1515] Problem 5

problem number 1515

Added May 26, 2019.

Problem Chapter 6.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos^n(\lambda x)\cos^m(\beta y)w_y + c\cos^k(\mu x)\cos^r(\gamma * z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[lambda*x]^n*Cos[beta*y]^m*D[w[x, y,z], y] +c*Cos[mu*x]^k*Co
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{n+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda x)\right)}{a\lambda n+a\lambda}+\frac{\sqrt{\sin^2}}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*cos(lambda*x)^n*cos(beta*y)^m*diff(w(x,y,z),y)+c*cos(mu*x)^k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int (\cos^n(\lambda x))\,dx\right) + \int \frac{a(\cos^{-m}(\beta y))}{b}dy, -\left(\int (\cos^k(\mu x))\,dx\right) + \int \frac{a(\cos^{-r}(\gamma z)}{c}\right.$$

7.6.16 6.3

Local contents

7.6.16.1 [1516] Problem 1

problem number 1516

Added May 26, 2019.

Problem Chapter 6.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\sin(\gamma z))}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right)-\gamma cx}{\gamma c}\right)$$

Hand solution

Solve

$$aw_x + bw_y + c\tan(\gamma z)\, w_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c\tan(\gamma z)} = \frac{dw}{0}$$

From first two pair of equation we obtain $\frac{b}{a}x - y = C_1$ and from $\frac{dx}{a} = \frac{dz}{c\tan(\gamma z)}$ we obtain $\frac{c}{a}x - \frac{1}{\gamma}\ln(\sin(\gamma z)) = C_2$. Since $dw = 0$ and $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(\frac{b}{a}x - y, \frac{c}{a}x - \frac{1}{\gamma}\ln(\sin(\gamma z))\right)$$

7.6.16.2 [1517] Problem 2

problem number 1517

Added May 26, 2019.

Problem Chapter 6.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\log(\cos(\lambda x))}{a\lambda}+z,\frac{\log(\sin(\beta y))}{\beta}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x+a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta},\frac{2a\lambda z-c\ln\left(\tan^2(\lambda x)+1\right)}{2a\lambda}\right)$$

Hand solution

Solve

$$aw_x+b\tan(\beta y)\,w_y+c\tan(\lambda x)\,w_z=0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a}=\frac{dy}{b\tan(\beta y)}=\frac{dz}{c\tan(\lambda x)}=\frac{dw}{0}$$

From first two pair of equations, integrating gives $\frac{b}{a}x-\frac{1}{\beta}\ln(\sin(\beta y))=C_1$ and from $\frac{dx}{a}=\frac{dz}{c\tan(\lambda x)}$ we obtain $\frac{c}{a}\tan(\lambda x)\,dx=dz$. Integrating gives $-\frac{c}{a\lambda}\ln(\cos(\lambda x))-z=C_2$. Since $dw=0$ then $w=C_3$. Where C_1,C_2,C_3 are constants. But $C_3=F(C_1,C_2)$ where F is arbitrary function. Hence

$$u(x,y,z)=F\left(\frac{b}{a}x-\frac{1}{\beta}\ln(\sin(\beta y)),-\frac{c}{a\lambda}\ln(\cos(\lambda x))-z\right)$$

7.6.16.3 [1518] Problem 3

problem number 1518

Added May 26, 2019.

Problem Chapter 6.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\sin(\beta y))}{\beta}-\frac{bx}{a},\frac{b\log\left(\sin^2(\gamma z)\right)}{\gamma}-\frac{2c\log(\sin(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right)-\gamma cx}{\gamma c}\right)$$

7.6.16.4 [1519] Problem 4

problem number 1519

Added May 26, 2019.

Problem Chapter 6.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\sin(\beta y))}{\beta}-\frac{bx}{a},\frac{b\log\left(\sin^2(\gamma z)\right)}{\gamma}-\frac{2c\log(\sin(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right)-\gamma c x}{\gamma c}\right)$$

7.6.16.5 [1520] Problem 5

problem number 1520

Added May 26, 2019.

Problem Chapter 6.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$\mu\nu\tan(\lambda x)w_x + \lambda\nu\tan(\mu y)w_y + \lambda\mu\tan(\nu z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  mu*nu*Tan[lambda*x]*D[w[x, y,z], x] + lambda*nu*Tan[mu*y]*D[w[x, y,z], y] +lambda*mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\log\left(\csc^2(\mu y)\sin^2(\nu z)\right),-4\sin(\lambda x)\csc(\mu y)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  mu*nu*tan(lambda*x)*diff(w(x,y,z),x)+ lambda*nu*tan(mu*y)*diff(w(x,y,z),y)+lambda*mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}\,\sin(\lambda x)}\right)}{\lambda},\frac{\ln\left(\frac{\tan(\nu z)}{\sqrt{\tan^2(\nu z)+1}\,\sin(\lambda x)}\right)}{\lambda}\right)$$

7.6.17 6.4

Local contents

7.6.17.1 [1521] Problem 1

problem number 1521

Added May 26, 2019.

Problem Chapter 6.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cot(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log(\sec(\gamma z))}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-ay+bx}{b},\frac{b\ln\left(\cot^2(\gamma z)+1\right)-2b\ln\left(\cot(\gamma z)\right)-2\gamma cy}{2\gamma c}\right)$$

7.6.17.2 [1522] Problem 2

problem number 1522

Added May 26, 2019.

Problem Chapter 6.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log(\sec(\beta y))}{\beta}-\frac{bx}{a},z-\frac{c\log(\sin(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2b\beta x-a\ln\left(\cot^2\left(\beta y\right)+1\right)+2a\ln\left(\cot\left(\beta y\right)\right)}{2b\beta},\frac{-2b\beta c\ln\left(\left(\left(-\left(\cot^2\left(\beta y\right)+1\right)^{\frac{ia\lambda}{b\beta}}\left(\mathrm{e}^{2i\beta y}\right.\right.\right.\right.}{}\right.$$

7.6.17.3 [1523] Problem 3

problem number 1523

Added May 26, 2019.

Problem Chapter 6.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx}{a}+\frac{\log(\cos(\beta y))}{\beta},\frac{2c\log(\cos(\beta y))}{\beta}-\frac{b\log\left(\cos^2(\gamma z)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2b\beta x-a\ln\left(\cot^2(\beta y)+1\right)+2a\ln\left(\cot(\beta y)\right)}{2b\beta},\frac{b\beta\ln\left(\frac{\cot^2(\gamma z)+1}{\cot(\gamma z)^2}\right)+\gamma c\ln\left(\cos^2(\beta y)\right)}{2\gamma\beta c}\right)$$

7.6.17.4 [1524] Problem 4

problem number 1524

Added May 26, 2019.

Problem Chapter 6.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cot(\beta y)w_y+c\cot(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx}{a}+\frac{\log(\cos(\beta y))}{\beta},\frac{2c\log(\cos(\beta y))}{\beta}-\frac{b\log\left(\cos^2(\gamma z)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2b\beta x-a\ln\left(\cot^2\left(\beta y\right)+1\right)+2a\ln\left(\cot\left(\beta y\right)\right)}{2b\beta},\frac{b\beta\ln\left(\frac{\cot^2(\gamma z)+1}{\cot(\gamma z)^2}\right)+\gamma c\ln\left(\cos^2\left(\beta y\right)\right)}{2\gamma\beta c}\right)$$

7.6.17.5 [1525] Problem 5

problem number 1525

Added May 26, 2019.

Problem Chapter 6.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$\mu\nu\cot(\lambda x)w_x+\lambda\nu\cot(\mu y)w_y+\lambda\mu\cot(\nu z)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  mu*nu*Cot[lambda*x]*D[w[x, y,z], x] + lambda*nu*Cot[mu*y]*D[w[x, y,z], y] +lambda*mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  mu*nu*cot(lambda*x)*diff(w(x,y,z),x)+ lambda*nu*cot(mu*y)*diff(w(x,y,z),y)+lambda*mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{\ln\left(\sqrt{\tan^2(\mu y)+1}\,\cos(\lambda x)\right)}{\lambda}, \frac{\ln\left(\sqrt{\tan^2(\nu z)+1}\,\cos(\lambda x)\right)}{\lambda}\right)$$

7.6.18 6.5

Local contents

7.6.18.1 [1526] Problem 1

problem number 1526

Added May 31, 2019.

Problem Chapter 6.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + (c\sin^n(\lambda x) + s\cos^k(\beta y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +(c*Sin[lambda*x]^n+s*Cos[beta*y]^k)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow c_1\left(y-\frac{bx}{a},-\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(\lambda x)\right)}{a\lambda n+a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+(c*sin(lambda*x)^n+s*cos(beta*y)^k)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c(\sin^n(\_a\lambda))+s\left(\cos^k\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)}{a}d\_a\right)\right)$$

7.6.18.2 [1527] Problem 2

problem number 1527

Added May 31, 2019.

Problem Chapter 6.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\beta y)w_y+c\cos(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Cos[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sin(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*cos(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\lambda z-c\sin(\lambda x)}{a\lambda}\right)$$

7.6.18.3 [1528] Problem 3

problem number 1528

Added May 31, 2019.

Problem Chapter 6.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\cos^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +b*Cos[beta*x]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{k+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(\beta x)\right)}{\beta k+\beta}+z,y-\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+b*cos(beta*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sin^n(\lambda x))\,dx\right),z-\left(\int b(\cos^k(\beta x))\,dx\right)\right)$$

7.6.18.4 [1529] Problem 4

problem number 1529

Added May 31, 2019.

Problem Chapter 6.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\sin^k(\beta y)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +b*Sin[beta*y]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+b*sin(beta*y)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sin^k\left(\left(a\left(\int (\sin^n(\_b\lambda))\,d\_b\right) + y - \left(\int a(\mathrm{si}\right.\right.\right.\right.\right.$$

7.6.18.5 [1530] Problem 5

problem number 1530

Added May 31, 2019.

Problem Chapter 6.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b\tan(\beta y) w_y + c\cot(\lambda x) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Cot[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(z - \frac{c\log(\sin(\lambda x))}{a\lambda}, \frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*cot(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{2a\lambda z + c\ln\left(\cot^2(\lambda x)+1\right)}{2a\lambda}\right)$$

7.6.18.6 [1531] Problem 6

problem number 1531

Added May 31, 2019.

Problem Chapter 6.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\lambda x)w_y + b\tan^k(\beta y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[lambda*x]^n*D[w[x, y,z], y] +b*Tan[beta*y]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*cot(lambda*x)^n*diff(w(x,y,z),y)+b*tan(beta*y)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cot^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\frac{-\tan\left(\left(y - \left(\int a(\cot^n(\lambda x))\,dx\right)\right)\beta\right) - \tan\left(a\beta\right.}{\tan\left(\left(y - \left(\int a\left(\cot^n(\lambda x)\right)dx\right)\right)\beta\right)\tan\left(a\beta\left(\int\right.\right.}\right.\right.\right.$$

7.6.19 7.1

Local contents

7.6.19.1 [1532] Problem 1

problem number 1532

Added May 31, 2019.

Problem Chapter 6.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\int_1^x\frac{c\sin^{-1}(\lambda K[1])^n}{a}dK[1]-\frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^k\text{Gamma}(1-\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda x 2^n 2^{-n}}{2} + \frac{(\lambda x-1)(\lambda x+1)(k\cdots}{\cdots}\right.}{\cdots}\right.$$

7.6.19.2 [1533] Problem 2

problem number 1533

Added May 31, 2019.

Problem Chapter 6.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\int_1^x\frac{c\sin^{-1}(\lambda K[1])^n\left(\left(\frac{a\sin^{-1}(\lambda K[1])^{-n}\text{InverseFunction[Inactive[Integrate],1,2]}\left[\int_1^x\frac{c\sin^{-1}}{}\right.}{c}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\left(\int^x\arcsin(\_a\lambda)^n\arcsin\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right)^m d\_a\right)-\frac{\left(-\gamma 1kz2^k\,\mathrm{L}\right.}{}\right.$$

7.6.19.3 [1534] Problem 3

problem number 1534

Added May 31, 2019.

Problem Chapter 6.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arcsin^n(\lambda x)w_y+c\arcsin^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] +c*ArcSin[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z+\frac{ic\sin^{-1}(\beta x)^k\left(\sin^{-1}(\beta x)^2\right)^{-k}\left(\left(i\sin^{-1}(\beta x)\right)^k\text{Gamma}\left(k+1,-i\sin^{-1}(\beta x)\right)-(-\right.}{2a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+c*arcsin(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{\left(-\arcsin(\lambda x)^n\arcsin(\lambda x)^{\frac{3}{2}}+\text{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)\arcsin(\lambda x)\right)\sqrt{-\lambda^2x^2}}{}\right.$$

7.6.19.4 [1535] Problem 4

problem number 1535

Added May 31, 2019.

Problem Chapter 6.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arcsin^n(\lambda x)w_y+c\arcsin^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] +c*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left(\left(-i\sin^{-1}(\beta z)\right)^k\operatorname{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-\left(i\sin^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Maple ✗

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+c*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.6.19.5 [1536] Problem 5

problem number 1536

Added May 31, 2019.

Problem Chapter 6.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda y)w_y + c\arcsin^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*y]^n*D[w[x, y,z], y] +c*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^k\operatorname{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-(i\sin^{-1}(\beta z))\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+c*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{\left(-\arcsin\left(\lambda y\right)^{-n}\arcsin\left(\lambda y\right)^{\frac{3}{2}}+\operatorname{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin\left(\lambda y\right)\right)\arcsin\left(\lambda y\right)\right)\sqrt{-}}{}\right.$$

7.6.20 7.2

Local contents

7.6.20.1 [1537] Problem 1

problem number 1537

Added May 31, 2019.

Problem Chapter 6.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\int_1^x\frac{c\cos^{-1}(\lambda K[1])^n}{a}dK[1]+\frac{\cos^{-1}(\beta z)^{-k}\left((-i\cos^{-1}(\beta z))^k\text{Gamma}\left(1-k\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)}+\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^{n-}}{\sqrt{\pi}\,(n+2)}\right.}{\lambda}\right.$$

7.6.20.2 [1538] Problem 2

problem number 1538

Added May 31, 2019.

Problem Chapter 6.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^m(\beta y)\arccos^k(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCos[lambda*x]^n*ArcCos[beta*y]^m*ArcCos[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\cos^{-1}(\gamma z)^{-k}\left(-2\gamma\cos^{-1}(\gamma z)^k\int_1^x\frac{c\cos^{-1}(\lambda K[1])^n\left(\left(\frac{a\cos^{-1}(\lambda K[1])^{-n}\text{InverseFunction[Inacti}}{}\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccos(lambda*x)^n*arccos(beta*y)^m*arccos(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{-(k-2)\,c\gamma 1\left(\int^{x} \arccos(\_a\lambda)^{n} \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^{m} d\_a\right) + \frac{((k-2)\gamma 1z\,\mathrm{Lon}}{}}{}\right.$$

7.6.20.3 [1539] Problem 3

problem number 1539

Added May 31, 2019.

Problem Chapter 6.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] +c*ArcCos[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{(\cos^{-1}(\beta x)^2)^{-k}\left(-c(i\cos^{-1}(\beta x))^{k}\cos^{-1}(\beta x)^{k}\,\mathrm{Gamma}\left(k+1,-i\cos^{-1}(\beta x)\right) - c(-i\right.}{2a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+c*arccos(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y + \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^{n+1}}{\sqrt{\pi}\,(n+2)} - \right.}{a\lambda}\right.$$

7.6.20.4 [1540] Problem 4

problem number 1540

Added May 31, 2019.

Problem Chapter 6.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] +c*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a} + \frac{\cos^{-1}(\beta z)^{-k}\left(\left(-i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right) + \left(i\cos^{-1}(\beta z)\right)^k\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+c*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{\frac{\left(-2(n+2)2^{-n-1}\operatorname{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)+\left(\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\arccos(\lambda x)-\arccos(\lambda x)^{n+1}\sqrt{\operatorname{arcc}}\right.\right.}{\sqrt{\arccos(\lambda x)}}}{}\right.$$

7.6.20.5 [1541] Problem 5

problem number 1541

Added May 31, 2019.

Problem Chapter 6.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda y)w_y + c\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*y]^n*D[w[x, y,z], y] +c*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}+\frac{\cos^{-1}(\beta z)^{-k}\left(\left(-i\cos^{-1}(\beta z)\right)^k\operatorname{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right)+\left(i\cos^{-1}(\beta z)\right)^{k}\right.\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*y)^n*diff(w(x,y,z),y)+c*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x + \frac{\sqrt{\pi}\left(\frac{\sqrt{-\lambda^2y^2+1}\,2^n\,\mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)}}{\sqrt{\pi}\,(n-2)} - \frac{\sqrt{-\lambda^2y^2+1}\,2^n\arccos(\lambda y)^{-n+1}}{\sqrt{\pi}\,(n-2)} + \frac{3(}{}\right.}{b\lambda}\right.$$

7.6.21 7.3

Local contents

7.6.21.1 [1542] Problem 1

problem number 1542

Added May 31, 2019.

Problem Chapter 6.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arctan^n(\lambda x)\arctan^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z\tan^{-1}(\beta K[1])^{-k}dK[1]-\int_1^x\frac{c\tan^{-1}(\lambda K[2])^n}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\left(\int\arctan\left(\lambda x\right)^n dx\right)+\int\frac{a\arctan\left(\beta z\right)^{-k}}{c}dz\right)$$

7.6.21.2 [1543] Problem 2

problem number 1543

Added May 31, 2019.

Problem Chapter 6.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^m(\beta y)\arctan^k(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*ArcTan[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z\tan^{-1}(\gamma K[1])^{-k}dK[1]-\int_1^x\frac{c\tan^{-1}(\lambda K[2])^n\left(\left(\frac{a\tan^{-1}(\lambda K[2])^{-n}\text{InverseFunct}}{}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arctan(lambda*x)^n*arctan(beta*y)^m*arctan(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\int\frac{a\arctan(\gamma 1z)^{-k}}{c}dz-\left(\int^x\arctan(\_a\lambda)^n\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right.\right.$$

7.6.21.3 [1544] Problem 3

problem number 1544

Added May 31, 2019.

Problem Chapter 6.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda x)w_y+c\arctan^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] +c*ArcTan[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1], z-\int_1^x \frac{c\tan^{-1}(\beta K[2])^k}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+c*arctan(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int \frac{b\arctan\left(\lambda x\right)^n}{a}dx\right), z-\left(\int \frac{c\arctan\left(\beta x\right)^k}{a}dx\right)\right)$$

7.6.21.4 [1545] Problem 4

problem number 1545

Added May 31, 2019.

Problem Chapter 6.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda x)w_y+c\arctan^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] +c*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1],\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+c*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int \frac{b\arctan\left(\lambda x\right)^n}{a}dx,\int \frac{b\arctan\left(\beta z\right)^{-k}}{c}dz-\left(\int^y \arctan\left(\lambda \operatorname{RootOf}\left(\_b-y+\right.\right.\right.\right.$$

7.6.21.5 [1546] Problem 5

problem number 1546

Added May 31, 2019.

Problem Chapter 6.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda y)w_y+c\arctan^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*y]^n*D[w[x, y,z], y] +c*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \tan^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a},\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*y)^n*diff(w(x,y,z),y)+c*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{a\left(\int \arctan\left(\lambda y\right)^{-n}dy\right)}{b}+x,-\left(\int \arctan\left(\lambda y\right)^{-n}dy\right)+\int \frac{b\arctan\left(\beta z\right)^{-k}}{c}dz\right)$$

7.6.22 7.4

Local contents

7.6.22.1 [1547] Problem 1

problem number 1547

Added May 31, 2019.

Problem Chapter 6.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c \operatorname{arccot}^n(\lambda x) \operatorname{arccot}^k(\beta z) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z \cot^{-1}(\beta K[1])^{-k}dK[1]-\int_1^x \frac{c\cot^{-1}(\lambda K[2])^n}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\left(\int\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n dx\right)+\int\frac{a\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}dz\right)$$

7.6.22.2 [1548] Problem 2

problem number 1548

Added May 31, 2019.

Problem Chapter 6.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c \operatorname{arccot}^n(\lambda x) \operatorname{arccot}^m(\beta y) \operatorname{arccot}^k(\gamma z) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z\cot^{-1}(\gamma K[1])^{-k}dK[1]-\int_1^x\frac{c\cot^{-1}(\lambda K[2])^n\left(\left(\frac{a\cot^{-1}(\lambda K[2])^{-n}\text{InverseFuncti}}{}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\int\frac{a\left(-\arctan\left(\gamma 1z\right)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\left(\int^x\left(-\arctan\left(\_a\lambda\right)+\frac{\pi}{2}\right)^n\left(-\arctan\left(\right.\right.\right.\right.\right.$$

7.6.22.3 [1549] Problem 3

problem number 1549

Added May 31, 2019.

Problem Chapter 6.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\,\text{arccot}^n(\lambda x)w_y+c\,\text{arccot}^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*x]^n*D[w[x, y,z], y] +c*ArcCot[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1],z-\int_1^x \frac{c\cot^{-1}(\beta K[2])^k}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+c*arccot(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int \frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx\right),z-\left(\int \frac{c\left(-\arctan(\beta x)+\frac{\pi}{2}\right)^k}{a}dx\right)\right)$$

7.6.22.4 [1550] Problem 4

problem number 1550

Added May 31, 2019.

Problem Chapter 6.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\,\text{arccot}^n(\lambda x)w_y+c\,\text{arccot}^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*x]^n*D[w[x, y,z], y] +c*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x\frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1],\int_1^z\cot^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+c*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int\frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx,\int\frac{b\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\int^y\left(-\arctan\left(\right.\right.\right.\right.$$

7.6.22.5 [1551] Problem 5

problem number 1551

Added May 31, 2019.

Problem Chapter 6.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\operatorname{arccot}^n(\lambda y)w_y+c\operatorname{arccot}^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*y]^n*D[w[x, y,z], y] +c*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \cot^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a},\int_1^z \cot^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*y)^n*diff(w(x,y,z),y)+c*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{a\left(\int\left(-\arctan\left(\lambda y\right)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x,-\left(\int\left(-\arctan\left(\lambda y\right)+\frac{\pi}{2}\right)^{-n}dy\right)+\int\frac{b(-\mathrm{ar}}{}\right.$$

7.6.23 8.1

Local contents

7.6.23.1 [1552] Problem 1

problem number 1552

Added May 31, 2019.

Problem Chapter 6.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + f(x)w_y + g(x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*D[w[x, y,z], y] +g[x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x f(K[1])dK[1],z-\int_1^x g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+g(x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y-\left(\int f(x)\,dx\right), z-\left(\int g(x)\,dx\right)\right)$$

7.6.23.2 [1553] Problem 2

problem number 1553

Added May 31, 2019.

Problem Chapter 6.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + f(x)(y+a)w_y + g(x)(z+b)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*(y+a)*D[w[x, y,z], y] +g[x]*(z+b)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right) - \int_1^x a\exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right) f(K[2])dK[2], z\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+g(x)*(z+b)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((a+y)\,\mathrm{e}^{-\left(\int f(x)dx\right)}, (b+z)\,\mathrm{e}^{-\left(\int g(x)dx\right)}\right)$$

7.6.23.3 [1554] Problem 3

problem number 1554

Added May 31, 2019.

Problem Chapter 6.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + f(x))w_y + (bz + g(x))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+f[x])*D[w[x, y,z], y] +(b*z+g[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1],ze^{-bx}-\int_1^x e^{-bK[2]}g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+(b*z+g(x))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right),z\,\mathrm{e}^{-bx}-\left(\int \mathrm{e}^{-bx}g(x)\,dx\right)\right)$$

7.6.23.4 [1555] Problem 4

problem number 1555

Added May 31, 2019.

Problem Chapter 6.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)y+g_2(x))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[x]*y+g2[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[3]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[3])dK[3],-\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(x)*y+g2(x))*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)}f2(x)\,dx\right),z-\left(\int^x\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}\mathrm{e}^{\int f1(\_f)d\_f}g1(\_f)+\right.\right.\right.$$

7.6.23.5 [1556] Problem 5

problem number 1556

Added May 31, 2019.

Problem Chapter 6.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)z+g_2(x))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[x]*z+g2[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x\exp\left(-\int_1^{K[2]}\mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2],z\,\mathrm{e}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(x)*z+g2(x))*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)}f2(x)\,dx\right),z\,\mathrm{e}^{-\left(\int g1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int g1(x)dx\right)}g2(x)\,dx\right)\right)$$

7.6.23.6 [1557] Problem 6

problem number 1557

Added May 31, 2019.

Problem Chapter 6.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_2(x)y+f_1(x)z+f_0(x))w_y+(g_2(x)y+g_1(x)z+g_0(x))w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (f2[x]*y+f1[x]*z+f0[x])*D[w[x, y,z], y] +(g2[x]*y+g1[x]*z+g0[x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (f2(x)*y+f1(x)*z+f0(x))*diff(w(x,y,z),y)+(g2(x)*y+g1(x)*z+g0(x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.23.7 [1558] Problem 7

problem number 1558

Added May 31, 2019.

Problem Chapter 6.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x))w_y + f(x) \sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\int_1^x f(K[2])dK[2],\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}}-\int_1^{e^{\lambda x}}\frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{He}\right.}{-(-\sinh(\lambda x)+i)\left(\sinh^2(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda\right.}{2}}\right.$$

7.6.23.8 [1559] Problem 8

problem number 1559

Added May 31, 2019.

Problem Chapter 6.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(x)z^m)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g1[x]*z+g2[x]*z^m)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2]+y^{1-k}\exp\left((k-1)\int\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g1(x)*z+g2(x)*z^m)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int g1(x)dx\right)} + (m\right.$$

7.6.23.9 [1560] Problem 9

problem number 1560

Added May 31, 2019.

Problem Chapter 6.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x) + g_2(x)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g1[x]+g2[x]*Exp[lambda*z])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g1(x)+g2(x)*exp(lambda*z))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g1(x)dx\right)} g2(x)\,d\right.}{\lambda}\right.$$

7.6.23.10 [1561] Problem 10

problem number 1561

Added May 31, 2019.

Problem Chapter 6.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x) + g_2(x)e^{\beta z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g1[x]+g2[x]*Exp[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g1(x)+g2(x)*exp(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int \mathrm{e}^{\lambda(\int f1(x)dx)} f2(x)\,dx\right) - \mathrm{e}^{(-y+\int f1(x)dx)\lambda}}{\lambda}, \frac{-\beta\left(\int \mathrm{e}^{\beta(\int g1(x)dx)} g2(x)\,dx\right) - \mathrm{e}^{-(z-(\int g1}}{\beta}\right.$$

7.6.24 8.2

Local contents

7.6.24.1 [1562] Problem 1

problem number 1562

Added May 31, 2019.

Problem Chapter 6.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x)g(y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z+f[x]*g[y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{z}{x}-\int_1^x\frac{f(K[1])g\left(\frac{yK[1]}{x}\right)}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+(z+f(x)*g(y))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{-x\left(\int^x\frac{f(\_a)g\left(\frac{\_ay}{x}\right)}{\_a^2}d\_a\right)+z}{x}\right)$$

7.6.24.2 [1563] Problem 2

problem number 1563

Added May 31, 2019.

Problem Chapter 6.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g_1(y)z + g_2(y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[y]*z+g2[y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right) - \int_1^x \exp\left(-\int_1^{K[2]} \text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK[2], z\,e\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+  (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(y)*z+g2(y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,e^{-\left(\int f1(x)dx\right)} - \left(\int e^{-\left(\int f1(x)dx\right)} f2(x)\,dx\right), z\,e^{-\left(\int^x g1\left(\left(y\,e^{-\left(\int f1(x)dx\right)} + \int e^{-\left(\int f1(\_f)d\_f\right)} f2(\_f)d\_f-\right.\right.\right.}\right.$$

7.6.24.3 [1564] Problem 3

problem number 1564

Added May 31, 2019.

Problem Chapter 6.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 - a^2 + a\lambda\sinh(\lambda x) - a^2\sinh^2(\lambda x))w_y + f(x)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^z \frac{1}{g(K[2])}dK[2]-\int_1^x f(K[3])dK[3],\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}}-\int_1^{e^{\lambda x}}\frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+  (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{He}\right.}{-(-\sinh(\lambda x)+i)\left(\sinh^2(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda}{2}\right.}\right.$$

7.6.24.4 [1565] Problem 4

problem number 1565

Added May 31, 2019.

Problem Chapter 6.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + z^k w_y + g(y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + z^k*D[w[x, y,z], y] +g[y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y,z),x)+  z^k*diff(w(x,y,z),y)+g(y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(z\,z^k + (-k-1)\left(\int g(y)\,dy\right), \int \frac{1}{f(x)}dx - \left(\int^y \left(\left(z^{k+1} + \int (-k-1)\,g(y)\,dy + \int\right.\right.\right.\right.$$

7.6.24.5 [1566] Problem 5

problem number 1566

Added May 31, 2019.

Problem Chapter 6.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + g[y]*D[w[x, y,z], y] +h[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1]-\int_1^x \frac{1}{f(K[2])}dK[2],\int_1^z \frac{1}{h(K[3])}dK[3]-\int_1^x \frac{1}{f(K[4])}dK[4]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y,z),x)+  g(y)*diff(w(x,y,z),y)+h(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy, -\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{h(z)}dz\right)$$

7.6.24.6 [1567] Problem 6

problem number 1567

Added May 31, 2019.

Problem Chapter 6.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y,z], x] + f2[x]*g[y]*D[w[x, y,z], y] +f3[x]*g[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1]-\int_1^x \frac{\text{f2}(K[2])}{\text{f1}(K[2])}dK[2],\int_1^z \frac{1}{g(K[3])}dK[3]-\int_1^x \frac{\text{f3}(K[4])}{\text{f1}(K[4])}dK[4]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y,z),x)+  f2(x)*g(y)*diff(w(x,y,z),y)+f3(x)*h(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\int \frac{1}{g(y)}dy-\left(\int \frac{f2(x)}{f1(x)}dx\right),\int \frac{1}{h(z)}dz-\left(\int \frac{f3(x)}{f1(x)}dx\right)\right)$$

7.6.24.7 [1568] Problem 7

problem number 1568

Added May 31, 2019.

Problem Chapter 6.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x+b\sinh(\gamma z)w_y+f_1(x)f_2(z)\sinh(\beta y)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[gamma*z]*D[w[x, y,z], y] +f1[x]*f2[z]*Sinh[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(\int_1^z \frac{1}{\text{f2}(K[1])}dK[1]-\int_1^x \frac{\text{f1}(K[2])}{a}dK[2],-\frac{\beta\int_1^x \frac{b\sinh\left(\gamma\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{\text{f2}(K[1])}dK[1]\&\right]\left[-\int_1\right.\right.}{}}{}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\int_1^z \frac{1}{\text{f2}(K[1])}dK[1]-\int_1^x \frac{\text{f1}(K[2])}{a}dK[2],\frac{\cosh(\beta y)}{\beta}-\int_1^x \frac{b\sinh\left(\gamma\text{InverseFunction}\left[\int_1^{\#}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+  b*sinh(gamma1*z)*diff(w(x,y,z),y)+f2(x)*f2(z)*sinh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\int\frac{a}{f2\left(z\right)}dz-\left(\int f2(x)\,dx\right),\frac{-b\beta\left(\int^x\sinh\left(\gamma 1\,\text{RootOf}\left(\int\frac{a}{f2(z)}dz+\int f2(\_f)\,d\_f-\left(\right.\right.\right.\right.}{b\beta}\right.$$

7.6.25 8.3

Local contents

7.6.25.1 [1569] Problem 1

problem number 1569

Added May 31, 2019.

Problem Chapter 6.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +f[x,y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\int_1^x\frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+f(x,y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{f\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a\right)\right)$$

7.6.25.2 [1570] Problem 2

problem number 1570

Added May 31, 2019.

Problem Chapter 6.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +f[x,y]*g[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z\frac{1}{g(K[1])}dK[1]-\int_1^x\frac{f\left(K[2],y+\frac{b(K[2]-x)}{a}\right)}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+f(x,y)*g(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\int\frac{a}{g(z)}dz-\left(\int^x f\left(\_a,\frac{ay-(-\_a+x)\,b}{a}\right)d\_a\right)\right)$$

7.6.25.3 [1571] Problem 3

problem number 1571

Added May 31, 2019.

Problem Chapter 6.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x, y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z+f[x,y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{z}{x}-\int_1^x\frac{f\left(K[1],\frac{yK[1]}{x}\right)}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+  y*diff(w(x,y,z),y)+(z+f(x,y))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{-x\left(\int^x\frac{f\left(\_a,\frac{\_ay}{x}\right)}{\_a^2}d\_a\right)+z}{x}\right)$$

7.6.25.4 [1572] Problem 4

problem number 1572

Added May 31, 2019.

Problem Chapter 6.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + f(x,y)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +f[x,y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},z-\int_1^x\frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+f(x,y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,x^{-\frac{b}{a}},z-\left(\int^x\frac{f\left(\_a,y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_aa}d\_a\right)\right)$$

7.6.25.5 [1573] Problem 5

problem number 1573

Added May 31, 2019.

Problem Chapter 6.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$axw_x + byw_y + f(x, y)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +f[x,y]*g[x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},z-\int_1^x\frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)g(K[1])}{aK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+f(x,y)*g(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,x^{-\frac{b}{a}},\int\frac{a}{g\left(z\right)}dz-\left(\int^x\frac{f\left(\_a,y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_a}d\_a\right)\right)$$

7.6.25.6 [1574] Problem 6

problem number 1574

Added May 31, 2019.

Problem Chapter 6.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g(x,y)z + h(x,y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g[x,y]*z+h[x,y])*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right)-\int_1^x\exp\left(-\int_1^{K[2]}\text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK[2],z\,e\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,e^{-\left(\int f1(x)dx\right)}-\left(\int e^{-\left(\int f1(x)dx\right)}f2(x)\,dx\right),z\,e^{-\left(\int^x g\left(\_f,\left(y\,e^{-\left(\int f1(x)dx\right)}+\int e^{-\left(\int f1(\_f)d\_f\right)}f2(\_f)d\_f\right.\right.\right.}\right.$$

7.6.25.7 [1575] Problem 7

problem number 1575

Added May 31, 2019.

Problem Chapter 6.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g(x,y)z + h(x,y)z^m)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*z^m)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK[2]+y^{1-k}\exp\left((k-1)\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int^x g\left(\_f,\left(y^{-k+}\right.\right.\right.}\right.$$

7.6.25.8 [1576] Problem 8

problem number 1576

Added May 31, 2019.

Problem Chapter 6.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g(x, y) + h(x, y)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*Exp[lambda*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*exp(lambda*z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.6.25.9 [1577] Problem 9

problem number 1577

Added May 31, 2019.

Problem Chapter 6.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g(x, y)z + h(x, y)z^k)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*z^k)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*z^k)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int \mathrm{e}^{\lambda(\int f1(x)dx)} f2(x)\,dx\right) - \mathrm{e}^{-(y-(\int f1(x)dx))\lambda}}{\lambda}, z^{-k+1}\mathrm{e}^{(k-1)\left(\int^x g\left(\_a, \frac{\lambda(\int f1(\_a)d\_a)+\ln\left(\frac{}{(-(\int \mathrm{e}}\right.}{}\right.\right.}\right.$$

7.6.25.10 [1578] Problem 10

problem number 1578

Added May 31, 2019.

Problem Chapter 6.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g(x,y) + h(x,y)e^{\beta z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g[x,y]+h[x,y]*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g(x,y)+h(x,y)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int f1(x)dx\right)} f2(x)\,dx\right) - e^{-\left(y-\left(\int f1(x)dx\right)\right)\lambda}}{\lambda}, \frac{-\beta\left(\int^{x} e^{\beta\left(\int g\left(\_g, \frac{\lambda(\int f1(\_g)d\_g)+\ln\left(\frac{}{(-(\int e^{\lambda(}}\right.}{}\right.\right.}\right.}{}\right.$$

7.6.25.11 [1579] Problem 11

problem number 1579

Added May 31, 2019.

Problem Chapter 6.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x,y) + h_2(x,y)z^m)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] +(h1[x,y]+h2[x,y]*z^m)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+(h1(x,y)+h2(x,y)*z^m)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.6.25.12 [1580] Problem 12

problem number 1580

Added May 31, 2019.

Problem Chapter 6.8.3.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x,y) + h_2(x,y)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] +(h1[x,y]+h2[x,y]*Exp[lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+(h1(x,y)+h2(x,y)*exp(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7 chapter 7

Local contents

7.7.1 2.1

Local contents

7.7.1.1 [1581] Problem 1

problem number 1581

Added May 31, 2019.

Problem Chapter 7.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==alpha*x+beta*y+gamma*z+delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{x(a(\alpha x+2\beta y+2\delta+2\gamma z)-x(b\beta+c\gamma))}{2a^2}+c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)= alpha*x+beta*y+gamma*z+del
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)+\frac{(\beta y+\gamma z+\delta)\,x}{a}+\frac{(a\alpha-b\beta-c\gamma)\,x^2}{2a^2}$$

7.7.1.2 [1582] Problem 2

problem number 1582

Added May 31, 2019.

Problem Chapter 7.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + azw_y + byw_z = cx + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c*x+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)= c*x+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}-\frac{\left(-\ln\left(\frac{\_aab+\sqrt{(a\,z^2+(\_a^2-y^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)+\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)\right)c+(-cx-s)\sqrt{ab}}{\sqrt{ab}\,\sqrt{(a\,z^2+(\_a^2-y^2)\,b)\,a}}d\_a+\_F1\Big($$

7.7.1.3 [1583] Problem 3

problem number 1583

Added May 31, 2019.

Problem Chapter 7.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1x + a_0)w_y + (b_1x + b_0)w_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x+a0)*D[w[x, y,z], y] +(b1*x+b0)*D[w[x,y,z],z]==alpha*x+beta*y+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{6}x\left(-3\text{a0}\beta x - 2\text{a1}\beta x^2 + 3\alpha x - 3\text{b0}\gamma x - 2\text{b1}\gamma x^2 + 6\beta y + 6\delta + 6\gamma z\right) + c_1\left(-\text{a0}x - \frac{\text{a1}x^2}{2}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a1*x+a0)*diff(w(x,y,z),y)+(b1*x+b0)*diff(w(x,y,z),z)=alpha*x+beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(-2a1\,\beta - 2\gamma b1)\,x^3}{6} + \frac{(-3\beta a0 - 3\gamma b0 + 3\alpha)\,x^2}{6} + \frac{(6\beta y + 6\gamma z + 6\delta)\,x}{6} + \_F1\left(-\frac{1}{2}a1\,x^2 - a0x\right.$$

7.7.1.4 [1584] Problem 4

problem number 1584

Added May 31, 2019.

Problem Chapter 7.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2y + a_1x + a_0)w_y + (b_2y + b_1x + b_0)w_z = c_2y + c_1z + c_0x + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*y+a1*x+a0)*D[w[x, y,z], y] +(b2*y+b1*x+b0)*D[w[x,y,z],z]==c2*y+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{6\mathrm{a2}^4c_1\left(\frac{e^{-\mathrm{a2}x}(\mathrm{a2}(\mathrm{a0}+\mathrm{a2}y)+\mathrm{a1a2}x+\mathrm{a1})}{\mathrm{a2}^2},\frac{e^{-\mathrm{a2}x}\left(\mathrm{a2}\left(2\mathrm{a0b2}\left(\mathrm{a2}xe^{\mathrm{a2}x}+1\right)-\mathrm{a2}\left(\mathrm{a2}e^{\mathrm{a2}x}\left(2\mathrm{b0}x+\mathrm{b1}x^2-2z\right)+2\mathrm{b2}y\left(e^{\mathrm{a2}x}-1\right)\right.\right.\right.}{2\mathrm{a2}^3}\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a2*y+a1*x+a0)*diff(w(x,y,z),y)+(b2*y+b1*x+b0)*diff(w(x,y,z),z)=c2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{6\left(-\frac{b1\,c1\,x^2}{3}+c1\,z+s+\left(-\frac{b0\,c1}{2}+\frac{c0}{2}\right)x\right)a2^4x+6a2^4\_F1\left(\frac{(a2^2y+a1+(a1x+a0)a2)\mathrm{e}^{-a2x}}{a2^2},\frac{-2a0\,a2\,b}{}\right.}{}$$

7.7.1.5 [1585] Problem 5

problem number 1585

Added May 31, 2019.

Problem Chapter 7.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x + k_0)w_y + (bz + s_1x + s_0)w_z = c_1x + c_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x+k0)*D[w[x, y,z], y] +(b*z+s1*x+s0)*D[w[x,y,z],z]==c1*x+c0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-ax}(a^2y+a(\text{k0}+\text{k1}x)+\text{k1})}{a^2},\frac{e^{-bx}(b^2z+b(\text{s0}+\text{s1}x)+\text{s1})}{b^2}\right)+\text{c0}x+\frac{\text{c1}x^2}{2}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a*y+k1*x+k0)*diff(w(x,y,z),y)+(b*z+s1*x+s0)*diff(w(x,y,z),z)=c1*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{c1\,x^2}{2}+c0x+\_F1\left(\frac{(a^2y+(k1\,x+k0)\,a+k1)\,\mathrm{e}^{-ax}}{a^2},\frac{(b^2z+(s1\,x+s0)\,b+s1)\,\mathrm{e}^{-bx}}{b^2}\right)$$

7.7.1.6 [1586] Problem 6

problem number 1586

Added May 31, 2019.

Problem Chapter 7.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==alpha*x+beta*y+gamma*z+de
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)+\frac{x(\alpha x+2\delta)}{2a}+\frac{\beta y}{b}+\frac{\gamma z}{c}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x+beta*y+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha\,x^2}{2a}+\frac{\delta x}{a}+\frac{\beta y}{b}+\frac{\gamma z}{c}+\_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}},z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

Hand solution

Solve

$$aw_x + byw_y + czw_z = \alpha x + \beta y + \gamma z + \delta$$

The parametrization invariant Lagrange-Charpit equations are

$$\frac{dx}{a}=\frac{dy}{by}=\frac{dz}{cz}=\frac{dw}{\alpha x+\beta y+\gamma z+\delta}$$

Solving $\frac{dx}{a}=\frac{dy}{by}$ gives

$$\begin{aligned}\frac{b}{a}dx &= \frac{dy}{y}\\ \frac{b}{a}x &= \ln y + C_1 \qquad (1)\\ \ln y &= \frac{b}{a}x - C_1\\ y &= C_1 e^{\frac{b}{a}x}\\ C_1 &= y e^{-\frac{b}{a}x}\end{aligned}$$

Equation $\frac{dx}{a}=\frac{dz}{cz}$ gives

$$\begin{aligned}\frac{c}{a}dx &= \frac{dz}{z}\\ \frac{c}{a}x &= \ln z + C_2\\ \ln z &= \frac{c}{a}x - C_2\\ z &= C_2 e^{\frac{c}{a}x}\\ C_2 &= z e^{-\frac{c}{a}x} \qquad (2)\end{aligned}$$

And $\frac{dx}{a}=\frac{dw}{\alpha x+\beta y+\gamma z+\delta}$ gives

$$\begin{aligned}\frac{\alpha x+\beta y+\gamma z+\delta}{a}dx &= dw\\ \left(\frac{\alpha}{a}x+\beta\frac{y}{a}+\gamma\frac{z}{a}+\frac{\delta}{a}\right)dx &= dw\end{aligned}$$

But from (1) $y=C_1e^{\frac{b}{a}x}$ and from (2) $z=C_2e^{\frac{c}{a}x}$. Hence the above becomes

$$\left(\frac{\alpha}{a}x+\frac{\beta}{a}C_1e^{\frac{b}{a}x}+\frac{\gamma}{a}C_2e^{\frac{c}{a}x}+\frac{\delta}{a}\right)dx = dw$$

Integrating

$$\frac{\alpha}{a}\frac{x^2}{2}+\frac{\beta}{b}C_1e^{\frac{b}{a}x}+\frac{\gamma}{c}C_2e^{\frac{c}{a}x}+\frac{\delta}{a}x = w + C_3$$

But $C_2=ze^{-\frac{c}{a}x}$ and $C_1=ye^{-\frac{b}{a}x}$, hence the above becomes

$$\begin{aligned}\frac{\alpha}{a}\frac{x^2}{2}+\frac{\beta}{b}ye^{-\frac{b}{a}x}e^{\frac{b}{a}x}+\frac{\gamma}{c}ze^{-\frac{c}{a}x}e^{\frac{c}{a}x}+\frac{\delta}{a}x &= w+C_3\\ \frac{\alpha}{a}\frac{x^2}{2}+\frac{\beta}{b}y+\frac{\gamma}{c}z+\frac{\delta}{a}x &= w+C_3\\ C_3 &= \left(\frac{\alpha}{a}\frac{x^2}{2}+\frac{\beta}{b}y+\frac{\gamma}{c}z+\frac{\delta}{a}x\right)-w\end{aligned}$$

Since $C_3 = F(C_1, C_2)$ then the solution is

$$\left(\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x\right) - w = F\left(ye^{-\frac{b}{a}x}, ze^{-\frac{c}{a}x}\right)$$

$$w(x,y,z) = F\left(ye^{-\frac{b}{a}x}, ze^{-\frac{c}{a}x}\right) + \left(\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x\right)$$

(sign change on F does not matter, since arbitrary function, can be renamed).

7.7.1.7 [1587] Problem 7

problem number 1587

Added June 1, 2019.

