Computer Algebra Report

Solving

Partial Differential Equations

Using

Maple 2020.2 and Mathematica 12.2

Nasser M. Abbasi

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CHAPTER 1

Introduction, Summary of results and lookup table

1.1 Introduction

This report gives the result of running a number of partial differential equations in Mathematica and Maple. This is work in progress as more PDE's are being added.

The following are the systems used

1. Mathematica 12.2 (64 bit).
2. Maple 2020.2 (64 bit)

The following are plain text files of the current collection of the PDE's used in this report.

Mathematica_PDE_IN_CAS_problems.txt

Maple_PDE_IN_CAS_problems.txt

10 minutes real time is used as the time limit to complete a problem. If CAS does not finish within this time limit, a failed score is given.

The PC used to run these tests is windows 10 professional 64 bit with 64 GB RAM running Intel core i7-8086K at 4 GHZ.

All possible options, assumptions and HINTS are tried to obtain a solution. The command `DSolve` is used in Mathematica and the command `pdsolve` in Maple.

In this version, all results are automatically simplified. This is done to reduce the size of the final solution.

It is possible I missed some option, assumption or HINT, which could help make the CAS able to solve a given PDE now marked as unsolved. Will correct such a case if found.

Most of the solutions returned are not verified. If a CAS returns a solution, it is assumed to be correct and that the problem was solved by the CAS.

These problems were collected from textbooks such as

1. Richard Haberman applied partial differential equations, 5th edition
2. David J Logan applied Partial differential equations.
3. Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.
4. Handbook of first order partial differential equations (HFOPDE), Volume 1, by Polyanin, Zaitsev, Moussiaux (2002).
5. Handbook of nonlinear partial differential equations (HNPDE), by Polyanin, Zaitsev (2004).
6. Nonlinear Partial Differential Equations by Lokenath Debnath, 3rd edition. (2012).
7. Introduction to Partial Differential Equations by Peter J. Olver, Springer. (2014).

PDE's from other text books will be added with time.

Some problems were also collected from Maple and Mathematica help pages, documentation and technical forums.

Some of these problems I solved by hand. I have tried to verify that all hand solutions are correct by verifying them using Maple's `pdetest()`. Will add more hand solutions in the future in order to compare with the computer solution.

The following are some PDF files used to collect some problems from

1. PDE_and_BC_during_december_2018.pdf
2. What_is_New_after_Maple_2018.pdf
3. Integral_Transforms_revamped_Oct_2019.pdf

1.2 Results

The current number of partial differential equations in this report is [2012].

Mathematica solved [1460] and Maple solved [1801].

Table 1.1: Percentage solved by each CAS for all problems

Mathematica	Maple
72.56%	89.51%

Table 1.2: Pecentage of solved by each CAS broken down by each chapter

chapter name	Number of Problems	Mathematica	Maple
Miscellaneous PDE's	149	75.17%	91.28%
Parabolic PDE's (Diffusion)	130	91.54%	86.15%
Elliptic PDE's (Laplace, Poisson, Helmholtz)	53	71.7%	75.47%
Hyperbolic PDE's (Wave)	83	91.57%	85.54%
Handbook of first order partial differential equations	1585	70.28%	90.66%
Handbook of nonlinear partial differential equations	12	8.33%	41.67%

Table 1.3: Percentage of failed due to time out among all problems that could not be solved

Mathematica	Maple
7.25%	11.85%

Table 1.4: Total real time used to solve all problems

Mathematica	Maple
14.47 (hours)	8.89 (hours)

Table 1.5: Average real time used to solve one problem

Mathematica	Maple
25.882 (sec)	15.904 (sec)

List of problems not solved by Maple nor by Mathematica [175] (8.7%)

32, 48, 57, 61, 119, 126, 131, 135, 136, 137, 169, 170, 272, 277, 278, 279, 307, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 332, 361, 362, 413, 454, 455, 456, 458, 474, 487, 488, 489, 492, 508, 516, 520, 521, 525, 526, 528, 530, 531, 543, 549, 557, 559, 561, 563, 564, 567, 620, 622, 623, 627, 628, 629, 650, 672, 673, 695, 698, 711, 712, 720, 723, 724, 735, 736, 746, 747, 748, 757, 758, 760, 761, 765, 766, 768, 770, 771, 772, 775, 776, 777, 779, 780, 783, 784, 785, 786, 787, 791, 802, 803, 808, 815, 816, 821, 822, 823, 824, 825, 826, 827, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 1029, 1172, 1179, 1194, 1366, 1377, 1378, 1380, 1383, 1384, 1385, 1386, 1387, 1413, 1416, 1417, 1421, 1428, 1429, 1450, 1451, 1452, 1453, 1457, 1462, 1463, 1555, 1574, 1577, 1578, 1628, 1629, 1632, 1726, 1736, 1748, 1749, 1800, 1801, 1906, 1907, 1914, 1915, 1916, 1918, 1924, 2001, 2003, 2008, 2009, 2010, 2011, 2012

List of problems solved by Maple but not by Mathematica [377] (18.74%)

16, 29, 34, 40, 67, 78, 100, 101, 102, 103, 104, 105, 107, 108, 116, 117, 118, 120, 121, 124, 127, 128, 130, 132, 133, 138, 140, 174, 182, 204, 214, 230, 297, 304, 310, 359, 394, 395, 408, 429, 449, 453, 457, 459, 460, 464, 465, 466, 469, 476, 480, 482, 484, 485, 486, 490, 493, 494, 499, 504, 505, 506, 507, 518, 519, 522, 524, 527, 529, 532, 537, 542, 551, 554, 558, 560, 570, 574, 576, 577, 578, 579, 583, 584, 589, 590, 591, 592, 593, 598, 599, 600, 601, 612, 615, 618, 621, 625, 634, 635, 636, 642, 643, 644, 646, 647, 648, 649, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 668, 669, 671, 675, 676, 677, 678, 679, 682, 683, 684, 691, 696, 697, 699, 700, 701, 702, 705, 707, 709, 710, 717, 719, 721, 722, 729, 731, 733, 741, 743, 745, 753, 755, 756, 759, 769, 774, 778, 782, 788, 790, 792, 793, 795, 796, 798, 800, 801, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 817, 818, 819, 820, 829, 830, 875, 884, 885, 901, 902, 907, 917, 929, 994, 1001, 1004, 1018, 1019, 1036, 1037, 1057, 1058, 1065, 1066, 1079, 1084, 1107, 1178, 1180, 1181, 1195, 1208, 1209, 1210, 1211, 1234, 1235, 1262, 1267, 1279, 1352, 1360, 1365, 1374, 1375, 1381, 1382, 1390, 1410, 1414, 1420, 1422, 1423, 1425, 1436, 1437, 1441, 1446, 1447, 1454, 1455, 1456, 1459, 1461, 1468, 1469, 1475, 1480, 1481, 1486, 1487, 1490, 1491, 1492, 1493, 1511, 1512, 1523, 1527, 1529, 1558, 1559, 1563, 1575, 1576, 1586, 1607, 1608, 1618, 1630, 1648, 1650, 1655, 1664, 1666, 1667, 1671, 1678, 1684, 1689, 1695, 1697, 1698, 1699, 1700, 1703, 1704, 1709, 1711, 1714, 1715, 1719, 1720, 1727, 1729, 1730, 1731, 1732, 1733, 1737, 1738, 1740, 1742, 1744, 1745, 1746, 1747, 1757, 1778, 1779, 1783, 1789, 1791, 1799, 1803, 1809, 1814, 1818, 1820, 1824, 1826, 1829, 1835, 1836, 1837, 1840, 1844, 1845, 1847, 1851, 1853, 1858, 1863, 1865, 1866, 1867, 1868, 1871, 1872, 1874, 1877, 1878, 1880, 1883, 1884, 1888, 1889, 1898, 1899, 1900, 1901, 1902, 1905, 1909, 1911, 1913, 1917, 1931, 1938, 1948, 1951, 1958, 1960, 1963, 1964, 1966, 1969, 1970, 1974, 1979, 1984, 1989, 1993, 1998, 1999, 2004, 2005, 2006, 2007

List of problems solved by Mathematica but not by Maple [36] (1.79%)

86, 94, 95, 166, 203, 226, 264, 265, 267, 268, 269, 270, 271, 275, 276, 300, 364, 393, 399, 401, 403, 409, 410, 411, 412, 1202, 1409, 1424, 1533, 1638, 1644, 1705, 1706, 1839, 1873,

1904

List of problems not solved by Mathematica [552] (27.44%)

16, 29, 32, 34, 40, 48, 57, 61, 67, 78, 100, 101, 102, 103, 104, 105, 107, 108, 116, 117, 118, 119, 120, 121, 124, 126, 127, 128, 130, 131, 132, 133, 135, 136, 137, 138, 140, 169, 170, 174, 182, 204, 214, 230, 272, 277, 278, 279, 297, 304, 307, 310, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 332, 359, 361, 362, 394, 395, 408, 413, 429, 449, 453, 454, 455, 456, 457, 458, 459, 460, 464, 465, 466, 469, 474, 476, 480, 482, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494, 499, 504, 505, 506, 507, 508, 516, 518, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 537, 542, 543, 549, 551, 554, 557, 558, 559, 560, 561, 563, 564, 567, 570, 574, 576, 577, 578, 579, 583, 584, 589, 590, 591, 592, 593, 598, 599, 600, 601, 612, 615, 618, 620, 621, 622, 623, 625, 627, 628, 629, 634, 635, 636, 642, 643, 644, 646, 647, 648, 649, 650, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 668, 669, 671, 672, 673, 675, 676, 677, 678, 679, 682, 683, 684, 691, 695, 696, 697, 698, 699, 700, 701, 702, 705, 707, 709, 710, 711, 712, 717, 719, 720, 721, 722, 723, 724, 729, 731, 733, 735, 736, 741, 743, 745, 746, 747, 748, 753, 755, 756, 757, 758, 759, 760, 761, 765, 766, 768, 769, 770, 771, 772, 774, 775, 776, 777, 778, 779, 780, 782, 783, 784, 785, 786, 787, 788, 790, 791, 792, 793, 795, 796, 798, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 875, 884, 885, 901, 902, 907, 917, 929, 994, 1001, 1004, 1018, 1019, 1029, 1036, 1037, 1057, 1058, 1065, 1066, 1079, 1084, 1107, 1172, 1178, 1179, 1180, 1181, 1194, 1195, 1208, 1209, 1210, 1211, 1234, 1235, 1262, 1267, 1279, 1352, 1360, 1365, 1366, 1374, 1375, 1377, 1378, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1390, 1410, 1413, 1414, 1416, 1417, 1420, 1421, 1422, 1423, 1425, 1428, 1429, 1436, 1437, 1441, 1446, 1447, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1459, 1461, 1462, 1463, 1468, 1469, 1475, 1480, 1481, 1486, 1487, 1490, 1491, 1492, 1493, 1511, 1512, 1523, 1527, 1529, 1555, 1558, 1559, 1563, 1574, 1575, 1576, 1577, 1578, 1586, 1607, 1608, 1618, 1628, 1629, 1630, 1632, 1648, 1650, 1655, 1664, 1666, 1667, 1671, 1678, 1684, 1689, 1695, 1697, 1698, 1699, 1700, 1703, 1704, 1709, 1711, 1714, 1715, 1719, 1720, 1726, 1727, 1729, 1730, 1731, 1732, 1733, 1736, 1737, 1738, 1740, 1742, 1744, 1745, 1746, 1747, 1748, 1749, 1757, 1778, 1779, 1783, 1789, 1791, 1799, 1800, 1801, 1803, 1809, 1814, 1818, 1820, 1824, 1826, 1829, 1835, 1836, 1837, 1840, 1844, 1845, 1847, 1851, 1853, 1858, 1863, 1865, 1866, 1867, 1868, 1871, 1872, 1874, 1877, 1878, 1880, 1883, 1884, 1888, 1889, 1898, 1899, 1900, 1901, 1902, 1905, 1906, 1907, 1909, 1911, 1913, 1914, 1915, 1916, 1917, 1918, 1924, 1931, 1938, 1948, 1951, 1958, 1960, 1963, 1964, 1966, 1969, 1970, 1974, 1979, 1984, 1989, 1993, 1998, 1999, 2001, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012

List of problems not solved by Maple [211] (10.49%)

32, 48, 57, 61, 86, 94, 95, 119, 126, 131, 135, 136, 137, 166, 169, 170, 203, 226, 264, 265,

267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 300, 307, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 332, 361, 362, 364, 393, 399, 401, 403, 409, 410, 411, 412, 413, 454, 455, 456, 458, 474, 487, 488, 489, 492, 508, 516, 520, 521, 525, 526, 528, 530, 531, 543, 549, 557, 559, 561, 563, 564, 567, 620, 622, 623, 627, 628, 629, 650, 672, 673, 695, 698, 711, 712, 720, 723, 724, 735, 736, 746, 747, 748, 757, 758, 760, 761, 765, 766, 768, 770, 771, 772, 775, 776, 777, 779, 780, 783, 784, 785, 786, 787, 791, 802, 803, 808, 815, 816, 821, 822, 823, 824, 825, 826, 827, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 1029, 1172, 1179, 1194, 1202, 1366, 1377, 1378, 1380, 1383, 1384, 1385, 1386, 1387, 1409, 1413, 1416, 1417, 1421, 1424, 1428, 1429, 1450, 1451, 1452, 1453, 1457, 1462, 1463, 1533, 1555, 1574, 1577, 1578, 1628, 1629, 1632, 1638, 1644, 1705, 1706, 1726, 1736, 1748, 1749, 1800, 1801, 1839, 1873, 1904, 1906, 1907, 1914, 1915, 1916, 1918, 1924, 2001, 2003, 2008, 2009, 2010, 2011, 2012

List of problems that are hand solved [195] (9.69%)

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24, 26, 27, 28, 29, 30, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 91, 92, 109, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 168, 171, 172, 175, 176, 177, 178, 180, 181, 183, 184, 185, 191, 194, 195, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 232, 234, 235, 236, 237, 247, 252, 253, 258, 259, 262, 263, 264, 267, 268, 269, 270, 271, 272, 274, 275, 280, 281, 282, 287, 289, 290, 291, 292, 306, 307, 309, 310, 312, 315, 326, 328, 332, 333, 334, 335, 336, 337, 342, 347, 348, 349, 352, 353, 354, 355, 356, 357, 358, 359, 363, 364, 365, 372, 373, 382, 387, 388, 390, 391, 392, 394, 395, 396, 397, 401, 404, 406, 409, 410, 411, 412, 413, 418, 420, 422, 423, 424, 425, 426, 427, 428, 1367, 1368, 1369, 1370, 1514, 1515, 1584, 1585

List of problems that are animated [60] (2.98%)

2, 3, 4, 5, 6, 7, 8, 9, 10, 151, 152, 154, 159, 161, 163, 176, 181, 191, 199, 202, 208, 209, 210, 219, 222, 227, 229, 235, 237, 259, 263, 264, 268, 269, 271, 272, 274, 275, 334, 335, 336, 349, 354, 356, 357, 358, 363, 372, 373, 382, 387, 388, 390, 391, 395, 396, 397, 406, 411, 412

1.3 Result lookup table

1.3.1 Miscellaneous PDE's

Table 1.6: Miscellaneous PDE's breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1	General first order	Transport equation $u_t + u_x = 0$	✓	0.046	✓	0.085	Yes	
2	General first order	Transport equation $u_t - 3u_x = 0$ IC $u(0,x) = e^{-x^2}$. Peter Olver textbook, 2.2.2 (a)	✓	0.007	✓	0.046	Yes	Yes
3	General first order	Transport equation $u_t + 2u_x = 0$ IC $u(-1,x) = \frac{x}{1+x^2}$. Peter Olver textbook, 2.2.2 (b)	✓	0.006	✓	0.02	Yes	Yes
4	General first order	Transport equation $u_t + u_x + \frac{1}{2}u = 0$ IC $u(0,x) = \arctan(x)$. Peter Olver textbook, 2.2.2 (c)	✓	0.005	✓	0.05	Yes	Yes
5	General first order	Transport equation $u_t - 4u_x + u = 0$ IC $u(0,x) = \frac{1}{1+x^2}$. Peter Olver textbook, 2.2.2 (d)	✓	0.005	✓	0.02	Yes	Yes
6	General first order	Transport equation $u_t + 2u_x = \sin x$ IC $u(0,x) = \sin x$. Peter Olver textbook, 2.2.5	✓	0.073	✓	0.046	Yes	Yes
7	General first order	Transport equation $u_t + \frac{1}{1+x^2}u_x = 0$ IC $u(x,0) = \frac{1}{1+(3+x)^2}$. Peter Olver textbook, page 27	✓	0.023	✓	0.077	Yes	Yes
8	General first order	Transport equation $u_t - xu_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.17	✓	0.006	✓	0.05	Yes	Yes
9	General first order	Transport equation $u_t + (1-2t)u_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.29	✓	0.032	✓	0.135	Yes	Yes

Continued on next page

Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
10	General first order	Transport equation $u_t+\frac{1}{x^2+4}u_x = 0$ IC $u(x,0)=e^{x^3+12x}$	✓	0.015	✓	0.072	Yes	Yes
11	General first order	$3u_x+5u_y=x$	✓	0.007	✓	0.011	Yes	
12	General first order	$xu_y+yu_x=-4xyu$ and $u(x,0)=e^{-x^2}$	✓	0.024	✓	0.058	Yes	
13	General first order	$u_t+u_x=0$ and $u(x,0)=\sin x$ and $u(0,t)=0$	✓	0.121	✓	0.303	Yes	
14	General first order	$u_t+cu_x=0$ and $u(x,0)=e^{-x^2}$	✓	0.005	✓	0.03	Yes	
15	General first order	(Haberman 12.2.2) $\omega_t-3\omega_x=0$ and $\omega(x,0)=\cos x$	✓	0.004	✓	0.022	Yes	
16	General first order	(Haberman 12.2.4) $\omega_t+c\omega_x=0$ and $\omega(x,0)=f(x)$ and $\omega(0,t)=h(t)$	✗	1.705	✓ Solution contains unresolved invlaplace calls	0.32	Yes	
17	General first order	(Haberman 12.2.5 (a)) $\omega_t+c\omega_x=e^{2x}$ and $\omega(x,0)=f(x)$	✓	0.048	✓	0.042	Yes	
18	General first order	(Haberman 12.2.5 (d)) $\omega_t+3t\omega_x=\omega(x,t)$ and $\omega(x,0)=f(x)$	✓	1.118	✓	0.062	Yes	
19	General first order	$2u_x+5u_y=u^2(x,y)+1$	✓	0.16	✓	0.039	Yes	
20	General first order	Clairaut equation $xu_x+yu_y+\frac{1}{2}((u_x)^2+(u_y)^2)=0$	✓	0.054	✓	0.126	Yes	
21	General first order	Clairaut equation. $xu_x+yu_y+\frac{1}{2}((u_x)^2+(u_y)^2)=0$ with $u(x,0)=\frac{1}{2}(1-x^2)$	✓	0.014	✓	0.724		

Continued on next page

Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
22	General first order	Clairaut equation. $u = xu_x + yu_y + \sin(u_x + u_y)$	✓	0.054	✓	0.134		
23	General first order	Recover a function from its gradient vector	✓	0.032	✓	0.085		
24	General first order	$xf_y - f_x = \frac{g(x)}{h(y)} f^2$	✓	0.055	✓	0.039	Yes	
25	General first order	$f_x + (f_y)^2 = f(x, y, z) + z$	✓	0.132	✓	0.562		
26	General first order	$xu_x + yu_y = u$ (Example 3.5.1 in Lokenath Debnath)	✓	0.017	✓	0.01	Yes	
27	General first order	$xu_x + yu_y = nu$ Example 3.5.2 in Lokenath Debnath	✓	0.018	✓	0.012	Yes	
28	General first order	$x^2u_x + y^2u_y = (x + y)u$ Example 3.5.3 in Lokenath Debnath	✓	0.12	✓	0.027	Yes	
29	General first order	$(y-z)u_x+(z-x)u_y+(x-y)u_z = 0$ (Example 3.5.4 in Lokenath Debnath)	✗ (Timed out)	600.	✓	2.163	Yes	
30	General first order	$u(x+y)u_x+u(x-y)u_y = x^2+y^2$ (Example 3.5.5 in Lokenath Debnath)	✓	0.489	✓	0.174	Yes	
31	General first order	$u_x - u_y = 1$ with $u(x, 0) = x^2$ Example 3.5.6 in Lokenath Debnath	✓	0.005	✓	0.018		
32	General first order	$yu_x + xu_y = u$ with $u(x, 0) = x^3$ and $u(0, y) = y^3$ Example 3.5.8 in Lokenath Debnath	✗	2.078	✗	0.679		
33	General first order	$xu_x + yu_y = xe^{-u}$ with $u = 0$ on $y = x^2$ Example 3.5.10 in Lokenath Debnath	✓	0.235	✓	0.06		
34	General first order	$u_t + uu_x = x$ with $u(x, 0) = f(x)$ Example 3.5.11 in Lokenath Debnath.	✗	3.678	✓	0.24		

Continued on next page

Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
35	General first order	$u_x = 0$ Problem 3.3(a) Lokenath Debnath	✓	0.004	✓	0.003		
36	General first order	$au_x + bu_y = 0$ Problem 3.3(b) Lokenath Debnath	✓	0.009	✓	0.009		
37	General first order	$u_x + yu_y = 0$ Problem 3.3(c) Lokenath Debnath	✓	0.029	✓	0.011		
38	General first order	$(1+x^2)u_x+u_y = 0$ Problem 3.3(d) Lokenath Debnath	✓	0.009	✓	0.009		
39	General first order	$2xyu_x + (x^2+y^2)u_y = 0$ Problem 3.3(e) Lokenath Debnath	✓	0.126	✓	0.024		
40	General first order	$(y+u)u_x + yu_y = x - y$ Problem 3.3(f) Lokenath Debnath	✗	105.34	✓	0.37		
41	General first order	$y^2u_x - xyu_y = x(u-2y)$ Problem 3.3(g) Lokenath Debnath	✓	0.137	✓	0.037		
42	General first order	$yu_y - xu_x = 1$ Problem 3.3(h) Lokenath Debnath	✓	0.02	✓	0.009		
43	General first order	$u_x + 2xy^2u_y = 0$ Problem 3.4 Lokenath Debnath	✓	0.126	✓	0.011		
44	General first order	$3u_x + 2u_y = 0$ with $u(x,0) = \sin x$. Problem 3.5(a) Lokenath Debnath	✓	0.005	✓	0.016		
45	General first order	$yu_x + xu_y = 0$ with $u(0,y) = e^{-y^2}$. Problem 3.5(b) Lokenath Debnath	✓	0.023	✓	0.028		
46	General first order	$xu_x + yu_y = 2xy$ with $u = 2$ on $y = x^2$. Problem 3.5(c) Lokenath Debnath	✓	0.017	✓	0.009		
47	General first order	$u_x + xu_y = 0$ with $u(0,y) = \sin y$. Problem 3.5(d) Lokenath Debnath	✓	0.006	✓	0.017		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
48	General first order	$yu_x + xu_y = xy$ with $u(0,y) = e^{-y^2}, u(x,0) = e^{-x^2}$. Problem 3.5(e) Lokenath Debnath	✗	3.803	✗	0.616		
49	General first order	$u_x + xu_y = (y - \frac{1}{2}x^2)^2$ with $u(0,y) = e^y$. Problem 3.5(f) Lokenath Debnath	✓	0.012	✓	0.079		
50	General first order	$xu_x + yu_y = u + 1$ with $u = x^2$ on $y = x^2$ Problem 3.5(g) Lokenath Debnath	✓	0.021	✓	0.012		
51	General first order	$uu_x - uu_y = u^2 + (x+y)^2$ with $u(x,0) = 1$ Problem 3.5(h) Lokenath Debnath	✓	0.065	✓	0.07		
52	General first order	$xu_x + (x+y)u_y = u + 1$ with $u(x,0) = x^2$ Problem 3.5(i) Lokenath Debnath	✓	0.027	✓	0.054		
53	General first order	$xu_x + yu_y + zu_z = 0$ Problem 3.8(a) .Lokenath Debnath	✓	0.032	✓	0.013		
54	General first order	$x^2u_x + y^2u_y + z(x+y)u_z = 0$ Problem 3.8(b) Lokenath Debnath	✓	0.12	✓	0.019		
55	General first order	$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$ Problem 3.8(c) Lokenath Debnath	✓	0.049	✓	0.934		
56	General first order	$yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$ Problem 3.8(d) Lokenath Debnath	✓	0.114	✓	0.044		
57	General first order	$x(y^2-z^2)u_x + y(z^2-y^2)u_y + z(x^2-y^2)u_z = 0$ Problem 3.8(e) Lokenath Debnath	✗	45.677	✗	0.256		
58	General first order	$u_x + xu_y = y$ with $u(0,y) = y^2$ Problem 3.9(a) Lokenath Debnath	✓	0.007	✓	0.021		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
59	General first order	$u_x + xu_y = y$ with $u(1,y) = 2y$ Problem 3.9(b) Lokenath Debnath	✓	0.007	✓	0.01		
60	General first order	$(u_x+u_y)^2-u^2=0$. Problem 3.10 Lokenath Debnath	✓	0.011	✓	0.021		
61	General first order	$(y+u)u_x + yu_y = x - y$ with $u(x,1) = 1+x$. Problem 3.11 Lokenath Debnath	✗	211.318	✗	0.913		
62	General first order	$2xu_x + (x+1)u_y = y$ with $u(1,y) = 2y$. Problem 3.14(d) Lokenath Debnath	✓	0.021	✓	0.154		
63	General first order	$xu_x+yu_y = x^2+y^2$ with $u(x,1) = x^2$. Problem 3.14(e) Lokenath Debnath	✓	0.034	✓	0.042		
64	General first order	$y^2u_x + (xy)u_y = x$ with $u(x,1) = x^2$. Problem 3.14(f) Lokenath Debnath	✓	0.035	✓	0.045		
65	General first order	$xu_x + yu_y = xy$ with $u = \frac{x^2}{2}$ at $y = x$. Problem 3.14(g) Lokenath Debnath	✓	0.019	✓	0.012		
66	General first order	$u_x + uu_y = 1$ with $u(0,y) = ay$. Problem 3.16(a) Lokenath Debnath	✓	0.084	✓	0.045		
67	General first order	$(y+u)u_x+(x+u)u_y = x+y$. Problem 3.17(a) Lokenath Debnath	✗ (Timed out)	600.	✓	1.518		
68	General first order	$xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$. Problem 3.17(b) Lokenath Debnath	✓	0.065	✓	0.044		
69	General first order	$(x+y)u_x+(x-y)u_y = 0$. Problem 3.17(c) Lokenath Debnath	✓	0.057	✓	0.047		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
70	General first order	$yu_x - xu_y = e^u$ with $u(0,y) = y^2 - 1$	✓	0.163	✓	0.098	Yes	
71	General first order	$yu_x - xu_y = e^u$	✓	0.074	✓	0.004	Yes	
72	General first order	$u_t + xu_x = 0$ with $u(x,0) = x^2$. Math 5587	✓	0.007	✓	0.023	Yes	
73	General first order	$u_t + tu_x = 0$ with $u(x,0) = e^x$	✓	0.02	✓	0.03	Yes	
74	General first order	$2u_x + 3u_y = 1$	✓	0.008	✓	0.009	Yes	
75	General first order	$xu_t - tu_x = 0$	✓	0.027	✓	0.011	Yes	
76	General first order	$u_t + u_x = 0$ with $u(x,1) = \frac{x}{1+x^2}$	✓	0.005	✓	0.01	Yes	
77	General first order	$u_x u_y = 1$	✓	0.003	✓	0.018	Yes	
78	General first order	$u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$	✗	1.493	✓	0.272	Yes	
79	Solved by factoring into two transport equations	$u_{xx} + u_{xt} - 6u_{tt} = 0$	✓	0.012	✓	0.102	Yes	
80	Solved by factoring into two transport equations	$u_{xx} - u_{xt} - 12u_{tt} = 0$	✓	0.013	✓	0.298	Yes	
81	Solved by factoring into two transport equations	$u_{xx} - 3u_{xt} - 4u_{tt} = 0$	✓	0.013	✓	2.132	Yes	

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

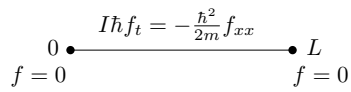
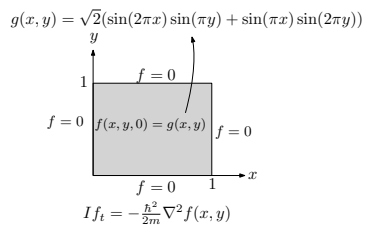
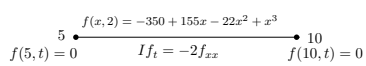
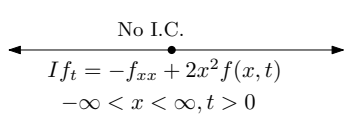
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
82	Solved by factoring into two transport equations	$u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$	✓	0.018	✓	1.895		
83	Schrodinger PDE	 Logan textbook, page 30	✓	0.527	✓	0.472		
84	Schrodinger PDE	 In a square, zero potential	✓	0.707	✓	4.106		
85	Schrodinger PDE	 From Mathematica help pages	✓	0.756	✓	0.929		
86	Schrodinger PDE	 From Mathematica help pages	✓	0.006	✗ Trivial solution. Maple does not support ∞ in boundary conditions	9.75		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

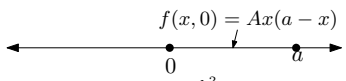
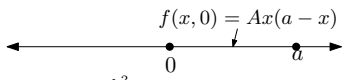
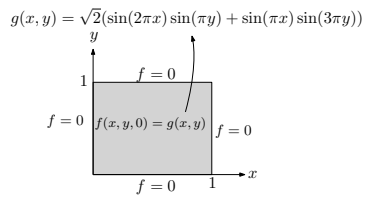
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
87	Schrodinger PDE	 $I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$ $-\infty < x < \infty, t > 0$ David Griffiths, page 47	✓	30.831	✓	2.01		
88	Schrodinger PDE	 $I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx} + V(x)f(x,t)$ $-\infty < x < \infty, t > 0$ David Griffiths, page 47	✓	0.678	✓	1.058		
89	Schrodinger PDE	 $I\hbar f_t = -\frac{\hbar^2}{2m}\nabla^2 f(x,y)$ In a square	✓	0.648	✓	6.622		
90	Beam PDE	Beam PDE $u_{tt} + u_{xxxx} = 0$	✓	1.711	✓	0.234		
91	Burger's PDE	Inviscid Burgers $u_x + uu_y = 0$	✓ Implicit solution	0.028	✓	0.026	Yes	
92	Burger's PDE	Inviscid Burgers with I.C. $u_x + uu_y = 0$ and $u(x,0) = \frac{1}{x+1}$	✓	0.011	✓	0.062	Yes	
93	Burger's PDE	$u_t + uu_x = \mu u_{xx}$	✓	0.039	✓	0.112		
94	Burger's PDE	$u_t + uu_x + \mu u_{xx}$ with IC	✓	10.918	✗	0.92		
95	Burger's PDE	$u_t + uu_x + \mu u_{xx}$ IC as UnitBox	✓	39.853	✗	0.658		
96	Black Scholes PDE	classic Black Scholes model from finance, European call version	✓	3.061	✓	1.014		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
97	Black Scholes PDE	Boundary value problem for the Black Scholes equation	✓	4.421	✓	2.492		
98	Korteweg-deVries PDE	$u_{xxx} + u_t - 6uu_x = 0$	✓	0.031	✓	0.193		
99	Tricomi PDE	$u_{xx} + yu_{yy} = 0$ with $u(x,0) = 0, u_y(x,0) = x^2$	✓	9.736	✓	3.933		
100	Tricomi PDE	$u_{xx} + xu_{yy} = 0$	✗	0.011	✓	3.122		
101	Keldysh equation	$xu_{xx} + u_{yy} = 0$	✗	0.007	✓	3.77		
102	Euler-Poisson-Darboux equation	$u_{xx} + u_{yy} + \frac{\beta}{x}u_x = 0$	✗	0.008	✓	0.117		
103	Euler-Poisson-Darboux equation	$u_{xx} - u_{yy} + \frac{\beta}{x}u_x = 0$	✗	0.008	✓	0.49		
104	Euler-Poisson-Darboux equation	$u_{tt} - u_{xx} - \frac{2}{x}u_x = 0$ with $u(x,0) = 0, u_t(x,0) = g(x)$	✗	2.613	✓	5.677		
105	Chaplygin's equation	$u_{\theta\theta} + \frac{v^2}{1-\frac{v^2}{c^2}}u_{vv} + vu_v = 0$	✗	0.027	✓	1.459		
106	Cauchy Riemann PDE's	Cauchy Riemann PDE with Prescribe the values of u and v on the x axis	✓	0.011	✓	0.177		
107	Cauchy Riemann PDE's	Cauchy Riemann PDE With extra term on right side	✗	0.003	✓	0.077		
108	Hamilton-Jacobi PDE	Hamilton-Jacobi type PDE	✗	0.01	✓	0.237		
109	Airy PDE	$u_t + u_{xxx} = 0$	✓	0.059	✓	0.104	Yes	
110	Nonlinear PDE's	Bateman-Burgers $u_t + uu_x = \nu u_{xx}$	✓	0.03	✓	0.081		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
111	Nonlinear PDE's	Benjamin Bona Mahony $u_t + u_x + uu + x - u_{xxt} = 0$	✓	0.036	✓	0.092		
112	Nonlinear PDE's	Benjamin Ono $u_t + Hu_{xx} + uu_x = 0$	✓	0.031	✓	0.078		
113	Nonlinear PDE's	Born Infeld $(1 - u_t^2)u_{xx} + 2u_x u_t u_{xt} - (1 + u_x^2)u_{tt} = 0$	✓	0.014	✓	0.163		
114	Nonlinear PDE's	Boussinesq $u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$	✓	0.058	✓	0.11		
115	Nonlinear PDE's	Boussinesq type $u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$	✓	0.048	✓	0.402		
116	Nonlinear PDE's	Buckmaster $u_t = (u^4)_{xx} + (u^3)_x$	✗	0.113	✓ Answer in terms of RootOf.	0.465		
117	Nonlinear PDE's	Camassa Holm $u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}$	✗	0.199	✓ Answer in terms of RootOf.	1.628		
118	Nonlinear PDE's	Chaffee Infante $u_t = u_{xx} + \lambda(u^3 - u) = 0$	✗	0.105	✓	0.226		
119	Nonlinear PDE's	Clarke. $\left(\theta_t - \gamma e^{\theta}\right)_{tt} = \left(\theta_t - e^{\theta}\right)_{xx}$	✗	0.013	✗	0.06		
120	Nonlinear PDE's	Degasperis Procesi $u_t - u_{xxt} + 4uu_x = 3u_x u_x x + uu_{xxx}$	✗	0.195	✓ But still has unresolved ODE's in solution	0.671		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
121	Nonlinear PDE's	Dym equation $u_t = u^3 u_{xxx}$	✗	0.096	✓ has RootOf	1.051		
122	Nonlinear PDE's	Estevez Mansfield Clarkson $u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$	✓	0.041	✓	0.15		
123	Nonlinear PDE's	Fisher's $u_t = u(1-u) + u_{xx}$	✓	0.063	✓	0.162		
124	Nonlinear PDE's	Hunter Saxton $(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$	✗	0.051	✓ with RootOf	0.155		
125	Nonlinear PDE's	Kadomtsev Petviashvili $\left(u_t + uu_x + \epsilon^2 u_{xxx}\right)_x + \lambda u_{yy} = 0$	✓	0.073	✓	0.132		
126	Nonlinear PDE's	Klein Gordon $u_{xx} + u_{yy} + \lambda u^p = 0$	✗	0.006	✗	0.025		
127	Nonlinear PDE's	Klein Gordon $u_{xx} + u_{yy} + u^2 = 0$	✗	0.237	✓	0.387		
128	Nonlinear PDE's	Khokhlov Zabolotskaya $u_{xt} - (uu_x)_x = u_{yy}$	✗	0.073	✓	0.275		
129	Nonlinear PDE's	Korteweg de Vries (KdV) $u_t + (u_x)^3 + 6uu_x = 0$	✓	0.038	✓	0.094		
130	Nonlinear PDE's	Lin Tsien $2u_{tx} + u_x u_{xx} - u_{yy} = 0$	✗	0.09	✓	0.265		
131	Nonlinear PDE's	Liouville $u_{xx} + u_{yy} + e^{\lambda u} = 0$	✗	0.006	✗	0.037		
132	Nonlinear PDE's	Plateau $(1+u_y^2)u_{xx} - 2u_x u_y y_{xy} + (1+u_x^2)u_{yy} = 0$	✗	0.039	✓	0.173		
133	Nonlinear PDE's	Rayleigh $u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$	✗	0.099	✓ Has RootOf	0.167		
134	Nonlinear PDE's	Sawada Kotera $u_t + 45u^2 u_x + 15u_x u_{xx} + 15uu_{xxx} + u_{xxxxx} = 0$	✓	0.093	✓	0.176		
135	Nonlinear PDE's	Sine Gordon $\phi_{tt} - \phi_{xx} + \sin\phi = 0$	✗	0.01	✗	0.026		
136	Nonlinear PDE's	Sinh Gordon $u_{xt} = \sinh u$	✗	0.01	✗	0.031		

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Table 1.6 – Miscellaneous PDE's. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
137	Nonlinear PDE's	Sinh Poisson $u_{xx}+u_{yy}+\sinh u=0$	✗	0.009	✗	0.016		
138	Nonlinear PDE's	Thomas equation $u_{xy}+\alpha u_x+\beta u_y+\nu u_x u_y=0$	✗	0.074	✓	0.45		
139	Nonlinear PDE's	phi equation $\phi_{tt}-\phi_{xx}-\phi+\phi^3=0$	✓	0.053	✓	0.165		
140	more miscellaneous	$SS_{xy}+S_xS_y=1$	✗	0.04	✓	0.052		
141	more miscellaneous	$u_{rr}+u_{\theta\theta}=0$	✓	30.952	✓	0.746		
142	more miscellaneous	$u_{xx}+yu_{yy}=0$	✓	8.842	✓	3.441		
143	more miscellaneous	$u_t+u_{xxx}=0$	✓	0.191	✓	15.76		
144	more miscellaneous	$u_{xy}=\sin(x)\sin(y)$	✓	5.14	✓	0.619		
145	more miscellaneous	$w_t=w_{x_1x_1}+w_{x_2x_2}+w_{x_3x_3}$	✓	2.907	✓	0.847		
146	more miscellaneous	Linear PDE, initial conditions at $t=t_0$	✓	3.923	✓	0.825		
147	more miscellaneous	second order in time, Linear PDE, initial conditions at $t=t_0$	✓	2.226	✓	2.962		
148	more miscellaneous	Einstein-Weiner $u_t=-\beta u_x+Du_{xx}$	✓	0.046	✓	0.362		
149	more miscellaneous	Using integral transforms.	✓	40.116	✓	2.906		

1.3.2 Parabolic PDE's (Diffusion)

Table 1.7: Parabolic PDE's (Diffusion) breakdown of results. Time in seconds

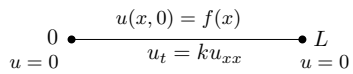
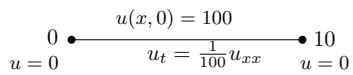
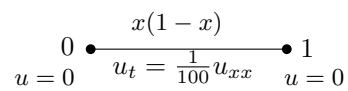
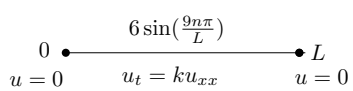
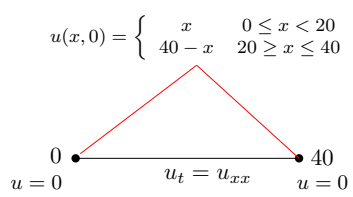
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
150	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 General initial conditions	✓	62.537	✓	2.083	Yes	
151	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Specific initial condition	✓	26.091	✓	1.237	Yes	Yes
152	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 IC $u=x(1-x)$	✓	27.383	✓	0.931	Yes	Yes
153	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.3 (a)	✓	0.42	✓	3.671	Yes	
154	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 IC hat function	✓	35.813	✓	2.606	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
155	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$ 0 $u=0$ — $u_t = ku_{xx}$ — L $u=0$ Haberman 2.3.3 (b)	✓	1.236	✓	4.116	Yes	
156	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$2\cos\frac{3\pi x}{L}$ 0 $u=0$ — $u_t = ku_{xx}$ — L $u=0$ Haberman 2.3.3 (c)	✓ but $n=3$ should be special case	65.187	✓ handled $n=3$ case correctly.	15.515	Yes	
157	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0) = \begin{cases} 1 & 0 < x \le \frac{L}{2} \\ 2 & \frac{L}{2} < x \le L \end{cases}$ 0 $u=0$ — $u_t = ku_{xx}$ — L $u=0$ Haberman 2.3.3 (d)	✓	545.655	✓	40.644	Yes	
158	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$f(x)$ -1 $u=0$ — $u_t = ku_{xx}$ — 1 $u=0$ domain from -1 to +1	✓	18.891	✓	1.61	Yes	
159	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$1-x^2$ -1 $u=0$ — $u_t = ku_{xx}$ — 1 $u=0$ domain from -1 to +1	✓	62.92	✓	1.042	Yes	Yes
160	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$u(x,0) = f(x)$ 0 $u=0$ — $u_t = ku_{xx} + Q(x)$ — L $u=0$ with source that depends on space only (general case)	✓	42.268	✓	2.872	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
161	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = \frac{1}{10}u_{xx} + x]{u(x,0) = x(5-x)} 5$, $u = 0$, $u = 0$ with source that depends on space only (special case)	✓	32.815	✓	2.741	Yes	Yes
162	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + Q(x,t)]{u(x,0) = f(x)} L$, $u = 0$, $u = 0$ with source that depends on space and time (general case)	✓	71.123	✓	4.79	Yes	
163	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + e^{-t}\sin(3x)]{u(x,0) = x(\pi - x)} \pi$, $u = 0$, $u = 0$ with source that depends on space and time (special case)	✓	35.558	✓	14.278	Yes	Yes
164	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = u_{xx} + t(\pi - x)]{0} \pi$, $u = 0$, $u = 0$ Math 4567 Exam	✓	18.115	✓	3.305	Yes	
165	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + Q(x,t)]{f(x)} 1$, $u = 0$, $u = 0$ With source	✓	67.959	✓	8.844		
166	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = u_{xx} - 9u_x]{e^{\frac{45}{10}}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))} 1$, $u = 0$, $u = 0$ special initial condition	✓	3.189	✗ (Timed out)	600.	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
167	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$1 \xrightarrow[u_t = x^2 u_{xx} + x u_x]{f(x)} b$, $u=0$, $u=0$ Diffusion Reaction and Euler-Cauchy Sturm-Liouville	✓	1.816	✓	7.57		
168	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = \frac{1}{10} u_{xx} + r u]{u(x,0)=1} 1$, $u=0$, $u=0$ Diffusion Reaction. Using growth form reaction term	✓	93.031	✓	1.13	Yes	
169	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = \frac{1}{100} u_{xx} + \frac{1}{10} u(1-\frac{u}{10})]{u(x,0)=1} 1$, $u=0$, $u=0$ Diffusion Reaction, using logistic form for reaction term	✗	4.709	✗	6.938		
170	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = \frac{1}{1000} u_{xx} + \frac{1}{100} u + \frac{1}{100} u^2 - \frac{5}{1000} u^3]{u(x,0)=1} 1$, $u=0$, $u=0$ Diffusion Reaction, using Aleee form for reaction term	✗	7.192	✗	11.236		
171	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = k u_{xx} - \alpha u,\ \alpha > 0]{f(x)} L$, $u=0$, $u=0$ Diffusion Reaction. Haberman 2.3.8	✓	34.92	✓	3.22	Yes	
172	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = u_{xx} - u]{f(x)} L$, $u=0$, $u=0$ Diffusion Reaction	✓	34.502	✓	1.977	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
173	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = 100u_{xx} - u]{\sin(2\pi x) - \sin(4\pi x)} 1$, $u=0$, $u=0$ Diffusion Reaction	✓	0.774	✓	2.005		
174	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx} - ux]{u(x,0) = \sin(x)} \pi$, $u=0$, $u=0$ Diffusion Reaction	✗	43.242	✓	1.68		
175	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + au_x,\ a>0]{u(x,0) = f(x)} L$, $u=0$, $u=0$ Diffusion convection (general case)	✓	3.081	✓	2.905	Yes	
176	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = u_{xx} + 5u_x]{u(x,0) = \sin(x)} \pi$, $u=0$, $u=0$ Diffusion convection (special case)	✓	63.055	✓	2.459	Yes	Yes
177	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx}]{f(x)} L$, $u_x=0$, $u=0$ Haberman 2.4.2 (general case)	✓	30.562	✓	4.834	Yes	
178	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow[u_t = ku_{xx}]{f(x)} L$, $u=0$, $u_x=0$ Left end zero, right end insulated, no source	✓	29.39	✓	2.623	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

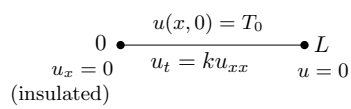
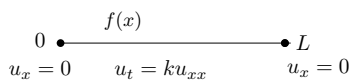
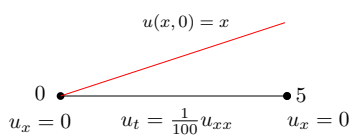
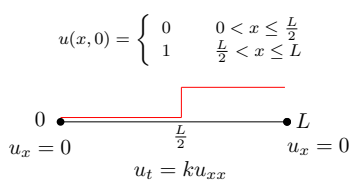
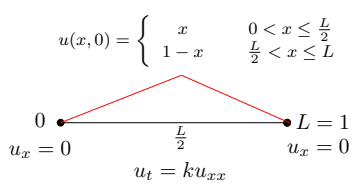
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
179	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 One end insulated	✓	32.45	✓	2.755		
180	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.3.7 (general case)	✓	32.108	✓	2.419	Yes	
181	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 specific case	✓	12.122	✓	1.513	Yes	Yes
182	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 2.4.1 (a)	✗ (Timed out)	600.	✓	39.665		
183	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Both ends insulated, no source	✓	33.1	✓	3.24	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
184	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{6+4\cos(\frac{3\pi x}{L})} L$, $u_x=0$, $u_t=ku_{xx}$, $u_x=0$ Haberman 2.4.1 (b) (special case)	✓	1.606	✓	5.717	Yes	
185	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{-2\sin(\frac{\pi x}{L})} L$, $u_x=0$, $u_t=ku_{xx}$, $u_x=0$ Haberman 2.4.1 (c) (special case)	✓	32.364	✓	19.146	Yes	
186	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{-3\cos(\frac{8\pi x}{L})} L$, $u_x=0$, $u_t=ku_{xx}$, $u_x=0$ Haberman 2.4.1 (d)	✓	1.387	✓	5.056		
187	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{\frac{x^2}{2}+x} 1$, $u_x=0$, $u_t=13u_{xx}$, $u_x=0$ both ends insulated	✓	91.769	✓	12.132		
188	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{1-\frac{x^3}{4}} 1$, $u_x=0$, $u_t=ku_{xx}$, $u_x+u=0$ convection heat loss	✓	32.721	✓	1.529		
189	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{x} L$, $u_x=0$, $u_t=ku_{xx}+\cos(\omega t)$, $u_x=0$ Pinchover and Rubinstein 6.25	✓	42.65	✓	13.906		
190	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	$0 \xrightarrow{f(x)} L$, $u_x=0$, $u_t=ku_{xx}+e^{ct}\sin(\frac{2\pi x}{L})$, $u_x=0$ external source	✓	18.726	✓	38.169		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

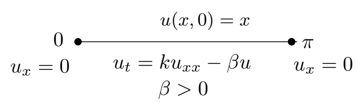
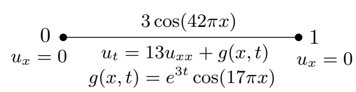
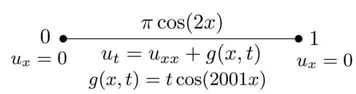
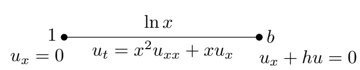
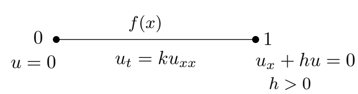
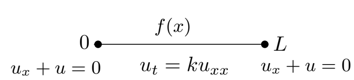
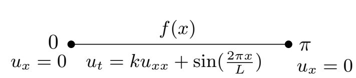
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
191	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Diffusion Reaction (general case)	✓	91.872	✓	1.319	Yes	Yes
192	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Pinchover and Rubinstein 6.23	✓	64.245	✓	24.371		
193	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Pinchover and Rubinstein 6.21	✓	63.132	✓	5.933		
194	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Diffusion Reaction. Euler-Cauchy Sturm-Liouville	✓	2.684	✓	38.371	Yes	
195	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 convection heat loss	✓	32.892	✓	2.715	Yes	
196	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Mixed BC	✓	33.75	✓	5.871		
197	Diffusion in 1D Finite domain (bar), Both ends homogeneous BC	 Haberman 8.2.1 (f)	✓	63.62	✓	7.951		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

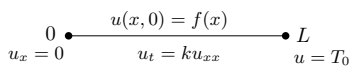
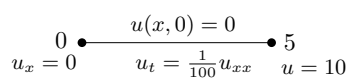
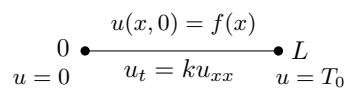
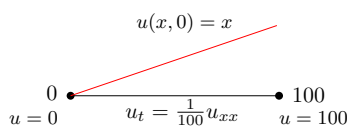
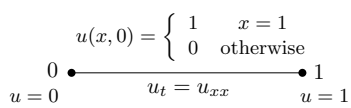
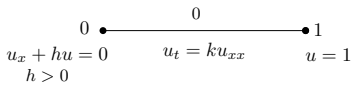
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
198	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 left end insulated (general case)	✓	33.393	✓	3.734	Yes	
199	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 left end insulated (special case)	✓	7.485	✓	2.226	Yes	Yes
200	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC (general case)	✓	66.082	✓	3.428	Yes	
201	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC (special case)	✓	0.01	✓	3.907	Yes	
202	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 right end nonhomogeneous BC, special case	✓	10.202	✓	3.405	Yes	Yes
203	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	 convection heat loss	✓	39.799	✗ (Timed out)	600.	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
204	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	$0 \xrightarrow[u_t = 13u_{xx}]{\frac{1}{2}x^2 + x} 1$, $u_x = 0$, $u_x = 1$ nonhomogeneous BC	✗	2.176	✓	10.863		
205	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	$0 \xrightarrow[u_t = u_{xx}]{0} \pi$, $u = 0$, $u_x = A$ nonhomogeneous BC	✓	92.738	✓	2.184		
206	Diffusion in 1D Finite domain (bar), left end homogeneous, right end not	$0 \xrightarrow[u_t = ku_{rr}]{rf(r)} a$, $u = 0$, $u = a\phi(t)$ nonhomogeneous BC	✓	34.188	✓	20.303	Yes	
207	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	$0 \xrightarrow[u_t = ku_{xx}]{u(x,0)=0} L$, $u_x = A(t)$, $u = 0$ left end BC depends on time (general case)	✓	66.569	✓	15.038	Yes	
208	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	$0 \xrightarrow[u_t = \frac{1}{100}u_{xx}]{u(x,0)=0} 5$, $u_x = e^t$, $u = 0$ left end BC depends on time (special case)	✓	88.627	✓	15.167	Yes	Yes
209	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	$0 \xrightarrow[u_t = \frac{1}{100}u_{xx}]{u(x,0)=0} 5$, $u_x = \sin(t)$, $u = 0$ left end BC depends on time (special case)	✓	86.742	✓	9.942	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
210	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	$u(x,0)=0$ 0 π $u=1$ $u_t = ku_{xx} + e^{-2t}\sin(5x)$ $u=0$ Haberman 8.3.6 (special case)	✓	95.024	✓	25.764	Yes	Yes
211	Diffusion in 1D Finite domain (bar), right end homogeneous, left end not	0 0 π $u=t$ $u_t = u_{xx}$ $u=0$ BC depends on time (special case)	✓	12.162	✓	5.486		
212	Diffusion in 1D Finite domain (bar), Periodic BC	$f(x)$ $-L$ L $u(-L,t)=u(L,t)$ $u_t = ku_{xx}$ $u_x(-L,t)=u_x(L,t)$ periodic B.C. Periodic BC (general case). IC given	✓	15.368	✓	9.892		
213	Diffusion in 1D Finite domain (bar), Periodic BC	$-\pi$ π $u(-\pi,t)=u(\pi,t)$ $u_t = ku_{xx}$ $u_x(-\pi,t)=u_x(\pi,t)$ periodic B.C. Periodic BC (general case). No IC given	✓	0.014	✓	1.958	Yes	
214	Diffusion in 1D Finite domain (bar), Periodic BC	$-\pi$ π $u(-\pi,t)=u(\pi,t)$ $u_t = ku_{xx} - u$ $u_x(-\pi,t)=u_x(\pi,t)$ periodic B.C. Periodic BC (general case). Damped heat PDE. No IC given	✗	0.071	✓	2.001	Yes	
215	Diffusion in 1D Finite domain (bar), Both ends nonhomoge-neous BC	$u(x,0)=f(x)$ 0 L $u=A$ $u_t = ku_{xx}$ $u=B$ both ends non-homogeneous BC (general case)	✓	63.746	✓	6.998	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
216	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=0$; 0 to 1; $u=20$, $u_t=u_{xx}$, $u=50$ non-homogeneous BC (special case)	✓	16.942	✓	3.002	Yes	
217	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=60x-50x^2+10$; 0 to 1; $u=10$, $u_t=\frac{1}{20}u_{xx}$, $u=20$ Articolo 8.4.1 (special case)	✓	63.188	✓	2.931	Yes	
218	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A$, $u_t=ku_{xx}+Q(x)$, $u=B$ With source that depends on space only (general case)	✓	52.613	✓	5.634	Yes	
219	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=60-2x$; 0 to 30; $u=20$, $u_t=ku_{xx}+\frac{x}{10}$, $u=50$ With source that depends on space only (special case)	✓	94.816	✓	3.601	Yes	Yes
220	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A$, $u_t=ku_{xx}+Q(x,t)$, $u=B$ With source that depends on space and time only (general case)	✓	69.048	✓	13.045	Yes	
221	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$u(x,0)=f(x)$; 0 to L; $u=A(t)$, $u_t=ku_{xx}$, $u=B(t)$ Both ends depend on time (general case)	✓	84.436	✓	29.074	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

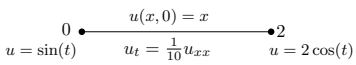
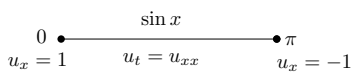
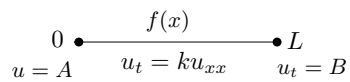
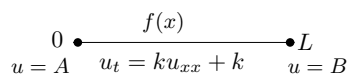
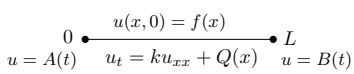
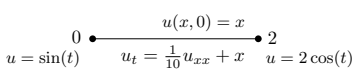
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
222	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time (special case)	✓	67.824	✓	34.132	Yes	Yes
223	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 both ends nonhomogeneous	✓	63.852	✓	33.68	Yes	
224	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Haberman 8.2.1 (a) (general case)	✓	38.949	✓	8.092	Yes	
225	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Haberman 8.2.1 (d) (general solution)	✓	54.039	✓	6.713	Yes	
226	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source that depends on space only (general solution)	✓	115.005	✗ (Timed out)	600.	Yes	
227	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	 Both ends depend on time with source that depends on space only (special case)	✓	104.862	✓	37.183	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
228	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + Q(x,t)]{u(x,0) = f(x)} L$, $u = A(t)$, $u = B(t)$ Both ends depend on time with source that depends on time and space (general solution)	✓	79.308	✓	18.269	Yes	
229	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = \frac{1}{10}u_{xx} + xte^{-t}\cos(t)]{u(x,0) = x} 2$, $u = \sin(t)$, $u = 2\cos(t)$ Both ends depend on time with source present (special case)	✓	117.548	✓	74.248	Yes	Yes
230	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = u_{xx} + 1 + x\cos(t)]{1 + \cos(2x)} 1$, $u_x = \sin(t)$, $u_x = \sin(t)$ Pinchover and Rubinstein 6.17	✗	21.971	✓	12.96		
231	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = ku_{xx} + x]{60 - 20x} \pi$, $u_x = \frac{t\sin t}{5}$, $u_x = \frac{t\cos t}{10}$ nonhomogeneous BC	✓	111.737	✓	31.023		
232	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = u_{xx} + Q(x,t)]{f(x)} L$, $u_x = A(t)$, $u_x = B(t)$ Haberman 8.2.2. (a)	✓	122.354	✓	25.014	Yes	
233	Diffusion in 1D Finite domain (bar), Both ends nonhomogeneous BC	$0 \xrightarrow[u_t = \frac{1}{20}u_{xx} + t]{\frac{-40x^2}{3} + \frac{45x}{2} + 5} 1$, $u = 5$, $u_x + u = 10$ Articolo 8.4.3	✓	5.486	✓	18.149		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
234	Diffusion in 1D Semi-infinite domain	$0 \xrightarrow[u_t = ku_{xx}]{u(x,0)=0} \infty$, $u = A$ left end constant (general case)	✓	60.035	✓	2.283	Yes	
235	Diffusion in 1D Semi-infinite domain	$0 \xrightarrow[u_t = \frac{1}{10}u_{xx}]{u(x,0)=0} \infty$, $u = 60$ left end constant (special case)	✓ It fail if assumption $x > 0$ is given. A bug	31.691	✓	3.029	Yes	Yes
236	Diffusion in 1D Semi-infinite domain	$0 \xrightarrow[u_t = u_{xx}]{0} \infty$, $u = f(t)$ Logan p. 76. Left end general function of time (general case)	✓	0.577	✓	7.363	Yes	
237	Diffusion in 1D Semi-infinite domain	$0 \xrightarrow[u_t = \frac{1}{10}u_{xx}]{u(x,0)=0} \infty$, $u = \sin(t)$ Left end function of time (special case)	✓	60.047	✓	12.418	Yes	Yes
238	Diffusion in 1D Semi-infinite domain	$0 \xrightarrow[u_t = ku_{xx}]{0} \infty$, $u = 1$ nonhomogeneous BC	✓	60.029	✓	2.499		
239	Diffusion in 1D Semi-infinite domain	$-x_0 \xrightarrow[u_t = \frac{1}{4}u_{xx}]{u(x,t_0)=10} \infty$, $u = 0$, $x > \lvert x_0\rvert, t > \lvert t_0\rvert$ I.C. not zero	✓ due to IC/BC not zero	13.357	✓	3.183		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

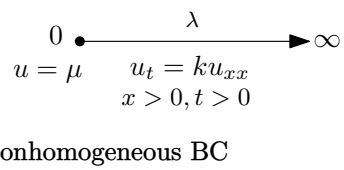
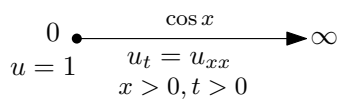
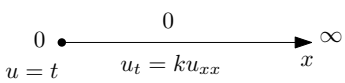
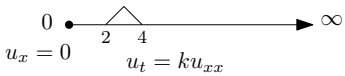
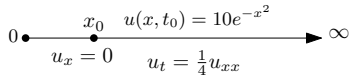
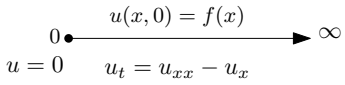
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
240	Diffusion in 1D Semi-infinite domain	 nonhomogeneous BC	✓	60.074	✓	2.483		
241	Diffusion in 1D Semi-infinite domain	 nonhomogeneous BC	✓	60.071	✓	6.834		
242	Diffusion in 1D Semi-infinite domain	 nonhomogeneous B.C.	✓	60.033	✓	3.126		
243	Diffusion in 1D Semi-infinite domain	 Unit triangle I.C.	✓	61.013	✓	4.208		
244	Diffusion in 1D Semi-infinite domain	 I.C. not at $t=0$	✓	61.517	✓	6.016		
245	Diffusion in 1D Semi-infinite domain	 Diffusion with advection	✓	1.75	✓	5.401		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

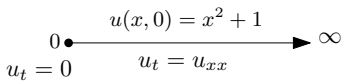
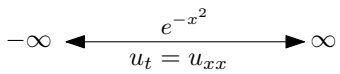
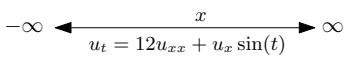
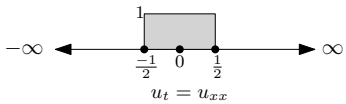
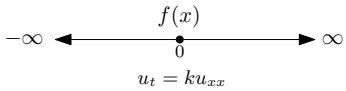
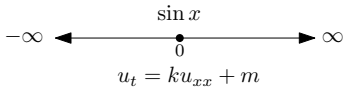
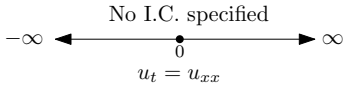
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
246	Diffusion in 1D Semi-infinite domain	 Practice exam problem	✓	60.108	✓	3.071		
247	Diffusion in 1D Infinite domain	 Inverse exponential I.C.	✓	0.869	✓	3.994	Yes	
248	Diffusion in 1D Infinite domain	 Advection term	✓	0.067	✓	2.934		
249	Diffusion in 1D Infinite domain	 UnitBox I.C.	✓	60.016	✓	6.347		
250	Diffusion in 1D Infinite domain	 No source	✓	0.4	✓	6.829		
251	Diffusion in 1D Infinite domain	 constant as source	✓	0.425	✓	3.457		
252	Diffusion in 1D Infinite domain	 No intial conditions	✓	0.004	✓	0.754	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

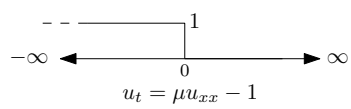
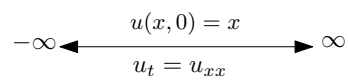
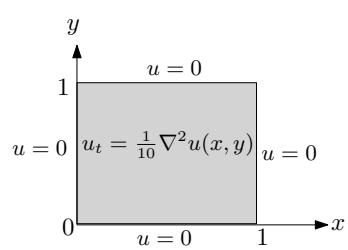
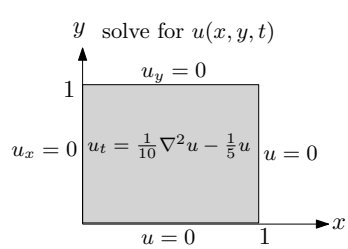
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
253	Diffusion in 1D Infinite domain	 piecewise I.C.	✓ due to i.c. not at zero	52.493	✓	5.05	Yes	
254	Diffusion in 1D Infinite domain	 Practice exam problem	✓	0.065	✓	1.716		
255	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 At $t = 0, u = x(1-x)(1-y)y$ No source	✓	4.249	✓	15.159		
256	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 At $t = 0, u = (1-x^2)(1-\frac{1}{2}y)y$ Internal source term	✓	10.246	✓	13.63		

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

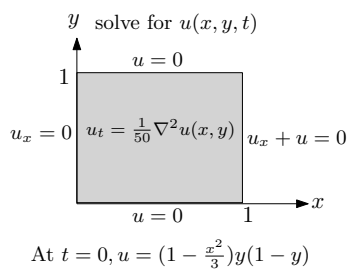
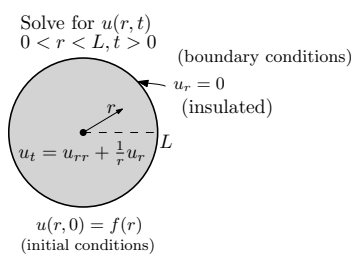
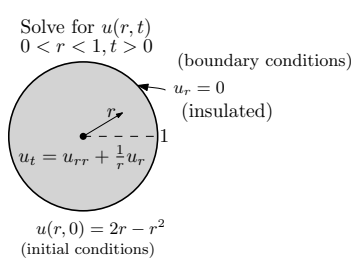
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
257	Diffusion in 2D Cartesian coordinates (Rectangle, Square)	 Articolo 6.6.3	✓	6.641	✓	22.307		
258	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency, insulated (General solution)	✓	0.97	✓ Cant get series solution	60.266	Yes	
259	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency, insulated (Specific solution)	✓	2.743	✓ Do not understand Maple solution	21.679	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

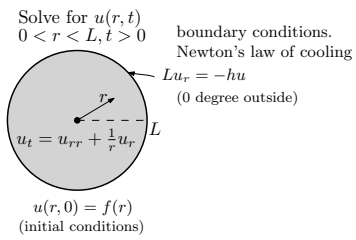
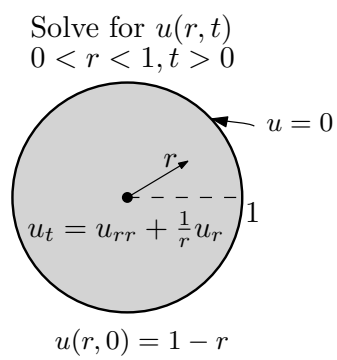
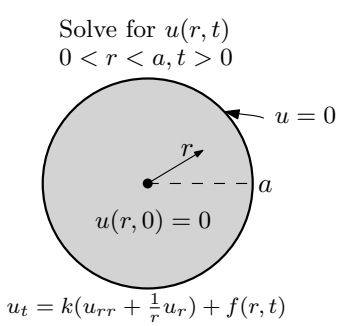
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
260	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency	✓	0.343	✓	6.467		
261	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 no θ dependency	✓	1.131	✓	1.321		
262	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 Haberman 8.3.5 (General solution)	✓	19.48	✓	5.786	Yes	

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

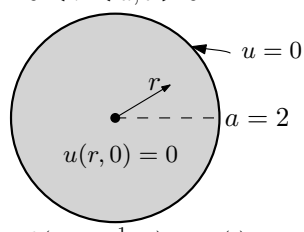
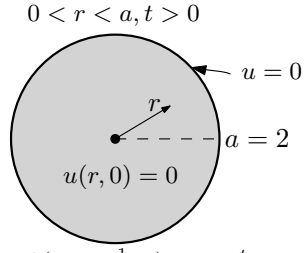
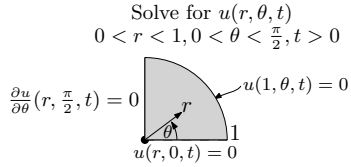
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
263	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,t)$ $0 < r < a, t > 0$  $u_t = k(u_{rr} + \frac{1}{r}u_r) + \sin(t)$ Where $k = \frac{1}{100}$ Specific example of the above	✓	3.518	✓	4.102	Yes	Yes
264	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,t)$ $0 < r < a, t > 0$  $u_t = k(u_{rr} + \frac{1}{r}u_r) + rte^{-t}$ Where $k = \frac{1}{100}$ Specific example of the above	✓	2.722	✗	6.169	Yes	Yes
265	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0 < r < 1, 0 < \theta < \frac{\pi}{2}, t > 0$  I.C. $u(r,\theta,0) = (r - r^3)\sin\theta$ $u_t = \frac{1}{50}(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$ Articolo 6.9.1	✓	4.127	✗	3.6		

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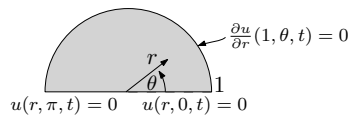
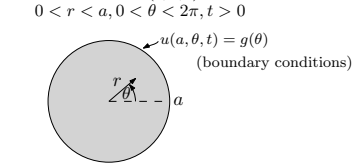
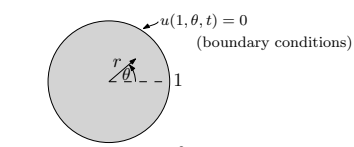
Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
266	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, 0<\theta<\pi, t>0$  $u(r,\pi,t)=0 \quad u(r,0,t)=0$ I.C. $u(r,\theta,0)=(r-\frac{r^3}{3})\sin\theta$ $u_t=\frac{1}{25}(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Articolo 6.9.2	✓	4.582	✓	41.933		
267	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<a, 0<\theta<2\pi, t>0$  I.C. $u(r,\theta,0)=f(r,\theta)$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Haberman 8.2.5 with θ dependency (General case)	✓	372.358	✗	1.783	Yes	
268	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, 0<\theta<2\pi, t>0$  I.C. $u(r,\theta,0)=1-r^2$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ With θ dependency (Specific example)	✓	8.97	✗	5.392	Yes	Yes

Continued on next page

Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

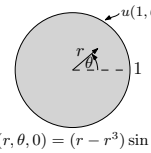
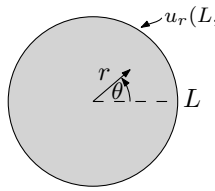
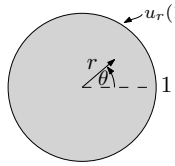
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
269	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, 0<\theta<2\pi, t>0$ $u(1,\theta,t)=0$ (boundary conditions)  I.C. $u(r,\theta,0)=(r-r^3)\sin\theta$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ With θ dependency (specific example)	✓	10.372	✗	8.127	Yes	Yes
270	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<L, -\pi<\theta<\pi, t>0$ $u_r(L,\theta,t)=0$ (Insulated)  I.C. $u(r,\theta,0)=f(r,\theta)$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Insulated with θ dependency (General solution)	✓	4.818	✗	8.399	Yes	
271	Diffusion in 2D Polar coordinates (disk, sector, annulus)	Solve for $u(r,\theta,t)$ $0<r<1, -\pi<\theta<\pi, t>0$ $u_r(1,\theta,t)=0$ (Insulated)  I.C. $u(r,\theta,0)=(2r-r^2)\cos\theta\sin\theta$ $u_t=u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}$ Insulated with θ dependency (Specific example)	✓	16.146	✗	7.529	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

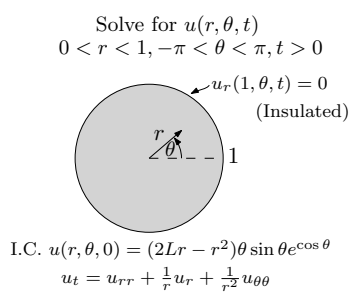
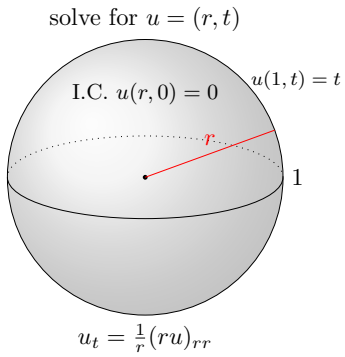
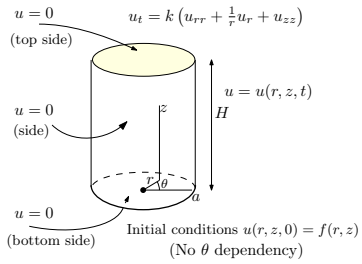
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
272	Diffusion in 2D Polar coordinates (disk, sector, annulus)	 Insulated with θ dependency (Specific example)	✗ (Timed out)	600.	✗	7.495	Yes	Yes
273	Diffusion in 3D Spherical coordinates	 No angle dependencies	✓	61.104	✓ Has unresolved Laplace integrals	3.881		
274	Diffusion in 3D Cylinderical coordinates	 Haberman 7.9.4 (a)	✓	1.883	✓	73.092	Yes	Yes

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Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

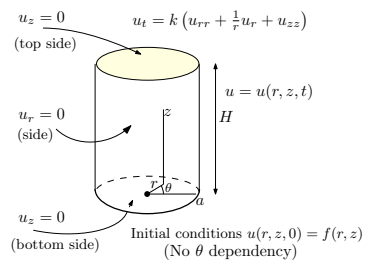
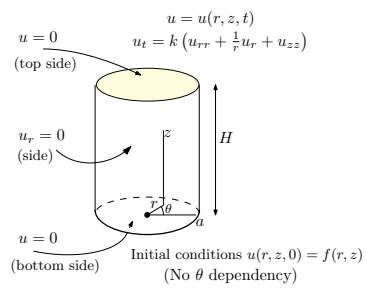
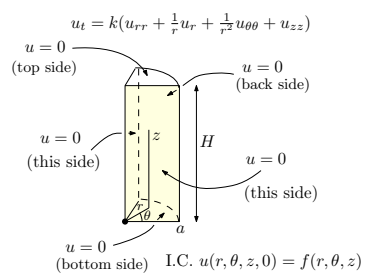
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
275	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.4 (b)	✓	6.469	✗	44.451	Yes	Yes
276	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.4 (c)	✓	3.162	✗	43.613		
277	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.3 (a)	✗	0.012	✗	4.304		

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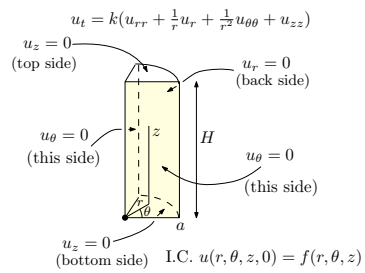
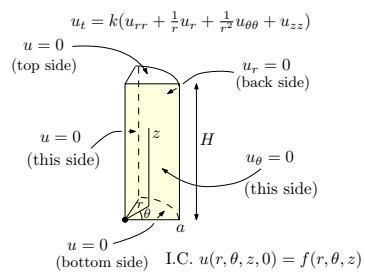
Table 1.7 – Parabolic PDE's (Diffusion). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
278	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.3 (b)	✗	0.008	✗	4.246		
279	Diffusion in 3D Cylinderical co-ordinates	 Haberman 7.9.3 (c)	✗	0.012	✗	4.19		

1.3.3 Elliptic PDE's (Laplace, Poisson, Helmholtz)

Table 1.8: Elliptic PDE's (Laplace, Poisson, Helmholtz) breakdown of results. Time in seconds

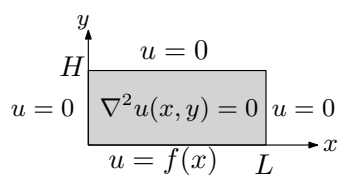
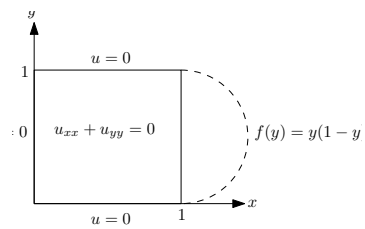
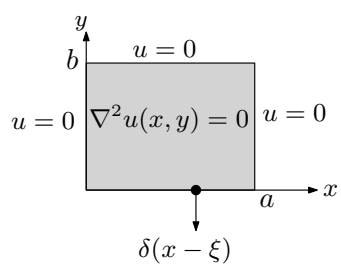
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
280	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, buttom edge not	✓	1.78	✓	78.191	Yes	
281	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, right edge not	✓	1.83	✓	8.617	Yes	
282	Laplace in 2D Cartesian coordinates	 Rectangle, 3 edges zero, buttom edge has impulse	✓	4.12	✓	27.954	Yes	

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

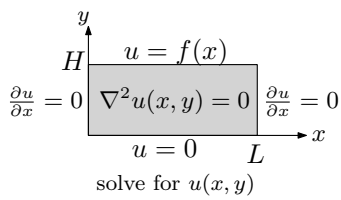
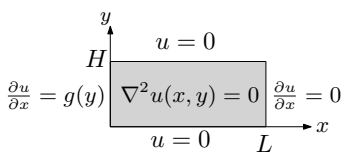
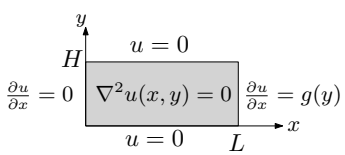
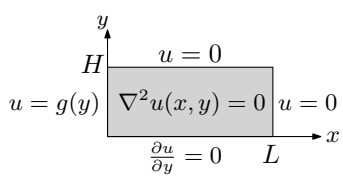
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
283	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (a)	✓	0.988	✓	64.358		
284	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (b)	✓	0.944	✓	65.826		
285	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (c)	✓	4.685	✓	54.033		
286	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (d)	✓	4.133	✓	82.87		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

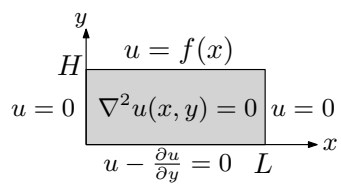
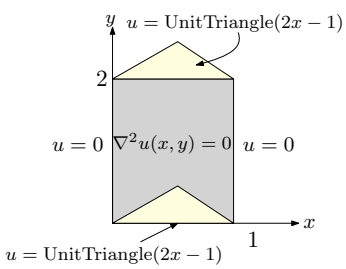
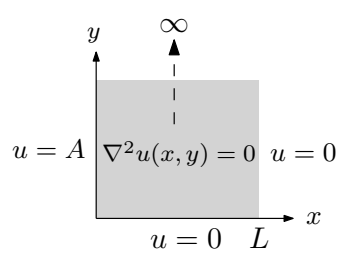
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
287	Laplace in 2D Cartesian coordinates	 Haberman 2.5.1 (e)	✓	19.674	✓	76.588	Yes	
288	Laplace in 2D Cartesian coordinates	 Unit triangle B.C.	✓	0.985	✓	21.389		
289	Laplace in 2D Cartesian coordinates	 Top edge at infinity	✓	3.586	✓	10.857	Yes	

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

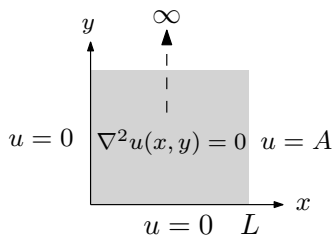
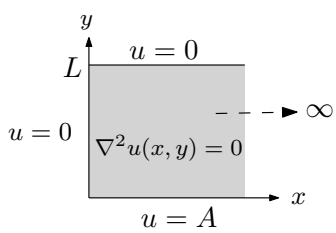
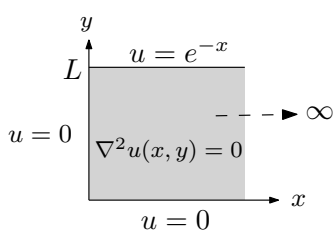
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
290	Laplace in 2D Cartesian coordinates	 Top edge at infinity	✓	2.813	✓	7.947	Yes	
291	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	3.514	✓	7.078	Yes	
292	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	41.351	✓	66.157	Yes	

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

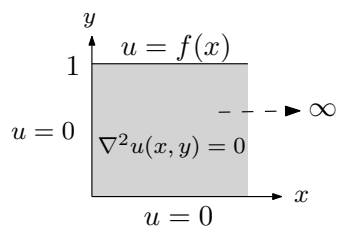
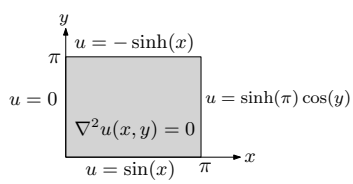
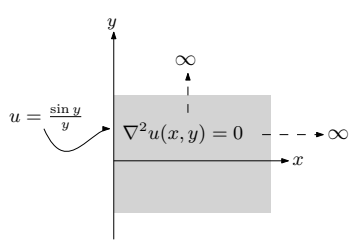
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
293	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✓	2.994	✓	85.598		
294	Laplace in 2D Cartesian coordinates	Laplace PDE in 2D Cartessian with boundary condition as Dirac function	✓	0.044	✓	3.888		
295	Laplace in 2D Cartesian coordinates	 One side homogeneous	✓	1.692	✓	114.204		
296	Laplace in 2D Cartesian coordinates	 In right half plane	✓	21.834	✓	4.683		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

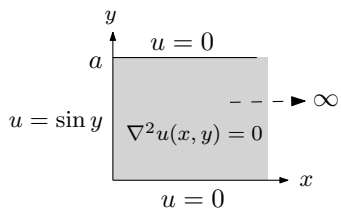
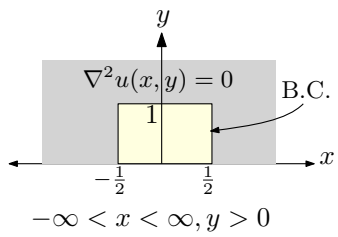
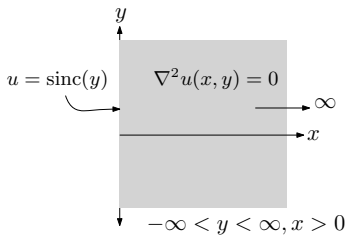
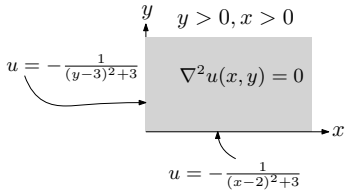
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
297	Laplace in 2D Cartesian coordinates	 Right edge at infinity	✗	2.434	✓	49.432		
298	Laplace in 2D Cartesian coordinates	 Dirichlet problem Upper half	✓	1.371	✓	5.602		
299	Laplace in 2D Cartesian coordinates	 Right half-plane	✓	19.623	✓	8.136		
300	Laplace in 2D Cartesian coordinates	 First quadrant	✓	14.632	✗	35.432		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

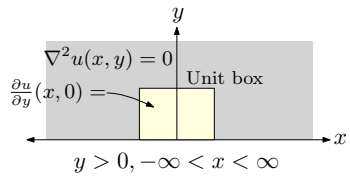
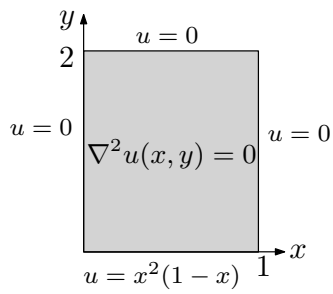
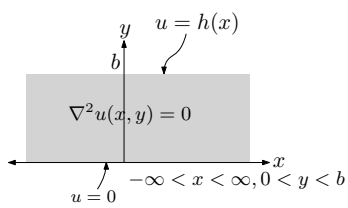
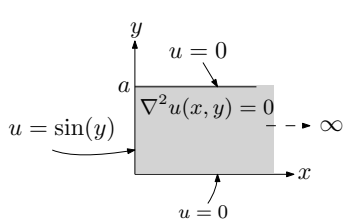
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
301	Laplace in 2D Cartesian coordinates	 Neumann problem upper half-plane	✓	4.77	✓ used convert(sol,Int).	9.987		
302	Laplace in 2D Cartesian coordinates	 Dirichlet problem in a rectangle	✓	2.303	✓	12.565		
303	Laplace in 2D Cartesian coordinates	 Strip in upper half	✓	60.258	✓	13.794		
304	Laplace in 2D Cartesian coordinates	 in Rectangle, right edge at infinity	✗	2.353	✓	48.434		

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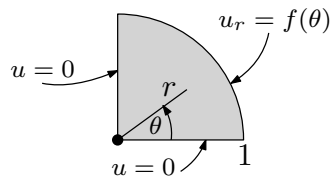
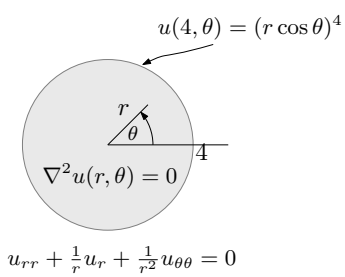
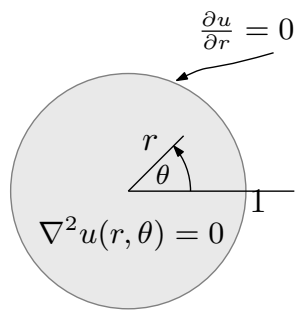
Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
305	Laplace in 2D Polar coordinates	$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$  Laplace PDE inside quarter disk, Neumann BC at edge	✓	3.459	✓	6.01		
306	Laplace in 2D Polar coordinates	 $r = 4$ and $u = x^4$ at boundary of disk	✓	150.086	✓	20.028	Yes	
307	Laplace in 2D Polar coordinates	 $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$ $r = 1$ and $u_r = x$ at boundary of disk	✗	2.589	✗	2.19	Yes	

Continued on next page

Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

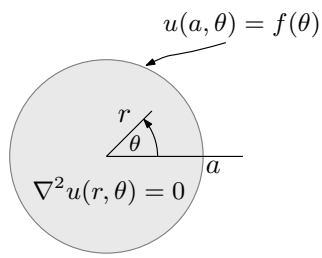
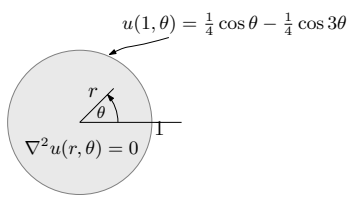
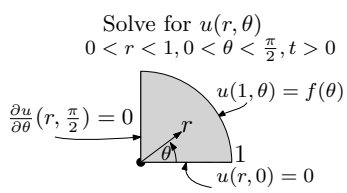
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
308	Laplace in 2D Polar coordinates	 $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$ Laplace inside disk. General solution	✓	0.624	✓	12.795		
309	Laplace in 2D Polar coordinates	 Laplace inside disk. Specific boundary conditions	✓	3.936	✓	11.652	Yes	
310	Laplace in 2D Polar coordinates	 $\nabla^2 u(r,\theta)=0$ $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$ Haberman 2.5.5 (c)	✗	3.989	✓	5.669	Yes	

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

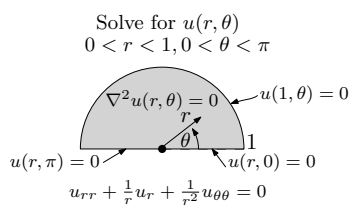
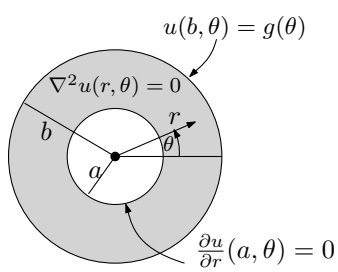
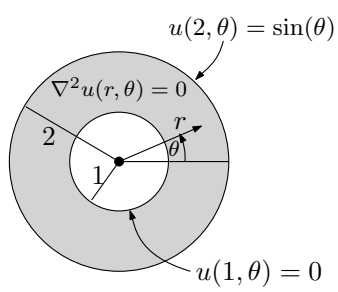
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
311	Laplace in 2D Polar coordinates	 semi-circle	✓	2.08	✓	2.38		
312	Laplace in 2D Polar coordinates	 Haberman 2.5.8 (b)	✓	8.91	✓	250.	Yes	
313	Laplace in 2D Polar coordinates	 Circular annulus	✓	0.519	✓	6.887		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

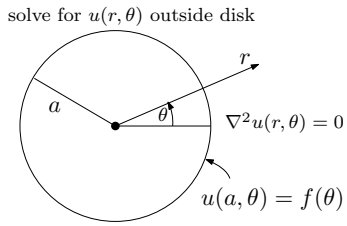
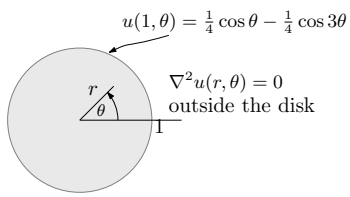
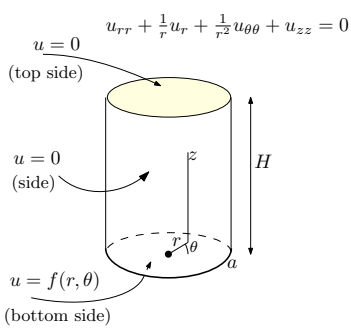
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
314	Laplace in 2D Polar coordinates	 Outside a disk	✓	6.915	✓	9.466		
315	Laplace in 2D Polar coordinates	 Outside a disk	✓	4.21	✓	16.356	Yes	
316	Laplace in 3D Spherical coordinates	In a sphere	✓	0.036	✓	3.632		
317	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (a)	✗	0.018	✗	1.206		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

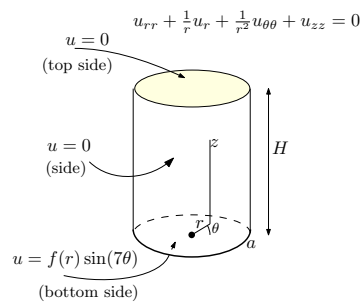
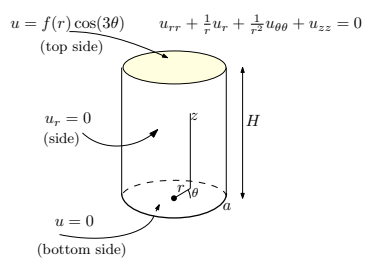
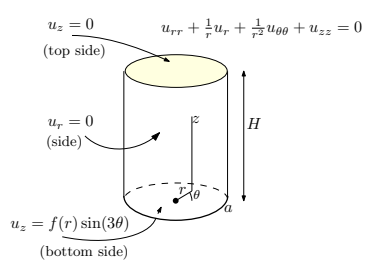
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
318	Laplace in 3D Cylinderical co-ordinates	 Haberman 7.9.1 (b)	✗	0.015	✗	0.948		
319	Laplace in 3D Cylinderical co-ordinates	 Haberman 7.9.1 (c)	✗	0.015	✗	0.984		
320	Laplace in 3D Cylinderical co-ordinates	 Haberman 7.9.1 (d)	✗	0.014	✗	1.03		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

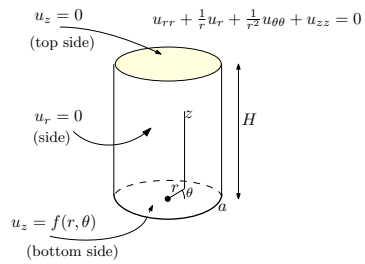
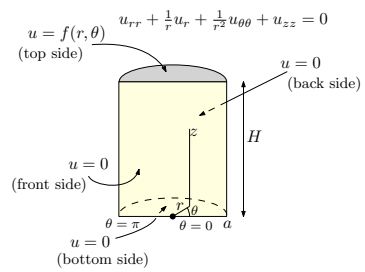
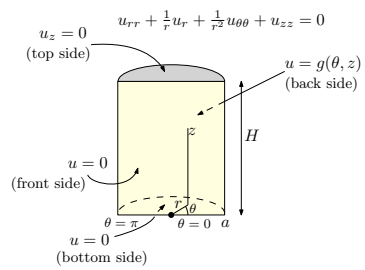
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
321	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.1 (e)	✗	0.015	✗	1.014		
322	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (a)	✗	0.015	✗	1.875		
323	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (b)	✗	0.015	✗	54.412		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

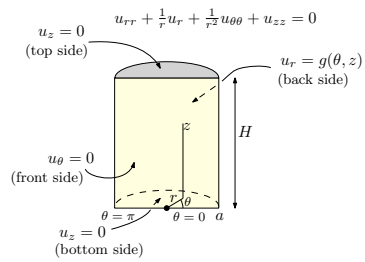
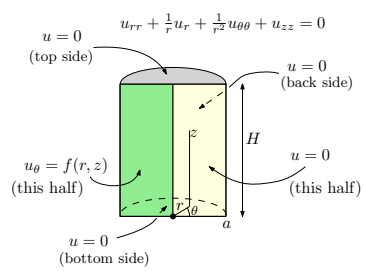
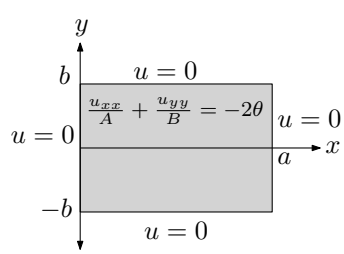
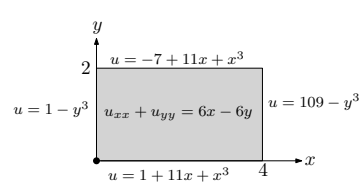
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
324	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (c)	✗	0.02	✗	55.061		
325	Laplace in 3D Cylinderical coordinates	 Haberman 7.9.2 (d)	✗	0.014	✗	1.882		
326	Poisson in 2D Cartesian coordinates	 All boundaries at zero	✗	0.015	✗	79.61	Yes	
327	Poisson in 2D Cartesian coordinates	 Dirichlet problem in a rectangle	✓	1.368	✓	0.457		

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Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

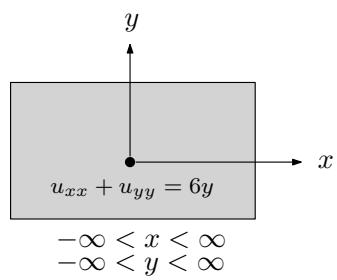
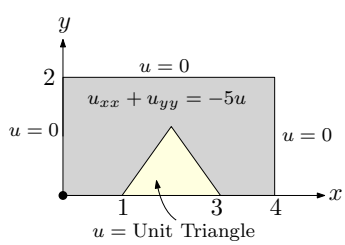
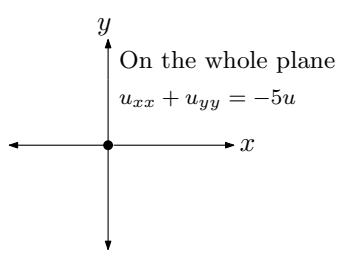
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
328	Poisson in 2D Cartesian coordinates	 Poisson PDE in whole 2D plane	✓	31.393	✓	0.07	Yes	
329	Helmholtz in 2D Cartesian coordinates	 In rectangle	✓	64.18	✓	35.915		
330	Helmholtz in 2D Cartesian coordinates	 On whole plane	✓ why? It solved earlier with BC?	0.004	✓	0.282		

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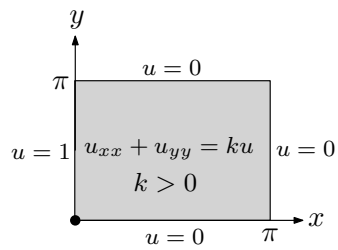
Table 1.8 – Elliptic PDE's (Laplace, Poisson, Helmholtz). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
331	Helmholtz in 2D Cartesian coordinates	 Reduced Helmholtz Inside square	✓	62.146	✓	98.688		
332	Helmholtz in 3D Spherical coordinates	Chain reaction PDE	✗	0.123	✗ trivial solution	2.657	Yes	

1.3.4 Hyperbolic PDE's (Wave)

Table 1.9: Hyperbolic PDE's (Wave) breakdown of results. Time in seconds

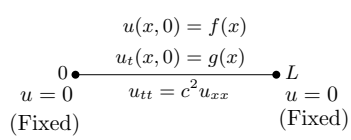
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
333	Wave PDE in 1D Finite length string	 General solution for both ends fixed. Domain is $0\ldots L$	✓	33.33	✓	27.737	Yes	

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
334	Wave PDE in 1D Finite length string	$u(x,0)=0$ $u_t(x,0)=\frac{8x(10-x)^2}{1000}$ 0 ● —— ● 10 $u=0$ (Fixed) $u_{tt}=4u_{xx}$ $u=0$ (Fixed) both ends fixed, inital position zero (special case)	✓	3.324	✓	15.475	Yes	Yes
335	Wave PDE in 1D Finite length string	$u(x,0)=\frac{8x(10-x)^2}{1000}$ $u_t(x,0)=0$ 0 ● —— ● 10 $u=0$ (Fixed) $u_{tt}=4u_{xx}$ $u=0$ (Fixed) both ends fixed, inital velocity zero (special case)	✓	2.518	✓	10.231	Yes	Yes
336	Wave PDE in 1D Finite length string	$u(x,0)=0$ $u_t(x,0)=\sin^2(x)$ $-\pi$ ● —— ● π $u=0$ (Fixed) $u_{tt}=c^2u_{xx}$ $u=0$ (Fixed) both ends fixed but domain is $-\pi\ldots\pi$. zero intial position, non zero initial velocity	✓	6.978	✓	85.345	Yes	Yes
337	Wave PDE in 1D Finite length string	$u(x,0)=\delta(x)$ $u_t(x,0)=0$ -1 ● —— ● 1 $u=0$ (Fixed) $u_{tt}=u_{xx}$ $u=0$ (Fixed) both ends fixed but domain is $-1\ldots1$. intial position is an impulse, zero initial velocity	✓	1.749	✓	23.163	Yes	

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
338	Wave PDE in 1D Finite length string	(No initial conditions) 0 — $u_{tt} = c^2 u_{xx}$ — L $u = 0$ (fixed end), $u = 0$ (fixed end) Logan book, page 28. Both ends fixed	✓	0.974	✓	2.69		
339	Wave PDE in 1D Finite length string	$u(x,0) = 0$ $u_t(x,0) = 1$ 0 — $u_{tt} = u_{xx} - u$ — π $u = 0$, $u = 0$ non-zero initial velocity. Both ends fixed	✓	86.507	✓	12.629		
340	Wave PDE in 1D Finite length string	$u(x,0) = 0$ $\frac{\partial u}{\partial t}(x,0) = 0$ 0 — $u_{tt} = c^2 u_{xx} + p(x,t)$ — π $u(0,t) = 0$ (fixed end), $u(\pi,t) = 0$ (fixed end) Logan book page 149)	✓	20.85	✓	13.871		
341	Wave PDE in 1D Finite length string	$u(x,0) = f(x)$ $\frac{\partial u}{\partial t}(x,0) = 0$ 0 — $u_{tt} = c^2 u_{xx} + Q(x,t)$ — L $u(0,t) = 0$ (fixed end), $u(L,t) = 0$ (fixed end) Haberman 8.5.2 (a)	✓	23.852	✓	57.95		
342	Wave PDE in 1D Finite length string	$u(x,0) = f(x)$ $\frac{\partial u}{\partial t}(x,0) = 0$ 0 — $u_{tt} = c^2 u_{xx} + g(x)\cos(\omega t)$ — L $u(0,t) = 0$ (fixed end), $u(L,t) = 0$ (fixed end) Haberman 8.5.2 (b)	✓	38.58	✓	37.831	Yes	
343	Wave PDE in 1D Finite length string	$v(x,0) = f(x)$ $\frac{\partial v}{\partial t}(x,0) = g(x)$ 0 — $v_{tt} = v_{xx}$ — 1 $v(0,t) = 0$ (fixed end), $v(1,t) = 0$ (fixed end) Both I.C. not zero	✓	22.264	✓	18.398		

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
344	Wave PDE in 1D Finite length string	$v(x,0)=f(x)$ $\frac{\partial v}{\partial t}(x,0)=g(x)$ 0 — $u_{tt}=c^2u_{xx}+1$ — L $u(0,t)=0$ (fixed end) $u(L,t)=0$ (fixed end) With constant source	✓	34.232	✓	28.917		
345	Wave PDE in 1D Finite length string	$u(x,0)=0$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 — $u_{tt}=c^2u_{xx}+Ax$ — L $u(0,t)=0$ (fixed end) $u(L,t)=0$ (fixed end) Logan page 213	✓	4.763	✓	20.611		
346	Wave PDE in 1D Finite length string	$u(x,0)=f(x)$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 — $u_{tt}+2u_t=c^2u_{xx}$ — π $u(0,t)=0$ (fixed end) $u(\pi,t)=0$ (fixed end) Telegraphy PDE	✓	86.878	✓	20.632		
347	Wave PDE in 1D Finite length string	$u(x,0)=f(x)$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 — $u_{tt}+\gamma^2u=c^2u_{xx}$ — π $u(0,t)=0$ (Fixed) $u(\pi,t)=0$ (Fixed) dispersion term Dispersion term present (general case)	✓ Due to adding dispersion term	25.235	✓	26.439	Yes	
348	Wave PDE in 1D Finite length string	$u(x,0)=\sin^2(x)$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 — $u_{tt}+\gamma^2u=c^2u_{xx}$ — π $u(0,t)=0$ (fixed end) $u(\pi,t)=0$ (fixed end) dispersion term Dispersion term present	✓ Due to adding dispersion term	92.86	✓	110.806	Yes	
349	Wave PDE in 1D Finite length string	$u(x,0)==\begin{cases} x-4 & 4\le x\le 5 \\ 6-x & 5\le x\le 6 \\ 0 & \text{otherwise}\end{cases}$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 — $u_{tt}+(\frac{1}{8})^2u=c^2u_{xx}$ — 10 $u(0,t)=0$ (fixed end) $u(10,t)=0$ (fixed end) dispersion term Dispersion term present (specific case)	✓ Due to adding dispersion term	94.897	✓	25.956	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

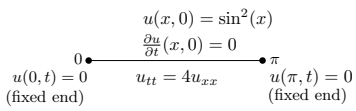
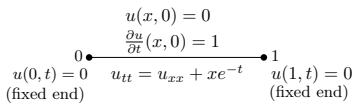
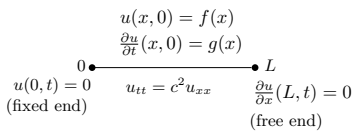
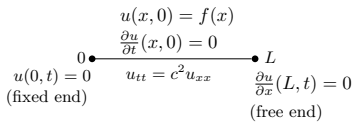
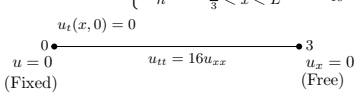
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
350	Wave PDE in 1D Finite length string	 non-zero initial position	✓ But sum should not include $n=2$	7.991	✓ Handled $n=2$ case correctly	86.579		
351	Wave PDE in 1D Finite length string	 With source	✓	10.847	✓	83.973		
352	Wave PDE in 1D Finite length string	 Right end free (general case)	✓	25.353	✓	36.437	Yes	
353	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity (general case)	✓	7.149	✓	26.58	Yes	
354	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity (special case)	✓	89.446	✓	20.785	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

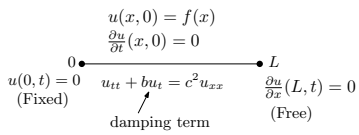
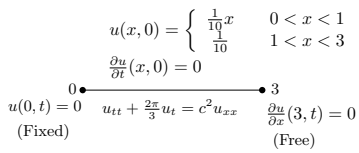
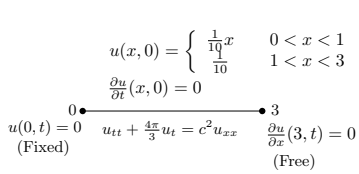
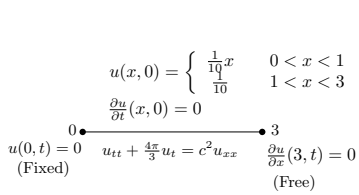
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
355	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (general case)	✓	52.43	✓	32.879	Yes	
356	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (special case, underdamped)	✓	114.95	✓	34.069	Yes	Yes
357	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (special case, critical damped)	✓	114.627	✓	45.159	Yes	Yes
358	Wave PDE in 1D Finite length string	 Right end free, zero initial velocity, damping present (special case, over damped)	✓	116.345	✓	32.581	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

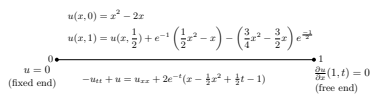
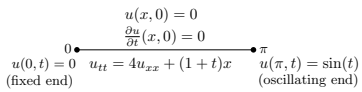
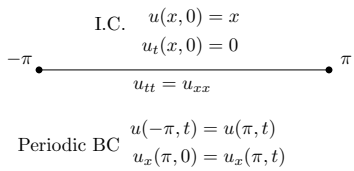
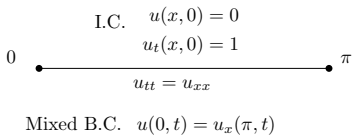
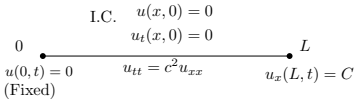
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
359	Wave PDE in 1D Finite length string	 I.C. at different times, right end free, with source	✗	40.028	✓	87.878	Yes	
360	Wave PDE in 1D Finite length string	 Right end oscillates	✓	66.861	✓	67.83		
361	Wave PDE in 1D Finite length string	 Perioidic B.C.	✗	2.412	✗	1.579		
362	Wave PDE in 1D Finite length string	 Mixed B.C.	✗	15.603	✗	1.116		
363	Wave PDE in 1D Finite length string	 Left end fixed, right end non-homogeneous Neumann BC. Zero initial conditions	✓	63.343	✓	27.013	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
364	Wave PDE in 1D Semi-infinite domain	 Left end fixed, (general case)	✓	3.996	✗	1.541	Yes	
365	Wave PDE in 1D Semi-infinite domain	 Left end fixed with specific initial position	✓	15.596	✓	19.95	Yes	
366	Wave PDE in 1D Semi-infinite domain	 Logan page 115, left end fixed with source	✓	7.298	✓	2.138		
367	Wave PDE in 1D Semi-infinite domain	 Left moving boundary condition	✓	1.94	✓	1.46		
368	Wave PDE in 1D Semi-infinite domain	 moving Left end	✓	9.201	✓	3.219		

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

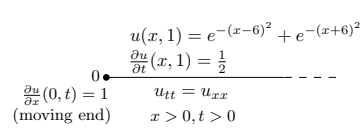
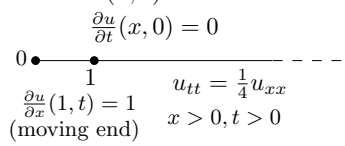
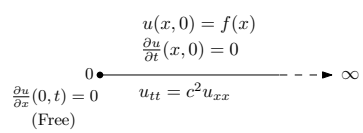
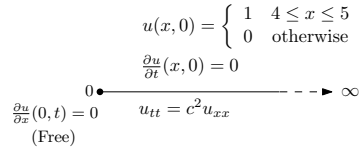
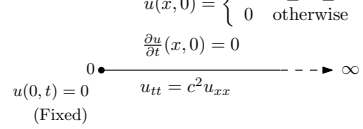
#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
369	Wave PDE in 1D Semi-infinite domain	 I.C. at $t=1$	✓	9.147	✓	1.544		
370	Wave PDE in 1D Semi-infinite domain	 B.C. at $x=1$	✓	76.551	✓	3.115		
371	Wave PDE in 1D Semi-infinite domain	 Left end free. zero initial velocity (general solution)	✓	7.123	✓	5.26		
372	Wave PDE in 1D Semi-infinite domain	 Left end free. zero initial velocity (Special solution)	✓	4.202	✓	3.402	Yes	Yes
373	Wave PDE in 1D Semi-infinite domain	 Left end fixed. zero initial velocity (Special solution)	✓	4.373	✓	3.247	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

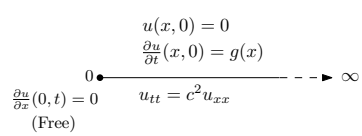
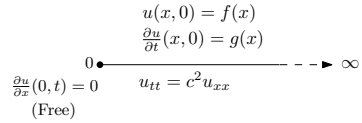
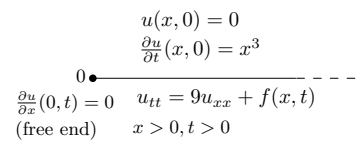
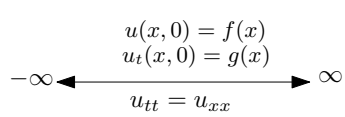
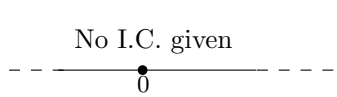
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
374	Wave PDE in 1D Semi-infinite domain	 Left end free. zero initial position (general solution)	✓	8.691	✓	1.785		
375	Wave PDE in 1D Semi-infinite domain	 Left end free. Non zero initial position and velocity (general solution)	✓	33.339	✓	1.563		
376	Wave PDE in 1D Semi-infinite domain	 Left end free with source	✓	1.122	✓	1.379		
377	Wave PDE in 1D Infinite domain	 General case. $u_{tt}=u_{xx}$ with $u(x,0)=f(x), u_t(x,0)=g(x)$	✓	0.019	✓	0.699		
378	Wave PDE in 1D Infinite domain	 $u_{tt}+u_{xt}=c^2u_{xx}$ $-\infty<x<\infty, t>0$ General case. No IC given. $u_{tt}+u_{xt}=c^2u_{xx}$	✓	0.005	✓	0.538		

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
379	Wave PDE in 1D Infinite domain	$u(x,1)=g(x)$ $\frac{\partial u}{\partial t}(x,1)=h(x)$ 0 $u_{tt}=c^2u_{xx}+f(x,t)$ $-\infty<x<\infty, t>0$ $u_{tt}=c^2u_{xx}+f(x,t)$, IC at $t=1$,$u(x,1)=g(x), u_t(x,1)=h(x)$	✓	0.158	✓	14.809		
380	Wave PDE in 1D Infinite domain	$u(x,0)=e^{-x^2}$ $\frac{\partial u}{\partial t}(x,0)=1$ 0 $u_{tt}=u_{xx}$ $-\infty<x<\infty, t>0$ No source. $u_{tt}=u_{xx}$, with $u(x,0)=e^{-x^2}, u_t(x,0)=1$	✓	0.002	✓	0.589		
381	Wave PDE in 1D Infinite domain	$u(x,0)=\sin x-\frac{\cos(3x)}{e^{\frac{\lvert x\rvert}{6}}}$ $\frac{\partial u}{\partial t}(x,0)=0$ 0 $u_{tt}=u_{xx}+m$ $-\infty<x<\infty, t>0$ With source term. $u_{tt}=u_{xx}+m$	✓	0.011	✓	0.437		
382	Wave PDE in 1D Infinite domain	No I.C. given 0 $u_t+6u+u_{xxx}=0$ $-\infty<x<\infty, t>0$ non-linear (Solitons) $u_t+6u(x,t)u_x+u_{xxx}=0$	✓	0.028	✓	0.458	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
383	Wave PDE in 1D Infinite domain	No I.C. given 0 $u_{tt} = 3u_{xx} + u_{xt} - 1$ $-\infty < x < \infty, t > 0$ Inhomogeneous PDE $3u_{xx} - u_{tt} + u_{xt} = 1$	✓	0.003	✓	0.329		
384	Wave PDE in 1D Infinite domain	$u_t(x,0) = \cos(x)$ $u(x,0) = \sin(x)$ $-\infty \longleftrightarrow \infty$ $u_{tt} = u_{xx}$ Practice exam problem Math 5587	✓	0.029	✓	0.388		
385	Wave PDE in 1D Infinite domain	$u_t(x,0) = x$ $u(x,0) = x^2$ $-\infty \longleftrightarrow \infty$ $u_{tt} = u_{xx}$ Practice exam problem Math 5587	✓	0.002	✓	0.304		
386	Wave PDE in 1D Infinite domain	$u_t(x,0) = \frac{4x}{1+x^2}$ $u(x,0) = 0$ $-\infty \longleftrightarrow \infty$ $u_{tt} = u_{xx}$ Practice exam problem Math 5587	✓	8.846	✓	0.407		
387	Wave PDE in 1D Infinite domain	$u_t(x,0) = 0$ $u(x,0) = \frac{1}{1+x^2}$ $-\infty \longleftrightarrow \infty$ $u_{tt} = u_{xx}$ zero initial velocity	✓	0.002	✓	0.309	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
388	Wave PDE in 1D Infinite domain	$u_t(x,0)=0$ $u(x,0)=\sin(x) \quad -\pi<x<\pi$ $-\infty \longleftarrow -\pi \quad \pi \longrightarrow \infty$ $u_{tt}=u_{xx}$ zero initial velocity	✓	0.004	✓	4.24	Yes	Yes
389	Wave PDE in 1D Infinite domain	$u(x,0)=\sin(x)$ $u_t(x,0)=-2xe^{-x^2}$ $-\infty \longleftarrow \longrightarrow \infty$ $u_{tt}=u_{xx}$ General case $u_{tt}=u_{xx}$ with $u(x,0)=\sin x, u_t(x,0)=-2xe^{-x^2}$	✓	0.1	✓	0.401		
390	Wave PDE in 1D Infinite domain	$u_t(x,0)=0$ $u(x,0)=1$ for $1<x<2$ and zero otherwise $-\infty \longleftarrow \longrightarrow \infty$ $u_{tt}=u_{xx}$ General case. $u_{tt}=u_{xx}$ dAlembert solution, box function as initial position	✓	0.003	✓	0.341	Yes	Yes
391	Wave PDE in 1D Infinite domain	$u_t(x,0)=\sin x$ $u(x,0)=\cos x$ $-\infty \longleftarrow \longrightarrow \infty$ $u_{tt}=4u_{xx}+\cos t$ $u_{tt}=4u_{xx}+\cos(t)$ dAlembert solution with $u(x,0)=\sin x, u_t(x,0)=\cos x$	✓	0.096	✓	0.737	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

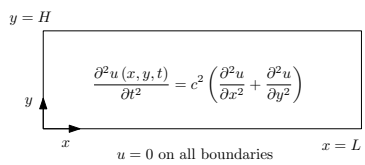
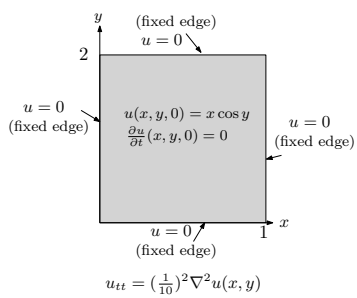
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
392	Wave PDE in 1D Infinite domain	$u_t(x,0)=0$ $u(x,0)=\delta(x-a)$ $-\infty \longleftrightarrow \infty$ $u_{tt}=c^2u_{xx}$ $u_{tt}=c^2u_{xx}$ dAlembert solution with $u(x,0)=\delta(x-a), u_t(x,0)=0$	✓	63.956	✓	0.737	Yes	
393	Wave PDE in 1D Infinite domain	system of 2 inhomogeneous linear hyperbolic system with constant coefficients	✓	0.267	✗	1.359		
394	Wave PDE in 2D Cartesian coordinates	 Rectangular membrane. Fixed on all edges, General solution	✗	2.271	✓	559.481	Yes	
395	Wave PDE in 2D Cartesian coordinates	 Rectangular membrane. Fixed on all edges, zero velocity. Specific example	✗	0.424	✓	30.771	Yes	Yes

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

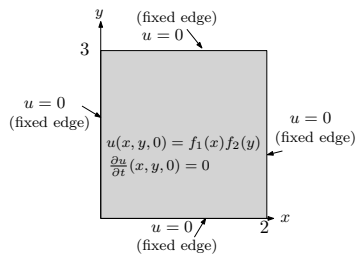
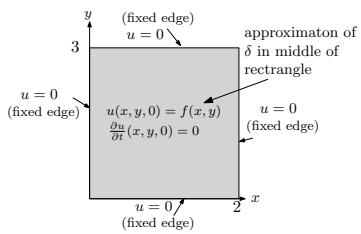
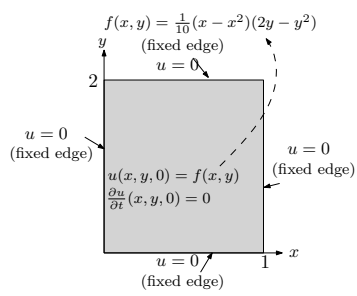
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
396	Wave PDE in 2D Cartesian coordinates	 $u_{tt}=(\frac{1}{3})^2\nabla^2 u(x,y)$ All 4 edges fixed, zero initial velocity, Specific example	✓	33.848	✓	53.935	Yes	Yes
397	Wave PDE in 2D Cartesian coordinates	 $u_{tt}=(\frac{1}{3})^2\nabla^2 u(x,y)$ All 4 edges fixed, zero initial velocity, Specific example, delta in center	✓	9.295	✓	53.67	Yes	Yes
398	Wave PDE in 2D Cartesian coordinates	 $u_{tt}=\nabla^2 u(x,y)$ All 4 edges fixed	✓	4.949	✓	29.946		

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

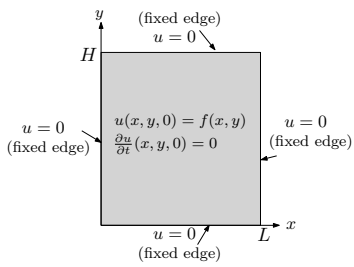
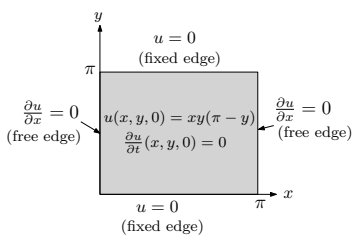
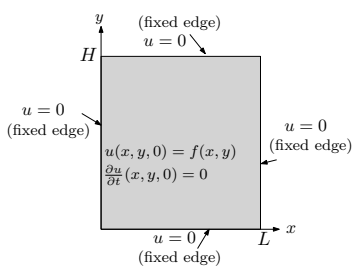
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
399	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = c^2\nabla^2 u(x,y) + Q(x,y,t)$ All edges fixed (Haberman 8.5.5 (a)	✓	27.715	✗	0.932		
400	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = \frac{1}{4}\nabla^2 u(x,y)$ 2 edgs fixed, 2 free, zero initial velocity	✓	5.952	✓	58.452		
401	Wave PDE in 2D Cartesian coordinates	 $u_{tt} = c^2\nabla^2 u(x,y)$ All 4 edges fixed, zero initial velocity, general solution	✓	1.411	✗ (Timed out)	600.	Yes	

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

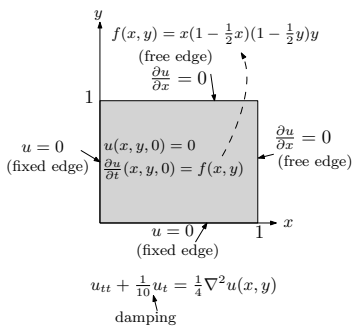
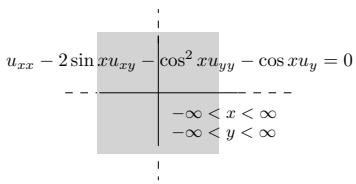
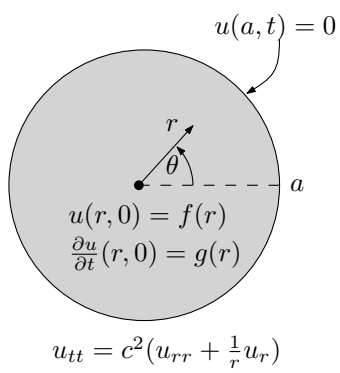
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
402	Wave PDE in 2D Cartesian coordinates	 With damping	✓	9.074	✓	87.58		
403	Wave PDE in 2D Cartesian coordinates	 On the whole plane	✓	0.171	✗	0.574		
404	Wave PDE in 2D Polar coordinates	 no θ dependency, fixed boundary, general case	✓	3.01	✓ Has unresolved Invlaplace calls	2.532	Yes	

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

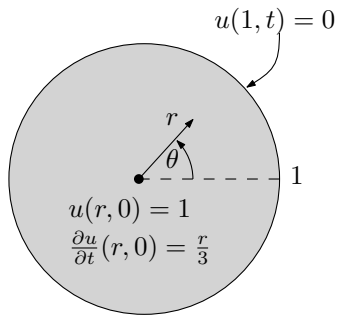
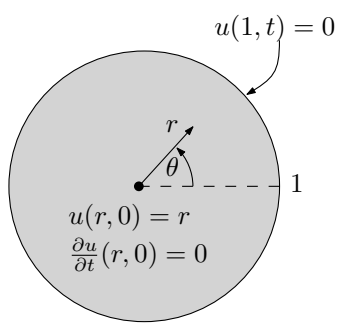
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
405	Wave PDE in 2D Polar coordinates	 $u_{tt}=c^2(u_{rr}+\frac{1}{r}u_r)$ no θ dependency. Specific example. Both initial conditions not zero	✓	2.684	✓ Has unresolved Invlaplace calls	93.789		
406	Wave PDE in 2D Polar coordinates	 $u_{tt}=c^2(u_{rr}+\frac{1}{r}u_r)$ $c=\frac{2}{10}$ no θ dependency. Specific example. Both initial conditions not zero	✓	2.549	✓ Has unresolved Invlaplace calls. How to get series solution?	39.994	Yes	Yes
407	Wave PDE in 2D Polar coordinates	no θ dependency. Using integral transforms. Source present. Specific example	✓	3.628	✓	18.789		
408	Wave PDE in 2D Polar coordinates	no θ dependency. Using integral transforms. Source present. Specific example	✗	2.511	✓	2.416		

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

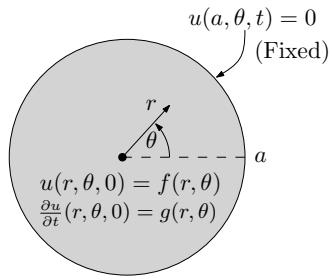
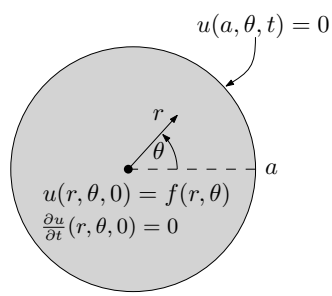
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
409	Wave PDE in 2D Polar coordinates	 $u_t = c^2(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$ θ dependency, fixed on edges, general solution	✓	9.409	✗	6.898	Yes	
410	Wave PDE in 2D Polar coordinates	 $u_t = c^2\nabla^2 u(r,\theta)$ θ dependency, fixed on edges, zero initial velocity, general solution	✓	4.687	✗	5.48	Yes	

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Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

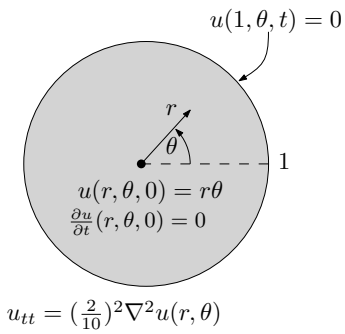
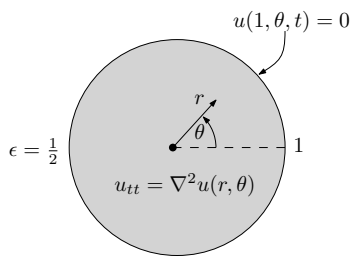
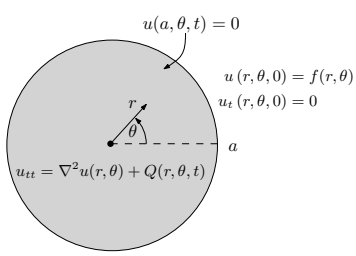
#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
411	Wave PDE in 2D Polar coordinates	 $u_{tt}=(\frac{2}{10})^2\nabla^2 u(r,\theta)$ θ dependency, fixed on edges, zero initial velocity, specific example	✓	15.449	✗	7.102	Yes	Yes
412	Wave PDE in 2D Polar coordinates	 $u(r,\theta,0)=0$ $u_t(r,\theta,0)=\begin{cases}\frac{1}{\pi\epsilon^2} & \text{if } r\le\epsilon\\ 0 & \text{otherwise}\end{cases}$ θ dependency, fixed on edges, zero initial position, specific example	✓	7.747	✗	6.869	Yes	Yes
413	Wave PDE in 2D Polar coordinates	 θ dependency, fixed on edges, zero initial position with internal source (Haberman 8.5.5. (b)	✗	0.042	✗	0.692	Yes	

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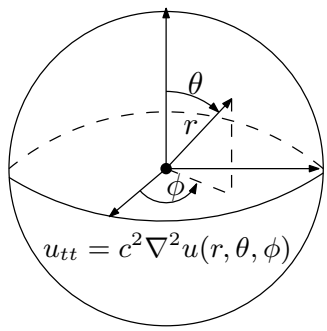
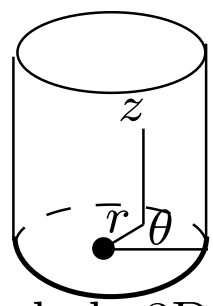
Table 1.9 – Hyperbolic PDE's (Wave). Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
414	Wave PDE in 3D Spherical coordinates	 No I.C. no B.C.	✓	0.039	✓	5.077		
415	Wave PDE in 3D Cylindrical coordinates	$u_{tt} = \nabla^2 u$  (whole 3D) No I.C. no B.C.	✓	0.006	✓	0.887		

1.3.5 Handbook of first order partial differential equations

Table 1.10: Handbook of first order partial differential equations breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
416	chapter 1	problem number 1	✓	0.005	✓	0.089		
417	chapter 1	problem number 2	✓	0.004	✓	0.007		
418	chapter 1	problem number 3	✓	0.017	✓	0.006	Yes	
419	chapter 1	problem number 4	✓	0.016	✓	0.004		
420	chapter 1	problem number 5	✓	0.032	✓	0.021	Yes	
421	chapter 1	problem number 6	✓	0.03	✓	0.018		
422	chapter 2 2.1	problem number 1	✓	0.006	✓	0.097	Yes	
423	chapter 2 2.1	problem number 2	✓	0.01	✓	0.094	Yes	
424	chapter 2 2.1	problem number 3	✓	0.054	✓	0.263	Yes	
425	chapter 2 2.1	problem number 4	✓	0.021	✓	0.161	Yes	
426	chapter 2 2.1	problem number 5	✓	0.024	✓	0.42	Yes	
427	chapter 2 2.1	problem number 6	✓	0.05	✓	0.317	Yes	
428	chapter 2 2.1	problem number 7	✓	0.055	✓	4.207	Yes	
429	chapter 2 2.1	problem number 8	✗	2.383	✓	7.648		
430	chapter 2 2.2	problem number 1	✓	0.009	✓	0.011		
431	chapter 2 2.2	problem number 2	✓	0.244	✓	0.467		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
432	chapter 2 2.2	problem number 3	✓	0.106	✓	0.101		
433	chapter 2 2.2	problem number 4	✓	0.385	✓	0.328		
434	chapter 2 2.2	problem number 5	✓	0.25	✓	1.324		
435	chapter 2 2.2	problem number 6	✓	0.33	✓	0.549		
436	chapter 2 2.2	problem number 7	✓	0.353	✓	0.937		
437	chapter 2 2.2	problem number 8	✓	0.511	✓	0.727		
438	chapter 2 2.2	problem number 9	✓	0.299	✓	0.65		
439	chapter 2 2.2	problem number 10	✓	0.171	✓	0.309		
440	chapter 2 2.2	problem number 11	✓	0.318	✓	0.417		
441	chapter 2 2.2	problem number 12	✓	0.822	✓	1.457		
442	chapter 2 2.2	problem number 13	✓	0.113	✓	0.017		
443	chapter 2 2.2	problem number 14	✓	0.27	✓	0.961		
444	chapter 2 2.2	problem number 15	✓	0.7	✓	0.774		
445	chapter 2 2.2	problem number 16	✓	1.564	✓	0.519		
446	chapter 2 2.2	problem number 17	✓	0.951	✓	0.966		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
447	chapter 2 2.2	problem number 18	✓	0.292	✓	0.121		
448	chapter 2 2.2	problem number 19	✓	0.294	✓	0.89		
449	chapter 2 2.2	problem number 20	✗	0.261	✓	1.122		
450	chapter 2 2.2	problem number 21	✓	0.196	✓	1.139		
451	chapter 2 2.2	problem number 22	✓	0.348	✓	1.8		
452	chapter 2 2.2	problem number 23	✓	0.438	✓	1.015		
453	chapter 2 2.2	problem number 24	✗	0.313	✓	0.85		
454	chapter 2 2.2	problem number 25	✗ (Timed out)	600.	✗ (Timed out)	600.		
455	chapter 2 2.2	problem number 26	✗	165.853	✗	33.849		
456	chapter 2 2.2	problem number 27	✗	101.598	✗	1.361		
457	chapter 2 2.2	problem number 28	✗	0.528	✓ solution contains RootOf	0.111		
458	chapter 2 2.2	problem number 29	✗	101.315	✗	1.211		
459	chapter 2 2.2	problem number 30	✗	0.305	✓	0.132		
460	chapter 2 2.2	problem number 31, Hesse's equation	✗	235.84	✓	9.247		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
461	chapter 2 2.3	problem number 1	✓ But it can't solve it when assuming $b > 0$ which is strange.	0.491	✓	0.53		
462	chapter 2 2.3	problem number 2	✓	0.074	✓	0.161		
463	chapter 2 2.3	problem number 3	✓	0.179	✓	0.486		
464	chapter 2 2.3	problem number 4	✗	0.277	✓	0.627		
465	chapter 2 2.3	problem number 5	✗	0.932	✓ Answer contains RootOf	0.075		
466	chapter 2 2.3	problem number 6	✗	0.232	✓ Answer contains RootOf	0.035		
467	chapter 2 2.3	problem number 7	✓	0.234	✓	0.083		
468	chapter 2 2.3	problem number 8	✓	0.265	✓	0.042		
469	chapter 2 2.3	problem number 9	✗ (Timed out)	600.	✓	0.036		
470	chapter 2 2.4	problem number 1	✓	0.037	✓	0.025		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
471	chapter 2 2.4	problem number 2	✓	0.268	✓	0.262		
472	chapter 2 2.4	problem number 3	✓	0.196	✓	0.194		
473	chapter 2 2.4	problem number 4	✓	0.467	✓	0.309		
474	chapter 2 2.4	problem number 5	✗	0.53	✗ (Timed out)	600.		
475	chapter 2 2.4	problem number 6	✓	1.952	✓	0.254		
476	chapter 2 2.4	problem number 7	✗	1.872	✓	3.05		
477	chapter 2 2.5	problem number 1	✓	0.058	✓	0.087		
478	chapter 2 2.5	problem number 2	✓	0.134	✓	0.57		
479	chapter 2 2.5	problem number 3	✓	0.198	✓	0.098		
480	chapter 2 2.5	problem number 4	✗	20.865	✓	0.613		
481	chapter 2 2.5	problem number 5	✓	0.933	✓	0.379		
482	chapter 2 2.5	problem number 6	✗	1.853	✓	0.662		
483	chapter 2 2.5	problem number 7	✓	0.458	✓	0.081		
484	chapter 2 2.5	problem number 8	✗	23.262	✓	1.414		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
485	chapter 2 2.5	problem number 9	✗	1.15	✓	0.698		
486	chapter 2 2.5	problem number 10	✗	2.964	✓	1.692		
487	chapter 2 2.5	problem number 11	✗	26.182	✗	6.795		
488	chapter 2 2.5	problem number 12	✗	25.253	✗	4.863		
489	chapter 2 2.5	problem number 13	✗	30.158	✗	9.931		
490	chapter 2 2.5	problem number 14	✗	0.36	✓ Solution contains RootOf	0.141		
491	chapter 2 2.5	problem number 15	✓	0.639	✓	2.027		
492	chapter 2 2.5	problem number 16	✗	78.509	✗	60.095		
493	chapter 2 2.5	problem number 17	✗	0.304	✓	0.506		
494	chapter 2 2.5	problem number 18	✗	0.578	✓	0.491		
495	chapter 2 2.5	problem number 19	✓	0.361	✓	2.204		
496	chapter 2 2.5	problem number 20	✓	0.298	✓	0.107		
497	chapter 2 2.5	problem number 21	✓	0.641	✓	0.132		
498	chapter 2 2.5	problem number 22	✓	0.59	✓	0.076		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
499	chapter 2 2.5	problem number 23	✗	26.472	✓	0.208		
500	chapter 2 2.5	problem number 24	✓	0.331	✓	0.076		
501	chapter 2 2.5	problem number 25	✓	0.614	✓	0.043		
502	chapter 2 2.5	problem number 26	✓	0.86	✓	0.077		
503	chapter 2 2.5	problem number 27	✓	0.413	✓	0.134		
504	chapter 2 2.5	problem number 28	✗	0.477	✓ Solution contains RootOf	0.07		
505	chapter 2 2.5	problem number 29	✗	0.365	✓ Solution contains RootOf	0.051		
506	chapter 2 2.5	problem number 30	✗	0.708	✓	1.33		
507	chapter 2 2.5	problem number 31	✗	18.477	✓	1.675		
508	chapter 2 2.5	problem number 32	✗	114.344	✗	51.341		
509	chapter 2 2.5	problem number 33	✓	0.562	✓	0.155		
510	chapter 2 2.5	problem number 34	✓	0.455	✓	0.063		
511	chapter 2 2.5	problem number 35	✓	0.242	✓	0.108		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
512	chapter 2 2.5	problem number 36	✓	0.213	✓	0.423		
513	chapter 2 2.5	problem number 37	✓	0.479	✓	0.641		
514	chapter 2 2.5	problem number 38	✓	2.637	✓	0.16		
515	chapter 2 2.5	problem number 39	✓	0.182	✓	0.125		
516	chapter 2 2.5	problem number 40	✗	103.396	✗	32.259		
517	chapter 2 2.5	problem number 41	✓	1.078	✓	0.225		
518	chapter 2 2.5	problem number 42	✗	4.276	✓	4.957		
519	chapter 2 2.5	problem number 43	✗	2.982	✓	3.824		
520	chapter 2 2.5	problem number 44	✗	31.136	✗	15.634		
521	chapter 2 2.5	problem number 45	✗	31.691	✗	25.436		
522	chapter 2 2.5	problem number 46	✗	12.09	✓	4.766		
523	chapter 2 2.5	problem number 47	✓	1.44	✓	0.135		
524	chapter 2 2.5	problem number 48	✗	33.94	✓	0.789		
525	chapter 2 2.5	problem number 49	✗	21.879	✗	17.779		
526	chapter 2 2.5	problem number 50	✗	51.312	✗	80.184		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
527	chapter 2 2.5	problem number 51	✗	0.992	✓	0.617		
528	chapter 2 2.5	problem number 52	✗	11.804	✗	3.492		
529	chapter 2 2.5	problem number 53	✗	0.833	✓	0.669		
530	chapter 2 2.5	problem number 54	✗	0.956	✗	2.882		
531	chapter 2 2.5	problem number 55	✗	19.752	✗	10.097		
532	chapter 2 2.5	problem number 56	✗	1.495	✓	1.217		
533	chapter 2 3.1	problem number 1	✓	0.021	✓	0.013		
534	chapter 2 3.1	problem number 2	✓	0.029	✓	0.01		
535	chapter 2 3.1	problem number 3	✓	0.215	✓	0.238		
536	chapter 2 3.1	problem number 4	✓	0.4	✓	0.318		
537	chapter 2 3.1	problem number 5	✗	1.053	✓	0.963		
538	chapter 2 3.1	problem number 6	✓	0.284	✓	0.048		
539	chapter 2 3.1	problem number 7	✓	0.235	✓	0.045		
540	chapter 2 3.1	problem number 8	✓	0.634	✓ Has RootOf	0.673		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
541	chapter 2 3.1	problem number 9	✓	0.499	✓	0.257		
542	chapter 2 3.1	problem number 10	✗	0.666	✓	0.671		
543	chapter 2 3.1	problem number 11	✗	2.633	✗	3.273		
544	chapter 2 3.2	problem number 1	✓	0.786	✓	0.315		
545	chapter 2 3.2	problem number 2	✓	1.149	✓	0.225		
546	chapter 2 3.2	problem number 3	✓	0.496	✓	0.104		
547	chapter 2 3.2	problem number 4	✓	0.625	✓	0.258		
548	chapter 2 3.2	problem number 5	✓	0.385	✓	0.073		
549	chapter 2 3.2	problem number 6	✗	23.202	✗	8.744		
550	chapter 2 3.2	problem number 7	✓	0.59	✓	0.099		
551	chapter 2 3.2	problem number 8	✗	1.051	✓	0.145		
552	chapter 2 3.2	problem number 9	✓	1.513	✓	0.366		
553	chapter 2 3.2	problem number 10	✓	1.289	✓	0.309		
554	chapter 2 3.2	problem number 11	✗	22.612	✓	0.46		
555	chapter 2 3.2	problem number 12	✓	3.572	✓	0.127		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
556	chapter 2 3.2	problem number 13	✓	0.76	✓	0.119		
557	chapter 2 3.2	problem number 14	✗	24.234	✗	7.086		
558	chapter 2 3.2	problem number 15	✗	2.973	✓	0.776		
559	chapter 2 3.2	problem number 16	✗	22.908	✗	6.699		
560	chapter 2 3.2	problem number 17	✗	1.804	✓	0.536		
561	chapter 2 3.2	problem number 18	✗	23.297	✗	4.619		
562	chapter 2 3.2	problem number 19	✓	0.513	✓	0.159		
563	chapter 2 3.2	problem number 20	✗	23.169	✗	4.096		
564	chapter 2 3.2	problem number 21	✗	25.852	✗	4.52		
565	chapter 2 3.2	problem number 22	✓	2.881	✓	0.19		
566	chapter 2 3.2	problem number 23	✓	0.787	✓	0.185		
567	chapter 2 3.2	problem number 24	✗	21.9	✗	4.456		
568	chapter 2 3.2	problem number 25	✓	0.485	✓	0.132		
569	chapter 2 3.2	problem number 26	✓	0.554	✓	0.135		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
570	chapter 2 3.2	problem number 27	✗	1.139	✓ Solution contains RootOf	0.156		
571	chapter 2 3.2	problem number 28	✓	0.606	✓	0.181		
572	chapter 2 3.2	problem number 29	✓	0.41	✓	0.098		
573	chapter 2 3.2	problem number 30	✓	3.889	✓	0.148		
574	chapter 2 3.2	problem number 31	✗	50.01	✓	20.971		
575	chapter 2 3.2	problem number 32	✓	0.28	✓	0.087		
576	chapter 2 3.2	problem number 33	✗	0.395	✓	0.171		
577	chapter 2 3.2	problem number 34	✗	1.469	✓	4.034		
578	chapter 2 3.2	problem number 35	✗	1.005	✓	3.255		
579	chapter 2 3.2	problem number 36	✗	1.093	✓	4.409		
580	chapter 2 4.1	problem number 1	✓	0.023	✓	0.015		
581	chapter 2 4.1	problem number 2	✓	0.14	✓	0.092		
582	chapter 2 4.1	problem number 3	✓	3.23	✓	0.93		
583	chapter 2 4.1	problem number 4	✗	43.098	✓	0.333		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
584	chapter 2 4.1	problem number 5	✗	57.21	✓	1.408		
585	chapter 2 4.1	problem number 6	✓	0.22	✓	0.049		
586	chapter 2 4.1	problem number 7	✓	0.041	✓	0.044		
587	chapter 2 4.2	problem number 1	✓	0.019	✓	0.012		
588	chapter 2 4.2	problem number 2	✓	0.115	✓	0.082		
589	chapter 2 4.2	problem number 3	✗	61.929	✓	1.036		
590	chapter 2 4.2	problem number 4	✗	64.619	✓	1.503		
591	chapter 2 4.2	problem number 5	✗	1.778	✓	2.435		
592	chapter 2 4.2	problem number 6	✗	2.923	✓	6.262		
593	chapter 2 4.2	problem number 7	✗	487.736	✓	1.779		
594	chapter 2 4.2	problem number 8	✓	0.212	✓	0.053		
595	chapter 2 4.3	problem number 1	✓	0.028	✓	0.016		
596	chapter 2 4.3	problem number 2	✓	0.185	✓	0.003		
597	chapter 2 4.3	problem number 3	✓	3.367	✓	0.589		
598	chapter 2 4.3	problem number 4	✗	43.425	✓	2.086		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
599	chapter 2 4.3	problem number 5	✗	2.778	✓	2.24		
600	chapter 2 4.3	problem number 6	✗	4.636	✓	4.019		
601	chapter 2 4.3	problem number 7	✗	75.718	✓	2.169		
602	chapter 2 4.3	problem number 8	✓	0.341	✓	0.114		
603	chapter 2 4.4	problem number 1	✓	0.029	✓	0.016		
604	chapter 2 4.4	problem number 2	✓	0.189	✓	0.323		
605	chapter 2 4.4	problem number 3	✓	3.102	✓	1.197		
606	chapter 2 4.4	problem number 4	✓	4.441	✓	0.517		
607	chapter 2 4.5	problem number 1	✓	0.28	✓	0.103		
608	chapter 2 4.5	problem number 2	✓	0.28	✓	0.063		
609	chapter 2 4.5	problem number 3	✓	2.838	✓	0.571		
610	chapter 2 4.5	problem number 4	✓	14.583	✓	5.267		
611	chapter 2 4.5	problem number 5	✓	0.192	✓	0.053		
612	chapter 2 4.5	problem number 6	✗	583.259	✓	4.686		
613	chapter 2 5.1	problem number 1	✓	0.061	✓	0.024		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
614	chapter 2 5.1	problem number 3	✓	1.078	✓	0.033		
615	chapter 2 5.1	problem number 4	✗	3.213	✓	0.076		
616	chapter 2 5.2	problem number 1	✓	0.252	✓	0.13		
617	chapter 2 5.2	problem number 2	✓	0.271	✓	0.067		
618	chapter 2 5.2	problem number 3	✗	1.315	✓	0.225		
619	chapter 2 5.2	problem number 4	✓	0.838	✓	0.219		
620	chapter 2 5.2	problem number 5	✗	23.747	✗	3.578		
621	chapter 2 5.2	problem number 6	✗	27.327	✓	1.08		
622	chapter 2 5.2	problem number 7	✗	27.996	✗	4.928		
623	chapter 2 5.2	problem number 8	✗	24.407	✗	3.554		
624	chapter 2 5.2	problem number 9	✓	0.937	✓	0.131		
625	chapter 2 5.2	problem number 10	✗	3.325	✓	0.924		
626	chapter 2 5.2	problem number 11	✓	0.218	✓	0.103		
627	chapter 2 5.2	problem number 12	✗	20.993	✗	1.659		
628	chapter 2 5.2	problem number 13	✗	23.817	✗	2.746		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
629	chapter 2 5.2	problem number 14	✗	22.633	✗	2.183		
630	chapter 2 5.2	problem number 15	✓	0.486	✓	0.466		
631	chapter 2 5.2	problem number 16	✓	0.319	✓	0.079		
632	chapter 2 5.2	problem number 17	✓	1.122	✓	0.101		
633	chapter 2 5.2	problem number 18	✓	0.808	✓	0.408		
634	chapter 2 5.2	problem number 19	✗	4.562	✓	1.341		
635	chapter 2 5.2	problem number 20	✗	4.582	✓	1.284		
636	chapter 2 5.2	problem number 21	✗	26.571	✓	0.119		
637	chapter 2 5.2	problem number 22	✓	0.721	✓	0.695		
638	chapter 2 5.2	problem number 23	✓	0.7	✓	0.202		
639	chapter 2 6.1	problem number 1	✓	0.497	✓	0.143		
640	chapter 2 6.1	problem number 2	✓ contains unre-solved integral	5.852	✓ contains unre-solved integral	0.927		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
641	chapter 2 6.1	problem number 3	✓	1.43	✓ contains unre-solved integral	1.581		
642	chapter 2 6.1	problem number 4	✗	11.481	✓ contains unre-solved integral	1.606		
643	chapter 2 6.1	problem number 5	✗	42.587	✓	0.902		
644	chapter 2 6.1	problem number 6	✗	4.735	✓ contains unre-solved integrals	0.279		
645	chapter 2 6.1	problem number 7	✓	19.363	✓	4.358		
646	chapter 2 6.1	problem number 8	✗	43.063	✓	0.448		
647	chapter 2 6.1	problem number 9	✗	44.379	✓	2.343		
648	chapter 2 6.1	problem number 10	✗	44.975	✓	1.377		
649	chapter 2 6.1	problem number 11	✗	37.431	✓	9.72		
650	chapter 2 6.1	problem number 12	✗	144.702	✗ (Timed out)	600.		
651	chapter 2 6.1	problem number 13	✓	0.761	✓	0.767		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
652	chapter 2 6.1	problem number 14	✗	13.412	✓	2.212		
653	chapter 2 6.2	problem number 1	✓	0.529	✓ Contains unre-solved integral	0.279		
654	chapter 2 6.2	problem number 2	✓	3.955	✓	0.43		
655	chapter 2 6.2	problem number 3	✓	0.819	✓	1.664		
656	chapter 2 6.2	problem number 4	✗	9.572	✓	0.792		
657	chapter 2 6.2	problem number 5	✗	42.291	✓	0.683		
658	chapter 2 6.2	problem number 6	✗	42.82	✓	0.518		
659	chapter 2 6.2	problem number 7	✗	45.317	✓	2.367		
660	chapter 2 6.2	problem number 8	✗	45.734	✓	1.367		
661	chapter 2 6.2	problem number 9	✗	135.562	✓	2.931		
662	chapter 2 6.2	problem number 10	✗	2.281	✓	3.102		
663	chapter 2 6.2	problem number 11	✗	1.857	✓	7.699		
664	chapter 2 6.2	problem number 12	✗	53.846	✓	3.14		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
665	chapter 2 6.3	problem number 1	✓	0.052	✓	0.016		
666	chapter 2 6.3	problem number 2	✓	0.434	✓	0.218		
667	chapter 2 6.3	problem number 3	✓	0.911	✓ Has un-resolved integrals	2.066		
668	chapter 2 6.3	problem number 4	✗	44.497	✓	0.617		
669	chapter 2 6.3	problem number 5	✗	46.942	✓	0.622		
670	chapter 2 6.3	problem number 6	✓	2.57	✓	1.32		
671	chapter 2 6.3	problem number 7	✗	38.765	✓	4.686		
672	chapter 2 6.3	problem number 8	✗	45.215	✗	20.774		
673	chapter 2 6.3	problem number 9	✗	166.671	✗ (Timed out)	600.		
674	chapter 2 6.3	problem number 10	✓	0.78	✓	0.611		
675	chapter 2 6.3	problem number 11	✗	94.742	✓	15.795		
676	chapter 2 6.3	problem number 12	✗	24.388	✓	3.168		
677	chapter 2 6.3	problem number 13	✗	2.08	✓	3.341		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
678	chapter 2 6.3	problem number 14	✗	2.011	✓	5.047		
679	chapter 2 6.3	problem number 15	✗	27.354	✓	4.43		
680	chapter 2 6.4	problem number 1	✓	0.42	✓ Has un-resolved integral	0.381		
681	chapter 2 6.4	problem number 2	✓	1.889	✓	0.426		
682	chapter 2 6.4	problem number 3	✗	10.658	✓	0.762		
683	chapter 2 6.4	problem number 4	✗	44.393	✓	0.597		
684	chapter 2 6.4	problem number 5	✗	46.966	✓	0.58		
685	chapter 2 6.4	problem number 6	✓	0.674	✓	0.164		
686	chapter 2 6.4	problem number 7	✓	0.616	✓	1.329		
687	chapter 2 6.4	problem number 8	✓	0.557	✓	0.502		
688	chapter 2 6.4	problem number 9	✓	2.124	✓	0.425		
689	chapter 2 6.4	problem number 10	✓	2.42	✓	4.456		
690	chapter 2 6.4	problem number 11	✓	0.636	✓	0.566		
691	chapter 2 6.4	problem number 12	✗	7.997	✓	1.376		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
692	chapter 2 6.5	problem number 1	✓	1.421	✓ Has un-resolved integrals	0.576		
693	chapter 2 6.5	problem number 2	✓	2.541	✓	0.697		
694	chapter 2 6.5	problem number 3	✓	0.992	✓ Mathematica answer is sim-pler	0.998		
695	chapter 2 6.5	problem number 4	✗	26.076	✗	35.38		
696	chapter 2 6.5	problem number 5	✗	44.173	✓	0.678		
697	chapter 2 6.5	problem number 6	✗	67.024	✓	1.395		
698	chapter 2 6.5	problem number 7	✗	134.91	✗	117.056		
699	chapter 2 6.5	problem number 8	✗	27.096	✓	0.695		
700	chapter 2 6.5	problem number 9	✗	25.423	✓	0.003		
701	chapter 2 6.5	problem number 10	✗	2.703	✓	1.561		
702	chapter 2 6.5	problem number 11	✗ (Timed out)	600.	✓	2.48		
703	chapter 2 7.1	problem number 1	✓	0.115	✓	0.159		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
704	chapter 2 7.1	problem number 2	✓	3.243	✓	0.172		
705	chapter 2 7.1	problem number 3	✗	0.642	✓	0.279		
706	chapter 2 7.1	problem number 4	✓	0.166	✓	0.23		
707	chapter 2 7.1	problem number 5	✗	1.937	✓	0.934		
708	chapter 2 7.1	problem number 6	✓	0.546	✓	1.879		
709	chapter 2 7.1	problem number 7	✗	27.74	✓	12.804		
710	chapter 2 7.1	problem number 8	✗	1.547	✓	0.907		
711	chapter 2 7.1	problem number 9	✗	66.878	✗	90.249		
712	chapter 2 7.1	problem number 10	✗	38.983	✗	132.413		
713	chapter 2 7.1	problem number 11	✓	2.214	✓	0.275		
714	chapter 2 7.1	problem number 12	✓	1.144	✓	1.199		
715	chapter 2 7.2	problem number 1	✓	0.111	✓	0.127		
716	chapter 2 7.2	problem number 2	✓	5.018	✓	0.192		
717	chapter 2 7.2	problem number 3	✗	15.525	✓	0.285		
718	chapter 2 7.2	problem number 4	✓	0.364	✓	0.279		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
719	chapter 2 7.2	problem number 5	✗	6.465	✓	57.786		
720	chapter 2 7.2	problem number 6	✗	30.645	✗	19.338		
721	chapter 2 7.2	problem number 7	✗	47.941	✓	5.516		
722	chapter 2 7.2	problem number 8	✗	8.379	✓	50.029		
723	chapter 2 7.2	problem number 9	✗	41.156	✗	97.47		
724	chapter 2 7.2	problem number 10	✗	41.363	✗	130.597		
725	chapter 2 7.2	problem number 11	✓	2.468	✓	0.331		
726	chapter 2 7.2	problem number 12	✓	1.31	✓	1.038		
727	chapter 2 7.3	problem number 1	✓	0.62	✓	0.293		
728	chapter 2 7.3	problem number 2	✓	4.24	✓	0.292		
729	chapter 2 7.3	problem number 3	✗	16.414	✓	0.551		
730	chapter 2 7.3	problem number 4	✓	0.477	✓	0.491		
731	chapter 2 7.3	problem number 5	✗	7.761	✓	2.298		
732	chapter 2 7.3	problem number 6	✓	2.485	✓	1.322		
733	chapter 2 7.3	problem number 7	✗	43.419	✓	23.664		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
734	chapter 2 7.3	problem number 8	✓	0.31	✓	0.343		
735	chapter 2 7.3	problem number 9	✗	40.246	✗	17.179		
736	chapter 2 7.3	problem number 10	✗	43.105	✗ (Timed out)	600.		
737	chapter 2 7.3	problem number 11	✓	1.069	✓	0.289		
738	chapter 2 7.3	problem number 12	✓	0.923	✓	2.878		
739	chapter 2 7.4	problem number 1	✓	0.696	✓	0.231		
740	chapter 2 7.4	problem number 2	✓	7.007	✓	0.145		
741	chapter 2 7.4	problem number 3	✗	21.042	✓	0.333		
742	chapter 2 7.4	problem number 4	✓	0.53	✓	0.33		
743	chapter 2 7.4	problem number 5	✗	7.845	✓	3.412		
744	chapter 2 7.4	problem number 6	✓	2.51	✓	1.249		
745	chapter 2 7.4	problem number 7	✗	43.597	✓	27.147		
746	chapter 2 7.4	problem number 8	✗	35.956	✗	13.193		
747	chapter 2 7.4	problem number 9	✗	40.981	✗	35.857		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
748	chapter 2 7.4	problem number 10	✗	43.309	✗ (Timed out)	600.		
749	chapter 2 7.4	problem number 11	✓	1.151	✓	0.413		
750	chapter 2 7.4	problem number 12	✓	1.212	✓	5.61		
751	chapter 2 8.1	problem number 1	✓	0.089	✓	0.056		
752	chapter 2 8.1	problem number 2	✓	0.343	✓	0.24		
753	chapter 2 8.1	problem number 3	✗	0.635	✓	0.134		
754	chapter 2 8.1	problem number 4	✓	0.288	✓	0.124		
755	chapter 2 8.1	problem number 5	✗	25.578	✓	0.903		
756	chapter 2 8.1	problem number 6	✗	0.951	✓	0.119		
757	chapter 2 8.1	problem number 7	✗	24.449	✗	5.854		
758	chapter 2 8.1	problem number 8	✗	24.683	✗	3.994		
759	chapter 2 8.1	problem number 9	✗	0.98	✓	0.113		
760	chapter 2 8.1	problem number 10	✗	29.935	✗	9.262		
761	chapter 2 8.1	problem number 11	✗	28.621	✗	14.006		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
762	chapter 2 8.1	problem number 12	✓	0.32	✓	0.214		
763	chapter 2 8.1	problem number 13	✓	2.757	✓	0.135		
764	chapter 2 8.2	problem number 1	✓	1.27	✓	0.623		
765	chapter 2 8.2	problem number 2	✗	22.195	✗	3.182		
766	chapter 2 8.2	problem number 3	✗	22.231	✗	3.627		
767	chapter 2 8.2	problem number 4	✓	0.356	✓	0.173		
768	chapter 2 8.2	problem number 5	✗	24.253	✗	3.854		
769	chapter 2 8.2	problem number 6	✗	1.4	✓	0.084		
770	chapter 2 8.2	problem number 7	✗	27.397	✗	9.035		
771	chapter 2 8.2	problem number 8	✗	28.142	✗	7.953		
772	chapter 2 8.2	problem number 9	✗	24.18	✗	5.314		
773	chapter 2 8.2	problem number 10	✓	0.394	✓	0.216		
774	chapter 2 8.2	problem number 11	✗	0.398	✓	0.381		
775	chapter 2 8.3	problem number 1	✗	27.712	✗	17.849		
776	chapter 2 8.3	problem number 2	✗	30.952	✗	119.76		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
777	chapter 2 8.3	problem number 3	✗	30.707	✗	11.339		
778	chapter 2 8.4	problem number 1	✗	22.104	✓	1.119		
779	chapter 2 8.4	problem number 2	✗	21.326	✗	4.121		
780	chapter 2 8.4	problem number 3	✗	21.383	✗	1.458		
781	chapter 2 8.4	problem number 4	✓	0.216	✓	0.088		
782	chapter 2 8.5	problem number 1	✗	27.023	✓	4.966		
783	chapter 2 8.5	problem number 2	✗	27.469	✗	15.424		
784	chapter 2 8.5	problem number 3	✗	27.157	✗	13.668		
785	chapter 2 8.5	problem number 4	✗	30.787	✗	33.838		
786	chapter 2 8.5	problem number 5	✗	31.085	✗	16.007		
787	chapter 2 8.6	problem number 1	✗	20.868	✗	3.438		
788	chapter 2 8.6	problem number 2	✗	21.009	✓	0.641		
789	chapter 2 8.6	problem number 3	✓	0.157	✓	0.047		
790	chapter 2 8.6	problem number 4	✗	0.489	✓	0.514		
791	chapter 2 8.6	problem number 5	✗	21.358	✗	2.316		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
792	chapter 2 8.6	problem number 6	✗	0.336	✓	0.099		
793	chapter 2 8.6	problem number 7	✗	0.389	✓	0.1		
794	chapter 2 8.6	problem number 8	✓	0.295	✓	0.13		
795	chapter 2 8.6	problem number 9	✗	0.785	✓	0.268		
796	chapter 2 8.6	problem number 10	✗	2.323	✓	0.423		
797	chapter 2 8.6	problem number 11	✓	0.396	✓	0.181		
798	chapter 2 8.6	problem number 12	✗	22.985	✓	1.155		
799	chapter 2 9.1	problem number 1	✓	0.087	✓	0.031		
800	chapter 2 9.1	problem number 2	✗	0.2	✓	0.276		
801	chapter 2 9.1	problem number 3	✗	0.513	✓	0.869		
802	chapter 2 9.1	problem number 4	✗	0.96	✗	2.84		
803	chapter 2 9.1	problem number 5	✗	3.074	✗	3.121		
804	chapter 2 9.2	problem number 1	✗	0.254	✓	0.066		
805	chapter 2 9.2	problem number 2	✗	0.08	✓	0.143		
806	chapter 2 9.2	problem number 3	✗	0.449	✓	0.272		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
807	chapter 2 9.2	problem number 4	✗	0.322	✓	0.187		
808	chapter 2 9.2	problem number 5	✗	0.666	✗	4.148		
809	chapter 2 9.2	problem number 6	✗	0.409	✓	0.224		
810	chapter 2 9.2	problem number 7	✗	0.585	✓	0.183		
811	chapter 2 9.2	problem number 8	✗	0.291	✓	0.19		
812	chapter 2 9.2	problem number 9	✗	0.349	✓	0.193		
813	chapter 2 9.2	problem number 10	✗	0.755	✓	0.73		
814	chapter 2 9.2	problem number 11	✗	0.404	✓	0.434		
815	chapter 2 9.2	problem number 12	✗	1.715	✗	3.088		
816	chapter 2 9.2	problem number 13	✗	2.636	✗	2.704		
817	chapter 2 9.2	problem number 14	✗	1.018	✓	0.777		
818	chapter 2 9.2	problem number 15	✗	0.406	✓	0.434		
819	chapter 2 9.2	problem number 16	✗	0.578	✓	0.244		
820	chapter 2 9.3	problem number 1	✗	5.761	✓	5.593		
821	chapter 2 9.3	problem number 2	✗	5.171	✗	5.184		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
822	chapter 2 9.3	problem number 3	✗	13.389	✗	6.655		
823	chapter 2 9.3	problem number 4	✗	2.527	✗	7.578		
824	chapter 2 9.3	problem number 5	✗	2.544	✗	6.623		
825	chapter 2 9.3	problem number 6	✗	23.376	✗	8.742		
826	chapter 2 9.3	problem number 7	✗	23.929	✗	6.257		
827	chapter 2 9.3	problem number 8	✗	13.317	✗	11.588		
828	chapter 2 9.3	problem number 9	✓	0.104	✓	0.059		
829	chapter 2 9.3	problem number 11	✗	0.382	✓	0.419		
830	chapter 2 9.3	problem number 12	✗	3.96	✓	6.998		
831	chapter 2 9.3	problem number 13	✗	11.486	✗	5.76		
832	chapter 2 9.3	problem number 14	✗	11.034	✗	3.727		
833	chapter 2 9.3	problem number 15	✗	18.354	✗	5.238		
834	chapter 2 9.3	problem number 16	✗	18.094	✗	5.404		
835	chapter 2 9.3	problem number 17	✗	4.589	✗	3.376		
836	chapter 2 9.3	problem number 18	✗	28.101	✗	6.614		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
837	chapter 2 9.3	problem number 19	✗	27.779	✗	4.787		
838	chapter 2 9.3	problem number 20	✗	34.12	✗	168.183		
839	chapter 2 9.3	problem number 21	✗	58.245	✗	54.284		
840	chapter 2 9.3	problem number 22	✗	12.827	✗	16.172		
841	chapter 2 9.3	problem number 23	✗	7.809	✗	91.033		
842	chapter 3 Examples	Example 1	✓	0.024	✓	0.055		
843	chapter 3 Examples	Example 2	✓	0.046	✓	0.06		
844	chapter 3 Examples	Example 3	✓	0.006	✓	0.014		
845	chapter 3 2.1	Problem 1	✓	0.009	✓	0.011		
846	chapter 3 2.1	Problem 2	✓	0.025	✓	0.025		
847	chapter 3 2.1	Problem 3	✓	0.027	✓	0.019		
848	chapter 3 2.1	Problem 4	✓	0.007	✓	0.008		
849	chapter 3 2.1	Problem 5	✓	0.316	✓	0.09		
850	chapter 3 2.1	Problem 6	✓	0.187	✓	0.105		
851	chapter 3 2.1	Problem 7	✓	0.044	✓	0.043		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
852	chapter 3 2.1	Problem 8	✓	0.067	✓	0.036		
853	chapter 3 2.2	Problem 1	✓	0.095	✓	0.026		
854	chapter 3 2.2	Problem 2	✓	0.22	✓	0.04		
855	chapter 3 2.2	Problem 3	✓	0.078	✓	0.033		
856	chapter 3 2.2	Problem 4	✓	0.395	✓	0.058		
857	chapter 3 2.2	Problem 5	✓	0.032	✓	0.038		
858	chapter 3 2.2	Problem 6	✓	0.254	✓ Contains unre-solved integral with RootOf	0.161		
859	chapter 3 2.2	Problem 7	✓	0.209	✓	0.028		
860	chapter 3 2.3	Problem 1	✓	0.026	✓	0.062		
861	chapter 3 2.3	Problem 2	✓	0.164	✓	0.056		
862	chapter 3 2.3	Problem 3	✓	0.151	✓	0.032		
863	chapter 3 2.3	Problem 4	✓	0.6	✓	0.056		
864	chapter 3 2.3	Problem 5	✓	0.024	✓	0.015		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
865	chapter 3 2.3	Problem 6	✓	0.126	✓	0.074		
866	chapter 3 2.4	Problem 1	✓	0.133	✓	0.027		
867	chapter 3 2.4	Problem 2	✓	0.056	✓	0.027		
868	chapter 3 2.4	Problem 3	✓	0.031	✓	0.026		
869	chapter 3 2.4	Problem 4	✓	0.045	✓	0.028		
870	chapter 3 2.4	Problem 5	✓	0.069	✓ Result has unresolved integral	0.089		
871	chapter 3 2.4	Problem 6	✓	0.117	✓ Result has unresolved integral	0.056		
872	chapter 3 2.4	Problem 7	✓	0.815	✓ Result has unresolved integral	1.253		
873	chapter 3 2.4	Problem 8	✓	2.198	✓ Result has unresolved integral	0.485		
874	chapter 3 2.4	Problem 9	✓	3.553	✓	0.416		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
875	chapter 3 2.4	Problem 10	✗	2.222	✓	10.174		
876	chapter 3 2.4	Problem 11	✓	0.505	✓	1.225		
877	chapter 3 3.1	Problem 1	✓	0.078	✓	0.047		
878	chapter 3 3.1	Problem 2	✓	0.063	✓	0.037		
879	chapter 3 3.1	Problem 3	✓	0.272	✓	0.081		
880	chapter 3 3.1	Problem 4	✓	0.292	✓	0.091		
881	chapter 3 3.1	Problem 5	✓	0.589	✓	0.198		
882	chapter 3 3.1	Problem 6	✓	0.732	✓	0.032		
883	chapter 3 3.1	Problem 7	✓	0.819	✓	0.172		
884	chapter 3 3.1	Problem 8	✗	3.636	✓	4.791		
885	chapter 3 3.1	Problem 9	✗	1.433	✓	2.537		
886	chapter 3 3.1	Problem 10	✓	1.86	✓	0.472		
887	chapter 3 3.1	Problem 11	✓	0.571	✓	0.739		
888	chapter 3 3.2	Problem 1	✓	0.195	✓	0.042		
889	chapter 3 3.2	Problem 2	✓	0.034	✓	0.074		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
890	chapter 3 3.2	Problem 3	✓	0.095	✓	0.058		
891	chapter 3 3.2	Problem 4	✓	0.6	✓	0.286		
892	chapter 3 3.2	Problem 5	✓	0.911	✓	1.434		
893	chapter 3 3.2	Problem 6	✓	0.051	✓	0.033		
894	chapter 3 3.2	Problem 7	✓	0.053	✓	0.029		
895	chapter 3 3.2	Problem 8	✓	0.436	✓	0.226		
896	chapter 3 3.2	Problem 9	✓	0.431	✓	0.643		
897	chapter 3 3.2	Problem 10	✓	0.402	✓	0.151		
898	chapter 3 3.2	Problem 11	✓	2.839	✓	0.311		
899	chapter 3 4.1	Problem 1	✓	0.119	✓	0.026		
900	chapter 3 4.1	Problem 2	✓	0.118	✓	0.029		
901	chapter 3 4.1	Problem 3	✗ (Timed out)	600.	✓	0.039		
902	chapter 3 4.1	Problem 4	✗ (Timed out)	600.	✓	7.128		
903	chapter 3 4.1	Problem 5	✓	16.09	✓	13.044		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
904	chapter 3 4.2	Problem 1	✓	0.117	✓	0.049		
905	chapter 3 4.2	Problem 2	✓	0.113	✓	0.025		
906	chapter 3 4.2	Problem 3	✓	0.385	✓	0.024		
907	chapter 3 4.2	Problem 4	✗ (Timed out)	600.	✓	0.803		
908	chapter 3 4.2	Problem 5	✓	153.043	✓	1.783		
909	chapter 3 4.3	Problem 1	✓	0.179	✓	0.032		
910	chapter 3 4.3	Problem 2	✓	0.097	✓	0.016		
911	chapter 3 4.3	Problem 3	✓	0.077	✓	0.018		
912	chapter 3 4.3	Problem 4	✓	538.891	✓	1.844		
913	chapter 3 4.3	Problem 5	✓	208.836	✓	1.301		
914	chapter 3 4.4	Problem 1	✓	0.271	✓	0.045		
915	chapter 3 4.4	Problem 2	✓	0.191	✓	0.017		
916	chapter 3 4.4	Problem 3	✓	0.093	✓	0.016		
917	chapter 3 4.4	Problem 4	✗ (Timed out)	600.	✓	1.729		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
918	chapter 3 4.4	Problem 5	✓	253.475	✓	1.303		
919	chapter 3 4.5	Problem 1	✓	0.126	✓	0.021		
920	chapter 3 4.5	Problem 2	✓	0.275	✓	0.073		
921	chapter 3 4.5	Problem 3	✓	0.11	✓	0.046		
922	chapter 3 4.5	Problem 4	✓	0.192	✓	0.094		
923	chapter 3 4.5	Problem 5	✓	0.177	✓	0.039		
924	chapter 3 5.1	Problem 1	✓	0.099	✓	0.019		
925	chapter 3 5.1	Problem 2	✓	0.029	✓	0.021		
926	chapter 3 5.1	Problem 3	✓	0.293	✓	0.484		
927	chapter 3 5.1	Problem 4	✓	1.482	✓	0.156		
928	chapter 3 5.1	Problem 5	✓	1.004	✓	0.21		
929	chapter 3 5.1	Problem 6	✗	0.326	✓	1.42		
930	chapter 3 5.2	Problem 1	✓	0.121	✓	0.023		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
931	chapter 3 5.2	Problem 2	✓	0.32	✓ Result has un-resolved integrals	0.683		
932	chapter 3 5.2	Problem 3	✓	0.149	✓	0.312		
933	chapter 3 5.3	Problem 4	✓	0.08	✓ Result has un-resolved integrals	0.114		
934	chapter 3 5.3	Problem 5	✓	0.09	✓	0.141		
935	chapter 3 5.3	Problem 6	✓	1.75	✓	1.15		
936	chapter 3 6.1	Problem 1	✓	0.129	✓	0.024		
937	chapter 3 6.1	Problem 2	✓	0.12	✓	0.017		
938	chapter 3 6.1	Problem 3	✓	0.087	✓	0.018		
939	chapter 3 6.1	Problem 4	✓	240.979	✓	3.549		
940	chapter 3 6.1	Problem 5	✓	92.342	✓ Result has un-resolved integrals	4.333		
941	chapter 3 6.2	Problem 1	✓	0.131	✓	0.041		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
942	chapter 3 6.2	Problem 2	✓	0.121	✓	0.027		
943	chapter 3 6.2	Problem 3	✓	0.085	✓	0.023		
944	chapter 3 6.2	Problem 4	✓	243.081	✓	1.94		
945	chapter 3 6.2	Problem 5	✓	53.544	✓	2.722		
946	chapter 3 6.3	Problem 1	✓	0.201	✓	0.032		
947	chapter 3 6.3	Problem 2	✓	0.107	✓	0.015		
948	chapter 3 6.3	Problem 3	✓	0.085	✓	0.015		
949	chapter 3 6.3	Problem 4	✓	73.994	✓	2.349		
950	chapter 3 6.3	Problem 5	✓	30.294	✓	3.259		
951	chapter 3 6.4	Problem 1	✓	0.295	✓	0.048		
952	chapter 3 6.4	Problem 2	✓	0.179	✓	0.016		
953	chapter 3 6.4	Problem 3	✓	0.102	✓	0.015		
954	chapter 3 6.4	Problem 4	✓	86.433	✓	2.27		
955	chapter 3 6.4	Problem 5	✓	29.022	✓	3.24		
956	chapter 3 6.5	Problem 1	✓	0.152	✓	0.034		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
957	chapter 3 6.5	Problem 2	✓	0.188	✓	0.048		
958	chapter 3 6.5	Problem 3	✓	0.417	✓	0.033		
959	chapter 3 6.5	Problem 4	✓	0.244	✓	0.17		
960	chapter 3 6.5	Problem 5	✓	0.309	✓	0.362		
961	chapter 3 6.5	Problem 6	✓	0.164	✓	0.128		
962	chapter 3 7.1	Problem 1	✓	1.178	✓	0.064		
963	chapter 3 7.1	Problem 2	✓	0.455	✓	0.049		
964	chapter 3 7.1	Problem 3	✓	0.148	✓	0.019		
965	chapter 3 7.1	Problem 4	✓	8.92	✓	0.93		
966	chapter 3 7.1	Problem 5	✓	1.269	✓	0.885		
967	chapter 3 7.2	Problem 1	✓	1.238	✓	0.03		
968	chapter 3 7.2	Problem 2	✓	0.764	✓	0.017		
969	chapter 3 7.2	Problem 3	✓	0.205	✓	0.017		
970	chapter 3 7.2	Problem 4	✓	13.428	✓	0.84		
971	chapter 3 7.2	Problem 5	✓	1.478	✓	0.598		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
972	chapter 3 7.3	Problem 1	✓	0.083	✓	0.194		
973	chapter 3 7.3	Problem 2	✓	0.106	✓	0.083		
974	chapter 3 7.3	Problem 3	✓	0.064	✓	0.04		
975	chapter 3 7.3	Problem 4	✓	0.548	✓	3.179		
976	chapter 3 7.3	Problem 5	✓	0.681	✓	1.381		
977	chapter 3 7.4	Problem 1	✓	0.058	✓	0.061		
978	chapter 3 7.4	Problem 2	✓	0.122	✓	0.023		
979	chapter 3 7.4	Problem 3	✓	0.07	✓	0.037		
980	chapter 3 7.4	Problem 4	✓	0.609	✓	2.039		
981	chapter 3 7.4	Problem 5	✓	0.744	✓	0.657		
982	chapter 3 8.1	Problem 1	✓	0.01	✓	0.013		
983	chapter 3 8.1	Problem 2	✓	0.041	✓	0.017		
984	chapter 3 8.1	Problem 3	✓	0.032	✓	0.027		
985	chapter 3 8.1	Problem 4	✓	0.03	✓	0.058		
986	chapter 3 8.1	Problem 5	✓	0.034	✓	0.027		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
987	chapter 3 8.1	Problem 6	✓	0.06	✓	0.021		
988	chapter 3 8.1	Problem 7	✓	0.082	✓	0.061		
989	chapter 3 8.1	Problem 8	✓	0.197	✓	0.04		
990	chapter 3 8.1	Problem 9	✓	0.185	✓	0.02		
991	chapter 3 8.1	Problem 10	✓	0.14	✓	0.014		
992	chapter 3 8.1	Problem 11	✓	0.213	✓	0.231		
993	chapter 3 8.1	Problem 12	✓	0.554	✓	0.147		
994	chapter 3 8.1	Problem 13	✗	0.433	✓	0.694		
995	chapter 3 8.1	Problem 14	✓	0.241	✓	0.19		
996	chapter 3 8.1	Problem 15	✓	0.731	✓	0.343		
997	chapter 3 8.1	Problem 16	✓	0.084	✓	0.019		
998	chapter 3 8.2	Problem 1	✓	0.018	✓	0.025		
999	chapter 3 8.2	Problem 2	✓	0.019	✓	0.015		
1000	chapter 3 8.2	Problem 3	✓	0.06	✓	0.039		
1001	chapter 3 8.2	Problem 4	✗	0.092	✓	0.058		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1002	chapter 3 8.2	Problem 5	✓	0.57	✓	0.19		
1003	chapter 3 8.2	Problem 6	✓	0.061	✓	0.018		
1004	chapter 3 8.2	Problem 7	✗	0.187	✓	0.069		
1005	chapter 3 8.3	Problem 1	✓	0.037	✓	0.021		
1006	chapter 3 8.3	Problem 2	✓	0.018	✓	0.011		
1007	chapter 3 8.3	Problem 3	✓	0.037	✓	0.02		
1008	chapter 3 8.3	Problem 4	✓	0.061	✓	0.022		
1009	chapter 3 8.3	Problem 5	✓	0.066	✓	0.191		
1010	chapter 3 8.3	Problem 6	✓	0.094	✓	0.082		
1011	chapter 3 8.3	Problem 7	✓	0.048	✓	0.023		
1012	chapter 3 8.3	Problem 8	✓	0.216	✓	0.363		
1013	chapter 3 8.4	Problem 1	✓	0.01	✓	0.016		
1014	chapter 3 8.4	Problem 2	✓	0.054	✓	0.02		
1015	chapter 3 8.4	Problem 3	✓	0.078	✓	0.042		
1016	chapter 3 8.4	Problem 4	✓	0.193	✓	0.123		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1017	chapter 3 8.4	Problem 5	✓	0.523	✓	0.269		
1018	chapter 3 8.4	Problem 6	✗	0.478	✓	0.497		
1019	chapter 3 8.4	Problem 7	✗	0.159	✓ Contains RootOf	0.033		
1020	chapter 4 1.1	Example 1	✓	0.033	✓	0.025		
1021	chapter 4 1.1	Example 2	✓	0.102	✓	0.022		
1022	chapter 4 1.1	Example 3	✓	0.007	✓	0.012		
1023	chapter 4 2.1	Problem 1	✓	0.007	✓	0.013		
1024	chapter 4 2.1	Problem 2	✓	0.03	✓	0.016		
1025	chapter 4 2.1	Problem 3	✓	0.018	✓	0.011		
1026	chapter 4 2.1	Problem 4	✓	0.016	✓	0.019		
1027	chapter 4 2.1	Problem 5	✓	0.018	✓	0.009		
1028	chapter 4 2.1	Problem 6	✓	0.027	✓	0.013		
1029	chapter 4 2.1	Problem 7	✗	0.221	✗ (Timed out)	600.		
1030	chapter 4 2.2	Problem 1	✓	0.042	✓	0.053		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1031	chapter 4 2.2	Problem 2	✓	0.03	✓	0.03		
1032	chapter 4 2.2	Problem 3	✓	0.146	✓	0.044		
1033	chapter 4 2.2	Problem 4	✓	0.164	✓	0.048		
1034	chapter 4 2.2	Problem 5	✓	0.158	✓	0.051		
1035	chapter 4 2.2	Problem 6	✓	0.094	✓	0.052		
1036	chapter 4 2.2	Problem 7	✗	15.1	✓	1.147		
1037	chapter 4 2.2	Problem 8	✗	1.928	✓	0.823		
1038	chapter 4 2.3	Problem 1	✓	0.023	✓	0.038		
1039	chapter 4 2.3	Problem 2	✓	0.03	✓	0.038		
1040	chapter 4 2.3	Problem 3	✓	0.023	✓	0.022		
1041	chapter 4 2.3	Problem 4	✓	0.149	✓	0.08		
1042	chapter 4 2.3	Problem 5	✓	0.056	✓	0.058		
1043	chapter 4 2.3	Problem 6	✓	0.135	✓	0.058		
1044	chapter 4 2.4	Problem 1	✓	0.183	✓	0.029		
1045	chapter 4 2.4	Problem 2 case $n \neq -1, n \neq -2$	✓	0.049	✓	0.071		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1046	chapter 4 2.4	Problem 2 case $n = -1$	✓	0.058	✓	0.029		
1047	chapter 4 2.4	Problem 2 case $n = -2$	✓	0.098	✓	0.032		
1048	chapter 4 2.4	Problem 3	✓	0.032	✓	0.022		
1049	chapter 4 2.4	Problem 4	✓	0.046	✓	0.024		
1050	chapter 4 2.4	Problem 5	✓	0.071	✓	0.021		
1051	chapter 4 2.4	Problem 6	✓	0.121	✓	0.029		
1052	chapter 4 2.4	Problem 7	✓	0.814	✓	1.683		
1053	chapter 4 2.4	Problem 8	✓	1.868	✓	0.756		
1054	chapter 4 2.4	Problem 9	✓	3.649	✓	4.667		
1055	chapter 4 2.4	Problem 10	✓	1.421	✓	12.338		
1056	chapter 4 2.4	Problem 11	✓	0.509	✓	1.318		
1057	chapter 4 2.4	Problem 12	✗	0.506	✓	0.936		
1058	chapter 4 2.4	Problem 13	✗	0.512	✓	0.467		
1059	chapter 4 3.1	Problem 1	✓	0.159	✓	0.03		
1060	chapter 4 3.1	Problem 2	✓	0.124	✓	0.032		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1061	chapter 4 3.1	Problem 3	✓	0.281	✓	0.063		
1062	chapter 4 3.1	Problem 4	✓	0.377	✓	0.079		
1063	chapter 4 3.1	Problem 5	✓	0.588	✓	0.361		
1064	chapter 4 3.1	Problem 6	✓	4.021	✓	0.14		
1065	chapter 4 3.1	Problem 7	✗	2.408	✓	2.438		
1066	chapter 4 3.1	Problem 8	✗	1.317	✓	1.672		
1067	chapter 4 3.1	Problem 9	✓	1.984	✓	1.121		
1068	chapter 4 3.1	Problem 10	✓	0.398	✓	0.024		
1069	chapter 4 3.2	Problem 1	✓	0.572	✓	0.037		
1070	chapter 4 3.2	Problem 2	✓	0.078	✓	0.024		
1071	chapter 4 3.2	Problem 3	✓	0.052	✓	0.024		
1072	chapter 4 3.2	Problem 4	✓	0.381	✓	0.14		
1073	chapter 4 3.2	Problem 5	✓	0.444	✓	0.688		
1074	chapter 4 3.2	Problem 6	✓	0.406	✓	0.149		
1075	chapter 4 3.2	Problem 7	✓	2.94	✓	0.224		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1076	chapter 4 4.1	Problem 1	✓	0.483	✓	0.031		
1077	chapter 4 4.1	Problem 2	✓	0.518	✓	0.025		
1078	chapter 4 4.1	Problem 3	✓	0.364	✓	0.021		
1079	chapter 4 4.1	Problem 4	✗ (Timed out)	600.	✓	1.463		
1080	chapter 4 4.1	Problem 5	✓	281.809	✓	0.964		
1081	chapter 4 4.2	Problem 1	✓	0.546	✓	0.05		
1082	chapter 4 4.2	Problem 2	✓	0.509	✓	0.036		
1083	chapter 4 4.2	Problem 3	✓	0.381	✓	0.035		
1084	chapter 4 4.2	Problem 4	✗ (Timed out)	600.	✓	0.512		
1085	chapter 4 4.2	Problem 5	✓	151.99	✓	0.727		
1086	chapter 4 4.3	Problem 1	✓	0.195	✓	0.096		
1087	chapter 4 4.3	Problem 2	✓	0.098	✓	0.099		
1088	chapter 4 4.3	Problem 3	✓	0.094	✓	0.074		
1089	chapter 4 4.3	Problem 4	✓	575.866	✓	0.877		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1090	chapter 4 4.3	Problem 5	✓	204.54	✓	0.903		
1091	chapter 4 4.4	Problem 1	✓	0.417	✓	0.105		
1092	chapter 4 4.4	Problem 2	✓	0.237	✓	0.081		
1093	chapter 4 4.4	Problem 3	✓	0.125	✓	0.067		
1094	chapter 4 4.4	Problem 4	✓	578.971	✓	1.006		
1095	chapter 4 4.4	Problem 5	✓	225.853	✓	1.012		
1096	chapter 4 4.5	Problem 1	✓	0.534	✓	0.025		
1097	chapter 4 4.5	Problem 2	✓	0.401	✓	0.048		
1098	chapter 4 4.5	Problem 3	✓	0.163	✓	0.126		
1099	chapter 4 4.5	Problem 4	✓	0.084	✓	0.039		
1100	chapter 4 4.5	Problem 5	✓	0.28	✓	0.359		
1101	chapter 4 4.5	Problem 6	✓	0.823	✓	1.751		
1102	chapter 4 5.1	Problem 1	✓	0.1	✓	0.688		
1103	chapter 4 5.1	Problem 2	✓	0.041	✓	0.059		
1104	chapter 4 5.1	Problem 3	✓	1.38	✓	0.147		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1105	chapter 4 5.1	Problem 4	✓	0.783	✓	0.254		
1106	chapter 4 5.1	Problem 5	✓	0.215	✓	0.268		
1107	chapter 4 5.1	Problem 6	✗	0.356	✓	1.338		
1108	chapter 4 5.2	Problem 1	✓	0.236	✓	1.434		
1109	chapter 4 5.2	Problem 2	✓	0.625	✓	0.385		
1110	chapter 4 5.2	Problem 3	✓	0.155	✓	0.023		
1111	chapter 4 5.2	Problem 4	✓	0.08	✓	0.043		
1112	chapter 4 5.2	Problem 5	✓	0.076	✓	0.196		
1113	chapter 4 5.2	Problem 6	✓	1.543	✓	6.035		
1114	chapter 4 6.1	Problem 1	✓	0.485	✓	0.03		
1115	chapter 4 6.1	Problem 2	✓	0.278	✓	0.031		
1116	chapter 4 6.1	Problem 3	✓	0.307	✓	0.024		
1117	chapter 4 6.1	Problem 4	✓	242.628	✓	2.787		
1118	chapter 4 6.1	Problem 5	✓	87.276	✓	1.783		
1119	chapter 4 6.2	Problem 1	✓	0.509	✓	0.026		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1120	chapter 4 6.2	Problem 2	✓	0.522	✓	0.054		
1121	chapter 4 6.2	Problem 3	✓	0.378	✓	0.018		
1122	chapter 4 6.2	Problem 4	✓	240.874	✓	1.084		
1123	chapter 4 6.2	Problem 5	✓	51.635	✓	1.277		
1124	chapter 4 6.3	Problem 1	✓	0.105	✓	0.053		
1125	chapter 4 6.3	Problem 2	✓	0.209	✓	0.067		
1126	chapter 4 6.3	Problem 3	✓	0.089	✓	0.044		
1127	chapter 4 6.3	Problem 4	✓	73.537	✓	1.22		
1128	chapter 4 6.3	Problem 5	✓	30.743	✓	1.144		
1129	chapter 4 6.4	Problem 1	✓	0.256	✓	0.047		
1130	chapter 4 6.4	Problem 2	✓	0.443	✓	0.084		
1131	chapter 4 6.4	Problem 3	✓	0.135	✓	0.04		
1132	chapter 4 6.4	Problem 4	✓	83.174	✓	2.312		
1133	chapter 4 6.4	Problem 5	✓	28.145	✓	1.356		
1134	chapter 4 6.5	Problem 1	✓	0.217	✓	0.044		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1135	chapter 4 6.5	Problem 2	✓	0.081	✓	0.077		
1136	chapter 4 6.5	Problem 3	✓	0.21	✓	0.163		
1137	chapter 4 6.5	Problem 4	✓	0.217	✓	0.536		
1138	chapter 4 6.5	Problem 5	✓	0.797	✓	0.249		
1139	chapter 4 6.5	Problem 6	✓	0.201	✓	0.316		
1140	chapter 4 7.1	Problem 1	✓	1.176	✓	0.058		
1141	chapter 4 7.1	Problem 2	✓	0.431	✓	0.061		
1142	chapter 4 7.1	Problem 3	✓	0.492	✓	0.05		
1143	chapter 4 7.1	Problem 4	✓	0.094	✓	3.616		
1144	chapter 4 7.1	Problem 5	✓	1.293	✓	2.93		
1145	chapter 4 7.2	Problem 1	✓	1.358	✓	0.07		
1146	chapter 4 7.2	Problem 2	✓	0.704	✓	0.027		
1147	chapter 4 7.2	Problem 3	✓	0.647	✓	0.034		
1148	chapter 4 7.2	Problem 4	✓	0.105	✓	2.917		
1149	chapter 4 7.2	Problem 5	✓	0.382	✓	2.892		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1150	chapter 4 7.3	Problem 1	✓	0.174	✓	0.157		
1151	chapter 4 7.3	Problem 2	✓	0.12	✓	0.127		
1152	chapter 4 7.3	Problem 3	✓	0.435	✓	0.148		
1153	chapter 4 7.3	Problem 4	✓	0.133	✓	1.181		
1154	chapter 4 7.3	Problem 5	✓	0.409	✓	1.016		
1155	chapter 4 7.4	Problem 1	✓	0.146	✓	0.053		
1156	chapter 4 7.4	Problem 2	✓	0.133	✓	0.039		
1157	chapter 4 7.4	Problem 3	✓	0.345	✓	0.087		
1158	chapter 4 7.4	Problem 4	✓	0.138	✓	0.871		
1159	chapter 4 7.4	Problem 5	✓	0.408	✓	0.94		
1160	chapter 4 8.1	Problem 1	✓	0.01	✓	0.029		
1161	chapter 4 8.1	Problem 2	✓	0.041	✓	0.018		
1162	chapter 4 8.1	Problem 3	✓	0.032	✓	0.019		
1163	chapter 4 8.1	Problem 4	✓	0.031	✓	0.019		
1164	chapter 4 8.1	Problem 5	✓	0.035	✓	0.023		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1165	chapter 4 8.1	Problem 6	✓	0.058	✓	0.017		
1166	chapter 4 8.1	Problem 7	✓	0.084	✓	0.045		
1167	chapter 4 8.1	Problem 8	✓	0.193	✓	0.054		
1168	chapter 4 8.1	Problem 9	✓	0.19	✓	0.055		
1169	chapter 4 8.1	Problem 10	✓	0.142	✓	0.036		
1170	chapter 4 8.1	Problem 11	✓	0.215	✓	0.114		
1171	chapter 4 8.1	Problem 12	✓	0.549	✓	0.188		
1172	chapter 4 8.1	Problem 13	✗	23.95	✗	4.932		
1173	chapter 4 8.1	Problem 14	✓	0.247	✓	0.081		
1174	chapter 4 8.1	Problem 15	✓	0.242	✓	0.051		
1175	chapter 4 8.2	Problem 1	✓	0.018	✓	0.02		
1176	chapter 4 8.2	Problem 2	✓	0.019	✓	0.017		
1177	chapter 4 8.2	Problem 3	✓	0.058	✓	0.021		
1178	chapter 4 8.2	Problem 4	✗	0.085	✓ contains RootOf	0.043		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1179	chapter 4 8.2	Problem 5	✗	22.035	✗	2.174		
1180	chapter 4 8.2	Problem 6	✗	0.178	✓ has RootOf	0.072		
1181	chapter 4 8.2	Problem 7	✗	0.05	✓ has RootOf	0.039		
1182	chapter 4 8.3	Problem 1	✓	0.034	✓	0.021		
1183	chapter 4 8.3	Problem 2	✓	0.018	✓	0.017		
1184	chapter 4 8.3	Problem 3	✓	0.034	✓	0.017		
1185	chapter 4 8.3	Problem 4	✓	0.096	✓	0.928		
1186	chapter 4 8.3	Problem 5	✓	0.09	✓	0.034		
1187	chapter 4 8.3	Problem 6	✓	0.045	✓	0.025		
1188	chapter 4 8.3	Problem 7	✓	0.053	✓	1.04		
1189	chapter 4 8.4	Problem 1	✓	0.009	✓	0.013		
1190	chapter 4 8.4	Problem 2	✓	0.051	✓	0.023		
1191	chapter 4 8.4	Problem 3	✓	0.072	✓	0.022		
1192	chapter 4 8.4	Problem 4	✓	0.185	✓	0.034		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1193	chapter 4 8.4	Problem 5	✓	0.529	✓	0.224		
1194	chapter 4 8.4	Problem 6	✗	23.902	✗	3.376		
1195	chapter 4 8.4	Problem 7	✗	0.056	✓ has RootOf	0.062		
1196	chapter 5 2.1	Problem 1	✓	0.018	✓	0.019		
1197	chapter 5 2.1	Problem 2	✓	0.031	✓	0.015		
1198	chapter 5 2.1	Problem 3	✓	0.051	✓	0.033		
1199	chapter 5 2.1	Problem 4	✓	0.058	✓	0.035		
1200	chapter 5 2.1	Problem 5	✓	0.378	✓	0.054		
1201	chapter 5 2.1	Problem 6	✓	0.098	✓	0.031		
1202	chapter 5 2.1	Problem 7	✓	5.912	✗ (Timed out)	600.		
1203	chapter 5 2.1	Problem 8	✓	1.624	✓	0.491		
1204	chapter 5 2.2	Problem 1	✓	0.121	✓	0.027		
1205	chapter 5 2.2	Problem 2	✓	0.149	✓	0.028		
1206	chapter 5 2.2	Problem 3	✓	0.023	✓	0.021		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1207	chapter 5 2.2	Problem 4	✓	0.27	✓	0.038		
1208	chapter 5 2.2	Problem 5	✗ (Timed out)	600.	✓	24.562		
1209	chapter 5 2.2	Problem 6	✗ (Timed out)	600.	✓	3.811		
1210	chapter 5 2.2	Problem 7	✗	42.122	✓	0.731		
1211	chapter 5 2.2	Problem 8	✗	27.288	✓	24.03		
1212	chapter 5 2.3	Problem 1	✓	0.402	✓	0.218		
1213	chapter 5 2.3	Problem 2	✓	0.428	✓	0.06		
1214	chapter 5 2.3	Problem 3	✓	3.345	✓	0.142		
1215	chapter 5 2.3	Problem 4	✓	3.251	✓	0.002		
1216	chapter 5 2.3	Problem 5	✓	0.545	✓	0.117		
1217	chapter 5 2.3	Problem 6	✓	0.17	✓	0.132		
1218	chapter 5 2.3	Problem 7	✓	1.32	✓ contains RootOf	0.422		
1219	chapter 5 2.4	Problem 1	✓	0.544	✓	0.189		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1220	chapter 5 2.4	Problem 2	✓	0.148	✓	0.109		
1221	chapter 5 2.4	Problem 3	✓	0.049	✓	0.131		
1222	chapter 5 2.4	Problem 4	✓	0.141	✓	0.082		
1223	chapter 5 2.4	Problem 5	✓	0.63	✓	1.004		
1224	chapter 5 2.4	Problem 6	✓	0.492	✓	1.019		
1225	chapter 5 2.4	Problem 7	✓	0.295	✓	0.155		
1226	chapter 5 2.4	Problem 8	✓	0.274	✓	0.124		
1227	chapter 5 2.4	Problem 9	✓	6.684	✓	8.126		
1228	chapter 5 2.4	Problem 10	✓	2.088	✓	2.791		
1229	chapter 5 2.4	Problem 11	✓	4.096	✓	3.054		
1230	chapter 5 2.4	Problem 12	✓	1.276	✓	0.976		
1231	chapter 5 3.1	Problem 1	✓	0.371	✓	0.194		
1232	chapter 5 3.1	Problem 2	✓	0.56	✓	0.252		
1233	chapter 5 3.1	Problem 3	✓	1.446	✓	0.403		
1234	chapter 5 3.1	Problem 4	✗	2.4	✓	1.591		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1235	chapter 5 3.1	Problem 5	✗	1.468	✓	1.271		
1236	chapter 5 3.1	Problem 6	✓	5.785	✓	0.597		
1237	chapter 5 3.1	Problem 7	✓	0.319	✓	0.054		
1238	chapter 5 3.1	Problem 8	✓	0.333	✓	0.144		
1239	chapter 5 3.2	Problem 1	✓	0.176	✓	0.105		
1240	chapter 5 3.2	Problem 2	✓	0.157	✓	0.049		
1241	chapter 5 3.2	Problem 3	✓	0.131	✓	0.05		
1242	chapter 5 3.2	Problem 4	✓	0.787	✓	0.975		
1243	chapter 5 3.2	Problem 5	✓	0.398	✓	0.199		
1244	chapter 5 3.2	Problem 6	✓	0.289	✓	0.109		
1245	chapter 5 3.2	Problem 7	✓	3.02	✓	0.421		
1246	chapter 5 3.2	Problem 8	✓	0.094	✓	0.033		
1247	chapter 5 3.2	Problem 9	✓	0.605	✓	0.473		
1248	chapter 5 3.2	Problem 10	✓	0.658	✓	0.202		
1249	chapter 5 4.1	Problem 1	✓	9.188	✓	11.318		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1250	chapter 5 4.1	Problem 2	✓	267.327	✓	0.232		
1251	chapter 5 4.1	Problem 3	✓	73.281	✓	5.817		
1252	chapter 5 4.1	Problem 4	✓	3.952	✓	6.87		
1253	chapter 5 4.1	Problem 5	✓	0.726	✓	1.381		
1254	chapter 5 4.2	Problem 1	✓	9.015	✓	9.096		
1255	chapter 5 4.2	Problem 2	✓	136.984	✓	0.211		
1256	chapter 5 4.2	Problem 3	✓	74.284	✓	4.123		
1257	chapter 5 4.2	Problem 4	✓	0.151	✓	0.082		
1258	chapter 5 4.2	Problem 5	✓	3.934	✓	5.727		
1259	chapter 5 4.2	Problem 6	✓	2.066	✓	1.402		
1260	chapter 5 4.3	Problem 1	✓	10.527	✓	1.108		
1261	chapter 5 4.3	Problem 2	✓	82.418	✓	0.242		
1262	chapter 5 4.3	Problem 3	✗ (Timed out)	600.	✓	2.259		
1263	chapter 5 4.3	Problem 4	✓	5.011	✓	3.106		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1264	chapter 5 4.3	Problem 5	✓	1.458	✓	1.614		
1265	chapter 5 4.4	Problem 1	✓	10.298	✓	1.142		
1266	chapter 5 4.4	Problem 2	✓	80.641	✓	0.246		
1267	chapter 5 4.4	Problem 3	✗ (Timed out)	600.	✓	2.331		
1268	chapter 5 4.4	Problem 4	✓	5.027	✓	2.056		
1269	chapter 5 4.4	Problem 5	✓	1.452	✓	1.685		
1270	chapter 5 4.5	Problem 1	✓	1.022	✓	11.838		
1271	chapter 5 4.5	Problem 2	✓	7.489	✓	8.512		
1272	chapter 5 4.5	Problem 3	✓	0.451	✓	1.711		
1273	chapter 5 4.5	Problem 4	✓	15.107	✓	0.833		
1274	chapter 5 4.5	Problem 5	✓	4.184	✓	7.995		
1275	chapter 5 4.5	Problem 6	✓	3.921	✓	4.434		
1276	chapter 5 5.1	Problem 1	✓	0.253	✓	0.648		
1277	chapter 5 5.1	Problem 2	✓	0.087	✓	0.095		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1278	chapter 5 5.1	Problem 3	✓	0.604	✓	1.319		
1279	chapter 5 5.1	Problem 4	✗	0.14	✓	0.283		
1280	chapter 5 5.1	Problem 5	✓	0.458	✓	590.		
1281	chapter 5 5.1	Problem 6	✓	0.192	✓	580.		
1282	chapter 5 5.2	Problem 1	✓	0.653	✓	0.41		
1283	chapter 5 5.2	Problem 2	✓	0.565	✓	0.39		
1284	chapter 5 5.2	Problem 3	✓	0.324	✓	0.206		
1285	chapter 5 5.2	Problem 4	✓	0.611	✓	0.226		
1286	chapter 5 5.2	Problem 5	✓	0.205	✓	0.427		
1287	chapter 5 5.2	Problem 6	✓	1.445	✓	0.563		
1288	chapter 5 5.2	Problem 7	✓	0.307	✓	15.351		
1289	chapter 5 6.1	Problem 1	✓	0.588	✓	0.06		
1290	chapter 5 6.1	Problem 2	✓	1.067	✓	39.069		
1291	chapter 5 6.1	Problem 3	✓	3.414	✓	31.454		
1292	chapter 5 6.1	Problem 4	✓	1.999	✓	0.293		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1293	chapter 5 6.1	Problem 5	✓	4.365	✓	13.929		
1294	chapter 5 6.1	Problem 6	✓	1.809	✓	34.265		
1295	chapter 5 6.1	Problem 7	✓	0.835	✓	29.138		
1296	chapter 5 6.2	Problem 1	✓	0.569	✓	0.076		
1297	chapter 5 6.2	Problem 2	✓	1.338	✓	13.931		
1298	chapter 5 6.2	Problem 3	✓	3.409	✓	25.575		
1299	chapter 5 6.2	Problem 4	✓	1.509	✓	0.39		
1300	chapter 5 6.2	Problem 5	✓	4.631	✓	0.464		
1301	chapter 5 6.2	Problem 6	✓	1.8	✓	24.834		
1302	chapter 5 6.3	Problem 1	✓	2.393	✓	0.233		
1303	chapter 5 6.3	Problem 2	✓	5.764	✓	1.664		
1304	chapter 5 6.3	Problem 3	✓	2.392	✓	1.365		
1305	chapter 5 6.3	Problem 4	✓	0.926	✓	1.762		
1306	chapter 5 6.3	Problem 5	✓	5.744	✓	0.24		
1307	chapter 5 6.3	Problem 6	✓	1.841	✓	23.539		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1308	chapter 5 6.3	Problem 7	✓	1.21	✓	11.926		
1309	chapter 5 6.4	Problem 1	✓	5.518	✓	0.291		
1310	chapter 5 6.4	Problem 2	✓	6.402	✓	1.45		
1311	chapter 5 6.4	Problem 3	✓	2.301	✓	1.403		
1312	chapter 5 6.4	Problem 4	✓	1.288	✓	20.55		
1313	chapter 5 6.4	Problem 5	✓	7.149	✓	0.294		
1314	chapter 5 6.4	Problem 6	✓	1.823	✓	9.826		
1315	chapter 5 6.4	Problem 7	✓	1.218	✓	25.722		
1316	chapter 5 6.5	Problem 1	✓	1.143	✓	33.25		
1317	chapter 5 6.5	Problem 2	✓	3.027	✓	12.651		
1318	chapter 5 6.5	Problem 3	✓	1.456	✓	0.475		
1319	chapter 5 6.5	Problem 4	✓	1.893	✓	0.156		
1320	chapter 5 6.5	Problem 5	✓	1.882	✓	1.702		
1321	chapter 5 6.5	Problem 6	✓	1.729	✓	28.343		
1322	chapter 5 6.5	Problem 7	✓	1.887	✓	1.788		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1323	chapter 5 7.1	Problem 1	✓	1.11	✓	0.448		
1324	chapter 5 7.1	Problem 2	✓	1.478	✓	0.054		
1325	chapter 5 7.1	Problem 3	✓	2.364	✓	0.471		
1326	chapter 5 7.1	Problem 4	✓	0.48	✓	19.264		
1327	chapter 5 7.1	Problem 5	✓	0.119	✓	5.595		
1328	chapter 5 7.2	Problem 1	✓	1.366	✓	0.164		
1329	chapter 5 7.2	Problem 2	✓	1.493	✓	0.049		
1330	chapter 5 7.2	Problem 3	✓	4.798	✓	0.195		
1331	chapter 5 7.2	Problem 4	✓	0.463	✓	3.644		
1332	chapter 5 7.2	Problem 5	✓	0.131	✓	5.156		
1333	chapter 5 7.3	Problem 1	✓	2.42	✓	0.905		
1334	chapter 5 7.3	Problem 2	✓	1.782	✓	260.		
1335	chapter 5 7.3	Problem 3	✓	2.437	✓	9.016		
1336	chapter 5 7.3	Problem 4	✓	0.431	✓	2.074		
1337	chapter 5 7.3	Problem 5	✓	0.162	✓	0.908		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1338	chapter 5 7.4	Problem 1	✓	2.588	✓	1.773		
1339	chapter 5 7.4	Problem 2	✓	1.769	✓	618.426		
1340	chapter 5 7.4	Problem 3	✓	3.26	✓	8.326		
1341	chapter 5 7.4	Problem 4	✓	0.477	✓	1.051		
1342	chapter 5 7.4	Problem 5	✓	0.166	✓	0.643		
1343	chapter 5 8.1	Problem 1	✓	0.039	✓	0.026		
1344	chapter 5 8.1	Problem 2	✓	0.085	✓	0.042		
1345	chapter 5 8.1	Problem 3	✓	0.1	✓	0.037		
1346	chapter 5 8.1	Problem 4	✓	0.08	✓	0.031		
1347	chapter 5 8.1	Problem 5	✓	0.153	✓	0.049		
1348	chapter 5 8.1	Problem 6	✓	0.059	✓	0.028		
1349	chapter 5 8.1	Problem 7	✓	0.229	✓	0.063		
1350	chapter 5 8.1	Problem 8	✓	0.213	✓	0.143		
1351	chapter 5 8.1	Problem 9	✓	0.616	✓	0.354		
1352	chapter 5 8.1	Problem 10	✗	0.487	✓	0.53		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1353	chapter 5 8.1	Problem 11	✓	0.268	✓	0.309		
1354	chapter 5 8.1	Problem 12	✓	0.335	✓	1.84		
1355	chapter 5 8.2	Problem 1	✓	0.044	✓	0.027		
1356	chapter 5 8.2	Problem 2	✓	0.179	✓	0.027		
1357	chapter 5 8.2	Problem 3	✓	0.044	✓	0.022		
1358	chapter 5 8.2	Problem 4	✓	0.104	✓	0.059		
1359	chapter 5 8.2	Problem 5	✓	0.071	✓	0.027		
1360	chapter 5 8.2	Problem 6	✗	0.087	✓	0.079		
1361	chapter 5 8.3	Problem 1	✓	0.043	✓	0.025		
1362	chapter 5 8.3	Problem 2	✓	0.101	✓	0.047		
1363	chapter 5 8.3	Problem 3	✓	0.062	✓	0.087		
1364	chapter 5 8.3	Problem 4	✓	0.217	✓	0.304		
1365	chapter 5 8.3	Problem 5	✗ (Timed out)	600.	✓	0.458		
1366	chapter 5 8.3	Problem 6	✗	24.019	✗	3.511		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1367	chapter 6 2.1	Problem 1	✓	0.009	✓	0.015	Yes	
1368	chapter 6 2.1	Problem 2	✓	0.01	✓	0.017	Yes	
1369	chapter 6 2.1	Problem 3	✓	0.054	✓	0.021	Yes	
1370	chapter 6 2.1	Problem 4	✓	0.123	✓	0.08	Yes	
1371	chapter 6 2.1	Problem 5	✓	0.033	✓	0.02		
1372	chapter 6 2.1	Problem 6	✓	0.123	✓	0.059		
1373	chapter 6 2.1	Problem 7	✓	0.077	✓	0.06		
1374	chapter 6 2.1	Problem 8	✗	0.031	✓	0.025		
1375	chapter 6 2.1	Problem 9	✗	0.025	✓	0.302		
1376	chapter 6 2.1	Problem 10	✓	0.192	✓	0.051		
1377	chapter 6 2.1	Problem 11	✗	6.979	✗	2.361		
1378	chapter 6 2.1	Problem 12	✗	3.292	✗	0.149		
1379	chapter 6 2.1	Problem 13	✓	0.041	✓	0.018		
1380	chapter 6 2.1	Problem 14	✗ (Timed out)	600.	✗	1.059		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1381	chapter 6 2.1	Problem 15	✗ (Timed out)	600.	✓	2.712		
1382	chapter 6 2.1	Problem 16	✗ (Timed out)	600.	✓	2.106		
1383	chapter 6 2.1	Problem 17	✗ (Timed out)	600.	✗	1.758		
1384	chapter 6 2.1	Problem 18	✗ (Timed out)	600.	✗	1.62		
1385	chapter 6 2.1	Problem 19	✗	2.034	✗	0.477		
1386	chapter 6 2.1	Problem 20	✗	232.551	✗	0.54		
1387	chapter 6 2.1	Problem 21	✗	0.093	✗	2.114		
1388	chapter 6 2.2	Problem 1	✓	0.3	✓	0.173		
1389	chapter 6 2.2	Problem 2	✓	0.193	✓	0.069		
1390	chapter 6 2.2	Problem 3	✗ (Timed out)	600.	✓	1.013		
1391	chapter 6 2.2	Problem 4	✓	0.257	✓	0.128		
1392	chapter 6 2.2	Problem 5	✓	0.043	✓	0.083		
1393	chapter 6 2.2	Problem 6	✓	0.04	✓	0.017		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1394	chapter 6 2.2	Problem 7	✓	0.028	✓	0.022		
1395	chapter 6 2.2	Problem 8	✓	0.039	✓	0.018		
1396	chapter 6 2.2	Problem 9	✓	0.225	✓	0.028		
1397	chapter 6 2.2	Problem 10	✓	0.036	✓	0.017		
1398	chapter 6 2.2	Problem 11	✓	0.156	✓	0.043		
1399	chapter 6 2.2	Problem 12	✓	0.012	✓	0.083		
1400	chapter 6 2.2	Problem 13	✓	0.034	✓	0.018		
1401	chapter 6 2.2	Problem 14	✓	0.136	✓	0.045		
1402	chapter 6 2.2	Problem 15	✓	0.044	✓	0.018		
1403	chapter 6 2.2	Problem 16	✓	0.036	✓	0.041		
1404	chapter 6 2.2	Problem 17	✓	0.041	✓	0.051		
1405	chapter 6 2.2	Problem 18	✓	0.041	✓	0.042		
1406	chapter 6 2.2	Problem 19	✓	0.062	✓	0.088		
1407	chapter 6 2.2	Problem 20	✓	0.042	✓	0.038		
1408	chapter 6 2.2	Problem 21	✓	0.047	✓	0.054		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1409	chapter 6 2.2	Problem 22	✓	0.047	✗	0.248		
1410	chapter 6 2.2	Problem 23	✗	9.231	✓	0.953		
1411	chapter 6 2.2	Problem 24	✓	0.083	✓	1.005		
1412	chapter 6 2.2	Problem 25	✓	0.594	✓	0.704		
1413	chapter 6 2.2	Problem 26	✗ (Timed out)	600.	✗	0.334		
1414	chapter 6 2.2	Problem 27	✗	0.769	✓	1.017		
1415	chapter 6 2.2	Problem 28	✓	1.628	✓	0.14		
1416	chapter 6 2.2	Problem 29	✗	0.423	✗	1.35		
1417	chapter 6 2.3	Problem 1	✗ (Timed out)	600.	✗	0.334		
1418	chapter 6 2.3	Problem 2	✓	0.082	✓	0.031		
1419	chapter 6 2.3	Problem 3	✓	0.03	✓	0.069		
1420	chapter 6 2.3	Problem 4	✗	0.024	✓	0.715		
1421	chapter 6 2.3	Problem 5	✗	56.178	✗	0.316		
1422	chapter 6 2.3	Problem 6	✗	0.548	✓	1.065		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1423	chapter 6 2.3	Problem 7	✗ (Timed out)	600.	✓	1.718		
1424	chapter 6 2.3	Problem 8	✓	0.258	✗	0.427		
1425	chapter 6 2.3	Problem 9	✗	0.025	✓	0.803		
1426	chapter 6 2.3	Problem 10	✓	0.04	✓	0.033		
1427	chapter 6 2.3	Problem 11	✓	0.319	✓	0.309		
1428	chapter 6 2.3	Problem 12	✗	0.051	✗	1.96		
1429	chapter 6 2.3	Problem 13	✗	0.522	✗ (Timed out)	600.		
1430	chapter 6 2.4	Problem 1	✓	0.559	✓	9.542		
1431	chapter 6 2.4	Problem 2	✓	1.183	✓	0.942		
1432	chapter 6 2.4	Problem 3	✓	0.231	✓	0.363		
1433	chapter 6 2.4	Problem 4	✓	0.755	✓	1.461		
1434	chapter 6 2.4	Problem 5	✓	0.729	✓	3.034		
1435	chapter 6 2.4	Problem 6	✓	0.408	✓	0.21		
1436	chapter 6 2.4	Problem 7	✗	0.583	✓	0.802		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1437	chapter 6 2.4	Problem 8	✗	0.031	✓	1.053		
1438	chapter 6 3.1	Problem 1	✓	0.192	✓	0.036		
1439	chapter 6 3.1	Problem 2	✓	0.212	✓	0.08		
1440	chapter 6 3.1	Problem 3	✓	0.258	✓	0.045		
1441	chapter 6 3.1	Problem 4	✗	0.971	✓	0.792		
1442	chapter 6 3.1	Problem 5	✓	0.492	✓	0.067		
1443	chapter 6 3.1	Problem 6	✓	0.45	✓	0.178		
1444	chapter 6 3.1	Problem 7	✓	1.037	✓	1.017		
1445	chapter 6 3.1	Problem 8	✓	1.052	✓	0.557		
1446	chapter 6 3.2	Problem 1	✗	0.61	✓	1.928		
1447	chapter 6 3.2	Problem 2	✗	0.399	✓	0.19		
1448	chapter 6 3.2	Problem 3	✓	1.88	✓	0.608		
1449	chapter 6 3.2	Problem 4	✓	1.624	✓	0.352		
1450	chapter 6 3.2	Problem 5	✗	38.384	✗	28.995		
1451	chapter 6 3.2	Problem 6	✗	37.789	✗	175.698		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1452	chapter 6 3.2	Problem 7	✗	13.661	✗	135.207		
1453	chapter 6 3.2	Problem 8	✗	23.146	✗	705.375		
1454	chapter 6 3.2	Problem 9	✗	8.759	✓	0.768		
1455	chapter 6 3.2	Problem 10	✗	2.909	✓	0.839		
1456	chapter 6 3.2	Problem 11	✗	2.154	✓	0.375		
1457	chapter 6 3.2	Problem 12	✗	28.853	✗	17.028		
1458	chapter 6 3.2	Problem 13	✓	1.63	✓	0.48		
1459	chapter 6 3.2	Problem 14	✗	1.396	✓	0.681		
1460	chapter 6 3.2	Problem 15	✓	1.847	✓	6.793		
1461	chapter 6 3.2	Problem 16	✗	3.102	✓	9.759		
1462	chapter 6 3.2	Problem 17	✗	31.866	✗	9.673		
1463	chapter 6 3.2	Problem 18	✗	21.55	✗ (Timed out)	600.		
1464	chapter 6 4.1	Problem 1	✓	0.032	✓	0.033		
1465	chapter 6 4.1	Problem 2	✓	0.192	✓	0.143		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1466	chapter 6 4.1	Problem 3	✓	0.252	✓	0.12		
1467	chapter 6 4.1	Problem 4	✓	0.461	✓	0.091		
1468	chapter 6 4.1	Problem 5	✗	11.171	✓	0.548		
1469	chapter 6 4.1	Problem 6	✗ (Timed out)	600.	✓	0.073		
1470	chapter 6 4.2	Problem 1	✓	0.032	✓	0.018		
1471	chapter 6 4.2	Problem 2	✓	0.048	✓	0.022		
1472	chapter 6 4.2	Problem 3	✓	0.188	✓	0.048		
1473	chapter 6 4.2	Problem 4	✓	0.248	✓	0.06		
1474	chapter 6 4.2	Problem 5	✓	0.491	✓	0.095		
1475	chapter 6 4.2	Problem 6	✗	176.322	✓	2.016		
1476	chapter 6 4.3	Problem 1	✓	0.269	✓	0.585		
1477	chapter 6 4.3	Problem 2	✓	0.085	✓	0.024		
1478	chapter 6 4.3	Problem 3	✓	0.743	✓	8.194		
1479	chapter 6 4.3	Problem 4	✓	0.91	✓	1.031		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1480	chapter 6 4.3	Problem 5	✗	2.001	✓	1.134		
1481	chapter 6 4.3	Problem 6	✗	0.65	✓	6.149		
1482	chapter 6 4.4	Problem 1	✓	0.258	✓	0.305		
1483	chapter 6 4.4	Problem 2	✓	0.086	✓	0.025		
1484	chapter 6 4.4	Problem 3	✓	0.782	✓	0.25		
1485	chapter 6 4.4	Problem 4	✓	0.837	✓	0.047		
1486	chapter 6 4.4	Problem 5	✗	4.917	✓	1.448		
1487	chapter 6 4.4	Problem 6	✗	0.649	✓	6.055		
1488	chapter 6 4.5	Problem 1	✓	0.502	✓	0.204		
1489	chapter 6 4.5	Problem 2	✓	0.294	✓	0.065		
1490	chapter 6 4.5	Problem 3	✗ (Timed out)	600.	✓	0.06		
1491	chapter 6 4.5	Problem 4	✗ (Timed out)	600.	✓	4.848		
1492	chapter 6 4.5	Problem 5	✗	0.703	✓	5.78		
1493	chapter 6 4.5	Problem 6	✗	0.668	✓	0.668		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1494	chapter 6 5.1	Problem 1	✓	0.24	✓	0.219		
1495	chapter 6 5.1	Problem 2	✓	0.01	✓	0.02		
1496	chapter 6 5.1	Problem 3	✓	0.283	✓	0.874		
1497	chapter 6 5.1	Problem 4	✓	0.216	✓	0.286		
1498	chapter 6 5.2	Problem 1	✓	0.041	✓	0.02		
1499	chapter 6 5.2	Problem 2	✓	0.215	✓	0.319		
1500	chapter 6 5.2	Problem 3	✓	0.263	✓	5.551		
1501	chapter 6 5.2	Problem 4	✓	0.309	✓	5.792		
1502	chapter 6 5.2	Problem 5	✓	0.121	✓	0.099		
1503	chapter 6 5.2	Problem 6	✓	0.595	✓	0.671		
1504	chapter 6 6.1	Problem 1	✓	0.182	✓	0.16		
1505	chapter 6 6.1	Problem 2	✓	0.2	✓	0.145		
1506	chapter 6 6.1	Problem 3	✓	0.159	✓	0.046		
1507	chapter 6 6.1	Problem 4	✓	0.46	✓	0.211		
1508	chapter 6 6.1	Problem 5	✓	3.111	✓	8.096		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1509	chapter 6 6.2	Problem 1	✓	0.464	✓	0.157		
1510	chapter 6 6.2	Problem 2	✓	0.443	✓	0.143		
1511	chapter 6 6.2	Problem 3	✗ (Timed out)	600.	✓	0.047		
1512	chapter 6 6.2	Problem 4	✗ (Timed out)	600.	✓	0.003		
1513	chapter 6 6.2	Problem 5	✓	3.096	✓	1.55		
1514	chapter 6 6.3	Problem 1	✓	0.209	✓	0.41	Yes	
1515	chapter 6 6.3	Problem 2	✓	0.671	✓	5.052	Yes	
1516	chapter 6 6.3	Problem 3	✓	0.96	✓	0.612		
1517	chapter 6 6.3	Problem 4	✓	0.214	✓	0.015		
1518	chapter 6 6.3	Problem 5	✓	1.441	✓	1.097		
1519	chapter 6 6.4	Problem 1	✓	0.214	✓	0.588		
1520	chapter 6 6.4	Problem 2	✓	0.664	✓	8.542		
1521	chapter 6 6.4	Problem 3	✓	0.836	✓	0.713		
1522	chapter 6 6.4	Problem 4	✓	0.257	✓	0.003		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1523	chapter 6 6.4	Problem 5	✗	1.799	✓	5.839		
1524	chapter 6 6.5	Problem 1	✓	162.609	✓	5.935		
1525	chapter 6 6.5	Problem 2	✓	0.096	✓	0.1		
1526	chapter 6 6.5	Problem 3	✓	0.528	✓	0.756		
1527	chapter 6 6.5	Problem 4	✗	67.888	✓	6.934		
1528	chapter 6 6.5	Problem 5	✓	0.494	✓	0.4		
1529	chapter 6 6.5	Problem 6	✗	20.608	✓	2.248		
1530	chapter 6 7.1	Problem 1	✓	0.325	✓	0.342		
1531	chapter 6 7.1	Problem 2	✓	0.392	✓	0.451		
1532	chapter 6 7.1	Problem 3	✓	0.087	✓	0.115		
1533	chapter 6 7.1	Problem 4	✓	0.252	✗ (Timed out)	600.		
1534	chapter 6 7.1	Problem 5	✓	0.29	✓	0.763		
1535	chapter 6 7.2	Problem 1	✓	0.287	✓	0.335		
1536	chapter 6 7.2	Problem 2	✓	0.403	✓	0.245		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1537	chapter 6 7.2	Problem 3	✓	0.073	✓	0.105		
1538	chapter 6 7.2	Problem 4	✓	0.242	✓	3.547		
1539	chapter 6 7.2	Problem 5	✓	0.274	✓	0.182		
1540	chapter 6 7.3	Problem 1	✓	0.789	✓	5.541		
1541	chapter 6 7.3	Problem 2	✓	0.689	✓	566.981		
1542	chapter 6 7.3	Problem 3	✓	0.398	✓	0.399		
1543	chapter 6 7.3	Problem 4	✓	0.215	✓	0.858		
1544	chapter 6 7.3	Problem 5	✓	0.586	✓	0.338		
1545	chapter 6 7.4	Problem 1	✓	0.939	✓	0.411		
1546	chapter 6 7.4	Problem 2	✓	0.723	✓	530.714		
1547	chapter 6 7.4	Problem 3	✓	0.427	✓	0.289		
1548	chapter 6 7.4	Problem 4	✓	0.199	✓	0.787		
1549	chapter 6 7.4	Problem 5	✓	0.558	✓	0.258		
1550	chapter 6 8.1	Problem 1	✓	0.01	✓	0.018		
1551	chapter 6 8.1	Problem 2	✓	0.193	✓	0.047		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1552	chapter 6 8.1	Problem 3	✓	0.099	✓	0.046		
1553	chapter 6 8.1	Problem 4	✓	0.306	✓	0.261		
1554	chapter 6 8.1	Problem 5	✓	0.189	✓	0.043		
1555	chapter 6 8.1	Problem 6	✗	0.006	✗	1.304		
1556	chapter 6 8.1	Problem 7	✓	4.141	✓	7.033		
1557	chapter 6 8.1	Problem 8	✓	0.845	✓	0.362		
1558	chapter 6 8.1	Problem 9	✗	0.567	✓	0.413		
1559	chapter 6 8.1	Problem 10	✗	0.734	✓	0.653		
1560	chapter 6 8.2	Problem 1	✓	0.045	✓	0.033		
1561	chapter 6 8.2	Problem 2	✓	0.476	✓	0.707		
1562	chapter 6 8.2	Problem 3	✓	3.228	✓	0.048		
1563	chapter 6 8.2	Problem 4	✗	0.239	✓	0.245		
1564	chapter 6 8.2	Problem 5	✓	0.157	✓	0.04		
1565	chapter 6 8.2	Problem 6	✓	0.191	✓	0.039		
1566	chapter 6 8.2	Problem 7	✓	0.151	✓	0.125		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1567	chapter 6 8.3	Problem 1	✓	0.021	✓	0.03		
1568	chapter 6 8.3	Problem 2	✓	0.028	✓	0.032		
1569	chapter 6 8.3	Problem 3	✓	0.039	✓	0.023		
1570	chapter 6 8.3	Problem 4	✓	0.069	✓	0.021		
1571	chapter 6 8.3	Problem 5	✓	0.072	✓	0.026		
1572	chapter 6 8.3	Problem 6	✓	0.47	✓	0.447		
1573	chapter 6 8.3	Problem 7	✓	2.493	✓	6.959		
1574	chapter 6 8.3	Problem 8	✗	133.566	✗ (Timed out)	600.		
1575	chapter 6 8.3	Problem 9	✗	0.351	✓	7.617		
1576	chapter 6 8.3	Problem 10	✗	0.27	✓	2.977		
1577	chapter 6 8.3	Problem 11	✗	45.363	✗	6.523		
1578	chapter 6 8.3	Problem 12	✗	1.548	✗	0.95		
1579	chapter 7 2.1	Problem 1	✓	0.041	✓	0.033		
1580	chapter 7 2.1	Problem 2	✓	0.154	✓	0.171		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1581	chapter 7 2.1	Problem 3	✓	0.038	✓	0.032		
1582	chapter 7 2.1	Problem 4	✓	0.583	✓	0.068		
1583	chapter 7 2.1	Problem 5	✓	0.116	✓	0.059		
1584	chapter 7 2.1	Problem 6	✓	0.125	✓	0.033	Yes	
1585	chapter 7 2.1	Problem 7	✓	0.173	✓	0.057	Yes	
1586	chapter 7 2.1	Problem 8	✗	0.023	✓	0.378		
1587	chapter 7 2.1	Problem 9	✓	0.861	✓	0.113		
1588	chapter 7 2.2	Problem 1	✓	0.093	✓	0.023		
1589	chapter 7 2.2	Problem 2	✓	0.033	✓	0.021		
1590	chapter 7 2.2	Problem 3	✓	0.136	✓	0.037		
1591	chapter 7 2.2	Problem 4	✓	0.078	✓	0.032		
1592	chapter 7 2.2	Problem 5	✓	0.2	✓	0.037		
1593	chapter 7 2.2	Problem 6	✓	0.22	✓	0.044		
1594	chapter 7 2.2	Problem 7	✓	0.147	✓	0.03		
1595	chapter 7 2.2	Problem 8	✓	0.148	✓	0.025		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1596	chapter 7 2.2	Problem 9	✓	0.175	✓	0.053		
1597	chapter 7 2.3	Problem 1	✓	0.024	✓	0.042		
1598	chapter 7 2.3	Problem 2	✓	0.036	✓	0.025		
1599	chapter 7 2.3	Problem 3	✓	0.06	✓	0.032		
1600	chapter 7 2.3	Problem 4	✓	0.114	✓	0.22		
1601	chapter 7 2.3	Problem 5	✓	0.187	✓	0.04		
1602	chapter 7 2.4	Problem 1	✓	0.325	✓	0.038		
1603	chapter 7 2.4	Problem 2	✓	0.196	✓	0.038		
1604	chapter 7 2.4	Problem 3	✓	0.101	✓	0.131		
1605	chapter 7 2.4	Problem 4	✓	0.161	✓	0.966		
1606	chapter 7 2.4	Problem 5	✓	0.091	✓	0.137		
1607	chapter 7 2.4	Problem 6	✗	0.026	✓	0.476		
1608	chapter 7 2.4	Problem 7	✗	1.871	✓	0.204		
1609	chapter 7 2.4	Problem 8	✓	3.208	✓	1.974		
1610	chapter 7 2.4	Problem 9	✓	1.135	✓	3.751		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1611	chapter 7 2.4	Problem 10	✓	0.766	✓	7.364		
1612	chapter 7 2.4	Problem 11	✓	10.748	✓	2.159		
1613	chapter 7 2.4	Problem 12	✓	0.05	✓	0.038		
1614	chapter 7 2.4	Problem 13	✓	0.409	✓	0.348		
1615	chapter 7 3.1	Problem 1	✓	0.044	✓	0.032		
1616	chapter 7 3.1	Problem 2	✓	0.477	✓	0.535		
1617	chapter 7 3.1	Problem 3	✓	2.421	✓	0.72		
1618	chapter 7 3.1	Problem 4	✗	0.906	✓	0.376		
1619	chapter 7 3.1	Problem 5	✓	0.56	✓	0.101		
1620	chapter 7 3.1	Problem 6	✓	0.363	✓	5.241		
1621	chapter 7 3.1	Problem 7	✓	1.107	✓	1.305		
1622	chapter 7 3.1	Problem 8	✓	1.064	✓	1.005		
1623	chapter 7 3.2	Problem 1	✓	0.988	✓	0.547		
1624	chapter 7 3.2	Problem 2	✓	0.86	✓	0.624		
1625	chapter 7 3.2	Problem 3	✓	0.365	✓	0.042		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1626	chapter 7 3.2	Problem 4	✓	0.373	✓	0.361		
1627	chapter 7 3.2	Problem 5	✓	0.382	✓	22.678		
1628	chapter 7 3.2	Problem 6	✗	37.92	✗	26.653		
1629	chapter 7 3.2	Problem 7	✗	14.713	✗	150.023		
1630	chapter 7 3.2	Problem 8	✗	23.568	✓	147.726		
1631	chapter 7 3.2	Problem 9	✓	1.885	✓	0.416		
1632	chapter 7 3.2	Problem 10	✗	15.035	✗	46.533		
1633	chapter 7 3.2	Problem 11	✓	2.318	✓	1.76		
1634	chapter 7 4.1	Problem 1	✓	0.553	✓	0.216		
1635	chapter 7 4.1	Problem 2	✓	30.778	✓	5.914		
1636	chapter 7 4.1	Problem 3	✓	1.61	✓	0.413		
1637	chapter 7 4.1	Problem 4	✓	19.909	✓	0.954		
1638	chapter 7 4.1	Problem 5	✓	30.965	✗ (Timed out)	600.		
1639	chapter 7 4.2	Problem 1	✓	0.557	✓	0.204		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1640	chapter 7 4.2	Problem 2	✓	26.371	✓	0.885		
1641	chapter 7 4.2	Problem 3	✓	1.649	✓	0.415		
1642	chapter 7 4.2	Problem 4	✓	14.672	✓	0.933		
1643	chapter 7 4.2	Problem 5	✓	0.32	✓	0.192		
1644	chapter 7 4.2	Problem 6	✓	27.166	✗ (Timed out)	600.		
1645	chapter 7 4.3	Problem 1	✓	0.429	✓	0.801		
1646	chapter 7 4.3	Problem 2	✓	142.248	✓	18.04		
1647	chapter 7 4.3	Problem 3	✓	0.974	✓	0.824		
1648	chapter 7 4.3	Problem 4	✗	0.846	✓	4.727		
1649	chapter 7 4.3	Problem 5	✓	1.292	✓	6.951		
1650	chapter 7 4.3	Problem 6	✗	2.011	✓	4.181		
1651	chapter 7 4.3	Problem 7	✓	43.572	✓	5.009		
1652	chapter 7 4.4	Problem 1	✓	0.448	✓	0.257		
1653	chapter 7 4.4	Problem 2	✓	140.61	✓	19.116		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1654	chapter 7 4.4	Problem 3	✓	0.984	✓	0.607		
1655	chapter 7 4.4	Problem 4	✗	0.856	✓	3.115		
1656	chapter 7 4.4	Problem 5	✓	1.969	✓	0.726		
1657	chapter 7 4.4	Problem 6	✓	44.356	✓	4.756		
1658	chapter 7 4.5	Problem 1	✓	25.839	✓	5.616		
1659	chapter 7 4.5	Problem 2	✓	1.212	✓	0.594		
1660	chapter 7 4.5	Problem 3	✓	190.089	✓	1.739		
1661	chapter 7 4.5	Problem 4	✓	0.708	✓	0.394		
1662	chapter 7 4.5	Problem 5	✓	0.567	✓	0.211		
1663	chapter 7 5.1	Problem 1	✓	0.048	✓ Contains unre-solve integral because maple can not inte-grate $\ln^n(x)$	0.039		
1664	chapter 7 5.1	Problem 2	✗	0.329	✓	0.448		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1665	chapter 7 5.1	Problem 3	✓	0.084	✓ Contains unre-solve integral because maple can not inte-grate $\ln^n(x)$	0.064		
1666	chapter 7 5.1	Problem 4	✗	0.058	✓	0.962		
1667	chapter 7 5.1	Problem 5	✗	2.112	✓ Contains RootOf and un-resolved integrals $\ln^n(x)$	0.646		
1668	chapter 7 5.2	Problem 1	✓	0.772	✓ Answer has un-resolved integrals	11.958		
1669	chapter 7 5.2	Problem 2	✓	3.871	✓ Answer has un-resolved integrals	3.329		