Problem Chapter 7.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c\log(x) + c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+  a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{c\ln\left(\frac{aby}{\sqrt{ab}}+\sqrt{a^2z^2}\right)+\sqrt{ab}\,\_F1\left(\frac{a\,z^2-b\,y^2}{a},x\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)}{\sqrt{ab}}$$

Hand solution

Solve

$$xw_x+azw_y+byw_z=c$$

The parametrization invariant Lagrange-Charpit equations are

$$\frac{dx}{x}=\frac{dy}{az}=\frac{dz}{by}=\frac{dw}{c}$$

Solving $\frac{dy}{az}=\frac{dz}{by}$ gives

$$\begin{aligned}\frac{b}{a}ydy&=zdz\\ \frac{b}{a}y^2&=z^2+C_1 \qquad (1)\\ C_1&=\frac{b}{a}y^2-z^2\\ &=\frac{by^2-az^2}{a}\end{aligned}$$

Equation $\frac{dx}{x}=\frac{dy}{az}$ gives

$$a\frac{dx}{x}=\frac{dy}{z}$$

But from (1) $z=\sqrt{\frac{b}{a}y^2-C_1}$, hence the above becomes

$$a\frac{dx}{x}=\frac{dy}{\sqrt{\frac{b}{a}y^2-C_1}}$$

Integrating gives

$$
\begin{aligned}
a \ln x &= \sqrt{\frac{a}{b}} \ln\left(\sqrt{\frac{b}{a}}y + \sqrt{\frac{b}{a}y^2 - C_1}\right) + C_2 \\
\ln x &= \sqrt{\frac{1}{ab}} \ln\left(\sqrt{\frac{b}{a}}y + \sqrt{\frac{b}{a}y^2 - C_1}\right) + \frac{C_2}{a}
\end{aligned}
$$

Let $\frac{C_2}{a} = C_3$ and the above becomes

$$
\begin{aligned}
x &= C_3\left(\sqrt{\frac{b}{a}}y + \sqrt{\frac{b}{a}y^2 - C_1}\right)^{\sqrt{\frac{1}{ab}}} \\
C_3 &= x\left(\sqrt{\frac{b}{a}}y + \sqrt{\frac{b}{a}y^2 - C_1}\right)^{-\sqrt{\frac{1}{ab}}}
\end{aligned}
$$

But from (1)

$$
C_1 = \frac{b}{a}y^2 - z^2 \tag{2}
$$

Hence C_3 simplifies to

$$
\begin{aligned}
C_3 &= x\left(\sqrt{\frac{b}{a}}y + \sqrt{\frac{b}{a}y^2 - \left(\frac{b}{a}y^2 - z^2\right)}\right)^{-\sqrt{\frac{1}{ab}}} \\
&= x\left(\sqrt{\frac{b}{a}}y + z\right)^{-\sqrt{\frac{1}{ab}}} \qquad (4)
\end{aligned}
$$

And $\frac{dx}{x} = \frac{dw}{c}$ gives

$$
\ln x = \frac{1}{c}w + C_4
$$

But $C_4 = F(C_1, C_3)$. Hence

$$
\begin{aligned}
\ln x - \frac{1}{c}w &= F\left(\frac{b}{a}y^2 - z^2, x\left(\sqrt{\frac{b}{a}}y + z\right)^{-\sqrt{\frac{1}{ab}}}\right) \\
-\frac{1}{c}w &= F\left(\frac{b}{a}y^2 - z^2, x\left(\sqrt{\frac{b}{a}}y + z\right)^{-\sqrt{\frac{1}{ab}}}\right) - \ln x \\
w(x, y, z) &= cF\left(\frac{b}{a}y^2 - z^2, x\left(\sqrt{\frac{b}{a}}y + z\right)^{-\sqrt{\frac{1}{ab}}}\right) + c\ln x
\end{aligned}
$$

Verified OK under the assumptions that $a > 0, b > 0, z > 0$.

7.7.1.8 [1588] Problem 8

problem number 1588

Added June 1, 2019.

Problem Chapter 7.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$abxw_x + b(ay + bz)w_y + a(ay - bz)w_z = c$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]==c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*b*x*diff(w(x,y,z),x)+  b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \frac{2\sqrt{-a^2y^2 + 2abyz + b^2z^2}\,\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}\,b\_F1\left(-\frac{1}{\sqrt{-a^2y^2+2abyz+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2abyz+b^2z^2} + \left(-\right.}{\sqrt{}}\right.\right.}{}$$

7.7.1.9 [1589] Problem 9

problem number 1589

Added June 1, 2019.

Problem Chapter 7.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1x+a_0)w_x+(b_1y+b_0)w_y+(c_1z+c_0)w_z=\alpha x+\beta y+\gamma z+\delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x+a0)*D[w[x, y,z], x] + (b1*y+b0)*D[w[x, y,z], y] +(c1*z+c0)*D[w[x,y,z],z]==alpha
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{(\mathrm{b0}+\mathrm{b1}y)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{b1}}{\mathrm{a1}}}}{\mathrm{b1}},\frac{(\mathrm{c0}+\mathrm{c1}z)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{c1}}{\mathrm{a1}}}}{\mathrm{c1}}\right)-\frac{\mathrm{a0}\alpha\log(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2}+\frac{\log(\mathrm{a0}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := (a1*x+a0)*diff(w(x,y,z),x)+  (b1*y+b0)*diff(w(x,y,z),y)+(c1*z+c0)*diff(w(x,y,z),z)=al
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1^2b1^2c1^2\_F1\left(\frac{(b1y+b0)(a1x+a0)^{-\frac{b1}{a1}}}{b1},\frac{(c1z+c0)(a1x+a0)^{-\frac{c1}{a1}}}{c1}\right)-(a0\alpha b1\,c1+(b0\beta c1+(\gamma c0-c}{a1^{}}$$

7.7.2 2.2

Local contents

7.7.2.1 [1590] Problem 1

problem number 1590

Added June 1, 2019.

Problem Chapter 7.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a w_x + b w_y + c w_z = \alpha x^2 + \beta y^2 + \gamma z^2 + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==alpha*x^2+beta*y^2+gamma*z^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{1}{3}\left(3c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)+\frac{\alpha x^3+3\delta x}{a}+\frac{\beta y^3}{b}+\frac{\gamma z^3}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=alpha*x^2+beta*y^2+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{3a^3\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)+3\left(\left(\frac{\alpha x^2}{3}+\beta y^2+\gamma z^2+\delta\right)a^2-(b\beta y+c\gamma z)ax+\frac{(b^2\beta+c^2\gamma)x^2}{3}\right)x}{3a^3}$$

7.7.2.2 [1591] Problem 2

problem number 1591

Added June 1, 2019.

Problem Chapter 7.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^2+a_0)w_y+(b_1x^2+b_0)w_z=\alpha x+\beta y+\gamma z+\delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^2+a0)*D[w[x, y,z], y] +(b1*x^2+b0)*D[w[x,y,z],z]==alpha*x+bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to-\frac{1}{4}x\left(2\text{a0}\beta x+\text{a1}\beta x^3-2\alpha x+2\text{b0}\gamma x+\text{b1}\gamma x^3-4\beta y-4\delta-4\gamma z\right)+c_1\left(-\text{a0}x-\frac{\text{a1}x^3}{3}+\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a1*x^2+a0)*diff(w(x,y,z),y)+(b1*x^2+b0)*diff(w(x,y,z),z)=alpha*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(-a1\,\beta - \gamma b1)\,x^4}{4} + \frac{(-2\beta a0 - 2\gamma b0 + 2\alpha)\,x^2}{4} + \frac{(4\beta y + 4\gamma z + 4\delta)\,x}{4} + \_F1\Bigg(-\frac{1}{3}a1\,x^3 - a0x +$$

7.7.2.3 [1592] Problem 3

problem number 1592

Added June 1, 2019.

Problem Chapter 7.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + s_1x^2 + s_0)w_z = c_1x^2 + c_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x^2+k0)*D[w[x, y,z], y] +(b*z+s1*x^2+s0)*D[w[x,y,z],z]==c1*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\Bigg\{\Bigg\{w(x,y,z) \to c_1\Bigg(\frac{e^{-ax}(a^2(\mathrm{k0} + \mathrm{k1}x^2) + a^3y + 2a\mathrm{k1}x + 2\mathrm{k1})}{a^3}, \frac{e^{-bx}(b^2(\mathrm{s0} + \mathrm{s1}x^2) + b^3z + 2b\mathrm{s1}x + 2\mathrm{s1})}{b^3}\Bigg)$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a*y+k1*x^2+k0)*diff(w(x,y,z),y)+(b*z+s1*x^2+s0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c1\,x^3}{3} + c0x + \_F1\left(\frac{(a^3y + 2ak1x + (k1\,x^2 + k0)\,a^2 + 2k1)\,e^{-ax}}{a^3}, \frac{(b^3z + 2bs1x + (s1\,x^2 + s0)}{b^3}\right.$$

7.7.2.4 [1593] Problem 4

problem number 1593

Added June 1, 2019.

Problem Chapter 7.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2xy + a_1x^2 + a_0)w_y + (b_2xy + b_1x^2 + b_0)w_z = c_2y + c_1z + c_0x + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*x+a1*x^2+a0)*D[w[x, y,z], y] +(b2*x*y+b1*x^2+b0)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{180}x\left(15\text{a0b2c1}x^3 - 90\text{a0c2}x + 10\text{a1b2c1}x^5 - 45\text{a1c2}x^3 + 12\text{a2b2c1}x^4 - 60\text{a2c2}x^2 - 90\text{b}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a2*x+a1*x^2+a0)*diff(w(x,y,z),y)+(b2*x*y+b1*x^2+b0)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a1\,b2\,c1\,x^6}{18}+\frac{a2\,b2\,c1\,x^5}{15}+\frac{(-45a1\,c2+(15b2\,a0-45b1)\,c1)\,x^4}{180}+\frac{(-60b2\,c1\,y-60a2\,c2)\,x^3}{180}+$$

7.7.2.5 [1594] Problem 5

problem number 1594

Added June 1, 2019.

Problem Chapter 7.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = x(\alpha x + \beta y + \gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==x*(alpha*x+beta*y+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{x(a^2(\alpha x+2\beta y+2\gamma z)+a\alpha x(b+c)+2a(b\gamma z+\beta cy)+\alpha bcx)}{2a(a+b)(a+c)}+c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=x*(alpha*x+beta*y+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha\,x^{2}}{2a}+\left(\frac{\beta y}{\left(\frac{b}{a}+1\right)a}+\frac{\gamma z}{\left(\frac{c}{a}+1\right)a}\right)x+\_F1\left(y\,x^{-\frac{b}{a}},z\,x^{-\frac{c}{a}}\right)$$

7.7.2.6 [1595] Problem 6

problem number 1595

Added June 1, 2019.

Problem Chapter 7.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+bxyw_y+cxzw_z=\alpha x+\beta y+\gamma z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]==alpha*x+beta*y+ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)+\frac{\alpha\log(x)}{a}+\frac{-a(\beta y+\gamma z)+b\gamma z+\beta cy}{x(a-b)(a-c)}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)=alpha*x+beta*y+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha\ln(x)}{a}+\_F1\left(y\,x^{-\frac{b}{a}},z\,x^{-\frac{c}{a}}\right)+\frac{-\frac{\beta y}{a-b}-\frac{\gamma z}{a-c}}{x}$$

7.7.2.7 [1596] Problem 7

problem number 1596

Added June 1, 2019.

Problem Chapter 7.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+bxyw_y+cz^2w_z=ky^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to-\frac{ky^2}{ax-2bx}+c_1\left(yx^{-\frac{b}{a}},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*x*y*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{-\frac{k\,y^2}{x} + (a-2b)\_F1\left(y\,x^{-\frac{b}{a}}, \frac{ax-cz}{axz}\right)}{a-2b}$$

7.7.2.8 [1597] Problem 8

problem number 1597

Added June 1, 2019.

Problem Chapter 7.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + by^2w_y + cz^2w_z = kxy$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*x*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{kxy\log\left(\frac{ax}{y}\right)}{ax-by} + c_1\left(\frac{b}{ax} - \frac{1}{y}, \frac{c}{ax} - \frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*x*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{kxy\ln\left(\frac{ax}{y}\right)}{-ax+by} + \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

7.7.2.9 [1598] Problem 9

problem number 1598

Added June 1, 2019.

Problem Chapter 7.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + by^2w_y + cz^2w_z = \alpha x^2 + \beta y^2 + \gamma z^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==alpha*x^2+beta*y^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a(ax-by)(ax-cz)c_1\left(\frac{b}{ax}-\frac{1}{y}, \frac{c}{ax}-\frac{1}{z}\right) + a^2x(\alpha x^2-\beta y^2-\gamma z^2) - a\alpha x^2(by+cz) + ayz}{a(ax-by)(ax-cz)}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=alpha*x^2+beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{\beta y^2}{ax-by} - \frac{\gamma z^2}{ax-cz} + \frac{\alpha x}{a} + \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

7.7.3 2.3

Local contents

7.7.3.1 [1599] Problem 1

problem number 1599

Added June 1, 2019.

Problem Chapter 7.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = xyz$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +b*D[w[x,y,z],z]==x*y*z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{12}x^2(ax(bx-2z) - 2bxy + 6yz) + c_1(y-ax, z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+b*diff(w(x,y,z),z)=x*y*z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{ab\,x^4}{12} + \frac{x^2yz}{2} + \frac{(-2az-2by)\,x^3}{12} + \_F1(-ax+y, -bx+z)$$

7.7.3.2 [1600] Problem 2

problem number 1600

Added June 1, 2019.

Problem Chapter 7.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = kx^3 + sy^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==k*x^3+s*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{x(3a^2(kx^3+4sy^2) - 12absxy + 4b^2sx^2)}{12a^3} + c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=k*x^3+s*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{k\,x^4}{4a} + \frac{sx\,y^2}{a} - \frac{bs\,x^2y}{a^2} + \frac{b^2s\,x^3}{3a^3} + \_F1\left(\frac{ay-bx}{a}, \frac{az-cx}{a}\right)$$

7.7.3.3 [1601] Problem 3

problem number 1601

Added June 1, 2019.

Problem Chapter 7.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = kx + s\sqrt{x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==k*x+s*Sqrt[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{6ac_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right) + 3kx^2 + 4sx^{3/2}}{6a}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=k*x+s*sqrt(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{k\,x^{2}}{2a}+\frac{2s\,x^{\frac{3}{2}}}{3a}+\_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}},z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

7.7.3.4 [1602] Problem 4

problem number 1602

Added June 1, 2019.

Problem Chapter 7.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=c\sqrt{x}+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c*Sqrt[x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*sqrt(x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}\frac{\sqrt{\frac{\sqrt{ab}\,x+\ln\left(\frac{\_aab+\sqrt{\left(a\,z^{2}+\left(\_a^{2}-y^{2}\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^{2}z^{2}}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}}\,c+s}{\sqrt{\left(a\,z^{2}+\left(\_a^{2}-y^{2}\right)b\right)a}}d\_a+\_F1\left(\frac{a\,z^{2}-b\,y^{2}}{a},-\frac{-\sqrt{}}{}\right.$$

7.7.3.5 [1603] Problem 5

problem number 1603

Added June 1, 2019.

Problem Chapter 7.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+by^2w_y+cz^2w_z=kxyz$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*x*y*z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{kxyz\left(by(ax-cz)\log\left(\frac{ax}{y}\right)+cz(by-ax)\log\left(\frac{ax}{z}\right)\right)}{(ax-by)(ax-cz)(by-cz)}+c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*x*y*z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left((ax-cz)\,by\ln\left(\frac{ax}{y}\right)-(ax-by)\,cz\ln\left(\frac{ax}{z}\right)\right)kxyz+(by-cz)\,(ax-cz)\,(ax-by)\_F1\left(\frac{ax-}{axy}\right.}{(by-cz)\,(ax-cz)\,(ax-by)}$$

7.7.4 2.4

Local contents

7.7.4.1 [1604] Problem 1

problem number 1604

Added June 10, 2019.

Problem Chapter 7.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = \alpha x^n + \beta y^m + \gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gamma*z^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)+\frac{\alpha x^{n+1}}{an+a}+\frac{\beta y^{m+1}}{bm+b}+\frac{\gamma z^{k+1}}{ck+c}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gamma*z^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{(m+1)\,(k+1)\,(n+1)\,abc\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)+(m+1)\,(k+1)\,\alpha bc\,x^{n+1}+(n+1)\,((m+1)}{(m+1)\,(k+1)\,(n+1)\,abc}$$

7.7.4.2 [1605] Problem 2

problem number 1605

Added June 10, 2019.

Problem Chapter 7.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+byw_y+czw_z=\alpha x^n+\beta y^m+\gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)+\frac{\alpha x^{n+1}}{an+a}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gamma
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha\,x^{n+1}}{(n+1)\,a}+\_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}},z\,\mathrm{e}^{-\frac{cx}{a}}\right)+\frac{\beta\,y^m}{bm}+\frac{\gamma\,z^k}{ck}$$

7.7.4.3 [1606] Problem 3

problem number 1606

Added June 10, 2019.

Problem Chapter 7.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=cx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^{n+1}}{n+1}+c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}\frac{c\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_aab+\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)^n}{\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}}d\_a+\_F1\left(\frac{a\,z^2-b\,y^2}{a},-\frac{-\sqrt{ab}}{}\right.$$

7.7.4.4 [1607] Problem 4

problem number 1607

Added June 10, 2019.

Problem Chapter 7.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+czw_z=\alpha x^n+\beta y^m+\gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)+\frac{\alpha x^n}{an}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{\alpha\_a^n+\beta\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m+\gamma\left(z\_a^{\frac{c}{a}}x^{-\frac{c}{a}}\right)^k}{\_aa}d\_a+\_F1\left(y\,x^{-\frac{b}{a}},z\,x^{-\frac{c}{a}}\right)$$

7.7.4.5 [1608] Problem 5

problem number 1608

Added June 10, 2019.

Problem Chapter 7.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+azw_y+byw_z=cx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^n}{n}+c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}},y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}\frac{c\left(x\left(\frac{\_aab+\sqrt{(a\,z^2+(\_a^2-y^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^n}{\sqrt{(a\,z^2+(\_a^2-y^2)\,b)\,a}}d\_a+\_F1\left(\frac{a\,z^2-b\,y^2}{a},x\left(az+\right.\right.$$

7.7.4.6 [1609] Problem 6

problem number 1609

Added June 10, 2019.

Problem Chapter 7.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x+b(ay+bz)w_y+a(ay-bz)w_z=cx^n$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]== c*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*b*x*diff(w(x,y,z),x)+b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=c*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\left(\int^{y} -\frac{c\left(x\left(\frac{\frac{\sqrt{2}\_a\,a^2}{-a^2y^2+2abyz+b^2z^2}+\sqrt{\frac{2\_a^2a^2}{-a^2y^2+2abyz+b^2z^2}+1}\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}{\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}\right)^{\frac{\sqrt{2}\,a}{2\sqrt{-a^2y^2+2abyz+b^2z^2}\sqrt{\frac{a^2}{-a^2y^2+2a}}}}}{\sqrt{-a^2y^2}}\right.$$

7.7.4.7 [1610] Problem 7

problem number 1610

Added June 10, 2019.

Problem Chapter 7.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = cx^k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*y^m*D[w[x, y,z], y] +b*x^nu*y^mu*z^lambda*D[w[x,y,z],z]== c*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)=c*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c\,x^{k+1} + (k+1)\_F1\left(\frac{(m-1)a\,x^{n+1}+(n+1)y^{-m+1}}{n+1}, (\lambda-1)\,b\left(\int^x \_a^{\nu}\left(\left(\frac{(-\_a^{n+1}+x^{n+1})(m-1)a+(n+1)y^{-}}{n+1}\right.\right.\right.\right.}{k+1}$$

7.7.4.8 [1611] Problem 8

problem number 1611

Added June 10, 2019.

Problem Chapter 7.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^{n_1}y + b_1x^{m_1})w_y + (a_2x^{n_2}y + b_2x^{m_2})w_z = c_2x^{k_2}y + c_1x^{k_1}z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n1*y+ b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\operatorname{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}+e^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}}y,(-1)^{-\frac{\mathrm{n2}+1}{\mathrm{n1}+1}}\mathrm{a2}e^{-\frac{\mathrm{a1}}{}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.7.4.9 [1612] Problem 9

problem number 1612

Added June 10, 2019.

Problem Chapter 7.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2x^{m_2})w_z=c_2x^{k_2}y+c_1x^{k_1}z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+ b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\frac{\mathrm{a2}^{-\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}\mathrm{c1}e^{\frac{\mathrm{a2}\left(K[1]^{\mathrm{n2}+1}-x^{\mathrm{n2}+1}\right)}{\mathrm{n2}+1}}\left((\mathrm{n2}+1)z\mathrm{a2}^{\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}+\mathrm{b2}e^{\frac{\mathrm{a2}x^{\mathrm{n2}+1}}{\mathrm{n2}+1}}(\mathrm{n2}+1)^{\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}\mathrm{Gamma}\left(\right.\right.}{\mathrm{n2}+1}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.7.4.10 [1613] Problem 10

problem number 1613

Added June 10, 2019.

Problem Chapter 7.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1y^k)w_y+(a_2x^{n_2}z+b_2z^m)w_z=cx^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+ b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^{s+1}}{s+1}+c_1\left(\mathrm{b1}(-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}}(\mathrm{n1}+1)^{-\frac{\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{1}{\mathrm{n1}+1}}(k-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{1}{\mathrm{n1}+1},-\frac{\mathrm{a1}(k-1)x^{\mathrm{n}}}{\mathrm{n1}+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{c\,x^{s+1}+(s+1)\_F1\left(\frac{-(n1+1)(n1+2)^2 b1\, x^{-n1}\left(-\frac{(k-1)a1\,x^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\frac{n1+2}{2n1+2},\frac{2n1+3}{2n1+2},-\frac{(k-1)a1\,x^{n1}}{n1+1}\right.}{}\right.}{}$$

7.7.4.11 [1614] Problem 11

problem number 1614

Added June 10, 2019.

Problem Chapter 7.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1y^k)w_y+(a_2x^{n_2}z+b_2z^m)w_z=c_1x^{s_1}+c_2y^{s_2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n1*y+ b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\text{c1}K[1]^{\text{s1}}+\text{c2}\left(\left(\left(\frac{(-1)^{-\frac{1}{\text{n1}+1}}\text{a1}^{-\frac{1}{\text{n1}+1}}\exp\left(-\frac{\text{a1}\left(x^{\text{n1}+1}+(k-1)K[1]^{\text{n1}+1}\right)}{\text{n1}+1}\right)(k-1)^{-\frac{1}{\text{n1}+1}}y^{-k}\right.\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c1\_a^{s1}+c2\left(\left(\frac{(n1+1)(n1+2)^2\,b1\_a^{-n1}\left(-\frac{(k-1)a1\_a^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}}\text{WhittakerM}\left(\frac{n1}{2n1\right.}\right.\right.\right.$$

7.7.4.12 [1615] Problem 12

problem number 1615

Added June 10, 2019.

Problem Chapter 7.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+yw_y+a\sqrt{x^2+y^2}w_z=bx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]== b*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{bx^n}{n}+c_1\left(\frac{y}{x},z-a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+a*sqrt(x^2+y^2)*diff(w(x,y,z),z)=  b*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{b\,x^n+n\_F1\left(\frac{y}{x},-\sqrt{x^2+y^2}\,a+z\right)}{n}$$

7.7.4.13 [1616] Problem 13

problem number 1616

Added June 10, 2019.

Problem Chapter 7.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+yw_y+(z-a\sqrt{x^2+y^2+z^2}w_z=bx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]== b*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{bx^n}{n}+c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+(z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)=  b*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{b\,x^n+n\_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)}{n}$$

7.7.5 3.1

Local contents

7.7.5.1 [1617] Problem 1

problem number 1617

Added June 11, 2019.

Problem Chapter 7.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ae^{\lambda x}w_y + be^{\beta x}w_z = ce^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{ce^{\gamma x}}{\gamma} + c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, z - \frac{be^{\beta x}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c\,e^{\gamma x} + \gamma\_F1\left(\frac{-a\,e^{\lambda x}+\lambda y}{\lambda}, \frac{-b\,e^{\beta x}+\beta z}{\beta}\right)}{\gamma}$$

7.7.5.2 [1618] Problem 2

problem number 1618

Added June 11, 2019.

Problem Chapter 7.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + be^{\beta y}w_z = ce^{\gamma y} + se^{\mu z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \left(e^{\gamma\left(\frac{a\left(-e^{\lambda x}+e^{\lambda K[1]}\right)}{\lambda}+y\right)}c+\exp\left(\frac{\mu\left(\lambda z-be^{\beta\left(y-\frac{ae^{\lambda x}}{\lambda}\right)}\mathrm{Ei}\left(\frac{a\beta e^{\lambda x}}{\lambda}\right)+be^{\beta\left(y-\frac{ae^{\lambda x}}{\lambda}\right)}\mathrm{Ei}\left(\frac{a\beta e^{\lambda K}}{\lambda}\right.\right.}{\lambda}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \left(c\,\mathrm{e}^{-\frac{\left(-a\,\mathrm{e}^{-a\lambda}+a\,\mathrm{e}^{\lambda x}-\lambda y\right)\gamma}{\lambda}}+s\,\mathrm{e}^{\frac{\left(\left(-\mathrm{expIntegral}\left(1,-\frac{a\beta\,\mathrm{e}^{-a\lambda}}{\lambda}\right)+\mathrm{expIntegral}\left(1,-\frac{a\beta\,\mathrm{e}^{\lambda x}}{\lambda}\right)\right)b\,\mathrm{e}^{-\frac{\left(a\,\mathrm{e}^{\lambda x}-\lambda y\right)\beta}{\lambda}}+\lambda z\right)\mu}{\lambda}}\right)d$$

7.7.5.3 [1619] Problem 3

problem number 1619

Added June 11, 2019.

Problem Chapter 7.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda y}w_y + be^{\beta y}w_z = ce^{\gamma x} + se^{\mu z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*y]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(e^{\gamma K[1]}c+\exp\left(-\frac{\mu\left(b\lambda(x-K[1])\left(a\lambda(x-K[1])+e^{-\lambda y}\right)^{-\frac{\beta}{\lambda}}+(\beta-\lambda)z+\frac{be^{-\lambda y}\left((c\right.}{}\right.}{\lambda-\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*y)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\,\mathrm{e}^{-a\gamma}+s\,\mathrm{e}^{\frac{\left(-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y}+(\beta-\lambda)az+\left((-\_a+x)a\lambda+\mathrm{e}^{-\lambda y}\right)b\left(\frac{1}{(-\_a+x)a\lambda+\mathrm{e}^{-\lambda y}}\right)^{\frac{\beta}{\lambda}}\right)\mu}{(\beta-\lambda)a}}\right)d\_a+\_F1\left(\frac{-a\lambda x}{a}\right.$$

7.7.5.4 [1620] Problem 4

problem number 1620

Added June 11, 2019.

Problem Chapter 7.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (A_1 e^{\alpha_1 x} + B_1 e^{\nu_1 x + \lambda y}) w_y + (A_2 e^{\alpha_2 x} + B_2 e^{\nu_2 x + \beta y}) w_z = k e^{\gamma z}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (A1*Exp[alpha1*x] +B1*Exp[nu1*x+lambda*y] )*D[w[x, y,z], y] +(A2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (A1*exp(alpha1*x) +B1*exp(nu1*x+lambda*y) )*diff(w(x,y,z),y)+(A2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \int^x k\, \mathrm{e}^{\left(z + \int \left(B2 \left(-B1\lambda \left(\int \mathrm{e}^{\frac{A1\lambda\, \mathrm{e}^{-f\alpha 1} + \_f\alpha 1\nu 1}{\alpha 1}} d\_f \right) + B1\lambda \left(\int \mathrm{e}^{\frac{A1\lambda\, \mathrm{e}^{\alpha 1 x}}{\alpha 1} + \nu 1 x} dx \right) + \mathrm{e}^{\frac{\left(A1\, \mathrm{e}^{\alpha 1 x} - \alpha 1 y\right)\lambda}{\alpha 1}} \right)^{-\frac{\beta}{\lambda}} \mathrm{e}^{\frac{A1\beta\, \mathrm{e}^{-f\alpha 1} + \_f}{\alpha 1}}\right.\right.}$$

7.7.5.5 [1621] Problem 5

problem number 1621

Added June 11, 2019.

Problem Chapter 7.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y + ce^{\gamma z}w_z = ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{ke^{x(\lambda-\alpha)}}{a(\alpha-\lambda)}+c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta},\frac{ce^{-\alpha x}}{a\alpha}-\frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*exp(alpha*x)*diff(w(x,y,z),x)+  b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{(-\alpha+\lambda)\,a\_F1\left(-\frac{(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y})\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta},\frac{(-a\alpha\,\mathrm{e}^{\alpha x}+c\gamma\,\mathrm{e}^{\gamma z})\mathrm{e}^{-\alpha x-\gamma z}}{\alpha c\gamma}\right)+k\,\mathrm{e}^{-(\alpha-\lambda)x}}{(-\alpha+\lambda)\,a}$$

7.7.5.6 [1622] Problem 6

problem number 1622

Added June 11, 2019.

Problem Chapter 7.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ae^{\beta y}w_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y,z], x] + b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{ke^{x(\lambda-\beta)}}{a(\beta-\lambda)} + c_1\left(\frac{ce^{-\beta x}}{a\beta} - \frac{e^{-\gamma z}}{\gamma}, y - \frac{be^{\alpha x-\beta x}}{a\alpha - a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*exp(beta*y)*diff(w(x,y,z),x)+  b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\alpha k\,\mathrm{e}^{-\_a\lambda}}{a\alpha\,\mathrm{e}^{\beta y} - \left(-\mathrm{e}^{\_a\alpha} + \mathrm{e}^{\alpha x}\right) b\beta} d\_a + \_F1\left(\frac{a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha x}}{\alpha b\beta}, -\frac{\left(\alpha c\gamma x - c\gamma \ln\left(\frac{a\alpha\,\mathrm{e}^{\beta y}}{b\beta}\right) + (a\alpha\,\mathrm{e}\right.}{(a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha x}}\right.$$

7.7.5.7 [1623] Problem 7

problem number 1623

Added June 11, 2019.

Problem Chapter 7.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1 + a_2e^{\alpha x})w_x + (b_1 + b_2e^{\beta y})w_y + (c_1 + c_2e^{\gamma z})w_z = k_1 + k_2e^{\alpha x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + (b1+b2*Exp[beta*y])*D[w[x, y,z], y] +(c1+c2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{(\text{a1k2}-\text{a2k1})\log\left(\text{a1}+\text{a2}e^{\alpha x}\right)+\text{a2}\alpha\text{k1}x}{\text{a1a2}\alpha}+c_1\left(\frac{\log\left(\frac{e^{\beta y}(\text{a1}+\text{a2}e^{\alpha x})^{\frac{\text{b1}\beta}{\text{a1}\alpha}}}{\text{b1}+\text{b2}e^{\beta y}}\right)}{\text{b1}\beta}-\frac{x}{\text{a1}},\frac{\log\left(\frac{e^{\gamma z}(\text{a1}}{\text{c}}\right.}{\text{c}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+ (b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1\,a2\alpha\_F1\left(\frac{-b1\beta x+a1\ln\left(\frac{\left(-b1+\mathrm{RootOf}\left(a1\alpha\beta y-a1\alpha\ln\left(\frac{(\_Z-b1)(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1\beta}{a1\alpha}}}{b2}\right)+b1\beta\ln(a2\,\mathrm{e}^{\alpha x}+a1)\right)\right)(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1}{a1}}}{b2\,\mathrm{RootOf}\left(a1\alpha\beta y-a1\alpha\ln\left(\frac{(\_Z-b1)(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1\beta}{a1\alpha}}}{b2}\right)+b1\beta\ln(a2\,\mathrm{e}^{\alpha x}+a1)\right)}\right.}{a1\,b1\beta}\right.}{}$$

7.7.5.8 [1624] Problem 8

problem number 1624

Added June 11, 2019.

Problem Chapter 7.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$e^{\beta y}(a_1+a_2e^{\alpha x})w_x+e^{\alpha x}(b_1+b_2e^{\beta y})w_y+ce^{\beta y+\gamma z}w_z=k_3e^{\beta y}(k_1+k_2e^{\alpha x})$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + Exp[alpha*x]*(b1+b2*Exp[beta*y])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{\text{k3}((\text{a1k2}-\text{a2k1})\log\left(\text{a1}+\text{a2}e^{\alpha x}\right)+\text{a2}\alpha\text{k1}x)}{\text{a1a2}\alpha}+c_1\left(\frac{c\log\left(\text{a1}+\text{a2}e^{\alpha x}\right)}{\text{a1}\alpha}-\frac{cx}{\text{a1}}-\frac{e^{-\gamma z}}{\gamma},\frac{\log}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+  exp(alpha*x)*(b1+b2*exp(beta*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(k1\,k3x+a1\_F1\left(\frac{\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}\operatorname{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{\_Z-b2}\right)-b2\beta\ln\left(a2\,e^{\alpha x}+a1\right)\right)}{-b2+\operatorname{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,e^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{\_Z-b2}\right)-b2\beta\ln\left(a2\,e^{\alpha x}+a1\right)\right)}\right)}{b2\beta},\frac{c\gamma\ln}{}\right.\right.}{a1\,a2\alpha}$$

7.7.6 3.2

Local contents

7.7.6.1 [1625] Problem 1

problem number 1625

Added June 11, 2019.

Problem Chapter 7.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + bx^m w_z = ce^{\lambda x} y + ke^{\beta x} z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +b*x^m*D[w[x,y,z],z]== c*Exp[lambda*x]*y+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, \frac{-bx^{m+1}+mz+z}{m+1}\right) - \frac{acx^n(-\lambda x)^{-n}\,\text{Gamma}(n+2,-\lambda x)}{\lambda^2(n+1)} - \frac{bkx^{\prime}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*x^n*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)=c*exp(lambda*x)*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{-(n+1)\,(m+1)\,(-\Gamma(m)+\Gamma(m,-\beta x))\,b\gamma k\,\lambda^2 m\,x^m(-\beta)^m\,(-\beta)^{-m}\,(-\beta x)^{-m} - ((n+1)\,(-}{}$$

7.7.6.2 [1626] Problem 2

problem number 1626

Added June 11, 2019.

Problem Chapter 7.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + bx^m w_z = cx^n y + ke^{\beta x}z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*x^m*D[w[x,y,z],z]== c*x^n*y+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, \frac{-bx^{m+1}+mz+z}{m+1}\right) + \frac{acx^n(-\lambda x)^{-n}\,\text{Gamma}(n+1,-\lambda x)}{\lambda^2} - \frac{bkx^m(-\beta x)^-}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*exp(lambda*x)*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)=c*x^n*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{bk\lambda\_a^{m+1}\mathrm{e}^{-a\beta} - bk\lambda\, x^{m+1}\mathrm{e}^{-a\beta} - (m+1)\left(-ac\_a^{n}\mathrm{e}^{-a\lambda} - k\lambda z\,\mathrm{e}^{-a\beta} - \lambda s\,\mathrm{e}^{-a\gamma} + \left(a\,\mathrm{e}^{\lambda x} -\right.\right.}{(m+1)\,\lambda}$$

7.7.6.3 [1627] Problem 3

problem number 1627

Added June 11, 2019.

Problem Chapter 7.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + byw_z = ke^{\beta x}z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== k*Exp[beta*x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, \frac{abe^{\lambda x}(\lambda x-1)}{\lambda^2} - bxy + z\right) + \frac{abke^{x(\beta+\lambda)}}{\beta^2(\beta+\lambda)} - \frac{bkye^{\beta x}}{\beta^2} + \frac{kze^{\beta x}}{\beta} + \frac{se^{\gamma x}}{\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*exp(lambda*x)*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=k*exp(beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{ab\,\beta^2\gamma k\,\mathrm{e}^{(\beta+\lambda)x} + (\beta+\lambda)\,\beta^2\gamma\,\lambda^2\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda}, \frac{(\lambda x-1)ab\,\mathrm{e}^{\lambda x}-(bxy-z)\lambda^2}{\lambda^2}\right) + \left(\beta^2\lambda^2 s\,\mathrm{e}^{\gamma x} + ((-\beta +}{(\beta+\lambda)\,\beta^2\gamma\,\lambda^2}$$

7.7.6.4 [1628] Problem 4

problem number 1628

Added June 11, 2019.

Problem Chapter 7.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ay^n w_y + bz^m w_z = ce^{\lambda x} + ke^{\beta y} + se^{\gamma z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*y^n*D[w[x, y,z], y] +b*z^m*D[w[x,y,z],z]== c*Exp[lambda*x]+k*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-ax - \frac{\left(\frac{1}{y}\right)^{n-1}}{n-1}, -bx - \frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right) + \frac{k\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{n}{n-1}}\left(-\beta\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{1}{1-n}}\right)^n \mathrm{Gamma}}{a\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+    a*y^n*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)=c*exp(lambda*x)+k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(c\,\mathrm{e}^{-\_a\lambda} + k\,\mathrm{e}^{\beta\left(\left((-\_a+x)(n-1)a\,y^{n}+y\right)y^{-n}\right)^{-\frac{1}{n-1}}} + s\,\mathrm{e}^{\gamma\left(\left((-\_a+x)(m-1)b\,z^{m}+z\right)z^{-m}\right)^{-\frac{1}{m-1}}} \right) d\_a + \_F$$

7.7.6.5 [1629] Problem 5

problem number 1629

Added June 11, 2019.

Problem Chapter 7.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\beta y}w_y + bz^m w_z = ce^{\lambda x} + ky^n + se^{\gamma z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[beta*y]*D[w[x, y,z], y] +b*z^m*D[w[x,y,z],z]== x*Exp[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{a\beta x + e^{-\beta y}}{\beta}, -bx - \frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right) + \frac{k\log^{n-1}\left(e^{-\beta y}\right)\left(-\frac{\log\left(e^{-\beta y}\right)}{\beta}\right)^{n+1}\left(-\log^{2}\left(e^{-\beta y}\right)\right)^{-n}}{a}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+    a*exp(beta*y)*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)=x*exp(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(\_a\,\mathrm{e}^{-a\lambda} + k \left(\frac{\ln\left(\frac{1}{(-\_a+x)a\beta+\mathrm{e}^{-\beta y}}\right)}{\beta} \right)^{n} + s\,\mathrm{e}^{\gamma\left(((-\_a+x)(m-1)b\,z^{m}+z)z^{-m}\right)^{-\frac{1}{m-1}}} \right) d\_a + \_F1\Big($$

7.7.6.6 [1630] Problem 6

problem number 1630

Added June 11, 2019.

Problem Chapter 7.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(y^2 + by + ae^{\alpha x}(y-b) - b^2\right) w_y + \left(z^2 + c(xz-1)e^{\beta x}\right) w_z = ke^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+b*y+a*Exp[alpha*x]*(y-b)-b^2 )*D[w[x, y,z], y] +( z^2+c*(x*z-
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   ( y^2+b*y+a*exp(alpha*x)*(y-b)-b^2 )*diff(w(x,y,z),y)+( z^2+c*(x*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.6.7 [1631] Problem 7

problem number 1631

Added June 11, 2019.

Problem Chapter 7.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(y^2 + ae^{\alpha x}(x+1)\right) w_y + \left(ce^{\beta x}z^2 + be^{-\beta x}\right) w_z = ke^{\lambda x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+a*Exp[alpha*x]*(x+1))*D[w[x, y,z], y] +( c*Exp[beta*x]*z^2+b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   ( y^2+a*exp(alpha*x)*(x+1))*diff(w(x,y,z),y)+( c*exp(beta*x)*z^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.6.8 [1632] Problem 8

problem number 1632

Added June 11, 2019.

Problem Chapter 7.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(ae^{\alpha x}y^2 + be^{-\alpha x}\right) w_y + \left(de^{\beta x}z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x})\right) w_z = ke^{\lambda x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a*Exp[alpha*x]*y^2+b*Exp[-alpha*x])*D[w[x, y,z], y] + ( d*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a*exp(alpha*x)*y^2+b*exp(-alpha*x))*diff(w(x,y,z),y)+ ( d*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.7.6.9 [1633] Problem 9

problem number 1633

Added June 11, 2019.

Problem Chapter 7.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k\right) w_y + \left(a_2 e^{\lambda_2 x} z + b_2 e^{\beta_2 x} z^m\right) w_z = c x^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a1*Exp[lambda1*x]*y+ b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +  ( a2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{cx^{s+1}}{s+1} + c_1\left((k-1)\int_1^x \text{b1}e^{\frac{\text{a1}e^{\text{lambda1}K[1]}(k-1)}{\text{lambda1}}+\text{beta1}K[1]}dK[1] + y^{1-k}e^{\frac{\text{a1}(k-1)e^{\text{lambda1}x}}{\text{lambda1}}}, (m-1)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a1*exp(lambda1*x)*y+ b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+ ( a2*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c\,x^{s+1} + (s+1)\_F1\left((k-1)\,b1\left(\int \mathrm{e}^{\frac{\beta1\lambda1x+(k-1)a1\,\mathrm{e}^{\lambda1x}}{\lambda1}}dx\right) + y^{-k+1}\mathrm{e}^{\frac{(k-1)a1\,\mathrm{e}^{\lambda1x}}{\lambda1}}, (m-1)\,b2\left(\int \mathrm{e}\right.\right.}{s+1}$$

7.7.6.10 [1634] Problem 10

problem number 1634

Added June 11, 2019.

Problem Chapter 7.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(a_1e^{\beta_1x}y + b_1e^{\gamma_1x}y^k\right)w_y + \left(a_2e^{\beta_2x}z + b_2e^{\gamma_1x+\lambda z}\right)w_z = cx^s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a1*Exp[beta1*x]*y+ b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +  ( a2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a1*exp(beta1*x)*y+ b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+ ( a2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.6.11 [1635] Problem 11

problem number 1635

Added June 11, 2019.

Problem Chapter 7.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^n + b_1 x^m e^{\lambda y}) w_y + \left(a_2 x^k + b_2 x^L e^{\beta z}\right) w_z = c x^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n+b1*x^m*Exp[lambda*y] )*D[w[x, y,z], y] +  ( a2*x^k+b2*x^L*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{c x^{s+1}}{s+1} + c_1\left(\frac{\text{b2}\beta x^{L+1}\left(-\frac{\text{a2}\beta x^{k+1}}{k+1}\right)^{-\frac{L+1}{k+1}} \text{Gamma}\left(\frac{L+1}{k+1}, -\frac{\text{a2}\beta x^{k+1}}{k+1}\right) - (k+1)e^{-\frac{\beta\left(-\text{a2}x^{k+1}+kz+\right.}{k+1}}}{\text{a2b2}\beta^2(k+1)(k-L)}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a1*x^n+b1*x^m*exp(lambda*y) )*diff(w(x,y,z),y)+ ( a2*x^k+b2*x^L*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c\,x^{s+1} + (s+1)\_F1\left(\frac{(n+1)^2\left(-a1\,\lambda\,x^{m+1} + (m+n+2)x^{m-n}\right)b1\left(-\frac{a1\,\lambda\,x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}} \text{WhittakerM}\left(\frac{m-n}{2n+2}, \frac{m+2n+3}{2n+2},\right.}{}\right.}{}$$

7.7.7 4.1

Local contents

7.7.7.1 [1636] Problem 1

problem number 1636

Added June 19, 2019.

Problem Chapter 7.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sinh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-bx) + \frac{c\sqrt{\cosh^2(\lambda x)}\operatorname{sech}(\lambda x)\sinh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{k+1}{2}, \frac{k+3}{2}\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\sinh^k(\lambda x)\right) dx + \_F1(-ax+y,-bx+z)$$

7.7.7.2 [1637] Problem 2

problem number 1637

Added June 19, 2019.

Problem Chapter 7.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = k\sinh(\beta y) + s\sinh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]== k*Sinh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{s\sinh\left(\frac{\gamma(a\lambda z - c\cosh(\lambda x) + c\cosh(\lambda K[1]))}{a\lambda}\right) + k\sinh\left(\beta\left(y + \frac{b(K[1]-x)}{a}\right)\right)}{a} dK[1] + c_1\left(y - \frac{bx}{a},\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(lambda*x)*diff(w(x,y,z),z)=k*sinh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{k\sinh\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right) + s\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}, \frac{a\lambda z-c}{}\right.$$

7.7.7.3 [1638] Problem 3

problem number 1638

Added June 19, 2019.

Problem Chapter 7.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sinh^n(\beta x)w_y + c\sinh^k(\lambda x)w_z = c\sinh^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[beta*x]^n*D[w[x, y,z], y] +  c*Sinh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{a\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{n+3}{2}, -\sinh^2(\beta x\right.\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*sinh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\sinh^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\sinh^n(\beta x))\,dx\right), z - \left(\int c(\sinh^k(\lambda x))\,dx\right)\right)$$

7.7.7.4 [1639] Problem 4

problem number 1639

Added June 19, 2019.

Problem Chapter 7.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\lambda x)w_z = k\sinh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{k\sinh\left(\frac{\gamma(a\lambda z - c\cosh(\lambda x) + c\cosh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1] + c_1\left(z - \frac{c\cosh(\lambda x)}{a\lambda}, \frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{b}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{k \sinh\left(\frac{(a\lambda z + c\cosh(\_a\lambda) - c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a} d\_a + \_F1\left(\frac{-b\beta x - 2a \operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta}, \frac{a\lambda z - c\cosh(\lambda x}{a\lambda}\right.$$

7.7.7.5 [1640] Problem 5

problem number 1640

Added June 19, 2019.

Problem Chapter 7.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sinh^{n_1}(\lambda_1 x) w_x + b_1 \sinh^{m_1}(\beta_1 y) w_y + c_1 \sinh^{k_1}(\gamma_1 z) w_z = a_2 \sinh^{n_2}(\lambda_2 x) + b_2 \sinh^{m_2}(\beta_2 y) w_y + c_2 \sinh^{k_2}(\gamma_2 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Sinh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Sinh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Sin
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{\sinh^{-\text{n1}}(\text{lambda1}K[3])\left(\text{c2}\sinh^{\text{k2}}(\text{gamma2}K[3]) + \text{b2}\sinh^{\text{m2}}(\text{beta2}K[3]) + \text{a2}\sinh^{\text{n2}}\right.}{\text{a1}}\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde := a1*sinh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*sinh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*sinh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.7.8 4.2

Local contents

7.7.8.1 [1641] Problem 1

problem number 1641

Added June 19, 2019.

Problem Chapter 7.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cosh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cosh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\sqrt{-\sinh^2(\lambda x)}\operatorname{csch}(\lambda x)\cosh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k}{}\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*cosh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\cosh^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

7.7.8.2 [1642] Problem 2

problem number 1642

Added June 19, 2019.

Problem Chapter 7.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cosh(\lambda x)w_z=k\cosh(\beta y)+s\cosh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]== k*Cosh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{k\cosh\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a},\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cosh(lambda*x)*diff(w(x,y,z),z)=k*cosh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{k\cosh\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)+s\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c}{}\right.$$

7.7.8.3 [1643] Problem 3

problem number 1643

Added June 19, 2019.

Problem Chapter 7.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\beta x)w_y+b\cosh^k(\lambda x)w_z=c\cosh^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[beta*x]^n*D[w[x, y,z], y] +  b*Cosh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{a\sinh(\beta x)\cosh^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cosh^2(\beta x)\right)}{(\beta n+\beta)\sqrt{-\sinh^2(\beta x)}}+y,\frac{b\sinh(\lambda x)}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*cosh(beta*x)^n*diff(w(x,y,z),y)+ b*cosh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\cosh^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\cosh^n(\beta x))\,dx\right),z-\left(\int b\left(\cosh^k(\lambda x)\right)dx\right)\right)$$

7.7.8.4 [1644] Problem 4

problem number 1644

Added June 19, 2019.

Problem Chapter 7.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\lambda x)w_z=k\cosh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{k\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sinh}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+ c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{k\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\sinh\left(\lambda x\right)}{a\lambda}\right.$$

7.7.8.5 [1645] Problem 5

problem number 1645

Added June 19, 2019.

Problem Chapter 7.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\gamma z)w_z=p\cosh(\lambda x)+q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{p\sinh(\lambda x)+\lambda qx}{a\lambda}+c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+ c*cosh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a\lambda\_F1\left(\frac{-b\beta x+2a\arctan(e^{\beta y})}{b\beta},\frac{-c\gamma x+2a\arctan(e^{\gamma z})}{c\gamma}\right)+\lambda qx+p\sinh(\lambda x)}{a\lambda}$$

7.7.8.6 [1646] Problem 6

problem number 1646

Added June 19, 2019.

Problem Chapter 7.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cosh^{n_1}(\lambda_1 x)w_x+b_1\cosh^{m_1}(\beta_1 y)w_y+c_1\cosh^{k_1}(\gamma_1 z)w_z=a_2\cosh^{n_2}(\lambda_2 x)+b_2\cosh^{m_2}(\beta_2 y)w_y+c_2\cosh^{k_2}(\gamma_2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Cosh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cosh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Cos
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{\cosh^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\cosh^{\mathrm{k2}}(\mathrm{gamma2}K[3])+\mathrm{b2}\cosh^{\mathrm{m2}}(\mathrm{beta2}K[3])+\mathrm{a2}\cosh^{\mathrm{n}}\right.}{\mathrm{a1}}\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde := a1*cosh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cosh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cosh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.7.9 4.3

Local contents

7.7.9.1 [1647] Problem 1

problem number 1647

Added June 20, 2019.

Problem Chapter 7.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tanh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\tanh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\tanh^2(\lambda x)\right)}{k\lambda+\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\tanh^k(\lambda x)\right)dx + \_F1(-ax+y,-bx+z)$$

7.7.9.2 [1648] Problem 2

problem number 1648

Added June 20, 2019.

Problem Chapter 7.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tanh(\lambda x)w_z = k\tanh(\beta y) + s\tanh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]== k*Tanh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{k\tanh\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\tanh\left(\frac{\gamma(a\lambda z-c\log(\cosh(\lambda x))+c\log(\cosh(\lambda K[1])))}{a\lambda}\right)}{a}dK[1]+c_1\left(\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tanh(lambda*x)*diff(w(x,y,z),z)=k*tanh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{(k-s)\sinh\left(\frac{c\gamma\ln(\tanh(\_a\lambda)-1)+c\gamma\ln(\tanh(\_a\lambda)+1)-c\gamma\ln(\tanh(\lambda x)-1)-c\gamma\ln(\tanh(\lambda x)+1)+2(-a\gamma z+(ay-(-\_}{2a\lambda}\right.}{\left(\cosh\left(\frac{c\gamma\ln(\tanh(\_a\lambda)-1)+c\gamma\ln(\tanh(\_a\lambda)+1)-c\gamma\ln(\tanh(\lambda x)-1)-c\gamma\ln(\tanh(\lambda x)+1)+2(-a\gamma z+(ay-(-\_}{2a\lambda}\right.\right.}$$

7.7.9.3 [1649] Problem 3

problem number 1649

Added June 19, 2019.

Problem Chapter 7.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\beta x)w_y+c\tanh^k(\lambda x)w_z=c\tanh^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[beta*x]^n*D[w[x, y,z], y] +  b*Tanh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{a\tanh^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\tanh^2(\beta x)\right)}{\beta n+\beta},z-\frac{b\tanh^{k+1}(\lambda x)\,\text{F}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*tanh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\tanh^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\tanh^k(\lambda x))\,dx\right)\right)$$

7.7.9.4 [1650] Problem 4

problem number 1650

Added June 19, 2019.

Problem Chapter 7.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\tanh(\beta y)w_y+c\tanh(\lambda x)w_z=k\tanh(\gamma z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{k \sinh\left(\frac{\left(a\lambda z - c\ln(\cosh(\lambda x)) + c\ln\left(\cosh\left(\frac{(2b\beta x - a\ln(\tanh(\_a\beta)-1) - a\ln(\tanh(\_a\beta)+1) + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(}{2b\beta}\right.\right.\right.}{a\lambda}\right.}{b\cosh\left(\frac{\left(a\lambda z - c\ln(\cosh(\lambda x)) + c\ln\left(\cosh\left(\frac{(2b\beta x - a\ln(\tanh(\_a\beta)-1) - a\ln(\tanh(\_a\beta)+1) + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1)+}{2b\beta}\right.\right.\right.}{a\lambda}\right.}$$

7.7.9.5 [1651] Problem 5

problem number 1651

Added June 19, 2019.

Problem Chapter 7.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = k\tanh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{k\log(\cosh(\lambda x))}{a\lambda} + c_1\left(\frac{1}{2}\left(\frac{\log(\sinh(\beta y))}{\beta} - \frac{bx}{a}\right), \frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\sinh(\beta y))}{\beta}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = \frac{a\lambda\_F1\left(\frac{2b\beta x+a\ln(\tanh(\beta y)-1)+a\ln(\tanh(\beta y)+1)-2a\ln(\tanh(\beta y))}{2b\beta}, \frac{b\beta\ln\left(\sqrt{-\left(-\frac{1}{\left(e^{2\beta y}-1\right)^2}\right)^{\frac{c\gamma}{b\beta}}}\sinh(\gamma z)\right)+(\beta y+\ln}{\beta c\gamma}\right.}{a\lambda}$$

7.7.9.6 [1652] Problem 6

problem number 1652

Added June 19, 2019.

Problem Chapter 7.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\tanh(\lambda x)w_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*tanh(lambda*x)*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = \frac{2a\lambda\_F1\left(\frac{a\lambda\ln\left(\sqrt{-\left(-\frac{1}{\left(e^{2\lambda x}-1\right)^2}\right)^{\frac{b\beta}{a\lambda}}}\sinh(\beta y)\right)+(\lambda x+\ln(2))b\beta}{b\beta\lambda}, \frac{a\lambda\ln\left(\sqrt{-\left(-\frac{1}{\left(e^{2\lambda x}-1\right)^2}\right)^{\frac{c\gamma}{a\lambda}}}\sinh(\gamma z)\right)+(\lambda x+\ln(2\right.}{c\gamma\lambda}}{2a\lambda}$$

7.7.9.7 [1653] Problem 7

problem number 1653

Added June 19, 2019.

Problem Chapter 7.4.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\tanh^{n_1}(\lambda_1 x)w_x+b_1\tanh^{m_1}(\beta_1 y)w_y+c_1\tanh^{k_1}(\gamma_1 z)w_z = a_2\tanh^{n_2}(\lambda_2 x)+b_2\tanh^{m_2}(\beta_2 y)w_y+c_2\tanh^{k_2}(\gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Tanh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Tanh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Tan
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{\tanh^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\tanh^{\mathrm{k2}}(\mathrm{gamma2}K[3])+\mathrm{b2}\tanh^{\mathrm{m2}}(\mathrm{beta2}K[3])+\mathrm{a2}\tanh\right.}{\mathrm{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a1*tanh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*tanh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*tanh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{b2\left(\tanh^{m2}(\beta 2x)\right)\left(\tanh^{-n1}(\lambda 1x)\right) + c2\left(\tanh^{k2}(\gamma 2x)\right)\left(\tanh^{-n1}(\lambda 1x)\right) + a2\left(\tanh^{-n1+n}\right.}{a1}$$

7.7.10 4.4

Local contents

7.7.10.1 [1654] Problem 1

problem number 1654

Added June 20, 2019.

Problem Chapter 7.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Coth[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\coth^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\lambda x)\right)}{k\lambda+\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*coth(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\coth^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

7.7.10.2 [1655] Problem 2

problem number 1655

Added June 20, 2019.

Problem Chapter 7.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\coth(\lambda x)w_z=k\coth(\beta y)+s\coth(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]== k*Coth[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow\int_1^x\frac{k\coth\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\coth\left(\frac{\gamma(a\lambda z-c\log(\sinh(\lambda x))+c\log(\sinh(\lambda K[1])))}{a\lambda}\right)}{a}dK[1]+c_1\Big(y$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*coth(lambda*x)*diff(w(x,y,z),z)=k*coth(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{(-k+s)\sinh\left(\frac{c\gamma\ln(\coth(\_a\lambda)-1)+c\gamma\ln(\coth(\_a\lambda)+1)-c\gamma\ln(\coth(\lambda x)-1)-c\gamma\ln(\coth(\lambda x)+1)+2(-a\gamma z+(ay-(-}{2a\lambda}\right.}{\left(-\cosh\left(\frac{c\gamma\ln(\coth(\_a\lambda)-1)+c\gamma\ln(\coth(\_a\lambda)+1)-c\gamma\ln(\coth(\lambda x)-1)-c\gamma\ln(\coth(\lambda x)+1)+2(-a\gamma z+(ay-(-}{2a\lambda}\right.\right.}$$

7.7.10.3 [1656] Problem 3

problem number 1656

Added June 20, 2019.

Problem Chapter 7.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\coth^n(\beta x)w_y+b\coth^k(\lambda x)w_z=c\coth^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Coth[beta*x]^n*D[w[x, y,z], y] +  b*Coth[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{b\coth^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\lambda x)\right)}{k\lambda+\lambda},y-\frac{a\coth^{n+1}(\beta x)\,\text{Hy}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*coth(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\coth^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\coth^n(\beta x))\,dx\right),z-\left(\int b\left(\coth^k(\lambda x)\right)dx\right)\right)$$

7.7.10.4 [1657] Problem 4

problem number 1657

Added June 20, 2019.

Problem Chapter 7.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta y)w_y+c\coth(\lambda x)w_z=k\coth(\gamma z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+ c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\left(\int^{x} \frac{k\cosh\left(\frac{(-2a\lambda z+c\ln(\coth(\_a\lambda)-1)+c\ln(\coth(\_a\lambda)+1)-c\ln(\coth(\lambda x)-1)-c\ln(\coth(\lambda x)+1))\gamma}{2a\lambda}\right)}{a\sinh\left(\frac{(-2a\lambda z+c\ln(\coth(\_a\lambda)-1)+c\ln(\coth(\_a\lambda)+1)-c\ln(\coth(\lambda x)-1)-c\ln(\coth(\lambda x)+1))\gamma}{2a\lambda}\right)} d\_a\right) + \_$$

7.7.10.5 [1658] Problem 5

problem number 1658

Added June 20, 2019.

Problem Chapter 7.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = p\coth(\lambda x) + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a\lambda q\log(\cosh(\beta y)) + b\beta p\log(-\tanh(\lambda x)) + b\beta p\log(\cosh(\lambda x))}{ab\beta\lambda} + c_1\left(\frac{a\log(\operatorname{sech}(\beta y)) + b}{2a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+ c*coth(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{2a\lambda\_F1\left(\frac{-2b\beta x+a\ln\left(\frac{(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-2}\right)-a\ln(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1)))}{2b\beta}, \frac{-2c\gamma x+a\ln\left(\frac{(\mathrm{RootOf}(\gamma z-a}{\mathrm{RootOf}(\gamma z-a}\right.}{}\right.}{2}$$

7.7.10.6 [1659] Problem 6

problem number 1659

Added June 20, 2019.

Problem Chapter 7.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\coth^{n_1}(\lambda_1 x)w_x+b_1\coth^{m_1}(\beta_1 y)w_y+c_1\coth^{k_1}(\gamma_1 z)w_z = a_2\coth^{n_2}(\lambda_2 x)+b_2\coth^{m_2}(\beta_2 y)w_y+c_2\coth^{k_2}(\gamma_2 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Coth[lambda1*x]^n1*D[w[x, y,z], x] + b1*Coth[beta1*x]^m1*D[w[x, y,z], y] +  c1*Cot
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{\coth^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\coth^{\mathrm{k2}}(\mathrm{gamma2}K[3])+\mathrm{b2}\coth^{\mathrm{m2}}(\mathrm{beta2}K[3])+\mathrm{a2}\coth^{\mathrm{n}}\right.}{\mathrm{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a1*coth(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*coth(beta1*x)^m1*diff(w(x,y,z),y)+ c1*coth
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{b2\left(\coth^{m2}(\beta 2x)\right)\left(\coth^{-n1}(\lambda 1x)\right) + c2\left(\coth^{k2}(\gamma 2x)\right)\left(\coth^{-n1}(\lambda 1x)\right) + a2\left(\coth^{-n1+n2}\right.}{a1}$$

7.7.11 4.5

Local contents

7.7.11.1 [1660] Problem 1

problem number 1660

Added June 20, 2019.

Problem Chapter 7.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh^n(\lambda y)w_z = s\cosh^m(\beta x) + k\sinh^r(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*y]^n*D[w[x,y,z],z]== s*Cosh[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sqrt{\cosh^2(\lambda y)}\text{sech}(\lambda y)\sinh^{n+1}(\lambda y)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\text{si}\right.}{b\lambda n+b\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(lambda*y)^n*diff(w(x,y,z),z)=s*cosh(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{k\left(\sinh^r\left(\frac{(ay-(-\_a+x)b)\gamma}{a}\right)\right)+s(\cosh^m(\_a\beta))}{a}d\_a+\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c\left(\sinh^n\left(\frac{(a}{}\right.\right.}{}\right.\right.$$

7.7.11.2 [1661] Problem 2

problem number 1661

Added June 20, 2019.

Problem Chapter 7.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\lambda x)w_y+b\cosh^m(\beta x)w_z=s\cosh^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[lambda*x]^n*D[w[x, y,z], y] +  b*Cosh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{s\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cosh^2(\gamma x)\right)}{\gamma k+\gamma}+c_1\left(\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sinh(lambda*x)^n*diff(w(x,y,z),y)+ b*cosh(beta*x)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int s\left(\cosh^k(\gamma x)\right)dx+\_F1\left(y-\left(\int a(\sinh^n(\lambda x))\,dx\right),z-\left(\int b(\cosh^m(\beta x))\,dx\right)\right)$$

7.7.11.3 [1662] Problem 3

problem number 1662

Added June 20, 2019.

Problem Chapter 7.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\lambda x)w_y+b\sinh^m(\beta y)w_z=s\sinh^k(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[lambda*x]^n*D[w[x, y,z], y] +  b*Sinh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x s\sinh^k\left(\frac{\gamma\left(-b\sqrt{\cosh^2(\beta x)}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\sinh^2(\beta x)\right)\operatorname{sech}(\beta x)\sinh^{m+1}(\beta x)+\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cosh(lambda*x)^n*diff(w(x,y,z),y)+ b*sinh(beta*y)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x s\left(\sinh^k\left(\left(\left(b\left(\int\left(\sinh^m\left(\left(\left(a\left(\int\left(\cosh^n\left(\_f\lambda\right)\right)d\_f\right)+y-\left(\int a\left(\cosh^n\left(\lambda x\right)\right)dx\right)\right)\beta\right)\right.\right.\right.\right.\right.\right.\right.\right.$$

7.7.11.4 [1663] Problem 4

problem number 1663

Added June 20, 2019.

Problem Chapter 7.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\lambda x)w_y+b\coth^m(\beta x)w_z=s\coth^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[lambda*x]^n*D[w[x, y,z], y] +  b*Coth[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{s\coth^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\gamma x)\right)}{\gamma k+\gamma}+c_1\left(z-\frac{b\coth^{m+1}(\beta x)\,\text{Hyp}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tanh(lambda*x)^n*diff(w(x,y,z),y)+ b*coth(beta*x)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int s\left(\coth^k(\gamma x)\right)dx+\_F1\left(y-\left(\int a\left(\tanh^n(\lambda x)\right)dx\right),z-\left(\int b\left(\coth^m(\beta x)\right)dx\right)\right)$$

7.7.11.5 [1664] Problem 5

problem number 1664

Added June 20, 2019.

Problem Chapter 7.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x+b\sinh(\beta y)w_y+c\sinh(\gamma z)w_z=k\cosh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{k\log(\sinh(\lambda x))}{a\lambda}+c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta},\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(gamma*z)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a\lambda\_F1\left(\frac{-2a\lambda\operatorname{arctanh}(e^{\beta y})+2b\beta\operatorname{arctanh}(e^{\lambda x})}{b\beta\lambda},\frac{-2a\lambda\operatorname{arctanh}(e^{\gamma z})+2c\gamma\operatorname{arctanh}(e^{\lambda x})}{c\gamma\lambda}\right)+k\ln(\sinh(\lambda x))}{a\lambda}$$

7.7.12 5.1

Local contents

7.7.12.1 [1665] Problem 1

problem number 1665

Added June 26, 2019.

Problem Chapter 7.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\ln^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Log[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\log^k(\lambda x)(-\log(\lambda x))^{-k}\operatorname{Gamma}(k+1,-\log(\lambda x))}{\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*ln(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\ln(\lambda x)^k\,dx+\_F1(-ax+y,-bx+z)$$

Contains unresolve integral because maple can not integrate $\ln^n(x)$

7.7.12.2 [1666] Problem 2

problem number 1666

Added June 26, 2019.

Problem Chapter 7.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\ln(\beta y)\ln(\gamma z)w_z = k\ln(\alpha x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Log[beta*y]*Log[gamma*z]*D[w[x,y,z],z]== k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*ln(beta*y)*ln(gamma*z)*diff(w(x,y,z),z)=k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \frac{a\_F1\left(\frac{-ay+bx}{b}, \frac{-(\ln(\beta y)-1)c\gamma y - b\,\mathrm{expIntegral}(1, -\ln(\gamma z))}{c\gamma}\right) + (\ln(\alpha x) - 1)\,kx}{a}$$

7.7.12.3 [1667] Problem 3

problem number 1667

Added June 26, 2019.

Problem Chapter 7.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = c\ln^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to x(c\log(\gamma x) - c + s) + c_1\left(y - \frac{a(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1, -\log(\beta x))}{\beta}, z - \right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = cx\ln(\gamma x) + (-c+s)\,x + \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int b\ln(\lambda x)^k\,dx\right)\right)$$

Contains unresolve integral because maple can not integrate $\ln^n(x)$

7.7.12.4 [1668] Problem 4

problem number 1668

Added June 26, 2019.

Problem Chapter 7.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\lambda x)w_y + b\ln^m(\beta y)w_z = c\ln^k(\gamma y) + s\ln^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*Log[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*ln(beta*y)^m*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\ln\left(\left(a\left(\int \ln(\_f\lambda)^n d\_f\right) + y - \left(\int a\ln(\lambda x)^n dx\right)\right)\gamma\right)^k + s\ln\left(\left(b\left(\int \ln\left(\left(a\left(\int\right.\right.\right.\right.\right.\right.$$

7.7.12.5 [1669] Problem 5

problem number 1669

Added June 26, 2019.

Problem Chapter 7.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\ln^{n_1}(\lambda_1 x)w_x + b_1\ln^{m_1}(\beta_1 y)w_y + c_1\ln^{k_1}(\gamma_1 z)w_z = a_2\ln^{n_2}(\lambda_2 x) + b_2\ln^{m_2}(\beta_2 y) + c_2\ln^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Log[lambda1*x]^n1*D[w[x, y,z], x] + b1*Log[beta1*y]^m1*D[w[x, y,z], y] +  c1*Log[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*ln(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*ln(beta1*y)^m1*diff(w(x,y,z),y)+ c1*ln(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{\left(a2 \ln\left(\_f\lambda 2\right)^{n2} + b2 \ln\left(\beta 2 \operatorname{RootOf}\left(a1 \left(\int^{-Z} \ln\left(\_a\beta 1\right)^{-m1} d\_a\right) - b1 \left(\int \ln\left(\_f\lambda 1\right)^{-n1}\right.\right.\right.\right.}{}$$

Contains RootOf and unresolved integrals $\ln^n(x)$

7.7.13 5.2

Local contents

7.7.13.1 [1670] Problem 1

problem number 1670

Added June 26, 2019.

Problem Chapter 7.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c x^n \ln^k(\lambda y) w_z = s y^m \ln^r(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*x^n*Log[lambda*y]*D[w[x,y,z],z]== s*y^m*Lo
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{s\left(y+\frac{b(K[1]-x)}{a}\right)^m\log^r(\beta K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{bcx^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{bx}{bx-ay}\right.}{a(n+1)(n+2)(ay-bx)}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*x^n*ln(lambda*y)^k*diff(w(x,y,z),z)=s*y^m*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{s\left(\frac{ay-(-\_a+x)b}{a}\right)^m\ln(\_a\beta)^m}{a}d\_a+\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c\_a^n\ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^k}{a}d\_\right.\right.$$

Answer has unresolved integrals

7.7.13.2 [1671] Problem 2

problem number 1671

Added June 26, 2019.

Problem Chapter 7.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ax^nw_y+bx^mw_z=cy\ln^k(\lambda x)+sz\ln^r(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +  b*x^m*D[w[x,y,z],z]== c*y*Log[lambda*x]^k+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{(m+2)(-\log(\beta x))^{-r}(-\log(\lambda x))^{-k}\left(-\beta c(m+1)\left(ax^{n+1}-(n+1)y\right)(-\log(\beta x))^{r}\log^{k}(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\lambda x))-\lambda(n+1\right.}{\beta\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)= c*y*ln(lambda*x)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{\left(a\_a^{n+1}-a\,x^{n+1}+ny+y\right)(m+1)\,c\ln\left(\_a\lambda\right)^{k}+(n+1)\left(b\_a^{m+1}-b\,x^{m+1}+mz+z\right)}{(n+1)\,(m+1)}$$

Answer has unresolved integrals

7.7.13.3 [1672] Problem 3

problem number 1672

Added June 26, 2019.

Problem Chapter 7.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\lambda x)w_y + by^m w_z = c\ln^k(\beta x) + s\ln^r(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*y^m*D[w[x,y,z],z]== c*Log[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\log^k(\beta K[3])+s\log^r\left(\gamma\left(z-\int_1^x b\left(y-\int_1^x a\log^n(\lambda K[1])dK[1]+\int_1^{K[2]} a\log^n(\lambda\right.\right.\right.\right.\right.\right.$$

Generated internal errors from solve : inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*y^m*diff(w(x,y,z),z)= c*ln(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\ln\left(\_b\beta\right)^k+s\ln\left(-\frac{\left(-(m+1)\,az-\left(\_ba+y\ln\left(\_a\lambda\right)^{-n}-\ln\left(\_a\lambda\right)^{-n}\left(\int a\ln\left(\lambda x\right)^n d\right.\right.\right.}{}\right.\right.$$

Answer has unresolved integrals and RootOf

7.7.13.4 [1673] Problem 4

problem number 1673

Added June 26, 2019.

Problem Chapter 7.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln^n(\lambda x)w_x+zw_y+b\ln^k(\beta y)w_z=cx^m+s\ln(\gamma y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y,z], x] + z*D[w[x, y,z], y] +  b*Log[beta*y]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*ln(lambda*x)^n*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ b*ln(beta*y)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{c \operatorname{RootOf}\left(b \ln\left(\_a\beta\right)^{k} \left(\int \frac{\ln(\lambda x)^{-n}}{a} dx\right) - b \ln\left(\_a\beta\right)^{k} \left(\int^{\_Z} \frac{\ln(\_a\lambda)^{-n}}{a} d\_a\right) - \sqrt{2by \ln\left(\_a\beta\right.}}{\sqrt{2b\left(\int \ln\left(\_f\beta\right)^{k} d\_f\right)}}$$

7.7.13.5 [1674] Problem 5

problem number 1674

Added June 26, 2019.

Problem Chapter 7.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax \ln^{n}(x) w_x + by \ln^{m}(y) w_y + cz \ln(z)^{r} w_z = k \ln^{s}(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +  c*z*Log[z]^r*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{k(m-1)^{\frac{1}{m-1}} \log(y) \left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m \log^s \left(\exp\left(\left(\left((m-1)^{\frac{m}{m-1}} \log(y) \left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m\right)^{\frac{1}{1-m}}\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+ b*y*ln(y)^m*diff(w(x,y,z),y)+ c*z*ln(z)^r*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(n-s-1)\,a\_F1\left(\frac{-(n-1)a\ln(y)^{-m+1}+(m-1)b\ln(x)^{-n+1}}{(n-1)(m-1)b}, \frac{-(n-1)a\ln(z)^{-r+1}+(r-1)c\ln(x)^{-n+1}}{(r-1)(n-1)c}\right) - k\ln(x)^{-}}{(n-s-1)\,a}$$

7.7.14 6.1

Local contents

7.7.14.1 [1675] Problem 1

problem number 1675

Added June 26, 2019.

Problem Chapter 7.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\sin^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Sin[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-cx)+\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sin(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\sin^k(\lambda x)\right)dx + \_F1(-ax+y,-bx+z)$$

7.7.14.2 [1676] Problem 2

problem number 1676

Added June 26, 2019.

Problem Chapter 7.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\sin(\gamma z)w_z = k\sin(\alpha x) + s\sin(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sin[gamma*z]*D[w[x,y,z],z]== k*Sin[alpha*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)-\frac{k\cos(\alpha x)}{a\alpha}-\frac{s\cos(\beta y)}{b\beta}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sin(gamma*z)*diff(w(x,y,z),z)= k*sin(alpha
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a\alpha b\beta\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\operatorname{RootOf}\left(\gamma z-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{c\gamma x}{a}}}{\_Z^2\mathrm{e}^{\frac{2c\gamma x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2c\gamma x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2c\gamma x}{a}}+1}\right)\right)\right)}{c\gamma}\right)-a\alpha s\cos\left(\beta y\right)-b\beta k\cos\left(\right.}{a\alpha b\beta}$$

7.7.14.3 [1677] Problem 3

problem number 1677

Added June 26, 2019.

Problem Chapter 7.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta x)w_z = c\sin^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{c\sqrt{\cos^2(\gamma x)}\sec(\gamma x)\sin^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(\gamma x)\right)}{\gamma k+\gamma}+c_1\left(z-\frac{b\sqrt{}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int c\left(\sin^k(\gamma x)\right)dx+\_F1\left(y-\left(\int a\left(\sin^n(\lambda x)\right)dx\right),z-\left(\int b\left(\sin^m(\beta x)\right)dx\right)\right)$$

7.7.14.4 [1678] Problem 4

problem number 1678

Added June 26, 2019.

Problem Chapter 7.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta y)w_z = c\sin^k(\gamma y) + s\sin^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\sin^k\left(\frac{\gamma\left(-a\sqrt{\cos^2(\lambda x)}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)\sec(\lambda x)\sin^{n+1}(\lambda x)+a\sqrt{\cos^2(\right.}{\lambda(n}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\left(\sin^k\left(\left(a\left(\int(\sin^n(\_f\lambda))\,d\_f\right)+y-\left(\int a(\sin^n(\lambda x))\,dx\right)\right)\gamma\right)\right)+s\left(\sin^r\left(\left(b\left(\int\right.\right.\right.\right.\right.$$

7.7.14.5 [1679] Problem 5

problem number 1679

Added June 26, 2019.

Problem Chapter 7.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sin(\beta x)w_y + c\sin(\lambda x)w_z = k\sin(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*x]*D[w[x, y,z], y] +  c*Sin[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{k\sin\left(\frac{\gamma(a\lambda z+c\cos(\lambda x)-c\cos(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(\frac{b\cos(\beta x)}{a\beta}+y,\frac{c\cos(\lambda x)}{a\lambda}+z\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*x)*diff(w(x,y,z),y)+ c*sin(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=-\left(\int^x-\frac{k\sin\left(\frac{(a\lambda z-c\cos(\_a\lambda)+c\cos(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a\right)+\_F1\left(\frac{a\beta y+b\cos(\beta x)}{a\beta},\frac{a\lambda z+c\cos(\lambda x)}{a\lambda}\right)$$

7.7.14.6 [1680] Problem 6

problem number 1680

Added June 26, 2019.

Problem Chapter 7.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sin^{n_1}(\lambda_1 x) w_x + b_1 \sin^{m_1}(\beta_1 y) w_y + c_1 \sin^{k_1}(\gamma_1 z) w_z = a_2 \sin^{n_2}(\lambda_2 x) + b_2 \sin^{m_2}(\beta_2 y) + c_2 \sin^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Sin[beta1*y]^m1*D[w[x, y,z], y] +  c1*Sin[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*sin(beta1*y)^m1*diff(w(x,y,z),y)+ c1*sin(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{\left(a2(\sin^{n2}(\_f\lambda 2)) + b2\left(\sin^{m2}\left(\beta 2 \operatorname{RootOf}\left(\int (\sin^{-n1}(\_f\lambda 1))\, d\_f - \left(\int (\sin^{-n1}(\lambda 1 x))\, d\right.\right.\right.\right.\right.}{}$$

7.7.15 6.2

Local contents

7.7.15.1 [1681] Problem 1

problem number 1681

Added June 26, 2019.

Problem Chapter 7.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cos^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Cos[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-cx)-\frac{c\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cos(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\cos^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

7.7.15.2 [1682] Problem 2

problem number 1682

Added June 26, 2019.

Problem Chapter 7.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\cos(\beta z)w_z = k\cos(\lambda x) + s\cos(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cos[beta*z]*D[w[x,y,z],z]== k*Cos[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\frac{\cosh^{-1}\left(-\frac{\sec(\beta z)\left(2\left(2\sec(\beta z)\sqrt{\sin^2(\beta z)\cos^2(\beta z)\sinh^2\left(\frac{\beta cx}{a}\right)\left(\cosh\left(\frac{4\beta cx}{a}\right)-\sinh\left(\frac{4\beta cx}{a}\right)\right)}+\sin\right.}{4\cosh(}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cos(beta*z)*diff(w(x,y,z),z)= k*cos(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{ab\gamma\lambda\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\mathrm{RootOf}\left(\beta z-\arctan\left(\frac{\_Z^2e^{\frac{2\beta cx}{a}}-1}{\_Z^2e^{\frac{2\beta cx}{a}}+1},\frac{2\_Ze^{\frac{\beta cx}{a}}}{\_Z^2e^{\frac{2\beta cx}{a}}+1}\right)\right)\right)}{\beta c}\right)+a\lambda s\sin(\gamma y)+b\gamma k\sin(\lambda x}{ab\gamma\lambda}$$

7.7.15.3 [1683] Problem 3

problem number 1683

Added June 26, 2019.

Problem Chapter 7.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\beta x)w_y + b\cos^k(\lambda x)w_z = c\cos^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{a\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\beta x)\right)}{\beta n+\beta} + y, \frac{b\sqrt{}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx+\int c(\cos^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\cos^n(\beta x))\,dx\right), z-\left(\int b\left(\cos^k(\lambda x)\right)dx\right)\right)$$

7.7.15.4 [1684] Problem 4

problem number 1684

Added June 26, 2019.

Problem Chapter 7.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\cos^n(\lambda x)w_y + b\cos^m(\beta y)w_z = c\cos^k(\gamma y) + s\cos^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(c\cos^k\left(\frac{\gamma\left(a\csc(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda x)\right)\sqrt{\sin^2(\lambda x)}\cos^{n+1}\right.}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int (\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\cos^r\left(\left(b\left(\right.\right.\right.\right.\right.$$

7.7.15.5 [1685] Problem 5

problem number 1685

Added June 26, 2019.

Problem Chapter 7.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\cos(\beta x)w_y + c\cos(\lambda x)w_z = k\cos(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*x]*D[w[x, y,z], y] +  c*Cos[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \frac{k\cos\left(\frac{\gamma(a\lambda z-c\sin(\lambda x)+c\sin(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\sin(\beta x)}{a\beta},z-\frac{c\sin(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*x)*diff(w(x,y,z),y)+ c*cos(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{k\cos\left(\frac{(a\lambda z+c\sin(\_a\lambda)-c\sin(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{a\beta y-b\sin(\beta x)}{a\beta},\frac{a\lambda z-c\sin(\lambda x)}{a\lambda}\right)$$

7.7.15.6 [1686] Problem 6

problem number 1686

Added June 26, 2019.

Problem Chapter 7.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \cos^{n_1}(\lambda_1 x) w_x + b_1 \cos^{m_1}(\beta_1 y) w_y + c_1 \cos^{k_1}(\gamma_1 z) w_z = a_2 \cos^{n_2}(\lambda_2 x) + b_2 \cos^{m_2}(\beta_2 y) + c_2 \cos^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cos[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cos[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cos(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2(\cos^{n2}(\_f\lambda 2)) + b2\left(\cos^{m2}\left(\beta 2 \operatorname{RootOf}\left(\int (\cos^{-n1}(\_f\lambda 1))\, d\_f - \left(\int (\cos^{-n1}(\lambda 1 x))\right.\right.\right.\right.\right.}{}$$

7.7.16 6.3

Local contents

7.7.16.1 [1687] Problem 1

problem number 1687

Added June 26, 2019.

Problem Chapter 7.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c\tan^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Tan[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-cx)+\frac{c\tan^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\tan^2(\lambda x)\right)}{k\lambda+\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tan(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\tan^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

7.7.16.2 [1688] Problem 2

problem number 1688

Added June 26, 2019.

Problem Chapter 7.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tan(\beta z)w_z = k\tan(\lambda x) + s\tan(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tan[beta*z]*D[w[x,y,z],z]== k*Tan[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\frac{\log(\sin(\beta z))}{\beta}-\frac{cx}{a}\right)-\frac{k\log(\cos(\lambda x))}{a\lambda}-\frac{s\log(\cos(\gamma y))}{b\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tan(beta*z)*diff(w(x,y,z),z)= k*tan(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{2ab\gamma\lambda\_F1\left(\frac{ay-bx}{a},\frac{-\beta cx+a\ln\left(\frac{\tan(\beta z)}{\sqrt{\tan^2(\beta z)+1}}\right)}{\beta c}\right)+a\lambda s\ln\left(\tan^2(\gamma y)+1\right)+b\gamma k\ln\left(\tan^2(\lambda x)+1\right)}{2ab\gamma\lambda}$$

7.7.16.3 [1689] Problem 3

problem number 1689

Added June 26, 2019.

Problem Chapter 7.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tan^n(\beta x)w_y + b\tan^k(\lambda x)w_z = c\tan^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[beta*x]^n*D[w[x, y,z], y] +  b*Tan[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{a\tan^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(\beta x)\right)}{\beta n+\beta},z-\frac{b\tan^{k+1}(\lambda x)\,\text{Hy}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tan(beta*x)^n*diff(w(x,y,z),y)+ b*tan(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx+\int c(\tan^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\tan^n(\beta x))\,dx\right),z-\left(\int b\left(\tan^k(\lambda x)\right)dx\right)\right)$$

7.7.16.4 [1690] Problem 4

problem number 1690

Added June 26, 2019.

Problem Chapter 7.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tan^n(\lambda x)w_y + b\tan^m(\beta y)w_z = c\tan^k(\gamma y) + s\tan^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[lambda*x]^n*D[w[x, y,z], y] +  b*Tan[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\tan^k\left(\frac{\gamma(-a\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(\lambda x)\right)\tan^{n+1}(\lambda x)+a\,\text{Hyperg}}{\lambda(n+}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tan(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\left(\frac{\tan\left(\left(y-\left(\int a(\tan^n(\lambda x))\,dx\right)\right)\gamma\right)+\tan\left(a\gamma\left(\int(\tan^n(\_f\lambda))\,d\_f\right)\right)}{-\tan\left(\left(y-\left(\int a(\tan^n(\lambda x))\,dx\right)\right)\gamma\right)\tan\left(a\gamma\left(\int(\tan^n(\_f\lambda))\,d\_f\right)\right)+1}\right)^k+s\left(\frac{\text{ta}}{-\text{ta}}\right.\right.$$

7.7.16.5 [1691] Problem 5

problem number 1691

Added June 26, 2019.

Problem Chapter 7.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 \tan^{n_1}(\lambda_1 x) w_x + b_1 \tan^{m_1}(\beta_1 y) w_y + c_1 \tan^{k_1}(\gamma_1 z) w_z = a_2 \tan^{n_2}(\lambda_2 x) + b_2 \tan^{m_2}(\beta_2 y) + c_2 \tan^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Tan[beta1*y]^m1*D[w[x, y,z], y] +  c1*Tan[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*tan(beta1*y)^m1*diff(w(x,y,z),y)+ c1*tan(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{\left(a2\left(\tan^{n2}\left(\_f\lambda 2\right)\right) + b2\left(\tan^{m2}\left(\beta 2\,\mathrm{RootOf}\left(\int\left(\tan^{-n1}\left(\_f\lambda 1\right)\right)d\_f - \left(\int\left(\tan^{-n1}\left(\lambda 1 x\right)\right)\right.\right.\right.\right.\right.}{}$$

7.7.17 6.4

Local contents

7.7.17.1 [1692] Problem 1

problem number 1692

Added June 26, 2019.

Problem Chapter 7.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a w_y + b w_z = c \cot^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Cot[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-cx)-\frac{c\cot^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^2(\lambda x)\right)}{k\lambda+\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cot(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\cot^k(\lambda x)\right)dx + \_F1(-ax+y, -bx+z)$$

7.7.17.2 [1693] Problem 2

problem number 1693

Added June 26, 2019.

Problem Chapter 7.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\cot(\gamma z)w_z = k\cot(\lambda x) + s\cot(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cot[gamma*z]*D[w[x,y,z],z]== k*Cot[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{a\lambda s\log(\tan(\beta y))+a\lambda s\log(\cos(\beta y))+b\beta k\log(\sin(\lambda x))}{ab\beta\lambda}+c_1\left(y-\frac{bx}{a},\frac{\log(\sec(\gamma z))}{\gamma}-\frac{cx}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cot(gamma*z)*diff(w(x,y,z),z)= k*cot(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{2ab\beta\lambda\_F1\left(\frac{-ay+bx}{b},\frac{-2c\gamma y+b\ln(\cot^2(\gamma z)+1)-2b\ln(\cot(\gamma z))}{2c\gamma}\right)-a\lambda s\ln\left(\cot^2\left(\beta y\right)+1\right)-b\beta k\ln\left(\cot^2\left(\lambda\right.\right.}{2ab\beta\lambda}$$

7.7.17.3 [1694] Problem 3

problem number 1694

Added June 26, 2019.

Problem Chapter 7.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\cot^n(\beta x)w_y + b\cot^k(\lambda x)w_z = c\cot^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[beta*x]^n*D[w[x, y,z], y] +  b*Cot[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\cot^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^2(\lambda x)\right)}{k\lambda+\lambda}+z,\frac{a\cot^{n+1}(\beta x)\,\text{Hyperge}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cot(beta*x)^n*diff(w(x,y,z),y)+ b*cot(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\cot^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\cot^n(\beta x))\,dx\right), z - \left(\int b(\cot^k(\lambda x))\,dx\right)\right)$$

7.7.17.4 [1695] Problem 4

problem number 1695

Added June 26, 2019.

Problem Chapter 7.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\lambda x)w_y + b\cot^m(\beta y)w_z = c\cot^k(\gamma y) + s\cot^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[lambda*x]^n*D[w[x, y,z], y] +  b*Cot[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\cot^k\left(\frac{\gamma\left(a\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(\lambda x)\right)\cot^{n+1}(\lambda x)+\lambda(n+1)y\right.}{\lambda(n+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cot(lambda*x)^n*diff(w(x,y,z),y)+ b*cot(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\left(\frac{\cot\left(\left(y-\left(\int a(\cot^n(\lambda x))\,dx\right)\right)\gamma\right)\cot\left(a\gamma\left(\int(\cot^n(\_f\lambda))\,d\_f\right)\right)-1}{\cot\left(\left(y-\left(\int a(\cot^n(\lambda x))\,dx\right)\right)\gamma\right)+\cot\left(a\gamma\left(\int(\cot^n(\_f\lambda))\,d\_f\right)\right)}\right)^k+s\left(\frac{\cot\left(\left(z\right.\right.}{\cot\left(\left(z\right.\right.}\right.\right.$$

7.7.17.5 [1696] Problem 5

problem number 1696

Added June 26, 2019.

Problem Chapter 7.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta x)w_y + c\cot(\lambda x)w_z = k\cot(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*x]*D[w[x, y,z], y] +  c*Cot[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{k\cot\left(\frac{\gamma(a\lambda z-c\log(\sin(\lambda x))+c\log(\sin(\lambda K[1])))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\log(\sin(\beta x))}{a\beta},z-\frac{c\log(\sin(}{a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*x)*diff(w(x,y,z),y)+ c*cot(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{\left(\cot\left(\frac{(2a\lambda z+c\ln(\cot^2(\lambda x)+1))\gamma}{2a\lambda}\right)\cot\left(\frac{c\gamma\ln(\cot^2(\_a\lambda)+1)}{2a\lambda}\right)+1\right)k}{\left(-\cot\left(\frac{(2a\lambda z+c\ln(\cot^2(\lambda x)+1))\gamma}{2a\lambda}\right)+\cot\left(\frac{c\gamma\ln(\cot^2(\_a\lambda)+1)}{2a\lambda}\right)\right)a}d\_a+\_F1\left(\frac{2a\beta y+b\ln\left(\cot^2\left(\right.\right.}{2a\beta}\right.$$

7.7.18 6.2

Local contents

7.7.18.1 [1697] Problem 6

problem number 1697

Added June 26, 2019.

Problem Chapter 7.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 \cot^{n_1}(\lambda_1 x) w_x + b_1 \cot^{m_1}(\beta_1 y) w_y + c_1 \cot^{k_1}(\gamma_1 z) w_z = a_2 \cot^{n_2}(\lambda_2 x) + b_2 \cot^{m_2}(\beta_2 y) + c_2 \cot^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cot[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cot[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cot(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{\left(a2(\cot^{n2}(\_f\lambda 2)) + b2\left(\cot^{m2}\left(\beta 2 \operatorname{RootOf}\left(\int (\cot^{-n1}(\_f\lambda 1))\, d\_f - \left(\int (\cot^{-n1}(\lambda 1 x))\right.\right.\right.\right.\right.}{}$$

7.7.19 6.5

Local contents

7.7.19.1 [1698] Problem 1

problem number 1698

Added June 26, 2019.

Problem Chapter 7.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\cos^m(\beta x)w_z = c\sin^k(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{m+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(\beta x)\right)}{\beta m+\beta}+z,y\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\sin^k(\gamma x))\,dx + \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int b(\cos^m(\beta x))\,dx\right)\right)$$

7.7.19.2 [1699] Problem 2

problem number 1699

Added June 26, 2019.

Problem Chapter 7.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\sin^m(\beta y)w_z = c\cos^k(\gamma y) + s\sin^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] + b*Sin[beta*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int (\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\sin^r\left(\left(b\left(\right.\right.\right.\right.\right.$$

7.7.19.3 [1700] Problem 3

problem number 1700

Added June 26, 2019.

Problem Chapter 7.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\tan^m(\beta y)w_z = c\cos^k(\gamma y) + s\tan^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] + b*Tan[beta*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int (\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\frac{\sin\left(\left(z+\right.\right.}{\cos\left(\left(z+\right.\right.}\right.\right.$$

7.7.19.4 [1701] Problem 4

problem number 1701

Added June 26, 2019.

Problem Chapter 7.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a1\sin^{n1}(\lambda_1 x)w_x + b1\cos^{m1}(\beta_1 y)w_y + c1\cos^{k1}(\gamma_1 z)w_z = a2\cos^{n2}(\lambda_2 x) + b2\sin^{m2}(\beta_2 y) + c2\cos^{k2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2(\cos^{n2}(\_f\lambda2)) + b2(\sin^{m2}(\_f\beta2)) + c2\left(\cos^{k2}\left(\gamma2\,\mathrm{RootOf}\left(\int(\sin^{-n1}(\_f\lambda1))\,d\_f -\right.\right.\right.\right.}{}$$

7.7.19.5 [1702] Problem 5

problem number 1702

Added June 26, 2019.

Problem Chapter 7.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a1\tan^{n1}(\lambda_1 x)w_x+b1\cot^{m1}(\beta_1 y)w_y+c1\cot^{k1}(\gamma_1 z)w_z=a2\cot^{n2}(\lambda_2 x)+b2\tan^{m2}(\beta_2 y)+c2\cot^{k2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{\left(a2\left(\frac{\cos(\_f\lambda 2)}{\sin(\_f\lambda 2)}\right)^{n2}+b2\left(\frac{\sin(\_f\beta 2)}{\cos(\_f\beta 2)}\right)^{m2}+c2\left(\frac{\cos\left(\gamma 2\operatorname{RootOf}\left(\int\left(\frac{\sin(\lambda 1x)}{\cos(\lambda 1x)}\right)^{-n1}dx-\left(\int(\tan^{-n1}(\_f\lambda 1))d\_f\right.\right.\right.}{\sin\left(\gamma 2\operatorname{RootOf}\left(\int\left(\frac{\sin(\lambda 1x)}{\cos(\lambda 1x)}\right)^{-n1}dx-\left(\int(\tan^{-n1}(\_f\lambda 1))d\_f\right.\right.\right.}\right.\right.}{a1}$$

7.7.20 7.1

Local contents

7.7.20.1 [1703] Problem 1

problem number 1703

Added June 26, 2019.