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#	PDE	description	Mathematica		Maple		hand solved?	animated?
			result	time	result	time		
1670	chapter 7 5.2	Problem 3	✓ Generated internal errors from solve : inconsistent or redundant transcendental equation	0.099	✓ Answer has unresolved integrals and RootOf	0.859		
1671	chapter 7 5.2	Problem 4	✗	0.506	✓	0.511		
1672	chapter 7 5.2	Problem 5	✓	2.744	✓	1.17		
1673	chapter 7 6.1	Problem 1	✓	0.615	✓	0.824		
1674	chapter 7 6.1	Problem 2	✓	0.368	✓	0.335		
1675	chapter 7 6.1	Problem 3	✓	1.222	✓	1.584		
1676	chapter 7 6.1	Problem 4	✓	134.918	✓	7.28		
1677	chapter 7 6.1	Problem 5	✓	3.707	✓	2.008		
1678	chapter 7 6.1	Problem 6	✗	2.262	✓	11.292		
1679	chapter 7 6.2	Problem 1	✓	0.572	✓	0.593		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1680	chapter 7 6.2	Problem 2	✓	1.449	✓	0.379		
1681	chapter 7 6.2	Problem 3	✓	1.267	✓	0.812		
1682	chapter 7 6.2	Problem 4	✓	118.275	✓	5.271		
1683	chapter 7 6.2	Problem 5	✓	2.903	✓	1.642		
1684	chapter 7 6.2	Problem 6	✗	2.192	✓	6.967		
1685	chapter 7 6.3	Problem 1	✓	0.439	✓	0.538		
1686	chapter 7 6.3	Problem 2	✓	0.478	✓	1.339		
1687	chapter 7 6.3	Problem 3	✓	0.865	✓	1.117		
1688	chapter 7 6.3	Problem 4	✓	39.998	✓	8.115		
1689	chapter 7 6.3	Problem 5	✗	1.544	✓	4.856		
1690	chapter 7 6.4	Problem 1	✓	0.438	✓	0.411		
1691	chapter 7 6.4	Problem 2	✓	0.706	✓	1.788		
1692	chapter 7 6.4	Problem 3	✓	0.859	✓	1.046		
1693	chapter 7 6.4	Problem 4	✓	44.145	✓	6.069		
1694	chapter 7 6.4	Problem 5	✓	3.04	✓	6.204		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1695	chapter 7 6.2	Problem 6	✗	1.563	✓	5.77		
1696	chapter 7 6.5	Problem 1	✓	1.505	✓	0.058		
1697	chapter 7 6.5	Problem 2	✗	68.895	✓	4.689		
1698	chapter 7 6.5	Problem 3	✗	33.256	✓	7.102		
1699	chapter 7 6.5	Problem 4	✗ (Timed out)	600.	✓	5.447		
1700	chapter 7 6.5	Problem 5	✗	12.338	✓	3.661		
1701	chapter 7 7.1	Problem 1	✓	0.102	✓	0.257		
1702	chapter 7 7.1	Problem 2	✓	0.754	✓	0.148		
1703	chapter 7 7.1	Problem 3	✗	1.071	✓	0.393		
1704	chapter 7 7.1	Problem 4	✗	0.818	✓	0.528		
1705	chapter 7 7.1	Problem 5	✓ Generates Solve::incnst: Inconsistent or redundant transcendental equation	2.22	✗ (Timed out)	600.		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1706	chapter 7 7.1	Problem 6	✓ Generates Solve::in-cnst: Incon-sistent or re-dundant transcen-dental equation	0.714	✗ (Timed out)	600.		
1707	chapter 7 7.2	Problem 1	✓	0.097	✓ Answer contains unre-solved integrals	0.122		
1708	chapter 7 7.2	Problem 2	✓	0.667	✓	0.031		
1709	chapter 7 7.2	Problem 3	✗	1.012	✓	0.303		
1710	chapter 7 7.2	Problem 4	✓	2.572	✓	4.802		
1711	chapter 7 7.2	Problem 5	✗	0.944	✓	1.419		
1712	chapter 7 7.3	Problem 1	✓	0.617	✓	0.283		
1713	chapter 7 7.3	Problem 2	✓	0.21	✓	0.442		
1714	chapter 7 7.3	Problem 3	✗	0.804	✓	0.703		
1715	chapter 7 7.3	Problem 4	✗	0.395	✓	227.		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1716	chapter 7 7.3	Problem 5	✓	0.314	✓	0.98		
1717	chapter 7 7.4	Problem 1	✓	0.699	✓	0.166		
1718	chapter 7 7.4	Problem 2	✓	0.299	✓	0.072		
1719	chapter 7 7.4	Problem 3	✗	0.91	✓	0.417		
1720	chapter 7 7.4	Problem 4	✗	0.656	✓	540.133		
1721	chapter 7 7.4	Problem 5	✓	0.241	✓	0.256		
1722	chapter 7 8.1	Problem 1	✓	0.022	✓	0.064		
1723	chapter 7 8.1	Problem 2	✓	0.196	✓	0.046		
1724	chapter 7 8.1	Problem 3	✓	0.103	✓	0.043		
1725	chapter 7 8.1	Problem 4	✓	0.331	✓	0.744		
1726	chapter 7 8.1	Problem 5	✗	0.278	✗	1.351		
1727	chapter 7 8.1	Problem 6	✗	24.744	✓ Gives Warning: Incom-plete separa-tion	28.9		
1728	chapter 7 8.1	Problem 7	✓	0.78	✓	0.319		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1729	chapter 7 8.1	Problem 8	✗	0.614	✓	0.433		
1730	chapter 7 8.1	Problem 9	✗	0.764	✓	0.626		
1731	chapter 7 8.2	Problem 1	✗	0.174	✓	0.052		
1732	chapter 7 8.2	Problem 2	✗	0.233	✓	0.198		
1733	chapter 7 8.2	Problem 3	✗	0.207	✓	0.043		
1734	chapter 7 8.2	Problem 4	✓	0.465	✓	0.244		
1735	chapter 7 8.2	Problem 5	✓	1.238	✓	0.766		
1736	chapter 7 8.2	Problem 6	✗	38.56	✗	93.916		
1737	chapter 7 8.2	Problem 7	✗	0.346	✓	0.987		
1738	chapter 7 8.2	Problem 8	✗	0.25	✓	1.434		
1739	chapter 7 8.3	Problem 1	✓ Kernel message inconsistent or redundant transcendental equation	0.05	✓	0.038		
1740	chapter 7 8.3	Problem 2	✗	0.047	✓	0.049		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1741	chapter 7 8.3	Problem 3	✓	0.054	✓	0.037		
1742	chapter 7 8.3	Problem 4	✗	0.122	✓	0.056		
1743	chapter 7 8.3	Problem 5	✓	0.656	✓	1.312		
1744	chapter 7 8.3	Problem 6	✗ (Timed out)	600.	✓	3.88		
1745	chapter 7 8.3	Problem 7	✗	3.425	✓	5.145		
1746	chapter 7 8.3	Problem 8	✗	0.243	✓	5.451		
1747	chapter 7 8.3	Problem 9	✗	0.233	✓	5.389		
1748	chapter 7 8.3	Problem 10	✗	46.048	✗	1.677		
1749	chapter 7 8.3	Problem 11	✗	0.121	✗	1.052		
1750	chapter 8 2.1	Problem 1	✓	0.081	✓	0.047		
1751	chapter 8 2.1	Problem 2	✓	0.133	✓	0.18		
1752	chapter 8 2.1	Problem 3	✓	0.151	✓	0.052		
1753	chapter 8 2.1	Problem 4	✓	1.468	✓	0.111		
1754	chapter 8 2.1	Problem 5	✓	0.116	✓	0.043		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1755	chapter 8 2.1	Problem 6	✓	0.117	✓	0.06		
1756	chapter 8 2.1	Problem 7	✓	0.153	✓	0.088		
1757	chapter 8 2.1	Problem 8	✗	0.024	✓	0.338		
1758	chapter 8 2.1	Problem 9	✓	0.883	✓	0.101		
1759	chapter 8 2.2	Problem 1	✓	0.093	✓	0.043		
1760	chapter 8 2.2	Problem 2	✓	0.154	✓	0.031		
1761	chapter 8 2.2	Problem 3	✓	0.126	✓	0.041		
1762	chapter 8 2.2	Problem 4	✓	2.553	✓	0.33		
1763	chapter 8 2.2	Problem 5	✓	0.234	✓	0.112		
1764	chapter 8 2.2	Problem 6	✓	0.196	✓	0.206		
1765	chapter 8 2.2	Problem 7	✓	0.157	✓	0.04		
1766	chapter 8 2.2	Problem 8	✓	0.15	✓	0.031		
1767	chapter 8 2.2	Problem 9	✓	0.23	✓	0.069		
1768	chapter 8 2.3	Problem 1	✓	0.022	✓	0.037		
1769	chapter 8 2.3	Problem 2	✓	0.055	✓	0.033		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1770	chapter 8 2.3	Problem 3	✓	0.066	✓	0.033		
1771	chapter 8 2.3	Problem 4	✓	0.069	✓	0.164		
1772	chapter 8 2.3	Problem 5	✓	0.231	✓	0.064		
1773	chapter 8 2.4	Problem 1	✓	0.491	✓	0.055		
1774	chapter 8 2.4	Problem 2	✓	0.278	✓	0.044		
1775	chapter 8 2.4	Problem 3	✓	0.1	✓	0.181		
1776	chapter 8 2.4	Problem 4	✓	0.153	✓	1.029		
1777	chapter 8 2.4	Problem 5	✓	0.098	✓	0.157		
1778	chapter 8 2.4	Problem 6	✗	0.027	✓	0.178		
1779	chapter 8 2.4	Problem 7	✗	1.922	✓	3.349		
1780	chapter 8 2.4	Problem 8	✓	3.086	✓	4.848		
1781	chapter 8 2.4	Problem 9	✓	1.368	✓	6.004		
1782	chapter 8 2.4	Problem 10	✓	0.817	✓	2.81		
1783	chapter 8 2.4	Problem 11	✗	22.571	✓	8.774		
1784	chapter 8 2.4	Problem 12	✓	0.048	✓	0.046		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1785	chapter 8 2.4	Problem 13	✓	0.394	✓	0.35		
1786	chapter 8 3.1	Problem 1	✓	0.04	✓	0.03		
1787	chapter 8 3.1	Problem 2	✓	0.541	✓	0.059		
1788	chapter 8 3.1	Problem 3	✓ Generates Solve::in-cnst: Incon-sistent or re-dundant transcen-dental equation	2.348	✓	0.79		
1789	chapter 8 3.1	Problem 4	✗	2.11	✓	1.836		
1790	chapter 8 3.1	Problem 5	✓	0.571	✓	0.106		
1791	chapter 8 3.1	Problem 6	✗	2.071	✓	0.263		
1792	chapter 8 3.1	Problem 7	✓	1.061	✓	1.077		
1793	chapter 8 3.1	Problem 8	✓	1.062	✓	0.582		
1794	chapter 8 3.2	Problem 1	✓	1.786	✓	1.141		
1795	chapter 8 3.2	Problem 2	✓	1.572	✓	0.639		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1796	chapter 8 3.2	Problem 3	✓	0.993	✓	0.165		
1797	chapter 8 3.2	Problem 4	✓	0.395	✓	0.437		
1798	chapter 8 3.2	Problem 5	✓	0.285	✓	3.016		
1799	chapter 8 3.2	Problem 6	✗	39.07	✓	140.53		
1800	chapter 8 3.2	Problem 7	✗	15.044	✗	225.072		
1801	chapter 8 3.2	Problem 8	✗	23.588	✗ (Timed out)	600.		
1802	chapter 8 3.2	Problem 9	✓	2.091	✓	0.527		
1803	chapter 8 3.2	Problem 10	✗	1.468	✓	0.692		
1804	chapter 8 3.2	Problem 11	✓	2.4	✓	2.383		
1805	chapter 8 4.1	Problem 1	✓	0.987	✓	0.212		
1806	chapter 8 4.1	Problem 2	✓	28.056	✓	0.249		
1807	chapter 8 4.1	Problem 3	✓	11.887	✓	0.401		
1808	chapter 8 4.1	Problem 4	✓	18.777	✓	0.361		
1809	chapter 8 4.1	Problem 5	✗	5.134	✓	4.425		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1810	chapter 8 4.2	Problem 1	✓	1.01	✓	0.172		
1811	chapter 8 4.2	Problem 2	✓	23.355	✓	0.201		
1812	chapter 8 4.2	Problem 3	✓	4.194	✓	0.021		
1813	chapter 8 4.2	Problem 4	✓	14.214	✓	0.435		
1814	chapter 8 4.2	Problem 5	✗	5.197	✓	1.935		
1815	chapter 8 4.3	Problem 1	✓	0.245	✓	0.214		
1816	chapter 8 4.3	Problem 2	✓	109.367	✓	2.683		
1817	chapter 8 4.3	Problem 3	✓	0.766	✓	1.508		
1818	chapter 8 4.3	Problem 4	✗	0.906	✓	4.774		
1819	chapter 8 4.3	Problem 5	✓	1.328	✓	3.007		
1820	chapter 8 4.3	Problem 6	✗	5.486	✓	1.539		
1821	chapter 8 4.4	Problem 1	✓	0.249	✓	0.25		
1822	chapter 8 4.4	Problem 2	✓	76.565	✓	3.019		
1823	chapter 8 4.4	Problem 3	✓	0.758	✓	1.485		
1824	chapter 8 4.4	Problem 4	✗	0.876	✓	2.064		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1825	chapter 8 4.4	Problem 5	✓	1.701	✓	0.49		
1826	chapter 8 4.4	Problem 6	✗	5.167	✓	2.977		
1827	chapter 8 4.5	Problem 1	✓	57.799	✓	5.677		
1828	chapter 8 4.5	Problem 2	✓	9.787	✓	0.467		
1829	chapter 8 4.5	Problem 3	✗	192.154	✓	1.326		
1830	chapter 8 4.5	Problem 4	✓	0.758	✓	0.402		
1831	chapter 8 4.5	Problem 5	✓	0.563	✓	0.286		
1832	chapter 8 5.1	Problem 1	✓	0.033	✓	0.033		
1833	chapter 8 5.1	Problem 2	✓	0.09	✓	0.07		
1834	chapter 8 5.1	Problem 3	✓	0.091	✓	0.059		
1835	chapter 8 5.1	Problem 4	✗	0.534	✓	0.151		
1836	chapter 8 5.1	Problem 5	✗	0.056	✓	0.516		
1837	chapter 8 5.2	Problem 1	✗	0.345	✓	12.321		
1838	chapter 8 5.2	Problem 2	✓	2.475	✓	2.752		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1839	chapter 8 5.2	Problem 3	✓	0.556	✗ (Timed out)	600.		
1840	chapter 8 5.2	Problem 4	✗	0.516	✓	0.386		
1841	chapter 8 5.2	Problem 5	✓	0.807	✓	0.851		
1842	chapter 8 6.1	Problem 1	✓	1.002	✓	1.781		
1843	chapter 8 6.1	Problem 2	✓	0.622	✓	0.247		
1844	chapter 8 6.1	Problem 3	✗	3.538	✓	5.063		
1845	chapter 8 6.1	Problem 4	✗	72.066	✓	10.123		
1846	chapter 8 6.1	Problem 5	✓	64.915	✓	3.892		
1847	chapter 8 6.1	Problem 6	✗	1.362	✓	9.376		
1848	chapter 8 6.2	Problem 1	✓	1.564	✓	0.404		
1849	chapter 8 6.2	Problem 2	✓	1.397	✓	0.274		
1850	chapter 8 6.2	Problem 3	✓	3.209	✓	0.717		
1851	chapter 8 6.2	Problem 4	✗	69.57	✓	10.787		
1852	chapter 8 6.2	Problem 5	✓	110.348	✓	2.995		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1853	chapter 8 6.2	Problem 6	✗	1.36	✓	7.395		
1854	chapter 8 6.3	Problem 1	✓	0.274	✓	0.543		
1855	chapter 8 6.3	Problem 2	✓	0.487	✓	0.78		
1856	chapter 8 6.3	Problem 3	✓	0.673	✓	2.057		
1857	chapter 8 6.3	Problem 4	✓	29.217	✓	1.839		
1858	chapter 8 6.3	Problem 5	✗	1.249	✓	4.678		
1859	chapter 8 6.4	Problem 1	✓	0.086	✓	0.474		
1860	chapter 8 6.4	Problem 2	✓	0.82	✓	2.227		
1861	chapter 8 6.4	Problem 3	✓	0.668	✓	1.148		
1862	chapter 8 6.4	Problem 4	✓	41.154	✓	18.149		
1863	chapter 8 6.4	Problem 5	✗	1.221	✓	5.306		
1864	chapter 8 6.5	Problem 1	✓	8.927	✓	1.075		
1865	chapter 8 6.5	Problem 2	✗	69.505	✓	9.524		
1866	chapter 8 6.5	Problem 3	✗	36.425	✓	13.905		
1867	chapter 8 6.5	Problem 4	✗	1.874	✓	5.072		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1868	chapter 8 6.5	Problem 5	✗	1.565	✓	4.843		
1869	chapter 8 7.1	Problem 1	✓	0.188	✓	0.133		
1870	chapter 8 7.1	Problem 2	✓	1.084	✓	0.632		
1871	chapter 8 7.1	Problem 3	✗	1.024	✓	0.493		
1872	chapter 8 7.1	Problem 4	✗	0.784	✓	0.658		
1873	chapter 8 7.1	Problem 5	✓	2.043	✗ (Timed out)	600.		
1874	chapter 8 7.1	Problem 6	✗	0.91	✓	2.681		
1875	chapter 8 7.2	Problem 1	✓	0.049	✓	0.126		
1876	chapter 8 7.2	Problem 2	✓	1.1	✓	0.102		
1877	chapter 8 7.2	Problem 3	✗	0.901	✓	0.217		
1878	chapter 8 7.2	Problem 4	✗	0.728	✓	0.48		
1879	chapter 8 7.2	Problem 5	✓	2.365	✓	0.594		
1880	chapter 8 7.2	Problem 6	✗	0.897	✓	0.22		
1881	chapter 8 7.3	Problem 1	✓	0.303	✓	0.306		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1882	chapter 8 7.3	Problem 2	✓	0.627	✓	0.461		
1883	chapter 8 7.3	Problem 3	✗	0.773	✓	0.086		
1884	chapter 8 7.3	Problem 4	✗	0.573	✓	600.		
1885	chapter 8 7.3	Problem 5	✓	0.284	✓	0.58		
1886	chapter 8 7.4	Problem 1	✓	0.339	✓	0.312		
1887	chapter 8 7.4	Problem 2	✓	0.587	✓	0.189		
1888	chapter 8 7.4	Problem 3	✗	0.819	✓	0.083		
1889	chapter 8 7.4	Problem 4	✗	0.596	✓	0.115		
1890	chapter 8 7.4	Problem 5	✓	0.286	✓	3.218		
1891	chapter 8 8.1	Problem 1	✓	0.019	✓	0.055		
1892	chapter 8 8.1	Problem 2	✓	0.191	✓	0.043		
1893	chapter 8 8.1	Problem 3	✓	0.1	✓	0.026		
1894	chapter 8 8.1	Problem 4	✓	0.298	✓	0.09		
1895	chapter 8 8.1	Problem 5	✓	0.178	✓	0.34		
1896	chapter 8 8.1	Problem 6	✓	3.506	✓	3.265		

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Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1897	chapter 8 8.1	Problem 7	✓	0.762	✓	0.341		
1898	chapter 8 8.1	Problem 8	✗	0.533	✓	0.191		
1899	chapter 8 8.1	Problem 9	✗	0.711	✓	0.197		
1900	chapter 8 8.2	Problem 1	✗	0.151	✓	0.121		
1901	chapter 8 8.2	Problem 2	✗	0.22	✓	0.129		
1902	chapter 8 8.2	Problem 3	✗	0.191	✓	0.063		
1903	chapter 8 8.2	Problem 4	✓	0.165	✓	0.253		
1904	chapter 8 8.2	Problem 5	✓	1.139	✗	6.942		
1905	chapter 8 8.2	Problem 6	✗	0.305	✓	2.151		
1906	chapter 8 8.2	Problem 7	✗	21.021	✗	4.934		
1907	chapter 8 8.2	Problem 8	✗	20.227	✗	5.582		
1908	chapter 8 8.3	Problem 1	✓	0.036	✓	0.036		
1909	chapter 8 8.3	Problem 2	✗	0.041	✓	0.039		
1910	chapter 8 8.3	Problem 3	✓	0.054	✓	0.026		
1911	chapter 8 8.3	Problem 4	✗	0.107	✓	0.037		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1912	chapter 8 8.3	Problem 5	✓	0.619	✓	1.618		
1913	chapter 8 8.3	Problem 6	✗ (Timed out)	600.	✓	4.698		
1914	chapter 8 8.3	Problem 7	✗	129.84	✗ (Timed out)	600.		
1915	chapter 8 8.3	Problem 8	✗	20.238	✗	3.136		
1916	chapter 8 8.3	Problem 9	✗	20.233	✗	4.5		
1917	chapter 8 8.3	Problem 10	✗	45.026	✓	1.193		
1918	chapter 8 8.3	Problem 11	✗	1.466	✗	3.914		
1919	chapter 9 2.1	Problem 1	✓	0.351	✓	0.06		
1920	chapter 9 2.1	Problem 2	✓	0.413	✓	0.529		
1921	chapter 9 2.1	Problem 3	✓	0.25	✓	0.069		
1922	chapter 9 2.1	Problem 4	✓	0.111	✓	0.036		
1923	chapter 9 2.1	Problem 5	✓	0.21	✓	0.06		
1924	chapter 9 2.1	Problem 6	✗ (Timed out)	600.	✗ (Timed out)	600.		
1925	chapter 9 2.1	Problem 7	✓	0.167	✓	0.182		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1926	chapter 9 2.1	Problem 8	✓	0.123	✓	0.023		
1927	chapter 9 2.1	Problem 9	✓	0.272	✓	0.595		
1928	chapter 9 2.2	Problem 1	✓	0.3	✓	0.054		
1929	chapter 9 2.2	Problem 2	✓	0.083	✓	0.043		
1930	chapter 9 2.2	Problem 3	✓	0.257	✓	0.063		
1931	chapter 9 2.2	Problem 4	✗ (Timed out)	600.	✓	578.		
1932	chapter 9 2.2	Problem 5	✓	0.053	✓	0.129		
1933	chapter 9 2.2	Problem 6	✓	0.05	✓	0.042		
1934	chapter 9 2.2	Problem 7	✓	0.317	✓	0.265		
1935	chapter 9 2.3	Problem 1	✓	0.133	✓	0.143		
1936	chapter 9 2.3	Problem 2	✓	0.139	✓	0.122		
1937	chapter 9 2.3	Problem 3	✓	0.144	✓	1.511		
1938	chapter 9 2.3	Problem 4	✗ (Timed out)	600.	✓	177.289		
1939	chapter 9 2.3	Problem 5	✓	0.304	✓	0.575		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1940	chapter 9 2.4	Problem 1	✓	0.185	✓	0.247		
1941	chapter 9 2.4	Problem 2	✓	0.177	✓	0.076		
1942	chapter 9 2.4	Problem 3	✓	0.267	✓	3.25		
1943	chapter 9 2.4	Problem 4	✓	0.143	✓	0.281		
1944	chapter 9 2.4	Problem 5	✓	0.044	✓	1.599		
1945	chapter 9 2.4	Problem 6	✓	0.142	✓	0.565		
1946	chapter 9 2.4	Problem 7	✓	3.95	✓	12.536		
1947	chapter 9 2.4	Problem 8	✓	2.038	✓	10.178		
1948	chapter 9 2.4	Problem 9	✗	23.655	✓	36.975		
1949	chapter 9 2.4	Problem 10	✓	0.074	✓	0.125		
1950	chapter 9 2.4	Problem 11	✓	0.185	✓	0.434		
1951	chapter 9 2.4	Problem 12	✗	0.031	✓	0.701		
1952	chapter 9 2.4	Problem 13	✓	2.958	✓	2.391		
1953	chapter 9 2.4	Problem 14	✓	0.537	✓	0.353		
1954	chapter 9 3.1	Problem 1	✓	0.073	✓	0.054		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1955	chapter 9 3.1	Problem 2	✓	0.102	✓	0.061		
1956	chapter 9 3.1	Problem 3	✓	0.226	✓	0.04		
1957	chapter 9 3.1	Problem 4	✓	0.194	✓	0.046		
1958	chapter 9 3.1	Problem 5	✗	2.677	✓	2.257		
1959	chapter 9 3.1	Problem 6	✓	1.455	✓	0.975		
1960	chapter 9 3.1	Problem 7	✗ (Timed out)	600.	✓	32.505		
1961	chapter 9 3.2	Problem 1	✓	0.078	✓	0.066		
1962	chapter 9 3.2	Problem 2	✓	0.104	✓	0.063		
1963	chapter 9 3.2	Problem 3	✗	0.167	✓	0.204		
1964	chapter 9 3.2	Problem 4	✗	2.52	✓	3.626		
1965	chapter 9 3.2	Problem 5	✓	2.575	✓	1.111		
1966	chapter 9 3.2	Problem 6	✗	2.641	✓	2.329		
1967	chapter 9 3.2	Problem 7	✓	0.347	✓	0.118		
1968	chapter 9 3.2	Problem 8	✓	1.351	✓	0.513		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1969	chapter 9 3.2	Problem 9	✗ (Timed out)	600.	✓	3.155		
1970	chapter 9 3.2	Problem 10	✗	1.863	✓	1.685		
1971	chapter 9 4.1	Problem 1	✓	127.643	✓	0.274		
1972	chapter 9 4.1	Problem 2	✓	3.164	✓	0.573		
1973	chapter 9 4.1	Problem 3	✓	0.162	✓	0.249		
1974	chapter 9 4.1	Problem 4	✗	167.253	✓	0.556		
1975	chapter 9 4.1	Problem 5	✓	150.807	✓	2.987		
1976	chapter 9 4.2	Problem 1	✓	130.254	✓	0.22		
1977	chapter 9 4.2	Problem 2	✓	3.226	✓	0.707		
1978	chapter 9 4.2	Problem 3	✓	0.143	✓	0.262		
1979	chapter 9 4.2	Problem 4	✗	176.196	✓	0.451		
1980	chapter 9 4.2	Problem 5	✓	155.26	✓	2.194		
1981	chapter 9 4.3	Problem 1	✓	74.218	✓	0.267		
1982	chapter 9 4.3	Problem 2	✓	3.212	✓	2.997		

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#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1983	chapter 9 4.3	Problem 3	✓	0.235	✓	0.378		
1984	chapter 9 4.3	Problem 4	✗	185.477	✓	0.892		
1985	chapter 9 4.3	Problem 5	✓	275.802	✓	1.291		
1986	chapter 9 4.4	Problem 1	✓	70.045	✓	0.266		
1987	chapter 9 4.4	Problem 2	✓	3.915	✓	0.958		
1988	chapter 9 4.4	Problem 3	✓	0.209	✓	0.355		
1989	chapter 9 4.4	Problem 4	✗	189.822	✓	0.94		
1990	chapter 9 4.4	Problem 5	✓	272.554	✓	1.332		
1991	chapter 9 4.5	Problem 1	✓	132.333	✓	0.063		
1992	chapter 9 4.5	Problem 2	✓	59.285	✓	0.204		
1993	chapter 9 4.5	Problem 3	✗	180.194	✓	0.184		
1994	chapter 9 4.5	Problem 4	✓	0.109	✓	0.03		
1995	chapter 9 4.5	Problem 5	✓	158.848	✓	0.996		
1996	chapter 9 5.1	Problem 1	✓	0.169	✓	0.092		
1997	chapter 9 5.1	Problem 2	✓	0.126	✓	0.223		

Continued on next page

Table 1.10 – Handbook of first order partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
1998	chapter 9 5.1	Problem 3	✗	0.523	✓	0.139		
1999	chapter 9 5.1	Problem 4	✗	0.27	✓	0.637		
2000	chapter 9 5.1	Problem 5	✓	1.416	✓	157.		

1.3.6 Handbook of nonlinear partial differential equations

Table 1.11: Handbook of nonlinear partial differential equations breakdown of results. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
2001	chapter 1 1.1	Problem 1	✗	0.361	✗	4.009		
2002	chapter 1 1.1	Problem 2	✓	0.108	✓	0.241		
2003	chapter 1 1.2	Problem 1	✗	0.08	✗	13.744		
2004	chapter 1 1.2	Problem 2	✗	0.094	✓	0.241		
2005	chapter 1 1.2	Problem 3	✗	0.182	✓	0.227		
2006	chapter 1 1.2	Problem 4	✗	0.175	✓	0.357		
2007	chapter 1 1.2	Problem 5	✗	0.891	✓	0.918		

Continued on next page

Table 1.11 – Handbook of nonlinear partial differential equations. Continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?	ani-mated?
			result	time	result	time		
2008	chapter 1 1.3	Problem 1	✗	0.008	✗	0.023		
2009	chapter 1 1.3	Problem 2	✗	0.009	✗	0.017		
2010	chapter 1 1.3	Problem 3	✗	0.015	✗	0.044		
2011	chapter 1 1.3	Problem 4	✗	0.015	✗	0.026		
2012	chapter 1 1.4	Problem 1	✗	0.012	✗	0.019		

CHAPTER 2

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2.1 General first order

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2.1.1 Transport equation $u_t+u_x=0$

problem number 1

Taken from Mathematica Symbolic PDE document

Solve for $u(x,t)$

$$u_t+u_x=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t)\to c_1(t-x)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\_F1(t-x)$$

Hand solution

$$u_t + u_x = 0 \tag{1}$$

Let $u \equiv u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1) to (2) then we see that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = 1 \tag{4}$$

(3) says that u is constant. Since no initial conditions are given, let $u = F(x(0))$ where F is arbitrary function. To find $x(0)$ we solve (4). The solution to (4) is $x = x(0) + t$. Hence $x(0) = x - t$. Therefore

$$u(x,t) = F(x-t)$$

2.1.2 Transport equation $u_t - 3u_x = 0$ IC $u(0,x) = e^{-x^2}$. Peter Olver textbook, 2.2.2 (a)

problem number 2

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve for $u(t,x)$ in $u_t - 3u_x = 0$ with IC $u(0,x) = e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] -3* D[u[t, x], {x}] == 0;
ic = u[0,x]==Exp[-x^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to e^{-(3t+x)^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) - 3*diff(u(t, x),x) =0;
ic:=u(0,x)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = \mathrm{e}^{-(3t+x)^2}$$

Hand solution

Solve

$$u_t - 3u_x = 0 \tag{1}$$

With initial conditions $u(x,0) = e^{-x^2}$

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = -3 \tag{4}$$

Eq (3) gives $u = u(x(0))$. Using the given initial conditions, this becomes

$$u = e^{-x(0)^2} \tag{5}$$

Eq (4) is now used to find $x(0)$. Soving (4) gives $x = x(0) - 3t$. Hence $x(0) = x - 3t$. Therefore (5) becomes

$$u(x,t) = e^{-(x-3t)^2}$$

The following is an animation of the solution

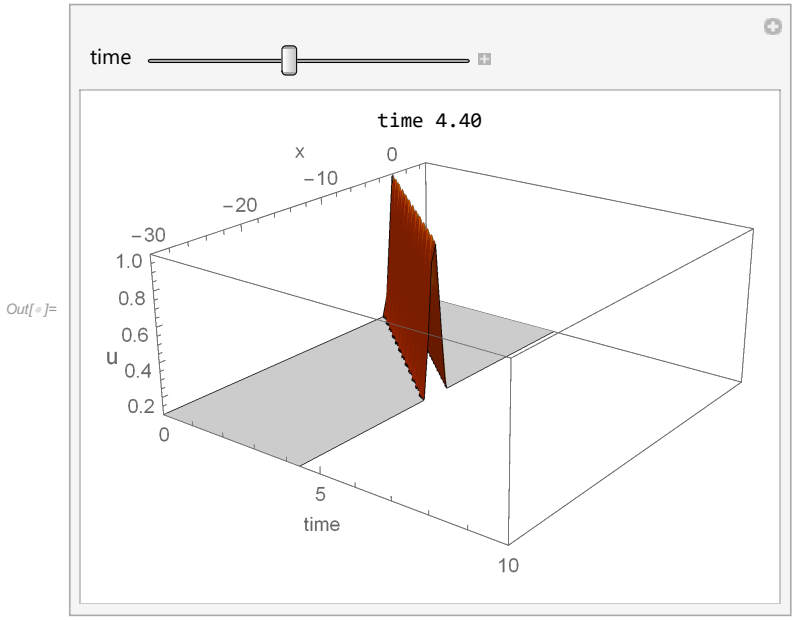

Figure 2.1: snap shot

Source code used for the above

```
(*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[- (x + 3 t) ^2];
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}],
         {Quiet@Plot3D[u[x, t], {x, -20, 2}, {t, 0, time},
                  PlotRange → {All, {0, 6}, {0.1, 1}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 40,
                  PlotStyle → plotStyle, Mesh → None,
                  ViewPoint → {-2.355, 1.645, 1.792},
                  Boxed → False]
         }}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 6, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.2: Source code 3D

```
(*2D*)
u[x_, t_] := Exp[- (x + 3 t) ^2];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {Quiet@Plot[u[x, time], {x, -20, 3},
                  PlotRange → {All, {0, 1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 6, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.3: Source code 2D

2.1.3 Transport equation $u_t + 2u_x = 0$ IC $u(-1, x) = \frac{x}{1+x^2}$. Peter Olver textbook, 2.2.2 (b)

problem number 3

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve for $u(t, x)$ in $u_t + 2u_x = 0$ with IC $u(-1, x) = \frac{x}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] +2* D[u[t, x], {x}] == 0;
ic = u[-1,x]==x/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to \frac{-2t + x - 2}{4t^2 - 4t(x-2) + x^2 - 4x + 5}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +2*diff(u(t, x),x) =0;
ic:=u(-1,x)=x/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = \frac{-2t + x - 2}{(-2t + x - 2)^2 + 1}$$

Hand solution

Solve

$$u_t + 2u_x = 0$$

With initial conditions $u(-1,x) = \frac{x}{1+x^2}$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = 2 \tag{4}$$

Eq (3) says that u is constant on the chatateristic lines, or $u = u(x(-1))$. Using the given initial conditions, this becomes

$$u(x(t), t) = \frac{x(-1)}{1 + x(-1)^2} \tag{5}$$

Eq (4) is now used to find $x(-1)$. Soving (4) gives $x = x(0) + 2t$. Hence $x(-1) = x(0) - 2$ or $x(0) = x(-1) + 2$. Therefore

$$x = x(-1) + 2 + 2t$$
$$x(-1) = x - 2 - 2t$$

Now that we found $x(-1)$, we substitute it in (5), giving the solution

$$u(x(t), t) = \frac{x - 2 - 2t}{1 + (x - 2 - 2t)^2}$$

<u>Alternative method</u>. Using Lagrange-charpit method

$$\frac{dt}{1} = \frac{dx}{2} = \frac{du}{0}$$

Which implies that $du = 0$ or $u = C_1$. A constant. Integrating $\frac{dt}{1} = \frac{dx}{2}$ gives $t = \frac{1}{2}x + C_2$ or $C_2 = t - \frac{1}{2}x$. But $C_1 = F(C_2)$ always, where F is arbitrary function. Since $C_1 = u$ then

$$\begin{aligned} u &= F(C_2) \\ u &= F\left(t - \frac{1}{2}x\right) \end{aligned} \tag{1}$$

At $t = -1$ the above becomes

$$\frac{x}{1+x^2} = F\left(-1 - \frac{1}{2}x\right)$$

Let $-1 - \frac{1}{2}x = z$ which implies $x = -2(1+z)$ The above can be written as

$$\begin{aligned} \frac{-2(1+z)}{1+(-2(1+z))^2} &= F(z) \\ F(z) &= -\frac{2(1+z)}{4z^2+8z+5} \end{aligned}$$

From the above then (1) can be written as

$$\begin{aligned} u(t,x) &= -\frac{2\left(1+\left(t-\frac{1}{2}x\right)\right)}{4\left(t-\frac{1}{2}x\right)^2 + 8\left(t-\frac{1}{2}x\right)+5} \\ &= \frac{x-2t-2}{4t^2-4tx+8t+x^2-4x+5} \\ &= \frac{x-2t-2}{1+(x-2-2t)^2} \end{aligned}$$

The following is an animation of the solution

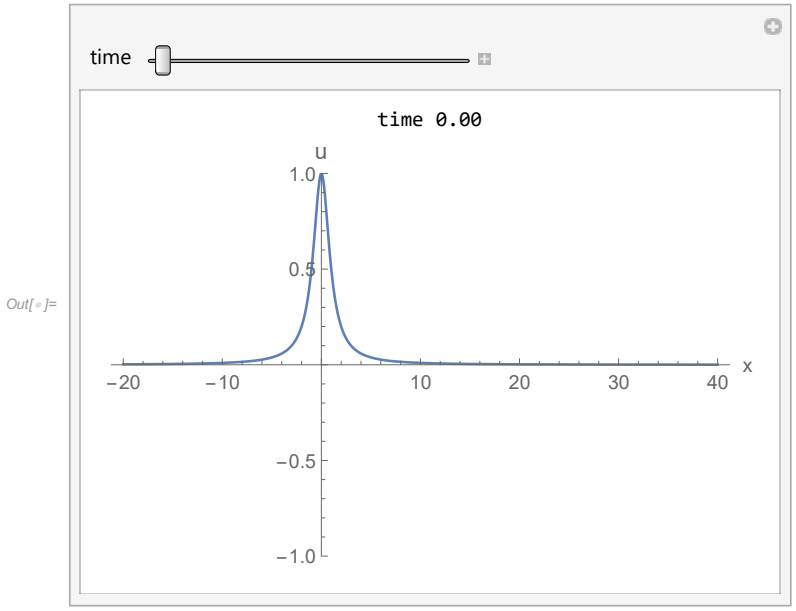

Figure 2.4: snap shot

Source code used for the above

```
(*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := (x - 2 t - 2)/(1 + (x - 2 - 2 t)^2);
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot3D[u[x, t], {x, -1, 30}, {t, 0, time},
                  PlotRange → {All, {0, 12}, {-.5, 1}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 75,
                  PlotStyle → plotStyle, Mesh → None,
                  ViewPoint → {1.765, 2.0924, 1.9887}, Boxed → True]
         }}
   ]
   ,
   {{time, 0.1, "time"}, 0.1, 12, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.5: Source code 3D

```
In[ ]:= (*2D*)
u[x_, t_] := (x - 2 t - 2)/(1 + (x - 2 - 2 t)^2);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {Quiet@Plot[u[x, time], {x, -20, 40},
                  PlotRange → {All, {-1, 1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 12, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.6: Source code 2D

2.1.4 Transport equation $u_t + u_x + \frac{1}{2}u = 0$ IC $u(0,x) = \arctan(x)$. Peter Olver textbook, 2.2.2 (c)

problem number 4

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t + u_x + \frac{1}{2}u = 0$ with IC $u(0,x) = \arctan(x)$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] + D[u[t, x], {x}]+1/2*u[t,x] == 0;
ic = u[0,x]==ArcTan[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to -e^{-t/2}\tan^{-1}(t-x)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +diff(u(t, x),x) +1/2*u(t,x)=0;
ic:=u(0,x)=arctan(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = -\arctan(t-x)\,\mathrm{e}^{-\frac{t}{2}}$$

Hand solution

Solve

$$u_t + u_x + \frac{1}{2}u = 0$$

With initial conditions $u(x,0) = \arctan(x)$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = -\frac{1}{2}u \tag{3}$$

$$\frac{dx}{dt} = 1 \tag{4}$$

Solving (3) gives

$$\begin{aligned}\frac{du}{u} &= \frac{-1}{2}dt\\ \ln|u| &= -\frac{1}{2}t + c\\ u &= u(x(0))\,e^{\frac{-t}{2}}\end{aligned}$$

Using the given initial conditions, this becomes

$$u(x,t) = \arctan(x(0))\,e^{\frac{-1}{2}t} \tag{5}$$

Now we just need to find $x(0)$. From (4)

$$\begin{aligned}x &= x(0) + t\\ x(0) &= x - t\end{aligned}$$

Substituting the above into (5) gives

$$u(x,t) = \arctan(x-t)\, e^{\frac{-1}{2}t}$$

The following is an animation of the solution

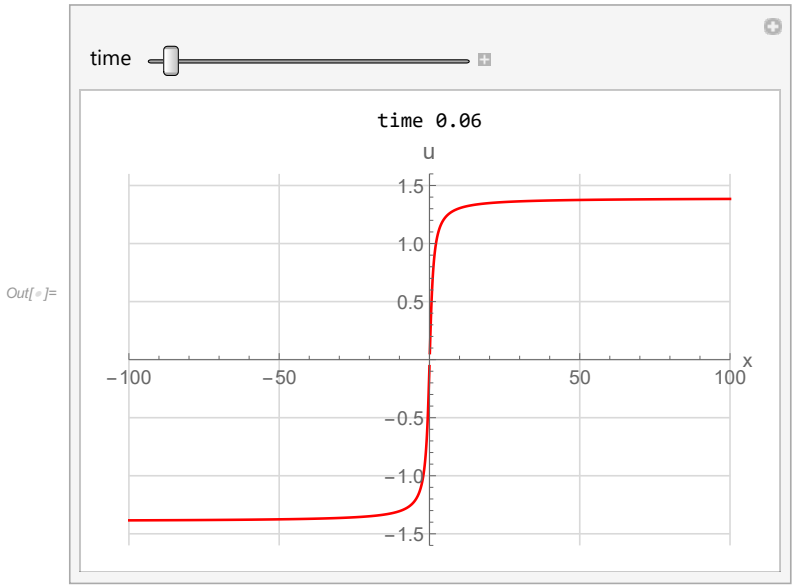

Figure 2.7: snap shot

Source code used for the above

```
In[ ]:= padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := ArcTan[x - t] Exp[-2 t];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
         {
            Quiet@Plot[u[x, time], {x, -100, 100},
                  PlotRange → {{-100, 100}, {-1.6, 1.6}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400,
                  PlotStyle → Red,
                  GridLines → Automatic, GridLinesStyle → LightGray
               ]
         }}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 1.8, .05},
   TrackedSymbols :→ {time}
]
```

Figure 2.8: Source code

2.1.5 Transport equation $u_t - 4u_x + u = 0$ IC $u(0,x) = \frac{1}{1+x^2}$. Peter Olver textbook, 2.2.2 (d)

problem number 5

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t - 4u_x + u = 0$ with IC $u(0,x) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] -4*D[u[t, x], {x}]+u[t,x] == 0;
ic = u[0,x]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to \frac{e^{-t}}{16t^2+8tx+x^2+1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) -4*diff(u(t, x),x) +u(t,x)=0;
ic:=u(0,x)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = \frac{\mathrm{e}^{-t}}{(4t+x)^2+1}$$

<u>Hand solution</u>

$$\begin{aligned} u_t - 4u_x + u &= 0 \qquad (1) \\ u(x,0) &= \frac{1}{1+x^2} \end{aligned}$$

<u>Solution</u>

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \qquad (2)$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = -u \tag{3}$$
$$\frac{dx}{dt} = -4 \tag{4}$$

Solving (3) gives

$$\begin{aligned}\frac{du}{u} &= -dt\\ \ln|u| &= -t + c\\ u &= u(x(0))\, e^{-t}\end{aligned}$$

Using the given initial conditions, this becomes

$$u = \frac{1}{1 + x(0)^2} e^{-t} \tag{5}$$

Now we just need to find $x(0)$. From (4)

$$\begin{aligned}x &= -4t + x(0)\\ x(0) &= x + 4t\end{aligned}$$

Substituting the above into (5) gives

$$u = \frac{e^{-t}}{1 + (x + 4t)^2}$$

The above is the method I prefer to solve these problems. Here are some alternative ways

<u>Alternative approach</u>

Let ξ be the characteristic variable defined such that $\xi = x - ct$. Where characteristic lines are given by $x = x_0 + ct$. But $c = -4$ in this problem. Hence characteristic lines are

$$x = x_0 - 4t$$

And

$$\xi = x + 4t$$

Then $u_t - 4u_x$ are transformed to $v(t, \xi)$ as was done in part (a) (will not be repeated) which results in

$$u_t - 4u_x = \frac{\partial v}{\partial t}$$

Substituting the above into (1) gives (where now v is used in place of u).

$$\frac{\partial v}{\partial t}+v=0$$

This is now first order ODE since it only depends on t. Therefore $v'+v=0$. This is linear in v. Hence the solution is $\frac{d}{dt}\left(ve^{\int dt}\right)=0$ or $ve^{t}=F(\xi)$ where F is arbitrary function of ξ. Hence

$$v(t,\xi)=e^{-t}F(\xi)$$

Converting to $u(t,x)$ gives

$$u(t,x)=e^{-t}F(x+4t) \tag{2}$$

At $u(0,x)=\frac{1}{1+x^2}$ the above becomes

$$\frac{1}{1+x_0^2}=F(x_0)$$

From the above then (2) can be written as

$$u(t,x)=\frac{e^{-t}}{1+(x+4t)^2}$$

An <u>alternative approach</u> to solve transport PDE is by using Lagrange-charpit method

$$\frac{dt}{1}=-\frac{dx}{4}=\frac{-du}{u}$$

Integrating $\frac{dt}{1}=\frac{-dx}{4}$ gives $t=-\frac{1}{4}x+C_2$ or

$$C_2=t+\frac{1}{4}x$$

Now either $\frac{dt}{1}=\frac{-du}{u}$ or $\frac{-dx}{4}=\frac{-du}{u}$ can be integrated. The choice is not important. Integrating $\frac{dt}{1}=\frac{-du}{u}$ gives $t=-\ln u+C_1$ or

$$C_1=t+\ln u$$

But $C_1=F(C_2)$ always, where F is arbitrary function therefore

$$\begin{aligned}
t+\ln u&=F(C_2)\\
t+\ln u&=F\left(t+\frac{1}{4}x\right)\\
\ln u&=F\left(t+\frac{1}{4}x\right)-t\\
u&=e^{-t}e^{F\left(t+\frac{1}{4}x\right)}
\end{aligned} \tag{1}$$

At $u(0,x)=\frac{1}{1+x^2}$ the above becomes

$$\frac{1}{1+x^2}=e^{F\left(\frac{1}{4}x\right)}$$
$$F\left(\frac{x}{4}\right)=\ln\left(\frac{1}{1+x^2}\right)$$

Let $z=\frac{x}{4}$, then $x=4z$. The above becomes

$$F(z)=\ln\left(\frac{1}{1+(4z)^2}\right)$$

From the above then (1) can be written as

$$\begin{aligned}u(t,x)&=e^{-t}e^{\ln\left(\frac{1}{1+\left(4\left(t+\frac{1}{4}x\right)\right)^2}\right)}\\&=\frac{e^{-t}}{1+\left(4\left(t+\frac{1}{4}x\right)\right)^2}\\&=\frac{e^{-t}}{1+(4t+x)^2}\end{aligned}$$

The following is an animation of the solution

Out[ ]=

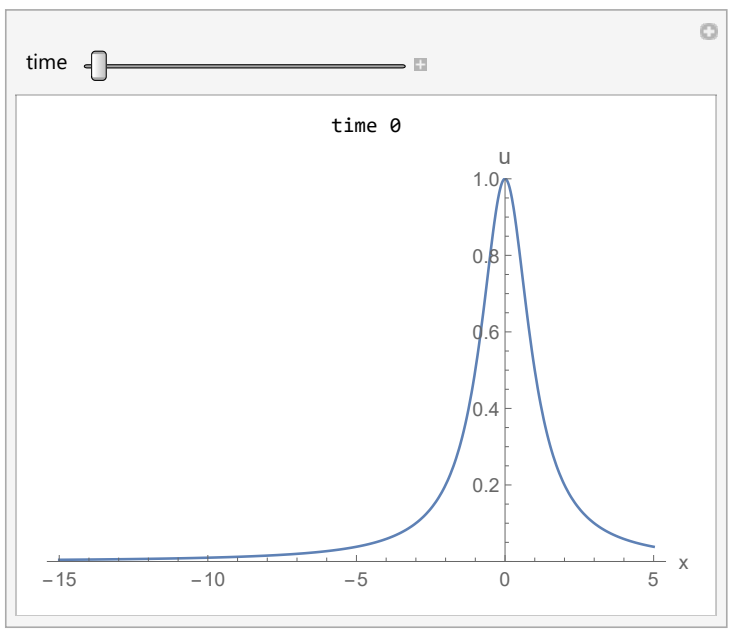

Figure 2.9: snap shot

Source code used for the above

```
In[•]:= (*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[-t]/(1 + (4 t + x)^2);
plotStyle = Automatic; (*Directive[Orange,Specularity[White,20]];*)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {Quiet@Plot3D[u[x, t], {x, -7, 5}, {t, 0, time},
                  PlotRange → {{-7, 5}, {0, 2}, {0, 2}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 30, Mesh → None,
                  PlotStyle → plotStyle,
                  ViewPoint → {1.996, 2.148, 1.6889}]}}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 2, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.10: Source code 3D

```
In[•]:=
(*2D*)
u[x_, t_] := Exp[-t]/(1 + (4 t + x)^2);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot[u[x, time], {x, -15, 5},
                  PlotRange → {All, {0, 1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 3.5, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.11: Source code 2D

2.1.6 Transport equation $u_t + 2u_x = \sin x$ IC $u(0, x) = \sin x$. Peter Olver textbook, 2.2.5

problem number 6

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations.

Solve $u_t + 2u_x = \sin x$ with IC $u(0,x) = \sin x$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[t, x], {t}] +2*D[u[t, x], {x}]== Sin[x];
ic = u[0,x]==Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[t, x], {t, x}], 60*10]];
```

$$\left\{\left\{u(t,x) \to \frac{1}{2}(-2\sin(2t-x)+\cos(2t-x)-\cos(x))\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(t, x), t) +2*diff(u(t, x),x) =sin(x);
ic:=u(0,x)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(t,x))),output='
```

$$u(t,x) = -\frac{\cos(x)}{2} + \frac{\cos(2t-x)}{2} - \sin(2t-x)$$

Hand solution

Solve

$$u_t + 2u_x = \sin x \tag{1}$$

With initial conditions $u(x,0) = \sin x$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = \sin x(t) \tag{3}$$

$$\frac{dx}{dt} = 2 \tag{4}$$

Solving (3) gives

$$\int du = \int \sin x(t)\, dt \tag{3A}$$

From (4)

$$x = 2t + x(0)$$

Substituting the above into (3A) gives

$$\begin{aligned}\int du &= \int \sin(2t + x(0))\, dt \\ u &= \frac{-\cos(2t + x(0))}{2} + C \qquad (3B)\end{aligned}$$

At $t = 0$ the above becomes

$$\begin{aligned}\sin(x(0)) &= \frac{-\cos(x(0))}{2} + C \\ C &= \sin(x(0)) + \frac{\cos(x(0))}{2}\end{aligned}$$

Hence (3B) becomes

$$u = \frac{-\cos(2t + x(0))}{2} + \sin(x(0)) + \frac{\cos(x(0))}{2}$$

But $x(0) = x - 2t$, therefore

$$\begin{aligned}u(x,t) &= \frac{-\cos(2t + x - 2t)}{2} + \sin(x - 2t) + \frac{\cos(x - 2t)}{2} \\ &= \frac{-\cos(x)}{2} + \sin(x - 2t) + \frac{\cos(x - 2t)}{2}\end{aligned}$$

An <u>alternative approach</u> to solve transport PDE is by using Lagrange-charpit method

$$\frac{dt}{1} = \frac{dx}{2} = \frac{du}{\sin x}$$

$\frac{dt}{1} = \frac{dx}{2}$ gives $\frac{dx}{dt} = 2$ or $x = 2t + C_1$. Hence

$$C_1 = x - 2t$$

And $\frac{dx}{2} = \frac{du}{\sin x}$ gives $\frac{du}{dx} = \frac{1}{2}\sin x$. Integrating gives $u = \frac{-1}{2}\cos x + C_2$. Therefore

$$C_2 = u + \frac{1}{2}\cos x$$

But $C_2 = F(C_1)$ where F is arbitrary function. Therefore

$$\begin{aligned}u + \frac{1}{2}\cos x &= F(x - 2t) \\ u(t,x) &= F(x - 2t) - \frac{1}{2}\cos x \qquad (1)\end{aligned}$$

When $t = 0$, $u(0, x) = \sin x$, therefore the above becomes

$$\begin{aligned}
\sin x &= F(x) - \frac{1}{2}\cos x \\
F(x) &= \sin x + \frac{1}{2}\cos x \\
F(z) &= \sin z + \frac{1}{2}\cos z
\end{aligned}$$

Therefore the solution (1) can now be written as

$$\begin{aligned}
u(t, x) &= \left(\sin(x - 2t) + \frac{1}{2}\cos(x - 2t)\right) - \frac{1}{2}\cos x \\
&= \sin(x - 2t) + \frac{1}{2}\cos(x - 2t) - \frac{1}{2}\cos x
\end{aligned}$$

The following is an animation of the solution

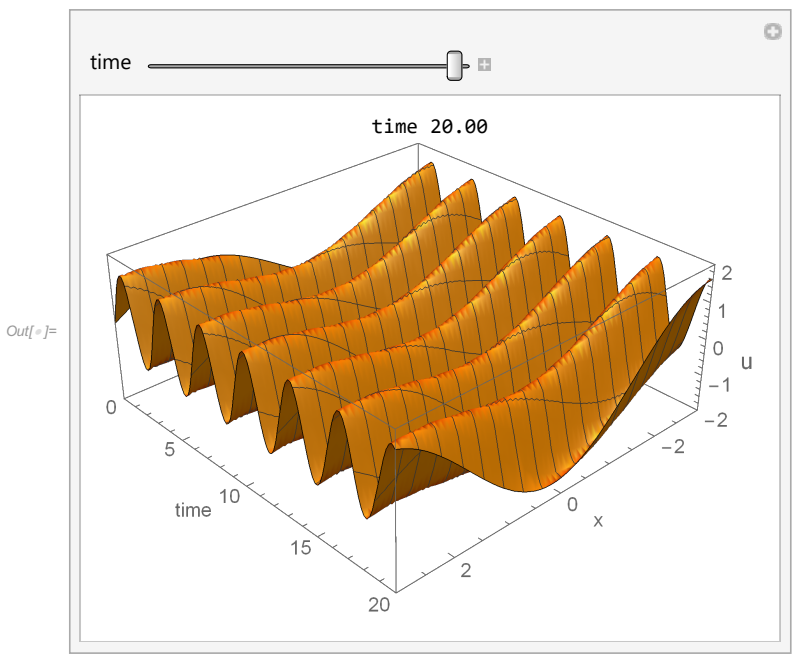

Figure 2.12: snap shot

Source code used for the above

```
In[•]:= (*3D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
         NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Sin[x - 2 t] + 1 / 2 Cos[x - 2 t] - 1 / 2 Cos[x];
plotStyle = Directive[Orange, Specularity[White, 20]];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot3D[u[x, t], {x, -3, 3}, {t, 0, time},
                  PlotRange → {{-3, 3}, {0, 20}, {-2, 2}},
                  AxesLabel → {Style["x", 12], Style["time", 12], Style["u", 14]},
                  BaseStyle → 12, PerformanceGoal → "Quality",
                  ImageSize → 400, PlotPoints → 30,
                  PlotStyle → plotStyle,
                  ViewPoint → {1.996, 2.148, 1.6889}]
         }}
   ]
   ,
   {{time, 0.01, "time"}, 0.01, 20, .1},
   TrackedSymbols :→ {time}
]
```

Figure 2.13: Source code 3D

```
In[•]:=
(*2D*)
u[x_, t_] := Sin[x - 2 t] + 1 / 2 Cos[x - 2 t] - 1 / 2 Cos[x];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot[u[x, time], {x, -15, 15},
                  PlotRange → {All, {-1.6, 1.6}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 15, .05},
   TrackedSymbols :→ {time}
]
```

Figure 2.14: Source code 2D

2.1.7 Transport equation $u_t + \frac{1}{1+x^2} u_x = 0$ IC $u(x,0) = \frac{1}{1+(3+x)^2}$. Peter Olver textbook, page 27

problem number 7

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. Example 2.4, page 27.

Solve $u_t + \frac{1}{1+x^2} u_x = 0$ with IC $u(x,0) = \frac{1}{1+(3+x)^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t],t] +1/(1+x^2)*D[u[x,t],x]== 0;
ic = u[x,0]==1/(1+(3+x)^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{}{\sqrt[3]{2}x^3\sqrt[3]{\sqrt{(-3t+x^3+3x)^2+4}-3t+x^3+3x}+3\sqrt[3]{2}x\sqrt[3]{\sqrt{(-3t+x^3+3x)^2+4}-3t+x}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +1/(1+x^2)*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+(3+x)^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{\left(\frac{\left(4x^3-12t+12x+4\sqrt{9\left(-\frac{1}{3}x^3+t-x\right)^2+4}\right)^{\frac{1}{3}}}{2} - \frac{2}{\left(4x^3-12t+12x+4\sqrt{9\left(-\frac{1}{3}x^3+t-x\right)^2+4}\right)^{\frac{1}{3}}}\right)^2 + 3\left(4x^3-12t+12x\right.}$$

<u>Hand solution</u>

Solve

$$u_t + \frac{1}{x^2+1} u_x = 0$$

With initial conditiomns $u(0,x) = \frac{1}{1+(x+3)^2}$

<u>Solution</u>

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = \frac{1}{x^2+1} \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1+(x(0)+3)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\begin{aligned} \left(x^2+1\right) dx &= dt \\ \frac{x^3}{3} + x &= t + C \end{aligned} \tag{6}$$

At $t = 0$

$$\frac{x(0)^3}{3} + x(0) = C$$

Hence (6) becomes

$$\frac{x^3}{3} + x - t = \frac{x(0)^3}{3} + x(0)$$

This is Cubic in $x(0)$. The solution is complicated and will not be given. All what is left is to substitute this solution back in (5) and this is what the computer did above.

The following is an animation of the solution obtained from CAS. Animation agrees with textbook screen shots.

Out[ ]=

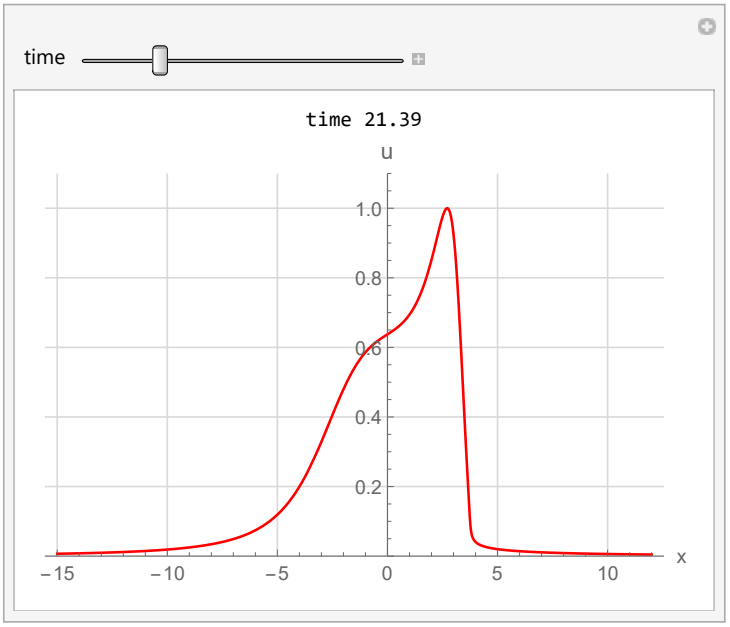

Figure 2.15: snap shot

Source code used for the above

In[ ]:=

```
u[x_, t_] := (2 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(2/3))/
    (2 × 2^(2/3) - 18 × 2^(2/3) t + 18 × 2^(2/3) x + 6 × 2^(2/3) x^3 + 6 × 2^(2/3) Sqrt[4 + (3 t - 3 x - x^3)^2] - 12 × 2^(1/3) (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) - 3 × 2^(1/3) t (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) +
      3 × 2^(1/3) x (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) + 2^(1/3) x^3 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) + 2^(1/3) Sqrt[4 + (3 t - 3 x - x^3)^2] (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(1/3) +
      16 (-3 t + 3 x + x^3 + Sqrt[4 + (3 t - 3 x - x^3)^2])^(2/3));
Manipulate[
  Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
     {
       Quiet@Plot[u[x, time], {x, -15, 12},
          PlotRange → {All, {0, 1.1}},
          AxesLabel → {Style["x", 12], Style["u", 14]},
          BaseStyle → 12,
          ImageSize → 400,
          PlotPoints → 40,
          GridLines → Automatic,
          GridLinesStyle → LightGray,
          PlotStyle → Red
        ]
     }}
  ]
  ,
  {{time, 0, "time"}, 0, 100, .01},
  TrackedSymbols :→ {time}
]
```

Figure 2.16: Source code

2.1.8 Transport equation $u_t - xu_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.17

problem number 8

Added Sept 12, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. problem 2.2.17

Solve $u_t - xu_x = 0$ with IC $u(x,0) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] -x*D[u[x,t], x]== 0;
ic = u[x,0]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{e^{2t}x^2+1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) -x*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{x^2 e^{2t}+1}$$

<u>Hand solution</u>

Solve the initial value problem

$$u_t - xu_x = 0$$

With initial conditions $u(0,x) = \frac{1}{1+x^2}$

<u>Solution</u>

Let $u = u(x(t),t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$
$$\frac{dx}{dt} = -x \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1 + x(0)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\begin{aligned} \ln|x| &= -t + C \\ x &= x(0)\, e^{-t} \\ x(0) &= xe^{t} \end{aligned} \tag{6}$$

Substituting (6) in (5) gives

$$u(x(t), t) = \frac{1}{1 + x^2 e^{2t}}$$

The following is an animation of the solution

Out[ ]=

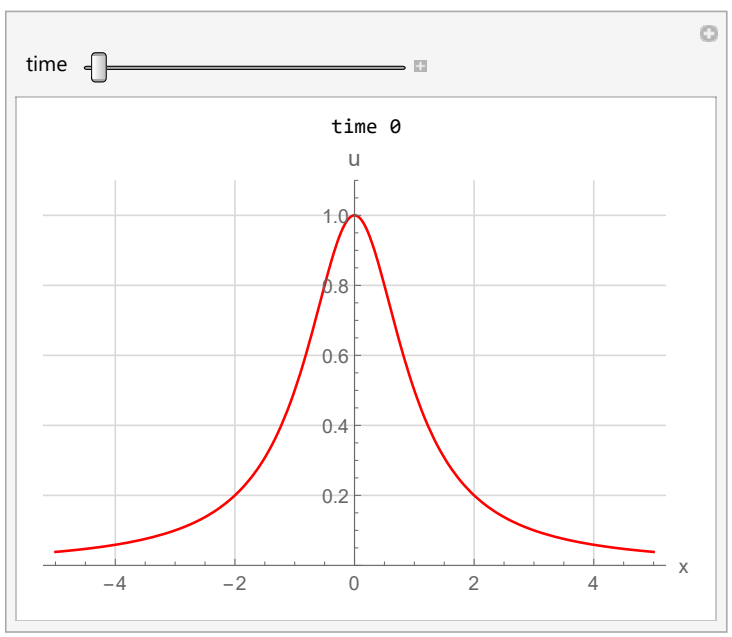

Figure 2.17: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}},
         {
            Quiet@Plot[u[x, time], {x, -5, 5},
                  PlotRange → {All, {0, 1.1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 5, .01},
   TrackedSymbols :→ {time}
]
```

Figure 2.18: Source code 2D

2.1.9 Transport equation $u_t + (1-2t)u_x = 0$ IC $u(x,0) = \frac{1}{1+x^2}$. Peter Olver textbook, problem 2.2.29

problem number 9

Added Sept 15, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. problem 2.2.29

Solve $u_t + (1-2t)u_x = 0$ with IC $u(x,0) = \frac{1}{1+x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] +(1-2*t)*D[u[x,t], x]== 0;
ic = u[x,0]==1/(1+x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2t^2x + t^4 - 2t^3 + t^2 - 2tx + x^2 + 1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +(1-2*t)*diff(u(x,t),x) =0;
ic:=u(x,0)=1/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{1}{(-t^2+t-x)^2+1}$$

Hand solution

Solve

$$u_t + (1-2t)\, u_x = 0$$

with initial conditions $u(0,x) = \frac{1}{1+x^2}$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = (1-2t) \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= \frac{1}{1+x(0)^2} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$x = t - t^2 + C$$

At $t = 0$

$$x(0) = C$$

Hence

$$\begin{aligned} x &= t - t^2 + x(0) \\ x(0) &= x - t + t^2 \end{aligned}$$

Substituting this back into (5) gives

$$u(x(t)\,,t) = \frac{1}{1+(x-t+t^2)^2}$$

The following is an animation of the solution

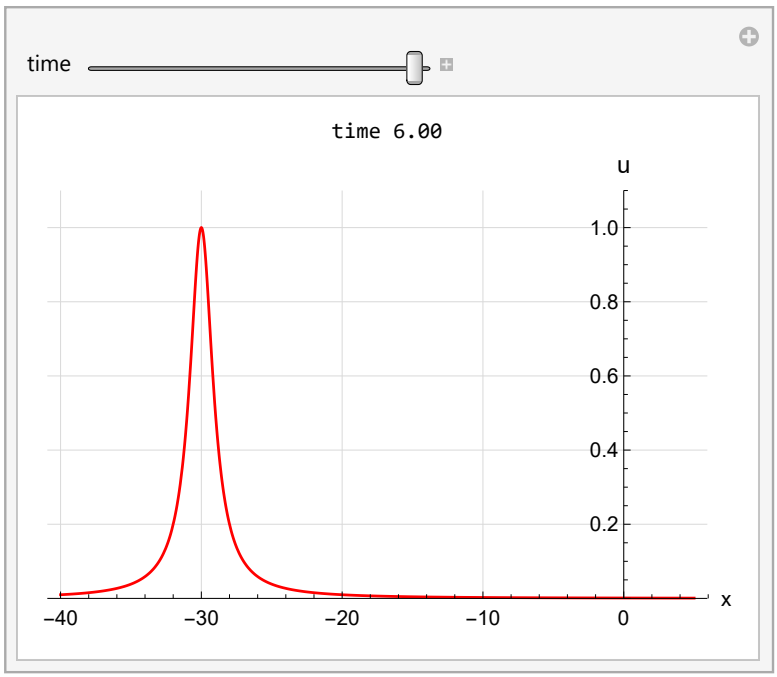

Figure 2.19: snap shot

Source code used for the above

```
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
         NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1/(1 + (x - t + t^2)^2)
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {3, 2}]}]},
         {
            Quiet@Plot[u[x, time], {x, -40, 5},
                  PlotRange → {All, {0, 1.1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotPoints → 40
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 6, .01},
   TrackedSymbols :→ {time}
]
```

Figure 2.20: Source code 2D

2.1.10 Transport equation $u_t + \frac{1}{x^2+4} u_x = 0$ IC $u(x,0) = e^{x^3+12x}$

problem number 10

Added Oct 8, 2019.

Exam problem. Math 5587, fall 2019. UMN

solve for $u(x,t)$ the PDE $u_t + \frac{1}{x^2+4} u_x = 0$ IC $u(x,0) = e^{x^3+12x}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x,t], t] +1/(x^2+4)*D[u[x,t], x]== 0;
ic = u[x,0]==Exp[x^3+12*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x,t], {x,t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{-3t+x^3+12x}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) +1/(x^2+4)*diff(u(x,t),x) =0;
ic:=u(x,0)=exp(x^3+12*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
sol:=simplify(expand(sol));
```

$$u(x,t) = \mathrm{e}^{x^3-3t+12x}$$

Hand solution

Solve

$$u_t + \frac{1}{x^2+4} u_x = 0$$

with initial conditions $u(x,0) = e^{x^3+12x}$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = \frac{1}{x^2+4} \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= e^{x(0)^3+12x(0)} \end{aligned} \tag{5}$$

We just need to find $x(0)$ to finish the solution. From (4)

$$\frac{x^3}{3} + 4x = t + C \tag{6}$$

At $t = 0$

$$\frac{x(0)^3}{3} + 4x(0) = C$$

Hence (6) becomes

$$
\begin{aligned}
\frac{x^3}{3} + 4x &= t + \frac{x(0)^3}{3} + 4x(0) \\
x(0)^3 &= 3\left(\frac{x^3}{3} + 4x - t - 4x(0)\right) \\
&= x^3 + 12x - 3t - 12x(0)
\end{aligned}
$$

Substituting this back into (5) gives

$$
\begin{aligned}
u(x(t), t) &= \exp\left(x^3 + 12x - 3t - 12x(0) + 12x(0)\right) \\
&= e^{x^3+12x-3t}
\end{aligned}
$$

The following is an animation of the solution

Out[ ]=

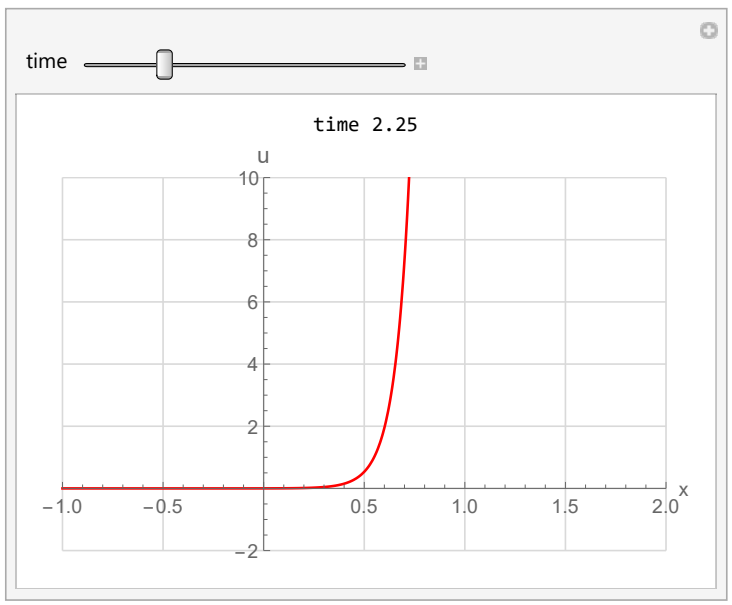

Figure 2.21: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := Exp[x^3 + 12 x - 3 t];

Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {3, 2}]}]}],
         {
            Quiet@Plot[u[x, time], {x, -1, 2},
                  PlotRange → {{-1, 2}, {-2, 10}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotPoints → 40
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 10, .01},
   TrackedSymbols :→ {time}
]
```

Figure 2.22: Source code 2D

2.1.11 $3u_x + 5u_y = x$

problem number 11

Taken from Mathematica help pages

Solve for $u(x, y)$

$$3u_x + 5u_y = x$$

Mathematica ✓

```
ClearAll["Global`*"];
sol = AbsoluteTiming[TimeConstrained[DSolve[3*D[u[x, y], x] + 5*D[u[x, y], y] == x, u[x, y],
```

$$\left\{\left\{u(x,y) \to \frac{x^2}{6} + c_1\left(y - \frac{5x}{3}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=3*diff(u(x, y), x) + 5*diff(u(x, y), y) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \frac{x^2}{6} + \_F1\left(-\frac{5x}{3} + y\right)$$

Hand solution

Solve

$$3u_x + 5u_y = x$$
$$u_x + \frac{5}{3}u_y = \frac{x}{3} \tag{1}$$

Solution

Let $u = u(y(x), x)$. Then

$$\frac{du}{dx} = \frac{\partial u}{\partial y}\frac{dy}{dx} + \frac{\partial u}{\partial x} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dx} = \frac{x}{3} \tag{3}$$
$$\frac{dy}{dx} = \frac{5}{3} \tag{4}$$

Solving (3) gives

$$u = \frac{x^2}{6} + C_1$$
$$C_1 = u - \frac{x^2}{6}$$

From (4)

$$y = \frac{5}{3}x + C_2$$
$$C_2 = y - \frac{5}{3}x$$

Let $C_1 = F(C_2)$ where F is arbutrary function. This gives

$$u - \frac{x^2}{6} = F\left(y - \frac{5}{3}x\right)$$
$$u(x,y) = F\left(y - \frac{5}{3}x\right) + \frac{x^2}{6}$$

2.1.12 $xu_y + yu_x = -4xyu$ and $u(x,0) = e^{-x^2}$

problem number 12

Taken from Mathematica help pages

Solve for $u(x,y)$

$$xu_y + yu_x = -4xyu$$

with initial value $u(x,0) = e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[u[x, y], y] + y*D[u[x, y], x] == -4*x*y*u[x, y];
ic  = u[x, 0] == Exp[-x^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{-x^2-y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(u(x, y), y) + y*diff(u(x, y), x) = -4*x*y*u(x, y);
ic  := u(x, 0) = exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, y))),outpu
```

$$u(x,y) = \mathrm{e}^{-x^2-y^2}$$

<u>Hand solution</u>

Solve

$$xu_y + yu_x = -4xyu$$

with $u(x,0) = e^{-x^2}$.

<u>Solution</u>

Let $u \equiv u(x(y), y)$. We've taken y as the independent variable for $x(y)$ here, since the initial conditions has $y(0)$ in it. The PDE can be written as

$$u_y + \frac{y}{x}u_x = -4yu \tag{1}$$

Then

$$\frac{du}{dy} = \frac{\partial u}{\partial x}\frac{dx}{dy} + \frac{\partial u}{\partial y} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dy} = -4yu \tag{3}$$
$$\frac{dx}{dy} = \frac{y}{x} \tag{4}$$

Solving (3) gives

$$\begin{aligned} \ln|u| &= -\frac{4y^2}{2} + C_1 \\ u &= C_1 e^{-2y^2} \end{aligned} \tag{5}$$

At $y = 0$, using initial conditions the above becomes

$$e^{-x(0)^2} = C_1$$

(5) becomes

$$\begin{aligned} u &= e^{-x(0)^2} e^{-2y^2} \\ &= e^{-x(0)^2 - 2y^2} \end{aligned} \tag{5A}$$

All what is left is to find $x(0)$ to finish the solution. From (4)

$$\frac{x^2}{2} = \frac{y^2}{2} + C_2 \tag{6}$$

At $y = 0$

$$\frac{x(0)^2}{2} = C_2$$

Hence (6) becomes

$$\begin{aligned} \frac{x^2}{2} &= \frac{y^2}{2} + \frac{x(0)^2}{2} \\ x(0)^2 &= x^2 - y^2 \end{aligned}$$

Substituting the above in (5A) gives

$$\begin{aligned} u(x(y), x) &= e^{-(x^2 - y^2) - 2y^2} \\ &= e^{-x^2 - y^2} \end{aligned}$$

The following is a plot of the above solution showing the initial conditions are red line

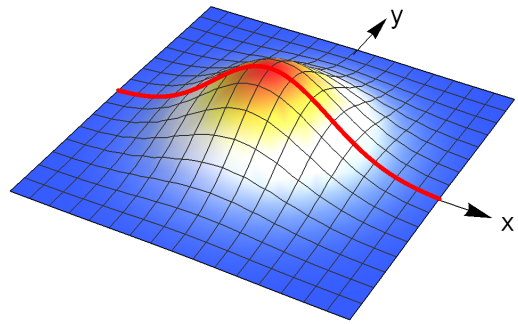

Figure 2.23: Solution $e^{-x^2-y^2}$

```
u[x_, y_] := Exp[-x^2 - y^2];
initialCurve = ParametricPlot3D[{x, 0, Exp[-x^2]}, {x, -2, 2},
      PlotStyle -> Red];
solution = Plot3D[u[x, y], {x, -2, 2}, {y, -2, 2},
      ColorFunction -> "TemperatureMap"];
Graphics3D[{
    First@solution,
    First@initialCurve,
    Arrow[{{0, 0, 0}, {2.6, 0, 0}}],
    Arrow[{{0, 0, 0}, {0, 2.8, 0}}],
    Text["x", {2.7, 0, 0}, {-1, 0}],
    Text["y", {0, 2.9, 0}, {-1, 0}]
  }, SphericalRegion -> True,
  Boxed → False, BaseStyle → 12,
  ImageSize → 300, PlotRange → All]
```

Figure 2.24: Code used for the plot

2.1.13 $u_t + u_x = 0$ and $u(x,0) = \sin x$ and $u(0,t) = 0$

problem number 13

Taken from Mathematica help pages

Solve for $u(x,t)$

$$u_t + u_x = 0$$

with initial value $u(x,0) = \sin x$ and boundary value $u(0,t) = 0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], x] == 0;
bc  = u[0, t] == 0;
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to (\theta(t-x)-1)\sin(t-x)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)=0;
bc  := u(0,t)=0;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = -\sin(t-x)\,\theta(-t+x)$$

Hand solution

Since initial and boundary conditions are given, the Laplace transform method will be used to solve this PDE. Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Applying Laplace transform to the PDE gives

$$\begin{aligned} sU - u(x,0) + \frac{dU}{dx} &= 0 \\ \frac{dU}{dx} + sU &= \sin x \end{aligned}$$

Integrating factor is $\mu = e^{\int s dx} = e^{sx}$. Multiplying the above by μ gives

$$\frac{d}{dx}(Ue^{sx}) = e^{sx}\sin x$$

Integrating

$$\begin{aligned} Ue^{sx} &= \int e^{sx}\sin x dx + C \\ &= \frac{e^{sx}(s\sin x - \cos x)}{1+s^2} + C \\ U(x,s) &= \frac{s\sin x - \cos x}{1+s^2} + Ce^{-sx} \end{aligned}$$

Applying boundary conditions $U(0,s)=0$ gives

$$
\begin{aligned}
0 &= \frac{-1}{1+s^2}+C \\
C &= \frac{1}{1+s^2}
\end{aligned}
$$

Hence

$$
\begin{aligned}
U(x,s) &= \frac{s\sin x-\cos x}{1+s^2}+\frac{e^{-sx}}{1+s^2} \\
&= \frac{s\sin x}{1+s^2}-\frac{\cos x}{1+s^2}+\frac{e^{-sx}}{1+s^2}
\end{aligned}
$$

Applying inverse Laplace transform gives

$$
\begin{aligned}
u(x,t) &= \cos t\sin x-\cos x\sin t+\text{Heaviside}\,(t-x)\sin\,(t-x) \\
&= -\sin\,(t-x)+\text{Heaviside}\,(t-x)\sin\,(t-x) \\
&= (\text{Heaviside}\,(t-x)-1)\sin\,(t-x)
\end{aligned}
$$

2.1.14 $u_t+cu_x=0$ and $u(x,0)=e^{-x^2}$

problem number 14

Taken from Mathematica help pages

Solve for $u(x,t)$

$$u_t+cu_x=0$$

With initial conditions $u(x,0)=e^{-x^2}$

Mathematica ✓

```
ClearAll["Global`*"];
ic  = u[x, 0] == Exp[-x^2];
pde =  D[u[x, t], {t}] + c*D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{-(x-ct)^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) + c* diff(u(x, t),x) =0;
ic  := u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \mathrm{e}^{-(tc-x)^2}$$

Hand solution

Solve

$$u_t + cu_x = 0 \tag{1}$$

with initial conditions $u(x,0) = e^{-x^2}$.

Solution

Let $u = u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = c \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= e^{-x(0)^2} \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= ct + x(0) \\ x(0) &= x - ct \end{aligned}$$

Then (5) becomes

$$u(x(t), t) = e^{-(x-ct)^2}$$

2.1.15 (Haberman 12.2.2) $\omega_t - 3\omega_x = 0$ and $\omega(x,0) = \cos x$

problem number 15

Added Nov 25, 2018.

Problem 12.2.2 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t - 3\omega_x = 0$$

With initial conditions $\omega(x,0) = \cos x$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] - 3*D[w[x, t], x] == 0;
ic  = w[x, 0] == Cos[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x, t], {x, t}], 60*10]];
```

$$\{\{w(x,t) \to \cos(3t+x)\}\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)-3*diff(w(x,t),x)=0;
ic:=w(x,0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t) = \cos(3t+x)$$

<u>Hand solution</u>

Solve

$$w_t - 3w_x = 0 \tag{1}$$

With I.C. $w(x,0) = \cos x$

<u>Solution</u>

Let $w = w(x(t), t)$. Then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{dw}{dt} = 0 \tag{3}$$
$$\frac{dx}{dt} = -3 \tag{4}$$

Solving (3) gives

$$\begin{aligned} w &= w(x(0)) \\ &= \cos(x(0)) \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= -3t + x(0) \\ x(0) &= x + 3t \end{aligned}$$

Hence (5) becomes

$$w(x(t), t) = \cos(x + 3t)$$

2.1.16 (Haberman 12.2.4) $\omega_t + c\omega_x = 0$ and $\omega(x,0) = f(x)$ and $\omega(0,t) = h(t)$

problem number 16

Added Nov 25, 2018.

Problem 12.2.4 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + c\omega_x = 0$$

With $c > 0$. For $x > 0, t > 0$ if $\omega(x,0) = f(x)$ and $\omega(0,t) = h(t)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] + c*D[w[x, t], x] == 0;
ic  = w[x, 0] == f[x];
bc  = w[0, t] == h[t];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, w[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,t),t)+c*diff(w(x,t),x)=0;
ic:=w(x,0)=f(x);
bc:=w(0,t)=h(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],w(x,t)) assumi
```

$$w(x,t) = \frac{ch\left(t-\frac{x}{c}\right)\theta\left(t-\frac{x}{c}\right)+\mathcal{L}^{-1}\left(\left(\int \mathrm{e}^{\frac{sx}{c}} f(x)\,dx\right)\mathrm{e}^{-\frac{sx}{c}},s,t\right)-\mathcal{L}^{-1}\left(\left(\int^{0} \mathrm{e}^{-\frac{as}{c}} f(\_a)\,d\_a\right)\mathrm{e}^{-\frac{sx}{c}},s,t\right)}{c}$$

Solution contains unresolved invlaplace calls

<u>Hand solution</u>

$$\frac{\partial w}{\partial t}+c\frac{\partial w}{\partial x}=0 \tag{1}$$

Let

$$w \equiv w(x(t),t)$$

Hence

$$\frac{dw}{dt}=\frac{\partial w}{\partial t}+\frac{\partial w}{\partial x}\frac{dx}{dt} \tag{2}$$

Comparing given (1) and (2), we see that if we let $\frac{dx}{dt}=c$ in (2), then we obtain (1). Hence we conclude that $\frac{dw}{dt}=0$. Therefore, $w(x(t),t)$ is constant. At $t=0$, we are given that

$$w(x(t),t)=f(x(0)) \qquad t=0 \tag{3}$$

We just now need to determine $x(0)$. This is found from $\frac{dx}{dt}=c$, which has the solution $x(t)=x(0)+ct$. Hence $x(0)=x(t)-ct$. Therefore (3) becomes

$$w(x,t)=f(x-ct)$$

This is valid for $x>ct$. We now start all over again, and look at Let

$$w \equiv w(x,t(x))$$

Hence

$$\frac{dw}{dx}=\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}\frac{dt}{dx} \tag{4}$$

Comparing (4) and (1), we see that if we let $\frac{dt}{dx}=\frac{1}{c}$ in (4), then we obtain (1). Hence we conclude that $\frac{dw}{dx}=0$. Therefore, $w(x,t(x))$ is constant. At $x=0$, we are given that

$$w(x,t(x))=h(t(0)) \qquad x=0 \tag{5}$$

We just now need to determine $t(0)$. This is found from $\frac{dt}{dx} = \frac{1}{c}$, which has the solution $t(x) = t(0) + \frac{1}{c}x$. Hence $t(0) = t(x) - \frac{1}{c}x$. Therefore (5) becomes

$$w(x,t) = h\left(t - \frac{1}{c}x\right)$$

Valid for $t > \frac{x}{c}$ or $x < ct$. Therefore, the solution is

$$w(x,t) = \begin{cases} f(x-ct) & x > ct \\ h\left(t - \frac{1}{c}x\right) & x < ct \end{cases}$$

2.1.17 (Haberman 12.2.5 (a)) $\omega_t + c\omega_x = e^{2x}$ and $\omega(x,0) = f(x)$

problem number 17

Added Nov 25, 2018.

Problem 12.2.5 (a) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + c\omega_x = e^{2x}$$

With $\omega(x,0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] + c*D[w[x, t], x] == Exp[2*x];
ic  = w[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}, Assumption
```

$$\left\{\left\{w(x,t) \to f(x-ct) + \frac{e^{2x}(1-e^{-2ct})}{2c}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)+c*diff(w(x,t),x)=exp(2*x);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t)) assuming
```

$$w(x,t) = \frac{2cf(-tc+x) + \mathrm{e}^{2x} - \mathrm{e}^{-2tc+2x}}{2c}$$

Hand solution

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = c \tag{2}$$
$$\frac{du}{ds} = e^{2x} \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = t_1, x(0) = t_2, u(0) = t_3$$

And $u(x,0) = f(x)$ becomes

$$t_3 = f(t_2)\,, t_1 = 0 \tag{4}$$

Equation (1) gives

$$\begin{aligned} t &= s + t_1 \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives

$$x = cs + t_2 \tag{6}$$

From (5,6) solving for t_2 gives

$$\begin{aligned} t_2 &= x - cs \\ &= x - ct \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} du &= e^{2x} ds \\ &= e^{2(cs+t_2)} ds \end{aligned}$$

Integrating

$$u = \frac{e^{2(cs+t_2)}}{2c} + t_3$$

Using (7,4,5) in the above gives the solution

$$\begin{aligned} u(x,t) &= \frac{e^{2(ct+(x-ct))}}{2c} + f(x-ct) \\ &= \frac{1}{2c}e^{2x} + f(x-ct) \end{aligned}$$

My solution is not the same as CAS, but it was verified OK using Maple pdetest.

2.1.18 (Haberman 12.2.5 (d)) $\omega_t + 3t\omega_x = \omega(x,t)$ and $\omega(x,0) = f(x)$

problem number 18

Added Nov 25, 2018.

Problem 12.2.5 (d) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + 3t\omega_x = \omega(x,t)$$

with $\omega(x,0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, t], t] + 3*t*D[w[x, t], x] == w[x, t];
ic  = w[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}]], 60*10]];
```

$$\left\{\left\{w(x,t) \to e^t f\left(x - \frac{3t^2}{2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,t),t)+3*t*diff(w(x,t),x)=w(x,t);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t) = \mathrm{e}^t f\left(-\frac{3t^2}{2} + x\right)$$

Hand solution

Solve

$$\frac{\partial w}{\partial t} + 3t\frac{\partial w}{\partial x} = w(x,t) \tag{1}$$

With initial conditions $w(x,0) = f(x)$

Solution

Let $w \equiv w(x(t), t)$ then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial t} \tag{2}$$

Comparing (1,2) shows that

$$\frac{dw}{dt} = w \tag{3}$$

$$\frac{dx}{dt} = 3t \tag{4}$$

Solving (3) gives

$$w = Ce^t$$

From initial conditions at $t = 0$, the above becomes $f(x(0)) = C$. Hence the above becomes

$$w(x,t) = f(x(0))\, e^t \tag{5}$$

From (4)

$$x = \frac{3}{2}t^2 + x(0)$$
$$x(0) = x - \frac{3}{2}t^2$$

Substituting the above in (5) gives

$$w(x(t), t) = f\left(x - \frac{3}{2}t^2\right) e^t$$

<u>Alternative solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = 3t \tag{2}$$
$$\frac{dw}{ds} = w \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = t_1, x(0) = t_2, w(0) = t_3$$

And $w(x, 0) = f(x)$ becomes

$$t_3 = f(t_2), t_1 = 0 \tag{4}$$

Equation (1) gives

$$\begin{aligned} t &= s + t_1 \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives, after replacing t by s from (5)

$$\begin{aligned} \frac{dx}{ds} &= 3s \\ x &= \frac{3}{2}s^2 + t_2 \end{aligned} \tag{6}$$

Solving for t_2 gives

$$t_2 = x - \frac{3}{2}s^2 \tag{7}$$

Equation (3) gives

$$\begin{aligned}\ln w &= s + t_3\\ w &= t_3 e^s\\ &= f(t_2)\, e^s\end{aligned}$$

Using (7,5) in the above gives the solution

$$w(x,t) = f\left(x - \frac{3}{2}t^2\right) e^t$$

2.1.19 $2u_x + 5u_y = u^2(x,y) + 1$

problem number 19

Taken from Mathematica help pages

General solution for a quasilinear first-order PDE

Solve for $u(x,y)$

$$2u_x + 5u_y = u^2(x,y) + 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*D[u[x, y], x] + 5*D[u[x, y], y] == u[x, y]^2 + 1;
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \tan\left(\frac{x}{2} + c_1\left(y - \frac{5x}{2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := 2* diff(u(x, y), x) + 5*diff(u(x, y), y) = u(x, y)^2 + 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, y))),output='rea
```

$$u(x,y) = \tan\left(\frac{x}{2} + \frac{\_F1\left(-\frac{5x}{2} + y\right)}{2}\right)$$

Hand solution

Solve for $u(x,y)$ in $2u_x + 5u_y = u^2 + 1$. Using the Lagrange-charpit method, the characteristic equations are

$$\frac{dx}{2} = \frac{dy}{5} = \frac{du}{u^2+1}$$

From the first pair of equation we obtain

$$\begin{aligned} 5dx &= 2dy \\ 5x &= 2y + C_1 \\ C_1 &= 5x - 2y \end{aligned}$$

Now we can pick the pair $\frac{dy}{5} = \frac{du}{u^2+1}$ or $\frac{dx}{2} = \frac{du}{u^2+1}$ to solve for u. It does not matter which. Using

$$\frac{dx}{2} = \frac{du}{u^2+1}$$

Integrating gives

$$\begin{aligned} \frac{1}{2}x &= \arctan(u) + C_2 \\ C_2 &= \frac{1}{2}x - \arctan(u) \end{aligned}$$

C_1 and C_2 are always related by $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$\begin{aligned} \frac{1}{2}x - \arctan(u) &= F(5x-2y) \\ \arctan(u) &= \frac{1}{2}x - F(5x-2y) \\ u(x,y) &= \tan\left(\frac{1}{2}x - F(5x-2y)\right) \end{aligned}$$

2.1.20 Clairaut equation $xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$

problem number 20

Taken from Mathematica Symbolic PDE document

nonlinear first-order PDE, the Clairaut equation

Solve for $u(x,y)$

$$xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1x+c_2y+\frac{1}{2}\left(c_1{}^2+c_2{}^2\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y)=-\frac{x^2}{2}-\frac{y^2}{2}-\_c_1\ln\left(x+\sqrt{x^2+2\_c_1}\right)+\_c_1\ln\left(y+\sqrt{y^2-2\_c_1}\right)+c_1+c_2-\frac{\sqrt{x^2+2\_c_1}\,x}{2}-\frac{\sqrt{y^2}}{}$$

Hand solution

Assuming the solution is $u(x,y)=X(x)+Y(y)$. Substituting this into the PDE gives

$$xX'+yY'+\frac{1}{2}\left((X')^2+(Y')^2\right)=0$$

$$\frac{1}{2}(X')^2+xX'=-\frac{1}{2}(Y')^2-yY'$$

The above is possible when each side is equal to same constant, say C_1. This gives two ODE's

$$\frac{1}{2}(X')^2+xX'=C_1 \tag{1}$$

$$\frac{1}{2}(Y')^2+yY'=-C_1 \tag{2}$$

ODE (1) becomes

$$(X')^2+2xX'-2C_1=0$$

$$\begin{aligned}X'&=\frac{-b}{2a}\pm\frac{1}{2a}\sqrt{b^2-4ac}\\&=\frac{-2x}{2}\pm\frac{1}{2}\sqrt{4x^2+8C_1}\\&=-x\pm\sqrt{x^2+2C_1}\end{aligned}$$

For the case $X' = -x + \sqrt{x^2 + 2C_1}$, the solution is

$$\begin{aligned} X(x) &= \int -x + \sqrt{x^2 + 2C_1} dx + C_2 \\ &= -\frac{x^2}{2} + \frac{x\sqrt{x^2 + 2C_1}}{2} + C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2 \end{aligned}$$

For the case $X' = -x - \sqrt{x^2 + 2C_1}$, the solution is

$$\begin{aligned} X(x) &= \int -x - \sqrt{x^2 + 2C_1} dx + C_2 \\ &= -\frac{x^2}{2} - \frac{x\sqrt{x^2 + 2C_1}}{2} - C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2 \end{aligned}$$

Combining the above two solutions to one gives

$$X(x) = -\frac{x^2}{2} \pm \frac{x\sqrt{x^2 + 2C_1}}{2} \pm C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2 \tag{3}$$

ODE (2) becomes

$$\begin{aligned} (Y')^2 + 2yY' + 2C_1 = 0 \\ Y' &= \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac} \\ &= \frac{-2y}{2} \pm \frac{1}{2}\sqrt{4y^2 - 8C_1} \\ &= -y \pm \sqrt{y^2 - 2C_1} \end{aligned}$$

For the case $Y' = -y + \sqrt{x^2 - 2C_1}$, the solution is

$$\begin{aligned} Y(y) &= \int -y + \sqrt{y^2 - 2C_1} dy + C_2 \\ &= \frac{-y^2}{2} + \frac{y\sqrt{y^2 - 2C_1}}{2} - C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3 \end{aligned}$$

For the case $Y' = -y - \sqrt{y^2 - 2C_1}$, the solution is

$$\begin{aligned} Y(y) &= \int -y - \sqrt{y^2 + 2C_1} dy + C_2 \\ &= -\frac{y^2}{2} - \frac{y\sqrt{x^2 - 2C_1}}{2} + C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3 \end{aligned}$$

Combining the above two solutions to one gives

$$Y(x) = -\frac{y^2}{2} \pm \frac{y\sqrt{y^2 - 2C_1}}{2} \pm C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3 \tag{4}$$

From (3,4) the final solution is

$$
\begin{aligned}
u(x,y) &= X(x) + Y(x) \\
&= \left(-\frac{x^2}{2} \pm \frac{x\sqrt{x^2+2C_1}}{2} \pm C_1 \ln\left(x+\sqrt{x^2+2C_1}\right) + C_2\right) + \left(-\frac{y^2}{2} \pm \frac{y\sqrt{y^2-2C_1}}{2} \pm C_1 \ln\left(y+\right.\right. \\
&= -\frac{x^2}{2} \pm \frac{x}{2}\sqrt{x^2+2C_1} \pm C_1 \ln\left(x+\sqrt{x^2+2C_1}\right) - \frac{y^2}{2} \pm \frac{y}{2}\sqrt{y^2-2C_1} \pm C_1 \ln\left(y+\sqrt{y^2-2C_1}\right)
\end{aligned}
$$

Where $C_4 = C_2 + C_3$.

2.1.21 Clairaut equation. $xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$ with $u(x,0) = \frac{1}{2}(1-x^2)$

problem number 21

Taken from Mathematica Symbolic PDE document

Clairaut equation with initial value

Solve for $u(x,y)$

$$xu_x + yu_y + \frac{1}{2}((u_x)^2 + (u_y)^2) = 0$$

With $u(x,0) = \frac{1}{2}(1-x^2)$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x,
ic  = u[x, 0] == (1*(1 - x^2))/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -\frac{x^2}{2} + y + \frac{1}{2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
ic  := u(x,0)=1/2*(1-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\frac{(x-y+1)(x-y-1)}{2}$$
$$u(x,y) = -\frac{(x+y+1)(x+y-1)}{2}$$

2.1.22 Clairaut equation. $u = xu_x + yu_y + \sin(u_x + u_y)$

problem number 22

Taken from Mathematica DSolve help pages

Another example of nonlinear Clairaut equation

Solve for $u(x,y)$

$$u = xu_x + yu_y + \sin(u_x + u_y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = u[x, y] == x*D[u[x, y], x] + y*D[u[x, y], y] + Sin[D[u[x, y], x] + D[u[x, y], y]];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1 x + c_2 y + \sin(c_1 + c_2)\}\}$$

Maple ✓

```
restart;
pde := u(x,y)= x*diff(u(x,y),x)+y*diff(u(x,y),y)+sin(diff(u(x,y),x)+diff(u(x,y),y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = x\_c_1 + y\_c_2 + \sin(\_c_1 + \_c_2)$$

2.1.23 Recover a function from its gradient vector

problem number 23

Taken from Mathematica DSolve help pages

Solve for $f(x,y)$

$$\frac{\partial f}{\partial x} = xy\cos(xy) + \sin(xy)$$
$$\frac{\partial f}{\partial y} = -e^{-y} + x^2\cos(xy)$$

Mathematica ✓

```
ClearAll["Global`*"];
eq1 = D[f[x, y], x] == x*y*Cos[x*y] + Sin[x*y];
 eq2 = D[f[x, y], y] == -E^(-y) + x^2*Cos[x*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{eq1, eq2}, f[x, y], {x, y}], 60*10]];
```

$$\{\{f(x,y) \to x\sin(xy) + e^{-y} + c_1\}\}$$

Maple ✓

```
restart;
eq1:=diff(f(x,y),x)=x*y*cos(x*y)+sin(x*y);
eq2:=diff(f(x,y),y)=-exp(-y)+x^2*cos(x*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({eq1,eq2},f(x,y))),output=
```

$$\{f(x,y) = x\sin(xy) + c_1 + \mathrm{e}^{-y}\}$$

2.1.24 $xf_y - f_x = \frac{g(x)}{h(y)} f^2$

problem number 24

Taken from Maple pdsolve help pages

General solution of a first order nonlinear PDE

Solve for $f(x, y)$

$$xf_y - f_x = \frac{g(x)}{h(y)} f^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[f[x, y], y] - D[f[x, y], x] == (f[x, y]^2*g[x])/h[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{f(x,y) \to -\frac{1}{\int_1^x -\frac{g(K[1])}{h\left(\frac{x^2}{2}-\frac{K[1]^2}{2}+y\right)} dK[1] + c_1\left(\frac{x^2}{2}+y\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(f(x,y),y)-diff(f(x,y),x)=f(x,y)^2*g(x)/h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y))),output='realt
```

$$f(x,y) = \frac{1}{\int^x \frac{g(\_a)}{h\left(-\frac{\_a^2}{2}+\frac{x^2}{2}+y\right)} d\_a + \_F1\left(\frac{x^2}{2}+y\right)}$$

<u>Hand solution</u>

Solve for $f(x, y)$ in $xf_y - f_x = \frac{g(x)}{h(y)} f^2$. Using the Lagrange-charpit method, the characteristic equations are

$$\frac{dy}{x} = \frac{-dx}{1} = \frac{df}{\frac{g(x)}{h(y)} f^2}$$

From the first pair of equation we obtain

$$\begin{aligned} dy &= -xdx \\ y &= -\frac{x^2}{2} + C_1 \\ C_1 &= y + \frac{x^2}{2} \end{aligned} \tag{1}$$

Using $-dx = \frac{df}{\frac{g(x)}{h(y)} f^2}$ as choice of the second pair of equations. Hence $\frac{df}{f^2} = -\frac{g(x)}{h(y)} dx$. But from (1) $y = C_1 - \frac{x^2}{2}$, therefore

$$\frac{df}{f^2} = -\frac{g(x)}{h\left(C_1 - \frac{x^2}{2}\right)} dx$$

Integrating gives

$$\begin{aligned} -\frac{1}{f} &= -\int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds + C_2 \\ C_2 &= -\frac{1}{f} + \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds \end{aligned}$$

But $C_2 = F(C_1)$ where F is arbitrary function. Therefore

$$\begin{aligned} -\frac{1}{f} + \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds &= F\left(y + \frac{x^2}{2}\right) \\ -\frac{1}{f} &= F\left(\frac{1}{2}\left(2y + x^2\right)\right) - \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds \\ \frac{1}{f} &= \int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds - F\left(\frac{1}{2}\left(2y + x^2\right)\right) \\ f &= \frac{1}{\int_0^x \frac{g(s)}{h\left(C_1 - \frac{s^2}{2}\right)} ds - F\left(\frac{1}{2}\left(2y + x^2\right)\right)} \end{aligned}$$

But $C_1 = \frac{1}{2}(2y + x^2)$ and the above becomes

$$f(x, y) = \frac{1}{\int_0^x \frac{g(s)}{h\left(\frac{1}{2}(2y + x^2) - \frac{s^2}{2}\right)} ds - F\left(\frac{1}{2}\left(2y + x^2\right)\right)}$$

2.1.25 $f_x + (f_y)^2 = f(x, y, z) + z$

problem number 25

Taken from Maple pdsolve help pages, probem 5

Nonlinear first order PDE

Solve for $f(x, y, z)$

$$f_x + (f_y)^2 = f(x, y, z) + z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[f[x, y, z], x] + D[f[x, y, z], y]^2 == f[x, y, z] + z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y, z], {x, y, z}], 60*10]];
```

$$\left\{\left\{f(x,y,z) \to \frac{1}{4}\left((c_1(z))^2 \operatorname{ProductLog}\left(-\frac{\exp\left(\frac{(x-1)c_1(z)+c_2(z)+y}{c_1(z)}\right)}{c_1(z)}\right)^2 + 2(c_1(z))^2 \operatorname{ProductLog}\left(-\frac{\exp}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(f(x,y,z),x) + (diff(f(x,y,z),y))^2 = f(x,y,z)+z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y,z),'build')),out
```

$$f(x,y,z) = \frac{(-c_5^2 z\,\mathrm{e}^{-x} - c_3^2 \mathrm{e}^{x} - c_5(c_3 y + c_4 z + c_1))\,\mathrm{e}^{x}}{c_5^2}$$

2.1.26 $xu_x + yu_y = u$ (Example 3.5.1 in Lokenath Debnath)

problem number 26

Added June 2, 2019.

From example 3.5.1, page 210 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = u$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[u[x, y], x] + y*D[u[x, y], y] ==u[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to xc_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=x\_F1\left(\frac{y}{x}\right)$$

<u>Hand solution</u>

Solve

$$xu_x+yu_y=u$$

Using the Lagrange-charpit method

$$\frac{dx}{x}=\frac{dy}{y}=\frac{du}{u}$$

The first pair of equations gives

$$x=C_1y$$

And $\frac{dx}{x}=\frac{du}{u}$ gives

$$x=C_2u$$

Since $C_2=G(C_1)$ then $\frac{x}{u}=G\left(\frac{x}{y}\right)$ or $u=xG^{-1}\left(\frac{x}{y}\right)$. Let $G^{-1}=F$. Then the solution

$$u(x,y)=xF\left(\frac{x}{y}\right)$$

2.1.27 $xu_x + yu_y = nu$ Example 3.5.2 in Lokenath Debnath

problem number 27

Added June 2, 2019.

From example 3.5.2, page 211 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = nu$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x, y], y] ==n*u[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to x^n c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=n*u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = x^n \_F1\left(\frac{y}{x}\right)$$

<u>Hand solution</u>

Solve

$$xu_x + yu_y = nu$$

Using the Lagrange-charpit method

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{nu}$$

The first pair of equations gives

$$x = C_1 y$$

And $\frac{dx}{x} = \frac{du}{nu}$ gives

$$\begin{aligned}\ln x &= \frac{1}{n}\ln u + C_2\\ x &= C_2 u^{\frac{1}{n}}\\ x^n &= C_3 u\end{aligned}$$

Since $C_3 = G(C_1)$ then $\frac{x^n}{u} = G\left(\frac{x}{y}\right)$ or $u = x^n G^{-1}\left(\frac{x}{y}\right)$. Let $G^{-1} = F$. Then the solution

$$u(x,y) = x^n F\left(\frac{x}{y}\right)$$

2.1.28 $x^2 u_x + y^2 u_y = (x+y)u$ Example 3.5.3 in Lokenath Debnath

problem number 28

Added June 2, 2019.

From example 3.5.3, page 211 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$x^2 u_x + y^2 u_y = (x+y)u$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[u[x, y], x] + y^2*D[u[x, y], y] ==(x+y)*u[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to xyc_1\left(\frac{1}{x} - \frac{1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(u(x,y),x)+y^2*diff(u(x,y),y)=(x+y)*u(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = xy\_F1\left(\frac{x-y}{xy}\right)$$

Hand solution

Solve

$$x^2 u_x + y^2 u_y = (x+y)\, u$$

Using the Lagrange-charpit method

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{du}{(x+y)\, u}$$

The first pair of equations gives

$$-\frac{1}{x} = -\frac{1}{y} + C_1 \tag{1}$$

And $\frac{dx}{x^2} = \frac{du}{(x+y)u}$ gives

$$\frac{(x+y)}{x^2} dx = \frac{du}{u}$$

But from (1) $\frac{1}{y} = \frac{1}{x} + C_1$ or $\frac{1}{y} = \frac{1+xC_1}{x}$ or $y = \frac{x}{1+xC_1}$Therefore the above becomes

$$\begin{aligned}
\frac{\left(x + \frac{x}{1+xC_1}\right)}{x^2} dx &= \frac{du}{u} \\
\frac{(x(1+xC_1)+x)}{x^2\,(1+xC_1)} dx &= \frac{du}{u} \\
\frac{2x + x^2 C_1}{x^2 + x^3 C_1} dx &= \frac{du}{u} \\
\frac{2 + xC_1}{x + x^2 C_1} dx &= \frac{du}{u} \\
2\ln x - \ln(1 + C_1 x) &= \ln u + C_2 \\
\ln \frac{x^2}{1 + C_1 x} &= \ln u + C_2 \\
\frac{x^2}{1 + C_1 x} &= C_2 u
\end{aligned}$$

But $C_1 = \frac{1}{y} - \frac{1}{x}$ hence the above becomes

$$\begin{aligned}
\frac{x^2}{1 + \left(\frac{1}{y} - \frac{1}{x}\right) x} &= C_2 u \\
\frac{x^2}{1 + \left(\frac{x-y}{yx}\right) x} &= C_2 u \\
\frac{yx^2}{y + (x-y)} &= C_2 u \\
yx &= C_2 u
\end{aligned}$$

Since $C_2 = G(C_1)$ then $\frac{yx}{u} = G\left(\frac{1}{y} - \frac{1}{x}\right)$ or

$$\begin{aligned} u &= yxG^{-1}\left(\frac{1}{y} - \frac{1}{x}\right) \\ &= yxG^{-1}\left(\frac{y-x}{yx}\right) \end{aligned}$$

Let $G^{-1} = F$

$$u = yxF\left(\frac{y-x}{yx}\right)$$

Where F is arbitrary function.

2.1.29 $(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$ (Example 3.5.4 in Lokenath Debnath)

problem number 29

From example 3.5.4, page 212 nonlinear pde's by Lokenath Debnath, 3rd edition.

First order PDE of three unknowns. Solve for $u(x, y, z)$

$$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y - z)*D[u[x, y, z], x] + (z - x)*D[u[x, y, z], y] + (x - y)*D[u[x, y, z], z] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, z], {x, y, z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=(y-z)*diff(u(x,y,z),x)+(z-x)*diff(u(x,y,z),y)+(x-y)*diff(u(x,y,z),z)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z),'build')),out
```

$$u(x, y, z) = c_3 c_4 c_5 e^{c_1 x} e^{c_1 y} e^{c_1 z} e^{\frac{c_2 x^2}{2}} e^{\frac{c_2 y^2}{2}} e^{\frac{c_2 z^2}{2}}$$

<u>Hand solution</u>

Solve

$$(y-z)\,u_x + (z-x)\,u_y + (x-y)\,u_z = 0$$

The following method works only when the sum of the coefficients of u_x, u_y, u_z is zero. This is the case here. Hence we write

$$du = 0 \tag{1}$$

$$dx + dy + dz = (y - z) + (z - x) + (x - y) = 0 \tag{2}$$

We need one more equation which is

$$xdx + ydy + ydz = x(y - z) + y(z - x) + z(x - y) = 0 \tag{3}$$

Integrating (1,2,3) gives

$$\begin{aligned} u &= C_1 \\ x + y + z &= C_2 \\ x^2 + y^2 + z^2 &= C_3 \end{aligned}$$

And since we know that $C_1 = F(C_2, C_3)$ where F is arbitrary function, then the solution is

$$u(x, y, z) = F\left(x + y + z, x^2 + y^2 + z^2\right)$$

Again, the above method only worked because of the special value of the coefficients. If the PDE was $(2y - z)\, u_x + (z - x)\, u_y + (x - y)\, u_z = 0$ for example, then the above method will not work and we have to use the Lagrange-Charpit equations. But since the equations will be coupled, this would make the solution harder.

2.1.30 $u(x + y)u_x + u(x - y)u_y = x^2 + y^2$ (Example 3.5.5 in Lokenath Debnath)

problem number 30

Added June 2, 2019.

From example 3.5.5, page 214 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u(x + y)u_x + u(x - y)u_y = x^2 + y^2$$

with $u = 0$ on $y = 2x$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x,y]*(x+y)*D[u[x, y], x] + u[x,y]*(x-y)*D[u[x, y], y] ==x^2+y^2;
ic  = u[x,2*x]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y)\to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to -\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$
$$\left\{u(x,y)\to \sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}$$

Maple ✓

```
restart;
pde :=u(x,y)*(x+y)*diff(u(x,y),x)+u(x,y)*(x-y)*diff(u(x,y),y)=x^2+y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=-\sqrt{-x^2+2xy+y^2}\sqrt{\frac{2xy+\_F1\left(-\frac{1}{\sqrt{-x^2+2xy+y^2}}\right)}{-x^2+2xy+y^2}}$$

<u>Hand solution</u>

Solve

$$u(x+y)\,u_x+u(x-y)\,u_y=x^2+y^2$$

With $u=0$ on $y=2x$. This solution follows the book method, but adds more details and more steps to make it more clear.

The following method works only when we can find two equations when the sum of the

coefficients is zero. We see that

$$\begin{aligned} ydx + xdy - udu &= y(u(x+y)) + x(u(x-y)) - u\left(x^2+y^2\right) \\ &= yux + y^2u + x^2u - yxu - ux^2 - y^2u \\ &= 0 \end{aligned} \tag{1}$$

But we need one more equation. We see that

$$\begin{aligned} xdx - ydy - udu &= x(u(x+y)) - y(u(x-y)) - u\left(x^2+y^2\right) \\ &= ux^2 + uxy - yux + y^2u - ux^2 - y^2u \\ &= 0 \end{aligned} \tag{2}$$

Now we just Integrate (1,2). Rewriting (1) as

$$\begin{aligned} ydx + xdy - udu &= 0 \\ d\left(xy - \frac{1}{2}u^2\right) &= 0 \end{aligned} \tag{1A}$$

And rewriting (2) as

$$\begin{aligned} xdx - ydy - udu &= 0 \\ d\left(\frac{1}{2}\left(x^2 - y^2 - u^2\right)\right) &= 0 \end{aligned} \tag{2A}$$

Then integrating (1A,2A) now gives

$$xy - \frac{1}{2}u^2 = C_1 \tag{1C}$$

$$\frac{1}{2}\left(x^2 - y^2 - u^2\right) = C_2 \tag{2C}$$

Since $u = 0$ on $y = 2x$, then the above becomes

$$\begin{aligned} x(2x) &= C_1 \\ \frac{1}{2}\left(x^2 - (2x)^2\right) &= C_2 \end{aligned}$$

Or

$$\begin{aligned} 2x^2 &= C_1 \\ -\frac{3}{2}x^2 &= C_2 \end{aligned}$$

Or $\frac{C_1}{C_2} = \frac{2x^2}{-\frac{3}{2}x^2} = -\frac{4}{3}$. Hence $-4C_2 = 3C_1$. Using this on (1C,2C) gives

$$
\begin{aligned}
-4\left(\frac{1}{2}(x^2 - y^2 - u^2)\right) &= 3\left(xy - \frac{1}{2}u^2\right) \\
-2(x^2 - y^2 - u^2) &= 3xy - \frac{3}{2}u^2 \\
-2x^2 + 2y^2 + 2u^2 &= 3xy - \frac{3}{2}u^2 \\
u^2\left(2 + \frac{3}{2}\right) &= 3xy + 2x^2 - 2y^2 \\
\frac{7}{2}u^2 &= 3xy + 2x^2 - 2y^2
\end{aligned}
$$

Hence the solution is

$$u(x, y) = \pm\sqrt{\frac{2}{7}(3xy + 2x^2 - 2y^2)}$$

2.1.31 $u_x - u_y = 1$ with $u(x, 0) = x^2$ Example 3.5.6 in Lokenath Debnath

problem number 31

Added June 2, 2019.

From example 3.5.6, page 214 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x - u_y = 1$$

with $u(x, 0) = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] - D[u[x, y], y] ==1;
ic  = u[x,0]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to x^2 + 2xy + (y - 1)y\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)-diff(u(x,y),y)=1;
ic  := u(x,2)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -y + (x+y-2)^2 + 2$$

2.1.32 $yu_x + xu_y = u$ with $u(x,0) = x^3$ and $u(0,y) = y^3$ Example 3.5.8 in Lokenath Debnath

problem number 32

Added June 2, 2019.

From example 3.5.8, page 216 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_x + xu_y = u$$

with with $u(x,0) = x^3$ and $u(0,y) = y^3$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] +x*D[u[x, y], y] ==u[x,y];
ic  = {u[x,0]==x^3,u[0,y]==y^3};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=y*diff(u(x,y),x)+x*diff(u(x,y),y)=u(x,y);
ic  := u(x,0)=x^3,u(0,y)=y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

sol=()

2.1.33 $xu_x + yu_y = xe^{-u}$ with $u = 0$ on $y = x^2$ Example 3.5.10 in Lokenath Debnath

problem number 33

Added June 2, 2019.

From example 3.5.10, page 218 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$xu_x + yu_y = xe^{-u}$$

with $u = 0$ on $y = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y] ==x*Exp[-u[x,y]];
ic  = u[x,x^2]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \log\left(-\frac{y}{x} + x + 1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+y*diff(u(x,y),y)=x*exp(-u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \ln\left(x + \_F1\left(\frac{y}{x}\right)\right)$$

2.1.34 $u_t + uu_x = x$ with $u(x, 0) = f(x)$ Example 3.5.11 in Lokenath Debnath.

problem number 34

Added June 2, 2019.

From example 3.5.11, page 219 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_t + uu_x = x$$

with $u(x,0) = f(x)$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] +u[x,t]*D[u[x, t], x] ==x;
ic  = u[x,0]==f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)+u(x,t)*diff(u(x,t),x)=x;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \text{RootOf}\left(\left(\_Z\,\mathrm{e}^{2t} - 2x\,\mathrm{e}^{t} + \mathrm{e}^{2t} f(\_Z) + \_Z - f(\_Z)\right)\left(\_Z + f(\_Z)\right)\right)\mathrm{e}^{t} + \mathrm{e}^{t} f\left(\text{RootOf}\left(\left(\_Z\,\mathrm{e}^{2t}\right.\right.\right.$$

2.1.35 $u_x = 0$ Problem 3.3(a) Lokenath Debnath

problem number 35

Added June 2, 2019.

Problem 3.3(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(y)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1(y)$$

2.1.36 $au_x + bu_y = 0$ Problem 3.3(b) Lokenath Debnath

problem number 36

Added June 2, 2019.

Problem 3.3(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$au_x + bu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[u[x, y], x] + b*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(u(x,y),x)+b*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{ya - bx}{a}\right)$$

2.1.37 $u_x + yu_y = 0$ Problem 3.3(c) Lokenath Debnath

problem number 37

Added June 2, 2019.

Problem 3.3(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + yu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + y*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to c_1(e^{-x}y)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+y*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x, y) = \_F1(y\,\mathrm{e}^{-x})$$

2.1.38 $(1 + x^2)u_x + u_y = 0$ Problem 3.3(d) Lokenath Debnath

problem number 38

Added June 2, 2019.

Problem 3.3(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(1 + x^2)u_x + u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (1+x^2)*D[u[x, y], x] + D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(y - \tan^{-1}(x))\}\}$$

Maple ✓

```
restart;
pde :=(1+x^2)*diff(u(x,y),x)+diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1(y - \arctan(x))$$

2.1.39 $2xyu_x + (x^2 + y^2)u_y = 0$ Problem 3.3(e) Lokenath Debnath

problem number 39

Added June 2, 2019.

Problem 3.3(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$2xyu_x + (x^2 + y^2)u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*x*y*D[u[x, y], x] + (x^2+y^2)*D[u[x, y], y] ==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to c_1\left(\frac{y^2}{x} - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=2*x*y*diff(u(x,y),x)+(x^2+y^2)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{-x^2+y^2}{x}\right)$$

2.1.40 $(y+u)u_x + yu_y = x - y$ Problem 3.3(f) Lokenath Debnath

problem number 40

Added June 3, 2019.

Problem 3.3(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(y+u)u_x + yu_y = x - y$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] + y*D[u[x, y], y] ==x-y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x)+y*diff(u(x,y),y)=x-y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \frac{-c_1 y - y + \sqrt{c_1^2 x^2 + y^2 + c_1 (2xy + 2c_2)}}{c_1}$$

2.1.41 $y^2u_x - xyu_y = x(u-2y)$ Problem 3.3(g) Lokenath Debnath

problem number 41

Added June 3, 2019.

Problem 3.3(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$y^2u_x - xyu_y = x(u-2y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[u[x, y], x] - x*y*D[u[x, y], y] ==x*(u[x,y]-2*y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y) \to \boxed{\frac{\sqrt{y^2}c_1\left(\frac{1}{2}(x^2+y^2)\right)-x^2\sqrt{-y^2}}{\sqrt{-y^4}} \text{ if } x=0 \vee y \geq 0}\right\}$$
$$\left\{u(x,y) \to \boxed{\frac{\sqrt{-y^2}x^2+\sqrt{y^2}c_1\left(\frac{1}{2}(x^2+y^2)\right)}{\sqrt{-y^4}} \text{ if } x=0 \vee y \leq 0}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(u(x,y),x)- x*y*diff(u(x,y),y)=x*(u(x,y)-2*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = -\frac{x^2}{y} + \frac{\_F1(x^2+y^2)}{\sqrt{-y^2}}$$

2.1.42 $yu_y - xu_x = 1$ Problem 3.3(h) Lokenath Debnath

problem number 42

Added June 3, 2019.

Problem 3.3(h) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$yu_y - xu_x = 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], y] - x*D[u[x, y], x] ==1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to -\log(x) + c_1(xy)\}\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),y)- x*diff(u(x,y),x)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x, y) = \_F1(xy) - \ln(x)$$

2.1.43 $u_x + 2xy^2u_y = 0$ Problem 3.4 Lokenath Debnath

problem number 43

Added June 3, 2019.

Problem 3.4 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + 2xy^2u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + 2*x*y^2*D[u[x,y],y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to c_1\left(-\frac{x^2y+1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ 2*x*y^2*diff(u(x,y),y)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y)=\_F1\left(\frac{x^2y+1}{y}\right)$$

2.1.44 $3u_x+2u_y=0$ with $u(x,0)=\sin x$. Problem 3.5(a) Lokenath Debnath

problem number 44

Added June 3, 2019.

Problem 3.5(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$3u_x+2u_y=0$$

with $u(x,0)=\sin x$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  3*D[u[x, y], x] + 2*D[u[x,y],y] == 0;
ic  = u[x,0]==Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \sin\left(x-\frac{3y}{2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=3*diff(u(x,y),x)+ 2*diff(u(x,y),y)= 0;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \sin\left(x - \frac{3y}{2}\right)$$

2.1.45 $yu_x + xu_y = 0$ with $u(0,y) = e^{-y^2}$. Problem 3.5(b) Lokenath Debnath

problem number 45

Added June 3, 2019.

Problem 3.5(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_x + xu_y = 0$$

with $u(0,y) = e^{-y^2}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] + x*D[u[x,y],y] == 0;
ic  = u[0,y]==Exp[-y^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{x^2-y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)+ x*diff(u(x,y),y)= 0;
ic  := u(0,y)=exp(-y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \mathrm{e}^{x^2-y^2}$$

2.1.46 $xu_x + yu_y = 2xy$ with $u = 2$ on $y = x^2$. Problem 3.5(c) Lokenath Debnath

problem number 46

Added June 3, 2019.

Problem 3.5(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = 2xy$$

with with $u = 2$ on $y = x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x,y],y] == 0;
ic  = u[x,x^2]==2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to 2\}\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+ y*diff(u(x,y),y)= 0;
ic  := u(x,x^2)=2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_F1\left(\frac{y}{x}\right)$$

2.1.47 $u_x + xu_y = 0$ with $u(0, y) = \sin y$. Problem 3.5(d) Lokenath Debnath

problem number 47

Added June 3, 2019.

Problem 3.5(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + xu_y = 0$$

with $u(0, y) = \sin y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + x*D[u[x,y],y] == 0;
ic  = u[0,y]==Sin[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -\sin\left(\frac{1}{2}(x^2 - 2y)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ x*diff(u(x,y),y)= 0;
ic  := u(0,y)=sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\sin\left(\frac{x^2}{2} - y\right)$$

2.1.48 $yu_x + xu_y = xy$ with $u(0,y) = e^{-y^2}, u(x,0) = e^{-x^2}$. Problem 3.5(e) Lokenath Debnath

problem number 48

Added June 3, 2019.

Problem 3.5(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$yu_x + xu_y = xy$$

with $u(0,y) = e^{-y^2}, u(x,0) = e^{-x^2}$ for $x > 0, y > 0$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[u[x, y], x] + x*D[u[x,y],y] == x*y;
ic  = {u[0,y]==Exp[-y^2],u[x,0]==Exp[-x^2]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y},Assumptions->{x>0,y>0
```

Failed

Maple ✗

```
restart;
pde :=y*diff(u(x,y),x)+ x*diff(u(x,y),y)= x*y;
ic  := u(0,y)=exp(-y^2),u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

sol=()

2.1.49 $u_x + xu_y = (y - \frac{1}{2}x^2)^2$ with $u(0,y) = e^y$. Problem 3.5(f) Lokenath Debnath

problem number 49

Added June 3, 2019.

Problem 3.5(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + xu_y = (y - \frac{x^2}{2})^2$$

with $u(0,y) = e^y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] + x*D[u[x,y],y] == (y-x^2/2)^2;
ic  = u[0,y]==Exp[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{1}{4}x\left(x^2-2y\right)^2+e^{y-\frac{x^2}{2}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)+ x*diff(u(x,y),y)= (y-x^2/2)^2;
ic  := u(0,y)=exp(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y)=\frac{x^5}{4}-x^3y+xy^2+\mathrm{e}^{-\frac{x^2}{2}+y}$$

2.1.50 $xu_x+yu_y=u+1$ with $u=x^2$ on $y=x^2$ Problem 3.5(g) Lokenath Debnath

problem number 50

Added June 3, 2019.

Problem 3.5(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x+yu_y=u+1$$

with $u=x^2$ on $y=x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] + y*D[u[x,y],y] == u[x,y]+1;
ic  = u[x,x^2]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{x^2}{y}+y-1\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+ y*diff(u(x,y),y)= u(x,y)+1;
ic  := u(x,x^2)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y)=x\_F1\left(\frac{y}{x}\right)-1$$

2.1.51 $uu_x - uu_y = u^2 + (x+y)^2$ with $u(x,0)=1$ Problem 3.5(h) Lokenath Debnath

problem number 51

Added June 3, 2019.

Problem 3.5(h) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$uu_x - uu_y = u^2 + (x+y)^2$$

With $u(x,0)=1$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  u[x,y]*D[u[x, y], x] - u[x,y]*D[u[x, y], y]== u[x,y]^2+ (x+y)^2;
ic  = u[x,0]==1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \sqrt{e^{-2y}\left(x^2+2xy+y^2+1\right)-(x+y)^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=u(x,y)*diff(u(x,y),x)- u(x,y)*diff(u(x,y),y)= u(x,y)^2+(x+y)^2;
ic  := u(x,0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=\sqrt{\left((x+y)^2+1\right)\mathrm{e}^{2x}\mathrm{e}^{-2x-2y}-(x+y)^2}$$

2.1.52 $xu_x+(x+y)u_y=u+1$ with $u(x,0)=x^2$ Problem 3.5(i) Lokenath Debnath

problem number 52

Added June 3, 2019.

Problem 3.5(i) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x+(x+y)u_y=u+1$$

With $u(x,0)=x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +(x+y)*D[u[x, y], y]== u[x,y]+1;
ic  = u[x,0]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to x^2e^{-\frac{y}{x}}+e^{\frac{y}{x}}-1\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x)+(x+y)*diff(u(x,y),y)= u(x,y)+1;
ic  := u(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=x\,\mathrm{e}^{\frac{-x\ln(x)+y}{x}}+x\,\mathrm{e}^{\frac{x\ln(x)-y}{x}}-1$$

2.1.53 $xu_x+yu_y+zu_z=0$ Problem 3.8(a) .Lokenath Debnath

problem number 53

Added June 3, 2019.

Problem 3.8(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$xu_x+yu_y+zu_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y,z], x] +y*D[u[x, y,z], y]+z*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x,y,z)\to c_1\left(\frac{y}{x},\frac{z}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y,z),x)+diff(u(x,y,z),y)+diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x,y,z) = \_F1(y - \ln(x), z - \ln(x))$$

2.1.54 $x^2u_x + y^2u_y + z(x+y)u_z = 0$ Problem 3.8(b) Lokenath Debnath

problem number 54

Added June 3, 2019.

Problem 3.8(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$x^2u_x + y^2u_y + z(x+y)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[u[x, y,z], x] +y^2*D[u[x, y,z], y]+z*(x+y)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x,y,z) \to c_1\left(\frac{1}{x} - \frac{1}{y}, \frac{z}{xy}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(u(x,y,z),x)+y^2*diff(u(x,y,z),y)+z*(x+y)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x,y,z) = \_F1\left(\frac{x-y}{xy}, \frac{z}{xy}\right)$$

2.1.55 $x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$ Problem 3.8(c) Lokenath Debnath

problem number 55

Added June 3, 2019.

Problem 3.8(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y, z)$

$$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(y-z)*D[u[x, y,z], x] +y*(z-x)*D[u[x, y,z], y]+z*(x-y)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\{\{u(x, y, z) \to c_1(-xyz, x+y+z)\}\}$$

Maple ✓

```
restart;
pde :=x*(y-z)*diff(u(x,y,z),x)+y*(z-x)*diff(u(x,y,z),y)+z*(x-y)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z),'build')),out
```

$$u(x, y, z) = \frac{c_4 c_5 x^{c_2} y^{c_2} z^{c_2} e^{c_2} e^{-c_1 x} e^{-c_1 y} e^{-c_1 z}}{c_3}$$

2.1.56 $yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$ Problem 3.8(d) Lokenath Debnath

problem number 56

Added June 3, 2019.

Problem 3.8(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y, z)$

$$yzu_x - xzu_y + xy(x^2+y^2)u_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*z*D[u[x, y,z], x] - x*z*D[u[x, y,z], y]+x*y*(x^2+y^2)*D[u[x, y,z], z]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{u(x,y,z)\to c_1\left(\frac{1}{2}\left(x^2+y^2\right),\frac{1}{2}\left(-x^2y^2-x^4+z^2\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*z*diff(u(x,y,z),x) - x*z*diff(u(x,y,z),y)+x*y*(x^2+y^2)*diff(u(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

$$u(x,y,z)=\_F1\left(x^2+y^2,-x^4-x^2y^2+z^2\right)$$

2.1.57 $x(y^2-z^2)u_x+y(z^2-y^2)u_y+z(x^2-y^2)u_z=0$ Problem 3.8(e) Lokenath Debnath

problem number 57

Added June 3, 2019.

Problem 3.8(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$x(y^2-z^2)u_x+y(z^2-y^2)u_y+z(x^2-y^2)u_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(y^2-z^2)*D[u[x, y,z], x] +y*(z^2-y^2)*D[u[x, y,z], y]+z*(x^2-y^2)*D[u[x, y,z], z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=x*(y^2-z^2)*diff(u(x,y,z),x) + y*(z^2-y^2)*diff(u(x,y,z),y)+z*(x^2-y^2)*diff(u(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z))),output='rea
```

sol=()

2.1.58 $u_x + xu_y = y$ with $u(0, y) = y^2$ Problem 3.9(a) Lokenath Debnath

problem number 58

Added June 3, 2019.

Problem 3.9(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + xu_y = y$$

With $u(0, y) = y^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +x*D[u[x, y], y]== y;
ic  = u[0,y]==y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -x^2y + \frac{x^4}{4} - \frac{x^3}{3} + xy + y^2\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + x*diff(u(x,y),y)= y;
ic  := u(0,y)=y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\frac{x^3}{3} + xy + \left(-\frac{x^2}{2} + y\right)^2$$

2.1.59 $u_x + xu_y = y$ with $u(1, y) = 2y$ Problem 3.9(b) Lokenath Debnath

problem number 59

Added June 3, 2019.

Problem 3.9(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$u_x + xu_y = y$$

With $u(1, y) = 2y$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +x*D[u[x, y], y]== y;
ic  = u[1,y]==2*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -\frac{x^3}{3} - \frac{x^2}{2} + xy + y + \frac{5}{6}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + x*diff(u(x,y),y)= y;
ic  := u(1,y)=2*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + xy + y + \frac{5}{6}$$

2.1.60 $(u_x + u_y)^2 - u^2 = 0$. Problem 3.10 Lokenath Debnath

problem number 60

Added June 3, 2019.

Problem 3.10 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(u_x + u_y)^2 - u^2 = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (D[u[x, y], x] +D[u[x, y], y])^2-u[x,y]^2== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\{u(x, y) \to e^{-x} c_1(y - x)\}$$
$$\{u(x, y) \to e^{x} c_1(y - x)\}$$

Maple ✓

```
restart;
pde :=(diff(u(x,y),x) + diff(u(x,y),y))^2-u(x,y)^2= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x, y) = c_1 \mathrm{e}^{\frac{y\_c_2 + x}{-c_2 + 1}}$$

2.1.61 $(y + u)u_x + yu_y = x - y$ with $u(x, 1) = 1 + x$. Problem 3.11 Lokenath Debnath

problem number 61

Added June 3, 2019.

Problem 3.11 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x, y)$

$$(y + u)u_x + yu_y = x - y$$

With $u(x,1) = 1 + x$.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] +y*D[u[x, y], y]== x-y;
 ic=u[x,1]==1+x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x) + y*diff(u(x,y),y)= x-y;
ic  := u(x,1)=1+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

sol=()

2.1.62 $2xu_x + (x+1)u_y = y$ with $u(1,y) = 2y$. Problem 3.14(d) Lokenath Debnath

problem number 62

Added June 3, 2019.

Problem 3.14(d) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$2xu_x + (x+1)u_y = y$$

With $u(1,y) = 2y$ with $x > 0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  2*x*D[u[x, y], x] +(x+1)*D[u[x, y], y]== y;
 ic=u[1,y]==2*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}, Assumptions->x>0], 6
```

$$\left\{\left\{u(x,y) \to \frac{1}{8}\left(-2(x-2y+4)\log(x) - 6x - \log^2(x) + 16y + 6\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=2*x*diff(u(x,y),x) + (x+1)*diff(u(x,y),y)= y;
ic  := u(1,y)=2*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

$$u(x,y) = -\frac{\ln(x)^2}{8} - \frac{3x}{4} + 2y + \frac{(-2x+4y-8)\ln(x)}{8} + \frac{3}{4}$$

2.1.63 $xu_x + yu_y = x^2 + y^2$ with $u(x,1) = x^2$. Problem 3.14(e) Lokenath Debnath

problem number 63

Added June 3, 2019.

Problem 3.14(e) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = x^2 + y^2$$

With $u(x,1) = x^2$ with $x > 0, y > 0$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y]== x^2+y^2;
 ic=u[x,1]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y},Assumptions->{x>0,y>0
```

$$\left\{\left\{u(x,y) \to \frac{x^2y^2 + x^2 + y^4 - y^2}{2y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x) + y*diff(u(x,y),y)= x^2+y^2;
ic  := u(x,1)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) assuming
```

$$u(x,y) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{x^2}{2y^2} - \frac{1}{2}$$

2.1.64 $y^2 u_x + (xy)u_y = x$ with $u(x,1) = x^2$. Problem 3.14(f) Lokenath Debnath

problem number 64

Added June 3, 2019.

Problem 3.14(f) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$y^2 u_x + (xy)u_y = x$$

With $u(x,1) = x^2$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[u[x, y], x] +(x*y)*D[u[x, y], y]== x;
 ic=u[x,1]==x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to x^2 - y^2 + \frac{\log(y^2)}{2} + 1\right\}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(u(x,y),x) + (x*y)*diff(u(x,y),y)= x;
ic  := u(x,1)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y) = x^2 - y^2 + \frac{\ln(y^2)}{2} + 1$$

2.1.65 $xu_x + yu_y = xy$ with $u = \frac{x^2}{2}$ at $y = x$. Problem 3.14(g) Lokenath Debnath

problem number 65

Added June 3, 2019.

Problem 3.14(g) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu_x + yu_y = xy$$

With $u = \frac{x^2}{2}$ at $y = x$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], x] +y*D[u[x, y], y]== x*y;
 ic=u[x,x]==x^2/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{xy}{2} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,y),x) + y*diff(u(x,y),y)= x*y;
ic  := u(x,x)=x^2/2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y) = \frac{xy}{2} + \_F1\left(\frac{y}{x}\right)$$

2.1.66 $u_x + uu_y = 1$ with $u(0,y) = ay$. Problem 3.16(a) Lokenath Debnath

problem number 66

Added June 3, 2019.

Problem 3.16(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$u_x + uu_y = 1$$

With $u(0,y) = ay$.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], x] +u[x,y]*D[u[x, y], y]== 1;
 ic=u[0,y]==a*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde,ic} ,u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{ax^2 + 2ay + 2x}{2ax + 2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x) + u(x,y)*diff(u(x,y),y)=1;
ic  := u(0,y)=a*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y)) ),output=
```

$$u(x,y) = \frac{(x^2+2y)\,a+2x}{2ax+2}$$

2.1.67 $(y+u)u_x + (x+u)u_y = x+y$. Problem 3.17(a) Lokenath Debnath

problem number 67

Added June 3, 2019.

Problem 3.17(a) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(y+u)u_x + (x+u)u_y = x+y$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y+u[x,y])*D[u[x, y], x] +(x+u[x,y])*D[u[x, y], y]== x+y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=(y+u(x,y))*diff(u(x,y),x) + (x+u(x,y))*diff(u(x,y),y)=x+y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') ),outp
```

$$u(x,y) = \frac{-\operatorname{RootOf}\left(\left(c_1 e^{\frac{3c_1c_2}{y}} + e^{\frac{3c_1c_2}{y}}\right)\_Z^9 - c_1^3x^3 - 3c_1^2x^2y - 3c_1xy^2 + (3c_1^3x^3 + 9c_1^2x^2y + 9c_1xy^2 + 3y^{}}{(c_1x+y)^2}$$

2.1.68 $xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$. Problem 3.17(b) Lokenath Debnath

problem number 68

Added June 3, 2019.

Problem 3.17(b) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$xu(u^2+xy)u_x - yu(u^2+xy)u_y = x^4$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*u[x,y]*(u[x,y]^2+x*y)*D[u[x, y], x] -y*u[x,y]*(u[x,y]^2+x*y)*D[u[x, y], y]== x^4;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y) \to -\sqrt{-xy - \sqrt{4c_1(xy) + x^2y^2 + x^4}}\right\}$$
$$\left\{u(x,y) \to \sqrt{-xy - \sqrt{4c_1(xy) + x^2y^2 + x^4}}\right\}$$
$$\left\{u(x,y) \to -\sqrt{-xy + \sqrt{4c_1(xy) + x^2y^2 + x^4}}\right\}$$
$$\left\{u(x,y) \to \sqrt{-xy + \sqrt{4c_1(xy) + x^2y^2 + x^4}}\right\}$$

Maple ✓

```
restart;
pde :=x*u(x,y)*(u(x,y)^2+x*y)*diff(u(x,y),x) -y*u(x,y)*(u(x,y)^2+x*y)*diff(u(x,y),y)=x^4;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y) = \sqrt{-xy - \sqrt{x^4 + x^2y^2 + 4\_F1\,(xy)}}$$

2.1.69 $(x+y)u_x + (x-y)u_y = 0$. Problem 3.17(c) Lokenath Debnath

problem number 69

Added June 3, 2019.

Problem 3.17(c) nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y)$

$$(x+y)u_x + (x-y)u_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x+y)*D[u[x, y], x] +(x-y)*D[u[x, y], y]== 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde ,u[x, y], {x, y}], 60*10]];
```

$$\left\{u(x,y) \to c_1\left(\log\left(-\sqrt{-x^2+2xy+y^2}\right)\right)\right\}$$
$$\left\{u(x,y) \to c_1\left(\frac{1}{2}\log\left(-x^2+2xy+y^2\right)\right)\right\}$$

Maple ✓

```
restart;
pde :=(x+y)*diff(u(x,y),x) +(x-y)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y)) ),output='real
```

$$u(x,y) = \_F1\left(-\frac{1}{\sqrt{-x^2+2xy+y^2}}\right)$$

2.1.70 $yu_x - xu_y = e^u$ with $u(0,y) = y^2 - 1$

problem number 70

Added May 21, 2019.

Characteristics, with IC

Taken from "the method of Characteristics" by Ryan C. Daileda. Page 16 `http://ramanujan.math.trinity.edu/rdaileda/teach/s15/m3357/lectures/lecture_1_22_slides.pdf`

Solve for $u(x,y)$

$$yu_x - xu_y = e^u$$

With initial conditions $u(0,y) = y^2 - 1$.

Mathematica ✓

```
ClearAll["Global`*"];
pde = y*D[u[x, y], x] - x*D[u[x, y], y] == Exp[u[x, y]];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, u[0, y] == y^2 - 1}, u[x, y], {x, y}], 60*
```

$$\left\{\left\{u(x,y) \to -\log\left(e^{-x^2-y^2+1} + \tan^{-1}\left(\frac{x}{\sqrt{y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)-x*diff(u(x,y),y)=exp(u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,u(0,y)=y^2-1],u(x,y))
```

$$u(x,y) = \ln\left(-\frac{1}{\arctan\left(\frac{x}{y}\right) - \mathrm{e}^{-x^2-y^2+1}}\right)$$

<u>Hand solution</u>

Solve

$$\begin{aligned} yu_x - xu_y &= e^u \\ u(0,y) &= y^2 - 1 \end{aligned}$$

The characteristic equations (using s as the parameter) are

$$\frac{dx}{ds} = y \tag{1}$$
$$\frac{dy}{ds} = -x \tag{2}$$
$$\frac{du}{ds} = e^u \tag{3}$$

With initial point at $s = 0$ which can be written as $u(0, \xi) = \xi^2 - 1$. The idea of this method is to use (1,2) to obtain expressions for s, ξ. These are the unknowns. Then using these in (3) to obtain the final solution $u(x, y)$. One problem that we see right away, is that (1,2) are coupled. When this happens, we must decouple them first. Differentiating (1) gives $\frac{d^2x}{ds^2} = y' = -x$. Hence (1) becomes $x''(s) + x = 0$. This has solution

$$x(s) = c_1 \cos s + c_2 \sin s \tag{4}$$

But from (1), we see that $x'(s) = y$. Therefore

$$y(s) = -c_1 \sin s + c_2 \cos s \tag{5}$$

We made some progress. Found $x(s), y(s)$. But need to solve for c_1, c_2. This is done using the initial point, which is always at $s = 0$. When $s = 0$, using $u(0, \xi) = \xi^2 - 1$, which says $x(0) = 0$ and $y(0) = \xi$. Using $x(0) = 0$ when $s = 0$ then from (4) we obtain $0 = c_1$. Now the solutions becomes

$$x(s) = c_2 \sin s$$
$$y(s) = c_2 \cos s$$

Now we use the second initial condition on $y(s)$ which says $y(0) = \xi$. Hence from the second equation above, $\xi = c_2$. Therefore the solution now becomes

$$x(s) = \xi \sin s \tag{6}$$
$$y(s) = \xi \cos s \tag{7}$$

This is as far as we can go. Remembering that we are after expressions for s and ξ. Dividing (6/7) gives

$$\frac{x}{y} = \tan(s)$$
$$s = \arctan\left(\frac{x}{y}\right) \tag{8}$$

Good. We obtained relation for s in terms of x, y. What about ξ?. By equation (6) and (7) and adding them we obtain

$$\begin{aligned} x^2(s) + y^2(s) &= \xi^2 \\ \xi &= \sqrt{x^2 + y^2} \end{aligned} \tag{9}$$

Good. (8,9) is what we wanted. Equations (1,2) have done their job. We used them to find s, ξ. Now we move on to (3) which is

$$\frac{du}{ds} = e^u$$

Solving it gives

$$-e^{-u} = s + C$$

But at $s = 0$ we know that $u(0) = \xi^2 - 1$. $-e^{-(\xi^2-1)} = C$. Then the above becomes

$$-e^{-u} = s - e^{-(\xi^2-1)}$$

We are almost there. We just need now to go back to x, y from s, ξ. By using (8,9) the above becomes

$$\begin{aligned} -e^{-u} &= \arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)} \\ e^{-u} &= -\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)} \end{aligned}$$

We can stop here. But if assume $u > 0$ then the above can be simplified more

$$\begin{aligned} -u &= \ln\left(-\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)}\right) \\ u(x,y) &= -\ln\left(-\arctan\left(\frac{x}{y}\right) + e^{-(x^2+y^2-1)}\right) \\ &= -\ln\left(-\left(\arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)}\right)\right) \\ &= \ln\left(\frac{1}{-\left(\arctan\left(\frac{x}{y}\right) - e^{-(x^2+y^2-1)}\right)}\right) \end{aligned}$$

I used document titled "The method of Characteristics" by Ryan C. Daileda for help which is a very useful document.

2.1.71 $yu_x - xu_y = e^u$

problem number 71

Added May 21, 2019.

This is same problem as above, but without I.C. given.

Solve for $u(x, y)$

$$yu_x - xu_y = e^u$$

No IC are given.

Mathematica ✓

```
ClearAll["Global`*"];
pde = y*D[u[x, y], x] - x*D[u[x, y], y] == Exp[u[x, y]];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to -\log\left(\tan^{-1}\left(\frac{x}{\sqrt{y^2}}\right) - c_1\left(\frac{1}{2}(x^2+y^2)\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(u(x,y),x)-x*diff(u(x,y),y)=exp(u(x,y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \ln\left(-\frac{1}{\_F1\left(x^2+y^2\right) + \arctan\left(\frac{x}{y}\right)}\right)$$

<u>Hand solution</u>

Solve

$$yu_x - xu_y = e^u$$

Since no initial conditions are given, I find using Lagrange-charpit method is better here than using characteristic equations with parameter s. The Lagrange-charpit equations for the above PDE are

$$\frac{dx}{y} = \frac{-dy}{x} = \frac{du}{e^u}$$

The first two equations gives

$$
\begin{aligned}
xdx &= -ydy \\
\frac{x^2}{2} &= -\frac{y^2}{2} + C_1 \\
C_1 &= \frac{1}{2}\left(x^2 + y^2\right) \\
C_1 &= \left(x^2 + y^2\right) \qquad (1)
\end{aligned}
$$

Where the 2 is absorbed by the constant. We now need to decide to either solve $\frac{-dy}{x} = \frac{du}{e^u}$ together or $\frac{dx}{y} = \frac{du}{e^u}$. It does not matter which pair to pick. Using the second pair gives

$$
\frac{dx}{y} = \frac{du}{e^u}
$$

But from (1), $y = \sqrt{C_1 - x^2}$ (taking only the positive root) and the above equation now becomes

$$
\frac{dx}{\sqrt{C_1 - x^2}} = \frac{du}{e^u}
$$

Integrating gives

$$
\begin{aligned}
\arctan\left(\frac{x}{\sqrt{C_1 - x^2}}\right) &= -e^{-u} + C_2 \\
\arctan\left(\frac{x}{y}\right) &= -e^{-u} + C_2 \\
C_2 &= \arctan\left(\frac{x}{y}\right) + e^{-u}
\end{aligned}
$$

In this method, the constants C_1, C_2 are always related by $C_2 = F(C_1)$ where F is an arbitrary function. Hence we obtain

$$
\begin{aligned}
\arctan\left(\frac{x}{y}\right) + e^{-u} &= F\left(x^2 + y^2\right) \\
e^{-u} &= F\left(x^2 + y^2\right) - \arctan\left(\frac{x}{y}\right)
\end{aligned}
$$

For positive u the above simplifies to

$$-u = \ln\left(F(x^2+y^2) - \arctan\left(\frac{x}{y}\right)\right)$$
$$u(x,y) = -\ln\left(F(x^2+y^2) - \arctan\left(\frac{x}{y}\right)\right)$$

2.1.72 $u_t + xu_x = 0$ with $u(x,0) = x^2$. Math 5587

problem number 72

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 3.

Solve for $u(x,t)$

$$u_t + xu_x = 0$$

With with $u(x,0) = x^2$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + x*D[u[x, t], x] == 0;
ic  = u[x,0]==x^2;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to e^{-2t}x^2\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+x*diff(u(x,t),x)=0;
ic :=u(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x^2 e^{-2t}$$

Hand solution

Solve $u_t + xu_x = 0$ with $u(x,0) = x^2$. Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = x \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = 0, x(0) = \xi, u(0) = \xi^2$$

Equation (1) gives

$$\begin{aligned} t &= s + t(0) \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives

$$\begin{aligned} \ln x &= s + x(0) \\ x &= \xi e^s \end{aligned} \tag{6}$$

From (5,6) solving for ξ gives

$$\begin{aligned} \xi &= xe^{-s} \\ &= xe^{-t} \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} u &= u(0) \\ &= \xi^2 \\ &= x^2 e^{-2t} \end{aligned}$$

2.1.73 $u_t + tu_x = 0$ with $u(x,0) = e^x$

problem number 73

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 4.

Solve for $u(x,t)$

$$u_t + tu_x = 0$$

With with $u(x,0)=e^x$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + t*D[u[x, t], x] == 0;
ic  = u[x,0]==Exp[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{x-\frac{t^2}{2}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+t*diff(u(x,t),x)=0;
ic :=u(x,0)=exp(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\mathrm{e}^{-\frac{t^2}{2}+x}$$

<u>Hand solution</u>

Solve $u_t+xu_x=0$ with $u(x,0)=e^x$. Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds}=1 \tag{1}$$
$$\frac{dx}{ds}=t \tag{2}$$
$$\frac{du}{ds}=0 \tag{3}$$

With initial conditions at $s=0$

$$t(0)=0,x(0)=\xi,u(0)=e^{\xi}$$

Equation (1) gives

$$\begin{aligned} t&=s+t(0)\\ &=s \end{aligned} \tag{5}$$

Equation (2) now becomes $\frac{dx}{ds} = s$, whose solution is

$$\begin{aligned} x &= \frac{s^2}{2} + x(0) \\ x &= \frac{s^2}{2} + \xi \end{aligned} \tag{6}$$

From (5,6) solving for ξ gives

$$\begin{aligned} \xi &= x - \frac{s^2}{2} \\ &= x - \frac{t^2}{2} \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} u &= u(0) \\ &= e^{\xi} \\ &= e^{\left(x - \frac{t^2}{2}\right)} \end{aligned}$$

2.1.74 $2u_x + 3u_y = 1$

problem number 74

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 5.

Solve for $u(x, y)$

$$2u_x + 3u_y = 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = 2 D[u[x, y], x] + 3*D[u[x, y], y] == 1;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{x}{2} + c_1\left(y - \frac{3x}{2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=2*diff(u(x,y),x)+3*diff(u(x,y),y)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \frac{x}{2} + \_F1\left(-\frac{3x}{2} + y\right)$$

Hand solution

Solve $2u_x + 3u_y = 1$. Using the method of characteristics

$$\frac{dx}{2} = \frac{dy}{3} = \frac{du}{1}$$

From the first pair of equations we obtain $\frac{1}{2}x = \frac{1}{3}y + C_1$ or $C_1 = \frac{1}{2}x - \frac{1}{3}y$. From the pair $\frac{dx}{2} = \frac{du}{1}$ we obtain

$\frac{1}{2}x = u + C_2$ or $C_2 = \frac{1}{2}x - u$. But $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$\frac{1}{2}x - u = F\left(\frac{1}{2}x - \frac{1}{3}y\right)$$
$$u = \frac{1}{2}x - F\left(\frac{1}{2}x - \frac{1}{3}y\right)$$

2.1.75 $xu_t - tu_x = 0$

problem number 75

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 6.

Solve for $u(x,t)$ with $x > 0, t > 0$ and $u(x,0) = x^2$

$$xu_t - tu_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x D[u[x, t], t] - t*D[u[x, t], x] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to c_1\left(\frac{1}{2}(t^2 + x^2)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(u(x,t),t)-t*diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \_F1\left(t^2 + x^2\right)$$

Hand solution

Solve $xu_t - tu_x = 0$. Using the method of characteristics

$$\frac{dt}{x} = -\frac{dx}{t} = \frac{du}{0}$$

From the first pair of equations we obtain $tdt = -xdx$ or $\frac{t^2}{2} = -\frac{x^2}{2} + C_1$ or $C_1 = \frac{t^2}{2} + \frac{x^2}{2}$. From $du = 0$ then $u = C_2$. But $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$u = F\left(\frac{t^2}{2} + \frac{x^2}{2}\right)$$

2.1.76 $u_t + u_x = 0$ with $u(x,1) = \frac{x}{1+x^2}$

problem number 76

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 3.

Solve for $u(x,t)$ with $x > 0, t > 0$ and initial conditions not zero $u(x,1) = \frac{x}{1+x^2}$

$$u_t + u_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] + D[u[x, t], x] == 0;
ic  = u[x,1]== x/(1+1+x^2);
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{-t + x + 1}{t^2 - 2t(x+1) + x^2 + 2x + 3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)=0;
ic  := u(x,1)= x/(1+x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{-t+x+1}{(-t+x+1)^2+1}$$

Hand solution

Solve $u_t + u_x = 0$ with initial conditions $u(x,1) = \frac{x}{1+x^2}$. The characteristic equations are

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = 1 \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

At $s=0$ we have $x(0)=\xi, t(1)=1, u(0)=\frac{\xi}{1+\xi^2}$. Solving (1) gives $t = s + t(1) = s+1$. Solving (2) gives $x = s + x(0) = s + \xi$. From these solutions we solve for ξ, which gives $\xi = x - s = x - (t-1)$. Hence

$$\xi = x - t + 1$$

Equation (3) gives

$$\begin{aligned} u &= u(0) \\ &= \frac{\xi}{1+\xi^2} \end{aligned}$$

Hence

$$u(x,y) = \frac{x-t+1}{1+(x-t+1)^2}$$

2.1.77 $u_x u_y = 1$

problem number 77

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 5(a).

Solve for $u(x, y)$

$$u_x u_y = 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], x] * D[u[x, y], y] == 1;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{x}{c_2} + c_2 y + c_1\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x)*diff(u(x,y),y)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),HINT=`+`,'build
```

$$u(x,y) = y\_c_2 + c_1 + c_2 + \frac{x}{\_c_2}$$

Hand solution

Solve $u_x u_y = 1$. Let $u = f(x) + g(y)$. The pde becomes $f'(x)\, g'(y) = 1$ or $f'(x) = \frac{1}{g'(y)}$. Hence both are constant, say λ. Therefore $f'(x) = \lambda$ or $f(x) = \lambda x + C_1$ and similarly, $\frac{1}{g'(y)} = \lambda$ or $g'(y) = \frac{1}{\lambda}$ or $g(y) = \frac{y}{\lambda} + C_2$. Therefore the solution becomes

$$\begin{aligned} u(x,y) &= f(x) + g(y) \\ &= \lambda x + C_1 + \frac{y}{\lambda} + C_2 \end{aligned}$$

2.1.78 $u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$

problem number 78

Added May 27, 2019.

From UMN Math 5587 HW2, Fall 2016, problem 5(b).

Solve for $u(x,y)$

$$u_x u_y = u$$

With $u(x,0) = 0, u(0,y) = 0$.

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, y], x] * D[u[x, y], y] == u[x,y];
bc={u[x,0]==0,u[0,y]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc},u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x)*diff(u(x,y),y)=u(x,y);
bc  := u(x,0)=0,u(0,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y),HINT=`*`))
```

$$u(x,y) = xy$$

<u>Hand solution</u>

Solve $u_x u_y = u$ with $u(x,0) = 0, u(0,y) = 0$.

Let $u = X(x)\,Y(y)$. The pde becomes $(X'Y)\,(Y'X) = XY$ or $X'Y' = 1$ or $X' = \frac{1}{Y'}$. Hence both sides are constant, say λ. Therefore we have 2 differential equations to solve

$$X' = \lambda$$

And

$$\frac{1}{Y'} = \lambda$$

The solution to the first one is $X(x) = \lambda x + C_1$. The solution to the second equation is $Y(y) = \frac{1}{\lambda} y + C_2$.

Since $u = X(x)\,Y(y)$ then the solution is

$$\begin{aligned} u(x,y) &= (\lambda x + C_1)\left(\frac{1}{\lambda}y + C_2\right) \\ &= xy + C_2\lambda x + \frac{C_1}{\lambda}y + C_1C_2 \end{aligned}$$

Using the condition $u(x,0) = 0$ gives

$$0 = C_2\lambda x + C_1C_2$$

Differentiating gives

$$0 = C_2\lambda$$

But $\lambda \neq 0$ for nontrivial solution, hence $C_2 = 0$. Therefore (1) reduces to

$$u(x,y) = xy + \frac{C_1}{\lambda}y$$

Using the condition $u(0,y) = 0$ gives

$$0 = \frac{C_1}{\lambda}y$$

Hence $C_1 = 0$. Therefore the solution becomes

$$u(x,y) = xy$$

2.2 Solved by factoring into two transport equations

Local contents

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2.2.1 $u_{xx} + u_{xt} - 6u_{tt} = 0$

problem number 79

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 8.

Solve for $u(x,t)$ with $u(x,0) = x$ and $u_t(x,0) = 0$ by factoring the PDE into two transport PDE

$$u_{xx} + u_{xt} - 6u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] + D[D[u[x,t],x],t] -  6*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==x, Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)+diff(diff(u(x,t),x),t) - 6 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=x, eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x$$

Hand solution

Solve $u_{xx} + u_{xt} - 6u_{tt} = 0$ with $u(x,0) = x$ and $u_t(x,0) = 0$. Writing the PDE as

$$\left(\frac{\partial}{\partial x} + 3\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - 2\frac{\partial}{\partial t}\right)u = 0$$

Let

$$\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial t} = w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x} + 3\frac{\partial w}{\partial t} = 0 \tag{2}$$

From (1),

$$w(x,0) = u_x(x,0) - 2u_t(x,0)$$

But $u(x,0) = x$ hence $u_x(x,0) = 1$, and $u_t(x,0) = 0$. Therefore the above gives

$$w(x,0) = 1$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds} &= 1\\ \frac{dt}{ds} &= 3\\ \frac{dw}{ds} &= 0\end{aligned}$$

With $x(0) = \xi, t(0) = 0, w(0) = 1$. The above equations give

$$\begin{aligned}x &= s + x(0) = s + \xi\\ t &= 3s\\ w &= u(0) = 1\end{aligned}$$

Hence

$$w(x,y) = 1$$

From (1)

$$\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial t} = 1 \tag{3}$$

with $u(x,0) = x$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds} &= 1\\ \frac{dt}{ds} &= -2\\ \frac{du}{ds} &= 1\end{aligned}$$

With $x(0) = \xi, t(0) = 0, u(0) = \xi$. The above equations give

$$\begin{aligned} x &= s + x(0) = s + \xi \\ t &= -2s \\ u &= s + u(0) = s + \xi \end{aligned}$$

But $s = -\frac{t}{2}$ and $\xi = x - s = x - \left(-\frac{t}{2}\right) = x + \frac{t}{2}$, hence

$$\begin{aligned} u(x,t) &= -\frac{t}{2} + x + \frac{t}{2} \\ &= x \end{aligned}$$

2.2.2 $u_{xx} - u_{xt} - 12u_{tt} = 0$

problem number 80

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 9.

Solve for $u(x,t)$ with $u(x,0) = 0$ and $u_t(x,0) = x$ by factoring the PDE into two transport PDE

$$u_{xx} - u_{xt} - 12u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] - D[D[u[x,t],x],t] -  12*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==0, Derivative[0,1][u][x,0]==x};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to tx - \frac{t^2}{24}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)-diff(diff(u(x,t),x),t) - 12 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = -\frac{1}{24}t^2 + tx$$

Hand solution

Solve $u_{xx} - u_{xt} - 12u_{tt} = 0$ with $u(x,0) = x$ and $u_t(x,0) = x$. Writing the PDE as

$$\left(\frac{\partial}{\partial x} + 3\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - 4\frac{\partial}{\partial t}\right)u = 0$$

Let

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x} + 3\frac{\partial w}{\partial t} = 0 \tag{2}$$

From (1),

$$w(x,0) = u_x(x,0) - 4u_t(x,0)$$

But $u(x,0) = 0$ hence $u_x(x,0) = 0$, and $u_t(x,0) = x$. Therefore the above gives

$$w(x,0) = -4x$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds} &= 1\\ \frac{dt}{ds} &= 3\\ \frac{dw}{ds} &= 0\end{aligned}$$

With $x(0) = \xi, t(0) = 0, w(0) = -4\xi$. The above equations give

$$\begin{aligned}x &= s + x(0) = s + \xi\\ t &= 3s\\ w &= u(0) = -4\xi\end{aligned}$$

Solving for ξ, s from the first 2 equations. $\xi = x - s = x - \frac{t}{3}$. From the last equation above

$$w(x,y) = -4\left(x - \frac{t}{3}\right)$$

Using the above into (1) gives

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = -4\left(x - \frac{t}{3}\right) \tag{3}$$

with $u(x,0)=0$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=-4\\ \frac{du}{ds}&=-4\left(x-\frac{t}{3}\right)\end{aligned}$$

With $x(0)=\xi, t(0)=0, u(0)=0$. The above equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=-4s\end{aligned}$$

Solving the above for s,ξ gives $\xi=x-s=x+\frac{t}{4}$. Therefore $\frac{du}{ds}=-4\left(x-\frac{t}{3}\right)$ becomes

$$\begin{aligned}\frac{du}{ds}&=-4\left(s+\xi-\frac{(-4s)}{3}\right)\\ &=-4\left(s+\xi+\frac{4s}{3}\right)\\ &=-\frac{28}{3}s-4\xi\end{aligned}$$

Solving the above gives

$$\begin{aligned}u&=-\frac{28}{3}\frac{s^2}{2}-4\xi s+u(0)\\ &=-\frac{28}{3}\frac{s^2}{2}-4\xi s\end{aligned}$$

Converting to x,t

$$\begin{aligned}u(x,y)&=-\frac{28}{3}\frac{1}{2}\left(-\frac{t}{4}\right)^2-4\left(x+\frac{t}{4}\right)\left(-\frac{t}{4}\right)\\ &=-\frac{1}{24}\left(t^2-24xt\right)\\ &=-\frac{1}{24}t^2+xt\end{aligned}$$

2.2.3 $u_{xx} - 3u_{xt} - 4u_{tt} = 0$

problem number 81

Added May 25, 2019.

From HW 3, UMN Math 5587, Fall 2016, problem 2.

Solve for $u(x,t)$ with $u(x,0) = e^x$ and $u_t(x,0) = 0$ by factoring the PDE into two transport PDE

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {x,2}] - 3*D[D[u[x,t],x],t] -  4*D[u[x, t], {t,2}] == 0;
ic  = {u[x,0]==Exp[x], Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{5}\left(4e^{5t/4}+1\right)e^{x-t}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$2)-3*diff(diff(u(x,t),x),t) - 4 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=exp(x), eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \frac{e^{-t+x}}{5} + \frac{4\,e^{\frac{t}{4}+x}}{5}$$

Hand solution

Solve $u_{xx} - 3u_{xt} - 4u_{tt} = 0$ with $u(x,0) = e^x$ and $u_t(x,0) = x$. Writing the PDE as

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - 4\frac{\partial}{\partial t}\right)u = 0$$

Let

$$\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial t} = w(x,t) \tag{1}$$

Then the PDE becomes

$$\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}=0 \tag{2}$$

From (1),

$$w(x,0)=u_x(x,0)-4u_t(x,0)$$

But $u(x,0)=e^x$ hence $u_x(x,0)=e^x$, and $u_t(x,0)=0$. Therefore the above gives

$$w(x,0)=e^x$$

Hence we need to solve (2) for $w(x,t)$ with the above initial condition. The characteristics for (2) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=1\\ \frac{dw}{ds}&=0\end{aligned}$$

With $x(0)=\xi, t(0)=0, w(0)=e^{\xi}$. The above equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=s\\ w&=u(0)=e^{\xi}\end{aligned}$$

Since $\xi=x-s=x-t$, then the last equation above gives

$$w(x,y)=e^{x-t}$$

Using the above into (1) gives

$$\frac{\partial u}{\partial x}-4\frac{\partial u}{\partial t}=e^{x-t} \tag{3}$$

with $u(x,0)=e^x$. The characteristics for (3) are

$$\begin{aligned}\frac{dx}{ds}&=1\\ \frac{dt}{ds}&=-4\\ \frac{du}{ds}&=e^{x-t}\end{aligned}$$

With $x(0)=\xi, t(0)=0, u(0)=e^{\xi}$. The above two equations give

$$\begin{aligned}x&=s+x(0)=s+\xi\\ t&=-4s\end{aligned}$$

Solving the above for s, ξ gives $\xi = x - s = x + \frac{t}{4}$. Therefore $\frac{du}{ds} = e^{x-t}$ becomes

$$\begin{aligned}\frac{du}{ds} &= e^{s+\xi-(-4s)}\\ &= e^{5s+\xi}\end{aligned}$$

Solving the above gives

$$\begin{aligned}u &= \frac{1}{5}e^{5s+\xi} + u(0)\\ &= \frac{1}{5}e^{5s+\xi} + e^{\xi}\end{aligned}$$

Converting to x, t, using $s = \frac{-t}{4}$ and $\xi = x + \frac{t}{4}$ gives

$$\begin{aligned}u(x,y) &= \frac{1}{5}e^{5\left(\frac{-t}{4}\right)+\left(x+\frac{t}{4}\right)} + e^{\left(x+\frac{t}{4}\right)}\\ &= \frac{1}{5}e^{x-t} + e^{\left(x+\frac{t}{4}\right)}\end{aligned}$$

2.2.4 $u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$

problem number 82

Added Oct 6, 2019.

Problem 2.4.19 Peter Olver, Into to Partial differential equations 4th edition

Solve $u_{tt} - 2u_{xt} - 3u_{xx} = 0$ with $u(0,x) = x^2, u_t(x,0) = e^x$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] - 2*D[D[u[x,t],x],t] -  3*D[u[x, t], {x,2}] == 0;
ic  = {u[x,0]==x^2, Derivative[0,1][u][x,0]==Exp[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to 3t^2 - \frac{e^{x-t}}{4} + \frac{1}{4}e^{3t+x} + x^2\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-2*diff(diff(u(x,t),x),t) - 3 * diff(u(x,t),t$2)=0;
ic  := u(x,0)=x^2, D[2](u)(x,0)=exp(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x^2 + \mathrm{e}^x - \mathrm{e}^{-t+x}$$

2.3 Schrodinger PDE

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2.3.1 Logan textbook, page 30

problem number 83

From page 30, David J Logan textbook, applied PDE textbook.

Schrodinger PDE with zero potential (Logan p. 30)

Solve

$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,0) = 0$$

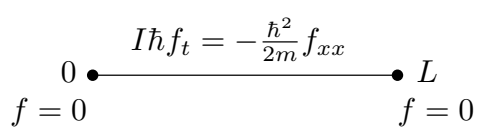

Figure 2.25: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
bc  = {f[0, t] == 0, f[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L >
sol =  sol /. K[1] -> n;
```

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Maple ✓

```
restart;
interface(showassumed=0);
pde :=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t)) assuming
```

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2.3.2 In a square, zero potential

problem number 84

With initial and boundary conditions. In a square, zero potential.

Solve for $f(x, y, t)$

$$I f_t = -\frac{\hbar^2}{2m} \nabla^2 f(x, y)$$

With boundary conditions

$$f(0, y, t) = 0$$
$$f(1, y, t) = 0$$
$$f(x, 1, t) = 0$$
$$f(x, 0, t) = 0$$

And initial conditions $f(x, y, 0) = \sqrt{2}(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y))$

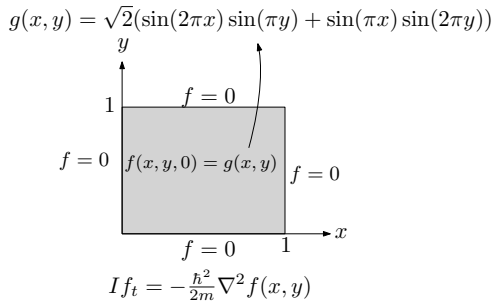

Figure 2.26: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, y, t], {t}] == -((hBar^2*Laplacian[f[x, y, t], {x, y}])/(2*m));
 initSum = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[2*Pi*y]);
 bcs = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcs, initSum}, f[x, y, t], {x, y, t}], 60
```

$$\left\{\left\{f(x,y,t) \to 2\sqrt{2}\sin(\pi x)\sin(\pi y)e^{-\frac{5i\pi^2 \text{hBar}^2 t}{2m}}(\cos(\pi x) + \cos(\pi y))\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  I* diff(f(x,y,t),t) = -hBar^2/(2*m) * (diff(f(x,y,t),x$2) +  diff(f(x,y,t),y$2));
ic  := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y) + sin(Pi*x)*sin(2*Pi*y));
bc  := f(0, y, t) = 0,
      f(1, y, t) = 0,
      f(x, 1, t) = 0,
      f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],f(x,y,t))),out
```

$$f(x,y,t) = \sqrt{2}\left(\sin(\pi x)\sin(2\pi y) + \sin(\pi y)\sin(2\pi x)\right)\mathrm{e}^{-\frac{5i\pi^2 hBar^2 t}{2m}}$$

2.3.3 From Mathematica help pages

problem number 85

Taken from Mathematica DSolve help pages

Initial value problem with Dirichlet boundary conditions. 1D, zero potential.

Solve for $f(x,t)$

$$If_t = -2f_{xx}$$

With boundary conditions

$$f(5,t) = 0$$
$$f(10,t) = 0$$

And initial conditions $f(x,2) = f(x)$ where $f(x) = -350 + 155x - 22x^2 + x^3$

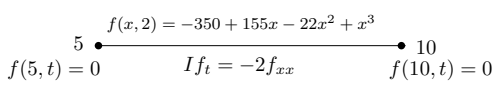

Figure 2.27: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, t], t] == -2*D[f[x, t], {x, 2}];
 g[x_] := -350 + 155*x - 22*x^2 + x^3;
ic  = f[x, 2] == g[x];
bc  = {f[5, t] == 0, f[10, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, f[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

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Maple ✓

```
restart;
pde :=I*diff(f(x,t),t)=-2*diff(f(x,t),x$2);
bc:=f(5,t)=0,f(10,t)=0;
g:=x->-350+155*x-22*x^2+x^3;
ic:=f(x,2)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],f(x,t))),outpu
```

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2.3.4 From Mathematica help pages

problem number 86

Taken from Mathematica DSolve help pages

Solve a Schrodinger equation with potential over the whole real line.

Solve for $f(x,t)$

$$If_t = -f_{xx} + 2x^2 f(x,t)$$

With boundary conditions

$$f(-\infty,t) = 0$$
$$f(\infty,t) = 0$$

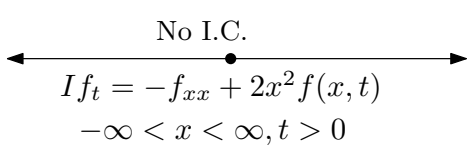

Figure 2.28: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  I*D[f[x, t], t] == -D[f[x, t], {x, 2}] + 2*x^2*f[x, t];
bc  = {f[-Infinity, t] == 0, f[Infinity, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

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Maple ✗

```
restart;
pde :=I*diff(f(x,t),t)=-diff(f(x,t),x$2)+2*x^2*f(x,t);
bc:=f(-infinity ,t)=0,f(infinity,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t))),output='
```

$$f(x,t)=0$$

Trivial solution. Maple does not support ∞ in boundary conditions

2.3.5 David Griffiths, page 47

problem number 87

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47.

Solve for $f(x,t)$

$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx}$$

With initial conditions $f(x,0) = Ax(a-x)$ for $0 \le x \le a$ and zero otherwise.

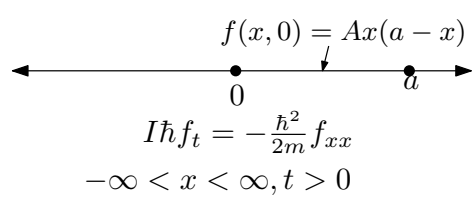


Figure 2.29: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = Piecewise[{{A*x*(a - x), 0 <= x <= a}, {0, True}}];
pde =  I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, f[x, 0] == ic}, f[x, t], {x, t}, Assumpti
```

$$\left\{\left\{f(x,t)\to \frac{\int_{-\infty}^{\infty}-\frac{A\exp\left(-\frac{iK[1](2am-2xm+htK[1])}{2m}\right)\left(aK[1]+e^{iaK[1]}(aK[1]+2i)-2i\right)}{K[1]^3}dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde :=I*h*diff(f(x,t),t) = -h^2/(2*m)*diff(f(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',dsolve([pde,ic],f(x,t)) assuming a
sol:=convert(sol,Int);
```

$$f(x,t)=-\frac{\left(a\left(\int_{-\infty}^{\infty}\frac{e^{-\frac{i\left(\frac{hst}{2}+(a-x)m\right)s}{m}}}{s^2}ds\right)+a\left(\int_{-\infty}^{\infty}\frac{e^{-\frac{ihs^2t}{2m}+isx}}{s^2}ds\right)-2i\left(\int_{-\infty}^{\infty}\frac{e^{-\frac{i\left(\frac{hst}{2}+(a-x)m\right)s}{m}}}{s^3}ds\right)+2i\left(\int_{-\infty}^{\infty}\right.\right.}{2\pi}$$

2.3.6 David Griffiths, page 47

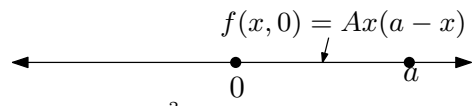
problem number 88

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47. This is the same as the above problem but has an extra $V(x)f(x,t)$ terms where $V(x)$ is the infinite square well potential defined by $V(x)=0$ if $0\le x\le a$ and $V(x)=\infty$ otherwise.

Solve for $f(x,t)$

$$I\hbar f_t=-\frac{\hbar^2}{2m}f_{xx}+V(x)f(x,t)$$

With initial conditions $f(x,0)=Ax(a-x)$ for $0\le x\le a$ and zero otherwise.



$$I\hbar f_t = -\frac{\hbar^2}{2m} f_{xx} + V(x) f(x,t)$$
$$-\infty < x < \infty, t > 0$$

Figure 2.30: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[3*Pi*y]);
bc  = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
pde =  I*h*D[f[x, y, t], t] == -((h^2*(D[f[x, y, t], {x, 2}] + D[f[x, y, t], {y, 2}]))/(2*m)
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, f[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{f(x,y,t) \to \sqrt{2} e^{-\frac{5i\pi^2 ht}{m}} \left(\sin(\pi x)\sin(3\pi y) + \sin(2\pi x)\sin(\pi y) e^{\frac{5i\pi^2 ht}{2m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
V:=x->piecewise(0<=x and x<=a,0,infinity);
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde :=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2) +V(x)*f(x,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],f(x,t)) assuming
```

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2.3.7 In a square

problem number 89

Added December 20, 2018.

Example 28, taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

In 2 space dimensions Solve for $f(x,y,t)$

$$I\hbar f_t = -\frac{\hbar^2}{2m} \nabla^2 f$$

With initial conditions $f(x,y,0)=\sqrt{2}(\sin(2\pi x)\sin(\pi y)+\sin(\pi x)\sin(3\pi y))$ and boundary conditions

$$f(0,y,t)=0$$
$$f(1,y,t)=0$$
$$f(x,1,t)=0$$
$$f(x,0,t)=0$$

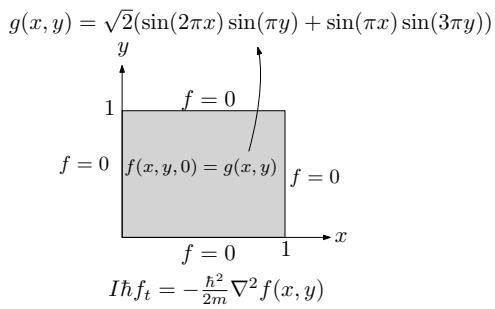

Figure 2.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ic  = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y]+Sin[Pi*x]*Sin[3*Pi*y]);
bc  = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
pde =  I*h*D[f[x, y,t], t] == -h^2/(2*m)*(D[f[x, y, t], {x, 2}]+D[f[x, y, t], {y, 2}]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, f[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{f(x,y,t)\to\sqrt{2}e^{-\frac{5i\pi^2 ht}{m}}\left(\sin(\pi x)\sin(3\pi y)+\sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2 ht}{2m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := I*hbar* diff(f(x, y, t), t) = - hbar^2/(2*m)*  (diff(f(x, y, t), x$2)+diff(f(x, y, t)
ic  := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y)+sin(Pi*x)*sin(3*Pi*y));
bc  := f(0, y, t) = 0, f(1, y, t) = 0, f(x, 1, t) = 0, f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],f(x,y,t))),ou
```

$$f(x,y,t)=\sqrt{2}\left(2\cos(\pi x)\,\mathrm{e}^{-\frac{5i\pi^2 hbart}{2m}}\sin(\pi y)+\mathrm{e}^{-\frac{5i\pi^2 hbart}{m}}\sin(3\pi y)\right)\sin(\pi x)$$

2.4 Beam PDE

Local contents

2.4.1 Beam PDE $u_{tt} + u_{xxxx} = 0$ 332

2.4.1 Beam PDE $u_{tt} + u_{xxxx} = 0$

problem number 90

Added January 20, 2018.

Beam PDE with zero initial velocity. Solve

$$u_{tt} + u_{xxxx} = 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= -12t^2 \\ f(1,t) &= 1 - 12t^2 \\ \frac{\partial^2 u}{\partial x^2} u(0,t) &= 0 \\ \frac{\partial^2 u}{\partial x^2} u(1,t) &= 12 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^4 \\ \frac{\partial u}{\partial t} u(x,0) &= 0 \end{aligned}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + D[u[x, t], {x, 4}] == 0;
bc  = {u[0, t] == -12*t^2, u[1, t] == 1 - 12*t^2, Derivative[2, 0][u][0, t] == 0, Derivative
ic  = {u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+diff(u(x,t),x$4)=0;
bc  := u(0,t)=-12*t^2,
    u(1,t)=1-12*t^2,D[1,1](u)(0,t)=0,
    D[1,1](u)(1,t)=12;
ic  := u(x,0)=x^4,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT=`+
```

$$u(x,t) = x^4 - 12t^2$$

2.5 Burger's PDE

Local contents

2.5.1 Inviscid Burgers $u_x + uu_y = 0$ 334
2.5.2 Inviscid Burgers with I.C. $u_x + uu_y = 0$ and $u(x,0) = \frac{1}{x+1}$ 335
2.5.3 $u_t + uu_x = \mu u_{xx}$ 337
2.5.4 $u_t + uu_x + \mu u_{xx}$ with IC 338
2.5.5 $u_t + uu_x + \mu u_{xx}$ IC as UnitBox 339

2.5.1 Inviscid Burgers $u_x + uu_y = 0$

problem number 91

Taken from Mathematica Symbolic PDE document

quasilinear first-order PDE, scalar conservation law

Solve for $u(x,y)$

$$u_x + uu_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Implicit solution

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
sol:=DEtools:-remove_RootOf(sol);
```

$$xu(x,y) - y + \_F1(u(x,y)) = 0$$

Hand solution

Solve for $u(x,y)$ in $u_x + u\, u_y = 0$. Using the Lagrange-Charpit method, the characteristic equations are

$$\frac{dx}{1} = \frac{dy}{u} = \frac{du}{0}$$

From the first pair of equation we obtain

$$u = \frac{dy}{dx}$$

But $du = 0$ or $u = C_2$. Hence the above becomes

$$\begin{aligned} \frac{dy}{dx} &= C_2 \\ y &= xC_2 + C_1 \\ C_1 &= y - xC_2 \end{aligned}$$

Since $C_2 = F(C_1)$ where F is arbitrary function, then

$$u(x,y) = F(y - ux)$$

2.5.2 Inviscid Burgers with I.C. $u_x + uu_y = 0$ and $u(x,0) = \frac{1}{x+1}$

problem number 92

Taken from Mathematica Symbolic PDE document

quasilinear first-order PDE, scalar conservation law with initial value

Solve for $u(x,y)$

$$u_x + uu_y = 0$$

With $u(x,0) = \frac{1}{x+1}$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
ic  = u[x, 0] == 1/(x + 1);
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{y+1}{x+1}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
ic  := u(x,0)=1/(x+1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \frac{y+1}{x+1}$$

Hand solution

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dx}{ds} = 1 \tag{1}$$
$$\frac{dy}{ds} = u \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$x(0) = t_1, y(0) = t_2, u(0) = t_3$$

We are given that $u(x,0) = \frac{1}{1+x}$. This initial condition translates to

$$t_3 = \frac{1}{1+t_1}, t_2 = 0 \tag{4}$$

Equation (1) gives

$$x = s + t_1 \tag{5}$$

Equation (2) gives

$$\begin{aligned} y &= su + t_2 \\ &= su \end{aligned} \tag{7}$$

Equation (3) gives

$$u = t_3$$

Hence the solution is

$$\begin{aligned} u &= t_3 \\ &= \frac{1}{1+t_1} \\ &= \frac{1}{1+(x-s)} \\ &= \frac{1}{1+\left(x-\frac{y}{u}\right)} \end{aligned}$$

Solving for u gives

$$\begin{aligned} u\left(1+\left(x-\frac{y}{u}\right)\right) &= 1 \\ u+xu-y &= 1 \\ u(1+x) &= 1+y \\ u &= \frac{1+y}{1+x} \end{aligned}$$

2.5.3 $u_t + uu_x = \mu u_{xx}$

problem number 93

From Mathematica symbolic PDE document.

viscous fluid flow with no initial conditions

Solve for $u(x,t)$

$$u_t + uu_x = \mu u_{xx}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -2c_1\mu\tanh(c_2t+c_1x+c_3) - \frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) + u(x, t)*diff(u(x, t), x) = mu* diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, t))),output='rea
```

$$u(x,t) = \frac{-2c_2^2\mu \tanh(c_3 t + c_2 x + c_1) - c_3}{c_2}$$

2.5.4 $u_t + uu_x + \mu u_{xx}$ with IC

problem number 94

From Mathematica symbolic PDE document.

Viscous fluid flow with initial conditions.

Solve for $u(x,t)$

$$u_t + uu_x + \mu u_{xx}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
ic  = u[x, 0] == Piecewise[{{1, x < 0}, {0, x >= 1}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> mu >
```

$$\left\{\left\{u(x,t) \to \frac{1}{\frac{e^{-\frac{t-2x}{4\mu}}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{\mu}\sqrt{t}}\right)+1\right)}{\operatorname{erf}\left(\frac{t-x}{2\sqrt{\mu}\sqrt{t}}\right)+1}+1}\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x))  = mu*(diff(u(x, t), x$2));
ic  := u(x, 0) =  piecewise(x>=0,0,x<0,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

2.5.5 $u_t + uu_x + \mu u_{xx}$ IC as UnitBox

problem number 95

From Mathematica DSolve help pages.