Problem Chapter 7.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c \arcsin^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcSin[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{(\sin^{-1}(\lambda x)^2)^{-k}\left(-ic(i\sin^{-1}(\lambda x))^k\sin^{-1}(\lambda x)^k\operatorname{Gamma}\left(k+1,-i\sin\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arcsin(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\arcsin(\lambda x)^k\,dx + \_F1(-ax+y,-bx+z)$$

7.7.20.2 [1704] Problem 2

problem number 1704

Added June 26, 2019.

Problem Chapter 7.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 w_x + a_2 w_y + a_3 w_z = b_1 \arcsin(\lambda_1 x) + b_2 \arcsin(\lambda_2 y) + b_3 \arcsin(\lambda_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcSin[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)+\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\sin^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}\sqrt{1-\text{laml}}}{\text{a2lambd}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arcsin(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1\,a2\,a3\,\lambda1\lambda2\lambda3\_F1\left(\frac{a1\,y-a2\,x}{a1},\frac{a1\,z-a3\,x}{a1}\right)+\sqrt{-\lambda1^2x^2+1}\,a2\,a3\,b1\,\lambda2\lambda3+\left(\sqrt{-\lambda2^2y^2+1}\,a1\,a3\right.}{}$$

7.7.20.3 [1705] Problem 3

problem number 1705

Added June 26, 2019.

Problem Chapter 7.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s\arcsin(\gamma x)^m}{a}dx + \_F1\left(\frac{ay-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda}{2}\right.}{}\right.$$

7.7.20.4 [1706] Problem 4

problem number 1706

Added June 26, 2019.

Problem Chapter 7.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{sx}{a} + \_F1\left(\frac{ay-bx}{a}, -\left(\int^x \arcsin(\_a\lambda)^n \arcsin\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^m d\_a\right) + \frac{(\gamma kz2^k}{}\right.$$

7.7.20.5 [1707] Problem 5

problem number 1707

Added June 26, 2019.

Problem Chapter 7.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\sin^{-1}\left(\frac{\gamma\left(ia\left(-i\sin^{-1}(\beta z)\right)^k \text{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)\sin^{-1}(\beta z)^{-k}-ia\left(i\sin^{-1}(\beta z)\right)^k \text{Gamma}\left(1-k,i\sin^{-1}\right.\right.}{}\right.}{}\right.\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.7.20.6 [1708] Problem 6

problem number 1708

Added June 26, 2019.

Problem Chapter 7.7.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left(\left(-i\sin^{-1}(\beta z)\right)^k\text{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-\left(i\sin^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.7.21 7.2

Local contents

7.7.21.1 [1709] Problem 1

problem number 1709

Added June 26, 2019.

Problem Chapter 7.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c \arccos^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcCos[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-bx) + \frac{(\cos^{-1}(\lambda x)^2)^{-k}\left(c(i\cos^{-1}(\lambda x))^k \cos^{-1}(\lambda x)^k \text{Gamma}\left(k+1, -i\cos^{-1}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccos(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c \arccos(\lambda x)^k\, dx + \_F1(-ax+y, -bx+z)$$

Answer contains unresolved integrals

7.7.21.2 [1710] Problem 2

problem number 1710

Added June 26, 2019.

Problem Chapter 7.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 w_x + a_2 w_y + a_3 w_z = b_1 \arccos(\lambda_1 x) + b_2 \arccos(\lambda_2 y) + b_3 \arccos(\lambda_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcCos[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)-\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\cos^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}x\sin^{-1}(\text{lar}}{\text{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arccos(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1\,a2\,a3\,\lambda1\lambda2\lambda3\_F1\left(\frac{a1y-a2x}{a1},\frac{a1z-a3x}{a1}\right)-\sqrt{-\lambda1^2x^2+1}\,a2\,a3\,b1\,\lambda2\lambda3-\left(\sqrt{-\lambda2^2y^2+1}\,a1\,a3\right.}{}$$

7.7.21.3 [1711] Problem 3

problem number 1711

Added June 26, 2019.

Problem Chapter 7.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arccos^n(\lambda x)\arccos^k(\beta z)w_z=s\arccos^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s\arccos(\gamma x)^m}{a}dx + \_F1\left(\frac{ay-bx}{a}, \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \right.}{}\right.$$

7.7.21.4 [1712] Problem 4

problem number 1712

Added June 26, 2019.

Problem Chapter 7.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = s\arccos^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] + c*ArcCos[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\cos^{-1}\left(\frac{\gamma\left(-a\left(-i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right)\cos^{-1}(\beta z)^{-k} - a\left(i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k, i\cos^{-1}\right.\right.}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s \arccos\left(\gamma \operatorname{RootOf}\left(\_Zb\lambda n \operatorname{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos\left(\_Z\lambda\right)\right)\arccos\left(\_Z\lambda\right)+2\_Zb\lambda \operatorname{L}\right.\right.}{}$$

7.7.21.5 [1713] Problem 5

problem number 1713

Added June 26, 2019.

Problem Chapter 7.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda y)w_y + c\arcsin^k(\beta z)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*y]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s \arcsin(\lambda y)^{-n}}{b} dy + \_F1\left(-\frac{\left(-\arcsin(\lambda y)^{-n} \arcsin(\lambda y)^{\frac{3}{2}} + \mathrm{LommelS1}\left(-n+\frac{3}{2}, \frac{1}{2}, \arcsin\right.\right.}{}\right.$$

7.7.22 7.3

Local contents

7.7.22.1 [1714] Problem 1

problem number 1714

Added June 26, 2019.

Problem Chapter 7.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c \arctan^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcTan[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\tan^{-1}(\lambda K[1])^k+s\right)dK[1]+c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arctan(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\arctan(\lambda x)^k\,dx+\_F1(-ax+y,-bx+z)$$

7.7.22.2 [1715] Problem 2

problem number 1715

Added June 26, 2019.

Problem Chapter 7.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=b_1\arctan(\lambda_1x)+b_2\arctan(\lambda_2y)+b_3\arctan(\lambda_3z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcTan[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)-\frac{\text{b2}\log\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)}{2\text{a2lambda2}}-\frac{\text{b3}\log\left(\text{a1}^2\left(\text{lambda3}^2z^2+1\right)\right)}{2\text{a3lambda3}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arctan(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{b1\,x\arctan(\lambda 1x)}{a1}+\frac{b2\,y\arctan(\lambda 2y)}{a2}+\frac{b3\,z\arctan(\lambda 3z)}{a3}+\frac{2a1\,a2\,a3\,\lambda 1\lambda 2\lambda 3\_F1\left(\frac{a1\,y-a2\,x}{a1},\frac{a1\,z-}{a}\right.}{}$$

7.7.22.3 [1716] Problem 3

problem number 1716

Added June 26, 2019.

Problem Chapter 7.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^k(\beta z)w_z=s\arctan^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s \arctan(\gamma x)^m}{a} dx + \_F1\left(\frac{ay-bx}{a}, -\left(\int \arctan(\lambda x)^n \, dx\right) + \int \frac{a \arctan(\beta z)^{-k}}{c} dz\right)$$

7.7.22.4 [1717] Problem 4

problem number 1717

Added June 26, 2019.

Problem Chapter 7.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arctan^n(\lambda x)\arctan^m(\beta y)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*y)^m*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{sx}{a} + \_F1\left(\frac{ay-bx}{a}, z - \left(\int^x \frac{c\arctan(\_a\lambda)^n \arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^m}{a} d\_a\right)\right)$$

7.7.22.5 [1718] Problem 5

problem number 1718

Added June 26, 2019.

Problem Chapter 7.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arctan^n(\lambda x)w_y + c\arctan^k(\beta z)w_z = s\arctan^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] + c*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\tan^{-1}(\beta K[3])^{-k}\tan^{-1}\left(\frac{\gamma\left(cx-a\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\tan^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)^m}{c} dK[3$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s\arctan\left(\gamma \operatorname{RootOf}\left(\_b - y + \int \frac{b\arctan(\lambda x)^{n}}{a}dx - \left(\int^{\_Z} \frac{b\arctan(\_a\lambda)^{n}}{a}d\_a\right)\right)\right)^{m}\arctan\left(\lambda \mathrm{R}\right.}{b}$$

7.7.23 7.4

Local contents

7.7.23.1 [1719] Problem 1

problem number 1719

Added June 26, 2019.

Problem Chapter 7.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c \operatorname{arccot}^{k}(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcCot[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\cot^{-1}(\lambda K[1])^k+s\right)dK[1]+c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccot(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^k dx+\_F1(-ax+y,-bx+z)$$

7.7.23.2 [1720] Problem 2

problem number 1720

Added June 26, 2019.

Problem Chapter 7.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=b_1\operatorname{arccot}(\lambda_1x)+b_2\operatorname{arccot}(\lambda_2y)+b_3\operatorname{arccot}(\lambda_3z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcCot[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)+\frac{1}{2}\left(\frac{\text{b2}\log\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)}{\text{a2lambda2}}+\frac{\text{b3}\log\left(\text{a1}^2\left(\text{lambda3}^2z^2+\right.\right.}{\text{a3lambda3}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arccot(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{2a1\,a2\,a3\,\lambda1\lambda2\lambda3\_F1\left(\frac{a1y-a2x}{a1},\frac{a1z-a3x}{a1}\right)+a2\,a3\,b1\,\lambda2\lambda3\ln\left(\lambda1^2x^2+1\right)+\left(a1\,a3\,b2\,\lambda3\ln\left(\lambda2^2y\right.\right.}{}$$

7.7.23.3 [1721] Problem 3

problem number 1721

Added June 26, 2019.

Problem Chapter 7.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\operatorname{arccot}^n(\lambda x)\operatorname{arccot}^k(\beta z)w_z=s\operatorname{arccot}^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int\frac{s\left(-\arctan\left(\gamma x\right)+\frac{\pi}{2}\right)^{m}}{a}dx+\_F1\left(\frac{ay-bx}{a},-\left(\int\left(-\arctan\left(\lambda x\right)+\frac{\pi}{2}\right)^{n}dx\right)+\int\frac{a\left(-\mathrm{arc}\right.}{}\right.$$

7.7.23.4 [1722] Problem 4

problem number 1722

Added June 26, 2019.

Problem Chapter 7.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\,\mathrm{arccot}^n(\lambda x)\,\mathrm{arccot}^m(\beta y)\,\mathrm{arccot}^k(\gamma z)w_z=s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{sx}{a} + \_F1\left(\frac{ay-bx}{a}, \int \frac{a\left(-\arctan(\gamma z)+\frac{\pi}{2}\right)^{-k}}{c}dz - \left(\int^{x}\left(-\arctan(\_a\lambda)+\frac{\pi}{2}\right)^{n}\left(-\arctan\right.\right.\right.$$

7.7.23.5 [1723] Problem 5

problem number 1723

Added June 26, 2019.

Problem Chapter 7.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\operatorname{arccot}^n(\lambda x)w_y + c\operatorname{arccot}^k(\beta z)w_z = s\operatorname{arccot}^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] + c*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_{1}^{z} \frac{s\cot^{-1}\left(\frac{\gamma\left(cx-a\int_{1}^{z}\tan^{-1}(\beta K[2])^{-k}dK[2]+a\int_{1}^{K[3]}\tan^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)^{m}\tan^{-1}(\beta K[3])^{-k}}{c}dK[3\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s\left(-\arctan\left(\gamma \operatorname{RootOf}\left(\_b - y + \int \frac{b \arctan(\lambda x)^n}{a} dx - \left(\int^{\_Z} \frac{b \arctan(\_a\lambda)^n}{a} d\_a\right)\right)\right) + \frac{\pi}{2}\right)^m \operatorname{arc}}{b}$$

7.7.24 8.1

Local contents

7.7.24.1 [1724] Problem 1

problem number 1724

Added June 27, 2019.

Problem Chapter 7.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + f(x)w_y + g(x)w_z = h_2(x)y + h_1(x) + h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*D[w[x, y,z], y] + g[x]*D[w[x,y,z],z]== h2[x]*y+h1[x]*z+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\text{h0}(K[3])+\text{h2}(K[3])\left(y-\int_1^x f(K[1])dK[1]+\int_1^{K[3]} f(K[1])dK[1]\right)+\text{h1}(K[3])\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+ g(x)*diff(w(x,y,z),z)= h2(x)*y+h1(x)*z+h0(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(\left(\left(\int f(\_f)\,d\_f\right)h2(\_f)+\left(\int g(\_f)\,d\_f\right)h1(\_f)+h0(\_f)+\left(z-\left(\int g(x)\,dx\right)\right)h1\right.\right.$$

7.7.24.2 [1725] Problem 2

problem number 1725

Added June 27, 2019.

Problem Chapter 7.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+f(x)(y+a)w_y+g(x)(z+b)w_z=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*(y+a)*D[w[x, y,z], y] + g[x]*(z+b)*D[w[x,y,z],z]== h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x h(K[5])dK[5]+c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right)-\int_1^x a\exp\left(-\int_1^{K[2]} f(K[1])dK[\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+ g(x)*(z+b)*diff(w(x,y,z),z)= h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int h(x)\,dx+\_F1\left((a+y)\,\mathrm{e}^{-\left(\int f(x)dx\right)},(b+z)\,\mathrm{e}^{-\left(\int g(x)dx\right)}\right)$$

7.7.24.3 [1726] Problem 3

problem number 1726

Added June 27, 2019.

Problem Chapter 7.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+f(x))w_y+(bz+g(x))w_z=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+f[x])*D[w[x, y,z], y] + (b*z+g[x])*D[w[x,y,z],z]== h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x h(K[3])dK[3]+c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1],ze^{-bx}-\int_1^x e^{-bK[2]}g(K[2])dK[\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+ (b*z+g(x))*diff(w(x,y,z),z)= h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int h(x)\,dx+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right),z\,\mathrm{e}^{-bx}-\left(\int \mathrm{e}^{-bx}g(x)\,dx\right)\right)$$

7.7.24.4 [1727] Problem 4

problem number 1727

Added June 27, 2019.

Problem Chapter 7.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)y+g_2(x))w_z=h_2(x)y+h_1(x)z+h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x]*y+g2[x])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)\left(\exp\left(\int_1^x \mathrm{f1}(K[1])dK[1]\right)\mathrm{h0}(K[5])+\exp\left(\int_1^{K[5]} \mathrm{f1}(K\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}\mathrm{e}^{\int f1(\_g)d\_g}h2(\_g)+\left(\int \mathrm{e}^{-\left(\int f1(\_g)d\_g\right)}f2(\_g)\,d\_g\right)\mathrm{e}^{\int f1(\_g)d\_g}h2(\_g)-\left(\right.\right.$$

7.7.24.5 [1728] Problem 5

problem number 1728

Added June 27, 2019.

Problem Chapter 7.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)z+g_2(x))w_z=h_2(x)y+h_1(x)z+h_0(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*z+f2[x])*D[w[x, y,z], y] + (g1[x]*y+g2[x])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*z+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.24.6 [1729] Problem 6

problem number 1729

Added June 27, 2019.

Problem Chapter 7.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x))w_y + f(x)\sinh(\gamma z)w_z = g(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2 + a *lambda*Sin[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2 + a *lambda*sin(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 - \frac{2\_c_3 \operatorname{arctanh}\left(e^{\gamma z}\right)}{\gamma} + \_F1(x) + \_F2(y) \textbf{ where } \left[\left\{-\frac{a^2 \cosh(2\lambda x)\left(\frac{d}{dy}\_F2(y)\right)}{2} + a\lambda\left(\frac{d}{dy}\right.\right.\right.$$

Gives Warning: Incomplete separation

7.7.24.7 [1730] Problem 7

problem number 1730

Added June 27, 2019.

Problem Chapter 7.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(x)z^m)w_z = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[x]*z^m)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x h(K[5])dK[5] + c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK[2] + \right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(x)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int h(x)\,dx + \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)(\int f1(x)dx)} + (k-1)\left(\int \mathrm{e}^{(k-1)(\int f1(x)dx)} f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)(\int$$

7.7.24.8 [1731] Problem 8

problem number 1731

Added June 27, 2019.

Problem Chapter 7.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x) + g_2(x)e^{\lambda z})w_z = h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]+g2[x]*Exp[lambda*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(lambda*z))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int h(x)\,dx+\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right),\frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g1\right.}\right.}{}\right.$$

7.7.24.9 [1732] Problem 9

problem number 1732

Added June 27, 2019.

Problem Chapter 7.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)+f_2(x)e^{\lambda y})w_y+(g_1(x)+g_2(x)e^{\beta z})w_z=h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]+g2[x]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int h(x)\,dx + \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int f1(x)dx\right)} f2(x)\,dx\right) - e^{-\left(y-\left(\int f1(x)dx\right)\right)\lambda}}{\lambda}, \frac{-\beta\left(\int e^{\beta\left(\int g1(x)dx\right)} g2(x)\,dx\right.}{\beta}\right.$$

7.7.25 8.2

Local contents

7.7.25.1 [1733] Problem 1

problem number 1733

Added June 27, 2019.

Problem Chapter 7.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = \Phi(x) + \Psi(x) + \chi(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f[x]*D[w[x, y,z], x] + g[y]*D[w[x, y,z], y] + h[z]*D[w[x,y,z],z]== phi[x]+psi[x]+chi[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ g(y)*diff(w(x,y,z),y)+ h(z)*diff(w(x,y,z),z)= phi(x)+psi(x)+c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{\chi(x)+\phi(x)+\psi(x)}{f(x)}dx + \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy, -\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{h(z)}d$$

7.7.25.2 [1734] Problem 2

problem number 1734

Added June 27, 2019.

Problem Chapter 7.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + zw_y + g(y)w_z = h_2(x) + h_1(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + z*D[w[x, y,z], y] + g[y]*D[w[x,y,z],z]== h2[x]+h1[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ g(y)*diff(w(x,y,z),z)= h2(x)+h1(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{h1(\_g) + h2\left(\mathrm{RootOf}\left(\int \frac{1}{\sqrt{z^2+2(\int g(\_g)d\_g)-2(\int g(y)dy)}}d\_g + \int \frac{1}{f(x)}dx - \left(\int^{y} \frac{}{\sqrt{z^2+2(\int g(\_b)d}}\right.\right.\right.}{\sqrt{z^2 + 2\left(\int g(\_g)\, d\_g\right) - 2\left(\int g(y)\, dy\right)}}$$

7.7.25.3 [1735] Problem 3

problem number 1735

Added June 27, 2019.

Problem Chapter 7.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = f_4(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y,z], x] + f2[x]*g[y]*D[w[x, y,z], y] + f3[x]*h[z]*D[w[x,y,z],z]== f4[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f1(x)*diff(w(x,y,z),x)+ f2(x)*g(y)*diff(w(x,y,z),y)+ f3(x)*h(z)*diff(w(x,y,z),z)= f4
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int \frac{f4(x)}{f1(x)}dx+\_F1\left(\int \frac{1}{g(y)}dy-\left(\int \frac{f2(x)}{f1(x)}dx\right),\int \frac{1}{h(z)}dz-\left(\int \frac{f3(x)}{f1(x)}dx\right)\right)$$

7.7.25.4 [1736] Problem 4

problem number 1736

Added June 27, 2019.

Problem Chapter 7.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)z+g_2(y))w_z=h_1(x)+h_2(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x]*z+g2[y])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \left(\text{h1}(K[5])+\text{h2}\left(\exp\left(\int_1^{K[5]}\text{f1}(K[1])dK[1]\right)\left(\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right)y-\int_1^x\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*z+g2(y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(h1(\_f) + h2\left(\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)} + \int \mathrm{e}^{-\left(\int f1(\_f)d\_f\right)} f2(\_f)\, d\_f - \left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)} f2(x)\, dx \right) \right.\right.\right.$$

7.7.25.5 [1737] Problem 5

problem number 1737

Added June 27, 2019.

Problem Chapter 7.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(y)z^m)w_z = h_1(x) + h_2(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[y]*z^m)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(\mathrm{h1}(K[5]) + \mathrm{h2}\left(\left(\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1] - (k-1)\int_1^{K[5]} \mathrm{f1}(K[1])dK[1] \right) y^{-k} \right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(h1(\_f)+h2\left(\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}f2(x)\,dx\right)+(-k+1\right.\right.\right.$$

7.7.25.6 [1738] Problem 6

problem number 1738

Added June 27, 2019.

Problem Chapter 7.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x)y^k)w_y+(g_1(x)z+g_2(y)e^{\lambda z})w_z=h_1(x)+h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[y]*Exp[lambda*z])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*exp(lambda*z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.25.7 [1739] Problem 7

problem number 1739

Added June 27, 2019.

Problem Chapter 7.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x)z + g_2(y)z^k)w_z = h_1(x) + h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]*z+g2[y]*z^k)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*z^k)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \int^x \left(h1(\_f) + h2 \left(\frac{\lambda\left(\int f1(\_f)\, d\_f\right) + \ln\left(\frac{1}{\left(-\left(\int e^{\lambda(\int f1(\_f)d\_f)} f2(\_f)d\_f\right) + \int e^{\lambda(\int f1(x)dx)} f2(x)dx\right)\lambda + e^{-(y-}} \right.}{\lambda} \right.\right.$$

7.7.25.8 [1740] Problem 8

problem number 1740

Added June 27, 2019.

Problem Chapter 7.8.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x) + g_2(y)e^{\beta z})w_z = h_1(x) + h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]+g2[y]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(y)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(h1(\_f) + h2 \left(\frac{\lambda\left(\int f1(\_f)\,d\_f\right) + \ln\left(\frac{1}{\left(-\left(\int e^{\lambda(\int f1(\_f)d\_f)} f2(\_f)d\_f\right) + \int e^{\lambda(\int f1(x)dx)} f2(x)dx\right)\lambda + e^{-(y-}}\right.}{\lambda}\right.\right.$$

7.7.26 8.3

Local contents

7.7.26.1 [1741] Problem 1

problem number 1741

Added June 27, 2019.

Problem Chapter 7.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)w_z = g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + f[x,y]*D[w[x,y,z],z]== g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\frac{g\left(K[2],y+\frac{b(K[2]-x)}{a}\right)}{a}dK[2]+c_1\left(y-\frac{bx}{a},z-\int_1^x\frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Kernel message inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*diff(w(x,y,z),z)=  g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{g\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a},z-\left(\int^{x}\frac{f\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a\right)\right)$$

7.7.26.2 [1742] Problem 2

problem number 1742

Added June 27, 2019.

Problem Chapter 7.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+f(x,y)g(z)w_z=h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + f[x,y]*g[z]*D[w[x,y,z],z]== h[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{h\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}, \int \frac{a}{g(z)} dz - \left(\int^{x} f\left(\_a, \frac{ay-(-\_a+x)\,b}{a}\right) d$$

7.7.26.3 [1743] Problem 3

problem number 1743

Added June 27, 2019.

Problem Chapter 7.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + (z + f(x,y))w_z = g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] + (z+f[x,y])*D[w[x,y,z],z]== g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{g\left(K[2], \frac{yK[2]}{x}\right)}{K[2]} dK[2] + c_1\left(\frac{y}{x}, \frac{z}{x} - \int_1^x \frac{f\left(K[1], \frac{yK[1]}{x}\right)}{K[1]^2} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+ (z+f(x,y))*diff(w(x,y,z),z)=  g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{g\left(\_a,\frac{\_ay}{x}\right)}{\_a}d\_a+\_F1\left(\frac{y}{x},\frac{-x\left(\int^{x}\frac{f\left(\_a,\frac{\_ay}{x}\right)}{\_a^{2}}d\_a\right)+z}{x}\right)$$

7.7.26.4 [1744] Problem 4

problem number 1744

Added June 27, 2019.

Problem Chapter 7.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+f(x,y)g(z)w_z=h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] + f[x,y]*g[z]*D[w[x,y,z],z]== h[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{h\left(\_a, y\,\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_aa} d\_a + \_F1\left(y\,x^{-\frac{b}{a}}, \int \frac{a}{g(z)} dz - \left(\int^{x} \frac{f\left(\_a, y\,\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_a} d\_a\right)\right)$$

7.7.26.5 [1745] Problem 5

problem number 1745

Added June 27, 2019.

Problem Chapter 7.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g_1(x,y)z + g_2(x,y))w_z = h(x,y,z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y])*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y))*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)} + \int \mathrm{e}^{-\left(\int f1(\_h)d\_h\right)} f2(\_h)\, d\_h - \left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)} f2(x)\, dx\right)\right) \mathrm{e}^{\int f1(\_$$

7.7.26.6 [1746] Problem 6

problem number 1746

Added June 27, 2019.

Problem Chapter 7.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x,y)z + g_2(x,y)z^m)w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y]*z^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
\
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y)*z^m)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\, dx\right) + (-k+1)\left(\left(\int \mathrm{e}^{(k-\right.\right.\right.$$

7.7.26.7 [1747] Problem 7

problem number 1747

Added June 27, 2019.

Problem Chapter 7.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x,y) + g_2(x,y)e^{\lambda z})w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x,y]+g2[x,y]*Exp[lambda*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x,y)+g2(x,y)*exp(lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x h\left(\_h, \left(y^{-k+1}\mathrm{e}^{(k-1)(\int f1(x)dx)} + (k-1)\left(\int \mathrm{e}^{(k-1)(\int f1(x)dx)} f2(x)\,dx\right) + (-k+1)\left(\int \mathrm{e}^{(k-\right.\right.$$

7.7.26.8 [1748] Problem 8

problem number 1748

Added June 27, 2019.

Problem Chapter 7.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x,y)z + g_2(x,y)z^k)w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y]*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y)*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}h\left(\_h,\frac{\lambda\left(\int f1(\_h)\,d\_h\right)+\ln\left(\frac{1}{\left(-\left(\int \mathrm{e}^{\lambda\left(\int f1(\_h)d\_h\right)}f2(\_h)d\_h\right)+\int \mathrm{e}^{\lambda\left(\int f1(x)dx\right)}f2(x)dx\right)\lambda+\mathrm{e}^{-\left(y-\left(\int f1(x)dx\right)\right.}}\right.}{\lambda}\right.$$

7.7.26.9 [1749] Problem 9

problem number 1749

Added June 27, 2019.

Problem Chapter 7.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)+f_2(x)e^{\lambda y})w_y+(g_1(x,y)+g_2(x,y)e^{\beta z})w_z=h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x,y]+g2[x,y]*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x,y)+g2(x,y)*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x h\left(\_h, \frac{\lambda\left(\int f1(\_h)\,d\_h\right) + \ln\left(\frac{1}{\left(-\left(\int e^{\lambda(\int f1(\_h)d\_h)} f2(\_h)d\_h\right)+\int e^{\lambda(\int f1(x)dx)} f2(x)dx\right)\lambda+e^{-(y-(\int f1(x)dx)}}\right.}{\lambda}\right.$$

7.7.26.10 [1750] Problem 10

problem number 1750

Added June 27, 2019.

Problem Chapter 7.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x,y) + h_2(x,y)z^m)w_z = h_3(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] + (h1[x,y]+h2[x,y]*z^m)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+ (h1(x,y)+h2(x,y)*z^m)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.7.26.11 [1751] Problem 11

problem number 1751

Added June 27, 2019.

Problem Chapter 7.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x,y) + h_2(x,y)e^{\lambda z})w_z = h_3(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] + (h1[x,y]+h2[x,y]*Exp[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+ (h1(x,y)+h2(x,y)*exp(lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8 chapter 8

Local contents

7.8.1 2.1

Local contents

7.8.1.1 [1752] Problem 1

problem number 1752

Added June 27, 2019.

Problem Chapter 8.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*D[w[x,y,z],z]== (alpha*x+beta*y+gamma*z+del
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\exp\left(\frac{x(a(\alpha x+2\beta y+2\delta+2\gamma z)-x(b\beta+c\gamma))}{2a^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=  (alpha*x+beta*y+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)\mathrm{e}^{\frac{\left(\left(\frac{\alpha x}{2}+\beta y+\gamma z+\delta\right)a-\frac{(b\beta+c\gamma)x}{2}\right)x}{a^2}}$$

7.8.1.2 [1753] Problem 2

problem number 1753

Added June 27, 2019.

Problem Chapter 8.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + azw_y + byw_z = (cx+s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== (c*x+s)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^2}{2}+sx}c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-\right.\right.}{2}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=  (c*x+s)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\,z^2-b\,y^2}{a},-\frac{-\sqrt{ab}\,x+\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)\mathrm{e}^{\int^y \frac{\left(\ln\left(\frac{\_aab+\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2}}{\sqrt{a}}\right.\right.}{\sqrt{ab}\,\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}}}$$

7.8.1.3 [1754] Problem 3

problem number 1754

Added June 27, 2019.

Problem Chapter 8.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x + a_0)w_y + (b_1x + b_0)w_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x+a0)*D[w[x, y,z], y] + (b1*x+b0)*D[w[x,y,z],z]== (alpha*x+beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\text{a0}x-\frac{\text{a1}x^2}{2}+y,-\text{b0}x-\frac{\text{b1}x^2}{2}+z\right)\exp\left(\frac{1}{6}x\left(-3\text{a0}\beta x-2\text{a1}\beta x^2+3\alpha x-3\text{b0}\gamma x-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a1*x+a0)*diff(w(x,y,z),y)+ (b1*x+b0)*diff(w(x,y,z),z)=  (alpha*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{1}{2}a1\,x^2-a0x+y,-\frac{1}{2}b1\,x^2-b0x+z\right)\mathrm{e}^{-\frac{\left(-\frac{3\alpha x}{2}+\left(a1\,x^2+\frac{3}{2}a0x-3y\right)\beta-3\delta+\left(b1\,x^2+\frac{3}{2}b0x-3z\right)\gamma\right)x}{3}}$$

7.8.1.4 [1755] Problem 4

problem number 1755

Added June 27, 2019.

Problem Chapter 8.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2y + a_1x + a_0)w_y + (b_2y + b_1x + b_0)w_z = (c_2y + c_1z + c_0x + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*y+a1*x+a0)*D[w[x, y,z], y] + (b2*y+b1*x+b0)*D[w[x,y,z],z]== (c2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\text{a2}x}(\text{a2}(\text{a0}+\text{a2}y)+\text{a1a2}x+\text{a1})}{\text{a2}^2},\frac{e^{-\text{a2}x}(\text{a2}(2\text{a0b2}(\text{a2}xe^{\text{a2}x}+1)-\text{a2}(\text{a2}e^{\text{a2}x}(2\text{b0}x+\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a2*y+a1*x+a0)*diff(w(x,y,z),y)+ (b2*y+b1*x+b0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(a2^2y+a1+(a1x+a0)\,a2)\,\mathrm{e}^{-a2x}}{a2^2},\frac{-2a0\,a2\,b2+(-b1\,x^2-2b0x+2z)\,a2^3+(a1\,x^2\right.$$

7.8.1.5 [1756] Problem 5

problem number 1756

Added June 27, 2019.

Problem Chapter 8.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x + k_0)w_y + (bz + s_1x + s_0)w_z = (c_1x + c_0)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x+k0)*D[w[x, y,z], y] + (b*z+s1*x+s0)*D[w[x,y,z],z]== (c1*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\text{c0}x+\frac{\text{c1}x^2}{2}}c_1\left(\frac{e^{-ax}(a^2y+a(\text{k0}+\text{k1}x)+\text{k1})}{a^2},\frac{e^{-bx}(b^2z+b(\text{s0}+\text{s1}x)+\text{s1})}{b^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a*y+k1*x+k0)*diff(w(x,y,z),y)+(b*z+s1*x+s0)*diff(w(x,y,z),z)=  (c1
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a^2y+(k1x+k0)\,a+k1)\,\mathrm{e}^{-ax}}{a^2},\frac{(b^2z+(s1x+s0)\,b+s1)\,\mathrm{e}^{-bx}}{b^2}\right)\mathrm{e}^{\frac{(c1x+2c0)x}{2}}$$

7.8.1.6 [1757] Problem 6

problem number 1757

Added June 27, 2019.

Problem Chapter 8.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$axw_x + byw_y + czw_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] + c*z*D[w[x,y,z],z]== (alpha*x+beta*y+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to x^{\frac{\beta}{a}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)e^{\frac{\alpha x}{a}+\frac{\beta y}{b}+\frac{\gamma z}{c}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (alpha*x+beta*y+ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = x^{\frac{\beta}{a}}\_F1\left(y\,x^{-\frac{b}{a}},z\,x^{-\frac{c}{a}}\right)\mathrm{e}^{\frac{ab\gamma z+(a\beta y+\alpha bx)c}{abc}}$$

7.8.1.7 [1758] Problem 7

problem number 1758

Added June 27, 2019.

Problem Chapter 8.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + azw_y + byw_z = cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== c*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to x^c c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}},y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{\sqrt{a}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*y*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\,z^2-2by}{a}, x\,\mathrm{e}^{\frac{2\,\mathrm{arctanh}\left(\frac{z}{\sqrt{\frac{a\,z^2-2by}{a}}}\right)}{\sqrt{\frac{a\,z^2-2by}{a}}\,a}}\right)\mathrm{e}^{-\frac{2c\,\mathrm{arctanh}\left(\frac{\sqrt{a^2z^2}}{\sqrt{\frac{a\,z^2-2by}{a}}\,a}\right)}{\sqrt{\frac{a\,z^2-2by}{a}}\,a}}$$

7.8.1.8 [1759] Problem 8

problem number 1759

Added June 27, 2019.

Problem Chapter 8.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + b(ay+bz)w_y + a(ay-bz)w_z = cw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] + a*(a*y-b*z)*D[w[x,y,z],z]== c*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a*b*x*diff(w(x,y,z),x)+ b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)= 
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\frac{\sqrt{2}\,a^2 y}{(-a^2y^2+2abyz+b^2z^2)\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}} + \frac{ay}{\sqrt{-a^2y^2+2abyz+b^2z^2}} + \frac{bz}{\sqrt{-a^2y^2+2abyz}}\right.$$

7.8.1.9 [1760] Problem 9

problem number 1760

Added June 27, 2019.

Problem Chapter 8.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1x+a_0)w_x + (b_1y+b_0)w_y + (c_1z+c_0)w_z = (\alpha x+\beta y+\gamma z+\delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x+a0)*D[w[x, y,z], x] + (b1*y+b0)*D[w[x, y,z], y] +(c1*z+c0)*D[w[x,y,z],z]== (alp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to (\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{a0}\alpha \mathrm{b1c1}+\mathrm{a1b0}\beta \mathrm{c1}+\mathrm{a1b1c0}\gamma-\mathrm{a1b1c1}\delta}{\mathrm{a1}^2\mathrm{b1c1}}}c_1\left(\frac{(\mathrm{b0}+\mathrm{b1}y)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{b1}}{\mathrm{a1}}}}{\mathrm{b1}},\frac{(\mathrm{c0}+\mathrm{c1}z)(\mathrm{a0}+\mathrm{a1}x)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   (a1*x+a0)*diff(w(x,y,z),x)+(b1*y+b0)*diff(w(x,y,z),y)+(c1*z+c0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=(a1x+a0)^{-\frac{a0\alpha}{a1^2}-\frac{b0\beta}{a1b1}-\frac{c0\gamma}{a1c1}+\frac{\delta}{a1}}\_F1\left(\frac{(b1y+b0)(a1x+a0)^{-\frac{b1}{a1}}}{b1},\frac{(c1z+c0)(a1x+a0)^{-\frac{c1}{a1}}}{c1}\right)$$

7.8.2 2.2

Local contents

7.8.2.1 [1761] Problem 1

problem number 1761

Added June 28, 2019.

Problem Chapter 8.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\lambda x^2 + \beta y^2 + \gamma z^2 + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (alpha*x^2+beta*y^2+gamma*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\exp\left(\frac{1}{3}\left(\frac{\alpha x^3+3\delta x}{a}+\frac{\beta y^3}{b}+\frac{\gamma z^3}{c}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (alpha*x^2+beta*y^2+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)\mathrm{e}^{\frac{\left(\left(\frac{\alpha x^2}{3}+\beta y^2+\gamma z^2+\delta\right)a^2-(b\beta y+c\gamma z)ax+\frac{\left(b^2\beta+c^2\gamma\right)x^2}{3}\right)x}{a^3}}$$

7.8.2.2 [1762] Problem 2

problem number 1762

Added June 28, 2019.

Problem Chapter 8.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^2 + a_0)w_y + (b_1x^2 + b_0)w_z = (\lambda x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^2+a0)*D[w[x, y,z], y] +(b1*x^2+b0)*D[w[x,y,z],z]== (alpha*x+b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\text{a0}x-\frac{\text{a1}x^3}{3}+y,-\text{b0}x-\frac{\text{b1}x^3}{3}+z\right)\exp\left(-\frac{1}{4}x\left(2\text{a0}\beta x+\text{a1}\beta x^3-2\alpha x+2\text{b0}\gamma x+\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a1*x^2+a0)*diff(w(x,y,z),y)+(b1*x^2+b0)*diff(w(x,y,z),z)=  (alpha
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{3}a1\,x^3 - a0x + y, -\frac{1}{3}b1\,x^3 - b0x + z\right)\mathrm{e}^{-\frac{\left(-2\alpha x+\left(a1\,x^3+2a0x-4y\right)\beta-4\delta+\left(b1\,x^3+2b0x-4z\right)\gamma\right)x}{4}}$$

7.8.2.3 [1763] Problem 3

problem number 1763

Added June 28, 2019.

Problem Chapter 8.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + s_1x^2 + s_0)w_z = (c_1x^2 + c_0)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x^2+k0)*D[w[x, y,z], y] +(b*z+s1*x^2+s0)*D[w[x,y,z],z]== (c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\text{c0}x+\frac{\text{c1}x^3}{3}}c_1\left(\frac{e^{-ax}(a^2(\text{k0}+\text{k1}x^2)+a^3y+2a\text{k1}x+2\text{k1})}{a^3},\frac{e^{-bx}(b^2(\text{s0}+\text{s1}x^2)+b^3z+2b\text{s1}x}{b^3}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a*y+k1*x^2+k0)*diff(w(x,y,z),y)+(b*z+s1*x^2+s0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a^3y + 2ak1x + (k1\,x^2 + k0)\,a^2 + 2k1)\,\mathrm{e}^{-ax}}{a^3}, \frac{(b^3z + 2bs1x + (s1\,x^2 + s0)\,b^2 + 2s1)\,\mathrm{e}^{-}}{b^3}\right.$$

7.8.2.4 [1764] Problem 4

problem number 1764

Added June 28, 2019.

Problem Chapter 8.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2xy + a_1x^2 + a_0)w_y + (b_2xy + b_1x^2 + b_0)w_z = (c_2y + c_1z + c_0x + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x*y+a1*x^2+a0)*D[w[x, y,z], y] +(b2*x*y+b1*x^2+b0)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\text{a0b2}x}{\text{a1}}-\frac{\text{b2}y}{\text{a1}}-\text{b0}x-\frac{\text{b1}x^3}{3}+\frac{\text{b2}x^3}{3}+z,e^{-\frac{\text{a1}x^2}{2}}(x+y)-\frac{\sqrt{\frac{\pi}{2}}(\text{a0}+1)\text{erf}\left(\frac{\sqrt{\text{a1}}x}{\sqrt{2}}\right)}{\sqrt{\text{a1}}}\right)e$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a1*x*y+a1*x^2+a0)*diff(w(x,y,z),y)+(b2*x*y+b1*x^2+b0)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(x+y)\sqrt{a1}\,\mathrm{e}^{-\frac{a1\,x^2}{2}}-\frac{\sqrt{\pi}\sqrt{2}\,(a0+1)\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}}{\sqrt{a1}},-\frac{3\left(-\frac{\sqrt{2}\,(a0+1)\left(\frac{\sqrt{\pi}\sqrt{\frac{a1}{\pi}}}{\sqrt{a1}}-1\right)\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}+\right.}{}\right.$$

7.8.2.5 [1765] Problem 5

problem number 1765

Added June 28, 2019.

Problem Chapter 8.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = x(\lambda x + \beta y + \gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== x*(lambda*x+beta*y+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)e^{\frac{\beta xy}{a+b}+\frac{\text{gamaxz}}{a+c}+\frac{\lambda x^2}{2a}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  x*(lambda*x+beta*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,x^{-\frac{b}{a}},z\,x^{-\frac{c}{a}}\right)\mathrm{e}^{\frac{\left(bc\lambda x+(2\beta y+2gamaz+\lambda x)a^2+(2bgamaz+2\beta cy+(b+c)\lambda x)a\right)x}{2(a+b)(a+c)a}}$$

7.8.2.6 [1766] Problem 6

problem number 1766

Added June 28, 2019.

Problem Chapter 8.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + bxyw_y + cxzw_z = (\lambda x + \beta y + \gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]== (lambda*x+beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to x^{\frac{\lambda}{a}}e^{-\frac{\frac{\beta y}{a-b}+\frac{\text{gamaz}}{a-c}}{x}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)=  (lambda*x+be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = x^{\frac{\lambda}{a}}\_F1\left(y\,x^{-\frac{b}{a}}, z\,x^{-\frac{c}{a}}\right)\mathrm{e}^{\frac{-(a-c)\beta y-(a-b)gamaz}{(a-b)(a-c)x}}$$

7.8.2.7 [1767] Problem 7

problem number 1767

Added June 28, 2019.

Problem Chapter 8.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + bxyw_y + cz^2w_z = ky^2w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*y^2*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{ky^2}{ax-2bx}}c_1\left(yx^{-\frac{b}{a}},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*x*y*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*y^2*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,x^{-\frac{b}{a}},\frac{ax-cz}{axz}\right)\mathrm{e}^{-\frac{k\,y^2}{(a-2b)x}}$$

7.8.2.8 [1768] Problem 8

problem number 1768

Added June 28, 2019.

Problem Chapter 8.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+by^2w_y+cz^2w_z=kxyw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*x*y*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\left(\frac{ax}{y}\right)^{\frac{kxy}{ax-by}}c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*x*y*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\frac{ax}{y}\right)^{\frac{kxy}{ax-by}} \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

7.8.2.9 [1769] Problem 9

problem number 1769

Added June 28, 2019.

Problem Chapter 8.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + by^2w_y + cz^2w_z = (\lambda x^2 + \beta y^2 + \gamma z^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== (lambda*x^2+beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b}{ax} - \frac{1}{y}, \frac{c}{ax} - \frac{1}{z}\right) \exp\left(\frac{\beta y^2}{by-ax} + \frac{\gamma z^2}{cz-ax} + \frac{\lambda x}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  (lambda*x^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right) e^{\frac{bc\lambda xyz+\left(-\beta\, y^2-\gamma\, z^2+\lambda\, x^2\right)a^2x+\left(\beta c\, y^2z-c\lambda\, x^2z-\left(-\gamma\, z^2+\lambda\, x^2\right)by\right)a}{(ax-cz)(ax-by)a}}$$

7.8.3 2.3

Local contents

7.8.3.1 [1770] Problem 1

problem number 1770

Added July 1, 2019.

Problem Chapter 8.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = xyzw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +b*D[w[x,y,z],z]== x*y*z*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{1}{12}x^2(ax(bx-2z)-2bxy+6yz)\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+b*diff(w(x,y,z),z)=  x*y*z*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,e^{\frac{\left(ab\,x^{2}+6yz+(-2az-2by)x\right)x^{2}}{12}}$$

7.8.3.2 [1771] Problem 2

problem number 1771

Added July 1, 2019.

Problem Chapter 8.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = (kx^3 + sy^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (k*x^3+s*y^2)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{x(3a^2(kx^3+4sy^2)-12absxy+4b^2sx^2)}{12a^3}\right)c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (k*x^3+s*y^2)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{az-cx}{a}\right) \mathrm{e}^{\frac{\left(a^2k\,x^3+4a^2s\,y^2-4absxy+\frac{4}{3}b^2s\,x^2\right)x}{4a^3}}$$

7.8.3.3 [1772] Problem 3

problem number 1772

Added July 1, 2019.

Problem Chapter 8.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = (kx + s\sqrt{x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (k*x+s*Sqrt[x])*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{3kx^2+4sx^{3/2}}{6a}} c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (k*x+s*sqrt(x))*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right) \mathrm{e}^{\frac{3k\,x^2+4s\,x^{\frac{3}{2}}}{6a}}$$

7.8.3.4 [1773] Problem 4

problem number 1773

Added July 1, 2019.

Problem Chapter 8.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + azw_y + byw_z = (c\sqrt{x} + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + b*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== (c*Sqrt[x]+s)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{2}{3}cx^{3/2}+sx}c_1\left(\frac{1}{2}e^{-bx}\left(y\left(-e^{2bx}\right)+ze^{2bx}+y+z\right),\frac{1}{2}e^{-bx}\left(ye^{2bx}-ze^{2bx}+y+z\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+b*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  (c*sqrt(x)+s)*w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y^2+z^2, \frac{bx-\ln(y+z)}{b}\right)\mathrm{e}^{\int^y \frac{\sqrt{\frac{bx+\ln\left(\_a+\sqrt{\_a^2-y^2+z^2}\right)-\ln(y+z)}{b}}\,c+s}{\sqrt{\_a^2-y^2+z^2}\,b}d\_a}$$

7.8.3.5 [1774] Problem 5

problem number 1774

Added July 1, 2019.

Problem Chapter 8.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2w_x + by^2w_y + cz^2w_z = kxyzw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*x*y*z*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\exp\left(\frac{kxyz\left(by(ax-cz)\log\left(\frac{ax}{y}\right)+cz(by-ax)\log\left(\frac{ax}{z}\right)\right)}{(ax-by)(ax-cz)(by-cz)}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*x*y*z*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\frac{ax}{y}\right)^{\frac{bkx\,y^2z}{(by-cz)(ax-by)}}\left(\frac{ax}{z}\right)^{-\frac{ckxy\,z^2}{(by-cz)(ax-cz)}}\_F1\left(\frac{ax-by}{axy},\frac{ax-cz}{axz}\right)$$

7.8.4 2.4

Local contents

7.8.4.1 [1775] Problem 1

problem number 1775

Added July 1, 2019.

Problem Chapter 8.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+gamma*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\exp\left(\frac{\lambda x^{n+1}}{an+a}+\frac{\beta y^{m+1}}{bm+b}+\frac{\gamma z^{k+1}}{ck+c}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (lambda*x^n+beta*y^m+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{az-cx}{a}\right) \mathrm{e}^{\frac{(n+1)(k+1)a\beta c\, y^{m+1}+\left((n+1)a\gamma\, z^{k+1}+(k+1)c\lambda\, x^{n+1}\right)(m+1)b}{(m+1)(k+1)(n+1)abc}}$$

7.8.4.2 [1776] Problem 2

problem number 1776

Added July 1, 2019.

Problem Chapter 8.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right) \exp\left(\frac{\lambda x^{n+1}}{an+a} + \frac{\beta y^m}{bm} + \frac{\gamma z^k}{ck}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (lambda*x^n+beta*y^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}},z\,\mathrm{e}^{-\frac{cx}{a}}\right)\mathrm{e}^{\frac{bck\lambda m\,x^{n+1}+(n+1)\left(b\gamma m\,z^{k}+\beta ck\,y^{m}\right)a}{(n+1)abckm}}$$

7.8.4.3 [1777] Problem 3

problem number 1777

Added July 1, 2019.

Problem Chapter 8.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=cx^nw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{n+1}}{n+1}}c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+\right.\right.}{2\cdot}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*x^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\,z^2 - b\,y^2}{a}, -\frac{-\sqrt{ab}\,x + \ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right) \mathrm{e}^{\int^y \frac{c\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_aab+\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)-\ln\left(\cdots\right.}{\sqrt{ab}}\right.}{\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}}}$$

7.8.4.4 [1778] Problem 4

problem number 1778

Added July 1, 2019.

Problem Chapter 8.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\frac{\lambda x^n}{an}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (lambda*x^n+beta*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,x^{-\frac{b}{a}}, z\,x^{-\frac{c}{a}}\right) \mathrm{e}^{\int^{x} \frac{\beta\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^{m}+\gamma\left(z\_a^{\frac{c}{a}}x^{-\frac{c}{a}}\right)^{k}+\lambda\_a^{n}}{\_aa}d\_a}$$

7.8.4.5 [1779] Problem 5

problem number 1779

Added July 1, 2019.

Problem Chapter 8.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = cx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{cx^n}{n}} c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   x*diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*x^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\,z^2-b\,y^2}{a}, x\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right) \mathrm{e}^{\int^y \frac{c\left(x\left(\frac{\_aab+\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^n}{\sqrt{\left(a\,z^2+\left(\_a^2-y^2\right)b\right)a}}d\_a}$$

7.8.4.6 [1780] Problem 6

problem number 1780

Added July 1, 2019.

Problem Chapter 8.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + b(ay+bz)w_y + a(ay-bz)w_z = cx^n w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]== c*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a*b*x*diff(w(x,y,z),x)+b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{\sqrt{-a^2y^2+2abyz+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2abyz+b^2z^2} + \left(\frac{ay}{\sqrt{-a^2y^2+2abyz+b^2z^2}} + \frac{bz}{\sqrt{-a^2y^2+2abyz+b^2z^2}}\right.}{\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}\right.\right.$$

7.8.4.7 [1781] Problem 7

problem number 1781

Added July 1, 2019.

Problem Chapter 8.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = cx^k w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*y^m*D[w[x, y,z], y] +b*x^nu*y^mu*z^lambda*D[w[x,y,z],z]== c*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(m-1)\,a\,x^{n+1} + (n+1)\,y^{-m+1}}{n+1}, (\lambda-1)\,b\left(\int^{x} \_a^{\nu}\left(\left(\frac{\left(-\_a^{n+1} + x^{n+1}\right)(m-1)\,a + (\right.}{n+1}\right.\right.\right.\right.$$

7.8.4.8 [1782] Problem 8

problem number 1782

Added July 1, 2019.

Problem Chapter 8.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^{n_1}y + b_1x^{m_1})w_y + (a_2x^{n_2}y + b_2x^{m_2})w_z = (c_1x^{k_2}y + c_1x^{k_1}z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\int_1^x \frac{(-1)^{-\frac{n2+1}{n1+1}} a1^{-\frac{m1+n2+2}{n1+1}} c1 e^{-\frac{a1 x^{n1+1}}{n1+1}} \left((-1)^{\frac{n2+1}{n1+1}} a1^{\frac{m1+n2+2}{n1+1}} e^{\frac{a1 x^{n1+1}}{n1+1}} zK[2]^{k1} + (-1)^{\frac{n2+1}{n1+1}} a1^{\frac{m1+n2+2}{n1+1}} e^{\frac{a1 x^{n1+1}}{n1+1}} n1^2 zK\right.}{}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.8.4.9 [1783] Problem 9

problem number 1783

Added July 1, 2019.

Problem Chapter 8.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{n_1} y + b_1 x^{m_1}) w_y + (a_2 x^{n_2} z + b_2 x^{m_2}) w_z = (c_1 x^{k_2} y + c_1 x^{k_1} z) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\text{b1}(\text{n1}+1)^{\frac{\text{m1}-\text{n1}}{\text{n1}+1}}\text{a1}^{-\frac{\text{m1}+1}{\text{n1}+1}}\text{Gamma}\left(\frac{\text{m1}+1}{\text{n1}+1},\frac{\text{a1}x^{\text{n1}+1}}{\text{n1}+1}\right)+ye^{-\frac{\text{a1}x^{\text{n1}+1}}{\text{n1}+1}},\text{b2}(\text{n2}+1)^{\frac{\text{m2}-\text{n2}}{\text{n2}+1}}\text{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.8.4.10 [1784] Problem 10

problem number 1784

Added July 1, 2019.

Problem Chapter 8.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{n_1} y + b_1 y^k) w_y + (a_2 x^{n_2} z + b_2 z^m) w_z = c x^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{s+1}}{s+1}}c_1\left(\text{b1}(-1)^{\frac{\text{n1}}{\text{n1}+1}}(\text{n1}+1)^{-\frac{\text{n1}}{\text{n1}+1}}\text{a1}^{-\frac{1}{\text{n1}+1}}(k-1)^{\frac{\text{n1}}{\text{n1}+1}}\text{Gamma}\left(\frac{1}{\text{n1}+1},-\frac{\text{a1}(k-1)x^{\text{n1}+1}}{\text{n1}+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-(n1+1)(n1+2)^2 b1\, x^{-n1}\left(-\frac{(k-1)a1\, x^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}} \text{WhittakerM}\left(\frac{n1+2}{2n1+2}, \frac{2n1+3}{2n1+2}, -\frac{(k-1)a}{n1}\right.}{}\right.$$

7.8.4.11 [1785] Problem 11

problem number 1785

Added July 1, 2019.

Problem Chapter 8.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^{n_1}y + b_1y^k)w_y + (a_2y^{n_2}z + b_2z^m)w_z = (c_1x^{s_1} + c_2y^{s_2} + c_3z^{s_3})w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*y^k)*D[w[x, y,z], y] +(a2*y^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*y^k)*diff(w(x,y,z),y)+(a2*y^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.8.4.12 [1786] Problem 12

problem number 1786

Added July 1, 2019.

Problem Chapter 8.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + a\sqrt{x^2+y^2}w_z = bx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]== b*x^n*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{bx^n}{n}}c_1\left(\frac{y}{x},z-a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+a*sqrt(x^2+y^2)*diff(w(x,y,z),z)= b*x^n*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},-\sqrt{x^2+y^2}\,a+z\right)\mathrm{e}^{\frac{b\,x^n}{n}}$$

7.8.4.13 [1787] Problem 13

problem number 1787

Added July 1, 2019.

Problem Chapter 8.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + (z - a\sqrt{x^2+y^2+z^2}w_z = bx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]== b*x^n*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{bx^n}{n}}c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+(z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)= b*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)\mathrm{e}^{\frac{b\,x^n}{n}}$$

7.8.5 3.1

Local contents

7.8.5.1 [1788] Problem 1

problem number 1788

Added July 1, 2019.

Problem Chapter 8.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + be^{\beta x}w_z = ce^{\gamma x}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\gamma x}}{\gamma}}c_1\left(y-\frac{ae^{\lambda x}}{\lambda},z-\frac{be^{\beta x}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)= c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{-b\,\mathrm{e}^{\beta x}+\beta z}{\beta}\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

7.8.5.2 [1789] Problem 2

problem number 1789

Added July 1, 2019.

Problem Chapter 8.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + be^{\beta x}w_z = (ce^{\gamma y} + se^{\mu z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== (c*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},z-\frac{be^{\beta x}}{\beta}\right)\exp\left(\frac{c\text{Ei}\left(\frac{ae^{\lambda x}\gamma}{\lambda}\right)e^{\gamma\left(y-\frac{ae^{\lambda x}}{\lambda}\right)}}{\lambda}+\frac{s\text{Ei}\left(\frac{be^{\beta x}\mu}{\beta}\right)e^{\mu\left(z-\frac{be^{\beta x}}{\beta}\right)}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)= (c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,e^{\lambda x}+\lambda y}{\lambda},\frac{-b\,e^{\beta x}+\beta z}{\beta}\right)e^{\frac{-\beta c\,\mathrm{expIntegral}\left(1,-\frac{a\gamma\,e^{\lambda x}}{\lambda}\right)e^{-\frac{\left(a\,e^{\lambda x}-\lambda y\right)\gamma}{\lambda}}-\lambda s\,\mathrm{expIntegral}\left(1,-\frac{b\mu\,e^{\beta x}}{\beta}\right)e^{\frac{\left(-b\,e^{\beta x}\cdots\right.}{\beta}}}{\beta\lambda}}$$

7.8.5.3 [1790] Problem 3

problem number 1790

Added July 2, 2019.

Problem Chapter 8.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\lambda y}w_y+be^{\beta y}w_z=(ce^{\gamma x}+se^{\mu z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*y]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== (c*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{a\lambda x+e^{-\lambda y}}{\lambda},\frac{b\left(e^{-\lambda y}\right)^{1-\frac{\beta}{\lambda}}}{a(\lambda-\beta)}+z\right)\exp\left(\int_1^x\left(e^{\gamma K[1]}c+\exp\left(-\frac{\mu\left(b\lambda(x-K[1])\left(a\lambda(\right.\right.}{}\right.\right.\right.\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*y)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)= (c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\lambda x-\mathrm{e}^{-\lambda y}}{a\lambda},\frac{-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y}+(\beta-\lambda)\,az}{(\beta-\lambda)\,a}\right)\mathrm{e}^{\int^x\left(c\,\mathrm{e}^{-a\gamma}+s\,\mathrm{e}^{\frac{\left(-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y}+(\beta-\lambda)az+\left((-\_a+x)a\lambda\right.\right.}{(\beta-\lambda)a}}\right.}$$

7.8.5.4 [1791] Problem 4

problem number 1791

Added July 2, 2019.

Problem Chapter 8.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(A_1e^{\alpha_1x}+B_1e^{\nu_1x+\lambda y})w_y+(A_2e^{\alpha_2x}+B_2e^{\nu_2x+\beta z})w_z=ke^{\gamma z}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (A1*Exp[alpha1*x] + B1*Exp[nu1*x+lambda*y])*D[w[x, y,z], y] + (A2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(A1*exp(alpha1*x) + B1*exp(nu1*x+lambda*y))*diff(w(x,y,z),y)+(A2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-B1\lambda\left(\int \mathrm{e}^{\frac{A1\lambda\,\mathrm{e}^{\alpha1 x}}{\alpha1}+\nu1 x}dx\right) - \mathrm{e}^{\frac{\left(A1\,\mathrm{e}^{\alpha1 x}-\alpha1 y\right)\lambda}{\alpha1}}}{\lambda}, \frac{-B2\beta\left(\int \mathrm{e}^{\frac{A2\beta\,\mathrm{e}^{\alpha2 x}}{\alpha2}+\nu2 x}dx\right) - \mathrm{e}^{\frac{\left(A2\,\mathrm{e}^{\alpha2 x}-\alpha2 z\right)\beta}{\alpha2}}}{\beta}\right.$$

7.8.5.5 [1792] Problem 5

problem number 1792

Added July 2, 2019.

Problem Chapter 8.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y + ce^{\gamma z}w_z = ke^{\lambda x}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] + c*Exp[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{ke^{x(\lambda-\alpha)}}{a(\lambda-\alpha)}}c_1\left(\frac{be^{-\alpha x}}{a\alpha} - \frac{e^{-\beta y}}{\beta}, \frac{ce^{-\alpha x}}{a\alpha} - \frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{(a\alpha\,\mathrm{e}^{\alpha x} - b\beta\,\mathrm{e}^{\beta y})\,\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}, -\frac{(a\alpha\,\mathrm{e}^{\alpha x} - c\gamma\,\mathrm{e}^{\gamma z})\,\mathrm{e}^{-\alpha x-\gamma z}}{\alpha c\gamma}\right)\mathrm{e}^{-\frac{k\,\mathrm{e}^{-(\alpha-\lambda)x}}{(\alpha-\lambda)a}}$$

7.8.5.6 [1793] Problem 6

problem number 1793

Added July 2, 2019.

Problem Chapter 8.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\beta y}w_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*y]*D[w[x, y,z], x] + b*Exp[alpha*x]*D[w[x, y,z], y] + c*Exp[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*exp(beta*y)*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha\,e^{\beta y}-b\beta\,e^{\alpha x}}{\alpha b\beta}, -\frac{\left(a\alpha\,e^{\beta y}e^{-\gamma z}+\alpha c\gamma x-b\beta\,e^{\alpha x}e^{-\gamma z}-c\gamma\ln\left(\frac{a\alpha\,e^{\beta y}}{b\beta}\right)\right)b\beta}{\left(a\alpha\,e^{\beta y}-b\beta\,e^{\alpha x}\right)\alpha c\gamma}\right)e^{\int^{x}\frac{\alpha k}{a\alpha\,e^{\beta y}-(-}}$$

7.8.5.7 [1794] Problem 7

problem number 1794

Added July 2, 2019.

Problem Chapter 8.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1+a_2e^{\alpha x})w_x+(b_1+b_2e^{\beta y})w_y+(c_1+c_2e^{\gamma z})w_z=(k_1+k_2e^{\alpha x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + (b1+b2*Exp[beta*y])*D[w[x, y,z], y] + (c1+c2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{\text{k1}x}{\text{a1}}}\left(\text{a1}+\text{a2}e^{\alpha x}\right)^{\frac{\text{a1k2}-\text{a2k1}}{\text{a1a2}\alpha}}c_1\left(\frac{\log\left(\frac{e^{\beta y}\left(\text{a1}+\text{a2}e^{\alpha x}\right)^{\frac{\text{b1}\beta}{\text{a1}\alpha}}}{\text{b1}+\text{b2}e^{\beta y}}\right)}{\text{b1}\beta}-\frac{x}{\text{a1}},\frac{\log\left(\frac{e^{\gamma z}\left(\text{a1}+\text{a2}e^{\alpha x}\right)^{\frac{\text{c1}\gamma}{\text{a1}\alpha}}}{\text{c1}+\text{c2}e^{\gamma z}}\right)}{\text{c1}\gamma}-\frac{x}{\text{a1}}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+(b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = (a2\,e^{\alpha x} + a1)^{-\frac{k1}{a1\,\alpha}+\frac{k2}{a2\,\alpha}} (e^{\alpha x})^{\frac{k1}{a1\,\alpha}} \_F1\left(\frac{-b1\,\beta x + a1\,\ln\left(\frac{\left(-b1+\operatorname{RootOf}\left(a1\,\alpha\beta y - a1\,\alpha\ln\left(\frac{(\_Z-b1)(a2\,e^{\alpha x}}{b2}\right.\right.\right.}{b2\,\operatorname{RootOf}\left(a1\,\alpha\beta y - a1\,\alpha\ln\left(\frac{(\_Z-b1)}{}\right.\right.}\right.}{a1\,b1\,\beta}\right.$$

7.8.5.8 [1795] Problem 8

problem number 1795

Added July 2, 2019.

Problem Chapter 8.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$e^{\beta y}(a_1 + a_2 e^{\alpha x})w_x + e^{\alpha x}(b_1 + b_2 e^{\beta y})w_y + c e^{\beta y + \gamma z} w_z = k_3 e^{\beta y}(k_1 + k_2 e^{\alpha x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + Exp[alpha*x]*(b1+b2*Exp[beta*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{\text{k3}((\text{a1k2} - \text{a2k1})\log(\text{a1} + \text{a2}e^{\alpha x}) + \text{a2}\alpha\text{k1}x)}{\text{a1a2}\alpha}\right) c_1\left(\frac{c\log(\text{a1} + \text{a2}e^{\alpha x})}{\text{a1}\alpha} - \frac{cx}{\text{a1}} - \frac{e^{-\gamma z}}{\gamma}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+exp(alpha*x)*(b1+b2*exp(beta*y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(a2\,\mathrm{e}^{\alpha x}+a1\right)^{-\frac{k1\,k3}{a1\,\alpha}+\frac{k2\,k3}{a2\,\alpha}} \left(\mathrm{e}^{\alpha x}\right)^{\frac{k1\,k3}{a1\,\alpha}} \_F1\left(\frac{\ln\left(\frac{b1\left(a2\,\mathrm{e}^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}\mathrm{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,\mathrm{e}^{\alpha x}+a1\right)^{-\frac{b2}{a2}}}{\_Z-b2}\right.\right.}{-b2+\mathrm{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,\mathrm{e}^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{\_Z-b2}\right)-b\right.}\right.}{b2\beta}\right.$$

7.8.6 3.2

Local contents

7.8.6.1 [1796] Problem 1

problem number 1796

Added July 2, 2019.

Problem Chapter 8.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ax^n w_y + bx^m w_z = (ce^{\lambda x} y + ke^{\beta x} z + se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] + b*x^m*D[w[x,y,z],z]== (c*Exp[lambda*x]*y+k*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1},\frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(-\frac{acx^n(-\lambda x)^{-n}\,\text{Gamma}(n+2,-\lambda x)}{\lambda^2(n+1)}-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*diff(w(x,y,z),y)+c*x^m*diff(w(x,y,z),z)= (c*exp(lambda*x)*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,x^{n+1}+(n+1)\,y}{n+1},\frac{-c\,x^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\frac{(n+1)(m+1)(\Gamma(n)-\Gamma(n,-\lambda x))a\,\beta^2c\gamma n\,x^n(-\lambda x)^{-n}+(n+1)(m}{}}$$

7.8.6.2 [1797] Problem 2

problem number 1797

Added July 2, 2019.

Problem Chapter 8.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + bx^m w_z = (cx^n y + ke^{\beta x}z + se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] + b*x^m*D[w[x,y,z],z]== (c*x^n*y+k*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},\frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(\frac{acx^n(-\lambda x)^{-n}\,\mathrm{Gamma}(n+1,-\lambda x)}{\lambda^2}-\frac{bkx^m(-\beta\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)= (c*x^n*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{-b\,x^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\int^x \frac{bk\lambda\_a^{m+1}\mathrm{e}^{\_a\beta}-bk\lambda x^{m+1}\mathrm{e}^{\_a\beta}-(m+1)\left(-ac\_a^n\mathrm{e}^{\_a\lambda}-k\lambda z\,\mathrm{e}^{\_a\beta}-\right.}{(m+1)\lambda}}$$

7.8.6.3 [1798] Problem 3

problem number 1798

Added July 2, 2019.

Problem Chapter 8.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + byw_z = (ke^{\beta x}z + se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== (k*Exp[beta*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},\frac{abe^{\lambda x}(\lambda x-1)}{\lambda^2}-bxy+z\right)\exp\left(\frac{abke^{x(\beta+\lambda)}}{\beta^2(\beta+\lambda)}+\frac{ke^{\beta x}(\beta z-by)}{\beta^2}+\frac{se^{\gamma x}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)= (k*exp(beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{(\lambda x-1)\,ab\,\mathrm{e}^{\lambda x}-(bxy-z)\,\lambda^2}{\lambda^2}\right)\mathrm{e}^{\frac{ab\,\beta^2\gamma k\,\mathrm{e}^{(\beta+\lambda)x}+(\beta+\lambda)\left(\beta^2\lambda^2 s\,\mathrm{e}^{\gamma x}+\left((-\beta+\lambda)ab\,\mathrm{e}^{\lambda x}-(by-\right.\right.}{(\beta+\lambda)\beta^2\gamma\,\lambda^2}}$$

7.8.6.4 [1799] Problem 4

problem number 1799

Added July 2, 2019.

Problem Chapter 8.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ay^n w_y + bz^m w_z = (ce^{\lambda x} + ke^{\beta y} + se^{\gamma z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*y^n*D[w[x, y,z], y] + b*z^m*D[w[x,y,z],z]== (c*Exp[lambda*x]+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-ax-\frac{\left(\frac{1}{y}\right)^{n-1}}{n-1},-bx-\frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right)\exp\left(\frac{k\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{n}{n-1}}\left(-\beta\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{1}{1-n}}\right)^{n}\text{Gam}}{a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*y^n*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)= (c*exp(lambda*x)+k*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(((n-1)\,ax\,y^n+y)\,y^{-n},((m-1)\,bx\,z^m+z)\,z^{-m}\right)\mathrm{e}^{\int^x\left(c\,\mathrm{e}^{-a\lambda}+k\,\mathrm{e}^{\beta\left(((-\_a+x)(n-1)a\,y^n+y)y^{-n}\right)^{-\frac{1}{n-}}}\right.}$$

7.8.6.5 [1800] Problem 5

problem number 1800

Added July 2, 2019.

Problem Chapter 8.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ae^{\beta y} w_y + bz^m w_z = (ce^{\lambda x} + ky^n + se^{\gamma z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[beta*y]*D[w[x, y,z], y] + b*z^m*D[w[x,y,z],z]== (c*Exp[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{a\beta x + e^{-\beta y}}{\beta}, -bx - \frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right) \exp\left(-\frac{k\left(-\log\left(e^{-\beta y}\right)\right)^{-n}\left(-\frac{\log\left(e^{-\beta y}\right)}{\beta}\right)^n \text{Gamma}\,(n}{a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(beta*y)*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)= (c*exp(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\beta x - e^{-\beta y}}{a\beta}, ((m-1)\,bx\,z^m + z)\,z^{-m}\right) e^{\int^x \left(c\,e^{-a\lambda} + k\left(\frac{\ln\left(\frac{1}{(-\_a+x)a\beta + e^{-\beta y}}\right)}{\beta}\right)^n + s\,e^{\gamma\left(((-\_a+x)(m\right.}\right.}$$

7.8.6.6 [1801] Problem 6

problem number 1801

Added July 2, 2019.

Problem Chapter 8.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 + by + ae^{\alpha y}(y-b) - b^2)w_y + (z^2 + c(xz-1)e^{\beta x})w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2+ b*y+ a*Exp[alpha*y]*(y-b)-b^2)*D[w[x, y,z], y] +  (z^2+c*(x*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(y^2+ b*y+ a*exp(alpha*y)*(y-b)-b^2)*diff(w(x,y,z),y)+(z^2+c*(x*z-1
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1\_F1(x)\_F3(z)\,\mathrm{e}^{-c_2\left(\int \frac{1}{(b-y)a\,\mathrm{e}^{\alpha y}+b^2-by-y^2}dy\right)}\textbf{where}\left[\left\{(xz-1)\,c\_F1(x)\left(\frac{d}{dz}\_F3(z)\right)\mathrm{e}^{\beta x} - k\right.\right.$$

7.8.6.7 [1802] Problem 7

problem number 1802

Added July 2, 2019.

Problem Chapter 8.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 + ae^{\alpha x}(x+1))w_y + (ce^{\beta x}z^2 + be^{-\beta x})w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2+ a*Exp[alpha*x]*(x+1))*D[w[x, y,z], y] + (c*Exp[beta*x]*z^2+b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(y^2+ a*exp(alpha*x)*(x+1))*diff(w(x,y,z),y)+(c*exp(beta*x)*z^2+b*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.8.6.8 [1803] Problem 8

problem number 1803

Added July 2, 2019.

Problem Chapter 8.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ae^{\alpha x}y^2 + be^{-\alpha x})w_y + (de^{\beta x}z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x}))w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[alpha*x]*y^2+b*Exp[-alpha*x])*D[w[x, y,z], y] + (d*Exp[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a*exp(alpha*x)*y^2+b*exp(-alpha*x))*diff(w(x,y,z),y)+(d*exp(beta*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.8.6.9 [1804] Problem 9

problem number 1804

Added July 2, 2019.

Problem Chapter 8.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k) w_y + (a_2 e^{\lambda_2 x} z + b_2 e^{\beta_2 x} z^m) w_z = c x^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*Exp[lambda1*x]*y+b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +  (a2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{s+1}}{s+1}}c_1\left((k-1)\int_1^x \mathrm{b1}e^{\frac{\mathrm{a1}e^{\mathrm{lambda1}K[1]}(k-1)}{\mathrm{lambda1}}+\mathrm{beta1}K[1]}dK[1]+y^{1-k}e^{\frac{\mathrm{a1}(k-1)e^{\mathrm{lambda1}x}}{\mathrm{lambda1}}},(m-1)\int_1^x\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*exp(lambda1*x)*y+b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+(a2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left((k-1)\,b1\left(\int \mathrm{e}^{\frac{\beta 1\lambda 1x+(k-1)a1\,\mathrm{e}^{\lambda 1x}}{\lambda 1}}dx\right)+y^{-k+1}\mathrm{e}^{\frac{(k-1)a1\,\mathrm{e}^{\lambda 1x}}{\lambda 1}},(m-1)\,b2\left(\int \mathrm{e}^{\frac{\beta 2\lambda 2x+(m-1)a2\,\mathrm{e}^{\lambda 2x}}{\lambda 2}}\right.\right.$$

7.8.6.10 [1805] Problem 10

problem number 1805

Added July 2, 2019.

Problem Chapter 8.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 e^{\beta_1 x} y + b_1 e^{\gamma_1 x} y^k) w_y + (a_2 e^{\beta_2 x} + b_2 e^{\gamma_2 x + \lambda_2 z}) w_z = c x^s w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*Exp[beta1*x]*y+b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +  (a2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*exp(beta1*x)*y+b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+ (a2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((k-1)\,b1\left(\int \mathrm{e}^{\frac{\beta 1 \gamma 1 x + (k-1) a1\, \mathrm{e}^{\beta 1 x}}{\beta 1}} dx\right) + y^{-k+1} \mathrm{e}^{\frac{(k-1) a1\, \mathrm{e}^{\beta 1 x}}{\beta 1}}, \frac{-b2 \lambda 2 \left(\int \mathrm{e}^{\frac{a2 \lambda 2\, \mathrm{e}^{\beta 2 x}}{\beta 2} + \gamma 2 x} dx\right) - \mathrm{e}}{\lambda 2}\right.$$

7.8.6.11 [1806] Problem 11

problem number 1806

Added July 2, 2019.

Problem Chapter 8.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1x^n + b_1x^m e^{\lambda y})w_y + (a_2x^k + b_2x^r e^{\beta z})w_z = cx^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n+b1*x^m*Exp[lambda*y] )*D[w[x, y,z], y] +   (a2*x^k+b2*x^r*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{s+1}}{s+1}}c_1\left(\frac{(n+1)e^{-\frac{\lambda\left(-\text{a1}x^{n+1}+ny+y\right)}{n+1}}-\text{b1}\lambda x^{m+1}\left(-\frac{\text{a1}\lambda x^{n+1}}{n+1}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},-\frac{\text{a1}\lambda x^{n+1}}{n+1}\right)}{\text{a1b1}\lambda^2(n+1)(m-n)}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n+b1*x^m*exp(lambda*y) )*diff(w(x,y,z),y)+ (a2*x^k+b2*x^r*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{\left(-a1\,\lambda\,x^{m+1}+(m+n+2)\,x^{m-n}\right)(n+1)^2\,b1\left(-\frac{a1\,\lambda\,x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}}\text{WhittakerM}\left(\frac{m-n}{2n+2},\right.}{}\right.$$

7.8.7 4.1

Local contents

7.8.7.1 [1807] Problem 1

problem number 1807

Added Oct 10, 2019.

Problem Chapter 8.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sinh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sinh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

7.8.7.2 [1808] Problem 2

problem number 1808

Added Oct 10, 2019.

Problem Chapter 8.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = (k\sinh(\beta x) + s\sinh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]== (k*Sinh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\cosh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{s\sinh\left(\frac{\gamma(a\lambda z-c\cosh(\lambda x)+c\cosh(\lambda K[1]))}{a\lambda}\right)+k\sinh(\beta K[}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*sinh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c\cosh(\lambda x)}{a\lambda}\right)e^{\int^x\frac{k\sinh(\_a\beta)+s\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

7.8.7.3 [1809] Problem 3

problem number 1809

Added Oct 10, 2019.

Problem Chapter 8.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\beta x)w_y+b\sinh^k(\lambda x)w_z=c\sinh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[beta*x]^n*D[w[x, y,z], y] +  b*Sinh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{c\sqrt{\cosh^2(\gamma x)}\operatorname{sech}(\gamma x)\sinh^{m+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\sinh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(y-\frac{a\sqrt{c\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*sinh(beta*x)^n*diff(w(x, y,z), y) +  b*sinh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\beta x))\,dx\right),z-\left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\sinh^m(\gamma x))dx}$$

7.8.7.4 [1810] Problem 4

problem number 1810

Added Oct 10, 2019.

Problem Chapter 8.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sinh(\beta y)w_y+c\sinh(\lambda x)w_z=k\sinh(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{c\cosh(\lambda x)}{a\lambda},\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{k\sinh\left(\frac{\gamma(a\lambda z-c\cosh(\lambda x)+c\cosh(\lambda K[1]))}{a\lambda}\right)}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*sinh(beta*y)*diff(w(x, y,z), y) +  c*sinh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x-2a\operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\cosh\left(\lambda x\right)}{a\lambda}\right)e^{\int^x\frac{k\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

7.8.7.5 [1811] Problem 5

problem number 1811

Added Oct 10, 2019.

Problem Chapter 8.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$a_1\sinh^{n_1}(\lambda_1 x)w_x+b_1\sinh^{m_1}(\beta_1 y)w_y+c_1\sinh^{k_1}(\gamma_1 z)w_z=\left(a_2\sinh^{n_2}(\lambda_2 x)w_x+b_2\sinh^{m_2}(\beta_2 y)w_y+c_2\sinh^{k}\right.$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sinh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Sinh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Sin
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sinh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*sinh(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\sinh^{-n1}(\lambda 1x)\right)dx\right) + \int \frac{a1\left(\sinh^{-m1}(\beta 1y)\right)}{b1}dy, \frac{a1\,z - c1\left(\int \left(\sinh^{k1}(\gamma 1x)\right)\left(\mathrm{si}\right.\right.}{a1}\right.$$

7.8.8 4.2

Local contents

7.8.8.1 [1812] Problem 1

problem number 1812

Added Oct 10, 2019.

Problem Chapter 8.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cosh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cosh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{-\sinh^2(\beta x)}\operatorname{csch}(\beta x)\cosh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(\beta x\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cosh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cosh^n(\beta x))dx}$$

7.8.8.2 [1813] Problem 2

problem number 1813

Added Oct 10, 2019.

Problem Chapter 8.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cosh(\lambda x)w_z=(k\cosh(\beta x)+s\cosh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]== (k*Cosh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{k\cosh(\beta K[1])+s\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1])}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*cosh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c\sinh(\lambda x)}{a\lambda}\right)e^{\int^x\frac{k\cosh(\_a\beta)+s\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

7.8.8.3 [1814] Problem 3

problem number 1814

Added Oct 10, 2019.

Problem Chapter 8.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\beta x)w_y+b\cosh^k(\lambda x)w_z=c\cosh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[beta*x]^n*D[w[x, y,z], y] +  b*Cosh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{m+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cosh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(\frac{a\sinh(\beta x\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*sinh(beta*x)^n*diff(w(x, y,z), y) +  b*sinh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\beta x))\,dx\right),z-\left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\sinh^m(\gamma x))dx}$$

7.8.8.4 [1815] Problem 4

problem number 1815

Added Oct 10, 2019.

Problem Chapter 8.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\lambda x)w_z=k\cosh(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{k\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*cosh(beta*y)*diff(w(x, y,z), y) +  c*cosh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\sinh\left(\lambda x\right)}{a\lambda}\right)e^{\int^x\frac{k\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

7.8.8.5 [1816] Problem 5

problem number 1816

Added Oct 10, 2019.

Problem Chapter 8.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cosh^{n_1}(\lambda_1x)w_x+b_1\cosh^{m_1}(\beta_1y)w_y+c_1\cosh^{k_1}(\gamma_1z)w_z=\left(a_2\cosh^{n_2}(\lambda_2x)w_x+b_2\cosh^{m_2}(\beta_2y)w_y+c_2\cosh\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cosh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cosh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cos
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cosh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*cosh(beta1*y)^m1*diff(w(x, y,z), y) + 
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cosh^{-n1}(\lambda 1x)\right) dx\right) + \int \frac{a1\left(\cosh^{-m1}(\beta 1y)\right)}{b1} dy, \frac{a1\,z - c1\left(\int \left(\cosh^{k1}(\gamma 1x)\right) (c\right.}{a1}\right.$$

7.8.9 4.3

Local contents

7.8.9.1 [1817] Problem 1

problem number 1817

Added Oct 10, 2019.

Problem Chapter 8.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tanh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tanh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\tanh^n(\beta x))dx}$$

7.8.9.2 [1818] Problem 2

problem number 1818

Added Oct 10, 2019.

Problem Chapter 8.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tanh(\lambda x)w_z = (k\tanh(\beta x) + s\tanh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]== (k*Tanh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\log(\cosh(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x\frac{k\tanh(\beta K[1])+s\tanh\left(\frac{\gamma(a\lambda z-c\log(\cosh(\lambda x))+c}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*tanh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{2a\lambda z+c\ln(\tanh(\lambda x)-1)+c\ln(\tanh(\lambda x)+1)}{2a\lambda}\right)e^{\int^x\frac{k\tanh(\_a\beta)-s\tanh\left(\frac{(-2a\lambda z+c\,l}{}\right.}{}}$$

7.8.9.3 [1819] Problem 3

problem number 1819

Added Oct 10, 2019.

Problem Chapter 8.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\beta x)w_y+b\tanh^k(\lambda x)w_z=c\tanh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[beta*x]^n*D[w[x, y,z], y] +  b*Tanh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{m+1}(\gamma x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\tanh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(y-\frac{a\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+}{2}\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*tanh(beta*x)^n*diff(w(x, y,z), y) +  b*tanh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a\left(\tanh^n\left(\beta x\right)\right)dx\right),z-\left(\int b\left(\tanh^k\left(\lambda x\right)\right)dx\right)\right)\mathrm{e}^{\int c\left(\tanh^m(\gamma x)\right)dx}$$

7.8.9.4 [1820] Problem 4

problem number 1820

Added Oct 10, 2019.

Problem Chapter 8.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\tanh(\beta y)w_y+c\tanh(\lambda x)w_z=k\tanh(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*tanh(beta*y)*diff(w(x, y,z), y) +  c*tanh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta}, \frac{a\lambda z - c\ln(\cosh}{a\lambda}\right.$$

7.8.9.5 [1821] Problem 5

problem number 1821

Added Oct 10, 2019.

Problem Chapter 8.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = k\tanh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \cosh^{\frac{k}{a\lambda}}(\lambda x)c_1\left(\frac{1}{2}\left(\frac{\log(\sinh(\beta y))}{\beta}-\frac{bx}{a}\right), \frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma}-\frac{2c\log(\sinh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*tanh(beta*y)*diff(w(x, y,z), y) +  c*tanh(gamma*z)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\cosh^{\frac{k}{a\lambda}}(\lambda x)\right)\_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta},\right.$$

7.8.9.6 [1822] Problem 6

problem number 1822

Added Oct 10, 2019.

Problem Chapter 8.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\tanh^{n_1}(\lambda_1 x)w_x + b_1\tanh^{m_1}(\beta_1 y)w_y + c_1\tanh^{k_1}(\gamma_1 z)w_z = \left(a_2\tanh^{n_2}(\lambda_2 x)w_x + b_2\tanh^{m_2}(\beta_2 y)w_y + c_2\tan\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tanh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Tanh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Tan
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tanh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*tanh(beta1*y)^m1*diff(w(x, y,z), y) + 
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\tanh^{-n1}(\lambda 1x)\right)dx\right) + \int \frac{a1\left(\tanh^{-m1}(\beta 1y)\right)}{b1}dy, \frac{a1\,z - c1\left(\int \left(\frac{\sinh(\gamma 1x)}{\cosh(\gamma 1x)}\right)^{k1}\left(\frac{\mathrm{si}}{\mathrm{co}}\right.\right.}{a1}\right.$$

7.8.10 4.4

Local contents

7.8.10.1 [1823] Problem 1

problem number 1823

Added Oct 10, 2019.

Problem Chapter 8.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Coth[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\beta x)\right)}{\beta n+\beta}\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*coth(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\coth^n(\beta x))dx}$$

7.8.10.2 [1824] Problem 2

problem number 1824

Added Oct 10, 2019.

Problem Chapter 8.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\coth(\lambda x)w_z=(k\coth(\beta x)+s\coth(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]== (k*Coth[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x\frac{k\coth(\beta K[1])+s\coth\left(\frac{\gamma(a\lambda z-c\log(\sinh(\lambda x))+c\,l}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*coth(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{2a\lambda z+c\ln(\coth(\lambda x)-1)+c\ln(\coth(\lambda x)+1)}{2a\lambda}\right)e^{-\left(\int^x\frac{-k\coth(\_a\beta)+s\coth\left(\frac{(-2a\lambda}{}\right.}{}\right.}$$

7.8.10.3 [1825] Problem 3

problem number 1825

Added Oct 10, 2019.

Problem Chapter 8.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\coth^n(\beta x)w_y+b\coth^k(\lambda x)w_z=c\coth^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Coth[beta*x]^n*D[w[x, y,z], y] +  b*Coth[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{m+1}(\gamma x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\coth^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(z-\frac{b\coth^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2}\right.}{k\lambda+\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*coth(beta*x)^n*diff(w(x, y,z), y) +  b*coth(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\coth^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\coth^m(\gamma x))dx}$$

7.8.10.4 [1826] Problem 4

problem number 1826

Added Oct 10, 2019.

Problem Chapter 8.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta y)w_y+c\coth(\lambda x)w_z=k\coth(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*coth(beta*y)*diff(w(x, y,z), y) +  c*coth(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-2}\right) - a\ln\left(\mathrm{RootOf}\left(\beta y - \mathrm{arccoth}(\_Z-1)\right)\right)}{2b\beta}, \right.$$

7.8.10.5 [1827] Problem 5

problem number 1827

Added Oct 10, 2019.

Problem Chapter 8.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = k\coth(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{k(\log(-\tanh(\lambda x))+\log(\cosh(\lambda x)))}{a\lambda}}c_1\left(\frac{a\log(\mathrm{sech}(\beta y))+b\beta x}{2a\beta}, \frac{2c\log(\mathrm{sech}(\beta y))}{\beta} - \frac{b\log\left(\mathrm{sech}^2(\gamma z)\right)}{\gamma}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*coth(beta*y)*diff(w(x, y,z), y) +  c*coth(gamma*z)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = (\coth(\lambda x)-1)^{-\frac{k}{2a\lambda}}(\coth(\lambda x)+1)^{-\frac{k}{2a\lambda}}\_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y-\operatorname{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y-\operatorname{arccoth}(\_Z-1))-2}\right) - }{2b\beta}\right.$$

7.8.10.6 [1828] Problem 6

problem number 1828

Added Oct 10, 2019.

Problem Chapter 8.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\coth^{n_1}(\lambda_1 x)w_x+b_1\coth^{m_1}(\beta_1 y)w_y+c_1\coth^{k_1}(\gamma_1 z)w_z = \left(a_2\coth^{n_2}(\lambda_2 x)w_x + b_2\coth^{m_2}(\beta_2 y)w_y + c_2\coth\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Coth[lambda1*x]^n1*D[w[x, y,z], x] + b1*Coth[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cot
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*coth(lambda1*x)^n1*diff(w(x, y,z), x) + b1*coth(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\coth^{-n1}(\lambda 1 x)\right) dx\right) + \int \frac{a1\left(\coth^{-m1}(\beta 1 y)\right)}{b1} dy, \frac{a1\, z - c1\left(\int \left(\frac{\cosh(\gamma 1 x)}{\sinh(\gamma 1 x)}\right)^{k1} \left(\frac{\text{co}}{\text{sin}}\right.\right.}{a1}\right.$$

7.8.11 4.5

Local contents

7.8.11.1 [1829] Problem 1

problem number 1829

Added Oct 10, 2019.

Problem Chapter 8.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c\sinh^n(\lambda y) w_z = (s\cosh^m(\beta x) + k\sinh^r(\gamma y))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*y]^n*D[w[x,y,z],z]== (s*Cosh[b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow c_1\left(y-\frac{bx}{a},z-\frac{c\sqrt{\cosh^2(\lambda y)}\text{sech}(\lambda y)\sinh^{n+1}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\lambda y)\right)}{b\lambda n+b\lambda}\right)\exp\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*sinh(lambda*y)^n*diff(w(x,y,z),z)= (s*cosh(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c\left(\sinh^n\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)\right)}{a}d\_a\right)\right)\mathrm{e}^{\int^x\frac{k\left(\sinh^r\left(\frac{(ay-(-\_a+x)b)\gamma}{a}\right)\right)+s(\cosh^m(}{a}}$$

7.8.11.2 [1830] Problem 2

problem number 1830

Added Oct 10, 2019.

Problem Chapter 8.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\lambda x)w_y+b\cosh^m(\beta x)w_z=s\cosh^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[lambda*x]^n*D[w[x, y,z], y] +  b*Cosh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{s\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{k+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cosh^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(\frac{b\sinh(\beta x)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sinh(lambda*x)^n*diff(w(x,y,z),y)+ b*cosh(beta*x)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\lambda x))\,dx\right),z-\left(\int b(\cosh^m(\beta x))\,dx\right)\right)e^{\int s(\cosh^k(\gamma x))dx}$$

7.8.11.3 [1831] Problem 3

problem number 1831

Added Oct 10, 2019.

Problem Chapter 8.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\lambda x)w_y+b\sinh^m(\beta y)w_z=s\sinh^k(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[lambda*x]^n*D[w[x, y,z], y] +  b*Sinh[beta*y]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cosh(lambda*x)^n*diff(w(x,y,z),y)+ b*sinh(beta*y)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cosh^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sinh^m\left(\left(a\left(\int (\cosh^n(\_b\lambda))\,d\_b\right) + y - \left(\int\right.\right.\right.\right.\right.\right.\right.$$

7.8.11.4 [1832] Problem 4

problem number 1832

Added Oct 10, 2019.

Problem Chapter 8.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tanh^n(\lambda x)w_y + b\coth^m(\beta x)w_z = s\coth^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[lambda*x]^n*D[w[x, y,z], y] +  b*Coth[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{s\coth^{k+1}(\gamma x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\coth^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(z - \frac{b\coth^{m+1}(\beta x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2}\right.}{\beta m+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*tanh(lambda*x)^n*diff(w(x,y,z),y)+ b*coth(beta*x)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\tanh^n(\lambda x))\,dx\right), z - \left(\int b(\coth^m(\beta x))\,dx\right)\right) e^{\int s(\coth^k(\gamma x))dx}$$

7.8.11.5 [1833] Problem 5

problem number 1833

Added Oct 10, 2019.

Problem Chapter 8.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = k\cosh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \sinh^{\frac{k}{a\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta}, \frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\sinh^{\frac{k}{a\lambda}}(\lambda x)\right)\_F1\left(\frac{-2a\lambda\,\mathrm{arctanh}\left(\mathrm{e}^{\beta y}\right)+2b\beta\,\mathrm{arctanh}\left(\mathrm{e}^{\lambda x}\right)}{b\beta\lambda}, \frac{-2a\lambda\,\mathrm{arctanh}\left(\mathrm{e}^{\gamma z}\right)+2c\gamma\,\mathrm{arct}}{c\gamma\lambda}\right.$$

7.8.12 5.1

Local contents

7.8.12.1 [1834] Problem 1

problem number 1834

Added Oct 17, 2019.

Problem Chapter 8.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\ln^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Log[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1,-\log(\beta x))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*ln(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,e^{\int c\ln(\beta x)^n dx}$$

7.8.12.2 [1835] Problem 2

problem number 1835

Added Oct 17, 2019.

Problem Chapter 8.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\ln^n(\beta x)w_z=s\ln^m(\lambda y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Log[beta*x]^n*D[w[x,y,z],z]== s*Log[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{s(-\log(\lambda y))^{-m}\log^m(\lambda y)\,\text{Gamma}(m+1,-\log(\lambda y))}{b\lambda}\right)c_1\left(y-\frac{bx}{a},z-\frac{c(-\log(\beta x)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*ln(beta*x)^n*diff(w(x,y,z),z)= s*ln(lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z - \left(\int \frac{c\ln(\beta x)^n}{a}dx\right)\right) e^{\int^x \frac{s\ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^m}{a}d\_a}$$

7.8.12.3 [1836] Problem 3

problem number 1836

Added Oct 17, 2019.

Problem Chapter 8.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = c\ln^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c(-\log(\gamma x))^{-m}\log^m(\gamma x)\,\text{Gamma}(m+1,-\log(\gamma x))}{\gamma}\right)c_1\left(y - \frac{a(-\log(\beta x))^{-n}\log^n\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int b\ln(\lambda x)^k\,dx\right)\right) e^{\int c\ln(\gamma x)^m dx}$$

7.8.12.4 [1837] Problem 4

problem number 1837

Added Oct 17, 2019.