Viscous fluid flow with initial conditions as UnitBox

Solve for $u(x,t)$

$$u_t + uu_x = \mu u_{xx}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{e^{\frac{t+1}{4\mu}}\left(\operatorname{erf}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)-\operatorname{erf}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)\right)}{e^{\frac{t+1}{4\mu}}\left(\operatorname{Erfc}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)-\operatorname{Erfc}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)\right)+e^{\frac{x}{2\mu}}\left(\operatorname{Erfc}\left(\frac{1-2x}{4\sqrt{\mu t}}\right)+e^{\frac{1}{2}/\mu}\operatorname{Erfc}\left(\frac{2x+1}{4\sqrt{\mu t}}\right)\right)}\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic  := u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

2.6 Black Scholes PDE

Local contents

2.6.1 classic Black Scholes model from finance, European call version 341
2.6.2 Boundary value problem for the Black Scholes equation 342

2.6.1 classic Black Scholes model from finance, European call version

problem number 96

From Mathematica symbolic PDE document.

Solve for $V(S,t)$ where V is the price of the option as a function of stock price S and time t. r is the risk-free interest rate, and σ is the volatility of the stock.

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}=rV-rS\frac{\partial V}{\partial S}$$

With boundary condition $V(S,T)=\max\{S-k,0\}$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation`
See the European call version at bottom of the page.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1*sigma^2*D[u[x, t], {x, 2}])/2;
ic  = u[x, 0] == k*Exp[x - 1]*HeavisideTheta[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> k >
```

$$\left\{\left\{u(x,t)\to\frac{1}{2}ke^{\frac{\sigma^2 t}{2}+x-1}\left(\operatorname{erf}\left(\frac{\sigma^2 t+x}{\sqrt{2}\sqrt{t}|\sigma|}\right)+1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t) = 1/2*sigma^2*diff(u(x,t),x$2);
ic  := u(x, 0) = k*exp(x - 1)*Heaviside(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \mathrm{e}^{-1}\left(-i\mathcal{F}^{-1}\left(\frac{\mathrm{e}^{-\frac{s^2\sigma^2 t}{2}}}{s+i}, s, x\right) + \mathcal{F}^{-1}\left(\mathrm{e}^{-\frac{s^2\sigma^2 t}{2}}\mathcal{F}(\mathrm{e}^x, x, s), s, x\right)\right) k$$

2.6.2 Boundary value problem for the Black Scholes equation

problem number 97

From Mathematica DSolve help pages.

Solve for $V(t,s)$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2} + (r-q)s\frac{\partial v}{\partial s} - rv(t,s) = 0$$

With boundary condition $v(T,s) = \psi(s)$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation`

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[v[t, s], t] + (1*sigma^2*s^2*D[v[t, s], {s, 2}])/2 + (r - q)*s*D[v[t, s], s] - r*v[
bc  = v[T, s] == psi[s];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, v[t, s], {t, s}], 60*10]];
```

$$\left\{\left\{v(t,s) \to \frac{e^{r(t-T)}\int_{-\infty}^{\infty}\psi\left(e^{K[1]}\right)\exp\left(-\frac{\left(-K[1]+\frac{1}{2}(t-T)\left(2q-2r+\sigma^2\right)+\log(s)\right)^2}{2\sigma^2(T-t)}\right)dK[1]}{\sqrt{2\pi}\sqrt{\sigma^2(T-t)}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(v(t, s), t) +s^2*(diff(v(t, s), s, s))/(2*sigma^2)+(r-q)*s*(diff(v(t, s), s))-r*
ic:=v(T, s) = psi(s);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],v(t,s))),output='
```

$$v(t,s) =$$

2.7 Korteweg-deVries PDE

Local contents

2.7.1 $u_{xxx} + u_t - 6uu_x = 0$ 344

2.7.1 $u_{xxx} + u_t - 6uu_x = 0$

problem number 98

From Mathematica symbolic PDE document.

Korteweg-deVries (waves on shallow water surfaces) with no initial conditions

Solve for $u(x,t)$

$$u_{xxx} + u_t - 6uu_x = 0$$

Reference `https://en.wikipedia.org/wiki/Korteweg%E2%80%93de_Vries_equation`

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {x, 3}] + D[u[x, t], {t}] - 6*u[x, t]*D[u[x, t], {x}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{12{c_1}^3 \tanh^2(c_2 t + c_1 x + c_3) - 8{c_1}^3 + c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),x$3)+ diff(u(x,t),t)-6*u(x,t)* diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 2c_2^2\left(\tanh^2\left(c_3 t + c_2 x + c_1\right)\right) + \frac{-8c_2^3 + c_3}{6c_2}$$

2.8 Tricomi PDE

Local contents

2.8.1 $u_{xx} + yu_{yy} = 0$ with $u(x,0) = 0, u_y(x,0) = x^2$ 345
2.8.2 $u_{xx} + xu_{yy} = 0$ 346

2.8.1 $u_{xx} + yu_{yy} = 0$ with $u(x,0) = 0, u_y(x,0) = x^2$

problem number 99

From Mathematica DSolve helps pages.

Boundary value problem for the Tricomi equation.

Solve for $u(x,y)$

$$u_{xx} + yu_{yy} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -y(y-x^2)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ y*diff(u(x,y),y$2)=0;
bc  := u(x,0)=0,(D[2](u))(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = (x^2 - y)\, y$$

2.8.2 $u_{xx} + xu_{yy} = 0$

problem number 100

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x, y)$

$$u_{xx} + xu_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + x*D[u[x, y], {y, 2}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ x*diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \left(c_1 \operatorname{AiryAi}\left(-(-\_c_1)^{\frac{1}{3}} x\right) + c_2 \operatorname{AiryBi}\left(-(-\_c_1)^{\frac{1}{3}} x\right)\right)\left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\sqrt{\_c_1}\right)\right)$$

2.9 Keldysh equation

Local contents

2.9.1 $xu_{xx} + u_{yy} = 0$ 347

2.9.1 $xu_{xx} + u_{yy} = 0$

problem number 101

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x, y)$

$$xu_{xx} + u_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \left(c_1 \operatorname{BesselJ}\left(1, 2\sqrt{-\_c_1}\,\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(1, 2\sqrt{-\_c_1}\,\sqrt{x}\right)\right)\left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\sqrt{\_c_1}\right)\right)$$

2.10 Euler-Poisson-Darboux equation

Local contents

2.10.1 $u_{xx} + u_{yy} + \frac{\beta}{x} u_x = 0$ 348
2.10.2 $u_{xx} - u_{yy} + \frac{\beta}{x} u_x = 0$ 349
2.10.3 $u_{tt} - u_{xx} - \frac{2}{x} u_x = 0$ with $u(x,0) = 0, u_t(x,0) = g(x)$ 349

2.10.1 $u_{xx} + u_{yy} + \frac{\beta}{x} u_x = 0$

problem number 102

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x,y)$

$$u_{xx} + u_{yy} + \frac{\beta}{x} u_x = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] + beta/x*D[u[x,y],x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}, Assumptions->beta>0], 60*
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2) + beta/x*diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') assumi
```

$$u(x,y) = \left(c_1 \operatorname{BesselJ}\left(\frac{\beta}{2} - \frac{1}{2}, \sqrt{-\_c_1}\, x\right) + c_2 \operatorname{BesselY}\left(\frac{\beta}{2} - \frac{1}{2}, \sqrt{-\_c_1}\, x\right)\right) \left(c_3 \sin\left(y\sqrt{\_c_1}\right) + c_4 \cos\left(y\right.\right.$$

2.10.2 $u_{xx} - u_{yy} + \frac{\beta}{x} u_x = 0$

problem number 103

Added June 20, 2019

Taken from `http://people.maths.ox.ac.uk/chengq/outreach/The%20Tricomi%20Equation.pdf`

Solve for $u(x, y)$

$$u_{xx} - u_{yy} + \frac{\beta}{x} u_x = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] - D[u[x, y], {y, 2}] + beta/x*D[u[x,y],x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}, Assumptions->beta>0], 60*
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)- diff(u(x,y),y$2) + beta/x*diff(u(x,y),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build') assumi
```

$$u(x, y) = c_3 \beta^{\frac{\beta}{2}} \left(-\frac{1}{c_1 (2x + 2y) + 2c_2} \right)^{\frac{\beta}{2}} (c_1 (x - y) - c_2)^{-\frac{\beta}{2}}$$

2.10.3 $u_{tt} - u_{xx} - \frac{2}{x} u_x = 0$ with $u(x, 0) = 0, u_t(x, 0) = g(x)$

problem number 104

Added Oct 6, 2019

Problem 2.4.18 from Peter Olver, Intoduction to Partial differential equations, 4th edition.

Solve for $u(x, t)$

$$u_{tt} - u_{xx} - \frac{2}{x} u_x = 0$$

With $u(x,0) = 0, u_t(x,0) = g(x)$. Note, in the book, it says to assume $g(x)$ is even function. In the code below, this assumption is not used. When I find the correct way to implement this assumption in CAS, will have to re-run these.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - 2/x*D[u[x,t],x] == 0;
ic = {u[x,0]==0,Derivative[0,1][u][x,0]==g[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)- diff(u(x,t),x$2) - 2/x*diff(u(x,t),x)=0;
ic:= u(x,0)=0, D[2](u)(x,0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) ),output=
```

Unable to process latex

2.11 Chaplygin's equation

Local contents

2.11.1 $u_{\theta\theta} + \frac{v^2}{1-\frac{v^2}{c^2}} u_{vv} + v u_v = 0$ 351

2.11.1 $u_{\theta\theta} + \frac{v^2}{1-\frac{v^2}{c^2}} u_{vv} + v u_v = 0$

problem number 105

Added June 20, 2019 From `https://en.wikipedia.org/wiki/Chaplygin%27s_equation`

Solve for $u(\theta, v)$

$$u_{\theta\theta} + \frac{v^2}{1-\frac{v^2}{c^2}} u_{vv} + v u_v = 0$$

Here c is the speed of sound.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[theta, v], {theta, 2}] + v^2/(1-v^2/c^2)* D[u[theta,v],{v,2}]+v*D[u[theta,v],v]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[theta, v], {theta, v}, Assumptions->c>0]
```

Failed

Maple ✓

```
restart;
pde := diff(u(theta,v),theta$2)+ v^2/(1-v^2/c^2)* diff(u(theta,v),v$2)+v*diff(u(theta,v),v)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(theta,v),'build') as
```

$$u(\theta, v) = \frac{\left(c_1 e^{2\theta\sqrt{-c_1}} + c_2\right)\left(c_3 \operatorname{WhittakerM}\left(-\frac{-c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2}\right) + c_4 \operatorname{WhittakerW}\left(-\frac{-c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2}\right.\right.}{v}$$

2.12 Cauchy Riemann PDE's

Local contents

2.12.1 Cauchy Riemann PDE with Prescribe the values of u and v on the x axis 352
2.12.2 Cauchy Riemann PDE With extra term on right side 353

2.12.1 Cauchy Riemann PDE with Prescribe the values of u and v on the x axis

problem number 106

From Mathematica DSolve helps pages.

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

With boundary conditions

$$u(x,0) = x^3$$
$$v(x,0) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x];
bc  = {u[x, 0] == x^3, v[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2, bc}, {u[x, y], v[x, y]}, {x, y}],
```

$$\{\{u(x,y) \to x^3 - 3xy^2, v(x,y) \to 3x^2y - y^3\}\}$$

Maple ✓

```
restart;
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y);
bc  := u(x,0)=x^3,v(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2,bc],[u(x,y),v(x
```

$$\{u(x,y) = x^3 - 3xy^2, v(x,y) = -3x^2y + y^3\}$$

2.12.2 Cauchy Riemann PDE With extra term on right side

problem number 107

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} + y$$

Mathematica ✗

```
ClearAll["Global`*"];
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x] + y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2}, {u[x, y], v[x, y]}, {x, y}], 60*1
```

Failed

Maple ✓

```
restart;
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y)+y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2],[u(x,y),v(x,y)
```

$$\left\{u(x,y) = xy + c_1 - i\_F1(-ix+y) + i\_F2(ix+y), v(x,y) = \frac{x^2}{2} + \_F1(-ix+y) + \_F2(ix+y)\right\}$$

2.13 Hamilton-Jacobi PDE

Local contents

2.13.1 Hamilton-Jacobi type PDE 355

2.13.1 Hamilton-Jacobi type PDE

problem number 108

Taken from Maple pdsolve help pages, which is taken from Landau, L.D. and Lifshitz, E.M. Translated by Sykes, J.B. and Bell, J.S. Mechanics. Oxford: Pergamon Press, 1969

Solve for $S(t,\xi,\eta,\phi)$

$$-\frac{\partial}{\partial t}S(t,\xi,\eta,\phi) = 1/2\,\frac{\left(\frac{\partial}{\partial\xi}S(t,\xi,\eta,\phi)\right)^2(\xi^2-1)}{\sigma^2 m\,(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial\eta}S(t,\xi,\eta,\phi)\right)^2(-\eta^2+1)}{\sigma^2 m\,(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial\phi}S(t,\xi,\eta\right.}{\sigma^2 m\,(\xi^2-1)\,(}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  -D[s[t, \[Zeta], \[Eta], \[Phi]], t] == ((\[Zeta]^2 - 1)*D[s[t, \[Zeta], \[Eta], \[Ph
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, s[t, \[Zeta], \[Eta], \[Phi]], {t, \[Zeta]
```

Failed

Maple ✓

```
restart;
pde := -diff(S(t,xi,eta,phi),t) =1/2*diff(S(t,xi,eta,phi),xi)^2*(xi^2-1)/sigma^2/m/(xi^2-eta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,'build')),output='real
```

$$S(t,\xi,\eta,\phi) = \phi\_c_4 + t\_c_1 + c_1 + c_2 + c_3 + c_4 - \left(\int \frac{\sqrt{(2\eta^2-2)\,m\sigma^2 b\,(\eta) - 2\,(\eta-1)\,(\eta+1)\,(\eta^2\_c_1 + \_c_3)\,m}}{\eta^2-1}\right.$$

2.14 Airy PDE

Local contents

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2.14.1 $u_t + u_{xxx} = 0$

problem number 109

Added May 30, 2019.

Airy PDE

Solve for $u(x,t)$

$$u_t + u_{xxx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], {x,3}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to x^3(c_{11}t + c_4) + x^2(c_3 - 60c_6t) + x(c_2 - 24c_5t) - 3t(c_{11}t + 2c_4) - \frac{c_{11}x^6}{120} + c_6x^5 + c_5x^4 + c_1\right\}\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x$3)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = c_4\left(c_1 e^{-\frac{i\sqrt{3}\,x\_c_1^{\frac{1}{3}}}{2}} e^{-\frac{x\_c_1^{\frac{1}{3}}}{2}} + c_2 e^{\frac{i\sqrt{3}\,x\_c_1^{\frac{1}{3}}}{2}} e^{-\frac{x\_c_1^{\frac{1}{3}}}{2}} + c_3 e^{x\_c_1^{\frac{1}{3}}}\right) e^{-t\_c_1}$$

Hand solution

Solve for $u_t + u_{xxx} = 0$ on the real line for $t > 0$. Let $u = T(t)\,X(t)$. The pde becomes

$$\begin{aligned} T'X + X'''T &= 0 \\ \frac{T'}{T} &= -\frac{X'''}{X} = -\lambda \end{aligned}$$

Hence $X''' + \lambda X = 0$. This ODE has solution

$$X(x) = C_1 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} - \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_2 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} + \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_3 e^{\lambda^{\frac{1}{3}}x}$$

The ODE $T' + \lambda T = 0$ has the solution $T(t) = C_4 e^{-\lambda t}$. Therefore the solution to the PDE is $T(t)\,X(t)$ given by

$$u(x,t) = C_4 e^{-\lambda t}\left(C_1 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} - \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_2 e^{\left(-\frac{\lambda^{\frac{1}{3}}}{2} + \frac{i}{2}\lambda^{\frac{1}{3}}\sqrt{3}\right)x} + C_3 e^{\lambda^{\frac{1}{3}}x}\right)$$

2.15 Nonlinear PDE's

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2.15.1 Bateman-Burgers $u_t + uu_x = \nu u_{xx}$

problem number 110

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Bateman-Burgers.

Solve for $u(x,t)$

$$u_t + uu_x = \nu u_{xx}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + u[x, t]*D[u[x, t], x] == v*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -2c_1 v \tanh(c_2 t + c_1 x + c_3) - \frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+u(x,t)*diff(u(x,t),x)=v*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{-2c_2^2 v \tanh(c_3 t + c_2 x + c_1) - c_3}{c_2}$$

2.15.2 Benjamin Bona Mahony $u_t + u_x + uu + x - u_{xxt} = 0$

problem number 111

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t + u_x + uu + x - u_{xxt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], x] + u[x, t]*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to 12c_1c_2\tanh^2(c_2t + c_1x + c_3) - 1 - 8c_1c_2 - \frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+diff(u(x,t),x)+u(x,t)*diff(u(x,t),x)-diff(u(x,t),x,x,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{12c_3c_2^2\left(\tanh^2\left(c_3t + c_2x + c_1\right)\right) - 8c_3c_2^2 - c_2 - c_3}{c_2}$$

2.15.3 Benjamin Ono $u_t + Hu_{xx} + uu_x = 0$

problem number 112

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Benjamin Ono. Solve for $u(x,t)$

$$u_t + Hu_{xx} + uu_x = 0$$

Important note. H above is meant to be Hilbert transform. `https://en.wikipedia.org/wiki/Benjamin%E2%80%93Ono_equation` However, here in the code below it is taken as just a scalar. Need to correct this when I have time.

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + h*D[u[x, t], {x, 2}] + u[x, t]*D[u[x, t], x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to 2c_1 h\tanh(c_2 t+c_1 x+c_3)-\frac{c_2}{c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+H*diff(u(x,t),x$2)+u(x,t)*diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\frac{2Hc_2^2\tanh\left(c_3t+c_2x+c_1\right)-c_3}{c_2}$$

2.15.4 Born Infeld $(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$

problem number 113

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Born Infeld. Solve for $u(x,t)$

$$(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (1 - D[u[x, t], t]^2)*D[u[x, t], {x, 2}] + 2*D[u[x, t], x]*D[u[x, t], t]*D[D[u[x, t],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to c_1(t+x) + c_2(t-x)\}\}$$

Maple ✓

```
restart;
pde :=(1-diff(u(x,t),t)^2)*diff(u(x,t),x$2)+2*diff(u(x,t),x)*diff(u(x,t),t)*diff(u(x,t),x,t)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = c_7\left(\tanh^3\left(c_1 + c_2(-t+x)\right)\right) + c_5 \tanh\left(c_1 + c_2(-t+x)\right) + c_4$$

2.15.5 Boussinesq $u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$

problem number 114

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Boussinesq. Solve for $u(x,t)$

$$u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{6}\left(-12{c_1}^2 \tanh^2(c_2 t + c_1 x + c_3) - 1 + 8{c_1}^2 + \frac{{c_2}^2}{{c_1}^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)-diff(u(x,t),x$4)- 3 * diff( u(x,t)^2, x$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = -2c_2^2\left(\tanh^2\left(c_3t + c_2x + c_1\right)\right) + \frac{8c_2^4 - c_2^2 + c_3^2}{6c_2^2}$$

2.15.6 Boussinesq type $u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$

problem number 115

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Boussinesq type PDE. Solve for $u(x,t)$

$$u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{6}\left(-12{c_1}^2\tanh^2(c_2t + c_1x + c_3) - 1 + 8{c_1}^2 + \frac{{c_2}^2}{{c_1}^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)-2*alpha*diff( (u(x,t)*diff(u(x,t),x)) ,x) - beta*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{-12c_2^2c_3^2\beta\left(\tanh^2\left(c_3t + c_2x + c_1\right)\right) + \left(8c_3^2\beta - 1\right)c_2^2 + c_3^2}{2c_2^2\alpha}$$

2.15.7 Buckmaster $u_t = (u^4)_{xx} + (u^3)_x$

problem number 116

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Buckmaster. Solve for $u(x,t)$

$$u_t = (u^4)_{xx} + (u^3)_x$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t]^4, {x, 2}] + D[u[x, t]^3, x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t)^4,x$2)+diff(u(x,t)^3,x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \text{RootOf}\left(c_1 x + c_2 t + c_3 + c_4 + \int^{\_Z} \frac{4c_1^2 \_f^3}{c_1 \_f^3 + 4c_3 c_1^2 - c_2 \_f} d\_f\right)$$

Answer in terms of RootOf.

2.15.8 Camassa Holm $u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}$

problem number 117

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Camassa Holm. Solve for $u(x,t)$

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 2*k*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] + 3*u[x, t]*D[u[x, t], x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)+2*k*diff(u(x,t),x)- diff(u(x,t),x,x,t)+3*u(x,t)*diff(u(x,t),x)=2*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{\text{RootOf}\left(c_1c_5 - c_1x + c_1\left(\int^{-\frac{-\_Z^2+c_2}{c_1}} \frac{\sqrt{c_1\_f+c_2}}{\sqrt{-c_4c_1^3\_f+c_1\_f^3+2c_1\_f^2k-c_1^2c_2c_4+c_2\_f^2-c_3c_1^2}}d\_f\right) - c_2t - c_3\right)^2}{c_1}$$

Answer in terms of RootOf.

2.15.9 Chaffee Infante $u_t = u_{xx} + \lambda(u^3 - u) = 0$

problem number 118

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Chaffee Infante equation. Solve for $u(x,t)$

$$u_t = u_{xx} + \lambda(u^3 - u) = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] - D[u[x, t], {x, 2}] + lambda*(u[x, t]^3 - u[x, t]) == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)-diff(u(x,t),x$2)+lambda*(u(x,t)^3-u(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{\tanh\left(-\frac{3\lambda t}{4} + \frac{\sqrt{2}\sqrt{\lambda}x}{4} + c_1\right)}{2} - \frac{1}{2}$$

2.15.10 Clarke. $\left(\theta_t - \gamma e^{\theta}\right)_{tt} = \left(\theta_t - e^{\theta}\right)_{xx}$

problem number 119

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Clarke equation. Solve for $\theta(x,t)$

$$\left(\theta_t - \gamma e^{\theta}\right)_{tt} = \left(\theta_t - e^{\theta}\right)_{xx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[theta[x, t], t] - gamma*Exp[theta[x, t]], {t, 2}] == D[D[theta[x, t], t] - Exp[th
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, theta[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(diff(theta(x,t),t)-g*exp(theta(x,t)),t$2) = diff( diff(theta(x,t),t)-exp(theta(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,theta(x,t))),output='r
```

sol=()

2.15.11 Degasperis Procesi $u_t - u_{xxt} + 4uu_x = 3u_x u_x x + uu_{xxx}$

problem number 120

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Degasperis Procesi. Solve for $u(x,t)$

$$u_t - u_{xxt} + 4uu_x = 3u_x u_x x + uu_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] - D[D[u[x, t], {x, 2}], t] + 4*u[x, t]*D[u[x, t], x] == 3*D[u[x, t], x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)-diff(u(x,t),x,x,t)+4*u(x,t)*diff(u(x,t),x)=3*diff(u(x,t),x)*diff(u(x,t
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{\_F1(x)}{-\_c_2 t + c_2} \textbf{where} \left[\left\{\left\{\_F1(x) = \_a \textbf{where} \left[\left\{\frac{\_a\_b(\_a)^2 \left(\frac{d^2}{d\_a^2}\_b(\_a)\right) + \_a\_b(\_a) \left(\frac{d}{d\_a}\right.}{}\right.\right.\right.\right.$$

But still has unresolved ODE's in solution

2.15.12 Dym equation $u_t = u^3 u_{xxx}$

problem number 121

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Dym equation. Solve for $u(x,t)$

$$u_t = u^3 u_{xxx}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == u[x, t]^3*D[u[x, t], {x, 3}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t)=u(x,t)^3 * diff(u(x,t),x$3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{\operatorname{RootOf}\left(c_3 + x - \left(\int^{\_Z} \frac{1}{\operatorname{RootOf}\left(c_2 + 2\left(\int^{\_Z} \frac{\_h}{\_h^2 + 2 2^{\frac{1}{3}} \left(-\_c_1^2\right)^{\frac{1}{3}} \operatorname{RootOf}\left(c_1 2^{\frac{1}{3}} \left(-\_c_1^2\right)^{\frac{1}{3}} \_h \operatorname{AiryBi}(\_Z) + 2 c_1 \_c_1 \operatorname{AiryBi}(1, \_Z\right.\right.}\right.\right.\right.}\right.}{(-3t\_c_1 + c_4)^{\frac{1}{3}}}$$

has RootOf

2.15.13 Estevez Mansfield Clarkson

$$u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$$

problem number 122

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Estevez Mansfield Clarkson equation. Solve for $u(x,y,t)$

$$u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], t], {y, 3}] + beta*D[u[x, y, t], y]*D[D[u[x, y, t], y], t] + beta*D[u
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to \frac{6c_1(x)\tanh\left(-4t(c_1(x))^3+yc_1(x)+c_3(x)\right)}{\beta}+c_4(x)\right\}\right\}$$

Maple ✓

```
restart;
beta='beta';
pde := diff(u(x,y,t),t,y,y,y)+ beta*diff(u(x,y,t),y)*diff(u(x,y,t),y,t) +  beta*diff(u(x,y,t
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t)=\frac{6c_3\tanh\left(-4c_3^3t+c_2x+c_3y+c_1\right)}{\beta}+c_5$$

2.15.14 Fisher's $u_t = u(1-u)+u_{xx}$

problem number 123

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Fisher's equation. Solve for $u(x,t)$

$$u_t = u(1-u)+u_{xx}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == u[x, t]*(1 - u[x, t]) + D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5t-\sqrt{6}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5t-i\sqrt{6}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5t-i\sqrt{6}x-12c_3\right)\right)\right)\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5t+i\sqrt{6}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5t+i\sqrt{6}x-12c_3\right)\right)\right)\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5t+\sqrt{6}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{5t}{12}-\frac{x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{5t}{12}-\frac{ix}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5t}{12}-\frac{ix}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{5t}{12}+\frac{ix}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5t}{12}+\frac{ix}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{u(x,t)\to \frac{1}{4}\left(1+\tanh\left(\frac{5t}{12}+\frac{x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= u(x,t)*(1-u(x,t))+ diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,u(x,t)))
```

$$\{u(x,t)=1\}$$

$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$

$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$

$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}-\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$

$$\left\{u(x,t)=\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}+\frac{\sqrt{6}\,x}{12}\right)}{2}+\frac{1}{4}\right\}$$

$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$

$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$

$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1-\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}-\frac{\tanh\left(c_1-\frac{5t}{12}-\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$

$$\left\{u(x,t)=-\frac{\left(\tanh^2\left(c_1+\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)\right)}{4}+\frac{\tanh\left(c_1+\frac{5t}{12}+\frac{i\sqrt{6}\,x}{12}\right)}{2}+\frac{3}{4}\right\}$$

2.15.15 Hunter Saxton $(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$

problem number 124

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Hunter Saxton. Solve for $u(x,t)$

$$(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[u[x, t], t] + u[x, t]*D[u[x, t], x], x] == (1*D[u[x, t], x]^2)/2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff( (diff(u(x,t),t)+ u(x,t)* diff(u(x,t),x)) , x) = 1/2* (diff(u(x,t),x))^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{2\,\mathrm{RootOf}\left(-c_2\_c_1^3 - x\_c_1^3 + 2c_1^2 \ln\left(\sqrt{\_Z}\,\_c_1 + c_1\right) + \_Z\_c_1^2 - 2c_1\sqrt{\_Z}\,\_c_1\right)}{t\_c_1 + 2c_3}$$

with RootOf

2.15.16 Kadomtsev Petviashvili $(u_t + uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$

problem number 125

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Kadomtsev Petviashvili. Solve for $u(x,y,t)$

$$\left(u_t + uu_x + \epsilon^2 u_{xxx}\right)_x + \lambda u_{yy} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], t] + u[x, y, t]*D[u[x, y, t], x] + eps^2*D[u[x, y, t], {x, 3}], t] +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to -\frac{12c_3{c_1}^3\text{eps}^2\tanh^2(c_3t+c_1x+c_2y+c_4)+c_3(c_3-8{c_1}^3\text{eps}^2)+{c_2}^2\lambda}{c_1c_3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff( diff(u(x,y,t),t)+u(x,y,t)*diff(u(x,y,t),x)+epsilon^2* diff(u(x,y,t),x$3),x)+ la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t)=\frac{-12c_2^4\epsilon^2\left(\tanh^2\left(c_4t+c_2x+c_3y+c_1\right)\right)+8c_2^4\epsilon^2-c_3^2\lambda-c_2c_4}{c_2^2}$$

2.15.17 Klein Gordon $u_{xx}+u_{yy}+\lambda u^p=0$

problem number 126

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Klein Gordon (nonlinear). Solve for $u(x,y)$

$$u_{xx}+u_{yy}+\lambda u^p=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + lambda*u[x, y]^p == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+lambda*u(x,y)^p=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

sol=()

2.15.18 Klein Gordon $u_{xx} + u_{yy} + u^2 = 0$

problem number 127

Added December 27, 2018.

Special case Klein Gordon (nonlinear). Solve for $u(x, y)$

$$u_{xx} + u_{yy} + u^2 = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + u[x, y]^2 == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+u(x,y)^2=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x, y) = -6\left(c_1^2 + c_2^2\right) \text{WeierstrassP}\left(c_1 x + c_2 y + 2c_3, 0, c_4\right)$$

2.15.19 Khokhlov Zabolotskaya $u_{xt} - (uu_x)_x = u_{yy}$

problem number 128

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Khokhlov Zabolotskaya. Solve for $u(x, y, t)$

$$u_{xt} - (uu_x)_x = u_{yy}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[D[u[x, y, t], x], t] - D[u[x, y, t]*D[u[x, y, t], x], x] == D[u[x, y, t], {y, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y,t),x,t)- diff( (u(x,y,t)* diff(u(x,y,t),x)) ,x ) = diff(u(x,y,t),y$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x, y, t) = \frac{c_1 c_3 - c_2^2 + \sqrt{c_1^2 c_3^2 - 2c_1 c_2^2 c_3 + c_2^4 + 2c_4 (c_3 t + c_1 x + c_2 y + c_4) c_1^2 + 2c_5 c_1^2}}{c_1^2}$$

2.15.20 Korteweg de Vries (KdV) $u_t + (u_x)^3 + 6uu_x = 0$

problem number 129

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Korteweg de Vries (KdV). Solve for $u(x, t)$

$$u_t + u_{xxx} + 6uu_x = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + D[u[x, t], {x,3}] + 6*u[x, t]*D[u[x, t], x] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{12{c_1}^3\tanh^2(c_2t+c_1x+c_3)-8{c_1}^3+c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+ diff( u(x,t),x$3 ) + 6 * u(x,t)* diff(u(x,t),x) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)+\frac{8c_2^3-c_3}{6c_2}$$

2.15.21 Lin Tsien $2u_{tx}+u_xu_{xx}-u_{yy}=0$

problem number 130

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Lin Tsien equation. Solve for $u(x,y,t)$

$$2u_{tx}+u_xu_{xx}-u_{yy}=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[u[x, y, t], t, x] + D[u[x, y, t], x]*D[u[x, y, t], {x, 2}] - D[u[x, y, t], {y, 2}
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=2*diff(u(x,y,t),t,x)+ diff(u(x,y,t),x)* diff(u(x,y,t),x$2) - diff(u(x,y,t),y$2) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = c_4 + c_5(c_3 t + c_1 x + c_2 y + c_4)$$

2.15.22 Liouville $u_{xx} + u_{yy} + e^{\lambda u} = 0$

problem number 131

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Liouville equation. Solve for $u(x,y)$

$$u_{xx} + u_{yy} + e^{\lambda u} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + Exp[lam*u[x, y]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)+exp(lambda*u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

2.15.23 Plateau $(1+u_y^2)u_{xx} - 2u_x u_y y_{xy} + (1+u_x^2)u_{yy} = 0$

problem number 132

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Plateau. Solve for $u(x,y)$

$$(1+u_y^2)u_{xx} - 2u_x u_y y_{xy} + (1+u_x^2)u_{yy} = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (1 + D[u[x, y], y]^2)*D[u[x, y], {x, 2}] - 2*D[u[x, y], x]*D[u[x, y], y]*D[u[x, y], x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(1+diff(u(x,y),y)^2)* diff(u(x,y),x$2)-2* diff(u(x,y),x)*
       diff(u(x,y),y)*diff(u(x,y),x,y)+
       (1+diff(u(x,y),x)^2)*diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = c_7\left(\tanh^3\left(c_1 + c_2(-iy+x)\right)\right) + c_5 \tanh\left(c_1 + c_2(-iy+x)\right) + c_4$$

2.15.24 Rayleigh $u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$

problem number 133

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Rayleigh. Solve for $u(x,t)$

$$u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] == epsilon*(D[u[x, t], t] - D[u[x, t], t]^3);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)-diff(u(x,t),x$2)=epsilon*(diff(u(x,t),t)-diff(u(x,t),t)^3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{x^2 \_c_1}{2} + c_1 x + c_2 + c_4 + \int \text{RootOf}\left(c_3 + t + \int^{\_Z} \frac{1}{\_f^3 \epsilon - \_f \epsilon - \_c_1} d\_f\right) dt$$

Has RootOf

2.15.25 Sawada Kotera

$$u_t + 45u^2 u_x + 15u_x u_{xx} + 15u u_{xxx} + u_{xxxxx} = 0$$

problem number 134

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sawada Kotera. Solve for $u(x,t)$

$$u_t + 45u^2 u_x + 15u_x u_{xx} + 15u u_{xxx} + u_{xxxxx} = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 45*u[x, t]^2*D[u[x, t], x] + 15*D[u[x, t], x]*D[u[x, t], {x, 2}] + 15
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{u(x,t)\to -\frac{4}{3}{c_1}^2\left(-2+3\tanh^2\left(-16{c_1}^5t+c_1x+c_3\right)\right)\right\}$$
$$\left\{u(x,t)\to \frac{-30{c_1}^{5/2}\tanh^2(c_2t+c_1x+c_3)+20{c_1}^{5/2}+\sqrt{20{c_1}^5-5c_2}}{15\sqrt{c_1}}\right\}$$
$$\left\{u(x,t)\to \frac{20{c_1}^{5/2}-\sqrt{20{c_1}^5-5c_2}}{15\sqrt{c_1}}-2{c_1}^2\tanh^2(c_2t+c_1x+c_3)\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)+45* u(x,t)^2* diff(u(x,t),x)+ 15* diff(u(x,t),x)*diff(u(x,t),x$2)+15*u
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,u(x,t)))
```

$$\{u(x,t)=c_4\}$$
$$\left\{u(x,t)=-4c_2^2\left(\tanh^2\left(-16c_2^5t+c_2x+c_1\right)\right)+\frac{8c_2^2}{3}\right\}$$
$$\left\{u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)-\frac{-20c_2^3+\sqrt{20c_2^6-5c_3c_2}}{15c_2}\right\}$$
$$\left\{u(x,t)=-2c_2^2\left(\tanh^2\left(c_3t+c_2x+c_1\right)\right)+\frac{20c_2^3+\sqrt{20c_2^6-5c_3c_2}}{15c_2}\right\}$$

2.15.26 Sine Gordon $\phi_{tt} - \phi_{xx} + \sin \phi = 0$

problem number 135

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sine Gordon. Solve for $u(x,t)$

$$\phi_{tt} - \phi_{xx} + \sin \phi = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[phi[x, t], {t, 2}] - D[phi[x, t], {x, 2}] + Sin[phi[x, t]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(phi(x,t),t$2)-diff(phi(x,t),x$2)+sin(phi(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,phi(x,t))),output='rea
```

sol=()

2.15.27 Sinh Gordon $u_{xt} = \sinh u$

problem number 136

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sinh Gordon. Solve for $u(x,t)$

$$u_{xt} = \sinh u$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], x, t] == Sinh[u[x, t]];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,t),x,t)=sinh(u(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

sol=()

2.15.28 Sinh Poisson $u_{xx} + u_{yy} + \sinh u = 0$

problem number 137

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Sinh Poisson. Solve for $u(x,t)$

$$u_{xx} + u_{yy} + \sinh u = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] + Sinh[u[x, y]] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+ sinh(u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

2.15.29 Thomas equation $u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$

problem number 138

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Thomas equation. Solve for $u(x,t)$

$$u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], x, y] + alpha*D[u[x, y], x] + beta*D[u[x, y], y] + nu*D[u[x, y], x]*D[u[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x,y),x,y)+alpha*diff(u(x,y),x)+beta*diff(u(x,y),y)
      +nu* diff(u(x,y),x)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \frac{-2\alpha y - 2\beta x - \ln\left(\frac{\alpha^2-2\alpha\beta+\beta^2-4\_c_1\nu}{\left(c_1 e^{(x-y)\sqrt{\alpha^2-2\alpha\beta+\beta^2-4\_c_1\nu}}-c_2\right)^2\nu^2}\right) - \ln\left(\frac{\alpha^2+2\alpha\beta+\beta^2-4\_c_1\nu}{\left(c_3 e^{(x+y)\sqrt{\alpha^2+2\alpha\beta+\beta^2-4\_c_1\nu}}-c_4\right)^2\nu^2}\right) + (}{2\nu}$$

2.15.30 phi equation $\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$

problem number 139

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

phi equation. Solve for $phi(x,t)$

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[phi[x, t], t, t] - D[phi[x, t], x, x] - phi[x, t] + phi[x, t]^3 == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

$$\left\{\phi(x,t) \to -\tanh\left(c_2 t - \sqrt{\frac{1}{2} + {c_2}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to \tanh\left(c_2 t - \sqrt{\frac{1}{2} + {c_2}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to -\tanh\left(c_2 t + \sqrt{\frac{1}{2} + {c_2}^2} x + c_3\right)\right\}$$
$$\left\{\phi(x,t) \to \tanh\left(c_2 t + \sqrt{\frac{1}{2} + {c_2}^2} x + c_3\right)\right\}$$

Maple ✓

```
restart;
pde := diff(phi(x,t),t$2)-diff(phi(x,t),x$2) - phi(x,t) + phi(x,t)^3=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',PDEtools:-TWSolutions(pde,phi(x,t)
```

$$\{\phi(x,t) = -1\}$$
$$\{\phi(x,t) = 1\}$$
$$\left\{\phi(x,t) = -\tanh\left(c_2 x + c_1 - \frac{\sqrt{4c_2^2 - 2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t) = \tanh\left(c_2 x + c_1 + \frac{\sqrt{4c_2^2 - 2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t) = \tanh\left(c_2 x + c_1 - \frac{\sqrt{4c_2^2 - 2}\,t}{2}\right)\right\}$$
$$\left\{\phi(x,t) = -\tanh\left(c_2 x + c_1 + \frac{\sqrt{4c_2^2 - 2}\,t}{2}\right)\right\}$$

2.16 more miscellaneous

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2.16.1 $SS_{xy} + S_x S_y = 1$

problem number 140

Taken from Maple pdsolve help pages, problem 4. A second order PDE

Solve for $S(x, y)$

$$SS_{xy} + S_x S_y = 1$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  s[x, y]*D[s[x, y], x, y] + D[s[x, y], x]*D[s[x, y], y] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, s[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := S(x,y)*diff(S(x,y),y,x) + diff(S(x,y),x)*diff(S(x,y),y) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,S(x,y),'build')),outpu
```

$$S(x, y) = \text{RootOf}\left(\_Z^2 - 2xy - 2\_F1(y) - \_F2(x)\right)$$

2.16.2 $u_{rr} + u_{\theta\theta} = 0$

problem number 141

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Second order PDE in Polar coordinates. Solve for $u(r, \theta)$

$$u_{rr} + u_{\theta\theta} = 0$$

With boundary conditions

$$u(2, \theta) = 3\sin(2\theta) + 1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + D[u[r, theta], {theta, 2}] == 0;
bc  = u[2, theta] == 3*Sin[2*theta] + 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta) \to \boxed{\begin{array}{lll} \{ & \begin{array}{cc} 6e^{4-2r}\cos(\theta)\sin(\theta)+1 & r \geq 2 \\ \text{Indeterminate} & \text{True} \end{array} & \text{if } r \neq 2 \end{array}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2)+diff(u(r, theta), theta$2) = 0;
bc  := u(2, theta) = 3*sin(2*theta)+1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r,theta),meth
```

$$u(r,\theta) = -\frac{3ie^{2i\theta-2r+4}}{2} + \frac{3ie^{-2i\theta+2r-4}}{2} + 1$$

2.16.3 $u_{xx} + yu_{yy} = 0$

problem number 142

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Laplace like PDE with polynomial solution. Solve for $u(x, y)$

$$u_{xx} + yu_{yy} = 0$$

With boundary conditions

$$u(x, 0) = 0$$
$$\frac{\partial u}{\partial y}(x, 0) = x^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to -y(y - x^2)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+y*(diff(u(x, y), y$2)) = 0;
bc  := u(x,0)=0, eval(diff(u(x,y),y),y=0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(x, y))),outpu
```

$$u(x, y) = (x^2 - y)\, y$$

2.16.4 $u_t + u_{xxx} = 0$

problem number 143

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Third oder PDE. Solve for $u(x, y)$

$$u_t + u_{xxx} = 0$$

With initial conditions

$$u(x, 0) = f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == -D[u[x, t], {x, 3}];
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{\int_{-\infty}^{\infty} e^{iK[1](tK[1]^2+x)} \int_{-\infty}^{\infty} e^{-ixK[1]} f(x)\, dx dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=- diff(u(x, t), x$3);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t))),output=
```

$$u(x,t) = \frac{\int_{-\infty}^{\infty} \frac{4\pi\sqrt{-\frac{x+\zeta}{(-t)^{\frac{1}{3}}}}\,\mathrm{BesselK}\left(\frac{1}{3}, \frac{2\sqrt{3}\left(-\frac{x+\zeta}{(-t)^{\frac{1}{3}}}\right)^{\frac{3}{2}}}{9}\right) f(-\zeta)}{3(-t)^{\frac{1}{3}}} d\zeta}{4\pi^2}$$

2.16.5 $u_{xy} = \sin(x)\sin(y)$

problem number 144

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

PDE solved by Laplace transform. Solve for $u(x, y)$

$$u_{xy} = \sin(x)\sin(y)$$

With boundary conditions

$$u(x, 0) = 1 + \cos(x)$$
$$\frac{\partial u}{\partial y}(0, y) = -2\sin y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], y, x] == Sin[x]*Sin[y];
bc  = {u[x, 0] == 1 + Cos[x], Derivative[0, 1][u][0, y] == -2*Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

$$\{\{u(x, y) \to (\cos(x) + 1)\cos(y)\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), y,x)=sin(x)*sin(y);
bc  := u(x,0)=1+cos(x),eval( diff(u(x,y),y),x=0)=-2*sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y))),output=
```

$$u(x, y) = (\cos(x) + 1)\cos(y)$$

2.16.6 $w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3}$

problem number 145

Added December 20, 2018.

Example 25, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Linear PDE, initial conditions at $t = 1$. Solve for $w(x_1, x_2, x_3, t)$

$$w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3}$$

With initial condition $w(x_1, x_2, x_3, 1) = e^a x_1^2 + x_2x_3$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], {x1, 2}] + D[w[x1, x2, x3, t], {x2, 2}]
ic  = w[x1, x2, x3, 1] == Exp[a]*x1^2 + x2*x3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\{\{w(\text{x1},\text{x2},\text{x3},t) \to e^a(2t + \text{x1}^2 - 2) + \text{x2x3}\}\}$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t) = diff(w(x1, x2, x3, t), x1$2)+diff(w(x1, x2, x3, t), x2$2)
ic  := w(x1, x2, x3, 1) = exp(a)*x1^2+x2*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = x2x3 + (x1^2 + 2t - 2)\,\mathrm{e}^a$$

2.16.7 Linear PDE, initial conditions at $t = t_0$

problem number 146

Added December 20, 2018.

Example 26, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w}{\partial t} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} + \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition $w(x_1, x_2, x_3, t_0) = e^{x_1} + x_2 - 3x_3$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1, x3] +
ic  = w[x1, x2, x3, t0] == Exp[x1] + x2 - 3*x3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\{\{w(\mathrm{x1}, \mathrm{x2}, \mathrm{x3}, t) \to e^{\mathrm{x1}} + \mathrm{x2} - 3\mathrm{x3}\}\}$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff(w
ic  := w(x1, x2, x3, t0) = exp(x1)+x2-3*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = x2 - 3x3 + \mathrm{e}^{x1}$$

2.16.8 second order in time, Linear PDE, initial conditions at $t = t_0$

problem number 147

Added December 20, 2018.

Example 27, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w^2}{\partial t^2} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} - \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition

$$w(x_1, x_2, x_3, t_0) = x_1^3 x_2^2 + x_3$$
$$\frac{\partial w}{\partial t}(x_1, x_2, x_3, t_0) = -x_2 x_3 + x_1$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x1, x2, x3, t], {t, 2}] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1,
ic  = {w[x1, x2, x3, t0] == x1^3*x2^2 + x3, Derivative[0, 0, 0, 1][w][x1, x2, x3, t0] == -(x
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

$$\left\{\left\{w(\mathrm{x1}, \mathrm{x2}, \mathrm{x3}, t) \to t^2\left(3\mathrm{t0}^2\mathrm{x1} - \frac{\mathrm{t0}}{2} + 3\mathrm{x1}^2\mathrm{x2}\right) + t^3\left(\frac{1}{6} - 2\mathrm{t0x1}\right) + \frac{t^4\mathrm{x1}}{2} + t\left(-2\mathrm{t0}^3\mathrm{x1} + \frac{\mathrm{t0}^2}{2} - 6\mathrm{t0x1}^2\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x1, x2, x3, t), t$2)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff
ic  := w(x1, x2, x3, t0) = x1^3*x2^2+x3, eval( diff( w(x1,x2,x3,t),t),t=t0)=-x2*x3+x1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = 3t^2 x1^2 x2 + \frac{t0^4 x1}{2} + x1^3 x2^2 + \frac{t^3}{6} - tx2x3 + \frac{(-12x1t - 1)\, t0^3}{6} + \frac{(18t^2 x1 + 18x1^2 x2 + 3t)\, t0}{6}$$

2.16.9 Einstein-Weiner $u_t = -\beta u_x + D u_{xx}$

problem number 148

Added January 2, 2018.

Einstein-Weiner PDE. Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_t = -\beta u_x + D u_{xx}$$

Assuming $\beta > 0, D > 0$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == beta*D[u[x, t], x] + d*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}, Assumptions -> {beta > 0,
```

$$\{\{u(x,t) \to \cosh(c_2(\beta t + c_2 dt + x) + c_1) + \sinh(c_2(\beta t + c_2 dt + x) + c_1) + 1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=-beta*diff(u(x,t),x)+d*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t) = c_3\left(c_1 e^{\frac{\sqrt{\beta^2-4d\_c_1}\,x}{2d}} + c_2 e^{-\frac{\sqrt{\beta^2-4d\_c_1}\,x}{2d}}\right)\sqrt{e^{\frac{\beta x}{d}}}\, e^{-t\_c_1}$$

2.16.10 Using integral transforms.

problem number 149

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$x^2 \frac{\partial^2}{\partial x^2} u(x,y) + x \frac{\partial}{\partial x} u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = 0$$

With boundary conditions

$$u(x,1) = \begin{cases} 1 & 0 \le x \text{ and } x < 1 \\ 0 & 1 < x \end{cases}$$

$$u(x,0) = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x^2*D[u[x, y], {x, 2}] + x*D[u[x, y], x] + D[u[x, y], {y, 2}] ==0;
bc = {u[x, 0] == 0, u[x, 1] == Piecewise[{{1, 0 <= x < 1}, {0, 1 < x}}]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to \frac{\text{Integrate}\left[\frac{x^{-\frac{149}{33}-iK[1]}\csc\left(\frac{149}{33}+iK[1]\right)\sin\left(y\left(\frac{149}{33}+iK[1]\right)\right)}{\frac{149}{33}+iK[1]},\{K[1],-\infty,\infty\},\text{Assumptions}\to K[1]\in\mathbb{R}\right.}{2\pi}\right.\right.$$

Maple ✓

```
restart;
pde := x^2*diff(u(x, y), x, x) + x*diff(u(x, y), x) + diff(u(x, y), y, y) = 0;
iv := u(x, 0) = 0, u(x, 1) = piecewise(0 <= x and x < 1, 1, 1 < x, 0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(x,y))),output='
```

$$u(x,y)=$$

CHAPTER 3

Parabolic PDE's (Diffusion)

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3.1 Diffusion in 1D

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3.1.1.47 [196] Mixed BC 532
3.1.1.48 [197] Haberman 8.2.1 (f) 533

3.1.1.1 [150] General initial conditions

problem number 150

Added June 9, 2019

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

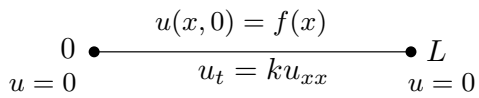

Figure 3.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. {K[1] -> n}
```

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Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(L,t) = 0;
ic  := u(x,0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

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<u>Hand solution</u>

Solve

$$u_t = ku_{xx}$$

BC

$$\begin{aligned} u(0,t) &= 0 \qquad t > 0 \\ u(L,t) &= 0 \qquad t > 0 \end{aligned}$$

Initial conditions

$$u(x,0) = f(x) \qquad 0 < x < L$$

Solution

Using separation of variables, let $u(x,t) = X(x)\,T(t)$. Substituting this back into the PDE gives

$$\begin{aligned} T'X &= kX''T \\ \frac{1}{k}\frac{T'}{T} &= \frac{X''}{X} = -\lambda \end{aligned}$$

Where the separation constant is some real value $-\lambda$. This gives the following two ODE's to solve

$$T' + \lambda kT = 0 \tag{1}$$
$$X'' + \lambda X = 0 \tag{2}$$

Starting with the spatial ODE in order to obtain the eigenvalues and eigenfunctions. The boundary conditions on the spatial ODE become

$$\begin{aligned} X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

There are three cases to consider, depending on if $\lambda < 0, \lambda = 0, \lambda > 0$. Each one of these cases give a different solution.

Case 1 Assuming $\lambda < 0$. Therefore $-\lambda$ is positive say μ. The solution (2) can now be written as

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) + c_2 \sinh\left(\sqrt{\mu}x\right) \tag{3B}$$

Applying left boundary conditions to (3B) gives

$$0 = c_1$$

The solution (3B) now reduces to

$$X(x) = c_2 \sinh\left(\sqrt{\mu}x\right)$$

Applying right side boundary conditions to the above results in

$$0 = c_2 \sinh\left(\sqrt{\mu}L\right)$$

But $\sinh\left(\sqrt{\mu}L\right) \neq 0$ since it was assumed μ is not zero and sinh is only zero when its argument is zero. The only possibility then is that $c_2 = 0$, which leads to trivial solution. Therefore $\lambda < 0$ is not an eigenvalue.

Case 2. Assuming $\lambda = 0$. The ODE becomes $X'' = 0$, which has the solution

$$X(x) = c_1 x + c_2$$

Applying left side B.C. gives

$$0 = c_2$$

The solution now reduces to

$$X(x) = c_1 x$$

Applying right side B.C. gives

$$0 = c_1 L$$

Leading to the trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case 3 Assuming $\lambda > 0$. In this case equation $\sqrt{-\lambda}$ is complex and solution to equation (2) can be expressed in terms of trig functions using Euler relation which results in

$$X(x) = c_1 \cos\left(\sqrt{\lambda} x\right) + c_2 \sin\left(\sqrt{\lambda} x\right) \tag{4}$$

Applying left side B.C. gives

$$0 = c_1$$

Solution (4) now reduces to

$$X(x) = C_2 \sin\left(\sqrt{\lambda} x\right) \tag{5}$$

Applying right side B.C. gives

$$0 = C_2 \sin\left(\sqrt{\lambda} L\right)$$

Non-trivial solution implies $\sin\left(\sqrt{\lambda} L\right) = 0$ or $\sqrt{\lambda} L = n\pi$ for $n = 1, 2, 3, \cdots$. Therefore the eigenvalues are

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunctions from (5) are

$$X_n(x) = C_n \sin\left(\sqrt{\lambda_n} x\right) \tag{6}$$

Now that the eigenvalues are known, the solution to the time ODE (1) can be found.

$$T' + \lambda_n k T = 0$$

This has the solution (using an integrating factor method)

$$T_n(t) = e^{-\lambda_n k t} \tag{7}$$

The constant of integration is not needed for (7) since it will be absorbed with the constant of integration coming from solution of the spatial ODE (6) when these solutions are multiplied with each others below. Therefore the fundamental solution is

$$u_n(x,t) = T_n(t)\, X_n(x)$$

Linear combination of fundamental solutions is also a solution (since this is a linear PDE). Therefore the general solution is given by

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} u_n \\ &= \sum_{n=1}^{\infty} T_n(t)\, X_n(x) \\ &= \sum_{n=1}^{\infty} C_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \end{aligned} \tag{8}$$

The initial condition is now used to determine c_n, . At $t=0$, $u(x,0)=f(x)$ and the above becomes

$$f(x) = \sum_{n=1}^{\infty} C_n \sin\left(\sqrt{\lambda_n} x\right)$$

Multiplying both sides of the above equation by eigenfunction $\sin\left(\sqrt{\lambda_m} x\right)$ and integrating over the domain of $f(x)$ gives

$$\int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx = \int_0^L \sum_{n=1}^{\infty} C_n \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

Interchanging the order of summation and integration gives

$$\int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx = \sum_{n=1}^{\infty} C_n \int_0^L \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

By the orthogonality of the sine functions, all terms in the right side vanish except when $n=m$, leading to

$$\begin{aligned} \int_0^L f(x) \sin\left(\sqrt{\lambda_m} x\right) dx &= C_m \int_0^L \sin^2\left(\sqrt{\lambda_m} x\right) dx \\ &= C_m \frac{L}{2} \end{aligned}$$

Therefore (replacing m back to n now, since it is arbitrary)

$$C_n = \frac{2}{L} \int_0^L f(x) \sin\left(\sqrt{\lambda_n} x\right) dx \qquad n = 1, 2, 3, \cdots$$

Summary of solution

$$u(x,t) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi}{L}\right)^2 \lambda_n t} \sin\left(\frac{n\pi}{L}x\right)$$

3.1.1.2 [151] Specific initial condition

problem number 151

Added June 20, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= 0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$ using the following specific values

$$\begin{aligned} f(x) &= 100 \\ k &= \frac{1}{100} \\ L &= 10 \end{aligned}$$

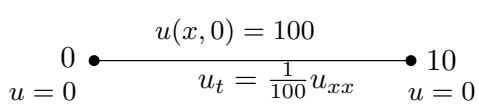

Figure 3.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
 L=10;
 f=100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n}
```

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Maple ✓

```
restart;
k:=1/100;
L:=10;
f:=100;
pde := diff(u(x,t), t) = k*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(L,t) = 0;
ic  := u(x,0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{200((-1)^n - 1)\,\mathrm{e}^{-\frac{\pi^2 n^2 t}{10000}} \sin\left(\frac{\pi n x}{10}\right)}{\pi n}$$

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi}{L}\right)^2 \lambda_n t} \sin\left(\frac{n\pi}{L}x\right)$$

$$C_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx \qquad n = 1, 2, 3, \cdots$$

In this case $L = 10, k = \frac{1}{100}, f(x) = 100$. Hence the solution becomes

$$\begin{aligned} C_n &= \frac{2}{10}\int_0^{10} 100 \sin\left(\frac{n\pi}{10}x\right) dx \\ &= \frac{200}{\pi n}(1 - \cos(n\pi)) \\ &= \frac{200}{\pi n}\left(1 + (-1)^{n+1}\right) \end{aligned}$$

Hence

$$u(x,t) = \frac{200}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left(1 + (-1)^{n+1}\right) e^{-\left(\frac{n\pi}{10}\right)^2 \lambda_n t} \sin\left(\frac{n\pi}{10}x\right)$$

The following is an animation of the solution

Out[ ]=

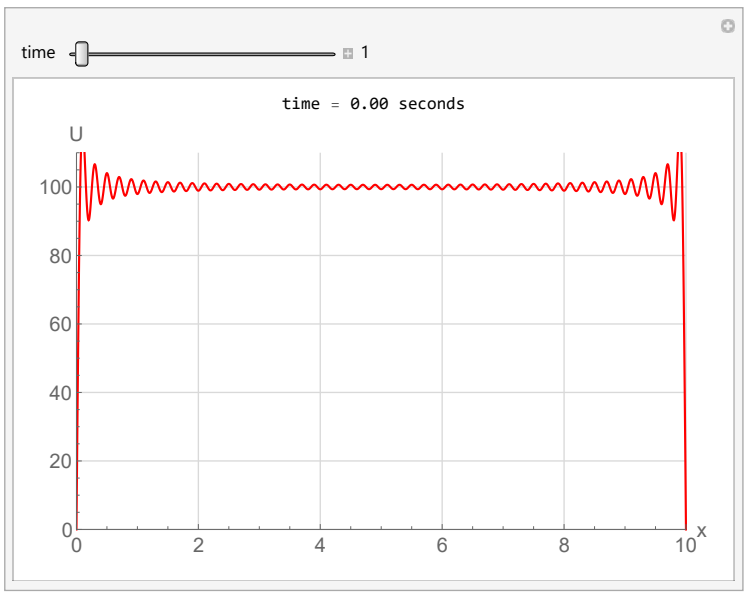

Figure 3.3: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, y, t, n, k]
L = 10;
k = 1 / 100;
f = 100;
numberOfTerms = 100;
mySol[x_, t_] = 200/π * Sum[1/n (1 + (-1)^(n+1)) Exp[- (n π/10)^2 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 3.4: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {2, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {0, 110}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 50, 0.1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.5: Code for animation

3.1.1.3 [152] IC $u = x(1-x)$

problem number 152

Added June 11, 2019

Solve the heat equation

$$u_t = \frac{1}{100} u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = x(1-x)$

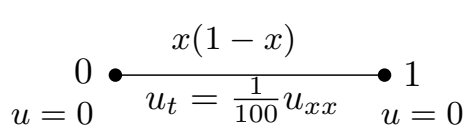

Figure 3.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 1/100*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == x*(1-x);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. {K[1] -> n}
```

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Maple ✓

```
restart;
pde := diff(u(x,t), t) = 1/100*diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(1,t) = 0;
ic  := u(x,0) = x*(1-x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{4((-1)^n - 1)\,\mathrm{e}^{-\frac{\pi^2 n^2 t}{100}} \sin(\pi n x)}{\pi^3 n^3}$$

Hand solution

Solve $u_t = \frac{1}{100} u_{xx}$ with $0 < x < 1$ and initial conditions $u(x,0) = x(1-x)$

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$ and $\sin\left(\sqrt{\lambda_n} x\right)$ are the eigenfunctions. Here $L = 1$ and $k = \frac{1}{100}$. Hence the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k(n\pi)^2 t} \sin(n\pi x)$$

At $t = 0$

$$x(1-x) = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

Multiplying both sides by $\sin(m\pi x)$ and integrating gives

$$\int_0^1 x(1-x)\sin(m\pi x)\,dx = \int_0^1 \sum_{n=1}^{\infty} B_n \sin(n\pi x)\sin(m\pi x)\,dx$$

By orthogonality of sin function the above simplifies to

$$\int_0^1 x(1-x)\sin(n\pi x)\,dx = B_n \int_0^1 \sin^2(n\pi x)\,dx$$
$$\int_0^1 x(1-x)\sin(n\pi x)\,dx = \frac{1}{2}B_n$$

But $\int_0^1 x(1-x)\sin(n\pi x)\,dx = \frac{2\left(1+(-1)^{n+1}\right)}{n^3\pi^3}$. Hence

$$B_n = \frac{4\left(1+(-1)^{n+1}\right)}{n^3\pi^3}$$

And the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4\left(1+(-1)^{n+1}\right)}{n^3\pi^3} e^{-k(n\pi)^2 t}\sin(n\pi x)$$

This is animation of the solution for 30 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
In[•]:= ClearAll[x, t, n, lam]
L0 = 1;
k = 1 / 100;
pde = D[u[x, t], {t, 1}] == k * D[u[x, t], {x, 2}];
ic = u[x, 0] == x (1 - x);
bc = {u[0, t] == 0, u[L0, t] == 0};
maxTime = 30;
lam[n_] := (n Pi / L0) ^2;
numberOfTerms = 10;
mySol = Sum[4 (1 + (-1)^(n+1)) / (n^3 π^3) Exp[- k lam[n] t] Sin[Sqrt[lam[n]] x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[ Evaluate[Quiet[N[mySol /. {x → x0, t → time}]]], {x0, 0, 1},
      PlotLabel → Row[{t, " sec,", " U at center=",
            padIt2@Evaluate[Quiet[N[mySol /. {x → 1 / 2, t → time}]], {3, 2}]}],
      PlotRange → {{0, 1}, {0, .3}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
    ]
  ],
  {{time, 0, "time"}, 0, 35, .01, Appearance → "Labeled"}]
```

```
In[•]:= ClearAll[x, t];
r = Table[
      Plot[ Evaluate[Quiet[mySol /. {x → x0, t → time}]], {x0, 0, 1},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " Temperature at center = ",
              padIt2@Evaluate[Quiet[mySol /. {x → 1 / 2, t → time}], {4, 3}]}],
        PlotRange → {{0, 1}, {0, .3}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
      ],
      {time, 0, 30, .08}];
```

```
In[•]:= Export["anim.gif", r, "DisplayDurations" → Table[0.04, {Length@r}]]
```

Figure 3.7: Source code

3.1.1.4 [153] Haberman 2.3.3 (a)

problem number 153

This is problem 2.3.3, part (a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 6\sin\left(\frac{9\pi x}{L}\right)$

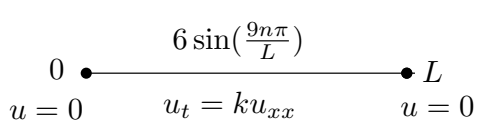

Figure 3.8: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
NumericQ[L] =. ;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 6*Sin[(9*Pi*x)/L];
NumericQ[L] = True;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t)\to 6e^{-\frac{81\pi^2 kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=6*sin(9*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=6\,\mathrm{e}^{-\frac{81\pi^2 kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)$$

<u>Hand solution</u>

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x,0) = 6\sin\left(\frac{9\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t)=\sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. At $t = 0$

$$6\sin\left(\frac{9\pi x}{L}\right)=\sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi}{L}x\right)$$

For $n = 9$

$$6\sin\left(\frac{9\pi x}{L}\right)=B_9\sin\left(\frac{9\pi}{L}x\right)$$

Hence $B_9 = 6$ and all other terms are zero. Therefore the solution is

$$\begin{aligned} u(x,t) &= B_9 \sin\left(\sqrt{\lambda_9}x\right) e^{-k\lambda_9 t} \\ &= 6\sin\left(\frac{9\pi}{L}x\right) e^{-k\left(\frac{9\pi}{L}\right)^2 t} \\ &= 6\sin\left(\frac{9\pi}{L}x\right) e^{-k\frac{81\pi^2}{L^2}t} \end{aligned}$$

3.1.1.5 [154] IC hat function

problem number 154

Added Feb 10, 2018.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(40,t) &= 0 \end{aligned}$$

Initial condition is

$$u(x,0) = \begin{cases} x & 0 \leq x < 20 \\ 40 - x & 20 \geq x \leq 40 \end{cases}$$

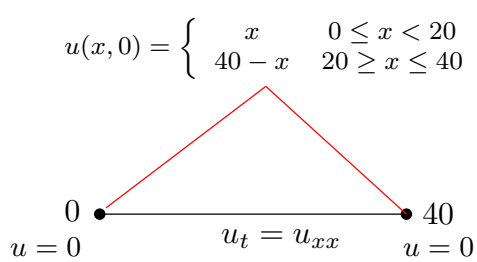

Figure 3.9: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f = Piecewise[{{x, Inequality[0, LessEqual, x, Less, 20]}, {40 - x, 20 <= x <= 40}}];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[40, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

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Maple ✓

```
restart;
interface(showassumed=0);
f:=piecewise(0<x and x<20,x,20<x and x<40,(40-x));
pde := diff(u(x,t),t)=diff(u(x,t),x$2);
ic  := u(x,0)=f;
bc  := u(0,t)=0,u(40,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{160\,\mathrm{e}^{-\frac{\pi^2 n^2 t}{1600}}\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi n x}{40}\right)}{\pi^2 n^2}$$

<u>Hand solution</u>

Solve $u_t = k u_{xx}$ with $0 < x < 40$ and initial conditions $u(x,0) = f(x)$ as hat function.

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t)=\sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Here $L = 40$ and $k = 1$. Hence the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{40}\right)^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{40}x\right)$$

Multiplying both sides by $\sin(m\pi x)$ and integrating gives

$$\int_0^{40} f(x)\sin\left(\frac{m\pi}{40}x\right)dx = \int_0^{40}\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{40}x\right)\sin\left(\frac{m\pi}{40}x\right)dx$$

By orthogonality of sin function the above simplifies to

$$\int_0^{40} f(x)\sin\left(\frac{n\pi}{40}x\right)dx = B_n\int_0^{40}\sin^2\left(\frac{n\pi}{40}x\right)dx$$

$$\int_0^{20} x\sin\left(\frac{n\pi}{40}x\right)dx + \int_{20}^{40}(40-x)\sin\left(\frac{n\pi}{40}x\right)dx = 20B_n$$

But $\int_0^{20} x\sin\left(\frac{n\pi}{40}x\right)dx + \int_{20}^{40}(40-x)\sin\left(\frac{n\pi}{40}x\right)dx = \frac{3200}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$. Hence

$$B_n = \frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

And the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty}\frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)e^{-\left(\frac{n\pi}{40}\right)^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

When n is even the terms are zero Hence

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty}\frac{160}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)e^{-\left(\frac{n\pi}{40}\right)^2 t}\sin\left(\frac{n\pi}{40}x\right)$$

Or

$$u(x,t) = \sum_{n=1}^{\infty}\frac{160}{(2n-1)^2\pi^2}\sin\left(\frac{(2n-1)\pi}{2}\right)e^{-\left(\frac{(2n-1)\pi}{40}\right)^2 t}\sin\left(\frac{(2n-1)\pi}{40}x\right)$$

This is animation of the solution for 400 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
ClearAll[x, t, n, lam]
L0 = 40;
k = 1;
pde = D[u[x, t], {t, 1}] == k*D[u[x, t], {x, 2}];

lam[n_] := ((2 n - 1) π / L0);
numberOfTerms = 15;
mySol = Sum[160/((2 n - 1)^2 π^2) Sin[(2 n - 1) π/2] Exp[- k lam[n]^2 t] Sin[lam[n] x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[ Quiet[mySol /. {x → x0, t → time}], {x0, 0, L0},
      PlotLabel → Row[{padIt2[t, {4, 2}], " sec,", " U at center=",
            padIt2@Evaluate[Quiet[N[mySol /. {x → 20, t → time}]], {4, 3}]}],
      PlotRange → {{0, L0}, {0, 20}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
    ]
  ],
  {{time, 0, "time"}, 0, 400, .01, Appearance → "Labeled"}]
```

```
In[•]:= ClearAll[x, t];
r = Table[
      Plot[Quiet[mySol /. {x → x0, t → time}], {x0, 0, L0},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " Temperature at center = ",
              padIt2@Evaluate[Quiet[mySol /. {x → L0/2, t → time}], {4, 3}]}],
        PlotRange → {{0, L0}, {0, L0/2}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
      ],
      {time, 0, 400, 1.0}];
```

```
In[•]:= Export["anim.gif", r, "DisplayDurations" → Table[0.01, {Length@r}]]
```

Figure 3.10: Source code

3.1.1.6 [155] Haberman 2.3.3 (b)

problem number 155

This is problem 2.3.3, part (b) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

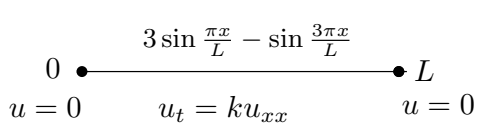

Figure 3.11: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
NumericQ[L] =. ;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 3*Sin[(Pi*x)/L] - Sin[(3*Pi*x)/L];
NumericQ[L] = True;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t)\to e^{-\frac{9\pi^2 kt}{L^2}}\sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2 kt}{L^2}}-2\cos\left(\frac{2\pi x}{L}\right)-1\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=-\mathrm{e}^{-\frac{9\pi^2 kt}{L^2}}\sin\left(\frac{3\pi x}{L}\right)+3\,\mathrm{e}^{-\frac{\pi^2 kt}{L^2}}\sin\left(\frac{\pi x}{L}\right)$$

<u>Hand solution</u>

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x,0) = 3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t)=\sum_{n=1}^{\infty}B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where the eigenvalues are $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions.

Initial conditions are now applied. Setting $t = 0$, the above becomes

$$u(x,0)=3\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)$$

As the series is unique, the terms coefficients must match for those shown only, and all other B_n terms vanish. This means that by comparing terms

$$3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right) = B_1 \sin\left(\frac{\pi x}{L}\right) + B_3 \sin\left(\frac{3\pi}{L}x\right)$$

Therefore

$$\begin{aligned} B_1 &= 3 \\ B_3 &= -1 \end{aligned}$$

And all other $B_n = 0$. The solution is

$$u(x,t) = 3\sin\left(\frac{\pi}{L}x\right) e^{-k\left(\frac{\pi}{L}\right)^2 t} - \sin\left(\frac{3\pi}{L}x\right) e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

3.1.1.7 [156] Haberman 2.3.3 (c)

problem number 156

This is problem 2.3.3, part (c) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 2\cos\frac{3\pi x}{L}$

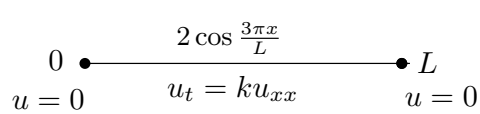

Figure 3.12: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == 2*Cos[(3*Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol/. K[1]->n;
```

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but $n = 3$ should be special case

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=2*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

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handled $n = 3$ case correctly.
Hand solution

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x, 0) = 2\cos\left(\frac{3\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$ and $\sin\left(\sqrt{\lambda_n} x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t = 0$, the above becomes

$$u(x,0) = 2\cos\left(\frac{3\pi}{L}x\right) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating

$$\begin{aligned}\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= \sum_{n=1}^{\infty} B_n \int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\end{aligned}$$

By orthogonality of sin functions the above simplifies to

$$\begin{aligned}\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= B_m\int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx\\ &= B_m\frac{L}{2}\\ B_m &= \frac{4}{L}\int_0^L \cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\end{aligned}$$

For $m=3, B_3=0$. For $m\neq 3$

$$\begin{aligned}B_m &= \frac{4}{L}\left(\frac{1+(-1)^m}{m^2-9}\frac{nL}{\pi}\right)\\ &= \frac{4n}{\pi}\left(\frac{1+(-1)^m}{m^2-9}\right)\end{aligned}$$

Hence the solution becomes

$$\begin{aligned}u(x,t) &= \sum_{n=1,n\neq 3}^{\infty}\frac{4n}{\pi}\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\\ &= \frac{4}{\pi}\sum_{n=1,n\neq 3}^{\infty} n\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\end{aligned}$$

When n is odd, all terms become zero, hence the above can be also be written as

$$u(x,t) = \frac{8}{\pi}\sum_{n=2,4,6,\cdots}^{\infty}\left(\frac{n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

3.1.1.8 [157] Haberman 2.3.3 (d)

problem number 157

This is problem 2.3.3, part (d) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = \begin{cases} 1 & 0 < x \leq \frac{L}{2} \\ 2 & \frac{L}{2} < x \leq L \end{cases}$

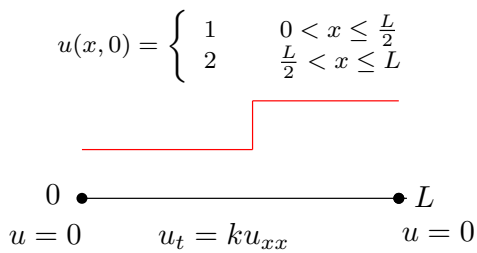

Figure 3.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == Piecewise[{{1, Inequality[0, Less, x, LessEqual, L/2]}, {2, L/2 < x < L}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

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Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{\left(-4(-1)^{n}+2\cos\left(\frac{\pi n}{2}\right)+2\right)\mathrm{e}^{-\frac{\pi^{2}kn^{2}t}{L^{2}}}\sin\left(\frac{\pi nx}{L}\right)}{\pi n}$$

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(x,t)=\sum_{n=1}^{\infty}B_{n}e^{-k\lambda_{n}t}\sin\left(\sqrt{\lambda_{n}}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t=0$, the above becomes

$$f(x)=\sum_{n=1}^{\infty}B_{n}\sin\left(\frac{n\pi}{L}x\right) \tag{3}$$

Where

$$f(x)=\begin{cases}1 & 0<x\leq\frac{L}{2}\\ 2 & \frac{L}{2}<x<L\end{cases}$$

Multiplying both sides of (3) by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating over the domain gives

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\int_{0}^{L}\left[\sum_{n=1}^{\infty}B_{n}\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)\right]dx$$

Interchanging the order of integration and summation

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\sum_{n=1}^{\infty}\left[B_{n}\left(\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx\right)\right]$$

But $\int_0^L\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx=0$ for $n\neq m$, hence only one term survives

$$\int_{0}^{L}\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=B_{m}\int_{0}^{L}\sin^{2}\left(\frac{m\pi}{L}x\right)dx$$

Renaming m back to n and since $\int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$ the above becomes

$$\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx = \frac{L}{2}B_n$$

$$\begin{aligned}
B_n &= \frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx + \int_{\frac{L}{2}}^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx\right) \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) dx + 2\int_{\frac{L}{2}}^L \sin\left(\frac{n\pi}{L}x\right) dx\right) \\
&= \frac{2}{L}\left(\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\Bigg|_0^{\frac{L}{2}} + 2\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\Bigg|_{\frac{L}{2}}^L\right) \\
&= \frac{2}{n\pi}\left(\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_0^{\frac{L}{2}} + 2\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_{\frac{L}{2}}^L\right) \\
&= \frac{2}{n\pi}\left(\left[-\cos\left(\frac{n\pi}{L}\frac{L}{2}\right) + \cos(0)\right] + 2\left[-\cos(n\pi) + \cos\left(\frac{n\pi}{2}\right)\right]\right) \\
&= \frac{2}{n\pi}\left(-\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi) + 2\cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi)\right)
\end{aligned}$$

Hence the solution is

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

With

$$\begin{aligned}
B_n &= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - 2\cos(n\pi) + 1\right) \\
&= \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(1 + 2(-1)^{n+1} + \cos\left(\frac{n\pi}{2}\right)\right)
\end{aligned}$$

Therefore

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

3.1.1.9 [158] domain from -1 to +1

problem number 158

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = f(x)$

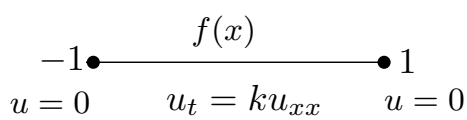

Figure 3.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[-1, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

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Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic  := u(x,0) = f(x);
bc  := u(-1,t)=0,  u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\int_{-1}^{1} f(x) \sin\left(\frac{\pi(x+1)\,n}{2}\right) dx \right) \mathrm{e}^{-\frac{\pi^2 n^2 t}{4}} \sin\left(\frac{\pi(x+1)\,n}{2}\right)$$

Hand solution

Solve $u_t = ku_{xx}$ with $-1 < x < 1$ and initial conditions $u(x,0) = f(x)$ and B.C. $u(-1,t) = u(1,t) = 0$.

Let $\xi = x + 1$ where $x = -1 \cdots 1$ Therefore $\xi = 0 \cdots 2$. This also implies mapping the initial temperature to $f(x+a)$. Now the ODE is solved

$$\begin{aligned} u_t &= u_{\xi\xi} \\ u(0,t) &= 0 \\ u(2,t) &= 0 \end{aligned}$$

The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$u(\xi,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}\xi\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}\xi\right)$ are the eigenfunctions. Here $L = 2$. Hence the solution becomes

$$u(\xi,t) = \sum_{n=1}^{\infty} B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}\xi\right)$$

Changing back to x

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}(x+1)\right)$$

At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} B_n e^{-k\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}(x+1)\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2}(x+1)\right)$ and integrating gives

$$\int_{-1}^{1} f(x) \sin\left(\frac{m\pi}{2}(x+1)\right) dx = \int_{-1}^{1} \sum_{n=1}^{\infty} B_n \sin\left(\frac{m\pi}{2}(x+1)\right) \sin\left(\frac{n\pi}{2}(x+1)\right) dx$$

By orthogonality of sin function the above simplifies to

$$\int_{-1}^{1} f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx = B_n\int_{-1}^{1}\sin^2\left(\frac{n\pi}{2}(x+1)\right)dx$$
$$\int_{-1}^{1} f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx = B_n$$

Hence

$$B_n = \int_{-1}^{1} f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx$$

And the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(\int_{-1}^{1} f(x)\sin\left(\frac{n\pi}{2}(x+1)\right)dx\right)e^{-k\left(\frac{n\pi}{2}\right)^2 t}\sin\left(\frac{n\pi}{2}(x+1)\right)$$

3.1.1.10 [159] domain from -1 to +1

problem number 159

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = 1 - x^2$

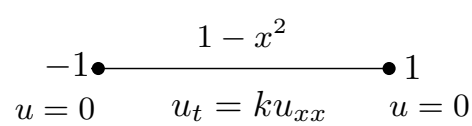


Figure 3.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == 1-x^2;
bc  = {u[-1, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Unable to display latex

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic  := u(x,0) = 1-x^2;
bc  := u(-1,t)=0,  u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{16((-1)^n - 1)\,\mathrm{e}^{-\frac{\pi^2 n^2 t}{4}} \sin\left(\frac{\pi(x+1)n}{2}\right)}{\pi^3 n^3}$$

<u>Hand solution</u>

Solve $u_t = u_{xx}$ with $-1 < x < 1$ and initial conditions $u(x,0) = f(x) = 1 - x^2$ and B.C. $u(-1,t) = u(1,t) = 0$.

The basic solution for this type of PDE was already given in problem 3.1.1.9 on page 424 as

$$u(x,t) = \sum_{n=1}^{\infty} \left(\int_{-1}^{1} f(x) \sin\left(\frac{n\pi}{2}(x+1)\right) dx \right) e^{-k\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}(x+1)\right)$$

Replacing $f(x) = 1 - x^2$ then

$$\begin{aligned} \int_{-1}^{1} f(x) \sin\left(\frac{n\pi}{2}(x+1)\right) dx &= \int_{-1}^{1} (1 - x^2) \sin\left(\frac{n\pi}{2}(x+1)\right) dx \\ &= \frac{16\left(1 + (-1)^{n+1}\right)}{n^3 \pi^3} \end{aligned}$$

Hence the solution becomes, using $k = 1$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{16\left(1+(-1)^{n+1}\right)}{n^3\pi^3} e^{-\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2}(x+1)\right)$$

This is animation of the solution for 2 seconds. (Animation will only show in the HTML version)

Source code used for the above

```
In[∘]:= ClearAll[x, t, n, lam]
L0 = 2;
k = 1;
pde = D[u[x, t], {t, 1}] == k*D[u[x, t], {x, 2}];

lam[n_] := (n π / L0);
numberOfTerms = 15;
mySol = Sum[16 (1 + (-1)^(n+1)) / (n^3 π^3) Exp[- lam[n]^2 t] Sin[lam[n] (x + 1)], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Module[{x0},
    Plot[ Quiet[mySol /. {x → x0, t → time}], {x0, -1, 1},
      PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " U at center = ",
            padIt2@Evaluate[Quiet[N[mySol /. {x → 0, t → time}]], {4, 3}]}],
      PlotRange → {{-1, 1}, {0, 1}},
      GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red, ImageSize → 300
    ]
  ],
  {{time, 0, "time"}, 0, 2, .01, Appearance → "Labeled"}]

In[∘]:= ClearAll[x, t];
r = Table[
      Plot[ Quiet[mySol /. {x → x0, t → time}], {x0, -1, 1},
        PlotLabel → Row[{padIt2[time, {4, 2}], " sec,", " U at center = ",
              padIt2@Evaluate[Quiet[N[mySol /. {x → 0, t → time}]], {4, 3}]}],
        PlotRange → {{-1, 1}, {0, 1}},
        GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red, ImageSize → 300
      ],
      {time, 0, 2, 0.1}];

In[∘]:= Export["anim.gif", r, "DisplayDurations" → Table[0.4, {Length@r}]]
```

Figure 3.16: Source code

3.1.1.11 [160] with source that depends on space only (general case)

problem number 160

Added July 3, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

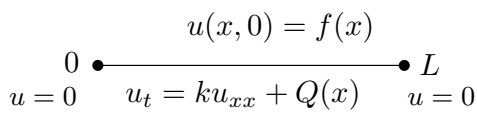


Figure 3.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\frac{\left(1-e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\right)\left(\int_0^L \frac{\sqrt{2}Q(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)L^2}{k\pi^2 K[1]^2} + e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\int_0^L \frac{\sqrt{2}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)\sin}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q(x);
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \frac{2Lk\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L \left(Lkf(\_a)+L\left(\int_0^{\_a}\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)-\_a\left(\int_0^L\int_0^{\_z1} Q(\_z1)d\_z1d\_z1\right)\right)\sin\left(\frac{\pi\_an}{L}\right)d\_a\right)e^{-\frac{\pi^2 kn^2 t}{L^2}}}{L^2k}\right.}{Lk}$$

<u>Hand solution</u>

Let the solution be

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x) \tag{1}$$

Where $\Phi_n(x)$ are the eigenfunctions of the eigenvalue problem for the pde $u_t = ku_{xx}$ with $u(0,x) = 0, u(L,x) = 0$. This was solved in problem 3.1.1.1 on page 400 and $\Phi_n(x)$ was found as

$$\begin{aligned}\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots\end{aligned}$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\sin\left(\sqrt{\lambda_n}x\right) \tag{1A}$$

Substituting this back into the original given PDE gives

$$\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) = k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n\Phi_n(x)$$

Where $Q(x) = \sum_{n=1}^{\infty} q_n\Phi_n(x)$. Since $\Phi_n''(x) = -\lambda_n\Phi_n(x)$, then the above simplifies to

$$\begin{aligned}\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) &= -k\sum_{n=1}^{\infty} a_n(t)\,\lambda_n\Phi_n(x) + \sum_{n=1}^{\infty} q_n\Phi_n(x)\\ a_n'(t) + k\lambda_n a_n(t) &= q_n \qquad (2)\end{aligned}$$

We now need to find q_n. By orthogonality

$$\int_0^L Q(x)\,\Phi_n(x)\,dx = \int_0^L q_n \Phi_n^2(x)\,dx$$

But $\int_0^L \Phi_n^2(x)\,dx = \int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx = \frac{L}{2}$ and the above becomes

$$q_n = \frac{2}{L}\int_0^L Q(x)\,\Phi_n(x)\,dx$$

Using the above in (2) gives

$$a_n'(t) + k\lambda_n a_n(t) = \frac{2}{L}\int_0^L Q(x)\,\Phi_n(x)\,dx$$

The integrating factor is $I = e^{k\lambda_n t}$. Hence the above becomes

$$\begin{aligned}
\frac{d}{dt}\left(a_n(t)\,e^{k\lambda_n t}\right) &= \frac{2}{L}e^{k\lambda_n t}\int_0^L Q(x)\,\Phi_n(x)\,dx\\
a_n(t)\,e^{k\lambda_n t} &= \int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x)\,\Phi_n(x)\,dx\right)ds + a_n(0)\\
a_n(t) &= a_n(0)\,e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x)\,\Phi_n(x)\,dx\right)ds
\end{aligned}$$

Therefore the solution (1A) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left[a_n(0)\,e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x) \quad (3)$$

At $t = 0$

$$\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} a_n(0)\,\Phi_n(x)\\
\int_0^L f(x)\,\Phi_n(x)\,dx &= \int_0^L a_n(0)\,\Phi_n^2(x)\,dx\\
a_n(0) &= \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx
\end{aligned}$$

The solution (3) becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\left[\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x)\\
&= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

3.1.1.12 [161] with source that depends on space only (special case)

problem number 161

Added July 4, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$ using the following values

$$L = 5$$
$$k = \frac{1}{10}$$
$$f(x) = x(5-x)$$
$$Q(x) = x$$

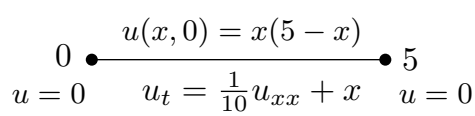


Figure 3.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5; k=1/10; f=x*(5-x); Q=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{100(-1+(-1)^n)\,e^{-\frac{1}{250}n^2\pi^2t}\sin\left(\frac{n\pi x}{5}\right)}{n^3\pi^3}+\sum_{n=1}^{\infty}-\frac{2500(-1)^n\left(1-e^{-\frac{1}{250}n^2\pi^2t}\right)\sin\left(\frac{n\pi x}{5}\right)}{n^3\pi^3}\right.\right.$$

Maple ✓

```
restart;
L=5;
k=1/10;
f=x*(5-x);
Q=x;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{(L-x)\,Qx+4k\left(\sum_{n=1}^{\infty}\frac{(-\pi^2fkn^2+L^2Q)((-1)^n-1)e^{-\frac{\pi^2kn^2t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{\pi^3kn^3}\right)}{2k}$$

<u>Hand solution</u>

Solve

$$\begin{aligned}u_t&=ku_{xx}+Q(x)\\u(0,t)&=0\\u(L,0)&=0\\u(x,0)&=f(x)\end{aligned}$$

With $k=\frac{1}{10},L=5,f(x)=x(5-x)\,,Q(x)=x.$

The general problem above was solved in 3.1.1.11 on page 429 and the solution is

$$u(x,t)=\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t\frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)$$

Where

$$\Phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n=\left(\frac{n\pi}{L}\right)^2\qquad n=1,2,3,\cdots$$

Substituting the specific values given above into this solution gives

$$\begin{aligned}u(x,t)&=\sum_{n=1}^{\infty}e^{-\frac{1}{10}\left(\frac{n\pi}{5}\right)^2 t}\sin\left(\frac{n\pi}{5}x\right)\left(\frac{2}{5}\int_0^5 s(5-s)\sin\left(\frac{n\pi}{5}s\right)ds\right)\\&+\sum_{n=1}^{\infty}e^{-\frac{1}{10}\left(\frac{n\pi}{5}\right)^2 t}\sin\left(\frac{n\pi}{5}x\right)\left(\int_0^t\frac{2}{5}e^{\frac{1}{10}\left(\frac{n\pi}{5}\right)^2\tau}\left(\int_0^5 s\sin\left(\frac{n\pi}{5}s\right)ds\right)d\tau\right)\end{aligned}$$

Animation is below

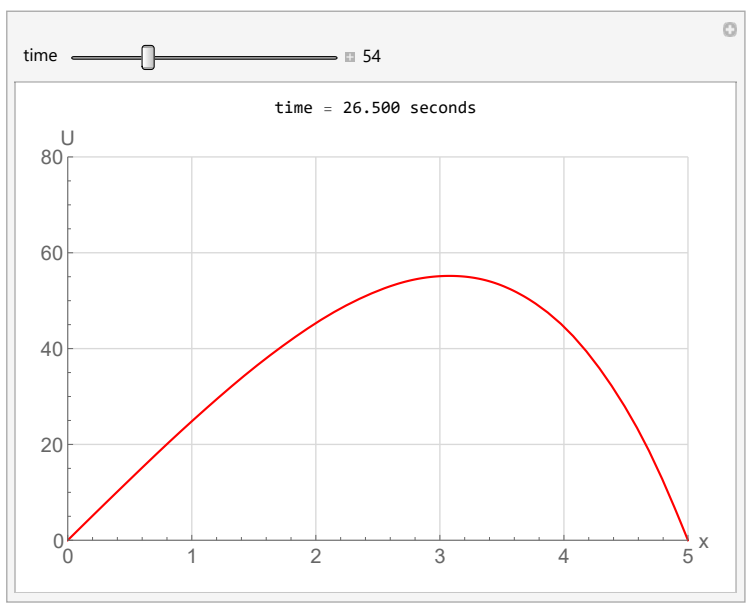


Figure 3.19: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
        L = 5;
        k = 1 / 10;
        f[x_] := x (5 - x);
        Q[x_] := x;
        ϕ[x_, n_] := Sin[√λ x];
        λ = (n π / L)^2;
        numberOfTerms = 15;
        padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[•]:= int1 = Integrate[Q[s] * ϕ[s, n], {s, 0, L}];
        int1 = Assuming[Element[n, Integers] && n > 0, Simplify[int1]];
        int2 = Integrate[2/L Exp[k λ τ] int1, {τ, 0, t}];
        int3 = Integrate[2/L f[s] × ϕ[s, n], {s, 0, L}];
        int3 = Assuming[Element[n, Integers] && n > 0, Simplify[int3]];
        mySol[x_, t_] = Chop@N@Sum[Exp[-k λ t] ϕ[x, n] int3, {n, 1, numberOfTerms}] + Chop@N@Sum[Exp[-k λ t] ϕ[x, n] int2, {n, 1, numberOfTerms}];
```

Figure 3.20: Source code

```
In[•]:= tab = Table[
            Grid[{
                {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
                {
                  Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                      BaseStyle → 15,
                      ImageMargins → 3,
                      PerformanceGoal → "Quality",
                      PlotRange → {{0, L}, {0, 80}},
                      ImageSize → 500,
                      AxesLabel → {"x", "U"},
                      GridLines → Automatic,
                      GridLinesStyle → LightGray,
                      PlotStyle → Red
                    ]

                }}],
            {t, 0, 100, .5}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.21: Code for animation

3.1.1.13 [162] with source that depends on space and time (general case)

problem number 162

Added June 23, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

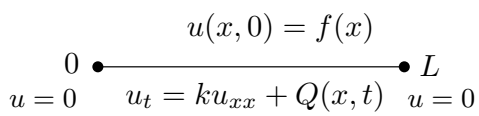


Figure 3.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2 K[1]^2 (t-K[2])}{L^2}} \text{Integrate}\left[\frac{\sqrt{2}Q(x,K[2])\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}, \{x,0,L\}, \text{Assumptions} \to k > \right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q(x,t);
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \int_0^t \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi n x}{L}\right) dx\right) \mathrm{e}^{-\frac{\pi^2 (t-\tau) k n^2}{L^2}} \sin\left(\frac{\pi n x}{L}\right)}{L} \right) d\tau + \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right) dx\right) \mathrm{e}}{L} \right.$$

<u>Hand solution</u>

Solving

$$\begin{aligned} u_t &= k v_{xx} + Q(x,t) \qquad (1) \\ u(0,t) &= 0 \\ u(L,t) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

Let the solution be

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x) \qquad (1)$$

Where $\Phi_n(x)$ are the eigenfunctions of the eigenvalue problem for the pde $u_t = k u_{xx}$ with $u(0,x) = 0, u(L,x) = 0$. This was solved in problem 3.1.1.1 on page 400 and $\Phi_n(x)$ was found as

$$\begin{aligned} \Phi_n(x) &= \sin\left(\sqrt{\lambda_n} x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots \end{aligned}$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t) \sin\left(\sqrt{\lambda_n} x\right) \qquad (1A)$$

Substituting this back into the original given PDE gives

$$\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) = k \sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)$$

Where $Q(x,t)=\sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)$. Since $\Phi_n''(x)=-\lambda_n\Phi_n(x)$, then the above simplifies to

$$\begin{aligned}\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) &= -k\sum_{n=1}^{\infty} a_n(t)\,\lambda_n\Phi_n(x)+\sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)\\ a_n'(t)+k\lambda_n a_n(t) &= q_n(t) \qquad (2)\end{aligned}$$

We now need to find $q_n(t)$. By orthogonality

$$\int_0^L Q(x,t)\,\Phi_n(x)\,dx=\int_0^L q_n(t)\,\Phi_n^2(x)\,dx$$

But $\int_0^L \Phi_n^2(x)\,dx=\int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx=\frac{L}{2}$ and the above becomes

$$q_n(t)=\frac{2}{L}\int_0^L Q(x,t)\,\Phi_n(x)\,dx$$

Using the above in (2) gives

$$a_n'(t)+k\lambda_n a_n(t)=\frac{2}{L}\int_0^L Q(x,t)\,\Phi_n(x)\,dx$$

The integrating factor is $I=e^{k\lambda_n t}$. Hence the above becomes

$$\begin{aligned}\frac{d}{dt}\left(a_n(t)\,e^{k\lambda_n t}\right) &= \frac{2}{L}e^{k\lambda_n t}\int_0^L Q(x,t)\,\Phi_n(x)\,dx\\ a_n(t)\,e^{k\lambda_n t} &= \int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x,s)\,\Phi_n(x)\,dx\right)ds+a_n(0)\\ a_n(t) &= a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n s}\left(\int_0^L Q(x,s)\,\Phi_n(x)\,dx\right)ds\end{aligned}$$

Therefore the solution (1A) becomes

$$u(x,t)=\sum_{n=1}^{\infty}\left[a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x) \qquad (3)$$

At $t=0$

$$\begin{aligned}f(x) &= \sum_{n=1}^{\infty} a_n(0)\,\Phi_n(x)\\ \int_0^L f(x)\,\Phi_n(x)\,dx &= \int_0^L a_n(0)\,\Phi_n^2(x)\,dx\\ a_n(0) &= \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx\end{aligned}$$

The solution (3) becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\left[\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right]\Phi_n(x)\\
&= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\right.
\end{aligned}
$$

Where

$$
\begin{aligned}
\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\
\lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots
\end{aligned}
$$

3.1.1.14 [163] with source that depends on space and time (special case)

problem number 163

Added July 5, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= 0\\
u(L,t) &= 0
\end{aligned}
$$

Initial condition is $u(x,0) = f(x)$. Using the following values

$$
\begin{aligned}
L &= \pi\\
k &= \frac{1}{300}\\
Q(x,t) &= e^{-t}\sin(3x)\\
f(x) &= x(\pi - x)
\end{aligned}
$$

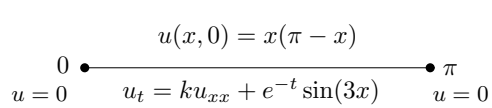


Figure 3.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=Pi; f=x*(Pi-x); k=1/300; Q=Exp[-t]*Sin[3*x];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol /. K[1] -> n
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{4(-1+(-1)^n)\,e^{-\frac{n^2t}{300}}\sin(nx)}{n^3\pi}+\frac{100}{97}e^{-t}\left(e^{97t/100}-1\right)\sin(3x)\right\}\right\}$$

Maple ✓

```
restart;
L:=Pi;
f:=x*(Pi-x);
k:=1/300;
Q:=exp(-t)*sin(3*x);
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2) + Q;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t)=-\frac{100\,\mathrm{e}^{-t}\sin(3x)}{97}+\frac{100\,\mathrm{e}^{-\frac{3t}{100}}\sin(3x)}{97}-4\left(\sum_{n=1}^{\infty}\frac{((-1)^n-1)\,\mathrm{e}^{-\frac{n^2t}{300}}\sin(nx)}{\pi n^3}\right)$$

<u>Hand solution</u>

Solving

$$\begin{aligned}u_t&=ku_{xx}+Q(x,t)\\u(0,t)&=0\\u(L,0)&=0\\u(x,0)&=f(x)\end{aligned}$$

With $k=\frac{1}{300}, L=\pi, f(x)=x(\pi-x)\,, Q(x)=e^{-t}\sin(3x)\,.$

The general problem above was solved in 3.1.6.4 on page 664 and the solution is

$$\begin{aligned}u(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)\\ &+\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right)\end{aligned}$$

Where

$$\begin{aligned}\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots\end{aligned}$$

Substituting the specific values given above into this solution gives

$$\begin{aligned}\Phi_n(x) &= \sin(nx)\\ \lambda_n &= n^2 \qquad n=1,2,3,\cdots\end{aligned}$$

And

$$\begin{aligned}u(x,t) &= \sum_{n=1}^{\infty} e^{-\frac{1}{300}n^2 t}\sin(nx)\left(\frac{2}{\pi}\int_0^{\pi} s(\pi-s)\sin(ns)\,ds\right)\\ &+\sum_{n=1}^{\infty} e^{-\frac{1}{300}n^2 t}\sin(nx)\left(\int_0^t \frac{2}{\pi}e^{\frac{1}{300}n^2\tau}e^{-\tau}\left(\int_0^{\pi}\sin(3s)\sin(ns)\,ds\right)d\tau\right)\end{aligned}$$

But $\int_0^{\pi} s(\pi-s)\sin(ns)\,ds = \frac{2-2(-1)^n}{n^3}$ and $\int_0^{\pi}\sin(3s)\sin(ns)\,ds = 0$ when $n\neq 3$ and for $n=3$ it is $\frac{\pi}{2}$. Hence the above reduces to

$$\begin{aligned}u(x,t) &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx)+e^{-\frac{1}{300}(3)^2 t}\sin(3x)\left(\int_0^t \frac{2}{\pi}e^{\frac{1}{300}(9)^2\tau}e^{-\tau}\left(\frac{\pi}{2}\right)d\tau\right)\\ &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx)+e^{-\frac{9t}{300}}\sin(3x)\left(\int_0^t e^{\frac{9\tau}{300}}e^{-\tau}d\tau\right)\end{aligned}$$

But $\int_0^t e^{\frac{9\tau}{300}}e^{-\tau}d\tau = \frac{100}{97}-\frac{100e^{\frac{-97}{100}t}}{97}$. Hence the above becomes

$$\begin{aligned}u(x,t) &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx)+e^{-\frac{9t}{300}}\sin(3x)\left(\frac{100}{97}-\frac{100e^{\frac{-97}{100}t}}{97}\right)\\ &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx)+\sin(3x)\left(\frac{100}{97}e^{-\frac{3t}{100}}-\frac{100e^{-t}}{97}\right)\\ &= 4\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^3}\right)e^{-\frac{1}{300}n^2 t}\sin(nx)-\frac{100e^{-t}}{97}\sin(3x)+\frac{100}{97}e^{-\frac{3t}{100}}\sin(3x)\end{aligned}$$

Animation is below

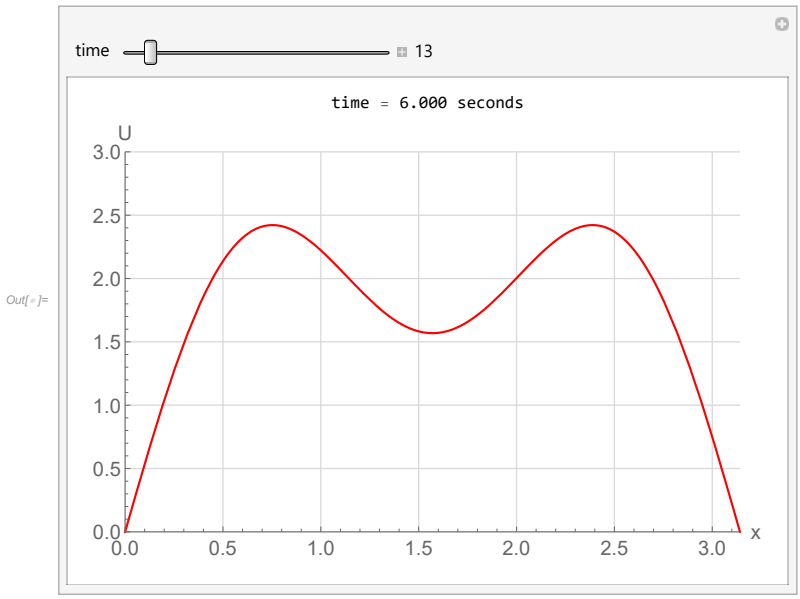


Figure 3.24: Initial state

Source code used for the above

```
ClearAll[x, t, n, f, A, B, s, mySol]
L = π;
k = 1 / 300;
f[x_] := x (π - x);
Q[x_, t_] := Exp[-t] Sin[3 x];
ϕ[x_, n_] := Sin[n x];
λ = n^2;
numberOfTerms = 15;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = N[4 * Sum[Exp[-k λ t] ϕ[x, n] (1 - (-1)^n / (π n^3)), {n, 1, numberOfTerms}]] - 100 Exp[-t] / 97 Sin[3 x] + 100 / 97 Exp[-3 t / 100] Sin[3 x];
```

Figure 3.25: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {0, 3}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 200, .5}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.26: Code for animation

3.1.1.15 [164] Math 4567 Exam

problem number 164

Added April 3, 2019.

Exam question. Math 4567 UMN. Spring 2019.

Solve the heat equation

$$u_t = u_{xx} + t(\pi - x)$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

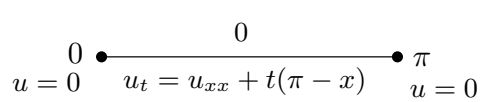


Figure 3.27: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + t*(Pi-x);
bc  = {u[0,t] == 0, u[Pi,t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> t
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{2\left(tn^2+e^{-n^2t}-1\right)\sin(nx)}{n^5}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+t*(Pi-x);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=0, u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=2\left(\sum_{n=1}^{\infty}\frac{\left(n^2t+\mathrm{e}^{-n^2t}-1\right)\sin(nx)}{n^5}\right)$$

<u>Hand solution</u>

Let $u(x,t)=\sum_{n=1}^{\infty}B_n(t)\,X_n(x)$, where $X_n(x)$ are the eigenfunctions of the homogeneous PDE $u_t=u_{xx}$. These are known to be $X_n(x)=\sin\left(\sqrt{\lambda_n}x\right)$ where $\lambda_n=n^2, n=1,2,3,\cdots$. Hence

$$X_n(x)=\sin(nx)\qquad n=1,2,3,\cdots$$

Therefore the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}B_n(t)\sin(nx)\tag{A}$$

Substituting this into the PDE $u_t=u_{xx}+t(\pi-x)$ gives

$$\sum_{n=1}^{\infty}B_n'(t)\,X_n(x)=\sum_{n=1}^{\infty}B_n(t)\,X_n''(x)+\sum_{n=1}^{\infty}q_n(t)\,X_n(x)$$

Where $\sum_{n=1}^{\infty} q_n(t)\, X_n(x)$ is the Fourier series expansion of $t(\pi - x)$. But $X_n''(x) = -\lambda_n X_n(x) = -n^2 X_n(x)$. Hence the above becomes

$$\begin{aligned}
\sum_{n=1}^{\infty} B_n'(t)\, X_n(x) &= -\sum_{n=1}^{\infty} n^2 B_n(t)\, X_n(x) + \sum_{n=1}^{\infty} q_n(t)\, X_n(x) \\
B_n'(t)\, X_n(x) + n^2 B_n(t)\, X_n(x) &= q_n(t)\, X_n(x) \\
B_n'(t) + n^2 B_n(t) &= q_n(t) \qquad (1)
\end{aligned}$$

This is first order ODE which is now solved for B_n. But first $q_n(t)$ is found. Since

$$t(\pi - x) = \sum_{n=1}^{\infty} q_n(t)\, X_n(x)$$

Applying orthogonality. Integrating both sides and multiplying by $X_m(x)$ gives

$$\begin{aligned}
\int_0^{\pi} t(\pi - x)\, X_m(x)\, dx &= \sum_{n=1}^{\infty} q_n(t) \int_0^{\pi} X_n(x)\, X_m(x)\, dx \\
&= \sum_{n=1}^{\infty} q_n(t) \int_0^{\pi} \sin(nx) \sin(mx)\, dx \\
&= q_m(t) \int_0^{\pi} \sin^2(mx)\, dx \\
&= \frac{\pi}{2} q_m(t)
\end{aligned}$$

Therefore

$$\begin{aligned}
q_n(t) &= \frac{2}{\pi} \int_0^{\pi} t(\pi - x) \sin(nx)\, dx \\
&= \frac{2t(\pi n - \sin(n\pi))}{\pi n^2}
\end{aligned}$$

But n is integer, hence the above simplifies to

$$q_n(t) = \frac{2t}{n}$$

Hence (1) becomes

$$B_n'(t) + n^2 B_n(t) = \frac{2t}{n}$$

Integrating factor is $I = e^{\int n^2 dt} = e^{n^2 t}$. The above becomes

$$\frac{d}{dt}\left(B_n e^{n^2 t}\right) = \frac{2t}{n} e^{n^2 t}$$

Integrating

$$
\begin{aligned}
B_n e^{n^2 t} &= \frac{2}{n}\int_0^t s e^{n^2 s} ds + C \\
B_n(t) &= \frac{2}{n} e^{-n^2 t}\int_0^t s e^{n^2 s} ds + C e^{-n^2 t} \qquad (2)
\end{aligned}
$$

At $t = 0, B_n(0) = 0$. The above reduces to

$$0 = C$$

Therefore (2) becomes

$$
\begin{aligned}
B_n(t) &= \frac{2}{n} e^{-n^2 t}\int_0^t s e^{n^2 s} ds \\
&= \frac{2}{n}\int_0^t s e^{n^2 (s-t)} ds \\
&= \frac{2\left(n^2 t + e^{-n^2 t} - 1\right)}{n^5}
\end{aligned}
$$

Hence Eq. (A) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\left(n^2 t + e^{-n^2 t} - 1\right)}{n^5} \sin(nx)$$

3.1.1.16 [165] With source

problem number 165

Taken from Maple PDE help pages

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx} + Q(x,t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= 0 \\
u(1,t) &= 0
\end{aligned}
$$

Initial condition is $u(x,0) = f(x)$

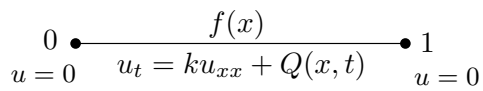


Figure 3.28: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {k
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \sqrt{2}\left(\int_0^t e^{-k\pi^2 K[1]^2 (t-K[2])}\text{Integrate}\left[\sqrt{2}Q(x,K[2])\sin(\pi x K[1]),\{x,0,1\},\text{Assumptions}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+Q(x, t);
bc  := u(0, t) = 0, u(l, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t)) as
```

$$u(x,t) = \int_0^t \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^l Q(x,\tau)\sin\left(\frac{\pi n x}{l}\right)dx\right)\mathrm{e}^{\frac{\pi^2(-t+\tau)kn^2}{l^2}}\sin\left(\frac{\pi n x}{l}\right)}{l}\right)d\tau + 2\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^l f(x)\sin\left(\frac{\pi n x}{l}\right)dx\right)\mathrm{e}^{-}}{l}\right.$$

3.1.1.17 [166] special initial condition

problem number 166

Added April 28, 2019.

Taken from `https://mathematica.stackexchange.com/questions/197155/solving-a-heat-equation-problem`

Solve $u(x,t)$

$$u_t = u_{xx} - 9u_x$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition $u(x,0) = e^{\frac{45}{10}}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$

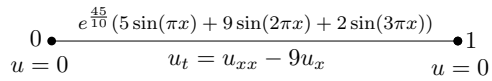


Figure 3.29: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - 9*D[u[x, t], x];
ic  = u[x, 0] == Exp[45/10 x]*(5 Sin[Pi*x] + 9 Sin[2*Pi*x] + 2 Sin[3*Pi*x]);
bc  = {u[0, t] == 0, u[1, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{\frac{9}{4}(2x-9t)} \sum_{K[1]=1}^{\infty} 0\right\}\right\}$$

Maple ✗

```
restart;
pde := diff(u(x,t),t)= diff(u(x, t), x$2) - 9*diff(u(x,t),x);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x, 0) = exp(45/10*x)*(5*sin(Pi*x) + 9*sin(2*Pi*x) + 2*sin(3*Pi*x));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

time expired

Hand solution

Solve

$$u_t = u_{xx} - 9u_x$$

IC

$$u(x,0) = e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$$

BC

$$u(0,x) = 0$$
$$u(1,x) = 0$$

Let $u = X(x)T(t)$, the PDE becomes

$$T'X = X''T - 9X'T$$
$$\frac{T'}{T} = \frac{X''}{X} - 9\frac{X'}{X} = -\lambda$$

Where λ is the seperation constant. From B.C. we know $\lambda > 0$. Hence the eigenvalue ODE is

$$X'' - 9X' + \lambda X = 0$$

The solution to the above is

$$\begin{aligned} X(x) &= C_1 e^{\frac{1}{2}(9-\sqrt{81-4\lambda})x} + C_2 e^{\frac{1}{2}(9+\sqrt{81-4\lambda})x} \\ &= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right) \end{aligned}$$

At $X(0) = 0$ this gives

$$0 = C_1 + C_2$$

And at $X(1)=0$

$$
\begin{aligned}
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= C_1 e^{\frac{9}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)
\end{aligned}
$$

For nontrivial solution we want

$$
\begin{aligned}
e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}} &= 0\\
e^{-\frac{1}{2}\sqrt{81-4\lambda}} &= e^{\frac{1}{2}\sqrt{81-4\lambda}} \qquad (1)
\end{aligned}
$$

<u>Case $81-4\lambda>0$</u>

This means $81-4\lambda$ must be zero or

$$\lambda = \frac{81}{4}$$

But using this eigenvalue makes the eigenfunction zero as shown below

$$
\begin{aligned}
X(x) &= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}(1-1)\\
&= 0
\end{aligned}
$$

Therefore $\lambda = \frac{81}{4}$ can not be used as eigenfunction.

<u>Case $81-4\lambda<0$</u>

Then (1) becomes

$$
\begin{aligned}
e^{-\frac{i}{2}\sqrt{4\lambda-81}} &= e^{\frac{i}{2}\sqrt{4\lambda-81}}\\
\cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) - i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= \cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) + i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right)\\
2i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\frac{1}{2}\sqrt{4\lambda-81} &= n\pi \qquad n=1,2,\cdots
\end{aligned}
$$

Hence

$$\begin{aligned}\frac{1}{4}(4\lambda - 81) &= n^2\pi^2\\ 4\lambda &= 81 + 4n^2\pi^2\\ \lambda_n &= \frac{81}{4} + n^2\pi^2\end{aligned}$$

The corresponding eigenfunctions are (and since $C_2 = -C_1$) then

$$\begin{aligned}X_n(x) &= C_n\left(e^{\frac{1}{2}(9-\sqrt{81-4\lambda_n})x} - e^{\frac{1}{2}(9+\sqrt{81-4\lambda_n})x}\right)\\ &= C_n\left(e^{\frac{1}{2}(9-i\sqrt{4\lambda_n-81})x} - e^{\frac{1}{2}(9+i\sqrt{4\lambda_n-81})x}\right)\\ &= C_n e^{\frac{9x}{2}}\left(e^{-\frac{i}{2}\sqrt{4\lambda_n-81}x} - e^{\frac{i}{2}\sqrt{4\lambda_n-81}x}\right)\\ &= C_n e^{\frac{9x}{2}}\left(\cos\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right) - i\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right) - \cos\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right) - \sin\left(\frac{1}{2}\sqrt{4\lambda_n -\right.\right.\\ &= C_n e^{\frac{9x}{2}}\left(-2i\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)\right)\\ &= A_n e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)\end{aligned}$$

Hence the solution is

$$u(x,t) = \sum_{n=1}^{\infty} X_n(t)\,T_n(t)$$

But $T' + \lambda_n T = 0$ has solution $T = e^{-\lambda_n t}$. Therefore the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-\lambda_n t} e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)$$

At $t = 0$

$$e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x)) = \sum_{n=1}^{\infty} A_n e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n-81}x\right)$$

But $\lambda_n = \frac{81}{4} + n^2\pi^2$. The above becomes

$$\begin{aligned}e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+\pi^2\right)-81}x\right)\\ &+ A_2 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+4\pi^2\right)-81}x\right)\\ &+ A_3 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4}+9\pi^2\right)-81}x\right)\\ &+ \cdots\end{aligned}$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\pi^2}x\right)\\
&+ A_2 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{16\pi^2}x\right)\\
&+ A_3 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{36\pi^2}x\right)\\
&+\cdots
\end{aligned}
$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin(\pi x)\\
&+ A_2 e^{\frac{9x}{2}}\sin(2\pi x)\\
&+ A_3 e^{\frac{9x}{2}}\sin(3\pi x)\\
&+\cdots
\end{aligned}
$$

By comparing coefficients, we see that $A_1 e^{\frac{9x}{2}} = 5e^{\frac{45}{10}x}$ or $A_1 = e^{(\frac{45}{10}-\frac{9}{2})x} = 5$ and $A_2 = 9$ and $A_3 = 2$ and and all other A_n for $n > 3$ are zero. Hence the solution becomes

$$
\begin{aligned}
u(x,t) &= 5e^{-\lambda_1 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_1-81}x\right)+9e^{-\lambda_2 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_2-81}x\right)+2e^{-\lambda_3 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_3-81}x\right.\\
&= e^{-(\frac{81}{4}+\pi^2)t}e^{\frac{9x}{2}}\sin(\pi x)+e^{-(\frac{81}{4}+4\pi^2)t}e^{\frac{9x}{2}}\sin(2\pi x)+e^{-(\frac{81}{4}+9\pi^2)t}e^{\frac{9x}{2}}\sin(3\pi x)\\
&= e^{-\frac{81}{4}t+\frac{9}{2}x}\left(5e^{-\pi^2 t}\sin(\pi x)+9e^{-4\pi^2 t}\sin(2\pi x)+2e^{-9\pi^2 t}\sin(3\pi x)\right)
\end{aligned}
$$

3.1.1.18 [167] Diffusion Reaction and Euler-Cauchy Sturm-Liouville

problem number 167

Added May 5, 2019.

Solve $u(x,t)$

$$u_t = x^2 u_{xx} + x u_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(1,t) &= 0\\
u(b,t) &= 0
\end{aligned}
$$

Initial condition $u(x,0) = f(x)$

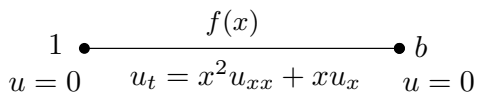


Figure 3.30: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic  = u[x, 0] == f[x];
bc  = {u[1, t] == 0, u[b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> b
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{2e^{-\frac{\pi^2 t K[1]^2}{\log^2(b)}} \left(\int_1^b \frac{f(x)\sin\left(\frac{\pi K[1]\log(x)}{\log(b)}\right)}{x}\,dx\right)\sin\left(\frac{\pi K[1]\log(x)}{\log(b)}\right)}{\log(b)}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff( u(x,t),t)= x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc  := u(1,t)=0,u(b,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\left(\int_1^b \frac{f(x)\sin\left(\frac{\pi n \ln(x)}{\ln(b)}\right)}{x}dx\right) \mathrm{e}^{-\frac{\pi^2 n^2 t}{\ln(b)^2}} \sin\left(\frac{\pi n \ln(x)}{\ln(b)}\right)}{\ln(b)}$$

3.1.1.19 [168] Diffusion Reaction. Using growth form reaction term

problem number 168

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = ku_{xx} + ru$$

with $k = \frac{1}{10}, r = 1$ and $0 < x < 1$ and $t > 0$. With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x,0) = 1$.

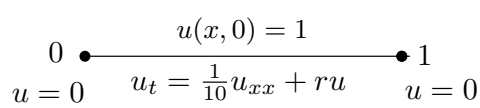


Figure 3.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/10;
 r = 1;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} -\frac{2\left(-1+(-1)^{K[1]}\right) e^{t-\frac{1}{10}\pi^2 t K[1]^2} \sin(\pi x K[1])}{\pi K[1]}\right\}\right\}$$

Maple ✓

```
restart;
k:=1/10;
r:=1;
pde := diff(u(x,t), t) = k*diff(u(x, t), x$2) + r*u(x,t);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x,t))),out
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{2((-1)^n - 1)\,\mathrm{e}^{-\frac{(\pi^2 n^2 - 10)t}{10}} \sin(\pi n x)}{\pi n}$$

<u>Hand solution</u>

Solution added 4/3/2019.

$$\begin{aligned} u_t &= k u_{xx} + r u \qquad t > 0, 0 < x < 1 \\ u(0,t) &= 0 \\ u(1,t) &= 0 \\ u(x,0) &= 1 \end{aligned}$$

Let $u = v e^{rt}$. Hence $u_t = v_t e^{rt} + v r e^{rt}$ and $u_{xx} = v_{xx} e^{rt}$. Hence the PDE becomes $v_t e^{rt} + v r e^{rt} = v_{xx} e^{rt} + r v e^{rt}$ which simplifies to

$$\begin{aligned} v_t &= k v_{xx} \qquad t > 0, 0 < x < 1 \\ v(0,t) &= 0 \\ v(1,t) &= 0 \end{aligned}$$

The above is now in canonical form, it is standard heat PDE with homogeneous B.C. This has the solution

$$v(x,t) = \sum_{n=1}^{\infty} B_n e^{-n^2 \pi^2 t} \sin(n\pi x)$$

Therefore

$$\begin{aligned} u &= v e^{rt} \\ u &= \sum_{n=1}^{\infty} B_n e^{-t(n^2\pi^2 - r)} \sin(n\pi x) \end{aligned}$$

At $t = 0$ the above becomes

$$1 = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

Hence B_n are sin Fourier coefficient of 1 which is

$$\begin{aligned} B_n &= 2\int_0^1 \sin(n\pi x)\,dx \\ &= 2\left(-\frac{1}{n\pi}\right)(\cos n\pi x)_0^1 \\ &= \frac{-2}{n\pi}((-1)^n - 1) \end{aligned}$$

Hence the solution becomes

$$u = \sum_{n=1}^{\infty} \frac{-2}{n\pi}((-1)^n - 1)\, e^{-t(n^2\pi^2 - r)} \sin(n\pi x)$$

But $r = \frac{1}{10}$, therefore

$$u(x,t) = -\frac{2}{\pi}\sum_{n=1}^{\infty} \frac{((-1)^n - 1)}{n} e^{-t(n^2\pi^2 - \frac{1}{10})} \sin(n\pi x)$$

3.1.1.20 [169] Diffusion Reaction, using logistic form for reaction term

problem number 169

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = ku_{xx} + ru\left(1 - \frac{u}{\alpha}\right)$$

with $k = \frac{1}{100}, r = \frac{1}{10}, \alpha = 10$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= 0 \end{aligned}$$

And initial conditions $u(x,0) = 1$.

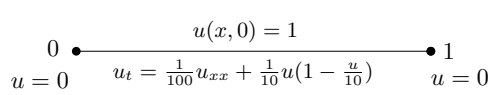


Figure 3.32: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
k = 1/100;
 r = 1/10;
 alpha = 10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t]*(1 - u[x, t]/alpha);
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
k := 1/100;
r := 1/10;
alpha := 10;
pde := diff(u(x, t), t)=   k*diff(u(x, t),x$2) + r*u(x, t)*(1- u(x,t)/alpha);
bc  := u(0, t) = 0, u(1, t) = 0;
ic  := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc], u(x, t))),o
```

sol=()

3.1.1.21 [170] Diffusion Reaction, using Aleee form for reaction term

problem number 170

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = ku_{xx} + \alpha u + \beta u^2 - \gamma u^3$$

with $k = \frac{1}{1000}, \alpha = \frac{1}{100}, \beta = \frac{1}{100}, \gamma = \frac{5}{1000}$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x,0) = 1$.

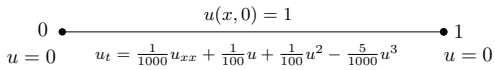


Figure 3.33: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
k = 1/1000;
 alpha = 1/10;
 beta = 1/100;
 gamma = 5/1000;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + alpha*u[x, t] + beta*u[x, t]^2 - gamma*u[x, t
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
k := 1/100;
alpha:=1/100;
beta:=1/1000;
g:=5/1000;
pde := diff(u(x, t), t)= k*diff(u(x, t),x$2) +
        alpha*u(x,t)+ beta*u(x,t)^2 - g*u(x,t)^3;
bc  := u(0, t) = 0, u(1, t) = 0;
ic  := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

sol=()

3.1.1.22 [171] Diffusion Reaction. Haberman 2.3.8

problem number 171

This is problem 2.3.8, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = ku_{xx} - \alpha u$$

This corresponds to a one-dimentional rod either with heat loss through the lateral sides with outside temperature zero degrees ($\alpha > 0$) or with insulated sides with a heat sink propertional to the temperature.

Suppose the boundary conditions are $u(0,t) = 0, u(L,t) = 0$, solve with the temperature initially $u(x,0) = f(x)$ if $\alpha > 0$

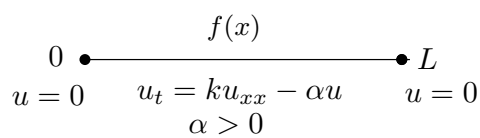


Figure 3.34: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - alpha*u[x, t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-t\left(\frac{k\pi^2 K[1]^2}{L^2}+\alpha\right)}\left(\int_0^L \frac{\sqrt{2}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-alpha*u(x,t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\left(L^2\alpha+\pi^2 k n^2\right)t}{L^2}}\sin\left(\frac{\pi n x}{L}\right)}{L}$$

<u>Hand solution</u>

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}-\alpha u$$
$$\frac{\partial u}{\partial t}+\alpha u=k\frac{\partial^2 u}{\partial x^2}$$

Assuming $u(x,t)=X(x)\,T(t)$ and substituting in the above gives

$$XT'+\alpha XT=kTX''$$

Dividing by $kXT\neq 0$

$$\frac{T'}{kT}+\frac{\alpha}{k}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T}+\frac{\alpha}{k}=\frac{X''}{X}=-\lambda$$

The two ODE's are

$$\frac{1}{k}\frac{T'}{T}+\frac{\alpha}{k}=-\lambda$$
$$\frac{X''}{X}=-\lambda$$

Or

$$T'+(\alpha+\lambda k)\,T=0$$
$$X''+\lambda X=0$$

The solution to the space ODE is the familiar (where $\lambda > 0$ is only possible case, As found in Haberman problem 2.3.3, part d. Since it has the same B.C.)

$$X_n = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2$. The time ODE is now solved.

$$\frac{dT_n}{dt} + (\alpha + \lambda_n k)\, T_n = 0$$

This has the solution

$$\begin{aligned} T_n(t) &= e^{-(\alpha+\lambda_n k)t} \\ &= e^{-\alpha t} e^{-\left(\frac{n\pi}{L}\right)^2 kt} \end{aligned}$$

For the same eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the B_n when combined in the following step below. Therefore the solution to the PDE is

$$u_n(x,t) = T_n(t)\, X_n(x)$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} u_n(x,t) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\alpha t} e^{-\left(\frac{n\pi}{L}\right)^2 kt} \\ &= e^{-\alpha t} \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt} \end{aligned}$$

Where $e^{-\alpha t}$ was moved outside since it does not depend on n. From initial condition

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

Applying orthogonality of sin as before to find B_n results in

$$B_n = \frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx$$

Hence the solution becomes

$$\begin{aligned} u(x,t) &= \frac{2}{L} e^{-\alpha t}\left(\sum_{n=1}^{\infty}\left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx\right) \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt}\right) \\ &= \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\, dx\right) \sin\left(\frac{n\pi}{L}x\right) e^{-t\left(\frac{n^2\pi^2 k + \alpha L^2}{L^2}\right)} \end{aligned}$$

Hence it is clear that in the limit as t becomes large $u(x,t) \to 0$ since $\alpha > 0$ and

$$\lim_{t\to\infty} u(x,t) = 0$$

3.1.1.23 [172] Diffusion Reaction

problem number 172

Solve the heat equation

$$u_t = u_{xx} - u(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

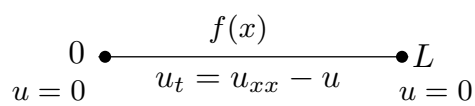


Figure 3.35: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + u[x, t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \sqrt{2} e^{t\left(-\frac{\pi^2 K[1]^2}{L^2}-1\right)} \sqrt{\frac{1}{L}} \left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}} f(x) \sin\left(\frac{\pi x K[1]}{L}\right) dx\right) \sin\left(\frac{\pi x K[1]}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)+u(x,t)=diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\left(L^2+\pi^2n^2\right)t}{L^2}}\sin\left(\frac{\pi n x}{L}\right)}{L}$$

<u>Hand solution</u>

$$\begin{aligned}u_t&=u_{xx}-u\\u(0,t)&=0\\u(L,t)&=0\\u(x,0)&=f(x)\end{aligned}$$

Let $u(x,t)=v(x,t)\,e^{-t}$, hence $u_t=v_te^{-t}-ve^{-t}$ and $u_{xx}=v_{xx}e^{-t}$. Therefore the above PDE becomes

$$\begin{aligned}v_te^{-t}-ve^{-t}&=v_{xx}e^{-t}-ve^{-t}\\v_t&=v_{xx}\end{aligned}$$

With boundary conditions

$$\begin{aligned}v(0,t)&=0\\v(L,t)&=0\end{aligned}$$

The solution to this PDE is known, since it has homogenouse BC and it is in standard form. The solution is

$$v(x,t)=\sum_{n=1}^{\infty}B_ne^{-\left(\frac{n\pi}{L}\right)^2t}\sin\left(\frac{n\pi}{L}x\right)$$

Hence

$$
\begin{aligned}
u(x,t) &= e^{-t}\sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t-t}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2}{L^2}+1\right)}\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\sin\left(\frac{n\pi}{L}x\right) \qquad (1)
\end{aligned}
$$

Applying initial conditions $u(x,0)=f(x)$ gives

$$
f(x)=\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)
$$

Hence B_n are the Fourier sine coefficients of $f(x)$

$$
B_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx
$$

Therefore the solution in (1) becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\frac{2}{L}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\sin\left(\frac{n\pi}{L}x\right)\\
&= \frac{2}{L}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{L}x\right)e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)
\end{aligned}
$$

3.1.1.24 [173] Diffusion Reaction

problem number 173

Added Feb 10, 2018.

Solve the heat equation

$$
u_t+u(x,t)=100u_{xx}
$$

For $0<x<1$ and $t>0$. The boundary conditions are

$$
\begin{aligned}
u(0,t)&=0\\
u(1,t)&=0
\end{aligned}
$$

Initial condition is $u(x,0) = \sin(2\pi x) - \sin(5\pi x)$

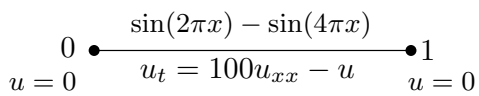


Figure 3.36: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 100*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 0};
ic  = u[x, 0] == Sin[2*Pi*x] - Sin[5*Pi*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to e^{-400\pi^2 t}\sin(2\pi x) - e^{-2500\pi^2 t}\sin(5\pi x)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=100*diff(u(x,t),x$2);
ic  := u(x,0)=sin(2*Pi*x)-sin(5*Pi*x);
bc  := u(0,t)=0,u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = -\mathrm{e}^{-2500\pi^2 t}\sin(5\pi x) + \mathrm{e}^{-400\pi^2 t}\sin(2\pi x)$$

3.1.1.25 [174] Diffusion Reaction

problem number 174

Added June 23, 2019.

Solve the heat equation

$$u_t = ku_{xx} - ux$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Initial condition $u(x,0) = \sin(x)$

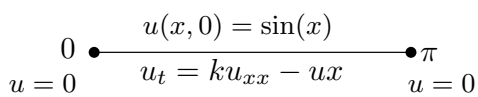


Figure 3.37: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - u[x,t]*x;
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)- u(x,t)*x;
ic  := u(x,0)=sin(x);
bc  := u(0,t)=0,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{(\cosh(t\lambda_n) + \sinh(t\lambda_n)) \left(-\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right) \operatorname{AiryBi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right) + \operatorname{AiryAi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right) \operatorname{AiryBi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)\right.}{\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right)^2 \left(\int_0^{\pi} \operatorname{AiryBi}\left(\frac{x+\lambda_n}{k^{\frac{1}{3}}}\right)^2 dx\right) - 2\operatorname{AiryAi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right) \operatorname{AiryBi}\left(\frac{\lambda_n+\pi}{k^{\frac{1}{3}}}\right.}$$

3.1.1.26 [175] Diffusion convection (general case)

problem number 175

Added June 23, 2019.

Solve the heat equation

$$u_t = ku_{xx} + au_x$$

Where $a > 0$. For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition $u(x,0) = f(x)$

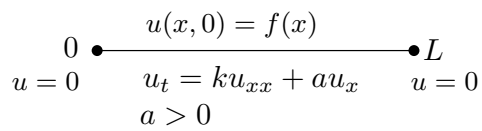


Figure 3.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + a*D[u[x, t], x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to e^{-\frac{a(at+2x)}{4k}} \sum_{K[1]=1}^{\infty} \frac{2e^{-\frac{k\pi^2 tK[1]^2}{L^2}} \left(\int_0^L e^{\frac{ax}{2k}} f(x) \sin\left(\frac{\pi x K[1]}{L}\right) dx\right) \sin\left(\frac{\pi x K[1]}{L}\right)}{L}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+ a*diff(u(x,t),x);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\cosh\left(\frac{(L^2a^2+4\pi^2k^2n^2)t}{4L^2k}\right)-\sinh\left(\frac{(L^2a^2+4\pi^2k^2n^2)t}{4L^2k}\right)\right)\left(\cosh\left(\frac{ax}{2k}\right)-\sinh\left(\frac{ax}{2k}\right)\right)\left(\int_0^L\left(\cosh\left(\frac{ax}{2k}\right)-\right.\right.}{L}$$

<u>Hand solution</u>

Solve

$$\begin{aligned}u_t&=ku_{xx}+au_x\\u(0,t)&=0\\u(L,0)&=0\\u(x,0)&=f(x)\end{aligned}$$

Trying separation of variables. Let $u=XT$, then the PDE becomes

$$\begin{aligned}T'X&=kX''T+aX'T\\\frac{1}{k}\frac{T'}{T}&=\frac{X''}{X}+\frac{a}{k}\frac{X'}{X}=-\lambda\end{aligned}$$

Where λ is separation constant. Hence

$$T'+k\lambda T=0$$

Which has solution $T(t)=e^{-k\lambda t}$ (the constant of integration is not needed, it will be combined with constant coming from the spatial ODE), and the spatial ODE is

$$\begin{aligned}\frac{X''}{X}+\frac{a}{k}\frac{X'}{X}&=-\lambda\\X''+\frac{a}{k}X'+\lambda X&=0\\X(0)&=0\\X(L)&=0\end{aligned}$$

The characteristic equation is $r^2+\frac{a}{k}r+\lambda=0$.

<u>Case $\lambda = 0$</u> Then $r^2 + \frac{a}{k}r = 0$ or $r\left(r + \frac{a}{k}\right) = 0$. Hence $r = 0$ or $r = -\frac{a}{k}$. So the solution is

$$X(x) = c_1 + c_2 e^{-\frac{a}{k}x}$$

At $X(0) = 0$ the above gives $0 = c_1 + c_2$. Therefore the solution is $X(x) = c_1\left(1 - e^{\frac{-a}{k}x}\right)$. At $X(L) = 0$ then $c_1\left(1 - e^{\frac{a}{k}L}\right) = 0$. This means this for non-trivial solution $e^{\frac{-a}{k}L} = 1$ or $\frac{-a}{k}L = 0$. Hence <u>$\lambda = 0$ is not an eigenvalue</u>.

<u>Case $\lambda < 0$</u> Then $r^2 + \frac{a}{k}r + \lambda = 0$ or $r = \frac{-b}{2} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-a}{2k} \pm \frac{\sqrt{\frac{a^2}{k^2}-4\lambda}}{2}$. Since $\lambda < 0$ then the term under the sqrt root is positive. Hence the solution will have real roots. Not complex conjugate. Hence the solution is pure exponentials. Let $\frac{a^2}{k^2} - 4\lambda = \alpha$ and $\frac{a}{2k} = \beta$ then

$$\begin{aligned} r_1 &= -\beta + \sqrt{\alpha} \\ r_2 &= -\beta - \sqrt{\alpha} \end{aligned}$$

Then $X(x) = c_1 \cosh\left((-\beta + \alpha)\, x\right) + c_2 \sinh\left((-\beta - \alpha)\, x\right)$. At $X(0) = 0$ this gives $0 = c_1$. Hence solution is $X(x) = c_2 \sinh\left((-\beta - \alpha)\, x\right)$. At $X(L) = 0$, this gives

$$0 = c_2 \sinh\left((-\beta - \alpha)\, L\right)$$

For non-trivial solution, $\sinh\left((-\beta - \alpha)\, L\right) = 0$. But this is zero only when $(-\beta - \alpha)\, L$ or $\beta = -\alpha$. Or $\frac{-a}{2k} = -\sqrt{\frac{a^2}{k^2} - 4\lambda}$. Or $\frac{a^2}{4k^2} = \frac{a^2}{k^2} - 4\lambda$ or

$$\begin{aligned} 4\lambda &= \frac{a^2}{k^2} - \frac{a^2}{4k^2} \\ \lambda &= \frac{3}{16}\frac{a^2}{k^2} \end{aligned}$$

But λ was assumed negative. Hence this is not possible. <u>$\lambda < 0$ is not eigenvalue</u>.

<u>Case $\lambda > 0$</u> Then $r^2 + \frac{a}{k}r + \lambda = 0$ or $r = \frac{-a}{2k} \pm \frac{1}{2}\sqrt{\frac{a^2}{k^2} - 4\lambda} = \frac{-a}{2k} \pm \sqrt{\frac{a^2}{4k^2} - \lambda}$. Since $\lambda > 0$ then the term under the sqrt root can be negative. Then only when $\lambda > \frac{a^2}{4k^2}$ will there be complex roots. Therefore assuming $\lambda > \frac{a^2}{4k^2}$, then

$$\begin{aligned} r_1 &= \frac{-a}{2k} + i\sqrt{\lambda - \frac{a^2}{4k^2}} \\ r_2 &= \frac{-a}{2k} - i\sqrt{\lambda - \frac{a^2}{4k^2}} \end{aligned}$$

The solution is

$$X(x) = e^{\frac{-a}{2k}x}\left(c_1 \cos\left(\sqrt{\lambda - \frac{a^2}{4k^2}}\,x\right) + c_2 \sin\left(\sqrt{\lambda - \frac{a^2}{4k^2}}\,x\right)\right)$$

At $X(0)=0$ this gives $0=c_1$. Hence solution is

$$X(x)=e^{\frac{-a}{2k}x}c_2\sin\left(\sqrt{\lambda-\frac{a^2}{4k^2}}x\right)$$

At $X(L)=0$, this gives

$$0=e^{\frac{-a}{2k}\pi}c_2\sin\left(\sqrt{\lambda-\frac{a^2}{4k^2}}L\right)$$

For non-trivial solution, $\sin\left(\sqrt{\lambda-\frac{a^2}{4k^2}}L\right)=0$ or $\sqrt{\lambda-\frac{a^2}{4k^2}}L=n\pi$. Hence

$$\begin{aligned}\lambda-\frac{a^2}{4k^2}&=\left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots\\ \lambda_n&=\left(\frac{n\pi}{L}\right)^2+\frac{a^2}{4k^2} \qquad n=1,2,3,\cdots\end{aligned}$$

And the corresponding eigenfunction is

$$X_n(x)=e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right) \qquad n=1,2,3,\cdots$$

Therefore the solution is

$$u(x,t)=\sum_{n=1}^{\infty}c_ne^{-k\lambda_nt}e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)$$

c_n is found from initial conditions. At $t=0$

$$\begin{aligned}f(x)&=\sum_{n=1}^{\infty}c_ne^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)\\ e^{\frac{a}{2k}x}f(x)&=\sum_{n=1}^{\infty}c_n\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

Therefore, by orthogonality

$$\begin{aligned}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx&=c_n\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx\\ \int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx&=c_n\frac{L}{2}\\ c_n&=\frac{2}{L}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx\end{aligned}$$

Hence the solution becomes

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-k\lambda_n t}e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2+\frac{a^2}{4k^2}\qquad n=1,2,3,\cdots$

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L e^{\frac{a}{2k}x}f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)e^{-k\left(\frac{n^2\pi^2}{L^2}+\frac{a^2}{4k^2}\right)t}e^{\frac{-a}{2k}x}\sin\left(\frac{n\pi}{L}x\right)$$

3.1.1.27 [176] Diffusion convection (special case)

problem number 176

Added June 23, 2019.

Solve the heat equation

$$u_t=ku_{xx}+au_x$$

Where $a>0$. For $0<x<L$ and $t>0$. The boundary conditions are

$$u(0,t)=0$$
$$u(L,t)=0$$

Initial condition $u(x,0)=f(x)$ using the following values

$$f(x)=\sin(x)$$
$$k=1$$
$$a=5$$
$$L=\pi$$

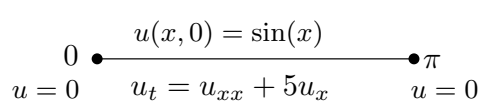


Figure 3.39: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f=Sin[x];
L=Pi;
k=1;
a=5;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + a*D[u[x, t], x];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{-\text{Integrate}\left[\frac{(1+e^{-i\pi K})e^{K(-Kt+5it+ix)}}{1-K^2},\{K,-\infty,\infty\},\text{Assumptions}\to K\in\mathbb{C}\right]+\int_{\infty}^{-\infty}\frac{\exp\left(it\sqrt{4}\right.}{}}{}\right.\right.$$

Maple ✓

```
restart;
f:=sin(x);
L:=Pi;
k:=1;
a:=5;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+ a*diff(u(x,t),x);
ic  := u(x,0)=f;
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}-\frac{160\left(\cosh\left(\frac{5x}{2}\right)-\sinh\left(\frac{5x}{2}\right)\right)\left(\cosh\left(n^2t+\frac{25}{4}t\right)-\sinh\left(n^2t+\frac{25}{4}t\right)\right)\left(e^{\frac{5\pi}{2}}(-1)^n+1\right)n\sin(nx}{\pi\left(16n^4+168n^2+841\right)}$$

Hand solution

Solve

$$\begin{aligned} u_t &= ku_{xx} + au_x \\ u(0,t) &= 0 \\ u(L,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

With $a = 5, k = 1, L = \pi, f(x) = \sin(x)$. The above problem was solved in 3.1.1.26 on page 467 and the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\lambda_n t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2 + \frac{a^2}{4k^2} \qquad n = 1, 2, 3, \cdots$

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} f(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

Substituting for $f(x) = \sin(x)$ the above becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L e^{\frac{a}{2k}x} \sin(x) \sin\left(\frac{n\pi}{L}x\right) dx \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

But

$$\int_0^{\pi} e^{\frac{a}{2k}x} \sin(x) \sin(nx)\, dx = \frac{-16nak^3\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2 - 1\right)^2 + 8a^2k^2\left(1 + n^2\right)}$$

Hence the solution becomes

$$u(x,t) = \frac{-32ak^3}{L} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2 - 1\right)^2 + 8a^2k^2\left(1 + n^2\right)} \right) e^{-k\left(\frac{n^2\pi^2}{L^2} + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin\left(\frac{n\pi}{L}x\right)$$

But $L = \pi$, hence

$$u(x,t) = \frac{-32ak^3}{\pi} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{\pi a}{2k}}\right)}{a^4 + 16k^4\left(n^2 - 1\right)^2 + 8a^2k^2\left(1 + n^2\right)} \right) e^{-k\left(n^2 + \frac{a^2}{4k^2}\right)t} e^{\frac{-a}{2k}x} \sin(nx)$$

And $a = 5$

$$u(x,t) = \frac{-160k^3}{\pi} \sum_{n=1}^{\infty} \left(\frac{n\left(1 + (-1)^n e^{\frac{5\pi}{2k}}\right)}{625 + 16k^4\left(n^2 - 1\right)^2 + 200k^2\left(1 + n^2\right)} \right) e^{-k\left(n^2 + \frac{25}{5k^2}\right)t} e^{\frac{-5}{2k}x} \sin(nx)$$

And $k = 1$

$$u(x,t) = \frac{-160}{\pi} \sum_{n=1}^{\infty} \left(\frac{n\left(1+(-1)^n e^{\frac{5\pi}{2}}\right)}{625+16\left(n^2-1\right)^2+200\left(1+n^2\right)} \right) e^{-\left(n^2+\frac{25}{4}\right)t} e^{\frac{-5}{2}x} \sin(nx)$$

Animation is below

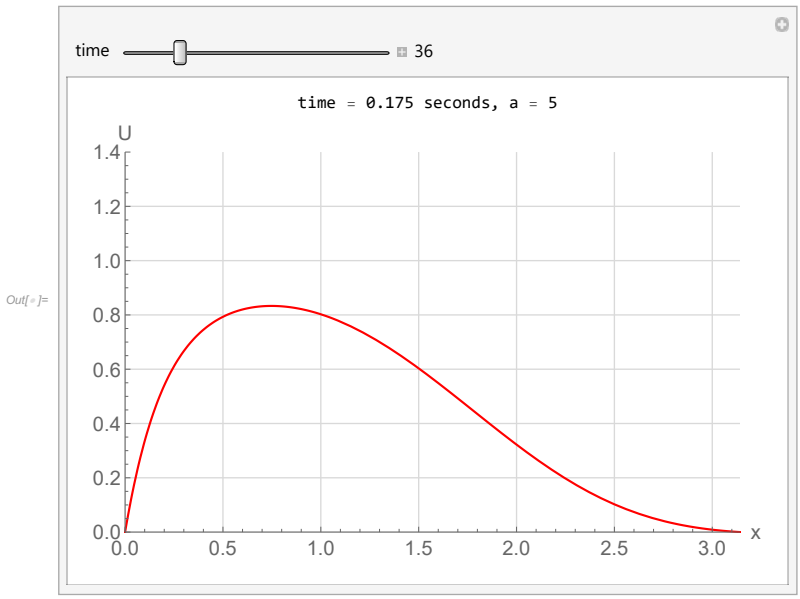


Figure 3.40: Initial state

Source code used for the above

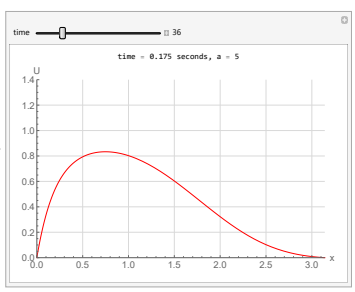
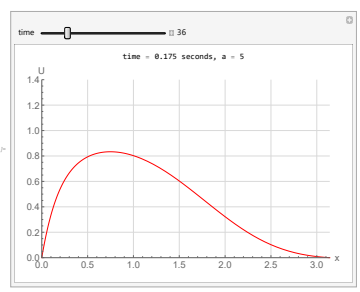
```
In[∘]:= ClearAll[x, y, t, n, k]
L = Pi;
numberOfTerms = 100;
a = 5;
k = 1;
mySol[x_, t_] = (-32 a k^3)/Pi Sum[(n (1 + (-1)^n Exp[(Pi a)/(2 k)]))/(a^4 + 16 k^4 (-1 + n^2)^2 + 8 a^2 k^2 (1 + n^2)) Exp[-k (n^2 + a^2/(4 k^2)) t] Exp[(-a)/(2 k) x] Sin[n x], {n, 1, numberOfTerms}];

padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 3.41: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds, a = ", 5}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {0, 1.4}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 0.95, .005}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.42: Code for animation

The following animations for $a = 1, a = 7$. Showing the effect of increasing the convection factor a on the solution.

$a = 1$	$a = 7$
	

3.1.1.28 [177] Haberman 2.4.2 (general case)

problem number 177

This is problem 2.4.2 from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

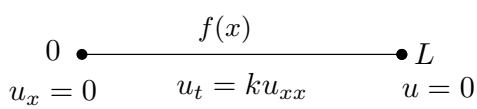


Figure 3.43: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t) \to \frac{2\sum_{n=0}^{\infty} e^{-\frac{k(2\pi n+\pi)^2 t}{4L^2}} \cos\left(\frac{(2n+1)\pi x}{2L}\right) \int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right) f(x)\,dx}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\int_0^L\cos\left(\frac{(2n+1)\pi x}{2L}\right)f(x)\,dx\right)\cos\left(\frac{(2n+1)\pi x}{2L}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2kt}{4L^2}}}{L}$$

<u>Hand solution</u>

Solve

$$\frac{\partial u}{\partial t}=\kappa\frac{\partial^2 u}{\partial x^2}$$

Let $u(x,t)=T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{\kappa}T'X=X''T$$

Dividing by XT

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is real.

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

The two ODE's are

$$T'+k\lambda T=0 \tag{1}$$
$$X''+\lambda X=0 \tag{2}$$

Per problem statement, $\lambda\geq 0$, so only two cases needs to be examined.

<u>Case $\lambda=0$</u>

The space equation becomes $X''=0$ with the solution

$$X=Ax+b$$

Hence left B.C. implies $X'(0) = 0$ or $A = 0$. Therefore the solution becomes $X = b$. The right B.C. implies $X(L) = 0$ or $b = 0$. Therefore this leads to $X = 0$ as the only solution. This results in trivial solution. Therefore $\underline{\lambda = 0 \text{ is not an eigenvalue}}$.

<u>Case $\lambda > 0$</u>

Starting with the space ODE, the solution is

$$\begin{aligned} X(x) &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\ X'(x) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right) \end{aligned}$$

Left B.C. gives

$$\begin{aligned} 0 &= X'(0) \\ &= B\sqrt{\lambda} \end{aligned}$$

Hence $B = 0$ since it is assumed $\lambda \neq 0$ and $\lambda > 0$. Solution becomes

$$X(x) = A\cos\left(\sqrt{\lambda}x\right)$$

Applying right B.C. gives

$$\begin{aligned} 0 &= X(L) \\ &= A\cos\left(\sqrt{\lambda}L\right) \end{aligned}$$

$A = 0$ leads to trivial solution. Therefore $\cos\left(\sqrt{\lambda}L\right) = 0$ or

$$\begin{aligned} \sqrt{\lambda} &= \frac{n\pi}{2L} \qquad n = 1, 3, 5, \cdots \\ &= \frac{(2n-1)\pi}{2L} \qquad n = 1, 2, 3\cdots \end{aligned}$$

Hence

$$\begin{aligned} \lambda_n &= \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1, 3, 5, \cdots \\ &= \frac{(2n-1)^2\pi^2}{4L^2} \qquad n = 1, 2, 3\cdots \end{aligned}$$

Therefore

$$X_n(x) = A_n\cos\left(\frac{n\pi}{2L}x\right) \qquad n = 1, 3, 5, \cdots$$

And the corresponding time solution

$$T_n = e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \qquad n = 1, 3, 5, \cdots$$

Hence

$$
\begin{aligned}
u_n(x,t) &= X_n T_n \\
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \sum_{n=1}^{\infty} A_n \cos\left(\frac{(2n-1)\pi}{2L}x\right) e^{-k\left(\frac{(2n-1)\pi}{2L}\right)^2 t}
\end{aligned}
$$

From initial conditions

$$
f(x) = \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right)
$$

Multiplying both sides by $\cos\left(\frac{m\pi}{2L}x\right)$ and integrating

$$
\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx = \int \left(\sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{m\pi}{2L}x\right)\cos\left(\frac{n\pi}{2L}x\right)\right) dx
$$

Interchanging order of summation and integration and applying orthogonality results in

$$
\begin{aligned}
\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx &= A_m \frac{L}{2} \\
A_n &= \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx
\end{aligned}
$$

Therefore the solution is

$$
\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty} \left[\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx\right] \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \frac{2}{L}\sum_{n=0}^{\infty} \left(\int_0^L f(x)\cos\left(\frac{(2n+1)\pi}{2L}x\right) dx\right) \cos\left(\frac{(2n+1)\pi}{2L}x\right) e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}
\end{aligned}
$$

3.1.1.29 [178] Left end zero, right end insulated, no source

problem number 178

Added January 13, 2020.

Problem 4.1.4, Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

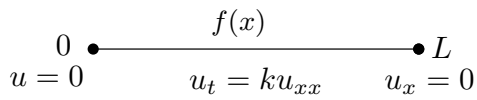


Figure 3.44: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t) \to \frac{2\sum_{n=1}^{\infty} e^{-\frac{k(1-2n)^2\pi^2 t}{4L^2}} \left(\int_0^L f(x)\sin\left(\frac{(2n-1)\pi x}{2L}\right) dx\right)\sin\left(\frac{(2n-1)\pi x}{2L}\right)}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right) dx\right) \mathrm{e}^{-\frac{\pi^2(2n+1)^2 k t}{4L^2}} \sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}$$

<u>Hand solution</u>

The problem to solve is to solve for $u(x,t)$ in

$$u_t = ku_{xx} \qquad 0 < x < L, t > 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u_x(L,t) &= 0 \end{aligned}$$

And initial conditions

$$u(x,0) = f(x)$$

Let $u(x,t) = T(t)\,X(x)$, then the PDE becomes

$$T'X = kX''T$$

Dividing by XT

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's are

$$T' + \lambda kT = 0 \tag{1}$$

And the eigenvalue ODE

$$\begin{aligned} X'' + \lambda X &= 0 \qquad (2) \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Now we solve (2) to find the eigenvalues and eigenfunctions.

<u>Case $\lambda < 0$</u>

Let $-\lambda = \omega^2$. Hence the ODE is $X'' - \omega^2 X = 0$ and the solution becomes

$$X(x) = C_1 \cosh(\omega x) + C_2 \sinh(\omega x)$$

At $x = 0$ the above gives

$$0 = C_1$$

Hence the solution now becomes

$$X(x) = C_2 \sinh(\omega x)$$

Taking derivative gives

$$X'(x) = \omega C_2 \sinh(\omega x)$$

At $x = L$

$$0 = \omega C_2 \cosh(\omega L)$$

But $\cosh(\omega L)$ is never zero. Therefore $C_2 = 0$ which leads to trivial solution. Therefore $\lambda < 0$ <u>is not eigenvalue</u>.

<u>Case $\lambda = 0$</u>

The space equation becomes $X'' = 0$ with the solution

$$X = Ax + B$$

At $x = 0$ the above gives $0 = B$. Therefore the solution is $X = Ax$. Taking derivative gives $X' = A$. At $x = L$ this gives $0 = A$. Which leads to trivial solutions. Therefore <u>$\lambda = 0$ is not an eigenvalue</u>.

<u>Case $\lambda > 0$</u>

Starting with the space ODE, the solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Left B.C. gives

$$0 = A$$

The solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivative gives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Applying right B.C. gives

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

For non trivial solution we want $\cos\left(\sqrt{\lambda}L\right) = 0$ or

$$\sqrt{\lambda} = \frac{n\pi}{2L} \qquad n = 1, 3, 5, \cdots$$

Hence the eigenvalues are

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1, 3, 5, \cdots$$

Therefore the eigenfunctions are

$$X_n(x) = \sin\left(\frac{n\pi}{2L}x\right) \qquad n = 1, 3, 5, \cdots$$

Now that we found the eigenvalues, we can solve the time ODE (1).

$$\begin{aligned} T_n' + k\lambda_n T &= 0 \\ T_n &= B_n e^{-k\lambda_n t} \\ &= B_n e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \end{aligned}$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,t) &= X_n T_n \\ u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \end{aligned} \tag{3}$$

From initial conditions

$$f(x) = \sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{n\pi}{2L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2L}x\right)$ and integrating

$$\int_0^L f(x)\sin\left(\frac{m\pi}{2L}x\right)dx = \int_0^L \left(\sum_{n=1,3,5,\cdots}^{\infty} B_n \sin\left(\frac{m\pi}{2L}xx\right)\sin\left(\frac{n\pi}{2L}x\right)\right)dx$$

Interchanging order of summation and integration and applying orthogonality between cos functions results in

$$\begin{aligned} \int_0^L f(x)\sin\left(\frac{m\pi}{2L}x\right)dx &= \int_0^L B_m \sin^2\left(\frac{m\pi}{2L}x\right)dx \\ &= B_m\frac{L}{2} \end{aligned}$$

Therefore

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx$$

Therefore the solution is (3) becomes

$$\begin{aligned} u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\sin\left(\frac{n\pi}{2L}x\right)e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\ &= \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)e^{-\left(\frac{(2n+1)\pi}{2L}\right)^2 t} \end{aligned}$$

3.1.1.30 [179] One end insulated

problem number 179

Added June 9, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = 0$$
$$u(L,t) = T_0$$

Where $T_0 > 0$ and initial condition is $u(x,0) = 0$

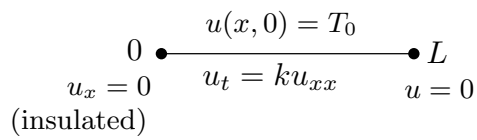


Figure 3.45: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == 0, u[1, t] == T0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{T0
 sol= sol/.{K[1]->n};
```

$$\left\{\left\{u(x,t) \to 2\sum_{n=0}^{\infty} -\frac{2e^{-\frac{1}{4}k(2\pi n+\pi)^2 t}\mathrm{T0}\cos(n\pi)\cos\left(\frac{1}{2}(2n+1)\pi x\right)}{2\pi n+\pi} + \mathrm{T0}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =0,u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = T0 - 4\left(\sum_{n=0}^{\infty} \frac{T0(-1)^n \cos\left(\frac{(2n+1)\pi x}{2L}\right) e^{-\frac{\pi^2(2n+1)^2 kt}{4L^2}}}{(2n+1)\pi}\right)$$

3.1.1.31 [180] Haberman 2.3.7 (general case)

problem number 180

This is problem 2.3.7, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u_x(0,t) = 0$, $u_x(L,t) = 0$ with initial conditions $u(x,0) = f(x)$

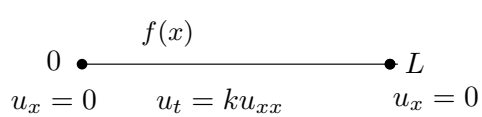


Figure 3.46: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty}e^{-\frac{kn^2\pi^2t}{L^2}}\cos\left(\frac{n\pi x}{L}\right)\int_0^L\cos\left(\frac{n\pi x}{L}\right)f(x)\,dx+\int_0^L f(x)\,dx}{L}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L\cos\left(\frac{\pi nx}{L}\right)f(x)\,dx\right)\cos\left(\frac{\pi nx}{L}\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}}{L}\right)+\frac{\int_0^L f(x)\,dx}{L}$$

<u>Hand solution</u>

$$\begin{aligned}\frac{\partial u}{\partial t}&=k\frac{\partial^2u}{\partial x^2}\\ u_x(0,t)&=0\\ u_x(L,t)&=0\\ u(x,0)&=f(x)\end{aligned}$$

Let $u(x,t)=T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{k}T'X=X''T$$

Dividing by $XT \neq 0$

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's generated are

$$T' + k\lambda T = 0 \tag{1}$$

$$X'' + \lambda X = 0 \tag{2}$$

Starting with the space ODE equation (2), with corresponding boundary conditions $\frac{dX}{dx}(0) = 0$, $\frac{dX}{dx}(L) = 0$. Assuming the solution is $X(x) = e^{rx}$, Then the characteristic equation is

$$\begin{aligned} r^2 + \lambda &= 0 \\ r^2 &= -\lambda \\ r &= \pm\sqrt{-\lambda} \end{aligned}$$

The following cases are considered.

<u>case $\lambda < 0$</u> In this case, $-\lambda$ and also $\sqrt{-\lambda}$ are positive. Hence the roots $\pm\sqrt{-\lambda}$ are both real. Let

$$\sqrt{-\lambda} = s$$

Where $s > 0$. This gives the solution

$$\begin{aligned} X(x) &= A\cosh(sx) + B\sinh(sx) \\ \frac{dX}{dx} &= A\sinh(sx) + B\cosh(sx) \end{aligned}$$

Applying the left B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(0) \\ &= B\cosh(0) \\ &= B \end{aligned}$$

The solution becomes $X(x) = A\cosh(sx)$ and hence $\frac{dX}{dx} = A\sinh(sx)$. Applying the right B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(L) \\ &= A\sinh(sL) \end{aligned}$$

$A = 0$ result in trivial solution. Therefore assuming $\sinh(sL) = 0$ implies $sL = 0$ which is not valid since $s > 0$ and $L \neq 0$. Hence only trivial solution results from this case. $\lambda < 0$ is not an eigenvalue.

case $\lambda = 0$

The ODE becomes

$$\frac{d^2X}{dx^2} = 0$$

The solution is

$$\begin{aligned} X(x) &= c_1 x + c_2 \\ \frac{dX}{dx} &= c_1 \end{aligned}$$

Applying left boundary conditions gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(0) \\ &= c_1 \end{aligned}$$

Hence the solution becomes $X(x) = c_2$. Therefore $\frac{dX}{dx} = 0$. Applying the right B.C. provides no information.

Therefore this case leads to the solution $X(x) = c_2$. Associated with this one eigenvalue, the time equation becomes $\frac{dT_0}{dt} = 0$ hence T_0 is constant, say α. Hence the solution $u_0(x,t)$ associated with this $\lambda = 0$ is

$$\begin{aligned} u_0(x,t) &= X_0 T_0 \\ &= c_2 \alpha \\ &= A_0 \end{aligned}$$

where constant $c_2\alpha$ was renamed to A_0 to indicate it is associated with $\lambda = 0$. $\lambda = 0$ is an eigenvalue.

case $\lambda > 0$

Hence $-\lambda$ is negative, and the roots are both complex.

$$r = \pm i\sqrt{\lambda}$$

The solution is

$$\begin{aligned} X(x) &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\ \frac{dX}{dx} &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right) \end{aligned}$$

Applying the left B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(0) \\ &= B\sqrt{\lambda}\cos(0) \\ &= B\sqrt{\lambda} \end{aligned}$$

Therefore $B = 0$ as $\lambda > 0$. The solution becomes $X(x) = A\cos\left(\sqrt{\lambda}x\right)$ and $\frac{dX}{dx} = -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)$. Applying the right B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(L) \\ &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}L\right) \end{aligned}$$

$A = 0$ gives a trivial solution. Selecting $\sin\left(\sqrt{\lambda}L\right) = 0$ gives

$$\sqrt{\lambda}L = n\pi \qquad n = 1, 2, 3, \cdots$$

Or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Therefore the space solution is

$$X_n(x) = A_n\cos\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time solution is found by solving

$$\frac{dT_n}{dt} + k\lambda_n T_n = 0$$

This has the solution

$$\begin{aligned} T_n(t) &= e^{-k\lambda_n t} \\ &= e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n = 1, 2, 3, \cdots \end{aligned}$$

For the same set of eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the A_n when combined in the following step below. Since for $\lambda = 0$ the time solution was found to be constant, and for $\lambda > 0$ the time solution is $e^{-k\left(\frac{n\pi}{L}\right)^2 t}$, then <u>no time solution will grow with time</u>. Time solutions always decay with time as the exponent $-k\left(\frac{n\pi}{L}\right)^2 t$ is negative quantity. The solution to the PDE for $\lambda > 0$ is

$$u_n(x,t) = T_n(t)\,X_n(x) \qquad n = 0, 1, 2, 3, \cdots$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned} u(x,t) &= u_{\lambda=0}(x,t) + \sum_{n=1}^{\infty} u_n(x,t) \\ &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned}$$

From the solution found above, setting $t=0$ gives

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides with $\cos\left(\frac{m\pi}{L}x\right)$ where in this problem $m = 0, 1, 2, \cdots$ (since there was an eigenvalue associated with $\lambda = 0$), and integrating over the domain gives

$$\begin{aligned} \int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right) dx &= \int_0^L \cos\left(\frac{m\pi}{L}x\right)\left(A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)\right) dx \\ &= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \int \cos\left(\frac{m\pi}{L}x\right) \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) dx \\ &= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \int_0^L \sum_{n=1}^{\infty} A_n \cos\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx \end{aligned}$$

Interchanging the order of summation and integration

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right) dx = \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx \tag{1}$$

<u>case $m = 0$</u>

When $m = 0$ then $\cos\left(\frac{m\pi}{L}x\right) = 1$ and the above simplifies to

$$\int_0^L f(x)\, dx = \int_0^L A_0 dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{n\pi}{L}x\right) dx$$

But $\int_0^L \cos\left(\frac{n\pi}{L}x\right) dx = 0$ and the above becomes

$$\begin{aligned} \int_0^L f(x)\, dx &= \int_0^L A_0 dx \\ &= A_0 L \end{aligned}$$

Therefore

$$A_0 = \frac{1}{L}\int_0^L f(x)\, dx$$

<u>case $m > 0$</u>

From (1), one term survives in the integration when only $n = m$, hence

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx = A_0\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx + A_m\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx = 0$ and the above becomes

$$\int_0^L f(x)\cos\left(\frac{m\pi}{L}x\right)dx = A_m\frac{L}{2}$$

Therefore

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

For $n = 1, 2, 3, \cdots$

Therefore the solution is

$$\begin{aligned} u(x,t) &= A_0 + \sum_{n=1}^{\infty} A_n\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \\ &= \frac{1}{L}\int_0^L f(x)\,dx + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned}$$

In the limit as $t \to \infty$ the term $e^{-k\left(\frac{n\pi}{L}\right)^2 t} \to 0$. What is left is A_0. But $A_0 = \frac{1}{L}\int_0^L f(x)\,dx$ from above. This quantity is the average of the initial temperature.

3.1.1.32 [181] specific case

problem number 181

Added June 21, 2019

Solve

$$u_t = ku_{xx}$$

Subject to boundary conditions $u_x(0,t) = 0$, $u_x(L,t) = 0$ with initial conditions $u(x,0) = f(x)$ using the following values

$$\begin{aligned} L &= 5 \\ k &= \frac{1}{100} \\ f(x) &= x \end{aligned}$$

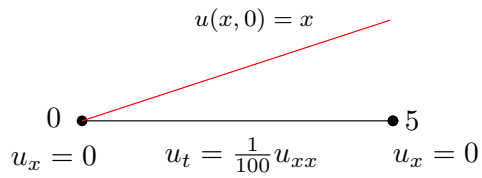


Figure 3.47: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
L=5;
f=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t) \to \frac{2}{5}\sum_{n=1}^{\infty}\frac{25(-1+(-1)^n)\,e^{-\frac{n^2\pi^2 t}{2500}}\cos\left(\frac{n\pi x}{5}\right)}{n^2\pi^2}+\frac{5}{2}\right\}\right\}$$

Maple ✓

```
restart;
L:=5;
k:=1/100;
f:=x;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 10\left(\sum_{n=1}^{\infty}\frac{((-1)^n-1)\cos\left(\frac{\pi n x}{5}\right)\mathrm{e}^{-\frac{\pi^2 n^2 t}{2500}}}{\pi^2 n^2}\right)+\frac{5}{2}$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 3.1.1.31 on page 485 as

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

In this problem $u(x,0) = f(x) = x, L = 5, k = \frac{1}{100}$. Hence the above becomes

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{5}x\right) e^{-\left(\frac{1}{100}\left(\frac{n\pi}{5}\right)^2 t\right)} \tag{2}$$

At $t = 0$ the above becomes

$$x = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{5}x\right)$$

For $n = 0$ orthogonality gives

$$\begin{aligned}
\int_0^5 x dx &= \int_0^5 A_0 dx \\
\frac{25}{2} &= 5A_0 \\
A_0 &= \frac{5}{2}
\end{aligned}$$

For $n > 0$

$$\begin{aligned}
\int_0^5 x \cos\left(\frac{n\pi}{5}x\right) dx &= \int_0^5 A_n \cos^2\left(\frac{n\pi}{5}x\right) dx \\
\frac{25}{n^2\pi^2}(-1 + (-1)^n) &= \frac{5}{2}A_n \\
A_n &= \frac{10}{n^2\pi^2}(-1 + (-1)^n)
\end{aligned}$$

Hence the solution (2) becomes

$$u(x,t) = \frac{5}{2} + \frac{10}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2}(-1 + (-1)^n) \cos\left(\frac{n\pi}{5}x\right) e^{-\frac{n^2\pi^2}{2500}t}$$

The following is an animation of the solution

Out[ ]=

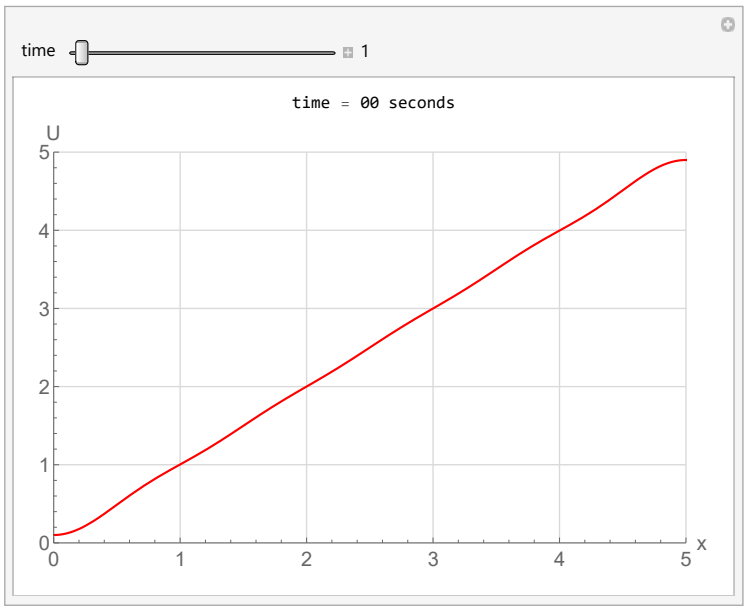


Figure 3.48: Initial state

Source code used for the above

In[ ]:=
```
ClearAll[x, y, t, n, k]
L = 5;
k = 1 / 100;
f = x;
numberOfTerms = 10;
mySol[x_, t_] = 5/2 + 10/π^2 Sum[1/n^2 (-1 + (-1)^n) Exp[- (n^2 π^2/2500) t] Cos[n π/5 x], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 3.49: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {2, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {0, 5}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 700, 2}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.50: Code for animation

3.1.1.33 [182] Haberman 2.4.1 (a)

problem number 182

This is problem 2.4.1 part(a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$. Initial conditions

$$u(x,0) = \begin{cases} 0 & x < \frac{L}{2} \\ 1 & x > \frac{L}{2} \end{cases}$$

$$u(x,0) = \begin{cases} 0 & 0 < x \leq \frac{L}{2} \\ 1 & \frac{L}{2} < x \leq L \end{cases}$$

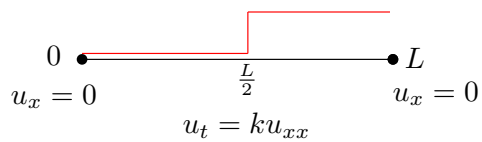


Figure 3.51: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[1] -> n};
```

$Aborted

Maple ✓

```
restart;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = -2\left(\sum_{n=1}^{\infty} \frac{\cos\left(\frac{\pi n x}{L}\right) e^{-\frac{\pi^2 k n^2 t}{L^2}} \sin\left(\frac{\pi n}{2}\right)}{\pi n}\right) + \frac{1}{2}$$

3.1.1.34 [183] Both ends insulated, no source

problem number 183

Added January 13, 2020

This is problem 4.1.7 Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$. Initial conditions

$$u(x,0) = \begin{cases} x & 0 < x < \frac{L}{2} \\ 1-x & \frac{L}{2} < x < L \end{cases} \quad Use\text{L=1.}$$

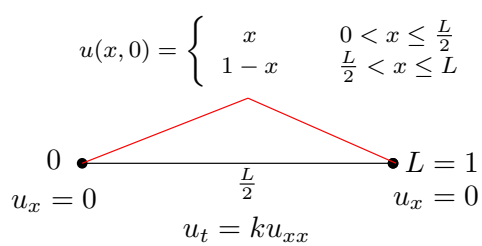


Figure 3.52: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=1;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == Piecewise[{{x, 0<x < L/2}, {1-x, L/2<=x<L}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to 2\sum_{n=1}^{\infty} \frac{4e^{-kn^2\pi^2 t}\cos\left(\frac{n\pi}{2}\right)\cos(n\pi x)\sin^2\left(\frac{n\pi}{4}\right)}{n^2\pi^2} + \frac{1}{4}\right\}\right\}$$

Maple ✓

```
restart;
L:=1;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=piecewise(0<x and x<=L/2,x,L/2<x and x<L,1-x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 2\left(\sum_{n=1}^{\infty}\left(-\frac{\left((-1)^n - 2\cos\left(\frac{\pi n}{2}\right)+1\right)\cos(\pi n x)\,\mathrm{e}^{-\pi^2 k n^2 t}}{\pi^2 n^2}\right)\right) + \frac{1}{4}$$

<u>Hand solution</u>

Let $u(x,t) = T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{k}T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

The two ODE's generated are

$$T' + k\lambda T = 0 \tag{1}$$

And the eigenvalue ODE

$$\begin{aligned} X'' + \lambda X &= 0 \qquad (2)\\ X'(0) &= 0\\ X'(L) &= 0 \end{aligned}$$

Starting with the eigenvalue ODE equation (2). The following cases are considered.

<u>case $\lambda < 0$</u>

In this case, $-\lambda$ is positive. Let $-\lambda = \omega^2$. Hence the ODE is $X'' - \omega^2 X = 0$ and the solution becomes

$$X(x) = C_1\cosh(\omega x) + C_2\sinh(\omega x)$$

Therefore

$$X' = C_1 \sinh(\omega x) + C_2 \cosh(\omega x)$$

Applying the left B.C. gives

$$0 = C_2$$

Therefore the solution becomes $X(x) = C_1 \cosh(\omega x)$ and $X'(x) = C_1 \sinh(\omega x)$. Applying the right B.C. gives

$$0 = C_1 \sinh(\omega L)$$

For non-trivial solution we want $\sinh(\omega L) = 0$. But this is not possible since sinh is zero when its argument is zero, which is not the case here. Hence only trivial solution results from this case. $\lambda < 0$ is not an eigenvalue.

<u>case $\lambda = 0$</u>

The solution is

$$\begin{aligned} X(x) &= c_1 x + c_2 \\ X'(x) &= c_1 \end{aligned}$$

Applying left boundary conditions gives

$$0 = c_1$$

Hence the solution becomes $X(x) = c_2$. Therefore $\frac{dX}{dx} = 0$. Applying the right B.C. provides no information. Any c_2 will work. Therefore this case leads to the solution $X(x) = c_2$. Associated with this one eigenvalue, the time equation becomes $T_0'(t) = 0$ hence $T_0(t)$ is a constant. Hence the solution $u_0(x,t)$ associated with this $\lambda = 0$ is

$$\begin{aligned} u_0(x,t) &= X_0 T_0 \\ &= A_0 \end{aligned}$$

where constant $c_2 T_0$ was renamed to $\frac{A_0}{2}$ to indicate it is associated with $\lambda = 0$. <u>$\lambda = 0$ is an eigenvalue</u> with eigenfunction constant $\frac{A_0}{2}$.

<u>case $\lambda > 0$</u>

The solution is

$$\begin{aligned} X(x) &= c_1 \cos\left(\sqrt{\lambda} x\right) + c_2 \sin\left(\sqrt{\lambda} x\right) \\ X'(x) &= -c_1 \sqrt{\lambda} \sin\left(\sqrt{\lambda} x\right) + c_2 \sqrt{\lambda} \cos\left(\sqrt{\lambda} x\right) \end{aligned}$$

Applying the left B.C. gives

$$0 = c_2 \sqrt{\lambda}$$

Therefore $c_2 = 0$ as $\lambda > 0$. The solution becomes

$$X(x) = c_1 \cos\left(\sqrt{\lambda}x\right)$$

And $X'(x) = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)$. Applying the right B.C. gives

$$0 = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}L\right)$$

$c_1 = 0$ gives a trivial solution. Selecting $\sin\left(\sqrt{\lambda}L\right) = 0$ gives

$$\sqrt{\lambda}L = n\pi \qquad n = 1, 2, 3, \cdots$$

Or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Therefore the eigenfunctions are

$$X_n(x) = \cos\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time solution is found by solving

$$T_n'(t) + k\lambda_n T_n(t) = 0$$

This has the solution

$$\begin{aligned} T_n(t) &= A_n e^{-k\lambda_n t} \\ &= A_n e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n = 1, 2, 3, \cdots \end{aligned}$$

The solution to the PDE is

$$u_n(x,t) = T_n(t)\, X_n(x) \qquad n = 0, 1, 2, 3, \cdots$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned} u(x,t) &= u_0(x,t) + \sum_{n=1}^{\infty} u_n(x,t) \\ &= \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned} \tag{1}$$

From the solution found above, setting $t = 0$ gives

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Hence A_0, A_n are the Fourier cos coefficients for the function $f(x)$. Doing an even extension of $f(x)$ from $[-L, L]$, then $\frac{A_0}{2}$ is the average of the function $f(x)$ over $[-L, L]$. But this average is seen as $\frac{2(\frac{1}{2}\times\frac{1}{2})}{2} = \frac{1}{4}$. The term $\frac{1}{2} \times \frac{1}{2}$ is the area of $f(x)$ from $[0, L]$.

$$\frac{A_0}{2} = \frac{1}{4}$$

For A_n

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

Replacing $L = 1$ and using the definition of $f(x)$ given above gives

$$A_n = \int_{-1}^{1} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

But $f(x)$ is even (after even extending) and cos is even, hence the above becomes

$$\begin{aligned}
A_n &= 2\int_0^1 f(x)\cos(n\pi x)\,dx \\
&= 2\left(\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx + \int_{\frac{1}{2}}^1 (1-x)\cos(n\pi x)\,dx\right) \\
&= 2\left(\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx + \int_{\frac{1}{2}}^1 \cos(n\pi x)\,dx - \int_{\frac{1}{2}}^1 x\cos(n\pi x)\,dx\right) \qquad (2)
\end{aligned}$$

But

$$\begin{aligned}
\int_a^b x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_a^b - \frac{1}{n\pi}\int_a^b \sin(n\pi x)\,dx \\
&= \frac{1}{n\pi}[x\sin(n\pi x)]_a^b + \frac{1}{n^2\pi^2}[\cos(n\pi x)]_a^b \qquad (3)
\end{aligned}$$

When $a = 0, b = \frac{1}{2}$ the above gives

$$\begin{aligned}
\int_0^{\frac{1}{2}} x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_0^{\frac{1}{2}} + \frac{1}{n^2\pi^2}[\cos(n\pi x)]_0^{\frac{1}{2}} \\
&= \frac{1}{n\pi}\left(\frac{1}{2}\sin\left(\frac{n\pi}{2}\right)\right) + \frac{1}{n^2\pi^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right) \\
&= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right) \\
&= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^2} \qquad (4)
\end{aligned}$$

And when $a=\frac{1}{2}, b=1$ (3) gives

$$\begin{aligned}\int_{\frac{1}{2}}^{1} x\cos(n\pi x)\,dx &= \frac{1}{n\pi}[x\sin(n\pi x)]_{\frac{1}{2}}^{1}+\frac{1}{n^2\pi^2}[\cos(n\pi x)]_{\frac{1}{2}}^{1}\\ &= \frac{1}{n\pi}\left[\sin(n\pi)-\frac{1}{2}\sin\left(\frac{n\pi}{2}\right)\right]+\frac{1}{n^2\pi^2}\left[\cos(n\pi)-\cos\left(\frac{n\pi}{2}\right)\right]\\ &= -\frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)+\frac{1}{n^2\pi^2}\cos(n\pi)-\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) \qquad (5)\end{aligned}$$

Substituting (4,5) into (2) gives

$$\begin{aligned}\frac{A_n}{2} &= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)+\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\\ &+\int_{\frac{1}{2}}^{1}\cos(n\pi x)\,dx\\ &-\left(-\frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)+\frac{1}{n^2\pi^2}\cos(n\pi)-\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)\right)\end{aligned}$$

Or

$$\begin{aligned}\frac{A_n}{2} &= \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)+\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\\ &+\frac{1}{n\pi}\overbrace{\sin(n\pi)}^{0}-\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)\\ &+\frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\cos(n\pi)+\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)\end{aligned}$$

Or

$$\begin{aligned}\frac{A_n}{2} &= \left(\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)+\frac{2}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\right)-\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\cos(n\pi)\\ &= \frac{2\cos\left(\frac{n\pi}{2}\right)-1-(-1)^n}{n^2\pi^2}\end{aligned}$$

Therefore the solution (1) becomes, after replacing $L=1$

$$u(x,t)=\frac{1}{4}+2\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi}{2}\right)-1-(-1)^n}{n^2\pi^2}\cos(n\pi x)\,e^{-kn^2\pi^2 t}$$

3.1.1.35 [184] Haberman 2.4.1 (b) (special case)

problem number 184

This is problem 2.4.1 part(b) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = 6 + 4\cos\left(\frac{3\pi x}{L}\right)$

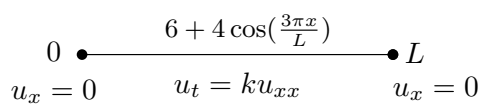


Figure 3.53: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
NumericQ[L] =. ;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == 6 + 4*Cos[(3*Pi*x)/L];
NumericQ[L] = True;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n};
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t) \to 4e^{-\frac{9\pi^2 k t}{L^2}} \cos\left(\frac{3\pi x}{L}\right) + 6\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0 and k>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=6+4*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 4\cos\left(\frac{3\pi x}{L}\right) \mathrm{e}^{-\frac{9\pi^2 kt}{L^2}} + 6$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 3.1.1.31 on page 485 as

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

In this example $u(x,0) = f(x) = 6 + 4\cos\frac{3\pi x}{L}$. Hence at $t = 0$ the above becomes

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$
$$6 + 4\cos\frac{3\pi x}{L} = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Comparing terms shows that

$$A_0 = 6$$
$$A_3 = 4$$

And all other $A_n = 0$. Hence the solution (1) is

$$u(x,t) = 6 + 4\cos\left(\frac{3\pi}{L}x\right) e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

3.1.1.36 [185] Haberman 2.4.1 (c) (special case)

problem number 185

This is problem 2.4.1 part(c) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = -2\sin\frac{\pi x}{L}$

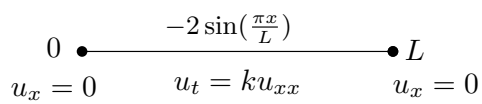


Figure 3.54: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == -2*Sin[(Pi*x)/L];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \frac{2\sum_{n=1}^{\infty} \frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2 t}{L^2}} L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L} - \frac{4}{\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
assume(L>0 and k>0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic  := u(x,0)=-2*sin(Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\left(\sum_{n=2}^{\infty}\frac{4((-1)^n+1)\cos\left(\frac{\pi n x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}}{\pi\left(n^2-1\right)}\right)-\frac{4}{\pi}$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 3.1.1.31 on page 485 as

$$u(x,t)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

At $t=0$ the above becomes

$$\begin{aligned}f(x)&=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\\-2\sin\frac{\pi x}{L}&=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\end{aligned}$$

Multiplying both sides by $\cos\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}-2\int_0^L\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx&=\int_0^L\left(A_0\cos\left(\frac{m\pi}{L}x\right)+\cos\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\right)dx\\&=\int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\int_0^L\sum_{n=1}^{\infty}A_n\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx\end{aligned}$$

Interchanging the order of integration and summation

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx=\int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\sum_{n=1}^{\infty}A_n\int_0^L\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

<u>Case $m=0$</u>

The above becomes

$$-2\int_0^L \sin\left(\frac{\pi x}{L}\right)dx = \int_0^L A_0 dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{n\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{n\pi}{L}x\right)dx = 0$ hence

$$\begin{aligned}
\int_0^L -2\sin\left(\frac{\pi x}{L}\right)dx &= \int_0^L A_0 dx \\
A_0 L &= -2\int_0^L \sin\left(\frac{\pi x}{L}\right)dx \\
A_0 L &= -2\left(-\frac{\cos\left(\frac{\pi x}{L}\right)}{\frac{\pi}{L}}\right)_0^L \\
&= -\frac{2L}{\pi}\left(-\cos\left(\frac{\pi L}{L}\right)+\cos\left(\frac{\pi 0}{L}\right)\right) \\
&= -\frac{2L}{\pi}(-(-1)+1) \\
&= -\frac{4L}{\pi}
\end{aligned}$$

Hence

$$A_0 = \frac{-4}{\pi}$$

<u>Case $m > 0$</u>

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

One term survives the summation resulting in

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \frac{-4}{\pi}\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx + A_m\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx = 0$ and $\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$, therefore

$$\begin{aligned}
\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx &= A_m\frac{L}{2} \\
A_n &= \frac{-4}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx
\end{aligned}$$

But

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{-L(1+\cos(n\pi))}{\pi(n^2-1)}$$

Therefore

$$\begin{aligned} A_n &= 4\frac{(1+\cos(n\pi))}{\pi(n^2-1)} \\ &= 4\frac{(-1)^n+1}{\pi(n^2-1)} \qquad n=1,2,3,\cdots \end{aligned}$$

Hence the solution becomes

$$u(x,t) = \frac{-4}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n+1}{(n^2-1)}\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

3.1.1.37 [186] Haberman 2.4.1 (d)

problem number 186

This is problem 2.4.1 part(d) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t)=0$ and $\frac{\partial u}{\partial x}(L,t)=0$ with the temperature initially $u(x,0)=-3\cos\frac{8\pi x}{L}$

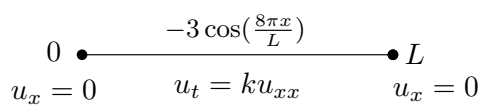


Figure 3.55: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
NumericQ[L] =. ;

pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == -3*Cos[(8*Pi*x)/L];
NumericQ[L] = True;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n};
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t) \to -3e^{-\frac{64\pi^2 kt}{L^2}} \cos\left(\frac{8\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0 and k>0);
ic  := u(x,0)=-3*cos(8*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = -3\cos\left(\frac{8\pi x}{L}\right) \mathrm{e}^{-\frac{64\pi^2 kt}{L^2}}$$

3.1.1.38 [187] both ends insulated

problem number 187

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve the heat equation for $u(x,t)$

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$

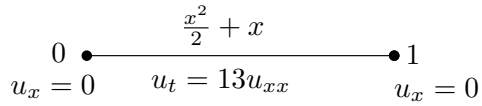


Figure 3.56: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 13*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
ic  = u[x, 0] == (1*x^2)/2 + x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(2\sum_{K[1]=1}^{\infty}\frac{2\left(-1+(-1)^{K[1]}\right)e^{-13\pi^2 t K[1]^2}\cos(\pi x K[1])}{\pi^2 K[1]^2} + 26t + x^2 + 1\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0 , eval( diff(u(x,t),x),x=1)=1;
ic  := u(x,0)=1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t))),out
```

$$u(x,t) = \frac{x^2}{2} + 13t + 2\left(\sum_{n=1}^{\infty}\frac{\left((-1)^n - 1\right)\cos(\pi n x)\,\mathrm{e}^{-13\pi^2 n^2 t}}{\pi^2 n^2}\right) + \frac{1}{2}$$

3.1.1.39 [188] convection heat loss

problem number 188

This problem is taken from Maple primes post

Left end insulated, right end has convection heat loss `https://www.mapleprimes.com/posts/209681-Solving-PDEs-With-Initial-And-Boundary`

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are, on the left end $\frac{\partial u}{\partial x}(0,t) = 0$ and on the right end $\frac{\partial u}{\partial x}(1,t) = -u(1,t)$ with the temperature initially $u(x,0) = 1 - \frac{1}{4}x^3$

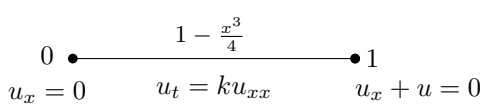


Figure 3.57: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == 1 - (1*x^3)/4;
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == -u[1, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \{ \sum_{K[1]=1}^{\infty} - \frac{e^{-ktK[2,K[1]]} \cos\left(x\sqrt{K[2,K[1]]}\right)\left(3\cos\left(\sqrt{K[2,K[1]]}\right)(K[2,K[1]]-2) - 3\sqrt{K[2,K[1]]}(K[2,K[1]]+2)\sin\left(\sqrt{\right.\right.}{\sqrt{2}\sqrt{3-\cos\left(2\sqrt{K[2,K[1]]}\right)}K[2,K[1]]^2\sqrt{\sin^2\left(\sqrt{K[2,K[1]]}\right)+1}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic  := u(x,0) = 1-(1/4)*x^3;
bc  := eval(diff(u(x,t), x), x = 0) = 0, eval(diff(u(x,t), x), x = 1)+u(1,t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 3\left(\sum_{n=0}^{\infty} \frac{(\lambda_n^3 \sin(\lambda_n) - \lambda_n^2 \cos(\lambda_n) + 2\lambda_n \sin(\lambda_n) + 2\cos(\lambda_n) - 2)\cos(x\lambda_n)\,\mathrm{e}^{-kt\lambda_n^2}}{(2\lambda_n + \sin(2\lambda_n))\,\lambda_n^3}\right) \textbf{where}\{\lambda_n \text{ ta}$$

3.1.1.40 [189] Pinchover and Rubinstein 6.25

problem number 189

Added July 2, 2018. Taken from Maple 2018.1 document, originally exercise 6.25 from Pinchover and Rubinstein.

Solve the heat equation

$$u_t = ku_{xx} + \cos(wt)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = x$.

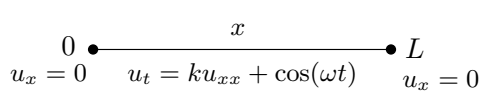


Figure 3.58: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Cos[w*t];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\frac{2\left(-1+(-1)^{K[1]}\right)e^{-\frac{k\pi^2 tK[1]^2}{L^2}}L\cos\left(\frac{\pi xK[1]}{L}\right)}{\pi^2K[1]^2}+\frac{L}{2}+\frac{\sin(tw)}{w}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+cos(w*t);
bc  := (D[1](u))(L, t) = 0, (D[1](u))(0, t) = 0;
ic  := u(x, 0) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{L}{2}+\left(\sum_{n=1}^{\infty}\frac{2((-1)^n-1)L\cos\left(\frac{\pi nx}{L}\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}}{\pi^2n^2}\right)+\frac{\sin(tw)}{w}$$

3.1.1.41 [190] external source

problem number 190

Added March 18, 2018.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2u}{\partial x^2}+\left(e^{-ct}\sin\left(\frac{2\pi x}{L}\right)\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

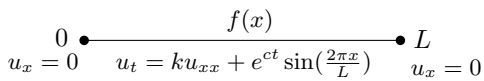


Figure 3.59: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Exp[-(c*t)]*Sin[(2*Pi*x)/L];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(e^{-\frac{k\pi^2 t K[1]^2}{L^2}}\int_0^L \frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)f(x)}{\sqrt{L}}\,dx + \left(\{\begin{array}{c} \frac{2\sqrt{2}(-1+(-1)^{K[1]})e^{-t\left(\frac{k\pi^2 K[1]^2}{L^2}\right.}}{\pi(K[1]^2-4)(k\pi} \\ 0 \end{array}\right.\right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+(exp(-c*t)*sin(2*Pi*x/L));
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L \cos\left(\frac{\pi n x}{L}\right) f(x)\,dx\right)\cos\left(\frac{\pi n x}{L}\right) \mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}}{L}\right) + \frac{\int_0^L f(x)\,dx}{L} + \frac{-\frac{8L^2\cos\left(\frac{\pi x}{L}\right)\mathrm{e}^{-ct}}{3} + \frac{8L^2\cos\left(\frac{\pi x}{L}\right)\mathrm{e}^{-\pi}}{3}}{}$$

3.1.1.42 [191] Diffusion Reaction (general case)

problem number 191

Added June 9, 2019

Consider the heat equation

$$u_t = k u_{xx} - \beta u$$

Suppose the boundary conditions are $u_x(0,t) = 0, u_x(\pi,t) = 0$, solve with the temperature initially $u(x,0) = x$

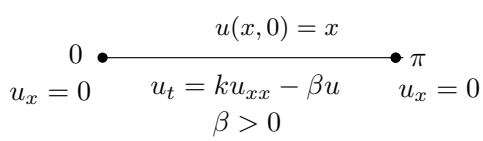


Figure 3.60: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] - beta*u[x, t];
bc  = {Derivative[1,0][u][0, t] == 0, Derivative[1,0][u][Pi, t] == 0};
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{2\left(-1+(-1)^{K[1]}\right)e^{-t\left(kK[1]^2+\beta\right)}\cos(xK[1])}{\pi K[1]^2}+\frac{1}{2}\pi e^{-\beta t}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-beta*u(x,t);
bc:=eval(diff(u(x,t),x),x=0)=0,eval(diff(u(x,t),x),x=Pi)=0;
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=2\left(\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)\cos(nx)\,\mathrm{e}^{-\left(kn^2+\beta\right)t}}{\pi n^2}\right)+\frac{\pi\,\mathrm{e}^{-\beta t}}{2}$$

<u>Hand solution</u>

Solve

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}-\beta u \qquad 0<x<\pi, t>0$$

with, $k>0, \beta>0$

$$\frac{\partial u(0,t)}{\partial x}=0$$
$$\frac{\partial u(\pi,t)}{\partial x}=0$$

And initial conditions

$$u(x,0)=x$$

<u>Solution</u>

Let $u = X(x)\,T(t)$. Substituting into the PDE gives

$$T'X = kX''T - \beta XT$$

Dividing by $XT \neq 0$ gives

$$\begin{aligned}
\frac{T'}{T} &= \frac{kX''}{X} - \beta \\
\frac{T'}{T} + \beta &= \frac{kX''}{X} \\
\frac{T'}{kT} + \frac{\beta}{k} &= \frac{X''}{X}
\end{aligned}$$

Since each sides depends on different variable and both are equal, they must be equal to same constant, say $-\lambda$

$$\frac{T'}{\alpha T} + \frac{\beta}{\alpha} = \frac{X''}{X} = -\lambda$$

This gives two ODE's to solve

$$\begin{aligned}
X'' + \lambda X &= 0 \qquad (1) \\
X'(0) &= 0 \\
X'(\pi) &= 0
\end{aligned}$$

And

$$\begin{aligned}
\frac{T'}{kT} + \frac{\beta}{k} &= -\lambda \qquad (2) \\
T' + \beta T &= -\lambda k T \\
T' + T(\lambda k + \beta) &= 0
\end{aligned}$$

Starting with (1).

<u>Assuming $\lambda < 0$</u>, the solution is

$$\begin{aligned}
X(x) &= A\cosh\left(\sqrt{-\lambda}x\right) + B\sinh\left(\sqrt{-\lambda}x\right) \\
X' &= \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}x\right) + \sqrt{-\lambda}B\cosh\left(\sqrt{-\lambda}x\right)
\end{aligned}$$

Applying first B.C. gives $0 = \sqrt{-\lambda}B$, hence $B = 0$. Therefore the solution becomes $X(x) = A\cosh\left(\sqrt{-\lambda}x\right)$ and $X' = \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}x\right)$. Applying second B.C. gives

$$0 = \sqrt{-\lambda}A\sinh\left(\sqrt{-\lambda}\pi\right)$$

But $\sinh\left(\sqrt{-\lambda}\pi\right) = 0$ only when its argument is zero, which is not the case here. This means $A = 0$, leading to trivial solution. Therefore $\lambda < 0$ is not eigenvalue.

Assuming $\lambda = 0$. The solution is $X = Ax + B$. Hence $X' = A$. Applying first B.C. Gives $A = 0$. Hence solution is $X = B$. Second boundary condition gives no additional information. Therefore $X = 1$ (constant) is the eigenfunction associated with $\lambda = 0$.

Assuming $\lambda > 0$. The solution is

$$\begin{aligned} X &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\ X' &= -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}x\right) + \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right) \end{aligned}$$

Applying first B.C. gives $0 = \sqrt{\lambda}B$, hence $B = 0$. and the solution becomes $X = A\cos\left(\sqrt{\lambda}x\right), X' = -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}x\right)$. Applying second B.C. gives

$$0 = -\sqrt{\lambda}A\sin\left(\sqrt{\lambda}\pi\right)$$

Therefore

$$\begin{aligned} \sin\left(\sqrt{\lambda}\pi\right) &= 0 \\ \sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\ \lambda &= n^2 \qquad n = 1, 2, 3, \cdots \end{aligned}$$

Therefore the space solution is

$$X(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) \tag{3}$$

Now the time ODE (2) is solved.

$$T' + T(\lambda k + \beta) = 0$$

Integrating factor is $e^{\int \lambda k + \beta dt} = e^{(\lambda k+\beta)t}$. Hence $\frac{d}{dt}\left(Te^{(\lambda k+\beta)t}\right) = 0$ or $Te^{(\lambda k+\beta)t} = c$ where c is constant. Therefore

$$T(t) = ce^{-(\lambda k+\beta)t}$$

For $\lambda = 0$, the solution is

$$T_0(t) = c_0 e^{-\beta t}$$

and for $\lambda > 0$, the solution is

$$T_n(t) = c_n e^{-(n^2 k+\beta)t}$$

Hence the time domain solution is

$$T(t) = c_0 e^{-\beta t} + \sum_{n=1}^{\infty} c_n e^{-(n^2 k+\beta)t} \tag{4}$$

Combining (3,4) the solution is

$$\begin{aligned} u_n &= X_n T_n \\ u(x,t) &= \sum_{n=0}^{\infty} X_n T_n \\ &= c_0 e^{-\beta t} + \sum_{n=1}^{\infty} b_n \cos(nx)\, e^{-(n^2 k+\beta)t} \end{aligned} \tag{5}$$

where both constants from space and time eigenfunctions are combined into one constant b_n above in the sum. Now initial conditions are used to find coefficients. At $t=0$

$$\begin{aligned} u(x,0) &= c_0 + \sum_{n=1}^{\infty} b_n \cos(nx) \\ x &= c_0 + \sum_{n=1}^{\infty} b_n \cos(nx) \end{aligned}$$

Multiplying both sides by $\cos(mx)$ and integrating gives

$$\int_0^{\pi} x \cos mx dx = \int_0^{\pi} \left(c_0 \cos mx + \sum_{n=1}^{\infty} b_n \cos mx \cos(nx) \right) dx$$

For $m=0$, all terms in the sum in RHS vanish, and only c_0 left

$$\begin{aligned} \int_0^{\pi} x dx &= \int_0^{\pi} c_0 dx \\ &= c_0 \pi \end{aligned}$$

Hence

$$\begin{aligned} c_0 &= \frac{1}{\pi} \int_0^{\pi} x dx \\ &= \frac{1}{\pi} \frac{\pi^2}{2} \\ &= \frac{\pi}{2} \end{aligned} \tag{6}$$

For $n>0$

$$\begin{aligned} \int_0^{\pi} x \cos mx dx &= \int_0^{\pi} \sum_{n=1}^{\infty} b_n \cos mx \cos(nx)\, dx \\ &= \sum_{n=1}^{\infty} b_n \int_0^{\pi} \cos mx \cos(nx)\, dx \end{aligned}$$

All terms in the sum vanish except for $n = m$. The above becomes

$$\int_0^\pi x\cos mx dx = b_n \frac{\pi}{2}$$

$$b_n = \frac{2}{\pi}\int_0^\pi x\cos mx dx \tag{7}$$

Using integration by part, $\int_0^\pi x\cos nx dx$, let $u = x, dv = \cos nx, \rightarrow du = 1, v = \frac{\sin nx}{n}$, therefore

$$\begin{aligned}\int_0^\pi x\cos nx dx &= \left(x\frac{\sin nx}{n}\right)_0^\pi - \frac{1}{n}\int_0^\pi \sin nx dx\\ &= \frac{1}{n}(\pi\sin n\pi - 0) + \frac{1}{n}\left(\frac{\cos nx}{n}\right)_0^\pi\\ &= 0 + \frac{1}{n^2}(\cos nx)_0^\pi\\ &= \frac{1}{n^2}(\cos n\pi - 1)\end{aligned}$$

Hence

$$\begin{aligned}b_n &= \frac{2}{\pi}\int_0^\pi x\cos mx dx\\ &= \frac{2}{\pi}\frac{(-1^n - 1)}{n^2}\end{aligned}$$

Therefore the final solution is, from (5), by combing all above results, becomes

$$\begin{aligned}u(x,t) &= c_0 e^{-\beta t} + \sum_{n=1}^{\infty} b_n \cos(nx)\, e^{-(n^2k+\beta)t}\\ &= \frac{\pi}{2}e^{-\beta t} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1^n - 1)}{n^2}\cos(nx)\, e^{-(n^2k+\beta)t}\end{aligned}$$

This is animation of the solution for $k = 1, \beta = 2$ for 2 seconds. (Animation will only show in the HTML version)

Out[ ]=

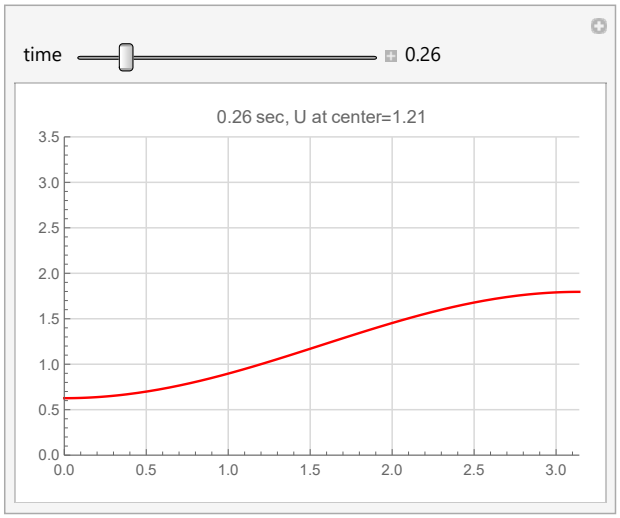


Figure 3.61: Screen shot

Source code used for the above

```
beta = 1; k = 2;
T0[x_, t_, m_] := Quiet[Pi/2 Exp[-beta t] + 2/Pi Sum[((-1)^n - 1)/n^2 Cos[n x] Exp[-(n^2 k + beta) t], {n, 1, m}]];
p = Plot3D[T0[x, t, 20], {x, 0, Pi}, {t, 0, .1}, PlotRange → All, AxesLabel → {x, "sec", u}, BaseStyle → 15]

In[ ]:= padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Plot[T0[x, t, 20], {x, 0, Pi},
    PlotLabel → Row[{t, " sec,", " U at center=", padIt2[N@T0[Pi / 2, t, 100], {3, 2}]}],
    PlotRange → {{0, Pi}, {0, 3.5}}, GridLines → Automatic, GridLinesStyle → LightGray, PlotStyle → Red
  ],
  {{t, 0, "time"}, 0, 2, .01, Appearance → "Labeled"}]

In[ ]:= r = Table[
      Plot[T0[x, t, 20], {x, 0, Pi},
        PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center = ", padIt2[N@T0[Pi / 2, t, 100], {3, 2}]}],
        PlotRange → {{0, Pi}, {0, 3.5}},
        GridLines → Automatic,
        GridLinesStyle → LightGray,
        PlotStyle → Red
      ],
      {t, 0, 2, .01}];

In[ ]:= Export["anim.gif", r, "DisplayDurations" → Table[0.1, {Length@r}]]

Out[ ]= anim.gif
```

Figure 3.62: Source code

3.1.1.43 [192] Pinchover and Rubinstein 6.23

problem number 192

Added July 2, 2018.

4th example from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.23 .

Solve the heat equation on bar

$$u_t = u_{xx} + g(x,t)$$

Where $g(x,t) = e^{3t}\cos(17\pi x)$ for $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = 3\cos(42\pi x)$.

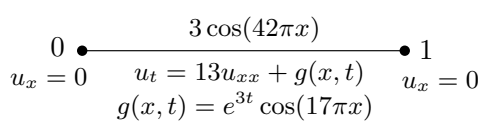


Figure 3.63: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f[x] := 3*Cos[42*Pi*x];
 g[x, t] := Exp[3*t]*Cos[17*x*Pi];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + g[x, t];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \sqrt{2}\cos(\pi x K[1])\left(\int_0^t e^{-\pi^2 K[1]^2 (t-K[2])}\left(\begin{cases} \frac{e^{3K[2]}}{\sqrt{2}} & K[1]=17 \\ 0 & \text{True} \end{cases}\right) dK[2] + e^{-\pi^2 t K[1]^2}\right.\right.$$

Maple ✓

```
restart;
f := x->3*cos(42*x*Pi);
g :=(x,t)->exp(3*t)*cos(17*x*Pi);
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + g(x, t);
bc  := (D[1](u))(0, t) = 0, (D[1](u))(1, t) = 0;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t) = \frac{(867\pi^2+9)\cos(42\pi x)\,\mathrm{e}^{-1764\pi^2 t} + \left(\mathrm{e}^{3t} - \mathrm{e}^{-289\pi^2 t}\right)\cos(17\pi x)}{289\pi^2+3}$$

3.1.1.44 [193] Pinchover and Rubinstein 6.21

problem number 193

added July 2, 2018.

Taken from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.21

Solve the heat equation on bar

$$u_t = u_{xx} + g(x,t)$$

Where $g(x,t) = t\cos(2001x)$ for $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = \pi\cos(2x)$.

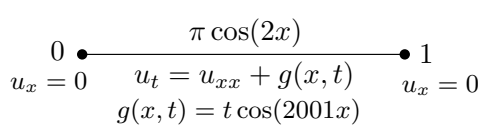


Figure 3.64: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + t*Cos[2001*x];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][Pi, t] == 0};
ic  = u[x, 0] == Pi*Cos[2*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \sum_{K[1]=1}^{\infty}\sqrt{\frac{2}{\pi}}\cos(xK[1])\left(\int_0^t e^{-K[1]^2(t-K[2])}\left(\begin{cases}\sqrt{\frac{\pi}{2}}K[2] & K[1]=2001\\ 0 & \text{True}\end{cases}\right)dK[2]+e^{-tK[1]}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + t*cos(2001*x);
bc  := (D[1](u))(0, t) = 0, (D[1](u))(Pi, t) = 0;
ic  := u(x, 0) = Pi*cos(2*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t)=\pi\cos(2x)\,\mathrm{e}^{-4t}+\frac{(4004001t+\mathrm{e}^{-4004001t}-1)\cos(2001x)}{16032024008001}$$

3.1.1.45 [194] Diffusion Reaction. Euler-Cauchy Sturm-Liouville

problem number 194

Added April 20, 2019.

Solve the parabolic pde for $u(x,t)$

$$u_t = x^2 u_{xx} + x u_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(1,t) &= 0 \\ hu(b,t) + u_x(b,t) &= 0 \end{aligned}$$

Where $h > 0$. Initial condition is $u(x,0) = \ln x$

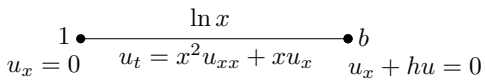


Figure 3.65: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic  = u[x, 0] == Log[x];
bc  = {Derivative[1, 0][u][1, t] == 0, h*u[b, t] + Derivative[1, 0][u][b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\frac{2be^{-tK[2,K[1]]}h\cos\left(\sqrt{K[2,K[1]]}\log(x)\right)\left(\cos\left(\sqrt{K[2,K[1]]}\log(b)\right)+\sqrt{K[2,K[1]]}\log(b)\sin\left(\sqrt{K[2,K[1]]}\log(b)\right.\right.}{K[2,K[1]]\left(\sin^2\left(\sqrt{K[2,K[1]]}\log(b)\right)+bh\log(b)\right)}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t),t)=x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc:=eval( diff(u(x,t),x),x=1)=0,h*u(b,t)+eval( diff(u(x,t),x),x=b)=0;
ic  := u(x,0)=ln(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

$$u(x,t)=\infty\,\mathrm{signum}\left(c_2\cos\left(\sqrt{\frac{b^2h^2\left(\cos^2\lambda\_Z65\right)}{\sin\left(\lambda\_Z65\right)^2}}\ln(x)\right)\mathrm{e}^{-\frac{b^2h^2t\left(\cos^2\lambda\_Z65\right)}{\sin\left(\lambda\_Z65\right)^2}}\right)\textbf{ where}\left\{\lambda\_Z65-\sqrt{\frac{b^2h^2}{\tan\left(\lambda\_Z6\right.}}\right.$$

<u>Hand solution</u>

Solve

$$u_t = x^2u_{xx} + xu_x \tag{1}$$

With $1 < x < b, t > 0$. BC

$$u_x(1,t) = 0$$
$$hu(b,t) + u_x(b,t) = 0$$

Where $h > 0$. And initial conditions

$$u(x,0) = \ln x$$

Let $u = X(x)\,T(x)$. Substituting into (1) gives

$$\begin{aligned} T'X &= x^2 X'' T + x X' T \\ \frac{T'}{T} &= x^2 \frac{X''}{X} + x \frac{X'}{X} = -\lambda \end{aligned}$$

Where λ is the separation constant. From the boundary conditions, we know that λ will be only positive. So we do not need to check for possibility of negative of zero eigenvalue. Letting $\lambda = \alpha^2$, then the above reduces to

$$x^2 X'' + x X' + \alpha^2 X = 0 \tag{2}$$

$$T' + \alpha^2 T = 0 \tag{3}$$

Equation (2) is Euler ODE. Assuming $X = x^r$, then $X' = r x^{r-1}, X'' = r(r-1)\,x^{r-2}$. Substituting back into (2) gives the characteristic equation

$$\begin{aligned} r(r-1)\,x^r + r x^r + \alpha^2 x^r &= 0 \\ r^2 + \alpha^2 &= 0 \\ r &= \pm i\alpha \end{aligned}$$

Hence the solution to (2) is

$$\begin{aligned} X(x) &= A x^{i\alpha} + B x^{-i\alpha} \\ &= A e^{\ln x^{i\alpha}} + B e^{\ln x^{-i\alpha}} \\ &= A e^{i\alpha \ln x} + B e^{-i\alpha \ln x} \end{aligned}$$

Which using Euler relation can be written as (using new constants, but the name of the constants kept the same for simplicity)

$$X(x) = A\cos(\alpha \ln x) + B\sin(\alpha \ln x)$$

Applying first BC. $X'(1,t) = 0$ gives

$$\begin{aligned} X'(x) &= -\frac{\alpha}{x} A \sin(\alpha \ln x) + B \frac{\alpha}{x} \cos(\alpha \ln x) \\ 0 &= -\alpha A \sin(\alpha \ln 1) + B\alpha \cos(\alpha \ln 1) \\ &= B\alpha \end{aligned}$$

Since $\alpha > 0$ then $B = 0$. Hence the solution becomes

$$X(x) = A\cos(\alpha \ln x)$$

Applying second BC. $hX(b) + X'(b) = 0$ gives

$$\begin{aligned} hA\cos(\alpha \ln b) - A\frac{\alpha}{b}\sin(\alpha \ln b) &= 0 \\ h - \frac{\alpha}{b}\tan(\alpha \ln b) &= 0 \\ \tan(\alpha \ln b) &= \frac{hb}{\alpha} \end{aligned} \tag{4}$$

There is no analytical solution to the above. The eigenvalues α_n are the solutions to the above nonlinear equation. Therefore the eigenfunctions are

$$X_n(x) = \cos(\alpha_n \ln x)$$

With eigenvalues $\alpha_n > 0$ given by solutions to (4). The solution to the time ODE is

$$\begin{aligned} T_n' + \alpha_n^2 T_n &= 0 \\ T_n(t) &= T_n(0)\, e^{-\alpha_n^2 t} \end{aligned}$$

Hence the solution to (1) is

$$u(x,t) = \sum_{n=1}^{\infty} T_n(0)\, e^{-\alpha_n^2 t}\cos(\alpha_n \ln x) \tag{5}$$

At $t = 0$

$$\ln x = \sum_{n=1}^{\infty} T_n(0)\cos(\alpha_n \ln x)$$

Applying orthogonality gives

$$\begin{aligned} \int_1^b \ln x \cos(\alpha_n \ln x)\, dx &= T_n(0)\int_1^b \cos^2(\alpha_n \ln x)\, dx \\ T_n(0) &= \frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx} \end{aligned}$$

Hence the solution (5) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(\frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx}\right) e^{-\alpha_n^2 t}\cos(\alpha_n \ln x)$$

3.1.1.46 [195] convection heat loss

problem number 195

Added April 28, 2019

Problem 4, section 74, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are, on the left end $u(0,t) = 0$ and on the right end $u_x(1,t) = -hu(1,t)$ with $h > 0$. Initial conditions $u(x,0) = f(x)$

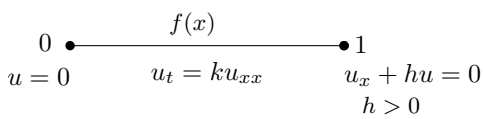


Figure 3.66: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
bc  = {u[0, t] == 0, Derivative[1, 0][u][1, t] == -h u[1, t]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t) \to \{ \sum_{K[1]=1}^{\infty} -\frac{\sqrt{2}e^{-ktK[2,K[1]]}\sqrt{h}\left(\int_0^1 -\frac{2\sqrt{h}f(x)\sin\left(x\sqrt{K[2,K[1]]}\right)}{\sqrt{2h+\cos\left(2\sqrt{K[2,K[1]]}\right)+1}}\,dx\right)\sin\left(x\sqrt{K[2,K[1]]}\right)}{\sqrt{\cos^2\left(\sqrt{K[2,K[1]]}\right)+h}} \quad h\tan\left(\sqrt{K[2,K[1]]}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic  := u(x,0) = f(x);
bc  := u(0,t)=0, eval(diff(u(x,t), x), x = 1)=-h*u(1,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} -\frac{2h\left(\int_0^1 f(x)\sin\left(\frac{hx\sin(\lambda_n)}{\cos(\lambda_n)}\right)dx\right)\mathrm{e}^{-\frac{h^2kt\left(\sin^2\lambda_n\right)}{\cos(\lambda_n)^2}}\operatorname{signum}(\cos(\lambda_n))\operatorname{signum}(\sin(\lambda_n))\sin(\lambda_n)}{-h\operatorname{signum}(\cos(\lambda_n))\operatorname{signum}(\sin(\lambda_n))\sin(\lambda_n)+\cos(\lambda_n)\cos\left(\frac{h\operatorname{signum}(\sin(\lambda_n))\sin(\lambda_n)}{|\cos(\lambda_n)|}\right)\sin\left(\frac{h}{}\right.}$$

Hand solution

To solve the PDE, we first check the boundary conditions by writing them as

$$a_1u(0,t)+a_2u_x(0,t)=0$$
$$b_1u(1,t)+b_2u_x(1,t)=0$$

Then $a_1=0, a_2=0$. Hence $a_1a_2=0$. And $b_1=1, b_2=h$. Then since it is assumed that $h>0$, then $b_1b_2\geq 0$. And since $q(x)=0$ from the PDE itself, then we know that eigenvalues must be $\lambda\geq 0$.

Let $u=X(x)\,T(t)$ then the PDE becomes

$$T'X=X''T$$
$$\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

Hence the Sturm Liouville problem is

$$X''+\lambda X=0$$
$$X(0)=0$$
$$X'(1)+hX(1)=0$$

Where $p(x)=1$.

Case $\lambda=0$

Solution is

$$X(x)=Ax+B$$

At $x=0$

$$0=B$$

Hence solution becomes

$$X(x) = Ax$$

At $x = 1$ the second boundary conditions gives

$$\begin{aligned} A + hA &= 0 \\ A(1+h) &= 0 \end{aligned}$$

For non trivial solution $1 + h = 0$ or $h = -1$. But we assumed that $h > 0$. Therefore $\lambda = 0$ is not eigenvalue.

<u>Case $\lambda > 0$</u>

Let $\lambda = \alpha^2, \alpha > 0$. Hence solution is

$$X(x) = A\cos(\alpha x) + B\sin(\alpha x)$$

At $X(0) = 0$

$$0 = A$$

The solution becomes

$$X(x) = B\sin(\alpha x)$$

At $x = 1$ the second boundary conditions gives

$$\begin{aligned} B\alpha\cos(\alpha) + hB\sin(\alpha) &= 0 \\ \alpha\cos(\alpha) + h\sin(\alpha) &= 0 \\ \tan(\alpha) &= -\frac{\alpha}{h} \end{aligned}$$

Therefore the eigenvalues are given by solution to

$$\tan(\alpha_n) = -\frac{\alpha_n}{h} \qquad n = 1, 2, 3, \cdots$$

And eigenfunctions are

$$X_n(x) = \sin(\alpha_n x)$$

The normalized eigenfunctions are

$$\phi_n(x) = \frac{X_n(x)}{\|X_n(x)\|}$$

But

$$
\begin{aligned}
\|X_n(x)\|^2 &= \int_0^1 p(x)\, X_n^2(x)\, dx \\
&= \int_0^1 \sin^2(\alpha_n x)\, dx \\
&= \frac{1}{2}\int_0^1 1 - \cos(2\alpha_n x)\, dx \\
&= \frac{1}{2}\left(1 - \left[\frac{\sin(2\alpha_n x)}{2\alpha_n}\right]_0^1\right) \\
&= \frac{1}{2}\left(1 - \frac{1}{2\alpha_n}[\sin(2\alpha_n x)]_0^1\right) \\
&= \frac{1}{2}\left(1 - \frac{\sin(2\alpha_n)}{2\alpha_n}\right) \\
&= \frac{1}{2} - \frac{\sin(2\alpha_n)}{4\alpha_n}
\end{aligned}
$$

But $\sin(2\alpha_n) = 2\sin\alpha_n\cos\alpha_n$ and $\alpha_n = -h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}$, therefore the above becomes

$$
\begin{aligned}
\|X_n(x)\|^2 &= \frac{1}{2} + \frac{2\sin\alpha_n\cos\alpha_n}{4h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}} \\
&= \frac{1}{2} + \frac{\cos^2\alpha_n}{2h} \\
&= \frac{h + \cos^2\alpha_n}{2h}
\end{aligned}
$$

Hence

$$
\begin{aligned}
\phi_n(x) &= \frac{X_n(x)}{\sqrt{\frac{h+\cos^2\alpha_n}{2h}}} \\
&= \sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n x)
\end{aligned}
$$

Now we use generalized Fourier series to find the solution. Let

$$
u(x,t) = \sum_{n=1}^{\infty} B_n(t)\,\phi_n(x) \tag{1}
$$

Substituting this back into the PDE gives

$$
\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) = k\sum_{n=1}^{\infty} B_n(t)\,\phi_n''(x)
$$

But $\phi_n''(x) = -\lambda_n \phi_n(x) = -\alpha_n^2 \phi_n(x)$. The above becomes

$$\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) = -k \sum_{n=1}^{\infty} B_n(t)\,\alpha_n^2 \phi_n(x)$$
$$B_n'(t) + k\alpha_n^2 B_n(t) = 0$$

The solution is

$$B_n(t) = B_n(0)\,e^{-k\alpha_n^2 t}$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-k\alpha_n^2 t}\phi_n(x)$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x)$$

Therefore

$$\begin{aligned} B_n(0) &= \langle f(x)\,,\phi_n(x)\rangle \\ &= \int_0^1 p(x)\,f(x)\,\phi_n(x)\,dx \\ &= \sqrt{\frac{2h}{h+\cos^2\alpha_n}} \int_0^1 f(x)\sin(\alpha_n x)\,dx \end{aligned}$$

Therefore

$$\begin{aligned} B_n(t) &= B_n(0)\,e^{-k\alpha_n^2 t} \\ &= \left(\sqrt{\frac{2h}{h+\cos^2\alpha_n}} \int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t} \end{aligned}$$

and solution (1) becomes

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} \sqrt{\frac{2h}{h+\cos^2\alpha_n}} \left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t} \sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n x) \\ &= \frac{2h}{h+\cos^2\alpha_n} \sum_{n=1}^{\infty} \left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right) e^{-k\alpha_n^2 t}\sin(\alpha_n x) \end{aligned}$$

3.1.1.47 [196] Mixed BC

problem number 196

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) + u(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) + u(L,t) = 0$$

And initial condition $u(x,0) = f(x)$

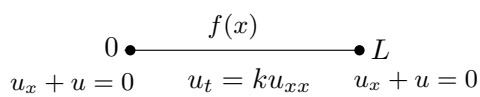


Figure 3.67: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] + u[0, t] == 0, Derivative[1, 0][u][L, t] + u[L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \{ \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-ktK[2,K[1]]}\left(\int_0^L \frac{\sqrt{2}f(x)\sqrt{K[2,K[1]]+1}\left(\cos\left(x\sqrt{K[2,K[1]]}\right)\sqrt{K[2,K[1]]}-\sin\left(x\sqrt{K[2,K[1]]}\right)\right)}{\sqrt{(K[2,K[1]]+1)\left(K[2,K[1]]L+L+\cos\left(2L\sqrt{K[2,K[1]]}\right)-1\right)}}\,dx\right)\sqrt{K[2,K[1]]+1}(}{\sqrt{2K[2,K[1]]\cos^2\left(L\sqrt{K[2,K[1]]}\right)+2\cos^2\left(L\sqrt{K[2,K[1]]}\right)+LK[2,K[1]]^2+L+2L}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(L\sin\left(\frac{\pi n x}{L}\right)-\pi n\cos\left(\frac{\pi n x}{L}\right)\right)\left(\int_0^L\left(L\sin\left(\frac{\pi n x}{L}\right)-\pi n\cos\left(\frac{\pi n x}{L}\right)\right)f(x)\,dx\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}}{\left(L^2+\pi^2 n^2\right)L}$$

3.1.1.48 [197] Haberman 8.2.1 (f)

problem number 197

added March 18, 2018.

This is problem 8.2.1, part(f) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+\sin\left(\frac{2\pi x}{L}\right)$$

For $0<x<L$ and $t>0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t)=0$$
$$\frac{\partial u}{\partial x}(L,t)=0$$

Initial condition is $u(x,0)=f(x)$.

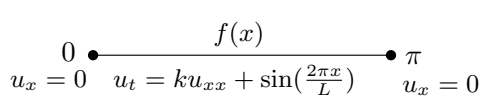


Figure 3.68: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Sin[(2*Pi*x)/L];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(\frac{\left(1-e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\right)\left(\begin{cases}0 & K[1]=2\\ \frac{2\sqrt{2}(-1+(-1)^{K[1]})\sqrt{L}}{\pi(K[1]^2-4)} & \text{True}\end{cases}\right)L^2}{k\pi^2K[1]^2}+e^{-\frac{k\pi^2 tK[1]}{L^2}}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+sin(2*Pi*x/L);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{L^2\sin\left(\frac{2\pi x}{L}\right)}{4\pi^2k}-\frac{Lx}{2\pi k}+c_2+\left(\sum_{n=1}^{\infty}-\frac{\left(\int_0^L\left(L^2\sin\left(\frac{2\pi x}{L}\right)-2\pi Lx+4c_2\pi^2k-4\pi^2kf(x)\right)\cos\left(\frac{\pi n x}{L}\right)dx\right)}{2\pi^2Lk}\right.$$

3.1.2 Finite domain (bar), left end homogeneous, right end not

Local contents

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3.1.2.1 [198] left end insulated (general case)

problem number 198

Added June 21, 2019

Solve the heat equation

$$u_t = ku_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$.

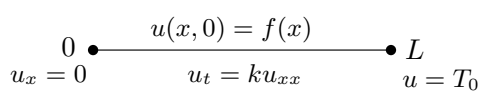

Figure 3.69: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] ;
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
sol =  sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=0}^{\infty}e^{-\frac{k(2\pi n+\pi)^2t}{4L^2}}\cos\left(\frac{(2n+1)\pi x}{2L}\right)\int_0^L\cos\left(\frac{(2n+1)\pi K[2]}{2L}\right)(f(K[2])-\mathrm{T0})\,dK[2]}{L}+\mathrm{T0}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=T0+2\left(\sum_{n=0}^{\infty}\frac{\left(\int_0^L(-T0+f(x))\cos\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos\left(\frac{(2n+1)\pi x}{2L}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2kt}{4L^2}}}{L}\right)$$

<u>Hand solution</u>

Solve $u_t=ku_{xx}$ with $u(x,0)=f(x)$ and $u_x(0,t)=0, u(L,t)=T_0$. Since the right end is not homogeneous, we need to find a reference function. Let $r(x)=Ax+B$. Then $r'(x)=A$. Since $u_x(0,t)=0$, then $A=0$. Hence $r(x)=B$. Since $u(L,0)=T_0$, then $r(L)=T_0$. Hence $B=T_0$. Therefore $r(x)=T_0$. Now let the solution be

$$u(x,t)=v(x,t)+r(x) \tag{1}$$

Where $v(x,t)$ solves the same pde but with homogeneous boundary conditions

$$\begin{aligned} v_t &= kv_{xx} \\ v_x(0,t) &= 0 \\ v(L,0) &= 0 \\ v(x,0) &= u(x,0) - r(x) \\ &= F(x) \end{aligned}$$

The above general PDE was solved in problem 3.1.1.28 on page 476 and the solution is

$$v(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L F(x)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}$$

Since here $F(x) = u(x,0) - r(x) = f(x) - T_0$ the above becomes

$$v(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t} \tag{2}$$

From (1,2) the final solution is

$$u(x,t) = T_0 + \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}$$

3.1.2.2 [199] left end insulated (special case)

problem number 199

Added June 21, 2019

Solve the heat equation

$$u_t = ku_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$. Using the following values

$$
\begin{aligned}
L &= 5 \\
T_0 &= 10 \\
k &= \frac{1}{100} \\
f(x) &= 0
\end{aligned}
$$

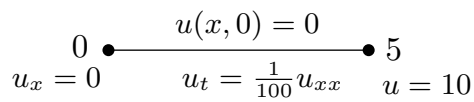

Figure 3.70: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
f=0;
T0=10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] ;
bc  = {Derivative[1, 0][u][0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol/.K[1]->n;
```

$$
\left\{\left\{u(x,t) \to \frac{2}{5}\left(\sum_{n=0}^{\infty} -\frac{100e^{-\frac{(2\pi n+\pi)^2 t}{10000}}\cos(n\pi)\cos\left(\frac{1}{10}(2n+1)\pi x\right)}{2\pi n+\pi} + 25\right)\right\}\right\}
$$

Maple ✓

```
restart;
L:=5;
k:=1/100;
f:=0;
T0:=10;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f;
bc  :=  D[1](u)(0,t)=0, u(L,t)=T0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) ),out
```

$$u(x,t) = -40\left(\sum_{n=0}^{\infty}\frac{(-1)^n\cos\left(\frac{(2n+1)\pi x}{10}\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2t}{10000}}}{(2n+1)\pi}\right)+10$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 3.1.1.28 on page 476 as

$$u(x,t) = T_0+\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L (f(x)-T_0)\cos\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{2L}x\right)e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2t}$$

In this problem $u(x,0) = f(x) = 0, L = 5, k = \frac{1}{100}$ and $T_0 = 10$, Hence the above becomes

$$u(x,t) = 10+\frac{2}{5}\sum_{n=0}^{\infty}\left(\int_0^5 -10\cos\left(\frac{(2n+1)\pi}{10}x\right)dx\right)\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2t}$$

But $\int_0^5 -10\cos\left(\frac{(2n+1)\pi}{10}x\right)dx = -\frac{100\cos(\pi n)}{\pi(1+2n)} = \frac{-100(-1)^n}{\pi(1+2n)}$ and the above becomes

$$\begin{aligned}u(x,t) &= 10+\frac{2}{5}\sum_{n=0}^{\infty}\frac{-100(-1)^n}{\pi(1+2n)}\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2t}\\ &= 10-\frac{40}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(1+2n)}\cos\left(\frac{(2n+1)\pi}{10}x\right)e^{-\frac{1}{100}\left(\frac{(2n+1)\pi}{10}\right)^2t}\end{aligned}$$

The following is an animation of the solution

Out[ ]=

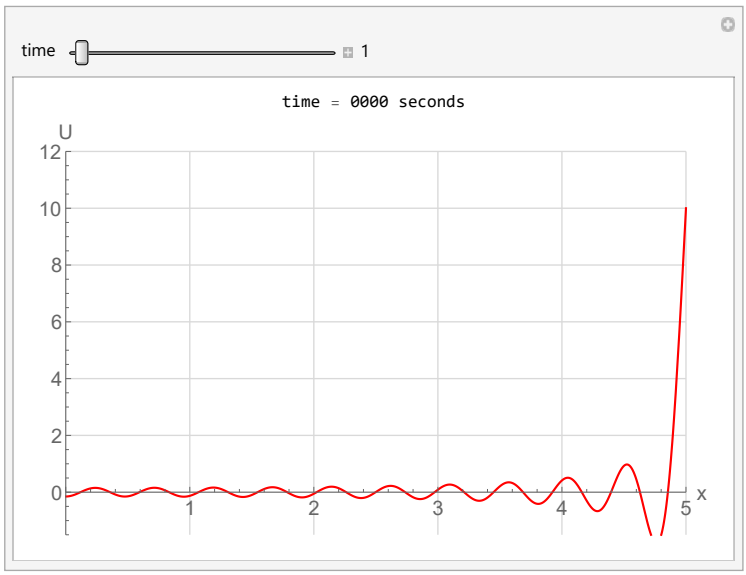

Figure 3.71: Initial state

Source code used for the above

In[ ]:=
```
ClearAll[x, y, t, n, k]
L = 5;
numberOfTerms = 20;
mySol[x_, t_] = 10 - 40/π Sum[(-1)^n/(1 + 2 n) Exp[-1/100 ((2 n + 1) π/10)^2 t] Cos[(2 n + 1) π/10 x], {n, 0, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 3.72: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-1.5, 12}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 2500, 10}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.73: Code for animation

3.1.2.3 [200] right end nonhomogeneous BC (general case)

problem number 200

Added June 20, 2019

Solve the heat equation

$$u_t = ku_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = T_0$$

Initial condition is $u(x,0) = f(x)$

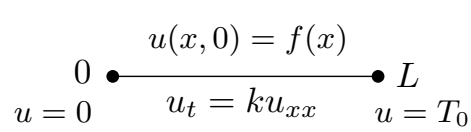

Figure 3.74: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->k>0
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{2e^{-\frac{kn^2\pi^2t}{L^2}}\left(\int_0^L\left(f(x)-\frac{\mathrm{T0}x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)\,dx\right)\sin\left(\frac{n\pi x}{L}\right)}{L}+\frac{\mathrm{T0}x}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = k*diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(L,t) = T0;
ic  := u(x,0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t)=\frac{T0x}{L}+\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L(Lf(x)-T0x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L^2}\right)$$

<u>Hand solution</u>

Since the right side boundary condition is not homogeneous, then we need to first find a reference function. Let $r(x)=Ax+B$. At $x=0, 0=B$. Hence $r(x)=Ax$. At $x=L, T_0=AL$, hence $A=\frac{T_0}{L}$. Therefore

$$r(x)=\frac{T_0}{L}x$$

Now let $u(x,t)=v(x,t)+r(x)$ where $v_t=v_{xx}$ but with homogeneous BC $v(0,t)=0, v(L,0)=0$. The basic solution for this type of PDE was already given in problem 3.1.1.1 on page 400 as

$$v(x,t)=\sum_{n=1}^{\infty}B_ne^{-k\lambda_nt}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Hence

$$\begin{aligned} u(x,t) &= r(x) + v(x,t) \\ &= \frac{T_0}{L}x + \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right) \end{aligned} \tag{1}$$

At $t = 0$

$$f(x) - \frac{T_0}{L}x = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right)$$

Multiplying both sides by $\sin\left(\sqrt{\lambda_{n'}}x\right)$ and integrating

$$\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_{n'}}x\right) dx = \int_0^L \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_{n'}}x\right) \sin\left(\sqrt{\lambda}x\right) dx$$

Moving integration inside summation and by orthogonality of sin function the above reduces to

$$\begin{aligned} \int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx &= B_n \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx \\ \int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx &= \frac{L}{2}B_n \\ B_n &= \frac{2}{L}\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx \end{aligned}$$

Therefore the solution from (1) is

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right) \sin\left(\sqrt{\lambda_n}x\right) dx\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n}x\right)$$

3.1.2.4 [201] right end nonhomogeneous BC (special case)

problem number 201

Added June 20, 2019

Solve the heat equation

$$u_t = ku_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= T_0 \end{aligned}$$

Initial condition is $u(x,0) = f(x)$ using these values

$$
\begin{aligned}
k &= \frac{1}{100} \\
L &= 100 \\
T_0 &= 100 \\
f(x) &= x
\end{aligned}
$$

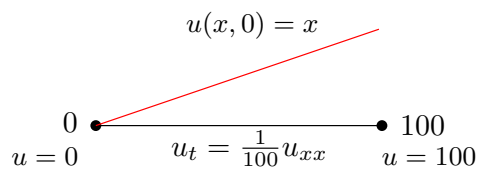

Figure 3.75: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/100;
L=100;
T0=100;
f=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == T0};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
L:=100;
k:=1/100;
T0:=100;
f:=x;
pde := diff(u(x,t), t) = k*diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(L,t) = T0;
ic  := u(x,0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ),o
```

$$u(x,t) = x$$

Hand solution

The general solution for this type of PDE is given in problem 3.1.2.3 on page 542 as

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

With $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. In this problem

$$\begin{aligned} L &= 100 \\ k &= \frac{1}{100} \\ T_0 &= 100 \\ f(x) &= x \end{aligned}$$

Hence the solution becomes

$$\begin{aligned} u(x,t) &= x + \frac{2}{100}\sum_{n=1}^{\infty}\left(\int_0^{100}(x-x)\sin\left(\frac{n\pi}{100}x\right)dx\right)e^{-\frac{1}{100}\left(\frac{n\pi}{100}\right)^2 t}\sin\left(\frac{n\pi}{100}x\right) \\ &= x + 0 \\ &= x \end{aligned}$$

3.1.2.5 [202] right end nonhomogeneous BC, special case

problem number 202

Added July 2, 2018. Can not find where I found this PDE.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 1$$

Initial condition is $u(x,0) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$

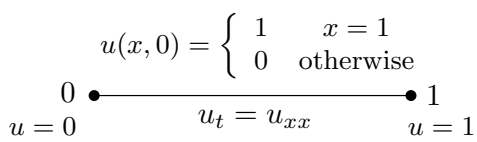

Figure 3.76: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[1, t] == 1};
ic  = u[x, 0] == Piecewise[{{1, x == 1}, {0, True}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{2(-1)^n e^{-n^2\pi^2 t} \sin(n\pi x)}{n\pi} + x\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t) = diff(u(x,t), x$2);
bc  := u(0,t) = 0, u(1,t) = 1;
ic  := u(x,0) = piecewise(x = 1, 1,  true,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = x + 2\left(\sum_{n=1}^{\infty} \frac{(-1)^n \mathrm{e}^{-\pi^2 n^2 t} \sin(\pi n x)}{\pi n}\right)$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 3.1.2.3 on page 542 as

$$u(x,t) = \frac{T_0}{L}x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \frac{T_0}{L}x\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

With $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. In this problem

$$\begin{aligned} L &= 1 \\ k &= 1 \\ T_0 &= 1 \\ f(x) &= \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Hence the solution becomes

$$u(x,t) = x + 2\sum_{n=1}^{\infty}\left(\int_0^1 (f(x) - x)\sin(n\pi x)\,dx\right)e^{-(n\pi)^2 t}\sin(n\pi x) \tag{1}$$

But

$$\begin{aligned} \int_0^1 (f(x) - x)\sin(n\pi x)\,dx &= \int_0^1 f(x)\sin(n\pi x)\,dx - \int_0^1 x\sin(n\pi x)\,dx \\ &= 0 - \int_0^1 x\sin(n\pi x)\,dx \end{aligned}$$

$\int_0^1 x\sin(n\pi x)\,dx = \frac{(-1)^{n+1}}{n\pi}$, hence

$$\begin{aligned} \int_0^1 (f(x) - x)\sin(n\pi x)\,dx &= -\frac{(-1)^{n+1}}{n\pi} \\ &= \frac{(-1)^n}{n\pi} \end{aligned}$$

Therefore (1) becomes

$$u(x,t) = x + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} e^{-(n\pi)^2 t} \sin(n\pi x)$$

This is animation of the solution for 0.3 seconds. (Animation will show only in the HTML version).

Out[ ]=

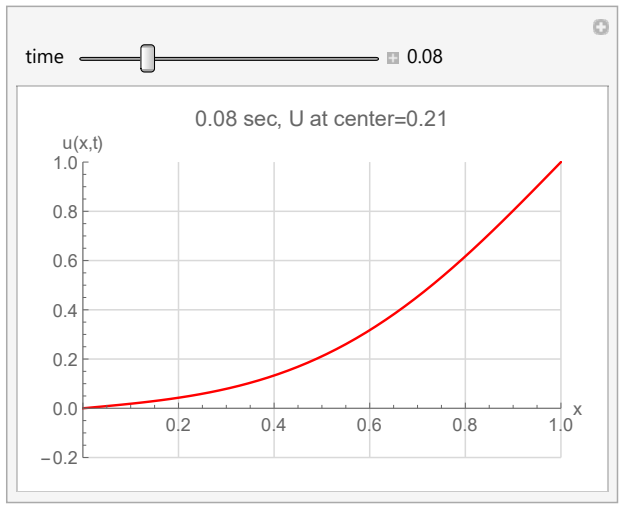

Figure 3.77: Screen shot

Source code used for the above

```
In[∘]:= L = 1;
u[x_, t_, m_] := x + 2/Pi Quiet[Sum[(-1)^n/n Sin[n π x] Exp[-(n π)^2 t], {n, 1, m}]];
Plot[u[x, 0, 20], {x, 0, L}, PlotRange → All]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
      NumberPadding → {"0", "0"}, SignPadding → True];
Manipulate[
  Plot[u[x, t, 20], {x, 0, L},
    PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center=", padIt2[N@u[L/2, t, 20], {3, 2}]}],
    PlotRange → {{0, L}, {-.2, 1}}, GridLines → Automatic,
    GridLinesStyle → LightGray,
    PlotStyle → Red, AxesLabel → {"x", "u(x,t)"},
    BaseStyle → 12
  ],
  {{t, 0, "time"}, 0, 0.4, .01, Appearance → "Labeled"}]
r = Table[
      Plot[u[x, t, 20], {x, 0, L},
        PlotLabel → Row[{padIt2[t, {3, 2}], " sec,", " U at center=", padIt2[N@u[L/2, t, 20], {3, 2}]}],
        PlotRange → {{0, L}, {-.2, 1}}, GridLines → Automatic,
        GridLinesStyle → LightGray,
        PlotStyle → Red, AxesLabel → {"x", "u(x,t)"}, BaseStyle → 12
      ],
      {t, 0, 0.3, .01}];
In[∘]:= Export["anim.gif", r, "DisplayDurations" → Table[If[i < 10, 0.5, 0.3], {i, Length@r}]]
Out[∘]= anim.gif
```

Figure 3.78: Source code

3.1.2.6 [203] convection heat loss

problem number 203

Added April 28, 2019

Problem 2, section 77, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are $u_x(0,t) = hu(0,t)$ and on the right end $u(1,t) = 1$ with $h > 0$. Initial conditions $u(x,0) = 0$

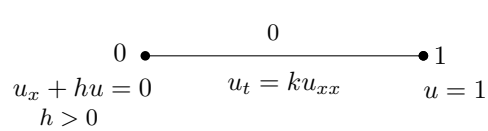

Figure 3.79: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == 0;
bc  = {Derivative[1, 0][u][0, t] == h * u[0,t], u[1, t] == 1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions->h>
```

$$\left\{\left\{u(x,t)\to\ \{\quad x^2+\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\sqrt{h}\left(\frac{\sqrt{2}e^{-tK[2,K[1]]}\left(2h+\cos\left(\sqrt{K[2,K[1]]}\right)((h-2)K[2,K[1]]-2h)-\sqrt{K[2,K[1]]}(2h+K[2,K[1]]-2)\sin\left(\sqrt{K[2,K}\right.\right.}{K[2,K[1]]^{3/2}\sqrt{h\left(-\cos^2\left(\sqrt{K[2,K[1]]}\right)+h+2\right)+\frac{\left(h-\cos^2\left(\sqrt{K[2,K[1]]}\right)\right)K[2,K[1]]}{h}}}\right.}{\sqrt{h^3-\cos^2\left(\sqrt{K[2,}\right.}\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(x,t), t) = (diff(u(x,t), x, x));
ic  := u(x,0) = 0;
bc  := eval(diff(u(x,t), x), x = 0) = h*u(0,t), u(1,t) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

time expired

Hand solution

Solve

$$u_t = u_{xx} \qquad 0 < x < 1, t > 0$$

With boundary conditions

$$\begin{aligned} u_x(0,t) - hu(0,t) &= 0 \\ u(1,t) &= 1 \end{aligned}$$

With $h > 0$. And initial conditions $u(x,0) = f(x)$.

Because the second B.C. is not zero, we need to introduce a reference function $r(x)$ which satisfies the nonhomogeneous boundary conditions.

Let $r(x) = Ax + B$. When $x = 0$ then the first BC gives

$$A - hB = 0$$

And the second BC gives

$$A + B = 1$$

From the first equation $A = hB$. Substituting in the second equation give $hB + B = 1$ or $B(1+h) = 1$ or $B = \frac{1}{1+h}$. Hence $A = \frac{h}{1+h}$. Therefore

$$\begin{aligned} r(x) &= Ax + B \\ &= \frac{h}{1+h}x + \frac{1}{1+h} \\ &= \frac{hx+1}{1+h} \end{aligned} \tag{1}$$

To verify. $r_x = \frac{h}{1+h}$. When $x = 0$ then $r(0) = \frac{1}{1+h}$. Hence $r_x(0) - hr(0) = \frac{h}{1+h} - h\frac{1}{1+h} = 0$ as expected. And when $x = 1$ then $r(1) = 1$ as expected. Now that we found $r(x)$ then we write

$$u(x,t) = v(x,t) + r(x)$$

Where $v(x,t)$ is the solution to the homogenous PDE

$$v_t = v_{xx} \qquad 0 < x < 1, t > 0$$

With boundary conditions

$$\begin{aligned} v_x(0,t) - hv(0,t) &= 0 \\ v(1,t) &= 0 \end{aligned}$$

We can now solve for $v(x,t)$ using separation of variables since boundary conditions are homogenous. Separation of variables gives

$$\begin{aligned} X'' + \lambda X &= 0 \\ X'(0) - hX(0) &= 0 \\ X(1) &= 0 \end{aligned}$$

The above is known eigenvalue problem which we found before. It has the following eigenfunctions and eigenvalues

$$\begin{aligned} \phi_n(x) &= \sqrt{\frac{2h}{h + \cos^2 \alpha_n}} \sin(\alpha_n(1-x)) \qquad n = 1, 2, \cdots \\ \tan(\alpha_n) &= \frac{-\alpha_n}{h} \end{aligned}$$

With $\alpha_n > 0$. Hence the solution $v(x,t)$ using generalized Fourier series is

$$v(x,t) = \sum_{n=1}^{\infty} B_n(t)\, \phi_n(x) \tag{2}$$

Substituting into the PDE $v_t = v_{xx}$ gives

$$\begin{aligned}\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) &= \sum_{n=1}^{\infty} B_n(t)\,\phi_n''(x)\\ &= -\sum_{n=1}^{\infty} B_n(t)\,\alpha_n^2\phi_n(x)\end{aligned}$$

Therefore the ODE is

$$B_n'(t) + \alpha_n^2 B_n(t) = 0$$

The solution is

$$B_n(t) = B_n(0)\,e^{-\alpha_n^2 t}$$

Hence (2) becomes

$$v(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x)$$

And since $u(x,t) = v(x,t) + r(x)$ then

$$u(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x) + \frac{hx+1}{1+h}$$

Now we find $B_n(0)$ from initial conditions. At $t = 0$ the above becomes

$$\begin{aligned}0 &= \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x) + \frac{hx+1}{1+h}\\ -\frac{hx+1}{1+h} &= \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x)\end{aligned}$$

Hence

$$\begin{aligned}B_n(0) &= \left\langle -\frac{hx+1}{1+h}, \phi_n(x)\right\rangle\\ &= -\int_0^1 p(x)\,\frac{hx+1}{1+h}\phi_n(x)\,dx\\ &= -\int_0^1 \frac{hx+1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)dx\\ &= -\frac{1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1 (hx+1)\sin\left(\alpha_n(1-x)\right)dx \qquad (3)\end{aligned}$$

But

$$\begin{aligned}
\int_0^1 (hx+1)\sin(\alpha_n(1-x))\,dx &= \int_0^1 \sin(\alpha_n(1-x))\,dx + h\int_0^1 x\sin(\alpha_n(1-x))\,dx \\
&= \left[\frac{\cos(\alpha_n(1-x))}{\alpha_n}\right]_0^1 + h\left[\frac{\alpha_n x\cos(\alpha_n(1-x))+\sin(\alpha_n(1-x))}{\alpha_n^2}\right]_0^1 \\
&= \frac{1-\cos(\alpha_n)}{\alpha_n} + \frac{h}{\alpha_n^2}\left[\alpha_n x\cos(\alpha_n(1-x))+\sin(\alpha_n(1-x))\right]_0^1 \\
&= \frac{1-\cos(\alpha_n)}{\alpha_n} + \frac{h}{\alpha_n^2}\left[\alpha_n - \sin\alpha_n\right] \\
&= \frac{\alpha_n - \alpha_n\cos(\alpha_n) + h\alpha_n - h\sin\alpha_n}{\alpha_n^2}
\end{aligned}$$

But $\frac{\sin(\alpha_n)}{\cos(\alpha_n)} = -\frac{\alpha_n}{h}$ or $h\sin(\alpha_n) = -\alpha_n\cos(\alpha_n)$ or $-h\sin\alpha_n = \alpha_n\cos(\alpha_n)$, hence the above simplifies to

$$\begin{aligned}
\int_0^1 (hx+1)\sin(\alpha_n(1-x))\,dx &= \frac{\alpha_n + h\alpha_n}{\alpha_n^2} \\
&= \frac{1+h}{\alpha_n}
\end{aligned}$$

Therefore (3) becomes

$$\begin{aligned}
B_n(0) &= \frac{-1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\left(\frac{1+h}{\alpha_n}\right) \\
&= -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}
\end{aligned}$$

Hence final solution becomes

$$\begin{aligned}
u(x,t) &= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x) \\
&= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n(1-x)) \\
&= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n(1-x)) \\
&= \frac{hx+1}{1+h} - 2h\sum_{n=1}^{\infty}\frac{\sin(\alpha_n(1-x))}{\alpha_n\left(h+\cos^2\alpha_n\right)}\exp\left(-\alpha_n^2 t\right)
\end{aligned}$$

3.1.2.7 [204] nonhomogeneous BC

problem number 204

Added July 2, 2018.

Second example from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$.

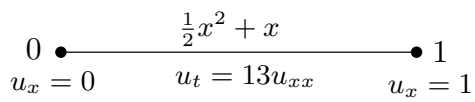

Figure 3.80: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, t], x] == 13*D[u[x, t], {x, 2}];
bc  = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
ic  = u[x, 0] == (1*x^2)/2 + x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x, x));
bc  := D[1](u)(0,t)=0,D[1](u)(1,t)=1;
ic  := u(x, 0) = 1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t) = \frac{x^2}{2} + 13t + 2\left(\sum_{n=1}^{\infty} \frac{((-1)^n - 1)\cos(\pi n x)\,\mathrm{e}^{-13\pi^2 n^2 t}}{\pi^2 n^2}\right) + \frac{1}{2}$$

3.1.2.8 [205] nonhomogeneous BC

problem number 205

Added March 31, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u_x(\pi,t) = A$$

Initial condition is $u(x,0) = 0$

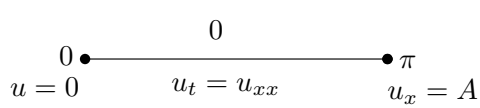

Figure 3.81: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] ;
ic  = u[x, 0] == 0;
bc  = {u[0,t] == 0, Derivative[1, 0][u][Pi, t] == A};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->A>0
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{8(-1)^{K[1]}Ae^{-\frac{1}{4}t(1-2K[1])^2}\sin\left(x\left(K[1]-\frac{1}{2}\right)\right)}{\pi(1-2K[1])^2}+Ax\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic  := u(x, 0) = 0:
bc  := u(0,t)=0,  eval( diff(u(x,t),x),x=Pi)=A:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t)=Ax+\left(\sum_{n=0}^{\infty}-\frac{8A(-1)^n\,\mathrm{e}^{-\frac{(2n+1)^2t}{4}}\sin\left(nx+\frac{1}{2}x\right)}{\pi\,(2n+1)^2}\right)$$

3.1.2.9 [206] nonhomogeneous BC

problem number 206

Added April 15, 2019.

Solve the heat equation for $u(x,t)$

$$u_t=ku_{rr}$$

For $0<r<a$ and $t>0$. The boundary conditions are

$$\begin{aligned}u(0,t)&=0\\u(a,t)&=a\phi(t)\end{aligned}$$

Initial condition is $u(r,0)=rf(r)$

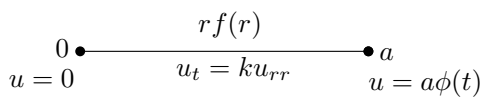

Figure 3.82: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*D[u[r, t], {r, 2}] ;
ic  = u[r, 0] == r*f[r];
bc  = {u[0, t] == 0, u[a, t] == a*phi[t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{a}}\left(e^{-\frac{k\pi^2 tK[1]^2}{a^2}} \int_0^a \sqrt{2}\sqrt{\frac{1}{a}} r(f(r)-\phi(0)) \sin\left(\frac{\pi r K[1]}{a}\right) dr + \int_0^t \frac{(-1)^{K[1]}\sqrt{2}e^{-\frac{k\pi}{}}}{\left(\frac{1}{a}\right)^{3/}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r, t), t) = k*diff(u(r, t), r$2):
ic  := u(r,0)=r*f(r);
bc  := u(0,t)=0,u(a,t)=a*phi(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(r, t))),
```

$$u(r,t) = r\phi(t) + \int_0^t \left(\sum_{n=1}^{\infty} \frac{2a(-1)^n \left(\frac{d}{d\tau}\phi(\tau)\right) \mathrm{e}^{-\frac{\pi^2(t-\tau)kn^2}{a^2}} \sin\left(\frac{\pi n r}{a}\right)}{\pi n}\right) d\tau + \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^a (f(r)-\phi(0))\, r \sin\left(\frac{\pi n}{a}\right.\right.}{a}\right.$$

<u>Hand solution</u>

Solve

$$u_t = ku_{rr} \qquad t > 0, 0 < r < a \tag{1}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(a,t) = a\phi(t)$$

And initial conditions

$$u(r,0) = rf(r)$$

Since the boundary conditions are not homogeneous, the first step is to convert them to homogeneous. This is done using a reference function which needs to only satisfy the boundary conditions. This reference function can be seen to be $v(r,t) = r\phi(t)$. Now we write

$$u(r,t) = w(r,t) + v(r,t)$$

Where $w(r,t)$ satisfies the PDE but with homogeneous B.C. Substituting the above into (1) gives

$$\begin{aligned} w_t(r,t) + r\phi'(t) &= kw_{rr} \\ w_t(r,t) &= kw_{rr} - r\phi'(t) \end{aligned} \tag{2}$$

With boundary conditions

$$\begin{aligned} w(0,t) &= 0 \\ w(a,t) &= 0 \end{aligned}$$

The solution to the homogeneous PDE $w_t(r,t) = kw_{rr}$ with the above boundary conditions is easily found and known. The eigenvalues are $\lambda_n = \left(\frac{n\pi}{a}\right)^2, n = 1, 2, \cdots$ and eigenfunctions $\Phi_n(r) = \sin\left(\sqrt{\lambda_n} r\right)$. Let the solution to (2), using eigenfunction expansion be

$$w(r,t) = \sum_{n=1}^{\infty} C_n(t)\,\Phi_n(r) \tag{2A}$$

Substituting the above back into (2) gives

$$\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) = k\sum_{n=1}^{\infty} C_n(t)\,\Phi_n''(r) - \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r) \tag{3}$$

Where $q_n(t)$ are the Fourier coefficients of $r\phi'(t)$ which are found by

$$r\phi'(t) = \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r)$$

Applying orthogonality using $\Phi_n(r)$ gives

$$\begin{aligned} \int_0^a r\phi'(t)\,\Phi_m(r)\,dr &= \int_0^a \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r)\,\Phi_m(r)\,dr \\ &= \sum_{n=1}^{\infty} q_n(t) \int_0^r \Phi_n(r)\,\Phi_m(r)\,dr \end{aligned}$$

But $\int_0^a \Phi_n(r)\,\Phi_m(r)\,dr = \int_0^a \sin\left(\frac{n\pi}{a}r\right)\sin\left(\frac{m\pi}{a}r\right)dr = \frac{a}{2}$ for $n=m$ only, and the above becomes

$$\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(s)\,dr = q_m(t)$$

Substituting the above back into (3) gives

$$\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) = k\sum_{n=1}^{\infty} C_n(t)\,\Phi_n''(r) - \sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)$$

But $\Phi_n''(r) = -\lambda_n\Phi_n(r)$ and above simplifies to

$$\begin{aligned}
\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) + k\sum_{n=1}^{\infty} C_n(t)\,\lambda_n\Phi_n(r) &= -\sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)\\
C_n'(t) + kC_n(t)\,\lambda_n &= -\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\\
&= -\frac{2}{a}\phi'(t)\int_0^a r\sin\left(\frac{n\pi}{a}r\right)dr\\
&= -\frac{2}{a}\phi'(t)\,\frac{(-1)^{n+1}\,a^2}{n\pi}\\
&= -2a\phi'(t)\,\frac{(-1)^{n+1}}{n\pi}
\end{aligned}$$

This is first order ODE in $C(t)$. The solution is

$$C_n(t) = e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau$$

From (2A)

$$w(r,t) = \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right)$$

Hence

$$\begin{aligned}
u(r,t) &= w(r,t) + v(r,t)\\
&= \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right) + r\phi(t)
\end{aligned} \tag{4}$$

At $t=0$ the above becomes

$$rf(r) = \sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) + r\phi(0)$$

$$\sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) = r(f(r) - \phi(0))$$

Hence $C_n(0)$ is the Fourier sine coefficients of $r(f(r)-\phi(0))$

$$\begin{aligned}\frac{a}{2}C_n(0) &= \int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\\ C_n(0) &= \frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\end{aligned}$$

Substituting this into (4) gives the final solution as

$$\begin{aligned}u(r,t) &= r\phi(t)+\sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)+2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n}{}\right.\\ &= r\phi(t)+\sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)+2a\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\right)\sin\left(\frac{n\pi}{a}\right.\\ &= r\phi(t)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\sin\left(\frac{n\pi}{a}r\right)+\sum_{n=1}^{\infty}2a\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}\end{aligned}$$

Or

$$\begin{aligned}u(r,t) &= r\phi(t)\\ &+\frac{2}{a}\sum_{n=1}^{\infty}e^{-k\lambda_n t}\sin\left(\frac{n\pi}{a}r\right)\left(\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\\ &+\frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi}{a}r\right)\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\end{aligned}$$

Where $\lambda_n=\left(\frac{n\pi}{a}\right)^2$.

3.1.3 Finite domain (bar), right end homogeneous, left end not

Local contents

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3.1.3.1 [207] left end BC depends on time (general case)

problem number 207

Added June 21, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = A(t)$$
$$u(L,t) = 0$$

And initial condition is $u(x,0) = 0$

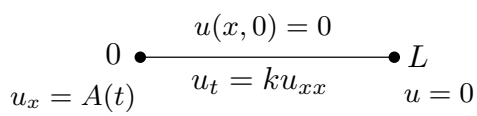

Figure 3.83: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == A[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\frac{4\sqrt{2}e^{-\frac{k\pi^2 t(1-2K[1])^2}{4L^2}}A(0)L^{3/2}}{\pi^2(1-2K[1])^2} + \int_0^t \frac{4\sqrt{2}\exp\left(-\frac{k\pi^2(1-2K[1])^2(t-K[2])}{4L^2}\right)L^{3/2}A'(K}{\pi^2(1-2K[1])^2}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =A(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = (-L+x)\,A(t) + \int_0^t \left(\sum_{n=0}^{\infty} \frac{8L\cos\left(\frac{(2n+1)\pi x}{2L}\right)\left(\frac{d}{d\tau}A(\tau)\right)\mathrm{e}^{-\frac{(t-\tau)\pi^2\left(n+\frac{1}{2}\right)^2 k}{L^2}}}{(2n+1)^2\pi^2} \right) d\tau + 8\left(\sum_{n=0}^{\infty} \frac{A(0)\,L\cos\left(\right.}{(2}\right.$$

Hand solution

This 1D heat PDE has left one end with boundary condition that is time dependent.

$$\begin{aligned} u_t &= k u_{xx} \qquad 0 < x < L, t > 0 \\ u_x(0,t) &= A(t) \\ u(L,t) &= 0 \\ u(x,0) &= 0 \end{aligned}$$

Solution

Since the boundary condition is not homogeneous, we need to first find a reference function $r(x,t)$. Let

$$r(x,t) = A(t)\,(x-L)$$

This function only needs to satisfy the nonhomogeneous boundary conditions given. i.e. $\frac{\partial r}{\partial x}(0,t) = A(t)\,, r(L,t) = 0$. Now we can write

$$u(x,t) = v(x,t) + r(x,t) \tag{1}$$

Since $r(x,t)$ satisfies the nonhomogeneous B.C's, then $v(x,t)$ satisfies the homogeneous boundary conditions. Substituting the above back into the original PDE gives

$$\begin{aligned} v_t + r_t &= k(v_{xx} + r_{xx}) \\ v_t + A'(t)\,(x-L) &= k v_{xx} \\ v_t &= k v_{xx} + A'(t)\,(L-x) \\ &= k v_{xx} + Q(x,t) \end{aligned} \tag{2}$$

The PDE $v_t = kv_{xx} + Q(x,t)$ has now homogeneous B.C. $v_x(0,t) = 0, v(L,0) = 0$. Where $Q(x,t) = A'(t)\,(L-x)$. The method of eigenfunction expansion is now used to solve (2). Let

$$v(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x)$$

Substituting this back into (2) gives

$$\begin{aligned}\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) &= k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + Q(x,t)\\ &= k\sum_{n=1}^{\infty} a_n(t)\,\Phi_n''(x) + \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)\end{aligned}$$

Where $Q(x,t) = \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)$. Now, since $\Phi_n''(x) = -\lambda_n\Phi_n(x)$ then the above reduces to

$$\begin{aligned}\sum_{n=1}^{\infty} a_n'(t)\,\Phi_n(x) + k\sum_{n=1}^{\infty} a_n(t)\,\lambda_n\Phi_n(x) &= \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(x)\\ a_n'(t) + k\lambda_n a_n(t) &= q_n(t) \qquad (3)\end{aligned}$$

The eigenfunctions $\Phi_n(x)$ come from solving the eigenvalue problem in $v_t = kv_{xx}$ with homogeneous boundary conditions $v_x(0,t) = 0, v(L,t) = 0$. This was solved before in problem 3.1.1.28 on page 476. The eigenfunctions were found to be $\Phi_n(x) = \cos\left(\sqrt{\lambda_n}x\right), n = 1,3,5,\cdots$ with eigenvalues $\lambda_n = \left(\frac{n\pi}{2L}\right)^2, n = 1,3,5,\cdots$. Before solving the ODE (3), we need to first find $q_n(t)$. Orthogonality is now used to find $q_n(t)$

$$\begin{aligned}Q(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} q_n(t)\cos\left(\sqrt{\lambda_n}x\right)\\ A'(t)\,(L-x) &= \sum_{n=1,3,5,\cdots}^{\infty} q_n(t)\cos\left(\sqrt{\lambda_n}x\right)\\ \int_0^L A'(t)\,(L-x)\cos\left(\sqrt{\lambda_n}x\right)dx &= \frac{L}{2}q_n(t)\\ q_n(t) &= \frac{2A'(t)}{L}\int_0^L (L-x)\cos\left(\frac{n\pi}{2L}x\right)dx\\ &= -\frac{2A'(t)}{L}\left(\frac{4L^2\left(\cos\left(\frac{n\pi}{2}\right)-1\right)}{\pi^2 n^2}\right) \qquad n = 1,3,5,\cdots\end{aligned}$$

But $\cos\left(\frac{n\pi}{2}\right)-1=-1$ for $n=1,3,5,\cdots$. Hence the above becomes

$$\begin{aligned} q_n(t) &= \frac{2A'(t)}{L}\left(\frac{4L^2}{\pi^2 n^2}\right) \qquad n=1,3,5,\cdots \\ &= \frac{8A'(t)\,L}{\pi^2 n^2} \end{aligned}$$

Hence (3) becomes

$$\begin{aligned} a_n'(t)+k\lambda_n a_n(t) &= q_n(t) \qquad n=1,2,3,\cdots \\ a_n'(t)+k\left(\frac{n\pi}{2L}\right)^2 a_n(t) &= \frac{8A'(t)\,L}{\pi^2 n^2} \\ a_n'(t)+k\lambda_n a_n(t) &= \frac{8A'(t)\,L}{\pi^2 n^2} \end{aligned}$$

Integrating factor is $e^{k\lambda_n t}$. Hence the above becomes

$$\frac{d}{dt}\left(a(t)\,e^{k\lambda_n t}\right)=\frac{8A'(t)\,L}{\pi^2 n^2}e^{k\lambda_n t}$$

Integrating gives

$$\begin{aligned} a_n(t)\,e^{k\lambda_n t} &= \int_0^t \frac{8A'(\tau)\,L}{\pi^2 n^2}e^{k\lambda_n \tau}d\tau + a_n(0) \\ a_n(t) &= a_n(0)\,e^{-k\lambda_n t}+e^{-k\lambda_n t}\int_0^t \frac{8A'(\tau)\,L}{\pi^2 n^2}e^{k\lambda_n \tau}d\tau \end{aligned}$$

Hence

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} a_n(t)\,\Phi_n(x) \\ &= \sum_{n=1,3,5,\cdots}^{\infty}\left(a_n(0)\,e^{-k\lambda_n t}+\frac{8L}{\pi^2 n^2}e^{-k\lambda_n t}\int_0^t A'(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right) \end{aligned}$$

Since $u(x,t)=v(x,t)+r(x,t)$ then

$$u(x,t)=A(t)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty}\left(a_n(0)\,e^{-k\lambda_n t}+\frac{8L}{\pi^2 n^2}e^{-k\lambda_n t}\int_0^t A'(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right) \tag{4}$$

At $t=0, u(x,0)=0$ and the above becomes

$$\begin{aligned} 0 &= A(0)\,(x-L)+\sum_{n=1,3,5,\cdots}^{\infty} a_n(0)\cos\left(\sqrt{\lambda_n}x\right) \\ A(0)\,(L-x) &= \sum_{n=1,3,5,\cdots}^{\infty} a_n(0)\cos\left(\sqrt{\lambda_n}x\right) \end{aligned}$$

Applying orthogonality

$$\int_0^L (L-x)\cos\left(\sqrt{\lambda_n}x\right)dx = a_n(0)\frac{L}{2}$$

$$\begin{aligned} a_n(0) &= \frac{2A(0)}{L}\int_0^L (L-x)\cos\left(\sqrt{\lambda_n}x\right)dx \\ &= \frac{2A(0)}{L}\left(-\frac{\cos\left(L\sqrt{\lambda_n}\right)-1}{\lambda_n}\right) \end{aligned}$$

But $\cos\left(L\sqrt{\lambda_n}\right)-1 = \cos\left(L\frac{n\pi}{2L}\right)-1 = \cos\left(\frac{n\pi}{2}\right)-1 = -1$ for $n = 1,3,5,\cdots$, and the above becomes

$$a_n(0) = \frac{2A(0)}{L\lambda_n}$$

Therefore the solution (4) is

$$u(x,t) = A(t)(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{2A(0)}{L\lambda_n} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)e^{k\lambda_n\tau}\right)\cos\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{2L}\right)^2$. Hence

$$u(x,t) = A(t)(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2A(0)}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)e^{k\left(\frac{n\pi}{2L}\right)^2\tau}d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

3.1.3.2 [208] left end BC depends on time (special case)

problem number 208

Added June 22, 2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u_x(0,t) &= A(t) \\ u(L,t) &= 0 \end{aligned}$$

And initial condition is $u(x,0) = 0$. Using the following values

$$\begin{aligned} L &= 5 \\ k &= \frac{1}{100} \\ A(t) &= e^t \end{aligned}$$

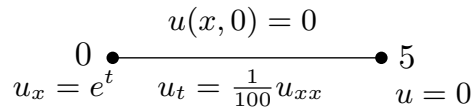

Figure 3.84: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == Exp[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{40e^{-\frac{\pi^2 t(1-2K[1])^2}{10000}}\cos\left(\frac{1}{10}\pi x(2K[1]-1)\right)\left(\pi^2(1-2K[1])^2+10000e^{\frac{\pi^2 t(1-2K[1])^2}{10000}+t}\right)}{\left(\pi^2(1-2K[1])^2+10000\right)(\pi-2\pi K[1])^2}+e^t(x\right.\right.$$

Maple ✓

```
restart;
k:=1/100;
L:=5;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =exp(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t)=\left(\sum_{n=0}^{\infty}\frac{10\left(2500\,\mathrm{e}^t+\pi^2\left(n+\frac{1}{2}\right)^2\mathrm{e}^{-\frac{\pi^2(2n+1)^2 t}{10000}}\right)\cos\left(\frac{(2n+1)\pi x}{10}\right)}{\left(2500+\left(n+\frac{1}{2}\right)^2\pi^2\right)\pi^2\left(n+\frac{1}{2}\right)^2}\right)+(x-5)\,\mathrm{e}^t$$

<u>Hand solution</u>

This PDE general solution was obtained in problem 3.1.3.1 on page 562 as

$$u(x,t) = A(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2A(0)}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\,e^{k\left(\frac{n\pi}{2L}\right)^2\tau}d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

When $A(t) = e^t$, the above becomes

$$u(x,t) = e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\int_0^t e(\tau)\,e^{k\left(\frac{n\pi}{2L}\right)^2\tau}d\tau\right)\cos\left(\frac{n\pi}{2L}x\right)$$

But

$$\int_0^t e(\tau)\,e^{k\left(\frac{n\pi}{2L}\right)^2\tau}d\tau = \frac{e^{k\left(\frac{n\pi}{2L}\right)^2 t+t} - 1}{k\left(\frac{n\pi}{2L}\right)^2 + 1}$$

And the general solution becomes

$$\begin{aligned}
u(x,t) &= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\frac{e^{k\left(\frac{n\pi}{2L}\right)^2 t+t} - 1}{k\left(\frac{n\pi}{2L}\right)^2 + 1}\right)\cos\left(\frac{n\pi}{2L}x\right)\\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2e^{-k\left(\frac{n\pi}{2L}\right)^2 t}}{L\left(\frac{n\pi}{2L}\right)^2} + \frac{8L}{\pi^2 n^2}\frac{e^t - e^{-k\left(\frac{n\pi}{2L}\right)^2 t}}{k\left(\frac{n\pi}{2L}\right)^2 + 1}\right)\cos\left(\frac{n\pi}{2L}x\right)\\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\left(\frac{n\pi}{2L}\right)^2 t}\left(\frac{2}{L\left(\frac{n\pi}{2L}\right)^2} - \frac{8L}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\right) + \frac{8Le^t}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\cos\left(\frac{n\pi}{2L}\right.\\
&= e^t(x-L) + \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{8L}{n^2\pi^2} - \frac{8L}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\right)e^{-k\left(\frac{n\pi}{2L}\right)^2 t} + \frac{8Le^t}{\left(k\left(\frac{n\pi}{2L}\right)^2 + 1\right)\pi^2 n^2}\cos\left(\frac{n\pi}{2L}x\right)
\end{aligned}$$

In this problem $L = 5, k = \frac{1}{100}$, hence the above becomes

$$u(x,t) = e^t(x-5) + \sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{40}{n^2\pi^2} - \frac{40}{\left(k\left(\frac{n\pi}{10}\right)^2 + 1\right)\pi^2 n^2}\right)e^{-\frac{n^2\pi^2}{10000}t} + \frac{40e^t}{\left(k\left(\frac{n\pi}{10}\right)^2 + 1\right)\pi^2 n^2}\right)\cos\left(\frac{n\pi}{10}x\right.$$

The following is an animation of the solution

Out[ ]=

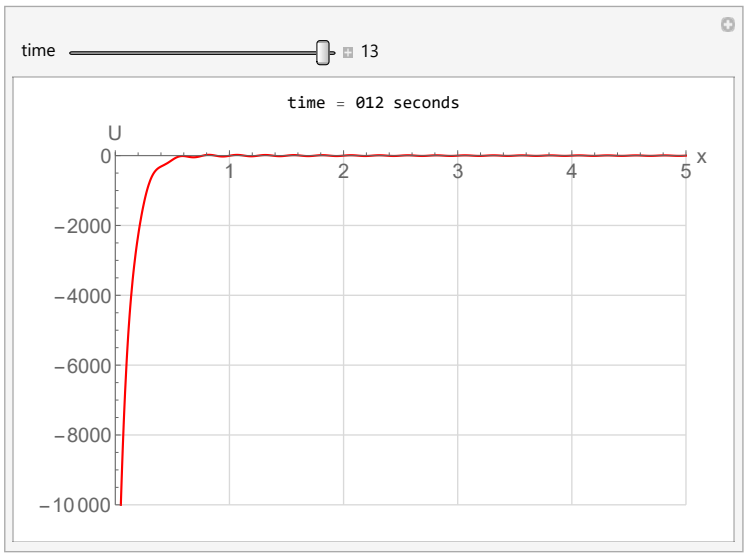

Figure 3.85: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, y, t, n, k]
L = 5;
k = 1 / 100;
numberOfTerms = 80;
mySol[x_, t_] =
  Exp[t] (x - L) + Sum[((8 L/(n^2 π^2) - 8 L/((k (n π/(2 L))^2 + 1) n^2 π^2)) Exp[-k (n π/(2 L))^2 t] + 8 L Exp[t]/((k (n π/(2 L))^2 + 1) n^2 π^2)) Cos[n π/(2 L) x],
    {n, 1, numberOfTerms, 2}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 3.86: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-1.5, 12}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 2500, 10}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.87: Code for animation

3.1.3.3 [209] left end BC depends on time (special case)

problem number 209

Added June 22, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = A(t)$$
$$u(L,t) = 0$$

And initial condition is $u(x,0) = 0$. Using the following values

$$L = 5$$
$$k = \frac{1}{100}$$
$$A(t) = \sin(t)$$

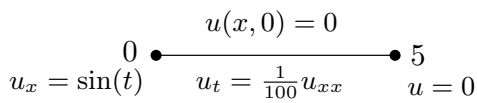

Figure 3.88: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=5;
k=1/100;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {Derivative[1,0][u][0, t] == Sin[t], u[L, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{400000\cos\left(\frac{1}{10}\pi x(2K[1]-1)\right)\left(\pi^2\cos(t)(1-2K[1])^2-e^{-\frac{\pi^2 t(1-2K[1])^2}{10000}}\pi^2(1-2K[1])^2+\right.}{\left(\pi^4(1-2K[1])^4+100000000\right)(\pi-2\pi K[1])^2}\right.\right.$$

Maple ✓

```
restart;
k:=1/100;
L:=5;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := D[1](u)(0,t) =sin(t),u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t)=-400000\left(\sum_{n=0}^{\infty}-\frac{\left(\pi^2\left(n+\frac{1}{2}\right)^2\cos(t)-\pi^2\left(n+\frac{1}{2}\right)^2 e^{-\frac{\pi^2(2n+1)^2 t}{10000}}+2500\sin(t)\right)\cos\left(\frac{(2n+1)\pi x}{10}\right)}{16\pi^2\left(n+\frac{1}{2}\right)^2\left(6250000+\left(n+\frac{1}{2}\right)^4\pi^4\right)}\right)+$$

Hand solution

This PDE general solution was obtained in problem 3.1.3.1 on page 562 as

$$u(x,t) = A(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{2A(0)}{L\lambda_n} + \frac{8L}{\pi^2 n^2}\int_0^t A'(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{2L}\right)^2, n = 1,3,5,\cdots$. When $A(t) = \sin(t)$, the above becomes

$$u(x,t) = \sin(t)\,(x-L) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{8L}{\pi^2 n^2}\int_0^t \cos(\tau)\,e^{k\lambda_n \tau}d\tau\right)\cos\left(\sqrt{\lambda_n}x\right)$$

But

$$\int_0^t \cos(\tau)\,e^{k\lambda_n \tau}d\tau = \frac{k\lambda_n e^{k\lambda_n t}\cos(t) + e^{k\lambda_n t}\sin(t) - k\lambda_n}{k^2\lambda_n^2 + 1}$$

And the general solution becomes

$$\begin{aligned} u(x,t) &= (x-L)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} e^{-k\lambda_n t}\left(\frac{8L}{\pi^2 n^2}\left(\frac{k\lambda_n e^{k\lambda_n t}\cos(t) + e^{k\lambda_n t}\sin(t) - k\lambda_n}{k^2\lambda_n^2+1}\right)\right)\cos\left(\sqrt{\lambda_n}x\right) \\ &= (x-L)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{8L}{\pi^2 n^2}\left(\frac{k\lambda_n\cos(t) + \sin(t) - k\lambda_n e^{-k\lambda_n t}}{k^2\lambda_n^2+1}\right)\cos\left(\sqrt{\lambda_n}x\right) \end{aligned}$$

In this problem $L = 5, k = \frac{1}{100}$, hence the above becomes

$$u(x,t) = (x-5)\sin(t) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{40}{\pi^2 n^2}\left(\frac{\frac{1}{100}\left(\frac{n\pi}{10}\right)^2\cos(t) + \sin(t) - \frac{1}{100}\left(\frac{n\pi}{10}\right)^2 e^{-\frac{1}{100}\left(\frac{n\pi}{10}\right)^2 t}}{\left(\frac{1}{100}\left(\frac{n\pi}{10}\right)^2\right)^2 + 1}\right)\cos\left(\frac{n\pi}{10}x\right)$$

The following is an animation of the solution

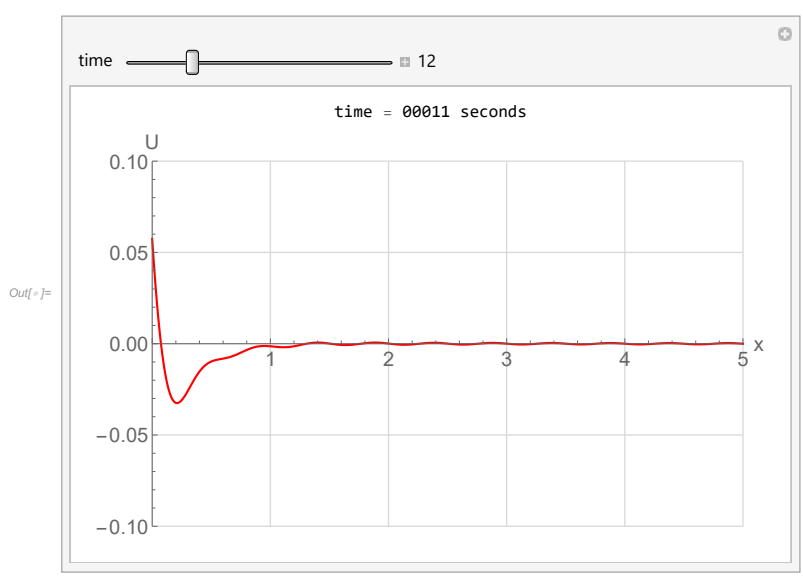

Figure 3.89: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, y, t, n, k]
L = 5;
k = 1 / 100;
λ = (n π / (2 L))^2;
numberOfTerms = 40;
mySol[x_, t_] =
  N[Sin[t] (x - L) + Sum[8 L / (n^2 π^2) ((k λ Cos[t] + Sin[t] - k λ Exp[-k λ t]) / (k^2 λ^2 + 1)) Cos[Sqrt[λ] x], {n, 1, numberOfTerms, 2}]];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"},
    SignPadding → True];
```

Figure 3.90: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {5, 1}], " seconds"}]},
              {
                Plot[Evaluate[mySol[x, t]], {x, 0, L},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  PerformanceGoal → "Quality",
                  PlotRange → {{0, L}, {-0.1, 0.1}},
                  ImageSize → 500,
                  AxesLabel → {"x", "U"},
                  GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  PlotStyle → Red
                ]

              }}],
          {t, 0, 50, 1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.1]
```

Figure 3.91: Code for animation

3.1.3.4 [210] Haberman 8.3.6 (special case)

problem number 210

Added Nov 25, 2018.

Problem 8.3.6 from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + \sin(5x)e^{-2t}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 1$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$

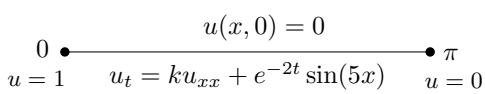

Figure 3.92: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + Sin[5*x]*Exp[-2*t];
bc  = {u[0, t] == 1, u[Pi, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty}\left(\sqrt{\frac{2}{\pi}}\int_0^t e^{-K[1]^2(t-K[2])}\left(\begin{cases} e^{-2K[2]}\sqrt{\frac{\pi}{2}} & K[1]=5 \\ 0 & \text{True}\end{cases}\right)dK[2] - \frac{2e^{-tK[1]^2}}{\pi K[1]}\right)\sin(xK[\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2)+ sin(5*x)*exp(-2*t);
ic  := u(x,0)=0;
bc  := u(0,t) =1,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t) = \frac{-23x - 46\pi\left(\sum_{n=1}^{\infty} \frac{e^{-n^2 t}\sin(nx)}{\pi n}\right) + \pi(-e^{-25t} + e^{-2t})\sin(5x) + 23\pi}{23\pi}$$

Hand solution

This problem has nonhomogeneous B.C. and non-homogenous in the PDE itself (source present). First step is to use reference function to remove the nonhomogeneous B.C. then use the method of eigenfunction expansion on the resulting problem. Let

$$r(x) = c_1 x + c_2$$

At $x = 0, r(x) = 1$, hence $1 = c_2$ and at $x = \pi, r(x) = 0$, hence $0 = c_1\pi + 1$ or $c_1 = -\frac{1}{\pi}$, therefore

$$r(x) = 1 - \frac{x}{\pi}$$

Therefore

$$u(x,t) = v(x,t) + r(x)$$

Where $v(x,t)$ solution for the given PDE but with homogeneous B.C., therefore

$$\begin{aligned} v_t &= v_{xx} + e^{-2t}\sin 5x \qquad (1)\\ v(0,t) &= 0\\ v(\pi,t) &= 0\\ v(x,0) &= f(x)\\ &= u(x,0) - r(x)\\ &= 0 - \left(1 - \frac{x}{\pi}\right)\\ &= \frac{x}{\pi} - 1 \end{aligned}$$

We now solve (1). This is a PDE with homogeneous B.C. of the form $v_t = v_{xx} + Q(x,t)$. The general solution to above PDE was solved in 3.1.6.4 on page 664 and the solution is

$$v(x,t)=\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)+\sum_{n=1}^{\infty}e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t\frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right. \tag{2}$$

Where

$$\Phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right) \tag{3}$$
$$\lambda_n=\left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots$$

Replacing $L=\pi, f(x)=\frac{x}{\pi}-1, Q(x,t)=e^{-2t}\sin(5x)$ into (3,2) gives

$$\Phi_n(x)=\sin(nx) \tag{3A}$$
$$\lambda_n=n^2 \qquad n=1,2,3,\cdots$$

And

$$v(x,t)=\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\frac{2}{\pi}\int_0^{\pi}\left(\frac{s}{\pi}-1\right)\sin(ns)\,ds\right)+\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\int_0^t\frac{2}{\pi}e^{kn^2\tau}e^{-2\tau}\left(\int_0^{\pi}\sin(5s\right.\right. \tag{2A}$$

But $\int_0^{\pi}\left(\frac{s}{\pi}-1\right)\sin(ns)\,ds=\frac{-1}{n}$ since n is integer. And $\int_0^{\pi}\sin 5s\sin(ns)\,ds=0$ when $n\neq 5$ and for $n=5$ it becomes $\frac{\pi}{2}$. Using these values in the above gives

$$v(x,t)=\sum_{n=1}^{\infty}e^{-kn^2t}\sin(nx)\left(\frac{-2}{\pi n}\right)+e^{-k(5)^2t}\sin(5x)\left(\int_0^t\frac{2}{\pi}e^{k(5)^2\tau}e^{-2\tau}\left(\frac{\pi}{2}\right)d\tau\right) \tag{2C}$$
$$=-\frac{2}{\pi}\sum_{n=1}^{\infty}e^{-kn^2t}\frac{\sin(nx)}{n}+e^{-25kt}\sin(5x)\left(\int_0^t e^{25k\tau}e^{-2\tau}d\tau\right) \tag{3.1}$$

But $\int_0^t e^{25k\tau}e^{-2\tau}d\tau=\frac{-1+e^{25kt-2t}}{25k-2}$ and the above becomes

$$v(x,t)=-\frac{2}{\pi}\sum_{n=1}^{\infty}e^{-kn^2t}\frac{\sin(nx)}{n}+e^{-25kt}\sin(5x)\left(\frac{-1+e^{25kt-2t}}{25k-2}\right)$$
$$=-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-kn^2t}\sin(nx)+\sin(5x)\left(\frac{-e^{-25kt}+e^{-2t}}{25k-2}\right)$$

Since $u(x,t) = v(x,t) + r(x)$ then the final solution is

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-kn^2t}\sin(nx)\right) + \sin(5x)\left(\frac{-e^{-25kt} + e^{-2t}}{25k - 2}\right)$$

Animation is below using $k = 1$, the solution becomes

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-n^2t}\sin(nx)\right) + \sin(5x)\left(\frac{e^{-2t} - e^{-25t}}{23}\right)$$

Out[ ]=

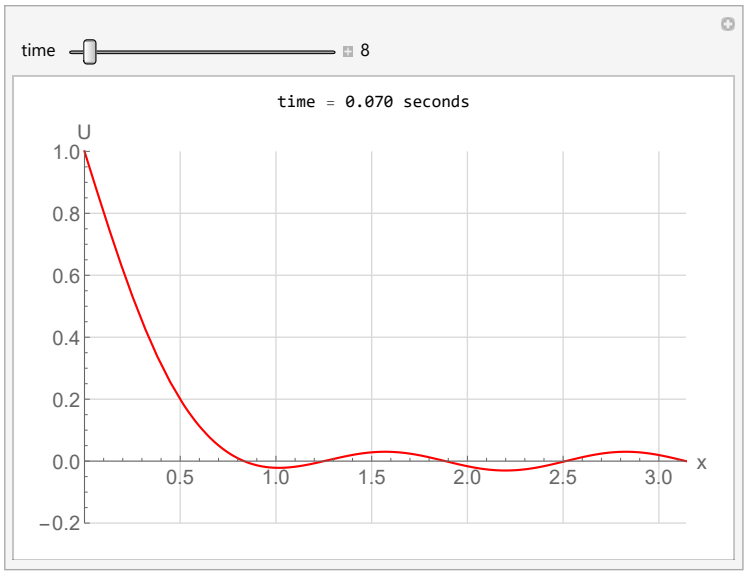

Figure 3.93: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = π;
k = 1;
f[x_] := 0;
Q[x_, t_] := Exp[-2 t] Sin[5 x];
ϕ[x_, n_] := Sin[n x];
λ = n^2;
numberOfTerms = 35;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = N[(1 - x/π) - 2/π Sum[1/n Exp[-k λ t] ϕ[x, n] , {n, 1, numberOfTerms}]] + Sin[5 x] ((Exp[-2 t] - Exp[-25 k t])/(25 k - 2));
```

Figure 3.94: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-.2, 1}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 2, .01}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.95: Code for animation

3.1.3.5 [211] BC depends on time (special case)

problem number 211

added March 8, 2018. Exam problem

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = t$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

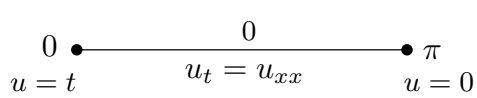

Figure 3.96: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == t, u[Pi, t] == 0};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{\left(2-2e^{-n^2t}\right)\sin(nx)}{n^3\pi}-\frac{tx}{\pi}+t\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=diff(u(x,t),x$2);
bc  := u(0,t)=t,u(Pi,t)=0;
ic  := u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))),outp
```

$$u(x,t)=\frac{12\pi\left(\sum_{n=1}^{\infty}\frac{e^{-n^2t}\sin(nx)}{\pi n^3}\right)-6\left(\frac{1}{6}x^2+t-\frac{1}{3}\pi x\right)(x-\pi)}{6\pi}$$

3.1.4 Finite domain (bar), Periodic BC

Local contents

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3.1.4.1 [212] Periodic BC (general case). IC given

problem number 212

Solve the heat equation

$$u_t = k u_{xx}$$

For $-L < x < L$ and $t > 0$. The boundary conditions are periodic

$$u(-L,t) = u(L,t)$$
$$\frac{\partial u}{\partial x}(-L,t) = \frac{\partial u}{\partial x}(L,t)$$

And initial conditions $u(x,0) = f(x)$

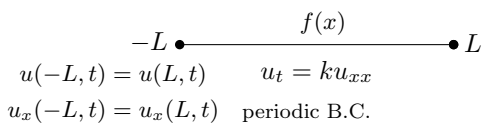

Figure 3.97: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[-L, t] == u[L, t], Derivative[1, 0][u][-L, t] == Derivative[1, 0][u][L, t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{e^{-\frac{k\pi^2 t K[1]^2}{L^2}} \left(\cos\left(\frac{\pi x K[1]}{L}\right) \int_{-L}^{L} \frac{\cos\left(\frac{\pi x K[1]}{L}\right) f(x)}{\sqrt{L}}\, dx + \left(\int_{-L}^{L} \frac{f(x) \sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\, dx\right) \sin\left(\frac{\pi x K[1]}{L}\right)\right)}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(-L,t)=u(L,t),eval(diff(u(r,t),r),r=-L)=eval(diff(u(r,t),r),r=L);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \frac{2L\left(\sum_{n=1}^{\infty} \frac{\left(\left(\int_{-L}^{L} \cos\left(\frac{\pi n x}{L}\right) f(x) dx\right) \cos\left(\frac{\pi n x}{L}\right) + \left(\int_{-L}^{L} f(x) \sin\left(\frac{\pi n x}{L}\right) dx\right) \sin\left(\frac{\pi n x}{L}\right)\right) e^{-\frac{\pi^2 k n^2 t}{L^2}}}{L}\right) + \int_{-L}^{L} f(x)\, dx}{2L}$$

3.1.4.2 [213] Periodic BC (general case). No IC given

problem number 213

Added Sept 21, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $-\pi < x < \pi$ and $t > 0$. The boundary conditions are periodic

$$u(-\pi, t) = u(\pi, t)$$
$$\frac{\partial u}{\partial x}(-\pi, t) = \frac{\partial u}{\partial x}(\pi, t)$$

No initial conditions give.

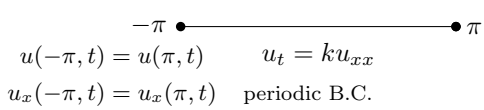

Figure 3.98: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[-Pi, t] == u[Pi, t], Derivative[1, 0][u][-Pi, t] == Derivative[1, 0][u][Pi, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to c_1\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc  := u(-Pi,t)=u(Pi,t),D[1](u)(-Pi,0)=D[1](u)(Pi,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) ),output=
```

$$u(x,t) = c_3\left(c_1 e^{ix} + c_2 e^{-ix}\right) e^{-kt}$$
$$u(x,t) = c_3(c_1 + c_2)$$

Hand solution

Solve the heat equation $u_t = ku_{xx}$ with periodic boundary conditions $u(t,-\pi) = u(t,\pi)\,, u_x(t,-\pi) = u_x(t,\pi)$

Solution

Using separation of variables, Let $u(x,t) = T(t)\,X(x)$. Substituting this into $u_t = ku_{xx}$ gives $T'X = TX''$. Dividing by $XT \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

Where λ is the seperation constant. This gives the following ODE's to solve

$$X''(x) + \lambda X(x) = 0$$
$$T'(t) + \lambda k T(t) = 0$$

Where λ is the separation constant. Eigenfunctions are solutions to the spatial ODE.

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

To determine the actual eigenfunctions and eigenvalues, boundary conditions are used. Starting with the spatial ODE above, and transferring the boundary condition to X, it becomes

$$\begin{aligned} X''(x) + \lambda X(x) &= 0 \\ X(-\pi) &= X(\pi) \\ X'(-\pi) &= X'(\pi) \end{aligned}$$

This is an eigenvalue boundary value problem. The solution is

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

<u>case $\lambda < 0$</u>

Since $\lambda < 0$, then $-\lambda$ is positive. Let $\mu = -\lambda$, where μ is now positive. The solution (1) becomes

$$X(x) = c_1 e^{\sqrt{\mu}x} + c_2 e^{-\sqrt{\mu}x}$$

The above can be written as

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) + c_2 \sinh\left(\sqrt{\mu}x\right) \tag{2}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using (2) gives

$$\begin{aligned} c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(\sqrt{\mu}\pi\right) \\ c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_2 \sinh\left(\sqrt{\mu}\pi\right) \end{aligned}$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_2 = 0$. The solution (2) now reduces to

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) \tag{3}$$

Taking derivative gives

$$X'(x) = c_1 \sqrt{\mu} \sinh\left(\sqrt{\mu}x\right) \tag{4}$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using (4) gives

$$c_1 \sqrt{\mu} \sinh\left(-\sqrt{\mu}\pi\right) = c_1 \sqrt{\mu} \sinh\left(\sqrt{\mu}x\right)$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_1 = 0$. This means a trivial solution. Therefore $\lambda < 0$ is not an eigenvalue.

<u>case $\lambda = 0$</u>

In this case the solution is $X(x) = c_1 + c_2 x$. Applying first BC $X(-\pi) = X(\pi)$ gives

$$\begin{aligned} c_1 - c_2\pi &= c_1 + c_2\pi \\ -c_2\pi &= c_2\pi \end{aligned}$$

This gives $c_2 = 0$. The solution now becomes $X(x) = c_1$ and $X'(x) = 0$. Applying the second boundary conditions $X'(-\pi) = X'(\pi)$ is not satisfies $(0 = 0)$. Therefore $\underline{\lambda = 0}$ is an eigenvalue with eigenfunction $X_0(0) = 1$ (selected $c_1 = 1$ since an arbitrary constant).

<u>case $\lambda > 0$</u>

The solution in this case is

$$\begin{aligned} X(x) &= c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \\ &= c_1 e^{i\sqrt{\lambda}x} + c_2 e^{-i\sqrt{\lambda}x} \end{aligned}$$

Which can be rewritten as (the constants c_1, c_2 below will be different than the above c_1, c_2, but kept the same name for simplicity).

$$X(x) = c_1 \cos\left(\sqrt{\lambda}x\right) + c_2 \sin\left(\sqrt{\lambda}x\right) \tag{5}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using the above gives

$$\begin{aligned} c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(\sqrt{\lambda}\pi\right) \\ c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_2 \sin\left(\sqrt{\lambda}\pi\right) \end{aligned}$$

There are two choices here. If $\sin\left(-\sqrt{\lambda}\pi\right) \neq \sin\left(\sqrt{\lambda}\pi\right)$, then this implies that $c_2 = 0$. If $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$ then $c_2 \neq 0$. Assuming for now that $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$. This happens when $\sqrt{\lambda}\pi = n\pi, n = 1, 2, 3, \cdots$, or

$$\lambda_n = n^2 \qquad n = 1, 2, 3, \cdots$$

Using this choice, we will now look to see what happens using the second BC. The solution (5) now becomes

$$X(x) = c_1 \cos(nx) + c_2 \sin(nx) \qquad n = 1, 2, 3, \cdots$$

Therefore

$$X'(x) = -c_1 n \sin(nx) + c_2 n \cos(nx)$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using the above gives

$$\begin{aligned} c_1 n \sin(n\pi) + c_2 n \cos(n\pi) &= -c_1 n \sin(n\pi) + c_2 n \cos(n\pi) \\ c_1 n \sin(n\pi) &= -c_1 n \sin(n\pi) \\ 0 &= 0 \end{aligned}$$

Since n is integer. Therefore this means that using $\lambda_n = n^2$ will satisfy both boundary conditions with $c_2 \neq 0, c_1 \neq 0$. This means the solution (5) becomes

$$X_n(x) = A_n \cos(nx) + B_n \sin(nx) \qquad n = 1, 2, 3, \cdots$$

The above says that there are two eigenfunctions in this case. They are

$$X_n(x) = \begin{cases} \cos(nx) \\ \sin(nx) \end{cases}$$

Since there is also zero eigenvalue, then the complete set of eigenfunctions become

$$X_n(x) = \begin{cases} 1 \\ \cos(nx) \\ \sin(nx) \end{cases}$$

Now that the eigenvalues are found, the solution to the time ODE can be found. Recalling that the time ODE from above was found to be

$$T'(t) + \lambda k T(t) = 0$$

For the zero eigenvalue case, the above reduces to $T'(t) = 0$ which has the solution $T_0(t) = C_0$. For non zero eigenvalues $\lambda_n = n^2$, the ODE becomes $T'(t) + n^2 T(t) = 0$, whose solution is $T_0(t) = C_n e^{-kn^2 t}$.

Putting all the above together, gives the fundamental solution as

$$u_n(x,t) = \begin{cases} C_0 & \\ C_n \cos(nx)\, e^{-kn^2 t} & n = 1, 2, 3, \cdots \\ B_n \sin(nx)\, e^{-kn^2 t} & n = 1, 2, 3, \cdots \end{cases}$$

Therefore the complete solution is the sum of the above solutions

$$u(x,t) = C_0 + \sum_{n=1}^{\infty} e^{-kn^2 t} (C_n \cos(nx) + B_n \sin(nx))$$

The constants C_0, C_n, B_n can be found from initial conditions.

3.1.4.3 [214] Periodic BC (general case). Damped heat PDE. No IC given

problem number 214

Added Sept 21, 2019

Solve the heat equation

$$u_t = ku_{xx} - u(x,t)$$

For $-\pi < x < \pi$ and $t > 0$. The boundary conditions are periodic

$$u(-\pi,t) = u(\pi,t)$$
$$\frac{\partial u}{\partial x}(-\pi,t) = \frac{\partial u}{\partial x}(\pi,t)$$

No initial conditions give.

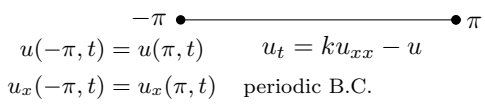

Figure 3.99: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] -u[x,t];
bc  = {u[-Pi, t] == u[Pi, t], Derivative[1, 0][u][-Pi, t] == Derivative[1, 0][u][Pi, t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)-u(x,t);
bc  := u(-Pi,t)=u(Pi,t),D[1](u)(-Pi,0)=D[1](u)(Pi,0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) ),output=
```

$$u(x,t) = c_3\left(c_1 e^{ix} + c_2 e^{-ix}\right) e^{-(k+1)t}$$
$$u(x,t) = c_3(c_1 + c_2)\, e^{-t}$$

Hand solution

Solve the heat equation $u_t = ku_{xx} - u$ with periodic boundary conditions $u(t,-\pi) = u(t,\pi)\,, u_x(t,-\pi) = u_x(t,\pi)$

Solution

Using separation of variables, Let $u(x,t) = T(t)\,X(x)$. Substituting this into $u_t = ku_{xx}$ gives $T'X = TX''$. Dividing by $XT \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} + XT = \frac{X''}{X} = -\lambda$$

Where λ is the seperation constant. This gives the following ODE's to solve

$$\begin{aligned} X''(x) + \lambda X(x) &= 0 \\ T'(t) + k(1+\lambda)\,T(t) &= 0 \end{aligned}$$

Where λ is the separation constant. Eigenfunctions are solutions to the spatial ODE.

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

To determine the actual eigenfunctions and eigenvalues, boundary conditions are used. Transferring the boundary condition to X, it becomes

$$\begin{aligned} X''(x) + \lambda X(x) &= 0 \\ X(-\pi) &= X(\pi) \\ X'(-\pi) &= X'(\pi) \end{aligned}$$

This is an eigenvalue boundary value problem. The solution is

$$X(x) = c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \tag{1}$$

case $\lambda < 0$

Since $\lambda < 0$, then $-\lambda$ is positive. Let $\mu = -\lambda$, where μ is now positive. The solution (1) becomes

$$X(x) = c_1 e^{\sqrt{\mu}x} + c_2 e^{-\sqrt{\mu}x}$$

The above can be written as

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) + c_2 \sinh\left(\sqrt{\mu}x\right) \tag{2}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using (2) gives

$$\begin{aligned} c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_1 \cosh\left(\sqrt{\mu}\pi\right) + c_2 \sinh\left(\sqrt{\mu}\pi\right) \\ c_2 \sinh\left(-\sqrt{\mu}\pi\right) &= c_2 \sinh\left(\sqrt{\mu}\pi\right) \end{aligned}$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_2 = 0$. The solution (2) now reduces to

$$X(x) = c_1 \cosh\left(\sqrt{\mu}x\right) \tag{3}$$

Taking derivative gives

$$X'(x) = c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right) \tag{4}$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using (4) gives

$$c_1\sqrt{\mu}\sinh\left(-\sqrt{\mu}\pi\right) = c_1\sqrt{\mu}\sinh\left(\sqrt{\mu}x\right)$$

But sinh is only zero when its argument is zero which is not the case here. Therefore the above implies that $c_1 = 0$. This means a trivial solution. Therefore $\lambda < 0$ is not an eigenvalue.

<u>case $\lambda = 0$</u>

In this case the solution is $X(x) = c_1 + c_2 x$. Applying first BC $X(-\pi) = X(\pi)$ gives

$$\begin{aligned} c_1 - c_2\pi &= c_1 + c_2\pi \\ -c_2\pi &= c_2\pi \end{aligned}$$

This gives $c_2 = 0$. The solution now becomes $X(x) = c_1$ and $X'(x) = 0$. Applying the second boundary conditions $X'(-\pi) = X'(\pi)$ is not satisfies $(0 = 0)$. Therefore <u>$\lambda = 0$</u> is an eigenvalue with eigenfunction $X_0(0) = 1$ (selected $c_1 = 1$ since an arbitrary constant).

<u>case $\lambda > 0$</u>

The solution in this case is

$$\begin{aligned} X(x) &= c_1 e^{\sqrt{-\lambda}x} + c_2 e^{-\sqrt{-\lambda}x} \\ &= c_1 e^{i\sqrt{\lambda}x} + c_2 e^{-i\sqrt{\lambda}x} \end{aligned}$$

Which can be rewritten as (the constants c_1, c_2 below will be different than the above c_1, c_2, but kept the same name for simplicity).

$$X(x) = c_1 \cos\left(\sqrt{\lambda}x\right) + c_2 \sin\left(\sqrt{\lambda}x\right) \tag{5}$$

Applying first B.C. $X(-\pi) = X(\pi)$ using the above gives

$$\begin{aligned} c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_1 \cos\left(\sqrt{\lambda}\pi\right) + c_2 \sin\left(\sqrt{\lambda}\pi\right) \\ c_2 \sin\left(-\sqrt{\lambda}\pi\right) &= c_2 \sin\left(\sqrt{\lambda}\pi\right) \end{aligned}$$

There are two choices here. If $\sin\left(-\sqrt{\lambda}\pi\right) \neq \sin\left(\sqrt{\lambda}\pi\right)$, then this implies that $c_2 = 0$. If $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$ then $c_2 \neq 0$. Assuming for now that $\sin\left(-\sqrt{\lambda}\pi\right) = \sin\left(\sqrt{\lambda}\pi\right)$. Then happens when $\sqrt{\lambda}\pi = n\pi, n = 1, 2, 3, \cdots$, or

$$\lambda_n = n^2 \qquad n = 1, 2, 3, \cdots$$

Using this choice, we will now look to see what happens using the second BC. The solution (5) now becomes

$$X(x) = c_1 \cos(nx) + c_2 \sin(nx) \qquad n = 1, 2, 3, \cdots$$

Therefore

$$X'(x) = -c_1 n \sin(nx) + c_2 n \cos(nx)$$

Applying the second BC $X'(-\pi) = X'(\pi)$ using the above gives

$$\begin{aligned} c_1 n \sin(n\pi) + c_2 n \cos(n\pi) &= -c_1 n \sin(n\pi) + c_2 n \cos(n\pi) \\ c_1 n \sin(n\pi) &= -c_1 n \sin(n\pi) \\ 0 &= 0 \end{aligned}$$

Since n is integer. Therefore this means that using $\lambda_n = n^2$ will satisfy both boundary conditions with $c_2 \neq 0, c_1 \neq 0$. This means the solution (5) becomes

$$X_n(x) = A_n \cos(nx) + B_n \sin(nx) \qquad n = 1, 2, 3, \cdots$$

The above says that there are two eigenfunctions in this case. They are

$$X_n(x) = \begin{cases} \cos(nx) \\ \sin(nx) \end{cases}$$

Since there is also zero eigenvalue, then the complete set of eigenfunctions become

$$X_n(x) = \begin{cases} 1 \\ \cos(nx) \\ \sin(nx) \end{cases}$$

Now that the eigenvalues are found, the solution to the time ODE can be found. Recalling that the time ODE from above was found to be

$$T'(t) + k(\lambda + 1)\, T(t) = 0$$

For the zero eigenvalue case, the above reduces to $T'(t) + kT(t) = 0$ which has the solution $T_0(t) = C_0 e^{-kt}$. For non zero eigenvalues $\lambda_n = n^2$, the ODE becomes $T'(t) + k(n^2 + 1)\, T(t) = 0$, whose solution is $T_0(t) = C_n e^{-k(n^2+1)t}$.

Putting all the above together, gives the fundamental solution as

$$u_n(t,x) = \begin{cases} C_0 e^{-kt} & \\ C_n \cos(nx)\, e^{-k(n^2+1)t} & n = 1,2,3,\cdots \\ B_n \sin(nx)\, e^{-k(n^2+1)t} & n = 1,2,3,\cdots \end{cases}$$

Therefore the complete solution is the sum of the above solutions

$$u(t,x) = C_0 e^{-kt} + \sum_{n=1}^{\infty} e^{-k(n^2+1)t}(C_n \cos(nx) + B_n \sin(nx))$$

The constants C_0, C_n, B_n can be found from initial conditions.

3.1.5 Finite domain (bar), Both ends nonhomogeneous BC

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problem number 215

Added June 22, 2019

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

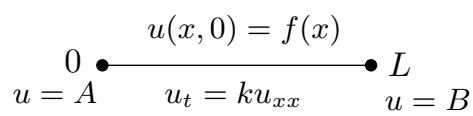

Figure 3.100: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{2e^{-\frac{kn^2\pi^2 t}{L^2}} \left(\int_0^L \left(-A + \frac{(A-B)x}{L} + f(x)\right) \sin\left(\frac{n\pi x}{L}\right) dx\right) \sin\left(\frac{n\pi x}{L}\right)}{L} + \frac{x(B-A)}{L} + A\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{AL - 2L\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L (Bx - Lf(x) + (L-x)A)\sin\left(\frac{\pi n x}{L}\right)dx\right)e^{-\frac{\pi^2 k n^2 t}{L^2}}\sin\left(\frac{\pi n x}{L}\right)}{L^2}\right) - (A-B)x}{L}$$

Hand solution

Solve

$$u_t = k u_{xx} \qquad t > 0, 0 < x < L \tag{1}$$

BC are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial conditions

$$u(x,0) = f(x)$$

Solution

Since boundary conditions are nonhomogeneous, then the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables (separation of variables can only be done on a PDE with homogeneous B.C.)

This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies homogeneous version of the B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$0 = k r''(x)$$
$$r(0) = A$$
$$r(L) = B$$

This has the solution $r(x) = c_1 x + c_2$. Using BC at $x = 0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1 x + A$. Using BC at $x = L$ gives $B = c_1 L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right) x + A$$

Therefore (1) becomes

$$u(x,t) = v(x,t) + \left(\frac{B-A}{L}\right) x + A$$

Substituting the above back in the original PDE $u_t = k u_{xx}$ gives

$$\begin{aligned} v_t &= k v_{xx} \\ v(0) &= 0 \\ v(L) &= 0 \\ v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\frac{B-A}{L} x + A\right) \end{aligned} \tag{3}$$

The above PDE was solved in problem 3.1.1.1 on page 400 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L F(s)\sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots \end{aligned} \tag{4}$$

Substituting (3) into (4) gives

$$v(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right)$$

From (2)

$$\begin{aligned} u(x,t) &= v(x,t) + r(x) \\ &= A + \left(\frac{B-A}{L}\right) x + \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\sqrt{\lambda_n} s\right) ds\right) e^{-k\lambda_n t} \sin\left(\sqrt{\lambda_n} x\right) \\ &= A + \left(\frac{B-A}{L}\right) x + \frac{2}{L}\sum_{n=1}^{\infty} \left(\int_0^L \left(f(s) - \left(\frac{B-A}{L} s + A\right)\right) \sin\left(\frac{n\pi}{L} s\right) ds\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

3.1.5.2 [216] non-homogeneous BC (special case)

problem number 216

Taken from Maple PDE help pages

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 20$$
$$u(1,t) = 50$$

Initial condition is $u(x,0) = 0$

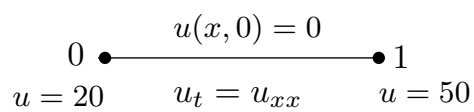

Figure 3.101: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 20, u[1, t] == 50};
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to -\frac{2\sum_{n=1}^{\infty}\frac{(20-50(-1)^n)e^{-n^2\pi^2 t}\sin(n\pi x)}{n}}{\pi} + 30x + 20\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=20, u(1,t)=50;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = 30x + 20\left(\sum_{n=1}^{\infty} \frac{(5(-1)^n - 2)\,\mathrm{e}^{-\pi^2 n^2 t}\sin(\pi n x)}{\pi n}\right) + 20$$

<u>Hand solution</u>

The general solution to

$$u_t = k u_{xx} \qquad t > 0, 0 < x < L \tag{1}$$

BC are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial conditions

$$u(x,0) = f(x)$$

Is given in problem 3.1.5.1 on page 591 as

$$u(x,t) = A + \left(\frac{B-A}{L}\right)x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \left(\frac{B-A}{L}x + A\right)\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. Substituting $A = 20, B = 50, L = 1, k = 1, f(x) = 0$ gives the solution as

$$u(x,t) = 20 + 30x + 2\sum_{n=1}^{\infty}\left(\int_0^1 -(30x+20)\sin(n\pi x)\,dx\right)e^{-kn^2\pi^2 t}\sin(n\pi x)$$

But $\int_0^1 -(30x+20)\sin(n\pi x)\,dx = \frac{50(-1)^n - 20}{n\pi}$ and the above becomes

$$\begin{aligned} u(x,t) &= 20 + 30x + 2\sum_{n=1}^{\infty}\left(\frac{50(-1)^n - 20}{n\pi}\right)e^{-kn^2\pi^2 t}\sin(n\pi x) \\ &= 20 + 30x + \sum_{n=1}^{\infty}\left(\frac{100(-1)^n - 40}{n\pi}\right)e^{-kn^2\pi^2 t}\sin(n\pi x) \end{aligned}$$

3.1.5.3 [217] Articolo 8.4.1 (special case)

problem number 217

Added December 20, 2018.

Example 8.4.1 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 10$$
$$u(1,t) = 20$$

Initial condition is $u(x,0) = 60x - 50x^2 + 10$ and $k = \frac{1}{20}$

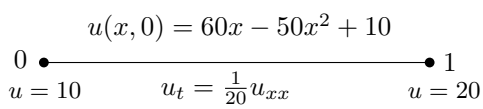

Figure 3.102: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/20;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 10, u[1, t] == 20};
ic  = u[x, 0] == 60*x - 50*x^2 + 10;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} -\frac{200(-1+(-1)^n)\, e^{-\frac{1}{20}n^2\pi^2 t} \sin(n\pi x)}{n^3\pi^3} + 10(x+1)\right\}\right\}$$

Maple ✓

```
restart;
k := 1/20;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = 10, u(1, t) = 20;
ic  := u(x, 0) = 60*x - 50*x^2 + 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t) = 10x - 200\left(\sum_{n=1}^{\infty} \frac{((-1)^n - 1)\,\mathrm{e}^{-\frac{\pi^2 n^2 t}{20}} \sin(\pi n x)}{\pi^3 n^3}\right) + 10$$

<u>Hand solution</u>

The general solution to

$$u_t = k u_{xx} \qquad t > 0, 0 < x < L \tag{1}$$

BC are

$$u(0,t) = A$$
$$u(L,t) = B$$

with Initial conditions

$$u(x,0) = f(x)$$

Is given in problem 3.1.5.1 on page 591 as

$$u(x,t) = A + \left(\frac{B-A}{L}\right)x + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L \left(f(x) - \left(\frac{B-A}{L}x + A\right)\right)\sin\left(\sqrt{\lambda_n}x\right)dx\right)e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$. Substituting $A = 10, B = 20, L = 1, k = \frac{1}{20}, f(x) = 60x - 50x^2 + 10$ gives the solution as

$$\begin{aligned} u(x,t) &= 10 + 10x + 2\sum_{n=1}^{\infty}\left(\int_0^1 \left(60x - 50x^2 + 10 - (10x + 10)\right)\sin(n\pi x x)\,dx\right)e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x) \\ &= 10 + 10x + 2\sum_{n=1}^{\infty}\left(\int_0^1 \left(50x - 50x^2\right)\sin(n\pi x x)\,dx\right)e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x) \end{aligned}$$

But $\int_0^1 (50x - 50x^2)\sin(n\pi x x)\,dx = \frac{100 - 100(-1)^n}{n^3\pi^3}$ and the above becomes

$$\begin{aligned} u(x,t) &= 10 + 10x + 2\sum_{n=1}^{\infty} 100\left(\frac{1-(-1)^n}{n^3\pi^3}\right)e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x) \\ &= 10 + 10x - \sum_{n=1}^{\infty} 200\left(\frac{(-1)^n - 1}{n^3\pi^3}\right)e^{-\frac{1}{20}n^2\pi^2 t}\sin(n\pi x x) \end{aligned}$$

3.1.5.4 [218] With source that depends on space only (general case)

problem number 218

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(1,t) = B$$

Initial condition is $u(x,0) = f(x)$

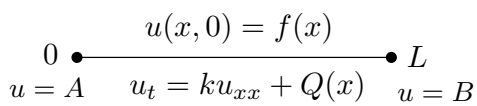

Figure 3.103: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\frac{\left(1-e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\right)\left(\int_0^L \frac{\sqrt{2}Q(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx\right)L^2}{k\pi^2 K[1]^2} + e^{-\frac{k\pi^2 tK[1]^2}{L^2}} \int_0^L \frac{\sqrt{2}(-Bx+A(x-L)+Lf(x))\sin\left(\right.}{L^{3/2}}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q(x);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t) = \frac{-2Lk\left(\sum_{n=1}^{\infty} \frac{\left(\int_0^L -\left(L\left(\int_0^{-a}\int_0^{-z1} Q(\_z1)d\_z1d\_z1\right)-\_a\left(\int_0^L\int_0^{-z1} Q(\_z1)d\_z1d\_z1\right)+(-AL+Lf(\_a)+(A-B)\_a)k\right) \mathrm{si}}{L^2k}\right.}{}$$

Hand solution

Solving

$$u_t = ku_{xx} + Q(x) \tag{1}$$

With initial conditions $u(x,0) = f(x)$ and boundary conditions $u(0,t) = A, u(L,t) = B$ with $0 < x < L, t > 0$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$\begin{aligned} 0 &= kr''(x) \\ r(0) &= A \\ r(L) &= B \end{aligned}$$

This has the solution $r(x) = c_1x + c_2$. Using BC at $x = 0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1x + A$. Using BC at $x = L$ gives $B = c_1L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right)x + A \tag{3}$$

Substituting (2) back in the original PDE $u_t = ku_{xx} + Q(x)$ gives

$$\begin{aligned} v_t &= kv_{xx} + Q(x) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \end{aligned} \tag{4}$$

The initial conditions are

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\left(\frac{B-A}{L}\right)x + A\right) \end{aligned}$$

The general solution to (4) was solved in 3.1.1.11 on page 429 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right. \\ \Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

But $F(x) = f(x) - \left(\left(\frac{B-A}{L}\right)x + A\right)$ in this case, hence the solution becomes

$$\begin{aligned} v(x,t) = &\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right)s + A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{L}e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right) \end{aligned}$$

Since $u(x,t) = v(x,t) + r(x)$, then the final solution is

$$\begin{aligned} u(x,t) = &\left(\frac{B-A}{L}\right)x + A + \\ &\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right)s + A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{L}e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right) \end{aligned}$$

3.1.5.5 [219] With source that depends on space only (special case)

problem number 219

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= A \\ u(1,t) &= B \end{aligned}$$

Initial condition is $u(x,0) = f(x)$, Using the following values

$$\begin{aligned} A &= 20 \\ B &= 50 \\ f(x) &= 60 - 2x \\ L &= 30 \\ k &= \frac{1}{10} \\ Q(x) &= \frac{x}{10} \end{aligned}$$

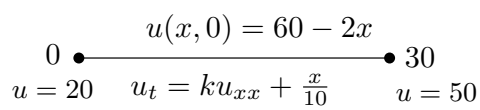

Figure 3.104: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
A=20; B=50; f=60-2*x; L=30; k=1/10; Q=x/10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}-\frac{20e^{-\frac{\pi^2 tK[1]^2}{9000}}\left(-\left(\left(4+5(-1)^{K[1]}\right)\pi^2K[1]^2\right)-2700(-1)^{K[1]}+2700(-1)^{K[1]}e^{\frac{\pi^2 tK[1]^2}{9000}}\right)}{\pi^3K[1]^3}\right.\right.$$

Maple ✓

```
restart;
A:=20;
B:=50;
f:=60-2*x;
L:=30;
k:=1/10;
Q:=x/10;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q;
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t)=-\frac{x^3}{6}+151x+20\left(\sum_{n=1}^{\infty}\frac{\left(5\pi^2n^2(-1)^n+4\pi^2n^2+2700(-1)^n\right)\mathrm{e}^{-\frac{\pi^2n^2t}{9000}}\sin\left(\frac{\pi nx}{30}\right)}{\pi^3n^3}\right)+20$$

<u>Hand solution</u>

Solving

$$\begin{aligned} u_t &= kv_{xx}+Q(x) \qquad (1)\\ u(0,t) &= A\\ u(L,t) &= B\\ u(x,0) &= f(x) \end{aligned}$$

Where $A=20, B=50, f(x)=60-2x, Q(x)=\frac{x}{10}, k=\frac{1}{10}, L=30$.

The general solution to above PDE was solved in 3.1.5.4 on page 598 and the solution is

$$\begin{aligned}
u(x,t) &= \left(\frac{B-A}{L}\right)x+A+\\
&\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L\left(f(s)-\left(\left(\frac{B-A}{L}\right)s+A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t\frac{2}{L}e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}$$

Replacing the specific values, the solution becomes

$$\begin{aligned}
u(x,t) &= x+20+\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{30}\int_0^{30}(60-2s-(s+20))\sin\left(\frac{n\pi}{30}s\right)ds\right)\\
&+\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t\frac{2}{30}e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^{30}\frac{s}{10}\sin\left(\frac{n\pi}{30}s\right)ds\right)d\tau\right)
\end{aligned}$$

Or

$$\begin{aligned}
u(x,t) &= x+20+\frac{1}{15}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^{30}(40-3s)\sin\left(\frac{n\pi}{30}s\right)ds\right)\\
&+\frac{1}{150}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^{30}s\sin\left(\frac{n\pi}{30}s\right)ds\right)d\tau\right)
\end{aligned}$$

But $\int_0^{30}(40-3s)\sin\left(\frac{n\pi}{30}s\right)ds=\frac{1500(-1)^n+1200}{n\pi}$ and $\int_0^{30}s\sin\left(\frac{n\pi}{30}s\right)ds=\frac{-900(-1)^n}{n\pi}$, hence the above becomes

$$\begin{aligned}
u(x,t) &= x+20+\frac{1}{15}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{1500(-1)^n+1200}{n\pi}\right)\\
&+\frac{1}{150}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\frac{-900(-1)^n}{n\pi}\right)d\tau\right)
\end{aligned}$$

Or

$$\begin{aligned}
u(x,t) &= x+20+\frac{300}{15}\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\\
&-\frac{900}{150}\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}d\tau\right)
\end{aligned}$$

But $\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}d\tau = \left[\frac{e^{k\left(\frac{n\pi}{L}\right)^2\tau}}{k\left(\frac{n\pi}{L}\right)^2}\right]_0^t = \frac{e^{k\left(\frac{n\pi}{L}\right)^2 t}-1}{k\left(\frac{n\pi}{L}\right)^2}$, therefore the above becomes

$$\begin{aligned}u(x,t) &= x+20+\frac{300}{15}\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\\ &\quad -\frac{900}{150}\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{e^{k\left(\frac{n\pi}{L}\right)^2 t}-1}{k\left(\frac{n\pi}{L}\right)^2}\right)\end{aligned}$$

Or

$$\begin{aligned}u(x,t) &= x+20+20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\\ &\quad -\frac{900}{150}\sum_{n=1}^{\infty}\frac{(-1)^n}{n\pi k\left(\frac{n\pi}{L}\right)^2}\sin\left(\frac{n\pi}{L}x\right)\left(1-e^{-k\left(\frac{n\pi}{L}\right)^2 t}\right)\end{aligned}$$

Finally replacing the remaining variables in the above for L,k gives

$$\begin{aligned}u(x,t) &= x+20+20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\sin\left(\frac{n\pi}{30}x\right)\\ &\quad -\frac{900}{150}\sum_{n=1}^{\infty}\frac{9000(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\right)\end{aligned}$$

Or

$$\begin{aligned}u(x,t) &= x+20+20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\sin\left(\frac{n\pi}{30}x\right)\\ &\quad -54000\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{1}{10}\left(\frac{n\pi}{30}\right)^2 t}\right)\end{aligned}$$

Or

$$u(x,t) = (x+20)+20\sum_{n=1}^{\infty}\left(\frac{5(-1)^n+4}{n\pi}\right)e^{-\frac{\pi^2 n^2}{9000}t}\sin\left(\frac{n\pi}{30}x\right)-54000\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3\pi^3}\sin\left(\frac{n\pi}{30}x\right)\left(1-e^{-\frac{\pi^2 n^2}{9000}t}\right)$$

Animation is below

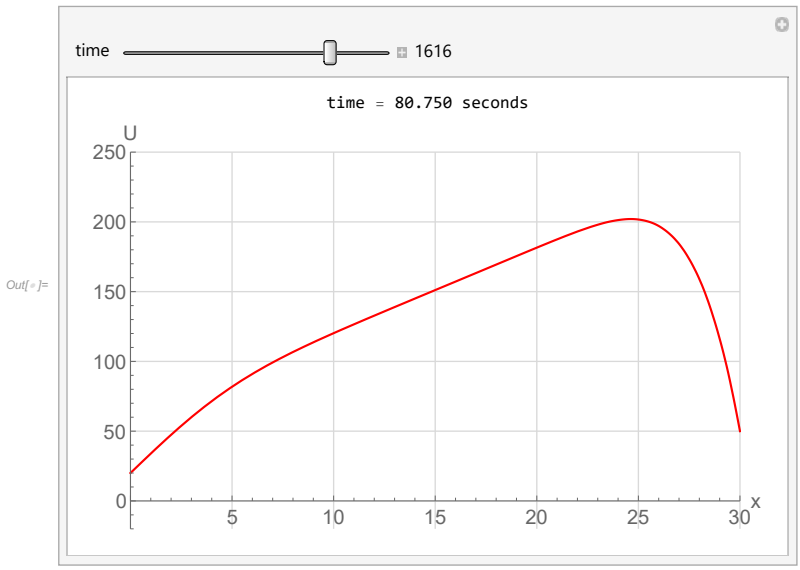

Figure 3.105: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 30;
k = 1 / 10;
f[x_] := 60 - 2 x;
Q[x_] := x / 10;
λ = (n π / L)^2;
ϕ[x_, n_] := Sin[√λ x];
numberOfTerms = 50;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = x + 20 + 20 Chop@N@Sum[ (5 (-1)^n + 4) / (n π) Exp[-k λ t] ϕ[x, n] , {n, 1, numberOfTerms}] - 54000 Chop@N@Sum[ (-1)^n / (n^3 π^3) (1 - Exp[-k λ t]) ϕ[x, n], {n, 1, numberOfTerms}];
```

Figure 3.106: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-20, 250}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 100, .1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.107: Code for animation

3.1.5.6 [220] With source that depends on space and time only (general case)

problem number 220

Added July 6,2019

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(1,t) = B$$

Initial condition is $u(x,0) = f(x)$

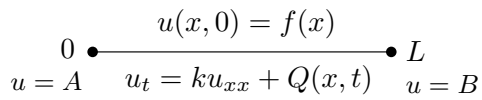

Figure 3.108: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\text{Integrate}\left[\frac{\sqrt{2}Q(x,K[2])\sin\left(\frac{\pi xK[1]}{L}\right)}{\sqrt{L}},\{x,0,L\},\text{Assumptions}\to k>\right.\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2)+Q(x,t);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{AL+L\left(\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi nx}{L}\right)dx\right)e^{-\frac{\pi^2(t-\tau)kn^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L}\right)d\tau\right)-2L\left(\sum_{n=1}^{\infty}\frac{\left(\int_0^L(AL-Lf(\tau)+(-\right.}{}\right.}{L}$$

<u>Hand solution</u>

Solving

$$u_t = ku_{xx} + Q(x,t) \tag{1}$$

With initial conditions $u(x,0) = f(x)$ and boundary conditions $u(0,t) = A, u(L,t) = B$ with $0 < x < L, t > 0$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using steady state solution. Let the total solution be

$$u(x,t) = v(x,t) + r(x) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. and $r(x)$ is the steady state solution which do not depend on time and just needs to satisfy the nonhomogeneous BC. Since $r(x)$ is the steady state solution, then the PDE becomes an ODE

$$\begin{aligned} 0 &= kr''(x) \\ r(0) &= A \\ r(L) &= B \end{aligned}$$

This has the solution $r(x) = c_1 x + c_2$. Using BC at $x = 0$ leads to $A = c_2$. Therefore the solution is $r(x) = c_1 x + A$. Using BC at $x = L$ gives $B = c_1 L + A$ or $c_1 = \frac{B-A}{L}$. Hence

$$r(x) = \left(\frac{B-A}{L}\right) x + A \tag{3}$$

Substituting (2) back in the original PDE $u_t = ku_{xx} + Q(x,t)$ gives

$$\begin{aligned} v_t &= kv_{xx} + Q(x,t) \qquad (4) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \end{aligned}$$

The initial conditions are

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x) \\ &= f(x) - \left(\left(\frac{B-A}{L}\right) x + A\right) \end{aligned}$$

The general solution to (4) was solved in 3.1.6.4 on page 664 and the solution is

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right) d\right. \\ \Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right) \\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

But $F(x) = f(x) - \left(\left(\frac{B-A}{L}\right)x + A\right)$ in this case, hence the solution becomes

$$v(x,t) = \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right)s + A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)$$
$$+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)$$

Since $u(x,t) = v(x,t) + r(x)$, then the final solution is

$$u(x,t) = \left(\frac{B-A}{L}\right)x + A +$$
$$\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\frac{2}{L}\int_0^L \left(f(s) - \left(\left(\frac{B-A}{L}\right)s + A\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)$$
$$+ \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t \frac{2}{L} e^{k\left(\frac{n\pi}{L}\right)^2 \tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)$$

3.1.5.7 [221] Both ends depend on time (general case)

problem number 221

Added June 23, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

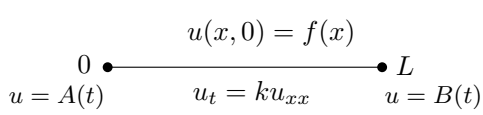

Figure 3.109: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{k>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(e^{-\frac{k\pi^2 tK[1]^2}{L^2}}\int_0^L\frac{\sqrt{2}(-LA(0)+xA(0)-xB(0)+Lf(x))\sin\left(\frac{\pi xK[1]}{L}\right)}{L^{3/2}}\,dx+\int_0^t\frac{\sqrt{2}e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\sqrt{L}((}{\pi K}\right.}{\sqrt{L}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = A(t), u(L, t) = B(t);
ic  := u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) a
```

$$u(x,t)=\frac{L\left(\int_0^t\left(\sum_{n=1}^{\infty}-\frac{2\left(-(-1)^n\left(\frac{d}{d\tau}B(\tau)\right)+\frac{d}{d\tau}A(\tau)\right)e^{-\frac{\pi^2(t-\tau)kn^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{\pi n}\right)d\tau\right)+L\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^L\frac{(Lf(x)-B(0)x+(-}{}\right.}{}\right.}{L}$$

<u>Hand solution</u>

$$\begin{aligned} u_t &= ku_{xx} \qquad (1)\\ u(0,t) &= A(t)\\ u(L,t) &= B(t)\\ u(x,0) &= f(x) \end{aligned}$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total

solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. We see that

$$\begin{aligned} r(x,t) &= A(t) + \frac{B(t) - A(t)}{L} x \\ &= A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L} x \end{aligned} \tag{3}$$

Satisfies the nonhomogeneous. Substituting (2) back into the original PDE (1) gives

$$\begin{aligned} \frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) \\ v_t(x,t) + r_t(x,t) &= kv_{xx}(x,t) + kr_{xx}(x,t) \end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t)\left(\frac{L-x}{L}\right) + \frac{B'(t)}{L}x$ and the above PDE becomes

$$v_t(x,t) = kv_{xx}(x,t) - r_t(x,t)$$

Let

$$\begin{aligned} Q(x,t) &= -r_t(x,t) \\ &= -\left(A'(t)\left(\frac{L-x}{L}\right) + \frac{B'(t)}{L}x\right) \end{aligned}$$

Therefore the problem has been transformed to

$$\begin{aligned} v_t &= kv_{xx} + Q(x,t) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \\ v(0,x) &= F(x) \\ &= u(x,0) - r(x,0) \\ &= f(x) - \left(A(0)\left(\frac{L-x}{L}\right) + \frac{B(0)}{L}x\right) \end{aligned}$$

The above problem was solved in 3.1.6.4 on page 664. The solution is

$$v(x,t) = \sum_{n=1}^{\infty}\left[\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right)e^{-k\lambda_n t} + e^{-k\lambda_n t}\int_0^t \frac{2}{L}e^{k\lambda_n \tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,dx\right)d\tau\right]\Phi_n(x) \tag{4}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence (4) becomes

$$\begin{aligned} v(x,t) &= \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L F(s)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)dx\right)d\tau \end{aligned}$$

Since $u(x,t) = r(x,t) + v(x,t)$ then the final solution becomes

$$\begin{aligned} u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L F(s)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\lambda_n t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau \end{aligned}$$

Or

$$\begin{aligned} u(x,t) &= \left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0)\left(\frac{L-s}{L}\right) + \frac{B(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right) \end{aligned}$$

Or

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)\,L-\left(A(0)\,(L-s)+B(0)\,s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L Q(s,\tau)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Or, since here $Q(x,t)=-\left(A'(t)\left(\frac{L-x}{L}\right)+\frac{B'(t)}{L}x\right)$ the above becomes

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)\,L-\left(A(0)\,(L-s)+B(0)\,s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(\frac{B'(\tau)}{L}s-A'(\tau)\left(\frac{L-s}{L}\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)-A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)-B(0)\,s\sin\left(\frac{n\pi}{L}s\right.\right.\\
&+\frac{2}{L}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\frac{B'(\tau)}{L}s\sin\left(\frac{n\pi}{L}s\right)-A'(\tau)\left(\frac{L-s}{L}\right)\sin\left(\frac{n\pi}{L}s\right)ds\right.\right.
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right) \qquad (5)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)-A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)-B(0)\,s\sin\left(\frac{n\pi}{L}s\right.\right.\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty}e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L B'(\tau)\,s\sin\left(\frac{n\pi}{L}s\right)-A'(\tau)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\right.
\end{aligned}
$$

But

$$\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right) - A(0)\,(L-s)\sin\left(\frac{n\pi}{L}s\right) - B(0)\,s\sin\left(\frac{n\pi}{L}s\right)ds =$$
$$L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds - B(0)\int_0^L s\sin\left(\frac{n\pi}{L}s\right)ds$$

Since $\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds = \frac{L^2}{n\pi}$ and $\int_0^L s\sin\left(\frac{n\pi}{L}s\right)dx = -\frac{L^2(-1)^n}{n\pi}$, then the above becomes

$$\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right) - A(0)\left(\frac{L-s}{L}\right)\sin\left(\frac{n\pi}{L}s\right) - \frac{B(0)}{L}s\sin\left(\frac{n\pi}{L}s\right)ds =$$
$$L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi} \quad (6)$$

And

$$\int_0^L B'(\tau)\,s\sin\left(\frac{n\pi}{L}s\right) - A'(\tau)\,(L-s)\sin\left(\frac{n\pi}{L}s\right)ds = -A'(\tau)\int_0^L (L-s)\sin\left(\frac{n\pi}{L}s\right)ds + B'(\tau)\int_0^L s\sin \quad (7)$$
$$= -A'(\tau)\,\frac{L^2}{n\pi} + B'(\tau)\,\frac{L^2(-1)^n}{n\pi}$$

Substituting (6,7) back into (5) gives

$$\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(L\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi}\right)\\
&-\frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(-B'(\tau)\,\frac{L^2(-1)^n}{n\pi} + A'(\tau)\,\frac{L^2}{n\pi}\right)d\tau\right)
\end{aligned}$$

Or

$$\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right) + \frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi} + B(0)\,\frac{L^2(-1)^n}{n\pi}\right)\\
&-2\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{1}{n\pi}\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}(-(-1)^n B'(\tau) + A'(\tau))\,d\tau\right)
\end{aligned}$$

3.1.5.8 [222] Both ends depend on time (special case)

problem number 222

Added June 26, 2019

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= A(t) \\ u(L,t) &= B(t) \end{aligned}$$

Initial condition is $u(x,0) = f(x)$ using the following values

$$\begin{aligned} L &= 2 \\ k &= \frac{1}{10} \\ f(x) &= x \\ A(t) &= \sin(t) \\ B(t) &= 2\cos(t) \end{aligned}$$

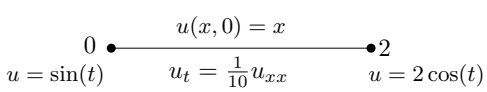

Figure 3.110: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f=x;
L=2;
k=1/10;
A=Sin[t];
B=2*Cos[t];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{80\left(e^{-\frac{1}{40}\pi^2 tK[1]^2}\pi^2K[1]^2-80(-1)^{K[1]}e^{-\frac{1}{40}\pi^2 tK[1]^2}+\cos(t)\left(80(-1)^{K[1]}-\pi^2K[1]^2\right)-2(\right.}{\pi K[1]\left(\pi^4K[1]^4+1600\right)}\right.$$

Maple ✓

```
restart;
f:=x;
L:=2;
k:=1/10;
A:=sin(t);
B:=2*cos(t);
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc  := u(0, t) = A, u(L, t) = B;
ic  := u(x, 0) = f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t)) )
```

$$u(x,t)=x\cos(t)-\frac{x\sin(t)}{2}-80\left(\sum_{n=1}^{\infty}\frac{\left(\pi^2n^2\cos(t)+\left(2\pi^2n^2\sin(t)-80\cos(t)\right)(-1)^n+(-\pi^2n^2+80(-1\right.}{\pi\left(\pi^4n^4+1600\right)n}\right.$$

Hand solution

The basic solution for this type of PDE was already given in problem 3.1.5.7 on page 609 as

$$
\begin{aligned}
u(x,t) = &\left(A(t)\left(\frac{L-x}{L}\right)+\frac{B(t)}{L}x\right)\\
&+\frac{2}{L^2}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L f(s)\,L\sin\left(\frac{n\pi}{L}s\right)ds - A(0)\,\frac{L^2}{n\pi}+B(0)\,\frac{L^2(-1)^n}{n\pi}\right)\\
&-2\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\frac{1}{n\pi}\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(-(-1)^n B'(\tau)+A'(\tau)\right)d\tau\right)
\end{aligned}
$$

In this problem we have

$$
\begin{aligned}
L &= 2\\
k &= \frac{1}{10}\\
f(x) &= x\\
A(t) &= \sin(t)\\
B(t) &= 2\cos(t)
\end{aligned}
$$

This is animation of the above solution using these specific values for 20 seconds. (Animation will only show in the HTML version)

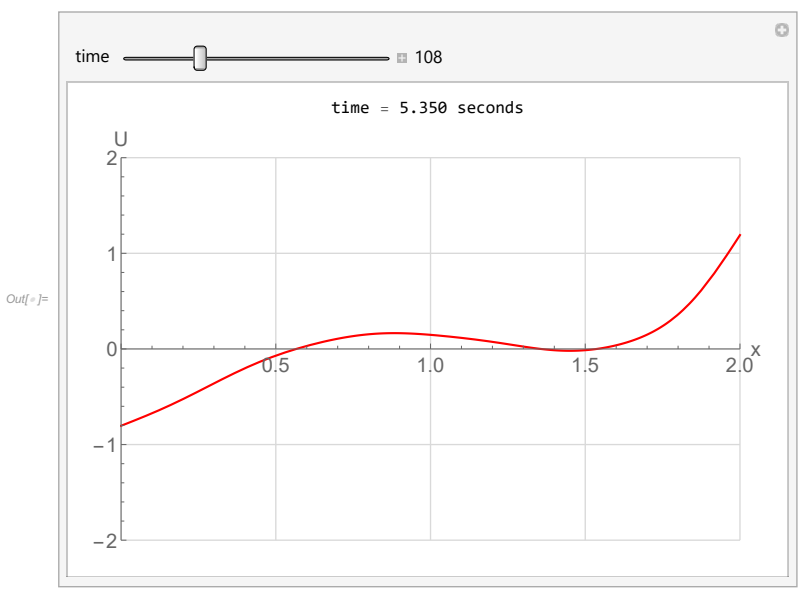

Figure 3.111: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1 / 10;
f[x_] := x;
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
firstIntegral = Assuming[Element[n, Integers], L Integrate[f[x] Sin[n Pi/L x], {x, 0, L}]];
secondIntegral = 1/(n Pi) Integrate[Exp[k n^2 Pi^2 s/L^2] (A'[s] - (-1)^n B'[s]), {s, 0, t}];
mySol[x_, t_] =
  N@Simplify[(A[t] (L - x)/L + B[t]/L x) + 2.0/L^2 Sum[Exp[-k (n Pi/L)^2 t] Sin[n Pi/L x] (firstIntegral - A[0] L^2/(n Pi) + (-1)^n B[0] L^2/(n Pi)), {n, 1, numberOfTerms}] -
    2.0 Sum[Exp[-k (n Pi/L)^2 t] Sin[n Pi/L x] (secondIntegral), {n, 1, numberOfTerms}]];
```

Figure 3.112: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 2}, {-2, 2}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 20, .05}];
In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.113: Code used for animation

3.1.5.9 [223] both ends nonhomogeneous

problem number 223

Added January 18, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = 1$$
$$u_x(1,t) = -1$$

Initial condition is $u(x,0) = \sin(x)$

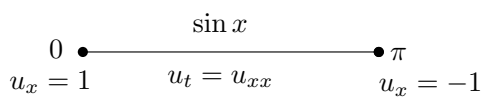

Figure 3.114: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + (x - (1*x^2)/Pi);
ic  = u[x, 0] == Sin[x];
bc  = {Derivative[1, 0][u][0, t] == 1, Derivative[1, 0][u][Pi, t] == -1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{e^{-tK[1]^2}\cos(xK[1])\left(\sqrt{2\pi}K[1]^4\left(\begin{cases} 0 & K[1]=1 \\ -\frac{(1+(-1)^{K[1]})\sqrt{\frac{2}{\pi}}}{K[1]^2(K[1]^2-1)} & \text{True}\end{cases}\right) - 2\left(1+(-1)^{K}\right.\right.}{\pi K[1]^4}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic  := u(x, 0) = sin(x):
bc  := eval(diff(u(x,t),x),x=0)=1,  eval( diff(u(x,t),x),x=Pi)=-1:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

$$u(x,t) = -\frac{x^2}{\pi} - \frac{2t}{\pi} + x + \left(\sum_{n=2}^{\infty} -\frac{2((-1)^n+1)\cos(nx)\,\mathrm{e}^{-n^2 t}}{(n^2-1)\,\pi n^2}\right) + \frac{2-\frac{\pi^2}{6}}{\pi}$$

Hand solution

Since the boundary conditions are not homogeneous, we can't use separation of variables. Let the solution be

$$u = v(x,t) + r(x)$$

Where $v(x,t)$ is the solution to $v_t = v_{xx}$ and homogenous B.C. $v_x(0,t) = 0, v_x(\pi,t) = 0$ and $r(x)$ is any reference solution which only needs to satisfy the nonhomogeneous boundary conditions: $r'(0) = 1, r'(\pi) = -1$. By guessing, let $r(x) = Ax + Bx^2$. Let see if this satisfies the boundary conditions. $r' = A + 2Bx$. At $x = 0$ this implies $1 = A$. Hence $r = x + Bx^2$. Now $r' = 1 + 2Bx$. At $x = \pi$ this gives $-1 = 1 + 2B\pi$ or $B = -\frac{1}{\pi}$. Therefore

$$r(x) = x - \frac{1}{\pi}x^2$$

Substituting $u = v(x,t) + r(x)$ into the PDE $u_t = u_{xx}$ and noting that $r''(x) = -\frac{2}{\pi}$ gives

$$v_t = v_{xx} - \frac{2}{\pi} \tag{1}$$

PDE (1) is now solved using eigenfunction expansion. We need to find eigenfunctions and eigenvalues of $v_t = v_{xx}$ with $v_x(0,t) = 0, v_x(\pi,t) = 0$. This is known PDE and have eigenfunctions and eigenvalues as follows. For zero eigenvalue, the eigenfunction is an arbitrary constant. Say β. let $\beta = 1$ since scale is not important.

$$\Phi_0(x) = 1$$

And for $n = 1, 2, 3, \cdots$

$$\begin{aligned}\Phi_n(x) &= \cos\left(\sqrt{\lambda_n}x\right)\\ &= \cos(nx)\end{aligned}$$

with eigenvalues $\lambda_n = n^2$ for $n = 1, 2, 3, \cdots$. Now we can eigenfunction expansion and assume the solution to (1) is

$$v(x,t) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n(x) \tag{2}$$

Plugging this into the PDE (1) gives

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n''(x) - \frac{2}{\pi}$$

But $\Phi_n''(x) = -\lambda_n \Phi_n(x)$ and the above simplifies to

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \frac{2}{\pi}$$

Since eigenfunctions are complete, we can expand $\frac{2}{\pi}$ using them and the above becomes

$$\begin{aligned}
\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) &= -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \sum_{n=0}^{\infty} C_n \Phi_n(x) \\
A_n'(t)\,\Phi_n(x) + A_n(t)\,\lambda_n \Phi_n(x) &= -C_n \Phi_n(x) \\
A_n'(t) + A_n(t)\,\lambda_n &= -C_n \qquad (3)
\end{aligned}$$

To find C_n

$$\sum_{n=0}^{\infty} C_n \Phi_n(x) = \frac{2}{\pi}$$

For $n = 0$

$$C_0 \Phi_0(x) = \frac{2}{\pi}$$

But $\Phi_0(x) = 1$, hence

$$C_0 = \frac{2}{\pi}$$

All other C_m for $m > 0$ are zero. Hence (3) becomes, for $n = 0$ (since $\lambda_0 = 0$)

$$\begin{aligned}
A_0'(t) &= -\frac{2}{\pi} \\
A_0(t) &= -\frac{2}{\pi} t + B_0
\end{aligned}$$

Where B_0 is integration constant. For $n > 0$ (3) becomes

$$A_n'(t) + A_n(t)\,n^2 = 0$$

This has the solution

$$A_n(t) = B_n e^{-n^2 t}$$

Where B_n is constant of integration. Hence from (2)

$$\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} A_n(t)\,\Phi_n(x) \\
&= A_0(t) + \sum_{n=1}^{\infty} A_n(t)\,\Phi_n(x) \\
&= -\frac{2}{\pi} t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t} \cos(nx)
\end{aligned}$$

Since $u = v(x,t) + r(x)$ then the solution becomes

$$u(x,t) = \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t}\cos(nx) \tag{4}$$

At $t = 0$

$$\sin(x) = \left(x - \frac{1}{\pi}x^2\right) + B_0 + \sum_{n=1}^{\infty} B_n \cos(nx) \tag{5}$$

case $n = 0$

$$\int_0^{\pi} \sin(x)\cos\left(\sqrt{\lambda_0}x\right)dx = \int_0^{\pi}\left(x - \frac{1}{\pi}x^2\right)\cos\left(\sqrt{\lambda_0}x\right)dx + \int_0^{\pi} B_0\cos\left(\sqrt{\lambda_0}x\right)dx$$

But $\lambda_0 = 0$ hence

$$\begin{aligned}
\int_0^{\pi}\sin(x)\,dx &= \int_0^{\pi}\left(x - \frac{1}{\pi}x^2\right)dx + \int_0^{\pi} B_0 dx\\
2 &= \frac{\pi^2}{6} + B_0\pi\\
B_0 &= \frac{2}{\pi} - \frac{\pi}{6}
\end{aligned}$$

For $n > 0$, Multiplying both sides of (5) by $\cos(mx)$ and integrating

$$\int_0^{\pi}\sin(x)\cos(mx)\,dx = \int_0^{\pi}\left(x - \frac{1}{\pi}x^2\right)\cos(mx)\,dx + \sum_{n=1}^{\infty} B_n\int_0^{\pi}\cos(nx)\cos(mx)\,dx$$

For $m = 1$

$$\begin{aligned}
0 &= 0 + B_1\frac{\pi}{2}\\
B_1 &= 0
\end{aligned}$$

For $m > 1$

$$\begin{aligned}
-\frac{1 + (-1)^m}{m^2(-1 + m^2)} &= \frac{\pi}{2}B_m\\
B_m &= \frac{-2}{\pi}\left(\frac{1}{m^2}\frac{(-1)^m + 1}{m^2 - 1}\right)
\end{aligned}$$

Hence solution (4) becomes

$$\begin{aligned}
u(x,t) &= \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=1}^{\infty} B_n e^{-n^2 t}\cos(nx)\\
u(x,t) &= \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=2}^{\infty}\frac{-2}{\pi}\left(\frac{1}{n^2}\frac{(-1)^n + 1}{n^2 - 1}\right)e^{-n^2 t}\cos(nx)
\end{aligned}$$

3.1.5.10 [224] Haberman 8.2.1 (a) (general case)

problem number 224

Added Nov 27, 2018

This is problem 8.2.1 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$\frac{\partial u}{\partial x}(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

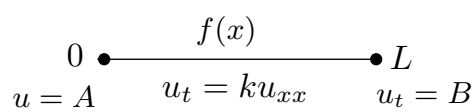

Figure 3.115: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = {u[0, t] == A, Derivative[1, 0][u][L, t] == B};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-\frac{k\pi^2 t(1-2K[1])^2}{4L^2}}\left(\int_0^L -\frac{\sqrt{2}(A+Bx-f(x))\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}} + A + Bx\right\}\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  := u(0,t)=A, eval( diff(u(x,t),x),x=L)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = Bx+A-2\left(\sum_{n=0}^{\infty}\frac{\left(\int_0^L (Bx+A-f(x))\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2(2n+1)^2kt}{4L^2}}\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}\right)$$

<u>Hand solution</u>

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

We can look for $u_E(x)$ which is the steady state solution that satisfies the non-homogenous boundary conditions. In (1) $v(x,t)$ satisfies the PDE itself but with homogenous boundary conditions. The first step is to find $u_E(x)$. We use the equilibrium solution in this case. At equilibrium $\frac{\partial u_E(x,t)}{\partial t} = 0$ and hence the solution is given $\frac{d^2u_E}{\partial x^2} = 0$ or

$$u_E(x) = c_1x + c_2$$

At $x = 0, u_E(x) = A$, Hence

$$c_2 = A$$

And solution becomes $u_E(x) = c_1x + A$. at $x = L$, $\frac{\partial u_E(x)}{\partial x} = c_1 = B$, Therefore

$$u_E(x) = Bx + A$$

Now we plug-in (1) into the original PDE, this gives

$$\frac{\partial v(x,t)}{\partial t} = k\left(\frac{\partial^2 v(x,t)}{\partial x} + \frac{\partial^2 u_E(x)}{\partial x}\right)$$

But $\frac{\partial^2 u_E(x)}{\partial x} = 0$, hence we need to solve

$$\frac{\partial v(x,t)}{\partial t} = k\frac{\partial^2 v(x,t)}{\partial x}$$

for $v(x,t) = u(x,t) - u_E(x)$ with homogenous boundary conditions $v(0,t) = 0$, $\frac{\partial v(L,t)}{\partial t} = 0$ and initial conditions

$$\begin{aligned} v(x,0) &= u(x,0) - u_E(x) \\ &= f(x) - (Bx + A) \end{aligned}$$

This PDE we already solved before and we know that it has the following solution

$$\begin{aligned} v(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right) e^{-k\lambda_n t} \\ \lambda_n &= \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots \end{aligned} \tag{2}$$

With b_n found from orthogonality using initial conditions $v(x,0) = f(x) - (Bx + A)$

$$v(x,0) = \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right)$$

$$\int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\lambda_m} x\right) dx = \int_0^L \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

$$\int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\lambda_m} x\right) dx = b_m \frac{L}{2}$$

Hence

$$b_n = \frac{2}{L} \int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\lambda_n} x\right) dx \qquad n = 1,3,5,\cdots \tag{3}$$

Therefore, from (1) the solution is

$$\begin{aligned} u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right) e^{-k\lambda_n t} + \overbrace{Bx + A}^{u_E(x)} \\ &= Bx + A + \sum_{n=1,3,5,\cdots}^{\infty} \left(\frac{2}{L} \int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\frac{n\pi}{L}} x\right) dx\right) \sin\left(\sqrt{\frac{n\pi}{L}} x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned}$$

Or

$$u(x,t) = Bx + A + \sum_{n=0}^{\infty} \left(\frac{2}{L} \int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\frac{(2n+1)\pi}{2L}} x\right) dx\right) \sin\left(\sqrt{\frac{(2n+1)\pi}{2L}} x\right) e^{-k\left(\frac{(2n}{}\right.}$$

3.1.5.11 [225] Haberman 8.2.1 (d) (general solution)

problem number 225

This is problem 8.2.1 part(d) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + k$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

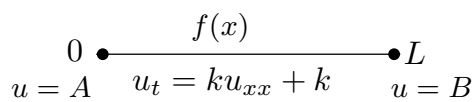

Figure 3.116: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + k;
bc  = {u[0, t] == A0, u[L0, t] == B0};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} -\frac{e^{-\frac{k\pi^2 tK[1]^2}{\mathrm{L0}^2}}\left(2\left(-1+(-1)^{K[1]}\right)\left(-1+e^{\frac{k\pi^2 tK[1]^2}{\mathrm{L0}^2}}\right)\mathrm{L0}^2 - \sqrt{2}\sqrt{\frac{1}{\mathrm{L0}}}\pi^3\left(\int_0^{\mathrm{L0}}\sqrt{2}\left(\frac{1}{\mathrm{L0}}\right)^{3/2}(\right.\right.}{\pi^3 K[1]^3}\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = -\frac{x^2}{2}+A+\sum_{n=1}^{\infty}\left(-\frac{\left(\int_0^L 2\left(\frac{L^2x}{2}-Lf(x)+\left(-\frac{x^2}{2}+A\right)L-(A-B)x\right)\sin\left(\frac{\pi n x}{L}\right)dx\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}\sin\left(\right.}{L^2}\right.$$

<u>Hand solution</u>

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

Where $u_E(x)$ is the equilibrium solution which needs to satisfy only the nonhomogeneous B.C. And $v(x,t)$ is transient solution to heat PDE with homogeneous B.C.

At equilibrium, $u_t = ku_{xx} + Q(x)$ becomes

$$\begin{aligned} 0 &= ku_E'' + Q(x) \\ &= ku_E'' + k \\ &= k(u_E'' + 1) \end{aligned}$$

Hence

$$u_E'' = -1$$

The solution to this ODE is

$$u_E = c_1 x + c_2 - \frac{1}{2}x^2$$

At $x = 0$, the above gives

$$A = c_2$$

And at $x = L$

$$\begin{aligned} B &= c_1 L + A - \frac{1}{2}L^2 \\ c_1 &= \frac{B - A + \frac{1}{2}L^2}{L} \\ &= \frac{B}{L} - \frac{A}{L} + \frac{1}{2}L \end{aligned}$$

Hence

$$u_E = \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right) x + A - \frac{1}{2}x^2$$

Hence from (1)

$$\begin{aligned} u(x,t) &= v(x,t) + u_E \\ &= v(x,t) + \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right) x + A - \frac{1}{2}x^2 \end{aligned} \tag{1A}$$

Substituting this in $u_t = ku_{xx} + k$ gives

$$\begin{aligned} v_t &= k(v_{xx} - 1) + k \\ &= kv_{xx} \end{aligned} \tag{2}$$

We need to solve the above for $v(x,t)$, but with homogeneous B.C. $v(0,t) = 0, v(L,t) = 0$. The eigenvalues for the homogeneous PDE $v_t = kv_{xx}$ with these boundary conditions is known to be $\lambda_n = \left(\frac{n\pi}{L}\right)^2$, for $n = 1, 2, \cdots$ and the corresponding eigenfunctions are $X_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$. Now, using eigenfunction expansion, let

$$v(x,t) = \sum_{n=1}^{\infty} b_n(t)\, X_n(x) \tag{3}$$

Substituting (3) into (2) gives

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) = k \sum_{n=1}^{\infty} b_n(t)\, X_n''(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, therefore the above becomes

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) + k \sum_{n=1}^{\infty} \lambda_n b_n(t)\, X_n(x) = 0$$

Since the above is true for each n and since eigenfunctions can not be zero, the above simplifies to

$$b_n'(t) + k\lambda_n b_n(t) = 0 \tag{4}$$

This is linear in $b(t)$. The solution using integrating factor is

$$b_n(t) = b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

Therefore (3) becomes

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} b_n(t)\, X_n(x) \\ &= \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

And from (1)

$$\begin{aligned} u(x,t) &= v(x,t) + u_E(x) \\ &= \overbrace{\left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2}^{u_E} + \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned} \tag{5}$$

At $t = 0$ the above becomes

$$f(x) = \frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2 + \sum_{n=1}^{\infty} b_0(0) \sin\left(\frac{n\pi}{L}x\right)$$

For $n > 0$, and applying orthogonality

$$\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx = \int_0^L \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \sin\left(\frac{n\pi}{L}x\right) dx + \int_0^L b_0(0) \sin^2\left(\frac{n\pi}{L}x\right) dx$$

Hence

$$\int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx = \frac{L}{2} b_0(0)$$

Therefore

$$b_0(0) = \frac{2}{L} \int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx$$

Substituting the above in (5) gives

$$\begin{aligned} u(x,t) = &\left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \\ &+ \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx\right) e^{-k\frac{n\pi}{L}t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

3.1.5.12 [226] Both ends depend on time with source that depends on space only (general solution)

problem number 226

Added July 3, 2019

Solve the heat equation

$$u_t = k u_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

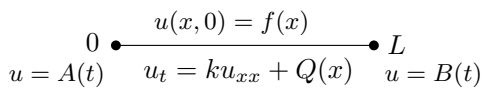

Figure 3.117: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{L>
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2 K[1]^2 (t-K[2])}{L^2}} \text{Integrate}\left[\frac{\sqrt{2}\sin\left(\frac{\pi x K[1]}{L}\right)(LQ(x)+(x-L)A'(K[2])-xB'(K[2]))}{L^{3/2}}, \{x,0,L\}, \text{A}\right.\right.}{}\right.\right.$$

Maple ✗

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q(x);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A(t), u(L,t)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

time expired Possible bug. Maple can solve with $Q(x,t)$ source but not with $Q(x)$

Hand solution

Solve

$$\begin{aligned} u_t &= ku_{xx} + Q(x) \\ u(0,t) &= A(t) \\ u(L,0) &= B(t) \\ u(x,0) &= f(x) \end{aligned}$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. One can easily see that the reference function is

$$r(x,t) = A(t) + \frac{B(t) - A(t)}{L}x \tag{3}$$

Substituting (1) back into the original PDE gives

$$\begin{aligned} \frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) + Q(x) \\ v_t(x,t) + r_t(x,t) &= kv_{xx}(x,t) + kr_{xx}(x,t) + Q(x) \end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t) + \frac{B'(t)-A'(t)}{L}x$ and PDE becomes

$$v_t(x,t) = kv_{xx}(x,t) - r_t(x,t) + Q(x)$$

Let

$$\begin{aligned} \tilde{Q}(x) &= Q(x) - r_t(x,t) \\ &= Q(x) - \left(A'(t) + \frac{B'(t) - A'(t)}{L}x\right) \end{aligned} \tag{4}$$

Therefore the problem has been transformed to

$$\begin{aligned} v_t &= kv_{xx} + \tilde{Q}(x) \\ v(0,t) &= 0 \\ v(L,t) &= 0 \\ v(0,x) &= F(x) \\ &= u(x,0) - r(x,0) \\ &= f(x) - \left(A(0) + \frac{B(0) - A(0)}{L}x\right) \end{aligned}$$

The basic solution for this type of PDE was already given in problem 3.1.1.11 on page 429 and the solution is

$$
\begin{aligned}
v(x,t) &= \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right)\\
&+\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s)\,\Phi_n(s)\,ds\right)d\tau\right)
\end{aligned}
$$

Where

$$
\begin{aligned}
\Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right)\\
\lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n=1,2,3,\cdots
\end{aligned}
$$

Hence, using our $\tilde{Q}(x)$ and $F(x)$ found above into this solution gives

$$
\begin{aligned}
v(x,t) &= \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)-\left(A(0)+\frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(Q(s)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau\right)
\end{aligned}
$$

Since $u(x,t)=v(x,t)+r(x,t)$ then the final solution is

$$
\begin{aligned}
u(x,t) &= A(t)+\frac{B(t)-A(t)}{L}x\\
&+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L\left(f(s)-\left(A(0)+\frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)\\
&+\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L\left(Q(s)-\left(A'(\tau)+\frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)\right.\right.
\end{aligned}
$$

3.1.5.13 [227] Both ends depend on time with source that depends on space only (special case)

problem number 227

Added July 4, 2019

Solve the heat equation

$$u_t = ku_{xx} + Q(x)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= A(t) \\ u(L,t) &= B(t) \end{aligned}$$

Initial condition is $u(x,0) = f(x)$ using these values

$$\begin{aligned} L &= 2 \\ k &= \frac{1}{10} \\ A(t) &= \sin(t) \\ B(t) &= 2\cos(t) \\ Q(x) &= x \\ f(x) &= x \end{aligned}$$

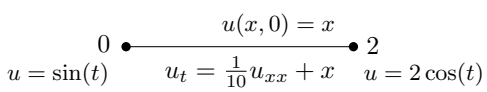

Figure 3.118: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
k=1/10;
A=Sin[t];
B=Cos[t];
f=x;
Q=x;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\left(-\frac{80\left(\pi^4\cos(t)K[1]^4+40\pi^2\sin(t)K[1]^2-e^{-\frac{1}{40}\pi^2tK[1]^2}\left(\pi^4K[1]^4+2(-1)^{K[1]}\left(\pi^4K[1]^{4}\right.\right.\right.}{\pi^3K[1]^3\left(\pi^{4}\right.}\right.\right.$$

Maple ✓

```
restart;
L:=2;
k:=1/10;
A:=sin(t);
B:=cos(t);
f:=x;
Q:=x;
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q;
ic  :=  u(x,0)=f;
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t)=-\frac{5x^3}{3}-\frac{(-3\cos(t)+3\sin(t)-40)\,x}{6}+\left(\sum_{n=1}^{\infty}-\frac{2\left(40\pi^2(\pi^2n^2\cos(t)+(\pi^2n^2\sin(t)-40\cos(t))\,(\right.}{}\right.$$

Hand solution

Solve

$$
\begin{aligned}
u_t &= k u_{xx} + Q(x) \\
u(0,t) &= A(t) \\
u(L,0) &= B(t) \\
u(x,0) &= f(x)
\end{aligned}
$$

With $k = \frac{1}{10}, L = 2, f(x) = x, Q(x) = x, A(t) = \sin(t), B(t) = 2\cos(t)$.

The general problem above was solved in 3.1.5.12 on page 629 and the solution is

$$
\begin{aligned}
u(x,t) = {} & A(t) + \frac{B(t) - A(t)}{L} x \\
& + \frac{2}{L} \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x\right) \left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0) - A(0)}{L} s \right) \right) \sin\left(\frac{n\pi}{L} s\right) ds \right) \\
& + \frac{2}{L} \sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x\right) \left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2 \tau} \left(\int_0^L \left(Q(s) - \left(A'(\tau) + \frac{B'(\tau) - A'(\tau)}{L} s \right) \right) \sin\left(\frac{n\pi}{L} s\right) ds \right) d\tau \right)
\end{aligned}
$$

Substituting the specific values given above into this solution gives

$$
\begin{aligned}
u(x,t) = {} & \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
& + \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \left(\int_0^2 \left(s - \left(A(0) + \frac{B(0) - A(0)}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right) \\
& + \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s - \left(A'(\tau) + \frac{B'(\tau) - A'(\tau)}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right) d\tau
\end{aligned}
$$

But $A(0) = 0, B(0) = 2, A'(t) = \cos(t), B'(t) = -2\sin(t)$ and the above becomes

$$
\begin{aligned}
u(x,t) = {} & \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
& + \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \left(\int_0^2 \left(s - \left(0 + \frac{2-0}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right) \\
& + \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s - \left(\cos(\tau) + \left(\frac{-2\sin(\tau) - \cos(\tau)}{2} \right) s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) = {} & \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
& + \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s - \left(\cos(\tau) - \left(\frac{2\sin(\tau) + \cos(\tau)}{2} \right) s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right)
\end{aligned}
$$

Animation is below

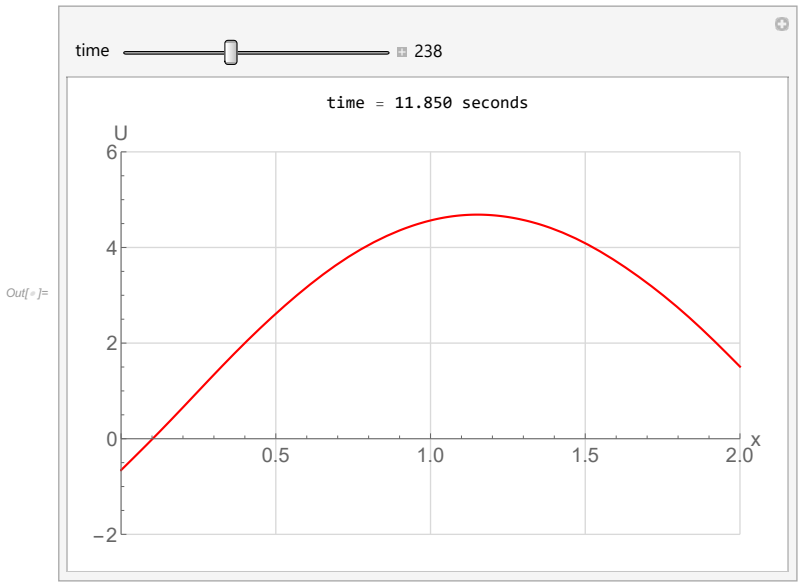

Figure 3.119: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1 / 10;
f[x_] := x;
Q[x_, t_] := x;
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[ ]:= sol = Integrate[(s - (Cos[tau] - ((2 Sin[tau] + Cos[tau])/L) * s)) Sin[(n Pi s)/L], {s, 0, L}];
sol2 = Assuming[Element[n, Integers] && n > 0, Simplify[sol]];
sol3 = N@Integrate[Exp[k ((n Pi)/L)^2 tau] * sol2, {tau, 0, t}];
mySol[x_, t_] = Chop@N@Simplify[((Sin[t] + ((2 Cos[t] - Sin[t])/L)) x) + Sum[Exp[-k ((n Pi)/L)^2 t] Sin[(n Pi)/L x] (sol3), {n, 1, numberOfTerms}]];
```

Figure 3.120: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 2}, {-2, 6}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 30, .05}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.121: Code for animation

3.1.5.14 [228] Both ends depend on time with source that depends on time and space (general solution)

problem number 228

Added June 27, 2019

Solve the heat equation

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

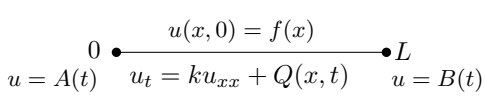

Figure 3.122: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x,t];
bc  = {u[0, t] == A[t], u[L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->{L>
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\left(\int_0^t e^{-\frac{k\pi^2K[1]^2(t-K[2])}{L^2}}\text{Integrate}\left[\frac{\sqrt{2}\sin\left(\frac{\pi xK[1]}{L}\right)(LQ(x,K[2])+(x-L)A'(K[2])-xB'(K[2]))}{L^{3/2}},\{x,0,L\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q(x,t);
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A(t), u(L,t)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\frac{L\left(\int_0^t\left(\sum_{n=1}^{\infty}-\frac{2\left(\int_0^L\left(-LQ(x,\tau)+x\left(\frac{d}{d\tau}B(\tau)\right)+(L-x)\left(\frac{d}{d\tau}A(\tau)\right)\right)\sin\left(\frac{\pi nx}{L}\right)dx\right)e^{-\frac{\pi^2(t-\tau)kn^2}{L^2}}\sin\left(\frac{\pi nx}{L}\right)}{L^2}\right)d\tau\right)-2L}{L}$$

<u>Hand solution</u>

Solve

$$\begin{aligned}u_t&=ku_{xx}+Q(x,t)\\u(0,t)&=A(t)\\u(L,0)&=B(t)\\u(x,0)&=f(x)\end{aligned}$$

Since boundary conditions are nonhomogeneous, the first step is to reduce the problem to one with homogeneous B.C. to be able to use separation of variables. This is done by using a reference solution $r(x,t)$ which only needs to satisfy the B.C. Let the total

solution be

$$u(x,t) = v(x,t) + r(x,t) \tag{2}$$

Where $v(x,t)$ is the transient solution which satisfies the homogeneous B.C. One can easily see that the reference function is

$$r(x,t) = A(t) + \frac{B(t) - A(t)}{L} x \tag{3}$$

Substituting (1) back into the original PDE gives

$$\begin{aligned}
\frac{\partial}{\partial t}(v(x,t) + r(x,t)) &= k\frac{\partial^2}{\partial x^2}(v(x,t) + r(x,t)) + Q(x,t) \\
v_t(x,t) + r_t(x,t) &= kv_{xx}(x,t) + kr_{xx}(x,t) + Q(x,t)
\end{aligned}$$

But $r_{xx}(x,t) = 0$ and $r_t = A'(t) + \frac{B'(t)-A'(t)}{L} x$ and PDE becomes

$$v_t(x,t) = kv_{xx}(x,t) - r_t(x,t) + Q(x,t)$$

Let

$$\begin{aligned}
\tilde{Q}(x,t) &= Q(x,t) - r_t(x,t) \\
&= Q(x,t) - \left(A'(t) + \frac{B'(t) - A'(t)}{L} x\right)
\end{aligned} \tag{4}$$

Therefore the problem has been transformed to

$$\begin{aligned}
v_t &= kv_{xx} + \tilde{Q}(x,t) \\
v(0,t) &= 0 \\
v(L,t) &= 0 \\
v(0,x) &= F(x) \\
&= u(x,0) - r(x,0) \\
&= f(x) - \left(A(0) + \frac{B(0) - A(0)}{L} x\right)
\end{aligned}$$

The basic solution for this type of PDE was already given in problem 3.1.6.4 on page 664 and the solution is

$$\begin{aligned}
v(x,t) = &\sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L F(s)\,\Phi_n(s)\,ds\right) \\
&+ \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L} e^{k\lambda_n \tau}\left(\int_0^L \tilde{Q}(s,\tau)\,\Phi_n(s)\,ds\right) d\tau\right)
\end{aligned}$$

Where

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$$
$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence, using our $\tilde{Q}(x,\tau)$ and $F(x)$ found above into this solution gives

$$\begin{aligned} v(x,t) = &\frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L \left(Q(s,\tau) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) d \end{aligned}$$

Since $u(x,t) = v(x,t) + r(x,t)$ then the final solution is

$$\begin{aligned} u(x,t) = &A(t) + \frac{B(t)-A(t)}{L}x \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L \left(Q(s,\tau) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right.\right.\right. \end{aligned}$$

3.1.5.15 [229] Both ends depend on time with source present (special case)

problem number 229

Added June 27, 2019

Solve the heat equation

$$u_t = ku_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A(t)$$
$$u(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$ using these values

$$
\begin{aligned}
L &= 2 \\
k &= \frac{1}{10} \\
A(t) &= \sin(t) \\
B(t) &= 2\cos(t) \\
Q(x,t) &= xte^{-t}\cos(t) \\
f(x) &= x
\end{aligned}
$$

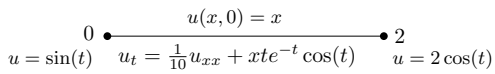

Figure 3.123: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
k=1/10;
A=Sin[t];
B=Cos[t];
f=x;
Q=x*t*Exp[-t]*Cos[t];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q;
bc  = {u[0, t] == A, u[L, t] == B};
ic  = u[x, 0] == f;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{2e^{-\frac{1}{40}\pi^2 t K[1]^2}\left(\frac{40\left(-80(-1)^{K[1]}\pi^2\left(\pi^2 K[1]^2-80\right)\left(\pi^4 K[1]^4+1600\right)K[1]^2-\left(40(-1)^{K[1]}-\pi^2 K[1]^2\right)\left(\pi^4 K[1]^4-80\pi^{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.\right.
$$

Maple ✓

```
restart;
L:=2;
k:=1/10;
A:=sin(t);
B:=cos(t);
f:=x;
Q:=x*t*exp(-t)*cos(t);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+Q;
ic  :=  u(x,0)=f;
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) ),outp
```

$$u(x,t) = \frac{x\cos(t)}{2} + \left(\sum_{n=1}^{\infty} - \frac{2\left(80(\pi^4 n^4 + 1600)\left(\left(\pi^6 n^6 t - 120\pi^4\left(t+\frac{1}{3}\right)n^4 + 6400\left(t+\frac{1}{2}\right)\pi^2 n^2 - 128000t\right)\right.\right.}{}\right.$$

<u>Hand solution</u>

Solve

$$\begin{aligned} u_t &= k u_{xx} + Q(x,t) \\ u(0,t) &= A(t) \\ u(L,0) &= B(t) \\ u(x,0) &= f(x) \end{aligned}$$

With $k = \frac{1}{10}, L = 2, f(x) = x, Q(x,t) = xte^{-t}\cos(t), A(t) = \sin(t), B(t) = 2\cos(t)$.

The general problem above was solved in 3.1.5.14 on page 637 and the solution is

$$\begin{aligned} u(x,t) &= A(t) + \frac{B(t)-A(t)}{L}x \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\left(\int_0^L \left(f(s) - \left(A(0) + \frac{B(0)-A(0)}{L}s\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right) \\ &+ \frac{2}{L}\sum_{n=1}^{\infty} e^{-k\left(\frac{n\pi}{L}\right)^2 t}\sin\left(\frac{n\pi}{L}x\right)\int_0^t e^{k\left(\frac{n\pi}{L}\right)^2\tau}\left(\int_0^L \left(Q(s,\tau) - \left(A'(\tau) + \frac{B'(\tau)-A'(\tau)}{L}x\right)\right)\sin\left(\frac{n\pi}{L}s\right)ds\right)d\tau \end{aligned}$$

Substituting the specific values given above into this solution gives

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \left(\int_0^2 \left(s - \left(A(0) + \frac{B(0) - A(0)}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right) \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s\tau e^{-\tau} \cos(\tau) - \left(A'(\tau) + \frac{B'(\tau) - A'(\tau)}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right.
\end{aligned}
$$

But $A(0) = 0, B(0) = 2, A'(t) = \cos(t), B'(t) = -2\sin(t)$ and the above becomes

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \left(\int_0^2 \left(s - \left(0 + \frac{2-0}{2} s \right) \right) \sin\left(\frac{n\pi}{2} s\right) ds \right) \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s\tau e^{-\tau} \cos(\tau) - \left(\cos(\tau) + \left(\frac{-2\sin(\tau) - \cos(\tau)}{2} \right) s \right) \right) \sin \right.
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= \sin(t) + \frac{2\cos(t) - \sin(t)}{2} x \\
&+ \sum_{n=1}^{\infty} e^{-\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 t} \sin\left(\frac{n\pi}{2} x\right) \int_0^t e^{\frac{1}{10}\left(\frac{n\pi}{2}\right)^2 \tau} \left(\int_0^2 \left(s\tau e^{-\tau} \cos(\tau) - \left(\cos(\tau) - \left(\frac{2\sin(\tau) + \cos(\tau)}{2} \right) s \right) \right) \sin \right.
\end{aligned}
$$

Animation is below

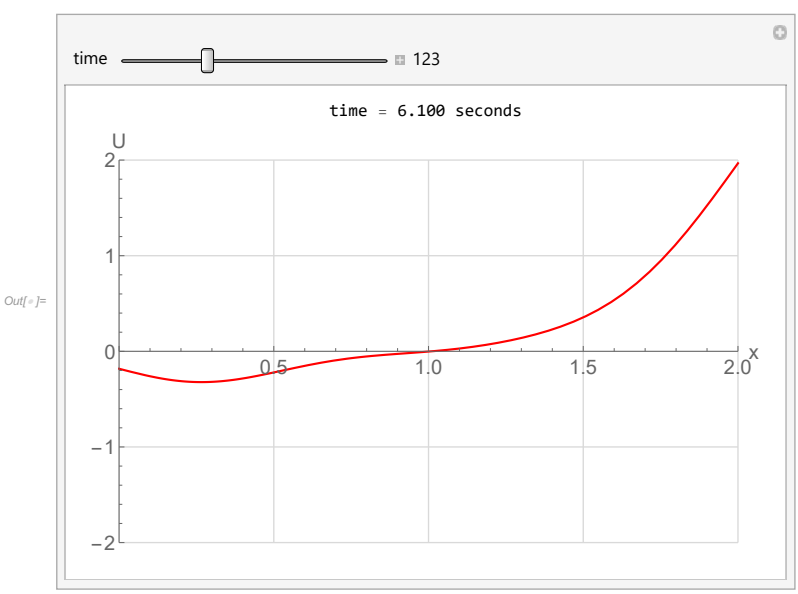

Figure 3.124: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
L = 2;
A[t_] := Sin[t];
B[t_] := 2 Cos[t];
k = 1 / 10;
f[x_] := x;
Q[x_, t_] := x t Exp[-t] Cos[t];
numberOfTerms = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

In[•]:= sol = Integrate[(s * tau * Exp[-tau] Cos[tau] - (Cos[tau] - (2 Sin[tau] + Cos[tau]) / L) * s)) Sin[n Pi s / L], {s, 0, L}];
sol2 = Assuming[Element[n, Integers] && n > 0, Simplify[sol]];
sol3 = N@Integrate[Exp[k (n Pi / L)^2 tau] * sol2, {tau, 0, t}];
mySol[x_, t_] = Chop@N@Simplify[(Sin[t] + ((2 Cos[t] - Sin[t]) / L) x) + Sum[Exp[-k (n Pi / L)^2 t] Sin[n Pi / L x] (sol3), {n, 1, numberOfTerms}]];
```

Figure 3.125: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 2}, {-2, 2}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 20, .05}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.126: Code for animation

3.1.5.16 [230] Pinchover and Rubinstein 6.17

problem number 230

Added July 2, 2018.

Pinchover and Rubinstein's exercise 6.17. Taken from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$\frac{\partial}{\partial t}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 1 + x\cos(t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = \sin(t)$$
$$\frac{\partial u}{\partial x}(1,t) = \sin(t)$$

Initial condition is $u(x,0) = 1 + \cos(2\pi x)$.

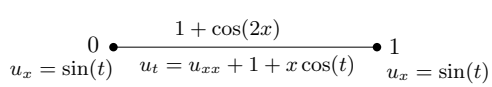

Figure 3.127: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, t], x] == D[u[x, t], {x, 2}] + 1 + x*Cos[t];
bc  = {Derivative[1, 0][u][0, t] == Sin[t], Derivative[1, 0][u][1, t] == Sin[t]};
ic  = u[x, 0] == 1 + Cos[2*Pi*x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic, bc}, u[x, t], x, t]], 60*10]
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + 1+x*cos(t);
bc  := (D[1](u))(0, t) = sin(t), (D[1](u))(1, t) = sin(t);
ic  := u(x, 0) = 1+cos(2*Pi*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = x\sin(t) + \cos(2\pi x)\,\mathrm{e}^{-4\pi^2 t} + t + 1$$

3.1.5.17 [231] nonhomogeneous BC

problem number 231

Added March 28, 2018. A problem from my PDE animation page.

Solve the heat equation

$$u_t = k u_{xx} + x$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = \frac{t\sin t}{5}$$
$$u(\pi,t) = \frac{t\cos t}{10}$$

Initial condition is $u(x,0) = 60 - 20x$.

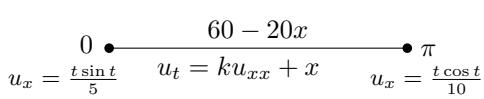

Figure 3.128: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}] + x;
bc  = {u[0, t] == (t*Sin[t])/5, u[Pi, t] == (t*Cos[t])/10};
ic  = u[x, 0] == 60 - 2*x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {t
```

$$\left\{\left\{u(x,t)\to \frac{10\pi \sum_{K[1]=1}^{\infty} \frac{e^{-tK[1]^2}\left(20\left(30+(-1)^{K[1]}(-30+\pi)\right)K[1]^2+\frac{(-1)^{K[1]+1}K[1]^8+10(-1)^{K[1]}\pi K[1]^8-10(-1)^{K[1]}e^{tK[1]^2}\pi K[1]^8-4K[1]^{6}}{}\right.}{}}{}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+x;
ic  :=  u(x,0)=(60-2*x);
bc  :=  u(0,t)=t/5*sin(t), u(Pi,t)=t/10*cos(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{(x\cos(t)+(-2x+2\pi)\sin(t))\,t+2\pi\left(\sum_{n=1}^{\infty}\frac{20\left(-\frac{\left(\left(n^4t-2n^2+t\right)n^2\cos(t)+\left(n^6+n^4t-n^2+t\right)\sin(t)\right)n^2}{10}+\left(-\frac{\left(n^4t-2n^{}\right.}{}\right.\right.}{}\right.}{}$$

3.1.5.18 [232] Haberman 8.2.2. (a)

problem number 232

Added Nov 27, 2018.

Problem 8.2.2 part(a) from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = A(t)$$
$$\frac{\partial u}{\partial x}(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

For hand solution see my HW9, Math 322, UW Madison. The text does not actually asks to solve this PDE but only to reduce the problem to one with homogeneous B.C.

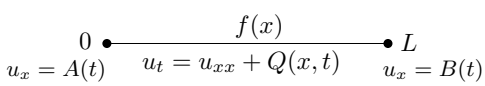

Figure 3.129: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {Derivative[1, 0][u][0, t] == A[t], Derivative[1, 0][u][L, t] == B[t]};
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}\cos\left(\frac{\pi x K[1]}{L}\right)\left(e^{-\frac{k\pi^2 t K[1]^2}{L^2}} \int_0^L \frac{\cos\left(\frac{\pi x K[1]}{L}\right)(x(x(A(0)-B(0))-2LA(0))+2Lf(x))}{\sqrt{2}L^{3/2}}\,dx + \int_0^t e^{-\frac{k\pi^2 K[1]}{}}\right.}{}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+Q(x,t);
ic  :=  u(x,0)=f(x);
bc  := eval( diff(u(x,t),x),x=0)=A(t), eval( diff(u(x,t),x),x=L)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = \frac{x^2 B(t)}{2L} + \left(-\frac{x^2}{2L} + x\right) A(t) + \int_0^t \left(\left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L -\frac{\left(\frac{x^2\left(\frac{d}{d\tau}B(\tau)\right)}{2} + LQ(x,\tau) + \left(L-\frac{x}{2}\right)x\left(\frac{d}{d\tau}A(\tau)\right) - (A(\tau)-B(\tau))k\right)}{L}\right.}{L}\right.\right.$$

<u>Hand solution</u>

Solve

$$\begin{aligned} u_t &= k u_{xx} + Q(x,t) \\ u_x(0,t) &= A(t) \\ u_x(L,0) &= B(t) \\ u(x,0) &= f(x) \end{aligned}$$

Let

$$u(x,t) = v(x,t) + r(x,t) \tag{1}$$

Since the problem has time dependent source function $Q(x,t)$ then $r(x,t)$ is now a reference function that only needs to satisfy the non-homogenous boundary conditions which in this problem are at both ends and $v(x,t)$ has homogenous boundary conditions. The first step is to find $r(x,t)$. Let

$$r(x,t) = c_1(t)\, x + c_2(t)\, x^2$$

Then

$$\frac{\partial r(x,t)}{\partial x} = c_1(t) + 2c_2(t)\, x$$

At $x = 0$

$$A(t) = c_1(t)$$

And at $x = L$

$$\begin{aligned} B(t) &= c_1(t) + 2c_2(t)\, L \\ c_2(t) &= \frac{B(t) - c_1(t)}{2L} \end{aligned}$$

Solving for c_1, c_2 gives

$$r(x,t) = A(t)\, x + \left(\frac{B(t) - A(t)}{2L}\right) x^2 \tag{2}$$

Replacing (1) into the original PDE $u_t = ku_{xx} + Q(x,t)$ gives

$$\begin{aligned} \frac{\partial}{\partial t}(v(x,t) - r(x,t)) &= k\frac{\partial^2}{\partial x}(v(x,t) - r(x,t)) + Q(x,t) \\ \frac{\partial v}{\partial t} - \frac{\partial r}{\partial t} &= k\frac{\partial^2 v}{\partial x^2} - k\frac{\partial^2 r}{\partial x^2} + Q(x,t) \end{aligned}$$

But $r_{xx} = \frac{B(t)-A(t)}{L}$, hence the above reduces to

$$v_t = kv_{xx} + Q(x,t) - k\frac{B(t) - A(t)}{L} + r_t \tag{3}$$

Let

$$\tilde{Q}(x,t) = Q(x,t) + r_t - k\frac{B(t) - A(t)}{L}$$

Then (3) becomes

$$\begin{aligned} v_t &= kv_{xx} + \tilde{Q}(x,t) \\ v_t(0,t) &= 0 \\ v_t(L,t) &= 0 \end{aligned} \tag{4}$$

And initial condition is

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x,0) \\ &= f(x) - \left(A(0)\, x + \left(\frac{B(0) - A(0)}{2L}\right) x^2\right) \end{aligned}$$

PDE (4) with its homogenous boundary conditions is standard one, its corresponding eigenvalue boundary value ODE $X'' + \lambda X = 0$ has $\lambda = 0$ as eigenvalue with corresponding eigenfunction $\Phi_0(x) = 1$ and $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1, 2, 3, \cdots$ with corresponding

eigenfunctions $\Phi_n(x) = \cos\left(\sqrt{\lambda_n} x\right)$. Using these, we can write the solution to (4) using eigenfunction expansion as

$$v(x,t) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n(x) \tag{4A}$$

Hence $v_t(x,t) = \sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x)$ and $v_{xx}(x,t) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x)$. Substituting these into (4) gives

$$\sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x) + \tilde{Q}(x,t)$$

Expanding $\tilde{Q}(x,t)$ using same eigenfunctions since they are complete, the above becomes

$$\sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x) + \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)$$

But $\Phi_n''(x) = -\lambda_n \Phi_n(x)$ and the above becomes

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) &= -\sum_{n=0}^{\infty} c_n(t)\,\lambda_n \Phi_n(x) + \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)\\
c_n'(t)\,\Phi_n(x) + c_n(t)\,\lambda_n \Phi_n(x) &= b_n(t)\,\Phi_n(x)\\
c_n'(t) + c_n(t)\,\lambda_n &= b_n(t)\\
c_n'(t) + c_n(t)\,\frac{n^2\pi^2}{L^2} &= b_n(t) \qquad (5)
\end{aligned}$$

To find $b_n(t)$, since $\tilde{Q}(x,t) = Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L}$ then

$$Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L} = \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)$$

Multiplying both sides by $\Phi_m(x)$ and integrating gives

$$\begin{aligned}
\int_0^L \left(Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L}\right)\Phi_m(x)\,dx &= \int_0^L \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)\,\Phi_m(x)\,dx\\
&= \sum_{n=0}^{\infty} b_n(t)\left(\int_0^L \Phi_n(x)\,\Phi_m(x)\,dx\right)
\end{aligned}$$

By orthogonality

$$\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\Phi_m(x)\,dx = b_m(t)\int_0^L \Phi_m^2(x)\,dx$$

When $m=0, \Phi_0(x)=1$ and the above gives

$$\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx = b_0(t)\int_0^L dx$$

$$b_0(t)=\frac{1}{L}\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx$$

When $m=1,2,3,\cdots$

$$\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = b_m(t)\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

$$\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = b_m(t)\frac{L}{2}$$

$$b_m(t)=\frac{2}{L}\int_0^L \left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)d$$

Therefore (5) is now solved. When $n=0$ (5) becomes

$$\begin{aligned}
c_0'(t)+c_0(t)\frac{n^2\pi^2}{L^2} &= b_0(t)\\
c_0'(t) &= b_0(t)\\
c_0'(t) &= \frac{1}{L}\int_0^L Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}dx
\end{aligned}$$

Hence

$$c_0(t)=\int_0^t\left(\frac{1}{L}\int_0^L Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}dx\right)dt+C_0$$

For $n=1,2,3,\cdots$ (5) becomes

$$\begin{aligned}
c_n'(t)+c_n(t)\frac{n^2\pi^2}{L^2} &= b_n(t)\\
&= \frac{2}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx
\end{aligned}$$

Integrating factor is $I=e^{\int\frac{n^2\pi^2}{L^2}dt}=e^{\frac{n^2\pi^2}{L^2}t}$ and the solution to the above becomes

$$\frac{d}{dt}\left(c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t}\right)=\frac{2e^{\frac{n^2\pi^2}{L^2}t}}{L}\int_0^L\left(Q(x,t)+r_t-k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t}=\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt+C_n$$

$$c_n(t)=e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L\left(Q(x,\tau)+r_\tau-k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt+C_ne^{-\frac{n^2\pi^2}{L^2}t}$$

Now that we found $c_n(t)$ for $n = 0, 1, 2, 3, \cdots$ the solution for $v(x,t)$ is found from 4A.

$$
\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} c_n(t)\,\Phi_n(x) \\
&= \int_0^t \left(\frac{1}{L} \int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L} dx \right) dt + C_0 + \sum_{n=1}^{\infty} c_n(t)\,\Phi_n(x)
\end{aligned}
$$

$$
\begin{aligned}
&= \int_0^t \left(\frac{1}{L} \int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L} dx \right) dt + C_0 \\
&+ \sum_{n=1}^{\infty} \left(e^{-\frac{n^2\pi^2}{L^2}t} \int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L} \int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L} \right) \cos\left(\frac{n\pi}{L}x\right) dx \right) dt + C_n e^{-\frac{n^2\pi^2}{L^2}t} \right) \cos
\end{aligned}
$$

But

$$
u(x,t) = v(x,t) + r(x,t)
$$

Hence

$$
\begin{aligned}
u(x,t) &= A(t)\,x + \left(\frac{B(t)-A(t)}{2L} \right) x^2 + \int_0^t \left(\frac{1}{L} \int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L} dx \right) dt + C_0 \\
&+ \sum_{n=1}^{\infty} \left(e^{-\frac{n^2\pi^2}{L^2}t} \int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L} \int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L} \right) \cos\left(\frac{n\pi}{L}x\right) dx \right) dt + C_n e^{-\frac{n^2\pi^2}{L^2}t} \right) \cos
\end{aligned}
$$

But

$$
\begin{aligned}
r_\tau &= \frac{d}{dt}\left(A(\tau)\,x + \left(\frac{B(\tau)-A(\tau)}{2L} \right) x^2 \right) \\
&= \frac{2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2}{2L}
\end{aligned}
$$

Hence

$$
\begin{aligned}
u(x,t) &= C_0 + A(t)\,x + \left(\frac{B(t)-A(t)}{2L} \right) x^2 \\
&+ \frac{1}{2L^2} \int_0^t \left(\int_0^L 2LQ(x,\tau) + 2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2 - 2k(B(\tau)-A(\tau))\,dx \right) dt + \\
&\sum_{n=1}^{\infty} \cos\frac{n\pi}{L}x \left(e^{-\frac{n^2\pi^2}{L^2}t} \int_0^t \left(\frac{e^{\frac{n^2\pi^2}{L^2}\tau}}{L^2} \int_0^L (2LQ(x,\tau) + 2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2 - 2k(B(\tau)-A(\tau))) \cos\left(\frac{n\pi}{L}x\right) dx \right) dt + C_n e^{-\frac{n^2\pi^2}{L^2}t} \right)
\end{aligned}
$$

The constants C_0, C_n are found from initial conditions $u(x,0) = f(x)$.

3.1.5.19 [233] Articolo 8.4.3

problem number 233

Added December 20, 2018.

Example 8.4.3 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + t$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 5 \\ u(1,t) + \frac{\partial u}{\partial x}(1,t) &= 10 \end{aligned}$$

Initial condition is $u(x,0) = \frac{-40x^2}{3} + \frac{45x}{2} + 5$ and $k = \frac{1}{20}$

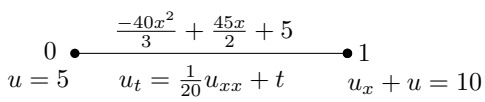

Figure 3.130: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/20;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + t;
bc  = {u[0, t] == 5, u[1, t] + Derivative[1, 0][u][1, t] == 10};
ic  = u[x, 0] == (-40*x^2)/3 + (45*x)/2 + 5;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to\{\quad \frac{5x}{2}+\sum_{K[1]=1}^{\infty}-\frac{\sqrt{2}\left(\frac{40e^{-\frac{1}{20}tK[2,K[1]]}\left(\cos\left(\sqrt{K[2,K[1]]}\right)-1\right)\left(e^{\frac{1}{20}tK[2,K[1]]}(tK[2,K[1]]-20)+20\right)}{\sqrt{\cos\left(2\sqrt{K[2,K[1]]}\right)+3K[2,K[1]]^{5/2}}}+\frac{40e^{-\frac{1}{20}tK[2,K[1]]}\left(\cos\right.}{}\right.}{\sqrt{\cos^2\left(\sqrt{K[2,K[1]]}\right)}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t) = (1/20)*(diff(u(x, t), x$2))+t;
bc  := u(0, t) = 5, (u(1, t)+ eval( diff(u(x,t),x),x=1)) = 10;
ic  := u(x, 0) = -40*x^2/3+45*x/2+5;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t))),o
```

$$u(x,t)=\frac{5x}{2}+\int_0^t\left(\sum_{n1=0}^{\infty}\begin{cases}0 & \lambda_n=0\\ \frac{4(\cos(\lambda_n)-1)\tau\,e^{-\frac{(t-\tau)\lambda_n^2}{20}}\sin(x\lambda_n)}{-2\lambda_n+\sin(2\lambda_n)} & otherwise\end{cases}\right)d\tau+\left(\sum_{n=0}^{\infty}\begin{cases}0\\ \frac{80(\lambda_n^2\cos(\lambda_n)+\lambda_n\sin(\lambda_n)+4\,c}{3(-2\lambda_n+\sin}\end{cases}\right.$$

3.1.6 Semi-infinite domain

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3.1.6.1 [234] left end constant (general case)

problem number 234

Added July 6, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = A$ and initial conditions $u(x,0) = 0$

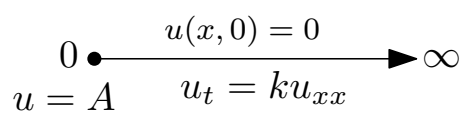

Figure 3.131: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == A;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \frac{x\text{Integrate}\left[\frac{A e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}}, \{K[2], 0, t\}, \text{Assumptions} \to \text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=A;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -A \operatorname{erf}\left(\frac{x}{2\sqrt{k}\sqrt{t}}\right) + A$$

Hand solution

Solving

$$\begin{aligned} u_t &= k u_{xx} \qquad t > 0, x > 0 \\ u(0,t) &= A \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

And $u(x,t) < \infty$ as $x \to \infty$. This means $u(x,t)$ is bounded. This conditions is always needed to solve these problems.

Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Defined as

$$\mathcal{L}(u,t) = \int_0^\infty e^{-st} u(x,t)\, dt$$

Applying Laplace transform to the original PDE (1) gives

$$sU(x,s) - u(x,0) = kU_{xx}(x,s)$$

But $u(x,0) = 0$, therefore the above becomes

$$U_{xx} - \frac{s}{k} U = 0$$

The solution to this differential equation is

$$U(x,s) = c_1 e^{\sqrt{\frac{s}{k}}x} + c_2 e^{-\sqrt{\frac{s}{k}}x}$$

Since $u(x,t)$ is bounded in the limit as $x \to \infty$ and $k > 0$, therefore it must be that $c_1 = 0$ to keep the solution bounded. The above simplifies to

$$U(x,s) = c_2 e^{-\sqrt{\frac{s}{k}}x} \tag{2}$$

At $x = 0\,, u(0,t) = A$. Therefore $U(0,s) = \mathcal{L}(u(0,t)) = \mathcal{L}(A) = \frac{1}{s}A$. Hence at $x = 0$ the above gives

$$\frac{1}{s}A = c_2$$

Therefore (2) becomes

$$U(x,s) = \frac{A}{s}e^{-\sqrt{\frac{s}{k}}x} \tag{3}$$

From tables, the inverse Laplace transform of the above is (since $x > 0, k > 0$)

$$\begin{aligned} u(x,t) &= A\,\mathrm{erfc}\left(\frac{x}{2\sqrt{kt}}\right) \\ &= A\left(1 - \mathrm{erf}\left(\frac{x}{2\sqrt{kt}}\right)\right) \end{aligned}$$

3.1.6.2 [235] left end constant (special case)

problem number 235

Added July 6, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = ku_{xx}$$

The boundary conditions are $u(0,t) = A$ and initial conditions $u(x,0) = 0$, using

$$\begin{aligned} A &= 60 \\ k &= \frac{1}{10} \end{aligned}$$

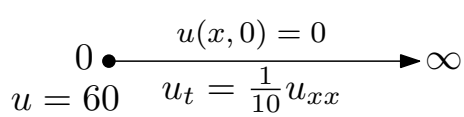

Figure 3.132: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/10; A=60;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == A;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to \boxed{\text{Indeterminate if } x \leq 0}\}\}$$

It fail if assumption $x > 0$ is given. A bug

Maple ✓

```
restart;
k:=1/10;
A:=60;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=A;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -60 \operatorname{erf}\left(\frac{\sqrt{10}\,x}{2\sqrt{t}}\right) + 60$$

<u>Hand solution</u>

Solving on semi-infinite domain

$$\begin{aligned} u_t &= k u_{xx} \qquad t > 0, x > 0 \\ u(0,t) &= A \\ u(x,0) &= 0 \end{aligned}$$

With $A = 60, k = \frac{1}{10}$

The general problem above was solved in 3.1.6.1 on page 656 and the solution is

$$u(x,t) = A\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{kt}}\right)\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = 60\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\frac{t}{10}}}\right)\right)$$

Animation is below

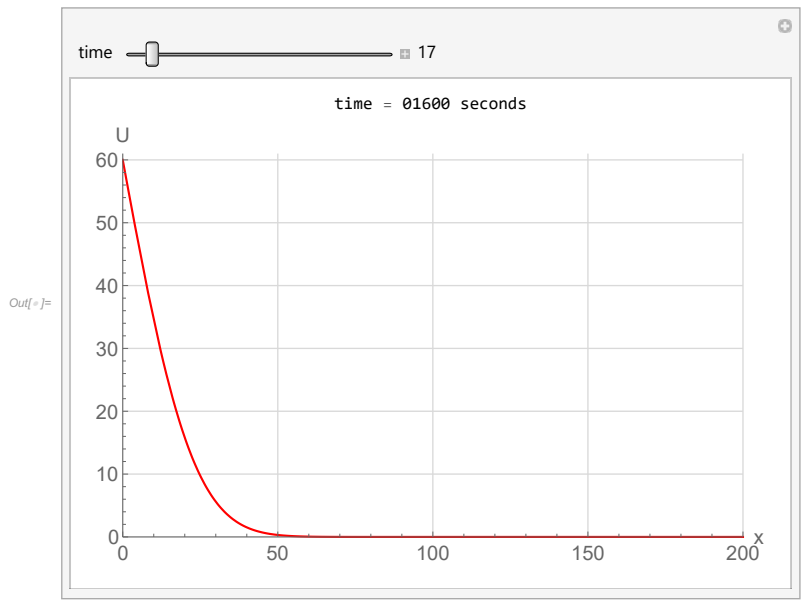

Figure 3.133: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
A = 60;
L = 200;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = A (1 - Erf[x / (2 Sqrt[k t])]);
```

Figure 3.134: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {5, 0}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {0, 61}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 30000, 100}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.135: Code for animation

3.1.6.3 [236] Logan p. 76. Left end general function of time (general case)

problem number 236

This is problem at page 76 from David J Logan text book.

Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = f(t)$ and initial conditions $u(x,0) = 0$

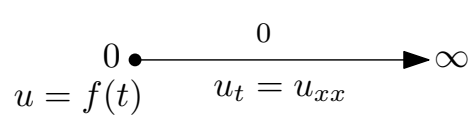

Figure 3.136: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == f[t];
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
sol =  sol /. {K[2] -> z}
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{f(z)e^{-\frac{x^2}{4kt-4kz}}}{(t-z)^{3/2}},\{z,0,t\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=f(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\frac{x\left(\int_0^t \frac{e^{\frac{x^2}{4(-t+\zeta)k}}f(\zeta)}{(t-\zeta)^{\frac{3}{2}}}d\zeta\right)}{2\sqrt{\pi}\sqrt{k}}$$

<u>Hand solution</u>

Solving on semi-infinite domain

$$\begin{aligned} u_t &= k u_{xx} \qquad t>0, x>0 \\ u(0,t) &= f(t) \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

With $k>0$ and $u(x,t)<\infty$ as $x\to\infty$. This means $u(x,t)$ is bounded. This conditions is always needed to solve these problems.

Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Defined as

$$\mathcal{L}(u,t)=\int_0^\infty e^{-st}u(x,t)\,dt$$

Applying Laplace transform to the original PDE (1) gives

$$sU(x,s) - u(x,0) = kU_{xx}(x,s)$$

But $u(x,0) = 0$, therefore the above becomes

$$U_{xx} - \frac{s}{k}U = 0$$

The solution to this differential equation is

$$U(x,s) = c_1 e^{\sqrt{\frac{s}{k}}x} + c_2 e^{-\sqrt{\frac{s}{k}}x}$$

Since $u(x,t)$ is bounded in the limit as $x \to \infty$ and $k > 0$, therefore it must be that $c_1 = 0$ to keep the solution bounded. The above simplifies to

$$U(x,s) = c_2 e^{-\sqrt{\frac{s}{k}}x} \tag{2}$$

At $x = 0\,, u(0,t) = f(t)$. Therefore $U(0,s) = \mathcal{L}(f(t)) = F(s)$. Hence at $x = 0$ the above gives

$$F(s) = c_2$$

Therefore (2) becomes

$$U(x,s) = F(s)\, e^{-\sqrt{\frac{s}{k}}x} \tag{3}$$

By convolution, the above becomes

$$u(x,t) = f(t) \circledast G(x,t) \tag{4}$$

Where $G(x,t)$ is the inverse transform of $e^{-\sqrt{\frac{s}{k}}x}$ which is $\frac{xe^{\frac{-x^2}{4kt}}}{2\sqrt{k\pi}t^{\frac{3}{2}}}$. Hence (4) becomes

$$\begin{aligned} u(x,t) &= f(t) \circledast \frac{xe^{\frac{-x^2}{4kt}}}{2\sqrt{k\pi}t^{\frac{3}{2}}} \\ &= \frac{x}{2\sqrt{k\pi}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4k(t-\tau)}} d\tau \end{aligned}$$

For $k = 1$

$$u(x,t) = \frac{x}{2\sqrt{\pi}} \int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4(t-\tau)}} d\tau$$

3.1.6.4 [237] Left end function of time (special case)

problem number 237

Added July 7, 2019 Solve the heat equation for $x > 0, t > 0$

$$u_t = k u_{xx}$$

The boundary conditions are $u(0,t) = \sin(t)$ and initial conditions $u(x,0) = 0$ using $k = \frac{1}{10}$

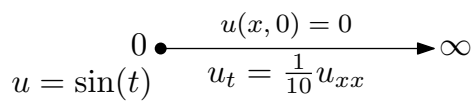

Figure 3.137: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1/10;
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == Sin[t];
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \sqrt{\frac{5}{2\pi}} x \text{Integrate}\left[\frac{\sin(K[2]) e^{-\frac{5x^2}{2(t-K[2])}}}{(t-K[2])^{3/2}}, \{K[2], 0, t\}, \text{Assumptions} \to \text{True}\right]\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
k:=1/10;
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=sin(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = -\frac{\sqrt{10}\,x\left(\int_0^t -\frac{e^{-\frac{5x^2}{2\zeta}}\sin(t-\zeta)}{\zeta^{\frac{3}{2}}}d\zeta\right)}{2\sqrt{\pi}}$$

<u>Hand solution</u>

Solving

$$\begin{aligned} u_t &= k u_{xx} \qquad t>0, x>0 \\ u(0,t) &= f(t) \\ u(x,0) &= 0 \end{aligned} \tag{1}$$

Using $k=\frac{1}{10}$ and $f(t)=\sin(t)$.

The general solution was solved in problem 3.1.6.3 on page 661 and the solution was found to be

$$u(x,t) = \frac{x}{2\sqrt{k\pi}}\int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-x^2}{4k(t-\tau)}} d\tau$$

Replacing the given values above, the solution becomes

$$u(x,t) = \sqrt{\frac{5}{2}}\frac{x}{\sqrt{\pi}}\int_0^t \frac{\sin(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{\frac{-5x^2}{2(t-\tau)}} d\tau$$

We could also use the following form of the solution

$$u(x,t) = \sqrt{\frac{5}{2}}\frac{x}{\sqrt{\pi}}\int_0^t \frac{\sin(t-\tau)}{\tau^{\frac{3}{2}}} e^{\frac{-5x^2}{2\tau}} d\tau$$

Animation is below

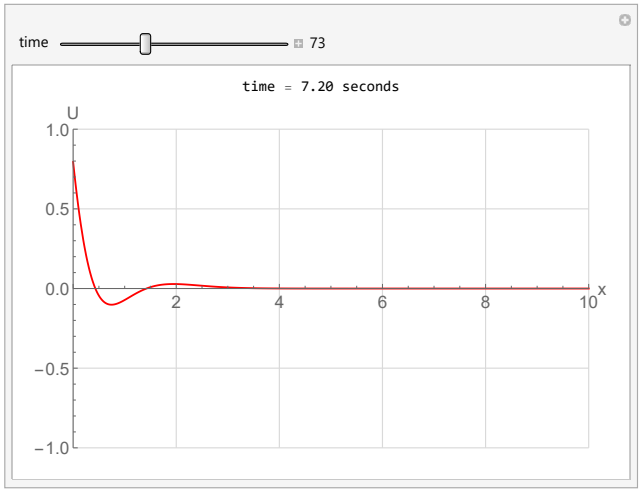

Figure 3.138: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];

mySol[x_?NumericQ, t_?NumericQ] := x/(2 Sqrt[Pi k]) NIntegrate[Exp[-x^2/(4 k s)]/s^(3/2) Sin[t - s], {s, 0, t}, Method → {"GlobalAdaptive", "SymbolicProcessing" → 0}]
```

Figure 3.139: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
          {
            Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, 10},
                BaseStyle → 15,
                ImageMargins → 3,
                PerformanceGoal → "Quality",
                PlotRange → {{0, 10}, {-1, 1}},
                ImageSize → 500,
                AxesLabel → {"x", "U"},
                GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotStyle → Red
              ]
          }}],
      {t, 0.001, 20, 0.1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 3.140: Code for animation

3.1.6.5 [238] nonhomogeneous BC

problem number 238

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = 1$ and And initial condition $u(x,0) = 0$

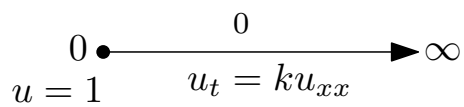

Figure 3.141: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == 1;
ic  = u[x, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \frac{x\text{Integrate}\left[\frac{e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}}, \{K[2],0,t\}, \text{Assumptions} \to \text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=0:
bc  := u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) = \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

3.1.6.6 [239] I.C. not zero

problem number 239

Added December 20, 2018.

From `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,t_0) = 10;$$

And boundary conditions

$$u(-x_0,t) = 0$$

For $x > |x_0|$ and $t > |t_0|$.

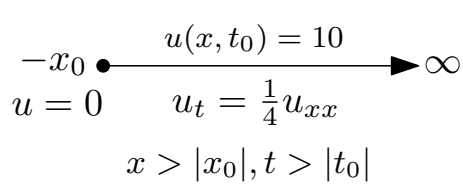

Figure 3.142: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1/4)*D[u[x, t], {x, 2}];
bc  = u[-x0, t] == 0;
ic  = u[x, t0] == 10;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t, Assumptions -> {t
```

$$\left\{\left\{u(x,t)\to \begin{cases} 10\text{erf}\left(\frac{x+\text{x0}}{\sqrt{t-\text{t0}}}\right) & x+\text{x0}>0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

due to IC/BC not zero

Maple ✓

```
restart;
pde := diff(u(x, t), t) = (1/4)*(diff(u(x, t), x$2));
bc  := u(-x0, t) = 0;
ic  := u(x, t0) = 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t)) assu
```

$$u(x,t)=10\,\text{erf}\left(\frac{x+x0}{\sqrt{t-t0}}\right)$$

3.1.6.7 [240] nonhomogeneous BC

problem number 240

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

For $x>0$ and $t>0$. The boundary conditions is $u(0,t)=\mu$ and And initial condition $u(x,0)=\lambda$

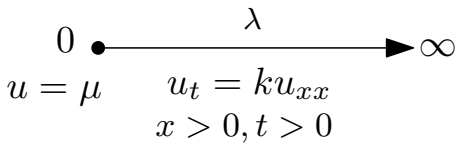

Figure 3.143: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc  = u[0, t] == lambda;
ic  = u[x, 0] == mu;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\sqrt{kt}\text{Integrate}\left[\frac{\lambda e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to\text{True}\right]+\sqrt{k}\text{Integrate}\left[\mu\left(e^{-\frac{(x-K[}{4kt}}\right.\right.}{2\sqrt{\pi}k\sqrt{t}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=mu:
bc  := u(0,t)=lambda;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t)=\mu+(\lambda-\mu)\operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

3.1.6.8 [241] nonhomogeneous BC

problem number 241

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x > 0$ and $t > 0$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0) = \cos x$$

And boundary conditions

$$u(0,t) = 1$$

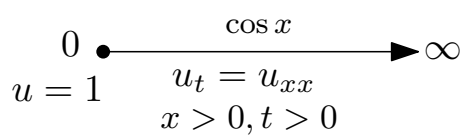

Figure 3.144: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == Cos[x];
bc  = u[0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{pde, ic, bc}, u[x, t], {x, t}]],
```

$$\left\{\left\{u(x,t) \to \{\frac{\text{Integrate}\left[\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)\cos(K[1]),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]+\sqrt{t}x\text{Integrate}\left[\frac{e^{-\frac{x^2}{4(t-K[2])}}}{(t-K[2])^{3/2}},\{\right.}{2\sqrt{\pi}\sqrt{t}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=cos(x);
bc  := u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = -\frac{\operatorname{erf}\left(\frac{2it-x}{2\sqrt{t}}\right)\mathrm{e}^{-ix-t}}{2} + \frac{\operatorname{erf}\left(\frac{2it+x}{2\sqrt{t}}\right)\mathrm{e}^{ix-t}}{2} - \operatorname{erf}\left(\frac{x}{2\sqrt{t}}\right) + 1$$

3.1.6.9 [242] nonhomogeneous B.C.

problem number 242

Solve the heat equation for $u(x,t)$ on half the line $x > 0$ and $t > 0$

$$u_t = k u_{xx}$$

With initial condition

$$u(x,0) = 0$$

And boundary conditions $u(0,t) = t$. Solution is bounded at infinity.