Problem Chapter 8.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda y)w_z = c\ln^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*y]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*y)^k*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int^x b\ln\left(\left(a\left(\int \ln(\_b\beta)^n\,d\_b\right) + y - \left(\int a\ln(\beta x)^n\,d\right.\right.\right.\right.\right.$$

7.8.12.5 [1838] Problem 5

problem number 1838

Added Oct 17, 2019.

Problem Chapter 8.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \ln^{n_1}(\lambda_1 x) w_x + b_1 \ln^{m_1}(\beta_1 y) w_y + c_1 \ln^{k_1}(\gamma_1 z) w_z = \left(a_2 \ln^{n_2}(\lambda_2 x) + b_2 \ln^{m_2}(\beta_2 y) + c_2 \ln^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   a1*Log[lambda1*x]^n1*D[w[x, y,z], x] + b1*Log[beta1*y]^m1*D[w[x, y,z], x]*D[w[x, y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a1*ln(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*ln(beta1*y)^m1*diff(w(x,y,z),y)+  c1*ln(ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \ln(\lambda 1 x)^{-n1} dx\right) + \int \frac{a1 \ln(\beta 1 y)^{-m1}}{b1} dy, -\left(\int \ln(\lambda 1 x)^{-n1} dx\right) + \int \frac{a1 \ln(\gamma 1 z}{c1}\right.$$

7.8.13 5.2

Local contents

7.8.13.1 [1839] Problem 1

problem number 1839

Added Oct 17, 2019.

Problem Chapter 8.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cx^n \ln^k(\lambda y) w_z = s y^m \ln^r(\beta x) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*x^n*Log[lambda*y]^k*D[w[x,y,z],z]== s*y^m*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*x^n*ln(lambda*y)^k*diff(w(x,y,z),z)= s*y^m*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z-\left(\int^x \frac{c\_a^n \ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^k}{a} d\_a\right)\right) e^{\int^x \frac{s\left(\frac{ay-(-\_a+x)b}{a}\right)^m \ln(\_a\beta)^r}{a} d\_a}$$

7.8.13.2 [1840] Problem 2

problem number 1840

Added Oct 17, 2019.

Problem Chapter 8.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ax^n w_y + bx^m w_z = \left(cy \ln^k(\lambda x) + sz \ln^r(\beta x)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +  b*x^m*D[w[x,y,z],z]== (c*y*Log[lambda*x]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, \frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(\frac{(n^2+3n+2)\,(-\log(\beta x))^{-r}(-\log(\lambda x)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)= (c*y*ln(lambda*x)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,x^{n+1}+(n+1)\,y}{n+1}, \frac{-b\,x^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\int^x \frac{\left(a\_a^{n+1}-a\,x^{n+1}+ny+y\right)(m+1)c\ln(\_a\lambda)^k+(n+1)\left(b\_a\right.}{(n+1)(m+1)}}$$

7.8.13.3 [1841] Problem 3

problem number 1841

Added Oct 17, 2019.

Problem Chapter 8.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\lambda x)w_y + by^m w_z = \left(c\ln^k(\beta x) + s\ln^r(\gamma z)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*y^m*D[w[x,y,z],z]== (c*Log[b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{a(-\log(\lambda x))^{-n}\log^n(\lambda x)\,\mathrm{Gamma}(n+1,-\log(\lambda x))}{\lambda}, z - \int_1^x b\left(\frac{-a\,\mathrm{Gamma}(n+\right.\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*y^m*diff(w(x,y,z),z)= (c*ln(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.8.13.4 [1842] Problem 4

problem number 1842

Added Oct 17, 2019.

Problem Chapter 8.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln^n(\lambda x)w_x + zw_y + b\ln^k(\beta y)w_z = \left(cx^m + s\ln(\gamma y)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y,z], x] + z*D[w[x, y,z], y] +  b*Log[lambda*y]^k*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*ln(lambda*x)^n*diff(w(x,y,z),x)+z*diff(w(x,y,z),y)+ b*ln(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-2b\left(\int \ln(\lambda y)^k\,dy\right) + z^2, \frac{b\left(\int \frac{\ln(\lambda x)^{-n}}{a}dx\right) - \sqrt{2by\ln(\_a\lambda)^k - 2b\left(\int \ln(\lambda y)^k\,dy\right) + z}}{b}\right.$$

7.8.13.5 [1843] Problem 5

problem number 1843

Added Oct 17, 2019.

Problem Chapter 8.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax(\ln x)^n w_x + by(\ln y)^m w_y + cz(\ln z)^r w_z = k(\ln x)^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +  c*z*Log[z]^r*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{k\log^{-n+s+1}(x)}{a(-n)+as+a}}c_1\left(\frac{b\log^{1-n}(x)}{a(n-1)}-(m-1)^{\frac{1}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m,\frac{c\log^{1-n}(x)}{a(n-1)}-(r-1\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+b*y*ln(y)^m*diff(w(x,y,z),y)+ c*z*ln(z)^r*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=x^{-\frac{k\ln(x)^{-n+s}}{(n-s-1)a}}\_F1\left(\frac{-(n-1)\,a\ln(y)^{-m+1}+(m-1)\,b\ln(x)^{-n+1}}{(n-1)\,(m-1)\,b},\frac{-(n-1)\,a\ln(z)^{-r+1}+(r-}{(r-1)\,(n-1)}\right.$$

7.8.14 6.1

Local contents

7.8.14.1 [1844] Problem 1

problem number 1844

Added Oct 18, 2019.

Problem Chapter 8.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sin^n(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sin[lambda*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)}{\lambda n+\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sin(lambda*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,e^{\int c(\sin^n(\lambda x))dx}$$

7.8.14.2 [1845] Problem 2

problem number 1845

Added Oct 18, 2019.

Problem Chapter 8.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sin(\lambda z)w_z = (k\sin(\gamma x) + s\sin(\beta y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sin[lambda*z]*D[w[x,y,z],z]== (k*Sin[gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{k\cos(\gamma x)}{a\gamma}-\frac{s\cos(\beta y)}{b\beta}}c_1\left(y-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\lambda z}{2}\right)\right)}{\lambda}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*sin(lambda*z)*diff(w(x,y,z),z)= (k*sin(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\operatorname{RootOf}\left(\lambda z-\arctan\left(\frac{2\_Z\mathrm{e}^{\frac{c\lambda x}{a}}}{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}+1}\right)\right)\right)}{c\lambda}\right)\mathrm{e}^{\frac{-a\gamma s\cos(\beta y)-b\beta k\cos(\gamma}{ab\beta\gamma}}$$

7.8.14.3 [1846] Problem 3

problem number 1846

Added Oct 18, 2019.

Problem Chapter 8.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\sin^m(\beta x)w_z=c\sin^k(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*z]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*z)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y+\int a(\sin^n(\lambda x))\,dx, \int \frac{a(\sin^{-m}(\beta z))}{b}dz - \left(\int^y \left(\sin^{-n}\left(\lambda \operatorname{RootOf}\left(\_b - y + \int \right.\right.\right.\right.\right.$$

7.8.14.4 [1847] Problem 4

problem number 1847

Added Oct 18, 2019.

Problem Chapter 8.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta y)w_z = \left(c\sin^k(\gamma y) + s\sin^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*y)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y-\left(\int a(\sin^n(\lambda x))\,dx\right), z-\left(\int^x b\left(\sin^m\left(\left(a\left(\int (\sin^n(\_b\lambda))\,d\_b\right)+y-\left(\int a(s\right.\right.\right.\right.\right.\right.$$

7.8.14.5 [1848] Problem 5

problem number 1848

Added Oct 18, 2019.

Problem Chapter 8.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sin(\beta y)w_y + c\sin(\lambda x)w_z = k\sin(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +  c*Sin[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+}{2}\right.}{a\lambda m+a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*sin(beta*y)*diff(w(x,y,z),y)+ c*sin(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},z-\left(\int\frac{c(\sin^m(\lambda x))}{a}dx\right)\right.$$

7.8.14.6 [1849] Problem 6

problem number 1849

Added Oct 18, 2019.

Problem Chapter 8.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\sin^{n_1}(\lambda_1 x)w_x + b_1\sin^{m_1}(\beta_1 y)w_y + c_1\sin^{k_1}(\gamma_1 z)w_z = \left(a_2\sin^{n_2}(\lambda_2 x) + b_2\sin^{m_2}(\beta_2 y) + c_2\sin^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Sin[beta1*y]^m1*D[w[x, y,z], y] + c1*Sin[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*sin(beta1*y)^m1*diff(w(x,y,z),y)+ c1*sin(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\sin^{-m1}(\beta 1 y)\right)dy\right) + \int \frac{b1\left(\sin^{-k1}(\gamma 1 z)\right)}{c1}dz, x - \left(\int^{y} \frac{a1\left(\sin^{-m1}(\_f\beta 1)\right)\left(\sin\right.}{}\right.\right.$$

7.8.15 6.2

Local contents

7.8.15.1 [1850] Problem 1

problem number 1850

Added Oct 18, 2019.

Problem Chapter 8.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c\cos^n(\beta x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cos[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(-\frac{c\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n}{}\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cos(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cos^n(\beta x))dx}$$

7.8.15.2 [1851] Problem 2

problem number 1851

Added Oct 18, 2019.

Problem Chapter 8.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c\cos(\beta z) w_z = (k\cos(\lambda x) + s\cos(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cos[beta*z]*D[w[x,y,z],z]== (k*Cos[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{k\sin(\lambda x)}{a\lambda}+\frac{s\sin(\gamma y)}{b\gamma}}c_1\left(y-\frac{bx}{a},-\frac{\cosh^{-1}\left(-\frac{\sec(\beta z)\left(2\left(2\sec(\beta z)\sqrt{\sin^2(\beta z)\cos^2(\beta z)\sinh^2\left(\frac{\beta cx}{a}\right)}\left(\cosh\left(\frac{4\beta cx}{a}\right.\right.\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*cos(beta*z)*diff(w(x,y,z),z)= (k*cos(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\mathrm{RootOf}\left(\beta z-\arctan\left(\frac{\_Z^2\mathrm{e}^{\frac{2\beta cx}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2\beta cx}{a}}+1},\frac{2\_Z\mathrm{e}^{\frac{\beta cx}{a}}}{\_Z^2\mathrm{e}^{\frac{2\beta cx}{a}}+1}\right)\right)\right)}{\beta c}\right)\mathrm{e}^{\frac{a\lambda s\sin(\gamma y)+b\gamma k\sin(\lambda x)}{ab\gamma\lambda}}$$

7.8.15.3 [1852] Problem 3

problem number 1852

Added Oct 18, 2019.

Problem Chapter 8.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cos^n(\beta x)w_y+b\cos^k(\lambda x)w_z=c\cos^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(-\frac{c\sqrt{\sin^2(\gamma x)}\csc(\gamma x)\cos^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\cos^n(\beta x))\,dx\right),z-\left(\int b\left(\cos^k(\lambda x)\right)dx\right)\right)\mathrm{e}^{\int c(\cos^m(\gamma x))dx}$$

7.8.15.4 [1853] Problem 4

problem number 1853

Added Oct 18, 2019.

Problem Chapter 8.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cos^n(\beta x)w_y+b\cos^m(\gamma y)w_z=\left(c\cos^k(\gamma y)+s\cos^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[gamma*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(gamma*y)^m*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cos^n(\beta x))\,dx\right), z - \left(\int^x b\left(\cos^m\left(\left(a\left(\int (\cos^n(\_b\beta))\,d\_b\right) + y - \left(\int a(\right.\right.\right.\right.\right.$$

7.8.15.5 [1854] Problem 5

problem number 1854

Added Oct 18, 2019.

Problem Chapter 8.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\lambda x)w_z = k\cos(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +  c*Cos[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.}{\beta}\right.\right.$$

$$\left\{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\cdots\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh\cdots}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*cos(beta*y)*diff(w(x,y,z),y)+ c*cos(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, z-\left(\int \frac{c(\cos^m(\lambda x))}{a}dx\right)\right)$$

7.8.15.6 [1855] Problem 6

problem number 1855

Added Oct 18, 2019.

Problem Chapter 8.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cos^{n_1}(\lambda_1 x)w_x+b_1\cos^{m_1}(\beta_1 y)w_y+c_1\cos^{k_1}(\gamma_1 z)w_z = \left(a_2\cos^{n_2}(\lambda_2 x)+b_2\cos^{m_2}(\beta_2 y)+c_2\cos^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cos[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cos(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cos^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\cos^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\cos^{-m1}(\_f\beta 1)\right)\left(\mathrm{co}\right.}{}\right.\right.$$

7.8.16 6.3

Local contents

7.8.16.1 [1856] Problem 1

problem number 1856

Added Oct 18, 2019.

Problem Chapter 8.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c\tan^n(\beta x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tan[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\tan^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(\beta x)\right)}{\beta n+\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tan(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\tan^n(\beta x))dx}$$

7.8.16.2 [1857] Problem 2

problem number 1857

Added Oct 18, 2019.

Problem Chapter 8.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\tan(\beta z)w_z=(k\tan(\lambda x)+s\tan(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tan[beta*z]*D[w[x,y,z],z]== (k*Tan[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\cos^{-\frac{k}{a\lambda}}(\lambda x)\cos^{-\frac{s}{b\gamma}}(\gamma y)c_1\left(y-\frac{bx}{a},\frac{\log(\sin(\beta z))}{\beta}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*tan(beta*z)*diff(w(x,y,z),z)= (k*tan(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\tan^2(\gamma y)+1\right)^{\frac{s}{2b\gamma}} \left(\tan^2(\lambda x)+1\right)^{\frac{k}{2a\lambda}} \_F1\left(\frac{ay-bx}{a}, \frac{-\beta c x + a\ln\left(\frac{\tan(\beta z)}{\sqrt{\tan^2(\beta z)+1}}\right)}{\beta c}\right)$$

7.8.16.3 [1858] Problem 3

problem number 1858

Added Oct 18, 2019.

Problem Chapter 8.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tan^n(\beta x)w_y + b\tan^k(\lambda x)w_z = c\tan^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[beta*x]^n*D[w[x, y,z], y] +  b*Tan[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c\tan^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\tan^2(\gamma x)\right)}{\gamma m+\gamma}\right) c_1\left(y - \frac{a\tan^{n+1}(\beta x}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*tan(beta*x)^n*diff(w(x,y,z),y)+ b*tan(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\tan^n(\beta x))\,dx\right), z - \left(\int b(\tan^k(\lambda x))\,dx\right)\right) e^{\int c(\tan^m(\gamma x))dx}$$

7.8.16.4 [1859] Problem 4

problem number 1859

Added Oct 18, 2019.

Problem Chapter 8.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\lambda x)w_z = k\tan(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +  c*Tan[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}, \frac{-c\tan^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\tan^2(\lambda x)\right) +}{a\lambda m + a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*tan(beta*y)*diff(w(x,y,z),y)+ c*tan(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, z - \left(\int \frac{c(\tan^m(\lambda x))}{a}dx\right)\right) e^{\int^x -\frac{k\tan\left(\left(-z-\left(\int \frac{c(\tan^m(\_b\lambda))}{a}d\_b\right)+\right.\right.}{a}}$$

7.8.16.5 [1860] Problem 5

problem number 1860

Added Oct 18, 2019.

Problem Chapter 8.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\tan^{n_1}(\lambda_1 x)w_x + b_1\tan^{m_1}(\beta_1 y)w_y + c_1\tan^{k_1}(\gamma_1 z)w_z = \left(a_2\tan^{n_2}(\lambda_2 x) + b_2\tan^{m_2}(\beta_2 y) + c_2\tan^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Tan[beta1*y]^m1*D[w[x, y,z], y] + c1*Tan[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*tan(beta1*y)^m1*diff(w(x,y,z),y)+ c1*tan(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\tan^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\tan^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\tan^{-m1}(\_f\beta 1)\right)\left(\mathrm{t}\right.}{}\right.\right.$$

7.8.17 6.4

Local contents

7.8.17.1 [1861] Problem 1

problem number 1861

Added Oct 18, 2019.

Problem Chapter 8.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c\cot^n(\beta x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cot[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(-\frac{c\cot^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(\beta x)\right)}{\beta n+\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cot(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cot^n(\beta x))dx}$$

7.8.17.2 [1862] Problem 2

problem number 1862

Added Oct 18, 2019.

Problem Chapter 8.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cot(\beta z)w_z=(k\cot(\lambda x)+s\cot(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cot[beta*z]*D[w[x,y,z],z]== (k*Cot[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\sin^{\frac{k}{a\lambda}}(\lambda x)e^{\frac{s(\log(\tan(\gamma y))+\log(\cos(\gamma y)))}{b\gamma}}c_1\left(y-\frac{bx}{a},\frac{\log(\sec(\beta z))}{\beta}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*cot(beta*z)*diff(w(x,y,z),z)= (k*cot(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\cot^2(\gamma y)+1\right)^{-\frac{s}{2b\gamma}} \left(\cot^2(\lambda x)+1\right)^{-\frac{k}{2a\lambda}} \_F1\left(\frac{-ay+bx}{b}, \frac{-2\beta cy+b\ln\left(\cot^2(\beta z)+1\right)-2b\ln}{2\beta c}\right.$$

7.8.17.3 [1863] Problem 3

problem number 1863

Added Oct 18, 2019.

Problem Chapter 8.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\beta x)w_y + b\cot^k(\lambda x)w_z = c\cot^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[beta*x]^n*D[w[x, y,z], y] +  b*Cot[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(-\frac{c\cot^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\gamma x)\right)}{\gamma m+\gamma}\right) c_1\left(\frac{b\cot^{k+1}(\lambda x)\,\text{H}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cot(beta*x)^n*diff(w(x,y,z),y)+ b*cot(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cot^n(\beta x))\,dx\right), z - \left(\int b(\cot^k(\lambda x))\,dx\right)\right) \mathrm{e}^{\int c(\cot^m(\gamma x))dx}$$

7.8.17.4 [1864] Problem 4

problem number 1864

Added Oct 18, 2019.

Problem Chapter 8.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\lambda x)w_z = k\cot(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +  c*Cot[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z) \to c_1\left(\frac{c\cot^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\lambda x)\right)+a\lambda mz+a\lambda z}{a\lambda m+a\lambda},\frac{\log(\sec(\beta}{\beta}\right.\right.$$

$$\left\{w(x,y,z) \to c_1\left(\frac{c\cot^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\lambda x)\right)+a\lambda mz+a\lambda z}{a\lambda m+a\lambda},\frac{\log(\sec(\beta}{\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*cot(beta*y)*diff(w(x,y,z),y)+ c*cot(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x - a\ln\left(\cot^2(\beta y)+1\right)+2a\ln\left(\cot(\beta y)\right)}{2b\beta}, z - \left(\int^y \frac{c\left(\frac{\cot\left(\frac{a\lambda\ln(\cot(\_a\beta))}{b\beta}\right)\cot\left(\frac{(2b\beta x - a\ln}{}\right.}{\cot\left(\frac{a\lambda\ln(\cot(\_}{b\beta}\right.}\right.}{}\right.\right.$$

7.8.17.5 [1865] Problem 5

problem number 1865

Added Oct 18, 2019.

Problem Chapter 8.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cot^{n_1}(\lambda_1 x)w_x + b_1\cot^{m_1}(\beta_1 y)w_y + c_1\cot^{k_1}(\gamma_1 z)w_z = \left(a_2\cot^{n_2}(\lambda_2 x) + b_2\cot^{m_2}(\beta_2 y) + c_2\cot^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cot[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cot(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cot^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\cot^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\cot^{-m1}(\_f\beta 1)\right)\left(\mathrm{co}\right.}{}\right.\right.$$

7.8.18 6.5

Local contents

7.8.18.1 [1866] Problem 1

problem number 1866

Added Oct 18, 2019.

Problem Chapter 8.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\cos^m(\beta x)w_z = c\sin^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c\sqrt{\cos^2(\gamma x)}\sec(\gamma x)\sin^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(\frac{b}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int b(\cos^m(\beta x))\,dx\right)\right) e^{\int c(\sin^k(\gamma x))dx}$$

7.8.18.2 [1867] Problem 2

problem number 1867

Added Oct 18, 2019.

Problem Chapter 8.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\sin^m(\beta y)w_z = \left(c\cos^k(\gamma y) + s\sin^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+b*cos(lambda*x)^n*diff(w(x,y,z),y)+  b*sin(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int b(\cos^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sin^m\left(\left(\left(b\left(\int (\cos^n(\_b\lambda))\,d\_b\right) + y - \left(\int b(c\right.\right.\right.\right.\right.\right.$$

7.8.18.3 [1868] Problem 3

problem number 1868

Added Oct 18, 2019.

Problem Chapter 8.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\tan^m(\beta y)w_z = \left(c\cos^k(\gamma y) + s\tan^k(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Tan[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*y)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cos^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\frac{-\tan\left(\left(y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\beta\right) - \tan\left(a\beta\right.}{\tan\left(\left(y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\beta\right)\tan\left(a\beta\left(\int\right.\right.}\right.\right.\right.$$

7.8.18.4 [1869] Problem 4

problem number 1869

Added Oct 18, 2019.

Problem Chapter 8.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sin^{n_1}(\lambda_1 x)w_x + b_1 \cot^{m_1}(\beta_1 y)w_y + c_1 \cos^{k_1}(\gamma_1 z)w_z = \left(a_2 \cos^{n_2}(\lambda_2 x) + b_2 \sin^{m_2}(\beta_2 y) + c_2 \cos^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\frac{\cos(\beta 1 y)}{\sin(\beta 1 y)}\right)^{-m1} dy\right) + \int \frac{b1\left(\cos^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\frac{\cos(\_f\beta 1)}{\sin(\_f\beta 1)}\right)^{-m1}\left(\mathrm{s}\right.}{}\right.\right.$$

7.8.18.5 [1870] Problem 5

problem number 1870

Added Oct 18, 2019.

Problem Chapter 8.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \tan^{n_1}(\lambda_1 x) w_x + b_1 \cot^{m_1}(\beta_1 y) w_y + c_1 \cot^{k_1}(\gamma_1 z) w_z = \left(a_2 \cot^{n_2}(\lambda_2 x) + b_2 \tan^{m_2}(\beta_2 y) + c_2 \cot^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cot^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\cot^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\frac{\cos(\_f\beta 1)}{\sin(\_f\beta 1)}\right)^{-m1}\left(\frac{\sin}{\cos}\right.}{}\right.\right.$$

7.8.19 7.1

Local contents

7.8.19.1 [1871] Problem 1

problem number 1871

Added Nov 30, 2019.

Problem Chapter 8.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c \arcsin^n(\beta x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcSin[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{ic\sin^{-1}(\beta x)^n\left(\sin^{-1}(\beta x)^2\right)^{-n}\left((-i\sin^{-1}(\beta x))^n\,\text{Gamma}\left(n+1,i\,\mathrm{s}\right.\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arcsin(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\arcsin(\beta x)^n dx}$$

7.8.19.2 [1872] Problem 2

problem number 1872

Added Nov 30, 2019.

Problem Chapter 8.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=\left(b_1\arcsin(\lambda_1x)+b_2\arcsin(\lambda_2y)+b_3\arcsin(\lambda_3z)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcSin[lambda1*x]+b2*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)\exp\left(\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\sin^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}\sqrt{1-\text{l}}}{\text{a2lar}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arcsin(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right)\mathrm{e}^{\frac{\sqrt{-y^2\lambda_2^2+1}\,a_1a_3b_2\lambda_1\lambda_3+\left(\sqrt{-z^2\lambda_3^2+1}\,a_1a_2b_3\lambda_1+\left(\sqrt{-\lambda_1^2x^2+1}\,a_2a_3b_1+(a_2a_3b_1x\arcsin(x\lambda\right.\right.}{a_1a_2a_3\lambda_1\lambda_2\lambda_3}}$$

7.8.19.3 [1873] Problem 3

problem number 1873

Added Nov 30, 2019.

Problem Chapter 8.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z=s\arcsin^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda x 2^n 2^{-n}}{2} + \frac{(\lambda x-1)(\lambda x+1)(k\cdots}{\cdots}\right.}{\cdots}\right.$$

7.8.19.4 [1874] Problem 4

problem number 1874

Added Nov 30, 2019.

Problem Chapter 8.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\left(\int^{x} \arcsin\left(\_a\lambda\right)^{n} \arcsin\left(\frac{\left(ay-\left(-\_a+x\right)b\right)\beta}{a}\right)^{m} d\_a\right) - \frac{\left(-\gamma k z 2^{k} \mathrm{Lo}\right.}{}\right.$$

7.8.19.5 [1875] Problem 5

problem number 1875

Added Nov 30, 2019.

Problem Chapter 8.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcSin[lambda*x]^n*D[w[x,y,z],y]+c*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} - \frac{i\sin^{-1}(\beta z)^{-k}\left(\left(-i\sin^{-1}(\beta z)\right)^{k}\mathrm{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right) - \left(i\sin^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.8.19.6 [1876] Problem 6

problem number 1876

Added Nov 30, 2019.

Problem Chapter 8.7.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda y)w_y + c\arcsin^k(\beta z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcSin[lambda*y]^n*D[w[x,y,z],y]+c*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left(-\arcsin(\lambda y)^{-n}\arcsin(\lambda y)^{\frac{3}{2}} + \mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda y)\right)\arcsin(\lambda y)\right)\sqrt{-}}{}\right.$$

7.8.20 7.2

Local contents

7.8.20.1 [1877] Problem 1

problem number 1877

Added Nov 30, 2019.

Problem Chapter 8.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arccos^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcCos[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\cos^{-1}(\beta x)^n\left(\cos^{-1}(\beta x)^2\right)^{-n}\left(\left(-i\cos^{-1}(\beta x)\right)^n\text{Gamma}\left(n+1,i\cos\right.\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccos(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\arccos(\beta x)^n dx}$$

7.8.20.2 [1878] Problem 2

problem number 1878

Added Nov 30, 2019.

Problem Chapter 8.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=\left(b_1\arccos(\lambda_1x)+b_2\arccos(\lambda_2y)+b_3\arccos(\lambda_3z)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcCos[lambda1*x]+b2*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\mathrm{a2}x}{\mathrm{a1}},z-\frac{\mathrm{a3}x}{\mathrm{a1}}\right)\exp\left(-\frac{\mathrm{b1}\sqrt{1-\mathrm{lambda1}^2x^2}}{\mathrm{a1lambda1}}+\frac{\mathrm{b1}x\cos^{-1}(\mathrm{lambda1}x)}{\mathrm{a1}}+\frac{\mathrm{b2}x\sin^{-}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arccos(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right)\mathrm{e}^{\frac{-\sqrt{-y^2\lambda_2^2+1}\,a_1a_3b_2\lambda_1\lambda_3+\left(-\sqrt{-z^2\lambda_3^2+1}\,a_1a_2b_3\lambda_1+\left(-\sqrt{-\lambda_1^2x^2+1}\,a_2a_3b_1+(a_2a_3b_1x\,\mathrm{arcc}\right.\right.}{a_1a_2a_3\lambda_1\lambda_2\lambda_3}}$$

7.8.20.3 [1879] Problem 3

problem number 1879

Added Nov 30, 2019.

Problem Chapter 8.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arccos^n(\lambda x)\arccos^k(\beta z)w_z=s\arccos^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^{n-}}{\sqrt{\pi}\,(n+2)}\right.}{\lambda}\right.$$

7.8.20.4 [1880] Problem 4

problem number 1880

Added Nov 30, 2019.

Problem Chapter 8.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^m(\beta y)\arccos^k(\gamma z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCos[lambda*x]^n*ArcCos[beta*y]^m*ArcCos[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*y)^m*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{-(k-2)\,c\gamma\left(\int^x \arccos(\_a\lambda)^n \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^m d\_a\right) + \frac{\left((k-2)\gamma z\,\mathrm{Lomm}\right.}{}}{}\right.$$

7.8.20.5 [1881] Problem 5

problem number 1881

Added Nov 30, 2019.

Problem Chapter 8.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = s\arccos^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCos[lambda*x]^n*D[w[x,y,z],y]+c*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} + \frac{\cos^{-1}(\beta z)^{-k}\left((-i\cos^{-1}(\beta z))^k\,\mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right) + (i\cos^{-1}(\beta z))^{k}\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\frac{\left(-\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\arccos(\lambda x)+\arccos(\lambda x)^{n+\frac{3}{2}}+(n+2)\operatorname{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)\right)\sqrt{-\lambda^2x^2+}}{\sqrt{\arccos(\lambda x)}}}{(n+}\right.$$

7.8.20.6 [1882] Problem 6

problem number 1882

Added Nov 30, 2019.

Problem Chapter 8.7.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda y)w_y + c\arccos^k(\beta z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCos[lambda*y]^n*D[w[x,y,z],y]+c*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*y)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x + \frac{\sqrt{\pi}\left(\frac{\sqrt{-\lambda^2y^2+1}\,2^n\,\mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)}}{\sqrt{\pi}\,(n-2)} - \frac{\sqrt{-\lambda^2y^2+1}\,2^n\arccos(\lambda y)^{-n+1}}{\sqrt{\pi}\,(n-2)} + \frac{3(}{}\right.}{b\lambda}\right.$$

7.8.21 7.3

Local contents

7.8.21.1 [1883] Problem 1

problem number 1883

Added Nov 30, 2019.

Problem Chapter 8.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arctan^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcTan[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\int_1^x c\tan^{-1}(\beta K[1])^n dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arctan(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\arctan(\beta x)^n dx}$$

7.8.21.2 [1884] Problem 2

problem number 1884

Added Nov 30, 2019.

Problem Chapter 8.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=(b_1\arctan(\lambda_1x)+b_2\arctan(\lambda_2y)+b_3\arctan(\lambda_3z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcTan[lambda1*x]+b2*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\left(\text{lambda1}^2x^2+1\right)^{-\frac{\text{b1}}{2\text{a1lambda1}}}\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)^{-\frac{\text{b2}}{2\text{a2lambda2}}}\left(\text{a1}^2\left(\text{lambda3}^2z^2+1\right)\right)^{-\frac{}{2\text{a3}}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arctan(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\lambda_1^2x^2+1\right)^{-\frac{b_1}{2a_1\lambda_1}}\left(y^2\lambda_2^2+1\right)^{-\frac{b_2}{2a_2\lambda_2}}\left(z^2\lambda_3^2+1\right)^{-\frac{b_3}{2a_3\lambda_3}}\_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right)\mathrm{e}^{\frac{a_1a_3b_2y\arctan}{}}$$

7.8.21.3 [1885] Problem 3

problem number 1885

Added Nov 30, 2019.

Problem Chapter 8.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^k(\beta z)w_z = s\arctan^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\gamma^2x^2+1\right)^{-\frac{s}{2a\gamma}}\_F1\left(\frac{ay-bx}{a},-\left(\int\arctan(\lambda x)^n\,dx\right)+\int\frac{a\arctan(\beta z)^{-k}}{c}dz\right)\mathrm{e}^{\frac{sx\arctan(\gamma x)}{a}}$$

7.8.21.4 [1886] Problem 4

problem number 1886

Added Nov 30, 2019.

Problem Chapter 8.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c \arctan^n(\lambda x) \arctan^m(\beta y) \arctan^k(\gamma z) w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*ArcTan[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*y)^m*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{ay - bx}{a}, \int \frac{a \arctan(\gamma z)^{-k}}{c} dz - \left(\int^x \arctan(\_a\lambda)^n \arctan\left(\frac{(ay - (-\_a + x)\, b)\, \beta}{a}\right)^n\right.\right.$$

7.8.21.5 [1887] Problem 5

problem number 1887

Added Nov 30, 2019.

Problem Chapter 8.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b \arctan^n(\lambda x) w_y + c \arctan^k(\beta z) w_z = s \arctan^m(\gamma x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcTan[lambda*x]^n*D[w[x,y,z],y]+c*ArcTan[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^z \frac{s \tan^{-1}(\beta K[3])^{-k} \tan^{-1}\left(\frac{\gamma\left(cx - a\int_1^z \tan^{-1}(\beta K[2])^{-k} dK[2] + a\int_1^{K[3]} \tan^{-1}(\beta K[2])^{-k} dK[2]\right)}{c}\right)}{c}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y + \int \frac{b \arctan(\lambda x)^n}{a} dx, \int \frac{b \arctan(\beta z)^{-k}}{c} dz - \left(\int^y \arctan\left(\lambda \operatorname{RootOf}\left(\_b - y + \right.\right.\right.\right.$$

7.8.22 7.4

Local contents

7.8.22.1 [1888] Problem 1

problem number 1888

Added Nov 30, 2019.

Problem Chapter 8.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c \operatorname{arccot}^n(\beta x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcCot[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\int_1^x c\cot^{-1}(\beta K[1])^n dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccot(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,e^{\int c\left(-\arctan(\beta x)+\frac{\pi}{2}\right)^n dx}$$

7.8.22.2 [1889] Problem 2

problem number 1889

Added Nov 30, 2019.

Problem Chapter 8.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 w_x + a_2 w_y + a_3 w_z = (b_1 \operatorname{arccot}(\lambda_1 x) + b_2 \operatorname{arccot}(\lambda_2 y) + b_3 \operatorname{arccot}(\lambda_3 z))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcCot[lambda1*x]+b2*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \left(\text{lambda1}^2x^2+1\right)^{\frac{\text{b1}}{2\text{a1lambda1}}} \left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)^{\frac{\text{b2}}{2\text{a2lambda2}}} \left(\text{a1}^2\left(\text{lambda3}^2z^2+1\right)\right)^{\frac{\text{b3}}{2\text{a3lambd}}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arccot(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\lambda_1^2x^2+1\right)^{\frac{b_1}{2a_1\lambda_1}} \left(y^2\lambda_2^2+1\right)^{\frac{b_2}{2a_2\lambda_2}} \left(z^2\lambda_3^2+1\right)^{\frac{b_3}{2a_3\lambda_3}} \_F1\left(\frac{ya_1-xa_2}{a_1}, \frac{za_1-a_3x}{a_1}\right) \mathrm{e}^{\frac{-2a_1a_3b_2y\arctan(y\lambda}{}}$$

7.8.22.3 [1890] Problem 3

problem number 1890

Added Nov 30, 2019.

Problem Chapter 8.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\,\mathrm{arccot}^n(\lambda x)\,\mathrm{arccot}^k(\beta z)w_z = s\,\mathrm{arccot}^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\gamma^2x^2+1\right)^{\frac{s}{2a\gamma}} \_F1\left(\frac{ay-bx}{a}, -\left(\int \left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n dx\right) + \int \frac{a\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}\right.$$

7.8.22.4 [1891] Problem 4

problem number 1891

Added Nov 30, 2019.

Problem Chapter 8.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\,\mathrm{arccot}^n(\lambda x)\,\mathrm{arccot}^m(\beta y)\,\mathrm{arccot}^k(\gamma z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \int \frac{a\left(-\arctan(\gamma z)+\frac{\pi}{2}\right)^{-k}}{c}dz - \left(\int^{x} \left(-\arctan(\_a\lambda)+\frac{\pi}{2}\right)^{n}\left(-\arctan\left(\frac{(a\right.\right.\right.\right.$$

7.8.22.5 [1892] Problem 5

problem number 1892

Added Nov 30, 2019.

Problem Chapter 8.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\,\mathrm{arccot}^n(\lambda x)w_y + c\,\mathrm{arccot}^k(\beta z)w_z = s\,\mathrm{arccot}^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCot[lambda*x]^n*D[w[x,y,z],y]+c*ArcCot[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^z \frac{s\cot^{-1}(\beta K[3])^{-k}\cot^{-1}\left(\frac{\gamma\left(cx-a\int_1^z \cot^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\cot^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)^{m}}{c}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+ c*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int\frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx,\int\frac{b\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\int^y\left(-\arctan\left(\right.\right.\right.\right.$$

7.8.23 8.1

Local contents

7.8.23.1 [1893] Problem 1

problem number 1893

Added December 1, 2019.

Problem Chapter 8.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + f(x)w_y + g(x)w_z = (h_2(x)y + h_1(x)z + h_0(x))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+f[x]*D[w[x,y,z],y]+g[x]*D[w[x,y,z],z]==(h2[x]*y+h1[x]*z+h0[x])*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x f(K[1])dK[1], z-\int_1^x g(K[2])dK[2]\right)\exp\left(\int_1^x\left(\text{h0}(K[3])+\text{h2}(K[3])\left(y-\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+ g(x)*diff(w(x,y,z),z)= (h2(x)*y+h1(x)*z+h0(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y-\left(\int f(x)\,dx\right), z-\left(\int g(x)\,dx\right)\right)e^{\int^x \left(\left(\int f(\_f)d\_f\right)h2(\_f)+\left(\int g(\_f)d\_f\right)h1(\_f)+h0(\_f)+(z-(\int\right.}$$

7.8.23.2 [1894] Problem 2

problem number 1894

Added December 1, 2019.

Problem Chapter 8.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + f(x)(y+a)w_y + g(x)(z+b)w_z = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+f[x]*(y+a)*D[w[x,y,z],y]+g[x]*(z+b)*D[w[x,y,z],z]==h[x]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x h(K[5])dK[5]\right)c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right)-\int_1^x a\exp\left(-\int_1^{K[2]} f(K[1]\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+ g(x)*(z+b)*diff(w(x,y,z),z)= h(x)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((a+y)\,\mathrm{e}^{-\left(\int f(x)dx\right)}, (b+z)\,\mathrm{e}^{-\left(\int g(x)dx\right)}\right)\mathrm{e}^{\int h(x)dx}$$

7.8.23.3 [1895] Problem 3

problem number 1895

Added December 1, 2019.

Problem Chapter 8.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ay + f(x))\, w_y + (bz + g(x))\, w_z = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+f[x])*D[w[x,y,z],y]+(b*z+g[x])*D[w[x,y,z],z]==h[x]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x h(K[3])dK[3]\right) c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]} f(K[1])dK[1], ze^{-bx} - \int_1^x e^{-bK[2]} g(K[2]\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+ (b*z+g(x))*diff(w(x,y,z),z)= h(x)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,e^{-ax} - \left(\int e^{-ax} f(x)\,dx\right), z\,e^{-bx} - \left(\int e^{-bx} g(x)\,dx\right)\right) e^{\int h(x)dx}$$

7.8.23.4 [1896] Problem 4

problem number 1896

Added December 1, 2019.

Problem Chapter 8.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x))\, w_y + (g_1(x)y + g_2(x))\, w_z = (h_2(x)y + h_1(x)z + h_0(x))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*y+g2[x])*D[w[x,y,z],z]==(h2[x]*y+h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right)-\int_1^x\exp\left(-\int_1^{K[3]}\text{f1}(K[1])dK[1]\right)\text{f2}(K[3])dK[3],-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)}f2(x)\,dx\right),z-\left(\int^x\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}\mathrm{e}^{\int f1(\_f)d\_f}g1(\_f)+\right.\right.\right.$$

7.8.23.5 [1897] Problem 5

problem number 1897

Added December 1, 2019.

Problem Chapter 8.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))\,w_y+(g_1(x)z+g_2(x))\,w_z=(h_2(x)y+h_1(x)z+h_0(x))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*z+g2[x])*D[w[x,y,z],z]==(h2[x]*y+h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2], z\,e\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*z+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)} f2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int g1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int g1(x)dx\right)} g2(x)\,dx\right)\right)$$

7.8.23.6 [1898] Problem 6

problem number 1898

Added December 1, 2019.

Problem Chapter 8.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(y^2-a^2+a\lambda\sinh(\lambda x)-a^2\sinh^2(\lambda x)\right)w_y+f(x)\sinh(\gamma z)w_z=g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x,y,z],y]+f[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x g(K[3])dK[3]\right)c_1\left(\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\int_1^x f(K[2])dK[2],\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}}-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{2\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda}\right.\right.}{-(-\sinh(\lambda x)+i)\left(\sinh^2(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda}{2}\right.}\right.$$

7.8.23.7 [1899] Problem 7

problem number 1899

Added December 1, 2019.

Problem Chapter 8.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(f_1(x)y+f_2(x)y^k\right)w_y+\left(g_1(x)z+g_2(x)z^m\right)w_z=h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]*z+g2[x]*z^m)*D[w[x,y,z],z]==h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\int_1^x h(K[5])dK[5]\right)c_1\left((k-1)\int_1^x\exp\left((k-1)\int_1^{K[2]}\text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(x)*z^m)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}+(k-1)\left(\int\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}f2(x)\,dx\right),z^{-m+1}\mathrm{e}^{(m-1)\left(\int g1(x)dx\right)}+(\right.$$

7.8.23.8 [1900] Problem 8

problem number 1900

Added December 1, 2019.

Problem Chapter 8.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(f_1(x)y+f_2(x)y^k\right)w_y+\left(g_1(x)+g_2(x)e^{\lambda z}\right)w_z=h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[lambda*z])*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(lambda*z))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}e^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int e^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), \frac{-\lambda\left(\int e^{\lambda\left(\int g1(x)dx\right)} g2(x)\,d\right.}{\lambda}\right.$$

7.8.23.9 [1901] Problem 9

problem number 1901

Added December 1, 2019.

Problem Chapter 8.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x) + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x) + g_2(x)e^{\beta z}\right) w_z = h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[beta*z])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int f1(x)dx\right)} f2(x)\, dx\right) - \mathrm{e}^{-\left(y-\left(\int f1(x)dx\right)\right)\lambda}}{\lambda}, \frac{-\beta\left(\int \mathrm{e}^{\beta\left(\int g1(x)dx\right)} g2(x)\, dx\right) - \mathrm{e}^{-\left(z-\left(\int\right.\right.}}{\beta}\right.$$

7.8.24 8.2

Local contents

7.8.24.1 [1902] Problem 1

problem number 1902

Added December 1, 2019.

Problem Chapter 8.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = (\varphi(z) + \psi(y) + \chi(z))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x,y,z],x]+g[y]*D[w[x,y,z],y]+h[z]*D[w[x,y,z],z]==(varphi[z]+psi[y]+chi[z])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ g(y)*diff(w(x,y,z),y)+ h(x)*diff(w(x,y,z),z)=(varphi(z)+psi(y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy, z - \left(\int \frac{h(x)}{f(x)}dx\right)\right) e^{\int^{x} \frac{\chi\left(z+\int \frac{h(\_g)}{f(\_g)}d\_g-\left(\int \frac{h(x)}{f(x)}dx\right)\right)+\psi\left(\text{RootOf}\left(\int\right.\right.}{}}$$

7.8.24.2 [1903] Problem 2

problem number 1903

Added December 1, 2019.

Problem Chapter 8.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + zw_y + g(y)w_z = (h_2(x) + h_1(y))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x,y,z],x]+z*D[w[x,y,z],y]+g[y]*D[w[x,y,z],z]==(h2[x]+h1[y])*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ g(y)*diff(w(x,y,z),z)=(h__2(x)+h__1(y))*w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(z^2 - 2\left(\int g(y)\,dy\right), \int \frac{1}{f(x)}dx - \left(\int^y \frac{1}{\sqrt{z^2 + 2\left(\int g(\_b)\,d\_b\right) - 2\left(\int g(y)\,dy\right)}}d\_b\right.\right.$$

7.8.24.3 [1904] Problem 3

problem number 1904

Added December 1, 2019.

Problem Chapter 8.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = f_4(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x,y,z],x]+f2[x]*g[y]*D[w[x,y,z],y]+f3[x]*h[z]*D[w[x,y,z],z]==f4[x]*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f__1(x)*diff(w(x,y,z),x)+ f__2(x)*g(y)*diff(w(x,y,z),y)+ f__3(x)*h(z)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\int \frac{1}{g(y)}dy - \left(\int \frac{f_2(x)}{f_1(x)}dx\right), \int \frac{1}{h(z)}dz - \left(\int \frac{f_3(x)}{f_1(x)}dx\right)\right) \mathrm{e}^{\int \frac{f_4(x)}{f_1(x)}dx}$$

7.8.24.4 [1905] Problem 4

problem number 1905

Added December 1, 2019.

Problem Chapter 8.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))\, w_y + (g_1(x)z + g_2(y))\, w_z = (h_1(x) + h_2(y))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*z+g2[x])*D[w[x,y,z],z]==(h1[x]+h2[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x \left(\mathrm{h1}(K[5]) + \mathrm{h2}\left(\exp\left(\int_1^{K[5]} \mathrm{f1}(K[1])dK[1]\right)\left(\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right) y\right.\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x))*diff(w(x,y,z),y)+ (g__1(x)*z+g__2(x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int g_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int g_1(x)dx\right)} g_2(x)\,dx\right)\right)$$

7.8.24.5 [1906] Problem 5

problem number 1906

Added December 1, 2019.

Problem Chapter 8.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right)w_y + \left(g_1(y)z + g_2(x)z^m\right)w_z = \left(h_1(x) + h_2(y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[y]*z+g2[x])*D[w[x,y,z],z]==(h1[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x \left(\mathrm{h1}(K[5]) + \mathrm{h2}\left(\left(\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1] - (k-1)\int_1^{K[5]} \mathrm{f1}(K[1])dK[1]\right)\right.\right.\right.\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(y)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(y)*z+g__2(x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8.24.6 [1907] Problem 6

problem number 1907

Added December 1, 2019.

Problem Chapter 8.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x) + g_2(y)e^{\lambda z}\right) w_z = (h_1(x) + h_2(y))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[lambda*z])*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x)+g__2(x)*exp(lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g_1(x)dx\right)} g_2(x)\,dx\right.}{\lambda}\right.$$

7.8.24.7 [1908] Problem 7

problem number 1908

Added December 1, 2019.

Problem Chapter 8.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(y)z + g_2(x)z^k\right) w_z = (h_1(x) + h_2(y))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[y]*z+g2[x]*z^k)*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(y)*z+g__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8.24.8 [1909] Problem 8

problem number 1909

Added December 1, 2019.

Problem Chapter 8.8.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x) + g_2(x)e^{\beta z}\right) w_z = (h_1(x) + h_2(y))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x]+g2[y]*Exp[beta*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x)+g__2(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8.25 8.3

Local contents

7.8.25.1 [1910] Problem 1

problem number 1910

Added Jan 1, 2020.