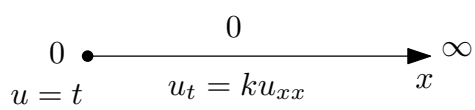

Figure 3.145: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == 0;
bc  = u[0, t] == t;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \frac{x\text{Integrate}\left[\frac{K[2]e^{-\frac{x^2}{4kt-4kK[2]}}}{(t-K[2])^{3/2}},\{K[2],0,t\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{k}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x, t), t)=k*diff(u(x, t), x$2);
ic  := u(x,0)=0;
bc  := u(0,t)=t;
assume(x>0);
assume(t>0);
assume(k>0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=-\frac{\sqrt{k}\,\sqrt{t}\,x\,\mathrm{e}^{-\frac{x^2}{4kt}}+\sqrt{\pi}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{k}\,\sqrt{t}}\right)-1\right)\left(kt+\frac{x^2}{2}\right)}{\sqrt{\pi}\,k}$$

3.1.6.10 [243] Unit triangle I.C.

problem number 243

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$u_t=u_{xx}$$

With initial condition

$$u(x,0)=\text{UnitTriagle[x-3]}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0$$

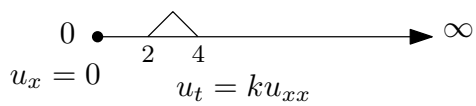

Figure 3.146: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitTriangle[x - 3];
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \Lambda(3-x) & t=0 \\ \frac{2\int_0^\infty \frac{4e^{-tK[1]^2}\cos(3K[1])\cos(xK[1])\sin^2\left(\frac{K[1]}{2}\right)}{K[1]^2}dK[1]}{\pi} & t>0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=piecewise( x>2 and x<3,-2+x, x>3 and x<4, 4-x, 0);
bc:=(D[1](u))(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{t\,\mathrm{e}^{-\frac{(x-4)^2}{4t}} - 2t\,\mathrm{e}^{-\frac{(x-3)^2}{4t}} + t\,\mathrm{e}^{-\frac{(x-2)^2}{4t}} + t\,\mathrm{e}^{-\frac{(x+2)^2}{4t}} - 2t\,\mathrm{e}^{-\frac{(x+3)^2}{4t}} + t\,\mathrm{e}^{-\frac{(x+4)^2}{4t}} + \frac{\left((x-4)\operatorname{erf}\left(\frac{x-4}{2\sqrt{t}}\right)+(-2x+6)\,\mathrm{er}\right.}{}}{\sqrt{\pi}\sqrt{t}}$$

3.1.6.11 [244] I.C. not at $t = 0$

problem number 244

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ for $t > 0, x > 0$

$$u_t = \frac{1}{4} u_{xx}$$

With initial condition

$$u(x, t_0) = 10e^{-x^2}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(x_0, t) = 0$$

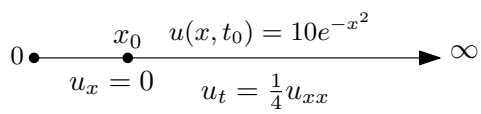

Figure 3.147: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == (1*D[u[x, t], {x, 2}])/4;
ic  = u[x, t0] == 10*Exp[-x^2];
bc  = Derivative[1, 0][u][x0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} 10e^{-x^2} & t - \\ \frac{2\int_0^\infty \frac{5}{2} e^{-\frac{1}{4}K[1](4ix0+(t-t0+1)K[1])}\sqrt{\pi}\cos((x-x0)K[1])\left(\text{Erfc}\left(x0-\frac{1}{2}iK[1]\right)+e^{2ix0K[1]}\,\text{Erfc}\left(x0+\frac{1}{2}iK[1]\right)\right)dK[1]}{\pi} & t - \\ \text{Indeterminate} & \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t) = 1/4*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=x0)=0;
ic  := u(x,t0)=10*exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = -\frac{5\left(-\operatorname{erf}\left(\frac{x+(-t+t0-1)x0}{\sqrt{t-t0+1}\,\sqrt{t-t0}}\right)+\left(\operatorname{erf}\left(\frac{-t0x0+x+(t-1)x0}{\sqrt{t-t0+1}\,\sqrt{t-t0}}\right)-1\right)\mathrm{e}^{\frac{4(-x+x0)x0}{-t+t0-1}}-1\right)\mathrm{e}^{\frac{x^2}{-t+t0-1}}}{\sqrt{t-t0+1}}$$

3.1.6.12 [245] Diffusion with advection

problem number 245

Added April 5, 2019.

Solve for $u(x,t)$ in

$$u_t = u_{xx} - u_x$$

For $t > 0, x > 0$. With boundary conditions $u(0,t) = 0$ and intitial conditions $u(x,0) = f(x)$

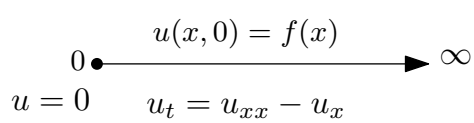

Figure 3.148: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - D[u[x, t], x];
ic  = u[x, 0] == f[x];
bc  = u[0, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

$$\left\{\left\{u(x,t)\to e^{\frac{x}{2}-\frac{t}{4}}\left(\begin{cases}\frac{\text{Integrate}\left[e^{-\frac{K[1]}{2}}\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)f(K[1]),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]}{2\sqrt{\pi}\sqrt{t}} & x>0\\ \text{Indeterminate} & \text{True}\end{cases}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)=diff(u(x,t),x$2)- diff(u(x,t),x);
ic  := u(x,0)=f(x);
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))as
```

$$u(x,t)=\left(\mathcal{L}^{-1}\left(\frac{\int\frac{\sqrt{e^{\sqrt{4s+1}\,x}}\,f(x)}{\sqrt{e^{x}}}dx}{\sqrt{4s+1}\,\sqrt{e^{\sqrt{4s+1}\,x}}},s,t\right)-\mathcal{L}^{-1}\left(\frac{\left(\int\frac{f(x)}{\sqrt{e^{x}}\,\sqrt{e^{\sqrt{4s+1}\,x}}}dx\right)\sqrt{e^{\sqrt{4s+1}\,x}}}{\sqrt{4s+1}},s,t\right)+\mathcal{L}^{-1}\left(\frac{\int^{0}\frac{f}{\sqrt{e^{-a}}\,\sqrt{}}}{\sqrt{4s+}}\right.\right.$$

3.1.6.13 [246] Problem exam problem

problem number 246

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 13.

Solve for $u(x,t)$ with IC $u(x,0)=x^2+1$ and BC $u_t(0,t)=1$ for $x>0,t>0$

$$u_t=u_{xx}$$

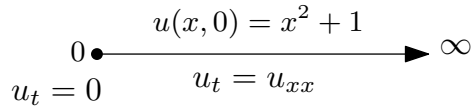

Figure 3.149: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x,t],{x,2}];
ic  = u[x,0]==x^2+1;
bc  = u[0,t]==1;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic,bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \frac{\text{Integrate}\left[\left(e^{-\frac{(x-K[1])^2}{4t}}-e^{-\frac{(x+K[1])^2}{4t}}\right)(K[1]^2+1),\{K[1],0,\infty\},\text{Assumptions}\to\text{True}\right]+\sqrt{t}x\text{Integrate}\left[\frac{e^{-\frac{x^2}{4(t-K[2])}}}{(t-K[2])^{3/2}},\right.}{2\sqrt{\pi}\sqrt{t}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2);
ic  := u(x,0)=x^2+1;
bc :=u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = x^2 + 2t - 2\mathcal{L}^{-1}\left(\frac{e^{-\sqrt{s}\,x}}{s^2}, s, t\right) + 1$$

3.1.7 Infinite domain

Local contents

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3.1.7.1 [247] Inverse exponential I.C.

problem number 247

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = u_{xx}$$

With initial condition

$$u(x,0) = e^{-x^2}$$

$$-\infty \xleftrightarrow[u_t = u_{xx}]{e^{-x^2}} \infty$$

Figure 3.150: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == E^(-x^2);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}$$

<u>Hand solution</u>

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

On $-\infty < x < \infty, t > 0$ with $u(x,0) = f(x) = e^{-x^2}$. The first step is to find Green function for the above PDE. Taking Fourier transform of both sides w.r.t. x, using $\hat{u}(k,t)$ as the Fourier transform of $u(x,t)$ gives

$$\begin{aligned}\frac{d}{dt}\hat{u}(k,t) &= (ik)^2\,\hat{u}(k,t)\\ &= -k^2\hat{u}(k,t)\\ \frac{d}{dt}\hat{u}(k,t) + k^2\hat{u}(k,t) &= 0\end{aligned}$$

The solution to the above is

$$\hat{u}(k,t) = Ce^{-k^2 t} \tag{1}$$

At $t = 0$,

$$\hat{u}(k,0) = \mathcal{F}(h(x))$$

Therefore

$$C = \mathcal{F}(h(x))$$

And (1) becomes

$$\hat{u}(k,t) = \mathcal{F}(h(x))\,e^{-k^2 t}$$

To find Green function, we replace $h(x)$ by $\delta(x-\xi)$ where ξ is the location of the pulse. But $\mathcal{F}(\delta(x-\xi)\,;k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta(x-\xi)\,e^{-ikx}dx = \frac{1}{\sqrt{2\pi}}e^{-i\xi k}$. Therefore the above becomes

$$\hat{G}(k,t) = \frac{1}{\sqrt{2\pi}}e^{-i\xi k}e^{-k^2 t}$$

The above is the Fourier transform of the Green function. Now we invert it

$$\begin{aligned} G(x,t) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}e^{-i\xi k}e^{-k^2t}\right)e^{ikx}dk \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi k-k^2t+ikx}dk \end{aligned} \tag{2}$$

We would like to use Gaussian as the integrand, hence we want to change $-i\xi k-k^2t+ikx$ to $-\left(k\sqrt{t}-A\right)^2$. We do this by completing the square.

$$\begin{aligned} -i\xi k-k^2t+ikx &= -\left(k\sqrt{t}-A\right)^2 \\ &= -\left(k^2t+A^2-2Ak\sqrt{t}\right) \\ &= -k^2t-A^2+2Ak\sqrt{t} \end{aligned}$$

Comparing sides then $2Ak\sqrt{t}=k(-i\xi+ix)$ or $A=\frac{-i\xi+ix}{2\sqrt{t}}$. Therefore

$$\begin{aligned} -i\xi k-k^2t+ikx &= -\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2+A^2 \\ &= -\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2+\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2 \end{aligned}$$

Hence

$$\begin{aligned} e^{-i\xi k-k^2t+ikx} &= e^{-\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2+\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \\ &= e^{-\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \end{aligned}$$

Substituting the above into (2) gives

$$\begin{aligned} G(x,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}dk \\ &= \frac{1}{2\pi}e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}\int_{-\infty}^{\infty}e^{-\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}dk \end{aligned}$$

To evaluate $\int_{-\infty}^{\infty}e^{-\left(k\sqrt{t}-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}dk$, let $u=k\sqrt{t}$, then $du=\sqrt{t}dk$. The above becomes

$$\begin{aligned} G(x,t) &= \frac{1}{2\pi}e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}\int_{-\infty}^{\infty}e^{-\left(u-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}\frac{du}{\sqrt{t}} \\ &= \frac{1}{2\pi\sqrt{t}}e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}\int_{-\infty}^{\infty}e^{-\left(u-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}du \end{aligned}$$

Now the integral is Gaussian. $\int_{-\infty}^{\infty} e^{-\left(u-\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} du = \sqrt{\pi}$ and the above becomes

$$\begin{aligned} G(x,t) &= \frac{\sqrt{\pi}}{2\pi\sqrt{t}} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \\ &= \frac{1}{2\sqrt{\pi t}} e^{\left(i\left(\frac{-\xi+x}{2\sqrt{t}}\right)\right)^2} \\ &= \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}} \end{aligned}$$

Now that we found the Green function for the PDE, we can find the solution as

$$\begin{aligned} u(x,t) &= \int_{-\infty}^{\infty} G(\xi,t)\, h(\xi)\, d\xi \\ &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}} h(\xi)\, d\xi \\ &= \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi \end{aligned}$$

But $\int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi = \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}}$, hence the above becomes

$$\begin{aligned} u(x,t) &= \frac{1}{\sqrt{4\pi t}} \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}} \\ &= \frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}} \end{aligned}$$

3.1.7.2 [248] Advection term

problem number 248

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = 12u_{xx} + u_x \sin t$$

With initial condition

$$u(x,0) = x$$

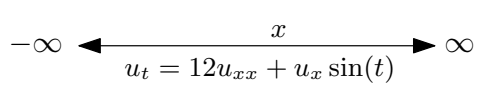

Figure 3.151: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == 12*D[u[x, t], {x, 2}] + Sin[t]*D[u[x, t], x];
ic  = u[x, 0] == x;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to t\sin(t) + x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= 12* diff(u(x,t),x$2)+sin(t)*diff(u(x,t),x);
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t))),output=
```

$$u(x,t) = x - \cos(t) + 1$$

3.1.7.3 [249] UnitBox I.C.

problem number 249

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = u_{xx}$$

With initial condition

$$u(x,0) = \text{UnitBox}[x]$$

Where UnitBox is equal to 1 if $|x| \leq \frac{1}{2}$ and zero otherwise.

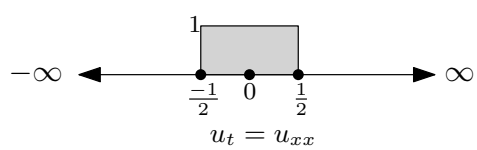

Figure 3.152: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
ic  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{\int_{-\infty}^{\infty} e^{K[1](ix-tK[1])}\text{sinc}\left(\frac{K[1]}{2}\right)dK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic  := u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t)=\frac{\text{erf}\left(\frac{2x+1}{4\sqrt{t}}\right)}{2}-\frac{\text{erf}\left(\frac{2x-1}{4\sqrt{t}}\right)}{2}$$

3.1.7.4 [250] No source

problem number 250

Solve the heat equation

$$u_t = ku_{xx}$$

For $-\infty < x < \infty$ and $t > 0$, and initial condition is $u(x,0) = f(x)$

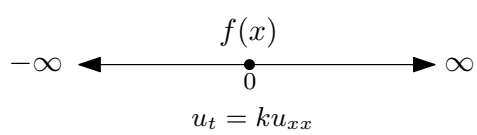

Figure 3.153: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic  = u[x, 0] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
 sol[[2]] = sol[[2]] /. K[1] -> s;
```

$$\left\{\left\{u(x,t)\to\int_{-\infty}^{\infty}\frac{f(s)e^{-\frac{(s-x)^2}{4kt}}}{2\sqrt{\pi}\sqrt{kt}}\,ds\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  := u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t)=\frac{\int_{-\infty}^{\infty}\frac{2\pi^{\frac{3}{2}}\mathrm{e}^{-\frac{(x+\zeta)^2}{4kt}}f(-\zeta)}{\sqrt{k}\sqrt{t}}d\zeta}{4\pi^2}$$

3.1.7.5 [251] constant as source

problem number 251

Solve the heat equation

$$u_t = ku_{xx} + m$$

For $-\infty < x < \infty$ and $t > 0$. Initial condition is $u(x,0) = \sin(x)$

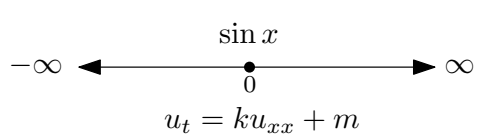

Figure 3.154: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == k*D[u[x, t], {x, 2}] + m;
ic  = u[x, 0] == Sin[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to e^{-kt}\sin(x) + mt\}\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic  := u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = mt + \mathrm{e}^{-kt}\sin(x)$$

3.1.7.6 [252] No intial conditions

problem number 252

Solve the heat equation for $u(x,t)$

$$u_t = u_x x$$

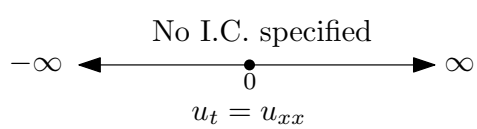

Figure 3.155: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] == D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to \cosh(c_2(x+c_2t)+c_1)+\sinh(c_2(x+c_2t)+c_1)+1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t) = c_3\left(c_1 \mathrm{e}^{2x\sqrt{\_c_1}} + c_2\right) \mathrm{e}^{t\_c_1} \mathrm{e}^{-x\sqrt{\_c_1}}$$

<u>Hand solution</u>

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $t > 0, -\infty < x < \infty$. Let $u = X(x)\,T(t)$ then we obtain

$$T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

(Only positive eigenvalues are possible). The two ODE's are

$$T' + \lambda T = 0 \tag{1}$$
$$X'' + \lambda X = 0 \tag{2}$$

Solution for (2) is $X(x) = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}$ and solution for (1) is $T(t) = C_3 e^{-\lambda t}$. Hence

$$\begin{aligned} u(x,t) &= C_3 e^{-\lambda t}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) \\ &= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + C_3 e^{-\lambda t} C_2 e^{-\sqrt{\lambda}x} \\ &= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + \frac{C_3 e^{-\lambda t} C_2}{e^{\sqrt{\lambda}x}} \end{aligned}$$

3.1.7.7 [253] piecewise I.C.

problem number 253

Added December 20, 2018.

From `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t = \mu u_{xx} - 1$$

With initial condition

$$u(x,1) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$$

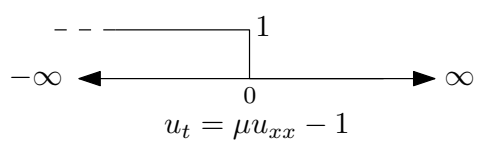

Figure 3.156: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 1 == mu*D[u[x, t], {x, 2}];
ic  = u[x, 1] == Piecewise[{{1, x <= 0}, {0, x > 0}}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], x, t, Assumptions -> mu > 0
```

$$\left\{\left\{u(x,t) \to \frac{\int_{-\infty}^{\infty} \frac{ie^{K[1](ix-\mu(t-1)K[1])}}{K[1]} dK[1]}{2\pi}\right\}\right\}$$

due to i.c. not at zero

Maple ✓

```
restart;
pde := diff(u(x, t), t)+1 = mu* diff(u(x, t), x$2);
ic  := u(x, 1) = piecewise(0 <= x, 0, x < 0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t) = -t - \frac{\operatorname{erf}\left(\frac{x}{2\sqrt{t-1}\sqrt{\mu}}\right)}{2} + \frac{3}{2}$$

<u>Hand solution</u>

Solve

$$u_t = \mu u_{xx} - 1$$

for $t > 0, -\infty < x < \infty$ with initial conditions $u(x,0) = f(x) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$

Let $v = u + t$. Hence $u = v - t$ and $u_t = v_t - 1$ and $u_x = v_x$ and $u_{xx} = v_{xx}$. The above PDE becomes

$$\begin{aligned} v_t - 1 &= \mu v_{xx} - 1 \\ v_t &= \mu v_{xx} \end{aligned} \tag{1}$$

Initial conditions do not change. They are $v(x,0) = u(x,0) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$. Using Green function for 1D heat PDE on the real line, (also called heat Kernel)

$$G(x,t) = \frac{1}{\sqrt{4\pi\mu t}} e^{\frac{-x^2}{4\mu t}}$$

Then the solution to (1) is

$$\begin{aligned} v(x,t) &= \int_{-\infty}^{\infty} f(x')\, G(x-x',t)\, dx' \\ &= \int_{-\infty}^{0} \frac{1}{\sqrt{4\pi\mu t}} e^{\frac{-\left(x-x'\right)^2}{4\mu t}} dx' \\ v(x,t) &= \frac{-1}{\sqrt{4\pi\mu t}} \int_{0}^{\infty} e^{\frac{-\left(x-x'\right)^2}{4\mu t}} dx' \end{aligned}$$

But $\int_0^\infty e^{\frac{-\left(x-x'\right)^2}{4\mu t}} dx' = \sqrt{\pi\mu t}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$, hence

$$v(x,t) = \frac{-1}{2}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$$

Since $u = v - t$ then

$$\begin{aligned} u(x,t) &= \frac{-1}{2}\left(1 + \operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right) - t \\ &= -\frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right) - t \end{aligned}$$

3.1.7.8 [254] Practice exam problem

problem number 254

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 14.

Solve for $u(x,t)$ with IC $u(x,0) = x$ for $-\infty < x < \infty, t > 0$

$$u_t = u_{xx}$$

$$-\infty \xleftarrow[u_t = u_{xx}]{u(x,0) = x} \infty$$

Figure 3.157: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], t] == D[u[x,t],{x,2}];
ic  = u[x,0]==x;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to x\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t)= diff(u(x,t),x$2);
ic  := u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = x$$

3.2 Diffusion in 2D

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3.2.2 Polar coordinates (disk, sector, annulus) 697

3.2.1 Cartesian coordinates (Rectangle, Square)

Local contents

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3.2.1.1 [255] No source

problem number 255

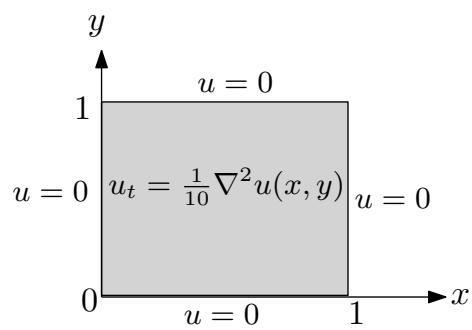
Taken from Maple help pages on PDE. Solve the heat equation for $u(x, y, t)$

$$u_t = \frac{1}{10}\nabla^2 u(x, y)$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0, y, t) &= 0 \\ u(1, y, t) &= 0 \\ u(x, 0, t) &= 0 \\ u(x, 1, t) &= 0 \end{aligned}$$

Initial condition is $u(x, y, 0) = x(1 - x)(1 - y)y$.



At $t = 0, u = x(1-x)(1-y)y$

Figure 3.158: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10;
ic  = u[x, y, 0] == x*(1 - x)*(1 - y)*y;
bc  = {u[0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, u[x, 1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{16\left(-1+(-1)^{K[1]}\right)\left(-1+(-1)^{K[3]}\right)e^{\frac{1}{10}t\left(-\pi^2K[1]^2-\pi^2K[3]^2\right)}\sin(\pi xK[1])\sin(\pi yK[3])}{\pi^6K[1]^3K[3]^3}\quad(K[1]|K[\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc  := u(0, y, t) = 0, u(1, y, t) = 0, u(x, 0, t) = 0, u(x, 1, t) = 0;
ic  := u(x, y, 0) = x*(1-x)*(1-y)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,y,t))),out
```

$$u(x,y,t)=\sum_{n1=1}^{\infty}\sum_{n=1}^{\infty}-\frac{16\left((-1)^n+(-1)^{n1}-(-1)^{n+n1}-1\right)\mathrm{e}^{-\frac{\pi^2\left(n^2+n1^2\right)t}{10}}\sin(\pi nx)\sin(\pi n1y)}{\pi^6n^3{n1}^3}$$

3.2.1.2 [256] Internal source term

problem number 256

Taken from Maple help pages on PDE

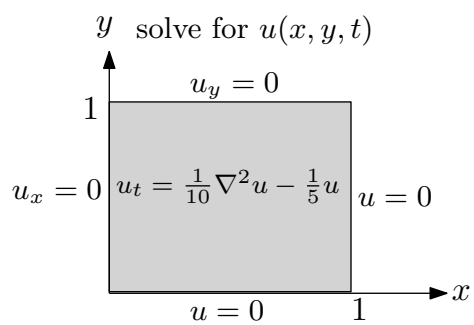
Solve the heat equation for $u(x, y, t)$

$$\frac{\partial u}{\partial t} = 1/10\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{5}u(x, y, t);$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned}
\frac{\partial u}{\partial x}u(0, y, t) &= 0 \\
u(1, y, t) &= 0 \\
u(x, 0, t) &= 0 \\
\frac{\partial u}{\partial y}u(x, 1, t) &= 0
\end{aligned}$$

Initial condition is $u(x, y, 0) = (1 - x^2)(1 - \frac{1}{2}y)y$.



At $t = 0, u = (1 - x^2)(1 - \frac{1}{2}y)y$

Figure 3.159: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10 - (1*u[x,
ic  = u[x, y, 0] == (-x^2 + 1)*(1 - (1/2)*y)*y;
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \{\quad \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}-\frac{512(-1)^{K[1]}\exp\left(t\left(\frac{1}{10}\left(-\frac{1}{4}\pi^2(2K[1]-1)^2-\frac{1}{4}\pi^2(2K[3]-1)^2\right)-\frac{1}{5}\right)\right)\cos\left(\frac{1}{2}\pi x(2K[1]-1)\right)\sin\left(\frac{1}{2}\pi y(\right.}{\pi^6(2K[1]-1)^3(2K[3]-1)^3}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2)) - 1/5 * u(x
ic  := u(x, y, 0) = (-x^2+1)*(1-(1/2)*y)*y;
bc  := (D[1](u))(0, y, t) = 0,
       u(1, y, t) = 0,
       u(x, 0, t) = 0,
       (D[2](u))(x, 1, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
```

$$u(x,y,t)=\sum_{n1=0}^{\infty}\sum_{n=0}^{\infty}\frac{512(-1)^n\cos\left(\frac{(2n+1)\pi x}{2}\right)e^{-\frac{\left(\pi^2n^2+\pi^2n1^2+\pi^2n+\pi^2n1+2+\frac{1}{2}\pi^2\right)t}{10}}\sin\left(\frac{(2n1+1)\pi y}{2}\right)}{\pi^6(2n+1)^3(2n1+1)^3}$$

3.2.1.3 [257] Articolo 6.6.3

problem number 257

Added December 20, 2018.

Example 6.6.3 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin rectangular plate over the finite two-dimensional domain $D=(x,y)$s.t.$0<x<1,0<y<1$. The lateral surfaces of the

plate are insulated. The boundaries $y = 0$ and $y = 1$ are fixed at temperature 0, the boundary $x = 0$ is insulated, and the boundary $x = 1$ is losing heat by convection into a surrounding medium at temperature 0. The initial temperature distribution f(x, y) is

$$u(x, y, 0) = \left(1 - \frac{x^2}{3}\right) y(1 - y)$$

The thermal diffusivity is $k = \frac{1}{50}$. Solve for $u(x, y, t)$ the heat PDE

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With $0 < x < 1, 0 < y < 1$ and $t > 0$. Boundary conditions are

$$\begin{aligned} \frac{\partial u}{\partial x}(0, y, t) &= 0 \\ \frac{\partial u}{\partial x}(1, y, t) + u(1, y, t) &= 0 \\ u(x, 0, t) &= 0 \\ u(x, 1, t) &= 0 \end{aligned}$$

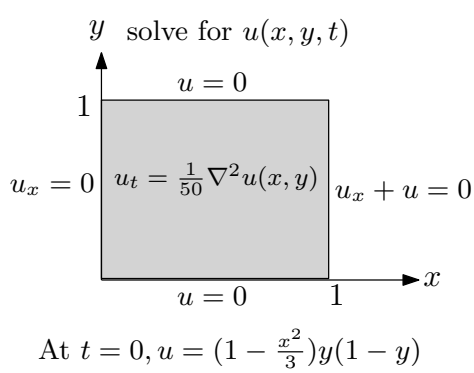


Figure 3.160: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/50;
pde =  D[u[x, y, t], t] == k*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]);
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] + u[1, y, t] ==
ic  = u[x, y, 0] == (1 - (1/3)*x^2)*y*(1 - y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{8\left(-1+(-1)^{K[3]}\right)e^{\frac{1}{50}t\left(-\pi^2K[3]^2-K[2,K[1]]\right)}\cos\left(x\sqrt{K[2,K[1]]}\right)\sin(\pi yK[3])\left(\frac{2\cos\left(\sqrt{K[2,K[1]]}\right)}{K[2,K[1]]}-\frac{2(}{}\right.}{3\pi^3K[3]^3\left(\sin^2\left(\sqrt{K[2,K[1]]}\right)+1\right)}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
k:=1/50;
pde := diff(u(x, y, t), t) = k*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc_left_edge:=eval( diff(u(x,y,t),x),x=0)=0;
bc_right_edge:= eval( diff(u(x,y,t),x),x=1)+u(1,y,t)=0;
bc_bottom_edge:=u(x,0,t)=0;
bc_top_edge:=u(x,1,t)=0;
bc:=bc_left_edge,bc_right_edge,bc_bottom_edge,bc_top_edge;
ic  := u(x, y, 0) = (1-(1/3)*x^2)*y*(1-y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x, y, t))
```

$$u(x,y,t)=\sum_{n1=1}^{\infty}\sum_{n=0}^{\infty}-\frac{32\left((-1)^{n1}-1\right)\left(\lambda_n^2\sin(\lambda_n)-\lambda_n\cos(\lambda_n)+\sin(\lambda_n)\right)\cos(x\lambda_n)\,\mathrm{e}^{-\frac{\left(\pi^2{n1}^2+\lambda_n^2\right)t}{50}}\sin(\pi n1}{3\left(2\lambda_n+\sin(2\lambda_n)\right)\pi^3{n1}^3\lambda_n^2}$$

3.2.2 Polar coordinates (disk, sector, annulus)

Local contents

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3.2.2.1 [258] no θ dependency, insulated (General solution)

problem number 258

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr} + \frac{1}{r}u_r)$$

For $0 < r < L$ and $t > 0$. The boundary conditions are such it is insulated

$$u_r(L,t) = 0$$

Initial condition is $u(r,0) = f(r)$.

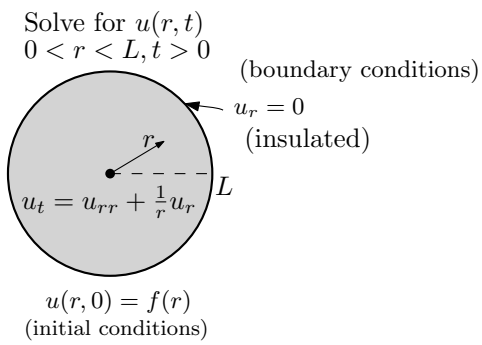

Figure 3.161: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == f[r];
bc  = Derivative[1,0][u][L, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions->{r
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2} e^{-\frac{kt\left(j_{1,K[1]}\right)^2}{L^2}} J_0\left(\frac{r j_{1,K[1]}}{L}\right) \int_0^L \frac{\sqrt{2} r J_0\left(\frac{r j_{1,K[1]}}{L}\right) f(r)}{L J_0\left(j_{1,K[1]}\right)} dr}{L J_0\left(j_{1,K[1]}\right)} + \frac{\sqrt{2} \int_0^L \frac{\sqrt{2} r f(r)}{L} dr}{L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)=f(r);
bc  := D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t) = \sqrt{k} \left(\int_{-L+r}^{0} \mathcal{L}^{-1} \left(\frac{\text{BesselJ}\left(0, \frac{\sqrt{-s}\,(r-\tau)}{\sqrt{k}}\right) \mathcal{L}\left(\frac{-D(f)(L+\tau)+(-L-\tau)D^{(2)}(f)(L+\tau)}{L+\tau}, t, s\right)}{\sqrt{-s}\ \text{BesselJ}\left(1, \frac{\sqrt{-s}\,L}{\sqrt{k}}\right)}, s, t \right) d\tau \right) - \sqrt{k}\,\mathcal{L}$$

Cant get series solution

Hand solution

Solve for $u(r,t)$

$$\begin{aligned} u_t &= k\left(u_{rr} + \frac{1}{r}u_r\right) \\ |u(0,t)| &< \infty \\ u_r(L,t) &= 0 \\ u(r,0) &= f(r) \end{aligned}$$

for $0 < r < L$. Let $u = RT$. Substituting in the PDE gives

$$\begin{aligned} \frac{T'R}{k} &= R''T + \frac{1}{r}R'T \\ \frac{T'}{kT} &= \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\lambda \end{aligned}$$

The above gives the following ODE's to solve

$$T' + \lambda kT = 0$$

And

$$\begin{aligned} R'' + \frac{1}{r}R' + \lambda R &= 0 \qquad (1) \\ R'(L) &= 0 \\ R(0) &< \infty \end{aligned}$$

Case $\lambda < 0$

Let $\lambda = -n^2$ for positive n. (1) becomes

$$\begin{aligned} R'' + \frac{1}{r}R' - n^2R &= 0 \\ r^2R'' + rR' - n^2r^2R &= 0 \end{aligned}$$

This is a Bessel ODE whose solution is $R(r) = C_1 \operatorname{BesselI}(0,nr) + C_2 \operatorname{Besselk}(0,nr)$. Since $R(r)$ is bounded at $r = 0$ then $C_2 = 0$ since Besselk blows up at $r = 0$. The solution becomes $R(r) = C_1 \operatorname{BesselI}(0,nr)$. Then $R'(r) = C_1 \operatorname{BesselI}(1,nr)$. Hence we need to solve for n in the following $R'(L) = 0 = \operatorname{BesselI}(1,nL)$. But BesselI has zero only at zero. Therefore $\lambda < 0$ is not possible eigenvalue.

Case $\lambda = 0$

Equation (1) becomes $R'' + \frac{1}{r}R' = 0$. The solution is $R(r) = C_1 \ln(r) + C_2$. Since bounded at $r = 0$ then $C_1 = 0$. The solution becomes $R(r) = C_2$ and $R'(r) = 0$. Which

satisfies the boundary condition for any constant C_2. Therefore $\underline{\lambda = 0 \text{ is an eigenvalue}}$ with eigenfunction $R_0(r) = 1$ as the choice of the constant.

<u>Case $\lambda > 0$</u>

Equation (1) becomes

$$r^2 R'' + rR' + \lambda r^2 R = 0$$

This is a Bessel ODE whose solution is $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$. Since $R(r)$ is bounded at $r = 0$ then $C_2 = 0$ since BesselY blows up at $r = 0$. The solution becomes $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$. Then $R'(r) = -C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} r\right)$. Absorbing the minus sign into the constant, hence $R'(r) = C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} r\right)$. At $r = L$ we want

$$0 = C_1 \operatorname{BesselJ}\left(1, \sqrt{\lambda} L\right)$$

For non-trivial solution, $\sqrt{\lambda} L$ are the $\underline{\text{zeros of the BesselJ}\,(1, x)}$. Let these zeros be Λ_n where $n = 1, 2, \cdots$. Hence

$$\begin{aligned} \sqrt{\lambda_n} L &= \Lambda_n \qquad n = 1, 2, \cdots \\ \lambda_n &= \left(\frac{\Lambda_n}{L}\right)^2 \end{aligned}$$

The corresponding eigenfunctions are

$$R_n(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right) \qquad n = 1, 2, \cdots$$

Now that we found the eigenvalues and eigenfunctions, we can solve the time ODE. For the zero eigenvalue $\lambda = 0$ the ODE $T' + \lambda k T = 0$ becomes $T' = 0$, hence $T_0(t)$ is a constant. For $\lambda > 0$ the time ODE has solution $T_n(t) = e^{-k\lambda_n t} = e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t}$. Therefore the complete solution is

$$u(r, t) = C_0 + \sum_{n=1}^{\infty} C_n e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t} \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right)$$

Now we find the constants from the initial conditions. At $t = 0$

$$f(r) = C_0 + \sum_{n=1}^{\infty} C_n \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{L} r\right)$$

For $n=0$, applying orthogonality gives

$$\begin{aligned}\int_0^L f(r)\,rdr &= \int_0^L C_0 rdr\\ &= C_0\frac{L^2}{2}\\ C_0 &= \frac{2}{L^2}\int_0^L f(r)\,rdr\end{aligned}$$

For $n>0$

$$\begin{aligned}\int_0^L f(r)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)rdr &= \int_0^L C_n\,\text{BesselJ}^2\left(0,\frac{\Lambda_n}{L}r\right)rdr\\ C_n &= \frac{\int_0^L f(r)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)rdr}{\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_n}{L}r\right)rdr}\end{aligned}$$

Hence the final solution is

$$u(r,t)=\frac{2}{L^2}\int_0^L f(r)\,rdr+\sum_{n=1}^{\infty}\left(\frac{\int_0^L f(r)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)rdr}{\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_n}{L}r\right)rdr}\right)e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t}\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)$$

Where Λ_n are the zeros of the $\text{BesselJ}(1,x)$.

3.2.2.2 [259] no θ dependency, insulated (Specific solution)

problem number 259

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr}+\frac{1}{r}u_r)$$

For $0<r<L$ and $t>0$ and $k=1, L=1$. The boundary conditions are such it is insulated

$$u_r(L,t)=0$$

Initial condition is $u(r,0)=2Lr-r^2$.

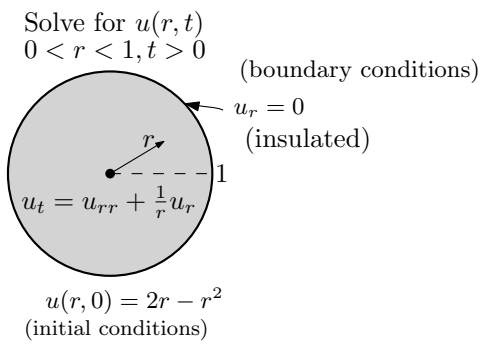

Figure 3.162: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
 L=1;
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == 2*L*r-r^2;
bc  = Derivative[1,0][u][L, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} -\frac{2e^{-t\left(j_{1,K[1]}\right)^2} J_0\left(r j_{1,K[1]}\right)\left(\frac{2J_2\left(j_{1,K[1]}\right)}{\left(j_{1,K[1]}\right)^2} - \frac{2}{3}\,{}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}\left(j_{1,K[1]}\right)^2\right) - \frac{J_3\left(j_{1,K[1]}\right)}{j_{1,K[1]}}\right)}{J_0\left(j_{1,K[1]}\right)^2} + \frac{5}{6}\right.\right.$$

Maple ✓

```
restart;
k:=1;
L:=1;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)= 2*L*r-r^2;
bc  := D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t))),outpu
```

$$u(r,t) = -r^2 + 2r - \left(\int_0^{r-1} \mathcal{L}^{-1}\left(\frac{\text{BesselJ}\left(0,\sqrt{-s}\,(r-\tau)\right)\mathcal{L}\left(\frac{4\tau+2}{\tau+1},t,s\right)}{\sqrt{-s}\,\text{BesselJ}\left(1,\sqrt{-s}\right)},s,t\right)d\tau\right)$$

Do not understand Maple solution
<u>Hand solution</u>

Solve for $u(r,t)$

$$\begin{aligned} u_t &= k\left(u_{rr} + \frac{1}{r}u_r\right) \\ |u(0,t)| &< \infty \\ u_r(1,t) &= 0 \\ u(r,0) &= 2Lr - r^2 \end{aligned}$$

for $0 < r < L$.

The basic solution for this type of PDE was already given in problem 3.2.2.1 on page 699 as

$$u(r,t) = \frac{2}{L^2}\int_0^L f(r)\,rdr + \sum_{n=1}^{\infty}\left(\frac{\int_0^L f(r)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)rdr}{\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_n}{L}r\right)rdr}\right)e^{-k\left(\frac{\Lambda_n}{L}\right)^2 t}\,\text{BesselJ}\left(0,\frac{\Lambda_n}{L}r\right)$$

Where Λ_n are the zeros of the $\text{BesselJ}(1,x)$. In this problem $L=1$ and $k=1$, $f(r) = 2Lr - r^2$. This is animation of the above solution using these specific values for for 0.2 seconds. (Animation will only show in the HTML version)

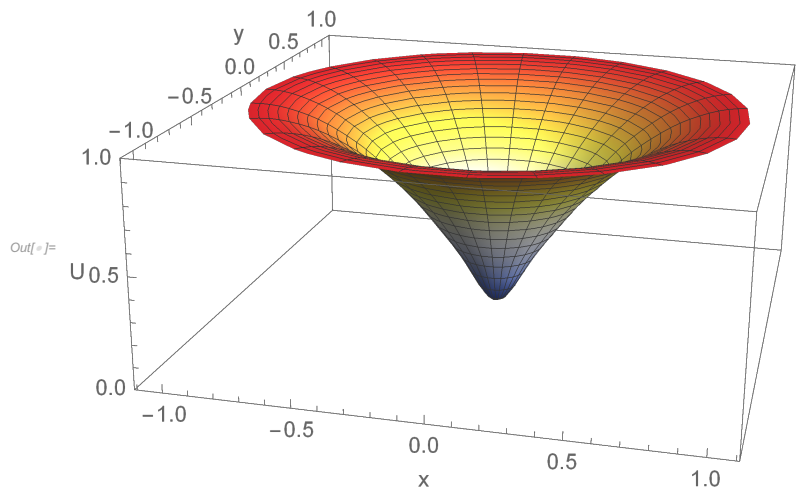

Figure 3.163: Initial state

Source code used for the above

```
In[ ]:= ClearAll[r, u, t, lam, mySol]
L = 1;
k = 1;
f = 2 L r - r^2;
maxTime = 30;
numberOfTerms = 10;
z = N@Table[ BesselJZero[1, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ1*)
An = Table[ Integrate[f BesselJ[0, z[[n]]/L r] r, {r, 0, L}] / Integrate[ BesselJ[0, z[[n]]/L r]^2 r, {r, 0, L}], {n, 1, numberOfTerms}];
mySol[r_, t_] = 2/L^2 Integrate[r f, {r, 0, L}] + Sum[An[[n]] Exp[- k (z[[n]]/L)^2 t] BesselJ[0, z[[n]]/L r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
(*display initial conditions*)
```

Figure 3.164: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]}},
              {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]}},
              {Row[{"Initial  temperature u = (2 L r-r^2)"}]}},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, L}, {θ, 0, 2 Pi},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  Mesh → 25,
                  PerformanceGoal → "Quality",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {0, (f /. r → L)}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, (f /. r → L)}}],
                  AxesLabel → {"x", "y", "U"},
                  ViewPoint → {0.623, -2.678, 0.896},
                  Boxed → True, Axes → True
                ]
              }}],
          {t, 0, 0.18, .001}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.05, {Length[tab]}]]
```

Figure 3.165: Code used for animation

3.2.2.3 [260] no θ dependency

problem number 260

Added June 1, 2019

Solve the heat equation in polar coordinates for $u(r,t)$

$$u_t = k(u_{rr} + \frac{1}{r}u_r)$$

For $0 < r < L$ and $t > 0$. The boundary conditions are Newton Law of cooling, where $h > 0$

$$Lu(L,t) = -hu(L,t)$$

Initial condition is $u(r,0) = f(r)$.

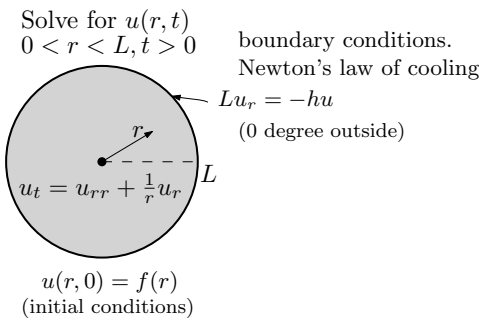

Figure 3.166: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = u[r, 0] == f[r];
bc  = L*Derivative[1,0][u][L, t] == -h*u[L,t];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions->{r
```

$$\left\{\left\{u(r,t) \to \{ \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}e^{-ktK[2,K[1]]^2} J_0(rK[2,K[1]]) \left(\int_0^L \frac{\sqrt{2}rJ_0(rK[2,K[1]])f(r)K[2,K[1]]}{J_0(LK[2,K[1]])\sqrt{h^2+L^2K[2,K[1]]^2}}\,dr\right)K[2,K[1]]}{LJ_0(LK[2,K[1]])\sqrt{\frac{h^2}{L^2}+K[2,K[1]]^2}} \quad hJ_0(LK[2,K[1]]\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r));
ic  := u(r,0)=f(r);
bc  := L*D[1](u)(L,t)=-h*u(L,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t) = hf(L)\ln(L) - hf(L)\ln(r) - 2\left(\sum_{n=0}^{\infty} \frac{\left(-\cosh\left(\frac{kt\lambda_n^2}{L^2}\right) + \sinh\left(\frac{kt\lambda_n^2}{L^2}\right)\right)\text{BesselJ}\left(0, \frac{r\lambda_n}{L}\right)\left(\int_0^L (-hf(L)\right.}{(\text{BesselJ}(0,\lambda_n)^2 + \text{B}}\right.$$

3.2.2.4 [261] no θ dependency

problem number 261

Taken from Mathematica DSolve help pages

Solve the heat equation in polar coordinates for $u(r,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

For $0 < r < 1$ and $t > 0$. The boundary conditions are

$$u(1,t) = 0$$

Initial condition is $u(r,0) = 1 - r$.

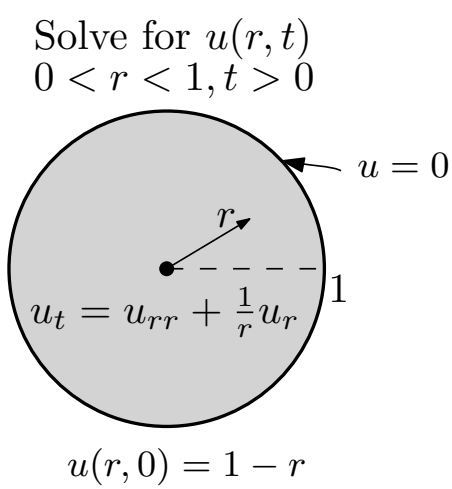

Figure 3.167: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r;
ic  = u[r, 0] == 1 - r;
bc  = u[1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{e^{-t(j_{0,n})^2}\pi J_0(r j_{0,n})\left(J_1(j_{0,n})\boldsymbol{H}_0(j_{0,n}) - J_0(j_{0,n})\boldsymbol{H}_1(j_{0,n})\right)}{J_1(j_{0,n})^2 (j_{0,n})^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r);
ic  := u(r,0)=1-r;
bc  := u(1,t) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t) = \sum_{n=1}^{\infty} \left(-\frac{\pi(\text{BesselJ}(0,\lambda_n)\,\text{StruveH}(1,\lambda_n) - \text{BesselJ}(1,\lambda_n)\,\text{StruveH}(0,\lambda_n))\,(\cosh(t\lambda_n^2) - \sinh(t\lambda}{(\text{BesselJ}(0,\lambda_n)^2 + \text{BesselJ}(1,\lambda_n)^2)\,\lambda_n^2}\right.$$

3.2.2.5 [262] Haberman 8.3.5 (General solution)

problem number 262

Added Nov 24, 2018.

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$.

One of the problems here, is how to tell CAS the implicit condition when solving this which is that $u(0,t) < \infty$.

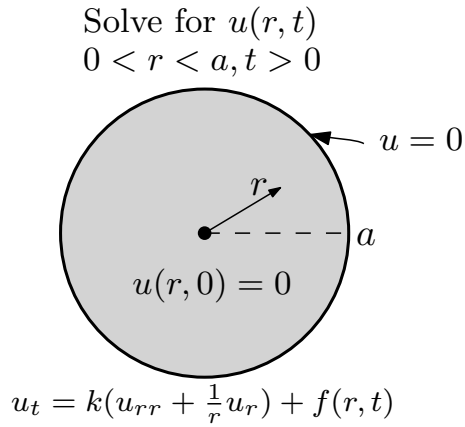

Figure 3.168: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f[r, t];
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \frac{\sqrt{2}J_0\left(\frac{rj_{0,K[1]}}{a}\right)\int_0^t e^{-\frac{k\left(j_{0,K[1]}\right)^2(t-K[2])}{a^2}}\text{Integrate}\left[\frac{\sqrt{2}rJ_0\left(\frac{rj_{0,K[1]}}{a}\right)f(r,K[2])}{aJ_1\left(j_{0,K[1]}\right)},\{r,0,a\},\text{Assumpti}\right.}{aJ_1\left(j_{0,K[1]}\right)}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f(r,t);
ic  := u(r,0)=0;
bc  := u(a,t) =0;
#do not use HINT=boundedseries below, Maple will  not solve it then
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t) = -\left(\int_{-a+r}^{0} \mathcal{L}^{-1}\left(\frac{\text{BesselJ}\left(0, \sqrt{-\frac{s}{k}}\,(r-\tau)\right)\mathcal{L}\left(-\frac{f(a+\tau,t)}{k}, t, s\right)}{\text{BesselJ}\left(0, \sqrt{-\frac{s}{k}}\,a\right)}, s, t\right) d\tau\right)$$

<u>Hand solution</u>

Since this problem has homogeneous B.C. but has time dependent source (i.e. non-homogenous in the PDE itself), then we will use the method of eigenfunction expansion. In this method, we first find the eigenfunctions $\phi_n(x)$ of the associated homogenous PDE without the source being present. Then use these $\phi_n(x)$ to expand the source $f(x,t)$ as generalized Fourier series. We now switch to the associated homogenous PDE in order to find the eigenfunctions. $u \equiv u(r,t)$. There is no θ. Hence

$$\begin{aligned} \frac{\partial u(r,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \qquad (1)\\ u(a,t) &= 0\\ |u(0,t)| &< \infty\\ u(r,0) &= 0 \end{aligned}$$

We need to solve the above in order to find the eigenfunctions $\phi_n(r)$. Let $u = R(r)\,T(t)$. Substituting this back into (1) gives

$$T'R = k\left(R''T + \frac{1}{r}R'T\right)$$

Dividing by RT

$$\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\lambda$$

Where λ is the separation constant. The above gives

$$T' + k\lambda T = 0$$

And

$$rR'' + R' + \lambda r R = 0$$

This is a singular Sturm-Liouville ODE. Standard form is

$$(rR')' = -\lambda r R$$

Hence

$$\begin{aligned} p &= r \\ q &= 0 \\ \sigma &= r \end{aligned}$$

The ODE $rR'' + R' + \lambda r R = 0$ is Bessel ODE whose solution is

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$$

Since $\operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$ blows up at $r = 0$, then $C_2 = 0$ and the solution becomes

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$$

At $r = a$ the above becomes $0 = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} a\right)$. Non trivial solution requires that $\sqrt{\lambda} a$ are the zeros of $\operatorname{BesselJ}(0, x)$. Let the zeros be called $\Lambda_n, n = 1, 2, 3, \cdots$. Therefore $\sqrt{\lambda_n} a = \Lambda_n$ or

$$\lambda_n = \left(\frac{\Lambda_n}{a}\right)^2 \qquad n = 1, 2, 3, \cdots$$

The corresponding eigenfunctions are $R_n(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$. Now that the eigenfunctions for the homogeneous PDE are found, eigenfunction expansion is used to find the general solution. Let

$$u(r, t) = \sum_{n=1}^{\infty} a_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \tag{2}$$

Where $a_n(t)$ is function of time since it includes the time solution in it. Substituting the above back into the original nonhomogeneous PDE

$$\begin{aligned} u_t &= k \nabla^2 u + f(r, t) \\ &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + f(r, t) \end{aligned} \tag{3}$$

Where $\nabla^2 u = -\lambda r u$. Substituting (2) into (3), and using $f(r, t) = \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$ gives

$$\sum_{n=1}^{\infty} a_n'(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) = ka_n(t)\left(\sum_{n=1}^{\infty}\text{BesselJ}''\left(0,\frac{\Lambda_n}{a}r\right)+\frac{1}{r}\,\text{BesselJ}'\left(0,\frac{\Lambda_n}{a}r\right)\right)+\sum_{n=1}^{\infty} b_n(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}\right.$$

But $\sum_{n=1}^{\infty}\text{BesselJ}''\left(0,\frac{\Lambda_n}{a}r\right)+\frac{1}{r}\,\text{BesselJ}'\left(0,\frac{\Lambda_n}{a}r\right) = -\lambda_n\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$. The above becomes

$$\sum_{n=1}^{\infty} a_n'(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) = -ka_n(t)\sum_{n=1}^{\infty}\left(\frac{\Lambda_n}{a}\right)^2\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)+\sum_{n=1}^{\infty} b_n(t)\,\text{BesselJ}\left(0\right.$$

$$\sum_{n=1}^{\infty}\left(a_n'(t)+k\left(\frac{\Lambda_n}{a}\right)^2 a_n(t)\right)\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) = \sum_{n=1}^{\infty} b_n(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

The above simplifies to

$$a_n'(t)+k\left(\frac{\Lambda_n}{a}\right)^2 a_n(t) = b_n(t)$$

The solution is

$$a_n(t) = e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau + a_n(0)\,e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}$$

Hence the solution (2) becomes

$$u(r,t) = \sum_{n=1}^{\infty}\left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)+a_n(0)\,e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\right)\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

To find $a_n(0)$, putting $t=0$ in the above gives

$$0 = \sum_{n=1}^{\infty} a_n(0)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

Hence $a_n(0)=0$. Therefore $a_n(t)$ becomes.

$$a_n(t) = e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau$$

Hence the solution from (2) now becomes

$$u(r,t) = \sum_{n=1}^{\infty}\left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)$$

And finally, to find $b_n(t)$, which is the generalized Fourier coefficient of the expansion of the source in (3) above, orthogonality is used as follows

$$\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr = b_n(t)\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr$$

$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}$$

<u>Summary of solution</u>

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} a_n(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) \\ &= \sum_{n=1}^{\infty}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) \end{aligned}$$

Where

$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}$$

3.2.2.6 [263] Specific example of the above

problem number 263

Added June 16,2019

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition, but added specific values for parameters in order to do animations

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$. Let $k = \frac{1}{100}, a = 2$. Let $f(r,t) = sin(t)$

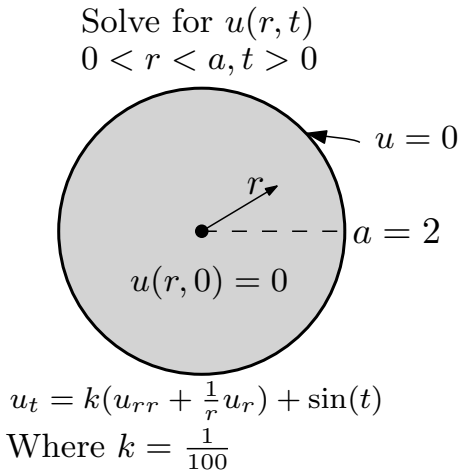

Figure 3.169: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=2;
 k=1/100;
 f=Sin[t];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f;
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \frac{800 J_0\left(\frac{1}{2} r j_{0,K[1]}\right)\left(\sin(t)\left(j_{0,K[1]}\right)^2 + 400\left(e^{-\frac{1}{400}t\left(j_{0,K[1]}\right)^2} - \cos(t)\right)\right)}{J_1\left(j_{0,K[1]}\right) j_{0,K[1]}\left(\left(j_{0,K[1]}\right)^4 + 160000\right)}\right\}\right\}$$

Maple ✓

```
restart;
a:=2;
k:=1/100;
f:=sin(t);
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f;
ic  := u(r,0)=0;
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

$$u(r,t) = -\cos(t) - \mathcal{L}^{-1}\left(\frac{\text{BesselJ}\left(0, 10\sqrt{-s}\, r\right)}{s\,\text{BesselJ}\left(0, 20\sqrt{-s}\right)}, s, t\right) + \mathcal{L}^{-1}\left(\frac{s\,\text{BesselJ}\left(0, 10\sqrt{-s}\, r\right)}{(s^2+1)\,\text{BesselJ}\left(0, 20\sqrt{-s}\right)}, s, t\right) + 1$$

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 3.2.2.5 on page 710 as

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} a_n(t)\,\text{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \\ &= \sum_{n=1}^{\infty} \left(\int_0^t b_n(\tau)\, e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau\right) e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\,\text{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \end{aligned}$$

Where

$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) r dr}{\int_0^a \text{BesselJ}^2\left(0, \frac{\Lambda_n}{a} r\right) r dr}$$

Where Λ_n are the nth zeros of $\text{BesselJ}(0, x)$. In this problem

$$\begin{aligned} a &= 2 \\ k &= \frac{1}{100} \\ f(r,t) &= \sin(t) \end{aligned}$$

This is animation of the above solution using these specific values for for 50 seconds. (Animation will only show in the HTML version)

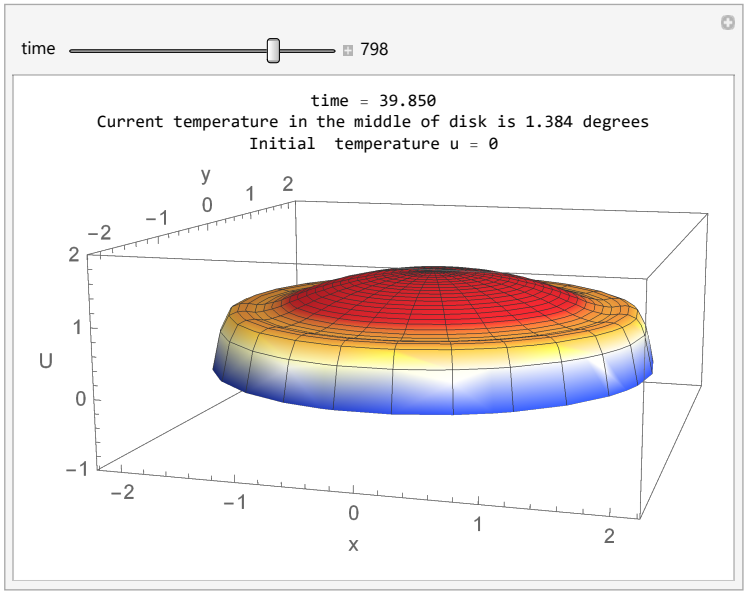

Figure 3.170: Initial state

Source code used for the above

```
In[ ]:= ClearAll[r, u, t, lam, mySol]
a = 2;
k = 1 / 100;
f = Sin[t];
maxTime = 30;
numberOfTerms = 20;
z = N@Table[ BesselJZero[0, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ0*)
bn = Table[ Integrate[f BesselJ[0, z[[n]]/a r] r, {r, 0, a}] / Integrate[ BesselJ[0, z[[n]]/a r]^2 r, {r, 0, a}], {n, 1, numberOfTerms}];
mySol[r_, t_] = Chop@Sum[Integrate[ (bn[[n]] /. t → s) Exp[ k (z[[n]]/a)^2 s], {s, 0, t}] Exp[- k (z[[n]]/a)^2 t] BesselJ[0, z[[n]]/a r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 3.171: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = 0"}]},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, a}, {θ, -Pi, Pi},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {-1, 2}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, 1}}],
                  AxesLabel → {"x", "y", "U"},
                  ViewPoint → {0.796 , -2.725 , 0.5471},
                  Boxed → True, Axes → True
                ]
              }}],
          {t, 0, 50, .05}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.05, {Length[tab]}]]
```

Figure 3.172: Code used for animation

3.2.2.7 [264] Specific example of the above

problem number 264

Added June 16,2019

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition, but added specific values for the parameters in order to do animations

Solve for $u(r,t)$

$$u_t = k(u_r r + 1/r u_r) + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$. Let $k = \frac{1}{100}, a = 2$. Let $f(r,t) = rte^{-t}$

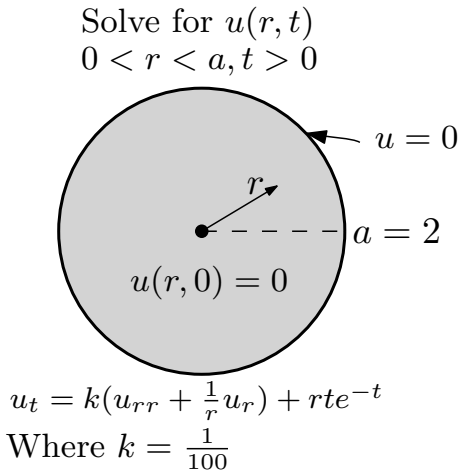

Figure 3.173: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=2;
 k=1/100;
 f=r*t*Exp[-t];
pde =  D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f;
ic  = u[r, 0] == 0;
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

$$\left\{\left\{u(r,t) \to \sum_{K[1]=1}^{\infty} \frac{1600e^{-\frac{1}{400}t\left(j_{0,K[1]}\right)^2} J_0\left(\frac{1}{2}rj_{0,K[1]}\right)\left(e^{\frac{1}{400}t\left(j_{0,K[1]}\right)^2-t}\left(t\left(j_{0,K[1]}\right)^2-400(t+1)\right)+400\right) {}_1F_2\left(\frac{3}{2}\right.}{3J_1\left(j_{0,K[1]}\right)^2\left(\left(j_{0,K[1]}\right)^2-400\right)^2}\right.$$

Maple ✗

```
restart;
a:=2;
k:=1/100;
f:=r*t*exp(-t);
pde := diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f;
ic  := u(r,0)=0;
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 3.2.2.5 on page 710 as

$$
\begin{aligned}
u(r,t) &= \sum_{n=1}^{\infty} a_n(t)\,\mathrm{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)\\
&= \sum_{n=1}^{\infty}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\,\mathrm{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)
\end{aligned}
$$

Where

$$
b_n(t) = \frac{\int_0^a f(r,t)\,\mathrm{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \mathrm{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}
$$

Where Λ_n are the nth zeros of $\mathrm{BesselJ}\,(0,x)$. In this problem

$$
\begin{aligned}
a &= 2\\
k &= \frac{1}{100}\\
f(r,t) &= rte^{-t}
\end{aligned}
$$

This is animation of the above solution using these specific values for for 100 seconds. (Animation will only show in the HTML version)

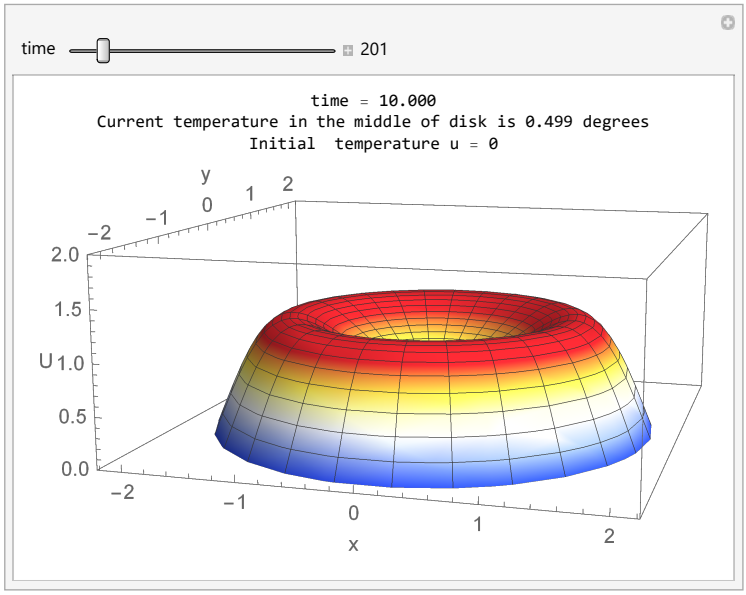

Figure 3.174: Initial state

Source code used for the above

```
In[•]:= ClearAll[r, u, t, lam, mySol]
a = 2;
k = 1 / 100;
f = r * t * Exp[-t];
maxTime = 30;
numberOfTerms = 10;
z = N@Table[ BesselJZero[0, n], {n, 1, numberOfTerms}]; (*zeros Of BesselJ0*)
bn = Table[ Integrate[f BesselJ[0, z[[n]]/a r] r, {r, 0, a}] / Integrate[ BesselJ[0, z[[n]]/a r]^2 r, {r, 0, a}], {n, 1, numberOfTerms}];
mySol[r_, t_] = Chop@Sum[Integrate[(bn[[n]] /. t → s) Exp[ k (z[[n]]/a)^2 s], {s, 0, t}] Exp[- k (z[[n]]/a)^2 t] BesselJ[0, z[[n]]/a r], {n, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 3.175: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[mySol[0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = 0"}]},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], Evaluate[mySol[r, t]]}, {r, 0, a}, {θ, -Pi, Pi},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {0, 2}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, 1}}],
                  AxesLabel → {"x", "y", "U"},
                  ViewPoint → {0.796 , -2.725 , 0.5471},
                  Boxed → True, Axes → True
                ]
              }}],
          {t, 0, 110, .05}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

        Export["anim.gif", tab, "DisplayDurations" → Table[.01, {Length[tab]}]]
```

Figure 3.176: Code used for animation

3.2.2.8 [265] Articolo 6.9.1

problem number 265

Added December 20, 2018.

Example 6.9.1 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution $u(r,\theta,t)$ in a thin circular plate over the two-dimensional domain $D = \{(r,\theta)|0 < r < 1, 0 < \theta < \frac{\pi}{2}\}$.

The lateral surfaces of the plate are insulated. The edges $r = 1$ and $\theta = 0$ are at a fixed temperature of 0, and the edge $\theta = \frac{\pi}{2}$ is insulated. The initial temperature distribution $u(r,\theta,0) = f(r,\theta)$ is $u(r,\theta,0) = (r - r^3)\sin(\theta)$.

The thermal diffusivity is $k = \frac{1}{50}$.

Solve for $u(r,\theta,t)$ the heat PDE

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$
\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u(1,\theta,t) &= 0 \\
u(r,0,t) &= 0 \\
\frac{\partial u}{\partial \theta}(1,\frac{\pi}{2},t) &= 0
\end{aligned}
$$

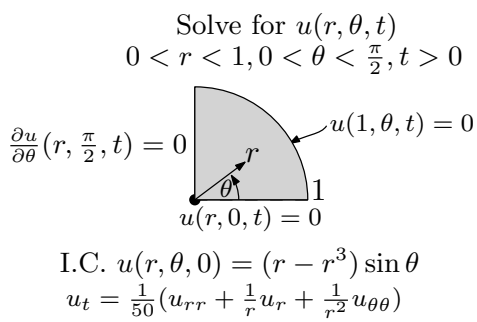

Figure 3.177: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/50;
pde =  D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + (1*D[
 bcOnR = u[1, theta, t] == 0;
 bcOnTheta = {u[r, 0, t] == 0, Derivative[0, 1, 0][u][r, Pi/2, t] == 0};
ic  = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r
```

$$
\left\{\left\{u(r,\theta,t) \to \begin{cases} \sum_{K[3]=1}^{\infty} \frac{4e^{-\frac{1}{50}t\left(j_{1,K[3]}\right)^2} J_1\left(rj_{1,K[3]}\right)\left(J_3\left(j_{1,K[3]}\right)+J_2\left(j_{1,K[3]}\right)j_{1,K[3]}\right)\sin(\theta)}{3J_0\left(j_{1,K[3]}\right)^2\left(j_{1,K[3]}\right)^2} & (K[1]|K[3]) \in \mathbb{Z} \wedge K[1] \geq \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right.
$$

Maple ✗

```
restart;
k:=1/50;
pde := diff(u(r, theta, t), t) = k*( diff(u(r, theta, t), r$2) + 1/r* diff(u(r, theta, t), r
bc_on_r:= u(1,theta,t)=0;
bc_on_theta:= u(r,0,t)=0, eval(diff(u(r,theta,t),theta),theta=Pi/2)=0;
ic  := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

sol=()

3.2.2.9 [266] Articolo 6.9.2

problem number 266

Added December 20, 2018.

Example 6.9.2 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin circular plate over the two-dimensional domain $D = \{(r,\theta)|0 < r < 1, 0 < \theta < \pi\}$. The lateral surfaces of the plate are insulated. The sides $\theta = 0$ and $\theta = \pi$ are at a fixed temperature of 0, and the edge $r = 1$ is insulated. The initial temperature distribution is $u(r,\theta,0) = \left(r - \frac{r^3}{3}\right)\sin\theta$.

The thermal diffusivity is $k = \frac{1}{25}$. Solve for $u(r,\theta,t)$ the heat PDE

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$\begin{aligned} |u(0,\theta,t) &< \infty \\ u(1,\theta,t) &= 0 \\ u(r,0,t) &= 0 \\ u(r,\pi,t)0 \end{aligned}$$

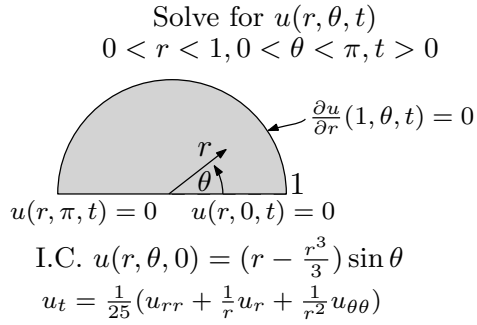

Figure 3.178: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k = 1/25;
pde =  D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + (1*D[
 bcOnR = Derivative[1, 0, 0][u][1, theta, t] == 0;
 bcOnTheta = {u[r, 0, t] == 0, u[r, Pi, t] == 0};
ic  = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r
```

$$\left\{\left\{u(r,\theta,t) \to \{ \sum_{K[3]=1}^{\infty} \frac{4e^{-\frac{1}{25}tK[2,1,K[3]]^2} J_1(rK[2,1,K[3]])(J_3(K[2,1,K[3]])+J_2(K[2,1,K[3]])K[2,1,K[3]])\sin(\theta)}{3K[2,1,K[3]]((J_0(K[2,1,K[3]])^2+J_1(K[2,1,K[3]])^2)K[2,1,K[3]]-2J_0(K[2,1,K[3]])J_1(K[2,1,K[3]]))} \quad (K[\right.\right.$$

Indeterminate

Maple ✓

```
restart;
k:=1/25;
pde := diff(u(r, theta, t), t) = k*( diff(u(r, theta, t), r$2) + 1/r* diff(u(r, theta, t), r
bc_on_r:= eval(diff(u(r,theta,t),r),r=1)=0;
bc_on_theta:= u(r,0,t)=0, u(r,Pi,t)=0;
ic  := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

$$u(r,\theta,t) = \sum_{n=0}^{\infty} -\frac{4(\lambda_n^3 \operatorname{BesselJ}(0,\lambda_n) - \lambda_n^2 \operatorname{BesselJ}(1,\lambda_n) + 4\lambda_n \operatorname{BesselJ}(0,\lambda_n) - 8 \operatorname{BesselJ}(1,\lambda_n)) \operatorname{Bessel}}{3\left(\lambda_n \operatorname{BesselJ}(0,\lambda_n)\right)^2 + \lambda_n \operatorname{BesselJ}(1,\lambda_n)^2 - 2 \operatorname{BesselJ}(0,\lambda_n) \operatorname{BesselJ}(1,\lambda}$$

3.2.2.10 [267] Haberman 8.2.5 with θ dependency (General case)

problem number 267

Added Feb 24, 2019.

Problem 8.2.5 from from Richard Haberman applied partial differential equations book, 5th edition.

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius a) $u_t = k\nabla^2 u$ with time-independent boundary conditions:

$$u(a,\theta,t) = g(\theta)$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

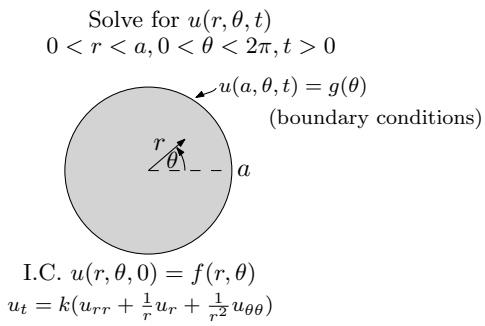

Figure 3.179: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = u[a, theta, t] == g[theta];
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == f[r, theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t) \to \left(\{ \sum_{K[3]=1}^{\infty} \frac{e^{-\frac{kt\left(j_{0,K[3]}\right)^2}{a^2}} J_0\left(\frac{r j_{0,K[3]}}{a}\right) \int_0^a \int_{-\pi}^{\pi} r J_0\left(\frac{r j_{0,K[3]}}{a}\right)\left(2\pi f(r,\theta) - \text{Integrate}[g(\theta),\{\theta,-\pi,\pi\},\text{Assumptions}\to\right.}{}\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= u(a,theta,t)=g(theta);
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=f(r,theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$
\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= g(\theta)\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

With initial conditions $u(r,\theta,0)=f(r,\theta)$.

Since the boundary conditions are not homogenous, and since there are no time dependent sources, then in this case we look for $u_E(r,\theta)$ which is solution at steady state which needs to satisfy the nonhomogeneous B.C., where $u(r,\theta,t)=\overbrace{v(r,\theta,t)}^{\text{transient}}+\overbrace{u_E(r,\theta)}^{\text{steady state}}$ and $v(r,\theta,t)$ solves the PDE but with homogenous B.C. Therefore, we need to find equilibrium (steady state) solution for Laplace PDE on disk, that only needs to satisfy the nonhomogeneous B.C.

$$
\begin{aligned}
\nabla^2 u_E &= 0\\
\frac{\partial^2 u_E}{\partial r^2}+\frac{1}{r}\frac{\partial u_E}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_E}{\partial \theta^2} &= 0
\end{aligned}
$$

With boundary condition

$$
\begin{aligned}
|u_E(0,\theta)| &< \theta\\
u_E(a,\theta) &= g(\theta)\\
u_E(r,-\pi) &= u_E(r,\pi)\\
\frac{\partial u_E}{\partial \theta}(r,-\pi) &= \frac{\partial u_E}{\partial \theta}(r,\pi)
\end{aligned}
$$

Let

$$u_E(r,\theta)=R(r)\,\Theta(\theta)$$

Where $R(r)$ is the solution in radial dimension and $\Theta(\theta)$ is solution in angular dimension. Substituting $u_E(r,\theta)$ in the PDE gives

$$R''\Theta+\frac{1}{r}R'\Theta+\frac{1}{r^2}\Theta''R=0$$

Dividing by $R(r)\,\Phi(\theta)$

$$
\begin{aligned}
\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}+\frac{1}{r^2}\frac{\Theta''}{\Theta} &= 0\\
r^2\frac{R''}{R}+r\frac{R'}{R} &= -\frac{\Theta''}{\Theta}
\end{aligned}
$$

Hence each side is equal to constant, say λ and we obtain

$$r^2\frac{R''}{R}+r\frac{R'}{R}=\lambda$$
$$-\frac{\Theta''}{\Theta}=\lambda$$

Or

$$r^2R''+rR'-\lambda R=0 \tag{1}$$
$$\Theta''+\lambda\Theta=0 \tag{2}$$

We start with Φ ODE. The boundary conditions on (3) are

$$\Theta(-\pi)=\Theta(\pi)$$
$$\frac{\partial\Theta}{\partial\theta}(-\pi)=\frac{\partial\Theta}{\partial\theta}(\pi)$$

<u>case $\lambda=0$</u> The solution is $\Phi=c_1\theta+c_2$. Hence we obtain, from first initial conditions

$$-\pi c_1+c_2=\pi c_1+c_2$$
$$c_1=0$$

Second boundary conditions just says that $c_2=c_2$, so any constant will do. Hence <u>$\lambda=0$ is an eigenvalue</u> with constant being eigenfunction.

<u>case $\lambda>0$</u> The solution is

$$\Theta(\theta)=c_1\cos\sqrt{\lambda}\theta+c_2\sin\sqrt{\lambda}\theta$$

The first boundary conditions gives

$$c_1\cos\left(-\sqrt{\lambda}\pi\right)+c_2\sin\left(-\sqrt{\lambda}\pi\right)=c_1\cos\left(\sqrt{\lambda}\pi\right)+c_2\sin\left(\sqrt{\lambda}\pi\right)$$
$$c_1\cos\left(\sqrt{\lambda}\pi\right)-c_2\sin\left(\sqrt{\lambda}\pi\right)=c_1\cos\left(\sqrt{\lambda}\pi\right)+c_2\sin\left(\sqrt{\lambda}\pi\right)$$
$$2c_2\sin\left(\sqrt{\lambda}\pi\right)=0 \tag{3}$$

From second boundary conditions we obtain

$$\Theta'(\theta)=-\sqrt{\lambda}c_1\sin\sqrt{\lambda}\theta+c_2\sqrt{\lambda}\cos\sqrt{\lambda}\theta$$

Therefore

$$-\sqrt{\lambda}c_1\sin\left(-\sqrt{\lambda}\pi\right)+c_2\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right)=-\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)+c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)$$
$$\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)+c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)=-\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)+c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)$$
$$\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)=-\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)$$
$$2c_1\sin\left(\sqrt{\lambda}\pi\right)=0 \tag{4}$$

Both (3) and (4) are satisfied if

$$\begin{aligned}\sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\ \lambda &= n^2 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

Therefore

$$\Theta_n(\theta) = \overbrace{A_0}^{\lambda=0} + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

Now we go back to the R ODE (1) given by $r^2R'' + rR' - \lambda_n R = 0$ and solve it. This is Euler ODE whose solution is found by substituting $R(r) = r^\alpha$. The solution comes out to be

$$R_n(r) = c_0 + \sum_{n=1}^{\infty} c_n r^n \tag{6}$$

Combining (5,6) we now find u_E as

$$\begin{aligned}u_{E_n}(r, \theta) &= R_n(r)\,\Theta_n(\theta) \\ u_E(r, \theta) &= A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta))\end{aligned} \tag{7}$$

Where c_0 was combined with A_0. Now the above equilibrium solution needs to satisfy the non-homogenous B.C. $u_E(a, \theta) = g(\theta)$. Using orthogonality on (7) to find A_n, B_n gives

$$g(\theta) = A_0 + \sum_{n=1}^{\infty} a^n (A_n \cos(n\theta) + B_n \sin(n\theta))$$

For $n = 0$

$$\begin{aligned}\int_0^{2\pi} g(\theta)\, d\theta &= A_0 \int_0^{2\pi} d\theta \\ A_0 &= \frac{1}{2\pi} \int_0^{2\pi} g(\theta)\, d\theta\end{aligned}$$

For $n > 0$, applying orthogonality using cosine to find A_n gives

$$\begin{aligned}\int_0^{2\pi} g(\theta) \cos(n\theta)\, d\theta &= A_n \int_0^{2\pi} \cos^2(n\theta)\, a^n d\theta \\ A_n &= \frac{1}{\pi} \int_0^{2\pi} g(\theta) \cos(n\theta)\, d\theta\end{aligned}$$

Similarly, applying orthogonality using sin to find B_n gives

$$B_n = \frac{1}{\pi} \int_0^{2\pi} g(\theta) \sin(n\theta)\, d\theta$$

Therefore, we have found $u_E(r,\theta)$ completely now. It is given by (7)

$$
\begin{aligned}
u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta)) \qquad (7A)\\
A_0 &= \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\, d\theta\\
A_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\, d\theta\\
B_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\, d\theta
\end{aligned}
$$

Now, since $u(r,\theta,t) = v(r,\theta,t) + u_E(r,\theta)$, then we need to solve now for $v(r,\theta,t)$ with homogeneous boundary conditions

$$
\begin{aligned}
v_t(r,\theta,t) &= k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}\right) \qquad (8)\\
|v(0,\theta,t)| &< \theta\\
v(a,\theta,t) &= 0\\
v(r,-\pi,t) &= v(r,\pi,t)\\
\frac{\partial v}{\partial \theta}(r,-\pi,t) &= \frac{\partial v}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

Let $v(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$. Substituting into (8) gives

$$
T' R\Theta = k\left(R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta'' RT\right)
$$

Dividing by $R(r)\,\Theta(\theta)\,T(t) \neq 0$ gives

$$
\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}
$$

Let first separation constant be $-\lambda$, hence the above becomes

$$
\begin{aligned}
\frac{1}{k}\frac{T'}{T} &= -\lambda\\
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= -\lambda
\end{aligned}
$$

Or

$$
\begin{aligned}
T' + \lambda k T &= 0\\
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= -\frac{\Theta''}{\Theta}
\end{aligned}
$$

We now separate the second equation above using μ giving

$$r^2\frac{R''}{R}+r\frac{R'}{R}+r^2\lambda=\mu$$
$$-\frac{\Theta''}{\Theta}=\mu$$

Or

$$R''+\frac{1}{r}R'+R\left(\lambda-\frac{\mu}{r^2}\right)=0 \tag{9}$$
$$\Theta''+\mu\Theta=0 \tag{10}$$

Equation (9) is Sturm-Liouville ODE with boundary conditions $R(a)=0$ and bounded at $r=0$ and (10) has periodic boundary conditions as was solved above. The solution to (10) is given in (5) above, no change for this part.

$$\Theta_n(\theta)=\overbrace{\alpha_0}^{\lambda=0}+\sum_{n=1}^{\infty}\alpha_n\cos(n\theta)+\beta_n\sin(n\theta) \tag{11}$$

Therefore (9) becomes $R''+\frac{1}{r}R'+R\left(\lambda-\frac{n^2}{r^2}\right)=0$ with $n=0,1,2,\cdots$. We found the solution to this Sturm-Liouville before, it is given by

$$R_{nm}(r)=J_n\left(\sqrt{\lambda_{nm}}r\right)\qquad n=0,1,2,\cdots,m=1,2,3,\cdots \tag{12}$$

Where $\sqrt{\lambda_{nm}}=\frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the mth zero of the Bessel function of order n. This is found numerically. We now just need to find the time solution from $T'+\lambda_{nm}kT=0$. For This has solution

$$T_{nm}(t)=e^{-k\lambda_{nm}t} \tag{13}$$

Now we combine (11,12,13) to find solution for $v(r,\theta,t)$, and combining constants gives

$$v_{nm}(r,\theta,t)=\Theta_n(\theta)\,R_{nm}(r)\,T_{nm}(t)$$
$$\begin{aligned}v(r,\theta,t)&=\alpha_{0,1}J_0\left(\sqrt{\lambda_{0,1}}r\right)+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta))\\&=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta))\end{aligned} \tag{14}$$

We now need to find $\alpha_0,\alpha_{nn},\beta_{nm}$, which are found from initial conditions on $v(r,\theta,0)$ which is given by

$$\begin{aligned}v(r,\theta,0)&=u(r,\theta,0)-u_E(r,\theta)\\&=f(r,\theta)-u_E(r,\theta)\end{aligned}$$

Hence from (14), at $t=0$

$$f(r,\theta)-u_E(r,\theta)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)\left(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta)\right) \tag{15}$$

For each n, inside the m sum, $\cos(n\theta)$ and $\sin(n\theta)$ will be constant. So we need to apply orthogonality twice in order to remove both sums. Multiplying (15) by $\cos(n'\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi}(f(r,\theta)-u_E(r,\theta))\cos(n'\theta)\,d\theta &= \int_{-\pi}^{\pi}\sum_{n=0}^{\infty}\left(\sum_{m=1}^{\infty}\alpha_{nn}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\cos(n\theta)\cos(n'\theta)\,d\theta\\ &+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\beta_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\sin(n\theta)\cos(n'\theta)\end{aligned}$$

The second sum in the RHS above goes to zero due to $\int_{-\pi}^{\pi}\sin(n\theta)\cos(n'\theta)\,d\theta$ and we end up with

$$\int_{-\pi}^{\pi}(f(r,\theta)-u_E(r,\theta))\cos(n\theta)\,d\theta=\alpha_{nn}\int_{-\pi}^{\pi}\cos^2(n\theta)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)d\theta$$

We now apply orthogonality again, but on Bessel functions and remembering to add the weight r, the above becomes

$$\begin{aligned}\int_0^a\int_{-\pi}^{\pi}(f(r,\theta)-u_E(r,\theta))\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm'}}r\right)r\,d\theta dr &= \alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)J_n\left(\sqrt{\lambda_{nm'}}r\right)r\\ &=\alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm'}}r\right)r\,d\theta dr\end{aligned}$$

Therefore

$$\alpha_{nn}=\frac{\int_0^a\int_{-\pi}^{\pi}(f(r,\theta)-u_E(r,\theta))\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

We will repeat the same thing to find β_{nm}. The only difference now is to use $\sin n\theta$. repeating these steps gives

$$\beta_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi}(f(r,\theta)-u_E(r,\theta))\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

This complete the solution.

<u>Summary of solution</u>

$$\begin{aligned}u(r,\theta,t)&=v(r,\theta,t)+u_E(r,\theta)\\ &=u_E(r,\theta)+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}(\alpha_{nn}\cos(n\theta)+\beta_{nm}\sin(n\theta))\end{aligned}$$

Where

$$
\begin{aligned}
u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta)) \\
A_0 &= \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\, d\theta \\
A_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\, d\theta \\
B_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\, d\theta
\end{aligned}
$$

And

$$
\alpha_{nn} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta)) \cos(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0, 1, 2, \cdots, m = 1, 2, 3, \cdots
$$

And

$$
\beta_{nm} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta)) \sin(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0, 1, 2, \cdots, m = 1, 2, 3, \cdots
$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the mth zero of the Bessel function of order n.

3.2.2.11 [268] With θ dependency (Specific example)

problem number 268

Added June 12, 2019

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius $a = 1$) $u_t = k\nabla^2 u$ with $k = 1$ with time-independent boundary conditions:

$$
u(1,\theta,t) = 0
$$

And initial conditions $u(r,\theta,0) = 1 - r^2$. There is an implied periodic boundary conditions on θ

$$
\begin{aligned}
u(r,-\pi,t) &= u(r,\pi,t) \\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

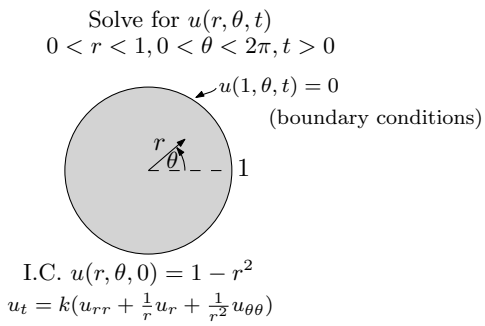

Figure 3.180: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
a=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = u[a, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == 1-r^2;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t) \to \begin{cases} \sum_{K[3]=1}^{\infty} \frac{4e^{-t\left(j_{0,K[3]}\right)^2} J_0\left(r j_{0,K[3]}\right) J_2\left(j_{0,K[3]}\right)}{J_1\left(j_{0,K[3]}\right)^2 \left(j_{0,K[3]}\right)^2} & (K[1]|K[3]) \in \mathbb{Z} \wedge K[1] \geq 1 \wedge K[3] \geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✗

```
restart;
a:=1;
k:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= u(a,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=1-r^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

<u>Hand solution</u>

Solve

$$\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= 0\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)\\
u(r,\theta,0) &= 1-r^2
\end{aligned}$$

With $t>0, 0<r<a$ where $a=1, k=1$.

This problem was solved in problem 3.2.2.10 on page 726 (since B.C. is zero, we just need to use $v(r,\theta,t)$ in the solution given in the above problem). Hence

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta))$$

Where

$$A_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

And

$$B_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

Where $\sqrt{\lambda_{nm}}=\frac{a}{z_{nm}}$ and z_{nm} is the mth zero of the Bessel function of order n. where a is the radius of the disk and z_{nm} is the mth zero of the Bessel function of order n. Since in this problem $k=1, a=1, f(r,\theta)=1-r^2$ then

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-\frac{t}{z_{nm}^2}}(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta))$$

$$A_{nm}=\frac{\int_0^1\int_{-\pi}^{\pi}(1-r^2)\cos(n\theta)\,J_n\left(\frac{r}{z_{nm}}\right)rd\theta dr}{\int_0^1\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\frac{r}{z_{nm}}\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

$$B_{nm}=\frac{\int_0^1\int_{-\pi}^{\pi}(1-r^2)\sin(n\theta)\,J_n\left(\frac{r}{z_{nm}}\right)rd\theta dr}{\int_0^1\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\frac{r}{z_{nm}}\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the mth zero of the Bessel function of order n.

This is animation of the solution for 0.4 seconds. (Animation will only show in the HTML version)

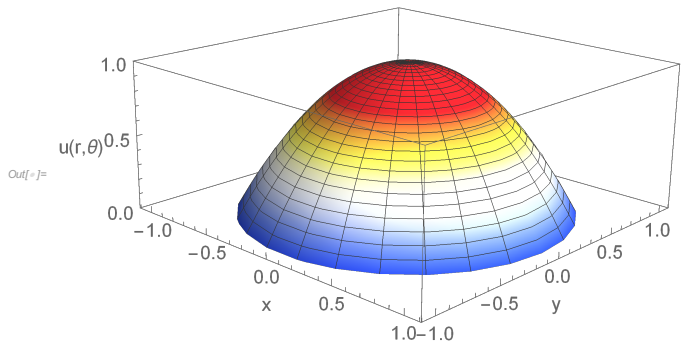

Figure 3.181: Initial state

Source code used for the above

```
In[ ]:= ClearAll[a, c, n, m, r, theta, f, g, θ, t, f, n, lam0, A0, A, B, B0, lam];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
a = 1;
k = 1;  (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (a - r^2);
A0[n_, m_] := A[[n + 1, m]];
B0[n_, m_] := B[[n + 1, m]];
lam0[n_, m_] := lam[[n + 1, m]];
getEigenvalue[n_, m_] := Module[{x}, x = BesselJZero[n, m]; N[(x / a)^2]];
lam = Table[getEigenvalue[n, m], {n, 0, maxN - 1}, {m, 1, maxM}];
A = Table[Integrate[f Cos[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Cos[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}], {n, 0, maxN - 1}, {m, 1, maxM}];
B = Table[If[n == 0, 0, Integrate[f Sin[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Sin[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}]], {n, 0, maxN - 1}, {m, 1, maxM}];
u[r_, θ_, t_] := Sum[Sum[BesselJ[n, r Sqrt[lam0[n, m]]] Exp[-k lam0[n, m] t] (A0[n, m] Cos[n θ] + B0[n, m] Sin[n θ]), {m, 1, maxM}], {n, 0, maxN - 1}]
```

Figure 3.182: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (1-r^2)"}]},
              {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
                  BaseStyle → 15,
                  ImageMargins → 5,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {0, 1}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, .8}}],
                  AxesLabel → {"x", "y", "U(r,θ)"},
                  ViewPoint → {2.17, -2.4, 1}
                ]
              }}],
          {t, 0, 0.4, .01}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.35, {Length[tab]}]]
```

Figure 3.183: Animation part of code

3.2.2.12 [269] With θ dependency (specific example)

problem number 269

Added June 12, 2019

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius $a = 1$) $u_t = k\nabla^2 u$ with $k = 1$ with time-independent boundary conditions:

$$u(1, \theta, t) = 0$$

And initial conditions $u(r, \theta, 0) = (r - r^3)\sin\theta$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r, -\pi, t) &= u(r, \pi, t) \\ \frac{\partial u}{\partial \theta}(r, -\pi, t) &= \frac{\partial u}{\partial \theta}(r, \pi, t) \end{aligned}$$

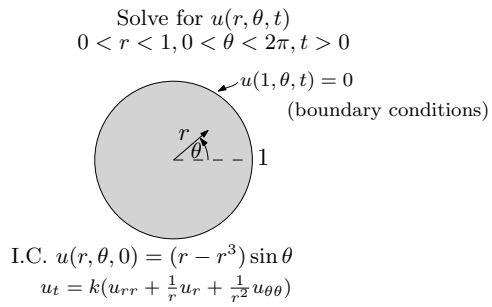

Figure 3.184: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
k=1;
a=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = u[a, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (r-r^3)*Sin[theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t) \to \begin{cases} \sum_{K[3]=1}^{\infty} \frac{4e^{-t\left(j_{1,K[3]}\right)^2} J_1\left(r j_{1,K[3]}\right) J_3\left(j_{1,K[3]}\right)\sin(\theta)}{J_0\left(j_{1,K[3]}\right)^2 \left(j_{1,K[3]}\right)^2} & (K[1]|K[3]) \in \mathbb{Z} \wedge K[1] \geq 1 \wedge K[3] \geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right.\right\}$$

Maple ✗

```
restart;
a:=1;
k:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= u(a,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=(r-r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= 0\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)\\
u(r,\theta,0) &= \left(r-r^3\right)\sin\theta
\end{aligned}$$

for $0<r<a$ where $a=1, k=1$.

This problem was solved in problem 3.2.2.10 on page 726 (since B.C. is zero, we just need to use $v(r,\theta,t)$ in the solution given in the above problem). Hence

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}\left(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta)\right)$$

Where

$$A_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

And

$$B_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi} f(r,\theta)\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\sqrt{\lambda_{nm}}r\right)r\,d\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

Where $\sqrt{\lambda_{nm}}=\frac{a}{z_{nm}}$ and z_{nm} is the mth zero of the Bessel function of order n and a is the radius of the disk and z_{nm} is the mth zero of the Bessel function of order n. Since in this problem $k=1, a=1, f(r,\theta)=(r-r^3)\sin\theta$ then the above solution becomes

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_n\cos(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-\frac{t}{z_{nm}^2}}+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_n\sin(n\theta)\,J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-\frac{t}{z_{nm}^2}}$$

$$A_n=\frac{\int_0^1\int_{-\pi}^{\pi}(r-r^3)\sin\theta\cos(n\theta)\,J_n\left(\frac{r}{z_{nm}}\right)r\,d\theta dr}{\int_0^1\int_{-\pi}^{\pi}\cos^2(n\theta)\,J_n^2\left(\frac{r}{z_{nm}}\right)r\,d\theta dr}$$

$$B_n=\frac{\int_0^1\int_{-\pi}^{\pi}(r-r^3)\sin\theta\sin(n\theta)\,J_n\left(\frac{r}{z_{nm}}\right)r\,d\theta dr}{\int_0^1\int_{-\pi}^{\pi}\sin^2(n\theta)\,J_n^2\left(\frac{r}{z_{nm}}\right)r\,d\theta dr}$$

This is animation of the solution for 0.11 seconds. (Animation will only show in the HTML version)

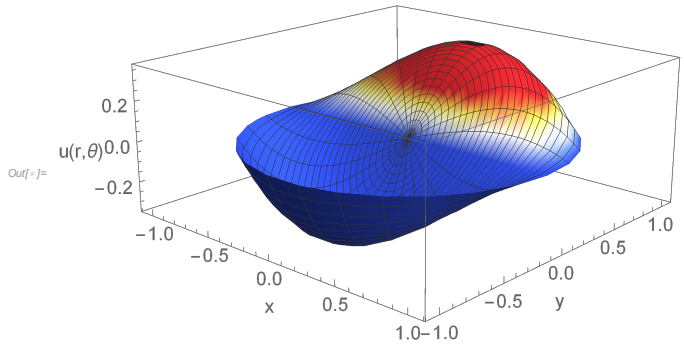

Figure 3.185: Initial state

Source code used for the above

```
In[ ]:= ClearAll[a, c, n, m, r, theta, f, g, θ, t, f, n, lam0, A0, A, B, B0, lam];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
a = 1;
k = 1;  (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (r - r^3) Sin[θ];
A0[n_, m_] := A[[n + 1, m]];
B0[n_, m_] := B[[n + 1, m]];
lam0[n_, m_] := lam[[n + 1, m]];
getEigenvalue[n_, m_] := Module[{x}, x = BesselJZero[n, m]; N[(x / a)^2]];
lam = Table[getEigenvalue[n, m], {n, 0, maxN - 1}, {m, 1, maxM}];
A = Table[Integrate[f Cos[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Cos[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}], {n, 0, maxN - 1}, {m, 1, maxM}];
B = Table[If[n == 0, 0, Integrate[f Sin[n θ] r BesselJ[n, r Sqrt[lam0[n, m]]], {r, 0, a}, {θ, -Pi, Pi}] / Integrate[Sin[n θ]^2 r BesselJ[n, r Sqrt[lam0[n, m]]]^2, {r, 0, a}, {θ, -Pi, Pi}]], {n, 0, maxN - 1}, {m, 1, maxM}];
u[r_, θ_, t_] := Sum[Sum[ BesselJ[n, r Sqrt[lam0[n, m]]] Exp[-k lam0[n, m] t] (A0[n, m] Cos[n θ] + B0[n, m] Sin[n θ]), {m, 1, maxM}], {n, 0, maxN - 1}]
```

Figure 3.186: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (1-r^2)"}]},
              {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
                  BaseStyle → 15,
                  ImageMargins → 5,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {-0.35, 0.38}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, .2}}],
                  AxesLabel → {"x", "y", "U(r,θ)"},
                  ViewPoint → {2.17, -2.4, 1}
                ]
              }}],
          {t, 0, 0.11, .005}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.5, {Length[tab]}]]
```

Figure 3.187: Animation part of code

3.2.2.13 [270] Insulated with θ dependency (General solution)

problem number 270

Added June 13, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

with time-independent boundary conditions:

$$u_r(a,\theta,t) = 0$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

The solution is bounded at $r = 0$.

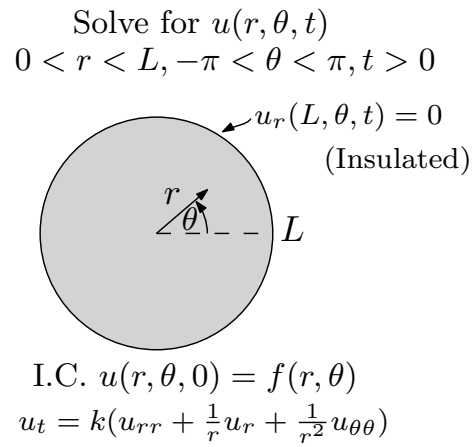

Figure 3.188: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == f[r,theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t) \to \{ \frac{\int_0^L \int_{-\pi}^{\pi} r f(r,\theta) d\theta dr}{L^2\pi} + \sum_{K[3]=1}^{\infty} \frac{e^{-ktK[2,0,K[3]]^2} J_0(rK[2,0,K[3]]) \left(\int_0^L \int_{-\pi}^{\pi} r J_0(rK[2,0,K[3]]) f(r,\theta) d\theta dr\right) \sqrt{K}}{L^2\pi\sqrt{J_0(LK[2,0,K[3]])^2+J_1(LK[2,0,K[3]])^2}\sqrt{(J_0(LK[2,0,K[3]])^2+J_1(LK[2,0,K[}}\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)=f(r,theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$\frac{\partial u(r,\theta,t)}{\partial t} = k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u_t(L,\theta,t) &= g(\theta) \\
u(r,-\pi,t) &= u(r,\pi,t) \\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. Let $y(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$. Substituting into (1) gives

$$T'R\Theta = k\left(R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT\right)$$

Dividing by $R(r)\,\Theta(\theta)\,T(t) \neq 0$ gives

$$\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}$$

Let first separation constant be $-\lambda$, hence the above becomes

$$\begin{aligned}
\frac{1}{k}\frac{T'}{T} &= -\lambda \\
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= -\lambda
\end{aligned}$$

Or

$$\begin{aligned}
T' + \lambda k T &= 0 \\
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= -\frac{\Theta''}{\Theta} = \mu
\end{aligned}$$

Hence

$$\begin{aligned}
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= \mu \\
-\frac{\Theta''}{\Theta} &= \mu
\end{aligned}$$

Or

$$r^2R''(r) + rR'(r) + R(r)\left(r^2\lambda - \mu\right) = 0 \tag{2}$$

$$\Theta''(\theta) + \mu\Theta(\theta) = 0 \tag{3}$$

Equation (2) is Sturm-Liouville ODE with boundary conditions $R'(a) = 0$ and bounded at $r = 0$ and (3) has periodic boundary conditions.

We start with Θ ODE. The boundary conditions on (3) are

$$\begin{aligned}\Theta(-\pi) &= \Theta(\pi)\\ \frac{\partial\Theta}{\partial\theta}(-\pi) &= \frac{\partial\Theta}{\partial\theta}(\pi)\end{aligned}$$

Negative μ is not possible, since this gives exponential and the boundary conditions on $\Theta(\theta)$ are periodic.

<u>case $\mu = 0$</u> The solution is $\Phi = c_1\theta + c_2$. Hence we obtain, from first initial conditions

$$\begin{aligned}-\pi c_1 + c_2 &= \pi c_1 + c_2\\ c_1 &= 0\end{aligned}$$

Second boundary conditions just says that $c_2 = c_2$, so any constant will do. <u>Hence $\mu = 0$ is an eigenvalue</u> with constant being eigenfunction $\Theta_0(\theta) = 1$.

<u>case $\mu > 0$</u> The solution is

$$\Theta(\theta) = c_1 \cos\sqrt{\mu}\theta + c_2 \sin\sqrt{\mu}\theta$$

The first boundary conditions gives

$$\begin{aligned}c_1 \cos\left(-\sqrt{\mu}\pi\right) + c_2 \sin\left(-\sqrt{\mu}\pi\right) &= c_1 \cos\left(\sqrt{\mu}\pi\right) + c_2 \sin\left(\sqrt{\mu}\pi\right)\\ c_1 \cos\left(\sqrt{\mu}\pi\right) - c_2 \sin\left(\sqrt{\mu}\pi\right) &= c_1 \cos\left(\sqrt{\mu}\pi\right) + c_2 \sin\left(\sqrt{\mu}\pi\right)\\ 2c_2 \sin\left(\sqrt{\mu}\pi\right) &= 0 \qquad (4)\end{aligned}$$

From second boundary conditions we obtain

$$\Theta'(\theta) = -\sqrt{\mu}c_1 \sin\sqrt{\mu}\theta + c_2\sqrt{\mu}\cos\sqrt{\mu}\theta$$

Therefore

$$\begin{aligned}-\sqrt{\mu}c_1 \sin\left(-\sqrt{\mu}\pi\right) + c_2\sqrt{\mu}\cos\left(-\sqrt{\mu}\pi\right) &= -\sqrt{\mu}c_1 \sin\left(\sqrt{\mu}\pi\right) + c_2\sqrt{\lambda}\cos\left(\sqrt{\mu}\pi\right)\\ \sqrt{\mu}c_1 \sin\left(\sqrt{\mu}\pi\right) + c_2\sqrt{\mu}\cos\left(\sqrt{\mu}\pi\right) &= -\sqrt{\mu}c_1 \sin\left(\sqrt{\mu}\pi\right) + c_2\sqrt{\mu}\cos\left(\sqrt{\mu}\pi\right)\\ \sqrt{\mu}c_1 \sin\left(\sqrt{\mu}\pi\right) &= -\sqrt{\mu}c_1 \sin\left(\sqrt{\mu}\pi\right)\\ 2c_1 \sin\left(\sqrt{\mu}\pi\right) &= 0 \qquad (5)\end{aligned}$$

Both (4) and (5) are satisfied if

$$\begin{aligned}\sqrt{\mu}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \qquad (5A)\\ \mu &= n^2 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

Therefore $\mu_n = 1, 4, 9, 16, \cdots$. Or $\sqrt{\mu_n} = n = 1, 2, 3, \cdots$

$$\Theta_n(\theta) = \overbrace{A_0}^{n=0} + \sum_{n=1}^{\infty} A_n \cos\left(\sqrt{\mu_n}\theta\right) + B_n \sin\left(\sqrt{\mu_n}\theta\right) \tag{6}$$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{3.2}$$

Now we go back to the $R(r)$ ODE (2) given by $r^2R'' + rR' + R(r^2\lambda - \mu) = 0$ or since $\mu_n = n^2$, we write it as $r^2R'' + rR' + R(r^2\lambda - n^2) = 0$ for $n = 0, 1, 2, \cdots$.

<u>Case $n = 0$</u>

The $R(r)$ ODE becomes $r^2R''+rR'+\lambda r^2R = 0$. This is a Bessel ODE. If $\lambda < 0$, this leads to solution (contains Besselk and BesselI) that can not satisfy the boundary conditions. If $\lambda = 0$ then the ODE becomes $r^2R'' + rR' = 0$ whose solution is $R(r) = C_1 \ln(r) + C_2$. Since bounded at $r = 0$. Then $R(r) = C_2$. Hence $R'(r) = 0$. Which satisfies the boundary condition at $r = L$ for any constant C_2. Therefore <u>$\lambda = 0$ is possible eigenvalue</u> with constant as its eigenfunction $R_0(r) = 1$.

If $\lambda > 0$ the ODE is a Bessel ODE whose solution is

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda}r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda}r\right)$$

We set $C_2 = 0$ since $\operatorname{BesselY}\left(0, \sqrt{\lambda}r\right)$ blows up at $r = 0$. Hence $R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda}r\right)$ and $R'(r) = -C_1\sqrt{\lambda}\operatorname{BesselJ}\left(1, \sqrt{\lambda}r\right)$. At $r = L$ we obtain

$R'(L) = -C_1\sqrt{\lambda}\operatorname{BesselJ}\left(1, \sqrt{\lambda}L\right) = 0$. Therefore for non-trivial solution we want $\sqrt{\lambda}L$ to be the zeros of $\operatorname{BesselJ}(1, x)$. Let these zeros be Λ_m, $m = 1, 2, 3, \cdots$. Hence

$$\sqrt{\lambda_{0,m}}L = \Lambda_m$$

$$\lambda_{0,m} = \left(\frac{\Lambda_m}{L}\right)^2 \qquad m = 1, 2, 3, \cdots$$

Where we added zero as subscript to $\lambda_{0,m}$ to indicate that this is associated with Case $n = 0$. Hence the eigenfunctions for $\mu = 0$ are

$$R_{0,m}(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_m}{L}r\right) \qquad n = 0, m = 1, 2, 3, \cdots$$

And $\Theta_0(\theta) = 1$.

<u>Case $n > 0$</u>

The $R(r)$ ODE becomes $r^2R'' + rR' + R(r^2\lambda - n^2) = 0$.

This is a Bessel ODE. If $\lambda < 0$, this leads to solution that can not satisfy the boundary conditions (contains Besselk and BesselI). If $\lambda = 0$ then the ODE becomes $r^2R'' + rR' - n^2R = 0$. This is Euler ODE whose solution is $R(r) = C_1r^n + C_2r^{-n}$. Since bounded at $r = 0$. Then $C_2 = 0$ and the solution becomes $R(r) = C_1r^n$. Hence $R'(r) = C_1nr^{n-1}$. At $r = L$ this becomes $C_1nL^{n-1} = 0$. Since $L \neq 0$, then this gives $C_1 = 0$, trivial solution. Hence <u>$\lambda = 0$ is not possible eigenvalue</u> when $\mu > 0$.

If $\lambda > 0$ the ODE becomes $r^2R'' + rR' + R(r^2\lambda - n^2) = 0$ with now λ positive. This is a Bessel ODE whose solution is

$$R_n(r) = C_1 \operatorname{BesselJ}\left(n, \sqrt{\lambda}r\right) + C_2 \operatorname{BesselY}\left(n, \sqrt{\lambda}r\right) \qquad n = 1, 2, 3, \cdots$$

Since $R(r)$ is bounded at $r = 0$ then $C_2 = 0$ and the above becomes

$$R_n(r) = C_1 \operatorname{BesselJ}\left(n, \sqrt{\lambda}r\right) \qquad n = 1, 2, 3, \cdots$$

Therefore

$$R_n'(r) = C_1 \frac{d}{dr} \operatorname{BesselJ}\left(n, \sqrt{\lambda}r\right)$$

At $r = L$

$$R_n'(L) = C_1 \frac{d}{dr} \operatorname{BesselJ}\left(n, \sqrt{\lambda}L\right)$$

For non-trivial solution we want

$$\frac{d}{dr} \operatorname{BesselJ}\left(n, \sqrt{\lambda}L\right) = 0 \tag{7}$$

The eigenvalues λ are solved for numerically by finding all zeros of $\frac{d}{dx} \operatorname{BesselJ}(n, x)$. Let these zeros be called Γ_{nm} for $n = 1, 2, 3, \cdots, m = 1, 2, 3, \cdots$. Hence

$$\begin{aligned}
\sqrt{\lambda_{nm}}L &= \Gamma_{nm} \\
\lambda_{nm} &= \left(\frac{\Gamma_{nm}}{L}\right)^2 \qquad n = 1, 2, 3, \cdots, m = 1, 2, 3, \cdots
\end{aligned}$$

With associated eigenfunctions $R_{n,m}(r) = \operatorname{BesselJ}\left(n, \frac{\Gamma_{nm}}{L}r\right)$. Now that we solved for $R_{n,m}(r)$ and $\Theta_{n,m}(\theta)$, what is left is to solve $\frac{1}{k}\frac{T'}{T} = -\lambda_{nm}$.

<u>For $n = 0$</u>

For $\lambda = 0$ the solution is constant. Say $T_{00}(t) = 1$. For $\lambda > 0$ the solution is $T_{0m}(t) = e^{-k\lambda_{0m}t}$. Where $\lambda_{0,m} = \left(\frac{\Lambda_m}{L}\right)^2, m = 1, 2, 3, \cdots$ and Λ_m are the zeros of $\operatorname{BesselJ}(1, x)$.

<u>For $n > 0$</u> In this case $\lambda > 0$ only. The solution is $T_{nm}(t) = e^{-k\lambda_{nm}t}$. Where $\lambda_{n,m}$ are now the solutions of (7). Hence $T_{nm}(t) = e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}$

The solution now becomes

$$u(r,\theta,t)=\overbrace{R_{0,0}(r)\,\Theta_{0,0}(\theta)\,T_{0,0}(t)}^{\text{arbitrary constant}}+\overbrace{R_{0,m}(r)\,\Theta_{0,m}(\theta)\,T_{0.m}(t)}^{n=0,m>0}+\overbrace{R_{n,m}(r)\,\Theta_{n,m}(\theta)\,T_{n.m}(t)}^{n>0,m>0}$$

Or

$$\begin{aligned}u(r,\theta,t)&=\beta_0\\&+\sum_{m=1}^{\infty}\alpha_{0,m}\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t}\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\left(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta)\right)\end{aligned}$$

Where Γ_{nm} are the mth zeros $\frac{d}{dx}\,\text{BesselJ}\,(n,x)$ and Λ_m are the mth zeros of $\text{BesselJ}\,(1,x)$. These have to be found numerically.

Now we find the constants $\beta_0,\alpha_m,A_{nm},B_{nm}$. For $n=0,m=0$, and at initial conditions at $t=0$, and applying orthogonality gives (all eigenfunctions are constants, assumed 1, in this case)

$$\begin{aligned}\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta&=\beta_0\int_{-\pi}^{\pi}\int_0^L rdrd\theta\\\beta_0&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta}{\pi L^2}\end{aligned}$$

For $n=0,m>0$

$$\begin{aligned}\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta&=\alpha_{0,m}\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta\\&=2\pi\alpha_{0,m}\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdr\\\alpha_{0,m}&=\frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta}{2\pi\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdr}\end{aligned}$$

For $n>0,m>0$, applying orthogonality on $\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)$ gives

$$\int_0^L f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)rdr=\sum_{n=1}^{\infty}\int_0^L \text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)rdr(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta))$$

applying orthogonality on $\cos(n\theta)$ the above becomes

$$\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta = A_{nm}\int_{-\pi}^{\pi}\int_{0}^{L}\text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta$$

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_{0}^{L}\text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta}$$

Similarly

$$B_{nm} = \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_{0}^{L}\text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,rdrd\theta}$$

This complete the solution. <u>In summary</u>, the solution is

$$\begin{aligned}
u(r,\theta,t) &= \beta_0 \\
&+ \sum_{m=1}^{\infty}\alpha_{0,m}\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} \\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\left(A_{nm}\cos(n\theta)+B_{nm}\sin(n\theta)\right)
\end{aligned}$$

Where

$$\begin{aligned}
\beta_0 &= \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,rdrd\theta}{\pi L^2} \\
\alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta}{2\pi\int_{0}^{L}\text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdr} \\
A_{nm} &= \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_{0}^{L}\text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta} \\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_{0}^{L} f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_{0}^{L}\text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,rdrd\theta}
\end{aligned}$$

And Γ_{nm} are the mth zeros of $\frac{d}{dx}\text{BesselJ}(n,x)$ and Λ_m are the mth zeros of $\text{BesselJ}(1,x)$. These have to be found numerically.

3.2.2.14 [271] Insulated with θ dependency (Specific example)

problem number 271

Added June 15, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

For $t > 0$ and $0 < r < L$ with $L = 1, k = 1$ and time-independent boundary conditions:

$$u_r(L,\theta,t) = 0$$

And initial conditions $u(r,\theta,0) = (2Lr - r^2)\cos\theta\sin\theta$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

The solution is bounded at $r = 0$.

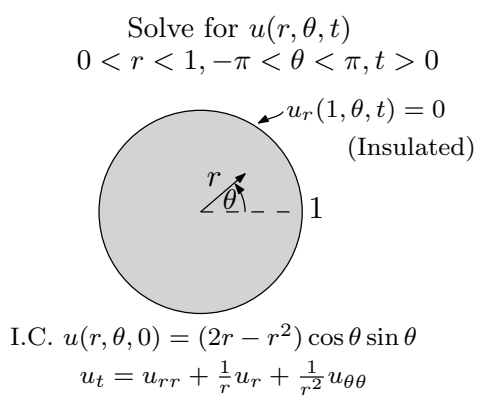

Figure 3.189: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=1;
k=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (2*L*r-r^2)*Cos[theta]*Sin[theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$$\left\{\left\{u(r,\theta,t)\to\{\sum_{K[3]=1}^{\infty}\frac{e^{-tK[2,2,K[3]]^2}J_2(rK[2,2,K[3]])(2\,{}_1F_2(\frac{5}{2};3,\frac{7}{2};-\frac{1}{4}K[2,2,K[3]]^2)-5\,{}_0\tilde{F}_1(;4;-\frac{1}{4}K[2,2,K[3]]^2))K[2,2,K[3]]^3\sin}{40((J_1(K[2,2,K[3]])^2+J_2(K[2,2,K[3]])^2)K[2,2,K[3]]-4J_1(K[2,2,K[3]])J_2(K[2,2,K[3]]))}\right.\right.$$

Indeterminate

Maple ✗

```
restart;
k:=1;
L:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)= (2*L*r-r^2)*cos(theta)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve for $u(r,\theta,t)$

$$u_t = k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}|u(0,\theta,t)| &< \infty\\ u_t(L,\theta,t) &= 0\\ u(r,-\pi,t) &= u(r,\pi,t)\\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)\end{aligned}$$

And initial conditions

$$
\begin{aligned}
u(r,\theta,0) &= f(r,\theta) = \left(2rL - r^2\right)\cos\theta\sin\theta \\
L &= 1 \\
k &= 1
\end{aligned}
$$

The basic solution for this type of PDE was already given in problem 3.2.2.13 on page 743 as

$$
\begin{aligned}
u(r,\theta,t) = \beta_0 & \\
&+ \sum_{m=1}^{\infty} \alpha_{0,m}\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right) e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} \\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\left(A_{nm}\cos(n\theta) + B_{nm}\sin(n\theta)\right)
\end{aligned}
$$

Where

$$
\begin{aligned}
\beta_0 &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,rdrd\theta}{\pi L^2} \\
\alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)rdrd\theta}{2\pi\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)rdr} \\
A_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,rdrd\theta} \\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,rdrd\theta}{\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,rdrd\theta}
\end{aligned}
$$

And Γ_{nm} are the mth zeros of $\frac{d}{dx}\text{BesselJ}(n,x)$ and Λ_m are the mth zeros of $\text{BesselJ}(1,x)$. These have to be found numerically. This is animation of the solution for 0.2 seconds. (Animation will only show in the HTML version)

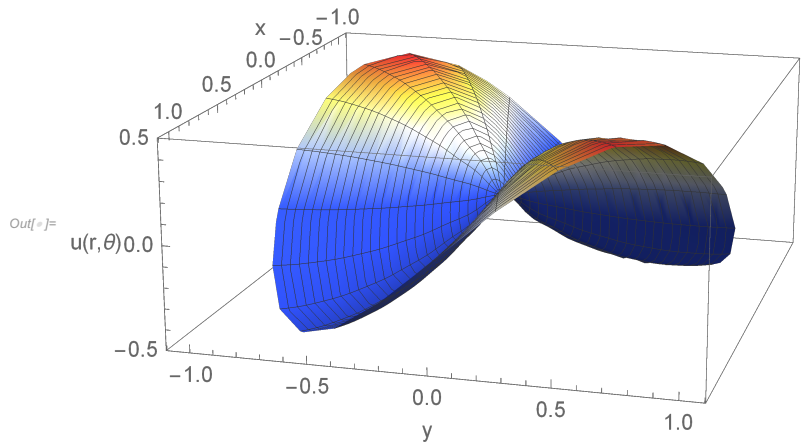

Figure 3.190: Initial state

Source code used for the above

```
Get["G:\\nabbasi\\data\\mathematica_stuff\\besselZeros_old_package\\BesselZeros.m"];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
L = 1;
k = 1; (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (2 * L * r - r^2)  Cos[θ] Sin[θ] ;
besselJ1Zero = N@Table[BesselJZero[1, m], {m, 1, maxM}];
besselJnPrimeZero = N@Table[BesselJPrimeZeros[n, m][[m]], {n, 1, maxN}, {m, 1, maxM}];
beta0 = 1/(π L^2) Integrate[f * r, {θ, -Pi, Pi}, {r, 0, L}];
alpha0M = Table[Integrate[f * r BesselJ[0, besselJ1Zero[[m]]/L r], {θ, -Pi, Pi}, {r, 0, L}] / (2 π Integrate[r BesselJ[0, besselJ1Zero[[m]]/L r]^2, {r, 0, L}]), {m, 1, maxM}];
Anm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Cos[n θ], {θ, -Pi, Pi}, {r, 0, L}] / Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Cos[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
Bnm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Sin[n θ], {θ, -Pi, Pi}, {r, 0, L}] / Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Sin[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
u[r_, θ_, t_] := beta0 +
    Sum[ alpha0M[[m]] BesselJ[0, besselJ1Zero[[m]]/L r] Exp[-k (besselJ1Zero[[m]]/L)^2 t] , {m, 1, maxM}] +
    Sum[Sum[Exp[-k (besselJnPrimeZero[[n, m]]/L)^2 t] BesselJ[n, besselJnPrimeZero[[n, m]]/L r]
        (Anm[[n, m]] Cos[n θ] + Bnm[[n, m]] Sin[n θ]), {n, 1, maxN}], {m, 1, maxM}];
```

Figure 3.191: Source code

```
In[∘]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (2 L r-r^2)Cos[θ]Sin[θ]"}]},
              {ParametricPlot3D[{r Cos[theta], r Sin[theta], Evaluate[u[r, theta, t]]}, {r, 0, 1}, {theta, 0, 2 Pi},
                  BaseStyle → 15,
                  ImageMargins → 5,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {-0.5, 0.5}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, .4}}],
                  AxesLabel → {"x", "y", "U(r,θ)"},
                  ViewPoint → {4.03, 1.133, 1.15}
                ]
              }}],
          {t, 0, 0.3, .005}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[∘]:= Export["anim.gif", tab, "DisplayDurations" → Table[.25, {Length[tab]}]]
```

Figure 3.192: Code for animation

3.2.2.15 [272] Insulated with θ dependency (Specific example)

problem number 272

Added June 15, 2019

Solve for $u(r,\theta,t)$, the initial value problem for a two-dimensional heat equation inside a circle of radius a

$$u_t = k\nabla^2 u$$

For $t > 0$ and $0 < r < L$ with $L = 1, k = 1$ and time-independent boundary conditions:

$$u_r(L,\theta,t) = 0$$

And initial conditions $u(r,\theta,0) = (2Lr - r^2)\theta \sin\theta e^{\cos\theta}$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

The solution is bounded at $r = 0$.

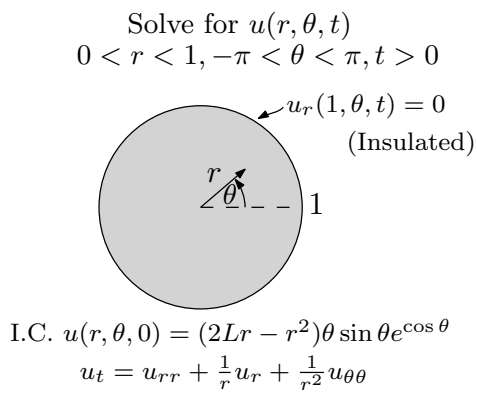

Figure 3.193: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
L=1;
k=1;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = Derivative[1,0,0][u][L, theta, t] == 0;
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic  = u[r, theta, 0] == (2*L*r - r^2)*theta*Sin[theta]*Exp[Cos[theta]];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

$Aborted

Maple ✗

```
restart;
k:=1;
L:=1;
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= D[1](u)(L,theta,t)=0;
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic  := u(r,theta,0)= (2*L*r -r^2)*theta*sin(theta)*exp(cos(theta));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()
Hand solution

Solve for $u(r,\theta,t)$

$$u_t = k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u_t(L,\theta,t) &= 0 \\
u(r,-\pi,t) &= u(r,\pi,t) \\
u_\theta(r,-\pi,t) &= u_\theta(r,\pi,t)
\end{aligned}$$

And initial conditions

$$\begin{aligned}
u(r,\theta,0) = f(r,\theta) &= \left(2rL - r^2\right)\theta\sin\theta e^{\cos\theta} \\
L &= 1 \\
k &= 1
\end{aligned}$$

The basic solution for this type of PDE was already given in problem 3.2.2.13 on page 743 as

$$u(r,\theta,t) = \beta_0 + \sum_{m=1}^{\infty}\alpha_{0,m}\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)e^{-k\left(\frac{\Lambda_m}{L}\right)^2 t} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} e^{-k\left(\frac{\Gamma_{nm}}{L}\right)^2 t}\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)(A_{nm}\cos(n$$

Where

$$\begin{aligned}
\beta_0 &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,r dr d\theta}{\pi L^2} \\
\alpha_{0,m} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(0,\frac{\Lambda_m}{L}r\right)r dr d\theta}{2\pi\int_0^L \text{BesselJ}^2\left(0,\frac{\Lambda_m}{L}r\right)r dr} \\
A_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos(n\theta)\,r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\cos^2(n\theta)\,r dr d\theta} \\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^L f(r,\theta)\,\text{BesselJ}\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin(n\theta)\,r dr d\theta}{\int_{-\pi}^{\pi}\int_0^L \text{BesselJ}^2\left(n,\frac{\Gamma_{nm}}{L}r\right)\sin^2(n\theta)\,r dr d\theta}
\end{aligned}$$

And Γ_{nm} are the mth zeros of $\frac{d}{dx}\text{BesselJ}(n,x)$ and Λ_m are the mth zeros of $\text{BesselJ}(1,x)$. These have to be found numerically. This is animation of the solution for0.18 seconds. (Animation will only show in the HTML version)

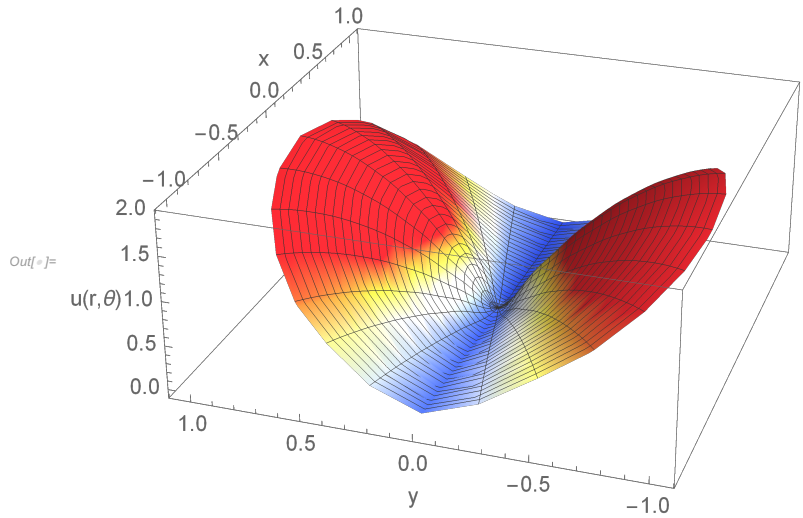

Figure 3.194: Initial state

Source code used for the above

```
In[ ]:= Get["G:\\nabbasi\\data\\mathematica_stuff\\besselZeros_old_package\\BesselZeros.m"];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
L = 1;
k = 1; (*Thermal diffusivity m^2/second*)
maxN = 10;
maxM = 10;
f = (2 * L * r - r^2)  θ Sin[θ] Exp[Cos[θ]] ;
besselJ1Zero = N@Table[BesselJZero[1, m], {m, 1, maxM}];
besselJnPrimeZero = N@Table[BesselJPrimeZeros[n, m][[m]], {n, 1, maxN}, {m, 1, maxM}];
beta0 = 1/(π L^2) Integrate[f * r, {θ, -Pi, Pi}, {r, 0, L}];
alpha0M = Table[Integrate[f * r BesselJ[0, besselJ1Zero[[m]]/L r], {θ, -Pi, Pi}, {r, 0, L}] / (2 π Integrate[r BesselJ[0, besselJ1Zero[[m]]/L r]^2, {r, 0, L}]), {m, 1, maxM}];
Anm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Cos[n θ], {θ, -Pi, Pi}, {r, 0, L}] / Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Cos[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
Bnm = Table[Integrate[f * r BesselJ[n, besselJnPrimeZero[[n,m]]/L r] Sin[n θ], {θ, -Pi, Pi}, {r, 0, L}] / Integrate[r BesselJ[n, besselJnPrimeZero[[n,m]]/L r]^2 Sin[n θ]^2, {θ, -Pi, Pi}, {r, 0, L}], {n, 1, maxN}, {m, 1, maxM}];
u[r_, θ_, t_] := beta0 +
   Sum[ alpha0M[[m]] BesselJ[0, besselJ1Zero[[m]]/L r] Exp[-k (besselJ1Zero[[m]]/L)^2 t] , {m, 1, maxM}] +
   Sum[Sum[Exp[-k (besselJnPrimeZero[[n, m]]/L)^2 t] BesselJ[n, besselJnPrimeZero[[n, m]]/L r] (Anm[[n, m]] Cos[n θ] + Bnm[[n, m]] Sin[n θ]), {n, 1, maxN}], {m, 1, maxM}];
```

Figure 3.195: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Row[{"Current temperature in the middle of disk is ", padIt2[u[0, 0, t], {4, 3}], " degrees"}]},
              {Row[{"Initial  temperature u = (2*L*r-r^2)   θ Sin[θ] Exp[Cos[θ]] "}]},
              {ParametricPlot3D[{r Cos[θ], r Sin[θ], u[r, θ, t]}, {r, 0, 1}, {θ, -Pi, Pi},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {Automatic, Automatic, {-.1, 2}},
                  ImageSize → 500,
                  ColorFunctionScaling → False,
                  ColorFunction → ColorData[{"TemperatureMap", {0, .8}}],
                  AxesLabel → {"x", "y", "u(r,θ)"},
                  ViewPoint → {-3.70, -1.245, 1.9}
                ]
              }}],
          {t, 0, 0.18, .001}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.12, {Length[tab]}]]

Out[•]= anim.gif
```

Figure 3.196: Code for animation

3.3 Diffusion in 3D

Local contents

3.3.1 Spherical coordinates 758
3.3.2 Cylinderical coordinates 759

3.3.1 Spherical coordinates

Local contents

3.3.1.1 [273] No angle dependencies 758

3.3.1.1 [273] No angle dependencies

problem number 273

Added March 28, 2019.

Problem 1, section 41, Fourier series and boundary value problems 8th edition by Brown and Churchill.

Solve $u_t = \nabla u$ where $\nabla u = \frac{1}{r}(ru)_{rr}$ in Spherical coordinates with initial conditions $u(r,0) = 0$ and boundary conditions $u(1,t) = t$

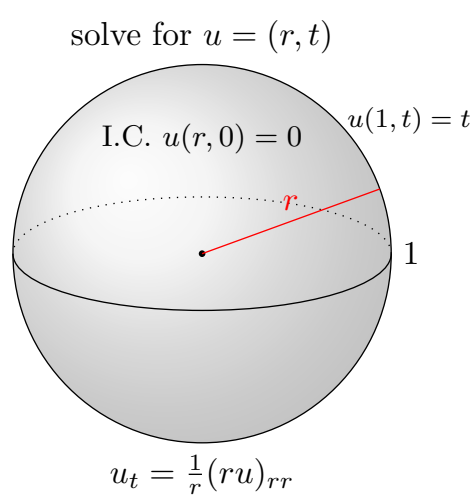

Figure 3.197: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], t] == (k*D[r*u[r, t], {r, 2}])/r;
ic  = u[r, 0] == 0;
bc  = u[1, t] == t;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

$$\left\{\left\{u(r,t)\to \sum_{K[1]=1}^{\infty}\frac{2(-1)^{K[1]}\left(1-e^{-k\pi^2 tK[1]^2}\right)\sin(\pi rK[1])}{k\pi^3 rK[1]^3}+t\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,t),t)= k/r*diff(r*u(r,t),r$2);
ic  := u(r,0)=0;
bc  := u(1,t) =t;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t)=\frac{\mathcal{L}^{-1}\left(\frac{\sinh\left(\frac{r\sqrt{s}}{\sqrt{k}}\right)}{s^2\sinh\left(\frac{\sqrt{s}}{\sqrt{k}}\right)},s,t\right)}{r}$$

Has unresolved Laplace integrals

3.3.2 Cylinderical coordinates

Local contents

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3.3.2.1 [274] Haberman 7.9.4 (a)

problem number 274

Added May 26, 2019.

Problem 7.9.4 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r,z)$ independent of θ if the boundary conditions are $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u(a,\theta,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of the full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

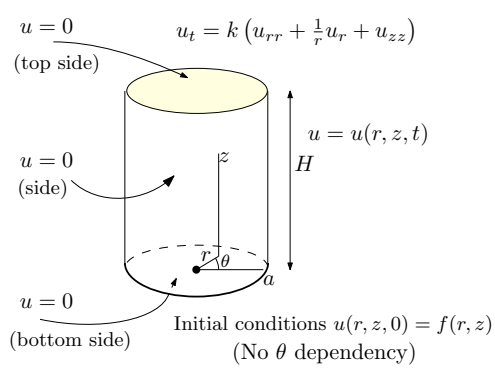

Figure 3.198: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r,  z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, 0, t] == 0, u[r, H, t] == 0, u[a, z,t] == 0};
ic  = u[r, z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r, z,t], {r, z
```

$$\left\{\left\{u(r,z,t)\to\{\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\exp\left(-kt\left(\frac{\left(j_{0,K[3]}\right)^2}{a^2}+\frac{\pi^2K[1]^2}{H^2}\right)\right)J_0\left(\frac{rj_{0,K[3]}}{a}\right)\left(\int_0^a\int_0^H rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,z)\sin\left(\frac{\pi zK[1]}{H}\right)dzdr\right)}{a^2HJ_1\left(j_{0,K[3]}\right)^2}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,0,t)=0,u(r,H,t)=0, u(a,z,t)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

$$u(r,z,t)=\sum_{n1=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\,\mathrm{BesselJ}\left(0,\frac{r\lambda_{n1}}{a}\right)\left(\left(\int_0^a r\,\mathrm{BesselJ}\left(0,\frac{r\lambda_{n1}}{a}\right)()\,dr\right)_{AllSolutions}\right)\mathrm{e}^{-\frac{\left(H^2\lambda_{n1}^2+\pi^2a^2n^2\right)kt}{H^2a^2}}\sin\left(\frac{\pi nz}{H}\right)}{Ha^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2}\right],[1,2],-\lambda_{n1}^2\right)}$$

<u>Hand solution</u>

Solve $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0)=f(r,z)$ independent of θ if the boundary conditions are $u(r,0,t)=0, u(r,H,t)=0, u(a,z,t)=0$

Let $u(r,z,t)=T(t)\,\Phi(r,z)$. Substituting into the PDE gives

$$T'(t)\,\Phi(r,z)=k\left(T(t)\,\nabla^2\Phi\right)$$
$$\frac{T'}{kT}=\frac{\nabla^2\Phi}{\Phi\,(r,z)}=-\lambda$$

Where λ is the separation constant assumed positive. This gives $T'+\lambda kT=0$ with solution $T(t)=Ce^{-k\lambda t}$. And

$$\nabla^2\Phi+\lambda\Phi(r,z)=0 \tag{1}$$

Where $\nabla^2\Phi(r,z) = \Phi_{rr} + \frac{1}{r}\Phi_r + \Phi_{zz}$ in this case, the Laplacian in cylindrical with no θ dependency. Let $\Phi(r,z) = R(r)\,Z(z)$. Substituting in (1) gives

$$R''Z + \frac{1}{r}R' + Z'' + \lambda RZ = 0$$

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = -\frac{Z''}{Z} = \upsilon \tag{2}$$

Where υ is the second separation constant assumed positive. This gives

$$Z'' + \upsilon Z = 0$$

$$Z(z) = A\cos\left(\sqrt{\upsilon}z\right) + B\sin\left(\sqrt{\upsilon}z\right)$$

At $z = 0, Z = 0$, hence $A = 0$ and the solution becomes $Z(z) = B\sin\left(\sqrt{\upsilon}z\right)$. At $z = H, Z = 0$, hence for non-trivial solution we want $\sqrt{\upsilon}H = n\pi, n = 1,2,3,\cdots$. Therefore

$$\upsilon_n = \left(\frac{n\pi}{H}\right)^2 \qquad n = 1,2,3,\cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \sin\left(\frac{n\pi}{H}z\right)$$

From (2) the radial equation becomes

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = \left(\frac{n\pi}{H}\right)^2$$

$$R'' + \frac{1}{r}R' + \left(\lambda - \left(\frac{n\pi}{H}\right)^2\right)R = 0$$

This is Bessel ODE. The solution is

$$R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right) + C_2 Y_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right)$$

Since Y_0 blows up at $r = 0$, it is discarded leaving $R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,r\right)$. At $r = a, R_n(a) = 0$. For non-trivial solution we want $J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,a\right) = 0$. Therefore $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\,a$ are the zeros of Bessel function $J_0(x)$. This allows us to determine all possible values of $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}$ (since a is given constant). Let the mth zero of $J_0(x)$ be

called Λ_m. Hence

$$\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} a = \Lambda_m \qquad m = 1, 2, 3, \cdots$$

$$\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} = \frac{\Lambda_m}{a}$$

$$\lambda_{nm} = \frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2 \qquad n = 1, 2, 3, \cdots, m = 1, 2, 3, \cdots$$

For each n, there are m values of λ_{nm}. The corresponding radial eigenfunction is

$$R_m(r) = J_0\left(\frac{\Lambda_m}{a} r\right) \qquad m = 1, 2, 3, \cdots$$

Therefore the complete solution is

$$\begin{aligned} u(r, z, t) &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} e^{-k\lambda_{nm} t} \sin\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} e^{-k\left(\frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2\right) t} \sin\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) \end{aligned}$$

What is left is to determine A_{nm}. This is done using initial conditions by using orthogonality. At $t = 0$

$$f(r, z) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right)$$

Multiplying both sides by $r J_0\left(\frac{\Lambda_{m'}}{a} r\right)$ and integrating

$$\int_0^a f(r, z) J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr = \int_0^a \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr$$

$$\int_0^a f(r, z) J_0\left(\frac{\Lambda_m}{a} r\right) r dr = \sum_{n=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{H} z\right) \int_0^a J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr$$

Multiplying both sides by $\sin\left(\frac{n'\pi}{H} z\right)$ and integrating

$$\int_0^H \left(\int_0^a f(r, z) J_0\left(\frac{\Lambda_m}{a} r\right) r dr\right) \sin\left(\frac{n'\pi}{H} z\right) dz = \int_0^H \sum_{n=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{H} z\right) \sin\left(\frac{n'\pi}{H} z\right) \left(\int_0^a J_0^2\left(\frac{\Lambda_m}{a} r\right) r$$

$$\int_0^H \int_0^a f(r, z) J_0\left(\frac{\Lambda_m}{a} r\right) \sin\left(\frac{n\pi}{H} z\right) r dr dz = A_{nm} \int_0^H \int_0^a \sin^2\left(\frac{n\pi}{H} z\right) J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr dz$$

$$A_{nm} = \frac{\int_0^H \int_0^a f(r, z) J_0\left(\frac{\Lambda_m}{a} r\right) \sin\left(\frac{n\pi}{H} z\right) r dr dz}{\int_0^H \int_0^a \sin^2\left(\frac{n\pi}{H} z\right) J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr dz}$$

Hence the final solution is

$$u(r,z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\sin\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a \sin^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}e^{-k\left(\frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2\right)t}\sin\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

Where Λ_m is the mth zero of $J_0(x)$. To verify the solution, it is compared to numerical solution, using the following values $a=1, H=3, k=\frac{1}{100}, f(r,z)=(a-r)\sin\left(\frac{z}{H}\pi\right)$. The summation was taken up to $n=10, m=10$. This animation only looks at cross section of the cylinder in the middle. The height indicates the amount of heat. As time passes the cylinder cools down. It runs for 15 seconds. (The animation will only show on the HTML version, not the PDE version of this report).

The following is the source code used to generate the above

```
ClearAll[t, r, m, n, f];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
      NumberPadding → {"0", "0"}, SignPadding → True];
nTerms = 10;
a = 1;
H = 3;
k = 1 / 100;
f[r_, z_, theta_] := (a - r) Sin[Pi / H * z];
(*f[r_,z_,theta_]:=Sin[r Pi]Sin[z/H *Pi];*)
lam = Table[ BesselJZero[0, m], {m, 1, nTerms}] // N;
A = Table[ Integrate[Integrate[f[r, z, 0] * BesselJ[0, lam[[m]] / a * r ] * r, {r, 0, a}] * Sin[n * Pi / H * z], {z, 0, H}] / Integrate[Integrate[BesselJ[0, lam[[m]] / a * r ]^2 * r, {r, 0, a}] Sin[n * Pi / H * z]^2, {z, 0, H}],
      {n, 1, nTerms}, {m, 1, nTerms}];
mySol[r_, z_, t_] := Sum[Sum[ A[[n, m]] * Exp[-k * (lam[[m]]^2 / a^2 + (n * Pi / H)^2) t] * Sin[n Pi / H z] BesselJ[0, lam[[m]] * r / a],
        {n, 1, nTerms}], {m, 1, nTerms}];
```

Figure 3.199: Code to generate solution

```
In[•]:= framesMySolVer2 = Table[
          Print["t=", t];
          Grid[{
              {Row[{"current time = ", padIt2[t, {3, 2}]}]},
              {Row[{"Tempreature in middle of cylinder = ", padIt2[mySol[0, 1.5, t], {5, 4}]}]},
              {Plot3D[mySol[Sqrt[x^2 + y^2], 1.5, t],
                  Element[{x, y}, Disk[{0, 0}, 1]],
                  PerformanceGoal → "Speed",
                  PlotRange → {All, All, {0, 1}}, Mesh → Full,
                  ImageSize → 400
                ]
              }}],
          {t, 0, 15, 0.1}];

In[•]:= Manipulate[
          framesMySolVer2[[i]],
          {{i, 1, "time"}, 1, Length@framesMySolVer2, 1, Appearance → "Labeled"}
        ]

In[•]:= Export["anim_plot3d_3.gif", framesMySolVer2,
          "DisplayDurations" → Table[0.05, {Length[framesMySolVer2]}]]
```

Figure 3.200: Code to make animation and export it

3.3.2.2 [275] Haberman 7.9.4 (b)

problem number 275

Added May 26, 2019.

Problem 7.9.4 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ independent of θ, subject to boundary conditions $u_z(r,0,t) = 0$, $u_z(r,H,t) = 0$, $u_r(a,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of the full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

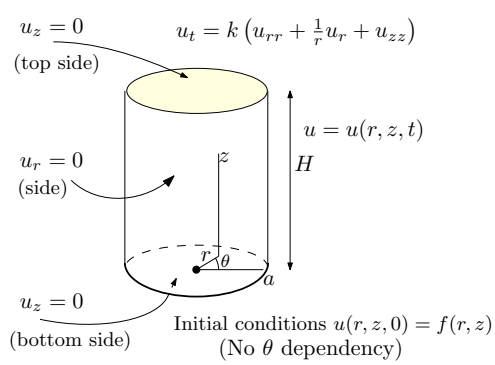

Figure 3.201: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, z, t], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,1,0][u][r,0, t] == 0, Derivative[0,1,0][u][r,  H, t] == 0, Derivative[1,
ic  = u[r,z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r,  z,t], {r,
```

$$\left\{\left\{u(r,z,t)\to\ \{\ \frac{2\int_0^a\int_0^H rf(r,z)dzdr}{a^2H}+\sum_{K[1]=1}^{\infty}\frac{4e^{-\frac{k\pi^2tK[1]^2}{H^2}}\cos\left(\frac{\pi zK[1]}{H}\right)\int_0^a\int_0^H r\cos\left(\frac{\pi zK[1]}{H}\right)f(r,z)dzdr}{a^2H}+\sum_{K[3]=1}^{\infty}\frac{2e^{-}}{a^2H\sqrt{J_0(}}\right.\right.$$

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc:=eval(diff(u(r,z,t),z),z=0)=0,eval(diff(u(r,z,t),z),z=H)=0, eval(diff(u(r,z,t),r),r=a)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

sol=()

Hand solution

Solve $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ independent of θ if the boundary conditions are $u_z(r,0,t) = 0, u_z(r,H,t) = 0, u_r(a,z,t) = 0$

Let $u(r,z,t) = T(t)\,\Phi(r,z)$. Substituting into the PDE gives

$$T'(t)\,\Phi(r,z) = k\big(T(t)\,\nabla^2\Phi\big)$$
$$\frac{T'}{kT} = \frac{\nabla^2\Phi}{\Phi\,(r,z)} = -\lambda$$

Where λ is the separation constant assumed positive. This gives $T' + \lambda kT = 0$ with solution $T(t) = Ce^{-k\lambda t}$. And

$$\nabla^2\Phi + \lambda\Phi(r,z) = 0 \tag{1}$$

Where $\nabla^2\Phi(r,z) = \Phi_{rr} + \frac{1}{r}\Phi_r + \Phi_{zz}$ in this case, the Laplacian in cylindrical with no θ

dependency. Let $\Phi(r,z) = R(r)\,Z(z)$. Substituting in (1) gives

$$R''Z + \frac{1}{r}R' + Z'' + \lambda RZ = 0$$

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = -\frac{Z''}{Z} = \upsilon \tag{2}$$

Where υ is the second separation. Then $Z'' + \upsilon Z = 0$

<u>case $\upsilon = 0$</u>

This gives

$$Z(z) = Az + B$$
$$Z' = A$$

At $z = 0, Z' = 0$, hence $A = 0$. The solution becomes $Z(z) = B$ and $Z'(z) = 0$. Which satisfies the boundary conditions at $z = H$. Hence $\upsilon = 0$ is eigenvalue with $Z_0 = 1$ as eigenfunction.

<u>case $\upsilon > 0$</u>

$$Z'' + \upsilon Z = 0$$
$$Z(z) = A\cos\left(\sqrt{\upsilon}z\right) + B\sin\left(\sqrt{\upsilon}z\right)$$
$$Z'(z) = -A\sqrt{\upsilon}\sin\left(\sqrt{\upsilon}z\right) + B\sqrt{\upsilon}\cos\left(\sqrt{\upsilon}z\right)$$

At $z = 0, Z' = 0$, hence $B = 0$ and the solution becomes $Z(z) = A\cos\left(\sqrt{\upsilon}z\right)$ and $Z'(z) = -A\sqrt{\upsilon}\sin\left(\sqrt{\upsilon}z\right)$. At $z = H, Z_t = 0$, hence for non-trivial solution we want $\sqrt{\upsilon}H = n\pi, n = 1,2,3,\cdots$. Therefore

$$\upsilon_n = \left(\frac{n\pi}{H}\right)^2 \qquad n = 1,2,3,\cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \cos\left(\frac{n\pi}{H}z\right)$$

From (2) the radial equation becomes

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda = \upsilon$$

<u>case $\upsilon = 0$</u>

The above becomes

$$R_0'' + \frac{1}{r}R_0' + R_0\lambda_0 = 0$$

This is Bessel ODE whose solution is

$$R_0(r) = c_1 J_0\left(\sqrt{\lambda_0} r\right) + c_2 Y_0\left(\sqrt{\lambda_0} r\right)$$

Since Y_0 blows at $r = 0$ the solution becomes

$$\begin{aligned} R_0(r) &= c_1 J_0\left(\sqrt{\lambda_0} r\right) \\ R_0'(r) &= c_1 J_0'\left(\sqrt{\lambda_0} r\right) \\ &= -c_1 \sqrt{\lambda_0} J_1\left(\sqrt{\lambda_0} r\right) \end{aligned}$$

At $r = a$ then $R_0'(a) = 0$. For nontrivial solution we want $J_1\left(\sqrt{\lambda} a\right) = 0$. Hence $\sqrt{\lambda} a$ are zeros of $J_1(x)$. This determines λ. Let Λ_m be the zeros of $J_1(x)$, therefore

$$\begin{aligned} \sqrt{\lambda_0} a &= \Lambda_m \qquad n = 0, m = 1, 2, 3, \cdots \\ \lambda_{0,m} &= \left(\frac{\Lambda_m}{a}\right)^2 \qquad m = 1, 2, 3, \cdots \end{aligned}$$

Hence the radial eigenfunction for $v = 0$ is given by

$$R_{0,m}(r) = c_1 J_0\left(\frac{\Lambda_m}{a} r\right) \qquad m = 1, 2, 3, \cdots$$

<u>case $v > 0$</u>

$$\begin{aligned} \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda &= \left(\frac{n\pi}{H}\right)^2 \\ R'' + \frac{1}{r} R' + \left(\lambda - \left(\frac{n\pi}{H}\right)^2\right) R &= 0 \end{aligned}$$

This is Bessel ODE. The solution is

$$R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right) + C_2 Y_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$$

Since Y_0 blows up at $r = 0$, it is discarded leaving $R_n(r) = C_1 J_0\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$. Hence $R_n'(r) = C_1 J_0'\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right) = -C_1 \sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} J_1\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} r\right)$

At $r = a$, $R_n'(a) = 0$. For non-trivial solution we want $J_1\left(\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} a\right) = 0$. Therefore $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} a$ are the zeros of Bessel function $J_1(x)$. This allows us to determine all

possible values of $\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}$ (since a is given constant). Let the mth zero of $J_1(x)$ be called Λ_m. Hence

$$\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2}\, a = \Lambda_m \qquad m = 1, 2, 3, \cdots$$

$$\sqrt{\lambda - \left(\frac{n\pi}{H}\right)^2} = \frac{\Lambda_m}{a}$$

$$\lambda_{nm} = \frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2 \qquad n = 1, 2, 3, \cdots, m = 1, 2, 3, \cdots$$

For each n, there are m values of λ_{nm}. The corresponding radial eigenfunction is

$$R_m(r) = J_0\left(\frac{\Lambda_m}{a} r\right) \qquad m = 1, 2, 3, \cdots$$

Now we look at the time solution again. $T(t) = Ce^{-k\lambda t}$. For $n = 0$ this gives $T_{0,m}(t) = C_{0,m} e^{-k\lambda_{0,m} t} = C_{0,m} e^{-k\left(\frac{\Lambda_m}{a}\right)^2 t}$. For $n > 0$ the solution becomes

$$T_{n,m}(t) = C_{n,m} e^{-k\lambda_{n,m} t} = Ce^{-k\left(\frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2\right) t}$$

Therefore the complete solution is

$$\begin{aligned} u(r, z, t) &= \sum_{m=1}^{\infty} B_m R_{0,m}(r)\, T_{0,m}(t) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} e^{-k\lambda_{nm} t} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) \\ &= \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) e^{-k\left(\frac{\Lambda_m}{a}\right)^2 t} + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} e^{-k\left(\frac{\Lambda_m^2}{a^2} + \left(\frac{n\pi}{H}\right)^2\right) t} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right) \end{aligned}$$

What is left is to determine A_{nm} and B_m. This is done using initial conditions by using orthogonality. At $t = 0$

$$f(r, z) = \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cos\left(\frac{n\pi}{H} z\right) J_0\left(\frac{\Lambda_m}{a} r\right)$$

For $n = 0$

$$f(r, z) = \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right)$$

$$\int_0^a f(r, z)\, J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr = \int_0^a \sum_{m=1}^{\infty} B_m J_0\left(\frac{\Lambda_m}{a} r\right) J_0\left(\frac{\Lambda_{m'}}{a} r\right) r dr$$

$$\int_0^a f(r, 0)\, J_0\left(\frac{\Lambda_m}{a} r\right) r dr = \int_0^a B_m J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr$$

$$B_m = \frac{\int_0^a f(r, 0)\, J_0\left(\frac{\Lambda_m}{a} r\right) r dr}{\int_0^a J_0^2\left(\frac{\Lambda_m}{a} r\right) r dr} \qquad m = 1, 2, 3, \cdots$$

Integrating again both sides over z gives (we can think of $\cos\left(\frac{n\pi}{H}z\right)=1$ with $n=0$ as the eigenfunction in this case).

$$\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz = B_m\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz$$

$$B_m=\frac{\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz}{\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

For $n>0$

$$f(r,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

Multiplying both sides by $rJ_0\left(\frac{\Lambda_{m'}}{a}r\right)$ and integrating

$$\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr=\int_0^a\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)J_0\left(\frac{\Lambda_{m'}}{a}r\right)rdr$$

$$\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr=\sum_{n=1}^{\infty}A_{nm}\cos\left(\frac{n\pi}{H}z\right)\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdr$$

Multiplying both sides by $\cos\left(\frac{n'\pi}{H}z\right)$ and integrating

$$\int_0^H\left(\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdr\right)\cos\left(\frac{n'\pi}{H}z\right)dz=\int_0^H\sum_{n=1}^{\infty}A_{nm}\sin\left(\frac{n\pi}{H}z\right)\cos\left(\frac{n'\pi}{H}z\right)\left(\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)r\right.$$

$$\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz=A_{nm}\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz$$

$$A_{nm}=\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

Hence the final solution is

$$u(r,z,t)=\sum_{m=1}^{\infty}B_mJ_0\left(\left(\frac{\Lambda_m}{a}\right)^2r\right)e^{-k\left(\frac{\Lambda_m}{a}\right)^2t}+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}e^{-k\left(\frac{\Lambda_m^2}{a^2}+\left(\frac{n\pi}{H}\right)^2\right)t}\cos\left(\frac{n\pi}{H}z\right)J_0\left(\frac{\Lambda_m}{a}r\right)$$

With

$$A_{nm}=\frac{\int_0^H\int_0^a f(r,z)\,J_0\left(\frac{\Lambda_m}{a}r\right)\cos\left(\frac{n\pi}{H}z\right)rdrdz}{\int_0^H\int_0^a\cos^2\left(\frac{n\pi}{H}z\right)J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

$$B_m=\frac{\int_0^H\int_0^a f(r,0)\,J_0\left(\frac{\Lambda_m}{a}r\right)rdrdz}{\int_0^H\int_0^a J_0^2\left(\frac{\Lambda_m}{a}r\right)rdrdz}$$

Where Λ_m is the mth zero of $J_1(x)$.

To verify the solution, it is compared to numerical solution, using the following values $a = 1, H = 3, k = \frac{1}{100}, f(r,z) = (a-r)\sin\left(\frac{z}{H}\pi\right)$. The summation was taken up to $n = 10, m = 10$. This animation only looks at cross section of the cylinder in the middle. The height indicates the amount of heat. As time passes the initial temperature averages inside (cylinder is insulated). It runs for 15 seconds. (The animation will only show on the HTML version, not the PDE version of this report).

The following is the source code used to generate the above

```
In[•]:= ClearAll[t, r, m, n, f, A, B];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
      NumberPadding → {"0", "0"}, SignPadding → True];
nTerms = 10;
a = 1;
H = 3;
k = 1 / 100;
f[r_, z_, theta_] := (a - r) Sin[Pi / H * z];
lam = Table[ BesselJZero[1, m], {m, 1, nTerms}] // N;

B = Table[ Integrate[Integrate[f[r, 0, 0] * BesselJ[0, lam[[m]]/a r] r, {r, 0, a}], {z, 0, H}] / Integrate[Integrate[BesselJ[0, lam[[m]]/a * r]^2 * r, {r, 0, a}], {z, 0, H}], {m, 1, nTerms}];
A = Table[ Integrate[Integrate[f[r, z, 0] * BesselJ[0, lam[[m]]/a r] r, {r, 0, a}] Cos[n Pi/H * z], {z, 0, H}] / Integrate[Integrate[BesselJ[0, lam[[m]]/a r]^2 r, {r, 0, a}] Cos[n Pi/H * z]^2, {z, 0, H}], {n, 1, nTerms}, {m, 1, nTerms}];
mySol[r_, z_, t_] := Sum[B[[m]] Exp[-k (lam[[m]]^2/a^2) t] BesselJ[0, (lam[[m]]/a)^2 r], {m, 1, nTerms}] +
    Sum[Sum[ A[[n, m]] * Exp[-k (lam[[m]]^2/a^2 + (n Pi/H)^2) t] Cos[n Pi/H * z] BesselJ[0, lam[[m]]/a r], {n, 1, nTerms}], {m, 1, nTerms}];
```

Figure 3.202: Code to generate solution

```
In[•]:= framesMySolVer2 = Table[
          Print["t=", t];
          Grid[{
              {Row[{"current time = ", padIt2[t, {3, 2}]}]},
              {Row[{"Tempreature in middle of cylinder = ", padIt2[mySol[0, 1.5, t], {5, 4}]}]},
              {Plot3D[mySol[Sqrt[x^2 + y^2], 1.5, t],
                  Element[{x, y}, Disk[{0, 0}, 1]],
                  PerformanceGoal → "Speed",
                  PlotRange → {All, All, {-0.125, .22}}, Mesh → Full,
                  ImageSize → 400
                ]
              }}],
          {t, 0, 15, 0.1}];

In[•]:= Manipulate[
        framesMySolVer2[[i]],
        {{i, 1, "time"}, 1, Length@framesMySolVer2, 1, Appearance → "Labeled"}
      ]

In[•]:= Export["anim.gif", framesMySolVer2,
        "DisplayDurations" → Table[0.1, {Length[framesMySolVer2]}]]

Out[•]= anim.gif
```

Figure 3.203: Code to make animation and export it

3.3.2.3 [276] Haberman 7.9.4 (c)

problem number 276

Added May 26, 2019.

Problem 7.9.4 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside cylinder with radius a and height H with initial conditions $u(r,z,0) = f(r,z)$ subject to boundary conditions $u(r,0,t) = 0$, $u(r,H,t) = 0$, $u_r(a,z,t) = 0$.

Since it says independent of θ, will use the PDE as

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + u_{zz}\right)$$

Instead of full Laplacian

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

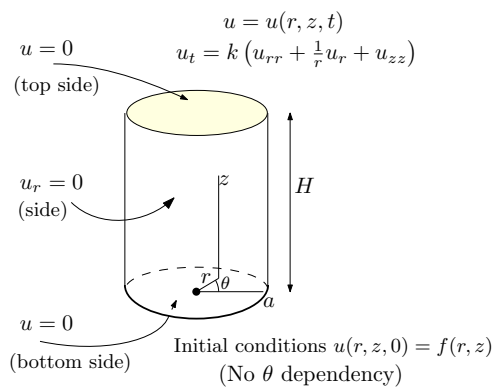

Figure 3.204: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, 0, t] == 0, u[r,  H, t] == 0, Derivative[1,0,0][u][a, z,t] == 0};
ic  = u[r, z, 0] == f[r,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,z,t],t]==k*lap, bc, ic}, u[r, z,t], {r, z
```

$$\left\{\left\{u(r,z,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\frac{4e^{-\frac{k\pi^2 tK[1]^2}{H^2}}\left(\int_0^a\int_0^H rf(r,z)\sin\left(\frac{\pi zK[1]}{H}\right)dzdr\right)\sin\left(\frac{\pi zK[1]}{H}\right)}{a^2H}+\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\exp\left(-kt\left(\frac{\pi^2K[1]^2}{H^2}+K[2\right.\right.}{a^2H\sqrt{J_0}}\right.\right.$$

Indetermi

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,0,t)=0,u(r,H,t)=0, eval(diff(u(r,z,t),r),r=a)=0;
ic  := u(r,z,0) = f(r,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,z,t),t) = k*lap,
```

sol=()

3.3.2.4 [277] Haberman 7.9.3 (a)

problem number 277

Added May 26, 2019.

Problem 7.9.3 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u(r,0,z,t) = 0$, $u(r,\frac{\pi}{2},z,t) = 0$, $u(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

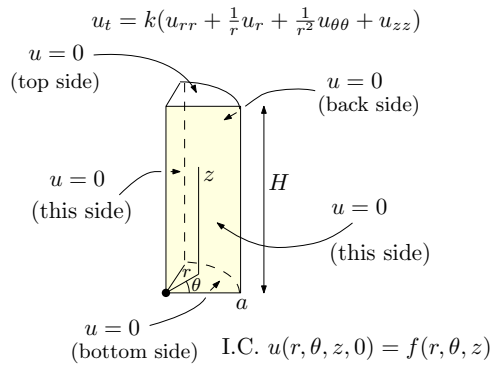

Figure 3.205: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0, t] == 0, u[r, theta, H, t] == 0, u[r, 0, z,t] == 0,u[r,Pi/2,z,t]==0,u[
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc, ic}, u[r, theta
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,theta,0,t)=0,u(r,theta,H,t)=0, u(r,0,z,t)=0, u(r,Pi/2,z,t)=0,u(a,Pi,z,t)=0;
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

3.3.2.5 [278] Haberman 7.9.3 (b)

problem number 278

Added May 26, 2019.

Problem 7.9.3 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u_z(r,\theta,0,t) = 0$, $u_z(r,\theta,H,t) = 0$, $u_\theta(r,0,z,t) = 0$, $u_\theta(r,\frac{\pi}{2},z,t) = 0$, $u_r(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

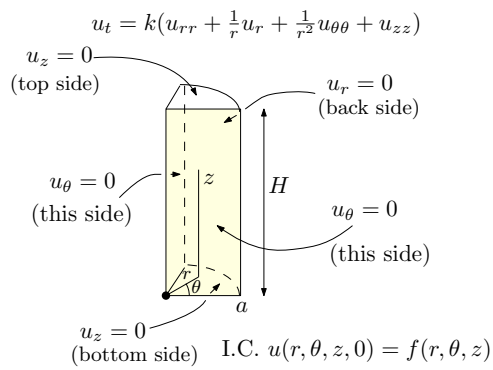

Figure 3.206: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1,0][u][r, theta, 0, t] == 0, Derivative[0,0,1,0][u][r, theta, H, t] =
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc}, u[r, theta, z,
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc:=eval(diff(u(r,theta,z,t),z),z=0)=0,eval(diff(u(r,theta,z,t),z),z=H)=0,eval(diff(u(r,thet
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

3.3.2.6 [279] Haberman 7.9.3 (c)

problem number 279

Added May 26, 2019.

Problem 7.9.3 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Heat PDE $u_t = k\nabla^2 u$ inside quarter circular cylinder $0 < \theta < \frac{\pi}{2}$ with radius a and height H with initial conditions $u(r,\theta,z,0) = f(r\theta,z)$ subject to boundary conditions $u(r,\theta,0,t) = 0$, $u(r,\theta,H,t) = 0$, $u_\theta(r,0,z,t) = 0$, $u(r,\frac{\pi}{2},z,t) = 0$, $u_r(a,\theta,z,t) = 0$.

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}\right)$$

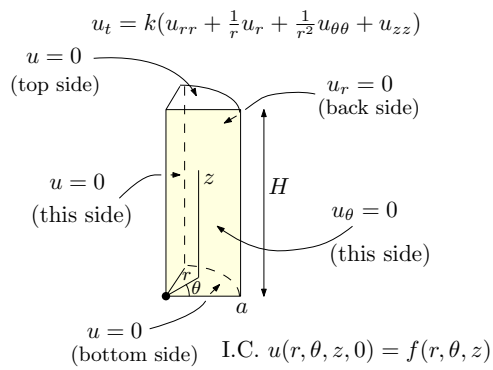

Figure 3.207: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z, t], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0, t] == 0, u[r, theta, H, t] == 0, u[r, 0, z,t] == 0,u[r,Pi/2,z,t]==0,u[
ic  = u[r, theta, z, 0] == f[r,theta,z];
sol = AbsoluteTiming[TimeConstrained[DSolve[{D[u[r,theta,z,t],t]==k*lap, bc}, u[r, theta, z,
```

Failed

Maple ✗

```
restart;
lap:=VectorCalculus:-Laplacian(u(r,theta,z,t),'cylindrical'[r,theta,z]);
bc  := u(r,theta,0,t)=0,u(r,theta,H,t)=0, eval(diff(u(r,theta,z,t),theta),theta=0)=0, u(r,Pi
ic  := u(r,theta,z,0) = f(r,theta,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([diff(u(r,theta,z,t),t) =
```

sol=()

CHAPTER 4

Elliptic PDE's (Laplace, Poisson, Helmholtz)

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4.1 Laplace in 2D

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4.1.1 Cartesian coordinates

Local contents

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problem number 280

Added Nov 20, 2019

Solve Laplace PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned} u(x,0) &= f(x) \\ u(x,H) &= 0 \\ u(0,y) &= 0 \\ u(L,y) &= 0 \end{aligned}$$

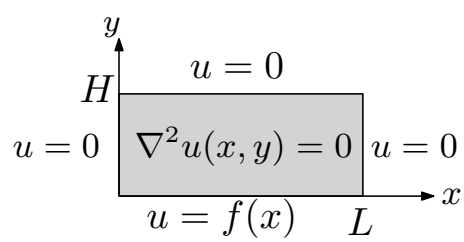


Figure 4.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[x,0]==f[x], u[x, H] == 0, u[0, y] == 0,u[L,y]==0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} \operatorname{csch}\left(\frac{Hn\pi}{L}\right) \text{FourierSinCoefficient}\left[f(x), x, n, \text{FourierParameters} \to \left\{1, \frac{\pi}{L}\right\}\right] \sin\left(\frac{n\pi x}{L}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(x,0)=f(x), u(x, H) = 0, u(0,y) = 0,u(L,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}-\frac{2\left(\mathrm{e}^{\frac{\pi n y}{L}}-\mathrm{e}^{-\frac{\pi(-2H+y)n}{L}}\right)\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\sin\left(\frac{\pi n x}{L}\right)}{\left(\mathrm{e}^{\frac{2\pi H n}{L}}-1\right)L}$$

Hand solution

Solve

$$\begin{aligned}\nabla^2 u &= 0 \qquad \text{on a rectangle} \qquad R=\{0<x<a, 0<y<b\}\\ u(x,0) &= f(x)\\ u(x,H) &= 0\\ u(0,y) &= 0\\ u(L,y) &= 0\end{aligned}$$

Solution

Let $u(x,y)=X(x)\,Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above.

$$\frac{X''}{X}=-\frac{Y''}{Y}=-\lambda$$

Two ODE's are obtained

$$X''+\lambda X=0 \tag{1}$$

With the boundary conditions

$$X(0) = 0$$
$$X(L) = 0$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$Y(0) = f(x)$$
$$Y(H) = 0$$

<u>Case $\lambda < 0$</u>

The solution to (1) is

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = L$ this gives $X = B\sinh\left(\sqrt{|\lambda|}L\right)$. But $\sinh\left(\sqrt{|\lambda|}L\right) = 0$ only at 0 and $\sqrt{|\lambda|}L \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

<u>Case $\lambda = 0$</u>

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = L$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

<u>Case $\lambda > 0$</u>

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = L$

$$0 = B\sin\left(\sqrt{\lambda}L\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$Y_n = C_n \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \tag{4}$$

Applying B.C. at $y = H$ gives

$$\begin{aligned} 0 &= C_n \cosh\left(\frac{n\pi}{L}H\right) + D_n \sinh\left(\frac{n\pi}{L}H\right) \\ C_n &= -D_n \frac{\sinh\left(\frac{n\pi}{L}H\right)}{\cosh\left(\frac{n\pi}{L}H\right)} \\ &= -D_n \tanh\left(\frac{n\pi}{L}H\right) \end{aligned}$$

Hence (4) becomes

$$\begin{aligned} Y_n &= -D_n \tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ &= D_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x,y) &= Y_n X_n \\ &= D_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right) B_n \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x,y) = B_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\sinh\left(\frac{n\pi}{L}y\right) - \tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right) \tag{5}$$

The nonhomogeneous boundary condition is now resolved. At $y = 0$

$$u(x,0) = f(x)$$

Therefore (5) becomes

$$f(x) = \sum_{n=1}^{\infty} -B_n \tanh\left(\frac{n\pi}{a}b\right)\sin\left(\frac{n\pi}{a}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx &= -\int_0^a \sin\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} B_n \tanh\left(\frac{n\pi}{L}H\right)\sin\left(\frac{n\pi}{L}x\right)dx\\ &= -\sum_{n=1}^{\infty} B_n \tanh\left(\frac{n\pi}{L}b\right)\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= -B_n \tanh\left(\frac{m\pi}{L}H\right)\left(\frac{L}{2}\right)\end{aligned}$$

Hence

$$B_n = -\frac{2}{L}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\tanh\left(\frac{n\pi}{L}H\right)}$$

The solution (5) becomes

$$\begin{aligned}u(x,y) &= -\frac{2}{L}\sum_{n=1}^{\infty}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\tanh\left(\frac{n\pi}{L}H\right)}\left(\sinh\left(\frac{n\pi}{L}y\right)-\tanh\left(\frac{n\pi}{L}H\right)\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)\\ &= -\frac{2}{L}\sum_{n=1}^{\infty}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\left(\frac{\sinh\left(\frac{n\pi}{L}y\right)}{\tanh\left(\frac{n\pi}{L}H\right)}-\cosh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

4.1.1.2 [281] Rectangle, 3 edges zero, right edge not

problem number 281

Added January 12, 2020

Solve Laplace PDE inside square $\nabla^2 u(x,y) = 0$ with $0 \le x \le 1, 0 \le y \le 1$, with following boundary conditions

$$\begin{aligned}u(x,0) &= 0\\ u(x,1) &= 0\\ u(0,y) &= 0\\ u(1,y) &= y(1-y)\end{aligned}$$

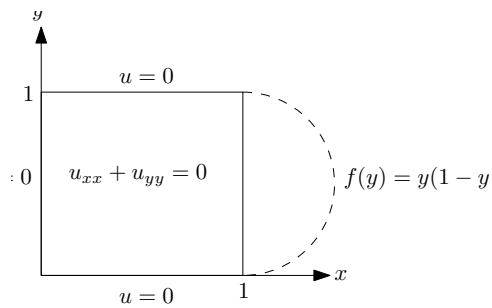


Figure 4.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1;b=1;
pde = Laplacian[u[x,y],{x,y}] == 0;
bc  = {u[x,0]==0, u[x, b] == 0, u[0, y] == 0,u[a,y]==y*(1-y)};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{4(-1+(-1)^n)\operatorname{csch}(n\pi)\sin(n\pi y)\sinh(n\pi x)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
a:=1;
b:=1;
pde := VectorCalculus:-Laplacian(u(x,y),'cartesian'[x,y])=0;
bc:=u(x,0)=0, u(x, b) = 0, u(0,y) = 0,u(a,y)=y*(1-y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) ),output
```

$$u(x,y)=\sum_{n=1}^{\infty}-\frac{4((-1)^n-1)\left(\mathrm{e}^{\pi nx}-\mathrm{e}^{-\pi nx}\right)\mathrm{e}^{\pi n}\sin(\pi ny)}{\pi^3\left(\mathrm{e}^{2\pi n}-1\right)n^3}$$

Hand solution

a is used for the length of the x dimension and b for the length of the y dimension.

Solution

Let $u(x,y) = X(x)\,Y(x)$. Substituting this into the PDE gives

$$X''Y + Y''X = 0$$

Dividing throughout by $XY \neq 0$ and simplifying gives

$$\frac{X''}{X} = -\frac{Y''}{Y} = \lambda$$

This gives the eigenvalue ODE

$$\begin{aligned} Y'' + \lambda Y &= 0 \\ Y(0) &= 0 \\ Y(b) &= 0 \end{aligned} \tag{1}$$

The solution to (1) gives the eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1, 2, 3\cdots$ and since $L = b$, this becomes

$$\lambda_n = \left(\frac{n\pi}{b}\right)^2 \qquad n = 1, 2, \cdots$$

And the corresponding eigenfunction

$$\begin{aligned} Y_n(y) &= c_n \sin\left(\sqrt{\lambda_n} y\right) \\ &= c_n \sin\left(\frac{n\pi}{b} y\right) \end{aligned}$$

Therefore the corresponding nonhomogeneous $X(x)$ ODE

$$\begin{aligned} X_n'' - \lambda_n X_n &= 0 \\ X_n(0) &= 0 \\ X_n(a) &= y - y^2 \end{aligned} \tag{2}$$

The solution to (2), since λ_n is positive is

$$\begin{aligned} X_n(x) &= A_n \cosh\left(\sqrt{\lambda_n} x\right) + B_n \sinh\left(\sqrt{\lambda_n} x\right) \\ &= A_n \cosh\left(\frac{n\pi}{b} x\right) + B_n \sinh\left(\frac{n\pi}{b} x\right) \end{aligned}$$

Boundary conditions $X(0) = 0$ gives

$$0 = A_n$$

The solution (3) now simplifies to

$$X_n(x) = B_n \sinh\left(\frac{n\pi}{b} x\right)$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,y) &= X_n Y_n \\ &= c_n \sinh\left(\frac{n\pi}{b}x\right)\sin\left(\frac{n\pi}{b}y\right) \end{aligned}$$

Where the constants B_n is merged with c_n. The solution is

$$u(x,y) = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}x\right)\sin\left(\frac{n\pi}{b}y\right) \tag{3}$$

c_n is now found by applying the boundary condition at $x = a$. The above becomes

$$y - y^2 = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}a\right)\sin\left(\frac{n\pi}{b}y\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{b}y\right)$ and integrating gives

$$\int_0^b (y-y^2)\sin\left(\frac{m\pi}{b}y\right)dy = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}a\right)\left(\int_0^b \sin\left(\frac{m\pi}{b}y\right)\sin\left(\frac{n\pi}{b}y\right)dy\right)$$

By orthogonality the above reduces to

$$\begin{aligned} \int_0^b (y-y^2)\sin\left(\frac{m\pi}{b}y\right)dy &= c_n \sinh\left(\frac{m\pi}{b}a\right)\int_0^b \sin^2\left(\frac{m\pi}{b}y\right)dy \\ &= \frac{b}{2}c_m \sinh\left(\frac{m\pi}{b}a\right) \end{aligned}$$

Therefore

$$c_n = \frac{2}{b}\frac{1}{\sinh\left(\frac{m\pi}{b}a\right)}\int_0^b (y-y^2)\sin\left(\frac{n\pi}{b}y\right)dy$$

Now replacing $\underline{a = 1, b = 1}$, the above becomes

$$\begin{aligned} c_n &= \frac{2}{\sinh(n\pi)}\int_0^1 (y-y^2)\sin(n\pi y)\,dy \\ &= \frac{2}{\sinh(n\pi)}\left(\frac{-2(-1+(-1)^n)}{n^3\pi^3}\right) \\ &= \frac{-4}{\sinh(n\pi)}\frac{(-1+(-1)^n)}{n^3\pi^3} \end{aligned}$$

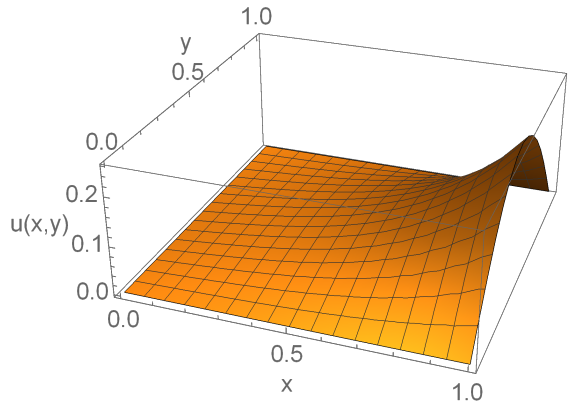
Hence the solution (3) becomes

$$u(x,y) = \frac{-4}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1+(-1)^n)}{n^3}\frac{\sinh(n\pi x)}{\sinh(n\pi)}\sin(n\pi y)$$

This is a 3D plot of the solution.

In[40]:=
```
mySol[x_, y_] := -4 / Pi^3 Sum[ (-1 + (-1)^n)/n^3 (Sinh[n Pi x]/Sinh[n Pi]) Sin[n Pi y] , {n, 1, 2}]
Plot3D[mySol[x, y], {x, 0, 1}, {y, 0, 1}, AxesLabel → {"x", "y", "u(x,y)"}, BaseStyle → 14]
```

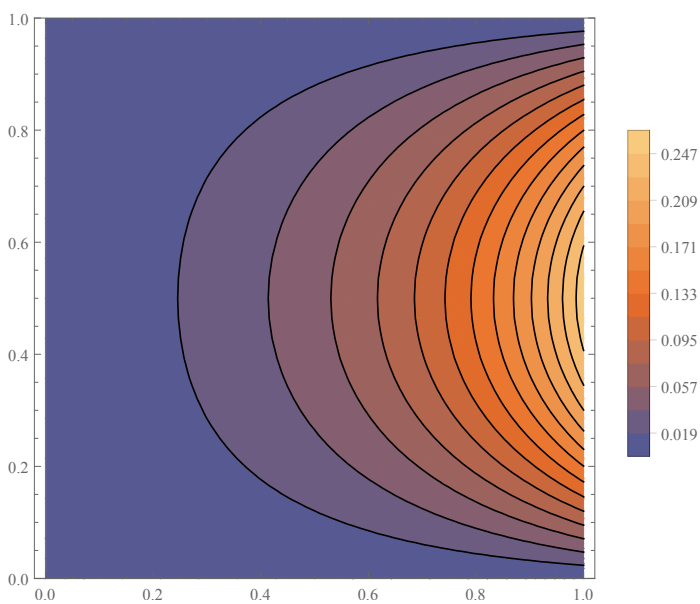
Out[41]=



This is a contour plot

```
ContourPlot[Evaluate[mySol[x, y]], {x, 0, 1}, {y, 0, 1}, AxesLabel → {x, y},
   PlotRange → {-1, 1}, Contours → 100, PlotTheme -> "Scientific", PlotLegends → Automatic]
```

Out[16]=



4.1.1.3 [282] Rectangle, 3 edges zero, buttom edge has impulse

problem number 282

Added Jan. 8, 2020.

This is Problem 6.3.10 from Introduction to Partial Differential Equations by Peter Olver ISBN 9783319020983.

Solve

$$
\begin{aligned}
\nabla^2 u &= 0 \qquad \text{on a rectangle} \qquad R = \{0 < x < a, 0 < y < b\} \\
u(x,0) &= f(x) \\
u(x,b) &= 0 \\
u(0,y) &= 0 \\
u(a,y) &= 0
\end{aligned}
$$

When the boundary data $f(x) = \delta(x - \xi)$ is a delta function at a point $0 < \xi < a$.

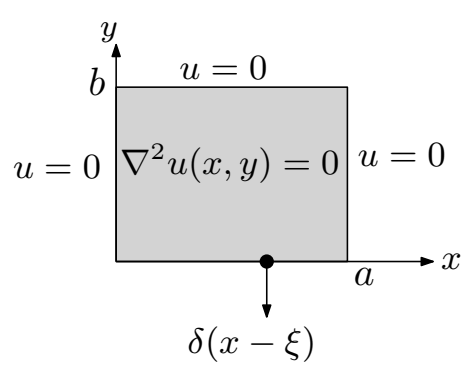


Figure 4.3: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = Laplacian[u[x, y], {x, y}] == 0;
f[x_] := DiracDelta[x - zeta];
bc = {u[x, 0] == f[x], u[x, b] == 0, u[0, y] == 0, u[a, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{2\operatorname{csch}\left(\frac{bn\pi}{a}\right)\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi\zeta}{a}\right)\sinh\left(\frac{n\pi(b-y)}{a}\right)}{a}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x->Dirac(x-zeta);
bc:=u(x,0)=f(x), u(x, b) = 0, u(0,y) = 0,u(a,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}-\frac{2\left(-\mathrm{e}^{\frac{\pi(2b-y)n}{a}}+\mathrm{e}^{\frac{\pi ny}{a}}\right)\sin\left(\frac{\pi nx}{a}\right)\sin\left(\frac{\pi n\zeta}{a}\right)}{\left(\mathrm{e}^{\frac{2\pi bn}{a}}-1\right)a}$$

<u>Hand solution</u>

Let $u(x,y)=X(x)\,Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above.

$$\frac{X''}{X}=-\frac{Y''}{Y}=-\lambda$$

Two ODE's are obtained

$$X'' + \lambda X = 0 \tag{1}$$

With the boundary conditions

$$\begin{aligned} X(0) &= 0 \\ X(a) &= 0 \end{aligned}$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$\begin{aligned} Y(0) &= f(x) \\ Y(b) &= 0 \end{aligned}$$

In all these cases λ will turn out to be positive. This is shown below.

<u>Case $\lambda < 0$</u>

The solution to (1) is

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = a$ this gives $X = B\sinh\left(\sqrt{|\lambda|}a\right)$. But $\sinh\left(\sqrt{|\lambda|}a\right) = 0$ only at 0 and $\sqrt{|\lambda|}a \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

<u>Case $\lambda = 0$</u>

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = a$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

<u>Case $\lambda > 0$</u>

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = a$

$$0 = B\sin\left(\sqrt{\lambda}a\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}a\right) = 0$ or $\sqrt{\lambda}a = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{a}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{a}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$Y_n = C_n \cosh\left(\frac{n\pi}{a}y\right) + D_n \sinh\left(\frac{n\pi}{a}y\right) \tag{4}$$

Applying B.C. at $y = b$ gives

$$\begin{aligned} 0 &= C_n \cosh\left(\frac{n\pi}{a}b\right) + D_n \sinh\left(\frac{n\pi}{a}b\right) \\ C_n &= -D_n \frac{\sinh\left(\frac{n\pi}{a}b\right)}{\cosh\left(\frac{n\pi}{a}b\right)} \\ &= -D_n \tanh\left(\frac{n\pi}{a}b\right) \end{aligned}$$

Hence (4) becomes

$$\begin{aligned} Y_n &= -D_n \tanh\left(\frac{n\pi}{a}b\right) \cosh\left(\frac{n\pi}{a}y\right) + D_n \sinh\left(\frac{n\pi}{a}y\right) \\ &= D_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right) \cosh\left(\frac{n\pi}{a}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x, y) &= Y_n X_n \\ &= D_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right) \cosh\left(\frac{n\pi}{a}y\right)\right) B_n \sin\left(\frac{n\pi}{a}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x, y) = B_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right) \cosh\left(\frac{n\pi}{a}y\right)\right) \sin\left(\frac{n\pi}{a}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x, y) = \sum_{n=1}^{\infty} B_n\left(\sinh\left(\frac{n\pi}{a}y\right) - \tanh\left(\frac{n\pi}{a}b\right) \cosh\left(\frac{n\pi}{a}y\right)\right) \sin\left(\frac{n\pi}{a}x\right) \tag{5}$$

The nonhomogeneous boundary condition is now resolved. At $y = 0$

$$u(x, 0) = f(x) = \delta(x - \xi)$$

Therefore (5) becomes

$$\delta(x-\xi)=\sum_{n=1}^{\infty}-B_n\tanh\left(\frac{n\pi}{a}b\right)\sin\left(\frac{n\pi}{a}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{a}x\right)$ and integrating gives

$$\begin{aligned}\int_0^a\delta(x-\xi)\sin\left(\frac{m\pi}{a}x\right)dx&=-\int_0^a\sin\left(\frac{m\pi}{a}x\right)\sum_{n=1}^{\infty}B_n\tanh\left(\frac{n\pi}{a}b\right)\sin\left(\frac{n\pi}{a}x\right)dx\\&=-\sum_{n=1}^{\infty}B_n\tanh\left(\frac{n\pi}{a}b\right)\int_0^a\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}x\right)dx\\&=-B_n\tanh\left(\frac{m\pi}{a}b\right)\left(\frac{a}{2}\right)\end{aligned}$$

Hence

$$B_n=-\frac{2}{a}\frac{\int_0^a\delta(x-\xi)\sin\left(\frac{n\pi}{a}x\right)dx}{\tanh\left(\frac{n\pi}{a}b\right)}$$

But $\int_0^a\delta(x-\xi)\sin\left(\frac{m\pi}{L}x\right)dx=\sin\left(\frac{m\pi}{L}\xi\right)$ by the property delta function. Therefore

$$B_n=-\frac{2}{a}\frac{\sin\left(\frac{n\pi}{a}\xi\right)}{\tanh\left(\frac{n\pi}{a}b\right)}$$

This completes the solution. (4) becomes

$$\begin{aligned}u(x,y)&=-\frac{2}{a}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{a}\xi\right)}{\tanh\left(\frac{n\pi}{a}b\right)}\left(\sinh\left(\frac{n\pi}{a}y\right)-\tanh\left(\frac{n\pi}{a}b\right)\cosh\left(\frac{n\pi}{a}y\right)\right)\sin\left(\frac{n\pi}{a}x\right)\\&=-\frac{2}{a}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{a}\xi\right)\sin\left(\frac{n\pi}{a}x\right)\left(\frac{\sinh\left(\frac{n\pi}{a}y\right)}{\tanh\left(\frac{n\pi}{a}b\right)}-\cosh\left(\frac{n\pi}{a}y\right)\right)\end{aligned}$$

Looking at the solution above, it is composed of functions that are all differentiable. Hence the solution is infinitely differentiable inside the rectangle.

Here is a plot of the above solution using $a=\pi,b=\frac{1}{2},\xi=1$.

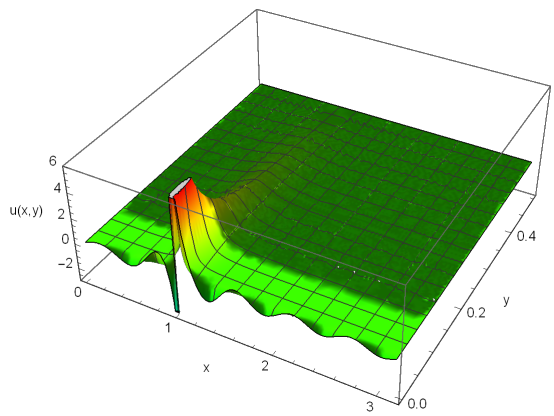


Figure 4.4: Plot of $u(x,y)$

```
u[x_, y_, ξ_] := -2/a Sum[Sin[n π/a ξ] Sin[n π/a x] (Sinh[n π/a y]/Tanh[n π/a b] - Cosh[n π/a y]), {n, 1, 300}];
a = Pi; b = 1/2; ξ = 1;
p = Plot3D[u[x, y, ξ], {x, 0, a}, {y, 0, b}, PlotRange → {Automatic, Automatic, {-3, 7}},
        PlotPoints → 40, AxesLabel → {"x", "y", "u(x,y)"},
        ColorFunction → Function[{x, y, z}, Hue[.45 (1 - z)]]];
```

Figure 4.5: Code used for the above plot

4.1.1.4 [283] Haberman 2.5.1 (a)

problem number 283

This is problem 2.5.1 part (a) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y) &= 0\\ \frac{\partial u}{\partial x}(L,y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= f(x)\end{aligned}$$

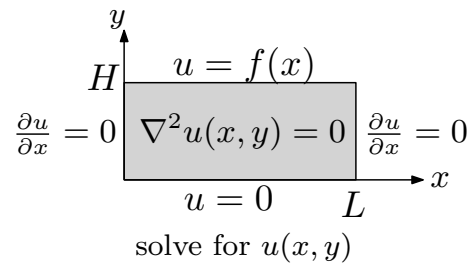


Figure 4.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == 0, Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} \frac{2\cos\left(\frac{n\pi x}{L}\right)\operatorname{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L} + \frac{y\int_0^L f(x)\,dx}{HL}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
#these simplifications below to convert answer to one that match standard;
sol:=convert(sol,trigh);
sol:=simplify(expand(sol));
```

$$u(x,y)=\frac{2H\left(\sum_{n=1}^{\infty}\frac{\left(\int_0^L\cos\left(\frac{\pi n x}{L}\right)f(x)dx\right)\cos\left(\frac{\pi n x}{L}\right)\sinh\left(\frac{\pi n y}{L}\right)}{\sinh\left(\frac{\pi H n}{L}\right)}\right)+y\left(\int_0^L f(x)\,dx\right)}{HL}$$

4.1.1.5 [284] Haberman 2.5.1 (b)

problem number 284

This is problem 2.5.1 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\le x\le L, 0\le y\le H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y)&=g(y)\\ \frac{\partial u}{\partial x}(L,y)&=0\\ u(x,0)&=0\\ u(x,H)&=0\end{aligned}$$

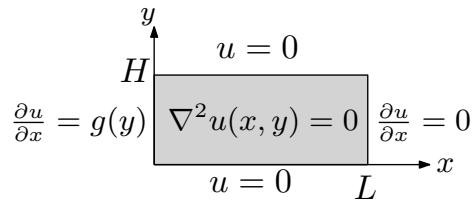


Figure 4.7: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == g[y], Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol =  sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)\,dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}-\frac{2\left(\cosh\left(\frac{\pi(-2L+x)n}{H}\right)+\cosh\left(\frac{\pi nx}{H}\right)-\sinh\left(\frac{\pi(-2L+x)n}{H}\right)+\sinh\left(\frac{\pi nx}{H}\right)\right)\left(\int_0^H g(y)\sin\left(\frac{\pi ny}{H}\right)dy\right.}{\pi\left(\cosh\left(\frac{2\pi Ln}{H}\right)+\sinh\left(\frac{2\pi Ln}{H}\right)-1\right)n}$$

4.1.1.6 [285] Haberman 2.5.1 (c)

problem number 285

This is problem 2.5.1 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial x}(0, y) &= 0 \\ u(L, y) &= g(y) \\ u(x, 0) &= 0 \\ u(x, H) &= 0 \end{aligned}$$

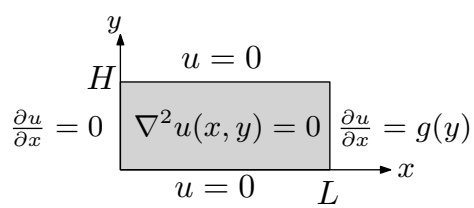


Figure 4.8: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {Derivative[1, 0][u][0, y] == 0, u[L, y] == g[y], u[x, 0] == 0, u[x, H] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

$$\left\{\left\{u(x,y) \to \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{H}} \cosh\left(\frac{\pi x K[1]}{H}\right) \left(\int_0^H \frac{\sqrt{2} g(y) \sin\left(\frac{\pi y K[1]}{H}\right)}{\sqrt{H}} dy\right) \operatorname{sech}\left(\frac{L\pi K[1]}{H}\right) \sin\left(\frac{\pi y K[1]}{H}\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{4\left(\cosh\left(\frac{\pi Ln}{H}\right)+\sinh\left(\frac{\pi Ln}{H}\right)\right)\left(\int_0^H g(y)\sin\left(\frac{\pi ny}{H}\right)dy\right)\cosh\left(\frac{\pi nx}{H}\right)\sin\left(\frac{\pi ny}{H}\right)}{\left(\cosh\left(\frac{2\pi Ln}{H}\right)+\sinh\left(\frac{2\pi Ln}{H}\right)+1\right)H}$$

4.1.1.7 [286] Haberman 2.5.1 (d)

problem number 286

This is problem 2.5.1 part (d) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\le x\le L, 0\le y\le H$, with following boundary conditions

$$\begin{aligned} u(0,y)&=g(y)\\ u(L,y)&=0\\ \frac{\partial u}{\partial y}u(x,0)&=0\\ u(x,H)&=0 \end{aligned}$$

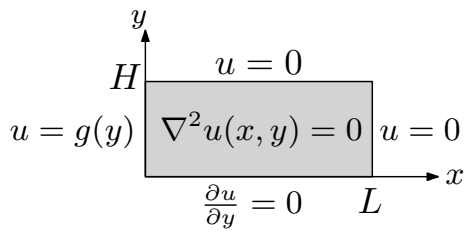


Figure 4.9: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[0, y] == g[y], u[L, y] == 0, Derivative[0, 1][u][x, 0] == 0, u[x, H] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{(2n-1)\pi y}{2H}\right)\operatorname{csch}\left(\frac{L(2n-1)\pi}{2H}\right)\left(\int_0^H\frac{\sqrt{2}\cos\left(\frac{(2n-1)\pi y}{2H}\right)g(y)}{\sqrt{H}}\,dy\right)\sinh\left(\frac{(2n-1)\pi(L-x)}{2H}\right)}{\sqrt{H}}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc  := u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=0}^{\infty}\frac{2\left(\cosh\left(\frac{(-2L+x)\pi\left(n+\frac{1}{2}\right)}{H}\right)-\cosh\left(\frac{(2n+1)\pi x}{2H}\right)+\sinh\left(\frac{(-2L+x)\pi\left(n+\frac{1}{2}\right)}{H}\right)+\sinh\left(\frac{(2n+1)\pi x}{2H}\right)\right)\left(\int_0^H\right.}{\left(\cosh\left(\frac{(2n+1)\pi L}{H}\right)-\sinh\left(\frac{(2n+1)\pi L}{H}\right)-1\right)H}$$

4.1.1.8 [287] Haberman 2.5.1 (e)

problem number 287

This is problem 2.5.1 part (e) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(L,y) &= 0 \\ u(x,0) - \frac{\partial u}{\partial y} u(x,0) &= 0 \\ u(x,H) &= f(x) \end{aligned}$$

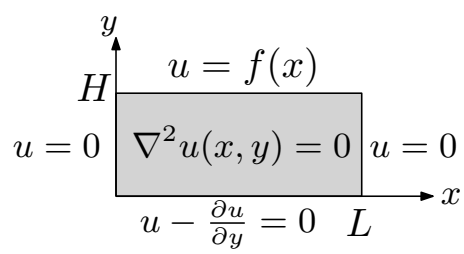


Figure 4.10: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[0, y] == 0, u[L, y] == 0, u[x, 0] - Derivative[0, 1][u][x, 0] == 0, u[x, H] == f[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{\sqrt{2}\sqrt{\frac{1}{L}}\left(\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{n\pi x}{L}\right)}{\sqrt{L}}\,dx\right)\sin\left(\frac{n\pi x}{L}\right)\left(n\pi\cosh\left(\frac{n\pi y}{L}\right)+L\sinh\left(\frac{n\pi y}{L}\right)\right)}{n\pi\cosh\left(\frac{Hn\pi}{L}\right)+L\sinh\left(\frac{Hn\pi}{L}\right)}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc  := u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{4\left(\cosh\left(\frac{\pi Hn}{L}\right)+\sinh\left(\frac{\pi Hn}{L}\right)\right)\left(L\sinh\left(\frac{\pi ny}{L}\right)+\pi n\cosh\left(\frac{\pi ny}{L}\right)\right)\left(\int_0^L f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi nx}{L}\right.}{\left(-L+\pi n+(L+\pi n)\cosh\left(\frac{2\pi Hn}{L}\right)+(L+\pi n)\sinh\left(\frac{2\pi Hn}{L}\right)\right)L}$$

Hand solution

Let $u(x,y)=X(x)\,Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above. Two ODE's (1,2) are obtained as follows

$$X''+\lambda X=0 \tag{1}$$

With the boundary conditions

$$X(0) = 0$$
$$X(L) = 0$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$Y(0) = Y'(0)$$
$$Y(H) = f(x)$$

In all these cases λ will turn out to be positive. This is shown for this problem only and not be repeated again.

<u>Case $\lambda < 0$</u>

The solution to (1) us

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = L$ this gives $X = B\sinh\left(\sqrt{|\lambda|}L\right)$. But $\sinh\left(\sqrt{|\lambda|}L\right) = 0$ only at 0 and $\sqrt{|\lambda|}L \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

<u>Case $\lambda = 0$</u>

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = L$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

<u>Case $\lambda > 0$</u>

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = L$

$$0 = B\sin\left(\sqrt{\lambda}L\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$\begin{aligned} Y_n &= C_n \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ Y_n' &= C_n \frac{n\pi}{L} \sinh\left(\frac{n\pi}{L}y\right) + D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) \end{aligned}$$

Applying B.C. at $y = 0$ gives

$$\begin{aligned} Y(0) &= Y'(0) \\ C_n \cosh(0) &= D_n \frac{n\pi}{L} \cosh(0) \\ C_n &= D_n \frac{n\pi}{L} \end{aligned}$$

The eigenfunctions Y_n are

$$\begin{aligned} Y_n &= D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ &= D_n\left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x, y) &= Y_n X_n \\ &= D_n\left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) B_n \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x, y) = B_n\left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x, y) = \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

The nonhomogeneous boundary condition is now resolved. At $y = H$

$$u(x, H) = f(x)$$

Therefore

$$f(x) = \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}H\right) + \sinh\left(\frac{n\pi}{L}H\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sin\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\sin\left(\frac{n\pi}{L}x\right)dx\\ &= \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= B_m\left(\frac{m\pi}{L}\cosh\left(\frac{m\pi}{L}H\right)+\sinh\left(\frac{m\pi}{L}H\right)\right)\frac{L}{2}\end{aligned}$$

Hence

$$B_n = \frac{2}{L}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)} \tag{4}$$

This completes the solution. In summary

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}y\right)+\sinh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)$$

With B_n given by (4).

4.1.1.9 [288] Unit triangle B.C.

problem number 288

Taken from Mathematica DSolve help pages.

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0 \le x \le 1, 0 \le y \le 2$, with following boundary conditions

$$\begin{aligned}u(0,y) &= 0\\ u(1,y) &= 0\\ u(x,0) &= \text{UnitTriagle(2 x-1)}\\ u(x,2) &= \text{UnitTriagle(2 x-1)}\end{aligned}$$

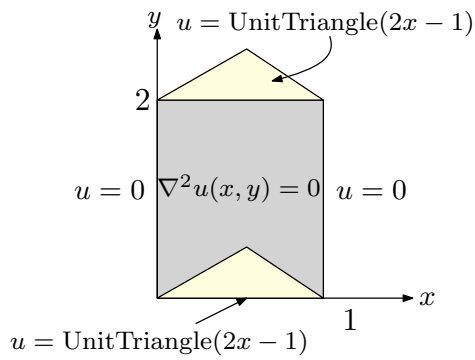


Figure 4.11: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
 L0 = 1;
 H0 = 2;
bc  = DirichletCondition[u[x, y] == Piecewise[{{UnitTriangle[2*x - L0], y == 0 || y == H0}},
 domain = Rectangle[{0, 0}, {L0, H0}];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], Element[{x, y}, do
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{8\operatorname{csch}(2n\pi)\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)(\sinh(n\pi(2-y))+\sinh(n\pi y))}{n^2\pi^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x-> piecewise(x>0 and x<1/2, 2*x, x>1/2 and x<1, 2-2*x);
bc  := u(0,y)=0,u(1,y)=0,u(x,0)=f(x),u(x,2)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{8\left(-\mathrm{e}^{\pi(y-2)n}+\mathrm{e}^{\pi ny}+\mathrm{e}^{-\pi(y-2)n}-\mathrm{e}^{-\pi ny}\right)\mathrm{e}^{2\pi n}\sin\left(\pi nx\right)\sin\left(\frac{\pi n}{2}\right)}{\pi^2\left(\mathrm{e}^{4\pi n}-1\right)n^2}$$

4.1.1.10 [289] Top edge at infinity

problem number 289

Added December 20, 2018.

Example 21, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \le x \le L, 0 \le y \le \infty$, with following boundary conditions

$$u(0, y) = A$$
$$u(L, y) = 0$$
$$u(x, 0) = 0$$

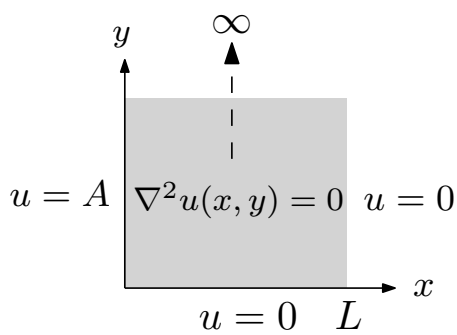


Figure 4.12: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == A, u[L, y] == 0, u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \frac{A\left(-iL\log\left(1-e^{\frac{i\pi(x+iy)}{L}}\right)+iL\log\left(1-e^{-\frac{\pi(y+ix)}{L}}\right)+\pi(L-x)\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = A;
bc_right_edge:= u(L, y) = 0;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y) = -\frac{Ax}{L} + A + \left(\sum_{n=1}^{\infty} -\frac{2A\,\mathrm{e}^{-\frac{\pi n y}{L}} \sin\left(\frac{\pi n x}{L}\right)}{\pi n}\right)$$

<u>Hand solution</u>

Let

$$u = U + v \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0) = A, v(L) = 0$. This implies

$$v(x) = A\left(1 - \frac{x}{L}\right)$$

Hence $u = U + A\left(1 - \frac{x}{L}\right)$. Substituting this back in $\nabla^2 u = 0$ gives

$$\nabla^2 U = 0$$

But with boundary condition on right edge being zero now. Let $U = X(x)\,Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$\begin{aligned} U(x,y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \end{aligned}$$

Using the above in (1) gives the solution

$$u(x,y) = A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}$$

At $y = 0$ the above gives

$$\begin{aligned} 0 &= A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\ A\left(\frac{x}{L} - 1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $\frac{A}{L} x$

$$\begin{aligned} B_n &= \frac{2}{L} \int_0^L A\left(\frac{x}{L} - 1\right) \sin\left(\frac{n\pi}{L} x\right) dx \\ &= \frac{2A}{L} \int_0^L \left(\frac{x}{L} - 1\right) \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L} \frac{L}{n\pi} \\ &= -\frac{2A}{n\pi} \end{aligned}$$

Hence the solution (2) becomes

$$u(x,y) = A\left(1 - \frac{x}{L}\right) - 2\frac{A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y}$$

4.1.1.11 [290] Top edge at infinity

problem number 290

Added March 19, 2019

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \le x \le L, 0 \le y \le \infty$, with following boundary conditions

$$u(0, y) = 0$$
$$u(L, y) = A$$
$$u(x, 0) = 0$$

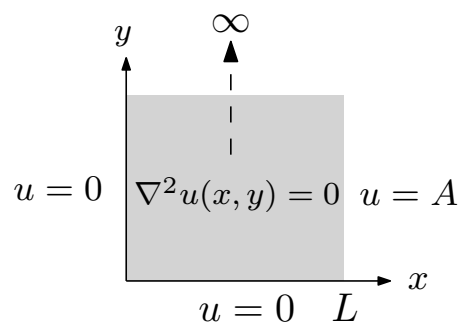


Figure 4.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[L, y] == A, u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \frac{A\left(iL\log\left(1+e^{\frac{i\pi(x+iy)}{L}}\right) - iL\log\left(1+e^{-\frac{\pi(y+ix)}{L}}\right) + \pi x\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_right_edge:= u(L, y) = A;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y) = \frac{Ax}{L} + \left(\sum_{n=1}^{\infty} \frac{2A(-1)^n \mathrm{e}^{-\frac{\pi n y}{L}} \sin\left(\frac{\pi n x}{L}\right)}{\pi n}\right)$$

<u>Hand solution</u>

Let

$$u = U + v \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0) = 0A, v(L) = A$. This implies

$$v(x) = A\frac{x}{L}$$

Hence $u = U + \frac{A}{L}x$. Substituting this back in $\nabla^2 u = 0$ gives

$$\nabla^2 U = 0$$

But with boundary condition on right edge being zero now. Let $U = X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$\begin{aligned} U(x,y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \end{aligned}$$

Using the above in (1) gives the solution

$$u(x,y) = \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}$$

At $y = 0$ the above gives

$$\begin{aligned} 0 &= \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\ -\frac{A}{L} x &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $-\frac{A}{L} x$

$$\begin{aligned} B_n &= -\frac{2}{L} \int_0^L \frac{A}{L} x \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L^2} \int_0^L x \sin\left(\frac{n\pi}{L} x\right) dx \\ &= -\frac{2A}{L^2} \frac{(-1)^{n+1} L^2}{n\pi} \\ &= \frac{2A}{n\pi} (-1)^n \end{aligned}$$

Hence the solution (2) becomes

$$u(x,y) = \frac{A}{L} x + \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y}$$

4.1.1.12 [291] Right edge at infinity

problem number 291

Added March 19, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \le y \le L, 0 \le x \le \infty$, with following boundary conditions

$$u(0, y) = 0$$
$$u(x, 0) = A$$
$$u(x, L) = 0$$

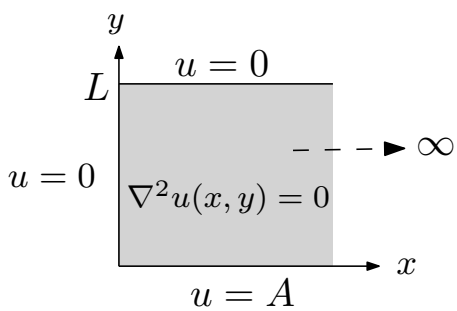


Figure 4.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, 0] == A, u[x, L] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \frac{A\left(-iL\log\left(1-e^{-\frac{\pi(x-iy)}{L}}\right)+iL\log\left(1-e^{-\frac{\pi(x+iy)}{L}}\right)+\pi(L-y)\right)}{\pi L}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = 0;
bc_bottom_edge:= u(x, 0) = A;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y) = -\frac{Ay}{L} + A + \left(\sum_{n=1}^{\infty} -\frac{2A\,\mathrm{e}^{-\frac{\pi n x}{L}} \sin\left(\frac{\pi n y}{L}\right)}{\pi n}\right)$$

<u>Hand solution</u>

Let

$$u(x,y) = U(x,y) + v(y) \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with bottom edge boundary conditions zero, and $v(y)$ satisfies the nonhomogeneous boundary conditions $v(0) = A, v(L) = 0$. This implies

$$v(y) = A\left(1 - \frac{y}{L}\right)$$

Substituting (1) back in $\nabla^2 u = 0$ results in

$$\nabla^2 U = 0$$

But with boundary condition on bottom edge as $U = 0$. Now we can use separation of variables. Let $U = X(x)\,Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the Y direction. Hence

$$\frac{Y''}{Y} = -\frac{X''}{X} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} Y'' + \lambda Y &= 0 \\ Y(0) &= 0 \\ Y(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $Y_n(x) = \sin\left(\sqrt{\lambda_n} y\right)$. The X ode is

$$\begin{aligned} X_n'' - \lambda_n X_n &= 0 \\ X_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $X_n(y) = c_{1_n} e^{\sqrt{\lambda_n} x} + c_{2_n} e^{-\sqrt{\lambda_n} x}$. Since $X_n(x)$ is bounded, then $c_{1_n} = 0$ and the $X_n(x) = c_{2_n} e^{-\sqrt{\lambda_n} x}$. Hence by superposition the solution is

$$\begin{aligned} U(x, y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} y\right) e^{-\sqrt{\lambda_n} x} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x} \end{aligned}$$

Substituting the above in (1) gives

$$u(x, y) = A\left(1 - \frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x} \tag{2}$$

At $x = 0$ the above gives

$$\begin{aligned} 0 &= A\left(1 - \frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) \\ A\left(\frac{y}{L} - 1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $A\left(\frac{y}{L} - 1\right)$

$$\begin{aligned} B_n &= \frac{2}{L} \int_0^L A\left(\frac{y}{L} - 1\right) \sin\left(\frac{n\pi}{L} y\right) dy \\ &= \frac{2A}{L} \int_0^L \left(\frac{y}{L} - 1\right) \sin\left(\frac{n\pi}{L} y\right) dy \\ &= -\frac{2A}{L} \frac{L}{n\pi} \\ &= -\frac{2A}{n\pi} \end{aligned}$$

Hence the solution (2) becomes

$$u(x, y) = A\left(1 - \frac{y}{L}\right) - \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x}$$

4.1.1.13 [292] Right edge at infinity

problem number 292

Added March 20, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \le y \le L, 0 \le x \le \infty$, with following boundary conditions

$$u(0, y) = 0$$
$$u(x, L) = e^{-x}$$
$$u(x, 0) = 0$$

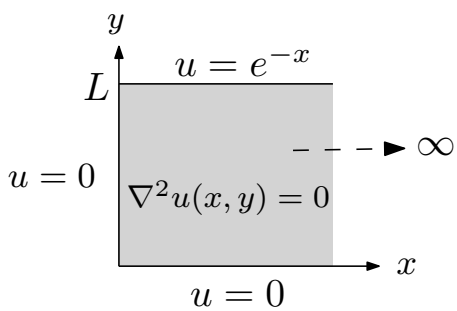


Figure 4.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, L] == Exp[-x], u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \left\{\begin{array}{cc} \int_0^\infty \frac{2\operatorname{csch}(LK[1])K[1]\sin(xK[1])\sinh(yK[1])}{\pi K[1]^2+\pi} dK[1] & x \ge 0 \land y \ge 0 \land L > 0 \\ \text{Indeterminate} & \text{True} \end{array}\right\}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = exp(-x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
#I need to find out how Maple obtained the above solution. It seems to have unknown constant
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
```

$$u(x,y) = \frac{y\,\mathrm{e}^{-x}}{L} + \sum_{n=1}^{\infty}\left(-\frac{\left(\left(L^2-\pi^2 n^2\right)\_F1(n)\,\mathrm{e}^{-\frac{\pi n x}{L}} - 2\left(-(-1)^n\,\mathrm{e}^{-x} + (L+\pi n)\,\_F1(n)\right)L + \left((-4L-\right.\right.}{\pi\,(L+\pi n)\,n}$$

Hand solution

Let $u = X(x)\,Y(x)$. Substituting this in $\nabla^2 u = 0$

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\frac{X''}{X} = -\frac{Y''}{Y} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ |X(x)| &< \infty \end{aligned}$$

case $\lambda < 0$

Solution is $X(x) = c_1 \cosh\left(\sqrt{-\lambda}x\right) + c_2 \sinh\left(\sqrt{-\lambda}x\right)$. Since $X(0) = 0$ then $c_1 = 0$. Solution becomes $X(x) = c_2 \sinh\left(\sqrt{-\lambda}x\right)$. Since sinh is not bounded on $x > 0$ as $x \to \infty$ then $c_2 = 0$. Therefore $\lambda < 0$ is not eigenvalue.

case $\lambda = 0$

Solution is $X(x) = c_1 x + c_2$. At $x = 0$ this gives $c_2 = 0$. Hence solution is $X(x) = c_1 x$. This is bounded as $x \to \infty$ only when $c_1 = 0$. Therefore $\lambda = 0$ is not eigenvalue.

case $\lambda > 0$

Let $\lambda = \alpha^2, \alpha > 0$. Then solution is $X(x) = c_1 \cos(\alpha x) + c_2 \sin(\alpha x)$. At $x = 0$ this results in $0 = c_1$. Hence the eigenvalues are $\lambda = \alpha^2$ for all real positive real numbers and eigenfunctions are

$$X_\alpha(x) = \sin(\alpha x)$$

For the Y ode,

$$\begin{aligned} Y'' - \alpha^2 Y &= 0 \\ Y(0) &= 0 \end{aligned}$$

The solution is $Y_\alpha(y) = c_1 e^{\alpha y} + c_2 e^{-\alpha y}$. Since $Y(0) = 0$ then $c_2 = -c_1$ and the solution becomes $Y_\alpha(y) = c_1(e^{\alpha y} - e^{-\alpha y}) = c_1 \sinh(\alpha y)$. Hence the solution is generalized linear combination of $Y(y)\,X(x)$ given by Fourier integral (since eigenvalues are continuous now and not discrete)

$$\begin{aligned} u(x,y) &= \int_0^\infty A(\alpha)\, Y_\alpha(y)\, X_\alpha(x)\, d\alpha \\ &= \int_0^\infty A(\alpha) \sinh(\alpha y) \sin(\alpha x)\, d\alpha \end{aligned} \tag{1}$$

When $y = L$, then above becomes

$$e^{-x} = \int_0^\infty (A(\alpha) \sinh(\alpha L)) \sin(\alpha x)\, d\alpha$$

Hence the coefficient $A(\alpha) \sinh(\alpha L)$ is given by

$$\begin{aligned} A(\alpha) \sinh(\alpha L) &= \frac{2}{\pi} \int_0^\infty e^{-x} \sin(\alpha x)\, dx \\ &= \frac{2}{\pi} \frac{\alpha}{1+\alpha^2} \end{aligned}$$

Therefore $A(\alpha) = \frac{2}{\pi \sinh(\alpha L)} \frac{\alpha}{1+\alpha}$. The solution (1) becomes

$$u(x,y) = \frac{2}{\pi} \int_0^\infty \frac{\alpha \sinh(\alpha y) \sin(\alpha x)}{(1+\alpha^2) \sinh(\alpha L)} d\alpha$$

4.1.1.14 [293] Right edge at infinity

problem number 293

Added April 4, 2019.

Second midterm exam problem, Math 4567, UMN. Spring 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq 1, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0, y) &= 0 \\ u(x, 1) &= f(x) \\ u(x, 0) &= 0 \end{aligned}$$

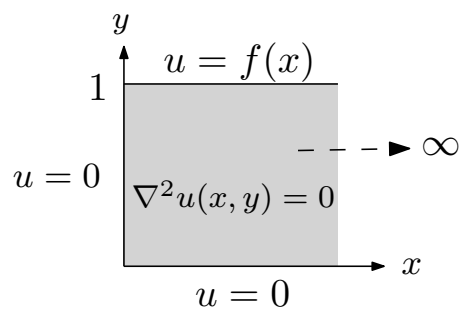


Figure 4.16: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[x, 1] == f[x], u[x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

$$\left\{\left\{u(x,y) \to \left\{ \begin{array}{cc} \int_0^\infty \frac{2\operatorname{csch}(K[1])\left(\int_0^\infty f(x)\sin(xK[1])\,dx\right)\sin(xK[1])\sinh(yK[1])}{\pi} dK[1] & x \geq 0 \wedge y \geq 0 \\ \text{Indeterminate} & \text{True} \end{array} \right\}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, 1) = f(x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
#Maple can not solve it when using boundedseries(x = infinity)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
```

$$u(x,y) = yf(x) + \int_0^x \left(\sum_{n=1}^{\infty} \left(\frac{2(-1)^n \left(\frac{d^2}{d\tau^2} f(\tau)\right) e^{\pi(-\tau+x)n}}{\pi n} - \_F1(n)\, e^{\pi(-\tau+x)n} + \_F1(n)\, e^{-\pi(-\tau+x)n} \right) \sin \right.$$

4.1.1.15 [294] Laplace PDE in 2D Cartesian with boundary condition as Dirac function

problem number 294

Added December 20, 2018

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary condition

$$u(x,0) = \delta(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[x, 0] == DiracDelta[x];
sol =  AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], x, y]], 60*10]];
```

$$\left\{\left\{u(x,y)\to \begin{cases} \frac{y}{\pi(x^2+y^2)} & y\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(x, 0) = Dirac(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y),method=Fo
sol:=convert(sol,Int);
```

$$u(x,y)=\frac{\int_{-\infty}^{\infty}\mathrm{e}^{(ix-y)s}ds}{2\pi}$$

4.1.1.16 [295] One side homogeneous

problem number 295

Added December 20, 2018

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation for $u(x,y)$

$$u_{xx}+u_{yy}=0$$

With boundary condition

$$\begin{aligned} u(0,y)&=0\\ u(\pi,y)&=\sinh(\pi)\cos(y)\\ u(x,0)&=\sin(x)\\ u(x,\pi)&=-\sinh(x) \end{aligned}$$

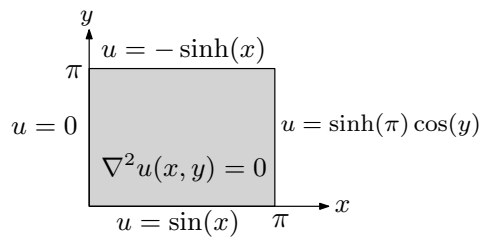


Figure 4.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[0, y] == 0, u[Pi, y] == Sinh[Pi]*Cos[y], u[x, 0] == Sin[x], u[x, Pi] == -Sinh[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

$$\left\{\left\{u(x,y) \to \sum_{K[1]=1}^{\infty} \operatorname{csch}(\pi K[1]) \left(\delta(K[1]-1)\sin(xK[1])\sinh((\pi-y)K[1]) + \frac{2K[1]\sinh(\pi)\left(\left(1+(-1)^{K}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_side   :=  u(0,y) = 0;
bc_right_side  := u(Pi,y) = sinh(Pi)*cos(y);
bc_bottom_side := u(x,0)  = sin(x);
bc_top_side    := u(x,Pi) = -sinh(x);
bc  := bc_left_side,bc_right_side,bc_bottom_side,bc_top_side;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = \frac{(\mathrm{e}^{2\pi}-1)\left(\sum_{n=1}^{\infty} \frac{(\mathrm{e}^{2\pi}-1)(\mathrm{e}^{2ny}-1)n(-1)^n \mathrm{e}^{-ny+(n-1)\pi}\sin(nx)}{\pi(n^2+1)(\mathrm{e}^{2\pi n}-1)}\right) + (2\,\mathrm{e}^{2\pi}-2)\left(\sum_{n=2}^{\infty} \frac{\sinh(\pi)((-1)^n+1)(\mathrm{e}^{2nx}-1)}{\pi(\mathrm{e}^{2\pi n}-1)(n}\right.}{\mathrm{e}^{2\pi}-1}$$

4.1.1.17 [296] In right half plane

problem number 296

PDE example 18 from Maple help page

see `march_20_2019_11_pm.tex` for start of solution. Not completed yet

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

With boundary conditions

$$u(0, y) = \frac{\sin y}{y}$$

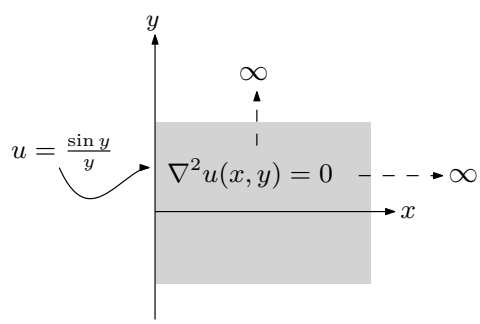


Figure 4.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = u[0, y] == Sin[y]/y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> 0 <
```

$$\left\{\left\{u(x, y) \to \frac{-2y \sinh(x) \sin(y) - 2x \cosh(x) \cos(y) + x}{x^2 + y^2}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(0,y)=sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y) = \_F2(ix+y) + \frac{(-ix+y)\,\_F2(-ix+y) + \sin(ix-y)}{ix-y}$$

4.1.1.18 [297] Right edge at infinity

problem number 297

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

With boundary conditions

$$\begin{aligned} u(0,y) &= \sin y \\ u(x,0) &= 0 \\ u(x,a) &= 0 \\ u(\infty,y) &= 0 \end{aligned}$$

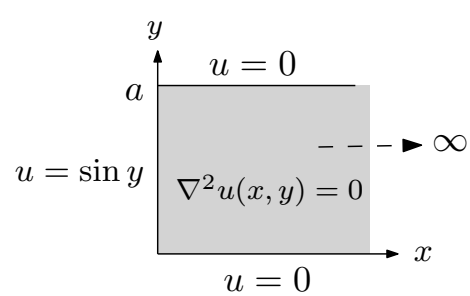


Figure 4.19: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc  = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc  := u(x,0)=0, u(x,a)=0, u(0,y)=sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({pde, bc}, u(x,y)) assumin
```

$$u(x,y) = \sum_{n=1}^{\infty} \frac{\left(-2\pi n\, e^{\frac{\pi n x}{a}} \left(\begin{cases} -1 & a = \pi n \\ -\frac{(-1)^n \sin(a)}{a-\pi n} & otherwise \end{cases}\right) + (a+\pi n)\left(-e^{\frac{\pi n x}{a}} + e^{-\frac{\pi n x}{a}}\right) \_F1(n)\right) \sin\left(\frac{\pi n y}{a}\right)}{a+\pi n}$$

4.1.1.19 [298] Dirichlet problem Upper half

problem number 298

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions $u(x,0) = 1$ for $-\frac{1}{2} \le x \le \frac{1}{2}$ and $x = 0$ otherwise. This is called UnitBox in Mathematica.

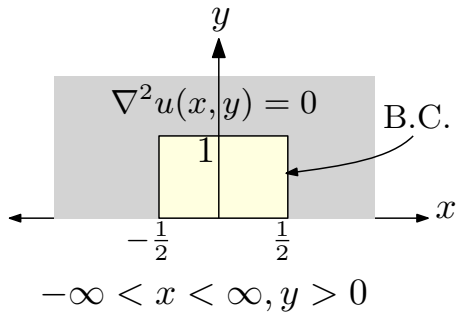


Figure 4.20: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {y >
```

$$\left\{\left\{u(x,y) \to \frac{\tan^{-1}\left(\frac{\frac{1}{2}-x}{y}\right)+\tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right)}{\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x,0) =piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y) = -\_F2(ix-y) + \_F2(ix+y) + \left(\begin{cases} 0 & iy+x < -\frac{1}{2} \\ 1 & iy+x \leq \frac{1}{2} \\ 0 & \frac{1}{2} < iy+x \end{cases}\right)$$

4.1.1.20 [299] Right half-plane

problem number 299

Taken from Mathematica DSolve help pages

Solve for $u(x, y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions $u(0, y) = \text{sinc}(y)$.

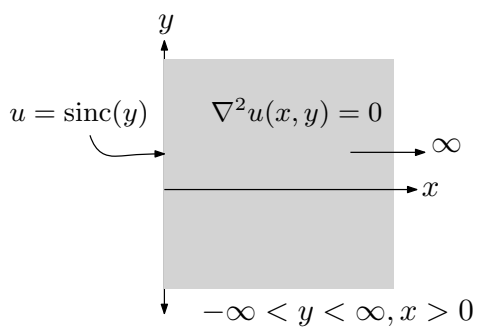


Figure 4.21: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = u[0, y] == Sinc[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {x >
```

$$\left\{\left\{u(x, y) \to \frac{-2y \sinh(x) \sin(y) - 2x \cosh(x) \cos(y) + x}{x^2 + y^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(0,y) =sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming x
```

$$u(x,y) = \_F2(ix+y) + \frac{(-ix+y)\_F2(-ix+y) + \sin(ix-y)}{ix-y}$$

4.1.1.21 [300] First quadrant

problem number 300

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions

$$u(x,0) = -\frac{1}{(x-2)^2+3}$$
$$u(0,y) = \frac{1}{(y-3)^2+1}$$

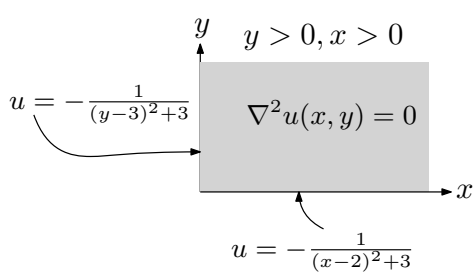


Figure 4.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == -((x - 2)^2 + 3)^(-1), u[0, y] == 1/((y - 3)^2 + 1)};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> {x >
```

$$\left\{\left\{u(x,y)\to \frac{2\left(\frac{3(y(3\pi(x+1)(2x^2(y^2+12)+x^4-4x^3-4x(y^2+10)+y^4-16y^2+100)+x(2x^2(y^2-10)+x^4+y^4+20y^2-260)\log(x^2+y^2)+x}{}\right.}{}\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x, 0) = (-1/((x - 2)^2 + 3)), u(0, y) = 1/((y - 3)^2 + 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming x
```

sol=()

4.1.1.22 [301] Neumann problem upper half-plane

problem number 301

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$\nabla^2 u(x,y) = 0$$

Boundary conditions $\frac{\partial u}{\partial y}(x,0) = \text{UnitBox[x]}$ where UnitBox[x] is 1 for $-\frac{1}{2} \le x \le \frac{1}{2}$ and 0 otherwise. This is called UnitBox in Mathematica.

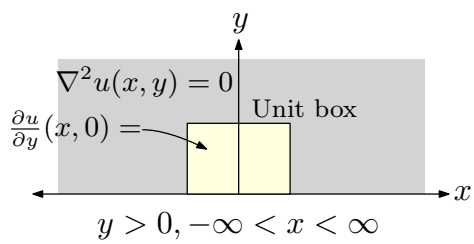


Figure 4.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = Derivative[0, 1][u][x, 0] == UnitBox[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y},Assumptions -> y>0],
```

$$\left\{\left\{u(x,y) \to \frac{-2x\log\left((1-2x)^2+4y^2\right)+\log\left((1-2x)^2+4y^2\right)+2x\log\left((2x+1)^2+4y^2\right)+\log\left((2x+1\right.}{4\pi}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=eval(diff(u(x,y),y),y=0)= piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))assuming y
sol:=convert(sol,Int);
```

$$u(x,y) = -\frac{i\left(\int_{-\infty}^{\infty} \frac{e^{\frac{(2ix-2y-i)s}{2}}}{s^2}\,ds - \left(\int_{-\infty}^{\infty} \frac{e^{\frac{(2ix-2y+i)s}{2}}}{s^2}\,ds\right)\right)}{2\pi}$$

used convert(sol,Int).

4.1.1.23 [302] Dirichlet problem in a rectangle

problem number 302

Taken from Mathematica DSolve help pages

Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(x, 0) = x^2(1 - x), u(x, 2) = 0, u(0, y) = 0, u(1, y) = 0$.

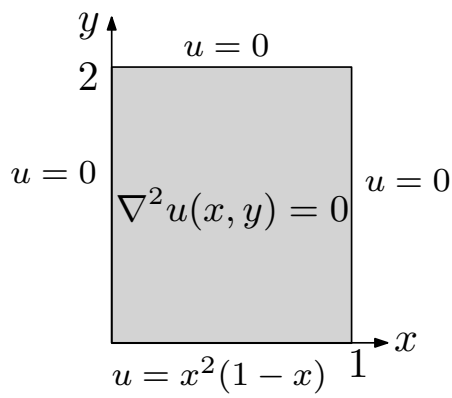


Figure 4.24: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == x^2*(1 - x), u[x, 2] == 0, u[0, y] == 0, u[1, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x, y) \to \sum_{n=1}^{\infty} -\frac{4(1 + 2(-1)^n)\operatorname{csch}(2n\pi)\sin(n\pi x)\sinh(n\pi(2 - y))}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc  := u(x, 0) = x^2*(1 - x),u(x, 2) = 0, u(0, y) = 0, u(1, y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{8\left((-1)^n+\frac{1}{2}\right)\left(\mathrm{e}^{\pi n y}-\mathrm{e}^{-\pi(y-4)n}\right)\sin\left(\pi n x\right)}{\pi^3\left(\mathrm{e}^{4\pi n}-1\right)n^3}$$

4.1.1.24 [303] Strip in upper half

problem number 303

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions

$$u(x,0)=0$$
$$u(x,b)=h(x)$$

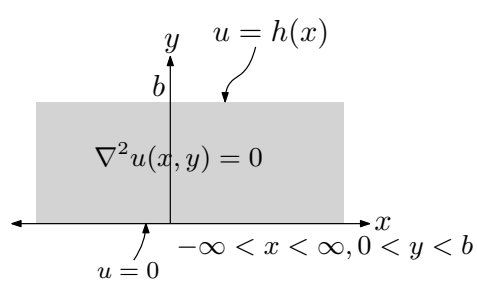


Figure 4.25: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == 0, u[x, b] == h[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

$$\left\{\left\{u(x,y)\to \frac{\int_{-\infty}^{\infty} e^{ixK[1]}\operatorname{csch}(bK[1])\sinh(yK[1])\int_{-\infty}^{\infty} e^{-ixK[1]}h(x)dxdK[1]}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2)=0;
bc  := u(x,0)=0,u(x,b)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
sol:=convert(sol,Int);
```

$$u(x,y)=\frac{-b\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h(x)dx\right)\mathrm{e}^{(ix+b-y)s}}{\mathrm{e}^{2bs}-1}ds\right)+b\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h(x)dx\right)\mathrm{e}^{(ix+b+y)s}}{\mathrm{e}^{2bs}-1}ds\right)-y\left(\int_{-\infty}^{\infty}\frac{\left(\int_{-\infty}^{\infty}\mathrm{e}^{-isx}h\right.}{\mathrm{e}^{2b}}\right.}{2\pi b}$$

4.1.1.25 [304] in Rectangle, right edge at infinity

problem number 304

Added December 20, 2018.

Example 23, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions

$$
\begin{aligned}
u(x,0) &= 0\\
u(x,a) &= 0\\
u(0,y) &= \sin(y)\\
u(\infty,y) &= 0
\end{aligned}
$$

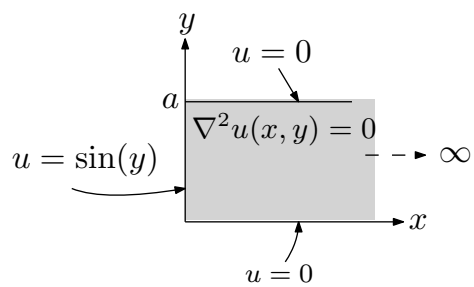


Figure 4.26: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 0;
bc  = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge:=u(0, y) = sin(y);
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,a)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

$$u(x,y) = \sum_{n=1}^{\infty} \frac{\left(-2\pi n\, \mathrm{e}^{\frac{\pi n x}{a}} \left(\begin{cases} -1 & a = \pi n \\ -\frac{(-1)^n \sin(a)}{a-\pi n} & \mathit{otherwise} \end{cases}\right) + (a+\pi n)\left(-\mathrm{e}^{\frac{\pi n x}{a}} + \mathrm{e}^{-\frac{\pi n x}{a}}\right) \_F1(n)\right) \sin\left(\frac{\pi n y}{a}\right)}{a+\pi n}$$

4.1.2 Polar coordinates

Local contents

4.1.2.1 [305] Laplace PDE inside quarter disk, Neumann BC at edge 837
4.1.2.2 [306] $r = 4$ and $u = x^4$ at boundary of disk 838
4.1.2.3 [307] $r = 1$ and $u_r = x$ at boundary of disk 846
4.1.2.4 [308] Laplace inside disk. General solution 848
4.1.2.5 [309] Laplace inside disk. Specific boundary conditions 850
4.1.2.6 [310] Haberman 2.5.5 (c) 855
4.1.2.7 [311] semi-circle 861
4.1.2.8 [312] Haberman 2.5.8 (b) 862
4.1.2.9 [313] Circular annulus 870
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4.1.2.1 [305] Laplace PDE inside quarter disk, Neumann BC at edge

problem number 305

Added December 20, 2018.

Example 20, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation in polar coordinates inside quarter disk with $0 < r < 1$ and $0 < \theta < \frac{\pi}{2}$

Solve for $u(r, \theta)$

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

Boundary conditions

$$\begin{aligned} u(r, 0) &= 0 \\ u(r, \frac{\pi}{2}) &= 0 \\ u_r(1, \theta) &= f(\theta) \end{aligned}$$

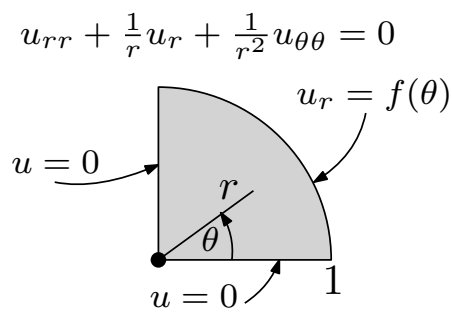


Figure 4.27: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bcOnR = {Derivative[1, 0][u][1, theta] == f[theta]};
 bcOnTheta = {u[r, 0] == 0, u[r, Pi/2] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta}, u[r, theta], {r, theta
```

$$\left\{\left\{u(r,\theta)\to\sum_{K[1]=1}^{\infty}\frac{r^{2K[1]}\left(\int_0^{\frac{\pi}{2}}\frac{2f(\theta)\sin(2\theta K[1])}{\sqrt{\pi}}\,d\theta\right)\sin(2\theta K[1])}{\sqrt{\pi}K[1]}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2)+1/r* diff(u(r, theta), r)+1/r^2* diff(u(r, theta), theta$2)= 0
bc_on_theta:=u(r, 0) = 0, u(r,Pi/2) = 0;
bc_on_r:= eval( diff(u(r,theta),r),r=1)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_theta,bc_on_r]
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{2r^{2n}\left(\int_0^{\frac{\pi}{2}}f(\theta)\sin(2n\theta)\,d\theta\right)\sin(2n\theta)}{\pi n}$$

4.1.2.2 [306] $r=4$ and $u=x^4$ at boundary of disk

problem number 306

Added Nov 10, 2019.

Problem 4.3.25 part c. Peter J. Olver, Introduction to Partial Differential Equations, 2014 edition.

Solve Laplace equation in polar coordinates inside a disk

Solve $\nabla^2 u=0$ with $x^2+y^2<4$ and boundary conditions $u=x^4, x^2+y^2=4$.

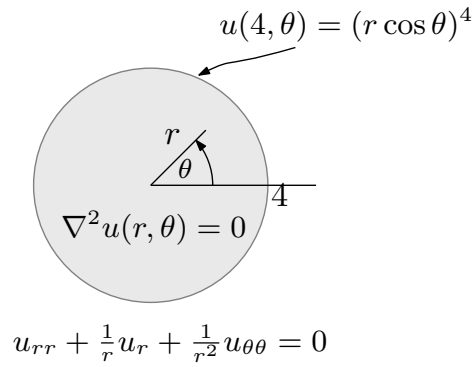


Figure 4.28: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (D[u[r, theta], r])/r + (D[u[r, theta], {theta, 2}])/r^2 ==
bc  = u[4, theta] == (4*Cos[theta])^4;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,\theta) \to \frac{1}{8}r^4\cos(4\theta) + 8r^2\cos(2\theta) + 96\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, theta), r$2) + diff(u(r, theta), r)/r + diff(u(r, theta), theta$2)/r^2 = 0;
bc  := u(4, theta) = (4*cos(theta))^4, u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta))),o
```

$$u(r,\theta) = \frac{r^4\cos(4\theta)}{8} + 8r^2\cos(2\theta) + 96$$

Hand solution

Solve the following boundary value problems $\nabla^2 u = 0, x^2 + y^2 < 4, u = x^4, x^2 + y^2 = 4$

Solution

In polar coordinates, where $x = r\cos\theta, y = r\sin\theta$, we need to solve for $u(r,\theta)$ inside

disk of radius $r_0 = 4$. The Laplace PDE in polar coordinates is

$$\begin{aligned} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \qquad & 0 < r < r_0, -\pi < \theta < \pi \\ u(r_0, \theta) = f(\theta) = (r_0 \cos\theta)^4 & \\ u(-\pi) = u(\pi) & \\ u_\theta(-\pi) = u_\theta(\pi) & \end{aligned}$$

Let the solution be

$$u(r, \theta) = R(r)\,\Theta(\theta)$$

Substituting this assumed solution back into the (A) gives

$$r^2 R''\Theta + rR'\Theta + R\Theta'' = 0$$

Dividing the above by $R\Theta$ gives

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + \frac{\Theta''}{\Theta} &= 0 \\ r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to the same constant. say λ.

$$r^2\frac{R''}{R} + r\frac{R'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

This results in the following two ODE's. The boundaries conditions in original PDE are transferred to each ODE which results in

$$\begin{aligned} \Theta'' + \lambda\Theta = 0 \qquad (1) \\ \Theta(-\pi) = \Theta(\pi) \\ \Theta'(-\pi) = \Theta'(\pi) \end{aligned}$$

And

$$r^2 R'' + rR' - \lambda R = 0 \qquad (2)$$

Starting with ODE (1) with periodic boundary conditions.

<u>Case $\lambda < 0$</u> The solution is

$$\Theta(\theta) = A\cosh\left(\sqrt{|\lambda|}\theta\right) + B\sinh\left(\sqrt{|\lambda|}\theta\right)$$

First B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
A\cosh\left(-\sqrt{|\lambda|}\pi\right) + B\sinh\left(-\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right)\\
A\cosh\left(\sqrt{|\lambda|}\pi\right) - B\sinh\left(\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right)\\
2B\sinh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}$$

But $\sinh = 0$ only at zero and $\lambda \neq 0$, hence $B = 0$ and the solution becomes

$$\begin{aligned}
\Theta(\theta) &= A\cosh\left(\sqrt{|\lambda|}\theta\right)\\
\Theta'(\theta) &= A\sqrt{\lambda}\cosh\left(\sqrt{|\lambda|}\theta\right)
\end{aligned}$$

Applying the second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi)\\
A\sqrt{|\lambda|}\cosh\left(-\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
2A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}$$

But cosh is never zero, hence $A = 0$. Therefore trivial solution and <u>$\lambda < 0$ is not an eigenvalue</u>.

<u>Case $\lambda = 0$</u> The solution is $\Theta = A\theta + B$. Applying the first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
-A\pi + B &= \pi A + B\\
2\pi A &= 0\\
A &= 0
\end{aligned}$$

And the solution becomes $\Theta = B_0$. A constant. Hence <u>$\lambda = 0$ is an eigenvalue</u>.

<u>Case $\lambda > 0$</u>

The solution becomes

$$\begin{aligned}
\Theta &= A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)\\
\Theta' &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\theta\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\theta\right)
\end{aligned}$$

Applying first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
A\cos\left(-\sqrt{\lambda}\pi\right) + B\sin\left(-\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
A\cos\left(\sqrt{\lambda}\pi\right) - B\sin\left(\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
2B\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)
\end{aligned}$$

Applying second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi) \\
-A\sqrt{\lambda}\sin\left(-\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) \\
2A\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)
\end{aligned}$$

Equations (3,4) can be both zero only if $A = B = 0$ which gives trivial solution, or when $\sin\left(\sqrt{\lambda}\pi\right) = 0$. Therefore taking $\sin\left(\sqrt{\lambda}\pi\right) = 0$ gives a non-trivial solution. Hence

$$\begin{aligned}
\sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\
\lambda_n &= n^2 \qquad n = 1, 2, 3, \cdots
\end{aligned}$$

Hence the eigenfunctions are

$$\{1, \cos(n\theta), \sin(n\theta)\} \qquad n = 1, 2, 3, \cdots \qquad (5)$$

<u>Now the R equation is solved</u>

The <u>case for $\lambda = 0$</u> gives from (2)

$$\begin{aligned}
r^2 R'' + rR' &= 0 \\
R'' + \frac{1}{r}R' &= 0 \qquad r \neq 0
\end{aligned}$$

The solution to this is

$$R_0(r) = A\ln r + C$$

Since u is bounded at $r = 0$ we want $A = 0$. Hence $R_0(r)$ is just a constant.

<u>Case $\lambda > 0$</u> The ODE (2) becomes

$$r^2R'' + rR' - n^2R = 0 \qquad n = 1, 2, 3, \cdots$$

Let $R = r^p$, the above becomes

$$\begin{aligned} r^2p(p-1)\,r^{p-2} + rpr^{p-1} - n^2r^p &= 0 \\ p(p-1)\,r^p + pr^p - n^2r^p &= 0 \\ p(p-1) + p - n^2 &= 0 \\ p^2 &= n^2 \\ p &= \pm n \end{aligned}$$

Hence the solution is

$$R_n(r) = C_nr^n + D_n\frac{1}{r^n} \qquad n = 1, 2, 3, \cdots$$

Since u is bounded at $r = 0$ we want $D = 0$. Hence $R_n(r) = C_nr^n$.

The complete solution for $R(r)$ is

$$R(r) = C_0 + \sum_{n=1}^{\infty} C_nr^n \tag{6}$$

Using (5),(6) gives

$$\begin{aligned} u_n(r,\theta) &= R_n\Theta_n \\ u(r,\theta) &= \left(C_0 + \sum_{n=1}^{\infty} C_nr^n\right)\left(A_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta) + B_n\sin(n\theta)\right) \end{aligned}$$

Combining constants to simplify things gives

$$u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r^n(A_n\cos(n\theta) + B_n\sin(n\theta)) \tag{7}$$

When $r = r_0$ the above becomes

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r_0^n(A_n\cos(n\theta) + B_n\sin(n\theta))$$

Hence

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta \\ r_0^nA_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta \\ A_n &= \frac{1}{\pi r_0^n}\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta \end{aligned}$$

And

$$
\begin{aligned}
r_0^n B_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta \\
B_n &= \frac{1}{\pi r_0^n}\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta
\end{aligned}
$$

Hence (7) becomes

$$
u(r,\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta + \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{r}{r_0}\right)^n\left(\cos(n\theta)\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,d\theta + \sin(n\theta)\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta\right) \tag{8}
$$

In this problem $f(\theta) = (r_0\cos\theta)^4$ where $r_0 = 4$, hence

$$
\begin{aligned}
A_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} 256\cos^4\theta d\theta \\
&= \frac{256}{\pi}\int_{-\pi}^{\pi}\cos^4\theta d\theta \\
&= \frac{256}{\pi}\left(\frac{3\theta}{8} + \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)\right)_{-\pi}^{\pi} \\
&= \frac{256}{\pi}\left(\frac{3\pi}{8} + \frac{3\pi}{8}\right) \\
&= \frac{256}{\pi}\left(\frac{3\pi}{4}\right) \\
&= 192
\end{aligned}
$$

And

$$
\begin{aligned}
A_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} 256\cos^4(\theta)\cos(n\theta)\,d\theta \\
&= \frac{256}{\pi}\int_{-\pi}^{\pi}\cos^4(\theta)\cos(n\theta)\,d\theta
\end{aligned}
$$

To evaluate the above integral, we will start by using the identity

$$
\cos^4(\theta) = \frac{3}{8} + \frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta)
$$

Therefore the integral now becomes

$$
\begin{aligned}
A_n &= \frac{256}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{8} + \frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta)\right)\cos(n\theta)\,d\theta \\
&= \frac{256}{\pi}\left[\frac{3}{8}\int_{-\pi}^{\pi}\cos(n\theta)\,d\theta + \frac{1}{8}\int_{-\pi}^{\pi}\cos(4\theta)\cos(n\theta)\,d\theta + \frac{1}{2}\int_{-\pi}^{\pi}\cos(2\theta)\cos(n\theta)\,d\theta\right]
\end{aligned} \tag{1}
$$

But $\int_{-\pi}^{\pi} \cos(n\theta)\, d\theta = 0$ and $\int_{-\pi}^{\pi} \cos(4\theta)\cos(n\theta)\, d\theta$ is not zero, only for $n = 4$ by orthogonality of cosine functions. Hence

$$\begin{aligned}\int_{-\pi}^{\pi} \cos(4\theta)\cos(n\theta)\, d\theta &= \int_{-\pi}^{\pi} \cos^2(4\theta)\, d\theta \\ &= \pi\end{aligned}$$

And similarly, $\int_{-\pi}^{\pi} \cos(2\theta)\cos(n\theta)\, d\theta$ is not zero, only for $n = 2$ by orthogonality of cosine functions. Hence

$$\begin{aligned}\int_{-\pi}^{\pi} \cos(2\theta)\cos(n\theta)\, d\theta &= \int_{-\pi}^{\pi} \cos^2(2\theta)\, d\theta \\ &= \pi\end{aligned}$$

Using these results in (1) gives, for $n = 2$

$$\begin{aligned}A_2 &= \frac{256}{\pi}\left[\frac{1}{2}\int_{-\pi}^{\pi} \cos^2(2\theta)\, d\theta\right] \\ &= \frac{256}{\pi}\left(\frac{\pi}{2}\right) \\ &= 128\end{aligned}$$

And for $n = 4$

$$\begin{aligned}A_4 &= \frac{256}{\pi}\left[\frac{1}{8}\int_{-\pi}^{\pi} \cos^2(4\theta)\, d\theta\right] \\ &= \frac{256}{\pi}\left(\frac{\pi}{8}\right) \\ &= 32\end{aligned}$$

And all other A_n are zero. Now that we found all A_n, and since $B_n = 0$ for all n (because $f(\theta)$ is even function) then the solution (8) becomes

$$\begin{aligned}u(r,\theta) &= \frac{192}{2} + a_2\left(\frac{r}{4}\right)^2 \cos(2\theta) + a_4\left(\frac{r}{4}\right)^4 \cos(4\theta) \\ &= 96 + 128\left(\frac{r^2}{16}\right)\cos(2\theta) + 32\frac{r^4}{256}\cos(4\theta)\end{aligned}$$

Therefore

$$u(r,\theta) = 96 + 8r^2\cos(2\theta) + \frac{1}{8}r^4\cos(4\theta)$$

Here is plot of the above solution.

```
sol = 96 + 8 r^2 Cos[2 θ] + 1/8 r^4 Cos[4 θ];
ParametricPlot3D[{r Cos[θ], r Sin[θ], sol}, {r, 0, 4}, {θ, 0, 2 Pi},
   AxesLabel → {x, y, "u(x,y"}, ImageSize → 400, BoxRatios → {1,  1,  1}, BaseStyle → 14]
```

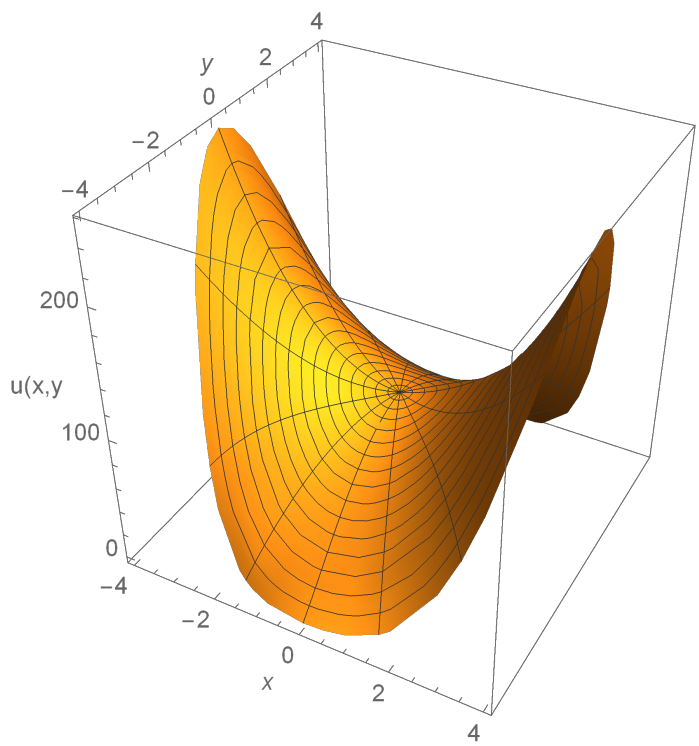


Figure 4.29: Solution plot to the above problem with code used

4.1.2.3 [307] $r = 1$ and $u_r = x$ at boundary of disk

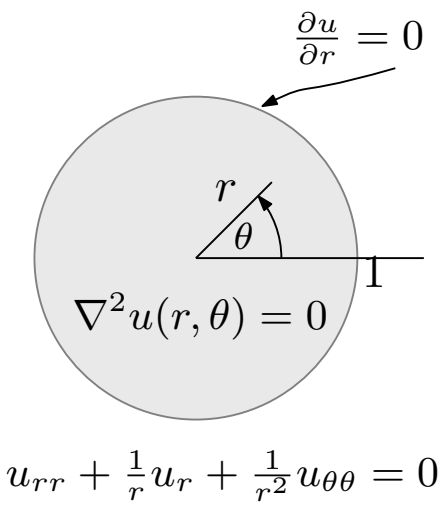
problem number 307

Added January 8, 2020.

Problem 4.3.25 part d, Peter J. Olver, Introduction to Partial Differential Equations, 2014 edition.

Solve Laplace equation in polar coordinates inside a disk of radius 1.

Solve $\nabla^2 u = 0$ and boundary conditions $\frac{\partial u}{\partial n} = x$.



$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

Figure 4.30: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
bc = {Derivative[1, 0][u][1, theta] == Cos[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde:=VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta])=0;
bc  := D[1](u)(1, theta) = cos(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta))),o
```

sol=()
<u>Hand solution</u>

In polar coordinates, where $x = r\cos\theta, y = r\sin\theta$, we need to solve for $u(r,\theta)$ inside

disk of radius $r_0 = 1$. The Laplace PDE in polar coordinates is

$$\begin{aligned} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \qquad 0 < r < 1, -\pi < \theta < \pi \\ u_r(1,\theta) = f(\theta) = \cos\theta \\ u(-\pi) = u(\pi) \\ u_\theta(-\pi) = u_\theta(\pi) \end{aligned}$$

Using separation of variables, let $u(r,\theta) = R(r)\,\Theta(\theta)$ the solution is given by

$$u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n r^n \cos(n\theta) + b_n r^n \sin(n\theta) \tag{1}$$

At $r = r_0 = 1$ we have that $\frac{\partial u(r,\theta)}{\partial r} = \cos\theta$ (since $x = r\cos\theta$ but $r = 1$ at boundary). The above becomes

$$\cos\theta = \sum_{n=1}^{\infty} n a_n r^{n-1} \cos(n\theta) + n b_n r^{n-1} \sin(n\theta)$$

Therefore $n = 1$ is only term that survives in the sum. Hence $a_1 = 1$ and all others are zero. The solution (1) becomes

$$u(r,\theta) = \frac{a_0}{2} + r\cos(\theta)$$

The solution is not unique as there is a_0 arbitrary constant.

4.1.2.4 [308] Laplace inside disk. General solution

problem number 308

Solve Laplace equation in polar coordinates inside a disk

Solve for $u(r,\theta)$

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

With $0 \le r \le a, 0 < \theta \le 2\pi$ Boundary conditions

$$\begin{aligned} u(a,\theta) &= f(\theta) \\ |u(0,\theta)| &< \infty \\ u(r,0) &= u(r,2\pi) \\ \frac{\partial u}{\partial \theta}(r,0) &= \frac{\partial u}{\partial \theta}(r,2\pi) \end{aligned}$$

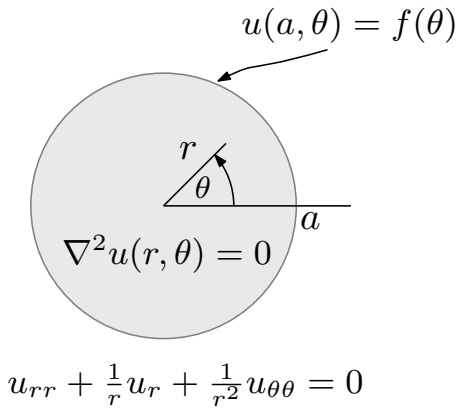


Figure 4.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
bc  = u[a, theta] == f[theta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,\theta) \to \sum_{n=1}^{\infty} \frac{\left(\frac{r}{a}\right)^n \left(\cos(n\theta) \int_{-\pi}^{\pi} \cos(n\theta) f(\theta)\, d\theta + \left(\int_{-\pi}^{\pi} f(\theta) \sin(n\theta)\, d\theta\right) \sin(n\theta)\right)}{\pi} + \frac{\int_{-\pi}^{\pi} f(\theta)\, d\theta}{2\pi}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r +(diff(u(r, theta), theta, theta))/r^2 = 0;
bc  := u(a, theta) = f(theta),
      u(r, -Pi) = u(r, Pi),
      (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta) = \frac{\int_{-\pi}^{\pi} f(\theta)\,d\theta + 2\pi\left(\sum_{n=1}^{\infty} \frac{\left(\left(\int_{-\pi}^{\pi} \cos(n\theta) f(\theta) d\theta\right)\cos(n\theta) + \left(\int_{-\pi}^{\pi} f(\theta)\sin(n\theta) d\theta\right)\sin(n\theta)\right)\left(\frac{a}{r}\right)^{-n}}{\pi}\right)}{2\pi}$$

4.1.2.5 [309] Laplace inside disk. Specific boundary conditions

problem number 309

Added January 12, 2020

Solve $u_{xx} + u_{yy} = 0$ on disk $x^2 + y^2 < 1$ with boundary condition xy^2 when $x^2 + y^2 = a$. Where $a = 1$ in this problem. Express solution in x, y

The first step is to convert the boundary condition to polar coordinates. Since $x = r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned} f(\theta) &= \cos\theta\sin^2\theta \\ &= \cos\theta\left(1-\cos^2\theta\right) \\ &= \cos\theta - \cos^3\theta \end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta) &= \cos\theta - \left(\frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta\right) \\ &= \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta \end{aligned} \tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

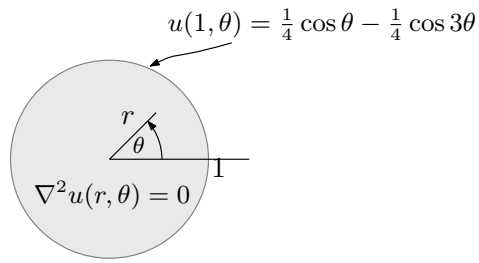


Figure 4.32: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a = 1;
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
f[theta_] := 1/4*(Cos[theta] - Cos[3*theta]);
bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Derivat
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta) \to \frac{1}{4}\left(r\cos(\theta) - r^3\cos(3\theta)\right)\right\}\right\}$$

Maple ✓

```
restart;
f:=theta-> 1/4*(cos(theta) - cos(3*theta));
a:=1;
pde := VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta]);
bc := u(a, theta) = f(theta),u(r, -Pi) = u(r, Pi),(D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
sol:=simplify(subs(cos(theta)^3=trigsubs(cos(theta)^3)[2],sol),size);
```

$$u(r,\theta) = \frac{\left(-r^2\cos(3\theta) + \cos(\theta)\right)r}{4}$$

Hand solution

Solve $u_{xx} + u_{yy} = 0$ on disk $x^2 + y^2 < 1$ with boundary condition xy^2 when $x^2 + y^2 = a$. Where $a = 1$ in this problem. Express solution in x, y

The first step is to convert the boundary condition to polar coordinates. Since $x =$

$r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned} f(\theta) &= \cos\theta\sin^2\theta \\ &= \cos\theta\left(1-\cos^2\theta\right) \\ &= \cos\theta - \cos^3\theta \end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta) &= \cos\theta - \left(\frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta\right) \\ &= \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta \end{aligned} \tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

The solution is known to be

$$u(r,\theta) = \frac{c_0}{2} + \sum_{n=1}^{\infty} r^n(c_n\cos(n\theta) + k_n\sin(n\theta)) \tag{2}$$

Since the above solution is the same as $f(\theta)$ when $r = 1$, then equating (2) when $r = 1$ to (1) gives

$$\frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta = \frac{c_0}{2} + \sum_{n=1}^{\infty}(c_n\cos(n\theta) + k_n\sin(n\theta))$$

By comparing terms on both sides, this shows by inspection that

$$\begin{aligned} c_0 &= 0 \\ c_1 &= \frac{1}{4} \\ c_3 &= \frac{-1}{4} \end{aligned}$$

And all other c_n, k_n are zero. Using the above result back in (2) gives the solution as

$$\boxed{u(r,\theta) = \tfrac{r}{4}\cos\theta - \tfrac{r^3}{4}\cos 3\theta} \tag{3}$$

This solution is now converted to xy using the formula

$$\begin{aligned} r^n\cos n\theta &= \sum_{\substack{k=0\\ even}}^{n}\binom{n}{k}x^{n-k}(-1)^{\frac{k}{2}}y^k \\ &= \sum_{\substack{k=0\\ even}}^{n}\frac{n!}{k!\,(n-k)!}x^{n-k}(-1)^{\frac{k}{2}}y^k \end{aligned}$$

For $n=1$ the above gives

$$\begin{aligned} r\cos\theta &= \frac{1!}{0!\,(1-0)!}x^{1-0}(-1)^0\,y^0 \\ &= x \end{aligned} \tag{4}$$

And for $n=3$

$$\begin{aligned} r^3\cos 3\theta &= \frac{3!}{0!\,(3-0)!}x^{3-0}(-1)^0\,y^0 + \frac{3!}{2!\,(3-2)!}x^{3-2}(-1)^1\,y^2 \\ &= x^3 - 3xy^2 \end{aligned} \tag{5}$$

Using (4,5) in (3) gives the solution in x, y

$$\boxed{u(x,y) = \tfrac{1}{4}x - \tfrac{1}{4}(x^3 - 3xy^2)} \tag{6}$$

This is now verified that is satisfies the PDE $u_{xx} + u_{yy} = 0$.

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{1}{4} - \frac{1}{4}\left(3x^2 - 3y^2\right) \\ \frac{\partial^2 u}{\partial x^2} &= -\frac{6}{4}x \end{aligned}$$

And

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{6}{4}xy \\ \frac{\partial^2 u}{\partial y^2} &= \frac{6}{4}x \end{aligned}$$

Therefore $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Now the boundary conditions $u(x,y) = xy^2$ are also verified. This condition applies when $x^2 + y^2 = 1$ or $y^2 = 1 - x^2$. Substituting this into (6) gives

$$u(x,y)_{@D} = \frac{1}{4}x - \frac{1}{4}\left(x^3 - 3x\overbrace{\left(1-x^2\right)}^{y2}\right)$$

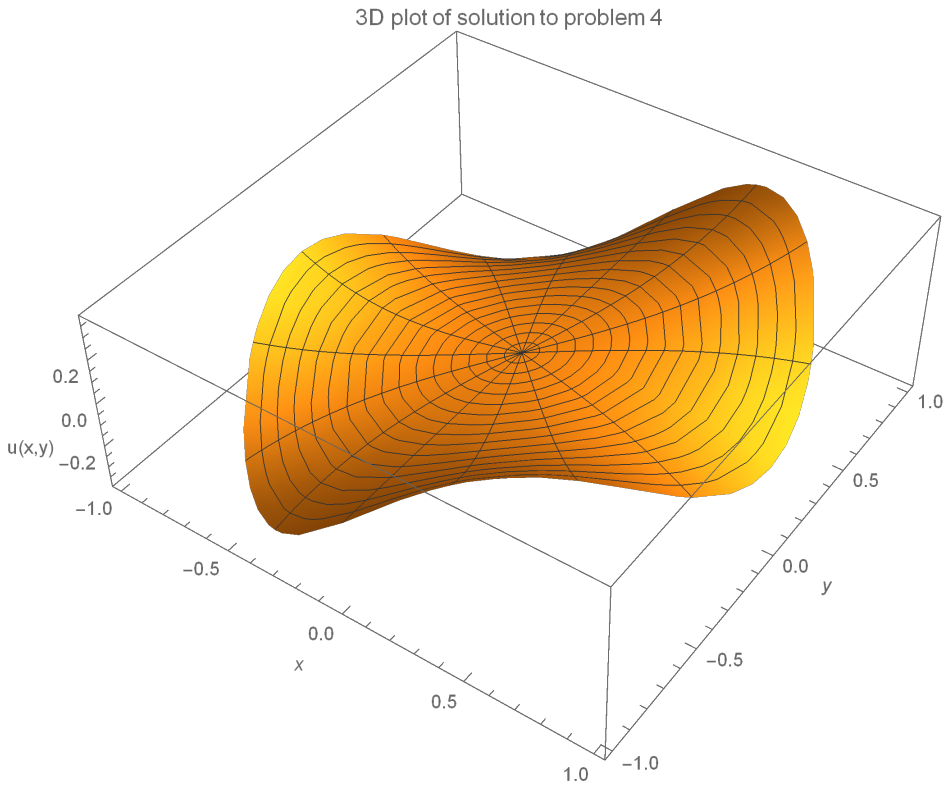
Simplifying gives

$$\begin{aligned} u(x,y)_{@D} &= \frac{1}{4}x - \frac{1}{4}\left(x^3 - \left(3x - 3x^3\right)\right) \\ &= \frac{1}{4}x - \frac{1}{4}x^3 + \frac{1}{4}\left(3x - 3x^3\right) \\ &= \frac{1}{4}x - \frac{1}{4}x^3 + \frac{3}{4}x - \frac{3}{4}x^3 \\ &= x - x^3 \\ &= x\left(1 - x^2\right) \\ &= xy^2 \end{aligned}$$

Verified. This is 3D plot of the solution

```
In[76]:= ParametricPlot3D[{r Cos[t], r Sin[t], r / 4 Cos[t] - r^3 / 4 Cos[3 t]},
           {r, 0, 1}, {t, 0, 2 Pi}, AxesLabel → {x, y, "u(x,y)"},
           PlotLabel → "3D plot of solution to problem 4", ImageSize → 500]
```

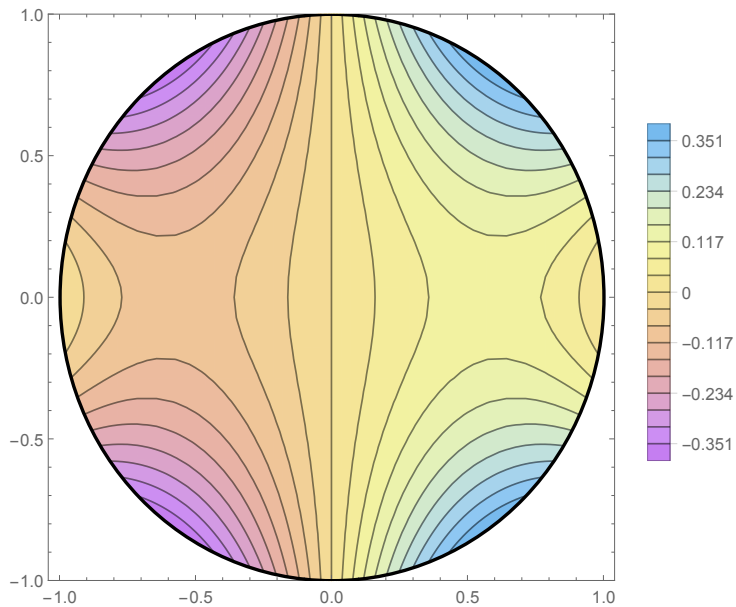
Out[76]=



This is a contour plot

In[96]:=
```
ContourPlot[1 / 4 x - 1 / 4 (x^3 - 3 x y^2), {x, -1, 1}, {y, -1, 1}, AxesLabel → {x, y},
  Contours → 50, PlotLegends → Automatic, ColorFunction → "Pastel",
  Epilog → {Thick, Circle[]},
  PlotRange → {-1, 1},
  RegionFunction → Function[{x, y, z}, Norm[{x, y}] < 1.]]
```

Out[96]=



4.1.2.6 [310] Haberman 2.5.5 (c)

problem number 310

This is problem 2.5.5 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

Inside quarter circle of radius 1 with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 1$, with following boundary conditions

$$u(r,0) = 0$$
$$u(r,\frac{\pi}{2}) = 0$$
$$\frac{\partial u}{\partial r}(1,\theta) = f(\theta)$$

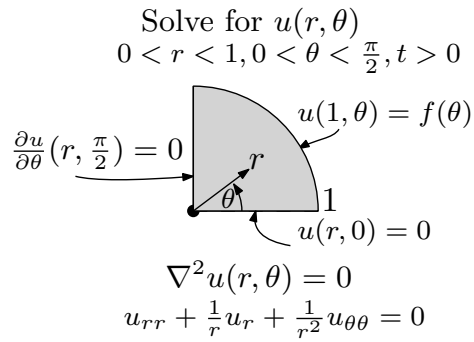


Figure 4.33: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) =
bc  = {Derivative[1, 0][u][1, theta] == f[theta], u[r, Pi/2] == 0, u[r, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r,theta),r$2)+ 1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta),HINT=b
```

$$u(r,\theta) = \sum_{n=1}^{\infty} \frac{2r^{2n}\left(\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta\right)\sin(2n\theta)}{\pi n}$$

<u>Hand solution</u>

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0 \tag{A}$$

With boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u\left(r,\frac{\pi}{2}\right) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta) = R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2R''\Theta + rR'\Theta + R\Theta'' = 0$$

Dividing the above by $R\Theta \neq 0$ gives

$$\begin{aligned} r^2\frac{R''}{R} + r\frac{R'}{R} + \frac{\Theta''}{\Theta} &= 0 \\ r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R} + r\frac{R'}{R} = -\frac{\Theta''}{\Theta} = \lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This gives

$$\begin{aligned} \Theta'' + \lambda\Theta &= 0 \\ \Theta(0) &= 0 \\ \Theta\left(\frac{\pi}{2}\right) &= 0 \end{aligned} \tag{1}$$

And

$$\begin{aligned} r^2R'' + rR' - \lambda R &= 0 \\ |R(0)| &< \infty \end{aligned} \tag{2}$$

Starting with (1). Consider the <u>Case $\lambda < 0$</u>. The solution in this case will be

$$\Theta = A\cosh\left(\sqrt{\lambda}\theta\right) + B\sinh\left(\sqrt{\lambda}\theta\right)$$

Applying first B.C. gives $A = 0$. The solution becomes $\Theta = B\sinh\left(\sqrt{\lambda}\theta\right)$. Applying second B.C. gives

$$0 = B\sinh\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

But sinh is zero only when $\sqrt{\lambda}\frac{\pi}{2} = 0$ which is not the case here. Therefore $B = 0$ and hence trivial solution. Hence $\underline{\lambda < 0 \text{ is not an eigenvalue}}$.

<u>Case $\lambda = 0$</u> The ODE becomes $\Theta'' = 0$ with solution $\Theta = A\theta + B$. First B.C. gives $0 = B$. The solution becomes $\Theta = A\theta$. Second B.C. gives $0 = A\frac{\pi}{2}$, hence $A = 0$ and trivial solution. Therefore $\underline{\lambda = 0 \text{ is not an eigenvalue}}$.

<u>Case $\lambda > 0$</u> The ODE becomes $\Theta'' + \lambda\Theta = 0$ with solution

$$\Theta = A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)$$

The first B.C. gives $0 = A$. The solution becomes

$$\Theta = B\sin\left(\sqrt{\lambda}\theta\right)$$

And the second B.C. gives

$$0 = B\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right) = 0$ or $\sqrt{\lambda}\frac{\pi}{2} = n\pi$ for $n = 1, 2, 3, \cdots$. Hence the eigenvalues are

$$\begin{aligned}\sqrt{\lambda_n} &= 2n \\ \lambda_n &= 4n^2 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

And the eigenfunctions are

$$\Theta_n(\theta) = B_n\sin(2n\theta) \qquad n = 1, 2, 3, \cdots \tag{3}$$

<u>Now the R ODE is solved</u>. There is one case to consider, which is $\lambda > 0$ based on the above. The ODE is

$$\begin{aligned}r^2R'' + rR' - \lambda_n R &= 0 \\ r^2R'' + rR' - 4n^2R &= 0 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

This is Euler ODE. Let $R(r) = r^p$. Then $R' = pr^{p-1}$ and $R'' = p(p-1)\,r^{p-2}$. This gives

$$\begin{aligned}r^2\left(p(p-1)\,r^{p-2}\right) + r\left(pr^{p-1}\right) - 4n^2r^p &= 0 \\ \left(\left(p^2 - p\right)r^p\right) + pr^p - 4n^2r^p &= 0 \\ r^pp^2 - pr^p + pr^p - 4n^2r^p &= 0 \\ p^2 - 4n^2 &= 0 \\ p &= \pm 2n\end{aligned}$$

Hence the solution is

$$R(r) = Cr^{2n} + D\frac{1}{r^{2n}}$$

Applying the condition that $|R(0)| < \infty$ implies $D = 0$, and the solution becomes

$$R_n(r) = C_n r^{2n} \qquad n = 1, 2, 3, \cdots \tag{4}$$

Using (3,4) the solution $u_n(r, \theta)$ is

$$\begin{aligned} u_n(r, \theta) &= R_n \Theta_n \\ &= C_n r^{2n} B_n \sin(2n\theta) \\ &= B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Where $C_n B_n$ was combined into one constant B_n. (No need to introduce new symbol). The final solution is

$$\begin{aligned} u(r, \theta) &= \sum_{n=1}^{\infty} u_n(r, \theta) \\ &= \sum_{n=1}^{\infty} B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Now the nonhomogeneous condition is applied to find B_n.

$$\frac{\partial}{\partial r} u(r, \theta) = \sum_{n=1}^{\infty} B_n (2n)\, r^{2n-1} \sin(2n\theta)$$

Hence $\frac{\partial}{\partial r} u(1, \theta) = f(\theta)$ becomes

$$f(\theta) = \sum_{n=1}^{\infty} 2B_n n \sin(2n\theta)$$

Multiplying by $\sin(2m\theta)$ and integrating gives

$$\begin{aligned} \int_0^{\frac{\pi}{2}} f(\theta) \sin(2m\theta)\, d\theta &= \int_0^{\frac{\pi}{2}} \sin(2m\theta) \sum_{n=1}^{\infty} 2B_n n \sin(2n\theta)\, d\theta \\ &= \sum_{n=1}^{\infty} 2nB_n \int_0^{\frac{\pi}{2}} \sin(2m\theta) \sin(2n\theta)\, d\theta \end{aligned} \tag{5}$$

When $n = m$ then

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}} \sin^2(2n\theta)\,d\theta \\
&= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos 4n\theta\right) d\theta \\
&= \frac{1}{2}[\theta]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin 4n\theta}{4n}\right]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{4} - \left(\frac{1}{8n}\left(\sin\frac{4n}{2}\pi\right) - \sin(0)\right)
\end{aligned}$$

And since n is integer, then $\sin\frac{4n}{2}\pi = \sin 2n\pi = 0$ and the above becomes $\frac{\pi}{4}$.

Now for the case when $n \neq m$ using $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ then

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos(2m\theta - 2n\theta) - \cos(2m\theta + 2n\theta))\,d\theta \\
&= \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2m\theta - 2n\theta)\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2m\theta + 2n\theta)\,d\theta \\
&= \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos((2m-2n)\theta)\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos((2m+2n)\theta)\,d\theta \\
&= \frac{1}{2}\left[\frac{\sin((2m-2n)\theta)}{(2m-2n)}\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin((2m+2n)\theta)}{(2m+2n)}\right]_0^{\frac{\pi}{2}} \\
&= \frac{1}{4(m-n)}[\sin((2m-2n)\theta)]_0^{\frac{\pi}{2}} - \frac{1}{4(m+n)}[\sin((2m+2n)\theta)]_0^{\frac{\pi}{2}} \\
&= \frac{1}{4(m-n)}\left[\sin\left((2m-2n)\frac{\pi}{2}\right) - 0\right] - \frac{1}{4(m+n)}\left[\sin\left((2m+2n)\frac{\pi}{2}\right) - 0\right]
\end{aligned}$$

Since $2m - 2n\frac{\pi}{2} = \pi(m-n)$ which is integer multiple of π and also $(2m+2n)\frac{\pi}{2}$ is integer multiple of π then the whole term above becomes zero. Therefore (5) becomes

$$\int_0^{\frac{\pi}{2}} f(\theta)\sin(2m\theta)\,d\theta = 2mB_m\frac{\pi}{4}$$

Hence

$$B_n = \frac{2}{\pi n}\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta$$

<u>Summary</u>: the final solution is

$$u(r,\theta) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\int_0^{\frac{\pi}{2}} f(\theta)\sin(2n\theta)\,d\theta\right]\left(r^{2n}\sin(2n\theta)\right)$$

4.1.2.7 [311] semi-circle

problem number 311

Solve Laplace equation

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

Inside semi-circle of radius 1 with $0 \le \theta \le \pi$ and $0 \le r \le 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned}$$

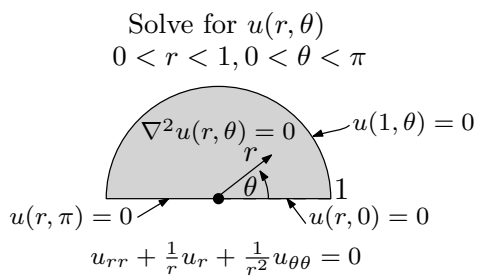


Figure 4.34: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta], {r, 2}] + (1/r) D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] ==
bc  = {u[r, 0] == 0, u[r, Pi] == 0, u[1, theta] == f[theta]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta) \to \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} r^{K[1]} \left(\int_0^{\pi} \sqrt{\frac{2}{\pi}} f(\theta) \sin(\theta K[1])\, d\theta\right) \sin(\theta K[1])\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(r,0)=0,u(r,Pi)=0,u(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta),HINT=b
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\frac{2r^n\left(\int_0^{\pi}f(\theta)\sin(n\theta)\,d\theta\right)\sin(n\theta)}{\pi}$$

4.1.2.8 [312] Haberman 2.5.8 (b)

problem number 312

This is problem 2.5.8 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation $\nabla^2 u(r,\theta)=0$ or

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

Inside circular annulus $a<r<b$ subject to the following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial r}(a,\theta)&=0\\ u(b,\theta)&=g(\theta)\end{aligned}$$

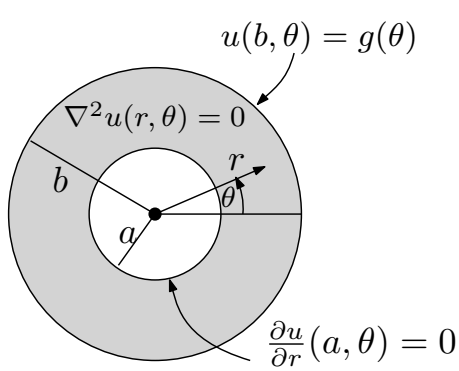


Figure 4.35: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, theta], {r, 2}] + 1/r*D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] == 0
bc  = {Derivative[1, 0][u][a, theta] == 0, u[b, theta] == g[theta]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta)\to\ \{\ \frac{\int_0^{2\pi} g(K[2])\,dK[2]}{2\pi}+\sum_{K[1]=1}^{\infty}\left(\cos(\theta K[1])\left(\frac{a^{2K[1]}b^{K[1]}\left(\int_0^{2\pi}\frac{\cos(K[1]K[2])g(K[2])}{\pi}\,dK[2]\right)r^{-K[1]}}{a^{2K[1]}+b^{2K[1]}}+\frac{b^{K[1]}\left(\int_0^{2\pi}\right.}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta)) assum
```

$$u(r,\theta)=-\mathcal{F}^{-1}\left(\frac{\mathcal{F}(g(\theta),\theta,s)}{e^{2(\ln(a)-\ln(b))s}+1},s,\theta\right)+\mathcal{F}^{-1}\left(\frac{\mathcal{F}(g(\theta),\theta,s)\,r^s a^{-s}e^{(\ln(a)-\ln(b))s}}{e^{2(\ln(a)-\ln(b))s}+1},s,\theta\right)+\mathcal{F}^{-1}\left(\frac{\mathcal{F}(g(\theta),\theta,s}{e^{2(\ln(}}\right.$$

Hand solution

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial\theta^2}=0 \tag{A}$$

With

$$\begin{aligned}\frac{\partial u}{\partial r}(a,\theta)&=0\\ u(b,\theta)&=g(\theta)\end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta)=R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2R''\Theta+rR'\Theta+R\Theta''=0$$

Dividing the above by $R\Theta$ gives

$$\begin{aligned} r^2\frac{R''}{R}+r\frac{R'}{R}+\frac{\Theta''}{\Theta}&=0\\ r^2\frac{R''}{R}+r\frac{R'}{R}&=-\frac{\Theta''}{\Theta} \end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R}+r\frac{R'}{R}=-\frac{\Theta''}{\Theta}=\lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This results in

$$\begin{aligned} \Theta''+\lambda\Theta&=0 \qquad (1)\\ \Theta(-\pi)&=\Theta(\pi)\\ \Theta'(-\pi)&=\Theta'(\pi) \end{aligned}$$

And

$$\begin{aligned} r^2R''+rR'-\lambda R&=0 \qquad (2)\\ R'(a)&=0 \end{aligned}$$

Starting with (1)

<u>Case $\lambda<0$</u> The solution is

$$\Theta(\theta)=A\cosh\left(\sqrt{|\lambda|}\theta\right)+B\sinh\left(\sqrt{|\lambda|}\theta\right)$$

First B.C. gives

$$\begin{aligned} \Theta(-\pi)&=\Theta(\pi)\\ A\cosh\left(-\sqrt{|\lambda|}\pi\right)+B\sinh\left(-\sqrt{|\lambda|}\pi\right)&=A\cosh\left(\sqrt{|\lambda|}\pi\right)+B\sinh\left(\sqrt{|\lambda|}\pi\right)\\ A\cosh\left(\sqrt{|\lambda|}\pi\right)-B\sinh\left(\sqrt{|\lambda|}\pi\right)&=A\cosh\left(\sqrt{|\lambda|}\pi\right)+B\sinh\left(\sqrt{|\lambda|}\pi\right)\\ 2B\sinh\left(\sqrt{|\lambda|}\pi\right)&=0 \end{aligned}$$

But $\sinh=0$ only at zero and $\lambda\neq 0$, hence $B=0$ and the solution becomes

$$\begin{aligned} \Theta(\theta)&=A\cosh\left(\sqrt{|\lambda|}\theta\right)\\ \Theta'(\theta)&=A\sqrt{\lambda}\cosh\left(\sqrt{|\lambda|}\theta\right) \end{aligned}$$

Applying the second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi)\\
A\sqrt{|\lambda|}\cosh\left(-\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right)\\
2A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= 0
\end{aligned}$$

But cosh is never zero, hence $A = 0$. Therefore trivial solution and <u>$\lambda < 0$ is not an eigenvalue</u>.

<u>Case $\lambda = 0$</u> The solution is $\Theta = A\theta + B$. Applying the first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
-A\pi + B &= \pi A + B\\
2\pi A &= 0\\
A &= 0
\end{aligned}$$

And the solution becomes $\Theta = B_0$. A constant. Hence <u>$\lambda = 0$ is an eigenvalue</u>.

<u>Case $\lambda > 0$</u>

The solution becomes

$$\begin{aligned}
\Theta &= A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)\\
\Theta' &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\theta\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\theta\right)
\end{aligned}$$

Applying first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi)\\
A\cos\left(-\sqrt{\lambda}\pi\right) + B\sin\left(-\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right)\\
A\cos\left(\sqrt{\lambda}\pi\right) - B\sin\left(\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right)\\
2B\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)
\end{aligned}$$

Applying second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi)\\
-A\sqrt{\lambda}\sin\left(-\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right)\\
2A\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)
\end{aligned}$$

Equations (3,4) can be both zero only if $A = B = 0$ which gives trivial solution, or when $\sin\left(\sqrt{\lambda}\pi\right) = 0$. Therefore taking $\sin\left(\sqrt{\lambda}\pi\right) = 0$ gives a non-trivial solution. Hence

$$\begin{aligned}\sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\ \lambda_n &= n^2 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

Hence the solution for Θ is

$$\Theta = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

<u>Now the R equation is solved</u>

The <u>case for $\lambda = 0$</u> gives

$$\begin{aligned}r^2 R'' + rR' &= 0 \\ R'' + \frac{1}{r}R' &= 0 \qquad r \neq 0\end{aligned}$$

As was done in last problem, the solution to this is

$$R(r) = A \ln|r| + C$$

Since $r > 0$ no need to keep worrying about $|r|$ and is removed for simplicity. Applying the B.C. gives

$$R' = A\frac{1}{r}$$

Evaluating at $r = a$ gives

$$0 = A\frac{1}{a}$$

Hence $A = 0$, and the solution becomes

$$R(r) = C_0$$

Which is a constant.

<u>Case $\lambda > 0$</u> The ODE in this case is

$$r^2 R'' + rR' - n^2 R = 0 \qquad n = 1, 2, 3, \cdots$$

Let $R = r^p$, the above becomes

$$\begin{aligned}r^2 p(p-1)\, r^{p-2} + rpr^{p-1} - n^2 r^p &= 0 \\ p(p-1)\, r^p + pr^p - n^2 r^p &= 0 \\ p(p-1) + p - n^2 &= 0 \\ p^2 &= n^2 \\ p &= \pm n\end{aligned}$$

Hence the solution is

$$R_n(r) = Cr^n + D\frac{1}{r^n} \qquad n = 1, 2, 3, \cdots$$

Applying the boundary condition $R'(a) = 0$ gives

$$\begin{aligned}
R_n'(r) &= nC_n r^{n-1} - nD_n \frac{1}{r^{n+1}} \\
0 &= R_n'(a) \\
&= nC_n a^{n-1} - nD_n \frac{1}{a^{n+1}} \\
&= nC_n a^{2n} - nD_n \\
&= C_n a^{2n} - D_n \\
D_n &= C_n a^{2n}
\end{aligned}$$

The solution becomes

$$\begin{aligned}
R_n(r) &= C_n r^n + C_n a^{2n} \frac{1}{r^n} \qquad n = 1, 2, 3, \cdots \\
&= C_n \left(r^n + \frac{a^{2n}}{r^n} \right)
\end{aligned}$$

Hence the complete solution for $R(r)$ is

$$R(r) = C_0 + \sum_{n=1}^{\infty} C_n \left(r^n + \frac{a^{2n}}{r^n} \right) \tag{6}$$

Using (5),(6) gives

$$\begin{aligned}
u_n(r, \theta) &= R_n \Theta_n \\
u(r, \theta) &= \left[C_0 + \sum_{n=1}^{\infty} C_n \left(r^n + \frac{a^{2n}}{r^n} \right) \right] \left[A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \right] \\
&= D_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) C_n \left(r^n + \frac{a^{2n}}{r^n} \right) + \sum_{n=1}^{\infty} B_n \sin(n\theta) C_n \left(r^n + \frac{a^{2n}}{r^n} \right)
\end{aligned}$$

Where $D_0 = C_0 A_0$. To simplify more, $A_n C_n$ is combined to A_n and $B_n C_n$ is combined to B_n. The full solution is

$$u(r, \theta) = D_0 + \sum_{n=1}^{\infty} A_n \left(r^n + \frac{a^{2n}}{r^n} \right) \cos(n\theta) + \sum_{n=1}^{\infty} B_n \left(r^n + \frac{a^{2n}}{r^n} \right) \sin(n\theta)$$

The final nonhomogeneous B.C. is applied.

$$\begin{aligned}
u(b, \theta) &= g(\theta) \\
g(\theta) &= D_0 + \sum_{n=1}^{\infty} A_n \left(b^n + \frac{a^{2n}}{b^n} \right) \cos(n\theta) + \sum_{n=1}^{\infty} B_n \left(b^n + \frac{a^{2n}}{b^n} \right) \sin(n\theta)
\end{aligned}$$

For $n=0$, integrating both sides give

$$\int_{-\pi}^{\pi} g(\theta)\,d\theta = \int_{-\pi}^{\pi} D_0 d\theta$$
$$D_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\theta)\,d\theta$$

For $n>0$, multiplying both sides by $\cos(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta\\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\cos(m\theta)\cos(n\theta)\,d\theta\\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\cos(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta\\ &+ \sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta\\ &+ \sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta \qquad (7)\end{aligned}$$

But

$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = \pi \qquad n=m\neq 0$$
$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = 0 \qquad n\neq m$$

And

$$\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta = 0$$

Then (7) becomes

$$\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta = \pi A_n\left(b^n+\frac{a^{2n}}{b^n}\right)$$
$$A_n = \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta}{b^n+\frac{a^{2n}}{b^n}} \qquad (8)$$

Again, multiplying both sides by $\sin(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\sin(m\theta)\,d\theta\\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\sin(m\theta)\cos(n\theta)\,d\theta\\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\sin(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0\sin(m\theta)\,d\theta\\ &+ \sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta\\ &+ \sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta \qquad (9)\end{aligned}$$

But

$$\begin{aligned}\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= \pi \qquad n=m\neq 0\\ \int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= 0 \qquad n\neq m\end{aligned}$$

And

$$\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\sin(m\theta)\,d\theta = 0$$

Then (9) becomes

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta &= \pi B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\\ B_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta}{b^n+\frac{a^{2n}}{b^n}}\end{aligned}$$

This complete the solution. Summary

$$
\begin{aligned}
u(r,\theta) &= D_0 + \sum_{n=1}^{\infty} A_n \left(r^n + \frac{a^{2n}}{r^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n \left(r^n + \frac{a^{2n}}{r^n}\right)\sin(n\theta)\\
D_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\theta)\,d\theta\\
A_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}}\\
B_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}}
\end{aligned}
$$

4.1.2.9 [313] Circular annulus

problem number 313

Solve Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

Inside circular annulus $1 < r < 2$ subject to the following boundary conditions

$$
\begin{aligned}
u(1,\theta) &= 0\\
u(2,\theta) &= \sin\theta
\end{aligned}
$$

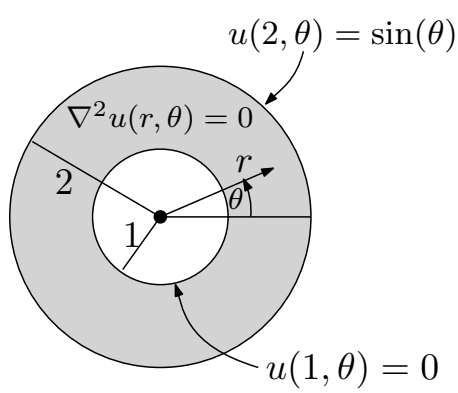


Figure 4.36: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
bc  = {u[1, theta] == 0, u[2, theta] == Sin[theta]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta) \to \begin{cases} \frac{2(r^2-1)\sin(\theta)}{3r} & 1 \le r \le 2 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc  := u(1,theta)=0,u(2,theta)=sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta))),outp
```

$$u(r,\theta) = \frac{2(r^2-1)\sin(\theta)}{3r}$$

4.1.2.10 [314] Outside a disk

problem number 314

Solve Laplace equation in polar coordinates outside a disk

Solve for $u(r,\theta)$

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$
$$a \le r$$
$$0 < \theta \le 2\pi$$

Boundary conditions

$$u(a,\theta) = f(\theta)$$
$$|u(0,\theta)| < \infty$$
$$u(r,0) = u(r,2\pi)$$
$$\frac{\partial u}{\partial \theta}(r,0) = \frac{\partial u}{\partial \theta}(r,2\pi)$$

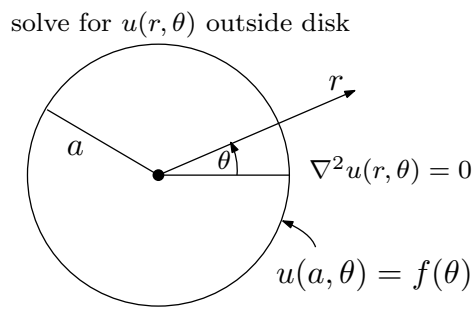


Figure 4.37: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta], {r, 2}] + 1/r*D[u[r, theta], r] + 1/r^2*D[u[r, theta], {theta, 2}] ==
bc  = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Deriva
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta) \to \sum_{K[1]=1}^{\infty} \frac{a^{-K[1]} r^{K[1]} \left(\cos(\theta K[1]) \int_{-\pi}^{\pi} \frac{\cos(\theta K[1]) f(\theta)}{\sqrt{\pi}}\, d\theta + \left(\int_{-\pi}^{\pi} \frac{f(\theta)\sin(\theta K[1])}{\sqrt{\pi}}\, d\theta\right) \sin(\theta K[1])\right)}{\sqrt{\pi}} + \frac{\int_{-\pi}^{\pi}}{\sqrt{}}\right.\right.$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(r, theta), r$2) + 1/r* diff(u(r,theta),r) + 1/r^2* diff(u(r, theta), theta$2)
bc  := u(a, theta) = f(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta) = \frac{\int_{-\pi}^{\pi} f(\theta)\, d\theta + 2\pi \left(\sum_{n=1}^{\infty} \frac{\left(\left(\int_{-\pi}^{\pi} \cos(n\theta) f(\theta) d\theta\right) \cos(n\theta) + \left(\int_{-\pi}^{\pi} f(\theta) \sin(n\theta) d\theta\right) \sin(n\theta)\right) \left(\frac{r}{a}\right)^{-n}}{\pi}\right)}{2\pi}$$

4.1.2.11 [315] Outside a disk

problem number 315

Added January 13, 2020

Laplace PDE in polar coordinates outside a disk

Solve $u_{xx} + u_{yy} = 0$ outside disk $x^2 + y^2 > 1$ with boundary condition xy^2 when $x^2 + y^2 = a$. Where $a = 1$ in this problem. Express solution in x, y

The first step is to convert the boundary condition to polar coordinates. Since $x = r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned} f(\theta) &= \cos\theta\sin^2\theta \\ &= \cos\theta\left(1-\cos^2\theta\right) \\ &= \cos\theta - \cos^3\theta \end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta) &= \cos\theta - \left(\frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta\right) \\ &= \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta \end{aligned} \tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

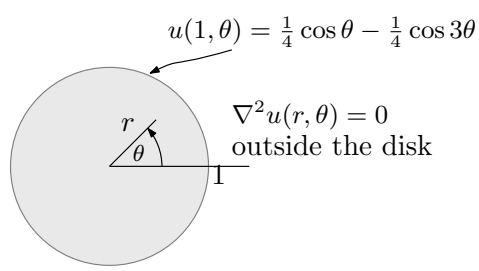


Figure 4.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1;
pde = Laplacian[u[r, theta], {r, theta}, "Polar"] == 0;
f[theta_] := 1/4*(Cos[theta] - Cos[3*theta]);
bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Derivat
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

$$\left\{\left\{u(r,\theta)\to\frac{1}{4}\left(r\cos(\theta)-r^3\cos(3\theta)\right)\right\}\right\}$$

Maple ✓

```
restart;
f:=theta-> 1/4*(cos(theta) - cos(3*theta));
a:=1;
pde := VectorCalculus:-Laplacian(u(r,theta),'polar'[r,theta]);
bc := u(a, theta) = f(theta),u(r, -Pi) = u(r, Pi),(D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
sol:=simplify(subs(cos(theta)^3=trigsubs(cos(theta)^3)[2],expand(sol)),size);
```

$$u(r,\theta)=\frac{r^2\cos(\theta)-\cos(3\theta)}{4r^3}$$

Hand solution

The first step is to convert the boundary condition to polar coordinates. Since $x = r\cos\theta, y = r\sin\theta$, then at the boundary $u(r,\theta) = r\cos\theta(r\sin\theta)^2$. But $r = 1$ (the radius). Hence at the boundary, $u(1,\theta) = f(\theta)$ where

$$\begin{aligned} f(\theta) &= \cos\theta\sin^2\theta \\ &= \cos\theta\left(1-\cos^2\theta\right) \\ &= \cos\theta-\cos^3\theta \end{aligned}$$

But $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$. Therefore the above becomes

$$\begin{aligned} f(\theta) &= \cos\theta-\left(\frac{3}{4}\cos\theta+\frac{1}{4}\cos 3\theta\right) \\ &= \frac{1}{4}\cos\theta-\frac{1}{4}\cos 3\theta \end{aligned} \tag{1}$$

The above is also seen as the Fourier series of $f(\theta)$. The PDE in polar coordinates is

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

The solution is known to be

$$u(r,\theta) = \frac{c_0}{2} + \sum_{n=1}^{\infty} r^{-n}(c_n \cos(n\theta) + k_n \sin(n\theta)) \tag{2}$$

Since the above solution is the same as $f(\theta)$ when $r = 1$, then equating (2) when $r = 1$ to (1) gives

$$\frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta = \frac{c_0}{2} + \sum_{n=1}^{\infty} (c_n \cos(n\theta) + k_n \sin(n\theta))$$

By comparing terms on both sides, this shows by inspection that

$$\begin{aligned} c_0 &= 0 \\ c_1 &= \frac{1}{4} \\ c_3 &= \frac{-1}{4} \end{aligned}$$

And all other c_n, k_n are zero. Using the above result back in (2) gives the solution as

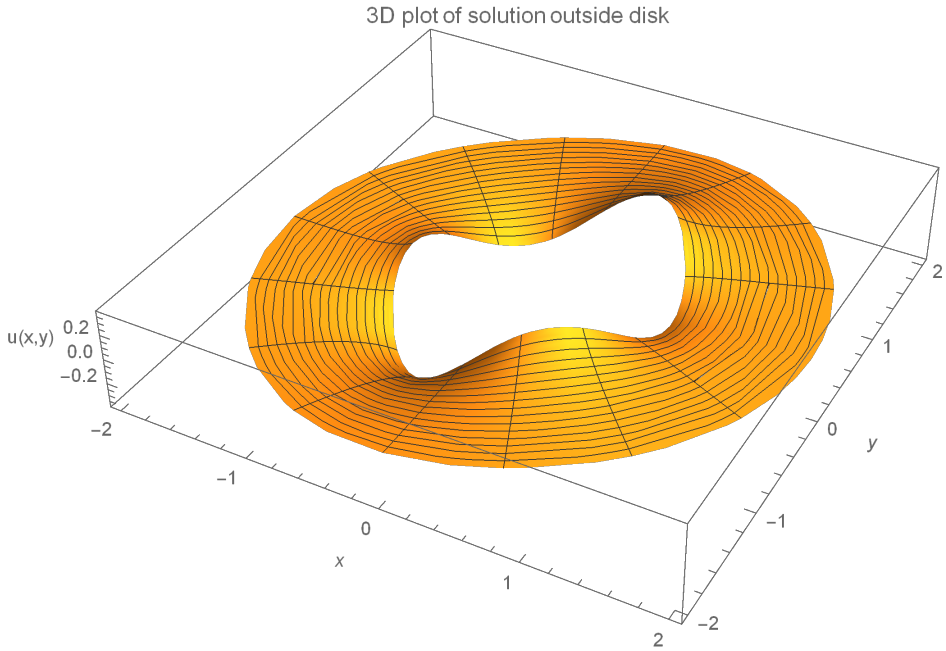
$$\begin{aligned} u(r,\theta) &= \frac{r^{-1}}{4}\cos\theta - \frac{r^{-3}}{4}\cos 3\theta \\ &= \frac{r^2\cos(\theta) - \cos(3\theta)}{4r^3} \end{aligned} \tag{3}$$

This is 3D plot of the solution

```
polarSolution[r_, phi_] := (r^-1) / 4 Cos[phi] - (r^-3) / 4 Cos[3 phi];
```

In[49]:=
```
ParametricPlot3D[{r Cos[t], r Sin[t], polarSolution[r, t]},
  {r, 1, 2}, {t, 0, 2 Pi}, AxesLabel → {x, y, "u(x,y)"},
  PlotLabel → "3D plot of solution outside disk", ImageSize → 500]
```

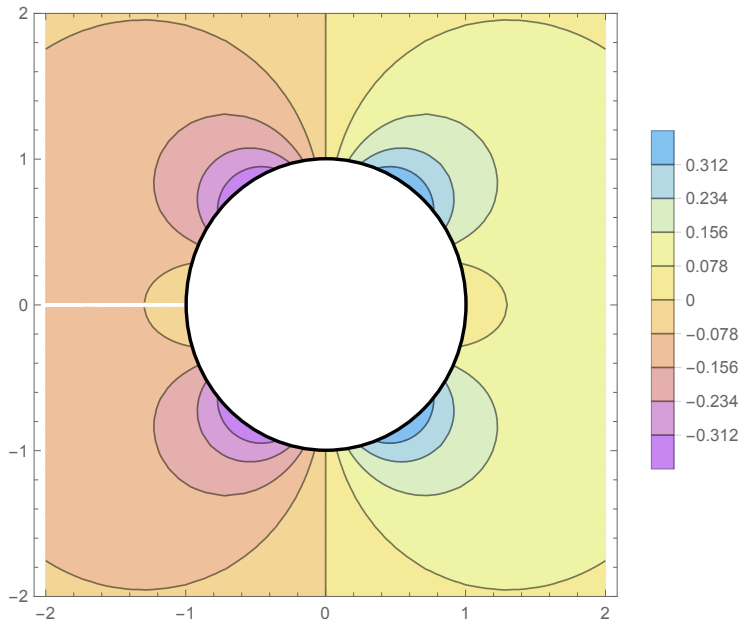
Out[49]=



This is a contour plot

In[52]:=
```
ContourPlot[polarSolution[Sqrt[x^2 + y^2], ArcTan[x, y]], {x, -2, 2}, {y, -2, 2},
  AxesLabel → {x, y},
  Contours → 50, PlotLegends → Automatic, ColorFunction → "Pastel",
  Epilog → {Thick, Circle[]},
  PlotRange → {-2, 2},
  RegionFunction → Function[{x, y, z}, Norm[{x, y}] > 1.]]
```

Out[52]=



4.2 Laplace in 3D

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4.2.2 Cylinderical coordinates 879

4.2.1 Spherical coordinates

Local contents

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4.2.1.1 [316] In a sphere

problem number 316

Taken from Maple pdsolve help pages

Solve for $u(r,\theta,\phi)$. Where θ is the polar angle and ϕ is the azimuthal angle. Hence $0<\theta<\pi$ and $-\pi<\phi<\pi$.

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2}=0$$

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[f[r, theta, phi], {r, theta, phi}, "Spherical"];
sol = AbsoluteTiming[TimeConstrained[DSolve[lap == 0, f[r, theta, phi], {r, theta, phi}], 60
```

$$\left\{\left\{f(r,\theta,\phi)\to\ \{\ \sqrt{2}r^{-\frac{1}{2}\sqrt{4c_7+1}-\frac{1}{2}}\left(c_1r^{\sqrt{4c_7+1}}+c_2\right)\left(c_4\,{}_2F_1\left(\tfrac{1}{4}\left(-\sqrt{4c_7+1}+2\sqrt{c_8}+1\right),\tfrac{1}{4}\left(\sqrt{4c_7+1}+\right.\right.\right.\right.$$

Maple ✓

```
restart;
PDE := diff(r^2*diff(F(r,theta,phi),r),r)+ 1/sin(theta)*diff(sin(theta)*diff(F(r,theta,phi),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(PDE,F(r,theta,phi),'build'
```

$$F(r,\theta,\phi) = \frac{\left(c_5 \sin\left(\phi\sqrt{\_c_2}\right) + c_6 \cos\left(\phi\sqrt{\_c_2}\right)\right)\left(c_1 r^{\frac{\sqrt{4\_c_1+1}}{2}} + c_2 r^{-\frac{\sqrt{4\_c_1+1}}{2}}\right)\left(c_4\sqrt{2}\ \text{hypergeom}\left(\left[\frac{\sqrt{\_c_2}}{2} +\right.\right.}{}$$

4.2.2 Cylinderical coordinates

Local contents

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problem number 317

Added May 25, 2019.

Problem 7.9.1 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0) = f(r,\theta)$, $u(r,\theta,H) = 0$, $u(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

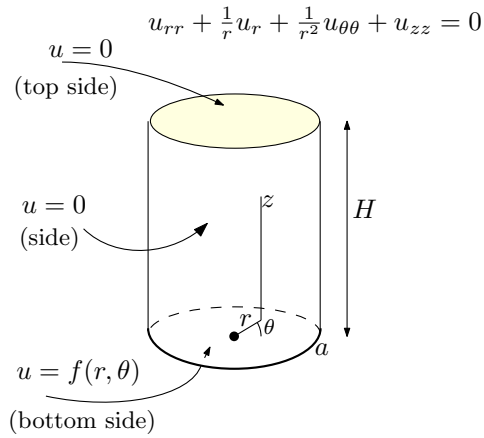

Figure 4.39: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == f[r, theta], u[r, theta, H] == 0, u[a, theta, z] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = f(r, theta), u(r, theta, H) = 0, u(a, theta, z) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.2 [318] Haberman 7.9.1 (b)

problem number 318

Added May 25, 2019.

Problem 7.9.1 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0) = f(r)\sin(7\theta)$, $u(r,\theta,H) = 0$, $u(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

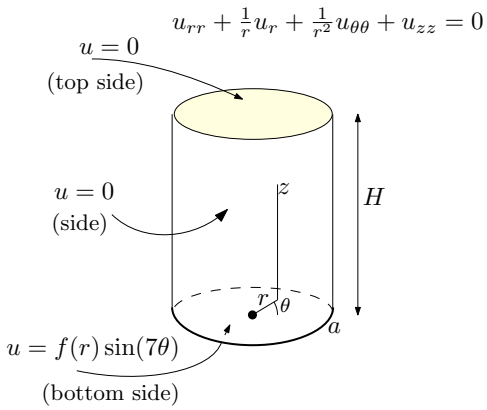

Figure 4.40: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == f[r]*Sin[7*theta], u[r, theta, H] == 0, u[a, theta, z] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = f(r)*sin(7*theta), u(r, theta, H) = 0, u(a, theta, z) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.3 [319] Haberman 7.9.1 (c)

problem number 319

Added May 25, 2019.

Problem 7.9.1 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u(r,\theta,0)=0$, $u(r,\theta,H)=f(r)\cos(3\theta)$, $u_r(a,\theta,z)=0$.

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}+u_{zz}=0$$

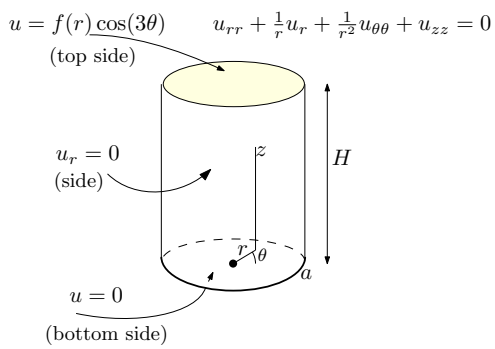

Figure 4.41: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, theta, H] == f[r]*Cos[3*theta], Derivative[1,0,0][u][a, the
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r, theta, 0) = 0, u(r, theta, H) = f(r)*cos(3*theta), eval(diff(u(r,theta,z),r),r=a
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.4 [320] Haberman 7.9.1 (d)

problem number 320

Added May 25, 2019.

Problem 7.9.1 (d) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u_z(r,\theta,0) = f(r)\sin(3\theta)$, $u_z(r,\theta,H) = 0$, $u_r(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

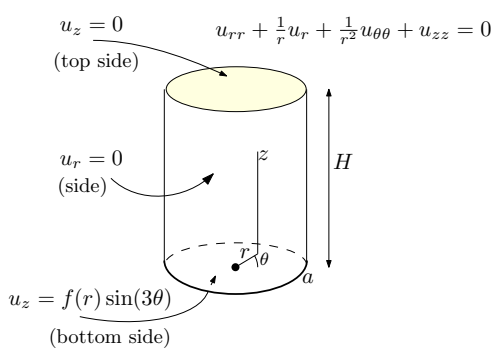

Figure 4.42: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == f[r]*Sin[3*theta], Derivative[0,0,1][u][r, theta
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=f(r)*sin(3*theta), eval(diff(u(r,theta,z),z),z=H)= 0, eva
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.5 [321] Haberman 7.9.1 (e)

problem number 321

Added May 25, 2019.

Problem 7.9.1 (e) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside circular cylinder subject to boundary conditions $u_z(r,\theta,0) = f(r,\theta)$, $u_z(r,\theta,H) = 0$, $u_r(a,\theta,z) = 0$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

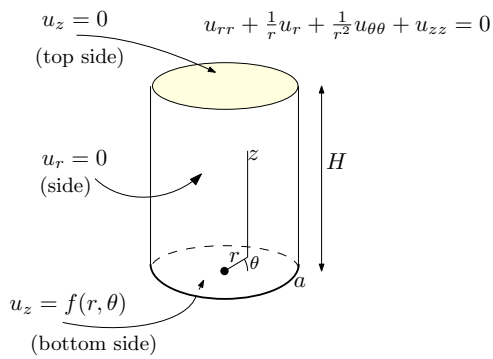

Figure 4.43: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == f[r,theta], Derivative[0,0,1][u][r, theta, H] ==
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=f(r,theta), eval(diff(u(r,theta,z),z),z=H)= 0, eval(diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.6 [322] Haberman 7.9.2 (a)

problem number 322

Added May 25, 2019.

Problem 7.9.2 (a) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r, \theta, 0) = 0$, $u(r, \theta, H) = f(r, \theta)$, $u(r, 0, z) = 0$, $u(r, \pi, z) = 0$, $u(a, \theta, z) = 0$.

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} + u_{zz} = 0$$

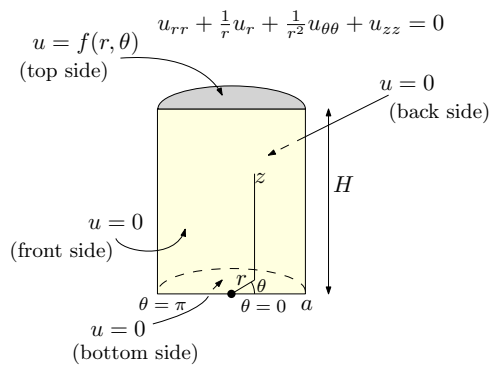

Figure 4.44: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, theta, H] == f[r,theta], u[r, 0, z] == 0,u[r,Pi,z]==0,u[a,t
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0, u(r,theta,H)= f(r,theta), u(r,0,z)=0, u(r,Pi,z)=0,u(a,theta,z)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.7 [323] Haberman 7.9.2 (b)

problem number 323

Added May 25, 2019.

Problem 7.9.2 (b) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r,\theta,0) = 0$, $u_z(r,\theta,H) = 0$, $u(r,0,z) = 0$, $u(r,\pi,z) = 0$, $u(a,\theta,z) = g(\theta,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

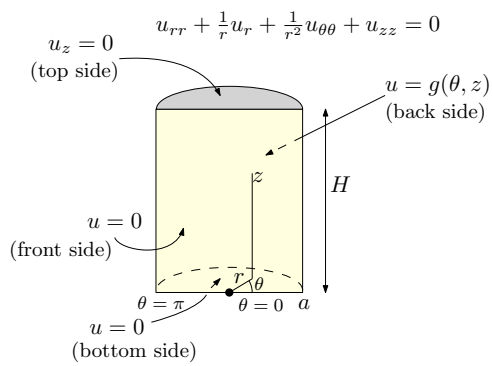

Figure 4.45: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, Derivative[0,0,1][u][r, theta, H] == 0, u[r, 0, z] == 0,u[r,Pi,z
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0, eval(diff(u(r,theta,z),z),z=H)=0, u(r,0,z)=0, u(r,Pi,z)=0,u(a,theta,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.8 [324] Haberman 7.9.2 (c)

problem number 324

Added May 25, 2019.

Problem 7.9.2 (c) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u_z(r,\theta,0) = 0$, $u_z(r,\theta,H) = 0$, $u_\theta(r,0,z) = 0$, $u_\theta(r,\pi,z) = 0$, $u_r(a,\theta,z) = g(\theta,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

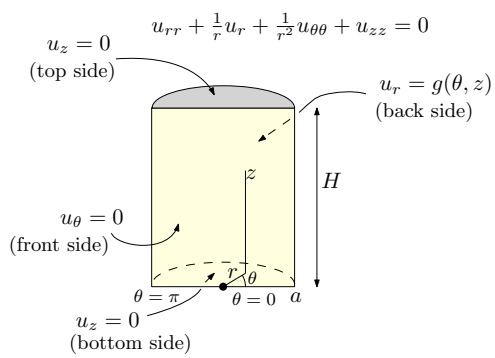

Figure 4.46: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {Derivative[0,0,1][u][r, theta, 0] == 0, Derivative[0,0,1][u][r, theta, H] == 0, Deriv
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc:=eval(diff(u(r,theta,z),z),z=0)=0, eval(diff(u(r,theta,z),z),z=H)=0, eval(diff(u(r,theta,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.2.2.9 [325] Haberman 7.9.2 (d)

problem number 325

Added May 26, 2019.

Problem 7.9.2 (d) from Richard Haberman Applied Partial Differential Equations, 4th edition.

Solve Laplace PDE inside semicircular cylinder subject to boundary conditions $u(r,\theta,0) = 0$, $u(r,0,z) = 0$, $u(a,\theta,z) = 0$, $u(r,\theta,H) = 0$, $u_\theta(r,\pi,z) = f(r,z)$.

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$$

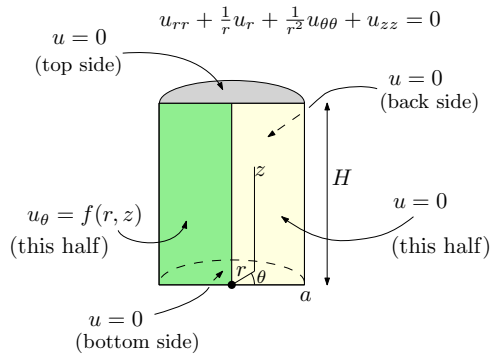

Figure 4.47: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, z], {r, theta, z}, "Cylindrical"];
bc  = {u[r, theta, 0] == 0, u[r, 0, z] == 0, u[a, theta, z] == 0,u[r,theta,H]==0,Derivative[
sol = AbsoluteTiming[TimeConstrained[DSolve[{lap == 0, bc}, u[r, theta, z], {r, theta, z}, A
```

Failed

Maple ✗

```
restart;
pde :=VectorCalculus:-Laplacian(u(r,theta,z),'cylindrical'[r,theta,z])=0;
bc  := u(r,theta,0)=0,u(r,0,z)=0, u(a,theta,z)=0, u(r,theta,H)=0,eval(diff(u(r,theta,z),thet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta,z)) ass
```

sol=()

4.3 Poisson in 2D

Local contents

4.3.1 Cartesian coordinates 891

4.3.1 Cartesian coordinates

Local contents

4.3.1.1 [326] All boundaries at zero 891
4.3.1.2 [327] Dirichlet problem in a rectangle 896
4.3.1.3 [328] Poisson PDE in whole 2D plane 897

4.3.1.1 [326] All boundaries at zero

problem number 326

Added March 13, 2019.

Solve for $u(x,y)$

$$\frac{u_{xx}}{A}+\frac{u_{xx}}{B}=-2\theta$$

Where A,B,θ are constants, and the boundary conditions are

$$\begin{aligned} u(x,-b)&=0\\ u(x,b)&=0\\ u(-a,y)&=0\\ u(a,y)&=0 \end{aligned}$$

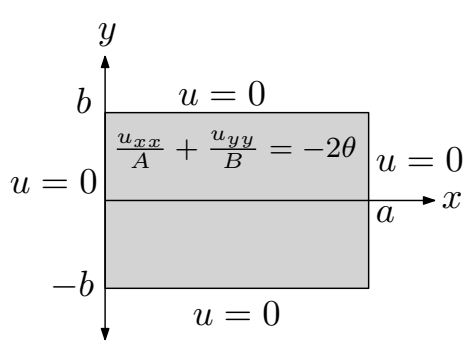

Figure 4.48: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, y], {x, 2}]/A + D[u[x, y], {y, 2}]/B == -2*theta;
bc  = {u[x, -b] == 0, u[x, b] == 0, u[-a, y] == 0, u[a, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x,y),x$2)/A+diff(u(x,y),y$2)/B  = -2*theta;
bc  := u(x,-b)=0, u(x,b)=0, u(-a,y)=0, u(a,y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

sol=()

Hand solution

solve

$$
\begin{aligned}
\frac{u_{xx}}{A}+\frac{u_{yy}}{B} &= -2\theta \\
Bu_{xx}+Au_{yy} &= -2\theta AB \\
&= C
\end{aligned}
$$

Where $C=-2\theta AB$ is a new constant. With boundary conditions

$$
\begin{aligned}
u(x,-b) &= 0 \\
u(x,b) &= 0 \\
u(-a,y) &= 0 \\
u(a,y) &= 0
\end{aligned}
$$

To simplify solution, shift the rectangle so its lower left corner on the origin. Let $\tilde{x}=x+a$, and $\tilde{y}=y+b$. The boundary conditions becomes

$$
\begin{aligned}
u(\tilde{x},0) &= 0 \\
u(\tilde{x},2b) &= 0 \\
u(0,\tilde{y}) &= 0 \\
u(2a,\tilde{y}) &= 0
\end{aligned}
$$

And the pde becomes $Bu_{\tilde{x}\tilde{x}}+Au_{\tilde{y}\tilde{y}}=C$. Instead of keep writing $\tilde{x},\tilde{y}$, will use x,y, but remember that these are shifted version. At the end, we shift back.

Hence the PDE to solve is $Bu_{xx} + Au_{yy} = C$ with BC

$$\begin{aligned} u(x,0) &= 0 \\ u(x,2b) &= 0 \\ u(0,y) &= 0 \\ u(2a,y) &= 0 \end{aligned}$$

Using eigenfunction expansion method. Let

$$u(x,y) = \sum_{n=1}^{\infty} b_n(y)\, X_n(x) \tag{1}$$

Where $X_n(x)$ is eigenfunctions for $X''(x) + \lambda_n X(x) = 0$ with boundary conditions $X(0) = X(2a) = 0$. This has eigenfunctions as $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$ with eigenvalues $\lambda_n = \left(\frac{n\pi}{2a}\right)^2$ for $n = 1, 2, \cdots$.

Substituting (1) into the PDE $Bu_{xx} + Au_{yy} = C$ gives

$$B\sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = C$$

Expanding C (a constant) as Fourier sine series the above becomes

$$B\sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = \sum_{n=1}^{\infty} q_n X_n(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, hence the above becomes

$$\begin{aligned} -B\sum_{n=1}^{\infty} \lambda_n b_n(y)\, X_n(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) &= \sum_{n=1}^{\infty} q_n X_n(x) \\ Ab_n''(y) - B\lambda_n b_n(y) &= q_n \end{aligned} \tag{1A}$$

But

$$\begin{aligned} C &= \sum_{n=1}^{\infty} q_n X_n(x) \\ \int_0^{2a} C X_n(x)\, dx &= q_n \int_0^{2a} X_n^2(x)\, dx \\ \int_0^{2a} C \sin\left(\sqrt{\lambda_n} x\right) dx &= q_n \int_0^{2a} \sin^2\left(\sqrt{\lambda_n} x\right) dx \\ \frac{-C}{\sqrt{\lambda_n}}((-1)^n - 1) &= q_n a \\ q_n &= \frac{-C}{a\sqrt{\lambda_n}}((-1)^n - 1) \end{aligned}$$

Hence (1A) becomes

$$Ab_n''(y) - B\lambda_n b_n(y) = \frac{-C}{a\sqrt{\lambda_n}}((-1)^n - 1)$$

This is standard second order linear ODE. The solution is

$$b_n(y) = D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1)$$

Using the above in (1) gives the solution

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1)\right)X_n(x) \tag{1A}$$

We now need to find D_n, E_n.

<u>Case n even</u>

When n is even $((-1)^n - 1) = 0$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y}\right)X_n(x)$$

At $y = 0$ the above gives

$$0 = \sum_{n=1}^{\infty}(D_n + E_n)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n + E_n = 0 \tag{2}$$

And at $y = 2b$

$$0 = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b} = 0 \tag{3}$$

From (2,3) we see that $D_n = E_n = 0$, Hence $u(x,y) = 0$ when n even.

<u>Case n odd</u>

When n is odd $((-1)^n - 1) = -2$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y} - \frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)X_n(x)$$

At $y=0$ the above gives

$$0=\sum_{n=1}^{\infty}\left(D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{4}$$

And at $y=2b$

$$0=\sum_{n=1}^{\infty}\left(D_ne^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_ne^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_ne^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_ne^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{5}$$

Solving (4,5) for D_n,E_n gives

$$D_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$

$$E_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$

Therefore the final solution from (1A) becomes

$$\begin{aligned}u(x,y)&=\sum_{n=1,3,5,\cdots}^{\infty}\left(D_ne^{\sqrt{\frac{B}{A}\lambda_n}y}+E_ne^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)X_n(x)\\&=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}y}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda}\right.\end{aligned}$$

Where $\lambda_n=\left(\frac{n\pi}{2a}\right)^2$. Switching back to original coordinates using $\tilde{x}=x+a$, and $\tilde{y}=y+b$, then the above is

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\left(-\sqrt{\frac{B}{A}\lambda_n}y+b\right)}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)$$

Where $C=-2\theta AB$, hence

$$\begin{aligned}u(x,y)&=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\right.\\&=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta A}{a\lambda_n^{\frac{3}{2}}}\right)\sin\end{aligned}$$

4.3.1.2 [327] Dirichlet problem in a rectangle

problem number 327

Taken from Mathematica DSolve help pages.

Solve for $u(x, y)$

$$u_{xx} + u_{yy} = 6x - 6y$$

Boundary conditions

$$\begin{aligned} u(x, 0) &= 1 + 11x + x^3 \\ u(x, 2) &= -7 + 11x + x^3 \\ u(0, y) &= 1 - y^3 \\ u(4, y) &= 109 - y^3 \end{aligned}$$

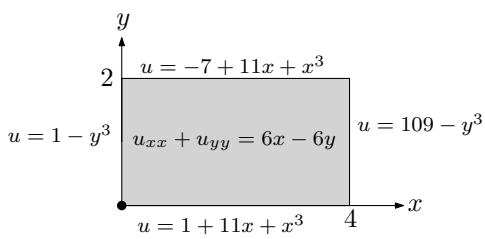

Figure 4.49: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 6*x - 6*y;
bc  = {u[x, 0] == 1 + 11*x + x^3, u[x, 2] == -7 + 11*x + x^3, u[0, y] == 1 - y^3, u[4, y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x, y) \to x^3 + 11x - y^3 + 1\}\}$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)=6*x-6*y;
bc  := u(x,0)=1+11*x+x^3,
    u(x,2)=-7+11*x+x^3,
    u(0,y)=1-y^3,
    u(4,y)=109-y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = x^3 - y^3 + 11x + 1$$

4.3.1.3 [328] Poisson PDE in whole 2D plane

problem number 328

Added January 13, 2020

Solve Poisson PDE

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 6y$$

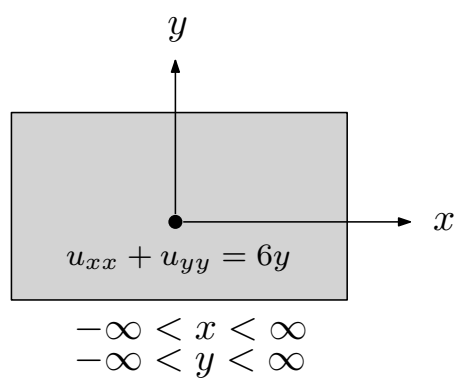

Figure 4.50: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] == 6*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(x-iy)+c_2(x+iy)+3x^2y\}\}$$

Maple ✓

```
restart;
pde:=VectorCalculus:-Laplacian(u(x,y),'cartesian'[x,y])=6*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = 3x^2y + \_F1(-ix+y) + \_F2(ix+y)$$

Hand solution

Solve Poisson PDE

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 6y$$

The solution is

$$f = f_h + f_p \tag{1}$$

Where f_h is the homogenous solution to Laplace $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ and f_p is a particular solution. The homogeneous solution is easily found as follows. Let

$$f_h = F(mx+y)$$

Then $\frac{\partial f_h}{\partial x} = mF'$ and $\frac{\partial^2 f_h}{\partial x^2} = m^2F''$ and $\frac{\partial f_h}{\partial y} = F'$ and $\frac{\partial^2 f_h}{\partial y^2} = F''$. Substituting these into $\frac{\partial^2 f_h}{\partial x^2} + \frac{\partial^2 f_h}{\partial y^2} = 0$ gives

$$\begin{aligned} m^2F'' + F'' &= 0 \\ m^2+1 &= 0 \\ m &= \pm i \end{aligned}$$

Since we assumed $f_h = F(mx+y)$, then the homogeneous solution is sum of two arbitrary functions (one for each root of m)

$$f_h = F_1(ix+y) + F_2(-ix+y)$$

To find particular solution, let

$$f_p = Ax^n y^m \tag{2}$$

Hence $\frac{\partial f_p}{\partial x} = Anx^{n-1}y^m$ and $\frac{\partial^2 f_p}{\partial x^2} = An(n-1)\,x^{n-2}y^m$ and $\frac{\partial f_p}{\partial y} = Ax^n m y^{m-1}$ and $\frac{\partial^2 f_p}{\partial y^2} = Ax^n m(m-1)\,y^{m-2}$. Substituting these into $\frac{\partial^2 f_p}{\partial x^2} + \frac{\partial^2 f_p}{\partial y^2} = 6y$ gives

$$\begin{aligned} An(n-1)\,x^{n-2}y^m + Ax^n m(m-1)\,y^{m-2} &= 6y \\ y^m\left(An(n-1)\,x^{n-2} + Ax^n m(m-1)\,y^{-2}\right) &= 6y \end{aligned}$$

Comparing terms, then $m = 1$ and

$$An(n-1)\,x^{n-2} + Ax^n m(m-1)\,y^{-2} = 6$$

Since $m = 1$ then the above simplifies to

$$An(n-1)\,x^{n-2} = 6$$

Since there is no x in RHS, then $n - 2 = 0$ or $n = 2$ and the above becomes

$$\begin{aligned} 2A(2-1) &= 6 \\ A &= 3 \end{aligned}$$

Hence (2) becomes

$$f_p = Ax^n y^m = 3x^2 y$$

And the complete solution (1) is

$$\begin{aligned} f(x,y) &= f_h + f_p \\ &= F_1(ix+y) + F_2(-ix+y) + 3x^2 y \end{aligned}$$

4.4 Helmholtz in 2D

Local contents

4.4.0.1 [329] In rectangle

problem number 329

Taken from Mathematica DSolve help pages.

Solve for $u(x, y)$

$$u_{xx} + u_{yy} + 5u(x, y) = 0$$

Boundary conditions

$$\begin{aligned} u(x, 0) &= \text{UnitTriangle[x-2]} \\ u(x, 2) &= 0 \\ u(0, y) &= 0 \\ u(4, y) &= 0 \end{aligned}$$

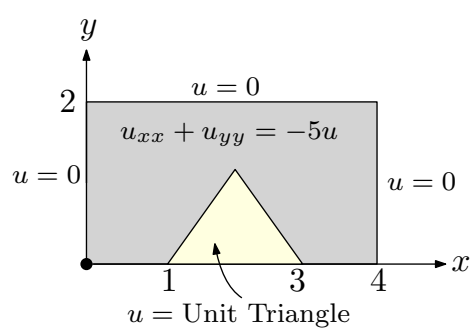

Figure 4.51: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
bc  = {u[x, 0] == Piecewise[{{-1 + x, x > 1 && x < 2}, {3 - x, x > 2 && x < 3}}], u[x, 2] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
sol =  sol /. K[1] -> n
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{64\left(\cos\left(\frac{n\pi}{8}\right)+\cos\left(\frac{3n\pi}{8}\right)\right)\operatorname{csch}\left(\frac{1}{2}\sqrt{n^2\pi^2-80}\right)\sin^3\left(\frac{n\pi}{8}\right)\sin\left(\frac{n\pi x}{4}\right)\sinh\left(\frac{1}{4}\sqrt{n^2\pi^2-80}(2-\right.}{n^2\pi^2}\right.$$

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
bc  := u(x,0)=piecewise( x>1 and x<2, -1+x,x>2 and x<3 ,3-x),
       u(x,2)=0,
       u(0,y)=0,
       u(4,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\sum_{n=1}^{\infty}\frac{32\left(\left(\cos\left(\frac{\pi n}{4}\right)-1\right)\cos\left(\frac{\sqrt{-\pi^2n^2+80}}{2}\right)\cos\left(\frac{\pi n}{4}\right)\sin\left(\frac{\pi n}{4}\right)\sin\left(\frac{\sqrt{-\pi^2n^2+80}\,y}{4}\right)+\frac{\left(\sin\left(\frac{\pi n}{2}\right)-\frac{\sin\left(\frac{\pi n}{4}\right)}{2}-\frac{\sin}{}\right.}{}\right.}{\pi^2n^2\sin\left(\frac{\sqrt{-\pi^2n^2+80}}{2}\right)}$$

4.4.0.2 [330] On whole plane

problem number 330

Added December 27, 2018.

Solve for $u(x,y)$

$$u_{xx}+u_{yy}+5u(x,y)=0$$

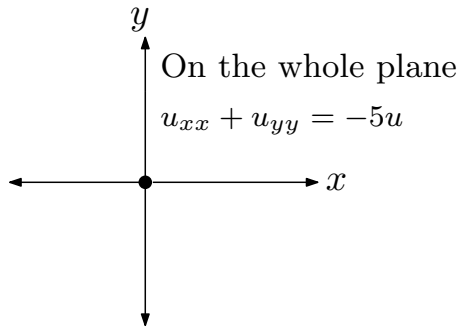

Figure 4.52: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{\sqrt{c_5}(-x)}\left(c_1 e^{2\sqrt{c_5}x} + c_2\right)\left(c_4 \cos\left(\sqrt{5+c_5}y\right) + c_3 \sin\left(\sqrt{5+c_5}y\right)\right)\right\}\right\}$$

why? It solved earlier with BC?

Maple ✓

```
restart;
pde := diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \left(c_1 \mathrm{e}^{2x\sqrt{\_c_1}} + c_2\right)\left(c_3 \sin\left(\sqrt{\_c_1 + 5}\, y\right) + c_4 \cos\left(\sqrt{\_c_1 + 5}\, y\right)\right) \mathrm{e}^{-x\sqrt{\_c_1}}$$

4.4.0.3 [331] Reduced Helmholtz Inside square

problem number 331

Added December 20, 2018.

Example 24, taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - ku(x,y) = 0$$

With $k > 0$. It is called reduced Helmholtz, because of the minus sign above. Otherwise, standard Helmholtz has a positive sign.

Boundary conditions

$$\begin{aligned} u(x,0) &= 0 \\ u(x,\pi) &= 0 \\ u(0,y) &= 1 \\ u(\pi,y) &= 0 \end{aligned}$$

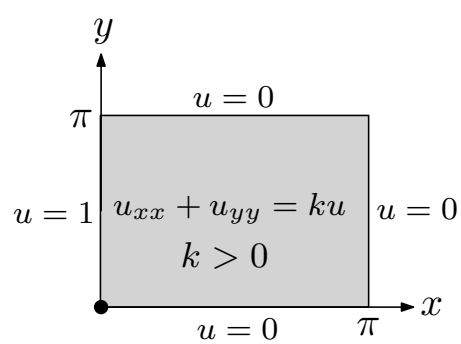

Figure 4.53: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Laplacian[u[x, y], {x, y}] - k*u[x, y] == 0;
bc  = {u[x, 0] == 0, u[x, Pi] == 0, u[0, y] == 1, u[Pi, y] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> k >
```

$$\left\{\left\{u(x,y)\to \sum_{K[1]=1}^{\infty}-\frac{2\left(-1+(-1)^{K[1]}\right)\operatorname{csch}\left(\pi\sqrt{K[1]^2+k}\right)\sin(yK[1])\sinh\left((\pi-x)\sqrt{K[1]^2+k}\right)}{\pi K[1]}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2)-k*u(x, y) = 0;
bc_left_edge:=u(0, y) = 1;
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,Pi)=0;
bc_right_edge:=u(Pi,y)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge,bc_right_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

$$u(x,y)=\sum_{n=1}^{\infty}-\frac{2((-1)^n-1)\left(-e^{\sqrt{n^2+k}\,x}+e^{-(x-2\pi)\sqrt{n^2+k}}\right)\sin(ny)}{\left(e^{2\sqrt{n^2+k}\,\pi}-1\right)\pi n}$$

4.5 Helmholtz in 3D

Local contents

4.5.1 Spherical coordinates 905

4.5.1 Spherical coordinates

Local contents

4.5.1.1 [332] Chain reaction PDE 905

4.5.1.1 [332] Chain reaction PDE

problem number 332

Added May 7, 2019.

Assume ϕ independence. Solve for $u(r,\theta,t)$

$$u_t = k(\lambda u + \nabla^2(u))$$

Where $\nabla^2(u) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)$ with $k > 0$.

Boundary conditions $u(R,\theta,t) = 0$.

Mathematica ✗

```
ClearAll["Global`*"];
U = u[r, theta, t];
pde = D[U, t] == k*(lambda*U + (1/r^2)*D[r^2*D[U, r], r] + (1/(r^2*Sin[theta]))*D[Sin[theta]
bc  = u[R, theta, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, U, {r, theta, t}, Assumptions -> {k >
```

Failed

Maple ✗

```
restart;
U   := u(r,theta,t);
pde := diff(U,t) = k*(lambda*U +1/r^2* diff(r^2*diff(U,r),r)+ 1/(r^2*sin(theta))*diff(sin(th
bc  := u(R,theta,t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc], U) assuming k>0,
```

$$u(r, theta, t) = 0$$

trivial solution
Hand solution

Solve for $u(r,\theta,t)$ in spherical coordinates (assuming ϕ independence) the chain reaction equation $\frac{1}{k}u_t = \lambda u + \nabla^2 u$ with boundary conditions $u(R,\theta,t) = 0$.

$$\begin{aligned}
\frac{1}{k}u_t &= \lambda u + \nabla^2 u \\
&= \lambda u + \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)\right) \\
&= \lambda u + \frac{1}{r^2}\left(2ru_r + r^2 u_{rr}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)
\end{aligned}$$

Let $u = R(r)\,\Theta(\theta)\,T(t)$. Substituting into the above gives

$$\begin{aligned}
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{1}{r^2}\left(2rR'\Theta T + r^2 R''\Theta T\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta(\Theta' RT)) \\
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{2}{r}R'\Theta T + R''\Theta T + \frac{RT}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

Dividing by $TR\Theta$ gives

$$\frac{1}{k}\frac{T'}{T} = \lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)$$

The left side depends on t only and the right depends on r,θ only. Let the separation variable be $-n$. This gives the following 2 equations

$$\frac{1}{k}\frac{T'}{T} = -n \tag{1}$$

$$\lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = -n \tag{2}$$

Now we consider (2). Multiplying both sides of (2) by r^2 gives

$$\begin{aligned}
\lambda r^2 + 2r\frac{R'}{R} + r^2\frac{R''}{R} + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) &= -nr^2 \\
2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda + n) &= -\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

The left side depends on r and the right side depends on θ. Let the separation variable be $l(l+1)$ where l is integer. Hence we obtain the following two equations

$$-\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = l(l+1) \tag{4}$$

$$2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda+n) = l(l+1) \tag{5}$$

Starting with (4)

$$\frac{\partial}{\partial\theta}(\Theta'\sin\theta) + l(l+1)\,\Theta\sin\theta = 0$$

$$\Theta''\sin\theta + \Theta'\cos\theta + l(l+1)\,\Theta\sin\theta = 0$$

Using the substitution $z = \cos\theta$ the above becomes

$$\left(1-z^2\right)\Theta'' - 2z\Theta' + l(l+1)\,\Theta = 0$$

This Legendre ODE. Solution is $P_l(\theta)$. The other solution to the above ODE is ignored as not bounded. Now back to solving (5). Writing it as

$$2rR' + r^2R'' + r^2(\lambda+n)\,R = l(l+1)\,R$$

$$r^2R'' + 2rR' + \left(r^2(\lambda+n) - l(l+1)\right)R = 0 \tag{6}$$

This can be converted to Bessel ODE using substitution. First let $v = r\sqrt{(\lambda+n)}$. Then $R'(r) = \sqrt{(\lambda+n)}R'(v)\,, R''(r) = (\lambda+n)\,R''(v)$ and (6) becomes

$$(\lambda+n)\,r^2R''(v) + 2r\sqrt{(\lambda+n)}R'(v) + \left(r^2(\lambda+n) - l(l+1)\right)R = 0$$

$$v^2R''(v) + 2vR'(v) + \left(v^2 - l(l+1)\right)R = 0 \tag{7}$$

Now, we apply second transformation $R(v) = \frac{Z(v)}{\sqrt{v}}$ Then

$$R'(v) = \frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}}$$

$$R''(v) = \frac{Z''(v)}{\sqrt{v}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}\left(-\frac{3}{2}\right)Z(v)\,\frac{1}{v^{\frac{5}{2}}}$$

$$= \frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}}$$

Hence (7) becomes

$$v^2\left(\frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}}\right) + 2v\left(\frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}}\right) + \left(v^2 - l(l+1)\right)\frac{Z(v)}{\sqrt{v}} = 0$$

Multiplying by $\sqrt{v}$ gives

$$
\begin{aligned}
v^2\left(Z''(v)-Z'(v)\frac{1}{v}+\frac{3}{4}Z(v)\frac{1}{v^2}\right)+2v\left(Z'(v)-\frac{1}{2}Z(v)\frac{1}{v}\right)+\left(v^2-l(l+1)\right)Z(v)&=0\\
\left(v^2Z''(v)-vZ'(v)+\frac{3}{4}Z(v)\right)+\left(2vZ'(v)-Z(v)\right)+\left(v^2-l(l+1)\right)Z(v)&=0\\
v^2Z''(v)+vZ'(v)+\frac{1}{4}Z(v)+\left(v^2-l(l+1)\right)Z(v)&=0\\
v^2Z''(v)+vZ'(v)+\left(v^2-l(l+1)+\frac{1}{4}\right)Z(v)&=0\\
v^2Z''(v)+vZ'(v)+\left(v^2-l^2+l+\frac{1}{4}\right)Z(v)&=0\\
v^2Z''(v)+vZ'(v)+\left(v^2-\left(l+\frac{1}{2}\right)^2\right)Z(v)&=0
\end{aligned}
$$

This is now in standard Bessel ODE form. Comparing it to $v^2Z''(v)+vZ'(v)+(v^2-d^2)\,Z(v)=0$ shows the order is $d=l+\frac{1}{2}$. The solutions are

$$Z(v)=c_1J_{l+\frac{1}{2}}(v)+c_2Y_{l+\frac{1}{2}}(v)$$

But $R(v)=\frac{Z(v)}{\sqrt{v}}$, hence

$$R(v)=c_1\frac{J_{l+\frac{1}{2}}(v)}{\sqrt{v}}+c_2\frac{Y_{l+\frac{1}{2}}(v)}{\sqrt{v}}$$

But $v=r\sqrt{\lambda+n}$ then above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}+c_2\frac{Y_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

The solution assumed bounded at $r=0$ hence $c_2=0$ and the above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

Let $m^2=(\lambda+n)$, hence

$$
\begin{aligned}
R(r)&=c_1\frac{J_{l+\frac{1}{2}}(mr)}{\sqrt{mr}}\\
&=j_l(mr)
\end{aligned}
$$

where $j_l(mr)$ are the spherical Bessel functions. Boundary conditions at $r = R$ gives

$$j_l(mR) = 0$$

Hence mR or $\sqrt{\lambda + n}R$ are the zeros of spherical Bessel functions $j_l(mR)$. There are infinite zeros for each l. Let the vth zero of j_l be called $Z_{l,v}$. Then $mR_{l,v} = Z_{l,v}$ or $\sqrt{\lambda + n}_{l,v} = \frac{Z_{l,v}}{R}$. or

$$n = \left(\frac{Z_{l,v}}{R}\right)^2 - \lambda$$

The solution to the time ODE is therefore

$$\begin{aligned} T_{l,v} &= A_{l,v} e^{-nkt} \\ &= A_{l,v} e^{-\left(\left(\frac{Z_{l,v}}{R}\right)^2 - \lambda\right)kt} \end{aligned}$$

Hence the complete solution is

$$\begin{aligned} u(r,\theta,t) &= e^{-i\alpha kt} P_l(\theta)\, j_l(mr) \\ &= \sum_{l=1}^{\infty} \sum_{v=0}^{\infty} A_{l,v} e^{-nkt} P_l(\theta)\, j_l\left(\frac{Z_{l,v}}{R} r\right) \end{aligned}$$

$A_{l.v}$ constants still need to be found from initial conditions. For each l, we have infinite sum over all $v's$ zeros of j_l.

CHAPTER 5

Hyperbolic PDE's (Wave)

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5.1 Wave PDE in 1D

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5.1.1.1 [333] General solution for both ends fixed. Domain is $0 \dots L$

problem number 333

Added July 7,2019

Solve for $u(x,t)$ for $t > 0$ and $0 < x < L$

$$u_{tt} = c^2 u_{xx}$$

With boundary condition both ends fixed

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

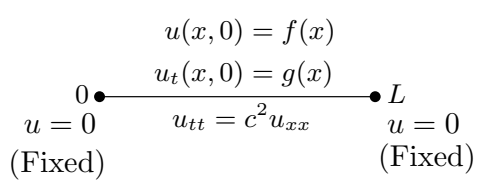

Figure 5.1: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions->L>
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\sqrt{2}\sqrt{\frac{1}{L}}\sin\left(\frac{\pi x K[1]}{L}\right)\left(\cos\left(\pi t\sqrt{\frac{c^2K[1]^2}{L^2}}\right)\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,dx+\frac{L\left(\int_0^L\frac{\sqrt{2}g(x)\sin\left(\frac{\pi x}{}\right.}{\sqrt{L}}\right.}{}\right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\pi cn\left(\int_0^L f(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\cos\left(\frac{\pi cnt}{L}\right)+L\left(\int_0^L g(x)\sin\left(\frac{\pi nx}{L}\right)dx\right)\sin\left(\frac{\pi cnt}{L}\right)\right)\sin\left(\frac{\pi nx}{L}\right)}{\pi Lcn}$$

<u>Hand solution</u>

Solving for $t>0, 0<x<L$

$$u_{tt}=c^2u_{xx}$$

With BC

$$u(0,t)=0$$
$$u(L,t)=0$$

And initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=g(x)$$

Let $u = X(x)\,T(t)$. The PDE becomes

$$\frac{T''X}{c^2} = X''T$$
$$\frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} = -\lambda$$

Where λ is separation constant. Hence the eigenvalue ODE is

$$X'' + \lambda X = 0$$
$$X(0) = 0$$
$$X(L) = 0$$

From the boundary conditions, we see that $\lambda > 0$ is the only possible value. Therefore the solution to the above ODE is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then $A = 0$ and the solution becomes $X(x) = B\sin\left(\sqrt{\lambda}x\right)$. Since $X(L) = 0$ then for non trivial solution we want $\sqrt{\lambda}L = n\pi$ or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE now becomes

$$T'' + c^2\left(\frac{n\pi}{L}\right)^2 T = 0$$

Which has the solution

$$T(t) = D_n\cos\left(c\frac{n\pi}{L}t\right) + E_n\sin\left(c\frac{n\pi}{L}t\right)$$

Therefore the complete solution becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(D_n\cos\left(c\frac{n\pi}{L}t\right) + E_n\sin\left(c\frac{n\pi}{L}t\right)\right)\Phi_n(x) \tag{1}$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} D_n\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx &= D_n\int_0^L \Phi_n^2(x)\,dx\\ &= \frac{L}{2}D_n\end{aligned}$$

Hence

$$D_n = \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t) = \sum_{n=1}^{\infty}\left(-c\frac{n\pi}{L}D_n\sin\left(c\frac{n\pi}{L}t\right) + E_n c\frac{n\pi}{L}\cos\left(c\frac{n\pi}{L}t\right)\right)\Phi_n(x)$$

At $t=0$ the above becomes

$$g(x) = \sum_{n=1}^{\infty} E_n c\frac{n\pi}{L}\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L g(x)\,\Phi_n(x)\,dx &= E_n c\frac{n\pi}{L}\int_0^L \Phi_n^2(x)\,dx\\ &= \frac{L}{2}E_n c\frac{n\pi}{L}\\ &= \frac{1}{2}E_n cn\pi\end{aligned}$$

Hence

$$E_n = \frac{2}{cn\pi}\int_0^L g(x)\,\Phi_n(x)\,dx \tag{3}$$

Using (2,3) in (1) gives the final solution as

$$\begin{aligned}u(x,t) = &\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\\ &+\frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

5.1.1.2 [334] both ends fixed, inital position zero (special case)

problem number 334

Added July 8,2019

Solve for $u(x,t)$ for $t>0$ and $0<x<L$

$$u_{tt}=c^2u_{xx}$$

With boundary condition both ends fixed

$$\begin{aligned}u(0,t)&=0\\u(L,t)&=0\end{aligned}$$

And initial conditions

$$\begin{aligned}u(x,0)&=f(x)\\u_t(x,0)&=g(x)\end{aligned}$$

Using the following values

$$\begin{aligned}L&=10\\c&=2\\f(x)&=0\\g(x)&=\frac{8x(L-x)^2}{L^3}\end{aligned}$$

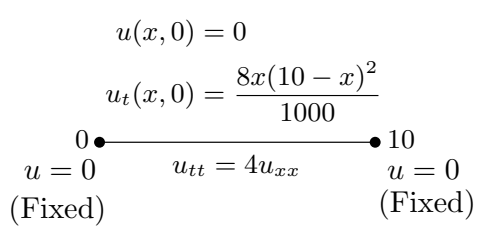

Figure 5.2: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=10; c=2; f=0; g=(8*x*(L-x)^2)/L^3;
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f, Derivative[0, 1][u][x, 0] == g};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{160(2+(-1)^n)\sin\left(\frac{n\pi t}{5}\right)\sin\left(\frac{n\pi x}{10}\right)}{n^4\pi^4}\right\}\right\}$$

Maple ✓

```
restart;
L:=10;
c:=2;
f:=0;
g:=(8*x*(L-x)^2)/L^3;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,eval(diff(u(x,t),t),t=0)=g;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{160((-1)^n+2)\sin\left(\frac{\pi nt}{5}\right)\sin\left(\frac{\pi nx}{10}\right)}{\pi^4n^4}$$

<u>Hand solution</u>

Solving the wave PDE on string with both ends fixed

$$u_{tt}=c^2u_{xx}\qquad t>0,x>0$$

With BC

$$u(0,t)=0$$
$$u(L,t)=0$$

And initial conditions

$$u(x,0) = f(x) = 0$$
$$u_t(x,0) = g(x) = \frac{8x(L-x)^2}{L^3}$$

Using $c = 2, L = 10$.

The general problem PDE was solved in 5.1.1.1 on page 914 and the solution is

$$u(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)$$
$$+\frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx\right)\sin\left(2\frac{n\pi}{10}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

But $\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx = \frac{160(2+(-1)^n)}{n^3\pi^3}$, hence the solution becomes

$$u(x,t) = \frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{160(2+(-1)^n)}{n^4}\sin\left(\frac{n\pi}{5}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

Animation is below

Out[ ]=

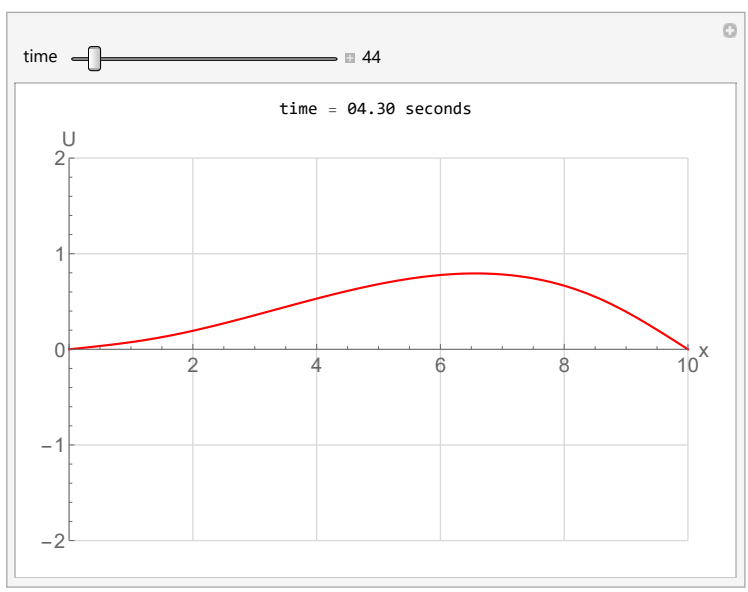

Figure 5.3: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 2;
L = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 15;
mySol[x_, t_] = 1/π^4 Sum[(160 (2 + (-1)^n))/n^4 Sin[n π/5 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 5.4: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-2, 2}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 100, 0.1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.5: Code for animation

5.1.1.3 [335] both ends fixed, inital velocity zero (special case)

problem number 335

Added July 8,2019

Solve for $u(x,t)$ for $t > 0$ and $0 < x < L$

$$u_{tt} = c^2 u_{xx}$$

With boundary condition both ends fixed

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

Using the following values

$$L = 10$$
$$c = 2$$
$$f(x) = \frac{8x(L-x)^2}{L^3}$$
$$g(x) = 0$$

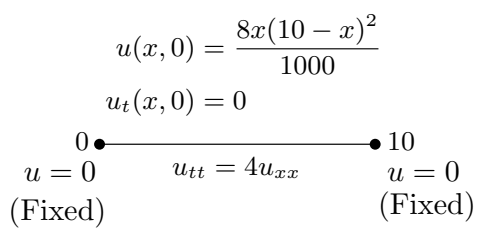

Figure 5.6: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=10; c=2; g=0; f=(8*x*(L-x)^2)/L^3;
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f, Derivative[0, 1][u][x, 0] == g};
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
sol = sol/.K[1]->n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{32(2+(-1)^n)\cos\left(\frac{n\pi t}{5}\right)\sin\left(\frac{n\pi x}{10}\right)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
restart;
L:=10;
c:=2;
g:=0;
f:=(8*x*(L-x)^2)/L^3;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,eval(diff(u(x,t),t),t=0)=g;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{32((-1)^n+2)\cos\left(\frac{\pi n t}{5}\right)\sin\left(\frac{\pi n x}{10}\right)}{\pi^3 n^3}$$

<u>Hand solution</u>

Solving the wave PDE on string with both ends fixed

$$u_{tt}=c^2u_{xx}\qquad t>0,x>0$$

With BC

$$\begin{aligned}u(0,t)&=0\\u(L,t)&=0\end{aligned}$$

And initial conditions

$$\begin{aligned}u(x,0)=f(x)&=\frac{8x(L-x)^2}{L^3}\\u_t(x,0)=g(x)&=0\end{aligned}$$

Using $c=2,L=10$.

The general problem PDE was solved in 5.1.1.1 on page 914 and the solution is

$$\begin{aligned}u(x,t)=&\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\\&+\frac{2}{c\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{L}x\right)dx\right)\sin\left(c\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)\end{aligned}$$

Substituting the specific values given above into this solution gives

$$u(x,t)=\frac{1}{5}\sum_{n=1}^{\infty}\left(\int_0^{10}\frac{8x(10-x)^2}{10^3}\sin\left(\frac{n\pi}{10}x\right)dx\right)\cos\left(\frac{n\pi}{5}t\right)\sin\left(\frac{n\pi}{10}x\right)$$

But $\int_0^{10} \frac{8x(10-x)^2}{10^3} \sin\left(\frac{n\pi}{10}x\right) dx = \frac{160(2+(-1)^n)}{n^3\pi^3}$, hence the solution becomes

$$u(x,t) = 32 \sum_{n=1}^{\infty} \frac{(2+(-1)^n)}{n^3\pi^3} \cos\left(\frac{n\pi}{5}t\right) \sin\left(\frac{n\pi}{10}x\right)$$

Animation is below

Out[ ]=

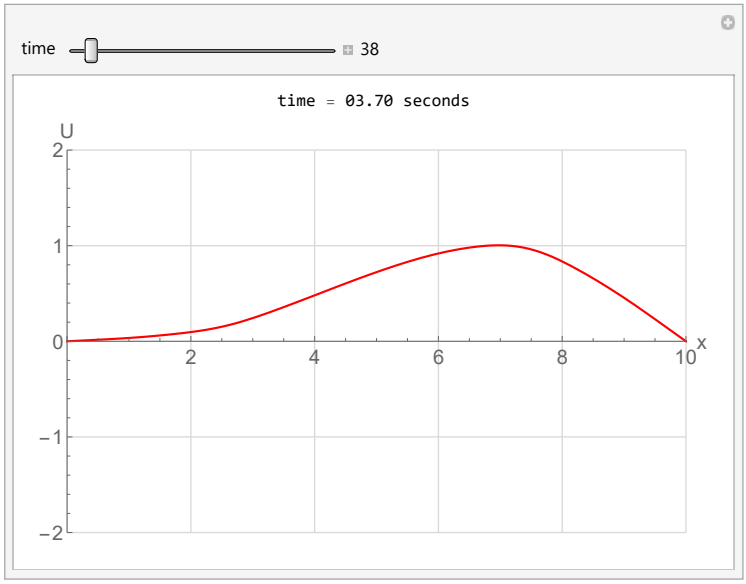

Figure 5.7: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 2;
L = 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 15;
mySol[x_, t_] = 32 Sum[(2 + (-1)^n)/(n^3 π^3) Sin[n π/5 t] Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 5.8: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-2, 2}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 100, 0.1}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.9: Code for animation

5.1.1.4 [336] both ends fixed but domain is $-\pi \dots \pi$. zero intial position, non zero initial velocity

problem number 336

Added sept 23 ,2019

Solve for $u(x,t)$ for $t > 0$ and $-\pi < x < \pi$

$$u_{tt} = c^2 u_{xx}$$

With boundary condition

$$\begin{aligned} u(-\pi, t) &= 0 \\ u(\pi, t) &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x, 0) &= 0 \\ u_t(x, 0) &= \sin(x)^2 \end{aligned}$$

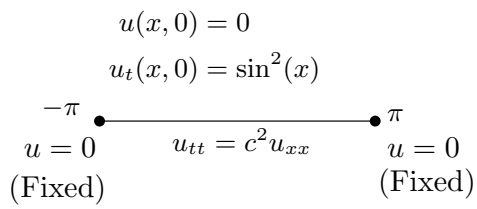

Figure 5.10: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == Sin[x]^2};
bc  = {u[-Pi, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \frac{32\left(-1+(-1)^{K[1]}\right)\sin\left(\frac{1}{2}\sqrt{c^2}tK[1]\right)\sin\left(\frac{1}{2}(x+\pi)K[1]\right)}{\sqrt{c^2}\pi K[1]^2\left(K[1]^2-16\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(-Pi,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=sin(x)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = \frac{32\pi c\left(\sum_{n=5}^{\infty} \frac{((-1)^n-1)\sin\left(\frac{(x+\pi)n}{2}\right)\sin\left(\frac{cnt}{2}\right)}{\pi(n^2-16)cn^2}\right) + \frac{64\cos\left(\frac{x}{2}\right)\sin\left(\frac{ct}{2}\right)}{15} - \frac{64\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3ct}{2}\right)}{63}}{\pi c}$$

Hand solution

Solve

$$u_{tt} = c^2 u_{xx}$$

With BC

$$\begin{aligned} u(-\pi,t) &= 0 \\ u(\pi,t) &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= 0 \\ u_t(x,0) &= \sin^2(x) \end{aligned}$$

Let $\xi = x+\pi$. When $x=-\pi, \xi=0$ and when $x=\pi, \xi=2\pi$. In terms of ξ, the new pde in $U(\xi,t)$ becomes

$$U_{tt} = c^2 U_{\xi\xi}$$

With BC

$$\begin{aligned} U(0,t) &= 0 \\ U(2\pi,t) &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} U(\xi,0) &= 0 \\ U_t(\xi,0) &= \sin^2(\xi) \end{aligned}$$

Let $U = X(\xi)\,T(t)$. The PDE becomes

$$\begin{aligned} \frac{T''X}{c^2} &= X''T \\ \frac{1}{c^2}\frac{T''}{T} &= \frac{X''}{X} = -\lambda \end{aligned}$$

Where λ is separation constant. Hence the eigenvalue ODE is

$$\begin{aligned} X''+\lambda X &= 0 \\ X(0) &= 0 \\ X(2\pi) &= 0 \end{aligned}$$

From the boundary conditions, we see that $\lambda > 0$ is the only possible value. Therefore the solution to the above ODE is

$$X(x) = A\cos\left(\sqrt{\lambda}\xi\right) + B\sin\left(\sqrt{\lambda}\xi\right)$$

Since $X(0) = 0$ then $A = 0$ and the solution becomes $X(\xi) = B\sin\left(\sqrt{\lambda}\xi\right)$. Since $X(2\pi) = 0$ then for non trivial solution we want $\sqrt{\lambda}2\pi = n\pi$ or

$$\lambda_n = \left(\frac{n}{2}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Hence the eigenfunctions are

$$X_n(\xi) = \sin\left(\frac{n}{2}\xi\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE now becomes

$$\begin{aligned} T_n'' + c^2\lambda_n T_n &= 0 \\ T_n'' + c^2\left(\frac{n}{2}\right)^2 T_n &= 0 \\ T_n'' + \frac{c^2n^2}{4}T_n &= 0 \end{aligned}$$

Which has the solution

$$T_n(t) = D_n\cos\left(\frac{cn}{2}t\right) + E_n\sin\left(\frac{cn}{2}t\right)$$

Therefore the complete solution becomes

$$U(\xi, t) = \sum_{n=1}^{\infty}\left(D_n\cos\left(\frac{cn}{2}t\right) + E_n\sin\left(\frac{cn}{2}t\right)\right)\sin\left(\frac{n}{2}\xi\right) \tag{1}$$

Switching back to x the above becomes

$$u(x, t) = \sum_{n=1}^{\infty}\left(D_n\cos\left(\frac{cn}{2}t\right) + E_n\sin\left(\frac{cn}{2}t\right)\right)\sin\left(\frac{n}{2}(x+\pi)\right) \tag{1A}$$

At $t = 0$ the above becomes

$$\begin{aligned} 0 &= \sum_{n=1}^{\infty} D_n\sin\left(\frac{n}{2}(x+\pi)\right) \\ D_n &= 0 \end{aligned}$$

The solution (1A) simplifies to

$$u(x, t) = \sum_{n=1}^{\infty} E_n\sin\left(\frac{cn}{2}t\right)\sin\left(\frac{n}{2}(x+\pi)\right) \tag{2}$$

Taking time derivative of (2) gives

$$u_t(x,t) = \sum_{n=1}^{\infty} \left(E_n \frac{cn}{2} \cos\left(\frac{cn}{2} t\right)\right) \sin\left(\frac{n}{2}(x+\pi)\right)$$

At $t=0$ the above becomes

$$\sin^2(x) = \sum_{n=1}^{\infty} E_n \frac{cn}{2} \sin\left(\frac{n}{2}(x+\pi)\right)$$

Applying orthogonality gives

$$\begin{aligned}\int_{-\pi}^{\pi} \sin^2(x) \sin\left(\frac{n}{2}(x+\pi)\right) dx &= E_n \frac{cn}{2} \int_{-\pi}^{\pi} \sin^2\left(\frac{n}{2}(x+\pi)\right) dx \\ &= E_n \frac{\pi c n}{2}\end{aligned}$$

For the LHS, for n even $\int_{-\pi}^{\pi} \sin^2(x) \sin\left(\frac{n}{2}(x+\pi)\right) dx = 0$. Hence $E_n = 0$ for all n even. For n odd

$$\begin{aligned}\int_{-\pi}^{\pi} \sin^2(x) \sin\left(\frac{n}{2}(x+\pi)\right) dx &= \frac{16(\cos(n\pi)-1)}{(n^2-16)\,n} \\ &= \frac{16((-1)^n-1)}{(n^2-16)\,n}\end{aligned}$$

But n is odd, hence the above simplifies more to

$$\int_{-\pi}^{\pi} \sin^2(x) \sin\left(\frac{n}{2}(x+\pi)\right) dx = \frac{-32}{(n^2-16)\,n}$$

Therefore

$$\begin{aligned}E_n &= \frac{2}{n\pi c} \frac{-32}{(n^2-16)\,n} \\ &= \frac{-64}{n^2 \pi c\,(n^2-16)}\end{aligned}$$

Therefore the final solution (2) now becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{-64}{n^2\pi c\,(n^2-16)} \sin\left(\frac{cn}{2} t\right) \sin\left(\frac{n}{2}(x+\pi)\right)$$

The following is an animation of the solution for $c=2$. Using $c=2$ then the solution above becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{-32}{n^2\pi\left(n^2-16\right)} \sin(nt) \sin\left(\frac{n}{2}(x+\pi)\right)$$

Out[ ]=

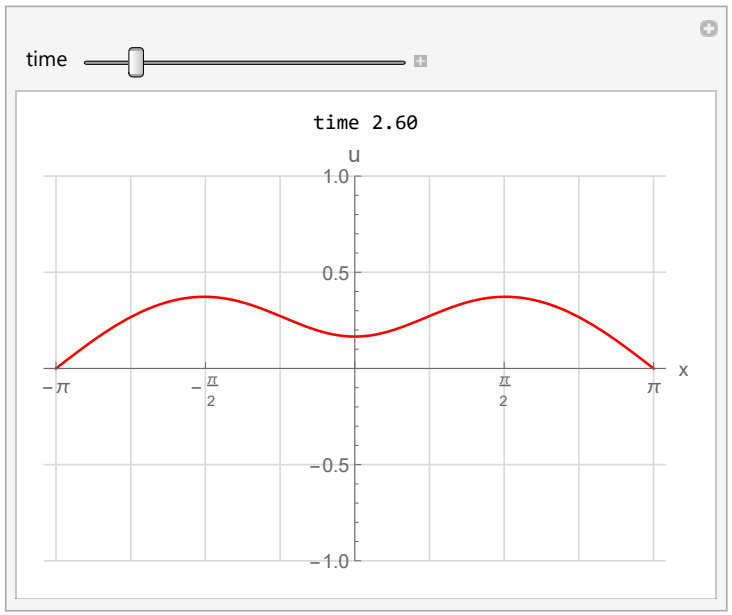

Figure 5.11: snap shot

Source code used for the above

```
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_, max_] := Sum[(-32)/(n^2 π (n^2 - 16)) Sin[n t] Sin[n/2 (x + π)], {n, 1, max, 2}]
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]},
         {
            Quiet@Plot[u[x, time, 7], {x, -Pi, Pi},
                  PlotRange → {All, {-1, 1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red,
                  GridLines → {Range[-Pi, Pi, Pi / 4], Automatic},
                  GridLinesStyle → LightGray,
                  Ticks → {Range[-Pi, Pi, Pi / 2], Automatic}
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 20, .1},
   TrackedSymbols :→ {time}
]
```

Figure 5.12: Source code 2D

5.1.1.5 [337] both ends fixed but domain is $-1\ldots 1$. intial position is an impulse, zero initial velocity

problem number 337

Added January 8 ,2020

Problem 6.3.31 Introduction to Partial Dfferential Equations by Peter Olver, ISBN 9783319020983.

Solve

$$
\begin{aligned}
u_{tt} &= u_{xx} \\
u(-1,t) &= 0 \\
u(1,t) &= 0 \\
u(x,0) &= \delta(x) \\
\frac{\partial u(x,0)}{\partial t} &= 0
\end{aligned}
$$

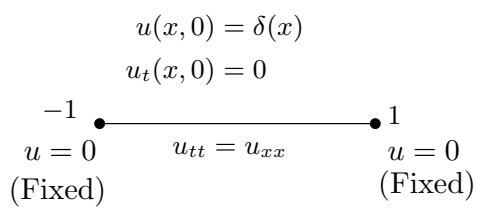

Figure 5.13: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
bc = {u[-1, t] == 0, u[1, t] == 0};
ic = {u[x, 0] == DiracDelta[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} \cos\left(\frac{1}{2}\pi t K[1]\right) \sin\left(\frac{1}{2}\pi K[1]\right) \sin\left(\frac{1}{2}\pi (x+1) K[1]\right)\right\}\right\}
$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=diff(u(x,t),x$2);
bc:=u(-1,t)=0,u(1,t)=0;
ic  := u(x,0)=Dirac(x), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t)=\sum_{n=1}^{\infty}\cos\left(\frac{\pi n t}{2}\right)\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi(x+1)n}{2}\right)$$

Hand solution

Since the boundary conditions are at $x=-1$ and at $x=1$, it is a little easier to solve this by first shifting the boundaries to $x=0$ and $x=2$. This is done by transformation. Let

$$z=x+1$$

When $x=-1$ then $z=0$ and when $x=1$ then $z=2$. The PDE in terms of z remains the same but the B.C. are shifted. Hence we want to solve for $v(z,t)$ in

$$\begin{aligned} v_{tt}&=v_{zz}\\ v(0,t)&=0\\ v(2,t)&=0 \end{aligned}$$

No need to worry about initial conditions now, since we will transform back to x before applying initial conditions and therefore will use the original initial conditions. This PDE is now solved by separation. Let $v=Z(z)\,T(t)$. Substituting into the PDE gives

$$\begin{aligned} T''Z&=Z''T\\ \frac{T''}{T}&=\frac{Z''}{Z}=-\lambda \end{aligned}$$

This gives the boundary value ODE

$$\begin{aligned} Z''+\lambda Z&=0 \qquad (1)\\ Z(0)&=0\\ Z(2)&=0 \end{aligned}$$

And the time ODE

$$T''+\lambda T=0 \qquad (2)$$

Solving (1). From the boundary conditions we know only $\lambda > 0$ is an eigenvalue. Hence for $\lambda > 0$ the solution is

$$Z(z) = A\cos\left(\sqrt{\lambda}z\right) + B\sin\left(\sqrt{\lambda}z\right)$$

At $z = 0$ this gives $A = 0$. Hence the solution now becomes $Z(z) = B\sin\left(\sqrt{\lambda}z\right)$. At $z = 2$ the above gives $0 = B\sin\left(2\sqrt{\lambda}\right)$. For non-trivial solution we want $\sin\left(2\sqrt{\lambda}\right) = 0$ which implies $2\sqrt{\lambda} = n\pi$ or

$$\lambda_n = \left(\frac{n\pi}{2}\right)^2 \qquad n = 1, 2, 3, \cdots$$

And the corresponding eigenfunctions

$$Z_n(z) = \sin\left(\frac{n\pi}{2}z\right) \qquad n = 1, 2, 3, \cdots$$

The time ODE (2) now becomes

$$T'' + \left(\frac{n\pi}{2}\right)^2 T = 0$$

Which has solution

$$T_n(t) = A_n\cos\left(\frac{n\pi}{2}t\right) + B_n\sin\left(\frac{n\pi}{2}t\right)$$

Hence the complete solution is

$$v(z,t) = \sum_{n=1}^{\infty}\left(A_n\cos\left(\frac{n\pi}{2}t\right) + B_n\sin\left(\frac{n\pi}{2}t\right)\right)\sin\left(\frac{n\pi}{2}z\right)$$

We are now ready to switch back from z to x. Since $z = x + 1$ then the above becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(A_n\cos\left(\frac{n\pi}{2}t\right) + B_n\sin\left(\frac{n\pi}{2}t\right)\right)\sin\left(\frac{n\pi}{2}(x+1)\right) \tag{3}$$

Now we apply initial conditions to find A_n, B_n. At $t = 0, u(x,0) = \delta(x)$. Hence the above gives

$$\delta(x) = \sum_{n=1}^{\infty} A_n\sin\left(\frac{n\pi}{2}(x+1)\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{2}(x+1)\right)$ and Integrating gives

$$\int_{-1}^{1}\delta(x)\sin\left(\frac{m\pi}{2}(x+1)\right)dx = \sum_{n=1}^{\infty} A_n\int_{-1}^{1}\sin\left(\frac{n\pi}{2}(x+1)\right)\sin\left(\frac{m\pi}{2}(x+1)\right)dx$$

By orthogonality of sin functions only term survives and the above simplifies to

$$\begin{aligned}\int_{-1}^{1}\delta(x)\sin\left(\frac{m\pi}{2}(x+1)\right)dx &= A_m\overbrace{\int_{-1}^{1}\sin^2\left(\frac{m\pi}{2}(x+1)\right)dx}^{1}\\ &= A_m\end{aligned}$$

But $\int_{-1}^{1}\delta(x)\sin\left(\frac{m\pi}{2}(x+1)\right)dx = \sin\left(\frac{m\pi}{2}\right)$ since that is where $x=0$. The above reduces to

$$A_n = \sin\left(\frac{n\pi}{2}\right) \qquad n = 1,2,3,\cdots$$

The solution (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2}t\right) + B_n\sin\left(\frac{n\pi}{2}t\right)\right)\sin\left(\frac{n\pi}{2}(x+1)\right) \tag{4}$$

Taking time derivatives

$$\frac{\partial}{\partial t}u(x,t) = \sum_{n=1}^{\infty}\left(-\frac{n\pi}{2}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}t\right) + \frac{n\pi}{2}B_n\cos\left(\frac{n\pi}{2}t\right)\right)\sin\left(\frac{n\pi}{2}(x+1)\right)$$

At $t=0$ the above becomes

$$0 = \sum_{n=1}^{\infty}\frac{n\pi}{2}B_n\sin\left(\frac{n\pi}{2}(x+1)\right)$$

Therefore $B_n = 0$. Hence the solution (4) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2}t\right)\sin\left(\frac{n\pi}{2}(x+1)\right) \tag{5}$$

Notice that $\sin\left(\frac{n\pi}{2}\right)$ is zero when n is even.

5.1.1.6 [338] Logan book, page 28. Both ends fixed

problem number 338

This is problem at page 28, David J Logan textbook, applied PDE textbook. No initial conditions given

$$u_{tt} = c^2 u_{xx}$$

With boundary condition

$$\begin{aligned}u(0,t) &= 0\\ u(L,t) &= 0\end{aligned}$$

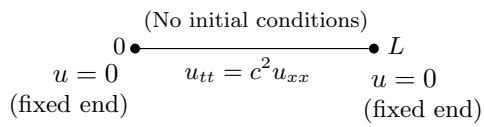

Figure 5.14: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}, Assumptions -> {L >
```

$$\left\{\left\{u(x,t) \to \{ \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}} \sin\left(\frac{\pi x K[1]}{L}\right) \left(\cos\left(\frac{\pi t|c|K[1]}{L}\right) \int_0^L \sqrt{2}\sqrt{\frac{1}{L}} \sin\left(\frac{\pi x K[1]}{L}\right) u(x,0)\, dx + \frac{L\left(\int_0^L \frac{\sqrt{2}\,\mathrm{s}}{}\right.}{}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) assuming
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\_F1(n) \sin\left(\frac{\pi c n t}{L}\right) + \_F2(n) \cos\left(\frac{\pi c n t}{L}\right)\right) \sin\left(\frac{\pi n x}{L}\right)$$

5.1.1.7 [339] non-zero initial velocity. Both ends fixed

problem number 339

Added Feb 25, 2019. Exam 1 problem, MATH 4567 Applied Fourier Analysis, University of Minnesota, Twin Cities.

Solve for $u(x,t)$

$$u_{tt} = u_{xx} - u$$

With boundary condition

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 1$$

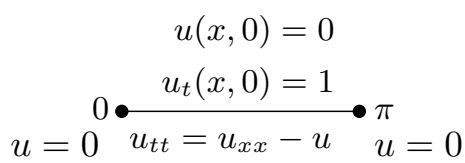

Figure 5.15: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] - u[x, t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 1};
bc  = {u[0, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \sum_{K[1]=1}^{\infty} \frac{2\left(1+(-1)^{K[1]+1}\right)\sin(xK[1])\sin\left(t\sqrt{K[1]^2+1}\right)}{\sqrt{\pi}K[1]\sqrt{\pi K[1]^2+\pi}} & K[1] \in \mathbb{Z} \wedge K[1] \geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=diff(u(x,t),x$2)-u(x,t);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{2((-1)^n - 1)\sin(nx)\sin\left(\sqrt{n^2+1}\,t\right)}{\sqrt{n^2+1}\,\pi n}$$

5.1.1.8 [340] Logan book page 149)

problem number 340

This is problem at page 149, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + p(x,t)$$

With boundary conditions

$$u(\pi,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

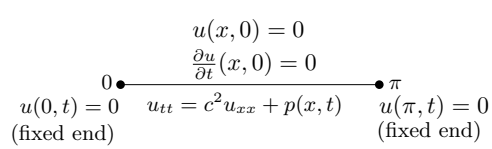

Figure 5.16: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + p[x, t];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\ \{\quad \sum_{K[1]=1}^{\infty}\sqrt{\frac{2}{\pi}}\left(\int_0^t \frac{\left(\int_0^{\pi}\sqrt{\frac{2}{\pi}}p(x,K[2])\sin(xK[1])\,dx\right)\sin\left(\sqrt{c^2K[1]^2}(t-K[2])\right)}{\sqrt{c^2K[1]^2}}\,dK[2]\right)\sin(xK[1])\quad K[1]\in\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\int_0^t\left(\sum_{n=1}^{\infty}\frac{2\left(\int_0^{\pi}p(x,\tau)\sin(nx)\,dx\right)\sin(nx)\sin((t-\tau)cn)}{\pi cn}\right)d\tau$$

5.1.1.9 [341] Haberman 8.5.2 (a)

problem number 341

Added Nov 25, 2018.

This is problem 8.5.2 (a), Richard Haberman applied partial differential equations book, 5th edition

Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space.

Consider a vibrating string with time-dependent forcing:

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+Q(x,t)$$

With boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

my hand solution in in the file `feb_24_2019_4_24_pm.tex`, but I need to go over my solution again to make sure it is correct.

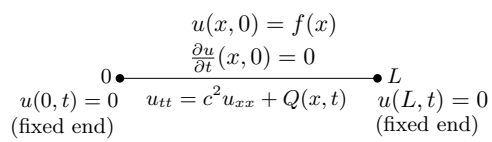

Figure 5.17: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + Q[x, t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \quad \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}}\left(\cos\left(\pi t\sqrt{\frac{c^2 K[1]^2}{L^2}}\right)\int_0^L \sqrt{2}\sqrt{\frac{1}{L}} f(x)\sin\left(\frac{\pi x K[1]}{L}\right)dx + \int_0^t \frac{\left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}} Q(x,K[2]\right.}{}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+Q(x,t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x), eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \int_0^t \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L Q(x,\tau)\sin\left(\frac{\pi n x}{L}\right) dx\right)\sin\left(\frac{\pi n x}{L}\right)\sin\left(\frac{\pi(t-\tau)cn}{L}\right)}{\pi c n} \right) d\tau + \left(\sum_{n=1}^{\infty} \frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right) dx\right.}{L}\right.$$

5.1.1.10 [342] Haberman 8.5.2 (b)

problem number 342

Added Nov 25, 2018.

This is problem 8.5.2 (b), Richard Haberman applied partial differential equations book, 5th edition.

Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space.

Consider a vibrating string with time-dependent forcing:

$$u_{tt} = c^2 u_{xx} + g(x)\cos(\omega t)$$

With boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

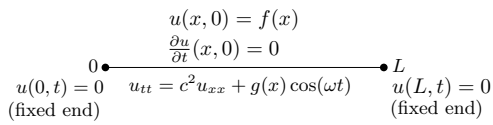

Figure 5.18: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + g[x]*Cos[w*t];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \quad \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}}\left(\cos\left(\pi t\sqrt{\frac{c^2 K[1]^2}{L^2}}\right)\int_0^L \sqrt{2}\sqrt{\frac{1}{L}} f(x)\sin\left(\frac{\pi x K[1]}{L}\right)\,dx + \int_0^t \frac{\left(\int_0^L \sqrt{2}\sqrt{\frac{1}{L}}\cos(wK[2\right.}{}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+ g(x)*cos(w*t);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \frac{-2\left(\sum_{n=1}^{\infty} \frac{\pi L c\left(\int_0^L g(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos(tw)\sin\left(\frac{\pi n x}{L}\right)}{L^2 w^2 - \pi^2 c^2 n^2}\right) + 2\left(\sum_{n=1}^{\infty} \frac{\pi L c\left(\int_0^L g(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos\left(\frac{\pi c n t}{L}\right)\sin\left(\frac{\pi n x}{L}\right)}{L^2 w^2 - \pi^2 c^2 n^2}\right)}{\pi c}$$

<u>Hand solution</u>

Let

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$$

Where we used $=$ instead of $\sim$ above, since the PDE given has homogeneous B.C. We know that $\phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$ for $n = 1, 2, 3, \cdots$ where $\lambda_n = \left(\frac{n\pi}{L}\right)^2$. Substituting the above in the given PDE gives

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = c^2 \sum_{n=1}^{\infty} A_n(t)\,\frac{d^2\phi_n(x)}{dx^2} + Q(x,t)$$

But $Q(x,t) = \sum_{n=1}^{\infty} q_n(t)\,\phi_n(x)$, hence the above becomes

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = c^2 \sum_{n=1}^{\infty} A_n(t)\,\frac{d^2\phi_n(x)}{dx^2} + \sum_{n=1}^{\infty} g_n(t)\,\phi_n(x)$$

But $\frac{d^2\phi_n(x)}{dx^2} = -\lambda_n\phi_n(x)$, hence

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = -c^2 \sum_{n=1}^{\infty} \lambda_n A_n(t)\,\phi_n(x) + \sum_{n=1}^{\infty} g_n(t)\,\phi_n(x)$$

Multiplying both sides by $\phi_m(x)$ and integrating gives

$$\int_0^L \sum_{n=1}^{\infty} A_n''(t)\,\phi_m(x)\,\phi_n(x)\,dx = -c^2 \int_0^L \sum_{n=1}^{\infty} \lambda_n A_n(t)\,\phi_m(x)\,\phi_n(x)\,dx + \int_0^L \sum_{n=1}^{\infty} g_n(t)\,\phi_m(x)\,\phi_n(x)\,dx$$

$$A_n''(t)\int_0^L \phi_n^2(x)\,dx = -c^2\lambda_n A_n(t)\int_0^L \phi_n^2(x)\,dx + g_n(t)\int_0^L \phi_n^2(x)\,dx$$

Hence

$$A_n''(t) + c^2\lambda_n A_n(t) = g_n(t)$$

Now we solve the above ODE. Let solution be

$$A_n(t) = A_n^h(t) + A_n^p(t)$$

Which is the sum of the homogenous and particular solutions. The homogenous solution is

$$A_n^h(t) = c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right)$$

And the particular solution depends on $q_n(t)$. Once we find $q_n(t)$, we plug-in everything back into $u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$ and then use initial conditions to find c_{1_n}, c_{2_n}, the two constant of integrations. Now we are given that $Q(x,t) = g(x)\cos(\omega t)$. Hence

$$g_n(t) = \frac{\int_0^L Q(x,t)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \frac{\cos(\omega t)\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \cos(\omega t)\,\gamma_n$$

Where

$$\gamma_n = \frac{\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx}$$

is constant that depends on n. Now we use the above in result found in part (a)

$$A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n \cos(\omega t) \tag{1}$$

We know the homogenous solution from part (a).

$$A_n^h(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right)$$

We now need to find the particular solution. Will solve using method of undetermined coefficients.

<u>Case 1</u> $\omega \neq c\sqrt{\lambda_n}$ (no resonance)

We can now guess

$$A_n^p(t) = z_1 \cos(\omega t) + z_2 \sin(\omega t)$$

Plugging this back into (1) gives

$$\begin{aligned}
(z_1 \cos(\omega t) + z_2 \sin(\omega t))'' + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t)\\
(-\omega z_1 \sin(\omega t) + \omega z_2 \cos(\omega t))' + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t)\\
-\omega^2 z_1 \cos(\omega t) - \omega^2 z_2 \sin(\omega t) + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t)
\end{aligned}$$

Collecting terms

$$\cos(\omega t)\left(-\omega^2 z_1 + c^2\lambda_n z_1\right) + \sin(\omega t)\left(-\omega^2 z_2 + c^2\lambda_n z_2\right) = \gamma_n \cos(\omega t)$$

Therefore we obtain two equations in two unknowns

$$\begin{aligned}
-\omega^2 z_1 + c^2\lambda_n z_1 &= \gamma_n\\
-\omega^2 z_2 + c^2\lambda_n z_2 &= 0
\end{aligned}$$

From the second equation, $z_2 = 0$ and from the first equation

$$\begin{aligned}
z_1\left(c^2\lambda_n - \omega^2\right) &= \gamma_n\\
z_1 &= \frac{\gamma_n}{c^2\lambda_n - \omega^2}
\end{aligned}$$

Hence

$$\begin{aligned}
A_n^p(t) &= z_1 \cos(\omega t) + z_2 \sin(\omega t)\\
&= \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Therefore

$$\begin{aligned}
A_n(t) &= A_n^h(t) + A_n^p(t)\\
&= c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Now we need to find c_{1_n}, c_{2_n}. Since

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x) \\ &= \sum_{n=1}^{\infty} \left(c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t) \right) \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

At $t = 0$ the above becomes

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} \left(c_{1_n} + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \right) \sin\left(\frac{n\pi}{L}x\right) \\ &= \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L}x\right) + \sum_{n=1}^{\infty} \frac{\gamma_n}{c^2\lambda_n - \omega^2} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Applying orthogonality

$$\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx = \int_0^L \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right) dx + \int_0^L \sum_{n=1}^{\infty} \frac{\gamma_n}{c^2\lambda_n - \omega^2} \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right.$$

$$\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx = c_{1_n} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx$$

Rearranging

$$\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx = c_{1_n} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$\begin{aligned} c_{1_n} &= \frac{\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx}{\int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx} - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \\ &= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \end{aligned}$$

We now need to find c_{2_n}. For this we need to differentiate the solution once.

$$\frac{\partial u(x,t)}{\partial t} = \sum_{n=1}^{\infty} \left(-c\sqrt{\lambda_n} c_{1_n} \sin\left(c\sqrt{\lambda_n}t\right) + c\sqrt{\lambda_n} c_{2_n} \cos\left(c\sqrt{\lambda_n}t\right) - \frac{\gamma_n}{c^2\lambda_n - \omega^2} \omega \sin(\omega t) \right) \sin\left(\frac{n\pi}{L}x\right)$$

Applying initial conditions $\frac{\partial u(x,0)}{\partial t} = 0$ gives

$$0 = \sum_{n=1}^{\infty} c\sqrt{\lambda_n} c_{2_n} \sin\left(\frac{n\pi}{L}x\right)$$

Hence

$$c_{2_n} = 0$$

Therefore the final solution is

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)$$

And

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t) \sin\left(\frac{n\pi}{L}x\right)$$

Where

$$c_{1_n} = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2}$$

<u>Case 2</u> $\omega = c\sqrt{\lambda_n}$ <u>Resonance case</u>. Now we can't guess $A_n^p(t) = z_1 \cos(\omega t) + z_2 \sin(\omega t)$ so we have to use

$$A_n^p(t) = z_1 t \cos(\omega t) + z_2 t \sin(\omega t)$$

Substituting this in $A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n \cos(\omega t)$ gives

$$(z_1 t \cos(\omega t) + z_2 t \sin(\omega t))'' + c^2\lambda_n(z_1 t \cos(\omega t) + z_2 t \sin(\omega t)) = \gamma_n \cos(\omega t) \tag{2}$$

But

$$\begin{aligned}
(z_1 t \cos(\omega t) + z_2 t \sin(\omega t))'' &= (z_1 \cos(\omega t) - z_1\omega t \sin(\omega t) + z_2 \sin(\omega t) + z_2\omega t \cos(\omega t))' \\
&= -z_1\omega \sin(\omega t) - \left(z_1\omega \sin(\omega t) + z_1\omega^2 t \cos(\omega t)\right) \\
&+ z_2\omega \cos(\omega t) + \left(z_2\omega \cos(\omega t) - z_2\omega^2 t \sin(\omega t)\right) \\
&= -2z_1\omega \sin(\omega t) - z_1\omega^2 t \cos(\omega t) + 2z_2\omega \cos(\omega t) - z_2\omega^2 t \sin(\omega t)
\end{aligned}$$

Hence (2) becomes

$$-2z_1\omega \sin(\omega t) - z_1\omega^2 t \cos(\omega t) + 2z_2\omega \cos(\omega t) - z_2\omega^2 t \sin(\omega t) + c^2\lambda_n(z_1 t \cos(\omega t) + z_2 t \sin(\omega t)) = \gamma_n \cos(\omega t$$

Comparing coefficients we see that $2z_2\omega = \gamma_n$ or

$$z_2 = \frac{\gamma_n}{2\omega}$$

And $z_1 = 0$. Therefore

$$A_n^p(t) = \frac{\gamma_n}{2\omega} t \sin(\omega t)$$

Therefore

$$\begin{aligned}
A_n(t) &= A_n^h(t) + A_n^p(t) \\
&= c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)
\end{aligned}$$

We now can find c_{1_n}, c_{2_n} from initial conditions.

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x) \\ &= \sum_{n=1}^{\infty} \left(c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t) \right) \sin\left(\frac{n\pi}{L}x\right) \quad (4) \end{aligned}$$

At $t = 0$

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L}x\right) \\ c_{1n} &= \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx \end{aligned}$$

Taking time derivative of (4) and setting it to zero will give c_{2n}. Since initial speed is zero then $c_{2_n} = 0$. Hence

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)$$

This completes the solution.

<u>Summary of solution</u>

The solution is given by

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$$

<u>Case $\omega \neq c\sqrt{\lambda_n}$</u>

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)$$

And

$$c_{1_n} = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{m\pi}{L}x\right) dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2}$$

And

$$\gamma_n = \frac{\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx}$$

And $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3,$

<u>Case $\omega = c\sqrt{\lambda_n}$</u> (resonance)

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)$$

And

$$c_{1_n} = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

5.1.1.11 [343] Both I.C. not zero

problem number 343

Added July 2, 2018.

Taken from Maple 2018.1 improvements to PDE's document. Solve

$$v_{tt} = v_{xx}$$

For $t > 0$ and $0 < x < 1$. With boundary conditions

$$v(0,t) = 0$$
$$v(1,0) = 0$$

With initial conditions

$$v(x,0) = f(x)$$
$$\frac{\partial v}{\partial t}(x,0) = g(x)$$

Where $f(x) = -\frac{e^2 x - e^{x+1} - x + e^{1-x}}{e^2 - 1}$ and $g(x) = 1 + \frac{e^2 x - e^{x+1} - x + e^{1-x}}{e^2 - 1}$

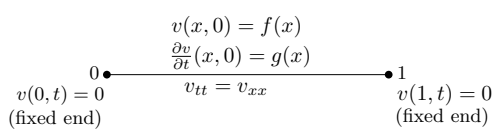

Figure 5.19: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[v[x, t], {t, 2}] == D[v[x, t], {x, 2}];
bc  = {v[0, t] == 0, v[1, t] == 0};
ic  = {v[x, 0] == -((Exp[2]*x - Exp[x + 1] - x + Exp[1 - x])/(Exp[2] - 1)), Derivative[0, 1]
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, v[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{v(x,t) \to \sum_{n=1}^{\infty} -\frac{2((-1)^{n+1} n\pi \cos(n\pi t) + ((-1+(-1)^n)\pi^2 n^2 + 2(-1)^n - 1)\sin(n\pi t))\sin(n\pi x)}{\pi^4 n^4 + \pi^2 n^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(v(x, t), t, t)=(diff(v(x, t), x, x));
bc  := v(0, t) = 0, v(1, t) = 0;
ic  := v(x, 0) = -(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1),
               (D[2](v))(x, 0) = 1+(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],v(x,t))),outp
```

$$v(x,t) = \sum_{n=1}^{\infty} -\frac{2(-\pi n(-1)^n \cos(\pi n t) + (\pi^2 n^2 (-1)^n - \pi^2 n^2 + 2(-1)^n - 1)\sin(\pi n t))\sin(\pi n x)}{\pi^2 (\pi^2 n^2 + 1) n^2}$$

5.1.1.12 [344] With constant source

problem number 344

Added July 2, 2018.

Third example, from Maple 2018.1 improvements to PDE's document. What_is_New_after_Maple_2018.pdf

Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + 1$$

For $t > 0$ and $0 < x < L$. With boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

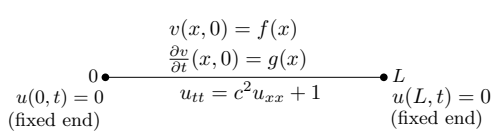

Figure 5.20: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + 1;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\{\quad\sum_{K[1]=1}^{\infty}\boxed{\sqrt{2}\sqrt{\frac{1}{L}}\sin\left(\frac{\pi x K[1]}{L}\right)\left(\frac{\sqrt{2}(1+(-1)^{K[1]+1})\left(L-L\cos\left(\frac{c\pi t K[1]}{L}\right)\right)L^{3/2}}{c^2\pi^3K[1]^3}+\frac{\left(\int_0^L\frac{\sqrt{2}g(x)\sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\,d\right.}{\pi|c||K}\right.}\right.$$

Indeter

Maple ✓

```
restart;
interface(showassumed=0);
f='f';
pde :=diff(u(x, t), t, t) = c^2* diff(u(x, t), x, x) + 1;
bc  := u(0, t) = 0, u(L, t) = 0;
ic  := u(x, 0) = f(x), (D[2](u))(x, 0) = g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t)=\left(\sum_{n=1}^{\infty}\frac{\left(2Lc\left(\int_0^L g(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\sin\left(\frac{\pi c n t}{L}\right)-\pi n\left(\int_0^L(-2c^2f(x)+Lx-x^2)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos\left(\right.\right.}{\pi L c^2 n}\right.$$

5.1.1.13 [345] Logan page 213

problem number 345

This is problem at page 213, David J Logan textbook, applied PDE textbook. Both ends fixed end, with source.

$$u_{tt} = c^2 u_{xx} + Ax$$

With boundary conditions

$$u(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

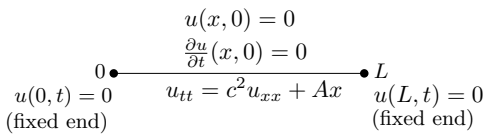

Figure 5.21: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + A*x;
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \quad -\frac{iAL^3\left(\text{PolyLog}\left(3,-e^{-\frac{i\pi(ct-x)}{L}}\right)-\text{PolyLog}\left(3,-e^{\frac{i\pi(ct-x)}{L}}\right)+2\,\text{PolyLog}\left(3,-e^{-\frac{i\pi x}{L}}\right)-2\,\text{PolyLog}\left(3,-e^{\frac{i\pi x}{L}}\right)-\text{Po}\right.}{2c^2\pi^3}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))  assu
```

$$u(x,t) = \frac{AL^2x}{6c^2} - \frac{Ax^3}{6c^2} + \left(\sum_{n=1}^{\infty} \frac{2AL^3(-1)^n \cos\left(\frac{\pi cnt}{L}\right) \sin\left(\frac{\pi nx}{L}\right)}{\pi^3 c^2 n^3}\right)$$

5.1.1.14 [346] Telegraphy PDE

problem number 346

Both ends fixed with damping Solve

$$u_{tt} + 2u_t = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

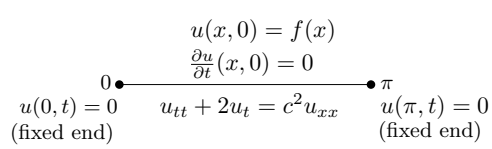

Figure 5.22: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] + 2*D[u[x, t], t] == D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t], 60*10]];
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty} e^{-t}\sqrt{\tfrac{2}{\pi}}\sin(xK[1])\left(\cos\left(\tfrac{1}{2}t\sqrt{4K[1]^2-4}\right)\int_0^{\pi}\sqrt{\tfrac{2}{\pi}}f(x)\sin(xK[1])\,dx+\frac{\sin\left(\frac{1}{2}t\sqrt{4K}\right.}{}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+2*diff(u(x,t),t)=diff(u(x,t),x$2);
ic  :=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=f(x);
bc  := u(0,t)=0,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\sum_{n=1}^{\infty}\begin{cases}\frac{2(t+1)e^{-t}\left(\int_0^{\pi}f(x)\sin(x)dx\right)\sin(x)}{\pi} & n=1\\ -\frac{\left(\left(i-\sqrt{n^2-1}\right)e^{\left(i\sqrt{n^2-1}-1\right)t}+\left(-i-\sqrt{n^2-1}\right)e^{-\left(i\sqrt{n^2-1}+1\right)t}\right)\left(\int_0^{\pi}f(x)\sin(nx)dx\right)\sin(nx)}{\sqrt{n^2-1}\,\pi} & otherwise\end{cases}$$

5.1.1.15 [347] Dispersion term present (general case)

problem number 347

Added July 12, 2019.

Solve

$$u_{tt}+\gamma^2u(x,t)=c^2u_{xx}$$

Dispersion term $\gamma^2u(x,t)$ causes the shape of the original wave to distort with time.

With $0 < x < L$ and $t > 0$ and with boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

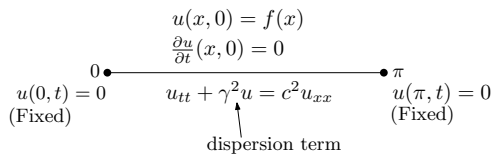

Figure 5.23: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2 D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t},Assumptions->L>0
```

$$\left\{\left\{u(x,t) \to \{ \quad \sum_{K[1]=1}^{\infty} \sqrt{2}\sqrt{\frac{1}{L}} \cos\left(t\sqrt{c^2\left(\frac{\gamma^2}{c^2} + \frac{\pi^2 K[1]^2}{L^2}\right)}\right)\left(\int_0^L \frac{\sqrt{2} f(x) \sin\left(\frac{\pi x K[1]}{L}\right)}{\sqrt{L}}\, dx\right) \sin\left(\frac{\pi x K[1]}{L}\right) \quad K[1]\right.\right.$$

Indeterminate

Due to adding dispersion term

Maple ✓

```
restart;
interface(showassumed=0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right)dx\right)\cos\left(\frac{\sqrt{\pi^2c^2n^2+\gamma^2L^2}\,t}{L}\right)\sin\left(\frac{\pi n x}{L}\right)}{L}$$

<u>Hand solution</u>

Solving for $t>0, 0<x<L$

$$\frac{\partial^2 u}{\partial t^2}+\gamma^2 u=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<L, t>0$$

With BC

$$\begin{aligned}u(0,t)&=0\\u(L,t)&=0\end{aligned}$$

And initial conditions

$$\begin{aligned}u(x,0)&=f(x)\\u_t(x,0)&=0\end{aligned}$$

Dispersion term $\gamma^2 u$ causes the shape of the original wave to distort with time. Using separation of variables, Let $u=X(x)T(t)$. Substituting this back in the PDE gives

$$\begin{aligned}T''X+\gamma^2XT&=c^2X''T\\\frac{1}{c^2}\left(\frac{T''}{T}+\gamma^2\right)&=\frac{X''}{X}=-\lambda\end{aligned}$$

The eigenvalue ODE is

$$\begin{aligned}X''+\lambda X&=0\\X(0)&=0\\X(L)&=0\end{aligned}$$

The eigenvalues are $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3, \cdots$ and the eigenfunctions are $X_n(x) = c_n \sin\left(\frac{n\pi}{L}x\right)$. The time ODE becomes

$$T'' + \left(\gamma^2 + c^2\lambda_n\right) T = 0$$

The solution is

$$T_n(t) = A_n \cos\left(\sqrt{\gamma^2 + c^2\lambda_n}t\right) + B_n \sin\left(\sqrt{\gamma^2 + c^2\lambda_n}t\right)$$

Taking time derivatives gives

$$T_n'(t) = -\sqrt{\gamma^2 + c^2\lambda_n}A_n \sin\left(\sqrt{\gamma^2 + c^2\lambda_n}t\right) + B_n\sqrt{\gamma^2 + c^2\lambda_n} \cos\left(\sqrt{\gamma^2 + c^2\lambda_n}t\right)$$

At time $t = 0$, the above is zero (initial velocity is zero), which gives

$$0 = B_n\sqrt{\gamma^2 + c^2\lambda_n}$$

Hence $B_n = 0$ and the time ODE solution becomes

$$T_n(t) = A_n \cos\left(\sqrt{\gamma^2 + c^2\lambda_n}t\right)$$

Hence the fundamental solution is

$$\begin{aligned} u_n(x,t) &= T_n X_n \\ &= c_n \cos\left(\sqrt{\gamma^2 + c^2\left(\frac{n\pi}{L}\right)^2}t\right) \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Therefore the solution is

$$u(x,t) = \sum_{n=1}^{\infty} c_n \cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2 + c^2n^2\pi^2)}t\right) \sin\left(\frac{n\pi}{L}x\right)$$

c_n is found from initial position. At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{L}x\right)$$

Applying orthogonality gives

$$\begin{aligned} \int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx &= c_n \frac{L}{2} \\ c_n &= \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx \end{aligned}$$

Hence solution is

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{L}\int_0^L f(s) \sin\left(\frac{n\pi}{L}s\right) ds\right) \cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2 + c^2n^2\pi^2)}t\right) \sin\left(\frac{n\pi}{L}x\right)$$

5.1.1.16 [348] Dispersion term present

problem number 348

Solve

$$u_{tt} + \gamma^2 u(x,t) = c^2 u_{xx}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time. With $0 < x < \pi$ and $t > 0$ and with boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

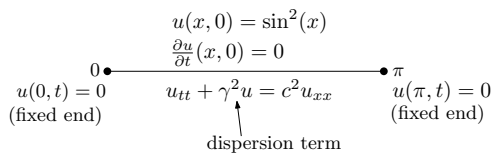

Figure 5.24: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[Pi, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{\sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} \cos\left(t\sqrt{c^2\left(\frac{\gamma^2}{c^2} + K[1]^2\right)}\right) \left(\left\{\begin{array}{cc} 0 & K[1]=2 \\ \frac{2(-1+(-1)^{K[1]})\sqrt{\frac{2}{\pi}}}{K[1]^3-4K[1]} & \text{True} \end{array}\right.\right) \sin(xK[1])\right.\right.\right.$$

Indeterminate

Due to adding dispersion term

Maple ✓

```
restart;
interface(showassumed=0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=sin(x)^2,(D[2](u))(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = \frac{8\cos\left(\sqrt{c^2+\gamma^2}\,t\right)\sin(x)}{3\pi} + \left(\sum_{n=3}^{\infty} \frac{4((-1)^n-1)\cos\left(\sqrt{c^2n^2+\gamma^2}\,t\right)\sin(nx)}{\pi\left(n^2-4\right)n}\right)$$

<u>Hand solution</u>

Solving for $t > 0, 0 < x < L$

$$\frac{\partial^2 u}{\partial t^2} + \gamma^2 u = \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < L, t > 0 \tag{1}$$

With BC

$$u(0,t) = 0$$
$$u(L,t) = 0$$

And initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Where now $L = \pi, f(x) = \sin^2(x)$.

The general solution for (1) was found in problem 5.1.1.15 on page 954 as

$$u(x,t) = \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\cos\left(\frac{t}{L}\sqrt{L^2\gamma^2+c^2n^2\pi^2}\right)\sin\left(\frac{n\pi}{L}x\right)$$

Using the above specific values for this problem, the solution becomes

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\int_0^{\pi} f(s)\sin\left(\frac{n\pi}{\pi}s\right)ds\right)\cos\left(\frac{t}{\pi}\sqrt{\pi^2\gamma^2+c^2n^2\pi^2}\right)\sin\left(\frac{n\pi}{\pi}x\right) \\ &= \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\int_0^{\pi} f(s)\sin(ns)\,ds\right)\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx) \end{aligned}$$

But

$$\begin{aligned}\int_0^\pi \sin^2(s)\sin(ns)\,ds &= \int_0^\pi \left(\frac{1}{2}-\frac{1}{2}\cos(2s)\right)\sin(ns)\,ds\\ &= \int_0^\pi \frac{1}{2}\sin(ns)\,ds-\frac{1}{2}\int_0^\pi \cos(2s)\sin(ns)\,ds\\ &= \frac{1-(-1)^n}{2n}-\frac{1}{2}\int_0^\pi \cos(2s)\sin(ns)\,ds\end{aligned}$$

$\int_0^\pi \cos(2s)\sin(ns)\,ds = -\frac{2}{3}$for $n=1$ and $\int_0^\pi \cos(2s)\sin(ns)\,ds = 0$ For $n=2$ and $\int_0^\pi \cos(2s)\sin(ns)\,ds = \frac{(1-(-1)^n)n}{n^2-4}$ for $n>2$. Hence

$$\int_0^\pi \sin^2(s)\sin(ns)\,ds = \begin{cases} \frac{1-(-1)}{2}+\frac{1}{3}=\frac{4}{3} & n=1\\ \frac{1-(-1)^n}{2n}=0 & n=2\\ \frac{1-(-1)^n}{2n}-\frac{1}{2}\frac{(1-(-1)^n)n}{n^2-4}=\frac{2}{n}\frac{(-1)^n-1}{n^2-4} & n>2\end{cases}$$

The solution becomes

$$\begin{aligned}u(x,t) &= \left(\frac{2}{\pi}\right)\frac{4}{3}\cos\left(\sqrt{\gamma^2+c^2}t\right)\sin(x)+\sum_{n=3}^{\infty}\left(\frac{2}{\pi}\frac{2}{n}\frac{(-1)^n-1}{n^2-4}\right)\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx)\\ &= \frac{8}{3\pi}\cos\left(\sqrt{\gamma^2+c^2}t\right)\sin(x)+\frac{4}{\pi}\sum_{n=3}^{\infty}\frac{(-1)^n-1}{n(n^2-4)}\cos\left(\sqrt{\gamma^2+c^2n^2}t\right)\sin(nx)\end{aligned}$$

5.1.1.17 [349] Dispersion term present (specific case)

problem number 349

Added July 12, 2019 Solve

$$u_{tt}+\gamma^2 u(x,t)=c^2u_{xx}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time. With $0<x<L$ and $t>0$ and with boundary conditions

$$\begin{aligned}u(0,t)&=0\\ u(\pi,0)&=0\end{aligned}$$

With initial conditions

$$\begin{aligned}\frac{\partial u}{\partial t}(x,0)&=0\\ u(x,0)&=f(x)\end{aligned}$$

Using the following values

$$
\begin{aligned}
L &= 10 \\
\gamma &= \frac{1}{8} \\
f(x) &= \begin{cases} x-4 & 4 \le x \le 5 \\ 6-x & 5 \le x \le 6 \\ 0 & \text{otherwise} \end{cases} \\
c &= 1
\end{aligned}
$$

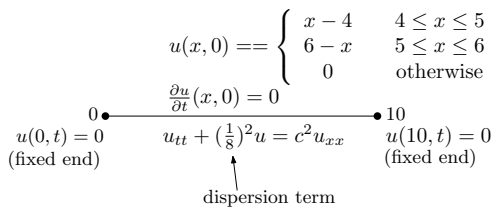

Figure 5.25: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
gamma=1/8; c=1; L=10;
f=Piecewise[{{x-4,4<=x<=5},{6-x,5<x<=6},{0,True}}];
pde =  D[u[x, t], {t, 2}] + gamma^2*u[x, t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, u[L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \{ \sum_{K[1]=1}^{\infty} -\frac{20\cos\left(t\sqrt{\frac{1}{100}\pi^2 K[1]^2+\frac{1}{64}}\right)\left(\sin\left(\frac{2}{5}\pi K[1]\right)-2\sin\left(\frac{1}{2}\pi K[1]\right)+\sin\left(\frac{3}{5}\pi K[1]\right)\right)\sin\left(\frac{1}{10}\pi x K[1]\right)}{\pi^2 K[1]^2} \quad K[1] \in \mathbb{Z} \right.\right.
$$

Indeterminate Tr

Due to adding dispersion term

Maple ✓

```
restart;
local gamma;
gamma:=1/8;
c:=1;
L:=10;
f:=piecewise(4<=x and x<=5, x-4,5<x and x<=6,6-x,true,0);
pde :=diff(u(x,t),t$2)+gamma^2*u(x,t)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(L,t)=0;
ic  := u(x,0)=f,(D[2](u))(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=1}^{\infty}-\frac{40\left(\cos\left(\frac{\pi n}{2}\right)-2\cos\left(\frac{\pi n}{5}\right)+2\cos\left(\frac{\pi n}{10}\right)-2\cos\left(\frac{2\pi n}{5}\right)+2\cos\left(\frac{3\pi n}{10}\right)-1\right)\cos\left(\frac{\sqrt{16\pi^2n^2+25}\,t}{40}\right)\mathrm{si}}{\pi^2n^2}$$

<u>Hand solution</u>

Solve for $0<x<L, t>0$

$$\frac{\partial^2 u}{\partial t^2}+\gamma^2 u=c^2\frac{\partial^2 u}{\partial x^2}$$

Boundary conditions, $t>0$ (both ends fixed)

$$u(0,t)=0$$
$$u(L,t)=0$$

Initial conditions, $t=0$

$$\frac{\partial u(x,0)}{\partial t}=0$$
$$u(x,0)=f(x)$$

Using $L=10, \gamma=1/8, c=1$ and initial position

$$f(x)=\begin{cases} x-4 & 4\le x\le 5\\ 6-x & 5\le x\le 6\\ 0 & \text{otherwise}\end{cases}$$

The general solution to the above PDE was given in problem 5.1.1.15 on page 954 as

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{L}s\right)ds\right)\cos\left(\frac{1}{L}\sqrt{(L^2\gamma^2+c^2n^2\pi^2)}t\right)\sin\left(\frac{n\pi}{L}x\right)$$

Replacing given values in the above solution results in

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty}\left(\frac{2}{10}\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{10}\sqrt{\left(100\left(\frac{1}{8}\right)^2+n^2\pi^2\right)}t\right)\sin\left(\frac{n\pi}{10}x\right)\\
&= \frac{2}{10}\sum_{n=1}^{\infty}\left(\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{10}\sqrt{\frac{1}{16}\left(\frac{100}{4}+16n^2\pi^2\right)}t\right)\sin\left(\frac{n\pi}{10}x\right) \qquad (1)\\
&= \frac{2}{10}\sum_{n=1}^{\infty}\left(\int_0^{10} f(s)\sin\left(\frac{n\pi}{10}s\right)ds\right)\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2}t\right)\sin\left(\frac{n\pi}{10}x\right) \qquad (5.1)
\end{aligned}
$$

But

$$
\int_0^{10} f(x)\sin\left(\frac{n\pi}{10}x\right)ds = \frac{100\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)}{n^2\pi^2}
$$

Hence the solution (1) becomes

$$
\begin{aligned}
u(x,t) &= \frac{2}{10}\sum_{n=1}^{\infty}\frac{100\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)}{n^2\pi^2}\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2}t\right)\sin\left(\frac{n\pi}{10}x\right)\\
&= \frac{20}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(2\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{2n\pi}{5}\right)-\sin\left(\frac{3n\pi}{5}\right)\right)\cos\left(\frac{1}{40}\sqrt{25+16n^2\pi^2}t\right)\sin\left(\frac{n\pi}{10}x\right)
\end{aligned}
$$

Animation is below. The left one uses $\gamma = \frac{1}{8}$ and the right one uses larger value of $\gamma = \frac{5}{8}$ in order to show the effect of larger dispersion.