Problem Chapter 8.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)w_z = g(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+f[x,y]*D[w[x,y,z],z]==g[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x \frac{g\left(K[2],y+\frac{b(K[2]-x)}{a}\right)}{a}dK[2]\right)c_1\left(y-\frac{bx}{a},z-\int_1^x \frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*diff(w(x,y,z),z)=g(x,y)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z-\left(\int^x \frac{f\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a\right)\right)e^{\int^x \frac{g\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a}$$

7.8.25.2 [1911] Problem 2

problem number 1911

Added Jan 1, 2020.

Problem Chapter 8.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + f(x, y)g(z)w_z = h(x, y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+f[x,y]*g[z]*D[w[x,y,z],z]==h[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=h(x,y)*w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{ay - bx}{a}, \int \frac{a}{g(z)}dz - \left(\int^x f\left(\_a, \frac{ay - (-\_a + x)b}{a}\right) d\_a\right)\right) e^{\int^x \frac{h\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a} d\_a}$$

7.8.25.3 [1912] Problem 3

problem number 1912

Added Jan 1, 2020.

Problem Chapter 8.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x, y))w_z = g(x, y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+y*D[w[x,y,z],y]+(z+f[x,y])*D[w[x,y,z],z]==g[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x \frac{g\left(K[2],\frac{yK[2]}{x}\right)}{K[2]}dK[2]\right)c_1\left(\frac{y}{x},\frac{z}{x}-\int_1^x \frac{f\left(K[1],\frac{yK[1]}{x}\right)}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+ (z+f(x,y))*diff(w(x,y,z),z)=g(x,y)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},\frac{-x\left(\int^x \frac{f\left(\_a,\frac{\_ay}{x}\right)}{\_a^2}d\_a\right)+z}{x}\right)e^{\int^x \frac{g\left(\_a,\frac{\_ay}{x}\right)}{\_a}d\_a}$$

7.8.25.4 [1913] Problem 4

problem number 1913

Added Jan 1, 2020.

Problem Chapter 8.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + f(x,y)g(z)w_z = h(x,y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+f[x,y]*g[z]*D[w[x,y,z],z]==h[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=h(x,y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,x^{-\frac{b}{a}}, \int \frac{a}{g(z)}dz - \left(\int^{x} \frac{f\left(\_a, y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_a}d\_a\right)\right) \mathrm{e}^{\int^{x} \frac{h\left(\_a, y\,\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_a a}d\_a}$$

7.8.25.5 [1914] Problem 5

problem number 1914

Added Jan 1, 2020.

Problem Chapter 8.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))\,w_y + (g_1(x,y)z + g_2(x,y))\,w_z = h(x,y,z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y])*D[w[x,y,z],z]==h[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int^x g_1\left(\_f, \left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)} + \int \mathrm{e}^{-\left(\int f_1(\_f)d\_f\right)} f_2(\_f)d\_\right.\right.\right.\right.}$$

7.8.25.6 [1915] Problem 6

problem number 1915

Added Jan 1, 2020.

Problem Chapter 8.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x,y)z + g_2(x,y)z^m\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*z^m)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y)*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} + (k-1)\left(\left(\int \mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int^x g_1\left(\_f, \left(y^{-k\cdot\right.\right.\right.}}\right.\right.$$

7.8.25.7 [1916] Problem 7

problem number 1916

Added Jan 1, 2020.

Problem Chapter 8.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x,y)z + g_2(x,y)e^{\lambda z}\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*Exp[lambda*z])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y)*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.8.25.8 [1917] Problem 8

problem number 1917

Added Jan 1, 2020.

Problem Chapter 8.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x,y)z + g_2(x,y)z^k\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*z^k)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8.25.9 [1918] Problem 9

problem number 1918

Added Jan 1, 2020.

Problem Chapter 8.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x,y)z + g_2(x,y)e^{\beta z}\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.8.25.10 [1919] Problem 10

problem number 1919

Added Jan 1, 2020.

Problem Chapter 8.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x, y) + h_2(x, y)z^m)\, w_z = h_3(x, y, z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x,y,z],x]+f2[x]*g2[y]*D[w[x,y,z],y]+(h1[x,y]+h2[x,y]*z^m)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f__1(x)*g__1(y)*diff(w(x,y,z),x)+ f__2(x)*g__2(y)*diff(w(x,y,z),y)+ (h__1(x,y)*z+h__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{f_2(x)}{f_1(x)}dx\right) + \int \frac{g_1(y)}{g_2(y)}dy, c_4\right) e^{\int^x \frac{h_3\left(\_h, \mathrm{RootOf}\left(\int \frac{f_2(\_h)}{f_1(\_h)}d\_h - \left(\int \frac{f_2(x)}{f_1(x)}dx\right) + \int \frac{g_1(y)}{g_2(y)}dy - \left(\int^{\_Z} \frac{g_1(\_a}{g_2(\_a}\right.\right.\right.}{}}$$

7.8.25.11 [1920] Problem 11

problem number 1920

Added Jan 1, 2020.

Problem Chapter 8.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + \left(h_1(x,y) + h_2(x,y)e^{\lambda z}\right)w_z = h_3(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x,y,z],x]+f2[x]*g2[y]*D[w[x,y,z],y]+(h1[x,y]+h2[x,y]*Exp[lambda*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f__1(x)*g__1(y)*diff(w(x,y,z),x)+ f__2(x)*g__2(y)*diff(w(x,y,z),y)+ (h__1(x,y)*z+h__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

7.9 chapter 9

Local contents

7.9.1 2.1

Local contents

7.9.1.1 [1921] Problem 1

problem number 1921

Added Jan 6, 2020.

Problem Chapter 9.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\alpha x + \beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{x(\alpha x+2\beta)}{2a}}c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\alpha x+\beta)^2}{2a\alpha}}(\alpha q-\beta p)\operatorname{erf}\left(\frac{\alpha x+\beta}{\sqrt{2}\sqrt{a}\sqrt{\alpha}}\right)}{\sqrt{a}\alpha^{3/2}}-\frac{p}{\alpha}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,y,z)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(-\frac{\sqrt{2}\sqrt{\pi}\left(-\alpha q+\beta p\right)\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\frac{\alpha}{a}}x}{2}+\frac{\sqrt{2}\beta}{2\sqrt{\frac{\alpha}{a}}a}\right)\mathrm{e}^{\frac{\beta^2}{2a\alpha}}}{2}+\sqrt{\frac{\alpha}{a}}\left(\alpha\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)-p\,\mathrm{e}^{-\frac{(\alpha x+2\beta)x}{2a}}\right)a\right)\mathrm{e}^{\frac{(\alpha x+2}{2a}}}{\sqrt{\frac{\alpha}{a}}\,a\alpha}$$

7.9.1.2 [1922] Problem 2

problem number 1922

Added Jan 6, 2020.

Problem Chapter 9.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=(cx+k)w+px+q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==(c*x+k)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(cx+2k)}c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=(c*x+k)*w(x,y,z)+p*x+q;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^{y}-\frac{\left(\left(-\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{(a\,z^2+(\_b^2-y^2)\,b)\,a}}{\sqrt{ab}}\right)+\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)\right)p+(-px-q)\sqrt{ab}\right)\mathrm{e}^{--}}{\sqrt{ab}\sqrt{(a\,z^2+(\_b^2-y^2)\,b}}\right.$$

7.9.1.3 [1923] Problem 3

problem number 1923

Added Jan 6, 2020.

Problem Chapter 9.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x+a_0)w_y+(b_1x+b_0)w_z=(c_1x+c_0)w+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x+a0)*D[w[x,y,z],y]+(b1*x+b0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,y,z]+s1*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(2\text{c0}+\text{c1}x)}c_1\left(-\text{a0}x-\frac{\text{a1}x^2}{2}+y,-\text{b0}x-\frac{\text{b1}x^2}{2}+z\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\text{c0}+\text{c1}x)^2}{2\text{c1}}}\text{erf}\left(\frac{\text{c0}+\text{c1}x}{\sqrt{2}\sqrt{\text{c1}}}\right)(\text{c1s0}-}{\text{c1}^{3/2}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x+a__0)*diff(w(x,y,z),y)+ (b__1*x+b__0)*diff(w(x,y,z),z)=(c_
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(2c_1^{\frac{5}{2}}\_F1\left(-\frac{1}{2}a_1x^2-a_0x+y,-\frac{1}{2}b_1x^2-b_0x+z\right)-2c_1^{\frac{3}{2}}s_1\mathrm{e}^{-\frac{1}{2}c_1x^2-c_0x}+\sqrt{2}\sqrt{\pi}\left(-c_0s_1+c_1s_0\right)\right.}{2c_1^{\frac{5}{2}}}$$

7.9.1.4 [1924] Problem 4

problem number 1924

Added Jan 6, 2020.

Problem Chapter 9.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(b_1x+b_0)w_y+(c_1y+c_0)w_z=aw+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x+b0)*D[w[x,y,z],y]+(c1*y+c0)*D[w[x,y,z],z]==a*w[x,y,z]+s1*x+s0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to-\frac{a^2(-e^{ax})\,c_1\left(-\text{b0}x-\frac{\text{b1}x^2}{2}+y,\frac{1}{2}\text{b0c1}x^2+\frac{1}{3}\text{b1c1}x^3-\text{c0}x-\text{c1}xy+z\right)+a\text{s0}+a\text{s1}x+}{a^2}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x+b__0)*diff(w(x,y,z),y)+ (c__1*x+c__0)*diff(w(x,y,z),z)=a*w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a^2\_F1\left(-\frac{1}{2}b_1x^2-b_0x+y,-\frac{1}{2}c_1x^2-c_0x+z\right)\mathrm{e}^{ax}+(-s_1x-s_0)\,a-s_1}{a^2}$$

7.9.1.5 [1925] Problem 5

problem number 1925

Added Jan 6, 2020.

Problem Chapter 9.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+k_1x+k_0)w_y+(bz+n_1x+n_0)w_z=(c_1x+c_0)w+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k1*x+k0)*D[w[x,y,z],y]+(b*z+n1*x+n0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(2c0+c1x)}c_1\left(\frac{e^{-ax}(a^2y+a(\mathrm{k0}+\mathrm{k1}x)+\mathrm{k1})}{a^2},\frac{e^{-bx}(b^2z+b(\mathrm{n0}+\mathrm{n1}x)+\mathrm{n1})}{b^2}\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(c0}{}}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k__1*x+k__0)*diff(w(x,y,z),y)+ (b*z+n__1*x+n__0)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(2c_1^{\frac{5}{2}}\_F1\left(\frac{(a^2y+(k_1x+k_0)a+k_1)\mathrm{e}^{-ax}}{a^2},\frac{(b^2z+(n_1x+n_0)b+n_1)\mathrm{e}^{-bx}}{b^2}\right)-2c_1^{\frac{3}{2}}s_1\mathrm{e}^{-\frac{1}{2}c_1x^2-c_0x}+\sqrt{2}\sqrt{\pi}\left(-c_0s_1\right.\right.}{2c_1^{\frac{5}{2}}}$$

7.9.1.6 [1926] Problem 6

problem number 1926

Added Jan 6, 2020.

Problem Chapter 9.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_2y+a_1x+a_0)w_y+(b_3z+b_2y+b_1x+b_0)w_z=(c_3z+c_2y+c_1x+c_0)w+s_3z+s_2y+s_1x+s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a2*y+a1*x+a0)*D[w[x,y,z],y]+(b3*z+b2*y+b1*x+b0)*D[w[x,y,z],z]==(c3*z+c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__2*y+a__1*x+a__0)*diff(w(x,y,z),y)+ (b__3*z+b__2*y+b__1*x+b__0)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

7.9.1.7 [1927] Problem 7

problem number 1927

Added Jan 6, 2020.

Problem Chapter 9.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + bxw_y + czw_z = (\alpha x + \beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*x*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{e^{\frac{\alpha x}{a}}\left(-\left(\frac{\alpha x}{a}\right)^{\frac{\beta}{a}}\left(ap\,\text{Gamma}\left(1-\frac{\beta}{a},\frac{\alpha x}{a}\right)+\alpha q\,\text{Gamma}\left(-\frac{\beta}{a},\frac{\alpha x}{a}\right)\right)+a\alpha x^{\frac{\beta}{a}}c_1\left(y-\frac{bx}{a},zx^{-\frac{c}{a}}\right)\right.}{a\alpha}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*x*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{3\left(\left(\alpha x+a-\beta\right)\left(a-\frac{\beta}{3}\right)a^2 q\, x^{-\frac{\beta}{a}-1}\left(\frac{\alpha}{a}\right)^{\frac{\beta}{a}}\left(\frac{\alpha}{a}\right)^{-\frac{\beta}{a}}\left(\frac{\alpha x}{a}\right)^{-\frac{a-\beta}{2a}}\operatorname{WhittakerM}\left(\frac{-a-\beta}{2a},\frac{2a-\beta}{2a},\frac{\alpha x}{a}\right)\mathrm{e}^{-\frac{\alpha}{2}}\right.}{}$$

7.9.1.8 [1928] Problem 8

problem number 1928

Added Jan 6, 2020.

Problem Chapter 9.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = (\alpha x + \beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{e^{\frac{\alpha x}{a}}\left(-\left(\frac{\alpha x}{a}\right)^{\frac{\beta}{a}}\left(ap\,\text{Gamma}\left(1-\frac{\beta}{a},\frac{\alpha x}{a}\right)+\alpha q\,\text{Gamma}\left(-\frac{\beta}{a},\frac{\alpha x}{a}\right)\right)+a\alpha x^{\frac{\beta}{a}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right)}{a\alpha}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{3\left(\left(\alpha x+a-\beta\right)\left(a-\frac{\beta}{3}\right)a^2 q\, x^{-\frac{\beta}{a}-1}\left(\frac{\alpha}{a}\right)^{\frac{\beta}{a}}\left(\frac{\alpha}{a}\right)^{-\frac{\beta}{a}}\left(\frac{\alpha x}{a}\right)^{-\frac{a-\beta}{2a}}\mathrm{WhittakerM}\left(\frac{-a-\beta}{2a},\frac{2a-\beta}{2a},\frac{\alpha x}{a}\right)\mathrm{e}^{-\frac{\alpha}{2}}\right.}{}$$

7.9.1.9 [1929] Problem 9

problem number 1929

Added Jan 6, 2020.

Problem Chapter 9.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = (cx+k)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==(c*x+k)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{e^{cx}\left(-(cx)^k(p\,\mathrm{Gamma}(1-k,cx)+cq\,\mathrm{Gamma}(-k,cx))+cx^k c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)\right.\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=(c*x+k)*w(x,y,z)+p*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{y} \frac{\left(px\left(\frac{\_bab+\sqrt{ab}\sqrt{(a\,z^2+(\_b^2-y^2)\,b)\,a}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}+q\right)\mathrm{e}^{-\left(\int \frac{cx\left(\frac{\_bab+\sqrt{ab}\sqrt{(a\,z^2+(\_b^2}{\sqrt{ab}}\right.}{\sqrt{(a\,z^2}}\right.}}{\sqrt{(a\,z^2+(\_b^2-y^2)\,b)\,a}} \right.$$

7.9.2 2.2

Local contents

7.9.2.1 [1930] Problem 1

problem number 1930

Added Jan 6, 2020.

Problem Chapter 9.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^2 + a_0)w_y + (b_1 x^2 + b_0)w_z = (c_1 x + c_0)w + s_1 x^2 + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^2+a0)*D[w[x,y,z],y]+(b1*x^2+b0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{1}{2}x(2\text{c0}+\text{c1}x)}c_1\left(-\text{a0}x - \frac{\text{a1}x^3}{3} + y, -\text{b0}x - \frac{\text{b1}x^3}{3} + z\right) + \frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\text{c0}+\text{c1}x)^2}{2\text{c1}}}\text{erf}\left(\frac{\text{c0}+\text{c1}x}{\sqrt{2}\sqrt{\text{c1}}}\right)(\text{c0}^2\text{s1} -}{\text{c1}^{5/2}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^2+a__0)*diff(w(x,y,z),y)+ (b__1*x^2+b__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(c_1^{\frac{9}{2}}\_F1\left(-\frac{1}{3}a_1x^3 - a_0x + y, -\frac{1}{3}b_1x^3 - b_0x + z\right) + \frac{\sqrt{\pi}\sqrt{2}\left(c_0^2s_1 + c_1^2s\_0 + c_1s_1\right)c_1^2\,\text{erf}\left(\frac{\sqrt{2}\left(\sqrt{c_1}\,x + \frac{c_0}{\sqrt{c_1}}\right)}{2}\right)e^{2}}{2}\right.}{c_1^{\frac{9}{2}}}$$

7.9.2.2 [1931] Problem 2

problem number 1931

Added Jan 6, 2020.

Problem Chapter 9.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (b_1x^2 + b_0)w_y + (c_1y^2 + c_0)w_z = aw + s_1x^2 + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x^2+b0)*D[w[x,y,z],y]+(c1*y^2+c0)*D[w[x,y,z],z]==a*w[x,y,z]+s1*x^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{a^3e^{ax}c_1\left(-\text{b0}x-\frac{\text{b1}x^3}{3}+y,-\frac{1}{3}\text{b0}^2\text{c1}x^3-\frac{3}{10}\text{b0b1c1}x^5+\text{b0c1}x^2y-\frac{1}{14}\text{b1}^2\text{c1}x^7+\frac{1}{2}\text{b1c1}x^4\right.}{a^3}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x^2+b__0)*diff(w(x,y,z),y)+ (c__1*y^2+c__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a^3\_F1\left(-\frac{1}{3}b_1x^3-b_0x+y,-c_0x-\frac{\left(b_1^2x^6+\frac{21}{5}b_0b_1x^4-7b_1x^3y+\frac{14}{3}b_0^2x^2-14b_0xy+14y^2\right)c_1x}{14}+z\right)\mathrm{e}^{ax}-2as_1x}{a^3}$$

7.9.2.3 [1932] Problem 3

problem number 1932

Added Jan 6, 2020.

Problem Chapter 9.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + n_1x^2 + n_0)w_z = (c_1x + c_0)w + s_1x + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k1*x^2+k0)*D[w[x,y,z],y]+(b*z+n1*x^2+n0)*D[w[x,y,z],z]==(c1*x+c0)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(2\text{c0}+\text{c1}x)}c_1\left(\frac{e^{-ax}(a^2(\text{k0}+\text{k1}x^2)+a^3y+2a\text{k1}x+2\text{k1})}{a^3},\frac{e^{-bx}(b^2(\text{n0}+\text{n1}x^2)+b^3z+2b\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k__1*x^2+k__0)*diff(w(x,y,z),y)+ (b*z+n__1*x^2+n__0)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{\left(-2c_1^{\frac{5}{2}}\_F1\left(\frac{(a^3y+2ak_1x+(k_1x^2+k_0)a^2+2k_1)\mathrm{e}^{-ax}}{a^3},\frac{(b^3z+2bn_1x+(n_1x^2+n_0)b^2+2n_1)\mathrm{e}^{-bx}}{b^3}\right)+2c_1^{\frac{3}{2}}s_1\mathrm{e}^{-\frac{1}{2}c_1x^2-c}\right.}{2c_1^{\frac{5}{2}}}$$

7.9.2.4 [1933] Problem 4

problem number 1933

Added Jan 6, 2020.

Problem Chapter 9.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_2xy+a_1x+a_0)w_y+(b_3yz+b_2y^2+b_1x^2+b_0)w_z=(c_3z+c_2y+c_1x+c_0)w+s_1xy+s_2xz$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a2*x*y+a1*x+a0)*D[w[x,y,z],y]+(b3*y*z+b2*y^2+b1*x^2+b0)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__2*x*y+a__1*x+a__0)*diff(w(x,y,z),y)+ (b__3*y*z+b__2*y^2+b__1*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.9.2.5 [1934] Problem 5

problem number 1934

Added Jan 6, 2020.

Problem Chapter 9.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+bxw_y+czw_z=kxw+sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*x*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x*w[x,y,z]+s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(a+kx)}{k^2}+e^{\frac{kx}{a}}c_1\left(y-\frac{bx}{a},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*x*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x*w(x,y,z)+s*x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{k^2\_F1\left(\frac{ay-bx}{a},z\,x^{-\frac{c}{a}}\right)\mathrm{e}^{\frac{kx}{a}}-(kx+a)\,s}{k^2}$$

7.9.2.6 [1935] Problem 6

problem number 1935

Added Jan 6, 2020.

Problem Chapter 9.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+czw_z=kxw+sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x*w[x,y,z]+s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(a+kx)}{k^2}+e^{\frac{kx}{a}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x*w(x,y,z)+s*x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{k^2 \_F1\left(y\,x^{-\frac{b}{a}}, z\,x^{-\frac{c}{a}}\right) e^{\frac{kx}{a}} - (kx+a)\,s}{k^2}$$

7.9.2.7 [1936] Problem 7

problem number 1936

Added Jan 6, 2020.

Problem Chapter 9.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = (kx+s)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x,y,z],x]+b*y^2*D[w[x,y,z],y]+c*z^2*D[w[x,y,z],z]==(k*x+s)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{e^{-\frac{s}{ax}}\left(-\frac{s}{ax}\right)^{-\frac{k}{a}}\left(asx^{\frac{k}{a}}\left(-\frac{s}{ax}\right)^{\frac{k}{a}} c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right) + ps\,\text{Gamma}\left(\frac{k}{a},-\frac{s}{ax}\right) - aq\,\text{Gamma}\left(\frac{a}{\ }\right.\right.}{as}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+ c*z^2*diff(w(x,y,z),z)=(k*x+s)*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{(px+q)\,x^{\frac{-2a-k}{a}}\,\mathrm{e}^{\frac{s}{ax}}}{a}dx+\_F1\left(\frac{ax-by}{axy},\frac{ax-cz}{axz}\right)\right)x^{\frac{k}{a}}\mathrm{e}^{-\frac{s}{ax}}$$

7.9.3 2.3

Local contents

7.9.3.1 [1937] Problem 1

problem number 1937

Added Jan 16, 2020.

Problem Chapter 9.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1\sqrt{x}+a_0)w_y+(b_1\sqrt{x}+b_0)w_z=cw+s_1\sqrt{x}+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*Sqrt[x]+a0)*D[w[x,y,z],y]+(b1*Sqrt[x]+b0)*D[w[x,y,z],z]==c*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{cx}c_1\left(-\text{a0}x-\frac{2}{3}\text{a1}x^{3/2}+y,-\text{b0}x-\frac{2}{3}\text{b1}x^{3/2}+z\right)+\frac{\sqrt{\pi}\text{s1}e^{cx}\text{erf}\left(\sqrt{c}\sqrt{x}\right)}{2c^{3/2}}-\frac{\text{s0}+\text{s1}\sqrt{x}}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*sqrt(x)+a__0)*diff(w(x,y,z),y)+ (b__1*sqrt(x)+b__0)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(2\sqrt{\frac{c}{\pi}}\,c\_F1\left(-\frac{2a_1x^{\frac{3}{2}}}{3}-a_0x+y,-\frac{2b_1x^{\frac{3}{2}}}{3}-b_0x+z\right)+s_1\,\mathrm{erf}\left(\sqrt{c}\,\sqrt{x}\right)\right)\mathrm{e}^{cx}-2\sqrt{\frac{c}{\pi}}\left(s_1\sqrt{x}+s\right.}{2\sqrt{\frac{c}{\pi}}\,c}$$

7.9.3.2 [1938] Problem 2

problem number 1938

Added Jan 16, 2020.

Problem Chapter 9.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(b_1x^2+b_0)w_y+(c_1y^3+c_0)w_z=aw+s_1x^3+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x^2+b0)*D[w[x,y,z],y]+(c1*y^3+c0)*D[w[x,y,z],z]==a*w[x,y,z]+ s1*x^3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{a^4(-e^{ax})\,c_1\left(-\mathrm{b0}x-\frac{\mathrm{b1}x^3}{3}+y,\mathrm{b0}^2\left(\frac{19}{60}\mathrm{b1c1}x^6-\mathrm{c1}x^3y\right)+\frac{1}{4}\mathrm{b0}^3\mathrm{c1}x^4+\frac{3}{280}\mathrm{b0c1}x^2\left(13\mathrm{b1}^2\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x^2+b__0)*diff(w(x,y,z),y)+ (c__1*y^3+c__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a^4\_F1\left(-\frac{1}{3}b_1x^3-b_0x+y,-c_0x+\frac{3\left(b_1^3x^9+\frac{13b_0b_1^2x^7}{2}-10b_1^2x^6y+\frac{133b_0^2b_1x^5}{9}-42b_0b_1x^4y-\frac{140b_0^2x^2y}{3}+70b_0x\,y^2+\left(\right.\right.}{140}\right.}{a^4}$$

7.9.3.3 [1939] Problem 3

problem number 1939

Added Jan 16, 2020.

Problem Chapter 9.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+kx^3)w_y+(bz+nx^3)w_z=cw+sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k*x^3)*D[w[x,y,z],y]+(b*z+n*x^3)*D[w[x,y,z],z]==c*w[x,y,z]+ s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(c^2x^2+2cx+2)}{c^3}+e^{cx}c_1\left(\frac{e^{-ax}(k(a^3x^3+3a^2x^2+6ax+6)+a^4y)}{a^4},\frac{e^{-bx}(n(b^3x^3+3b^2\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k*x^3)*diff(w(x,y,z),y)+ (b*z+n*x^3)*diff(w(x,y,z),z)=c*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{c^3\_F1\left(\frac{(k\,x^3a^3+y\,a^4+3k\,x^2a^2+6kax+6k)\,\mathrm{e}^{-ax}}{a^4},\frac{(n\,x^3b^3+z\,b^4+3n\,x^2b^2+6bnx+6n)\,\mathrm{e}^{-bx}}{b^4}\right)\mathrm{e}^{cx}-(x^2c^2+2cx+2)\,s}{c^3}$$

7.9.3.4 [1940] Problem 4

problem number 1940

Added Jan 16, 2020.

Problem Chapter 9.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+a_2x^3)w_y+(b_1yz+b_2y^3)w_z=(c_1z+c_2y)w+s_1x^2y+s_2xz^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x*y+a2*x^3)*D[w[x,y,z],y]+(b1*y*z+b2*y^3)*D[w[x,y,z],z]==(c1*z+c2*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x*y+a__2*x^3)*diff(w(x,y,z),y)+ (b__1*y*z+b__2*y^3)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.9.3.5 [1941] Problem 5

problem number 1941

Added Jan 16, 2020.

Problem Chapter 9.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^3 w_x + by^3 w_y + cz^3 w_z = xw + kx + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*y^3*D[w[x,y,z],y]+c*z^3*D[w[x,y,z],z]==x*w[x,y,z]+ k*x+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{x^2}{2a}}c_1\left(-\frac{bx}{a}-\frac{1}{2y^2},-\frac{cx}{a}-\frac{1}{2z^2}\right)+\frac{\sqrt{\frac{\pi}{2}}se^{\frac{x^2}{2a}}\text{erf}\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-k\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*y^3*diff(w(x,y,z),y)+ c*z^3*diff(w(x,y,z),z)=x*w(x,y,z)+ k*x+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\sqrt{2}\, s \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2\sqrt{a}}\right) e^{\frac{x^2}{2a}} + 2\sqrt{\frac{a}{\pi}}\, \_F1\left(\frac{2bx\, y^2+a}{a\, y^2}, \frac{2cx\, z^2+a}{a\, z^2}\right) e^{\frac{x^2}{2a}} - 2\sqrt{\frac{a}{\pi}}\, k}{2\sqrt{\frac{a}{\pi}}}$$

7.9.4 2.4

Local contents

7.9.4.1 [1942] Problem 1

problem number 1942

Added Jan 16, 2020.

Problem Chapter 9.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a w_x + b w_y + c w_z = k x^n w + s x^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*D[w[x,y,z],z]==k*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{kx^{n+1}}{an+a}}\left(-\frac{sx^{m+1}\left(\frac{kx^{n+1}}{an+a}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{kx^{n+1}}{an+a}\right)}{a(n+1)}+c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=k*x^n*w(x,y,z)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left((n+1)^2\left(k\,x^{n+1}+(m+n+2)\,a\right)s\,x^{m-n}\left(\frac{k}{(n+1)a}\right)^{\frac{-m-1}{n+1}}\left(\frac{k}{(n+1)a}\right)^{\frac{m+1}{n+1}}\left(\frac{k\,x^{n+1}}{(n+1)a}\right)^{\frac{-m-n-2}{2n+2}}\text{Whitta}}{}$$

7.9.4.2 [1943] Problem 2

problem number 1943

Added Jan 16, 2020.

Problem Chapter 9.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+byw_y+czw_z=kx^nw+sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{kx^{n+1}}{an+a}}\left(-\frac{sx^{m+1}\left(\frac{kx^{n+1}}{an+a}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{kx^{n+1}}{an+a}\right)}{a(n+1)}+c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x^n*w(x,y,z)+ s*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left((n+1)^2\left(k\,x^{n+1}+(m+n+2)\,a\right)s\,x^{m-n}\left(\frac{k}{(n+1)a}\right)^{\frac{-m-1}{n+1}}\left(\frac{k}{(n+1)a}\right)^{\frac{m+1}{n+1}}\left(\frac{k\,x^{n+1}}{(n+1)a}\right)^{\frac{-m-n-2}{2n+2}}\text{Whitta}\right.}{}$$

7.9.4.3 [1944] Problem 3

problem number 1944

Added Jan 16, 2020.

Problem Chapter 9.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=cx^nw+sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{n+1}}{n+1}}\left(-\frac{sx^{m+1}\left(\frac{cx^{n+1}}{n+1}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{cx^{n+1}}{n+1}\right)}{n+1}+c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)\right.}{2\sqrt{b}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^{y}\frac{s\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{\left(a\,z^{2}+\left(\_b^{2}-y^{2}\right)b\right)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^{2}z^{2}}\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)^{m}\mathrm{e}^{-c\left(\int\frac{\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{\left(a\,z^{2}+\left(\right.\right.}}{\sqrt{ab}}\right.}{}\right.}{\sqrt{\left(a\,z^{2}\right.}}\right.}}{\sqrt{\left(a\,z^{2}+\left(\_b^{2}-y^{2}\right)b\right)a}}\right.$$

7.9.4.4 [1945] Problem 4

problem number 1945

Added Jan 16, 2020.

Problem Chapter 9.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + bx^m w_z = cx^k w + sx^r$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*x^n*D[w[x,y,z],y]+b*x^m*D[w[x,y,z],z]==c*x^k*w[x,y,z]+ s*x^r;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{k+1}}{k+1}}\left(-\frac{sx^{r+1}\left(\frac{cx^{k+1}}{k+1}\right)^{-\frac{r+1}{k+1}}\text{Gamma}\left(\frac{r+1}{k+1},\frac{cx^{k+1}}{k+1}\right)}{k+1}+c_1\left(\frac{-ax^{n+1}+ny+y}{n+1},\frac{-bx^{m+1}+m\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)=c*x^k*w(x,y,z)+ s*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left((k+1)\left((k+1)\left(c\,x^{k+1}+k+r+2\right)\text{WhittakerM}\left(\frac{-k+r}{2k+2},\frac{2k+r+3}{2k+2},\frac{c\,x^{k+1}}{k+1}\right)+(k+r+2)^2\,\text{Wh}\right.\right.}{}$$

7.9.4.5 [1946] Problem 5

problem number 1946

Added Jan 16, 2020.

Problem Chapter 9.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + bx^n w_y + cy^m w_z = aw + sx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+b*x^n*D[w[x,y,z],y]+c*x^m*D[w[x,y,z],z]==a*w[x,y,z]+ s*x^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{ax}\left(-\frac{sx^k(ax)^{-k}\,\text{Gamma}(k+1,ax)}{a}+c_1\left(\frac{-bx^{n+1}+ny+y}{n+1},\frac{-cx^{m+1}+mz+z}{m+1}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ b*x^n*diff(w(x,y,z),y)+ c*x^m*diff(w(x,y,z),z)=a*w(x,y,z)+ s*x^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(s\,x^k(ax)^{-\frac{k}{2}}\,\text{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},ax\right)\mathrm{e}^{-\frac{ax}{2}}+(k+1)\,a\_F1\left(\frac{-bx\,x^n+(n+1)y}{n+1},\frac{-cx\,x^m+(m+1)z}{m+1}\right)\right)}{(k+1)\,a}$$

7.9.4.6 [1947] Problem 6

problem number 1947

Added Jan 16, 2020.

Problem Chapter 9.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + \beta x^n)w_y + (bz + \gamma x^m)w_z = cx^k w + sx^r$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+beta*x^n)*D[w[x,y,z],y]+(b*z+gamma*x^m)*D[w[x,y,z],z]==c*x^k*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{k+1}}{k+1}}\left(-\frac{sx^{r+1}\left(\frac{cx^{k+1}}{k+1}\right)^{-\frac{r+1}{k+1}}\text{Gamma}\left(\frac{r+1}{k+1},\frac{cx^{k+1}}{k+1}\right)}{k+1}+c_1\left(\gamma b^{-m-1}\text{Gamma}(m+1,bx)+ze\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+beta*x^n)*diff(w(x,y,z),y)+ (b*z+gamma*x^m)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left((k+1)\left((k+1)\left(c\,x^{k+1}+k+r+2\right)\text{WhittakerM}\left(\frac{-k+r}{2k+2},\frac{2k+r+3}{2k+2},\frac{c\,x^{k+1}}{k+1}\right)+(k+r+2)^2\,\text{Wh}\right.\right.}{}$$

7.9.4.7 [1948] Problem 7

problem number 1948

Added Jan 16, 2020.

Problem Chapter 9.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^{n_1} y + a_2 x^{n_2}) w_y + (b_1 y^{m_1} z + b_2 y^{m_2}) w_z = cw + s_1 x y^{k_1} + s_2 x^{k_2} z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^n1*y + a2*x^n2)*D[w[x,y,z],y]+(b1*y^m1*z + b2*y^m2)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left(\left(\left(\right. \right. \right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(n__1)*y + a__2*x^(n__2))*diff(w(x,y,z),y)+ (b__1*y^(m__1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.9.4.8 [1949] Problem 8

problem number 1949

Added Jan 16, 2020.

Problem Chapter 9.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{\lambda_1} y + a_2 x^{\lambda_2} y^k) w_y + (b_1 x^{\beta_1} z + b_2 x^{\beta_2} z^m) w_z = c_1 x^{\gamma_1} w + c_2 y^{\gamma_2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^lambda1*y + a2*x^lambda2*y^k)*D[w[x,y,z],y]+(b1*x^beta1*z + b2*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{\text{c1}x^{\text{gamma1}+1}}{\text{gamma1}+1}} \left(\int_1^x \text{c2} e^{-\frac{\text{c1}K[1]^{\text{gamma1}+1}}{\text{gamma1}+1}} \left(\left(\left(\frac{(-1)^{-\frac{\text{lambda2}+1}{\text{lambda1}+1}} \text{a1}^{-\frac{\text{lambda2}+1}{\text{lambda1}+1}} \exp\left(-\frac{\text{a1}\left(x^{\text{lambda1}+1}+(k-1)\right.}{\text{lambda1}} \right.}{} \right.\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(lambda__1)*y + a__2*x^(lambda__2)*y^k)*diff(w(x,y,z),y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^x c_2 \left(\left(\frac{-(\lambda_1+1)^2\left((k-1)\,a_1\_a^{\lambda_2+1} + (-\lambda_1-\lambda_2-2)\_a^{-\lambda_1+\lambda_2}\right) a_2 \left(-\frac{(k-1)a_1\_a^{\lambda_1+1}}{\lambda_1+1}\right)^{\frac{-\lambda}{}}}{}\right.\right.\right.$$

7.9.4.9 [1950] Problem 9

problem number 1950

Added Jan 16, 2020.

Problem Chapter 9.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{\lambda_1} y + a_2 x^{\lambda_2} y^k) w_y + (b_1 y^{\beta_1} z + b_2 y^{\beta_2} z^m) w_z = c_1 x^{\gamma_1} w + c_2 z^{\gamma_2}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^lambda1*y + a2*x^lambda2*y^k)*D[w[x,y,z],y]+(b1*y^beta1*z + b2*y^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(lambda__1)*y + a__2*x^(lambda__2)*y^k)*diff(w(x,y,z),y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

7.9.4.10 [1951] Problem 10

problem number 1951

Added Jan 16, 2020.

Problem Chapter 9.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + ayw_y + bzw_z = cx^n w + kx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*y*D[w[x,y,z],y]+b*z*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ k*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^n}{n}}\left(-\frac{kx^m\left(\frac{cx^n}{n}\right)^{-\frac{m}{n}}\text{Gamma}\left(\frac{m}{n},\frac{cx^n}{n}\right)}{n}+c_1\left(yx^{-a},zx^{-b}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*y*diff(w(x,y,z),y)+ b*z*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ k*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(\left(c\,x^{n}+m+n\right)k\,n^{2}x^{m-n}\left(\frac{c}{n}\right)^{\frac{m}{n}}\left(\frac{c}{n}\right)^{-\frac{m}{n}}\left(\frac{c\,x^{n}}{n}\right)^{-\frac{m+n}{2n}}\mathrm{WhittakerM}\left(\frac{m-n}{2n},\frac{m+2n}{2n},\frac{c\,x^{n}}{n}\right)\mathrm{e}^{-\frac{c\,x^{n}}{2n}}+\left(\left(\right.\right.\right.}{}$$

7.9.4.11 [1952] Problem 11

problem number 1952

Added Jan 16, 2020.

Problem Chapter 9.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = cx^n w + kx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ k*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^n}{n}}\left(-\frac{kx^m\left(\frac{cx^n}{n}\right)^{-\frac{m}{n}}\mathrm{Gamma}\left(\frac{m}{n},\frac{cx^n}{n}\right)}{n}+c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{\right.}\right.}{\sqrt{b}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ k*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{y} \frac{k\left(x\left(\frac{\_bab+\sqrt{ab}\sqrt{(a\,z^2+(\_b^2-y^2)b)a}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^{m} \mathrm{e}^{-c\left(\int \frac{\left(x\left(\frac{\_bab+\sqrt{ab}\sqrt{(a\,z^2+(\_b^2-}{\sqrt{ab}}\right.\right.}{\sqrt{(a\,z^2+}}\right.}}{\sqrt{(a\,z^2+(\_b^2-y^2)\,b)\,a}} \right.$$

7.9.4.12 [1953] Problem 12

problem number 1953

Added Jan 16, 2020.

Problem Chapter 9.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxw_x + c(by+cz)w_y + b(by-cz)w_z = kx^n w + sx^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*x*D[w[x,y,z],x]+ c*(b*y + c*z)*D[w[x,y,z],y]+b*(b*y - c*z)*D[w[x,y,z],z]==k*x^n*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  b*c*x*diff(w(x,y,z),x)+ c*(b*y + c*z)*diff(w(x,y,z),y)+ b*(b*y - c*z)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(-\left(\int^{y} -\frac{s\left(x\left(\frac{\frac{\sqrt{2}\,\_b\,b^2}{-b^2y^2+2bczy+c^2z^2}+\sqrt{\frac{2\_b^2b^2}{-b^2y^2+2bczy+c^2z^2}+1}\sqrt{\frac{b^2}{-b^2y^2+2bczy+c^2z^2}}}{\sqrt{\frac{b^2}{-b^2y^2+2bczy+c^2z^2}}}\right)^{\frac{\sqrt{2}\,b}{2\sqrt{-b^2y^2+2bczy+c^2z^2}\sqrt{-b^2y^2+2\cdots}}}\right.}{\cdots}\right.\right.$$

7.9.4.13 [1954] Problem 13

problem number 1954

Added Jan 16, 2020.

Problem Chapter 9.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1x^{n_1}w_x + b_2y^{n_2}w_y + b_3z^{n_3}w_z = aw + c_1x^{k_1} + c_2y^{k_2} + c_3x^{k_3}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*x^n1*D[w[x,y,z],x]+ b2*y^n2*D[w[x,y,z],y]+b3*z^n3*D[w[x,y,z],z]==a*w[x,y,z]+ c1*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ax^{1-\mathrm{n1}}}{\mathrm{b1}-\mathrm{b1n1}}}\left(\int_1^x \frac{e^{\frac{aK[1]^{1-\mathrm{n1}}}{\mathrm{b1}(\mathrm{n1}-1)}}K[1]^{-\mathrm{n1}}\left(\mathrm{c1}K[1]^{\mathrm{k1}}+\mathrm{c3}K[1]^{\mathrm{k3}}+\mathrm{c2}\left(\left(\frac{\mathrm{b2}(\mathrm{n2}-1)x^{-\mathrm{n1}}\left(x^{\mathrm{n1}}K[1]-xK[1]^{\mathrm{n1}}\right)}{\mathrm{b1}(\mathrm{n1}-1)}\right.\right.\right.}{\mathrm{b1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   b__1*x^(n__1)*diff(w(x,y,z),x)+ b__2*y^(n__2)*diff(w(x,y,z),y)+ b__3*z^(n__3)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x \frac{\left(c_{2\_\_}a^{-n_1}\left(\left(\frac{(n_1-1)b_1y^{-n_2+1}+(n_2-1)b_{2\_\_}a^{-n_1+1}-(n_2-1)b_2x^{-n_1+1}}{(n_1-1)b_1}\right)^{-\frac{1}{n_2-1}}\right)^{k_2}+c_{1\_\_}a^{k_1-n_1}+c_{3\_\_}a\right.}{b_1}\right.$$

7.9.4.14 [1955] Problem 14

problem number 1955

Added Jan 16, 2020.

Problem Chapter 9.2.4.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1x^{n_1}w_x+a_2y^{n_2}w_y+a_3z^{n_3}w_z=bx^kw+cx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*x^n1*D[w[x,y,z],x]+ a2*y^n2*D[w[x,y,z],y]+a3*z^n3*D[w[x,y,z],z]==b*x^k*w[x,y,z]+ c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{bx^{k-\mathrm{n1}+1}}{\mathrm{a1}k-\mathrm{a1n1}+\mathrm{a1}}}\left(-\frac{cx^{m-\mathrm{n1}+1}\left(\frac{bx^{k-\mathrm{n1}+1}}{\mathrm{a1}k-\mathrm{a1n1}+\mathrm{a1}}\right)^{\frac{-m+\mathrm{n1}-1}{k-\mathrm{n1}+1}}\mathrm{Gamma}\left(\frac{m-\mathrm{n1}+1}{k-\mathrm{n1}+1},\frac{bx^{k-\mathrm{n1}+1}}{\mathrm{a1}k-\mathrm{a1n1}+\mathrm{a1}}\right)}{\mathrm{a1}(k-\mathrm{n1}+1)}+c_1\left(\frac{\mathrm{a2}}{\mathrm{a1}(\mathrm{n}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   a__1*x^(n__1)*diff(w(x,y,z),x)+ a__2*y^(n__2)*diff(w(x,y,z),y)+ a__3*z^(n__3)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{6\left(-\frac{\left(-4\left(-\frac{k}{2}-\frac{m}{2}+n_1-\frac{3}{2}\right)^2 a_1\,\mathrm{WhittakerM}\left(\frac{k+m-2n_1+3}{2k-2n_1+2},\frac{2k+m-3n_1+4}{2k-2n_1+2},\frac{b\,x^{k-n_1+1}}{(k-n_1+1)a_1}\right)+\left(b\,x^{k-n_1+1}-2\left(-\frac{k}{2}-\frac{m}{2}+n_1-\frac{3}{2}\right)a_1\right.\right.}{}\right.\right.}{}$$

7.9.5 3.1

Local contents

7.9.5.1 [1956] Problem 1

problem number 1956

Added Jan 18, 2020.

Problem Chapter 9.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = ce^{\beta x}w + ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Exp[beta*x]*w[x,y,z]+ k*Exp[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\beta x}}{\beta}}\left(\int_1^x e^{\lambda K[1]-\frac{ce^{\beta K[1]}}{\beta}}kdK[1]+c_1(y-ax,z-bx)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*exp(beta*x)*w(x,y,z)+ k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k\,\mathrm{e}^{\frac{\beta\lambda x-c\,\mathrm{e}^{\beta x}}{\beta}}dx+\_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\beta x}}{\beta}}$$

7.9.5.2 [1957] Problem 2

problem number 1957

Added Jan 18, 2020.

Problem Chapter 9.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\beta x}w_y + be^{\lambda x}w_z = ce^{\gamma x}w + se^{\mu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Exp[beta*x]*D[w[x,y,z],y]+b*Exp[lambda*x]*D[w[x,y,z],z]==c*Exp[gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\gamma x}}{\gamma}}\left(\int_1^x e^{\mu K[1]-\frac{ce^{\gamma K[1]}}{\gamma}}sdK[1]+c_1\left(y-\frac{ae^{\beta x}}{\beta},z-\frac{be^{\lambda x}}{\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(beta*x)*diff(w(x,y,z),y)+ b*exp(lambda*x)*diff(w(x,y,z),z)=c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s\,\mathrm{e}^{\frac{\gamma\mu x-c\,\mathrm{e}^{\gamma x}}{\gamma}}dx+\_F1\left(\frac{-a\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{-b\,\mathrm{e}^{\lambda x}+\lambda z}{\lambda}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

7.9.5.3 [1958] Problem 3

problem number 1958

Added Jan 18, 2020.

Problem Chapter 9.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+be^{\beta x}w_y+ce^{\lambda y}w_z=aw+se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Exp[beta*x]*D[w[x,y,z],y]+c*Exp[lambda*y]*D[w[x,y,z],z]==a*w[x,y,z]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{e^{ax}\left(se^{x(\gamma-a)}+(\gamma-a)c_1\left(y-\frac{be^{\beta x}}{\beta},z-\frac{c\text{Ei}\left(\frac{be^{\beta x}\lambda}{\beta}\right)e^{\lambda\left(y-\frac{be^{\beta x}}{\beta}\right)}}{\beta}\right)\right)}{a-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*exp(beta*x)*diff(w(x,y,z),y)+ c*exp(lambda*y)*diff(w(x,y,z),z)=a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(-s\,\mathrm{e}^{-(a-\gamma)x}+(a-\gamma)\_F1\left(\frac{-b\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{c\,\mathrm{expIntegral}\left(1,-\frac{b\lambda\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{-\frac{\left(b\,\mathrm{e}^{\beta x}-\beta y\right)\lambda}{\beta}}+\beta z}{\beta}\right)\right)\mathrm{e}^{ax}}{a-\gamma}$$

7.9.5.4 [1959] Problem 4

problem number 1959

Added Jan 18, 2020.