Out[ ]=

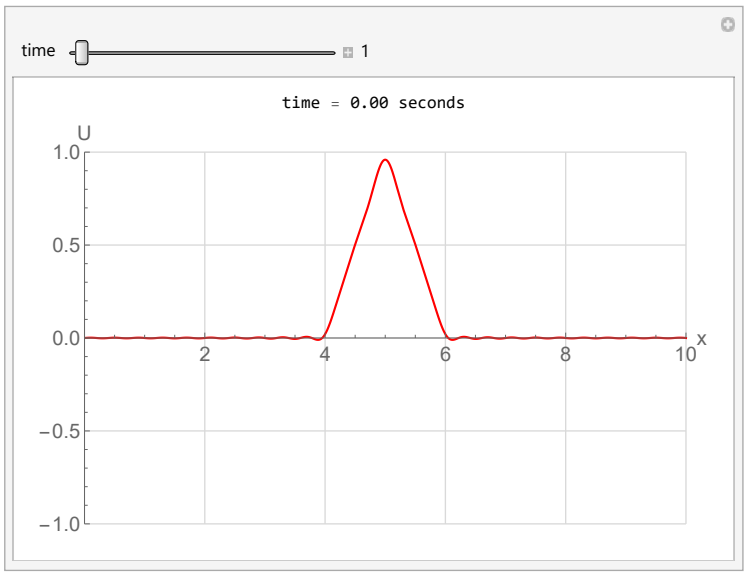

Figure 5.26: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, mySol]
gamma = 1 / 8; c = 1; L = 10;
f = Piecewise[{{x - 4, 4 ≤ x ≤ 5}, {6 - x, 5 < x ≤ 6}, {0, True}}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 50;
mySol[x_, t_] = 20/π^2 Sum[1/n^2 (2 Sin[n π/2] - Sin[2 n π/5] - Sin[3 n π/5]) (Cos[1/10 Sqrt[50/32 + n^2 π^2] t]) Sin[n π/10 x], {n, 1, numberOfTerms}];
```

Figure 5.27: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
          {
            Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                BaseStyle → 15,
                ImageMargins → 3,
                PerformanceGoal → "Quality",
                PlotRange → {{0, L}, {-1, 1}},
                ImageSize → 500,
                AxesLabel → {"x", "U"},
                GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotStyle → Red
              ]

          }}],
      {t, 0, 80, 0.1}];
In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.28: Code for animation

5.1.1.18 [350] non-zero initial position

problem number 350

Added March 9, 2018. Solve

$$u_{tt} = 4u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

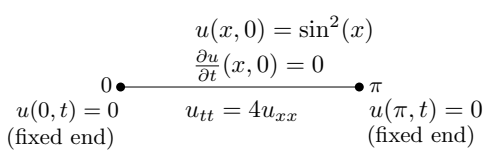

Figure 5.29: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}];
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
bc  = {u[0, t] == 0, u[Pi, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4(\cos(n\pi)-1)\cos(2nt)\sin(nx)}{(n^3-4n)\pi}\right\}\right\}$$

But sum should not include $n=2$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)= 4*diff(u(x,t),x$2);
bc  := u(0,t)=0,u(Pi,t)=0;
ic  := u(x,0)=sin(x)^2,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\frac{8\cos(2t)\sin(x)}{3\pi}+\left(\sum_{n=3}^{\infty}\frac{4((-1)^n-1)\cos(2nt)\sin(nx)}{\pi(n^2-4)n}\right)$$

Handled $n=2$ case correctly

5.1.1.19 [351] With source

problem number 351

Added December 20, 2018.

Example 18, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $0<x<1$ and $t>0$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+xe^{-t}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(1,0) = 0$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

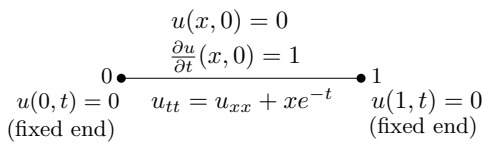

Figure 5.30: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + x*Exp[-t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[0, t] == 0, u[1, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \frac{e^{-i((-i+\pi)t+\pi x)}\left(ie^{2i\pi t+t}\pi^2(i+\pi)\,{}_2F_1\left(1,\frac{-i+\pi}{\pi};2-\frac{i}{\pi};-e^{i\pi(t-x)}\right)+e^{i\pi t}(1-i\pi)\pi^2\,{}_2F_1\left(1,\frac{-i+\pi}{\pi};2-\frac{i}{\pi};-e^{-i\pi x}\right)+ie^{i\pi(t+}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2)+x*exp(-t);
bc  := u(0,t)=0,u(1,t)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{2(\pi(-\cos(\pi n t) + \mathrm{e}^{-t})\, n(-1)^n + (\pi^2 n^2 (-1)^n - \pi^2 n^2 + 2(-1)^n - 1)\sin(\pi n t))\sin(\pi n x)}{\pi^2 (\pi^2 n^2 + 1)\, n^2}$$

5.1.1.20 [352] Right end free (general case)

problem number 352

Added July 8, 2019

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$

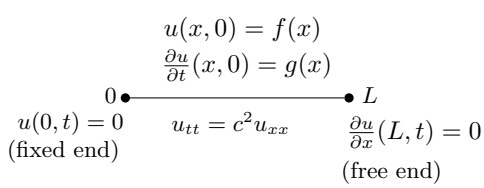

Figure 5.31: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == g[x], u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\cos\left(\frac{1}{2}\pi t\sqrt{\frac{c^2(2K[1]-1)^2}{L^2}}\right)\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx+\frac{2L\left(\int_0^L\frac{\sqrt{2}g(x)\sin\left(\frac{\pi x(2K[1]}{2L}\right.}{\sqrt{L}}\right.}{\pi|}\right.}{\sqrt{L}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=g(x),u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{4\left(L\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\sin\left(\frac{(2n+1)\pi ct}{2L}\right)+\pi\left(n+\frac{1}{2}\right)c\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos}{\pi\left(2n+1\right)Lc}$$

Hand solution

Solving for $0<x<L$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<L,t>0$$

Boundary conditions, $t>0$

$$u(0,t)=0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t = 0$

$$\begin{aligned} u_t(x,0) &= g(x) \\ u(x,0) &= f(x) \end{aligned}$$

Separation of variables gives the eigenvalue ODE

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Only $\lambda > 0$ gives non-trivial solution from the nature of the boundary conditions. Hence solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then the above gives $0 = A$ and the solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivatives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Since $X'(L) = 0$ the above becomes

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

Which implies $\sqrt{\lambda}L = \frac{n\pi}{2}$ for $n = 1, 3, 5, \cdots$ or

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1, 3, 5, \cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right) \qquad n = 1, 3, 5, \cdots$$

The time ODE now becomes

$$T'' + c^2\left(\frac{n\pi}{2L}\right)^2 T = 0$$

Which has the solution

$$T(t) = D_n\cos\left(c\frac{n\pi}{2L}t\right) + E_n\sin\left(c\frac{n\pi}{2L}t\right)$$

Therefore the complete solution becomes

$$u(x,t)=\sum_{n=1,3,5,\cdots}^{\infty}\left(D_n\cos\left(c\frac{n\pi}{2L}t\right)+E_n\sin\left(c\frac{n\pi}{2L}t\right)\right)\Phi_n(x) \tag{1}$$

At $t=0$ the above becomes

$$f(x)=\sum_{n=1,3,5,\cdots}^{\infty}D_n\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx&=D_n\int_0^L\Phi_n^2(x)\,dx\\&=\frac{L}{2}D_n\end{aligned}$$

Hence

$$D_n=\frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t)=\sum_{n=1,3,5,\cdots}^{\infty}\left(-c\frac{n\pi}{2L}D_n\sin\left(c\frac{n\pi}{2L}t\right)+E_nc\frac{n\pi}{L}\cos\left(c\frac{n\pi}{2L}t\right)\right)\Phi_n(x)$$

At $t=0$ the above becomes

$$g(x)=\sum_{n=1,3,5,\cdots}^{\infty}E_nc\frac{n\pi}{2L}\Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L g(x)\,\Phi_n(x)\,dx&=E_nc\frac{n\pi}{L}\int_0^L\Phi_n^2(x)\,dx\\&=\frac{L}{2}E_nc\frac{n\pi}{2L}\\&=\frac{1}{4}E_ncn\pi\end{aligned}$$

Hence

$$E_n=\frac{4}{cn\pi}\int_0^L g(x)\,\Phi_n(x)\,dx \tag{3}$$

Using (2,3) in (1) gives the final solution as

$$u(x,t)=\frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\cos\left(c\frac{n\pi}{2L}t\right)\sin\left(\frac{n\pi}{2L}x\right)$$
$$+\frac{4}{c\pi}\sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}\left(\int_0^L g(x)\sin\left(\frac{n\pi}{2L}x\right)dx\right)\sin\left(c\frac{n\pi}{2L}t\right)\sin\left(\frac{n\pi}{2L}x\right)$$

Or

$$u(x,t)=\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$
$$+\frac{4}{c\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

5.1.1.21 [353] Right end free, zero initial velocity (general case)

problem number 353

Added July 8, 2019

$$u_{tt}=c^2u_{xx}$$

With boundary conditions

$$u(0,t)=0$$
$$u_x(L,t)=0$$

With initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=0$$

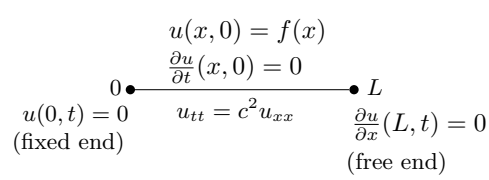

Figure 5.32: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \{\sum_{K[1]=1}^{\infty}\frac{\sqrt{2}\cos\left(\frac{1}{2}\pi t\sqrt{\frac{c^2(2K[1]-1)^2}{L^2}}\right)\left(\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}\quad K[1]\in\mathbb{Z}\wedge((L<\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{2\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi x}{2L}\right)dx\right)\cos\left(\frac{(2n+1)\pi ct}{2L}\right)\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{L}$$

<u>Hand solution</u>

Solving for $0<x<L$

$$u_{tt}=c^2u_{xx}\qquad 0<x<L,t>0$$

Boundary conditions, $t>0$

$$u(0,t)=0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$$

Initial conditions, $t=0$

$$u_t(x,0)=g(x)=0$$
$$u(x,0)=f(x)$$

The general PDE was solved in 5.1.1.20 on page 967 and the solution is

$$u(x,t)=\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$
$$+\frac{4}{c\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left(\int_0^L g(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\sin\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

But here $g(x)=0$, hence the above reduces to

$$u(x,t)=\frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx\right)\cos\left(c\frac{(2n+1)\pi}{2L}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

5.1.1.22 [354] Right end free, zero initial velocity (special case)

problem number 354

Added July 8, 2019

$$u_{tt}=c^2u_{xx}$$

With boundary conditions

$$u(0,t)=0$$
$$u_x(L,t)=0$$

With initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=0$$

Using the following values

$$c=4$$
$$L=3$$
$$h=\frac{1}{10}$$
$$f(x)=\begin{cases}\frac{3h}{L}x & 0<x<\frac{L}{3}\\ h & \frac{L}{3}<x<L\end{cases}$$

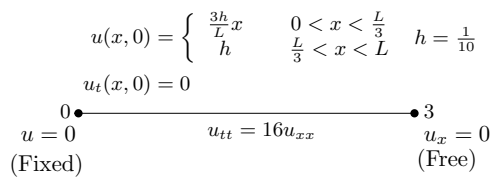

Figure 5.33: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \sum_{K[1]=1}^{\infty} -\frac{12\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{2}{3}\pi t\sqrt{(2K[1]-1)^2}\right)\sin\left(\frac{1}{6}\pi x(2K[1]-1)\right)}{5\pi^2(1-2K[1])^2} & K[1]\in\mathbb{Z}\wedge K[1]\geq 1 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{6\left(\cos\left(\frac{\pi n}{3}\right) + \sqrt{3}\sin\left(\frac{\pi n}{3}\right)\right)\cos\left(\frac{(4n+2)\pi t}{3}\right)\sin\left(\frac{(2n+1)\pi x}{6}\right)}{5\pi^2(2n+1)^2}$$

Hand solution

Solving the wave PDE on string

$$u_{tt} = c^2 u_{xx} \qquad t > 0, x > 0$$

Boundary conditions, $t > 0$

$$u(0,t) = 0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

Initial conditions, $t = 0$

$$u(x,0) = f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$
$$u_t(x,0) = 0$$

Using $c = 4, L = 3, h = \frac{1}{10}$. Hence $f(x) = \begin{cases} \frac{1}{10}x & 0 < x < 1 \\ \frac{1}{10} & 1 < x < 3 \end{cases}$

The general problem PDE was solved in 5.1.1.21 on page 971 and the solution is

$$u(x,t) = \frac{2}{L}\sum_{n=0}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)\cos\left(\frac{(2n+1)\pi}{2L}ct\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

Substituting the specific values given above into this solution gives

$$u(x,t) = \frac{2}{3}\sum_{n=0}^{\infty}\left(\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)\cos\left(4\frac{(2n+1)\pi}{6}t\right)\sin\left(\frac{(2n+1)\pi}{6}x\right)$$

But

$$\begin{aligned}\int_0^L f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx &= \int_0^{\frac{L}{3}} f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx + \int_{\frac{L}{3}}^L f(x)\sin\left(\frac{(2n+1)\pi}{6}x\right)dx \\ &= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx + \frac{1}{10}\int_1^3 \sin\left(\frac{(2n+1)\pi}{6}x\right)dx \\ &= \frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\end{aligned}$$

Hence the solution becomes

$$\begin{aligned}u(x,t) &= \frac{2}{3}\sum_{n=0}^{\infty}\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\cos\left(\frac{2}{3}(2n+1)\pi t\right)\sin\left(\frac{1}{6}(2n+1)\pi x\right) \\ &= \frac{36}{15\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)\cos\left(\frac{2}{3}(2n+1)\pi t\right)\sin\left(\frac{1}{6}(2n+1)\pi x\right)\end{aligned}$$

Animation is below

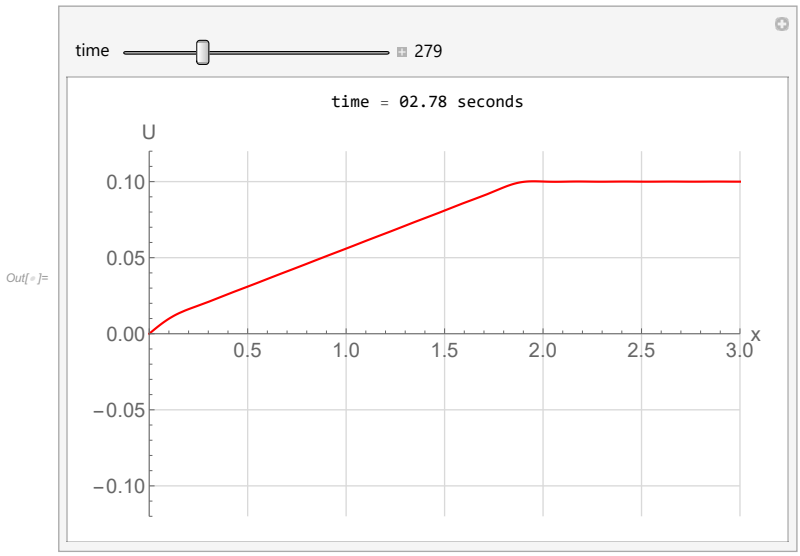

Figure 5.34: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, n, f, A, B, s, mySol]
c = 4;
L = 3;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 25;
mySol[x_, t_] = 36/(15 π^2) Sum[1/(2 n + 1)^2 Sin[(2 n + 1) π/6] Cos[2/3 (2 n + 1) π t] Sin[1/6 (2 n + 1) π x], {n, 0, numberOfTerms}];
```

Figure 5.35: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-0.12, 0.12}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 10, 0.01}];
In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.36: Code for animation

5.1.1.23 [355] Right end free, zero initial velocity, damping present (general case)

problem number 355

Added July 9, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

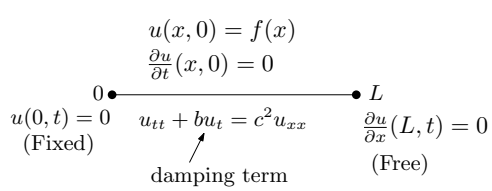

Figure 5.37: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions->{b
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\frac{\sqrt{2}e^{-\frac{bt}{2}}\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)\left(\cos\left(\frac{1}{2}t\sqrt{\frac{c^2\pi^2(2K[1]-1)^2}{L^2}-b^2}\right)\int_0^L\frac{\sqrt{2}f(x)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\sqrt{L}}dx+\frac{2\left(\int_0^L\frac{bf(x)\sin\left(\frac{\pi}{}\right.}{\sqrt{}}\right.}{}\right.}{\sqrt{L}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=0}^{\infty}\frac{\left(\left(Lb+\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi^2c^2}\right)e^{-\frac{\left(Lb-\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi^2c^2}\right)t}{2L}}-\left(Lb-\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi}\right.\right.}{\sqrt{L^2b^2-4\left(n+\frac{1}{2}\right)^2\pi}}$$

Hand solution

Solving for $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L, t>0$$

Boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ \left.\frac{\partial u}{\partial x}\right|_{x=L} &= 0 \end{aligned}$$

Initial conditions, $t = 0$

$$\begin{aligned} u_t(x,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

Separation of variables gives

$$\begin{aligned} T''X + bT'X &= c^2 X''T \\ \frac{1}{c^2}\left(\frac{T''}{T} + b\frac{T'}{T}\right) &= \frac{X''}{X} = -\lambda \end{aligned}$$

The eigenvalue ODE is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Only $\lambda > 0$ gives non-trivial solution from the nature of the boundary conditions. Hence solution is

$$X(x) = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

Since $X(0) = 0$ then the above gives $0 = A$ and the solution becomes

$$X(x) = B\sin\left(\sqrt{\lambda}x\right)$$

Taking derivatives

$$X'(x) = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}x\right)$$

Since $X'(L) = 0$ the above becomes

$$0 = \sqrt{\lambda}B\cos\left(\sqrt{\lambda}L\right)$$

Which implies $\sqrt{\lambda}L = \frac{n\pi}{2}$ for $n = 1, 3, 5, \cdots$ or

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1, 3, 5, \cdots$$

Hence the eigenfunctions are

$$\Phi_n(x) = \sin\left(\sqrt{\lambda_n}x\right) \qquad n = 1, 3, 5, \cdots$$

The time ODE now becomes

$$\begin{aligned}\frac{1}{c^2}\left(\frac{T''}{T} + b\frac{T'}{T}\right) &= -\lambda_n \\ \frac{T''}{T} + b\frac{T'}{T} &= -c^2\lambda_n \\ T'' + bT' + c^2\lambda_n T &= 0\end{aligned}$$

The characteristic equation $r^2 + br + c^2\lambda_n = 0$ has the roots $r = \frac{-B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC} \rightarrow r = \frac{-b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4c^2\lambda_n}$ or

$$r = \frac{-b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4c^2\lambda_n}$$

<u>Case</u> $b^2 < 4c^2\lambda_n$ for all n. This is called the underdamped case, which generates damped oscillations. The roots becomes

$$r = \frac{-b}{2} \pm \frac{1}{2}i\sqrt{4c^2\lambda_n - b^2}$$

Let $\beta_n = \frac{1}{2}\sqrt{4c^2\lambda_n - b^2}$, then

$$r = \frac{-b}{2} \pm i\beta_n$$

Hence the solution is

$$T_n(t) = e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))$$

Therefore the complete solution becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))\,\Phi_n(x) \tag{1}$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1,3,5,\cdots}^{\infty} D_n \Phi_n(x)$$

Applying orthogonality gives

$$\begin{aligned}\int_0^L f(x)\,\Phi_n(x)\,dx &= D_n \int_0^L \Phi_n^2(x)\,dx \\ &= \frac{L}{2}D_n\end{aligned}$$

Hence

$$D_n = \frac{2}{L}\int_0^L f(x)\,\Phi_n(x)\,dx \tag{2}$$

Taking time derivative of (1) gives

$$u_t(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{-b}{2} e^{\frac{-b}{2}t}(D_n\cos(\beta_n t) + E_n\sin(\beta_n t))\,\Phi_n(x) + e^{\frac{-b}{2}t}(-\beta_n D_n\sin(\beta_n t) + E_n\beta_n\cos(\beta_n t))\,\Phi_n$$

At $t=0$ since $g(x)=0$ then the above becomes

$$\begin{aligned}
0 &= \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{-b}{2}D_n + E_n\beta_n\right)\Phi_n(x)\\
0 &= \frac{-b}{2}D_n + E_n\beta_n\\
E_n &= \frac{bD_n}{2\beta_n} \qquad (3)
\end{aligned}$$

Using (2,3), the solution (1) now becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}\left(D_n\cos(\beta_n t) + \frac{bD_n}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)\\
&= \sum_{n=1,3,5,\cdots}^{\infty} D_n e^{\frac{-b}{2}t}\left(\cos(\beta_n t) + \frac{b}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)
\end{aligned}$$

Or

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right)e^{\frac{-b}{2}t}\left(\cos(\beta_n t) + \frac{b}{2\beta_n}\sin(\beta_n t)\right)\Phi_n(x)$$

But $\beta_n = \frac{1}{2}\sqrt{4c^2\lambda_n - b^2}$,$\lambda_n = \left(\frac{n\pi}{2L}\right)^2$ and $\Phi_n(x) = \sin\left(\frac{n\pi}{2L}x\right)$, hence the above becomes

$$\begin{aligned}
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2}\,t\right)\sin\left(\frac{n\pi}{2L}x\right)\\
&+ \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2}\,t\right)}{\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2 - b^2}}\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}$$

Or

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)$$

<u>Case</u> $b^2=4c^2\left(\frac{n\pi}{2L}\right)^2$. We see that for $n=1$ it becomes critical, because then $b=\frac{\pi c}{L}$ and now and the discriminant is zero in this case. This is called the critical damped case. For $n>1$, it becomes underdamped, which is the above case. So we only need to find solution for $n=1$. In this case, the solution to $T''+bT'+c^2\lambda_1 T=0$ is

$$T_1(t)=D_1e^{\frac{-b}{2}t}+E_1te^{\frac{-b}{2}t}$$

Therefore the complete solution becomes

$$u(x,t)=\left(D_1e^{\frac{-b}{2}t}+E_1te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)+\sum_{n=3,5,\cdots}^{\infty}\left(D_ne^{\frac{-b}{2}t}+E_nte^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)\quad(4)$$

For $n=1$, At $t=0$, from intial conditions (4) becomes

$$f(x)=D_1\sin\left(\frac{\pi}{2L}x\right)$$

By orthognailty the above gives

$$D_1=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\quad(5)$$

Taking time derivative of (4) for $n=1$, gives

$$u(x,t)=\left(\frac{-b}{2}D_1e^{\frac{-b}{2}t}+E_1\left(e^{\frac{-b}{2}t}-\frac{b}{2}te^{\frac{-b}{2}t}\right)\right)\sin\left(\frac{\pi}{2L}x\right)$$

At $t=0$ and since $g(x)=0$, then the above becomes

$$0=\left(\frac{-b}{2}D_1+E_1\right)\sin\left(\frac{\pi}{2L}x\right)$$

$$\frac{-b}{2}D_1+E_1=0$$

$$E_1=\frac{b}{2}D_1\quad(6)$$

Using (5,6) then (4) becomes

$$
\begin{aligned}
u(x,t) &= D_1\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)+\sum_{n=3,5,\cdots}^{\infty}\left(D_n e^{\frac{-b}{2}t}+E_n te^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)\\
&= \left(\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\right)\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)\\
&+\sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\sqrt{\lambda_n}s\right)ds\right)\left(e^{\frac{-b}{2}t}+\frac{bt}{2}e^{\frac{-b}{2}t}\right)\sin\left(\sqrt{\lambda_n}x\right)
\end{aligned}
$$

For the case of $n>1$, the solution was found in the above underdamped case. Putting all these together, gives the solution as

$$
\begin{aligned}
u(x,t) &= \left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{\pi}{2L}s\right)ds\right)\left(e^{\frac{-b}{2}t}+\frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right)\\
&+\sum_{n=2}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right.\\
&+\sum_{n=2}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

<u>Case</u> $b^2>4c^2\left(\frac{n\pi}{2L}\right)^2$. Will consider only the case when this is true for $n=1$ only. If this is true for larger n, then same solution needs to be summed for each mode. But for simplicity, will consider $n=1$ here. In this case, the roots are

$$
r=\frac{-b}{2}\pm\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}
$$

Where now $b^2-4c^2\lambda_1$ is positive. Hence we get $r_1=\frac{-b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}, r_2=\frac{-b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\lambda_1}$ or

$$
\begin{aligned}
r_1 &= \frac{-b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}\\
r_1 &= \frac{-b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}
\end{aligned}
$$

And the solution to $T_1''+bT_1'+c^2\lambda_1 T=0$ is

$$
T_1(t)=e^{\frac{-b}{2}t}\left(D_1e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}+E_1e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)
$$

For the rest of the modes, the solution is from above

$$T_n(t) = e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t)) \qquad n = 3, 5, 7, \cdots$$

Hence the complete solution becomes

$$\begin{aligned} u(x,t) &= e^{\frac{-b}{2}t}\left(D_1 e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t} + E_1 e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)\sin\left(\frac{\pi}{2L}x\right) \\ &+ \sum_{n=3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}(D_n \cos(\beta_n t) + E_n \sin(\beta_n t))\sin\left(\frac{n\pi}{2L}x\right) \end{aligned} \tag{7}$$

At $t = 0$ and for $n = 1$, the above becomes

$$f(x) = (D_1 + E_1)\sin\left(\frac{\pi}{2L}x\right)$$

Hence

$$(D_1 + E_1) = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx \tag{8}$$

Taking time derivative of (7) and for $n = 1$ at $t = 0$ it gives

$$0 = \left(\frac{-b}{2}(D_1 + E_1) + \left(\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}D_1 - E_1\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right)\right)\sin\left(\frac{\pi}{2L}x\right)$$

Hence

$$\begin{aligned} \frac{-b}{2}(D_1 + E_1) + \left(\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}D_1 - E_1\frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right) &= 0 \\ -E_1\left(\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right) &= \left(\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}\right)D_1 \\ E_1 &= \frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}D_1 \end{aligned} \tag{9}$$

From (8,9)

$$
\begin{aligned}
D_1 &= \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx - E_1\\
D_1 - \frac{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}D_1 &= \frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\\
D_1 &= \frac{\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx}{1-\frac{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}}\\
&= \frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx
\end{aligned}
$$

And therefore

$$
\begin{aligned}
E_1 &= \frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\left(\frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx\\
&= \frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\frac{2}{L}\int_0^L f(x)\sin\left(\frac{\pi}{2L}x\right)dx
\end{aligned}
$$

For $n>1$ the solution is the same as the underdamped case above. Hence the complete solution becomes from (7)

$$
\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(D_1 e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t} + E_1 e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\right)\sin\left(\frac{\pi}{2L}x\right)\\
&+ \sum_{n=3,5,\cdots}^{\infty} e^{\frac{-b}{2}t}\left(D_n\cos(\beta_n t) + E_n\sin(\beta_n t)\right)\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ \sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{n\pi}{2L}x\right)\\
&+ \sum_{n=3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{n\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{n\pi}{2L}\right)^2-b^2}}\sin\left(\frac{n\pi}{2L}x\right)
\end{aligned}
$$

Or

$$
\begin{aligned}
u(x,t) &= e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right)\\
&+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right.\\
&+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

5.1.1.24 [356] Right end free, zero initial velocity, damping present (special case, underdamped)

problem number 356

Added July 9, 2019

$$u_{tt}+bu_t=c^2u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ u_x(L,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,0) &= f(x) \\ u_t(x,0) &= 0 \end{aligned}$$

Using the following values

$$\begin{aligned} L &= 3 \\ c &= 4 \\ f(x) &= \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases} \\ h &= \frac{1}{10} \\ b &= \frac{1}{2}\frac{\pi c}{L} \end{aligned}$$

Hence $b = \frac{2\pi}{3}$

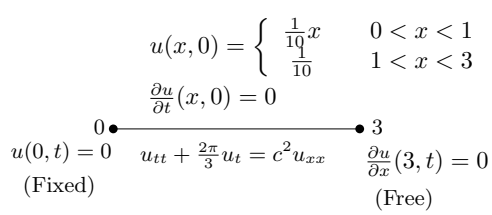

Figure 5.38: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=1/2*(Pi*c/L);
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\sqrt{\tfrac{2}{3}}e^{-\frac{\pi t}{3}}\sin\left(\tfrac{1}{6}\pi x(2K[1]-1)\right)\left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2-\frac{4\pi^2}{9}}\right)}{5\pi^2(1-2K[1])^2}-\frac{6}{}\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=1/2*(Pi*c/L);
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t)=\sum_{n=0}^{\infty}-\frac{3i\left(\cos\left(\frac{\pi n}{3}\right)+\sqrt{3}\sin\left(\frac{\pi n}{3}\right)\right)\left(\left(i\sqrt{16n^2+16n+3}+1\right)e^{\frac{\pi\left(i\sqrt{16n^2+16n+3}-1\right)t}{3}}+\left(i\sqrt{16n^2+16n\right.}{5\sqrt{16n^2+16n+3}\,\pi^2\left(2n+1\right)^2}$$

<u>Hand solution</u>

Solving the wave PDE on string underdamped case $t>0, 0<x<L$

$$u_{tt}+bu_t=c^2u_{xx}\qquad 0<x<L, t>0$$

Boundary conditions

$$
\begin{aligned}
u(0,t) &= 0\\
\left.\frac{\partial u}{\partial x}\right|_{x=L} &= 0
\end{aligned}
$$

Initial conditions, $t=0$

$$
\begin{aligned}
u_t(x,0) &= 0\\
u(x,0) &= f(x)
\end{aligned}
$$

Using

$$
\begin{aligned}
f(x) &= \begin{cases} \frac{3h}{L}x & 0<x<\frac{L}{3}\\ h & \frac{L}{3}<x<L \end{cases}\\
b &= \frac{2\pi}{3}\\
c &= 4\\
L &= 3
\end{aligned}
$$

Hence the PDE becomes $u_{tt}+\frac{2\pi}{3}u_t = 16u_{xx}$. The general solution to the above PDE was given in problem 5.1.1.23 on page 977. The eigenvalues are given as

$$
\lambda_n = \left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n=0,1,2,\cdots
$$

And the discriminant is $b^2-4c^2\lambda_n = b^2-4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 = b^2-(4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n=0$ this gives $b^2-\frac{16}{9}\pi^2$. But $b=\frac{2\pi}{3}$. Hence discriminant is $\left(\frac{2\pi}{3}\right)^2-\frac{16}{9}\pi^2 = -\frac{4}{3}\pi^2$. Since discriminant is negative, then this is underdamped wave with damped oscillations as the solution given from the above problem as

$$
\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)\\
&+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2-b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}
$$

Replacing given values in the above solution results in

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{3}\int_{0}^{3}f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-\pi}{3}t}\left(\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}t\right)\right)\sin\left(\frac{(2n+1)}{6}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{3}\int_{0}^{3}f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-\pi}{3}t}\left(\frac{2\pi}{3}\frac{\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}}\right)\sin\left(\frac{(2n+1)\pi}{6}x\right. \tag{1}$$

But

$$\begin{aligned}\int_{0}^{3}f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)&=\frac{1}{10}\int_{0}^{1}x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx+\frac{1}{10}\int_{1}^{3}\sin\left(\frac{(2n+1)\pi}{6}x\right)dx\\&=\frac{18}{5\pi^{2}(2n+1)^{2}}\sin\left(\frac{(2n+1)\pi}{6}\right)\end{aligned}$$

Hence the solution (1) becomes

$$u(x,t)=\sum_{n=0}^{\infty}\left(\frac{2}{3}\frac{18}{5\pi^{2}(2n+1)^{2}}\sin\left(\frac{(2n+1)\pi}{6}\right)\right)e^{\frac{-\pi}{3}t}\left(\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}t\right)\right)\sin\left(\frac{(2n+1}{6}\right.$$

$$+\sum_{n=0}^{\infty}\left(\frac{2}{3}\frac{18}{5\pi^{2}(2n+1)^{2}}\sin\left(\frac{(2n+1)\pi}{6}\right)\right)e^{\frac{-\pi}{3}t}\left(\frac{2\pi}{3}\frac{\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^{2}-\left(\frac{2\pi}{3}\right)^{2}}}\right)\sin\left(\frac{(2n+1)\pi}{6}\right. \tag{2}$$

Animation is below

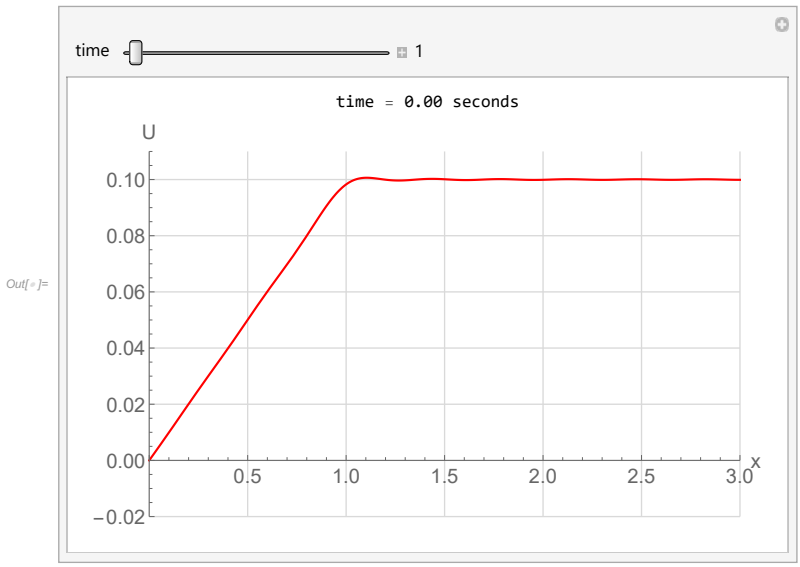

Figure 5.39: Initial state

Source code used for the above

```
ClearAll[x, t, n, f, A, B, s, mySol]
c = 4;
L = 3;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 16;
z = 2/3 18/(5 π^2 (2 n + 1)^2) Sin[(2 n + 1) π/6];
mySol[x_, t_] = Sum[z Exp[-π t/3] Sin[(2 n + 1) π/6 x] (Cos[1/2 Sqrt[64 ((2 n + 1) π/6)^2 - (2 π/3)^2] t]), {n, 0, numberOfTerms}]
+ Sum[z Exp[-π t/3] Sin[(2 n + 1) π/6 x] (2 π/3 Sin[1/2 Sqrt[64 ((2 n+1) π/6)^2 - (2 π/3)^2] t] / Sqrt[64 ((2 n+1) π/6)^2 - (2 π/3)^2]), {n, 0, numberOfTerms}];
```

Figure 5.40: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-0.02, 0.11}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 4, 0.01}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.41: Code for animation

5.1.1.25 [357] Right end free, zero initial velocity, damping present (special case, critical damped)

problem number 357

Added July 10, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Using the following values

$$
\begin{aligned}
L &= 3\\
c &= 4\\
f(x) &= \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3}\\ h & \frac{L}{3} < x < L \end{cases}\\
h &= \frac{1}{10}\\
b &= \frac{\pi c}{L}
\end{aligned}
$$

Hence $b = \frac{4\pi}{3}$

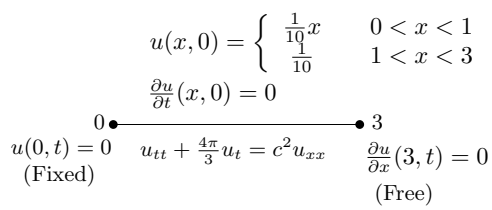

Figure 5.42: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=Pi*c/L;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \{ \quad \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{3}} e^{-\frac{2\pi t}{3}} \sin\left(\frac{1}{6}\pi x(2K[1]-1)\right) \left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2-\frac{16\pi^2}{9}}\right)}{5\pi^2(1-2K[1])^2} - \right.\right.
$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=Pi*c/L;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t) = \sum_{n=0}^{\infty} \begin{cases} \frac{4}{5\pi^2} e^{-\frac{2\pi t}{3}} \sin\left(\frac{\pi x}{6}\right)\left(\pi t + \frac{3}{2}\right) \\ \frac{3}{10\pi^2(1+2n)^2}\left(\sqrt{3}\sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{3}\right)\right)\sin\left(\frac{(1+2n)\pi x}{6}\right)\left(\left(2\sqrt{n}\sqrt{n+1}+i\right)e^{\frac{2i}{3}\pi\left(-2\sqrt{n}\sqrt{n+1}+i\right)t}\right. \end{cases}$$

<u>Hand solution</u>

Solving the wave PDE on string underdamped case $t > 0, 0 < x < L$

$$u_{tt} + bu_t = c^2 u_{xx} \qquad 0 < x < L, t > 0$$

Boundary conditions

$$\begin{aligned} u(0,t) &= 0 \\ \left.\frac{\partial u}{\partial x}\right|_{x=L} &= 0 \end{aligned}$$

Initial conditions, $t = 0$

$$\begin{aligned} u_t(x,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

Using

$$f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$

$$b = \frac{4\pi}{3}$$

$$c = 4$$

$$L = 3$$

$$h = \frac{1}{10}$$

Hence the PDE becomes $u_{tt} + \frac{4\pi}{3}u_t = 16u_{xx}$. The general solution to the above PDE was given in problem 5.1.1.23 on page 977. The eigenvalues are given as

$$\lambda_n = \left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n = 0, 1, 2, \cdots$$

And the discriminant is $b^2 - 4c^2\lambda_n = b^2 - 4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 = b^2 - (4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n = 0$ this gives $b^2 - \left(\frac{4}{3}\pi\right)^2$. But $b = \frac{4\pi}{3}$. Hence discriminant is zero for $n = 0$. This means this is critically damped in first mode. Using the the solution for this case from the above general solution as

$$\begin{aligned} u(x,t) = &\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{\pi}{2L}s\right)ds\right)\left(e^{\frac{-b}{2}t} + \frac{b}{2}te^{\frac{-b}{2}t}\right)\sin\left(\frac{\pi}{2L}x\right) \\ &+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right. \\ &+ \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right) \end{aligned}$$

Replacing given values in the above solution results in

$$
\begin{aligned}
u(x,t) = & \left(\frac{2}{3}\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx\right)\left(e^{\frac{-2\pi t}{3}}+\frac{b}{2}te^{\frac{-2\pi t}{3}}\right)\sin\left(\frac{\pi}{6}x\right) \qquad (1)\\
& +\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-2\pi t}{3}}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)\sin\left(\frac{(2n}{}\right.\\
& +\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{\frac{-b}{2}t}\frac{\left(\frac{4\pi}{3}\right)\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}}\sin\left(\frac{(2n+1}{6}\right.
\end{aligned}
$$

But

$$
\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx = \frac{9}{5\pi^2}
$$

And

$$
\begin{aligned}
\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right) &= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx+\frac{1}{10}\int_1^3\sin\left(\frac{(2n+1)\pi}{6}x\right)dx\\
&= \frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)
\end{aligned}
$$

Hence the solution (1) becomes

$$
\begin{aligned}
u(x,t) = & \frac{6}{5\pi^2}\left(e^{\frac{-2\pi t}{3}}+\frac{2}{3}\pi te^{\frac{-2\pi t}{3}}\right)\sin\left(\frac{\pi}{6}x\right) \qquad (1)\\
& +\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{\frac{-2\pi t}{3}}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)\sin\left(\frac{(2n+1}{6}\right.\\
& +\sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{\frac{-2\pi t}{3}}\frac{\left(\frac{4\pi}{3}\right)\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}t\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-\left(\frac{4\pi}{3}\right)^2}}\sin\left(\frac{(2n+1)\pi}{6}\right.
\end{aligned}
$$

Animation is below

Out[ ]=

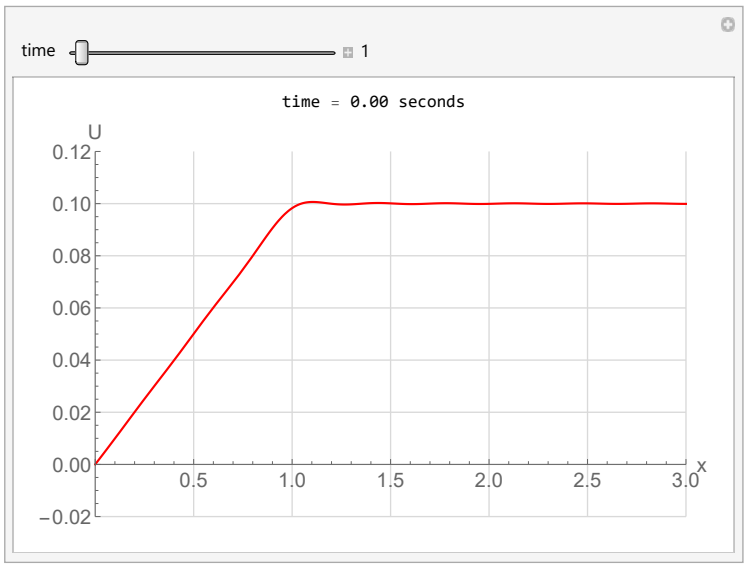

Figure 5.43: Initial state

Source code used for the above

In[ ]:=
```
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 50;
z = 12/(5 π^2 (2 n + 1)^2) Sin[(2 n + 1) π/6];
mySol[x_, t_] = 6/(5 π^2) (Exp[(-2 π t)/3] + 2/3 π t Exp[(-2 π t)/3]) Sin[π/6 x] + Sum[z Exp[(-2 π t)/3] Sin[(2 n + 1) π/6 x] (Cos[1/2 Sqrt[64 ((2 n + 1) π/6)^2 - (4 π/3)^2] t]), {n, 1, numberOfTerms}] +
    Sum[z Exp[(-2 π t)/3] Sin[(2 n + 1) π/6 x] (4 π/3 Sin[1/2 Sqrt[64 ((2 n+1) π/6)^2 - (4 π/3)^2] t] / Sqrt[64 ((2 n+1) π/6)^2 - (4 π/3)^2]), {n, 1, numberOfTerms}];
```

Figure 5.44: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, L}, {-0.01, 0.11}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]

              }}],
          {t, 0, 2.9, 0.01}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.45: Code for animation

5.1.1.26 [358] Right end free, zero initial velocity, damping present (special case, over damped)

problem number 358

Added July 11, 2019

$$u_{tt} + bu_t = c^2 u_{xx}$$

For $t > 0$ and $0 < x < L$ and boundary conditions

$$u(0,t) = 0$$
$$u_x(L,t) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

Using the following values

$$
\begin{aligned}
L &= 3\\
c &= 4\\
f(x) &= \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3}\\ h & \frac{L}{3} < x < L \end{cases}\\
h &= \frac{1}{10}\\
b &= \frac{3}{2}\frac{\pi c}{L}
\end{aligned}
$$

Hence $b = 2\pi$

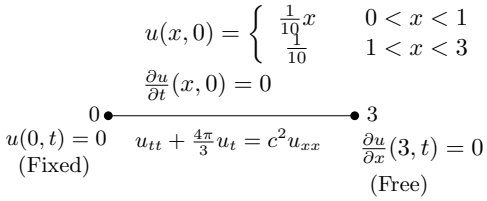

Figure 5.46: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=3; c=4; h=1/10; b=3/2*Pi*c/L;
f=Piecewise[{{3*h/L*x,0<x<L/3},{h,L/3<x<L}}];
pde =  D[u[x, t], {t, 2}] + b*D[u[x,t],t] == c^2*D[u[x, t], {x, 2}];
bc  = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic  = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \{ \quad \sum_{K[1]=1}^{\infty} \sqrt{\tfrac{2}{3}} e^{-\pi t} \sin\left(\tfrac{1}{6}\pi x(2K[1]-1)\right) \left(-\frac{6\sqrt{6}\cos\left(\frac{1}{3}\pi(K[1]+1)\right)\cos\left(\frac{1}{2}t\sqrt{\frac{16}{9}\pi^2(2K[1]-1)^2-4\pi^2}\right)}{5\pi^2(1-2K[1])^2} - \frac{18\sqrt{}}{}\right.\right.\right.
$$

Indeterminate

Maple ✓

```
restart;
L:=3;
c:=4;
h:=1/10;
b:=3/2*Pi*c/L;
f:=piecewise(0<x and x<L/3,3*h/L*x,L/3<x and x<L,h);
pde := diff(u(x,t),t$2) + b*diff(u(x,t),t) = c^2*diff(u(x,t),x$2);
bc  := u(0,t)=0,D[1](u)(L,t)=0;
ic  := D[2](u)(x,0)=0,u(x,0)=f;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) ),outp
```

$$u(x,t) = \sum_{n=0}^{\infty} \frac{3\left(\cos\left(\frac{\pi n}{3}\right) + \sqrt{3}\sin\left(\frac{\pi n}{3}\right)\right)\left(\left(\sqrt{-16n^2-16n+5}+3\right)\mathrm{e}^{\frac{\pi\left(\sqrt{-16n^2-16n+5}-3\right)t}{3}} + \left(\sqrt{-16n^2-16n}\right.\right.}{5\sqrt{-16n^2-16n+5}\,\pi^2\left(2n+1\right)^2}$$

<u>Hand solution</u>

Solving the wave PDE on string underdamped case $t > 0, 0 < x < L$

$$u_{tt} + bu_t = c^2 u_{xx} \qquad 0 < x < L, t > 0$$

Boundary conditions

$$u(0,t) = 0$$
$$\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

Initial conditions, $t = 0$

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

Using

$$f(x) = \begin{cases} \frac{3h}{L}x & 0 < x < \frac{L}{3} \\ h & \frac{L}{3} < x < L \end{cases}$$
$$b = 2\pi$$
$$c = 4$$
$$L = 3$$
$$h = \frac{1}{10}$$

Hence the PDE becomes $u_{tt} + 2\pi u_t = 16u_{xx}$. The general solution to the above PDE was given in problem 5.1.1.23 on page 977. The eigenvalues are given as

$$\lambda_n = \left(\frac{(2n+1)\pi}{2L}\right)^2 \qquad n = 0, 1, 2, \cdots$$

And the discriminant is $b^2 - 4c^2\lambda_n = b^2 - 4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 = b^2 - (4)(16)\left(\frac{(2n+1)^2\pi^2}{36}\right)$. For $n = 0$ this gives $b^2 - \left(\frac{4}{3}\pi\right)^2$. But $b = 2\pi$. Hence discriminant is positive for $n = 0$. This means this is critically damped in first mode. Using the the solution for this case from the above general solution as

$$\begin{aligned}
u(x,t) = {} & e^{\frac{-b}{2}t}\left(\frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right) \\
& + e^{\frac{-b}{2}t}\left(\frac{-\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}{\sqrt{b^2 - 4c^2\left(\frac{\pi}{2L}\right)^2}}\right)\frac{2}{L}\left(\int_0^L f(s)\sin\left(\sqrt{\frac{\pi}{2L}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{b^2-4c^2\left(\frac{\pi}{2L}\right)^2}t}\sin\left(\frac{\pi}{2L}x\right) \\
& + \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\cos\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)\sin\left(\frac{(2n+1)}{2L}\right. \\
& + \sum_{n=1}^{\infty}\left(\frac{2}{L}\int_0^L f(s)\sin\left(\frac{(2n+1)\pi}{2L}s\right)ds\right)e^{\frac{-b}{2}t}\frac{b\sin\left(\frac{1}{2}\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}t\right)}{\sqrt{4c^2\left(\frac{(2n+1)\pi}{2L}\right)^2 - b^2}}\sin\left(\frac{(2n+1)\pi}{2L}x\right)
\end{aligned}$$

Replacing given values in the above solution results in

$$
\begin{aligned}
u(x,t) &= e^{-\pi t}\left(\frac{\pi+\frac{1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}}{\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}}\right)\frac{2}{3}\left(\int_0^3 f(s)\sin\left(\sqrt{\frac{\pi}{6}}s\right)ds\right)e^{\frac{1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right) \qquad (1)\\
&+ e^{-\pi t}\left(\frac{-\pi+\frac{1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}}{\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}}\right)\frac{2}{3}\left(\int_0^3 f(s)\sin\left(\sqrt{\frac{\pi}{6}}s\right)ds\right)e^{\frac{-1}{2}\sqrt{4\pi^2-64\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right)\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{-\pi t}\cos\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-4\pi^2 t}\right)\sin\left(\frac{(2n+1)}{6}\right.\\
&+\sum_{n=1}^{\infty}\left(\frac{2}{3}\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right)ds\right)e^{-\pi t}\frac{2\pi\sin\left(\frac{1}{2}\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-4\pi^2 t}\right)}{\sqrt{64\left(\frac{(2n+1)\pi}{6}\right)^2-4\pi^2}}\sin\left(\frac{(2n+1)\pi}{6}x\right.
\end{aligned}
$$

But

$$
\int_0^3 f(x)\sin\left(\frac{\pi}{6}x\right)dx=\frac{9}{5\pi^2}
$$

And

$$
\begin{aligned}
\int_0^3 f(s)\sin\left(\frac{(2n+1)\pi}{6}s\right) &= \frac{1}{10}\int_0^1 x\sin\left(\frac{(2n+1)\pi}{6}x\right)dx+\frac{1}{10}\int_1^3\sin\left(\frac{(2n+1)\pi}{6}x\right)dx\\
&=\frac{18}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)
\end{aligned}
$$

Hence the solution (1) becomes

$$
\begin{aligned}
u(x,t) = {} & e^{-\pi t}\left(\frac{\pi+\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}{2\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}\right)\left(\frac{6}{5\pi^2}\right)e^{\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right) \\
& + e^{-\pi t}\left(\frac{-\pi+\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}{2\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}}\right)\left(\frac{6}{5\pi^2}\right)e^{-\sqrt{\pi^2-16\left(\frac{\pi}{6}\right)^2}t}\sin\left(\frac{\pi}{6}x\right) \\
& + \sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{-\pi t}\cos\left(\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2 t}\right)\sin\left(\frac{(2n+1)\pi}{6}x\right) \\
& + \sum_{n=1}^{\infty}\frac{12}{5\pi^2(2n+1)^2}\sin\left(\frac{(2n+1)\pi}{6}\right)e^{-\pi t}\frac{\pi\sin\left(\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2 t}\right)}{\sqrt{16\left(\frac{(2n+1)\pi}{6}\right)^2-\pi^2}}\sin\left(\frac{(2n+1)\pi}{6}x\right)
\end{aligned}
\tag{2}
$$

Animation is below

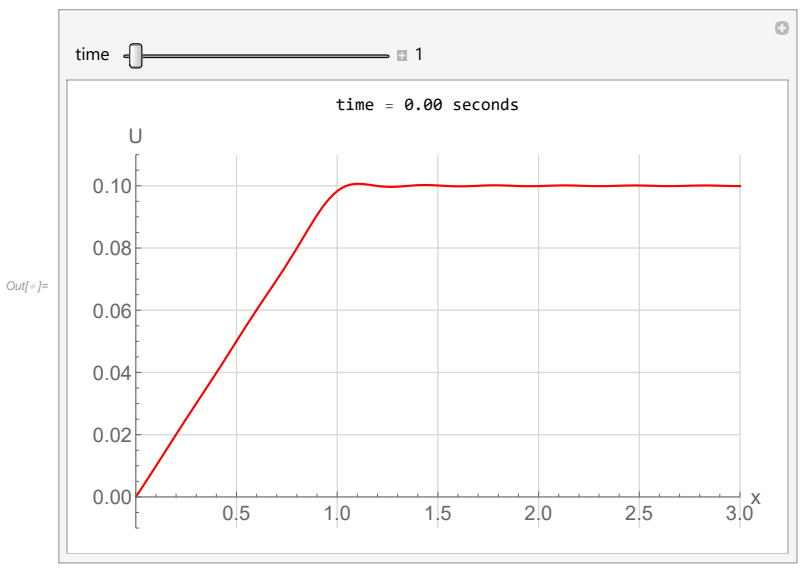

Figure 5.47: Initial state

Source code used for the above

```
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
numberOfTerms = 16;
z = 12/(5 π^2 (2 n + 1)^2) Sin[(2 n + 1) π/6];
mySol[x_, t_] = Exp[-π t] ((π + Sqrt[π^2 - 16 (π/6)^2])/(2 Sqrt[π^2 - 16 (π/6)^2])) 6/(5 π^2) Exp[Sqrt[π^2 - 16 (π/6)^2] t] Sin[π/6 x] + Exp[-π t] ((-π + Sqrt[π^2 - 16 (π/6)^2])/(2 Sqrt[π^2 - 16 (π/6)^2])) 6/(5 π^2) Exp[-Sqrt[π^2 - 16 (π/6)^2] t] Sin[π/6 x] +
    Sum[z Exp[- π t] Cos[Sqrt[16 ((2 n + 1) π/6)^2 - π^2] t] Sin[(2 n + 1) π/6 x], {n, 1, numberOfTerms}] +
    Sum[z Exp[- π t] (π Sin[Sqrt[16 ((2 n+1) π/6)^2 - π^2] t])/Sqrt[16 ((2 n+1) π/6)^2 - π^2] Sin[(2 n + 1) π/6 x], {n, 1, numberOfTerms}];
```

Figure 5.48: Source code

```
tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
          {
            Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                BaseStyle → 15,
                ImageMargins → 3,
                PerformanceGoal → "Quality",
                PlotRange → {{0, L}, {-0.01, 0.11}},
                ImageSize → 500,
                AxesLabel → {"x", "U"},
                GridLines → Automatic,
                GridLinesStyle → LightGray,
                PlotStyle → Red
              ]

          }}],
      {t, 0, 4, 0.01}];
Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.49: Code for animation

5.1.1.27 [359] I.C. at different times, right end free, with source

problem number 359

Added July 2, 2018. This is Example 2 (pde 10) taken from Maple document What_is_New_after_Maple_2018.pdf

Solve

$$-u_{tt} + u(x,t) = u_{xx} + 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)$$

With boundary condition

$$u(0,t) = 0$$
$$\frac{\partial u(1,t)}{\partial x} = 0$$

And initial conditions

$$u(x,0) = x^2 - 2x$$
$$u(x,1) = u(x,\frac{1}{2}) + e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}}$$

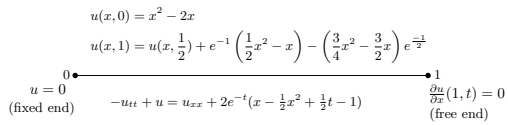

Figure 5.50: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  -D[u[x, t], {t, 2}] + u[x, t] == D[u[x, t], {x, 2}] + 2*Exp[-t]*(x - (1/2)*x^2 + (1/2
bc  = {u[0, t] == 0, Derivative[1, 0][u][1, t] == 0};
ic  = {u[x, 0] == x^2 - 2*x, u[x, 1] == u[x, 1/2] + ((1/2)*x^2 - x)*Exp[-1] - ((3*x^2)/4 - (
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := -diff(u(x, t), t, t) + u(x, t) = diff(u(x, t), x, x)+ 2*exp(-t)*(x-(1/2)*x^2+(1/2)*t-
ic  := u(x, 0) = x^2-2*x,
       u(x, 1) = u(x, 1/2)+((1/2)*x^2-x)*exp(-1)-(3/4*(x^2)-3/2*x)*exp(-1/2);
bc  := u(0, t) = 0, eval(diff(u(x, t), x), {x = 1}) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = -\frac{(x-2)(t-2)x\,\mathrm{e}^{-t}}{2}$$

Hand solution

Solve

$$-u_{tt}+u=u_{xx}+2e^{-t}\left(x-\frac{1}{2}x^2+\frac{1}{2}t-1\right) \qquad t>0, 0<x<1 \tag{1}$$

Boundary conditions

$$\begin{aligned} u(0,t)&=0\\ \left.\frac{\partial u}{\partial x}\right|_{x=1}&=0 \end{aligned}$$

Initial conditions, $t=0$

$$\begin{aligned} u(x,0)&=x^2-2x\\ u(x,1)&=u\left(x,\frac{1}{2}\right)+e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Since boundary conditions are homogeneous, we can directly use eigenfunction expansion method. Let the solution be

$$u(x,t)=\sum_{n=1}^{\infty}c_n(t)\,\Phi_n(x) \tag{2}$$

Where $\Phi_n(x)$ are the eigenfunctions of the corresponding homogeneous PDE $-u_{tt}+u=u_{xx}$. Using separation of variables, Let $u=X(x)\,T(t)$. Substituting this back in $-u_{tt}+u=u_{xx}$ gives

$$\begin{aligned} -T''X+XT&=X''T\\ -\frac{T''}{T}+1&=\frac{X''}{X}=-\lambda \end{aligned}$$

The eigenvalue ODE is

$$\begin{aligned} X''+\lambda X&=0\\ X(0)&=0\\ X'(1)&=0 \end{aligned}$$

This is known to have the eigenvalues are $\lambda_n=\left(\frac{n\pi}{2L}\right)^2=\left(\frac{n\pi}{2}\right)^2$, since $L=1$. This is for $n=1,3,5,\cdots$ and the eigenfunctions are $\Phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right)=\sin\left(\frac{n\pi}{2}x\right)$. Therefore the solution (2) is

$$\begin{aligned} u(x,t)&=\sum_{n=1,3,5,\cdots}^{\infty}c_n(t)\sin\left(\frac{n\pi}{2}x\right)\\ &=\sum_{n=1,3,5,\cdots}^{\infty}c_n(t)\,\Phi_n(x) \end{aligned}$$

Substituting this back into (1) gives

$$-\sum_{n=1,3,5,\cdots}^{\infty} c_n''(t)\,\Phi_n(x)+\sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n(x)=\sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n''(x)+\sum_{n=1,3,5,\cdots}^{\infty} b_n(t)\,\Phi_n(x) \tag{3}$$

Where $\sum_{n=1,3,5,\cdots}^{\infty} b_n(t)\,\Phi(x)=2e^{-t}\left(x-\frac{1}{2}x^2+\frac{1}{2}t-1\right)$. By orthogonality this becomes

$$\int_0^1 2e^{-t}\left(x-\frac{1}{2}x^2+\frac{1}{2}t-1\right)\Phi_n(x)\,dx=b_n(t)\int_0^1 \Phi_n^2(x)\,dx$$

$$2e^{-t}\int_0^1\left(x-\frac{1}{2}x^2+\frac{1}{2}t-1\right)\sin\left(\frac{n\pi}{2}x\right)dx=\frac{1}{2}b_n(t)$$

But $2e^{-t}\int_0^1\left(x-\frac{1}{2}x^2+\frac{1}{2}t-1\right)\sin\left(\frac{n\pi}{2}x\right)dx=\frac{2e^{-t}(8+(t-2)n^2\pi^2)}{n^3\pi^3}$. Hence the above gives

$$b_n(t)=\frac{4e^{-t}(8+(t-2)\,n^2\pi^2)}{n^3\pi^3}$$

Substituting the above into (3) gives

$$-\sum_{n=1,3,5,\cdots}^{\infty} c_n''(t)\,\Phi_n(x)+\sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n(x)=\sum_{n=1,3,5,\cdots}^{\infty} c_n(t)\,\Phi_n''(x)+\sum_{n=1,3,5,\cdots}^{\infty}\frac{4e^{-t}(8+(t-2)\,n^2\pi^2)}{n^3\pi^3}\Phi_n(x)$$

But $\Phi_n''(x)=-\lambda_n\Phi_n(x)$, hence the above simplifies to

$$\begin{aligned}
-c_n''(t)+c_n(t)&=-\lambda_n c_n(t)+\frac{4e^{-t}(8+(t-2)\,n^2\pi^2)}{n^3\pi^3}\\
-c_n''(t)+(1+\lambda_n)\,c_n(t)&=\frac{4e^{-t}(8+(t-2)\,n^2\pi^2)}{n^3\pi^3}\\
c_n''(t)-\left(1+\frac{n^2\pi^2}{4}\right)c_n(t)&=-\frac{4e^{-t}(8+(t-2)\,n^2\pi^2)}{n^3\pi^3}
\end{aligned}$$

The solution to this second order ODE can be found to be

$$c_n(t)=A_n e^{\frac{-\sqrt{n^2\pi^2+4}t}{2}}+B_n e^{\frac{\sqrt{n^2\pi^2+4}t}{2}}+\frac{16(t-2)\,e^{-t}}{n^3\pi^3}$$

Hence (2) becomes

$$u(x,t)=\sum_{n=1,3,5,\cdots}^{\infty}\left(A_n e^{\frac{-\sqrt{n^2\pi^2+4}t}{2}}+B_n e^{\frac{\sqrt{n^2\pi^2+4}t}{2}}+\frac{16(t-2)\,e^{-t}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right) \tag{4}$$

At $t=0$ the above becomes

$$x^2-2x=\sum_{n=1,3,5,\cdots}^{\infty}\left(A_n+B_n-\frac{32}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

Applying orthogonality gives

$$\int_0^1 (x^2-2x)\sin\left(\frac{n\pi}{2}x\right)dx = \left(A_n + B_n - \frac{32}{n^3\pi^3}\right)\frac{1}{2}$$

But $\int_0^1 (x^2-2x)\sin\left(\frac{n\pi}{2}x\right)dx = -\frac{16}{n^3\pi^3}$, hence the above gives

$$\begin{aligned} -\frac{32}{n^3\pi^3} &= A_n + B_n - \frac{32}{n^3\pi^3} \\ A_n &= -B_n \end{aligned} \tag{5}$$

Therefore the solution (4) now becomes

$$u(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}t}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}t}{2}} + \frac{16(t-2)\,e^{-t}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right) \tag{6}$$

The second initial conditions is $u(x,1) = u\left(x,\frac{1}{2}\right) + e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}}$. At $t=1$ the above gives

$$u(x,1) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}}{2}} - \frac{16e^{-1}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

At $t=\frac{1}{2}$ Eq (6) gives

$$u\left(x,\frac{1}{2}\right) = \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{4}} - e^{\frac{\sqrt{n^2\pi^2+4}}{4}} - \frac{24e^{-\frac{1}{2}}}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{2}x\right)$$

Hence the second initial conditions implies

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{2}} - e^{\frac{\sqrt{n^2\pi^2+4}}{2}} - \frac{16}{n^3\pi^3}e^{-1}\right)\sin\left(\frac{n\pi}{2}x\right) \\ &\quad - \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(e^{\frac{-\sqrt{n^2\pi^2+4}}{4}} - e^{\frac{\sqrt{n^2\pi^2+4}}{4}} - \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Or

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right) - \frac{16}{n^3\pi^3}e^{-1}\right)\sin\left(\frac{n\pi}{2}x\right) \\ &\quad - \sum_{n=1,3,5,\cdots}^{\infty} A_n\left(-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right) - \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Simplifying gives

$$\begin{aligned} &\sum_{n=1,3,5,\cdots}^{\infty} A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right) - 2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right) - \frac{16}{n^3\pi^3}e^{-1} + \frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right) = \\ &\qquad e^{-1}\left(\frac{1}{2}x^2 - x\right) - \left(\frac{3}{4}x^2 - \frac{3}{2}x\right)e^{\frac{-1}{2}} \end{aligned}$$

Applying orthogonality gives

$$A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)-\frac{16}{n^3\pi^3}e^{-1}+\frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)\frac{1}{2}=\int_0^1\left(e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right)dx$$

But $\int_0^1\left(e^{-1}\left(\frac{1}{2}x^2-x\right)-\left(\frac{3}{4}x^2-\frac{3}{2}x\right)e^{\frac{-1}{2}}\right)\sin\left(\frac{n\pi}{2}x\right)dx=-\frac{8e^{-\frac{1}{2}}-12e^{-1}}{n^3\pi^3}$, hence the above becomes

$$A_n\left(2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-2\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)-\frac{16}{n^3\pi^3}e^{-1}+\frac{24}{n^3\pi^3}e^{-\frac{1}{2}}\right)=-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}$$

$$2A_n\left(\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)\right)+A_n\left(-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}\right)=-\frac{16e^{-1}}{n^3\pi^3}+\frac{24e^{-\frac{1}{2}}}{n^3\pi^3}$$

Since this is true for all $n=1,3,5,\cdots$ then

$$A_n\left(\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)\right)=0$$

$$A_n=1$$

Which implies $\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{4}\right)-\sinh\left(\frac{\sqrt{n^2\pi^2+4}}{2}\right)=0$ but this is not possible for $n=1,3,5,\cdots$. Something went wrong. I need to look at this again.

5.1.1.28 [360] Right end oscillates

problem number 360

Added December 20, 2018.

Left end fixed, right end oscillates, initially at rest. With source that depends on time and space.

Example 19, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $0<x<\pi$ and $t>0$

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}+(1+t)x$$

With boundary conditions

$$u(0,t)=0$$
$$u(\pi,0)=\sin(t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

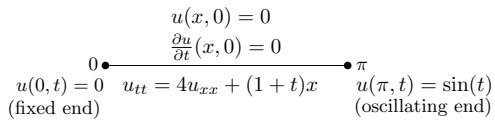

Figure 5.51: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}] + (1 + t)*x;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[0, t] == 0, u[Pi, t] == Sin[t]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \{ \quad \frac{x\sin(t)}{\pi} + \sum_{K[1]=1}^{\infty} \sqrt{\frac{2}{\pi}} \sin(xK[1]) \left(\frac{(-1)^{K[1]}(-8\sin(t)K[1]^3+2\pi(-t+\cos(2tK[1])-1)(4K[1]^2-1)K[1]+(4(1+}{4\sqrt{2\pi}K[1]^4(4K[1]^2-1)} \right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = 4*diff(u(x, t), x$2)+(1+t)*x;
bc  := u(0,t)=0,u(Pi,t)=sin(t);
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \frac{x\sin(t)}{\pi} + \left(\sum_{n=1}^{\infty} \frac{4\left(-\frac{(n^2\sin(t)+(-\pi n^2+\frac{1}{4}\pi)\cos(2nt)+(n-\frac{1}{2})\pi(n+\frac{1}{2})(t+1))n}{2} + (n^4+\frac{1}{4}\pi n^2-\frac{1}{16}\pi)\sin(2nt)\right)}{\pi(4n^2-1)n^4} \right.$$

5.1.1.29 [361] Perioidic B.C.

problem number 361

Added May 26, 2019.

Taken from midterm 2 sample exam. UMN Math 5587, Fall 2016. Problem 8

Solve for $u(x,t)$ with $-\pi < x < \pi$ and $t > 0$

$$u_{tt} = u_{xx}$$

With boundary conditions

$$u(-\pi,t) = u(\pi,t)$$
$$u_x(-\pi,0) = u_x(\pi,t)$$

With initial conditions

$$u(x,0) = x$$
$$u_t(x,0) = 0$$

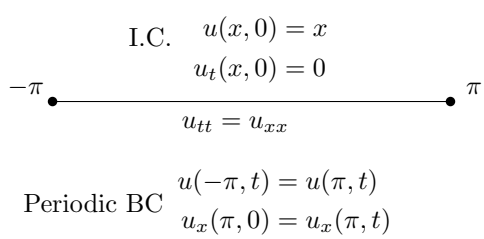

Figure 5.52: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] ;
ic  = {u[x, 0] == x, Derivative[0, 1][u][x, 0] == 0};
bc  = {u[-Pi, t] == u[Pi,t], Derivative[1, 0][u][-Pi, t]  ==Derivative[1, 0][u][Pi, t] };
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
bc  := u(-Pi,t)=u(Pi,t),eval(diff(u(x,t),x),x=-Pi)=eval(diff(u(x,t),x),x=Pi);
ic  := u(x,0)=x,eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

sol=()

5.1.1.30 [362] Mixed B.C.

problem number 362

Added May 26, 2019.

Taken from midterm 2 sample exam. UMN Math 5587, Fall 2016. Problem 10

Solve for $u(x,t)$ with $0 < x < \pi$ and $t > 0$

$$u_{tt} = u_{xx}$$

With boundary conditions $u(0,t) = u_t(\pi,t)$ and initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 1$$

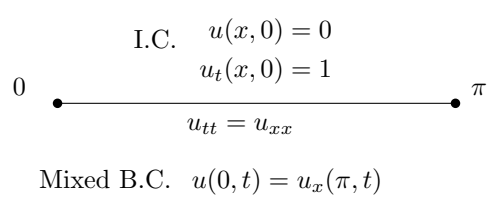

Figure 5.53: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] ;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 1};
bc  = u[0, t] == Derivative[1, 0][u][Pi, t] ;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
bc  := u(0,t)=eval(diff(u(x,t),x),x=Pi);
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

sol=()

5.1.1.31 [363] Left end fixed, right end non-homogeneous Neumann BC. Zero initial conditions

problem number 363

Added January 12, 2020.

Solve for $u(x,t)$ with $0 < x < L$ and $t > 0$

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions $u(0,t) = 0, u_x(L,t) = C$ and zero initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 0$$

For animations use $L = 10, c = 1, C = 5$

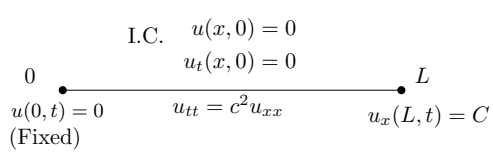

Figure 5.54: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == C0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t},Assumptions->L>0
```

$$\left\{\left\{u(x,t)\to\ \{\quad \mathrm{C0}x+\sum_{K[1]=1}^{\infty}\frac{8(-1)^{K[1]}\mathrm{C0}L\cos\left(\frac{\pi t\sqrt{c^2(2K[1]-1)^2}}{2L}\right)\sin\left(\frac{\pi x(2K[1]-1)}{2L}\right)}{\pi^2(1-2K[1])^2}\quad K[1]\in\mathbb{Z}\wedge((c<0\wedge K[1]\geq\right.\right.$$

Indeterminate True

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*diff(u(x, t), x$2);
bc  := u(0,t)=0, D[1](u)(L,t)=C0;
ic  := u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=C0x-8\left(\sum_{n=0}^{\infty}\frac{C0L(-1)^n\cos\left(\frac{(2n+1)\pi ct}{2L}\right)\sin\left(\frac{(2n+1)\pi x}{2L}\right)}{(2n+1)^2\pi^2}\right)$$

<u>Hand solution</u>

Let

$$u(x,t)=v(x,t)+u_E(x) \tag{2}$$

$u_E(x)$ is the steady state solution which only needs to satisfy the non-homogeneous boundary conditions. At equilibrium $\frac{\partial^2 u(x,t)}{\partial t^2}=0$ and the PDE becomes $\frac{\partial^2 u_E(x,t)}{\partial t^2}=0$ or the ODE $\frac{d^2 u_E(x,t)}{dx^2}=0$ with B.C. $u_E(0)=0, u_E'(L)=C$. The solution to this ODE is

$$u_E(x)=c_1x+c_2$$

At first B.C.

$$0=c_2$$

Solution becomes $u_E(x)=c_1x$. At second B.C. $u_E'(x)=c_1=C$. Therefore solution is

$$u_E(x)=Cx$$

Hence

$$u(x,t) = v(x,t) + Cx$$

$v(x,t)$ is the solution to the PDE but with homogeneous B.C. Plugging (2) into (1) gives

$$\frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial^2 u_E(x,t)}{\partial t^2} = c\left(\frac{\partial^2 v(x,t)}{\partial x^2} + \frac{\partial^2 u_E(x,t)}{\partial x^2}\right)$$

But $\frac{\partial^2 u_E(x,t)}{\partial x^2} = 0$ and also $\frac{\partial^2 u_E(x,t)}{\partial t^2} = 0$, hence above becomes

$$\frac{\partial^2 v(x,t)}{\partial t^2} = c\frac{\partial^2 v(x,t)}{\partial x^2}$$

With $v(x,t)$ having now homogeneous B.C.

$$\begin{aligned} v(0,t) &= 0 \\ \frac{\partial v(L,t)}{\partial x} &= 0 \end{aligned}$$

And initial conditions given by

$$\begin{aligned} v(x,0) &= u(x,0) - u_E(x) \\ &= 0 - Cx \\ &= -Cx \end{aligned}$$

And

$$\begin{aligned} \frac{\partial v(x,0)}{\partial t} &= \frac{\partial u(x,0)}{\partial t} - \frac{\partial u_E(x)}{\partial t} \\ &= 0 \end{aligned}$$

In summary, the PDE to solve for $v(x,t)$ is

$$\begin{aligned} \frac{\partial^2 v(x,t)}{\partial t^2} &= c\frac{\partial^2 v(x,t)}{\partial x^2} \\ v(0,t) &= 0 \\ \frac{\partial v(L,t)}{\partial x} &= 0 \\ v(x,0) &= -Cx \\ \frac{\partial v(x,0)}{\partial t} &= 0 \end{aligned} \tag{3}$$

Now we solve for PDE (3) for $v(x,t)$ using separation of variables since the boundary conditions in space are now homogeneous. Let $v(x,t) = X(x)\,T(t)$ and the PDE becomes

$$\frac{1}{c}T''X = X''T$$

Dividing by $XT \neq 0$ gives

$$\frac{1}{c}\frac{T''}{T} = \frac{X''}{X} = -\lambda \tag{4}$$

Where λ is some real positive constant. The space ODE becomes

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Case $\lambda < 0$: Let The solution is

$$X(x) = c_1 \cosh\left(\sqrt{\lambda}x\right) + c_2 \sinh\left(\sqrt{\lambda}x\right)$$

At $x = 0$

$$0 = c_1$$

Hence solution becomes

$$X(x) = c_2 \sinh\left(\sqrt{\lambda}x\right)$$

Taking derivative

$$X'(x) = \sqrt{\lambda}c_2 \cosh\left(\sqrt{\lambda}x\right)$$

Using second boundary conditions gives

$$0 = \sqrt{\lambda}c_2 \cosh\left(\sqrt{\lambda}L\right)$$

Since cosh is zero only when its argument is zero. But we assumed $\sqrt{\lambda}$ not zero here, then $c_2 = 0$ in only other choice. Hence this gives trivial solution. Therefore $\lambda < 0$ is not possible.

Case $\lambda = 0$

$$\begin{aligned} X'' &= 0 \\ X(0) &= 0 \\ X'(L) &= 0 \end{aligned}$$

Solution is $X(x) = c_1 x + c_2$. First B.C. gives $0 = c_2$. Solution becomes $X(x) = c_1 x$. Second B.C. gives $c_1 = 0$. This gives trivial solution again. Hence $\lambda = 0$ is not possible eigenvalue.

Case $\lambda > 0$: The solution becomes

$$X(x) = B_1 \cos\left(\sqrt{\lambda}x\right) + B_2 \sin\left(\sqrt{\lambda}x\right)$$

AT first B.C.

$$0 = B_1$$

Hence solution becomes

$$X(x) = B_2 \sin\left(\sqrt{\lambda} x\right)$$

Taking derivative

$$X'(x) = \sqrt{\lambda} B_2 \cos\left(\sqrt{\lambda} x\right)$$

At second B.C.

$$0 = \sqrt{\lambda} B_2 \cos\left(\sqrt{\lambda} L\right)$$

To avoid trivial solution, take $\cos\left(\sqrt{\lambda} L\right) = 0$ or $\sqrt{\lambda} L = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \cdots$ or

$$\begin{aligned}
\sqrt{\lambda} &= \frac{\pi}{2L}, \frac{3\pi}{2L}, \frac{5\pi}{2L}, \cdots \\
\sqrt{\lambda_n} &= \left(\frac{n\pi}{2L}\right) \qquad n = 1, 3, 5, \cdots \\
&= \frac{(2n+1)\pi}{2L} \qquad n = 0, 1, 2, \cdots
\end{aligned} \tag{5}$$

Hence the space solution is

$$X_n(x) = B_n \sin\left(\sqrt{\lambda_n} x\right) \qquad n = 1, 3, 5, \cdots \tag{6}$$

Now we solve the time ODE $T(t)$ from (4), which is

$$T'' + \lambda c T = 0$$

The solution is

$$T_n(t) = D_n \cos\left(\sqrt{\lambda_n} c t\right) + E_n \sin\left(\sqrt{\lambda_n} c t\right)$$

Therefore

$$\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} T_n(t)\, X_n(x) \\
&= \sum_{n=0}^{\infty} \left(D_n \cos\left(\sqrt{\lambda_n} c t\right) + E_n \sin\left(\sqrt{\lambda_n} c t\right)\right) \sin\left(\sqrt{\lambda_n} x\right)
\end{aligned}$$

Where constant B_n merged with the other constants. Now At $t = 0$

$$-Cx = \sum_{n=0}^{\infty} D_n \sin\left(\sqrt{\lambda_n} x\right)$$

Applying orthogonality

$$
\begin{aligned}
-\int_0^L Cx\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx &= D_n\int_0^L \sin^2\left(\frac{(2n+1)\pi}{2L}x\right)dx\\
-C\int_0^L x\sin\left(\frac{(2n+1)\pi}{2L}x\right)dx &= D_n\frac{L}{2}\\
-C\left(\frac{4(-1)^n L^2}{(\pi+2n\pi)^2}\right) &= D_n\frac{L}{2}\\
D_n &= -C\frac{8(-1)^n L}{(\pi+2n\pi)^2}
\end{aligned}
$$

Therefore

$$
v(x,t)=\sum_{n=0}^{\infty}\left(-C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\sqrt{\lambda_n}ct\right)+E_n\sin\left(\sqrt{\lambda_n}ct\right)\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

Taking time derivative

$$
\frac{\partial v(x,t)}{\partial t}=\sum_{n=0}^{\infty}\left(C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\frac{(2n+1)\pi}{2L}\sin\left(\sqrt{\lambda_n}ct\right)+E_n\sqrt{\lambda_n}c\cos\left(\sqrt{\lambda_n}ct\right)\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

At $t=0$

$$
0=\sum_{n=0}^{\infty}E_n\sqrt{\lambda_n}c\sin\left(\sqrt{\lambda_n}x\right)
$$

Hence $E_n=0$. Therefore solution becomes

$$
v(x,t)=\sum_{n=0}^{\infty}-C\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\sqrt{\lambda_n}ct\right)\sin\left(\sqrt{\lambda_n}x\right)
$$

Therefore, since $u(x,t)=v(x,t)+u_E(x)$ then

$$
u(x,t)=Cx-C\sum_{n=0}^{\infty}\frac{8(-1)^n L}{(\pi+2n\pi)^2}\cos\left(\frac{(2n+1)\pi}{2L}\sqrt{c}t\right)\sin\left(\frac{(2n+1)\pi}{2L}x\right)\tag{7}
$$

This animation runs for 40 seconds for $L=10, c=1, C=5$. The solution becomes

$$
u(x,t)=5x-5\sum_{n=0}^{\infty}\frac{80(-1)^n}{(\pi+2n\pi)^2}\cos\left(\frac{(2n+1)\pi}{20}\sqrt{10}t\right)\sin\left(\frac{(2n+1)\pi}{20}x\right)\tag{7}
$$

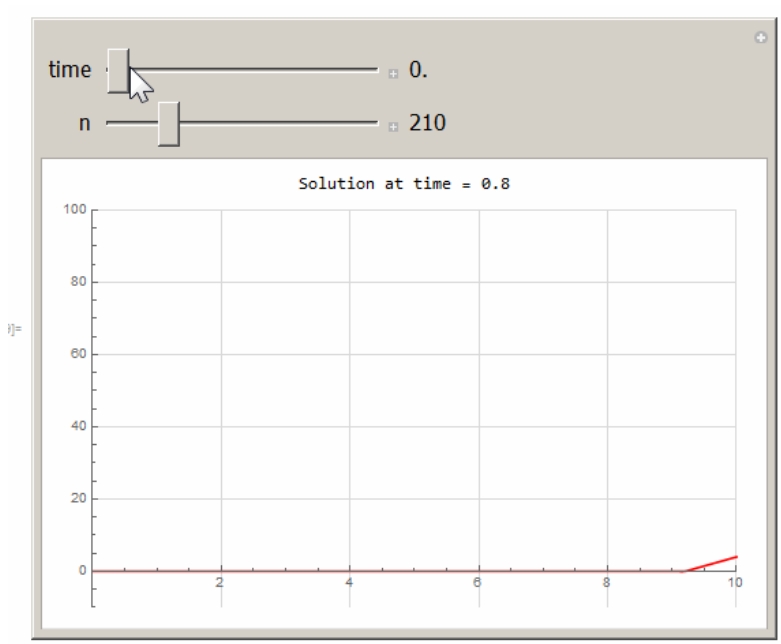

Figure 5.55: snap shot

Code used for the above is

```
L=10;c=1; C0=5;
sqrtLam= (2 n +1) Pi/(2 L);
Manipulate[
mysol=C0 x-C0 Sum[(8.0(-1)^n L)/(Pi+2n \[Pi])^2 Sin[sqrtLam  x] Cos[sqrtLam  Sqrt[c] t],{n,0
];
Grid[{{Row[{"Solution at time = ",i}]},
{Plot[mysol/.t->i,{x,0,10},PlotRange->{{0,10},{-10,100}}%
,ImageSize->500,GridLines->Automatic,GridLinesStyle->LightGray,PlotStyle->Red]}%
}],
{{i,0,"time"},0,100,.1,Appearance->"Labeled"},
{{numberOfTerms,1,"n"},1,1000,1,Appearance->"Labeled"}
]
```

5.1.2 Semi-infinite domain

Local contents

5.1.2.1 [364] Left end fixed, (general case) 1019
5.1.2.2 [365] Left end fixed with specific initial position 1022
5.1.2.3 [366] Logan page 115, left end fixed with source 1025
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5.1.2.1 [364] Left end fixed, (general case)

problem number 364

Added July 12, 2019 Solve for $u(x,t)$ with $t > 0$ and $x > 0$

$$u_{tt} = c^2 u_{xx}$$

With boundary conditions

$$u(0,t) = 0$$

With initial conditions

$$\begin{aligned} u_t(x,0) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$

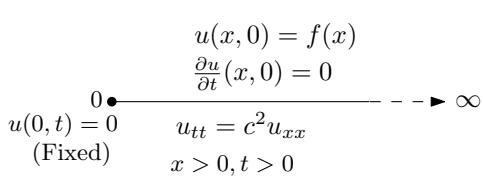

Figure 5.56: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = u[0, t] == 0;
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\begin{cases}\frac{1}{2}(f(x-ct)+f(ct+x)) & x>ct\\ \frac{1}{2}(f(ct+x)-f(ct-x)) & x\le ct\\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT =
```

sol=()

Hand solution

Solving on semi-infinite domain for $u(x,t)$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\qquad 0<x<\infty, t>0 \tag{1}$$

With BC

$$u(0,t)=0$$
$$u(\infty,t)<\infty$$

And initial conditions

$$u(x,0)=f(x)$$
$$u_t(x,0)=0$$

Seperation of variables method

Let $u(x,t) = X(x)\,T(t)$. The PDE in (1) becomes

$$\frac{T''}{c^2 T} = \frac{X''}{X} = -\lambda$$

Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(\infty) &< \infty \end{aligned}$$

It is clear that λ can not be negative because it gives a solution that blows up. For $\lambda = 0$, the solution is $X(x) = Ax + B$ and because $X(0) = 0$ this implies $B = 0$. Hence solution is $X(x) = Ax$. And this blows up as x increases unless $A = 0$. Hence $\lambda = 0$ is not valid eigenvalue. Therefore $\lambda > 0$. Let $\lambda = \alpha^2, \alpha > 0$ and the solution becomes

$$X(x) = A_\alpha \cos(\alpha x) + B_\alpha \sin(\alpha x)$$

At $x = 0$ the above gives

$$0 = A_\alpha$$

Therefore the solution becomes

$$X_\alpha(x) = B_\alpha \sin(\alpha x) \qquad \alpha > 0 \tag{1}$$

The time domain ODE becomes

$$\begin{aligned} T'' + c^2\alpha^2 T &= 0 \\ T &= C_\alpha \cos(\alpha c t) + D_\alpha \sin(\alpha c t) \\ T'(t) &= -c\alpha C_\alpha \sin(\alpha c t) + c\alpha D_\alpha \cos(\alpha c t) \end{aligned}$$

And at $t = 0, T'(0) = 0$, hence the above becomes

$$0 = c\alpha D_\alpha$$

Which means $D_\alpha = 0$. Therefore

$$T = C_\alpha \cos(\alpha c t) \qquad \alpha > 0 \tag{2}$$

From (1,2) the complete solution is therefore

$$u(x,t) = \int_0^\infty A_\alpha \cos(\alpha c t) \sin(\alpha x)\, d\alpha \tag{3}$$

Where A_α, C_α are merged into one constant. Now the last initial condition is applied, which is $u(x,0) = f(x)$ to the above which gives

$$f(x) = \int_0^\infty A_\alpha \sin(\alpha x)\, d\alpha$$

Hence

$$A_\alpha = \frac{2}{\pi} \int_0^\infty f(x) \sin(\alpha x)\, dx$$

Using the above in (3) gives the final solution as

$$u(x,t) = \frac{2}{\pi} \int_0^\infty \left(\int_0^\infty f(s) \sin(\alpha s)\, ds \right) \cos(\alpha c t) \sin(\alpha x)\, d\alpha$$

<u>D'Alambert's formula method</u>

For the half line, the D'Alambert's is given by, using $v_0(x) = u_t(x,0)$ as the initial velocity

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct) + f(x-ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} v_0(s)\, ds & x > ct \geq 0 \\ \\ \frac{1}{2}(f(x+ct) - f(ct-x)) + \frac{1}{2c}\int_{ct-x}^{x+ct} v_0(s)\, ds & x \leq ct \end{cases}$$

Hence, since $u_t(x,0) = 0$ in this problem

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct) + f(x-ct)) & x > ct \geq 0 \\ \\ \frac{1}{2}(f(x+ct) - f(ct-x)) & x \leq ct \end{cases}$$

5.1.2.2 [365] Left end fixed with specific initial position

problem number 365

Taken from Mathematica DSolve help pages.

Solve for $u(x,t)$ initial value wave PDE on infinite domain with $t > 0$ and $x > 0$.

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$\begin{aligned} u(x,0) &= \sin^2(x) \qquad \pi < x < 2\pi \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

And boundary conditions $u(0,t) = 0$

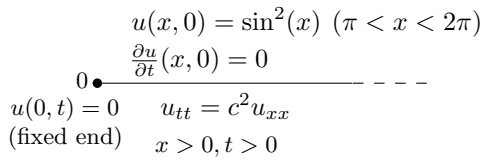

Figure 5.57: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == Piecewise[{{Sin[x]^2, Pi < x < 2*Pi}}], Derivative[0, 1][u][x, 0] == 0};
bc  = u[0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->c>0
```

$$\left\{\left\{u(x,t) \to \left\{\begin{array}{l} \frac{1}{2}\left(\left(\begin{cases} \sin^2(ct-x) & \pi < x - ct < 2\pi \\ 0 & \text{True} \end{cases}\right) + \left(\begin{cases} \sin^2(ct+x) & \pi < ct + x < 2\pi \\ 0 & \text{True} \end{cases}\right.\right. \\ \frac{1}{2}\left(\left(\begin{cases} \sin^2(ct+x) & \pi < ct + x < 2\pi \\ 0 & \text{True} \end{cases}\right) - \left(\begin{cases} \sin^2(ct-x) & \pi < ct - x < 2\pi \\ 0 & \text{True} \end{cases}\right.\right. \end{array}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2);
ic  := u(x,0)= piecewise(Pi<x and x<2*Pi,sin(x)^2),(D[2](u))(x,0)=0;
bc  := u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{\left(\begin{cases} -\left(\begin{cases} 0 & ct-x \le \pi \\ \sin^2(ct-x) & ct-x < 2\pi \\ 0 & 2\pi \le ct-x \end{cases}\right) + \left(\begin{cases} 0 & ct+x \le \pi \\ \sin^2(ct+x) & ct+x < 2\pi \\ 0 & 2\pi \le ct+x \end{cases}\right) & x < ct \\ \left(\begin{cases} 0 & -ct+x \le \pi \\ \sin^2(ct-x) & -ct+x < 2\pi \\ 0 & 2\pi \le -ct+x \end{cases}\right) + \left(\begin{cases} 0 & ct+x \le \pi \\ \sin^2(ct+x) & ct+x < 2\pi \\ 0 & 2\pi \le ct+x \end{cases}\right) & ct < x \end{cases}\right)}{2}$$

<u>Hand solution</u>

Solving on semi-infinite domain

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \qquad t > 0, x > 0 \\ u(0,t) &= 0 \\ u(x,0) &= f(x) = \sin^2(x) \qquad \pi < x < 2\pi \end{aligned} \tag{1}$$

With $k > 0$ and $u(x,t) < \infty$ as $x \to \infty$. This means $u(x,t)$ is bounded. The general solution to the above PDE was given in problem 5.1.2.1 on page 1020 as (using the D'Alambert's solution and not the Fourier integral solution)

$$u(x,t) = \begin{cases} \frac{1}{2}(f(x+ct) + f(x-ct)) & x > ct \ge 0 \\ \\ \frac{1}{2}(f(x+ct) - f(ct-x)) & x \le ct \end{cases}$$

But $f(x) = \sin^2(x)$ and the above becomes

$$u(x,t) = \begin{cases} \frac{1}{2}(\sin^2(x+ct) + \sin^2(x-ct)) & x > ct \ge 0 \\ \\ \frac{1}{2}(\sin^2(x+ct) - \sin^2(ct-x)) & x \le ct \end{cases}$$

But here $f(x)$ is restricted to $\pi < x < 2\pi$. Hence the above solution is modified as

follows

$$u(x,t)=\begin{cases}\begin{cases}\frac{1}{2}\sin^2(x+ct) & \pi<x+ct<2\pi\\ 0 & \text{otherwise}\end{cases}+\begin{cases}\frac{1}{2}\sin^2(x-ct) & \pi<x-ct<2\pi\\ 0 & \text{otherwise}\end{cases} & x>ct\geq\\ \\ \begin{cases}\frac{1}{2}\sin^2(x+ct) & \pi<x+ct<2\pi\\ 0 & \text{otherwise}\end{cases}-\begin{cases}\frac{1}{2}\sin^2(ct-x) & \pi<ct-x<2\pi\\ 0 & \text{otherwise}\end{cases} & 0<x\leq\end{cases}$$

5.1.2.3 [366] Logan page 115, left end fixed with source

problem number 366

This is problem at page 115, David J Logan textbook, applied PDE textbook.

Falling cable lying on a table that is suddenly removed.

$$u_{tt}=c^2u_{xx}-g$$

With boundary condition

$$u(0,t)=0$$

And initial conditions

$$u(x,0)=0$$
$$\frac{\partial u}{\partial t}(x,0)=0$$

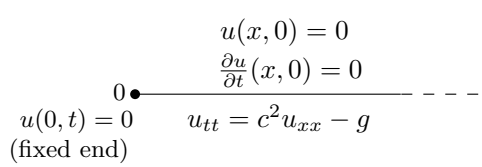

Figure 5.58: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] - g;
bc  = u[0, t] == 0;
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \begin{cases} -\frac{gt^2}{2} & ct\leq x \\ \frac{gx(x-2ct)}{2c^2} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic  :=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t),HINT = bou
```

$$u(x,t)=\frac{\left(-c^2t^2+(ct-x)^2\,\theta\left(t-\frac{x}{c}\right)\right)g}{2c^2}$$

5.1.2.4 [367] Left moving boundary condition

problem number 367

Solve for $u(x,t)$ with $t>0$ and $x>0$

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t)=g(t)$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=0$$

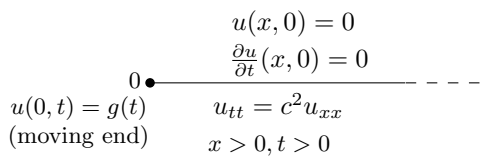

Figure 5.59: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
bc  = u[0, t] == g[t];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \begin{cases} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\le ct \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=0,D[2](u)(x,0)=0;
bc  := u(0,t)=g(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t)=$$

5.1.2.5 [368] moving Left end

problem number 368

Taken from Mathematica DSolve help pages. Initial value problem with a Neumann condition on the half-line.

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = \sin^3(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 1 - e^{-\frac{x}{10}}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

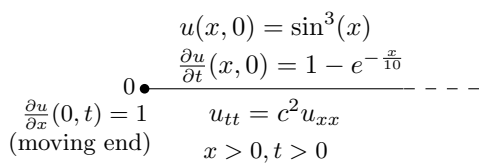

Figure 5.60: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == Sin[x]^3, Derivative[0, 1][u][x, 0] == 1 - E^(-(x/10))};
bc  = Derivative[1, 0][u][0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t},Assumptions
```

$$\left\{\left\{u(x,t) \to \begin{cases} 0 & x > ct \\ g\left(t - \frac{x}{c}\right) & x \le ct \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2);
ic  := u(x,0)= sin(x)^3,(D[2](u))(x,0)=1-exp(-x/10);
bc:=(D[1](u))(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{\begin{cases} -c(\sin^3(ct-x)) + c(\sin^3(ct+x)) + 2ct + 10\,\mathrm{e}^{-\frac{ct}{10}-\frac{x}{10}} - 10\,\mathrm{e}^{\frac{ct}{10}-\frac{x}{10}} & ct < \\ c(\sin^3(ct-x)) + c(\sin^3(ct+x)) - 2c^2t + (2t+2x)\,c + 10\,\mathrm{e}^{-\frac{ct}{10}-\frac{x}{10}} + 10\,\mathrm{e}^{-\frac{ct}{10}+\frac{x}{10}} - 20 & x < c \end{cases}}{2c}$$

5.1.2.6 [369] I.C. at $t = 1$

problem number 369

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018` Solve

$$u_{tt} = u_{xx}$$

With initial conditions

$$u(x,1) = e^{-(x-6)^2} + e^{-(x+6)^2}$$
$$\frac{\partial u}{\partial t}(x,1) = \frac{1}{2}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

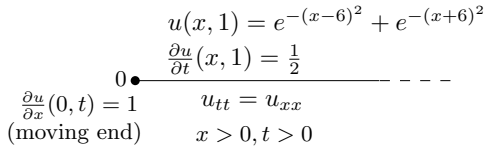

Figure 5.61: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 1] == Sin[x]^3, Derivative[0, 1][u][x, 1] == 1 - E^(-(x/10))};
bc  = Derivative[1, 0][u][0, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t}, Assumption
```

$$\left\{\left\{u(x,t)\to \begin{cases} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\le ct \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
ic  := u(x, 1) = exp(-(x-6)^2)+exp(-(x+6)^2), eval(diff(u(x,t),t),t=1)= 1/2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t)=\frac{t}{2}+\frac{e^{-(t+x-7)^2}}{2}+\frac{e^{-(t-x+5)^2}}{2}+\frac{e^{-(t-x-7)^2}}{2}+\frac{e^{-(t+x+5)^2}}{2}-\frac{1}{2}$$

5.1.2.7 [370] B.C. at $x=1$

problem number 370

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0)=e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0)=0$$

And Boundary conditions $\frac{\partial u}{\partial x}(1,t)=0$

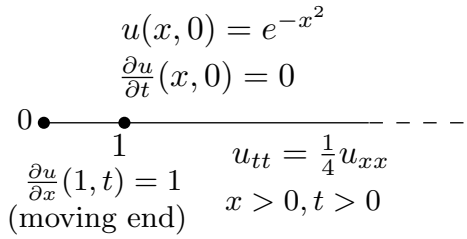

Figure 5.62: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == (1*D[u[x, t], {x, 2}])/4;
ic  = {u[x, 0] == Exp[-x^2], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} e^{-x^2} & x-1 \\ \frac{2\int_0^\infty \frac{1}{4} e^{-\frac{1}{4}K[1](K[1]+4i)}\sqrt{\pi}\cos\left(\frac{1}{2}tK[1]\right)\cos((x-1)K[1])\left(\operatorname{Erfc}\left(1-\frac{1}{2}iK[1]\right)+e^{2iK[1]}\operatorname{Erfc}\left(\frac{1}{2}iK[1]+1\right)\right)dK[1]}{\pi} & x-1 \\ \text{Indeterminate} & \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2)=(1/4)*(diff(u(x, t), x$2));
bc  := eval(diff(u(x,t),x),x=1)=0;
ic  :=  u(x, 0) = exp(-x^2), eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],u(x,t)) assum
```

$$u(x,t) = \frac{\left(\begin{cases} e^{-\frac{(t-2x)^2}{4}} + e^{-\frac{(t+2x)^2}{4}} & \frac{t}{2} < x-1 \\ e^{-\frac{(t+2x)^2}{4}} + e^{-\frac{(t-2x+4)^2}{4}} & x-1 < \frac{t}{2} \end{cases}\right)}{2}$$

5.1.2.8 [371] Left end free. zero initial velocity (general solution)

problem number 371

Added July 13, 2019.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

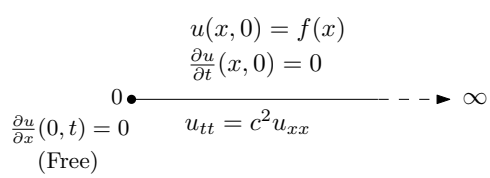

Figure 5.63: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} f(x) & x \geq 0 \wedge t = 0 \\ \frac{2\int_0^\infty \cos(ctK[1])\cos(xK[1])\int_0^\infty \cos(xK[1])f(x)dxdK[1]}{\pi} & x \geq 0 \wedge t > 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \frac{\left(\begin{cases} f(-ct+x)+f(ct+x) & ct<x \\ f(ct-x)+f(ct+x) & x<ct \end{cases}\right)}{2}$$

5.1.2.9 [372] Left end free. zero initial velocity (Special solution)

problem number 372

Added July 14, 2019.

Solve for $u(x,t)$ with $x>0, t>0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$ using

$$c = 3$$
$$f(x) = \begin{cases} 1 & 4 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

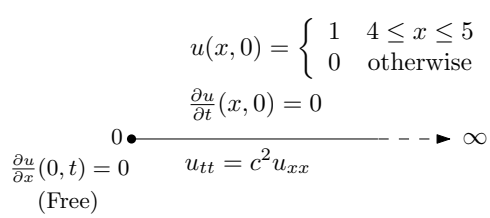

Figure 5.64: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=3;
f[x_]:=Piecewise[{{1,4<x<5},{0,True}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(\left(\begin{cases}1 & 4<x-3t<5\\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases}1 & 4<3t+x<5\\ 0 & \text{True}\end{cases}\right)\right) & x>3t\\ \frac{1}{2}\left(\left(\begin{cases}1 & 4<3t-x<5\\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases}1 & 4<3t+x<5\\ 0 & \text{True}\end{cases}\right)\right) & x\le 3t\end{cases}\right\}\right\}$$

Maple ✓

```
restart;
c:=3;
f:=piecewise(4<x and x<5,1,true,0);
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\begin{cases}\left(\begin{cases}1 & 4<6t+x\wedge 6t+x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t+x)+\left(\begin{cases}1 & 4<x-6t\wedge x-6t<5\\ 0 & \text{otherwise}\end{cases}\right)(x-3t) & 3t\\ \left(\begin{cases}1 & 4<6t+x\wedge 6t+x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t+x)+\left(\begin{cases}1 & 4<x\wedge x<5\\ 0 & \text{otherwise}\end{cases}\right)(3t-x) & x\end{cases}}{2}$$

Hand solution

Solve $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ for $x\ge 0, t\ge 0$ with initial conditions $u(x,0)=f(x)=\begin{cases}1 & 4\le x\le 5\\ 0 & \text{otherwise}\end{cases}$

and $\frac{\partial u(x,0)}{\partial t} = 0$ and boundary condition $\frac{\partial u(0,t)}{\partial x} = 0$ (Free end)

The general solution by D'Almbert's is given by

$$u(x,t) = \frac{1}{2}\begin{cases} f(x+ct) + f(x-ct) & x > ct \geq 0 \\ f(x+ct) + f(ct-x) & x \leq ct \end{cases}$$

But $f(x)$ is defined for $4 \leq x \leq 5$ only, hence the solution becomes

$$u(x,t) = \begin{cases} \begin{cases} \frac{1}{2}f(x+ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{2}f(x-ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} & x > ct \geq 0 \\ \\ \begin{cases} \frac{1}{2}f(x+ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{2}f(ct-x) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} & 0 < x \leq ct \end{cases}$$

Animation is below

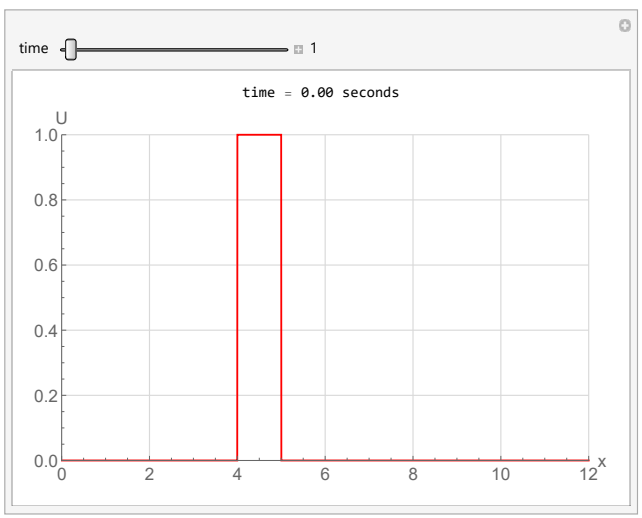

Figure 5.65: Initial state

Source code used for the above

```
In[•]:= ClearAll[x, t, n, f, A, B, s, mySol]
k = 1 / 10;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
c = 3;
f[x_] := Piecewise[{{1, 4 < x < 5}, {0, True}}];
mySol[x_, t_] := Piecewise[{
      {(f[x + c t] + f[x - c t]) / 2, x > c t},
      {(f[x + c t] + f[c t - x]) / 2, 0 ≤ x ≤ c t},
      {0, True}}]
```

Figure 5.66: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, 12},
                     Exclusions → None,
                    BaseStyle → 15,
                    ImageMargins → 3,
                    PerformanceGoal → "Quality",
                    PlotRange → {{0, 12}, {0, 1}},
                    ImageSize → 500,
                    AxesLabel → {"x", "U"},
                    GridLines → Automatic,
                    GridLinesStyle → LightGray,
                    PlotStyle → Red
                  ]
              }}],
          {t, 0, 5.5, 0.01}];
In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.67: Code for animation

5.1.2.10 [373] Left end fixed. zero initial velocity (Special solution)

problem number 373

Added January 9, 2020.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary condition $u(0,t) = 0$ using

$$c = 3$$
$$f(x) = \begin{cases} 1 & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

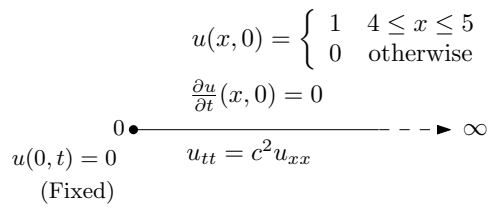

Figure 5.68: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=3;
f[x_]:=Piecewise[{{1,4<x<5},{0,True}}];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
bc  = u[0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(\left(\begin{cases} 1 & 4 < x - 3t < 5 \\ 0 & \text{True} \end{cases}\right) + \left(\begin{cases} 1 & 4 < 3t + x < 5 \\ 0 & \text{True} \end{cases}\right)\right) & x > 3t \\ \frac{1}{2}\left(\left(\begin{cases} 1 & 4 < 3t + x < 5 \\ 0 & \text{True} \end{cases}\right) - \left(\begin{cases} 1 & 4 < 3t - x < 5 \\ 0 & \text{True} \end{cases}\right)\right) & x \le 3t \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
c:=3;
f:=piecewise(4<x and x<5,1,true,0);
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := u(0,t)=0;
ic  := u(x,0)=f(x),D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \frac{\begin{cases} \left(\begin{cases} 1 & 4 < 6t + x \wedge 6t + x < 5 \\ 0 & \text{otherwise} \end{cases}\right)(3t + x) - \left(\begin{cases} 1 & 4 < x \wedge x < 5 \\ 0 & \text{otherwise} \end{cases}\right)(3t - x) & x \\ \left(\begin{cases} 1 & 4 < 6t + x \wedge 6t + x < 5 \\ 0 & \text{otherwise} \end{cases}\right)(3t + x) + \left(\begin{cases} 1 & 4 < x - 6t \wedge x - 6t < 5 \\ 0 & \text{otherwise} \end{cases}\right)(x - 3t) & 3t \end{cases}}{2}$$

<u>Hand solution</u>

Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for $x \geq 0, t \geq 0$ with initial conditions $u(x,0) = f(x) = \begin{cases} 1 & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$ and $\frac{\partial u(x,0)}{\partial t} = 0$ and boundary condition $u(0,t) = 0$ (Fixed end)

The general solution by D'Almbert's is given by

$$u(x,t) = \frac{1}{2} \begin{cases} f(x+ct) + f(x-ct) & x > ct \geq 0 \\ f(x+ct) - f(ct-x) & x \leq ct \end{cases}$$

But $f(x)$ is defined for $4 \leq x \leq 5$ only, hence the solution becomes

$$u(x,t) = \begin{cases} \begin{cases} \frac{1}{2} f(x+ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{2} f(x-ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} & x > ct \geq 0 \\ \begin{cases} \frac{1}{2} f(x+ct) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{2} f(ct-x) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} & 0 < x \leq ct \end{cases}$$

Animation is below

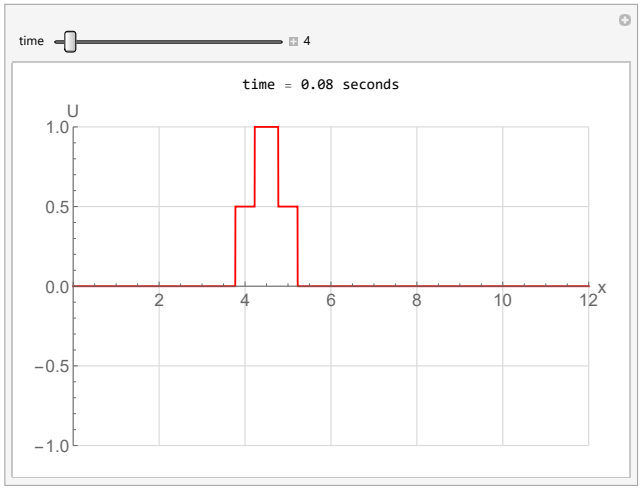

Figure 5.69: Initial state

Source code used for the above

```
In[ ]:= ClearAll[x, t, n, f, A, B, s, u]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
c = 3;
f[x_] := Piecewise[{{1, 4 < x < 5}, {0, True}}];
g[x_] := 0
u[x_, t_] := Piecewise[{
        {(f[x + c t] + f[x - c t]) / 2 + 1 / (2 c) Integrate[g[s], {s, -x - c t, x + c t}], t ≤ x / c},
        {(-f[c t - x] + f[x + c t]) / 2 + 1 / (2 c) Integrate[g[s], {s, c t - x, x + c t}], t > x / c}, {0, True}}
   ]


In[ ]:= tab = Table[
        Grid[{
              {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
              {
                 Quiet@Plot[Evaluate[u[x, t]], {x, 0, 12},
                       Exclusions → None,
                      BaseStyle → 15,
                      ImageMargins → 3,
                      PerformanceGoal → "Quality",
                      PlotRange → {{0, 12}, {-1, 1}},
                      ImageSize → 500,
                      AxesLabel → {"x", "U"},
                      GridLines → Automatic,
                      GridLinesStyle → LightGray,
                      PlotStyle → Red
                   ]
              }}],
        {t, 0, 3, 0.025}];

In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 5.70: Source code

5.1.2.11 [374] Left end free. zero initial position (general solution)

problem number 374

Added July 14, 2019.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

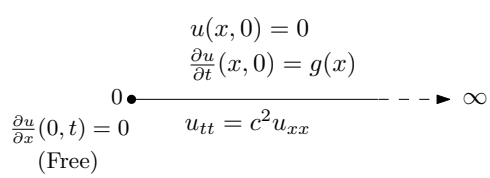

Figure 5.71: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == g[x]};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} 0 & x \geq 0 \wedge t = 0 \\ \frac{2\int_0^\infty \frac{\cos(xK[1])\sin(ctK[1])\int_0^\infty \cos(xK[1])g(x)dx}{cK[1]}dK[1]}{\pi} & x \geq 0 \wedge t > 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \frac{\begin{cases} \int_{-ct+x}^{ct+x} g(\zeta)\,d\zeta & ct < x \\ \int_0^{ct-x} g(\zeta)\,d\zeta + \int_0^{ct+x} g(\zeta)\,d\zeta & x < ct \end{cases}}{2c}$$

5.1.2.12 [375] Left end free. Non zero initial position and velocity (general solution)

problem number 375

Added July 14, 2019.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_{tt} = c^2 u_{xx}$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

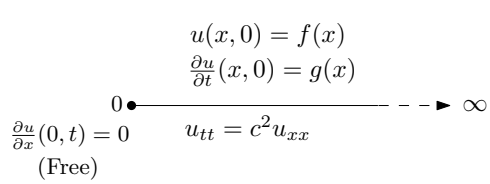

Figure 5.72: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\begin{cases} f(x) & x\geq 0\wedge t \\ \frac{2\int_0^\infty \frac{\sqrt{\frac{\pi}{2}}\cos(xK[1])\left(c\sqrt{\frac{2}{\pi}}\cos(ctK[1])K[1]\int_0^\infty \cos(xK[1])f(x)dx+\sqrt{\frac{2}{\pi}}\sin(ctK[1])\int_0^\infty \cos(xK[1])g(x)dx\right)}{cK[1]}dK[1]}{\pi} & x\geq 0\wedge t \\ \text{Indeterminate} & \text{True}\end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=f(x),eval(diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t)=\frac{\left(\begin{cases} f(-ct+x)+f(ct+x)+\frac{\int_{-ct+x}^{ct+x} g(\zeta)d\zeta}{c} & ct<x \\ \frac{cf(ct-x)+cf(ct+x)+\int_0^{ct-x} g(\zeta)d\zeta+\int_0^{ct+x} g(\zeta)d\zeta}{c} & x<ct\end{cases}\right)}{2}$$

5.1.2.13 [376] Left end free with source

problem number 376

Added December 20, 2018.

Example 17, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Left end free with initial position and velocity given.

Solve for $u(x,t)$ with $x>0, t>0$

$$\frac{\partial^2 u}{\partial t^2}=9\frac{\partial^2 u}{\partial x^2}+f(x,t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = x^3$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

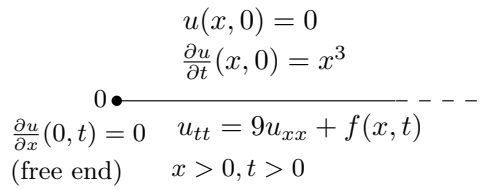

Figure 5.73: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == 9*D[u[x, t], {x, 2}];
ic  = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^3};
bc  = Derivative[1, 0][u][0, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \begin{cases} tx(9t^2+x^2) & x > 3t \\ \frac{1}{12}(81t^4+54x^2t^2+x^4) & x \le 3t \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = 9*(diff(u(x, t), x$2));
bc  := eval( diff(u(x,t),x),x=0)=0;
ic  := u(x,0)=0,eval(diff(u(x,t),t),t=0)=x^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = \frac{\left(\begin{cases} 108t^3x + 12tx^3 & 3t < x \\ 81t^4 + 54t^2x^2 + x^4 & x < 3t \end{cases}\right)}{12}$$

5.1.3 Infinite domain

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5.1.3.1 [377] General case. $u_{tt} = u_{xx}$ with $u(x,0) = f(x), u_t(x,0) = g(x)$

problem number 377

Added May 26, 2019.

Solve for $u(x,t)$ for all x and $t > 0$ with $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$

$$u_{tt} = u_{xx}$$

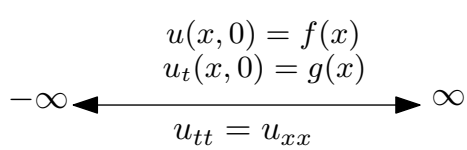


Figure 5.74: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==f[x], Derivative[0,1][u][x,0]==g[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}(f(x-t)+f(t+x))+\frac{1}{2}\int_{x-t}^{t+x} g(K[1])dK[1] & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=f(x), eval( diff(u(x,t),t),t=0)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\frac{\left(\int_{-t+x}^{t+x} g(x1)\,dx1\right)}{2}+\frac{f(-t+x)}{2}+\frac{f(t+x)}{2}$$

5.1.3.2 [378] General case. No IC given. $u_{tt}+u_{xt}=c^2u_{xx}$

problem number 378

From Mathematica DSolve help pages (slightly modified)

Solve for $u(x,t)$ with $t>0$ on real line

$$u_{tt}+u_{xt}=c^2u_{xx}$$

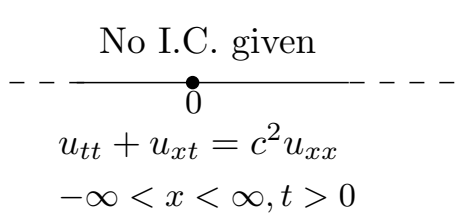


Figure 5.75: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] + D[u[x, t], x, t] == c^2*D[u[x, t], {x, 2}];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t},Assumptions->c>0], 60*10]]
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{\left(\sqrt{4c^2+1}-1\right)x}{2c^2}\right)+c_2\left(\frac{2c^2t+\sqrt{4c^2+1}x+x}{2c^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
interface(showassumed=0);
pde := diff(u(x,t),t$2)+diff(u(x,t),t,x)=c^2*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t)) assuming t>0,x
```

$$u(x,t)=\_F1\left(\frac{2c^2t+\sqrt{4c^2+1}\,x+x}{2c^2}\right)+\_F2\left(\frac{2c^2t-\sqrt{4c^2+1}\,x+x}{2c^2}\right)$$

5.1.3.3 [379] $u_{tt}=c^2u_{xx}+f(x,t)$, IC at $t=1$,$u(x,1)=g(x),u_t(x,1)=h(x)$

problem number 379

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$u_{tt}=c^2u_{xx}+f(x,t)$$

With initial conditions not at zero

$$u(x,1)=g(x)$$
$$\frac{\partial u}{\partial t}(x,1)=h(x)$$

$$u(x,1) = g(x)$$
$$\frac{\partial u}{\partial t}(x,1) = h(x)$$
$$0$$
$$u_{tt} = c^2 u_{xx} + f(x,t)$$
$$-\infty < x < \infty, t > 0$$

Figure 5.76: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic  = {u[x, 1] == g[x], Derivative[0, 1][u][x, 1] == h[x]};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(g\left(x-\sqrt{c^2}(t-1)\right)+g\left(\sqrt{c^2}(t-1)+x\right)\right)+\frac{\int_{x-\sqrt{c^2}(t-1)}^{\sqrt{c^2}(t-1)+x} h(K[1])dK[1]}{2\sqrt{c^2}} & t-1\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2)) + f(x, t);
ic  := u(x, 1) = g(x),eval(diff(u(x,t),t),t=1)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t)) assumi
```

$$u(x,t) = \frac{2cg(x) + (2t-2)\,ch(x) + \int_0^{t-1}\int_{(-t+\tau+1)c+x}^{(t-\tau-1)c+x}\left(c^2\tau\left(\frac{d^2}{d\zeta^2}h(\zeta)\right) + c^2\left(\frac{d^2}{d\zeta^2}g(\zeta)\right) + f(\zeta,\tau+1)\right)d\zeta d\tau}{2c}$$

5.1.3.4 [380] No source. $u_{tt} = u_{xx}$, with $u(x,0) = e^{-x^2}, u_t(x,0) = 1$

problem number 380

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$u_{tt} = u_{xx}$$

With initial conditions

$$u(x,0) = e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

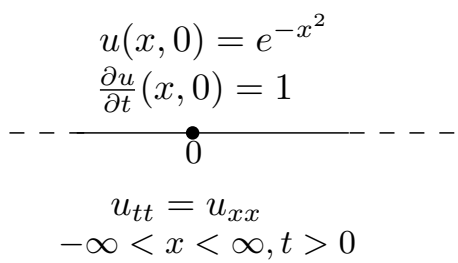


Figure 5.77: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
ic  = {u[x, 0] == E^(-x^2), Derivative[0, 1][u][x, 0] == 1};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(e^{-(x-t)^2} + e^{-(t+x)^2}\right) + t & t \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t), t$2) = diff(u(x,t), x$2);
ic  := u(x, 0) = exp(-x^2), (D[2](u))(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = t + \frac{e^{-(t-x)^2}}{2} + \frac{e^{-(t+x)^2}}{2}$$

5.1.3.5 [381] With source term. $u_{tt} = u_{xx} + m$

problem number 381

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$u_{tt} = u_{xx} + m$$

With initial conditions

$$u(x,0) = \sin x - \frac{\cos 3x}{e^{\frac{abs(x)}{6}}}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

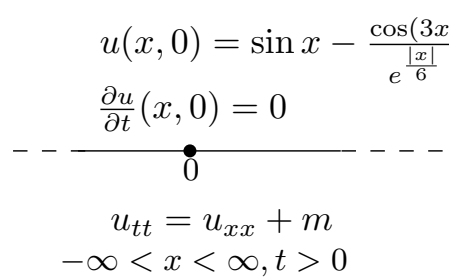


Figure 5.78: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  {D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + m};
ic  = {u[x, 0] == Sin[x] - Cos[3*x]/E^(Abs[x]/6), Derivative[0, 1][u][x, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(mt^2 - e^{-\frac{|t-x|}{6}}\cos(3t-3x) - e^{-\frac{|t+x|}{6}}\cos(3(t+x)) + 2\cos(t)\sin(x)\right) & t \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2) + m;
ic  := u(x, 0) = sin(x) - cos(3*x)/exp(abs(x)/6), (D[2](u))(x, 0) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = \frac{\left(-\cos(3t-3x)\,\mathrm{e}^{\frac{|t+x|}{6}} + \left(-\cos(3t+3x) + (mt^2 - \sin(t-x) + \sin(t+x))\,\mathrm{e}^{\frac{|t+x|}{6}}\right)\mathrm{e}^{\frac{|t-x|}{6}}\right)\mathrm{e}^{-\frac{|t-x}{6}}}{2}$$

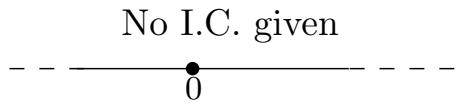
5.1.3.6 [382] non-linear (Solitons) $u_t + 6u(x,t)u_x + u_{xxx} = 0$

problem number 382

This was first solved analytically by (Krvskal, Zabrsky 1965).

Solve

$$u_t + 6u(x,t)u_x + u_{xxx} = 0$$



$$u_t + 6u + u_{xxx} = 0$$
$$-\infty < x < \infty, t > 0$$

Figure 5.79: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, t], t] + 6*u[x, t]*D[u[x, t], x] + D[u[x, t], {x, 3}] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -\frac{12{c_1}^3 \tanh^2(c_2 t + c_1 x + c_3) - 8{c_1}^3 + c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(u(x,t),t)+6*u(x,t)*diff(u(x,t),x)+diff(u(x,t),x$3)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t)) assuming t>0,x
```

$$u(x,t) = -2c_2^2\left(\tanh^2\left(c_3 t + c_2 x + c_1\right)\right) + \frac{8c_2^3 - c_3}{6c_2}$$

<u>Hand solution</u>

Solve

$$u_t + 6u\, u_x + u_{xxx} = 0$$

Assuming special solution $u = f(\xi)$ where $\xi = x - ct$, this PDE is transformed to non-linear first order ODE

$$-c\frac{f^2}{2} + f^3 + \frac{1}{2}\left(\frac{df}{d\xi}\right)^2 = 0$$

The above is solved analytically (Krvskal, Zabrsky 1965) and the solution is

$$f(x,c,t) = \left(\frac{1}{2}c\right)\operatorname{sech}^2\left(\frac{\sqrt{c}}{2}(x - ct)\right)$$

Tall waves move fast but have smaller period, while short wave move slow. Tall wave pass through short wave and leave in same shape they entered. Here are two animations and the above solution. This first animation has one tall wave passing though short wave

Out[ ]=

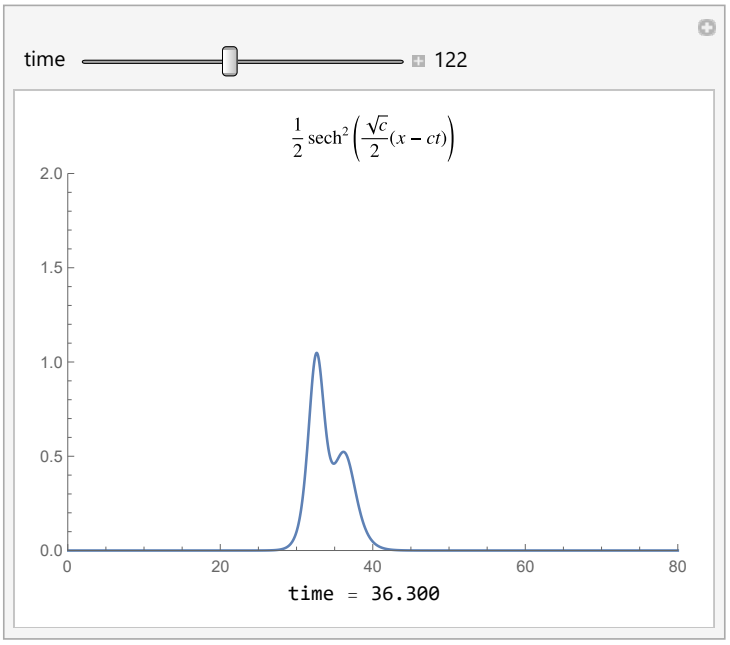


Figure 5.80: Screen shot

Source code used for the above

```
f[x_, c_, t_] := 1/2 c Sech[Sqrt[c]/2 (x - c t)]^2;
tab = Table[Grid[{
          {
            Plot[
              If[t > 20, f[x, 2, t - 20] + f[x, 1, t], f[x, 1, t]], {x, 0, 80},
              PlotRange → {{0, 80}, {0, 2}},
              ImageSize → 400,
              Frame → False,
              PlotLabel → title]
          },
          {Row[{"time = ", NumberForm[t, {5, 3}]}]}
        }
      ], {t, 0, 80, .3}
    ];
```

Figure 5.81: Source code

This animation shows three waves interacting

Out[ ]=

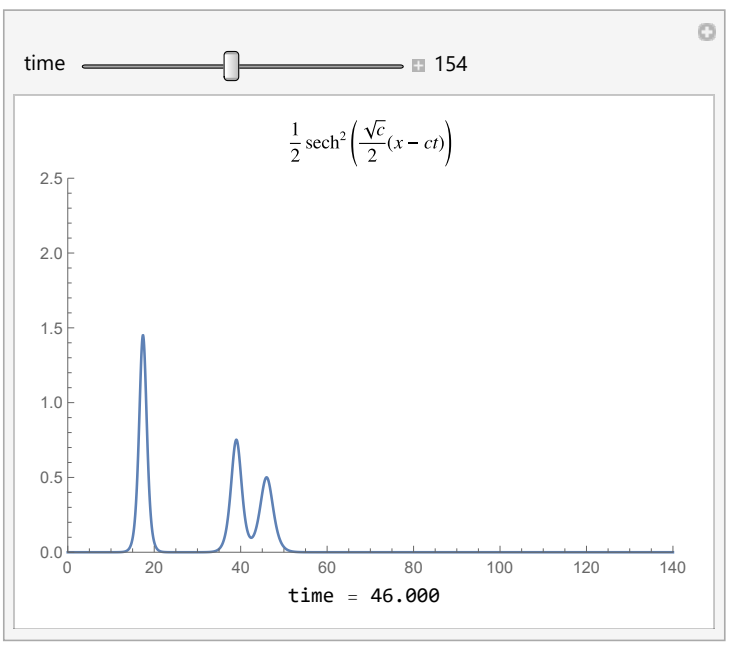


Figure 5.82: Screen shot

Source code used for the above

```
In[•]:= tab = Table[Grid[{
                {
                  Plot[
                    If[t > 40, f[x, 2.9, t - 40] + f[x, 1.5, t - 20] + f[x, 1, t],
                      If[t > 20, f[x, 1.5, t - 20] + f[x, 1, t],
                        f[x, 1, t]
                      ]
                    ], {x, 0, 140},
                    PlotRange → {{0, 140}, {0, 2.5}},
                    ImageSize → 400,
                    Frame → False,
                    PlotLabel → title]
                },
                {Row[{"time = ", NumberForm[t, {5, 3}]}]}
              }
            ],
            {t, 0.1, 100, .3}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim_2.gif", tab, "DisplayDurations" → 0.06]
```

Figure 5.83: Code for animation

5.1.3.7 [383] Inhomogeneous PDE $3u_{xx} - u_{tt} + u_{xt} = 1$

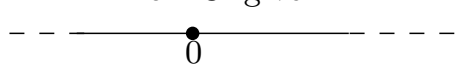
problem number 383

From Mathematica DSolve help pages. Inhomogeneous hyperbolic PDE with constant coefficients.

Solve for $u(x,t)$

$$3u_{xx} - u_{tt} + u_{xt} = 1$$

No I.C. given



$$u_{tt} = 3u_{xx} + u_{xt} - 1$$
$$-\infty < x < \infty, t > 0$$

Figure 5.84: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ode = 3*D[u[x, t], {x, 2}] - D[u[x, t], {t, 2}] + D[u[x, t], x, t] == 1;
sol =  AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{1}{6}\left(1+\sqrt{13}\right)x\right)+c_2\left(t+\frac{1}{6}\left(\sqrt{13}-1\right)x\right)+\frac{x^2}{6}\right\}\right\}$$

Maple ✓

```
restart;
pde := 3*diff(u(x, t), x$2) - diff(u(x, t),t$2)+diff(u(x, t), x,t) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, t))),output='rea
```

$$u(x,t)=-\frac{3t^2}{13}+\frac{tx}{13}+\frac{x^2}{13}+\_F1\left(\frac{x}{2}+\frac{(-6t+x)\sqrt{13}}{26}\right)+\_F2\left(t+\frac{\left(-1+\sqrt{13}\right)x}{6}\right)$$

5.1.3.8 [384] Practice exam problem Math 5587

problem number 384

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 10.

Solve for $u(x,t)$ with $u(x,0)=\sin(x)$ and $u_t(x,0)=\cos(x)$

$$u_{tt}=u_{xx}$$

$$\begin{array}{ccc} & u_t(x,0)=\cos(x) & \\ & u(x,0)=\sin(x) & \\ -\infty & \longleftrightarrow & \infty \\ & u_{tt}=u_{xx} & \end{array}$$

Figure 5.85: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==Sin[x], Derivative[0,1][u][x,0]==Cos[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \sin(t+x) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=sin(x), eval( diff(u(x,t),t),t=0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\sin(t+x)$$

5.1.3.9 [385] Practice exam problem Math 5587

problem number 385

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 11.

Solve for $u(x,t)$ with $u(x,0)=x^2$ and $u_t(x,0)=x$

$$u_{tt}=u_{xx}$$

$$\begin{array}{c} u_t(x,0)=x \\ u(x,0)=x^2 \\ -\infty \longleftarrow\!\longrightarrow \infty \\ u_{tt}=u_{xx} \end{array}$$

Figure 5.86: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==x^2, Derivative[0,1][u][x,0]==x};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} t^2+xt+x^2 & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=x^2, eval( diff(u(x,t),t),t=0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=t^2+tx+x^2$$

5.1.3.10 [386] Practice exam problem Math 5587

problem number 386

Added May 23, 2019.

From Math 5587 midterm I, Fall 2016, practice exam, problem 12.

Solve for $u(x,t)$ with $u(x,0)=0$ and $u_t(x,0)=\frac{4x}{x^2+1}$

$$u_{tt}=u_{xx}$$

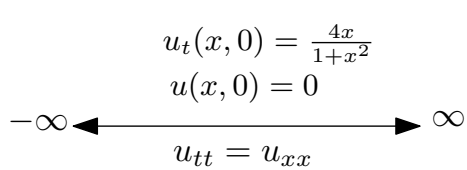


Figure 5.87: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==0, Derivative[0,1][u][x,0]==4*x/(x^2+1)};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \log\left((t+x)^2+1\right)-\log\left((t-x)^2+1\right) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=0, eval( diff(u(x,t),t),t=0)=4*x/(x^2+1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=-\ln\left(t^2-2tx+x^2+1\right)+\ln\left(t^2+2tx+x^2+1\right)$$

5.1.3.11 [387] zero initial velocity

problem number 387

Added January 9, 2020

Solve for $u(x,t)$ with $u(x,0)=\frac{1}{x^2+1}$ and $u_t(x,0)=0$

$$u_{tt}=u_{xx}$$

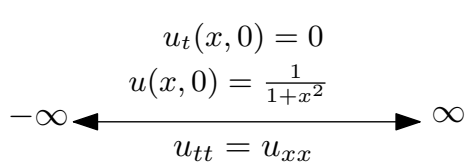


Figure 5.88: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==1/(x^2+1), Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{t^2+x^2+1}{t^4-2(x^2-1)t^2+(x^2+1)^2} & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=1/(1+x^2), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=\frac{1}{2\left(-t+x\right)^2+2}+\frac{1}{2\left(t+x\right)^2+2}$$

Hand solution

Solve the wave equation $u_{tt}=u_{xx}$ for infinite domain $-\infty<x<\infty$ with initial position $u(x,0)=\frac{1}{1+x^2}$ and zero initial velocity $g(x)=0$.

The solution for wave PDE $u_{tt}=a^2u_{xx}$ on infinite domain can be given as either series solution, or using D'Alembert solution. Using D'Alembert, the solution is

$$u(x,t)=\frac{1}{2}(f(x-at)+f(x+at))+\frac{1}{2a}\int_{x-at}^{x+at}g(s)\,ds$$

But here $c=1$ and $g(x)$ is zero. Therefore the above simplifies to

$$u(x,t)=\frac{1}{2}(f(x-t)+f(x+t))$$

Since $f(x)=\frac{1}{1+x^2}$, the above becomes

$$u(x,t)=\frac{1}{2}\left(\frac{1}{1+(x-t)^2}+\frac{1}{1+(x+t)^2}\right)$$

Animation is below

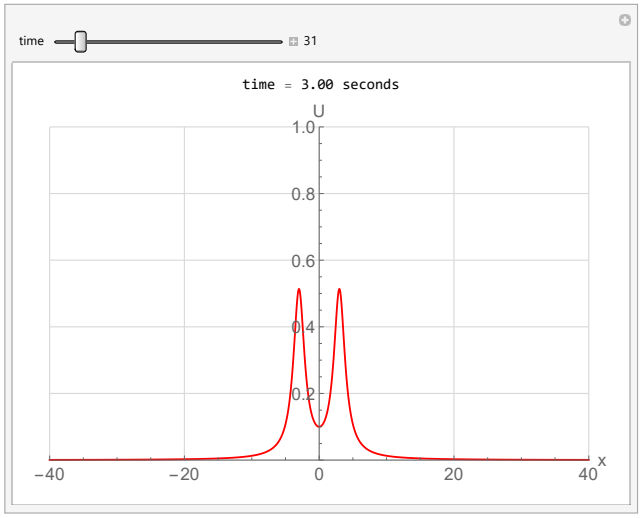


Figure 5.89: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, u]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
fLeft[x_, t_] := 1 / (1 + (x + t) ^2)
fRight[x_, t_] := 1 / (1 + (x - t) ^2)
u[x_, t_] := 1 / 2 (fRight[x, t] + fLeft[x, t]);

In[∘]:= tab = Table[
          Grid[{
                {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
                {
                   Quiet@Plot[Evaluate[u[x, t]], {x, -40, 40},
                         Exclusions → None,
                         BaseStyle → 15,
                         ImageMargins → 3,
                         PerformanceGoal → "Quality",
                         PlotRange → {{-40, 40}, {0, 1}},
                         ImageSize → 500,
                         AxesLabel → {"x", "U"},
                         GridLines → Automatic,
                         GridLinesStyle → LightGray,
                         PlotStyle → Red
                      ]
                }}],
          {t, 0, 40, 0.1}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 5.90: Source code

5.1.3.12 [388] zero initial velocity

problem number 388

Added January 10, 2020

Solve for $u(x,t)$ with $u(x,0) = \sin(x)$ from $-\pi < x < \pi$ and zero everywhere else and $u_t(x,0) = 0$

$$u_{tt} = u_{xx}$$

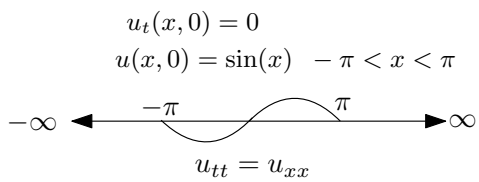


Figure 5.91: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
ic = {u[x, 0] == Piecewise[{{Sin[x], -Pi < x < Pi}, {0, True}}], Derivative[0, 1][u][x, 0] =
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{\frac{1}{2}\left(\left(\left\{\begin{array}{cc} -\sin(t-x) & t < x+\pi \wedge x < t+\pi \\ 0 & \text{True} \end{array}\right.\right) + \left(\left\{\begin{array}{cc} \sin(t+x) & -\pi < t+x < \\ 0 & \text{True} \end{array}\right.\right.\right.\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)= piecewise(-Pi<x and x<Pi,sin(x), true,0) , D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = -\frac{\left(\begin{cases} 0 & -t+x \leq -\pi \\ \sin(t-x) & -t+x < \pi \\ 0 & \pi \leq -t+x \end{cases}\right)}{2} + \frac{\left(\begin{cases} 0 & t+x \leq -\pi \\ \sin(t+x) & t+x < \pi \\ 0 & \pi \leq t+x \end{cases}\right)}{2}$$

Hand solution

The solution for wave PDE $u_{tt} = u_{xx}$ on infinite domain using D'Alembert solution with zero initial velocity is

$$\begin{aligned} u(x,t) &= \frac{1}{2}(f(x-t) + f(x+t)) \\ &= \frac{1}{2}(\sin(x-t) + \sin(x+t)) \end{aligned}$$

The following is an animation Here is animation for 10 seconds.

Animation is below

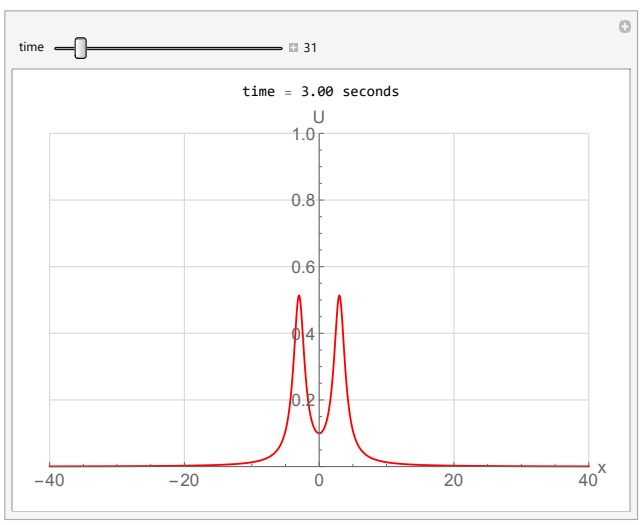


Figure 5.92: Initial state

Source code used for the above

```
In[∘]:= ClearAll[x, t, u]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
fLeft[x_, t_] := 1 / (1 + (x + t) ^2)
fRight[x_, t_] := 1 / (1 + (x - t) ^2)
u[x_, t_] := 1 / 2 (fRight[x, t] + fLeft[x, t]);

In[∘]:= tab = Table[
          Grid[{
                {Row[{"time = ", padIt2[t, {3, 2}], " seconds"}]},
                {
                   Quiet@Plot[Evaluate[u[x, t]], {x, -40, 40},
                         Exclusions → None,
                         BaseStyle → 15,
                         ImageMargins → 3,
                         PerformanceGoal → "Quality",
                         PlotRange → {{-40, 40}, {0, 1}},
                         ImageSize → 500,
                         AxesLabel → {"x", "U"},
                         GridLines → Automatic,
                         GridLinesStyle → LightGray,
                         PlotStyle → Red
                      ]
                }}],
          {t, 0, 40, 0.1}];

In[∘]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
```

Figure 5.93: Source code

5.1.3.13 [389] General case $u_{tt} = u_{xx}$ with $u(x,0) = \sin x, u_t(x,0) = -2xe^{-x^2}$

problem number 389

Added May 26, 2019.

Taken from Final exam, Math 5587 UMN, Fall 2016.

Solve for $u(x,t)$ for all x and $t > 0$ with $u(x,0) = \sin(x)$ and $u_t(x,0) = -2xe^{-x^2}$

$$u_{tt} = u_{xx}$$

$$\begin{array}{c} u(x,0) = \sin(x) \\ u_t(x,0) = -2xe^{-x^2} \\ -\infty \xleftrightarrow[u_{tt} = u_{xx}]{} \infty \end{array}$$

Figure 5.94: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0]==Sin[x], Derivative[0,1][u][x,0]==-2*x*Exp[-x^2]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(2\cos(t)\sin(x)-e^{-(t-x)^2}+e^{-(t+x)^2}\right) & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic  := u(x,0)=sin(x), eval( diff(u(x,t),t),t=0)=-2*x*exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=-\frac{\mathrm{e}^{-(t-x)^2}}{2}+\frac{\mathrm{e}^{-(t+x)^2}}{2}-\frac{\sin(t-x)}{2}+\frac{\sin(t+x)}{2}$$

5.1.3.14 [390] General case. $u_{tt}=u_{xx}$ dAlembert solution, box function as initial position

problem number 390

Added Sept, 15, 2019.

Taken from Peter Olver textbook, Introduction to Partial differential equations. Problem 2.4.2

Solve for $u(x,t)$ for all x and $t>0$ with $u(x,0)=1$ for $1<x<2$ and zero otherwise. $u_t(x,0)=0$

$$u_{tt}=u_{xx}$$

$$
\begin{array}{c}
u_t(x,0) = 0 \\
u(x,0) = 1 \text{ for } 1 < x < 2 \text{ and zero otherwise} \\
-\infty \xleftrightarrow[u_{tt} = u_{xx}]{} \infty
\end{array}
$$

Figure 5.95: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == D[u[x,t],{x,2}];
ic  = {u[x,0] == Piecewise[{{1, 1 < x < 2}, {0, True}}], Derivative[0,1][u][x,0]==0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$
\left\{\left\{u(x,t) \to \begin{cases} \frac{1}{2}\left(\left(\begin{cases} 1 & 1 < x - t < 2 \\ 0 & \text{True} \end{cases}\right) + \left(\begin{cases} 1 & 1 < t + x < 2 \\ 0 & \text{True} \end{cases}\right)\right) & t \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}
$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= diff(u(x,t),x$2);
ic:=u(x,0)=piecewise(1<x and x<2,1,true,0),eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$
u(x,t) = \left(\begin{cases} 0 & -t + x \leq 1 \\ \frac{1}{2} & -t + x < 2 \\ 0 & 2 \leq -t + x \end{cases}\right) + \left(\begin{cases} 0 & t + x \leq 1 \\ \frac{1}{2} & t + x < 2 \\ 0 & 2 \leq t + x \end{cases}\right)
$$

<u>Hand solution</u>

Solve the wave equation $u_{tt} = u_{xx}$ when the initial displacement is the box function $u(0,x) = \begin{cases} 1 & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$, while the initial velocity is zero.

<u>Solution</u>

d'Alembert solution of the wave equation is

$$u(t,x) = \frac{1}{2}(f(x-ct)+f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds$$

Where c is the wave speed which is $c = 1$ in this problem and $f(x) = u(0,x)$ and $g(x) = u_t(0,x) = 0$ in this problem. Hence the above simpifies to

$$\begin{aligned} u(t,x) &= \frac{1}{2}(f(x-t)+f(x+t)) \\ &= \frac{1}{2}\left(\begin{cases} 1 & 1 < x-t < 2 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 1 & 1 < x+t < 2 \\ 0 & \text{otherwise} \end{cases}\right) \end{aligned}$$

The following is an animation of the solution

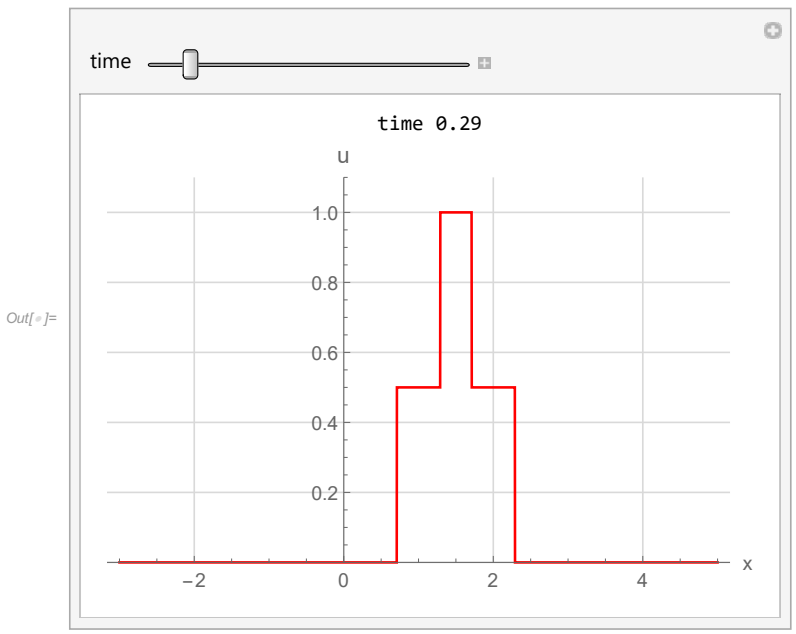


Figure 5.96: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
        NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1/2 (Piecewise[{{1, 1 < x - t < 2}, {0, True}}] + Piecewise[{{1, 1 < x + t < 2}, {0, True}}]);
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}],
         {
            Quiet@Plot[u[x, time], {x, -3, 5},
                  PlotRange → {All, {0, 1.1}},
                  AxesLabel → {Style["x", 12], Style["u", 14]},
                  BaseStyle → 12,
                  ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
                  GridLinesStyle → LightGray,
                  Exclusions → None
               ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 3, .01},
   TrackedSymbols :→ {time}
]
```

Figure 5.97: Source code 2D

5.1.3.15 [391] $u_{tt} = 4u_{xx} + \cos(t)$ dAlembert solution with $u(x,0) = \sin x, u_t(x,0) = \cos x$

problem number 391

Added Oct 8, 2019

Exam 1 problem, math 5587, fall 2019, UMN.

Solve for $u(x,t)$

$$u_{tt} = 4u_{xx} + \cos(t)$$

With initial conditions $u(x,0) = \sin x, u_t(x,0) = \cos x$

$$\begin{array}{c} u_t(x,0) = \sin x \\ u(x,0) = \cos x \\ -\infty \xleftrightarrow[u_{tt} = 4u_{xx} + \cos t]{} \infty \end{array}$$

Figure 5.98: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t,2}] == 4* D[u[x,t],{x,2}]+ Cos[t];
ic  = {u[x,0]== Sin[x], Derivative[0,1][u][x,0]==Cos[x]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde,ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \begin{cases} \sin(x)\cos^2(t)+(\cos(x)\sin(t)-1)\cos(t)-\sin^2(t)\sin(x)+1 & t\geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)= 4*diff(u(x,t),x$2)+cos(t);
ic:=u(x,0)=sin(x),D[2](u)(x,0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t)=-\cos(t)-\frac{\sin(2t-x)}{4}+\frac{3\sin(2t+x)}{4}+1$$

Hand solution

Solve the wave equation $u_{tt}=4u_{xx}+\cos t$ when initial conditions $u(x,0)=\sin x, u_t(x,0)=\cos x$

Solution

d'Alembert solution of the wave equation is

$$u(t,x)=\frac{1}{2}(f(x-ct)+f(x+ct))+\frac{1}{2c}\int_{x-ct}^{x+ct}g(s)\,ds+\frac{1}{2c}\int_0^t\int_{x-c(t-s)}^{x+c(t-s)}F(s)\,dyds$$

Where c is the wave speed which is $c=2$ in this problem and $f(x)=u(0,x)=\sin x$ and $g(x)=u_t(0,x)=\cos x$ and the force $F(t)=\cos t$ in this problem. Hence the above simpifies to

$$u(t,x)=\frac{1}{2}(\sin(x-2t)+\sin(x+2t))+\frac{1}{4}\int_{x-2t}^{x+2t}\cos(s)\,ds+\frac{1}{4}\int_0^t\int_{x-2(t-s)}^{x+2(t-s)}\cos(s)\,dyds$$

But $\frac{1}{4}\int_{x-2t}^{x+2t}\cos(s)\,ds=\frac{1}{4}[\sin(s)]_{x-2t}^{x+2t}=\frac{1}{4}(\sin(x+2t)-\sin(x-2t))$. Hence the above

becomes

$$
\begin{aligned}
u(t,x) &= \frac{1}{2}(\sin(x-2t)+\sin(x+2t)) + \frac{1}{4}(\sin(x+2t)-\sin(x-2t)) + \frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds \\
&= \frac{1}{4}\sin(x-2t) + \frac{3}{4}\sin(x+2t) + \frac{1}{4}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} \cos(s)\,dyds \qquad (1A)
\end{aligned}
$$

But

$$
\begin{aligned}
\frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds &= \frac{1}{4}\int_0^t \cos(s) \int_{x-2(t-s)}^{x+2(t-s)} dyds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(x+2(t-s)-x+2(t-s))\,ds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(2(t-s)+2(t-s))\,ds \\
&= \frac{1}{4}\int_0^t \cos(s)\,(2t-2s+2t-2s)\,ds \\
&= \int_0^t \cos(s)\,(t-s)\,ds \\
&= \int_0^t t\cos(s)\,ds - \int_0^t s\cos(s)\,ds \\
&= t[\sin(s)]_0^t - \int_0^t s\cos(s)\,ds \\
&= t\sin t - \int_0^t s\cos(s)\,ds \qquad (1)
\end{aligned}
$$

Integration by parts. $udv = uv - \int vdu$. Let $u = s, dv = \cos s$, then $du = 1, v = \sin(s)$, then

$$
\begin{aligned}
\int_0^t s\cos(s)\,ds &= [s\sin(s)]_0^t - \int_0^t \sin s ds \\
&= t\sin t - [-\cos s]_0^t \\
&= t\sin t + (\cos t - 1) \qquad (2)
\end{aligned}
$$

Using (2) in (1) gives

$$
\begin{aligned}
\frac{1}{4}\int_0^t \int_{x-2(t-s)}^{x+2(t-s)} \cos(s)\,dyds &= t\sin t - (t\sin t + (\cos t - 1)) \\
&= 1 - \cos t
\end{aligned}
$$

Substituting the above in (1A) gives

$$
u(x,t) = \frac{1}{4}\sin(x-2t) + \frac{3}{4}\sin(x+2t) + 1 - \cos t
$$

The following is an animation of the solution

Out[ ]=

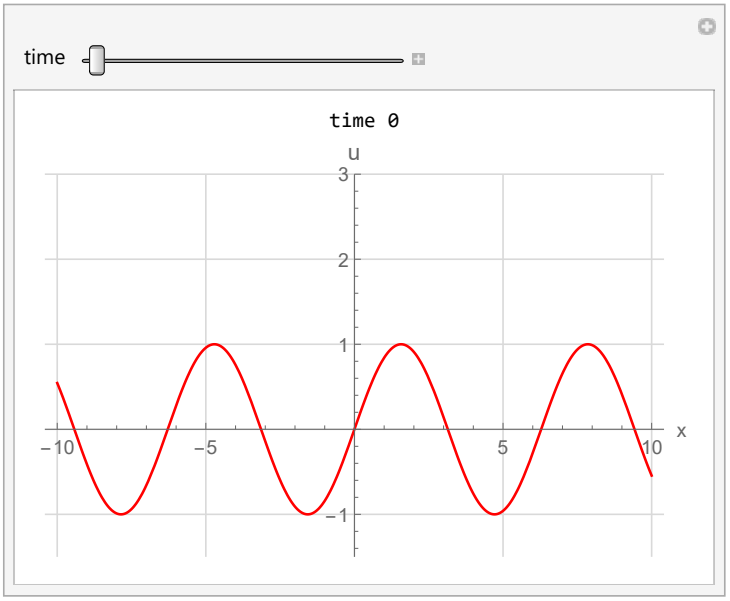


Figure 5.99: snap shot

Source code used for the above

```
In[ ]:=
(*2D*)
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""},
         NumberPadding → {"0", "0"}, SignPadding → True];
u[x_, t_] := 1 / 4 * Sin[x - 2 t] + 3 / 4 * Sin[x + 2 t] + 1 - Cos[t];
Manipulate[
   Grid[{{Row[{"time ", NumberForm[time, {4, 2}]}]}],
         {
            Plot[u[x, time], {x, -10, 10},
               PlotRange → {All, {-1.5, 3}},
               AxesLabel → {Style["x", 12], Style["u", 14]},
               BaseStyle → 12,
               ImageSize → 400, PlotStyle → Red, GridLines → Automatic,
               GridLinesStyle → LightGray,
               Exclusions → None
            ]
         }}
   ]
   ,
   {{time, 0, "time"}, 0, 30, .01},
   TrackedSymbols :→ {time}
]
```

Figure 5.100: Source code 2D

5.1.3.16 [392] $u_{tt} = c^2 u_{xx}$ dAlembert solution with $u(x,0) = \delta(x-a), u_t(x,0) = 0$

problem number 392

Added January 8, 2020

Problem 6.3.27 Introduction to Partial Differential Equations by Peter Olver, ISBN 9783319020983.

Consider the wave equation $u_{tt} = c^2 u_{xx}$ on the line $-\infty < x < \infty$. Use the d'Alembert formula to solve the initial value problem $u(x,0) = \delta(x-a)\,, u_t(x,0) = 0$.

$$u(x,t) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds \tag{2.82}$$

$$u_t(x,0) = 0$$
$$u(x,0) = \delta(x-a)$$
$$-\infty \longleftrightarrow \infty$$
$$u_{tt} = c^2 u_{xx}$$

Figure 5.101: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
ic = {u[x, 0] == DiracDelta[x - a], Derivative[0, 1][u][x, 0] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> a
```

$$\left\{\left\{u(x,t) \to \begin{cases} \delta(x-a) & x \geq 0 \wedge t = 0 \\ \frac{2\int_0^\infty \cos(aK[1])\cos\left(\sqrt{c^2}tK[1]\right)\cos(xK[1])dK[1]}{\pi} & x \geq 0 \wedge t > 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic  := u(x,0)=Dirac(x-a), D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \frac{\delta(-ct+a-x)}{2} + \frac{\delta(ct+a-x)}{2}$$

<u>Hand solution</u>

$$u(x,t) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds \tag{2.82}$$

In (2.82), the function f is the initial conditions and the function g is the initial velocity. Hence the above becomes

$$u(x,t) = \frac{1}{2}(\delta((x-a)-ct) + \delta((x-a)+ct))$$

But $\delta((x-a)-ct) = \delta(x-a-ct) = \delta(x-(a+ct))$ and $\delta((x-a)+ct) = \delta(x-a+ct) = \delta(x-(a-ct))$. Hence the above becomes

$$u(x,t) = \frac{1}{2}\delta(x-(a+ct)) + \frac{1}{2}\delta(x-(a-ct)) \tag{1}$$

The above is two half strength delta pulses, one traveling to the left and one traveling to the right from the starting position.

5.1.3.17 [393] system of 2 inhomogeneous linear hyperbolic system with constant coefficients

problem number 393

From Mathematica DSolve help pages

Solve for $u(x,t), v(x,t)$

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial v}{\partial x} + 1\\ \frac{\partial v}{\partial t} &= -\frac{\partial u}{\partial x} - 1\end{aligned}$$

With initial conditions

$$\begin{aligned}u(x,0) &= \cos^2 x\\ v(x,0) &= \sin x\end{aligned}$$

Mathematica ✓

```
ClearAll["Global`*"];
eqns = {D[u[x, t], t] == D[v[x, t], x] + 1, D[v[x, t], t] == -D[u[x, t], x] - 1};
ic  = {u[x, 0] == Cos[x]^2, v[x, 0] == Sin[x]};
sol =  AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{eqns, ic}, {u[x, t], v[x, t]}, {x
```

$$\left\{\left\{u(x,t) \to \sinh(t)\cos(x) + \frac{1}{2}\cosh(2t)\cos(2x) + t + \frac{1}{2}, v(x,t) \to \cosh(t)\sin(x)(2\sinh(t)\cos(x)+1) - \right.\right.$$

Maple ✗

```
restart;
pde1 := diff(u(x, t), t) = diff(v(x, t), x) + 1;
pde2 := diff(v(x, t), t) = -diff(u(x, t), x) - 1;
ic  := u(x, 0) = cos(x)^2, v(x, 0) = sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2, ic], {u(x, t),
```

sol=()

5.2 Wave PDE in 2D

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5.2.1 Cartesian coordinates

Local contents

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5.2.1.1 [394] Rectangular membrane. Fixed on all edges, General solution

problem number 394

Added January 10, 2020.

Solve

$$u_{tt} = c^4(u_{xx} + u_{yy})$$

$$0 < x < L$$
$$0 < y < H$$

Boundary conditions on x

$$u(0, y, t) = 0$$
$$u(L, y, t) = 0$$

And boundary conditions on y

$$u(x,0,t) = 0$$
$$u(x,H,t) = 0$$

Initial conditions

$$u(x,y,0) = f(x,y)$$
$$\frac{\partial u}{\partial t}(x,y,0) = g(x,y)$$

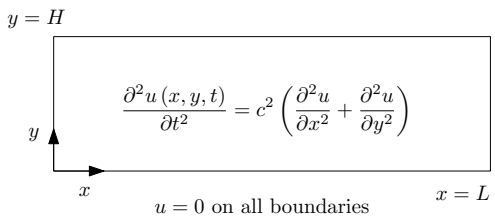


Figure 5.102: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}];
ic = {Derivative[0, 0, 1][u][x, y, 0] == g[x, y], u[x, y, 0] == f[x, y]};
bc = {u[0, y, t] == 0, u[0, H, t] == 0, u[x, 0, t] == 0, u[x, L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t},Assumption
```

Failed

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) = c^2*VectorCalculus:-Laplacian(u(x,y,t),[x,y]);
bc  := u(0,y,t)=0,
       u(L,y,t)=0,
       u(x, 0, t) = 0,
       u(x, H, t) = 0;
ic  := u(x, y, 0) = f(x,y),(D[3](u))(x, y, 0) = g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))assum
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\left(HL\left(\left(\int_0^H()\sin\left(\frac{\pi m y}{H}\right)dy\right)_{AllSolutions}\right)\sin\left(\frac{\pi\sqrt{H^2n^2+L^2m^2}\,ct}{HL}\right)+\pi\sqrt{H^2n^2+L^2m^2}\,c\left(\left(\right.\right.\right.}{\sqrt{H^2n^2+L^2m^2}\,\pi}$$

Hand solution

Assuming $u=X(x)\,Y(y)\,T(t)$ and substituting into the PDE gives

$$\frac{1}{c^2}T''XY=X''YT+Y''XT$$
$$\frac{1}{c^2}\frac{T''}{T}=\frac{X''}{X}+\frac{Y''}{Y}$$

Therefore

$$\frac{1}{c^2}\frac{T''}{T}=-\lambda$$
$$\frac{X''}{X}+\frac{Y''}{Y}=-\lambda$$

The time ODE becomes

$$T''+c^2\lambda T=0$$

And the space ODE is

$$\frac{X''}{X}+\frac{Y''}{Y}=-\lambda$$

Separating this again gives

$$\frac{X''}{X}=-\lambda-\frac{Y''}{Y}$$

Let the second separation variable be μ. This gives two new ODE's to solve

$$\frac{X''}{X}=-\mu$$
$$-\lambda-\frac{Y''}{Y}=-\mu$$

Or

$$X'' + \mu X = 0$$
$$Y'' + Y(\lambda - \mu) = 0$$

Solving for $X(x)$ ODE first, and knowing that only $\mu > 0$ will give non trivial solutions (from the nature of the boundary conditions), gives the solution as

$$X(x) = A\cos\left(\sqrt{\mu}x\right) + B\sin\left(\sqrt{\mu}x\right)$$

Applying B.C. at $x = 0$ results in

$$0 = A$$

Therefore $X(x) = B\sin\left(\sqrt{\mu}x\right)$. Applying the B.C. at $x = L$ gives

$$0 = B\sin\left(\sqrt{\mu}L\right)$$

For non trivial solution

$$\sqrt{\mu}L = n\pi$$
$$\mu = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Therefore the $X_n(x)$ eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Now, solving the $Y(y)$ ODE above

$$Y'' + Y\left(\lambda - \left(\frac{n\pi}{L}\right)^2\right) = 0$$

The solution is

$$Y_n(y) = A\cos\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right) + B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right)$$

Applying first B.C. gives

$$0 = A$$

Hence

$$Y_n(y) = B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right)$$

Applying the second B.C. gives

$$0 = B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}H\right)$$

For non trivial solution

$$\begin{aligned}
\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}H &= m\pi \qquad m = 1,2,3,\cdots \\
\lambda_{nm} - \left(\frac{n\pi}{L}\right)^2 &= \left(\frac{m\pi}{H}\right)^2 \\
\lambda_{nm} &= \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots
\end{aligned}$$

Hence the $Y_{nm}(y)$ solution is

$$Y_{nm} = B_{nm}\sin\left(\frac{m\pi}{H}y\right) \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots$$

The time ode $T(t)$ is now solved

$$\begin{aligned}
T''_{nm} + c^2\lambda_{nm}T_{nm} &= 0 \\
T_{nm}(t) &= A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)
\end{aligned}$$

Combining all solutions, and merging all constants into two results in

$$\begin{aligned}
u_{nm}(x,y,t) &= X_n(x)\,Y_{nm}(y)\,T_{nm}(t) \\
u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_m(x)\,Y_{mn}(y)\,T_{mn}(t) \\
&= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\lambda_{nm}}t\right) \\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\lambda_{nm}}t\right) \qquad (1)
\end{aligned}$$

Initial conditions are now used to find A_{nm}, B_{nm}. At $t = 0$

$$\begin{aligned}
u(x,y,0) &= f(x,y) \\
\frac{\partial u}{\partial t}(x,y,0) &= g(x,y)
\end{aligned}$$

Applying first initial condition to (1) gives

$$f(x,y) = \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{m\pi}{H}y\right)\right)\sin\left(\left(\frac{n\pi}{L}\right)x\right)$$

Applying 2D orthogonality gives

$$\int_0^L \int_0^H f(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy = A_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right)$$

$$A_{nm} = \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy$$

Taking time derivative of (1) gives

$$\begin{aligned}\frac{\partial u}{\partial t}(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}}A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\lambda_{nm}}t\right)\\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\lambda_{nm}}t\right)\end{aligned}$$

At $t=0$ the above becomes

$$g(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\sin\left(\left(\frac{n\pi}{L}\right)x\right)\sin\left(\frac{m\pi}{H}y\right)$$

Applying 2D orthogonality gives

$$\int_0^L \int_0^H g(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy = B_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right)$$

$$B_{nm} = \frac{4}{LH}\int_0^L \int_0^H g(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy$$

<u>Summary of solution</u>

$$u(x,y,t) = \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\lambda_{nm}}t\right)\right)\sin\left(\frac{n\pi}{L}x\right) + \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\lambda_{n}}\right.\right.$$

$$A_{nm} = \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\mathrm{si}$$

$$B_{nm} = \frac{4}{LH}\int_0^L \int_0^H g(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\mathrm{si}$$

$$\lambda_{nm} = \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2$$

5.2.1.2 [395] Rectangular membrane. Fixed on all edges, zero velocity. Specific example

problem number 395

Added January 10, 2020.

Solve

$$u_{tt} = c^4(u_{xx} + u_{yy})$$

$$0 < x < L$$
$$0 < y < H$$

Boundary conditions on x

$$u(0, y, t) = 0$$
$$u(L, y, t) = 0$$

And boundary conditions on y

$$u(x, 0, t) = 0$$
$$u(x, H, t) = 0$$

Initial conditions

$$u(x, y, 0) = f(x, y)$$
$$\frac{\partial u}{\partial t}(x, y, 0) = g(x, y)$$

Using $L = 1, H = 2, c = \frac{1}{10}, f(x, y) = x\cos(y), g(x, y) = 0$.

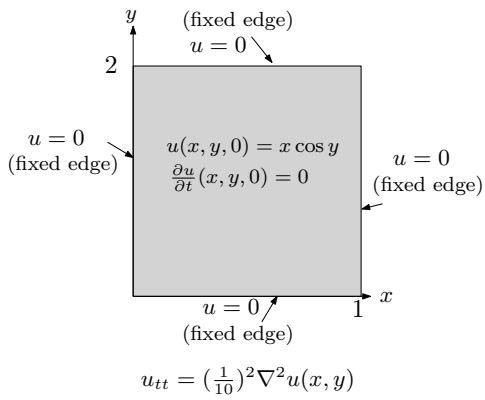


Figure 5.103: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
L=1;H=2;c=1/10;
f[x_,y_]:=x*Cos[y];
g[x_,y_]:=0;
pde = D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}];
ic = {Derivative[0, 0, 1][u][x, y, 0] == g[x, y], u[x, y, 0] == f[x, y]};
bc = {u[0, y, t] == 0, u[0, H, t] == 0, u[x, 0, t] == 0, u[x, L, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
L:=1;
H:=2;
c:=1/10;
f:=(x,y)->x*cos(y);
g:=(x,y)->0;
pde := diff(u(x, y, t), t$2) = c^2*VectorCalculus:-Laplacian(u(x,y,t),[x,y]);
bc  := u(0,y,t)=0,
       u(L,y,t)=0,
       u(x, 0, t) = 0,
       u(x, H, t) = 0;
ic  := u(x, y, 0) = f(x,y),(D[3](u))(x, y, 0) = g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))),out
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\left(-(-1)^{n}+\cos(2)(-1)^{m+n}\right)m\cos\left(\frac{\pi\sqrt{m^{2}+4n^{2}}\,t}{20}\right)\sin(\pi nx)\sin\left(\frac{\pi my}{2}\right)}{(\pi^{2}m^{2}-4)\,n}$$

Hand solution

The basic solution for this type of PDE was already given in problem 5.2.1.1 on page 1077 as

$$u(x,y,t)=\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}A_{nm}\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\lambda_{nm}}t\right)\right)\sin\left(\frac{n\pi}{L}x\right)+\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}B_{nm}\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\lambda_{n}}\right.\right.$$

$$A_{nm}=\frac{4}{LH}\int_{0}^{L}\int_{0}^{H}f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\right.$$

$$B_{nm}=\frac{4}{LH}\int_{0}^{L}\int_{0}^{H}g(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\right.$$

$$\lambda_{nm}=\left(\frac{m\pi}{H}\right)^{2}+\left(\frac{n\pi}{L}\right)^{2}\qquad n=1,2,3,\cdots$$

In this problem

$$
\begin{aligned}
L &= 1\\
H &= 2\\
c &= \frac{1}{10}\\
f(x,y) &= x\cos y\\
g(x,y) &= 0
\end{aligned}
$$

Hence the solution becomes

$$
\begin{aligned}
u(x,y,t) = \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{m\pi}{2}y\right)\cos\left(\frac{1}{10}\sqrt{\lambda_{nm}}t\right)\right)\sin(n\pi x) + \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{m\pi}{2}y\right)\sin\left(\frac{1}{10}\sqrt{}\right.\right.\\
A_{nm} = 2\int_0^1\int_0^2 x\cos y\sin(n\pi x)\sin\left(\frac{m\pi}{2}y\right.\\
B_{nm} = 0\\
\lambda_{nm} = \left(\frac{m\pi}{2}\right)^2 + (n\pi)^2 \qquad n = 1,2,3,\cdots
\end{aligned}
$$

But

$$
\begin{aligned}
A_{nm} &= 2\int_0^1\int_0^2 x\cos y\sin(n\pi x)\sin\left(\frac{m\pi}{2}y\right)\,dxdy\\
&= \frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)}
\end{aligned}
$$

Hence the solution simplifies to

$$
\begin{aligned}
u(x,y,t) &= \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)}\sin\left(\frac{m\pi}{2}y\right)\cos\left(\frac{1}{10}\sqrt{\left(\frac{m\pi}{2}\right)^2+(n\pi)^2}t\right)\right)\sin(n\pi x\\
&= \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\frac{4(-1)^n m(-1+(-1)^m\cos(2))}{n(m^2\pi^2-4)}\cos\left(\pi\frac{\sqrt{m^2+4n^2}}{20}t\right)\sin\left(\frac{m\pi}{2}y\right)\right)\sin(n\pi x)
\end{aligned}
$$

Animation is below

Out[ ]=

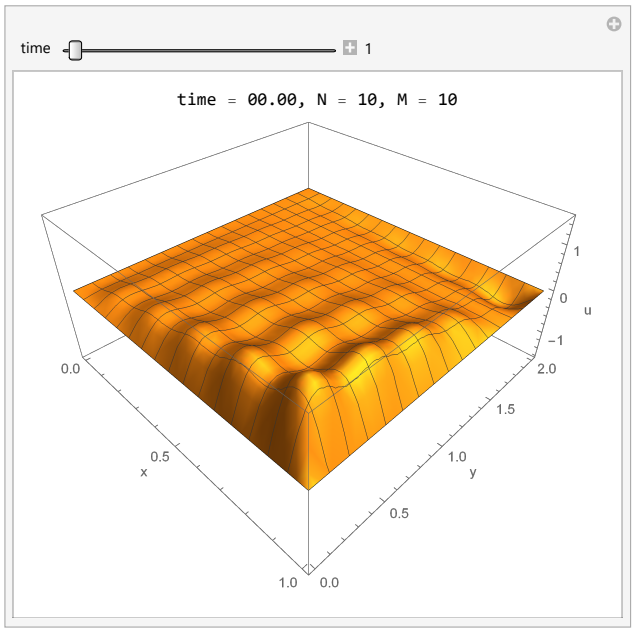


Figure 5.104: Initial state

Source code used for the above

```
ClearAll[x, t, u]
max = 10;
L0 = 1;
H0 = 2;
u[x_, y_, t_] := Sum[Sum[4 (-1)^n m (-1 + (-1)^m Cos[2]) / (n (m^2 π^2 - 4)) Cos[π Sqrt[m^2 + 4 n^2] / 20 t] Sin[m π / 2 y] * Sin[n π x], {m, 1, max}],
    {n, 1, max}]
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 5.105: Source code

```
tab = Table[
        Grid[{
              {Row[{"time = ", padIt2[t, {4, 2}], ", N = ", 10, ", M = ", 10}]},
              {Plot3D[Evaluate[u[x, y, t]], {x, 0, L0}, {y, 0, H0},
                    PlotRange → {Automatic, Automatic, {-1.65, 1.6}},
                    AxesLabel → {"x", "y", "u"}, BaseStyle → 10,
                    ImageMargins → 5, ViewPoint → {1, -1, 1},
                    BoxRatios → {1, 1, .5}, ImageSize → 400,
                    PerformanceGoal → "Quality"]}}
        ],
        {t, 0, 15, .1}
      ];
In[ ]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]
In[ ]:= Export["anim.gif", tab, "DisplayDurations" → 0.15] (*,"AnimationRepetitions"→Infinity]*)
```

Figure 5.106: Source code

5.2.1.3 [396] All 4 edges fixed, zero initial velocity, Specific example

problem number 396

Added June 17, 2019

Solve for $u(x,y,t)$ with $0 < x < L$ and $0 < y < H$ and $t > 0$.

Solve

$$u_{tt} = c^2 \nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned} u(x,0,t) &= 0 \\ u(0,y,t) &= 0 \\ u(L,y,t) &= 0 \\ u(x,H,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= 3 f_1(x) f_2(y) \\ u_t(x,y,0) &= 0 \end{aligned}$$

And

$$f_1(x) = \begin{cases} x & 0 < x < \frac{L}{2} \\ L - x & \frac{L}{2} < x < L \end{cases}$$

Where

$$f_2(y) = \begin{cases} y & 0 < y < \frac{H}{2} \\ H - y & \frac{H}{2} < y < H \end{cases}$$

And $L = 2, H = 3$ and $c = \frac{1}{3}$.

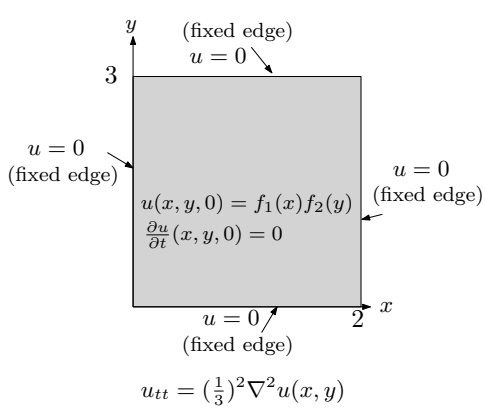


Figure 5.107: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L=2;
H=3;
c=1/3;
f1[x_] :=Piecewise[{{x, x < L/2}, {L - x, x > L/2}}];
f2[y_] := Piecewise[{{y, y < H/2}, {H - y, y > H/2}}];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == 3*f1[x]*f2[y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to\begin{cases}\sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{288\cos\left(\frac{1}{18}\pi t\sqrt{9n^2+4K[3]^2}\right)\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{2}\right)\sin\left(\frac{1}{2}\pi K[3]\right)\sin\left(\frac{1}{3}\pi yK[3]\right)}{n^2\pi^4K[3]^2} & (n|K[3])\in\mathbb{Z}\wedge n \\ \text{Indeterminate} & \text{Tru}\end{cases}\right.\right.$$

Maple ✓

```
restart;
L   := 2;
H   := 3;
c   := 1/3;
f1  := x-> piecewise(x < L/2,x, x > L/2,L - x);
f2  := y-> piecewise(y < H/2,y, y > H/2,H - y);
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=3*f1(x)*f2(y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{288\cos\left(\frac{\pi\sqrt{4m^2+9n^2}\,t}{18}\right)\sin\left(\frac{\pi m}{2}\right)\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi my}{3}\right)\sin\left(\frac{\pi nx}{2}\right)}{\pi^4m^2n^2}$$

Hand solution

The basic solution for this type of PDE was already given in problem 5.2.1.8 on page

1103 as

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right)$$

$$A_{nm}=\frac{4}{LH}\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\,dxdy$$

In this problem

$$L=2$$
$$H=3$$
$$c=\frac{1}{3}$$
$$f(x,y)=3f_1(x)\,f_2(y)$$

And

$$f_1(x)=\begin{cases}x & 0<x<\frac{L}{2}\\ L-x & \frac{L}{2}<x<L\end{cases}$$

And

$$f_2(y)=\begin{cases}y & 0<y<\frac{H}{2}\\ H-y & \frac{H}{2}<y<H\end{cases}$$

This is animation of the above solution using these specific values for for 40 seconds. (Animation will only show in the HTML version)

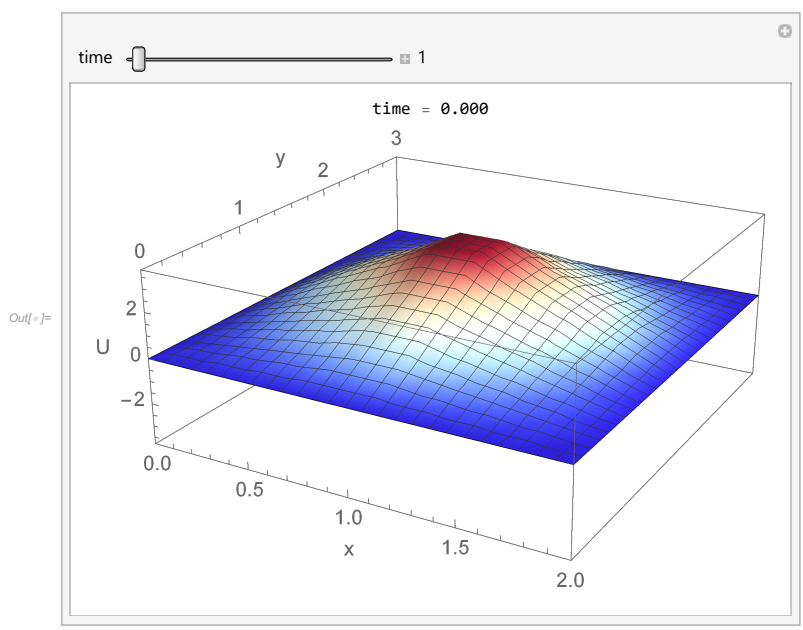


Figure 5.108: Initial state

Source code used for the above

```
ClearAll[x, y, t, mySol]
L = 2;
H = 3;
c = 0.3;
f1[x_] := Piecewise[{{x, 0 ≤ x ≤ L / 2}, {L - x, L / 2 < x ≤ L}}];
f2[y_] := Piecewise[{{y, 0 ≤ y ≤ H / 2}, {H - y, H / 2 < y ≤ H}}];
numberOfTerms = 10;
Anm = N[4/(L H) * Table[Print["n=", n, ", m=", m]; Integrate[3 f1[x] * f2[y] Sin[n π/L x] Sin[m π/H y], {x, 0, L}, {y, 0, H}],
      {n, 1, numberOfTerms}, {m, 1, numberOfTerms}]];
mySol[x_, y_, t_] = Chop@Sum[Anm[[n, m]] Sin[n π/L x] Sin[m π/H y] Cos[c t Sqrt[(m π/H)^2 + (n π/L)^2]],
      {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 5.109: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}]}]},
              {Plot3D[Evaluate[mySol[x, y, t]], {x, 0, L}, {y, 0, H},
                  BaseStyle → 15,
                  ImageMargins → 3,
                  Mesh → 25,
                  PerformanceGoal → "Speed",
                  BoxRatios → {1, 1, 0.4},
                  PlotRange → {{0, L}, {0, H}, {-3.8, 3.8}},
                  ImageSize → 500,
                  ColorFunctionScaling → True,
                  (*ColorFunction→ColorData[{"TemperatureMap",{0,.5}}],*)
                  AxesLabel → {"x", "y", "U"},
                  ColorFunction → Function[{x, y, z}, ColorData["ThermometerColors"][z]],
                  ViewPoint → {1.542, -2.736, 1.258}
                ]
              }}],
          {t, 0, 40, .3}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → Table[.3, {Length[tab]}]]
```

Figure 5.110: Code used for animation

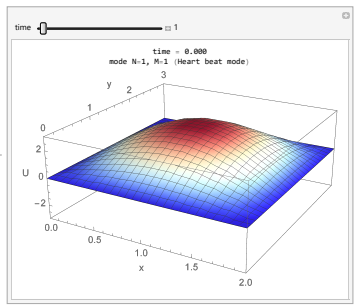
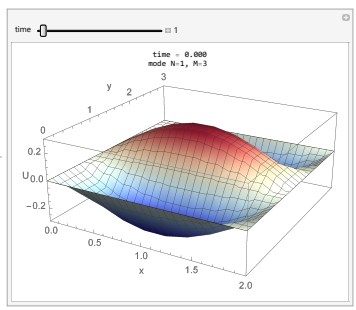
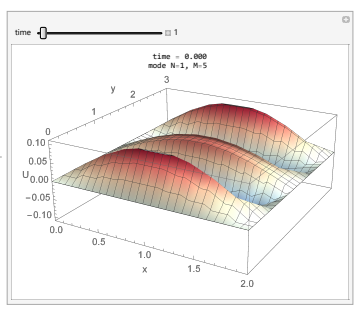
The following shows selected modes. For example, for $n = 1, m = 1$ the solution becomes

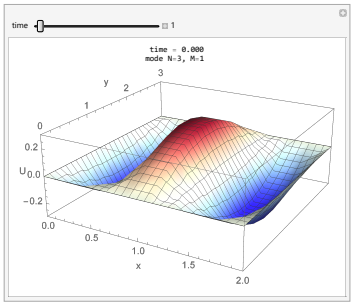
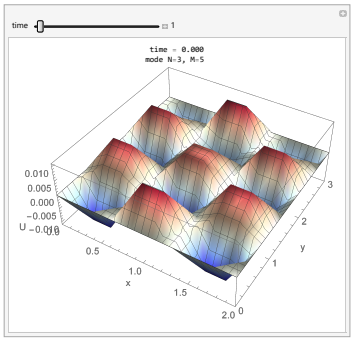
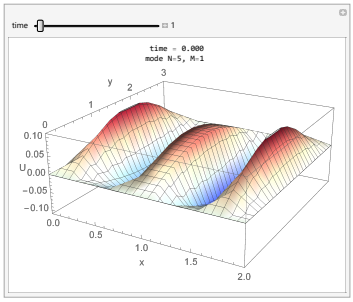
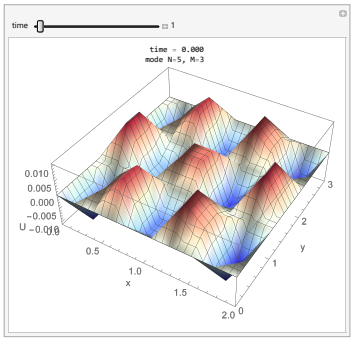
$$u(x, y, t) = A_{1,1} \sin\left(\frac{\pi}{L}x\right) \sin\left(\frac{\pi}{H}y\right) \cos\left(c\sqrt{\left(\frac{\pi}{H}\right)^2 + \left(\frac{1\pi}{L}\right)^2}\, t\right)$$

And for $n = 1, m = 5$ then the solution becomes

$$u(x,y,t) = A_{1,5} \sin\left(\frac{\pi}{L}x\right) \sin\left(\frac{5\pi}{H}y\right) \cos\left(c\sqrt{\left(\frac{5\pi}{H}\right)^2 + \left(\frac{\pi}{L}\right)^2}\, t\right)$$

And so on.

n	m	animation
1	1	
1	3	
1	5	

3	1	 time 1 time = 0.000 mode N=3, M=1 y U x
3	5	 time 1 time = 0.000 mode N=3, M=5 U x y
5	1	 time 1 time = 0.000 mode N=5, M=1 y U x
5	3	 time 1 time = 0.000 mode N=5, M=3 U x y

5.2.1.4 [397] All 4 edges fixed, zero initial velocity, Specific example, delta in center

problem number 397

Added June 18, 2019

Solve for $u(x,y,t)$ with $0 < x < L$ and $0 < y < H$ and $t > 0$.

Solve

$$u_{tt} = c^2 \nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned} u(x,0,t) &= 0 \\ u(0,y,t) &= 0 \\ u(L,y,t) &= 0 \\ u(x,H,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= f(x,y) \\ u_t(x,y,0) &= 0 \end{aligned}$$

And

$$\begin{aligned} L &= 20 \\ H &= 30 \\ c &= \frac{1}{3} \\ f(x,y) &= f_1(x)\, f_2(y) \end{aligned}$$

Where $f(x,y)$ is an approximation of delta in the middle of the membrane

$$f_1(x) = \begin{cases} 1 & \frac{45}{100}L < x < \frac{55}{100}L \\ 0 & \text{otherwise} \end{cases}$$

And

$$f_2(y) = \begin{cases} 1 & \frac{45}{100}H < y < \frac{55}{100}H \\ 0 & \text{otherwise} \end{cases}$$

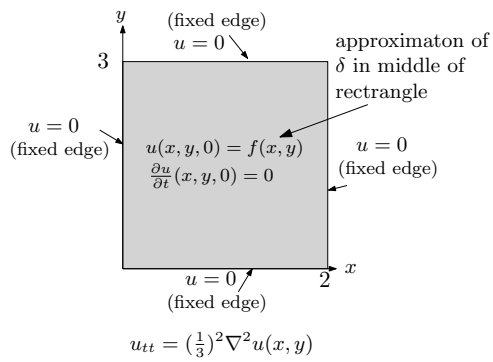


Figure 5.111: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
L  = 20;
H  = 30;
c  = 1/3;
f1[x] := Piecewise[{{1,45/100*L <= x<= 55/100*L},{0,True}}];
f2[y] := Piecewise[{{1,45/100*H<= y<= 55/100*H},{0,True}}];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == f1[x]*f2[y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\left(\cos\left(\frac{9n\pi}{20}\right)-\cos\left(\frac{11n\pi}{20}\right)\right)\left(\cos\left(\frac{9}{20}\pi K[3]\right)-\cos\left(\frac{11}{20}\pi K[3]\right)\right)\cos\left(\frac{1}{180}\pi t\sqrt{9n^2+4K[3]^2}\right)\sin\left(\frac{n\pi x}{20}\right)\sin\left(\frac{1}{30}\pi\right.}{n\pi^2K[3]}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
L   := 20;
H   := 30;
c   := 1/3;
f1  := x-> piecewise(x>45/100*L and x< 55/100*L,1, true,0);
f2  := y-> piecewise(y>45/100*H and y< 55/100*H,1, true,0);
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=f1(x)*f2(y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{16\left(-2\cos\left(\frac{\pi n}{5}\right)+2\cos\left(\frac{\pi n}{10}\right)-1\right)\left(-2\cos\left(\frac{\pi n}{10}\right)+2\cos\left(\frac{\pi n}{20}\right)-1\right)\left(2\cos\left(\frac{\pi n}{10}\right)+2\cos\left(\right.\right.}{}$$

Hand solution

The basic solution for this type of PDE was already given in problem 5.2.1.8 on page 1103 as

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}\,t\right)$$

$$A_{nm} = \frac{4}{LH}\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\,dxdy$$

In this problem

$$L = 20$$
$$H = 30$$
$$c = \frac{1}{3}$$
$$f(x,y) = f_1(x)\,f_2(y)$$

Where $f(x,y)$ is an approximation of delta in the middle of the membrane

$$f_1(x) = \begin{cases} 1 & \frac{45}{100}L < x < \frac{55}{100}L \\ 0 & \text{otherwise} \end{cases}$$

And

$$f_2(y) = \begin{cases} 1 & \frac{45}{100}H < y < \frac{55}{100}H \\ 0 & \text{otherwise} \end{cases}$$

This is animation of the above solution using these specific values for for 40 seconds. (Animation will only show in the HTML version)

Out[ ]=

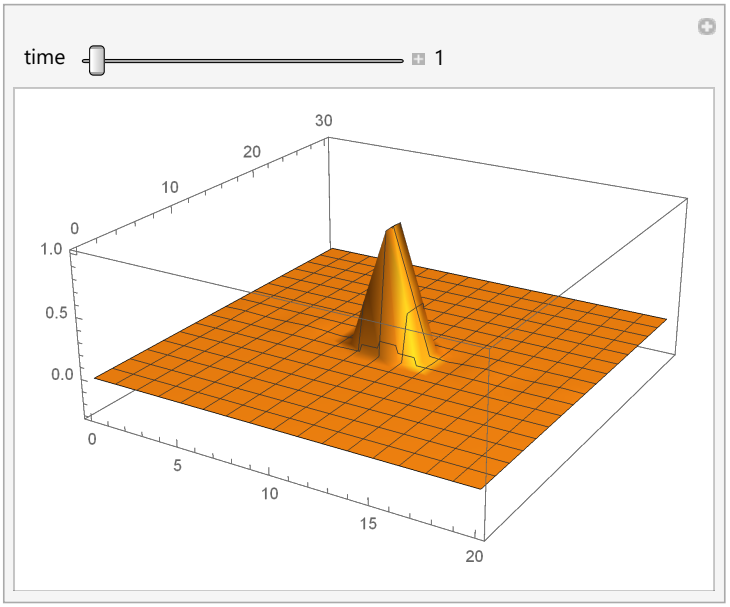


Figure 5.112: Initial state

Source code used for the above

```
ClearAll[x, y, t, n, m]
L = 20;
H = 30;
c = 3 / 10;
f1[x_] := Piecewise[{{1, 45 / 100 L ≤ x ≤ 55 / 100 L}, {0, True}}];
f2[y_] := Piecewise[{{1, 45 / 100 H ≤ y ≤ 55 / 100 H}, {0, True}}];
numberOfTerms = 150;
res[n_, m_] = N@Integrate[f1[x] * f2[y] * Sin[(n Pi) / L x] * Sin[(m Pi) / H y], {x, 0, L}, {y, 0, H}]

Anm = Table[ If[Mod[m, 20] == 0, Print["n=", n, ", m=", m]; Pause[0.05]];
      Evaluate[res[n, m]], {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
mySol[x_, y_, t_] = 4/(L H) * Sum[Anm[[n, m]] Sin[n π/L x] Sin[m π/H y] Cos[c t Sqrt[(m π/H)^2 + (n π/L)^2]], {n, 1, numberOfTerms}, {m, 1, numberOfTerms}];
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
```

Figure 5.113: Source code

```
In[=]:=
data = Reap@Do[
        Print["n=", n]; Pause[0.05];
        t = n;
        Sow[Table[Evaluate[{x, y, Chop[N[mySol[x, y, t]]]}], {x, 0, L, 1}, {y, 0, H, 1}]]
        ,
        {n, 0, 100, 0.1}
      ];
data = data[[2, 1]];

In[=]:= Manipulate[
  f = Flatten[data[[n, All]], 1];
  ListPlot3D[f, PlotRange → {{0, L}, {0, H}, {-0.3, .7}}, PerformanceGoal → "Quality", ImageSize → 400, ViewPoint → {1.604, -2.6836, 1.2937}],
  {{n, 1, "time"}, 1, Length@data, 1, Appearance → "Labeled"},
  TrackedSymbols :→ {n}
]

In[=]:= tab = Table[
      Grid[{
          {Row[{"time = ", padIt2[(n - 1) * 0.1, {3, 2}], " seconds"}]},
          {
            f = Flatten[data[[n, All]], 1];
            ListPlot3D[f, PlotRange → {{0, L}, {0, H}, {-0.3, .7}}, PerformanceGoal → "Quality", ImageSize → 400, ViewPoint → {1.604, -2.6836, 1.2937}
            ]
          }}],
      {n, 1, Length@data, 2}
    ];

In[=]:= Export["anim.gif", tab, "DisplayDurations" → Table[0.06, {Length[tab]}]]
```

Figure 5.114: Code used for animation

5.2.1.5 [398] All 4 edges fixed

problem number 398

Taken from Mathematica helps pages on DSolve

Solve for $u(x, y, t)$ with $0 < x < 1$ and $0 < y < 2$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

With boundary conditions

$$\begin{aligned} u(x, 0, t) &= 0 \\ u(0, y, t) &= 0 \\ u(1, y, t) &= 0 \\ u(x, 2, t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x, y, 0) &= \frac{1}{10}(x - x^2)(2y - y^2) \\ \frac{\partial u}{\partial t}(x, y, 0) &= 0 \end{aligned}$$

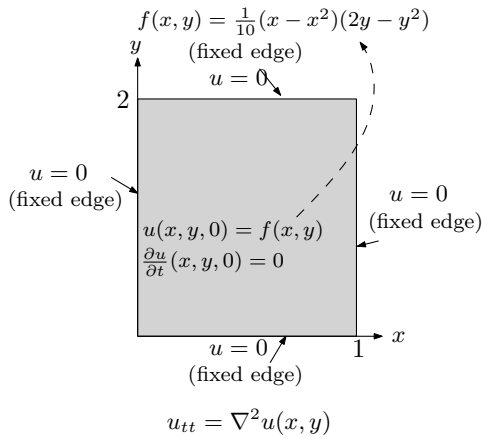


Figure 5.115: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == (1/10)*(x - x^2)*(2*y - y^2), Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[1, y, t] == 0, u[x, 2, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
sol =  Assuming[Element[{n, m}, Integers], FullSimplify[sol]];
```

$$\left\{\left\{u(x,y,t) \to \begin{cases} \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty} \frac{32(-1+(-1)^n)(-1+(-1)^{K[3]})\cos\left(\frac{1}{2}\pi t\sqrt{4n^2+K[3]^2}\right)\sin(n\pi x)\sin\left(\frac{1}{2}\pi y K[3]\right)}{5n^3\pi^6 K[3]^3} & K[3] \in \mathbb{Z} \wedge n \geq \\ \text{Indeterminate} & \text{Tru} \end{cases}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) =  VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y]);
ic  := u(x,y,0)=(1/10)*(x-x^2)*(2*y-y^2),(D[3](u))(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(1,y,t)=0,u(x,2,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\frac{32\left((-1)^m+(-1)^n-(-1)^{m+n}-1\right)\cos\left(\frac{\pi\sqrt{m^2+4n^2}\,t}{2}\right)\sin\left(\pi n x\right)\sin\left(\frac{\pi m y}{2}\right)}{5\pi^6 m^3 n^3}$$

5.2.1.6 [399] All edges fixed (Haberman 8.5.5 (a)

problem number 399

Added Nov 27, 2018.

This is problem 8.5.5 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the initial value problem for membrane with time-dependent forcing and fixed boundaries $u=0$.

$$u_{tt}=c^2\nabla^2 u+Q(x,y,t)$$

If the memberane is rectangle $(0<x<L,0<y<H)$. With initial conditions

$$u(x,y,0)=f(x,y)$$
$$\frac{\partial u}{\partial t}(x,y,0)=0$$

See my HW9, Math 322, UW Madison.

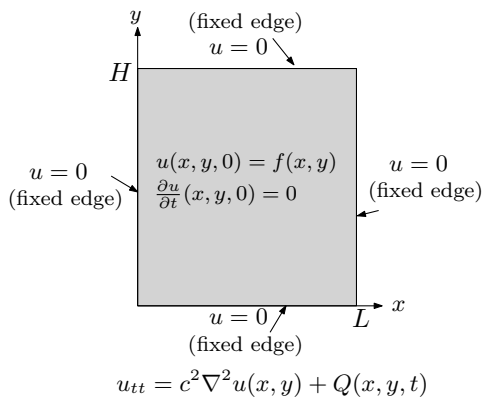


Figure 5.116: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}] + Q[x, y, t];
ic  = {u[x, y, 0] == f[x, y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[0, y, t] == 0, u[L, y, t] == 0, u[x, 0, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}, Assumptio
```

$$\left\{\left\{u(x,y,t)\to\{\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}2\sqrt{\frac{1}{H}}\sqrt{\frac{1}{L}}\left(\int_0^t\frac{2\left(\int_0^L\int_0^H Q(x,y,K[4])\sin\left(\frac{\pi x K[1]}{L}\right)\sin\left(\frac{\pi y K[3]}{H}\right)dydx\right)\sin\left(c\pi\sqrt{\frac{K[1]^2}{L^2}+\frac{K[3]^2}{H^2}}(t\right.}{c\pi\sqrt{\frac{HK[1]^2}{L}+\frac{LK[3]^2}{H}}}\right.\right.\right.$$

I

Maple ✗

```
restart;
interface(showassumed=0);
pde := diff(u(x,y,t),t$2)=c^2*(diff(u(x,y,t),x$2)+diff(u(x,y,t),y$2))+Q(x,y,t);
bc  := u(0,y,t)=0,u(L,y,t)=0,u(x,0,t)=0,u(x,H,t)=0;
ic  := u(x,y,0)=f(x,y), eval( diff(u(x,y,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t)) assu
```

sol=()

5.2.1.7 [400] 2 edgs fixed, 2 free, zero initial velocity

problem number 400

Taken from Maple PDE help pages. This wave PDE inside square with free to move on left edge and right edge, and top and bottom edges are fixed. It has zero initial velocity, but given a non-zero initial position. Where $0 < x < \pi$ and $0 < y < \pi$ and $t > 0$.

Solve

$$u_{tt} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}u(0,y,t) &= 0\\ \frac{\partial u}{\partial x}u(\pi,y,t) &= 0\\ u(x,0,t) &= 0\\ u(x,\pi,0) &= 0\end{aligned}$$

With initial conditions

$$\begin{aligned}\frac{\partial u}{\partial t}(x,y,0) &= 0\\ u(x,0) &= xy(\pi - y)\end{aligned}$$

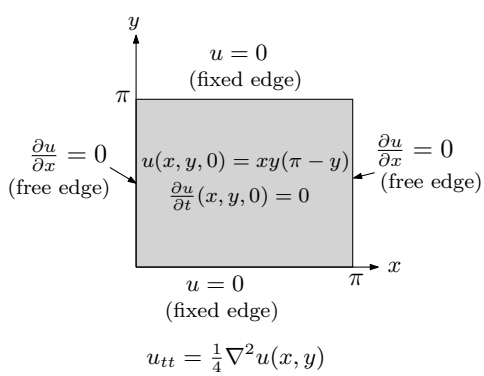


Figure 5.117: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4;
ic  = {Derivative[0, 0, 1][u][x, y, 0] == 0, u[x, y, 0] == x*y*(Pi - y)};
bc  = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][Pi, y, t] == 0, u[x, 0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{K[3]=1}^{\infty}-\frac{2\left(-1+(-1)^{K[3]}\right)\cos\left(\frac{1}{2}tK[3]\right)\sin(yK[3])}{K[3]^3}+\sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}-\frac{8\left(-1+(-1)^{K[1]}\right)\left(-1+(-1)^{K[3]}\right)\cos(x}{\pi^2K[}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, y, t), t, t) = (1/4)*(diff(u(x, y, t), x, x))+(1/4)*(diff(u(x, y, t), y, y)
bc  := (D[1](u))(0, y, t) = 0,
       (D[1](u))(Pi, y, t) = 0,
       u(x, 0, t) = 0,
       u(x, Pi, t) = 0;
ic  := u(x, y, 0) = x*y*(Pi-y),(D[3](u))(x, y, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))),out
sol := subs(n1=m,sol);
```

$$u(x,y,t)=-2\left(\sum_{n=1}^{\infty}\frac{\left((-1)^n-1\right)\cos\left(\frac{nt}{2}\right)\sin(ny)}{n^3}\right)+\left(\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{8\left((-1)^m+(-1)^n-(-1)^{m+n}-1\right)\cos(m}{\pi^2m^2n^3}\right.$$

5.2.1.8 [401] All 4 edges fixed, zero initial velocity, general solution

problem number 401

Added June 16, 2019

Solve for $u(x,y,t)$ with $0<x<L$ and $0<y<H$ and $t>0$.

Solve

$$u_{tt}=c^2\nabla^2u(x,y)$$

With boundary conditions

$$u(x,0,t)=0$$
$$u(0,y,t)=0$$
$$u(L,y,t)=0$$
$$u(x,H,t)=0$$

With initial conditions

$$u(x,y,0)=f(x,y)$$
$$u_t(x,y,0)=0$$

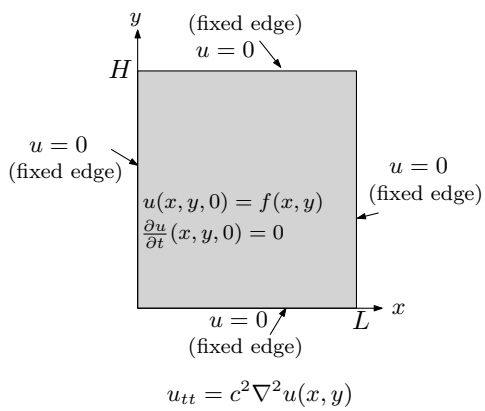


Figure 5.118: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == c^2 * Laplacian[u[x, y, t], {x, y}];
ic  = {u[x, y, 0] == f[x,y], Derivative[0, 0, 1][u][x, y, 0] == 0};
bc  = {u[x, 0, t] == 0, u[0, y, t] == 0, u[L, y, t] == 0, u[x, H, t] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
sol =  sol /. {K[1] -> n, K[2] -> m};
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{n=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{4\cos\left(\pi t\sqrt{c^2\left(\frac{n^2}{L^2}+\frac{K[3]^2}{H^2}\right)}\right)\left(\int_0^L\int_0^H f(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\pi y K[3]}{H}\right)dydx\right)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{\pi y K[3]}{H}\right)}{HL}\quad(n\right.\right.$$

Indeterminate

Maple ✗

```
restart;
pde := diff(u(x, y, t), t$2) = c^2* VectorCalculus:-Laplacian(u(x, y, t), 'cartesian'[x, y])
ic  := u(x,y,0)=f(x,y),D[3](u)(x,y,0)=0;
bc  := u(x,0,t)=0,u(0,y,t)=0,u(L,y,t)=0,u(x,H,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

time expired

Hand solution

Solve for $u(r,\theta,t)$

$$u_{tt} = c^2\nabla^2 u(x,y)$$

With boundary conditions such that all edges are fixed, and initial conditions $u(x,y,0) = f(x,y)$ and initial velocity $g(x,y) = 0$.

Let $u = X(x)\,Y(y)\,T(t)$. Substituting into the above PDE gives

$$\frac{1}{c^2}T''XY = X''YT + Y''XT$$
$$\frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} + \frac{Y''}{Y}$$

Hence

$$\frac{1}{c^2}\frac{T''}{T} = -\lambda$$
$$\frac{X''}{X} + \frac{Y''}{Y} = -\lambda$$

The time ODE becomes

$$T'' + c^2\lambda T = 0$$

And the space ODE is

$$\frac{X''}{X} + \frac{Y''}{Y} = -\lambda$$
$$\frac{X''}{X} = -\lambda - \frac{Y''}{Y}$$

Using a new separation variable μ gives the following two ODE's

$$\frac{X''}{X} = -\mu$$
$$-\lambda - \frac{Y''}{Y} = -\mu$$

Or

$$\begin{aligned} X'' + \mu X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

And

$$\begin{aligned} Y'' + Y(\lambda - \mu) &= 0 \\ Y(0) &= 0 \\ Y(H) &= 0 \end{aligned}$$

Solving first for the $X(x)$ ODE, and knowing that μ must be positive only here from the nature of the boundary conditions gives

$$X = A\cos\left(\sqrt{\mu}x\right) + B\sin\left(\sqrt{\mu}x\right)$$

Applying B.C. at $x = 0$

$$0 = A$$

Hence solution becomes $X(x) = B\sin\left(\sqrt{\mu}x\right)$. Applying the B.C. at $x = L$ gives

$$0 = B\sin\left(\sqrt{\mu}L\right)$$

Non trivial solution requires that

$$\begin{aligned} \sqrt{\mu}L &= n\pi \qquad n = 1, 2, 3, \cdots \\ \mu_n &= \left(\frac{n\pi}{L}\right)^2 \end{aligned}$$

Therefore the eigenfunctions $X_n(x)$ are

$$X_n(x) = \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots$$

Solving the $Y(y)$ ODE

$$Y_n'' + \left(\lambda - \left(\frac{n\pi}{L}\right)^2\right) Y_n = 0 \qquad n = 1, 2, 3, \cdots$$

The nature of the boundary conditions on $Y(y)$ suggests that $\left(\lambda - \left(\frac{n\pi}{L}\right)^2\right)$ must be positive (if $\lambda - \left(\frac{n\pi}{L}\right)^2 = 0$ or $\lambda - \left(\frac{n\pi}{L}\right)^2 < 0$, trivial solutions result).

Hence the solution for $Y_n(y)$ becomes

$$Y_n(y) = A\cos\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right) + B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right)$$

Applying first B.C. $Y(0) = 0$ gives

$$0 = A$$

The solution becomes

$$Y_n(y) = B_n\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,y\right)$$

Applying second B.C. $Y(H) = 0$ gives

$$0 = B\sin\left(\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,H\right)$$

Non trivial solution requires that

$$\begin{aligned}
\sqrt{\lambda - \left(\frac{n\pi}{L}\right)^2}\,H &= m\pi \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots\\
\lambda_{nm} - \left(\frac{n\pi}{L}\right)^2 &= \left(\frac{m\pi}{H}\right)^2\\
\lambda_{nm} &= \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2
\end{aligned}$$

Hence the $Y_{nm}(y)$ eigenfunctions are

$$Y_{nm}(y) = \sin\left(\frac{m\pi}{H}y\right) \qquad n = 1,2,3,\cdots, m = 1,2,3,\cdots$$

Now the time $T(t)$ ode is solved, and since λ_{nm} is positive, then

$$T''_{nm} + c^2\lambda_{nm}T_{nm} = 0$$

$$\begin{aligned}
T_{nm}(t) &= A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\\
&= A_{nm}\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}\,t\right) + B_{nm}\sin\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}\,t\right)
\end{aligned}$$

Combining all solution , and merging all constants into two results in

$$\begin{aligned}
u_{nm}(x,y,t) &= X_n(x)\,Y_{nm}(y)\,T_{nm}(t)\\
u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_n(x)\,Y_{nm}(y)\,T_{nm}(t)\\
&= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2}\,t\right) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin
\end{aligned} \tag{1}$$

Initial conditions are now used to find A_{nm}, B_{nm}. At $t=0$

$$\begin{aligned} u(x,y,0) &= f(x,y) \\ \frac{\partial u}{\partial t}(x,y,0) &= 0 \end{aligned}$$

Applying first initial condition to (1) gives

$$f(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)$$

Applying 2D orthogonality gives

$$\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy = A_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right)$$

$$A_{nm} = \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy$$

Taking time derivative of (1) gives

$$\begin{aligned} \frac{\partial u}{\partial t}(x,y,t) = &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\sin\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right) \\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right) \end{aligned}$$

AT $t=0$ the above becomes

$$\int_0^H g(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2} B_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)$$

Applying 2D orthogonality gives

$$\int_0^L \int_0^H g(x,y)\sin\left(\left(\frac{n\pi}{L}\right)x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy = B_{nm}\left(\frac{L}{2}\right)\left(\frac{H}{2}\right)$$

But the initial velocity $g(x,y)=0$. Hence $B_{nm}=0$ for all n,m.

<u>Summary of solution</u>

$$\begin{aligned} u(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\cos\left(c\sqrt{\left(\frac{m\pi}{H}\right)^2+\left(\frac{n\pi}{L}\right)^2}t\right) \\ A_{nm} &= \frac{4}{LH}\int_0^L \int_0^H f(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{H}y\right)\ dxdy \end{aligned}$$

5.2.1.9 [402] With damping

problem number 402

Taken from Maple PDE help pages. This wave PDE inside square with damping present.

Membrane is free to move on the right edge and also on top edge. But fixed at left edge and bottom edge.