Problem Chapter 9.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\beta x}w_y+be^{\lambda z}w_z=cw+ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Exp[beta*x]*D[w[x,y,z],y]+b*Exp[lambda*z]*D[w[x,y,z],z]==c*w[x,y,z]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(-\frac{b\lambda x+e^{-\lambda z}}{\lambda},y-\frac{ae^{\beta x}}{\beta}\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(beta*x)*diff(w(x,y,z),y)+ b*exp(lambda*z)*diff(w(x,y,z),z)=c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(-\frac{k\,\mathrm{e}^{-(c-\gamma)x}}{c-\gamma}+\_F1\left(\frac{-a\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{-b\lambda x-\mathrm{e}^{-\lambda z}}{b\lambda}\right)\right)\mathrm{e}^{cx}$$

7.9.5.5 [1960] Problem 5

problem number 1960

Added Jan 18, 2020.

Problem Chapter 9.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\sigma x}+a_2e^{\lambda y})w_y+(b_1e^{\mu y}+b_2e^{\beta z})w_z=c_1w+c_2e^{\nu x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[sigma*x]+ a2*Exp[lambda*y] )*D[w[x,y,z],y]+(b1*Exp[mu*y]+ b2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(sigma*x)+ a__2*exp(lambda*y) )*diff(w(x,y,z),y)+ (b__1*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(-c_2 \mathrm{e}^{(-c_1+\nu)x} + (c_1-\nu)\_F1\left(\frac{a_2\lambda\,\mathrm{expIntegral}\left(1,-\frac{a_1\lambda\,\mathrm{e}^{\sigma x}}{\sigma}\right)-\sigma\,\mathrm{e}^{\frac{(a_1\mathrm{e}^{\sigma x}-\sigma y)\lambda}{\sigma}}}{\lambda\sigma}, \frac{-b_2\beta\left(\int^x \mathrm{e}^{b_1\beta\left(\int\left(\frac{-\left(-\,\mathrm{expInteg}\right.}{}\right.\right.}\right.}{}\right.\right.}{}$$

7.9.5.6 [1961] Problem 6

problem number 1961

Added Jan 18, 2020.

Problem Chapter 9.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1 e^{\lambda_1 x} w_x + b_2 e^{\lambda_2 y} w_y + b_3 e^{\lambda_3 z} w_z = aw + c_1 e^{\beta_1 x} + c_2 e^{\beta_2 y} + c_3 e^{\beta_3 z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Exp[lambda1*x]*D[w[x,y,z],x]+ b2*Exp[lambda2*y]*D[w[x,y,z],y]+b3*Exp[lambda3*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{ae^{-\text{lambda1}x}}{\text{b1lambda1}}}\left(\int_1^x \frac{e^{\frac{ae^{-\text{lambda1}K[1]}}{\text{b1lambda1}}-\text{lambda1}K[1]}\left(\text{c2}\left(\frac{\text{b2}\left(-e^{-\text{lambda1}x}+e^{-\text{lambda1}K[1]}\right)\text{lambda2}}{\text{b1lambda1}}+e^{-\text{lamb}}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*exp(lambda__1*x)*diff(w(x,y,z),x)+ b__2*exp(lambda__2*y)*diff(w(x,y,z),y)+ b__3*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x \frac{c_2\left(\frac{b_1\lambda_1}{b_2\lambda_2 \mathrm{e}^{-\_a\lambda_1}+\left(b_1\lambda_1\mathrm{e}^{x\lambda_1}-b_2\lambda_2\mathrm{e}^{y\lambda_2}\right)\mathrm{e}^{-x\lambda_1-y\lambda_2}}\right)^{\frac{\beta_2}{\lambda_2}}\mathrm{e}^{\frac{-\_ab_1\lambda_1^2+a\,\mathrm{e}^{-\_a\lambda_1}}{b_1\lambda_1}}+c_3\left(\frac{b_1\lambda_1}{b_3\lambda_3\mathrm{e}^{-\_a\lambda_1}+\left(b_1\lambda_1\mathrm{e}^{x\lambda_1}-b_3\lambda_3\right.}\right.}{b_1}\right.$$

7.9.5.7 [1962] Problem 7

problem number 1962

Added Jan 18, 2020.

Problem Chapter 9.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1e^{\sigma_1x+\beta_1y}w_x+a_2e^{\sigma_2y+\beta_2y}w_y+\left(b_1e^{\nu_1x+\mu_1y}+b_2e^{\nu_2x+\mu_2y+\lambda z}\right)w_z=c_1w+c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Exp[sigma1*x+beta1*y]*D[w[x,y,z],x]+ a2*Exp[sigma2*y+beta2*y]*D[w[x,y,z],y]+( b1*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde := a__1*exp(sigma__1*x+beta__1*y)*diff(w(x,y,z),x)+ a__2*exp(sigma__2*y+beta__2*y)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{x} \frac{c_2 \left(\frac{a_1\sigma_1 \mathrm{e}^{-x\sigma_1} \mathrm{e}^{x\sigma_1+(\beta_1-\beta_2-\sigma_2)y} + (\beta_1-\beta_2-\sigma_2)\left(-\mathrm{e}^{-\_a\sigma_1}+\mathrm{e}^{-x\sigma_1}\right)a_2}{a_1\sigma_1} \right)^{-\frac{\beta_1}{\beta_1-\beta_2-\sigma_2}} \mathrm{e}^{\frac{-(\beta_2+\sigma_2)\_a a_2\sigma_1+c_1\left(\frac{a_1\sigma_1\mathrm{e}^{-x}}{}\right.}{}}}{a_1} \right.$$

7.9.6 3.2

Local contents

7.9.6.1 [1963] Problem 1

problem number 1963

Added Jan 19, 2020.

Problem Chapter 9.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = ce^{\beta x}w + kx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Exp[beta*x]*w[x,y,z]+ k*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\beta x}}{\beta}}\left(\int_1^x e^{-\frac{ce^{\beta K[1]}}{\beta}}kK[1]^n dK[1]+c_1(y-ax,z-bx)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*exp(beta*x)*w(x,y,z)+ k*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k\,x^n \mathrm{e}^{-\frac{c\,\mathrm{e}^{\beta x}}{\beta}}dx+\_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\beta x}}{\beta}}$$

7.9.6.2 [1964] Problem 2

problem number 1964

Added Jan 19, 2020.

Problem Chapter 9.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + be^{\lambda x} w_z = ce^{\gamma x} w + sx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*x^n*D[w[x,y,z],y]+b*Exp[lambda*x]*D[w[x,y,z],z]==c*Exp[gamma*x]*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\gamma x}}{\gamma}}\left(\int_1^x e^{-\frac{ce^{\gamma K[1]}}{\gamma}} sK[1]^k dK[1] + c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, z-\frac{be^{\lambda x}}{\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*exp(lambda*x)*diff(w(x,y,z),z)=c*exp(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s\,x^k \mathrm{e}^{-\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}} dx + \_F1\left(\frac{-ax\,x^n+(n+1)\,y}{n+1}, \frac{-b\,\mathrm{e}^{\lambda x}+\lambda z}{\lambda}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

7.9.6.3 [1965] Problem 3

problem number 1965

Added Jan 19, 2020.

Problem Chapter 9.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + be^{\beta x} w_y + cy^n w_z = aw + se^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Exp[beta*x]*D[w[x,y,z],y]+c*y^n*D[w[x,y,z],z]==a*w[x,y,z]+ s*Exp[gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*exp(beta*x)*diff(w(x,y,z),y)+ c*y^n*diff(w(x,y,z),z)=a*w(x,y,z)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(-s\,\mathrm{e}^{-(a-\gamma)x} + (a-\gamma)\_F1\left(\frac{-b\,\mathrm{e}^{\beta x}+\beta y}{\beta}, z - \left(\int^x c\left(\frac{b\,\mathrm{e}^{\_a\beta}-b\,\mathrm{e}^{\beta x}+\beta y}{\beta}\right)^n d\_a\right)\right)\right)\mathrm{e}^{ax}}{a-\gamma}$$

7.9.6.4 [1966] Problem 4

problem number 1966

Added Jan 19, 2020.

Problem Chapter 9.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 y + a_2 x y^k) w_y + (b_1 x + b_2 e^{\beta y + \lambda z}) w_z = c_1 w + c_2 e^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*y+a2*x*y^k)*D[w[x,y,z],y]+(b1*x+b2*Exp[beta*y+lambda*z])*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*y+a__2*x*y^k)*diff(w(x,y,z),y)+ (b__1*x+b__2*exp(beta*y+lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(-c_2 e^{-(c_1-\gamma)x} + (c_1-\gamma)\_F1\left(\frac{((k-1)a_1^2 y+((k-1)a_1 x-1)a_2 y^k)y^{-k} e^{(k-1)a_1 x}}{(k-1)a_1^2}, \frac{-b_2\lambda\left(\int^x e^{\frac{-a^2 b_1 \lambda}{2}+\beta\left(\frac{(-((k-1)\ldots}{\ldots}\right.}\right.}{\ldots}\right.\right.}{c_1-\gamma}$$

7.9.6.5 [1967] Problem 5

problem number 1967

Added Jan 19, 2020.

Problem Chapter 9.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x + a_2e^{\lambda y})w_y + (b_1z + b_2e^{\beta y}z^k)w_z = c_1w + c_2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*x+a2*Exp[lambda*y])*D[w[x,y,z],y]+(b1*z+b2*Exp[beta*y]*z^k)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{\text{c2}}{\text{c1}} + e^{\text{c1}x}c_1\left(-\frac{\frac{\sqrt{2\pi}\text{a2}\sqrt{\lambda}\,\text{Erfi}\left(\frac{\sqrt{\text{a1}}\sqrt{\lambda}x}{\sqrt{2}}\right)}{\text{a1}^{3/2}} + \frac{2e^{\frac{1}{2}\text{a1}\lambda x^2-\lambda y}}{\text{a1}}}{2\text{a2}\lambda^2}, (k-1)\int_1^x \text{b2}\exp\left(\frac{1}{2}\text{a1}\beta K[1]^2 + \text{b1}\ldots\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*x+a__2*exp(lambda*y))*diff(w(x,y,z),y)+ (b__1*z+b__2*exp(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1\_F1\left(\frac{\sqrt{-a_1\lambda}\,\sqrt{\pi}\,\sqrt{2}\,a_2\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{-a_1\lambda}\,x}{2}\right)\sqrt{\mathrm{e}^{-\left(a_1x^2-2y\right)\lambda}}+2a_1}{2a_1\lambda\sqrt{\mathrm{e}^{-\left(a_1x^2-2y\right)\lambda}}}, (k-1)\,b_2 2^{\frac{\beta}{2\lambda}}\pi^{-\frac{\beta}{2\lambda}}\left(\int^x \pi^{\frac{\beta}{\lambda}}\left(-\frac{}{\left(2\pi a_1 a_2\lambda\,\mathrm{erf}\right.}\right.\right.\right.}{}$$

7.9.6.6 [1968] Problem 6

problem number 1968

Added Jan 19, 2020.

Problem Chapter 9.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1e^{\mu x} + a_2e^{\lambda y})w_y + (b_1e^{\nu y} + b_2e^{\beta z})w_z = c_1w + c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[mu*x]+a2*Exp[lambda*y])*D[w[x,y,z],y]+(b1*Exp[nu*y]+b2*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(mu*x)+a__2*exp(lambda*y))*diff(w(x,y,z),y)+ (b__1*exp(nu*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1\_F1\left(\frac{a_2\lambda\,\text{expIntegral}\left(1,-\frac{a_1\lambda\,e^{\mu x}}{\mu}\right)-\mu\,e^{-\frac{(-a_1 e^{\mu x}+\mu y)\lambda}{\mu}}}{\lambda\mu}, \frac{-b_2\beta\left(\int^x e^{b_1\beta\left(\int\left(\frac{-\left(-\text{expIntegral}\left(1,-\frac{a_1\lambda\,e^{b\mu}}{\mu}\right)+\text{expIntegral}\cdots\right.}{\mu}\right.\right.}\right.}{\cdots}\right.}{\cdots}$$

7.9.6.7 [1969] Problem 7

problem number 1969

Added Jan 19, 2020.

Problem Chapter 9.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\lambda_1 x} y + a_2 e^{\lambda_2 x}) w_y + (b_1 e^{\beta_1 x} z + b_2 e^{\beta_2 x}) w_z = c_1 e^{\gamma_1 x} w + c_2 e^{\gamma_2 x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x])*D[w[x,y,z],y]+(b1*Exp[beta1*x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{\text{c1}e^{\text{gamma1}x}}{\text{gamma1}}}\left(\int_1^x \text{c2}e^{\text{gamma2}K[3]-\frac{\text{c1}e^{\text{gamma1}K[3]}}{\text{gamma1}}}dK[3]+c_1\left(ye^{-\frac{\text{a1}e^{\text{lambda1}x}}{\text{lambda1}}}-\int_1^x \text{a2}e^{\text{lambda2}K[1]-\frac{\text{a1}}{}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x))*diff(w(x,y,z),y)+ (
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int c_2\mathrm{e}^{\frac{x\gamma_1\gamma_2-c_1\mathrm{e}^{x\gamma_1}}{\gamma_1}}dx+\_F1\left(-a_2\left(\int \mathrm{e}^{\frac{x\lambda_1\lambda_2-a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}dx\right)+y\,\mathrm{e}^{-\frac{a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}},\frac{-b_1\beta_2\mathrm{e}^{\beta_1x}-b_2\beta_1\mathrm{e}^{\beta_2x}+}{\beta_1\beta_2}\right.\right.$$

7.9.6.8 [1970] Problem 8

problem number 1970

Added Jan 19, 2020.

Problem Chapter 9.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\lambda_1x}y+a_2e^{\lambda_2x}y^k)w_y+(b_1e^{\beta_1x}z+b_2e^{\beta_2x}z^m)w_z=c_1e^{\gamma_1x}w+c_2e^{\gamma_2y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x]*y^k)*D[w[x,y,z],y]+(b1*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{\text{c1}e^{\text{gamma1}x}}{\text{gamma1}}}\left(\int_1^x \text{c2}\exp\left(\text{gamma2}\left(e^{-\frac{\text{a1}\left(e^{\text{lambda1}K[3]}(k-1)+e^{\text{lambda1}x}\right)}{\text{lambda1}}}y^{-k}\left(e^{\frac{\text{a1}e^{\text{lambda1}x}}{\text{lambda1}}}(k-1)\int_1^{x}\right.\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x)*y^k)*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x c_2 \mathrm{e}^{\frac{\gamma_1\gamma_2\left((k-1)a_2\left(\int \mathrm{e}^{\frac{x\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}dx\right)-(k-1)a_2\left(\int \mathrm{e}^{\frac{\_b\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{\_b\lambda_1}}{\lambda_1}}d\_b\right)+y^{-k+1}\mathrm{e}^{\frac{(k-1)a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}\right)^{-\frac{1}{k-1}}\mathrm{e}^{\frac{a}{}}}{\gamma_1}}\right.$$

7.9.6.9 [1971] Problem 9

problem number 1971

Added Jan 19, 2020.

Problem Chapter 9.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\lambda_1 x}y+a_2e^{\lambda_2 x}y^k)w_y+(b_1e^{\beta_1 y}z+b_2e^{\beta_2 y}z^m)w_z=c_1e^{\gamma_1 x}w+c_2e^{\gamma_2 z}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x]*y^k)*D[w[x,y,z],y]+(b1*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x)*y^k)*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^x c_2 \mathrm{e}^{\frac{\gamma_1\gamma_2 \left(z^{-m+1} \mathrm{e}^{(m-1)b_1 \left(\int^x \mathrm{e}^{\beta_1 \left((k-1)a_2 \left(\int \mathrm{e}^{\frac{x\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}} dx \right) -(k-1)a_2 \left(\int \mathrm{e}^{\frac{\_b\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{\_b\lambda_1}}{\lambda_1}} d\_b \right) +y^{-k+} \right.}\right.}\right)}{}} \right.$$

7.9.6.10 [1972] Problem 10

problem number 1972

Added Jan 19, 2020.

Problem Chapter 9.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 e^{\beta y} w_x + a_2 e^{sigmax} w_y + (b_1 x^n e^{\mu y} + b_2 y^m e^{\nu x + \lambda z}) w_z = c_1 w + c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Exp[beta*y]*D[w[x,y,z],x]+ a2*Exp[sigma*x]*D[w[x,y,z],y]+(b1*x^n*Exp[mu*y]+b2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a__1*exp(beta*y)*diff(w(x,y,z),x)+ a__1*exp(sigma*x)*diff(w(x,y,z),y)+ (b__1*x^n*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1 \left(\frac{\sigma\,e^{\beta y}}{\beta}\right)^{\frac{c_1}{\left(\beta\,e^{\sigma x}-\sigma\,e^{\beta y}\right)a_1}} \_F1\left(\frac{-\beta\,e^{\sigma x}+\sigma\,e^{\beta y}}{\beta\sigma}, \frac{-b_2\lambda\sigma\left(\int^x \frac{e^{\frac{\_f a_1\beta\nu+b_1\lambda\sigma\left(\int \frac{\_f^n\left(-\frac{-\sigma\,e^{\beta y}+\left(-e^{\_f\sigma}+e^{\sigma x}\right)\beta}{\sigma}\right)^{\frac{\mu}{\beta}}}{e^{\_f\sigma}+\frac{-\beta\,e^{\sigma x}+\sigma\,e^{\beta y}}{\beta}}d\_f\right)}{a_1\beta}}}{e^{\_f\sigma}+\frac{-\beta\,e^{\sigma x}+\sigma\,e^{\beta y}}{\beta}}\right.}{\cdots}\right.}{\cdots}$$

7.9.7 4.1

Local contents

7.9.7.1 [1973] Problem 1

problem number 1973

Added Jan 19, 2020.

Problem Chapter 9.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w + k\sinh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Sinh[beta*x]^n*w[x,y,z]+ k*Sinh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\sinh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\sinh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

7.9.7.2 [1974] Problem 2

problem number 1974

Added Jan 19, 2020.

Problem Chapter 9.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\beta z)w_z = (p\sinh(\lambda x) + q)\,w + k\sinh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Sinh[beta*z]*D[w[x,y,z],z]==(p*Sinh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{p\cosh(\lambda x)+\lambda qx}{a\lambda}}\left(\int_1^x\frac{e^{-\frac{p\cosh(\lambda K[1])+\lambda qK[1]}{a\lambda}}k\sinh(\gamma K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{\log\left(\tanh\left(\frac{\beta z}{2}\right)\right)}{\beta}-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(beta*z)*diff(w(x,y,z),z)=(p*sinh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{k\,\mathrm{e}^{\frac{-\lambda qx-p\cosh(\lambda x)}{a\lambda}}\sinh(\gamma x)}{a}dx+\_F1\left(\frac{ay-bx}{a},\frac{-\beta cx-2a\operatorname{arctanh}\left(\mathrm{e}^{\beta z}\right)}{\beta c}\right)\right)\mathrm{e}^{\frac{\lambda qx+p\cosh(\lambda x)}{a\lambda}}$$

7.9.7.3 [1975] Problem 3

problem number 1975

Added Jan 19, 2020.

Problem Chapter 9.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sinh^n(\beta x)w_y + b\sinh^k(\lambda x)w_z = cw + s\sinh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Sinh[beta*x]^n*D[w[x,y,z],y]+b*Sinh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{k(e^{2\mu x}-1)\sinh^m(\mu x)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{c}{\mu}+m+2\right), -\frac{c+(m-2)\mu}{2\mu}, e^{2\mu x}\right)}{c+m\mu} + e^{cx}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*sinh(beta*x)^n*diff(w(x,y,z),y)+ b*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\sinh^m(\mu x))\,\mathrm{e}^{-cx}dx + \_F1\left(y - \left(\int a(\sinh^n(\beta x))\,dx\right), z - \left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\right)$$

7.9.7.4 [1976] Problem 4

problem number 1976

Added Jan 19, 2020.

Problem Chapter 9.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + b\sinh^n(\beta x)w_y + c\sinh^k(\lambda y)w_z = aw + s\sinh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Sinh[beta*x]^n*D[w[x,y,z],y]+c*Sinh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*sinh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \left(\int s(\sinh^m(\mu x))\,e^{-ax}dx + \_F1\left(y - \left(\int b(\sinh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\sinh^k\left(\left(b\left(\int (\mathrm{si}\right.\right.\right.\right.\right.\right.$$

7.9.7.5 [1977] Problem 5

problem number 1977

Added Jan 19, 2020.

Problem Chapter 9.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b_1\sinh^{n_1}(\lambda_1 x)w_x + b_2\sinh^{n_2}(\lambda_2 y)w_y + b_3\sinh^{n_3}(\lambda_3 z)w_z = aw + c_1\sinh^{k_1}(\beta_1 x) + c_2\sinh^{k_2}(\beta_2 y) + c_3\sinh^{k_3}(\beta_3 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Sinh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Sinh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Sinh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\sqrt{\cosh^2(\text{lambda1}x)}\text{sech}(\text{lambda1}x)\sinh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};-\sinh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*sinh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*sinh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\sinh^{k_1}(\beta_1x)\right)+c_2\left(\sinh^{k_2}(\beta_2x)\right)+c_3\left(\sinh^{k_3}(\beta_3x)\right)\right)\left(\sinh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\sinh^{-n_1}(x\lambda_1)\right.\right.}{b_1}}}{b_1}\right.$$

7.9.8 4.2

Local contents

7.9.8.1 [1978] Problem 1

problem number 1978

Added Jan 19, 2020.

Problem Chapter 9.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cosh^n(\beta x)w + k\cosh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Cosh[beta*x]^n*w[x,y,z]+ k*Cosh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{-\sinh^2(\beta x)}\operatorname{csch}(\beta x)\cosh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(\frac{c}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*cosh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k(\cosh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\cosh^n(\beta x))dx\right)}dx+\_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c(\cosh^n(\beta x))dx}$$

7.9.8.2 [1979] Problem 2

problem number 1979

Added Jan 19, 2020.

Problem Chapter 9.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cosh(\beta z)w_z = (p\cosh(\lambda x) + q)\,w + k\cosh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Cosh[beta*z]*D[w[x,y,z],z]==(p*Cosh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{p\sinh(\lambda x)+\lambda qx}{a\lambda}}\left(\int_1^x \frac{e^{-\frac{\lambda qK[1]+p\sinh(\lambda K[1])}{a\lambda}}k\cosh(\gamma K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta z}{2}\right)\right)}{\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cosh(beta*z)*diff(w(x,y,z),z)=(p*cosh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int \frac{k\cosh(\gamma x)\,\mathrm{e}^{\frac{-\lambda qx-p\sinh(\lambda x)}{a\lambda}}}{a}dx + \_F1\left(\frac{ay-bx}{a}, \frac{-\beta cx+2a\arctan\left(\mathrm{e}^{\beta z}\right)}{\beta c}\right)\right)\mathrm{e}^{\frac{\lambda qx+p\sinh(\lambda x)}{a\lambda}}$$

7.9.8.3 [1980] Problem 3

problem number 1980

Added Jan 19, 2020.

Problem Chapter 9.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cosh^n(\beta x)w_y + b\cosh^k(\lambda x)w_z = cw + s\cosh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Cosh[beta*x]^n*D[w[x,y,z],y]+b*Cosh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{k(e^{2\mu x}+1)\cosh^m(\mu x)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{c}{\mu}+m+2\right),-\frac{c+(m-2)\mu}{2\mu},-e^{2\mu x}\right)}{c+m\mu} +\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*cosh(beta*x)^n*diff(w(x,y,z),y)+ b*cosh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\mu x))\,\mathrm{e}^{-cx}dx + \_F1\left(y - \left(\int a(\cosh^n(\beta x))\,dx\right), z - \left(\int b(\cosh^k(\lambda x))\,dx\right)\right)\right.$$

7.9.8.4 [1981] Problem 4

problem number 1981

Added Jan 19, 2020.

Problem Chapter 9.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\cosh^n(\beta x)w_y + c\cosh^k(\lambda y)w_z = aw + s\cosh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Cosh[beta*x]^n*D[w[x,y,z],y]+c*Cosh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*cosh(beta*x)^n*diff(w(x,y,z),y)+ c*cosh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\left(\int s(\cosh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\cosh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\cosh^k\left(\left(b\left(\int (c\right.\right.\right.\right.\right.\right.$$

7.9.8.5 [1982] Problem 5

problem number 1982

Added Jan 19, 2020.

Problem Chapter 9.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\cosh^{n_1}(\lambda_1 x)w_x + b_2\cosh^{n_2}(\lambda_2 y)w_y + b_3\cosh^{n_3}(\lambda_3 z)w_z = aw + c_1\cosh^{k_1}(\beta_1 x) + c_2\cosh^{k_2}(\beta_2 y) + c_3\cosh^{k_3}(\beta_3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Cosh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Cosh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Cosh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{a\sqrt{-\sinh^2(\text{lambda1}x)}\operatorname{csch}(\text{lambda1}x)\cosh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};\cosh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*cosh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*cosh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\cosh^{k_1}(\beta_1x)\right)+c_2\left(\cosh^{k_2}(\beta_2x)\right)+c_3\left(\cosh^{k_3}(\beta_3x)\right)\right)\left(\cosh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\cosh^{-n_1}(x\lambda\right.\right.}{b_1}}}{b_1}\right.$$

7.9.9 4.3

Local contents

7.9.9.1 [1983] Problem 1

problem number 1983

Added Jan 19, 2020.

Problem Chapter 9.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\tanh^n(\beta x)w + k\tanh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Tanh[beta*x]^n*w[x,y,z]+ k*Tanh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\right.}{n\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\tanh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\tanh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c(\tanh^n(\beta x))dx}$$

7.9.9.2 [1984] Problem 2

problem number 1984

Added Jan 19, 2020.

Problem Chapter 9.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tanh(\beta z)w_z = (p\tanh(\lambda x) + q)\,w + k\tanh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Tanh[beta*z]*D[w[x,y,z],z]==(p*Tanh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{qx}{a}}\cosh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[1]}{a}}k\cosh^{-\frac{p}{a\lambda}}(\lambda K[1])\tanh(\gamma K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{\log(\sinh(}{\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tanh(beta*z)*diff(w(x,y,z),z)=(p*tanh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^z \frac{k\left(\tanh\left(\frac{(2\beta cx-a\ln(\tanh(\_a\beta)-1)-a\ln(\tanh(\_a\beta)+1)+a\ln(\tanh(\beta z)-1)+a\ln(\tanh(\beta z)+1)+2a\ln(\tanh(\_a\beta))}{2\beta c}\right.\right.}{}\right.$$

7.9.9.3 [1985] Problem 3

problem number 1985

Added Jan 19, 2020.

Problem Chapter 9.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tanh^n(\beta x)w_y + b\tanh^k(\lambda x)w_z = cw + s\tanh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Tanh[beta*x]^n*D[w[x,y,z],y]+b*Tanh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\rightarrow -\frac{k(e^{-2\mu x}-1)^m\left(e^{-2\mu x}+1\right)^m\left(-e^{-4\mu x}(e^{2\mu x}-1)^2\right)^{-m}\tanh^m(\mu x)F_1\left(\frac{c}{2\mu};m,-m;\frac{c}{2\mu}+1;\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*tanh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k(\tanh^m(\mu x))\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\tanh^k(\lambda x))\,dx\right)\right)\right.$$

7.9.9.4 [1986] Problem 4

problem number 1986

Added Jan 19, 2020.

Problem Chapter 9.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\tanh^n(\beta x)w_y + c\tanh^k(\lambda y)w_z = aw + s\tanh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Tanh[beta*x]^n*D[w[x,y,z],y]+c*Tanh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*tanh(beta*x)^n*diff(w(x,y,z),y)+ c*tanh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\left(\int s(\tanh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\tanh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\frac{\sinh\left(\left(b\left(\int (\tan\right.\right.\right.}{\cosh\left(\left(b\left(\int (\tan\right.\right.\right.}\right.\right.\right.\right.\right.$$

7.9.9.5 [1987] Problem 5

problem number 1987

Added Jan 19, 2020.

Problem Chapter 9.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\tanh^{n_1}(\lambda_1 x)w_x + b_2\tanh^{n_2}(\lambda_2 y)w_y + b_3\tanh^{n_3}(\lambda_3 z)w_z = aw + c_1\tanh^{k_1}(\beta_1 x) + c_2\tanh^{k_2}(\beta_2 y) + c_3\tanh^{k_3}(\ldots$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Tanh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Tanh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Tanh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\tanh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(1,\frac{1}{2}-\frac{\text{n1}}{2};\frac{3}{2}-\frac{\text{n1}}{2};\tanh^2(\text{lambda1}x)\right)}{\text{b1lambda1}-\text{b1lambda1n1}}\right)\left(\int_1^x \exp\left(\frac{a\,{}_2F_1(\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*tanh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*tanh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{\left(c_1\left(\tanh^{k_1}\left(\beta_1 x\right)\right)+c_2\left(\tanh^{k_2}\left(\beta_2 x\right)\right)+c_3\left(\tanh^{k_3}\left(\beta_3 x\right)\right)\right)\left(\tanh^{-n_1}\left(x\lambda_1\right)\right)\mathrm{e}^{-\frac{a\left(\int\left(\tanh^{-n_1}(x\right.\right.}{b_1}}}{b_1}\right.$$

7.9.10 4.4

Local contents

7.9.10.1 [1988] Problem 1

problem number 1988

Added Jan 19, 2020.

Problem Chapter 9.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^n(\beta x)w + k\coth^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Coth[beta*x]^n*w[x,y,z]+ k*Coth[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\coth^{n+1}(\beta K[1])\,{}_2F_1\left(1,\right.}{n\beta\,-}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*coth(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\coth^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\coth^n(\beta x))dx\right)}dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c(\coth^n(\beta x))dx}$$

7.9.10.2 [1989] Problem 2

problem number 1989

Added Jan 19, 2020.

Problem Chapter 9.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\coth(\beta z)w_z = (p\coth(\lambda x) + q)\,w + k\coth(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Coth[beta*z]*D[w[x,y,z],z]==(p*Coth[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z) \to e^{\frac{qx}{a}} \sinh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[1]}{a}} k\coth(\gamma K[1])\sinh^{-\frac{p}{a\lambda}}(\lambda K[1])}{a} dK[1] + c_1\left(y - \frac{bx}{a}, \frac{\log(\cosh(\beta z}{\beta}\right.\right.\right.$$

$$\left\{w(x,y,z) \to e^{\frac{qx}{a}} \sinh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[2]}{a}} k\coth(\gamma K[2])\sinh^{-\frac{p}{a\lambda}}(\lambda K[2])}{a} dK[2] + c_1\left(y - \frac{bx}{a}, \frac{\log(\cosh(\beta z}{\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*coth(beta*z)*diff(w(x,y,z),z)=(p*coth(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\left(\int \frac{k(\coth(\lambda x) - 1)^{\frac{p}{2a\lambda}}(\coth(\lambda x) + 1)^{\frac{p}{2a\lambda}}\coth(\gamma x)\,\mathrm{e}^{-\frac{qx}{a}}}{a} dx + \_F1\left(\frac{ay - bx}{a}, \frac{-2\beta cx + a\ln}{}\right.\right.\right.$$

7.9.10.3 [1990] Problem 3

problem number 1990

Added Jan 19, 2020.

Problem Chapter 9.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\coth^n(\beta x)w_y + b\coth^k(\lambda x)w_z = cw + s\coth^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Coth[beta*x]^n*D[w[x,y,z],y]+b*Coth[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{k(e^{-2\mu x}-1)^{-m}(e^{-2\mu x}+1)^{-m}\left(-e^{-4\mu x}(e^{2\mu x}-1)^2\right)^m\coth^m(\mu x)F_1\left(\frac{c}{2\mu};-m,m;\frac{c}{2\mu}+1\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*coth(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k(\coth^m(\mu x))\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a(\coth^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)\right.$$

7.9.10.4 [1991] Problem 4

problem number 1991

Added Jan 19, 2020.

Problem Chapter 9.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\coth^n(\beta x)w_y + c\coth^k(\lambda y)w_z = aw + s\coth^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Coth[beta*x]^n*D[w[x,y,z],y]+c*Coth[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*coth(beta*x)^n*diff(w(x,y,z),y)+ c*coth(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s(\coth^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\coth^n(\beta x))\,dx\right), z - \left(\int^x c\left(\frac{\cosh\left(\left(b\left(\int(\cot\right.\right.\right.}{\sinh\left(\left(b\left(\int(\cot\right.\right.\right.}\right.\right.\right.\right.$$

7.9.10.5 [1992] Problem 5

problem number 1992

Added Jan 19, 2020.

Problem Chapter 9.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\coth^{n_1}(\lambda_1 x)w_x + b_2\coth^{n_2}(\lambda_2 y)w_y + b_3\coth^{n_3}(\lambda_3 z)w_z = aw + c_1\coth^{k_1}(\beta_1 x) + c_2\coth^{k_2}(\beta_2 y) + c_3\coth^{k_3}(\beta_3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Coth[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Coth[lambda2*x]^n2*D[w[x,y,z],y]+b3*Coth[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\coth^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(1,\frac{1}{2}-\frac{\text{n1}}{2};\frac{3}{2}-\frac{\text{n1}}{2};\coth^2(\text{lambda1}x)\right)}{\text{b1lambda1}-\text{b1lambda1n1}}\right)\left(\int_1^x \frac{\exp\left(\frac{a\coth^{1}\cdots}{\cdots}\right.}{\cdots}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*coth(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*coth(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{\left(c_1\left(\coth^{k_1}\left(\beta_1 x\right)\right)+c_2\left(\coth^{k_2}\left(\beta_2 x\right)\right)+c_3\left(\coth^{k_3}\left(\beta_3 x\right)\right)\right)\left(\coth^{-n_1}\left(x\lambda_1\right)\right)\mathrm{e}^{-\frac{a\left(\int\left(\coth^{-n_1}(x\lambda\cdots\right.\right.}{b_1}}}{b_1}\right.$$

7.9.11 4.5

Local contents

7.9.11.1 [1993] Problem 1

problem number 1993

Added Jan 19, 2020.

Problem Chapter 9.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w + k\cosh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Sinh[beta*x]^n*w[x,y,z]+ k*Cosh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\sinh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

7.9.11.2 [1994] Problem 2

problem number 1994

Added Jan 19, 2020.

Problem Chapter 9.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c\tanh^n(\beta x)w + k\coth^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Tanh[beta*x]^n*w[x,y,z]+ k*Coth[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\right.}{n\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^m \mathrm{e}^{-c\left(\int\left(\frac{\sinh(\beta x)}{\cosh(\beta x)}\right)^n dx\right)}dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c\left(\frac{\sinh(\beta x)}{\cosh(\beta x)}\right)^n dx}$$

7.9.11.3 [1995] Problem 3

problem number 1995

Added Jan 19, 2020.

Problem Chapter 9.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + b\cosh^n(\beta x)w_y + c\sinh^k(\lambda y)w_z = aw + s\cosh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Cosh[beta*x]^n*D[w[x,y,z],y]+c*Sinh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*cosh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \left(\int k(\cosh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\cosh^n(\beta x))\,dx\right), z - \left(\int c(\sinh^k(\lambda x))\,dx\right)\right)\right.$$

7.9.11.4 [1996] Problem 4

problem number 1996

Added Jan 19, 2020.

Problem Chapter 9.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\tanh^n(\beta x)w_y + b\coth^k(\lambda x)w_z = cw + s\tanh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Tanh[beta*x]^n*D[w[x,y,z],y]+b*Coth[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(e^{-2\mu x}-1)^m\left(e^{-2\mu x}+1\right)^m\left(-e^{-4\mu x}(e^{2\mu x}-1)^2\right)^{-m}\tanh^m(\mu x)F_1\left(\frac{c}{2\mu};m,-m;\frac{c}{2\mu}+1;\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s(\tanh^m(\mu x))\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)\right.$$

7.9.11.5 [1997] Problem 5

problem number 1997

Added Jan 19, 2020.

Problem Chapter 9.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\sinh^{n_1}(\lambda_1 x)w_x+b_2\cosh^{n_2}(\lambda_2 y)w_y+b_3\sinh^{n_3}(\lambda_3 z)w_z=aw+c_1\cosh^{k_1}(\beta_1 x)+c_2\sinh^{k_2}(\beta_2 y)+c_3\sinh^{k_3}(\beta_3 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Sinh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Cosh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Sinh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{a\sqrt{\cosh^2(\text{lambda1}x)}\operatorname{sech}(\text{lambda1}x)\sinh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};-\sinh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*sinh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*cosh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\cosh^{k_1}(\beta_1x)\right)+c_2\left(\sinh^{k_2}(\beta_2x)\right)+c_3\left(\sinh^{k_3}(\beta_3x)\right)\right)\left(\sinh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\sinh^{-n_1}(x\lambda_1)\right.\right.}{b_1}}}{b_1}\right.$$

7.9.12 5.1

Local contents

7.9.12.1 [1998] Problem 1

problem number 1998

Added Jan 19, 2020.

Problem Chapter 9.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\ln^n(\beta x)w + k\ln^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Log[beta*x]^n*w[x,y,z]+ k*Log[lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1,-\log(\beta x))}{\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,\text{Gamma}(n+1\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*ln(beta*x)^n*w(x,y,z)+ k*ln
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k\ln(\lambda x)^m \mathrm{e}^{-c\left(\int \ln(\beta x)^n dx\right)} dx + \_F1(-ax+y,-bx+z)\right)\mathrm{e}^{\int c\ln(\beta x)^n dx}$$

7.9.12.2 [1999] Problem 2

problem number 1999

Added Jan 19, 2020.

Problem Chapter 9.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = cw + s\ln^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Log[beta*x]^n*D[w[x,y,z],y]+b*Log[lambda*x]^k*D[w[x,y,z],z]==c*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{cx}\left(\int_1^x e^{-cK[1]}s\log^m(\mu K[1])dK[1]+c_1\left(y-\frac{a(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1,-\right.\right.\right.\right.}{\beta}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)=c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s\ln(\mu x)^m\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a\ln(\beta x)^n\,dx\right),z-\left(\int b\ln(\lambda x)^k\,dx\right)\right)\right)\mathrm{e}^{cx}$$

7.9.12.3 [2000] Problem 3

problem number 2000

Added Jan 19, 2020.

Problem Chapter 9.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\ln^n(\beta x)w_y + c\ln^k(\lambda y)w_z = aw + s\ln^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Log[beta*x]^n*D[w[x,y,z],y]+c*Log[lambda*y]^k*D[w[x,y,z],z]==a*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+b*ln(beta*x)^n*diff(w(x,y,z),y)+ c*ln(lambda*y)^k*diff(w(x,y,z),z)=a
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s\ln(\mu x)^m \mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b\ln(\beta x)^n\,dx\right), z - \left(\int^x c\ln\left(\left(b\left(\int \ln(\_b\beta)^n\,d\_b\right.\right.\right.\right.\right.\right.$$

7.9.12.4 [2001] Problem 4

problem number 2001

Added Jan 19, 2020.

Problem Chapter 9.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln(\alpha x)w_x + b\ln(\beta y)w_y + c\ln(\gamma z)w_z = pw + q\ln(\lambda x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[alpha*x]*D[w[x,y,z],x]+ b*Log[beta*y]*D[w[x,y,z],y]+c*Log[gamma*z]*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*ln(alpha*x)*diff(w(x,y,z),x)+b*ln(beta*y)*diff(w(x,y,z),y)+ c*ln(gamma*z)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int \frac{q\,\mathrm{e}^{\frac{p\,\mathrm{expIntegral}(1,-\ln(\alpha x))}{a\alpha}}\ln(\lambda x)}{a\ln(\alpha x)}dx + \_F1\left(\frac{-a\alpha\,\mathrm{expIntegral}(1,-\ln(\beta y)) + b\beta\,\mathrm{expIntegral}(}{\alpha b\beta}\right.\right.$$

7.9.12.5 [2002] Problem 5

problem number 2002

Added Jan 19, 2020.

Problem Chapter 9.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\ln^{n_1}(\lambda_1 x)w_x + b_2\ln^{n_2}(\lambda_2 y)w_y + b_3\ln^{n_3}(\lambda_3 z)w_z = aw + c_1\ln^{k_1}(\beta_1 x) + c_2\ln^{k_2}(\beta_2 y) + c_3\ln^{k_3}(\beta_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Log[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Log[lambda2*x]^n2*D[w[x,y,z],y]+b3*Log[lambda3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{a(-\log(\mathrm{lambda1}x))^{\mathrm{n1}}\log^{-\mathrm{n1}}(\mathrm{lambda1}x)\,\mathrm{Gamma}(1-\mathrm{n1},-\log(\mathrm{lambda1}x))}{\mathrm{b1lambda1}}\right)\left(\int\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*ln(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*ln(lambda__2*x)^(n__2)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int \frac{\left(c_1 \ln\left(\beta_1 x\right)^{k_1} + c_2 \ln\left(\beta_2 x\right)^{k_2} + c_3 \ln\left(\beta_3 x\right)^{k_3}\right) \ln\left(x\lambda_1\right)^{-n_1} e^{-\frac{a\left(\int \ln(x\lambda_1)^{-n_1} dx\right)}{b_1}}}{b_1} dx + \_F1\left(-\frac{b\right.\right.$$

CHAPTER 8

Handbook of nonlinear partial differential equations

Local contents

8.1 chapter 1

Local contents

8.1.1 1.1

Local contents

8.1.1.1 [2003] Problem 1

problem number 2003

Added March 23, 2019.

Problem Chapter 1.1.1.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + bw^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) + b*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.1.2 [2004] Problem 2

problem number 2004

Added March 23, 2019.

Problem Chapter 1.1.1.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw(1-w)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]*(1 - w[x, t]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5at-\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5at-i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5at-i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\right.$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5at+i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5at+i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\right.$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5at+\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{5at}{12}-\frac{\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{5at}{12}-\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5at}{12}-\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{5at}{12}+\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5at}{12}+\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{5at}{12}+\frac{\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) + a*w(x,t)*(1-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = -\frac{\left(\tanh^2\left(-\frac{5at}{12}+c_1+\frac{\sqrt{-6a}\,x}{12}\right)\right)}{4} - \frac{\tanh\left(-\frac{5at}{12}+c_1+\frac{\sqrt{-6a}\,x}{12}\right)}{2} + \frac{3}{4}$$

8.1.2 1.2

Local contents

8.1.2.1 [2005] Problem 1

problem number 2005

Added March 23, 2019.

Problem Chapter 1.1.2.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} - bw^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.2.2 [2006] Problem 2

problem number 2006

Added March 23, 2019.

Problem Chapter 1.1.2.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw - bw^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] - b*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)- b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{\sqrt{ab}\left(\tanh\left(-\frac{3at}{4} + \frac{\sqrt{2}\,\sqrt{a}\,x}{4} + c_1\right) - 1\right)}{2b}$$

8.1.2.3 [2007] Problem 3

problem number 2007

Added March 23, 2019.

Problem Chapter 1.1.2.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = a w_{xx} - b w^3 - c w^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3 - c*w[x, t]^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3- c*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{\left(\tanh\left(-\frac{c^2 t}{4b} + \frac{\sqrt{2}\, c x}{4\sqrt{ab}} + c_1\right) - 1\right) c}{2b}$$

8.1.2.4 [2008] Problem 4

problem number 2008

Added March 23, 2019.

Problem Chapter 1.1.2.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} - w(1-w)(a-w)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] - w[x, t]*(1 - w[x, t])*(a - w[x, t]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) - w(x,t)*(1-w(x,t))*(a-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t)=\frac{\tanh\left(c_1+\frac{(-2a+1)t}{4}+\frac{\sqrt{2}\,x}{4}\right)}{2}+\frac{1}{2}$$

8.1.2.5 [2009] Problem 5

problem number 2009

Added March 23, 2019.

Problem Chapter 1.1.2.5, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t=aw_{xx}+b_0+b_1w+b_2w^2+b_3w^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b0 + b1*w[x, t] + b2*w[x, t]^2 + b3*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b0+b1*w(x,t)+b2*w(x,t)^2+b3*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{-2304\Big(ab3 \operatorname{RootOf}\left(512\_Z^6 a^3 b3^2 - 27b0^2 b3^3 + 18b0\, b1\, b2\, b3^2 - 4b0\, b2^3 b3 - 4b1^3 b3^2 + b1^2\right.}{}$$

8.1.3 1.3

Local contents

8.1.3.1 [2010] Problem 1

problem number 2010

Added March 23, 2019.

Problem Chapter 1.1.3.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + bw^k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b*w(x,t)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.3.2 [2011] Problem 2

problem number 2011

Added March 23, 2019.

Problem Chapter 1.1.3.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.3.3 [2012] Problem 3

problem number 2012

Added March 23, 2019.

Problem Chapter 1.1.3.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m + cw^{2m-1}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m + c*w[x, t]^(2*m - 1);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m+c*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.3.4 [2013] Problem 4

problem number 2013

Added March 23, 2019.

Problem Chapter 1.1.3.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw^{m-1} + bmw^m - mb^2w^{2m-1}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]^(m - 1) + b*m*w[x, t]^m - m*b^2*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)^(m-1)+b*m*w(x,t)^m-m*b^2*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

8.1.4 1.4

Local contents

8.1.4.1 [2014] Problem 1

problem number 2014

Added March 23, 2019.

Problem Chapter 1.1.4.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + s_1(bx+ct)^k + s_2w^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + s1*(b*x + c*t)^k + s2*w[x, t]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +s1*(b*x+c*t)^k+s2*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

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