It has zero initial position, but given a non-zero initial velocity. Where $0 < x < 1$ and $0 < y < 1$ and $t > 0$. Solve

$$u_{tt} + \frac{1}{10}u_t = \frac{1}{4}\nabla^2 u(x,y)$$

With boundary conditions

$$\begin{aligned} u(0,y,t) &= 0 \\ \frac{\partial u}{\partial x}u(1,y,t) &= 0 \\ u(x,0,t) &= 0 \\ \frac{\partial u}{\partial y}u(x,1,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= 0 \\ \frac{\partial u}{\partial t}(x,y,0) &= x(1-\frac{1}{2}x)(1-\frac{1}{2}y)y \end{aligned}$$

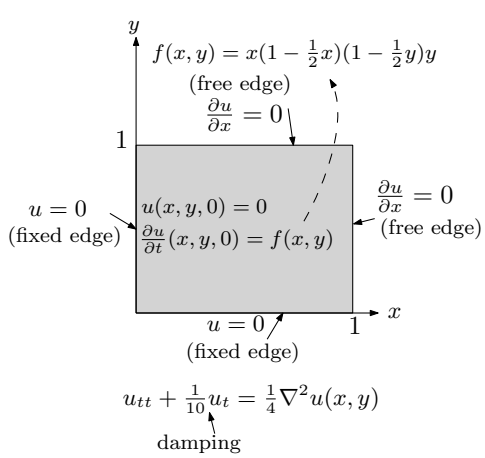


Figure 5.119: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4 - (1*D
ic  = {u[x, y, 0] == 0, Derivative[0, 0, 1][u][x, y, 0] == x*(1 - (1/2)*x)*(1 - (1/2)*y)*y};
bc  = {u[0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to\ \{\ \sum_{K[1]=1}^{\infty}\sum_{K[3]=1}^{\infty}\frac{5120e^{-t/20}\sin\left(\frac{1}{2}\pi x(2K[1]-1)\right)\sin\left(\frac{1}{2}\pi y(2K[3]-1)\right)\sin\left(\frac{1}{20}t\sqrt{50\pi^2(2K[1]^2-2K[1]+2K[3]^2-2K[3]}\right.}{\pi^6(2K[1]-1)^3(2K[3]-1)^3\sqrt{50\pi^2(2K[1]^2-2K[1]+2K[3]^2-2K[3]+1)-1}}\right.\right.$$

Indeterminate

Maple ✓

```
restart;
pde := diff(u(x, y, t), t$2) = 1/4*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2))-(1/10)*(dif
bc  := u(0, y, t) = 0,
      (D[1](u))(1, y, t) = 0,
      u(x, 0, t) = 0,
      (D[2](u))(x, 1, t) = 0;
ic  := u(x, y, 0) = 0, (D[3](u))(x, y, 0) = x*(1-(1/2)*x)*(1-(1/2)*y)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol := subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{5120\,\mathrm{e}^{-\frac{t}{20}}\sin\left(\frac{\sqrt{-1+(100m^2+100n^2+100m+100n+50)\pi^2}\,t}{20}\right)\sin\left(\frac{(2n+1)\pi x}{2}\right)\sin\left(\frac{(2m+1)\pi y}{2}\right)}{\sqrt{-1+(100m^2+100n^2+100m+100n+50)\,\pi^2}\,\pi^6\,(2m+1)^3\,(2n+1)^3}$$

5.2.1.10 [403] On the whole plane

problem number 403

From Mathematica DSolve help pages.

Hyperbolic partial differential equation with non-rational coefficients.

Solve for $u(x,y)$

$$u_{xx}-2\sin x u_{xy}-\cos^2 x u_{yy}-\cos x u_y=0$$

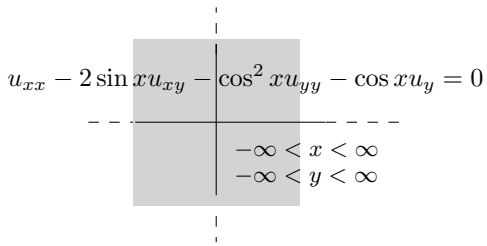


Figure 5.120: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
ode = D[u[x, y], {x, 2}] - 2*Sin[x]*D[u[x, y], x, y] - Cos[x]^2*D[u[x, y], {y, 2}] - Cos[x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(x - \cos(x) + y) + c_2(-x - \cos(x) + y)\}\}$$

Maple ✗

```
restart;
interface(showassumed=0);
ode := diff(u(x, y), x$2) - 2*sin(x)*diff(u(x, y),x,y)-cos(x)^2*diff(u(x, y), y$2) - cos(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(ode, u(x, y))),output='rea
```

sol=()

5.2.2 Polar coordinates

Local contents

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5.2.2.1 [404] no θ dependency, fixed boundary, general case

problem number 404

Added January 12, 2020

Circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0 < r < a$ and $t > 0$.

$$u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r\right)$$

With boundary conditions

$$u(a,t) = 0$$

With initial conditions

$$\begin{aligned} u(r,0) &= f(r) \\ \frac{\partial u}{\partial t}(r,0) &= g(r) \end{aligned}$$

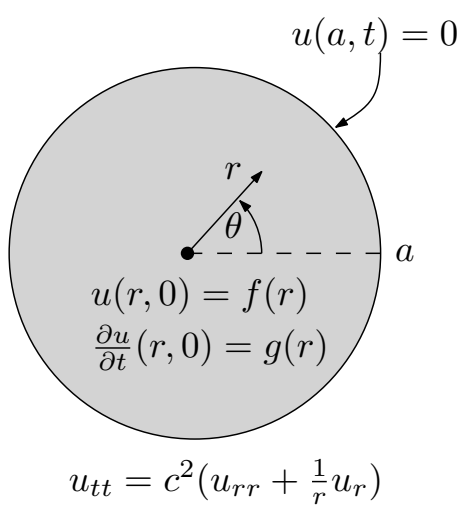

Figure 5.121: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == f[r], Derivative[0, 1][u][r, 0] == g[r]};
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t},Assumptions->{t>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t)\to\sum_{n=1}^{\infty}\boxed{\frac{2J_0\left(\frac{rj_{0,n}}{a}\right)\left(\sqrt{c^2}j_{0,n}\cos\left(\frac{\sqrt{c^2}tj_{0,n}}{a}\right)\int_0^a rJ_0\left(\frac{rj_{0,n}}{a}\right)f(r)\,dr+a\left(\int_0^a rJ_0\left(\frac{rj_{0,n}}{a}\right)g(r)\,dr\right)\sin\left(\frac{\sqrt{c^2}tj_{0,n}}{a}\right)\right)}{a^2\sqrt{c^2}(J_0(j_{0,n})^2+J_1(j_{0,n})^2)j_{0,n}}\text{ if }j_{0,n}\geq}\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=f(r), D[2](u)(r,0)=g(r);
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t),HINT
```

$$u(r,t)=\frac{-\mathcal{L}^{-1}\left(\text{BesselK}\left(0,\frac{rs}{c}\right)\left(\int\left(sf(a)+g(a)\right)a\,\text{BesselI}\left(0,\frac{as}{c}\right)da\right),s,t\right)+\mathcal{L}^{-1}\left(\text{BesselK}\left(0,\frac{rs}{c}\right)\left(\int\left(sf(\right.\right.}{}$$

Has unresolved Invlaplace calls

<u>Hand solution</u>

Assuming $u=T(t)\,R(r)$. Substituting in the PDE gives

$$\frac{1}{c^2}T''R=R''T+\frac{1}{r}R'T$$

Dividing by RT

$$\frac{1}{c^2}\frac{T''}{T}=\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}$$

Hence

$$\frac{1}{c^2}\frac{T''}{T}=-\lambda$$
$$\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}=-\lambda$$

The time ODE is

$$T'' + c^2 \lambda T = 0$$

And the r ODE is (Sturm-Liouville)

$$rR'' + R' + \lambda r R = 0$$

Where $p = r, q = 0, \sigma = r$. This is singular SL. The solution turns out to be

$$R_n(r) = A_n J_0\left(\sqrt{\lambda_n} r\right) \qquad n = 1, 2, 3, \cdots$$

Where λ_n is found from roots of $0 = J_n\left(\sqrt{\lambda_n} a\right)$ giving the eigenvalues. Now the time ODE is solved

$$\begin{aligned} T_n'' + c^2 \lambda_n T_n &= 0 \\ T_n &= B_n \cos\left(c\sqrt{\lambda_n} t\right) + C_n \sin\left(c\sqrt{\lambda_n} t\right) \qquad n = 1, 2, 3, \ldots, \end{aligned}$$

Hence the solution is

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} T_n R_n \\ &= \sum_{n=1}^{\infty} A_n \cos\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) + B_n \sin\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) \qquad (1) \end{aligned}$$

Now initial conditions $u(r,0) = f(r)$ is used to find A_n using orthogonality. At $t = 0$ the solution simplifies to

$$u(r,0) = \sum_{n=1}^{\infty} A_n J_0\left(\sqrt{\lambda_n} r\right)$$

Hence

$$\begin{aligned} f(r) &= \sum_{n=1}^{\infty} A_n J_0\left(\sqrt{\lambda_n} r\right) \\ \int_0^a f(r) J_0\left(\sqrt{\lambda_n} r\right) r dr &= A_n \int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr \\ A_n &= \frac{\int_0^a f(r) J_0\left(\sqrt{\lambda_n} r\right) r dr}{\int_0^a J_0^2\left(\sqrt{\lambda_n} r\right) r dr} \end{aligned}$$

Now we will look at the second initial conditions $\frac{\partial u}{\partial t}(r,0) = g(r)$. Taking derivative w.r.t. time t of the solution in (1) gives

$$\frac{\partial u}{\partial t}(r,t) = \sum_{n=1}^{\infty} -c\sqrt{\lambda_n} A_n \sin\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right) + B_n c\sqrt{\lambda_n} \cos\left(c\sqrt{\lambda_n} t\right) J_0\left(\sqrt{\lambda_n} r\right)$$

At time $t=0$ the above becomes

$$g(r)=\sum_{n=1}^{\infty}B_n c\sqrt{\lambda_n}J_0\left(\sqrt{\lambda_n}r\right)$$

Now orthogonality is used. The above becomes

$$B_n=\frac{\int_0^a g(r)\,J_0\left(\sqrt{\lambda_n}r\right)rdr}{c\sqrt{\lambda_n}\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right)rdr}$$

Summary of solution

$$u(r,t)=\sum_{n=1}^{\infty}A_n\cos\left(c\sqrt{\lambda_n}t\right)J_0\left(\sqrt{\lambda_n}r\right)+B_n\sin\left(c\sqrt{\lambda_n}t\right)J_0\left(\sqrt{\lambda_n}r\right)$$

$$A_n=\frac{\int_0^a f(r)\,J_0\left(\sqrt{\lambda_n}r\right)rdr}{\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right)rdr}$$

$$B_n=\frac{\int_0^a g(r)\,J_0\left(\sqrt{\lambda_n}r\right)rdr}{c\sqrt{\lambda_n}\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right)rdr}$$

With λ_n being the solutions for $0=J_0\left(\sqrt{\lambda_n}a\right)$. We have infinite number of zeros. This generates all the needed λ_n. Hence $\sqrt{\lambda_n}a=BesselJZero(0,n)$, therefore $\sqrt{\lambda_n}=\frac{a}{BesselJZero(0,n)}$

5.2.2.2 [405] no θ dependency. Specific example. Both initial conditions not zero

problem number 405

Taken from Mathematica helps pages on DSolve

In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0<r<1$ and $t>0$.

$$u_{tt}=c^2\left(u_{rr}+\frac{1}{r}u_r\right)$$

With boundary conditions

$$u(1,t)=0$$

With initial conditions

$$u(r,0) = 1$$
$$\frac{\partial u}{\partial t}(r,0) = \frac{r}{3}$$

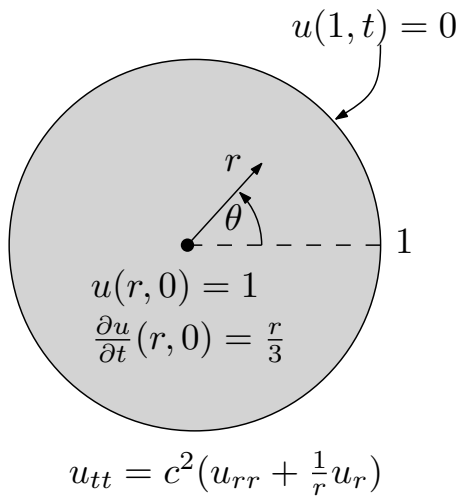

Figure 5.122: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == 1, Derivative[0, 1][u][r, 0] == r/3};
bc  = u[1, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
sol =  sol /. K[1] -> n;
sol =  FullSimplify[sol];
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2J_0(rj_{0,n})\left(9\sqrt{c^2}J_1(j_{0,n})\cos(ctj_{0,n}) + {}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}(j_{0,n})^2\right)\sin\left(\sqrt{c^2}tj_{0,n}\right)\right)}{9\sqrt{c^2}\left(J_0(j_{0,n})^2 + J_1(j_{0,n})^2\right)j_{0,n}}\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=1, eval( diff(u(r,t),t),t=0)=r/3;
bc  := u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t)) ass
```

$$u(r,t) = \frac{rt}{3} + \frac{\pi\left(-\mathcal{L}^{-1}\left(\frac{\mathrm{StruveL}(0,\frac{rs}{c})}{s^3}, s, t\right) + \mathcal{L}^{-1}\left(\frac{\mathrm{BesselI}(0,\frac{rs}{c})\,\mathrm{StruveL}(0,\frac{s}{c})}{s^3\,\mathrm{BesselI}(0,\frac{s}{c})}, s, t\right)\right)c}{6} - \frac{\mathcal{L}^{-1}\left(\frac{\mathrm{BesselI}(0,\frac{rs}{c})}{s^2\,\mathrm{BesselI}(0,\frac{s}{c})}, s, t\right)}{3} - \mathcal{L}$$

Has unresolved Invlaplace calls

5.2.2.3 [406] no θ dependency. Specific example. Both initial conditions not zero

problem number 406

Added January 12, 2020.

In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

Solve for $u(r,t)$ with $0 < r < a$ and $t > 0$.

$$u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r\right)$$

With boundary conditions

$$u(a,t) = 0$$

With initial conditions

$$u(r,0) = f(r)$$
$$\frac{\partial u}{\partial t}(r,0) = g(r)$$

Using $a = 1, c = \frac{2}{10}, g(r) = 0, f(r) = r$.

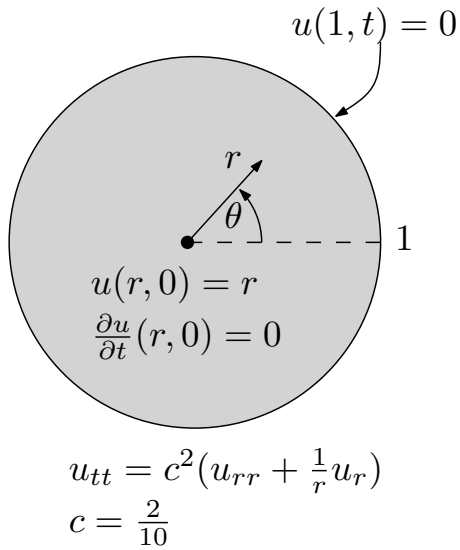

Figure 5.123: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
c=2/10; a=1;
g[r_]:=0;
f[r_]:=r;
pde =  D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + 1/r*D[u[r, t], r]);
ic  = {u[r, 0] == f[r], Derivative[0, 1][u][r, 0] == g[r]};
bc  = u[a, t] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t},Assumptions->{t>
sol =  sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2J_0(rj_{0,n})\cos\left(\frac{tj_{0,n}}{5}\right) {}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}(j_{0,n})^2\right)}{3\left(J_0\left(j_{0,n}\right)^2 + J_1\left(j_{0,n}\right)^2\right)}\right\}\right\}$$

Maple ✓

```
restart;
c:=2/10;
a:=1;
g:=r->0;
f:=r->r;
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  );
ic  := u(r,0)=f(r), D[2](u)(r,0)=g(r);
bc  := u(a,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t)) ass
```

$$u(r,t) = r + \int_0^{r-1} \mathcal{L}^{-1}\left(\frac{\text{BesselI}\left(0,5(r-\tau)\,s\right)\mathcal{L}\left(-\frac{1}{\tau+1},t,s\right)}{\text{BesselI}\left(0,5s\right)},s,t\right)d\tau + \mathcal{L}^{-1}\left(\frac{\text{BesselI}\left(0,5rs\right)\mathcal{L}(-1,t,s)}{\text{BesselI}\left(0,5s\right)},s,\right.$$

Has unresolved Invlaplace calls. How to get series solution?
<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 5.2.2.1 on page 1111 as

$$u(r,t) = \sum_{n=1}^{\infty} A_n \cos\left(c\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right) + B_n \sin\left(c\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right)$$

$$A_n = \frac{\int_0^a f(r)\, J_0\left(\sqrt{\lambda_n}r\right) r dr}{\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right) r dr}$$

$$B_n = \frac{\int_0^a g(r)\, J_0\left(\sqrt{\lambda_n}r\right) r dr}{c\sqrt{\lambda_n}\int_0^a J_0^2\left(\sqrt{\lambda_n}r\right) r dr}$$

With λ_n being the solutions for $0 = J_0\left(\sqrt{\lambda_n}a\right)$. We have infinite number of zeros. This generates all the needed λ_n. Hence $\sqrt{\lambda_n}a = BesselJZero(0,n)$, therefore $\sqrt{\lambda_n} = \frac{a}{BesselJZero(0,n)}$.

In this problem $c = \frac{2}{10}, a = 1, g(r) = 0$ and $f(r) = r$, hence the solution becomes

$$u(r,t) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{2}{10}\sqrt{\lambda_n}t\right) J_0\left(\sqrt{\lambda_n}r\right)$$

Where $\sqrt{\lambda_n} = \frac{1}{BesselJZero(0,n)}$.

This animation runs for 40 seconds.

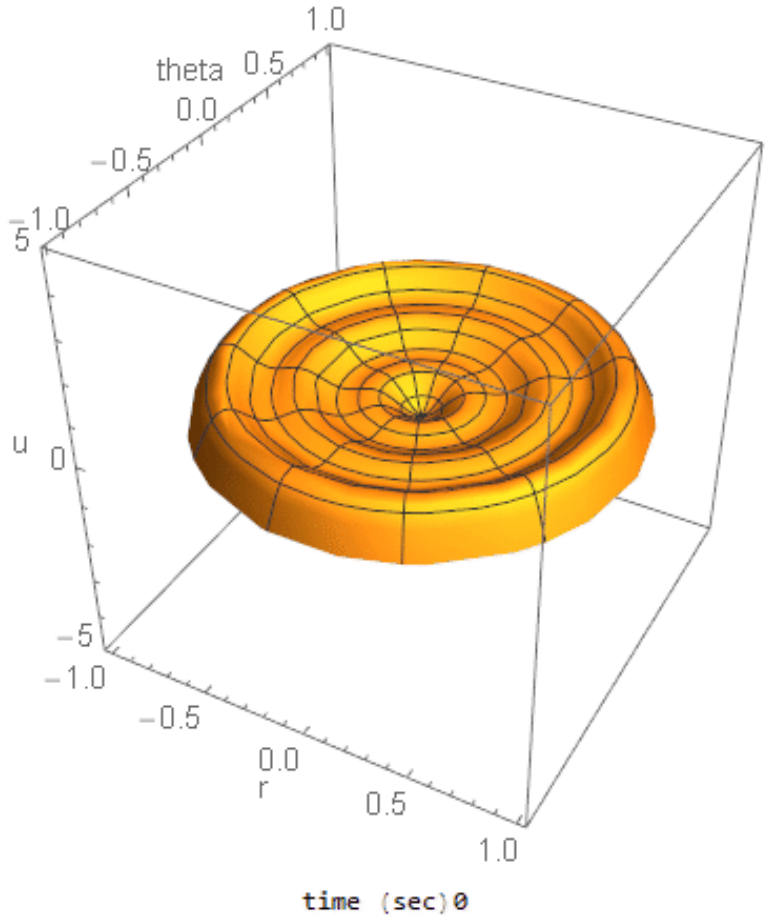

Figure 5.124: snap shot

Source code for all the above animation

```
(*By Nasser M Abbasi*)
SetDirectory[NotebookDirectory[]]

(*Axis symmetric*)
(*definitions*)
ClearAll[a,c,n,m,r,theta,f,g,u]
A0[n_,a_,lam0_]:= Module[{num,den,theta,r,f},
   f=r;
   num=N[(BesselJ[1,lam0] (2 lam0-Pi StruveH[0,lam0])+
        Pi BesselJ[0,lam0] StruveH[1,lam0])/(2 lam0^2)];
```

```
    den=0.5 (BesselJ[0,lam0]^2+BesselJ[1,lam0]^2);
    num/den
];

u[r_,theta_,t_]:=Sum[A0tbl[[n]]*Cos[c lamtbl[[n]] t] *BesselJ[0,lamtbl[[n]]*r],{n,1,maxN}];

maxN=6;
a=1;
c=.2;

lam[n_,a_]:=Module[{x},
   x=BesselJZero[0,n];
   N[(x/a)]
];


lamtbl=Table[lam[n,a],{n,1,maxN}];
A0tbl=Table[A0[n,a,lamtbl[[n]]],{n,1,maxN}];


t=.1
ParametricPlot3D[{r Cos[theta],r Sin[theta],Evaluate[u[r,theta,t]]},{r,0,1},
      {theta,0,2 Pi},AxesLabel->{"r","theta","u"},ImageMargins->5,
      PerformanceGoal->"Quality",BoxRatios->{1,1,1},
      PlotRange->{Automatic,Automatic,{-5,5}},Mesh->10,MaxRecursion->1]

Animate[ParametricPlot3D[{r Cos[theta],r Sin[theta],
        Evaluate[u[r,theta,t]]},{r,0,1},{theta,0,2 Pi},
        AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
        PerformanceGoal->"Speed",BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{-5,5}},
        Mesh->10,MaxRecursion->1],{t,0,50,.01}]

r=Table[
      Labeled[ParametricPlot3D[{r Cos[theta],r Sin[theta],
           Evaluate[u[r,theta,t]]},{r,0,1},{theta,0,2 Pi},AxesLabel->{"r","theta","u"},
           BaseStyle->15,ImageMargins->5,PerformanceGoal->"Speed",BoxRatios->{1,1,1},
           PlotRange->{Automatic,Automatic,{-5,5}},Mesh->10,MaxRecursion->1],
           Row[{"time (sec)",Round@t}]],{t,0,40,.1}];

Export["anim_axis.gif",r,"DisplayDurations"->Table[.05,{Length[r]}]]
```

5.2.2.4 [407] no θ dependency. Using integral transforms. Source present. Specific example

problem number 407

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$\frac{\partial^2}{\partial r^2}u(r,t)+\frac{\frac{\partial}{\partial r}u(r,t)}{r}+\frac{\partial^2}{\partial t^2}u(r,t)=-Q_0\,q(r)$$

With initial conditions

$$u(r,0)=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[u[r, t], {r, 2}] + D[u[r, t], r]/r + D[u[r, t], {t, 2}] == -Q0*q[r];
ic = u[r, 0] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[r, t], {r, t}], 60*10]];
```

$$\left\{\left\{u(r,t)\to \text{Q0}\left(\int_0^\infty \frac{J_0(rK[1])\int_0^\infty rq(r)J_0(rK[1])\,dr}{K[1]}\,dK[1]-\int_0^\infty \frac{e^{-tK[1]}J_0(rK[1])\int_0^\infty rq(r)J_0(rK[1])}{K[1]}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(u(r, t), r$2) + diff(u(r, t), r)/r + diff(u(r, t), t$2) = -Q__0*q(r);
iv := u(r, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(r,t))),output='
sol:=convert(sol,Int,only = hankel);
```

$$u(r,t)=\left(\int_0^\infty \frac{\text{BesselJ}\,(0,rs)\left(\int_0^\infty r\,\text{BesselJ}\,(0,rs)\,q(r)\,dr\right)}{s}ds-\left(\int_0^\infty \frac{\text{BesselJ}\,(0,rs)\left(\int_0^\infty r\,\text{BesselJ}\,(0,rs\right.}{s}\right.\right.$$

5.2.2.5 [408] no θ dependency. Using integral transforms. Source present. Specific example

problem number 408

Added Oct 6, 2019.

Taken from `https://www.mapleprimes.com/posts/211274-Integral-Transforms-revamped-And-PDE`

Solve

$$c^2\left(\frac{\partial^2}{\partial r^2}u(r,t)+\frac{\frac{\partial}{\partial r}u(r,t)}{r}\right)=\frac{\partial^2}{\partial t^2}u(r,t)$$

With initial conditions

$$u(r,0)=\frac{Aa}{\sqrt{a^2+r^2}}$$
$$\frac{\partial u(r,0)}{\partial t}=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = c^2*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) == D[u[r, t], {t, 2}];
ic = {u[r, 0] == A*a*(a^2 + r^2)^(-1/2), Derivative[0, 1][u][r, 0] == 0};
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[r, t], {r, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := c^2*(diff(u(r, t), r, r) + diff(u(r, t), r)/r) = diff(u(r, t), t, t);
iv := u(r, 0) = A*a*(a^2 + r^2)^(-1/2), D[2](u)(r, 0) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,iv],u(r,t),method = H
```

$$u(r,t)=\frac{\left(\sqrt{2iact-c^2t^2+a^2+r^2}+\sqrt{-2iact-c^2t^2+a^2+r^2}\right)Aa}{2\sqrt{-2iact-c^2t^2+a^2+r^2}\,\sqrt{2iact-c^2t^2+a^2+r^2}}$$

5.2.2.6 [409] θ dependency, fixed on edges, general solution

problem number 409

Added January 11, 2020

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) \\ \frac{\partial u}{\partial t}(r,\theta,0) &= g(r,\theta) \end{aligned}$$

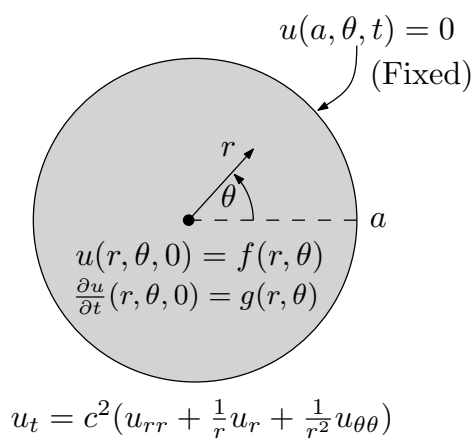

Figure 5.125: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == g[r,theta]};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to\{\sum_{K[3]=1}^{\infty}-\frac{\sqrt{\frac{2}{\pi}}J_0\left(\frac{rj_{0,K[3]}}{a}\right)\left(-\frac{\sqrt{\frac{2}{\pi}}\cos\left(\frac{\sqrt{c^2}tj_{0,K[3]}}{a}\right)\int_0^a\int_{-\pi}^{\pi}rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,\theta)d\theta dr}{aJ_1\left(j_{0,K[3]}\right)}-\frac{\sqrt{\frac{2}{\pi}}\left(\int_0^a\int_{-\pi}^{\pi}rJ_0\left(\frac{rj_{0,K[3]}}{a}\right.\right.}{|c|J_1\left(j_{0\right.}}\right.}{aJ_1\left(j_{0,K[3]}\right)}\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = g(r,theta);
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

<u>Hand solution</u>

Assuming $u = T(t)\,R(r)\,\Theta(\theta)$ and substituting in the PDE gives

$$\frac{1}{c^2}T''R\Theta = R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT$$

Dividing by $RT\Theta$

$$\frac{1}{c^2}\frac{T''}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}$$

Hence

$$\frac{1}{c^2}\frac{T''}{T} = -\lambda$$

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} = -\lambda$$

The time ODE is

$$T'' + c^2 \lambda T = 0$$

Now we separate again the space ODE's (remember to move the λ with the R and not the Θ)

$$\begin{aligned}
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \lambda &= -\frac{1}{r^2}\frac{\Theta''}{\Theta} \\
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= -\frac{\Theta''}{\Theta}
\end{aligned}$$

Let the new separation constant be μ, therefore

$$\begin{aligned}
-\frac{\Theta''}{\Theta} &= \mu \\
\Theta'' + \mu\Theta &= 0
\end{aligned}$$

With periodic boundary conditions and

$$\begin{aligned}
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= \mu \\
r^2R'' + rR' + \lambda r^2 R - \mu R &= 0 \\
rR'' + R' - \frac{\mu}{r}R &= -\lambda r R
\end{aligned}$$

Now it is in Sturm Liouville form, where $p = r, q = -\frac{\mu}{r}, \sigma = r$. This is singular SL. Can be written as

$$R'' + \frac{1}{r}R' + \left(\lambda - \frac{\mu}{r^2}\right)R = 0$$

Before we solve the above R ODE, we solve the $\Theta'' + \mu\Theta = 0$ to find μ Eigenvalues. The solution is

$$\Theta = A\cos\left(\sqrt{\mu}\theta\right) + B\sin\left(\sqrt{\mu}\theta\right)$$

With B.C $\Theta(-\pi) = \Theta(\pi)$ and $\Theta'(-\pi) = \Theta'(\pi)$. From first B.C. we obtain

$$\begin{aligned}
A\cos\left(\sqrt{\mu}\pi\right) - B\sin\left(\sqrt{\mu}\pi\right) &= A\cos\left(\sqrt{\mu}\pi\right) + B\sin\left(\sqrt{\mu}\pi\right) \\
2B\sin\left(\sqrt{\mu}\pi\right) &= 0 \qquad (1)
\end{aligned}$$

Looking at second B.C. $\Theta'(-\pi) = \Theta'(\pi)$

$$\Theta'(\theta) = -A\sqrt{\mu}\sin\left(\sqrt{\mu}\theta\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\theta\right)$$

Hence

$$\begin{aligned}
A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\pi\right) &= -A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) + \sqrt{\mu}B\cos\left(\sqrt{\mu}\pi\right) \\
A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) &= -A\sqrt{\mu}\sin\left(\sqrt{\mu}\pi\right) \\
2A\sin\left(\sqrt{\mu}\pi\right) &= 0 \qquad (2)
\end{aligned}$$

From (1,2), we see that both are satisfied if

$$\begin{aligned}\sqrt{\mu}\pi &= n\pi \qquad n = 1, 2, 3, \ldots \\ \mu &= n^2\end{aligned}$$

Hence

$$\Theta_n = A_n \cos(n\theta) + B_n \sin(n\theta)$$

There is another solution for $\mu = 0$ which is constant (that is why one of the sums below starts from $n = 0$). We can combine the zero eigenvalue with the above and write

$$\Theta_n = A_n \cos(n\theta) + B_n \sin(n\theta) \qquad n = 0, 1, 2, 3, \ldots$$

Since at $n = 0$ the above reduces to constant A_0.

Now that we know $\mu_n = n^2$, from solving the θ part, we go and solve the r ODE. For each n, the solution to the r (Bessel) ode

$$R'' + \frac{1}{r}R' + \left(\lambda - \frac{n^2}{r^2}\right) R = 0$$

The solution turns out to be

$$R_{nm}(r) = J_n\left(\sqrt{\lambda_{nm}}r\right) \qquad m = 1, 2, 3, \cdots$$

Where λ_{nm} is found from roots of $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$ giving the eigenvalues. Now the time ODE is solved

$$\begin{aligned}T''_{nm} + c^2\lambda_{nm}T_{nm} &= 0 \\ T_{nm} &= C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) \qquad n = 0, 1, 2, 3, \ldots, m = 1, 2, 3, \cdots\end{aligned}$$

Hence the solution is

$$\begin{aligned}u(r, \theta, t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} T_{nm}R_{nm}\Theta_n \\ &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\left(A_n\cos(n\theta) + B_n\sin(n\theta)\right)\end{aligned}$$

We now break this sum as follows

$$\begin{aligned}u(r, \theta, t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right) J_n\left(\sqrt{\lambda_{nm}}r\right) A_n\cos(n\theta) \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) + D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)\right) J_n\left(\sqrt{\lambda_{nm}}r\right) B_n\sin(n\theta)\end{aligned}$$

Or

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)A_n\cos(n\theta)+D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)A_n\cos$$
$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)B_n\sin(n\theta)+D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)B_n\sin$$

Then we break the above into 4 sums

$$\begin{aligned}u(r,\theta,t)&=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)A_n\cos(n\theta)\\&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)A_n\cos(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)B_n\sin(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)B_n\sin(n\theta)\end{aligned}$$

Finally, we merge constants in the above as follows

$$\begin{aligned}A_nC_{nm}&\equiv A_{nm}\\A_nD_{nm}&\equiv B_{nm}\\B_nC_{nm}&\equiv C_{nm}\\B_nD_{nm}&\equiv D_{nm}\end{aligned}$$

Hence the final solution now becomes

$$\begin{aligned}u(r,\theta,t)&=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}\tag{3}$$

Now initial conditions $u(r,\theta,0)=f(r,\theta)$ is used to find A_{nm},C_{nm} using orthogonality.

At $t=0$ the solution simplifies to (all terms with $\sin\left(c\sqrt{\lambda_{nm}}t\right)$ vanish giving

$$\begin{aligned} u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta) \end{aligned}$$

Hence

$$f(r,\theta) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta) \tag{4}$$

When iterating over m index, the terms $\cos(n\theta)$ and $\sin(n\theta)$ will be constant. So for each n, we have $\sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)$ and $\sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right)$. So orthogonality is carried out on the m index on the Bessel functions. Multiplying (4) by $J_n\left(\sqrt{\lambda_{nk}}r\right)$ and integrating

$$\begin{aligned} \int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nk}}r\right) r dr &= \sum_{n=0}^{\infty}\left(\int_0^a \sum_{m=1}^{\infty} A_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right) J_n\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\cos(n\theta) \\ &+ \sum_{n=1}^{\infty}\left(\int_0^a \sum_{m=1}^{\infty} C_{nm} J_n\left(\sqrt{\lambda_{nm}}r\right) J_n\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\sin(n\theta) \end{aligned}$$

Or

$$\begin{aligned} \int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nk}}r\right) r dr &= \sum_{n=0}^{\infty} A_{nk}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\cos(n\theta) \\ &+ \sum_{n=1}^{\infty} C_{nk}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nk}}r\right) r dr\right)\sin(n\theta) \end{aligned}$$

Replacing k back with m, the above becomes

$$\begin{aligned} \int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r dr &= \sum_{n=0}^{\infty} A_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\cos(n\theta) \\ &+ \sum_{n=1}^{\infty} C_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\sin(n\theta) \end{aligned} \tag{5}$$

We now apply orthogonality on n using the $\cos(n\theta)$ results in

$$\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\, dr\right)\cos(n\theta)\, d\theta = A_{nm}\int_{-\pi}^{\pi}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr\right)\cos^2(n\theta)\, d\theta$$

But $\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr$ does not depend on θ, therefore the above becomes

$$\begin{aligned}\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta &= A_{nm}\left(\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr\right)\int_{-\pi}^{\pi}\cos^2(n\theta)\, d\theta\\ &= A_{nm}\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr\end{aligned}$$

Therefore

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

Similarly for $\sin(n\theta)$, which gives

$$C_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

Now we will look at the second initial conditions $\frac{\partial u}{\partial t}(r,\theta,0) = g(r,\theta)$. Taking derivative w.r.t. time t of the solution in (3) gives

$$\begin{aligned}\frac{\partial u}{\partial t}(r,\theta,t) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}}A_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} -c\sqrt{\lambda_{nm}}C_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

At time $t=0$ the above becomes (all terms with $\sin\left(c\sqrt{\lambda_{nm}}t\right)$ vanish).

$$\begin{aligned}g(r,\theta) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

Now orthogonality is used. At $t=0$ the above becomes

$$\begin{aligned}g(r,\theta) = &\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}B_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\ &+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} c\sqrt{\lambda_{nm}}D_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

Similarly to the above we now find B_{nm} and D_{nm}. The only difference, is that now we have extra $c\sqrt{\lambda_{nm}}$ terms that show up. The final result will be

$$B_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\; d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\; dr}$$

And

$$D_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\; d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\; dr}$$

<u>Summary of solution</u>

$$\begin{aligned}
u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+ \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}$$

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\; dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}$$

$$C_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\; dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}$$

$$B_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\; d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\; dr}$$

$$D_{nm} = \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\; d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\; dr}$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$

5.2.2.7 [410] θ dependency, fixed on edges, zero initial velocity, general solution

problem number 410

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) \\ \frac{\partial u}{\partial t}(r,\theta,0) &= 0 \end{aligned}$$

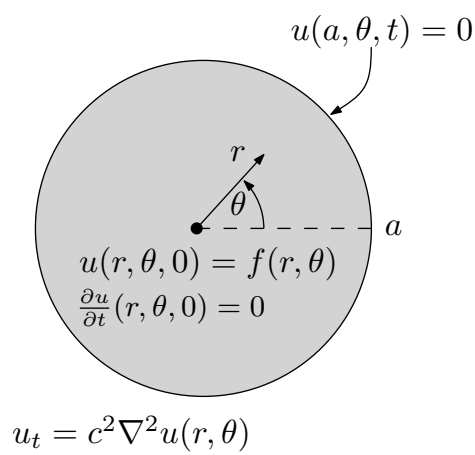

Figure 5.126: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to\ \{\ \sum_{K[3]=1}^{\infty}\frac{2J_0\left(\frac{rj_{0,K[3]}}{a}\right)\cos\left(\frac{\sqrt{c^2}tj_{0,K[3]}}{a}\right)\int_0^a\int_{-\pi}^{\pi}rJ_0\left(\frac{rj_{0,K[3]}}{a}\right)f(r,\theta)d\theta dr}{a^2\pi J_1\left(j_{0,K[3]}\right)^2}+\sum_{K[3]=1}^{\infty}\left(\sum_{K[1]=1}^{\infty}\left(\frac{2J_{K[1]}\left(\frac{rj_{K[1}}{}\right.}{}\right.\right.\right.\right.$$

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) =c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,theta
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

<u>Hand solution</u>

The basic solution for this type of PDE was already given in problem 5.2.2.6 on page 1123 as

$$\begin{aligned}u(r,\theta,t)&=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right)J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\end{aligned}$$

$$A_{nm} = \frac{\int_{-\pi}^{\pi} \left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r\ dr\right) \cos(n\theta)\, d\theta}{\pi \int_0^a J_n^2\left(\sqrt{\lambda_{nm}} r\right) r dr}$$

$$C_{nm} = \frac{\int_{-\pi}^{\pi} \left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r\ dr\right) \sin(n\theta)\, d\theta}{\pi \int_0^a J_n^2\left(\sqrt{\lambda_{nm}} r\right) r dr}$$

$$B_{nm} = \frac{\int_{-\pi}^{\pi} \int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) \cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}} \int_0^a J_n^2\left(\sqrt{\lambda_{nm}} r\right) r\ dr}$$

$$D_{nm} = \frac{\int_{-\pi}^{\pi} \int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) \sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}} \int_0^a J_n^2\left(\sqrt{\lambda_{nm}} r\right) r\ dr}$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}} a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}} a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$. Since $g(r,\theta) = 0$ in this case, then $B_{nm} = 0, D_{nm} = 0$ and the solution simplifies to

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} \cos\left(c\sqrt{\lambda_{nm}} t\right) J_n\left(\sqrt{\lambda_{nm}} r\right) \cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm} \cos\left(c\sqrt{\lambda_{nm}} t\right) J_n\left(\sqrt{\lambda_{nm}} r\right) \sin$$

5.2.2.8 [411] θ dependency, fixed on edges, zero initial velocity, specific example

problem number 411

Added January 11 2020.

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$u(r,\theta,0) = f(r,\theta)$$
$$\frac{\partial u}{\partial t}(r,\theta,0) = 0$$

Using $a = 1, c = 0.2, f(r,\theta) = r\theta$.

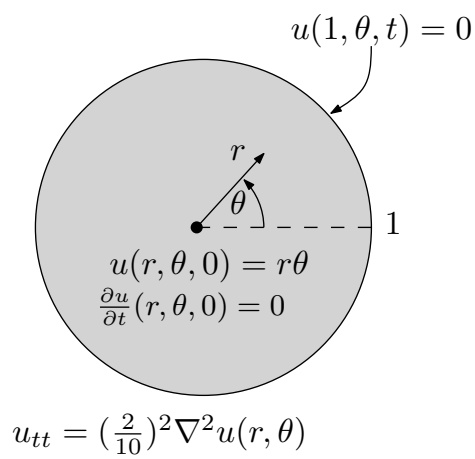

Figure 5.127: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
f[r_,theta_]:=r*theta;
c=2/10; a=1;
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t) \to \{ \quad \sum_{K[3]=1}^{\infty}\left(\sum_{K[1]=1}^{\infty} \frac{(-1)^{K[1]+1} J_{K[1]}(r j_{K[1],K[3]}) j_{K[1],K[3]} \cos(\frac{1}{5} t j_{K[1],K[3]}) \operatorname{Gamma}(\frac{1}{2}(K[1]+3))\, {}_1\tilde{F}_2(\frac{1}{2}(K[1]+3);\frac{1}{2}}{J_{K[1]-1}(j_{K[1],K[3]})\, {}_0\tilde{F}_1(;K[1];-\frac{1}{4}(j_{K[1],K[3]})^2) K[1}\right.\right.\right.$$

Indeterminate

Maple ✗

```
restart;
f:=(r,theta)->r*theta;
c:=2/10;
a:=1;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 5.2.2.6 on page 1123 as

$$
\begin{aligned}
u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}
$$

$$
\begin{aligned}
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}
\end{aligned}
$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number

of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$.

In this problem $g(r,\theta) = 0, f(r,\theta) = r\theta, a = 1, c = \frac{2}{10}$, then $B_{nm} = 0, D_{nm} = 0$ and the solution simplifies to

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(\frac{2}{10}\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(\frac{2}{10}\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}\right.$$

Where

$$A_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a r\theta J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$
$$C_{nm} = \frac{\int_{-\pi}^{\pi}\left(\int_0^a r\theta J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\,d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) rdr}$$

The following animations run for 80 seconds. They are for different n, m modes. (This only show in the HTML version)

5.2.2.8.1 Cases for $n = 0$

5.2.2.8.2 Cases for $n = 1$

5.2.2.8.3 Cases for $n = 2$

5.2.2.8.4 Cases for $n = 3$

Source code for all the above animations

```
(*By Nasser M. Abbasi*)
SetDirectory[NotebookDirectory[]]
 X:\data\public_html\my_notes\PDE_animations\problems\4

(*definitions*)
ClearAll[a,c,n,m,r,theta,f,g,u,maxM,maxN,t]
maxN=4;
maxM=4;
a=1;
c=.2;
minZ=-8;
```

```
maxZ=10;
A0[n_,m_]:= Module[{num,den,r,theta,f},
f=r*theta;
num=Integrate[f* BesselJ[n,lamtbl[[n+1,m]]  r] Cos[n theta] r,{r,0,a},{theta,0,2Pi}];
den=Integrate[(BesselJ[n,lamtbl[[n+1,m]] r])^2  (Cos[n theta])^2 r,{r,0,a},{theta,0,2Pi}];
num/den
];

C0[n_,m_]:= Module[{num,den,f,r,theta},
   f=r*theta;
   num=Integrate[f* BesselJ[n,lamtbl[[n+1,m]]  r] Sin[n theta] r,{r,0,a},{theta,0,2Pi}];
   den=Integrate[(BesselJ[n,lamtbl[[n+1,m]] r])^2  (Sin[n theta])^2 r,{r,0,a},{theta,0,2Pi}]
   num/den
   ];

lam[n_,m_]:=Module[{x},
    x=BesselJZero[n,m];
    N[(x/a)]
   ];

u[r_,theta_,t_,maxN_,maxM_]:=Module[{tmp,n,m},
     tmp=Sum[A0tbl[[n+1,m]]*Cos[c lamtbl[[n+1,m]] t] *BesselJ[n,lamtbl[[n+1,m]]*r]*
         Cos[n*theta],{n,0,maxN},{m,1,maxM}];

     tmp=tmp+Sum[C0tbl[[n+1,m]]*Cos[c lamtbl[[n+1,m]] t] *BesselJ[n,lamtbl[[n+1,m]]*r]*
        Sin[n*theta],{n,1,maxN},{m,1,maxM}]
   ];

lamtbl=Table[lam[n,m],{n,0,maxN},{m,1,maxN}];

A0tbl=Table[A0[n,m],{n,0,maxN},{m,1,maxM}]
C0tbl=Table[C0[n,m],{n,1,maxN},{m,1,maxM}]

(*n=0,m=1*)
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,12,0,1]]},
   {r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
   PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
   PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
     Evaluate[u[r0,theta0,t,0,1]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
```

```
     BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
     MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
     Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_0_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]


(*n=0,m=2*)
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,0,0,2]]},{r0,0,1},
   {theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
   PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
   PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

(*to speed it up, make Z in {t,0,100,Z} larger, and then make Z in Table[Z,{Length[r]}] smal
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
     Evaluate[u[r0,theta0,t,0,2]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
     BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
     MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
     Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_0_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=0,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
          Evaluate[u[r0,theta0,t,0,3]]},{r0,0,1},{theta0,0,2 Pi},
          AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
          PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
          PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",3}]],{t,0,80,.5}];

Export["anim_n_0_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]


(*n=0,m=4*)
t=1;
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],Evaluate[u[r0,theta0,t,0,4]]},
        {r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},BaseStyle->15,
        ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
        BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
```

```
          Evaluate[u[r0,theta0,t,0,4]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"
          BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
          BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 0, ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_0_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=1*)
t=1;
ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
       Evaluate[u[r0,theta0,t,1,1]]},{r0,0,1},{theta0,0,2 Pi},
       AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
       PerformanceGoal->"Speed",Mesh->10, BoxRatios->{1,1,1},
       PlotRange->{Automatic,Automatic,{minZ,maxZ}}]

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
            Evaluate[u[r0,theta0,t,1,1]]},{r0,0,1},{theta0,0,2 Pi},
            AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
            PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
            BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
            Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_1_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=2*)

r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,1,2]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
         BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_1_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
    Evaluate[u[r0,theta0,t,1,3]]},{r0,0,1},{theta0,0,2 Pi},
    AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
    PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
    PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
```

```
    Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",3}]],{t,0,80,.5}];

Export["anim_n_1_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=1,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
           Evaluate[u[r0,theta0,t,1,4]]},{r0,0,1},{theta0,0,2 Pi},
           AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
           PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
           BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
           Row[{"time (sec)",Round@t,", N = ", 1, ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_1_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=1*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
          Evaluate[u[r0,theta0,t,2,1]]},{r0,0,1},{theta0,0,2 Pi},
          AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
          PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
          PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
          Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",1}]]
          ,{t,0,80,.5}];

Export["anim_n_2_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=2*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,2,2]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
         PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_2_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
       Evaluate[u[r0,theta0,t,2,3]]},{r0,0,1},{theta0,0,2 Pi},AxesLabel->{"r","theta","u"},
       BaseStyle->15,ImageMargins->5,PerformanceGoal->"Quality",Mesh->10,
       MaxRecursion->1, BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
       Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",3}]],{t,0,80,.5}];
```

```
Export["anim_n_2_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=2,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
         Evaluate[u[r0,theta0,t,2,4]]},{r0,0,1},{theta0,0,2 Pi},
         AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
         PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
         PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
         Row[{"time (sec)",Round@t,", N = ", 2 ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_2_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=1*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
      Evaluate[u[r0,theta0,t,3,1]]},{r0,0,1},{theta0,0,2 Pi},
      AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
      PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
      PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
      Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",1}]],{t,0,80,.5}];

Export["anim_n_3_m_1.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=2*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
            Evaluate[u[r0,theta0,t,3,2]]},{r0,0,1},{theta0,0,2 Pi},
            AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
            PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1,
            BoxRatios->{1,1,1},PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
            Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",2}]],{t,0,80,.5}];

Export["anim_n_3_m_2.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]

(*n=3,m=3*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
           Evaluate[u[r0,theta0,t,3,3]]},{r0,0,1},{theta0,0,2 Pi},
           AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
           PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
           PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
           Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",3}]],{t,0,80,.5}];
Export["anim_n_3_m_3.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]
```

```
(*n=3,m=4*)
r=Table[Labeled[ParametricPlot3D[{r0 Cos[theta0],r0 Sin[theta0],
           Evaluate[u[r0,theta0,t,3,4]]},{r0,0,1},{theta0,0,2 Pi},
           AxesLabel->{"r","theta","u"},BaseStyle->15,ImageMargins->5,
           PerformanceGoal->"Quality",Mesh->10,MaxRecursion->1, BoxRatios->{1,1,1},
           PlotRange->{Automatic,Automatic,{minZ,maxZ}}],
           Row[{"time (sec)",Round@t,", N = ", 3 ", M = ",4}]],{t,0,80,.5}];

Export["anim_n_3_m_4.gif",r,"DisplayDurations"->Table[.25,{Length[r]}]]
```

5.2.2.9 [412] θ dependency, fixed on edges, zero initial position, specific example

problem number 412

Added January 11, 2020

Math 322 UW exam problem. 2018.

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= 0 \\ u_t(r,\theta,0) &= \begin{cases} \frac{1}{\pi\epsilon^2} & \text{if } r \leq \epsilon \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Where $0 < \epsilon < 1$

Using $a = 1, c = 1, \epsilon = \frac{1}{2}$.

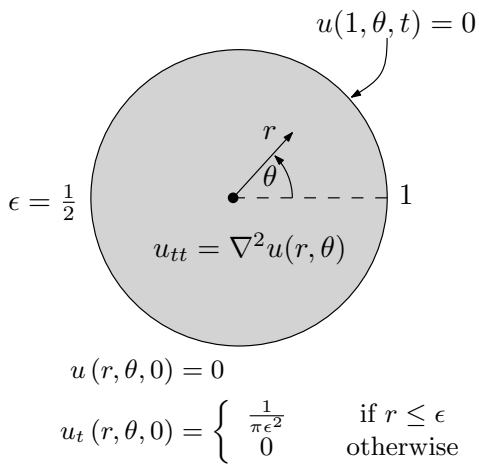

Figure 5.128: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
a=1; c=1; epsilon=1/2;
f[r_,theta_]:=0;
g[r_,theta_]:=Piecewise[{{1/(Pi*epsilon^2),r<epsilon},{0,True}}];
c=1; a=1;
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"];
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == g[r,theta]};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

$$\left\{\left\{u(r,\theta,t)\to\ \{\ \sum_{K[3]=1}^{\infty}\frac{2J_0\left(rj_{0,K[3]}\right)\,{}_0\tilde{F}_1\left(;2;-\frac{1}{16}\left(j_{0,K[3]}\right)^2\right)\sin\left(tj_{0,K[3]}\right)}{\pi J_1\left(j_{0,K[3]}\right)^2 j_{0,K[3]}}\quad (K[1]|K[3])\in\mathbb{Z}\wedge K[1]\ge 1\wedge K[3]\ge 1\right.\right.$$

Indeterminate True

Maple ✗

```
restart;
c:=1;
a:=1;
epsilon:=1/2;
f:=(r,theta)->r*theta;
g:=(r,theta)->piecewise(r<epsilon,1/(Pi*epsilon^2),true,0);
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = g(r,theta);
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

Hand solution

The basic solution for this type of PDE was already given in problem 5.2.2.6 on page 1123 as

$$
\begin{aligned}
u(r,\theta,t) &= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}\cos\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\\
&+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)
\end{aligned}
$$

$$
\begin{aligned}
A_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\cos(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
C_{nm} &= \frac{\int_{-\pi}^{\pi}\left(\int_0^a f(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right) r\ dr\right)\sin(n\theta)\, d\theta}{\pi\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r dr}\\
B_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}\\
D_{nm} &= \frac{\int_{-\pi}^{\pi}\int_0^a g(r,\theta)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)\, r\ d\theta dr}{c\pi\sqrt{\lambda_{nm}}\int_0^a J_n^2\left(\sqrt{\lambda_{nm}}r\right) r\ dr}
\end{aligned}
$$

With λ_{nm} being the solutions for $0 = J_n\left(\sqrt{\lambda_{nm}}a\right)$. For each n, we find $\lambda_{n,1}, \lambda_{n,2}, \lambda_{n,3}, \cdots$, which are the zeros of the Bessel $J_n(x)$ function. So for each n, there are infinite number of zeros. This generates all eigenvalues λ_{nm}. Hence $\sqrt{\lambda_{nm}}a = BesselJZero(n,m)$, therefore $\sqrt{\lambda_{nm}} = \frac{a}{BesselJZero(n,m)}$.

In this problem $f(r,\theta) = 0, a = 1, c = 1$, then $A_{nm} = 0, C_{nm} = 0$ and the solution simplifies to

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin\left(c\sqrt{\lambda_{nm}}t\right) J_n\left(\sqrt{\lambda_{nm}}r\right)\sin$$

Taking time derivative gives

$$u_t(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\cos(n\theta)\lambda_{nm}\cos(\lambda_{nm}t) J_n(\lambda_{nm}r) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin(n\theta)\lambda_{nm}\cos(\lambda_{nm}t) J_n(\lambda_{nm}r)$$

Applying the second initial condition at $t = 0$ gives

$$\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{nm}\cos(n\theta)\lambda_{nm}J_n(\lambda_{nm}r) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}\sin(n\theta)\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} \frac{1}{\pi\epsilon^2} & \text{if } r \le \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

<u>Case $n = 0$</u> (9) becomes

$$\sum_{m=1}^{\infty} B_{0m}\lambda_{0m}J_0(\lambda_{0m}r) = \begin{cases} \frac{1}{\pi\epsilon^2} & \text{if } r \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Applying orthogonality on $J_0(\lambda_{0m}r)$ results in

$$B_{0m}\lambda_{0m}\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr = \frac{1}{\pi\epsilon^2}\int_0^{\epsilon} rJ_0(\lambda_{0m}r)\,dr$$

$$B_{0m} = \frac{1}{\pi\epsilon^2\lambda_{0m}}\frac{\int_0^{\epsilon} rJ_0(\lambda_{0m}r)\,dr}{\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr} \tag{9A}$$

<u>Case $n > 1$</u> Applying orthogonality on $\cos(n\theta)$, equation (9) becomes

$$\sum_{m=1}^{\infty} B_{nm}\left(\int_{-\pi}^{\pi}\cos^2(n\theta)\,d\theta\right)\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} \frac{1}{\pi\epsilon^2}\int_{-\pi}^{\pi}\cos(n\theta)\,d\theta & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{m=1}^{\infty} \pi B_{nm}\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} 0 & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Hence <u>$B_{nm} = 0$ for all $n > 0$</u>.

The same is now done to find $\bar{D}_{nm}$. Applying orthogonality on $\sin(n\theta)$, equation (9) becomes

$$\sum_{m=1}^{\infty} D_{nm}\left(\int_{-\pi}^{\pi} \sin^2(n\theta)\,d\theta\right)\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} \frac{1}{\pi\epsilon^2}\int_{-\pi}^{\pi}\sin(n\theta)\,d\theta & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{m=1}^{\infty} D_{nm}\left(\int_{-\pi}^{\pi} \sin^2(n\theta)\,d\theta\right)\lambda_{nm}J_n(\lambda_{nm}r) = \begin{cases} 0 & \text{if } r^2 \le \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Hence all $\underline{D_{nm} = 0 \text{ for all } n > 0}$.

Therefore the solution (8) reduces to only using $n = 0, m = 1, 2, 3, \cdots$. The solution can now be written as

$$u(r,\theta,t) = \sum_{m=1}^{\infty} B_{0m}\sin(\lambda_{0m}t)\,J_0(\lambda_{0m}r) \tag{10}$$

Where $B_{0m} = \frac{1}{\pi\epsilon^2\lambda_{0m}}\frac{\int_0^{\epsilon} rJ_0(\lambda_{0m}r)dr}{\int_0^1 rJ_0^2(\lambda_{0m}r)dr}$ And λ_{0m} are all the positive zeros of $J_0(z)$, $m = 1, 2, 3, \cdots$.

B_{0m} is now simplified more. Considering first the numerator of B_{0m} which is $\int_0^{\epsilon} rJ_0(\lambda_{0m}r)\,dr$. The hint given says that

$$\frac{d}{dr}(rJ_1(r)) = rJ_0(r)$$

This is the same as saying

$$rJ_1(r) = \int rJ_0(r)\,dr \tag{10A}$$

However the integral in B_{0m} is $\int rJ_0(\lambda_{0m}r)\,dr$ and not $\int rJ_0(r)\,dr$. To transform it so that the hint can be used, let $\lambda_{0m}r = \bar{r}$, then $\frac{dr}{d\bar{r}} = \frac{1}{\lambda_{0m}}$ or $dr = \frac{d\bar{r}}{\lambda_{0m}}$. Now $\int rJ_0(\lambda_{0m}r)\,dr$ becomes $\int \frac{\bar{r}}{\lambda_{0m}}J_0(\bar{r})\frac{d\bar{r}}{\lambda_{0m}}$ or $\frac{1}{\lambda_{0m}^2}\int \bar{r}J_0(\bar{r})\,d\bar{r}$ and now the hint (10A) can be used on this integral giving

$$\frac{1}{\lambda_{0m}^2}\left(\int \bar{r}J_0(\bar{r})\,d\bar{r}\right) = \frac{1}{\lambda_{0m}^2}(\bar{r}J_1(\bar{r}))$$

Replacing $\bar{r}$ back by $\lambda_{0m}r$, gives the result needed

$$\begin{aligned}\frac{1}{\lambda_{0m}^2}(\bar{r}J_1(\bar{r})) &= \frac{1}{\lambda_{0m}^2}(\lambda_{0m}rJ_1(\lambda_{0m}r)) \\ &= \frac{1}{\lambda_{0m}}rJ_1(\lambda_{0m}r)\end{aligned}$$

Now the limits are applied, using the fundamental theory of calculus

$$\begin{aligned}\int_0^{\epsilon} rJ_0(\lambda_{0m}r)\,dr &= \frac{1}{\lambda_{0m}}[rJ_1(\lambda_{0m}r)]_0^{\epsilon} \\ &= \frac{\epsilon}{\lambda_{0m}}J_1(\lambda_{0m}\epsilon)\end{aligned} \tag{10B}$$

This completes finding the numerator integral in B_{0m}. The denominator integral in B_{0m} is $\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr$. This was found before which is

$$\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr = \frac{1}{2}[J_0'(\lambda_{0m})]^2$$

But $J_0'(\lambda_{0m}) = -J_1(\lambda_{0m})$, hence the above becomes

$$\int_0^1 rJ_0^2(\lambda_{0m}r)\,dr = \frac{1}{2}J_1^2(\lambda_{0m}) \tag{10C}$$

Applying (10B) and (10C), B_{0m} simplifies to the following expression

$$\begin{aligned} B_{0m} &= \frac{1}{\pi\epsilon^2\lambda_{0m}}\frac{\frac{\epsilon}{\lambda_{0m}}J_1(\lambda_{0m}\epsilon)}{\frac{1}{2}J_1^2(\lambda_{0m})} \\ &= \frac{2}{\pi\epsilon\lambda_{0m}^2}\frac{J_1(\lambda_{0m}\epsilon)}{J_1^2(\lambda_{0m})} \end{aligned}$$

Therefore the final solution becomes

$$\begin{aligned} u(r,\theta,t) &= \sum_{m=1}^{\infty} B_{0m}\sin(\lambda_{0m}t)\,J_0(\lambda_{0m}r) \\ u(r,\theta,t) &= \frac{2}{\pi\epsilon}\sum_{m=1}^{\infty}\frac{1}{\lambda_{0m}^2}\frac{J_1(\lambda_{0m}\epsilon)}{J_1^2(\lambda_{0m})}J_0(\lambda_{0m}r)\sin(\lambda_{0m}t) \end{aligned} \tag{11}$$

When $\epsilon = \frac{1}{2}$, the above solution (11) becomes

$$u(r,\theta,t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{\lambda_{0m}^2}\frac{J_1\left(\frac{1}{2}\lambda_{0m}\right)}{J_1^2(\lambda_{0m})}J_0(\lambda_{0m}r)\sin(\lambda_{0m}t) \tag{11A}$$

Here is animation for 5 seconds made in Mathematica

Mathematica Source code for all the above animations

```
(*By Nasser M. Abbasi. Animation of problem 4 solution*)

ClearAll[t,r,m];
padIt2[v_,f_List]:=AccountingForm[v,f,NumberSigns->{"",""},
                          NumberPadding->{"0","0"},SignPadding->True];
nTerms=40;
lam = Table[ BesselJZero[0,m],{m,1,nTerms}]//N;
c   = Table[1/lam[[m]]^2 BesselJ[1,lam[[m]]/2]/BesselJ[1,lam[[m]] ]^2,{m,1,nTerms}];
mySol[r_,t_]:=4/Pi   Sum[c[[m]]BesselJ[0,lam[[m]] r] Sin[lam[[m]] t],{m,1,nTerms}];
```

```
frames=Table[
  Print["t=",t];
  Grid[{
   {Row[{"time = ",padIt2[t,{3,2}]}]},
   {ParametricPlot3D[{r Cos[theta],r Sin[theta],mySol[r,t]},{r,0,1},{theta,0,2 Pi},
    PlotRange->{Automatic,Automatic,{-0.6,0.6}},
    PerformanceGoal->"Speed",Boxed->True,
    Axes->True,Mesh->20,
    ViewPoint->{2.17,-2.4,1},
    ImageSize->400,
    BoxRatios->{1, 1, 1}]
  }}],
    {t,0,5,0.05}
];

Manipulate[
  frames[[i]],
  {{i,1,"time"},1,Length@frames,1,Appearance->"Labeled"}
]

Export["anim.gif",frames,"DisplayDurations"->Table[.2,{Length@frames}]]
```

Here is the same animation made in Maple 2018

Maple source code for all the above animations

```
#by Nasser M. Abbasi, May 23,2018

restart;
currentdir("X:/data/public_html/my_notes/PDE_animations/problems/wave_disk_exam_problem_4");
nTerms := 20:
lam    := evalf([BesselJZeros(0,1..nTerms)]):
c      := seq(1/lam[n]^2*BesselJ(1,lam[n]/2)/BesselJ(1,lam[n])^2,n=1..nTerms):
mySol  := proc(r,t)
  local n;
  4/Pi*sum(c[n]*BesselJ(0,lam[n]*r)*sin(lam[n]*t),n=1..nTerms);
end proc:

maxTime := 5: (*seconds*)
delay   := 0.05:
nFrames := round(maxTime/delay):
```

```
frames  := [seq( plot3d([ r, theta, mySol(r,(i*delay)) ],
                   r      = 0..1,
                   theta  = 0..2*Pi,
                   coords = cylindrical,
                   axes   = none,
                   title  = sprintf("%s %3.2f %s","time ",(i*delay),"seconds")
                ),
             i=0..nFrames-1)
           ]:
plots:-display(convert(frames,list),insequence=true);
```

Here is the same animation made in Matlab 2016a

Matlab source code for all the above animations

```
function nma_HW4_math_322
%By Nasser M. Abbasi, May 23, 2018

close all;

GENERATE_GIF=true; %turn to false to not generate animated gif

lam = zeros(80,1); %eigenvalues
for i = 1:80
  lam(i) = fzero(@(x)besselj(0,x),i);
end;

lam    = uniquetol(lam); %must use  uniquetol
nTerms = 20;
c      = zeros(nTerms,1);

for i = 1:nTerms
  c(i) = 1/lam(i)^2*besselj(1,lam(i)/2)/besselj(1,lam(i))^2;
end

  %------------ inner function --------
  function tot = mySol(r,t)
    tot = 0;
    for ii =1:nTerms
        tot = tot + (c(ii)*besselj(0,lam(ii).*r).*sin(lam(ii)*t));
    end;
    tot = 4/pi*tot;
```

```
  end
  %-------------------

maxTime = 5; %seconds
delay   = 0.05;
nFrames = round(maxTime/delay);

r   = 0:.05:1;
phi = 0:pi/20:2*pi;
[R,PHI] = meshgrid(r,phi);

fig_handle   = figure();
set(fig_handle,'Name',....
   'Math 322, Final exam problem 4 animations by Nasser M. Abbasi');

for i=1:nFrames
  Z = mySol(R,((i-1)*delay));
  surf(R.*cos(PHI), R.*sin(PHI), Z);
  set(gca,'nextplot','replacechildren','visible','on');
  colormap cool ;
  title(sprintf('time = %3.2f', (i-1)*delay));
  zlim([-0.6 0.6]);
  drawnow;
  pause(.01);
  if GENERATE_GIF
      frame = getframe(gcf);
      im = frame2im(frame);
      [imind,cm] = rgb2ind(im,256);
      if i == 1
          imwrite(imind,cm,'matlab_animations.gif','gif', ...
                                  'DelayTime',0.1,'LoopCount',0);
      else
          imwrite(imind,cm,'matlab_animations.gif','gif',...
                 'WriteMode','append','DelayTime',0.1,'LoopCount',0);
      end
  end
end

end
```

5.2.2.10 [413] θ dependency, fixed on edges, zero initial position with internal source (Haberman 8.5.5. (b)

problem number 413

Added January 15, 2020

Problem 8.5.5. (b) Richard Haberman applied partial di fb00erential equations book, 5th edition

Solve wave PDE inside circular membrane for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$u_{tt} = c^2 \nabla^2 u(r,\theta) + Q(r,\theta,t)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0) &= f(r,\theta) \\ u_t(r,\theta,0) &= 0 \end{aligned}$$

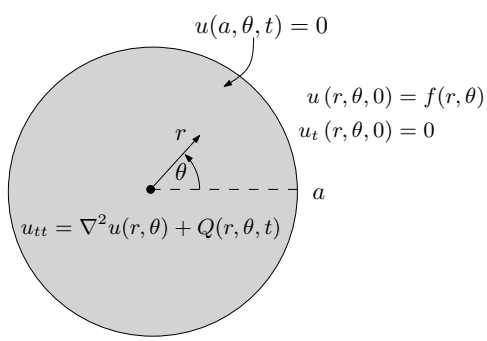

Figure 5.129: PDE specification

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[u[r, theta, t], {t, 2}] == c^2*Laplacian[u[r,theta,t],{r,theta},"Polar"]+Q[r,theta,
ic  = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
bc  = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

Failed

Maple ✗

```
restart;
pde := diff(u(r, theta, t), t$2) = c^2*VectorCalculus:-Laplacian(u(r,theta,t),'polar'[r,thet
ic  := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc  := u(a, theta, t) = 0,
       u(r, -Pi, t) = u(r, Pi, t),
       (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

<u>Hand solution</u>

The solution to the corresponding homogeneous wave PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2$$

Is known to be

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} a_n(t)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\cos(n\theta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_n(t)\, J_n\left(\sqrt{\lambda_{nm}}r\right)\sin(n\theta)$$

Where λ_{nm} are found by solving roots of $J_n\left(\sqrt{\lambda_{nm}}a\right) = 0$. To make things simpler, we will write

$$u(r,\theta,t) = \sum_i a_i(t)\,\Phi_i(r,\theta)$$

Where the above means the double sum of all eigenvalues λ_i. So $\Phi_i(r,\theta)$ represents $J_n\left(\sqrt{\lambda_{nm}}r\right)\{\cos(n\theta),\sin(\theta)\}$ combined. So double sum is implied everywhere. Given this, we now expand the source term

$$Q(r,\theta,t) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

And the original PDE becomes

$$\sum_i a_i''(t)\,\Phi(\lambda_i) = c^2 \sum_i a_i(t)\,\nabla^2(\Phi_i(r,\theta)) + \sum_i q_i(t)\,\Phi_i(r,\theta) \tag{1}$$

But

$$\nabla^2(\Phi_i(r,\theta)) = -\lambda_i \Phi_i(r,\theta)$$

Hence (1) becomes

$$\sum_i a_i''(t)\,\Phi_i(r,\theta) + c^2\lambda_i a_i(t)\,\Phi_i(r,\theta) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

$$\sum_i \left(a_i''(t) + c^2\lambda_i a_i(t)\right)\Phi_i(r,\theta) = \sum_i q_i(t)\,\Phi_i(r,\theta)$$

Applying orthogonality gives

$$a_i''(t) + c^2\lambda_i a_i(t) = q_i(t)$$

Where

$$q_i(t) = \frac{\int_0^a \int_{-\pi}^{\pi} Q(r,\theta,t)\,\Phi_i(r,\theta)\,r dr d\theta}{\int_0^a \int_{-\pi}^{\pi} \Phi_i^2(r,\theta)\,r dr d\theta}$$

The solution to the homogenous ODE is

$$a_i^h(t) = A_i \cos\left(c\sqrt{\lambda_i}t\right) + B_i \sin\left(c\sqrt{\lambda_i}t\right)$$

And the particular solution is found if we know what $Q(r,\theta,t)$ and hence $q_i(t)$. For now, lets call the particular solution as $a_i^p(t)$. Hence the solution for $a_i(t)$ is

$$a_i(t) = A_i \cos\left(c\sqrt{\lambda_i}t\right) + B_i \sin\left(c\sqrt{\lambda_i}t\right) + a_i^p(t)$$

Plugging the above into the $u(r,\theta,t) = \sum_i a_i(t)\,\Phi_i(r,\theta)$, gives

$$u(r,\theta,t) = \sum_i \left(A_i \cos\left(c\sqrt{\lambda_i}t\right) + B_i \sin\left(c\sqrt{\lambda_i}t\right) + a_i^p(t)\right)\Phi_i(r,\theta) \tag{2}$$

We now find A_i, B_i from initial conditions. At $t=0$

$$f(r,\theta) = \sum_i \left(A_i + a_i^p(0)\right)\Phi_i(r,\theta)$$

Applying orthogonality

$$\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta = \int_0^a \int_{-\pi}^{\pi} \sum_i \left(A_i + a_i^p(0)\right)\Phi_i(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta$$

$$\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_j(r,\theta)\,r dr d\theta = \left(A_j + a_j^p(0)\right)\int_0^a \int_{-\pi}^{\pi} \Phi_j^2(r,\theta)\,r dr d\theta$$

$$\left(A_i + a_i^p(0)\right) = \frac{\int_0^a \int_{-\pi}^{\pi} f(r,\theta)\,\Phi_i(r,\theta)\,r dr d\theta}{\int_0^a \int_{-\pi}^{\pi} \Phi_i^2(r,\theta)\,r dr d\theta}$$

Taking time derivative of (2)

$$\frac{\partial u(r,\theta,t)}{\partial t}=\sum_i\left(-A_ic\sqrt{\lambda_i}\sin\left(c\sqrt{\lambda_i}t\right)+c\sqrt{\lambda_i}B_i\cos\left(c\sqrt{\lambda_i}t\right)+\frac{da_i^p(t)}{dt}\right)\Phi_i(r,\theta)$$

At $t=0$

$$0=\sum_i\left(c\sqrt{\lambda_i}B_i+\frac{da_i^p(0)}{dt}\right)\Phi_i(r,\theta)$$

Hence $B_i=0$. Therefore the final solution is

$$u(r,\theta,t)=\sum_i\left(A_i\cos\left(c\sqrt{\lambda_i}t\right)+a_i^p(t)\right)\Phi_i(r,\theta)$$

Where

$$(A_i+a_i^p(0))=\frac{\int_0^a\int_{-\pi}^{\pi}f(r,\theta)\,\Phi_i(r,\theta)\,rdrd\theta}{\int_0^a\int_{-\pi}^{\pi}\Phi_i^2\,(r,\theta)\,rdrd\theta}$$

This complete the solution.

5.3 Wave PDE in 3D

Local contents

5.3.1 Spherical coordinates 1154
5.3.2 Cylindrical coordinates 1155

5.3.1 Spherical coordinates

Local contents

5.3.1.1 [414] No I.C. no B.C. 1154

5.3.1.1 [414] No I.C. no B.C.

problem number 414

Added Jan 10, 2019.

Solve for $u(r,\theta,\phi,t)$ the wave PDE in 3D

$$u_{tt} = c^2\nabla^2 u$$

Using the Physics convention for Spherical coordinates system.

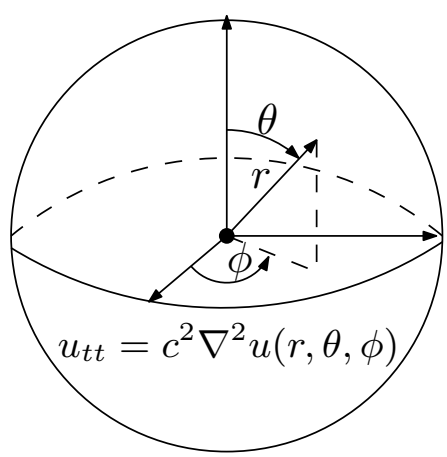

Figure 5.130: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, theta, phi, t], {r, theta, phi}, "Spherical"];
pde =  D[u[r, theta, phi, t], {t, 2}] == c^2*lap;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, theta, phi, t], {r, theta, phi, t}, A
```

$$\left\{\left\{u(r,\theta,\phi,t)\to\ \{\ \frac{\sqrt{2}e^{-\frac{1}{2}\sqrt{c_{10}}(2\phi+\pi)-t\sqrt{c_{11}}}\left(J_{\frac{1}{2}\sqrt{\frac{4c_9}{c^2}+1}}\left(\frac{r\sqrt{-c_{11}}}{\sqrt{c^2}}\right)c_1+Y_{\frac{1}{2}\sqrt{\frac{4c_9}{c^2}+1}}\left(\frac{r\sqrt{-c_{11}}}{\sqrt{c^2}}\right)c_2\right)\left(e^{2\phi\sqrt{c_{10}}}c_5+c_6\right)\left(e^{2t\sqrt{c_{11}}}c_7+c\right.}{}\right.\right.$$

Maple ✓

```
restart;
lap:=VectorCalculus:-Laplacian( u(r,theta,phi,t), 'spherical'[r,theta,phi] );
pde := diff(u(r,theta,phi,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,theta,phi,t),'buil
sol := simplify(sol);
```

$$u(r,\theta,\phi,t)=\frac{\left(c_7e^{2t\sqrt{-c_4}}+c_8\right)\left(c_5e^{2\phi\sqrt{-c_3}}+c_6\right)\left(c_1\,\mathrm{BesselJ}\left(\frac{\sqrt{\frac{c^2-4\_c_1}{c^2}}}{2},\frac{\sqrt{-\_c_4}\,r}{c}\right)+c_2\,\mathrm{BesselY}\left(\frac{\sqrt{\frac{c^2-4\_c_1}{c^2}}}{2},\right.\right.}{}$$

5.3.2 Cylindrical coordinates

Local contents

5.3.2.1 [415] No I.C. no B.C. 1156

5.3.2.1 [415] No I.C. no B.C.

problem number 415

Added Jan 10, 2019.

Solve for $u(r,\phi,z,t)$ the wave PDE in 3D

$$u_{tt} = c^2 \nabla^2 u$$

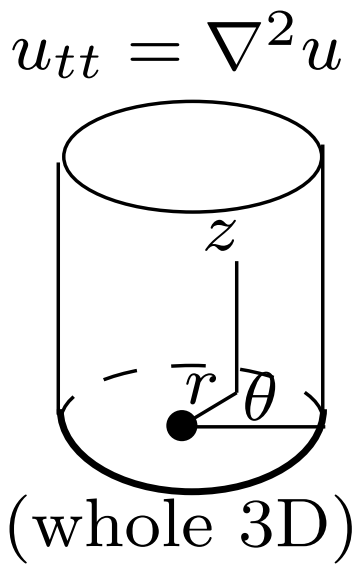

Figure 5.131: PDE specification

Mathematica ✓

```
ClearAll["Global`*"];
lap = Laplacian[u[r, phi, z, t], {r, phi, z}, "Cylindrical"];
pde =  D[u[r, phi, z, t], {t, 2}] == c^2*lap;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, phi, z, t], {r, phi, z, t}], 60*10]];
```

$$\left\{\left\{u(r,\phi,z,t) \to \{ \quad e^{-\sqrt{c_9}\phi - z\sqrt{c_{10}} - t\sqrt{c_{11}}} \left(J_{\sqrt{-c_9}}\left(\frac{r\sqrt{c^2 c_{10} - c_{11}}}{\sqrt{c^2}}\right) c_1 + Y_{\sqrt{-c_9}}\left(\frac{r\sqrt{c^2 c_{10} - c_{11}}}{\sqrt{c^2}}\right) c_2\right) \left(e^{2\phi\sqrt{c_9}} c_3 + c_4\right)\right.\right.$$

Indeterminate

Maple ✓

```
restart;
lap :=VectorCalculus:-Laplacian( u(r,phi,z,t), 'cylindrical'[r,phi,z] );
pde := diff(u(r,phi,z,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,phi,z,t),'build'))
```

$$u(r,\phi,z,t) = \left(c_3 e^{2\phi\sqrt{-\_c_2}} + c_4\right)\left(c_1 \operatorname{BesselJ}\left(\sqrt{-\_c_2}, \frac{\sqrt{\_c_3 c^2 - \_c_4}\, r}{c}\right) + c_2 \operatorname{BesselY}\left(\sqrt{-\_c_2}, \frac{\sqrt{\_c_3 c^2 -}}{c}\right.\right.$$

CHAPTER 6

Handbook of first order partial differential equations

Local contents

6.1 chapter 1 1160
6.2 chapter 2 1166
6.3 chapter 3 1526
6.4 chapter 4 1696
6.5 chapter 5 1858
6.6 chapter 6 2039
6.7 chapter 7 2242
6.8 chapter 8 2419
6.9 chapter 9 2584

6.1 chapter 1

Local contents

6.1.1 problem number 1 1160
6.1.2 problem number 2 1161
6.1.3 problem number 3 1161
6.1.4 problem number 4 1162
6.1.5 problem number 5 1163
6.1.6 problem number 6 1165

6.1.1 problem number 1

problem number 416

Added January 2, 2019.

Problem 1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x = f(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x f(K[1],y)dK[1] + c_1(y)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \int f(x,y)\,dx + \_F1(y)$$

6.1.2 problem number 2

problem number 417

Added January 2, 2019.

Problem 1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_y = f(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to \int_1^y f(x, K[1])dK[1] + c_1(x)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),y)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \int f(x, y)\, dy + \_F1(x)$$

6.1.3 problem number 3

problem number 418

Added January 2, 2019.

Problem 1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x = wf(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == w[x, y]*f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y)\exp\left(\int_1^x f(K[1],y)dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(y)\,\mathrm{e}^{\int f(x,y)dx}$$

Hand solution

$$\frac{\partial w}{\partial x}=wf(x,y)$$
$$\frac{1}{w}\frac{\partial w}{\partial x}=f(x,y)$$

Integrating both sides w.r.t. x gives

$$\ln(w)=\int_0^x f(s,y)\,ds+G(y)$$
$$w=e^{\int_0^x f(s,y)ds+G(y)}$$
$$=F(y)\,e^{\int_0^x f(s,y)ds}$$

Where $F(y)=e^{G(y)}$

6.1.4 problem number 4

problem number 419

Added January 2, 2019.

Problem 1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_y = wf(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == w[x, y]*f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1(x) \exp\left(\int_1^y f(x, K[1]) dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1(y)\, \mathrm{e}^{\int f(x,y)dx}$$

6.1.5 problem number 5

problem number 420

Added January 2, 2019.

Problem 1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x = wf(x, y) + g(x, y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] == w[x, y]*f[x, y] + g[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x f(K[1],y)dK[1]\right)\left(\int_1^x \exp\left(-\int_1^{K[2]} f(K[1],y)dK[1]\right)g(K[2],y)dK[2]+c_1(y)\right)\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \mathrm{e}^{-\left(\int f(x,y)dx\right)}g(x,y)\,dx+\_F1(y)\right)\mathrm{e}^{\int f(x,y)dx}$$

<u>Hand solution</u>

$$\frac{\partial w}{\partial x}=wf(x,y)+g(x,y)$$
$$\frac{\partial w}{\partial x}-wf(x,y)=g(x,y)$$

We can treat this similar to linear ODE and use an integrating factor $I=e^{-\int f(x,y)dx}$ hence the above becomes

$$\frac{\partial}{\partial x}\left(we^{-\int f(x,y)dx}\right)=e^{-\int wdx}g(x,y)$$
$$we^{-\int f(x,y)dx}=\int e^{-\int f(x,y)dx}g(x,y)\,dx+G(y)$$
$$w=\left(\int e^{-\int f(x,y)dx}g(x,y)\,dx+G(y)\right)e^{\int f(x,y)dx}$$

6.1.6 problem number 6

problem number 421

Added January 2, 2019.

Problem 1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = wf(x,y) + g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], y] == w[x, y]*f[x, y] + g[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^y f(x,K[1])dK[1]\right)\left(\int_1^y \exp\left(-\int_1^{K[2]} f(x,K[1])dK[1]\right) g(x,K[2])dK[2] + c_1(x)\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),y)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \mathrm{e}^{-\left(\int f(x,y)dy\right)} g(x,y)\,dy + \_F1(x)\right) \mathrm{e}^{\int f(x,y)dy}$$

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6.2.1.1 [422] problem number 1

problem number 422

Added January 2, 2019.

Problem 2.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya - bx}{a}\right)$$

Hand solution

$$aw_x + bw_y = 0$$

The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dw}{0}$$

The first pair of equations results in $bdx = ady$ or $bx = ay + C_1$. Hence

$$C_1 = bx - ay$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$ where F is arbitrary function, therefore the solution is

$$w(x, y) = F(bx - ay)$$

6.2.1.2 [423] problem number 2

problem number 423

Added January 2, 2019.

Problem 2.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + (bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + (b*x + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(-\frac{-2ay + bx^2 + 2cx}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+(b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-bx^2+2ya-2cx}{2a}\right)$$

Hand solution

Solve $aw_x + (bx+c)\,w_y = 0$. The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{(bx+c)} = \frac{dw}{0}$$

The first pair of equations gives $\frac{(bx+c)}{a}dx = dy$. Integrating results in

$$\frac{1}{a}\left(\frac{bx^2}{2}+cx\right) = y + C_1$$
$$C_1 = \frac{1}{a}\left(\frac{bx^2}{2}+cx\right) - y$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Where F is arbitray function. Therefore

$$w(x,y) = F\left(\frac{bx^2}{2a}+\frac{c}{a}x - y\right)$$

6.2.1.3 [424] problem number 3

problem number 424

Added January 2, 2019.

Problem 2.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax+by+c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x + b*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-bx}(abx+a+b(by+c))}{b^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(b^2y+a+(ax+c)\,b)\,\mathrm{e}^{-bx}}{b^2}\right)$$

<u>Hand solution</u>

Solve

$$w_x+(ax+by+c)\,w_y=0 \tag{1}$$

The Lagrange-charpit equations are

$$dx=\frac{dy}{(ax+by+c)}=\frac{dw}{0}$$

The first pair of equations gives

$$\frac{dy}{dx}=ax+by+c$$
$$\frac{dy}{dx}-by=ax+c$$

This is linear. Integrating factor is $I = e^{-bx}$. Hence the above becomes

$$\begin{aligned}
\frac{d}{dx}\left(ye^{-bx}\right) &= (ax+c)\,e^{-bx} \\
ye^{-bx} &= a\int xe^{-bx} + c\int e^{-bx} + C_1 \\
ye^{-bx} &= a\left(-\frac{(1+bx)\,e^{-bx}}{b^2}\right) - c\frac{e^{-bx}}{b} + C_1 \\
y &= -a\frac{(1+bx)}{b^2} - \frac{c}{b} - C_1 e^{bx} \\
C_1 &= -\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right)e^{-bx}
\end{aligned}$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$\begin{aligned}
w(x,y) &= F\left(-\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right)e^{-bx}\right) \\
&= F\left(\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right)e^{-bx}\right)
\end{aligned}$$

6.2.1.4 [425] problem number 4

problem number 425

Added January 2, 2019.

Problem 2.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)$$

Hand solution

Solve

$$axw_x + byw_y = 0 \tag{1}$$

The Lagrange-charpit equations are

$$\frac{dx}{ax} = \frac{dy}{by} = \frac{dw}{0}$$

The first pair of equations gives

$$\begin{aligned}
\frac{b}{a}\frac{dx}{x} &= \frac{dy}{y} \\
\frac{b}{a}\ln x &= \ln y + C_1 \\
x^{\frac{b}{a}} &= C_1 y \\
C_1 &= \frac{x^{\frac{b}{a}}}{y}
\end{aligned}$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$w(x,y) = F\left(\frac{x^{\frac{b}{a}}}{y}\right)$$

6.2.1.5 [426] problem number 5

problem number 426

Added January 2, 2019.

Problem 2.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y^2a-bx^2}{a}\right)$$

<u>Hand solution</u>

$$ayw_x+bxw_y=0$$

Using method of charaterstics the lagrange-Charpit equations are

$$\frac{dx}{ay}=\frac{dy}{bx}=\frac{du}{0}$$

The first two equations give $aydy=bxdx$. Hence

$$a\frac{y^2}{2}=\frac{bx^2}{2}+C_1$$
$$C_1=a\frac{y^2}{2}-\frac{bx^2}{2}$$

And $du=0$. This gives $u=C_2$. Now let $C_2=F(C_1)$. This gives

$$u=F\left(\frac{ay^2-bx^2}{2}\right)$$

6.2.1.6 [427] problem number 6

problem number 427

Added January 2, 2019.

Problem 2.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$yw_x + (y + a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + (y + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(a\left(-\log\left(e^{-\frac{a+y}{a}}(a+y)\right)\right) - a - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=y*diff(w(x,y),x)+(y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1(-a \ln(a + y) - x + y)$$

Hand solution

$$yw_x + (y + a)\, w_y = 0$$

Using method of charaterstics the lagrange-Charpit equations are

$$\frac{dx}{y} = \frac{dy}{y + a} = \frac{du}{0}$$

The first two equations give $dx = \frac{y}{y+a} dy$. Hence by integrating

$$x = y - a \ln(y + a) + C_1$$
$$C_1 = x - y + a \ln(y + a)$$

And $du = 0$. This gives $u = C_2$. Now let $C_2 = F(C_1)$. This gives

$$u = F(x - y + a \ln(y + a))$$

6.2.1.7 [428] problem number 7

problem number 428

Added January 2, 2019.

Problem 2.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ay + bx + c)w_x - (by + kx + s)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y + b*x + c)*D[w[x, y], x] - (b*y + k*x + s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y(ay+2bx+2c)+kx^2+2sx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=(a*y+b*x+c)*diff(w(x,y),x)-(b*y+k*x+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{\sqrt{a^3k^2y^2 - 2a^2b^2ky^2 + 2a^2bk^2xy + a^2k^3x^2 + ab^4y^2 - 4ab^3kxy - 2ab^2k^2x^2 + 2b^5xy + b^4k}}\right.$$

Hand solution

Solve

$$(ay + bx + c)\, w_x - (by + kx + s)\, w_y = 0$$

The lagrange-Charpit equations are

$$\frac{dx}{ay + bx + c} = \frac{dy}{-(by + kx + s)} = \frac{dw}{0}$$

The first two equations give

$$\frac{dy}{dx} = \frac{-(by + kx + s)}{ay + bx + c}$$

Need to solve the above, then solve for C_1 to finish the solution.

6.2.1.8 [429] problem number 8

problem number 429

Added January 2, 2019.

Problem 2.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(a_1x + b_1y + c_1)w_x + (a_2x + b_2y + c_2)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a1*x + b1*y + c1)*D[w[x, y], x] + (a1*x + b2*y + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a1*x+b1*y+c1)*diff(w(x,y),x)+(a2*x+b2*y+c2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2(a1+b2)\arctan\left(\frac{-2a2b1^2y+(-2a2c1+(-a2x+c2)a1+(2a1y+a2x+c2)b2)b1+(a1-b2)(a1x+c1)b2}{\sqrt{-a1^2+2a1b2-4a2b1-b2^2}\,((-a2x-c2)b1+(a1x+c1)b2)}\right)}{}\right.$$

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6.2.2.1 [430] problem number 1

problem number 430

Added January 2, 2019.

Problem 2.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (ax^2 + bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2 + b*x + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{ax^3}{3}-\frac{bx^2}{2}-cx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x^2+b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{3}ax^3-\frac{1}{2}bx^2-cx+y\right)$$

6.2.2.2 [431] problem number 2

problem number 431

Added January 2, 2019.

Problem 2.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay^2+by+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^2 + b*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2ay+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*y^2+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sqrt{4ca-b^2}\,x - 2\arctan\left(\frac{2ya+b}{\sqrt{4ca-b^2}}\right)}{\sqrt{4ca-b^2}}\right)$$

6.2.2.3 [432] problem number 3

problem number 432

Added January 2, 2019.

Problem 2.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^2 + cx)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^2 + c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-ax}(b(a^2x^2+2ax+2)+a(a^2y+acx+c))}{a^3}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*y+b*x^2+c*x)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(a^3y + (bx+c)\,a^2x + (2bx+c)\,a + 2b\right)\mathrm{e}^{-ax}}{a^3}\right)$$

6.2.2.4 [433] problem number 4

problem number 433

Added January 2, 2019.

Problem 2.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (axy + bx^2 + cx + ky + s)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x*y + b*x^2 + c*x + k*y + s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-\frac{1}{2}x(ax+2k)}\left(2\sqrt{a}(a^2y + a(bx+c) - bk) - \sqrt{2\pi}e^{\frac{(ax+k)^2}{2a}}\operatorname{erf}\left(\frac{ax+k}{\sqrt{2}\sqrt{a}}\right)(a^2s + a(b-ck) + bk\right.}{2a^{5/2}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*x*y+b*x^2+c*x+k*y+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left((a^2s - ack + (k^2+a)\,b)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2}\,(ax+k)}{2\sqrt{a}}\right)e^{\frac{2a^2x^2+4akx+k^2}{2a}} + 2\left(-a^{\frac{5}{2}}y + \sqrt{a}\,bk + (-\right.\right.}{2a^{\frac{5}{2}}}\right.$$

6.2.2.5 [434] problem number 5

problem number 434

Added January 2, 2019.

Problem 2.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 - a^2x^2 + 3a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2*x^2 + 3*a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(ax-y)\,\text{ParabolicCylinderD}\left(-2, i\sqrt{2}\sqrt{a}x\right) + i\sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(-1, i\sqrt{2}\sqrt{\right.}}{(ax+y)\,\text{ParabolicCylinderD}\left(1, \sqrt{2}\sqrt{a}x\right) - \sqrt{2}\sqrt{a}\,\text{ParabolicCylinderD}\left(2, \sqrt{2}\sqrt{a}x\right.}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2*x^2+3*a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-ax^2 + xy + 1}{-\sqrt{\pi}\left((-a)^{\frac{3}{2}}x^2 + \sqrt{-a}\,xy + \sqrt{-a}\right)\text{erf}\left(\sqrt{-a}\,x\right) + (ax-y)\,e^{ax^2}}\right)$$

6.2.2.6 [435] problem number 6

problem number 435

Added January 2, 2019.

Problem 2.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (y^2 - a^2x^2 + a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2*x^2 + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\pi}(y-ax)\operatorname{Erfi}\left(\sqrt{a}x\right)+2\sqrt{a}e^{ax^2}}{2\sqrt{a}(ax-y)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2*x^2+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(ax-y)\sqrt{\pi}}{\sqrt{\pi}\,(ax-y)\operatorname{erf}\left(\sqrt{-a}\,x\right)-2\sqrt{-a}\,\mathrm{e}^{ax^2}}\right)$$

6.2.2.7 [436] problem number 7

problem number 436

Added January 2, 2019.

Problem 2.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (y^2 + axy + a)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*y + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)-\frac{ye^{\frac{ax^2}{2}}}{\sqrt{2}\sqrt{a}(xy+1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+a*x*y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(xy+1)\,a\,\mathrm{erf}\left(\frac{\sqrt{-2a}\,x}{2}\right)-\sqrt{-\frac{2a}{\pi}}\,y\,\mathrm{e}^{\frac{ax^2}{2}}}{\sqrt{-\frac{2a}{\pi}}\,(xy+1)}\right)$$

6.2.2.8 [437] problem number 8

problem number 437

Added January 2, 2019.

Problem 2.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(y^2+axy-abx-b^2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*y - a*b*x - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{2b^2}{a}}\left(\sqrt{2\pi}(y-b)\operatorname{Erfi}\left(\frac{ax+2b}{\sqrt{2}\sqrt{a}}\right)+2\sqrt{a}e^{\frac{(ax+2b)^2}{2a}}\right)}{2\sqrt{a}(b-y)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+a*x*y-a*b*x-b^2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(\sqrt{\pi}\,(b-y)\operatorname{erf}\left(\frac{(ax+2b)\sqrt{2}}{2\sqrt{-a}}\right)+\sqrt{2}\,\sqrt{-a}\,\mathrm{e}^{\frac{(ax+2b)^2}{2a}}\right)\sqrt{2}\,\mathrm{e}^{-\frac{2b^2}{a}}}{\sqrt{-a}\,(2b-2y)}\right)$$

6.2.2.9 [438] problem number 9

problem number 438

Added January 2, 2019.

Problem 2.2.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+k(ax+by+c)^2w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*(a*x + a*y + c)^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-2ia\sqrt{k}x}\left(ia\sqrt{k}(x+y)+ic\sqrt{k}+1\right)}{2a\sqrt{k}\left(a\sqrt{k}(x+y)+c\sqrt{k}+i\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+k*(a*x+a*y+c)^2*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\sqrt{k}\,x-\arctan\left(\left((x+y)\,a+c\right)\sqrt{k}\right)}{a\sqrt{k}}\right)$$

6.2.2.10 [439] problem number 10

problem number 439

Added January 2, 2019.

Problem 2.2.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+(ay^2+cx^2+y)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + c*x^2 + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tan^{-1}\left(\frac{\sqrt{a}y}{\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+(a*y^2+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\sqrt{ac}\,x+\arctan\left(\frac{ay}{\sqrt{ac}\,x}\right)}{\sqrt{ac}}\right)$$

6.2.2.11 [440] problem number 11

problem number 440

Added January 2, 2019.

Problem 2.2.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+(ay^2+bxy+cx^2+y)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + b*x*y + c*x^2 + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2ay+bx}{x\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+(a*y^2+b*x*y+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\sqrt{4ac-b^2}\,x-2\arctan\left(\frac{2ya+bx}{\sqrt{4ac-b^2}\,x}\right)}{\sqrt{4ac-b^2}}\right)$$

6.2.2.12 [441] problem number 12

problem number 441

Added January 2, 2019.

Problem 2.2.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+c)w_x+\left(\alpha(ay+bx)^2+\beta(ay+bx)-bx+\gamma\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + c)*D[w[x, y], x] + (alpha*(a*y + b*x)^2 + beta*(a*y + b*x) - b*x + gamma)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(2\tan^{-1}\left(\frac{2\alpha(ay+bx)+\beta}{a\alpha\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}}\right)-a\alpha\log(ax+c)\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x + c)*diff(w(x,y),x)+(alpha*(a*y+b*x)^2+beta*(a*y+b*x)-b*x+g)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-2a^2\arctan\left(\frac{(2a\alpha y+2\alpha bx+\beta)a^2}{\sqrt{4a^4\alpha g-a^4\beta^2+4a^3\alpha bc}}\right)+\sqrt{4a^3\alpha bc+(4g\alpha-\beta^2)\,a^4}\,\ln(ax+c)}{\sqrt{4a^3\alpha bc+(4g\alpha-\beta^2)\,a^4}}\right)$$

6.2.2.13 [442] problem number 13

problem number 442

Added January 2, 2019.

Problem 2.2.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x+by^2w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{b}{ax}-\frac{1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ax-by}{axy}\right)$$

6.2.2.14 [443] problem number 14

problem number 443

Added January 2, 2019.

Problem 2.2.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2+b)w_x-\left(y^2-2xy+(1-a)x^2-b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^2 + b)*D[w[x, y], x] - (y^2 - 2*x*y + (1 - a)*x^2 - b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\frac{(y-x)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}-1}{x-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x^2+b)*diff(w(x,y),x)-(y^2-2*x*y+(1-a)*x^2-b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(x-y)\arctan\left(\frac{ax}{\sqrt{ab}}\right)+\sqrt{ab}}{\sqrt{ab}\,(x-y)}\right)$$

6.2.2.15 [444] problem number 15

problem number 444

Added January 2, 2019.

Problem 2.2.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2+b_1x+x_1)w_x+(a_2y^2+b_2y+c_2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x^2 + b1*x + c1)*D[w[x, y], x] + (a2*y^2 + b2*y + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\frac{2\text{a2}y+\text{b2}}{\sqrt{4\text{a2c2}-\text{b2}^2}}\right)}{\sqrt{4\text{a2c2}-\text{b2}^2}}-\frac{2\tan^{-1}\left(\frac{2\text{a1}x+\text{b1}}{\sqrt{4\text{a1c1}-\text{b1}^2}}\right)}{\sqrt{4\text{a1c1}-\text{b1}^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a1*x^2+b1*x+c1)*diff(w(x,y),x)+ (a2*y^2+b2*y+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(\sqrt{4c2\,a2-b2^2}\arctan\left(\frac{2a1\,x+b1}{\sqrt{4c1\,a1-b1^2}}\right)-\sqrt{4c1\,a1-b1^2}\arctan\left(\frac{2a2\,y+b2}{\sqrt{4c2\,a2-b2^2}}\right)\right)}{\sqrt{4c1\,a1-b1^2}\sqrt{4c2\,a2-b2^2}}\right)$$

6.2.2.16 [445] problem number 16

problem number 445

Added January 2, 2019.

Problem 2.2.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)(x-b)w_x - \left(y^2+k(y+x-a)(y+x-b)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*(x - b)*D[w[x, y], x] - (y^2 + k*(y + x - a)*(y + x - b))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-a)-\log(x-b))}{2(a-b)}-\tan^{-1}\left(\frac{ak+bk-2(k(x+y)+y)}{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde := (x-a)*(x-b)*diff(w(x,y),x)- (y^2+k*(y+x-a)*(y+x-b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{((a-x-y)\,k-y)\,(a-x)^{k}\,(b-x)^{-k}}{(b-x-y)\,k-y}\right)$$

6.2.2.17 [446] problem number 17

problem number 446

Added January 2, 2019.

Problem 2.2.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1y^2+b_1y+c_1)w_x+(a_2x^2+b_2x+c_2)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*y^2 + b1*y + c1)*D[w[x, y], x] + (a2*x^2 + b2*x + c2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{6}\left(2\text{a1}y^3-2\text{a2}x^3+3\text{b1}y^2-3\text{b2}x^2+6\text{c1}y-6\text{c2}x\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a1*y^2+b1*y+c1)*diff(w(x,y),x)+ (a2*x^2+b2*x+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{3}a1y^3-\frac{1}{3}a2x^3+\frac{1}{2}b1y^2-\frac{1}{2}b2x^2+c1y-c2x\right)$$

6.2.2.18 [447] problem number 18

problem number 447

Added January 2, 2019.

Problem 2.2.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$y(ax+b)w_x + (ay^2 - cx)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y*(a*x + b)*D[w[x, y], x] + (a*y^2 - c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{a(ay^2-2cx)-bc}{a^2(ax+b)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  y*(a*x+b)*diff(w(x,y),x)+ (a*y^2-c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a^2y^2-2acx-bc}{(ax+b)^2a^2}\right)$$

6.2.2.19 [448] problem number 19

problem number 448

Added January 2, 2019.

Problem 2.2.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ay^2 + bx)w_x - (cx^2 + by)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x)*D[w[x, y], x] - (x*x^2 + b*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^3}{3}+bxy+\frac{x^4}{4}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+b*x)*diff(w(x,y),x)- (x*x^2+b*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{1}{3}ay^3-\frac{1}{4}x^4-bxy\right)$$

6.2.2.20 [449] problem number 20

problem number 449

Added January 2, 2019.

Problem 2.2.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx^2)w_x+2bxw_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(bx^2 + (y^2 + 2y + 2)\,a)\,\mathrm{e}^{-y}}{b}\right)$$

6.2.2.21 [450] problem number 21

problem number 450

Added January 2, 2019.

Problem 2.2.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2 + bx^2)w_x + 2bxyw_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\log\left(\frac{bx^2}{y} - ay\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{y^2a - bx^2}\right)$$

6.2.2.22 [451] problem number 22

problem number 451

Added January 2, 2019.

Problem 2.2.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ay^2 + x^2)w_x + (bx^2 + c - 2xy)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*y^2 + x^2)*D[w[x, y], x] + (b*x^2 + c - 2*x*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\frac{1}{3}(ay^3 - bx^3 - 3cx + 3x^2y)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*y^2+x^2)*diff(w(x,y),x)+(b*x^2+c-2*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\frac{1}{3}ay^3 + \frac{1}{3}bx^3 - x^2y + cx\right)$$

6.2.2.23 [452] problem number 23

problem number 452

Added January 2, 2019.

Problem 2.2.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Ay^2 + Bx^2 - a^2B)w_x + (Cy^2 + 2Bxy)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x^2 - a^2*B)*D[w[x, y], x] + (C0*y^2 + 2*B*x*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a^2(-B)+y(\text{C0}x-Ay)+Bx^2}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (A*y^2+B*x^2-a^2*B)*diff(w(x,y),x)+(C*y^2+2*B*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-Ay^2-a^2B+Bx^2+Cxy}{y}\right)$$

6.2.2.24 [453] problem number 24

problem number 453

Added January 2, 2019.

Problem 2.2.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx^2+cy)w_x+2bxw_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^2 + b*x^2 + c*y)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*y^2+b*x^2+c*y)*diff(w(x,y),x)+2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(ay^2+bx^2+2a+c+(2a+c)\,y)\,\mathrm{e}^{-y}}{b}\right)$$

6.2.2.25 [454] problem number 25

problem number 454

Added January 2, 2019.

Problem 2.2.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Axy+Bx^2+kx)w_x+(Dy^2+Exy+Fx^2+ky)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x*y + B*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F*x^2 + k*y)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  (A*x*y+B*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.2.26 [455] problem number 26

problem number 455

Added January 2, 2019.

Problem 2.2.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Axy + Aky + Bx^2 + Bkx)w_x + (Cy^2 + Dxy + k(D - B)y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x*y + A*k*y + B*x^2 + B*k*x)*D[w[x, y], x] + (C0*y^2 + D0*x*y + k*(D0 - B)*y)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*x*y+A*k*y+B*x^2+B*k*x)*diff(w(x,y),x)+(C0*y^2+D0*x*y+k*(D0-B)*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.2.27 [456] problem number 27

problem number 456

Added January 2, 2019.

Problem 2.2.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Ay^2 + Bxy + Cx^2 + kx)w_x + (Dy^2 + Exy + Fx^2 + ky)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x*y + C0*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2 + k*y)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*y^2+B*x*y+C0*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.2.28 [457] problem number 28

problem number 457

Added January 2, 2019.

Problem 2.2.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(Ay^2 + Bxy + Cx^2)w_x + (Dy^2 + Exy + Fx^2)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + B*x*y + C0*x^2)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (A*y^2+B*x*y+C0*x^2)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\ln(x) - \frac{\left(A\,\mathrm{RootOf}\left(A\_Z^3 + (B-D0)\_Z^2 - F0 + (C0-E0)\_Z\right)^2 + B\,\mathrm{RootOf}\left(\right.\right.}{3A\,\mathrm{RootOf}\left(A\_Z^3 + (B-D0)\_Z^2 - F0 + (C0-E0)\_Z\right.}\right.$$

solution contains RootOf

6.2.2.29 [458] problem number 29

problem number 458

Added January 2, 2019.

Problem 2.2.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + 2Bxy + Dx^2 + a)w_x - (Dy^2 + 2Dxy - Ex^2 - b)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*y^2 + 2*B*x*y + D0*x^2 + a)*D[w[x, y], x] - (D0*y^2 + 2*D0*x*y - E0*x^2 - b)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (A*y^2+2*B*x*y+D0*x^2+a)*diff(w(x,y),x)-(D0*y^2+2*D0*x*y-E0*x^2-b)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.2.30 [459] problem number 30

problem number 459

Added January 2, 2019.

Problem 2.2.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(y^2 - 2xy + x^2 + ay)w_x + ayw_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y^2 - 2*x*y + x^2 + a*y)*D[w[x, y], x] + a*y*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (y^2-2*x*y+x^2+a*y)*diff(w(x,y),x)+a*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{a + (x - y)\ln(y)}{x - y}\right)$$

6.2.2.31 [460] problem number 31, Hesse's equation

problem number 460

Added January 2, 2019.

Problem 2.2.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke (1965).

Solve for $w(x, y)$

$$(xf_1 - f_2)w_x + (yf_1 - f_3)w_y = 0$$

Where $f_n = a_n + b_n x + c_n y$.

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (x*(a1 + b1*x + c1*y) - (a2 + b2*x + c2*y))*D[w[x, y], x] + (y*(a1 + b1*x + c1*y) - (
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (x *(a1+b1*x+c1*y)-(a2+b2*x+c2*y))*diff(w(x,y),x)+(y*(a1+b1*x+c1*y)-(a3+b3*x+c3*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

6.2.3 2.3

Local contents

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6.2.3.1 [461] problem number 1

problem number 461

Added January 2, 2019.

Problem 2.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (y^2 + bx^2y - a^2 - abx^2)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + b*x^2*y - a^2 - a*b*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{e^{\frac{bx^3}{3}}\left((bx^2+y)\,\mathrm{HeunT}[a^2,-((a-2)b),0,0,b,x]+\mathrm{HeunTPrime}[a^2,-((a-2)b),0,0,b\right.}{y\mathrm{HeunT}\left[a^2,-ab,0,0,-b,x\right]+\mathrm{HeunTPrime}\left[a^2,-ab,0,0,-b,x\right]}\right.\right.\right.$$

But it can't solve it when assuming $b>0$ which is strange.

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+b*x^2*y-a^2-a*b*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{3\left((bx^2\mathrm{csgn}(b)-bx^2-2y)\,HT\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)^{\frac{1}{3}}},-\frac{3(a-1)\sqrt{b^2}}{b},0,\frac{3^{\frac{2}{3}}(b^2)}{3}\right.\right.}{-3\left((bx^2\mathrm{csgn}\left(b\right)-bx^2-2y)\,HT\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)^{\frac{1}{3}}},-\frac{3(a-1)\sqrt{b^2}}{b},0,\frac{3^{\frac{2}{3}}(b^2)^{\frac{1}{6}}x}{3}\right)-\frac{23^{\frac{2}{3}}(b^2)^{\frac{1}{6}}HT'\left(-\frac{3^{\frac{2}{3}}a^2}{(b^2)^{\frac{1}{3}}}\right.}{}\right.}\right.$$

6.2.3.2 [462] problem number 2

problem number 462

Added January 2, 2019.

Problem 2.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ax^2y+bx^3+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2*y + b*x^3 + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt[3]{3}b\,\text{Gamma}\left(\frac{4}{3},\frac{ax^3}{3}\right)}{a^{4/3}}+\frac{c\,\text{Gamma}\left(\frac{1}{3},\frac{ax^3}{3}\right)}{3^{2/3}\sqrt[3]{a}}+ye^{-\frac{ax^3}{3}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*x^3+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{3\left(3^{\frac{1}{6}}(ac+b)\,x\,\text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},\frac{ax^3}{3}\right)\mathrm{e}^{\frac{ax^3}{6}}+\frac{4(ax^3)^{\frac{1}{6}}(cx-y)a}{3}\right)\mathrm{e}^{-\frac{ax^3}{3}}}{4\,(ax^3)^{\frac{1}{6}}\,a}\right)$$

6.2.3.3 [463] problem number 3

problem number 463

Added January 2, 2019.

Problem 2.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ax^2y+by^3)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^2*y + b*y^3)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{2ax^3}{3}}}{y^2}+\frac{i\left(\sqrt{3}+i\right)b\,\text{Gamma}\left(\frac{1}{3},-\frac{2ax^3}{3}\right)}{\sqrt[3]{2}3^{2/3}\sqrt[3]{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*y^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{-22^{\frac{2}{3}}3^{\frac{5}{6}}Obxy^2+32^{\frac{2}{3}}3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)bxy^2\Gamma\left(\frac{1}{3},-\frac{2ax^3}{3}\right)-9\Gamma\left(\frac{2}{3}\right)O\,\mathrm{e}^{\frac{2ax^3}{3}}}{9\Gamma\left(\frac{2}{3}\right)Oy^2}\right)$$

6.2.3.4 [464] problem number 4

problem number 464

Added January 2, 2019.

Problem 2.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(axy+b)y^2w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x*y + b)*y^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x*y+b)*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\sqrt{b^2-4a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{b^2-4a}\,(2ayx+b)}{-b^2+4a}\right) - 8\left(\ln(xy) - \frac{\ln\left((ay^2x^2+byx+1)x^2\right)}{2}\right)\left(-\frac{b^2}{4}+a\right)}{-2b^2+8a}\right)$$

6.2.3.5 [465] problem number 5

problem number 465

Added January 2, 2019.

Problem 2.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + A(ax+by+c)^3y^2w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + A*(a*x + b*y + c)^3*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  A*(a*x+b*y+c)^3*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \frac{\ln\left(\frac{ax+(-\operatorname{RootOf}(Ab^4\_Z^3+3Ab^3c\_Z^2+3Ab^2c^2\_Z+Abc^3+a)+y)b}{b}\right)}{3Ab\left(\operatorname{RootOf}\left(Ab^4\_Z^3+3Ab^3c\_Z^2+3Ab^2c^2\_Z+Abc^3+a\right)b+c\right)^2}\right)$$

Answer contains RootOf

6.2.3.6 [466] problem number 6

problem number 466

Added January 2, 2019.

Problem 2.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ax^4y^3 + (bx^2-1)y + cx)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^4*y^3 + (b*x^2 - 1)*y + c*x)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+  (a*x^4*y^3+(b*x^2-1)*y+c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-byx-\mathrm{RootOf}\left(c^2 a\_Z^3+b^3\_Z-b^3\right)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2 a\_Z^3+b^3\_Z-b^3\right)^2 ac^2+b^3}-\frac{bx^2}{2}\right)$$

Answer contains RootOf

6.2.3.7 [467] problem number 7

problem number 467

Added January 2, 2019.

Problem 2.2.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + (ax^2y^2 + bxy + c)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + (a*x^2*y^2 + b*x*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{\sqrt{-4ac+b^2+2b+1}}\left(\sqrt{-4ac+b^2+2b+1}+2axy+b+1\right)}{\sqrt{-4ac+b^2+2b+1}-2axy-b-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+  (a*x^2*y^2+b*x*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2ayx+b+1}{\sqrt{4ac-b^2-2b-1}}\right)+\sqrt{4ac-b^2-2b-1}\,\ln(x)}{\sqrt{4ac-b^2-2b-1}}\right)$$

6.2.3.8 [468] problem number 8

problem number 468

Added January 2, 2019.

Problem 2.2.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2y+b)w_x-(axy^2+c)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^2*y + b)*D[w[x, y], x] - (a*x*y^2 + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ax^2y^2+2by+2cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x^2*y+b)*diff(w(x,y),x)- (a*x*y^2+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{2}ax^2y^2-by-cx\right)$$

6.2.3.9 [469] problem number 9

problem number 469

Added January 2, 2019.

Problem 2.2.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax + by^3)w_x - (cx^3 + ay)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x + b*y^3)*D[w[x, y], x] - (c*x^3 + a*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  (a*x+b*y^3)*diff(w(x,y),x)- (c*x^3+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\frac{1}{4}by^4 - \frac{1}{4}cx^4 - axy\right)$$

6.2.4 2.4

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6.2.4.1 [470] problem number 1

problem number 470

Added January 2, 2019.

Problem 2.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (a\sqrt{x}y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-\frac{2}{3}ax^{3/2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{2ax^{\frac{3}{2}}}{3}}\right)$$

6.2.4.2 [471] problem number 2

problem number 471

Added January 2, 2019.

Problem 2.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (a\sqrt{x}y + b\sqrt{y})w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y + b*Sqrt[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{b\,\text{Gamma}\left(\frac{2}{3},\frac{1}{3}ax^{3/2}\right)}{\sqrt[3]{3}a^{2/3}}-\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(\frac{b\,\text{Gamma}\left(\frac{2}{3},\frac{1}{3}ax^{3/2}\right)}{\sqrt[3]{3}a^{2/3}}+\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(33^{\frac{1}{3}}bx\,\text{WhittakerM}\left(\frac{1}{3},\frac{5}{6},\frac{ax^{\frac{3}{2}}}{3}\right)\mathrm{e}^{\frac{ax^{\frac{3}{2}}}{6}}+5\left(ax^{\frac{3}{2}}\right)^{\frac{1}{3}}bx-10\left(ax^{\frac{3}{2}}\right)^{\frac{1}{3}}\sqrt{y}\right)\mathrm{e}^{-\frac{ax^{\frac{3}{2}}}{3}}}{10\left(ax^{\frac{3}{2}}\right)^{\frac{1}{3}}}\right)$$

6.2.4.3 [472] problem number 3

problem number 472

Added January 2, 2019.

Problem 2.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(a\sqrt{x}y+bx\sqrt{y})w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sqrt[x]*y + b*x*Sqrt[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt[3]{3}b\,\text{Gamma}\left(\frac{4}{3},\frac{1}{3}ax^{3/2}\right)}{a^{4/3}}+\sqrt{y}e^{-\frac{1}{3}ax^{3/2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*x*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(33^{\frac{1}{6}}b\sqrt{x}\,\text{WhittakerM}\left(\frac{1}{6},\frac{2}{3},\frac{ax^{\frac{3}{2}}}{3}\right)\mathrm{e}^{\frac{ax^{\frac{3}{2}}}{6}}-4\left(ax^{\frac{3}{2}}\right)^{\frac{1}{6}}a\sqrt{y}\right)\mathrm{e}^{-\frac{ax^{\frac{3}{2}}}{3}}}{4\left(ax^{\frac{3}{2}}\right)^{\frac{1}{6}}a}\right)$$

6.2.4.4 [473] problem number 4

problem number 473

Added January 2, 2019.

Problem 2.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+A\sqrt{ax+by+c}w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + A*Sqrt[a*x + b*y + c]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(x-\frac{a\log\left(\frac{e^{\frac{2\sqrt{A^2b^2(ax+by+c)}}{a}}}{\left(\sqrt{A^2b^2(ax+by+c)}+a\right)^2}\right)}{A^2b^2}\right)\right\}$$

$$\left\{w(x,y)\to c_1\left(x-\frac{a\log\left(\frac{e^{-\frac{2\sqrt{A^2b^2(ax+by+c)}}{a}}}{\left(a-\sqrt{A^2b^2(ax+by+c)}\right)^2}\right)}{A^2b^2}\right)\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ A*sqrt(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{A^2b^2x-2\sqrt{ax+by+c}\,Ab-a\ln\left(\sqrt{ax+by+c}\,Ab-a\right)+a\ln\left(\sqrt{ax+by+c}\,Ab+a\right)+}{A^2b^2}\right.$$

6.2.4.5 [474] problem number 5

problem number 474

Added January 2, 2019.

Problem 2.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ay+b\sqrt{y^2+cx^2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y + b*Sqrt[y^2 + c*x^2])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*diff(w(x,y),x)+ ( a*y + b *sqrt(y^2+c*x^2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.4.6 [475] problem number 6

problem number 475

Added January 2, 2019.

Problem 2.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax + b\sqrt{y})\, w_x - \left(c\sqrt{x} + ay\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b*Sqrt[y])*D[w[x, y], x] - (c*Sqrt[x] + a*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{3a^3x^3}{8b^2}+\frac{2}{3}cx^{3/2}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(axy-\frac{2}{3}by^{3/2}+\frac{2}{3}cx^{3/2}\right)\right\}$$
$$\left\{w(x,y)\to c_1\left(axy+\frac{2}{3}by^{3/2}+\frac{2}{3}cx^{3/2}\right)\right\}$$

Maple ✓

```
restart;
pde := (a*x+b*sqrt(y))* diff(w(x,y),x)- (c*sqrt(x)+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\text{RootOf}\left(3a^4y^4+8abc^2y^{\frac{5}{2}}-2\left(-a^3y^3-4bc^2y^{\frac{3}{2}}-6\_Zc^2+2\sqrt{2a^3by^{\frac{9}{2}}+3\_Za^3y^3+4b^2}\right.\right.\right.$$

6.2.4.7 [476] problem number 7

problem number 476

Added January 2, 2019.

Problem 2.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sqrt{f(x)}w_x+\sqrt{f(y)}w_y=0$$

Where $f(t)=\sum_{n=0}^{4}a_nt^n$

Mathematica ✗

```
ClearAll["Global`*"];
f[t_] := Sum[a[n]*t^n, {n, 1, 4}];
pde =  Sqrt[f[x]]*D[w[x, y], x] + Sqrt[f[y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
f:=t->sum(a[n]*t^n,n=1..4);
pde := sqrt(f(x))* diff(w(x,y),x)+ sqrt(f(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

6.2.5 2.5

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6.2.5.1 [477] problem number 1

problem number 477

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay + bx^k\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ba^{-k-1}\,\text{Gamma}(k+1,ax) + ye^{-ax}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y+b*x^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-bx^k(ax)^{-\frac{k}{2}}\,\text{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}} + (k+1)\,ay\right)\mathrm{e}^{-ax}}{(k+1)\,a}\right)$$

6.2.5.2 [478] problem number 2

problem number 478

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^k y + bx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^k*y + b*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(b(k+1)^{\frac{n-k}{k+1}}a^{-\frac{n+1}{k+1}}\,\text{Gamma}\left(\frac{n+1}{k+1},\frac{ax^{k+1}}{k+1}\right)+ye^{-\frac{ax^{k+1}}{k+1}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^k*y+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-(k+1)^2\left(ax^{n+1}+(k+n+2)\,x^{-k+n}\right)b\left(\frac{ax^{k+1}}{k+1}\right)^{\frac{-k-n-2}{2k+2}}\text{WhittakerM}\left(\frac{-k+n}{2k+2},\frac{2k+n+3}{2k+2},\frac{ax^{k+}}{k+1}\right.}{}\right.$$

6.2.5.3 [479] problem number 3

problem number 479

Added January 2, 2019.

Problem 2.2.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ay^2+bx^n\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^2 + b*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol=Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-2axy\,\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\,\text{BesselJ}\left(\frac{1}{n+2}-1,\frac{2\sqrt{}}{}\right.}{(2axy+1)\,\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)+\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\left(\text{BesselJ}\left(-\frac{n+3}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)-\text{B}\right.}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y^2+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{axy\,\text{BesselY}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,xx^{\frac{n}{2}}}{n+2}\right)-\sqrt{ab}\,xx^{\frac{n}{2}}\,\text{BesselY}\left(\frac{n+3}{n+2},\frac{2\sqrt{ab}\,xx^{\frac{n}{2}}}{n+2}\right)+\text{BesselY}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,xx}{n+2}\right.}{-axy\,\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,xx^{\frac{n}{2}}}{n+2}\right)+\sqrt{ab}\,xx^{\frac{n}{2}}\,\text{BesselJ}\left(\frac{n+3}{n+2},\frac{2\sqrt{ab}\,xx^{\frac{n}{2}}}{n+2}\right)-\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{ab}\,xx}{n+2}\right.}\right.$$

6.2.5.4 [480] problem number 4

problem number 480

Added January 2, 2019.

Problem 2.2.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+anx^{n-1}-a^2x^{2n}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*n*x^(n - 1) - a^2*x^(2*n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*n*x^(n-1)-a^2*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{}{-3\left(\left(n+\frac{4}{3}\right)ax^{n+1}+\frac{(n+2)(-xy+n+1)}{3}\right)(n+2)\,\mathrm{WhittakerM}\left(\frac{n+2}{2n+2},\frac{2n+3}{2n+2},-\frac{2ax^{n+1}}{n+1}\right)\mathrm{e}^{-\frac{ax^{n+}}{n+1}}}\right.$$

6.2.5.5 [481] problem number 5

problem number 481

Added January 2, 2019.

Problem 2.2.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y + ax^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x^n*y + a*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left((-1)^{\frac{1}{n+1}}(n+1)^{-\frac{n+2}{n+1}}a^{\frac{1}{n+1}}\,\mathrm{Gamma}\left(-\frac{1}{n+1},-\frac{ax^{n+1}}{n+1}\right)-\frac{e^{\frac{ax^{n+1}}{n+1}}}{x^2y+x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*x^n*y+a*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(n+1)^2\left(axy - nyx^{-n} - nx^{-n-1} + a\right)\left(-\frac{ax^{n+1}}{n+1}\right)^{-\frac{n}{2n+2}} \text{WhittakerM}\left(\frac{-n-2}{2n+2}, \frac{2n+1}{2n+2}, -\frac{ax^{n+}}{n+1}\right.}{(xy}\right.$$

6.2.5.6 [482] problem number 6

problem number 482

Added January 2, 2019.

Problem 2.2.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y - abx^n - b^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x^n*y - a*b*x^n - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2+a*x^n*y-a*b*x^n-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-b+y)\left(\int e^{\frac{(ax^n+2(n+1)b)x}{n+1}}dx\right)+e^{\frac{(ax^n+2(n+1)b)x}{n+1}}}{b-y}\right)$$

6.2.5.7 [483] problem number 7

problem number 483

Added January 2, 2019.

Problem 2.2.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^(-n - 2))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{\sqrt{(n+1)^2-4ab}}\left(\sqrt{(n+1)^2-4ab}+2ayx^{n+1}+n+1\right)}{\sqrt{(n+1)^2-4ab}-2ayx^{n+1}-n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2axyx^n+n+1}{\sqrt{4ab-n^2-2n-1}}\right)+\sqrt{4ab-n^2-2n-1}\,\ln(x)}{\sqrt{4ab-n^2-2n-1}}\right)$$

6.2.5.8 [484] problem number 8

problem number 484

Added January 2, 2019.

Problem 2.2.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bmx^{m-1} - ab^2 x^{n+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(n + 2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*m*x^(m-1) -a*b^2*x^(n+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{-3\left(\left(m+\frac{4n}{3}+\frac{4}{3}\right)abx^{m+n+1}-\frac{(m+2n+2)(ayx^{n+1}-m-n-1)}{3}\right)(m+2n+2)\,\text{WhittakerM}\left(\frac{m+}{2m+}\right.}\right.$$

6.2.5.9 [485] problem number 9

problem number 485

Added January 2, 2019.

Problem 2.2.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left((n+1)x^n y^2 - ax^{n+m+1} y + ax^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((n + 1)*x^n*y^2 - a*x^(n + m + 1)*y + a*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((n+1)*x^n*y^2 - a*x^(n+m+1)* y + a*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left((m-n)\,axx^m \operatorname{hypergeom}\left(\left[\frac{2m+2}{m+n+2}\right], \left[\frac{2m+n+3}{m+n+2}\right], \frac{ax^2x^mx^n}{m+n+2}\right) - (m+1)\,(axx^m\right.}{\left((m+1)\,ax^2x^mx^n \operatorname{hypergeom}\left(\left[\frac{2m+n+3}{m+n+2}\right], \left[\frac{2m+3n+5}{m+n+2}\right], \frac{ax^2x^mx^n}{m+n+2}\right) - (m+2n+3)\,((axx^m\right.}\right.$$

6.2.5.10 [486] problem number 10

problem number 486

Added January 2, 2019.

Problem 2.2.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 + bx^m y + bcx^m - ac^2 x^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + b*c*x^m - a*c^2*x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*x^m*y+ b*c*x^m -a*c^2*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-c-y)\left(\int ax^n e^{\frac{-2(m+1)acx^{n+1}+(n+1)bx^{m+1}}{(m+1)(n+1)}}dx\right) - e^{\frac{-2(m+1)acx^{n+1}+(n+1)bx^{m+1}}{(m+1)(n+1)}}}{c+y}\right)$$

6.2.5.11 [487] problem number 11

problem number 487

Added January 2, 2019.

Problem 2.2.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - ax^n(bx^m + c)y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*x^m + c)*y + b*m*x^(m - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2-a*x^n*(b*x^m +c)*y+ b*m*x^(m-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.12 [488] problem number 12

problem number 488

Added January 2, 2019.

Problem 2.2.5.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left(anx^{n-1}y^2 - cx^m(ax^n + b) + cx^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (a*n*x^(n - 1)*y^2 - c*x^m*(a*x^n + b) + c*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)- (a*n*x^(n-1)*y^2 - c*x^m*(a*x^n+b) + c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.13 [489] problem number 13

problem number 489

Added January 2, 2019.

Problem 2.2.5.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 + bx^m y + ckx^{k-1} - bcx^{m+k} - ac^2 x^{n+2k}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + c*k*x^(k - 1) - b*c*x^(m + k) - a*c^2*x^(n + 2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^m*y+ c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.14 [490] problem number 14

problem number 490

Added January 2, 2019.

Problem 2.2.5.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^{2n+1} y^3 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + b*x^(-n - 2))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^(2*n+1)*y^3 + b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\ln(x) - \frac{\ln\left(xyx^n - \text{RootOf}\left(\_Z^3 a + (n+1)\_Z + b\right)\right)}{3\,\text{RootOf}\left(\_Z^3 a + (n+1)\_Z + b\right)^2 a + n + 1}\right)$$

Solution contains RootOf

6.2.5.15 [491] problem number 15

problem number 491

Added January 2, 2019.

Problem 2.2.5.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^3 + 3abx^{n+m}y^2 - bmx^{m-1} - 2ab^3x^{n+3m}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 - b*m*x^(m - 1) - 2*a*b^3*x^(n + 3*m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{6^{-\frac{n+1}{2m+n+1}}(2m+n+1)^{-\frac{2m}{2m+n+1}}b^{-\frac{2(n+1)}{2m+n+1}}e^{-\frac{6ab^2x^{2m+n+1}}{2m+n+1}}\left(6^{\frac{n+1}{2m+n+1}}(2m+n+1)^{\frac{2m}{2m+n+1}}b^{\frac{2(n+}{2m+}}\right.}{(bx^m+y)^2}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2 - b*m*x^(m-1) - 2*a*b^3*x^(n+3*m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left((n+1)\left(n^2-n+1\right)\left(y^2x^{-2m}+b^2\right)2^{\frac{m}{2m+n+1}}+\left(4\left(m^3+\left(\frac{5n}{2}+\frac{5}{2}\right)m^2+\frac{3n^2}{2}+2(n+1)^2\right.\right.\right.}{}\right.$$

6.2.5.16 [492] problem number 16

problem number 492

Added January 2, 2019.

Problem 2.2.5.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^3 + 3abx^{n+m}y^2 + cx^ky - 2ab^3x^{n+3m} + bcx^{m+l} - bmx^{m-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 + c*x^k*y - 2*a*b^3*x^(n + 3*m) + b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2+ c*x^k*y-2*a*b^3*x^(n+3*m) + b*c*x^(m+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.17 [493] problem number 17

problem number 493

Added January 2, 2019.

Problem 2.2.5.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay^n + bx^{\frac{n}{1-n}}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y^n + b*x^(n/(1 - n)))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*y^n+b*x^(n/(1-n)))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-n\left(\int_{\_b}^{y} \frac{x^{\frac{n}{n-1}}}{(n-1)\,bx + ((n-1)\,ax\_a^n + \_a)\,x^{\frac{n}{n-1}}} d\_a\right) + \int_{\_b}^{y} \frac{x^{\frac{n}{n-1}}}{(n-1)\,bx + ((n-1)\,ax\_}$$

6.2.5.18 [494] problem number 18

problem number 494

Added January 2, 2019.

Problem 2.2.5.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^{m-n-(mn)}y^n + bx^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^(m - n - m*n)*y^n + b*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^(m-n-(m*n))*y^n + b*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} -\frac{x^n x^{mn}}{a\_a^n x^m + (bxx^m - (m+1)\_a)\, x^n x^{mn}} d\_a + \ln(x)\right)$$

6.2.5.19 [495] problem number 19

problem number 495

Added January 2, 2019.

Problem 2.2.5.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^k + bx^m y\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^k + b*x^m*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(a(-1)^{\frac{m-n}{m+1}}(m+1)^{\frac{n-m}{m+1}} b^{-\frac{n+1}{m+1}}(k-1)^{\frac{m-n}{m+1}} \text{Gamma}\left(\frac{n+1}{m+1}, -\frac{b(k-1)x^{m+1}}{m+1}\right) + y^{1-k} e^{\frac{b(k-1}{n}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*x^n*y^k + b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(m+1)^2\left((k-1)bx^{n+1}+(-m-n-2)x^{-m+n}\right)a\left(-\frac{(k-1)bx^{m+1}}{m+1}\right)^{\frac{-m-n-2}{2m+2}}\text{WhittakerM}\left(\frac{}{2}\right.}{}\right.$$

6.2.5.20 [496] problem number 20

problem number 496

Added January 2, 2019.

Problem 2.2.5.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^2 + by + cx^{2b}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + b*y + c*x^(2*b))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{a}y\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)+\sqrt{c}x^b\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{c}x^b\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)-\sqrt{a}y\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^2 + b*y+ c*x^(2*b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\sqrt{a}\,\sqrt{c}\,x^{b} - b\arctan\left(\frac{\sqrt{a}\,y x^{-b}}{\sqrt{c}}\right)}{b}\right)$$

6.2.5.21 [497] problem number 21

problem number 497

Added January 2, 2019.

Problem 2.2.5.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + \left(a y^2 + (n + b x^n) y + c x^{2n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^2 + (n + b*x^n)*y + c*x^(2*n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}\left(x^n\sqrt{b^2-4ac}+2ay+bx^n\right)}{x^n\sqrt{b^2-4ac}-2ay-bx^n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^2+(n+b*x^n)*y + c*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2bn\arctan\left(\frac{2abyx^{-n}+b^2}{\sqrt{4acb^2-b^4}}\right)+\sqrt{4acb^2-b^4}\,x^n\right)b}{\sqrt{4acb^2-b^4}\,n}\right)$$

6.2.5.22 [498] problem number 22

problem number 498

Added January 2, 2019.

Problem 2.2.5.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + by + cx^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + b*y + c/x^n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{x^{\sqrt{-4ac+b^2+2bn+n^2}}\left(\sqrt{-4ac+b^2+2bn+n^2}+2ayx^n+b+n\right)}{-\sqrt{-4ac+b^2+2bn+n^2}+2ayx^n+b+n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^n*y^2+b*y+c*x^(-n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2\arctan\left(\frac{2ayx^n+b+n}{\sqrt{4ac-b^2-2bn-n^2}}\right)+\sqrt{4ac-b^2-2bn-n^2}\,\ln(x)}{\sqrt{4ac-b^2-2bn-n^2}}\right)$$

6.2.5.23 [499] problem number 23

problem number 499

Added January 2, 2019.

Problem 2.2.5.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + my - ab^2 x^{x+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + m*y - a*b^2*x^(x + 2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+  (a*x^n*y^2+ m*y- a*b^2*x^(x+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Failed to convert to latex

6.2.5.24 [500] problem number 24

problem number 500

Added January 2, 2019.

Problem 2.2.5.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x^{2n}y^2 + (m-n)y + x^{2m}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x^(2*n)*y^2 + (m - n)*y + x^(2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(yx^{n-m}\right) - \frac{x^{m+n}}{m+n}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+  (x^(2*n)*y^2+(m-n)*y+ x^(2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{m+n} + (-m-n)\arctan\left(yx^{-m+n}\right)}{m+n}\right)$$

6.2.5.25 [501] problem number 25

problem number 501

Added January 2, 2019.

Problem 2.2.5.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n}y^2 + (bx^n - n)y + c\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*y^2 + (b*x^n - n)*y + c)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{x^n\sqrt{b^2-4ac}}{n}}\left(\sqrt{b^2-4ac}+2ayx^n+b\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n)*y^2+ (b*x^n -n)*y + c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(-2bn\arctan\left(\frac{2abyx^n+b^2}{\sqrt{4acb^2-b^4}}\right)+\sqrt{4acb^2-b^4}\,x^n\right)b}{\sqrt{4acb^2-b^4}\,n}\right)$$

6.2.5.26 [502] problem number 26

problem number 502

Added January 2, 2019.

Problem 2.2.5.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n+m}y^2+(bx^{n+m}-n)y+cx^m\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n + m)*y^2 + (b*x^(n + m) - n)*y + c*x^m)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\frac{\sqrt{b^2-4ac}x^{m+n}}{m+n}}\left(\sqrt{b^2-4ac}+2ayx^n+b\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n + m)*y^2 +(b*x^(n+m)-n)*y+ c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(-2(m+n)\,b\arctan\left(\frac{2abyx^n+b^2}{\sqrt{4acb^2-b^4}}\right)+\sqrt{4acb^2-b^4}\,x^m x^n\right)b}{\sqrt{4acb^2-b^4}\,(m+n)}\right)$$

6.2.5.27 [503] problem number 27

problem number 503

Added January 2, 2019.

Problem 2.2.5.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ay^3+3abx^ny^2-bnx^n-2ab^3x^{3n}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y^3 + 3*a*b*x^n*y^2 - b*n*x^n - 2*a*b^3*x^(3*n))*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{3ab^2x^{2n}}{n}}\left(ae^{\frac{3ab^2x^{2n}}{n}}(bx^n+y)^2\operatorname{Ei}\left(-\frac{3ab^2x^{2n}}{n}\right)+n\right)}{n\,(bx^n+y)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*y^3+3*a*b*x^n*y^2 - b*n*x^n -2*a*b^3*x^(3*n) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-(b^2x^{2n}+2byx^n+y^2)\,a\,\mathrm{expIntegral}\left(1,\frac{3ab^2x^{2n}}{n}\right)+n\,\mathrm{e}^{-\frac{3ab^2x^{2n}}{n}}}{(b^2x^{2n}+2byx^n+y^2)\,n}\right)$$

6.2.5.28 [504] problem number 28

problem number 504

Added January 2, 2019.

Problem 2.2.5.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n+1}y^3+(bx-n)y+cx^{1-n}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + (b*x - n)*y + c*x^(1 - n))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(2*n +1)*y^3 + (b*x-n)*y + c*x^(1-n) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-byx^n - \mathrm{RootOf}\left(c^2 a\_Z^3 + \_Zb^3 - b^3\right)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2 a\_Z^3 + \_Zb^3 - b^3\right)^2 ac^2 + b^3} - bx\right)$$

Solution contains RootOf

6.2.5.29 [505] problem number 29

problem number 505

Added January 2, 2019.

Problem 2.2.5.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{n+2}y^3 + (bx^n - 1)y + cx^{n-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(n + 2)*y^3 + (b*x^n - 1)*y + c*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ (a*x^(n+2)*y^3+ (b*x^n-1)*y + c*x^(n-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\frac{b^2 n \ln\left(\frac{-byx-\mathrm{RootOf}\left(c^2 a\_Z^3+\_Zb^3-b^3\right)c}{c}\right)}{3\,\mathrm{RootOf}\left(c^2 a\_Z^3+\_Zb^3-b^3\right)^2 ac^2+b^3} - x^n\right) b}{n}\right)$$

Solution contains RootOf

6.2.5.30 [506] problem number 30

problem number 506

Added January 2, 2019.

Problem 2.2.5.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(y + ax^{n-m}y^m + bx^{n-k}y^k\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (y + a*x^(n - m)*y^m + b*x^(n - k)*y^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+ ( y+a*x^(n - m)*y^m+b*x^(n-k)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)\,x\left(\int_{\_b}^{y} -\frac{x^k x^m}{(a\_a^m x^k + b\_a^k x^m)\,x}\,d\_a\right) + x^n}{(n-1)\,x}\right)$$

6.2.5.31 [507] problem number 31

problem number 507

Added January 2, 2019.

Problem 2.2.5.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y w_x + \left(x^{n-1}((1+2n)x + an)y - n x^{2n}(x+a)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + (x^(n - 1)*((1 + 2*n)*x + a*n)*y - n*x^(2*n)*(x + a))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := y*diff(w(x,y),x)+ ( x^(n-1)*((1+2*n)*x+a*n)*y-n*x^(2*n)*(x+a) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(a+\frac{x}{2}\right)n\,\mathrm{e}^{\frac{2\arctan\left(\frac{\left(2ax^{n}-y+x^{n+1}\right)n}{\sqrt{-n^{2}}\left(-y+x^{n+1}\right)}\right)}{\sqrt{-n^{2}}}} - \sqrt{-n^{2}}\,x\left(\int^{-\frac{2\arctan\left(\frac{\left(2ax^{n}-y+x^{n+1}\right)n}{\sqrt{-n^{2}}\left(-y+x^{n+1}\right)}\right)}{\sqrt{-n^{2}}}}\mathrm{e}^{-\_a}\tan\left(\frac{\sqrt{-n^{2}}}{2}\_\right.\right.}{2x}\right.$$

6.2.5.32 [508] problem number 32

problem number 508

Added January 2, 2019.

Problem 2.2.5.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + \left((a(2n+k)x^k+b)x^{n-1}y - (a^2nx^{2k}+abx^k-c)x^{2n-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + ((a*(2*n + k)*x^k + b)*x^(n - 1)*y - (a^2*n*x^(2*k) + a*b*x^k - c)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := y*diff(w(x,y),x)+ ( (a*(2*n+k)*x^k+b)*x^(n-1)*y -(a^2*n*x^(2*k)+ a*b*x^k-c)*x^(2*n-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.33 [509] problem number 33

problem number 509

Added January 2, 2019.

Problem 2.2.5.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2axy+b)w_x - \left(a(m+3)xy^2 + b(m+2)y - cx^m\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(2*a*x*y + b)*D[w[x, y], x] - (a*(m + 3)*x*y^2 + b*(m + 2)*y - c*x^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{x^{m+2}(2(m+1)y(axy+b)-cx^m)}{2a(m+1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*(2*a*x*y+b)*diff(w(x,y),x)- ( a*(m+3)*x*y^2+b*(m+2)*y-c*x^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\left(-\frac{cx^m}{2} + (m+1)(axy+b)y\right)x^2x^m}{2m+2}\right)$$

6.2.5.34 [510] problem number 34

problem number 510

Added January 2, 2019.

Problem 2.2.5.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x^2(2axy + b)w_x - \left(4ax^2y^2 + 3bxy - cx^2 - k\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*(2*a*x*y + b)*D[w[x, y], x] - (4*a*x^2*y^2 + 3*b*x*y - c*x^2 - k)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\frac{x^2(x(4y(axy + b) - cx) - 2k)}{4a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*(2*a*x*y+b)*diff(w(x,y),x)- ( 4*a*x^2*y^2 + 3*b*x*y-c*x^2 - k )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-ax^4y^2 - bx^3y + \frac{1}{4}cx^4 + \frac{1}{2}kx^2\right)$$

6.2.5.35 [511] problem number 35

problem number 511

Added January 2, 2019.

Problem 2.2.5.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ax^m w_x + by^n w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^m*D[w[x, y], x] + b*y^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-m}}{a(m-1)}-\frac{y^{1-n}}{n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^m*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(m-1)\,ay^{-n+1}-(n-1)\,bx^{-m+1}}{(m-1)\,a}\right)$$

6.2.5.36 [512] problem number 36

problem number 512

Added January 2, 2019.

Problem 2.2.5.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x+(by+cx^m)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*y + c*x^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{\frac{bx^{1-n}}{a(n-1)}}-\frac{c(a-an)^{\frac{-m+n-1}{n-1}}b^{\frac{m-n+1}{n-1}}\,\text{Gamma}\left(\frac{-m+n-1}{n-1},\frac{bx^{1-n}}{a-an}\right)}{a(n-1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+ (b*y+c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)(m-2n+2)^2 acx^m \left(-\frac{bx^{-n+1}}{(n-1)a}\right)^{\frac{m-2n+2}{2n-2}} \text{WhittakerM}\left(\frac{-m+2n-2}{2n-2}, \frac{-m+3n-3}{2n-2}, -\frac{bx^{-n+1}}{(n-1)a}\right)}{}\right.$$

6.2.5.37 [513] problem number 37

problem number 513

Added January 2, 2019.

Problem 2.2.5.37 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + (y^n + bx^m y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + (y^n + b*x^m*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> {n != 1}],
```

$$\left\{\left\{w(x,y) \to c_1\left((k-m-1)^{\frac{m}{k-m-1}} a^{\frac{m}{k-m-1}} b^{\frac{1-k}{k-m-1}} (n-1)^{\frac{m}{-k+m+1}} \text{Gamma}\left(\frac{k-1}{k-m-1}, \frac{b(n-1)x^{-k+m+1}}{a(k-m-1)}\right)\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^k*diff(w(x,y),x)+ (y^n+b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n<>1)
```

$$w(x,y) = \_F1\left(\frac{\left(-4(k-m-1)\left(k-\frac{m}{2}-1\right)^2 a x^{-m} y^{\frac{1}{k-m-1}} y^{\frac{m}{k-m-1}} y^{\frac{kn}{k-m-1}} \left(\frac{(n-1)b}{(k-m-1)a}\right)^{\frac{-k+1}{k-m-1}} \left(\frac{(n-1)b}{(k-m-1)a}\right)^{\overline{k}}}{}\right.$$

6.2.5.38 [514] problem number 38

problem number 514

Added January 2, 2019.

Problem 2.2.5.38 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^k+b)w_x + \left(\alpha x^n y^2 + (\beta - anx^k)y + \gamma x^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(a*x^k + b)*D[w[x, y], x] + (alpha*x^n*y^2 + (beta - a*n*x^k)*y + gamma/x^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\left(\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}+2\alpha y x^n+bn+\beta\right)\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}\left(k\log(x)-\log\left(ax^k+b\right)\right)\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}}{bk}\right)}{-\sqrt{\alpha}\sqrt{\gamma}\sqrt{\frac{(bn+\beta)^2}{\alpha\gamma}-4}+2\alpha y x^n+bn+\beta}\right)\right.\right.$$

Maple ✓

```
restart;
pde :=x*(a*x^k+b)*diff(w(x,y),x)+ (alpha*x^n*y^2+(beta-a*n*x^k)*y+g*x^(-n))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(2(bn+\beta)\,bk\,\operatorname{arctanh}\left(\frac{(bn+\beta)(2\alpha y x^n+bn+\beta)}{\sqrt{(bn+\beta)^2(b^2n^2+2b\beta n-4\alpha g+\beta^2)}}\right)+\sqrt{(bn+\beta)^2\left(b^2n^2+2b\beta n-4\alpha g\right.}}{\sqrt{(bn+\beta)^2\left(b^2n^2+2b\beta n-4\alpha g+\beta^2\right)}\,bk}\right.$$

6.2.5.39 [515] problem number 39

problem number 515

Added January 2, 2019.

Problem 2.2.5.39 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y+Ax^n+a)w_x-\left(nAx^{n-1}y+kx^m+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (y + A*x^n + a)*D[w[x, y], x] - (n*A*x^(n - 1)*y + k*x^m + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y(2a+2Ax^n+y)+2bx+\frac{2kx^{m+1}}{m+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=(y+ A*x^n + a)*diff(w(x,y),x)- ( n*A*x^(n-1)*y + k*x^m + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2kxx^m - 2(m+1)\left(Ayx^n + ay + bx + \frac{y^2}{2}\right)}{2m+2}\right)$$

6.2.5.40 [516] problem number 40

problem number 516

Added January 2, 2019.

Problem 2.2.5.40 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y + ax^{n+1} + bx^n)w_x + \left(anx^n + cx^{n-1}\right)yw_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y + a*x^(n + 1) + b*x^n)*D[w[x, y], x] + (a*n*x^n + c*x^(n - 1))*y*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(y+ a*x^(n+1)+b*x^n)*diff(w(x,y),x)+ ( a*n*x^n + c*x^(n-1))*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.41 [517] problem number 41

problem number 517

Added January 2, 2019.

Problem 2.2.5.41 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2ax^n y+b)w_x-\left(a(3n+m)x^n y^2+b(2n+m)y-Ax^m-Cx^{-n}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(2*a*x^n*y + b)*D[w[x, y], x] - (a*(3*n + m)*x^n*y^2 + b*(2*n + m)*y - A*x^m - C0/x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{m+n}(x^n(2y(m+n)(ayx^n+b)-Ax^m)-2\text{C0})}{2a(m+n)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*(2*a*x^n*y+b)*diff(w(x,y),x)- ( a*(3*n+m)*x^n*y^2+b*(2*n+m)*y-A*x^m -C*x^(-n))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-2(m+n)\,ay^2x^{m+3n}-2(m+n)\,byx^{m+2n}+Ax^{2m+2n}+2Cx^{m+n}}{2m+2n}\right)$$

6.2.5.42 [518] problem number 42

problem number 518

Added January 2, 2019.

Problem 2.2.5.42 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n+bx^2+xy)w_x+\left(cx^n+bxy+y^2\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^2 + x*y)*D[w[x, y], x] + (c*x^n + b*x*y + y^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n+b*x^2+ x*y)*diff(w(x,y),x)+ ( c*x^n + b*x*y+ y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left((-n+1)\ln\left(\frac{9(a^2x^n+(ab+c)x^2)(n^2-3n+3)}{(n-3)(ax^n+(bx+y)x)a}\right)+\ln\left(\frac{9((n-1)a^2x^n+(cx+(-bx+(bx+y)n-2y)a)x)(n^2-3n+3}{(2n-3)(bx^2+ax^n+xy)a}\right.\right.}{3(n-}\right.$$

6.2.5.43 [519] problem number 43

problem number 519

Added January 2, 2019.

Problem 2.2.5.43 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^n + bx^2 + cxy)w_x + \left(ky^n + bxy + cy^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^n + b*x^2 + c*x*y)*D[w[x, y], x] + (k*y^n + b*x*y + c*y^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*y^n+b*x^2+c*x*y)*diff(w(x,y),x)+ ( k*y^n+ b*x*y+c*y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n^2-3n+3)\left((-n+1)\ln\left(\frac{9(n^2-3n+3)(k^2y^n+(ab+ck)y^2)}{(n-3)(bxy+cy^2+ky^n)k}\right)+\ln\left(\frac{9(n^2-3n+3)((n-1)k^2y^n+(aby+((n}{2(n-\frac{3}{2})(bxy+cy^2+}\right.\right.}{3}\right.$$

6.2.5.44 [520] problem number 44

problem number 520

Added January 2, 2019.

Problem 2.2.5.44 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n+bx^m+c)w_x+\left(cy^2-bx^{m-1}y+ax^{n-2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (c*y^2 - b*x^(m - 1)*y + a*x^(n - 2))*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( c*y^2-b*x^(m-1)*y+ a*x^(n-2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.45 [521] problem number 45

problem number 521

Added January 2, 2019.

Problem 2.2.5.45 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx^m + c)w_x + \left(ax^{n-2}y^2 + bx^{m-1}y + c\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (a*x^(n - 2)*y^2 + b*x^(m - 1)*y + c)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( a*x^(n-2)*y^2 + b*x^(m-1)*y + c)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.46 [522] problem number 46

problem number 522

Added January 2, 2019.

Problem 2.2.5.46 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx^m + c)w_x + \left(\alpha x^k y^2 + \beta x^s y - \alpha\lambda^2 x^k + \beta\lambda x^s\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(\lambda+y)\,\alpha\left(\int \frac{x^k \mathrm{e}^{-\left(\int \frac{2\alpha\lambda x^k-\beta x^s}{ax^n+bx^m+c}dx\right)}}{ax^n+bx^m+c}dx\right) - \mathrm{e}^{-\left(\int \frac{2\alpha\lambda x^k-\beta x^s}{ax^n+bx^m+c}dx\right)}}{\lambda+y}\right)$$

6.2.5.47 [523] problem number 47

problem number 523

Added January 2, 2019.

Problem 2.2.5.47 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n+bx^m+c)w_x - \left(sx^k y^2 - (ax^n+bx^m+c)y - s\lambda x^{k+2}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(a*x^n + b*x^m + c)*D[w[x, y], x] - (s*x^k*y^2 - (a*x^n + b*x^m + c)*y - s*lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tanh^{-1}\left(\frac{y}{\sqrt{\lambda}x}\right)}{\sqrt{\lambda}}-\int_1^x\frac{sK[1]^k}{bK[1]^m+aK[1]^n+c}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*(a*x^n + b*x^m + c)*diff(w(x,y),x)- (s*x^k*y^2 -(a*x^n + b*x^m+c)*y - s*lambda*x^(k+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\sqrt{\lambda}\,s\left(\int\frac{x^k}{ax^n+bx^m+c}dx\right)+\operatorname{arctanh}\left(\frac{y}{\sqrt{\lambda}\,x}\right)}{\sqrt{\lambda}\,s}\right)$$

6.2.5.48 [524] problem number 48

problem number 524

Added January 2, 2019.

Problem 2.2.5.48 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n+bx^m+c)w_x+\left((ax^n+bx^m+c)y^2-an(n-1)x^{n-2}-bm(m-1)x^{m-2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x^m + c)*D[w[x, y], x] + ((a*x^n + b*x^m + c)*y^2 - a*n*(n - 1)*x^(n - 2)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ((a*x^n+b*x^m + c)*y^2-a*n*(n-1)*x^(n-2)-b*m*(m-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a^2nx^{2n} + a^2yx^{2n+1} + 2abyx^{m+n+1} + 2acnx^n + 2acyx^{n+1} + b^2mx^{2m} + b^2yx^{2m+1} + 2bcyx}{}\right.$$

6.2.5.49 [525] problem number 49

problem number 525

Added January 2, 2019.

Problem 2.2.5.49 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + by^n + x)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*y^n + x)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n - m) + y)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n + b*y^n + x)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k) +beta*x^m*y^(n-m) + y )*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.50 [526] problem number 50

problem number 526

Added January 2, 2019.

Problem 2.2.5.50 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + by^n + Ax^2 + Bxy)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + Axy + By^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*y^n + A*x^2 + B*x*y)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n + b*y^n + A*x^2 + B*x*y)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k)+beta*x^m*y^(n-m) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.51 [527] problem number 51

problem number 527

Added January 2, 2019.

Problem 2.2.5.51 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ay^m + bx^n + s)w_x - \left(\alpha x^k + bnx^{n-1}y + \beta\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*y^m + b*x^n + s)*D[w[x, y], x] - (alpha*x^k + b*n*x^(n - 1)*y + beta)*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*y^m + b*x^n + s)*diff(w(x,y),x)- (alpha*x^k + b*n*x^(n-1)* y + beta)*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(m+1)\,\alpha x x^k - (k+1)\,(a y y^m + (m+1)\,b y x^n + (\beta x + s y)\,(m+1))}{(m+1)\,(k+1)}\right)$$

6.2.5.52 [528] problem number 52

problem number 528

Added January 2, 2019.

Problem 2.2.5.52 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a x^n y^m + x) w_x + \left(b x^k y^{n+m-k} + y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + x)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n*y^m +x)*diff(w(x,y),x)+  (b*x^k*y^(n+m-k) + y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.53 [529] problem number 53

problem number 529

Added January 2, 2019.

Problem 2.2.5.53 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n y^m + \alpha)w_x - y(bx^n y^m + \beta)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*x^n*y^m + alpha)*D[w[x, y], x] - y*(b*x^n*y^m + beta)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*(a*x^n*y^m +alpha)*diff(w(x,y),x)-  y*( b*x^n*y^m + beta )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{(an-bm)\beta m}(y^m)^{(an-bm)\alpha}\left(\alpha n - \beta m + (an-bm)\, x^n y^m\right)^{-(a\beta-\alpha b)m}\right)$$

6.2.5.54 [530] problem number 54

problem number 530

Added January 2, 2019.

Problem 2.2.5.54 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(anx^k y^{n+k} + s)w_x - y\left(bmx^{m+k}y^k + s\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*n*x^k*y^(n + k) + s)*D[w[x, y], x] - y*(b*m*x^(m + k)*y^k + s)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(a*n*x^k*y^(n+k) + s)*diff(w(x,y),x)- y*( b*m*x^(m+k)*y^k + s )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.55 [531] problem number 55

problem number 531

Added January 2, 2019.

Problem 2.2.5.55 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n y^m + Ax^2 + Bxy)w_x + \left(bx^k y^{n+m-k} + Axy + By^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + A*x^2 + B*x*y)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + A*x*y + B*y^2)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (a*x^n*y^m + A*x^2 + B*x*y)*diff(w(x,y),x)+ (b*x^k*y^(n+m-k) + A*x*y+ B*y^2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.5.56 [532] problem number 56

problem number 532

Added January 2, 2019.

Problem 2.2.5.56 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n y^m + bxy^k)w_x + (\alpha y^s + \beta)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + (alpha*y^s + beta)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*y^m + b*x*y^k)*diff(w(x,y),x)+ (alpha*y^s + beta)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \frac{y^m e^{(n-1)b\left(\int \frac{y^k}{\alpha y^s+\beta}dy\right)}}{\alpha y^s+\beta}dy\right) + x^{-n+1}e^{(n-1)b\left(\int \frac{y^k}{\alpha y^s+\beta}dy\right)}\right)$$

6.2.6 3.1

Local contents

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6.2.6.1 [533] problem number 1

problem number 533

Added January 2, 2019.

Problem 2.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ae^{\lambda x}w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Exp[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*exp(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda}\right)$$

6.2.6.2 [534] problem number 2

problem number 534

Added January 7, 2019.

Problem 2.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{ae^{\lambda x}}{\lambda}-bx+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\,\mathrm{e}^{\lambda x} - (bx-y)\,\lambda}{\lambda}\right)$$

6.2.6.3 [535] problem number 3

problem number 535

Added January 7, 2019.

Problem 2.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\log\left(\frac{e^{\lambda y}}{ae^{\lambda y}+b}\right)}{b\lambda} - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\ln\left(a\,\mathrm{e}^{b\lambda x} + b\,\mathrm{e}^{(bx-y)\lambda}\right)}{b\lambda}\right)$$

6.2.6.4 [536] problem number 4

problem number 536

Added January 7, 2019.

Problem 2.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ae^{\lambda y+\beta x} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\frac{\log\left(a\lambda e^{x(b\lambda+\beta)} + \beta e^{\lambda(bx-y)} + b\lambda e^{\lambda(bx-y)}\right)}{b\lambda + \beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{(bx - y)\lambda - \ln\left(\frac{1}{a\lambda e^{\beta x+\lambda y}+b\lambda+\beta}\right)}{b\lambda + \beta}\right)$$

6.2.6.5 [537] problem number 5

problem number 537

Added January 7, 2019.

Problem 2.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ae^{\lambda y+\beta x} + be^{\gamma x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b*Exp[gamma*x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b*exp(g*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda\left(\int e^{\frac{b\lambda e^{gx}}{g}+\beta x}dx\right) - e^{\frac{(b e^{gx}-gy)\lambda}{g}}}{\lambda}\right)$$

6.2.6.6 [538] problem number 6

problem number 538

Added January 7, 2019.

Problem 2.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(a\lambda\,\mathrm{e}^{\lambda x} - b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)$$

6.2.6.7 [539] problem number 7

problem number 539

Added January 7, 2019.

Problem 2.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x} + b\right) w_x + \left(ce^{\beta x} + d\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\beta(c+d)\log\left(ae^{\lambda x}+b\right) - \lambda e^{\beta x}\,{}_2F_1\left(1,\frac{\beta}{\lambda};\frac{\beta+\lambda}{\lambda};-\frac{ae^{\lambda x}}{b}\right) - \beta\lambda(-by+cx+dx)}{b\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{c+d+\mathrm{e}^{\beta x}}{a\,\mathrm{e}^{\lambda x}+b}dx\right)\right)$$

6.2.6.8 [540] problem number 8

problem number 540

Added January 7, 2019.

Problem 2.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x}+b\right)w_x+\left(ce^{\beta y}+d\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*y] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\log\left(\left(e^{\beta y}+c+d\right)e^{\frac{\beta x(c+d)}{b}-\beta y}\left(ae^{\lambda x}+b\right)^{-\frac{\beta(c+d)}{b\lambda}}\right)}{\beta(c+d)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*y)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-b\lambda\,\mathrm{RootOf}\left(\left(-\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{\beta c^2}{(c+d)b\lambda}}\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{\beta d^2}{(c+d)b\lambda}}\left(a\,\mathrm{e}^{\lambda x}+b\right)^{\frac{2\beta cd}{(c+d)b\lambda}}\mathrm{e}^{\frac{\beta cy}{c+d}}\mathrm{e}^{\frac{\beta dy}{c+d}}+\left(a\,\mathrm{e}^{\lambda x}+\right.\right.\right.}{}\right.$$

Has RootOf

6.2.6.9 [541] problem number 9

problem number 541

Added January 7, 2019.

Problem 2.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda y}+b\right)w_x+\left(ce^{\beta x}+d\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*y] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ae^{\lambda y}}{\lambda}+by-\frac{e^{\beta x}}{\beta}-cx-dx\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*exp(lambda*y)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}+\left(\left(by+(-c-d)\,x\right)\beta-\mathrm{e}^{\beta x}\right)\lambda}{\beta\lambda}\right)$$

6.2.6.10 [542] problem number 10

problem number 542

Added January 7, 2019.

Problem 2.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x}+be^{\beta y}\right)w_x+a\lambda e^{\lambda x}w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x] + b*Exp[beta*y])*D[w[x, y], x] + a*lambda*Exp[lambda*x]*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),x)+ a*lambda*exp(lambda*x)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda x - y + \ln\left(-b\,\mathrm{e}^{\beta y-\lambda x} + (\beta-1)\,a\right)}{\beta-1}\right)$$

6.2.6.11 [543] problem number 11

problem number 543

Added January 7, 2019.

Problem 2.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x+\beta y} + c\mu\right) w_x - \left(be^{\gamma x+muy} + c\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[lambda*x + beta*y] + c*mu)*D[w[x, y], x] - (b*Exp[gamma*x + mu*y] + c*lambda)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (a*exp(lambda*x+beta*y)+c*mu)*diff(w(x,y),x)- (b*exp(g*x+ mu*y)+c*lambda)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7 3.2

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6.2.7.1 [544] problem number 1

problem number 544

Added January 7, 2019.

Problem 2.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(\frac{\mathrm{Ei}\left(\frac{2ae^{\lambda x}}{\lambda}\right)\left(y - ae^{\lambda x}\right) + \lambda e^{\frac{2ae^{\lambda x}}{\lambda}}}{ae^{\lambda x} - y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+a*lambda*exp(lambda*x)- a^2*exp(2*lambda *x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\,\mathrm{e}^{\lambda x} + y}{\lambda\,\mathrm{e}^{\frac{2a\,\mathrm{e}^{\lambda x}}{\lambda}} + \left(a\,\mathrm{e}^{\lambda x} - y\right)\mathrm{expIntegral}\left(1, -\frac{2a\,\mathrm{e}^{\lambda x}}{\lambda}\right)}\right)$$

6.2.7.2 [545] problem number 2

problem number 545

Added January 7, 2019.

Problem 2.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + by + a(\lambda - b)e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + b*y + a*(lambda - b)*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2^{b/\lambda}\lambda^{-\frac{b}{\lambda}}e^{bx}a^{b/\lambda}\left(\left(a\left(-e^{\lambda x}\right)+b+y\right)\text{LaguerreL}\left(-\frac{b}{\lambda},\frac{b}{\lambda},\frac{2ae^{\lambda x}}{\lambda}\right)-2ae^{\lambda x}\,\text{LaguerreL}\left(-\frac{b+}{\lambda}\right.\right.}{ae^{\lambda x}-y}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+b*y+ a*(lambda-b)*exp(lambda*x) - a^2*exp(2*lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-a\,\mathrm{e}^{\lambda x}+y\right)\left(\int \mathrm{e}^{\frac{b\lambda x+2a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)+\mathrm{e}^{\frac{b\lambda x+2a\,\mathrm{e}^{\lambda x}}{\lambda}}}{a\,\mathrm{e}^{\lambda x}-y}\right)$$

6.2.7.3 [546] problem number 3

problem number 546

Added January 7, 2019.

Problem 2.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ae^{\lambda x}y - abe^{\lambda x} - b^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Exp[lambda*x]*y - a*b*Exp[lambda*x] - b^2)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(2b(-1)^{-\frac{b}{\lambda}}\left(-\frac{\text{Gamma}\left(\frac{2b}{\lambda},0,-\frac{ae^{\lambda x}}{\lambda}\right)}{\lambda} + \frac{\lambda^{-\frac{2b}{\lambda}}a^{\frac{2b}{\lambda}}e^{\frac{ae^{\lambda x}+2b\lambda x+2i\pi b}{\lambda}}}{b-y}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (y^2+a*exp(lambda*x)*y-a*b*exp(lambda*x)- b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b-y)\left(\int \mathrm{e}^{\frac{2b\lambda x+a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right) - \mathrm{e}^{\frac{2b\lambda x+a\,\mathrm{e}^{\lambda x}}{\lambda}}}{b-y}\right)$$

6.2.7.4 [547] problem number 4

problem number 547

Added January 7, 2019.

Problem 2.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(y^2 - axe^{\lambda x}y + ae^{\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (y^2 - a*x*Exp[lambda*x]*y + a*Exp[lambda*x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\frac{ae^{\lambda x}(\lambda x-1)}{\lambda^2}}}{x(xy-1)} - \int_1^x \frac{e^{\frac{ae^{\lambda K[1]}(\lambda K[1]-1)}{\lambda^2}}}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-  (y^2-a*x*exp(lambda*x)*y + a*exp(lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(yx^2-x)\left(\int \frac{e^{\frac{(\lambda x-1)a\,e^{\lambda x}}{\lambda^2}}}{x^2}dx\right) - e^{\frac{(\lambda x-1)a\,e^{\lambda x}}{\lambda^2}}}{(xy-1)\lambda^2 x}\right)$$

6.2.7.5 [548] problem number 5

problem number 548

Added January 7, 2019.

Problem 2.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + be^{-\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{x\left(-\sqrt{\lambda^2-4ab}\right)}\left(\sqrt{\lambda^2-4ab}-2aye^{\lambda x}-\lambda\right)}{a\left(2ye^{\lambda x}\sqrt{\lambda^2-4ab}-4b\right)+\lambda\left(\sqrt{\lambda^2-4ab}+\lambda\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(2\lambda\arctan\left(\frac{2a\lambda y\,e^{\lambda x}+\lambda^2}{\sqrt{4\lambda^2ab-\lambda^4}}\right)-\sqrt{4\lambda^2ab-\lambda^4}\,x\right)\lambda}{\sqrt{4\lambda^2ab-\lambda^4}}\right)$$

6.2.7.6 [549] problem number 6

problem number 549

Added January 7, 2019.

Problem 2.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + b\mu e^{\mu x} - ab^2e^{(\lambda+2\mu)x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*mu*Exp[mu*x] - a*b^2*Exp[(lambda + 2*mu)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*mu*exp(mu*x) - a*b^2*exp((lambda + 2*mu)*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.7 [550] problem number 7

problem number 550

Added January 7, 2019.

Problem 2.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + by + ce^{-\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*y + c*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{x\left(-\sqrt{-4ac+b^2+2b\lambda+\lambda^2}\right)}\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2} - 2aye^{\lambda x} - b -\right.}{a\left(2ye^{\lambda x}\sqrt{-4ac+b^2+2b\lambda+\lambda^2} - 4c\right) + b\left(\sqrt{-4ac+b^2+2b\lambda+\lambda^2} + 2\lambda\right) + \lambda\left(\sqrt{-4a}\right.}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*y +c*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2(b+\lambda)\left(-\frac{\sqrt{(b+\lambda)^2(4ac-b^2-2b\lambda-\lambda^2)}\,x}{2}+(b+\lambda)\arctan\left(\frac{(b+\lambda)(2ay\,\mathrm{e}^{\lambda x}+b+\lambda)}{\sqrt{(b+\lambda)^2(4ac-b^2-2b\lambda-\lambda^2)}}\right)\right)}{\sqrt{(b+\lambda)^2(4ac-b^2-2b\lambda-\lambda^2)}}\right)$$

6.2.7.8 [551] problem number 8

problem number 551

Added January 7, 2019.

Problem 2.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + \mu y - ab^2e^{(\lambda+2\mu)x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + mu*y - a*b^2*Exp[(lambda + 2*mu)*x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + mu*y - a*b^2*exp((lambda+2*mu)*x))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-b\cosh\left(\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}}{\lambda+\mu}\right)\mathrm{e}^{(\lambda+\mu)x} - y\,\mathrm{e}^{\lambda x}\sinh\left(\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}}{\lambda+\mu}\right)}{b\,\mathrm{e}^{(\lambda+\mu)x}\sinh\left(\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}}{\lambda+\mu}\right) + y\cosh\left(\frac{ab\,\mathrm{e}^{(\lambda+\mu)x}}{\lambda+\mu}\right)\mathrm{e}^{\lambda x}}\right)$$

6.2.7.9 [552] problem number 9

problem number 552

Added January 7, 2019.

Problem 2.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x} y^2 + a e^{\mu x} y + a\lambda e^{(\mu - lambda)x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda)*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left((-1)^{\lambda/\mu}\mu^{-\frac{\lambda}{\mu}} a^{\lambda/\mu}\,\text{Gamma}\left(-\frac{\lambda}{\mu}, -\frac{a e^{\mu x}}{\mu}\right) - \frac{\mu e^{\frac{a e^{\mu x}}{\mu} - \lambda x}}{y e^{\lambda x} + \lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (exp(lambda*x)*y^2  + a*exp(mu*x)*y+a*lambda*exp((mu-lambda)*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-\lambda+\mu)\left(y\,\mathrm{e}^{\lambda x} + \lambda\right)\mathrm{e}^{\lambda x}}{a\lambda\,\text{hypergeom}\left(\left[\frac{-\lambda+\mu}{\mu}\right], \left[\frac{-\lambda+2\mu}{\mu}\right], \frac{a\,\mathrm{e}^{\mu x}}{\mu}\right)\mathrm{e}^{\mu x} - (-\lambda+\mu)\,y\,\text{hypergeom}\left(\left[-\frac{\lambda}{\mu}\right], \left[\frac{-\lambda+\mu}{\mu}\right], \frac{a}{}\right.}\right.$$

6.2.7.10 [553] problem number 10

problem number 553

Added January 7, 2019.

Problem 2.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(\lambda e^{\lambda x} y^2 - a e^{\mu x} y + a e^{(\mu - lambda)x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (lambda*Exp[lambda*x]*y^2 - a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\mu\left(a e^{\mu x}\,\text{LaguerreL}\left(-\frac{-\lambda^2+\lambda+\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a e^{\mu x}}{\mu}\right) + \lambda\left(y e^{\lambda x} - 1\right)\text{LaguerreL}\left(\frac{(\lambda-}{\mu}\right.\right.}{\lambda\left(a(\lambda-1)e^{\mu x}\,\text{HypergeometricU}\left(\frac{-\lambda^2+\lambda+\mu}{\mu}, \frac{\lambda}{\mu}+2, \frac{a e^{\mu x}}{\mu}\right) + (\mu - \mu y e^{\lambda x})\,\text{Hypergeometr}\right.}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-  (lambda*exp(lambda*x)*y^2  - a*exp(mu*x)*y + a*lambda*exp((mu-lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-\lambda^2-\mu)\,M\left(\frac{-\lambda^2+\lambda-\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a\,e^{\mu x}}{\mu}\right) - \left(-\lambda y\,e^{\lambda x} + a\,e^{\mu x} - \lambda^2 + \lambda - \mu\right) M\left(-\frac{(\lambda-1)\lambda}{\mu}, \frac{\lambda+\mu}{\mu},\right.}{-\mu U\left(\frac{-\lambda^2+\lambda-\mu}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a\,e^{\mu x}}{\mu}\right) + \left(-\lambda y\,e^{\lambda x} + a\,e^{\mu x} - \lambda^2 + \lambda - \mu\right) U\left(-\frac{(\lambda-1)\lambda}{\mu}, \frac{\lambda+\mu}{\mu}, \frac{a\,e^{\mu x}}{\mu}\right.}\right)$$

6.2.7.11 [554] problem number 11

problem number 554

Added January 7, 2019.

Problem 2.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + abe^{(\lambda+\mu)x}y - b\mu e^{\mu x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*b*Exp[(lambda + mu)*x]*y - b*mu*Exp[mu*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2+ a*b*exp((lambda +mu)*x)*y - b*mu*exp(mu*x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{}{2\left(\frac{3\lambda}{2}+\mu\right)(2\lambda+\mu)^2 \operatorname{WhittakerM}\left(\frac{4\lambda+3\mu}{2\lambda+2\mu}, \frac{3\lambda+2\mu}{2\lambda+2\mu}, \frac{ab\,e^{(\lambda+\mu)x}}{\lambda+\mu}\right) e^{\frac{ab\,e^{(\lambda+\mu)x}-3(\lambda+\mu)\left(\frac{2\lambda}{3}+\mu\right)x}{2\lambda+2\mu}} + 2\left(\right.}\right.$$

6.2.7.12 [555] problem number 12

problem number 555

Added January 7, 2019.

Problem 2.3.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{(2\lambda+\mu)x}y^2 + \left(be^{(\lambda+\mu)x} - \lambda\right)y + ce^{\mu x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[(2*lambda + mu)*x]*y^2 + (b*Exp[(lambda + mu)*x] - lambda)*y +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{i\pi e^{-\frac{\sqrt{b^2-4ace}x(\lambda+\mu)}{2(\lambda+\mu)}}\left(\sqrt{b^2-4ac} - 2aye^{\lambda x} - b\right)}{2\left(\left(2aye^{\lambda x}+b\right)\cosh\left(\frac{\sqrt{b^2-4ace}x(\lambda+\mu)}{2(\lambda+\mu)}\right) + \sqrt{b^2-4ac}\sinh\left(\frac{\sqrt{b^2-4ace}x(\lambda+\mu)}{2(\lambda+\mu)}\right)\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (a*exp((2*lambda +mu)*x)*y^2+ (b*exp((lambda +mu)*x) -lambda)*y + c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2(\lambda+\mu)\,b\arctan\left(\frac{2aby\,\mathrm{e}^{\lambda x}+b^2}{\sqrt{4acb^2-b^4}}\right) + \sqrt{4acb^2-b^4}\,\mathrm{e}^{(\lambda+\mu)x}\right)b}{\sqrt{4acb^2-b^4}\,(\lambda+\mu)}\right)$$

6.2.7.13 [556] problem number 13

problem number 556

Added January 7, 2019.

Problem 2.3.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}(y - be^{\mu x})^2 + b\mu e^{\mu x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*(y - b*Exp[mu*x])^2 + b*mu*Exp[mu*x])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{b\left(-e^{x(\lambda+\mu)}\right) + ye^{\lambda x} + \lambda}{\lambda\left(be^{\mu x} - y\right)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( exp(lambda*x) *(y- b*exp(mu*x))^2 + b*mu*exp(mu*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b\,\mathrm{e}^{(\lambda+\mu)x} - y\,\mathrm{e}^{\lambda x} - \lambda}{\left(b\,\mathrm{e}^{\mu x} - y\right)\lambda}\right)$$

6.2.7.14 [557] problem number 14

problem number 557

Added January 7, 2019.

Problem 2.3.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + bnx^{n-1} - ab^2e^{\lambda x}x^{2n}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*n*x^(n - 1) - a*b^2*Exp[lambda*x]*x^(2*n))*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^2+ b*n*x^(n-1) - a*b^2*exp(lambda*x)*x^(2*n))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.15 [558] problem number 15

problem number 558

Added January 7, 2019.

Problem 2.3.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(e^{\lambda x} y^2 + a x^n y + a \lambda x^n e^{-\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*x^n*y + a*lambda*x^n*Exp[-(lambda*x)])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( exp(lambda*x)*y^2+ a*x^n*y + a*lambda*x^n*exp(-lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-y\,\mathrm{e}^{\lambda x}-\lambda\right)\left(\int \mathrm{e}^{\frac{(ax^n-(n+1)\lambda)x}{n+1}}dx\right)-\mathrm{e}^{\frac{(ax^n-(n+1)\lambda)x}{n+1}}}{y\,\mathrm{e}^{\lambda x}+\lambda}\right)$$

6.2.7.16 [559] problem number 16

problem number 559

Added January 7, 2019.

Problem 2.3.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda e^{\lambda x}y^2 + ax^n e^{\lambda x}y - ax^n e^{2\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Exp[lambda*x]*y^2 + a*x^n*Exp[lambda*x]*y - a*x^n*Exp[2*lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+  ( lambda*exp(lambda*x)*y^2+ a*x^n*exp(lambda*x)*y - a*x^n*exp(2*lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.17 [560] problem number 17

problem number 560

Added January 7, 2019.

Problem 2.3.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 - abx^n e^{\lambda x}y + bnx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 - a*b*x^n*Exp[lambda*x]*y + b*n*x^(n - 1))*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  ( a*exp(lambda*x)*y^2- a*b*x^n*exp(lambda*x)*y + b*n*x^(n-1))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(bx^n - y)\,a}{(bx^n - y)\,a\left(\int \lambda\,\mathrm{e}^{\frac{(-\Gamma(n)+\Gamma(n,-\lambda x))abnx^n(-\lambda)^n(-\lambda)^{-n}(-\lambda x)^{-n}+abx^n(-\lambda)^n(-\lambda)^{-n}\mathrm{e}^{\lambda x}+\lambda^2 x}{\lambda}}dx\right) - \lambda\,\mathrm{e}^{\left((-\Gamma(n\right.}}\right.$$

6.2.7.18 [561] problem number 18

problem number 561

Added January 7, 2019.

Problem 2.3.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + b\lambda e^{\lambda x} - ab^2 x^n e^{2\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + b*lambda*Exp[lambda*x] - a*b^2*x^n*Exp[2*lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 + b*lambda*exp(lambda*x) - a*b^2*x^n*exp(2*lambda*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.19 [562] problem number 19

problem number 562

Added January 7, 2019.

Problem 2.3.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 + \lambda y - ab^2 x^n e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 + lambda*y - a*b^2*x^n*Exp[2*lambda*x])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-i\left(ab(-1)^{-n}\lambda^{-n-1}\,\mathrm{Gamma}(n+1,-\lambda x) + \tanh^{-1}\left(\frac{ye^{-\lambda x}}{b}\right)\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  ( a*x^n*y^2 + lambda*y - a*b^2*x^n*exp(2*lambda*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-i\left(\left(n\Gamma(n,-\lambda x)-\Gamma(n+1)\right)(-\lambda x)^{-n}+\mathrm{e}^{\lambda x}\right)abx^{n}-i\lambda\operatorname{arctanh}\left(\frac{y\,\mathrm{e}^{-\lambda x}}{b}\right)}{\lambda}\right)$$

6.2.7.20 [563] problem number 20

problem number 563

Added January 7, 2019.

Problem 2.3.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - abx^n e^{\lambda x} y + b\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*b*x^n*Exp[lambda*x]*y + b*lambda*Exp[lambda*x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*b*x^n*exp(lambda*x)*y + b*lambda*exp(lambda*x) )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.21 [564] problem number 21

problem number 564

Added January 7, 2019.

Problem 2.3.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 - ax^n \left(be^{\lambda x} + c\right) y + b\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*Exp[lambda*x] + c)*y + b*lambda*Exp[lambda*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*x^n*(b*exp(lambda*x) + c )*y + b*lambda*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.22 [565] problem number 22

problem number 565

Added January 7, 2019.

Problem 2.3.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n e^{2\lambda x} y^2 + \left(bx^n e^{\lambda x} - \lambda\right) y + cx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*Exp[2*lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - lambda)*y + c*x^n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\sqrt{a}\sqrt{c}\left((-1)^{1-n}\lambda^{-n-1}\,\text{Gamma}(n+1,-\lambda x)-\frac{2\tan^{-1}\left(\frac{b-2aye^{\lambda x}}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*x^n*exp(2*lambda*x)*y^2 + (b*x^n*exp(lambda*x) - lambda)*y + c*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2\left(b\lambda\arctan\left(\frac{2aby\,\mathrm{e}^{\lambda x}+b^2}{\sqrt{4acb^2-b^4}}\right)-\frac{\sqrt{4acb^2-b^4}\left((n\Gamma(n,-\lambda x)-\Gamma(n+1))(-\lambda x)^{-n}+\mathrm{e}^{\lambda x}\right)x^n}{2}\right)b}{\sqrt{4acb^2-b^4}\,\lambda}\right)$$

6.2.7.23 [566] problem number 23

problem number 566

Added January 10, 2019.

Problem 2.3.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}(y-bx^n-c)^2+bnx^{n-1}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*(y - b*x^n - c)^2 + b*n*x^(n - 1))*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ae^{\lambda x}}{\lambda}-\frac{1}{bx^n+c-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (  a*exp(lambda*x)*(y- b*x^n - c)^2 +b*n*x^(n-1))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(bx^n+c-y)\,a\,\mathrm{e}^{\lambda x}-\lambda}{(bx^n+c-y)\,\lambda}\right)$$

6.2.7.24 [567] problem number 24

problem number 567

Added January 10, 2019.

Problem 2.3.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+2a\lambda xe^{\lambda x^2}-a^2e^{2\lambda x^2}\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*Exp[2*lambda*x^2])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+ (   y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*exp(2*lambda*x^2))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.7.25 [568] problem number 25

problem number 568

Added January 10, 2019.

Problem 2.3.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{-\lambda x^2}y^2 + \lambda xy + ab^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[-(lambda*x^2)]*y^2 + lambda*x*y + a*b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right) - \frac{\sqrt{\frac{\pi}{2}}ab\operatorname{erf}\left(\frac{\sqrt{\lambda}x}{\sqrt{2}}\right)}{\sqrt{\lambda}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (    a*exp(-lambda*x^2)*y^2 + lambda*x*y + a*b^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\sqrt{\pi}\sqrt{2}\,ab\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\lambda}x}{2}\right) - 2\sqrt{\lambda}\arctan\left(\frac{y\,\mathrm{e}^{-\frac{\lambda x^2}{2}}}{b}\right)}{2\sqrt{\lambda}}\right)$$

6.2.7.26 [569] problem number 26

problem number 569

Added January 10, 2019.

Problem 2.3.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + \lambda xy + ab^2 x^n e^{\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (a*x^n*y^2 + lambda*x*y + a*b^2*x^n*Exp[lambda*x^2])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right) - iabi^{-n}2^{\frac{n-1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}\,\text{Gamma}\left(\frac{n+1}{2},-\frac{\lambda x^2}{2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (   a*x^n*y^2 + lambda*x*y + a*b^2*x^n*exp(lambda*x^2) )*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(-\lambda x^2\right)^{-\frac{n}{2}-\frac{1}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2}\right) - ab2^{\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(-\lambda x^2\right)^{-\frac{n}{2}-\frac{1}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2},-\frac{\lambda x^2}{2}\right) - \text{a}$$

6.2.7.27 [570] problem number 27

problem number 570

Added January 10, 2019.

Problem 2.3.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{2\lambda x}y^3 + be^{\lambda x}y^2 + cy + de^{-\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[2*lambda*x]*y^3 + b*Exp[lambda*x]*y^2 + c*y + d*Exp[-(lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (  a*exp(2*lambda*x)*y^3 + b*exp(lambda*x)*y^2 + c*y+ d*exp(-lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \frac{\ln\left(y\,\mathrm{e}^{\lambda x} - \mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\,\_Z + d\right)\right)}{3\,\mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\,\_Z + d\right)^2 a + 2\,\mathrm{RootOf}\left(\_Z^3 a + \_Z^2 b + (c+\lambda)\right.}\right.$$

Solution contains RootOf

6.2.7.28 [571] problem number 28

problem number 571

Added January 10, 2019.

Problem 2.3.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^3 + 3abe^{\lambda x}y^2 + cy - 2ab^3e^{\lambda x} + bc\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^3 + 3*a*b*Exp[lambda*x]*y^2 + c*y - 2*a*b^3*Exp[la
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{6ab^2e^{\lambda x}}{\lambda}}\left(2(b+y)^2e^{\frac{6ab^2e^{\lambda x}}{\lambda}}\int_1^x ae^{(2c+\lambda)K[1]-\frac{6ab^2e^{\lambda K[1]}}{\lambda}}dK[1]+e^{2cx}\right)}{(b+y)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^3 + 3*a*b*exp(lambda*x)*y^2 + c*y- 2*a*b^3*exp(l
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2(b+y)^2a\left(\int e^{\frac{-6ab^2e^{\lambda x}+2\left(c+\frac{\lambda}{2}\right)\lambda x}{\lambda}}dx\right)+e^{-\frac{6ab^2e^{\lambda x}}{\lambda}+2cx}}{(b+y)^2}\right)$$

6.2.7.29 [572] problem number 29

problem number 572

Added January 10, 2019.

Problem 2.3.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ae^{\lambda x}y^2+ky+ab^2x^{2k}e^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + k*y + a*b^2*x^(2*k)*Exp[lambda*x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\sqrt{b^2}x^k(-\lambda x)^{-k}\,\mathrm{Gamma}(k,-\lambda x)+\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ ( a*exp(lambda*x)* y^2 + k*y + a*b^2*x^(2*k)*exp(lambda*x) )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(abx^k(-\lambda x)^{-k}\,\Gamma(k)-abx^k(-\lambda x)^{-k}\,\Gamma(k,-\lambda x)-\arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.7.30 [573] problem number 30

problem number 573

Added January 10, 2019.

Problem 2.3.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n}e^{\lambda x}y^2+(bx^ne^{\lambda x}-n)y+ce^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*Exp[lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - n)*y + c*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(c(-\lambda x)^{-n}\sqrt{\frac{ax^{2n}}{c}}\,\text{Gamma}(n,-\lambda x)-\frac{2\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{\frac{b^2}{ac}}-2y\sqrt{\frac{ax^{2n}}{c}}\right)}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ (  a*x^(2*n)*exp(lambda*x)*y^2 + (b*x^n*exp(lambda*x) - n)*y + c*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2\left(b\arctan\left(\frac{2abyx^n+b^2}{\sqrt{4acb^2-b^4}}\right)+\frac{\sqrt{4acb^2-b^4}\,(-\Gamma(n)+\Gamma(n,-\lambda x))x^n(-\lambda x)^{-n}}{2}\right)b}{\sqrt{4acb^2-b^4}}\right)$$

6.2.7.31 [574] problem number 31

problem number 574

Added January 10, 2019.

Problem 2.3.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x+e^{\lambda x}\left((2a\lambda x+a+b)y-e^{\lambda x}(a^2\lambda x^2+abx-c)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y*D[w[x, y], x] + Exp[lambda*x]*((2*a*lambda*x + a + b)*y - Exp[lambda*x]*(a^2*lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  y*diff(w(x,y),x)+ exp(lambda*x)* ( (2*a*lambda*x+a + b)*y - exp(lambda*x)*(a^2*lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\sqrt{\frac{-b^2-4\lambda c}{a^2}}\,a\left(\int \frac{\mathrm{e}^{\frac{2\arctan\left(\frac{2a\lambda x+b-2\lambda y\,\mathrm{e}^{-\lambda x}}{a\sqrt{\frac{-b^2-4\lambda c}{a^2}}}\right)}{2}}}{\sqrt{\frac{-b^2-4\lambda c}{a^2}}}\,\mathrm{e}^{-\_a}\tan\left(\frac{\sqrt{\frac{-b^2-4\lambda c}{a^2}}\,\_a}{2}\right)d\_a\right) - (2a\lambda x+b)\,\mathrm{e}^{-\frac{2\,\mathrm{ar}}{}}}{2a}\right.$$

6.2.7.32 [575] problem number 32

problem number 575

Added January 10, 2019.

Problem 2.3.2.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^m w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^m*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{y^{1-m}}{m-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*exp(lambda*x)*diff(w(x,y),x)+  b*y^m*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\lambda y^{-m+1}-(m-1)\,b\,\mathrm{e}^{-\lambda x}}{a\lambda}\right)$$

6.2.7.33 [576] problem number 33

problem number 576

Added January 10, 2019.

Problem 2.3.2.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^y+bx)w_x+w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Exp[y] + b*x)*D[w[x, y], x] + D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*exp(y)+b*x)*diff(w(x,y),x)+ diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(a\,\mathrm{e}^{by} + (b-1)\,x\,\mathrm{e}^{(b-1)y}\right)\mathrm{e}^{(-2b+1)y}}{b-1}\right)$$

6.2.7.34 [577] problem number 34

problem number 577

Added January 10, 2019.

Problem 2.3.2.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n e^{\lambda y} + bxy^m)w_x + e^{\mu y} w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Exp[lambda*y] + b*x*y^m)*D[w[x, y], x] + Exp[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*exp(lambda*y)+ b*x*y^m)*diff(w(x,y),x)+ exp(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{\frac{1}{m+1}} x^{\frac{m}{m+1}} x^{-\frac{n}{m+1}} x^{-\frac{mn}{m+1}} \mathrm{e}^{\frac{bny^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)\mathrm{e}^{-\frac{\mu y}{2}}}{(m+1)\mu}} \mathrm{e}^{-\frac{by^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)\mathrm{e}^{-}}{(m+1)\mu}}\right.$$

6.2.7.35 [578] problem number 35

problem number 578

Added January 10, 2019.

Problem 2.3.2.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxe^{\lambda y})w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*Exp[lambda*y])*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+ b *x*exp(lambda*y))*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(xx^{-n}\mathrm{e}^{\frac{bny^{-k}\mathrm{e}^{\lambda y}}{\lambda}}\mathrm{e}^{\frac{by^{-k}(-\lambda y)^k\Gamma(-k+1)}{\lambda}}\mathrm{e}^{\frac{bky^{-k}(-\lambda y)^k\Gamma(-k,-\lambda y)}{\lambda}}\mathrm{e}^{-\frac{by^{-k}\mathrm{e}^{\lambda y}}{\lambda}}\mathrm{e}^{-\frac{bny^{-k}(-\lambda y)^k\Gamma(-k+1)}{\lambda}}\mathrm{e}^{-\frac{bkny^{-k}(-\lambda y)^k\Gamma}{\lambda}}\right.$$

6.2.7.36 [579] problem number 36

problem number 579

Added January 10, 2019.

Problem 2.3.2.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxy^k)w_x + e^{\lambda y}w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + Exp[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+ b *x*y^k)*diff(w(x,y),x)+ exp(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{\frac{1}{k+1}} x^{\frac{k}{k+1}} x^{-\frac{n}{k+1}} x^{-\frac{kn}{k+1}} e^{\frac{bny^k(\lambda y)^{-\frac{k}{2}} \operatorname{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},\lambda y\right) e^{-\frac{\lambda y}{2}}}{(k+1)\lambda}} e^{-\frac{by^k(\lambda y)^{-\frac{k}{2}} \operatorname{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},\lambda y\right) e^{-\frac{\lambda y}{2}}}{(k+1)\lambda}} + c\right.$$

6.2.8 4.1

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6.2.8.1 [580] problem number 1

problem number 580

Added January 10, 2019.

Problem 2.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\cosh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\cosh(\lambda x)+\lambda y}{\lambda}\right)$$

6.2.8.2 [581] problem number 2

problem number 581

Added January 10, 2019.

Problem 2.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\sinh(\mu y)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu x - 2\operatorname{arctanh}\left(\mathrm{e}^{\mu y}\right)}{a\mu}\right)$$

6.2.8.3 [582] problem number 3

problem number 582

Added January 10, 2019.

Problem 2.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\sinh(\lambda x) - a^2\sinh^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sinh[lambda*x] - a^2*Sinh[lambda*x]^2)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}} - \int_1^{e^{\lambda x}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ (y^2-a^2 + a*lambda*sinh(lambda*x) - a^2*sinh(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{Heu}}{-\left(\sinh^2(\lambda x)+1\right)(-\sinh(\lambda x)+i)\,\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda x)}{2}\right.}\right.$$

6.2.8.4 [583] problem number 4

problem number 583

Added January 10, 2019.

Problem 2.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \lambda\left(\sinh(\lambda x)y^2 - \sinh^3(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + lambda*(Sinh[lambda*x]*y^2 - Sinh[lambda*x]^3)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ lambda*(sinh(lambda*x)*y^2 - sinh(lambda*x)^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y-\cosh(\lambda x))\sqrt{\pi}}{-\sqrt{\pi}\,y\,\mathrm{erfi}(\cosh(\lambda x)) + \sqrt{\pi}\,\mathrm{erfi}(\cosh(\lambda x))\cosh(\lambda x) - 2\,\mathrm{e}^{\cosh^2(\lambda x)}}\right)$$

6.2.8.5 [584] problem number 5

problem number 584

Added January 10, 2019.

Problem 2.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left((a\sinh^2(\lambda x) - \lambda)y^2 - a\sinh^2(\lambda x) + \lambda - a\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((a*Sinh[lambda*x]^2 - lambda)*y^2 - a*Sinh[lambda*x]^2 + lambda - a)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((a*sinh(lambda*x)^2-lambda)*y^2 - a*sinh(lambda*x)^2 + lambda - a)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2(\cosh(2\lambda x) - 1)\left(-(a(\sinh^2}{4\sqrt{\cosh(2\lambda x) - 1}\,(a\cosh(2\lambda x) - a - 2\lambda)\,\lambda\,\mathrm{e}^{\frac{a\cosh(2\lambda x)}{2\lambda}}\sinh(2\lambda x) + 2(\cosh(2\lambda x) - 1)}\right.$$

6.2.8.6 [585] problem number 6

problem number 585

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\sinh(\lambda x)w_x + a(\sinh(\mu y))\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[lambda*x]*D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\tanh^{-\frac{a\mu}{\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(lambda*x)*diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{2a\mu\operatorname{arctanh}\left(e^{\lambda x}\right)-2\lambda\operatorname{arctanh}\left(e^{\mu y}\right)}{a\lambda\mu}\right)$$

6.2.8.7 [586] problem number 7

problem number 586

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\mu y)w_x+a(\sinh(\lambda x))\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[mu*yx]*D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\cosh(\lambda x)\operatorname{csch}(\mu \mathrm{yx})}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(mu*y)*diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu\cosh(\lambda x) + \lambda\cosh(\mu y)}{a\lambda\mu}\right)$$

6.2.9 4.2

Local contents

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6.2.9.2 [588] problem number 2 1313
6.2.9.3 [589] problem number 3 1314
6.2.9.4 [590] problem number 4 1315
6.2.9.5 [591] problem number 5 1315
6.2.9.6 [592] problem number 6 1316
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6.2.9.8 [594] problem number 8 1318

6.2.9.1 [587] problem number 1

problem number 587

Added January 10, 2019.

Problem 2.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a(\cosh(\lambda x))\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\sinh(\lambda x)+\lambda y}{\lambda}\right)$$

6.2.9.2 [588] problem number 2

problem number 588

Added January 10, 2019.

Problem 2.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a(\cosh(\lambda x))\,w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ a*cosh(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda x + 2\arctan\left(e^{\lambda y}\right)}{a\lambda}\right)$$

6.2.9.3 [589] problem number 3

problem number 589

Added January 10, 2019.

Problem 2.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((a\cosh^2(\lambda x) - \lambda)y^2 - a\cosh^2(\lambda x) + \lambda + a\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((a*Cosh[lambda*x]^2 - lambda)*y^2 - a*Cosh[lambda*x]^2 + lambda + a)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ( (a *cosh(lambda*x)^2-lambda)*y^2 - a*cosh(lambda*x)^2+ lambda + a)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{8\sqrt{\cosh(2\lambda x) - 1}\left(\left(a\left(\cosh^2(\right.\right.\right.}{4\sqrt{\cosh(2\lambda x) + 1}\,(a\cosh(2\lambda x) + a - 2\lambda)\,\lambda\,e^{\frac{a\cosh(2\lambda x)}{2\lambda}}\sinh(2\lambda x) + 8\sqrt{\cosh(2\lambda x) - 1}}\right.$$

6.2.9.4 [590] problem number 4

problem number 590

Added January 10, 2019.

Problem 2.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2w_x + \left((a - \lambda + a\cosh(\lambda x))y^2 + a + \lambda - a\cosh(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((a - lambda + a*Cosh[lambda*x])*y^2 + a + lambda - a*Cosh[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+ ( (a - lambda + a*cosh(lambda*x))*y^2 + a+ lambda- a *cosh(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\sqrt{\cosh(\lambda x) - 1}\,(\cosh(\lambda x) + 1)^{\frac{3}{2}}\,(y\cosh(\lambda x}{-2(\cosh(\lambda x) + 1)\lambda\, e^{\frac{a\cosh(\lambda x)}{\lambda}}\sinh(\lambda x) + \sqrt{\cosh(\lambda x) - 1}\,\left(-(\cosh(\lambda x) + 1)^{\frac{5}{2}}\,y + (\cosh\right.}\right.$$

6.2.9.5 [591] problem number 5

problem number 591

Added January 10, 2019.

Problem 2.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx\cosh^m(y))\, w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cosh^{m} y)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cosh^{m} y)dy\right)}\right)$$

6.2.9.6 [592] problem number 6

problem number 592

Added January 10, 2019.

Problem 2.4.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cosh^m(y))\,w_x + \cosh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\cosh^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int (\cosh^{m} y)\left(\cosh^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\cosh^{m} y)(\cosh\right.}$$

6.2.9.7 [593] problem number 7

problem number 593

Added January 10, 2019.

Problem 2.4.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)\,w_x + \cosh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*y^m+ b*x)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m\left(\cosh^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int \left(\cosh^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int \left(\cosh^{-k}(\lambda y)\right)dy\right)}\right)$$

6.2.9.8 [594] problem number 8

problem number 594

Added January 10, 2019.

Problem 2.4.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(\cosh(\mu y))\, w_x + a\cosh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cosh[mu*y]*D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sinh(\mu y)}{\mu} - \frac{a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := cosh(mu*y)*diff(w(x,y),x)+a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu\sinh(\lambda x) + \lambda\sinh(\mu y)}{a\lambda\mu}\right)$$

6.2.10 4.3

Local contents

6.2.10.1 [595] problem number 1 1319
6.2.10.2 [596] problem number 2 1319
6.2.10.3 [597] problem number 3 1320
6.2.10.4 [598] problem number 4 1321
6.2.10.5 [599] problem number 5 1322
6.2.10.6 [600] problem number 6 1322
6.2.10.7 [601] problem number 7 1323
6.2.10.8 [602] problem number 8 1324

6.2.10.1 [595] problem number 1

problem number 595

Added January 10, 2019.

Problem 2.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \tanh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tanh[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1 \left(y - \frac{a \log(\cosh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1 \left(\frac{a \ln(\tanh(\lambda x) - 1) + a \ln(\tanh(\lambda x) + 1) + 2\lambda y}{2\lambda}\right)$$

6.2.10.2 [596] problem number 2

problem number 596

Added January 10, 2019.

Problem 2.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \tanh(\lambda y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tanh[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log(\sinh(\lambda y))}{\lambda}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{a\ln\left(\tanh\left(\lambda x\right)-1\right)+a\ln\left(\tanh\left(\lambda x\right)+1\right)+2\lambda y}{2\lambda}\right)$$

6.2.10.3 [597] problem number 3

problem number 597

Added January 10, 2019.

Problem 2.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+a\lambda-a(a+\lambda)\tanh^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Tanh[lambda*x]^2)*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\lambda e^{-2ax}\left({}_2F_1\left(-\frac{2a}{\lambda},-\frac{a}{\lambda};1-\frac{a}{\lambda};-e^{2\lambda x}\right)\left(a\left(e^{2\lambda x}-1\right)-y\left(e^{2\lambda x}+1\right)\right)+2a\left(e^{2\lambda x}+1\right)^{\frac{2a}{\lambda}+1}\right.}{2a\left(a\left(-e^{2\lambda x}\right)+a+ye^{2\lambda x}+y\right)}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2+a*lambda - a*(a+lambda)*tanh(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh\left(\lambda x\right)\right)-\left(y+\left(a+\lambda\right)\tanh\left(\lambda x\right)\right)\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh\left(\lambda x\right)\right)}{-\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\tanh\left(\lambda x\right)\right)+\left(y+\left(a+\lambda\right)\tanh\left(\lambda x\right)\right)\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\tanh\left(\lambda x\right)\right.}\right.$$

6.2.10.4 [598] problem number 4

problem number 598

Added January 10, 2019.

Problem 2.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + 3a\lambda - \lambda^2 - a(a+\lambda)\tanh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Tanh[lambda*x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2+3*a*lambda - lambda^2 -a*(a+lambda)*tanh(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\lambda+\sqrt{-a^2-4a\lambda+\lambda^2}\right)\left(-(a-\lambda+y)\left(\cosh\left(\lambda x\right)+\sinh\left(\lambda x\right)\right)\left(\lambda+\frac{\sqrt{-a^2-4a\lambda+\lambda^2}}{2}\right)a\,\mathrm{hy}\right.}{\left(a+\lambda-\sqrt{-a^2-4a\lambda+\lambda^2}\right)\left(\lambda-\sqrt{-a^2-4a\lambda+\lambda^2}\right)\left((a-\right.}\right.$$

6.2.10.5 [599] problem number 5

problem number 599

Added January 10, 2019.

Problem 2.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx \tanh^m(y))\, w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left((n-1)\, a\left(\int y^{-k} e^{(n-1)b\left(\int y^{-k}(\tanh^m y)dy\right)} dy\right) + x^{-n+1} e^{(n-1)b\left(\int y^{-k}(\tanh^m y)dy\right)}\right)$$

6.2.10.6 [600] problem number 6

problem number 600

Added January 10, 2019.

Problem 2.4.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx \tanh^m(y))\, w_x + \tanh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\tanh^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int (\tanh^{m} y)\left(\tanh^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\tanh^{m} y)(\tanh\right.}$$

6.2.10.7 [601] problem number 7

problem number 601

Added January 10, 2019.

Problem 2.4.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)\, w_x + \tanh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := ( a*x^n*y^m + b*x)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m\left(\tanh^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int\left(\tanh^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int\left(\tanh^{-k}(\lambda y)\right)dy\right)}\right)$$

6.2.10.8 [602] problem number 8

problem number 602

Added January 10, 2019.

Problem 2.4.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n\tanh^m y + bx)\,w_x + y^k w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x^n*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y K[1]^{-k}\tanh^m(K[1])dK[1] + \frac{x^{1-n}}{a(n-1)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := ( a*x^n*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}(\tanh^m y)\,dy\right) + x^{-n+1}\right)$$

6.2.11 4.4

Local contents

6.2.11.1 [603] problem number 1 1325
6.2.11.2 [604] problem number 2 1326
6.2.11.3 [605] problem number 3 1326
6.2.11.4 [606] problem number 4 1327

6.2.11.1 [603] problem number 1

problem number 603

Added January 10, 2019.

Problem 2.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \coth(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Coth[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{a \log(\sinh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*coth(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a \ln(\coth(\lambda x) - 1) + a \ln(\coth(\lambda x) + 1) + 2\lambda y}{2\lambda}\right)$$

6.2.11.2 [604] problem number 2

problem number 604

Added January 10, 2019.

Problem 2.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \coth(\lambda y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Coth[lambda*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\log(\cosh(\lambda y))}{\lambda} - ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*coth(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2a\lambda x + \ln(\coth(\lambda y) - 1) + \ln(\coth(\lambda y) + 1) - 2\ln(\coth(\lambda y))}{2a\lambda}\right)$$

6.2.11.3 [605] problem number 3

problem number 605

Added January 10, 2019.

Problem 2.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + a\lambda - a(a+\lambda)\coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\lambda e^{-2ax}\left(\frac{{}_2F_1\left(-\frac{2a}{\lambda},-\frac{a}{\lambda};1-\frac{a}{\lambda};e^{2\lambda x}\right)}{a}-\frac{2\left(1-e^{2\lambda x}\right)^{\frac{2a}{\lambda}+1}}{ae^{2\lambda x}+a-ye^{2\lambda x}+y}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 + a*lambda - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\lambda\,\mathrm{LegendreP}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth\left(\lambda x\right)\right)-\left(y+\left(a+\lambda\right)\coth\left(\lambda x\right)\right)\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth\left(\lambda x\right)\right)}{-\lambda\,\mathrm{LegendreQ}\left(\frac{a+\lambda}{\lambda},\frac{a}{\lambda},\coth\left(\lambda x\right)\right)+\left(y+\left(a+\lambda\right)\coth\left(\lambda x\right)\right)\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth\left(\lambda x\right)\right)}\right.$$

6.2.11.4 [606] problem number 4

problem number 606

Added January 10, 2019.

Problem 2.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+3a\lambda-\lambda^2-a(a+\lambda)\coth^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda e^{2x(\lambda-a)}\left((a-2\lambda)F_1\left(1-\frac{a}{\lambda};-\frac{2a}{\lambda},2;2-\frac{a}{\lambda};e^{2\lambda x},-e^{2\lambda x}\right)\left(a\left(2e^{2\lambda x}+3e^{4\lambda x}-1\right)-\left(e^{2\lambda x}\right.\right.\right.}{2\left(a^2-3a\right.}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 + a*lambda  -lambda^2 - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreP}\left(\frac{a}{\lambda},\frac{\sqrt{a^2+\lambda^2}}{\lambda},\coth(\lambda x)\right)+\left(-a-\lambda+\sqrt{a^2+\lambda^2}\right)\mathrm{Leger}}{-(y+(a+\lambda)\coth(\lambda x))\,\mathrm{LegendreQ}\left(\frac{a}{\lambda},\frac{\sqrt{a^2+\lambda^2}}{\lambda},\coth(\lambda x)\right)+\left(a+\lambda-\sqrt{a^2+\lambda^2}\right)\mathrm{Lege}}\right.$$

6.2.12 4.5

Local contents

6.2.12.1 [607] problem number 1 1329
6.2.12.2 [608] problem number 2 1329
6.2.12.3 [609] problem number 3 1330
6.2.12.4 [610] problem number 4 1331
6.2.12.5 [611] problem number 5 1332
6.2.12.6 [612] problem number 6 1333

6.2.12.1 [607] problem number 1

problem number 607

Added January 10, 2019.

Problem 2.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \sinh(\lambda x) \cosh(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[lambda*x]*Cosh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(\frac{4 \tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{2a \cosh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(lambda*x)*cosh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu \cosh(\lambda x) + 2\lambda \arctan\left(e^{\mu y}\right)}{a\lambda\mu}\right)$$

6.2.12.2 [608] problem number 2

problem number 608

Added January 10, 2019.

Problem 2.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \cosh(\lambda x) \sinh(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cosh[lambda*x]*Sinh[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh^2\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{2a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cosh(lambda*x)*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\mu\,\mathrm{e}^{\lambda x}-a\mu\,\mathrm{e}^{-\lambda x}+4\lambda\,\mathrm{arctanh}\left(\mathrm{e}^{\mu y}\right)}{2a\lambda\mu}\right)$$

6.2.12.3 [609] problem number 3

problem number 609

Added January 10, 2019.

Problem 2.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2-2\lambda^2\tanh^2(\lambda x)-2\lambda^2\coth^2(\lambda x)\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*lambda^2*Tanh[lambda*x]^2 - 2*lambda^2*Coth[lambda*x]^2)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-4\lambda x}\left(16\lambda^2xe^{4\lambda x}\left(e^{4\lambda x}+1\right)+y\left(e^{4\lambda x}+1\right)\left(e^{4\lambda x}-1\right)^2+2\lambda\left(e^{4\lambda x}-1\right)\left(-2e^{4\lambda x}(2xy+3\right.\right.}{2\left(-ye^{4\lambda x}+2\lambda\left(e^{4\lambda x}+1\right)+y\right)}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 -2 *lambda^2*tanh(lambda*x)^2 - 2*lambda^2*coth(lambda*x)^2)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{2\left(-4\sinh\left(\lambda x\right)-3\sinh\left(3\lambda x\right)+\sinh\left(5\lambda x\right)\right)\lambda\coth\left(\lambda x\right)+\left(-\lambda\left(\coth^2\left(\lambda x\right)\right)-y\coth\left(\lambda x\right.\right.}\right.$$

6.2.12.4 [610] problem number 4

problem number 610

Added January 10, 2019.

Problem 2.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) - 2ab - a(a+\lambda)\tanh^2(\lambda x) - b(b+\lambda)\coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*(a + b) - 2*a*b - a*(a + lambda)*Tanh[lambda*x]^2 - b*(
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda e^{-2x(a+b)}\left((a+b-\lambda)F_1\left(-\frac{a+b}{\lambda};-\frac{2b}{\lambda},-\frac{2a}{\lambda};-\frac{a+b-\lambda}{\lambda};e^{2\lambda x},-e^{2\lambda x}\right)\left(a\left(-2e^{2\lambda x}+3e^{4\lambda x}-1\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2 +lambda*(a+b)-2*a*b -a*(a+lambda)*tanh(lambda*x)^2 - b*(b+lambda)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(2a+3\lambda)\left(a\left(\cosh^2(\lambda x)\right)+b\left(\cosh^2(\lambda x)\right.\right.}{4\left(b-\frac{\lambda}{2}\right)\lambda\,\mathrm{hypergeom}\left(\left[2,-\frac{2b-3\lambda}{2\lambda}\right],\left[\frac{2a+5\lambda}{2\lambda}\right],\frac{\cosh^2(\lambda x)}{\sinh(\lambda x)^2}\right)\left(\cosh^2(\lambda x)\right)+2\left(\cosh(\lambda x)+1\right)}\right.$$

6.2.12.5 [611] problem number 5

problem number 611

Added January 10, 2019.

Problem 2.4.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\lambda y)w_x + a\cosh(\beta x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sinh[lambda*y]*D[w[x, y], x] + a*Cosh[beta*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\cosh(\lambda y)}{\lambda}-\frac{a\sinh(\beta x)}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := sinh(lambda*y)*diff(w(x,y),x)+a*cosh(beta*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\lambda\sinh(\beta x)+\beta\cosh(\lambda y)}{a\beta\lambda}\right)$$

6.2.12.6 [612] problem number 6

problem number 612

Added January 10, 2019.

Problem 2.4.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \cosh^m(\lambda y) + bx)\, w_x + \sinh^k(\beta y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Cosh[lambda*y]^m + b*x)*D[w[x, y], x] + Sinh[beta*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*x^n*cosh(lambda*y)^m+b*x)*diff(w(x,y),x)+sinh(beta*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\, a\left(\int (\cosh^m(\lambda y))\left(\sinh^{-k}(\beta y)\right) \mathrm{e}^{(n-1)b\left(\int \left(\sinh^{-k}(\beta y)\right) dy\right)} dy\right) + x^{-n+1} \mathrm{e}^{(n-1)b\left(\int \left(\sinh^{-k}\right.\right.}$$

6.2.13 5.1

Local contents

6.2.13.1 [613] problem number 1 1334
6.2.13.2 [614] problem number 3 1334
6.2.13.3 [615] problem number 4 1335

6.2.13.1 [613] problem number 1

problem number 613

Added January 14, 2019.

Problem 2.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \ln^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{a \log^k(\lambda x)(-\log(\lambda x))^{-k} \operatorname{Gamma}(k+1,-\log(\lambda x))}{\lambda} - bx + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*ln(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a \ln\left(\lambda x\right)^k dx\right)\right)$$

6.2.13.2 [614] problem number 3

problem number 614

Added January 14, 2019.

Problem 2.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \ln^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\log^k(\lambda K[1])+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*ln(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(x-\left(\int \frac{1}{a\ln(\lambda y)^k+b}dy\right)\right)$$

6.2.13.3 [615] problem number 4

problem number 615

Added January 14, 2019.

Problem 2.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\ln^k(x+\lambda y)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Log[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*ln(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{a\lambda \ln(\_a\lambda)^k + 1} d\_a\right) + x\right)$$

6.2.14 5.2

Local contents

6.2.14.1 [616] problem number 1 1337
6.2.14.2 [617] problem number 2 1337
6.2.14.3 [618] problem number 3 1338
6.2.14.4 [619] problem number 4 1339
6.2.14.5 [620] problem number 5 1340
6.2.14.6 [621] problem number 6 1341
6.2.14.7 [622] problem number 7 1341
6.2.14.8 [623] problem number 8 1342
6.2.14.9 [624] problem number 9 1343
6.2.14.10 [625] problem number 10 1343
6.2.14.11 [626] problem number 11 1344
6.2.14.12 [627] problem number 12 1345
6.2.14.13 [628] problem number 13 1346
6.2.14.14 [629] problem number 14 1347
6.2.14.15 [630] problem number 15 1347
6.2.14.16 [631] problem number 16 1348
6.2.14.17 [632] problem number 17 1349
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6.2.14.21 [636] problem number 21 1353
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6.2.14.23 [638] problem number 23 1355

6.2.14.1 [616] problem number 1

problem number 616

Added January 14, 2019.

Problem 2.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + ax^n \ln^k(\lambda y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*x^n*Log[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(\frac{(-\log(\lambda y))^k \log^{-k}(\lambda y) \operatorname{Gamma}(1-k, -\log(\lambda y))}{\lambda} - \frac{ax^{n+1}}{n+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*x^n*ln(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ax^{n+1} + (-n-1)\left(\int \ln(\lambda y)^{-k} dy\right)}{a}\right)$$

6.2.14.2 [617] problem number 2

problem number 617

Added January 14, 2019.

Problem 2.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + ay^n \ln^k(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y^n*Log[lambda*x]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{a\log^k(\lambda x)(-\log(\lambda x))^{-k}\,\text{Gamma}(k+1,-\log(\lambda x))}{\lambda}-\frac{y^{1-n}}{n-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*y^n*ln(lambda*x)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\left((n-1)\,ay^n\left(\int\ln(\lambda x)^k\,dx\right)+y\right)y^{-n}\right)$$

6.2.14.3 [618] problem number 3

problem number 618

Added January 14, 2019.

Problem 2.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+a\ln(\beta x)y-ab\ln(\beta x)-b^2\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Log[beta*x]*y - a*b*Log[beta*x] - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+ a*ln(beta*x)* y - a*b*ln(beta*x) - b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(\beta x)^{ax}\,\mathrm{e}^{-(a-2b)x} + (b-y)\left(\int (\beta x)^{ax}\,\mathrm{e}^{-(a-2b)x}dx\right)}{b-y}\right)$$

6.2.14.4 [619] problem number 4

problem number 619

Added January 14, 2019.

Problem 2.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\ln^m(bx)y + a\ln^m(bx)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Log[b*x]^m*y + a*Log[b*x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\int_1^x \frac{\exp\left(\frac{2^{-m-1}a\,\mathrm{Gamma}(m+1,-2(\log(b)+\log(K[1])))(-\log(b)-\log(K[1]))^{-m}(\log(b)+\log(K[1]))^m}{b^2}\right)}{K[1]^2}dK[1]\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2+ a*x*ln(b*x)^m * y + a *ln(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{ax^2 \ln(bx)^m - 2}{x} dx} dx\right) + x\, e^{\int \frac{ax^2 \ln(bx)^m - 2}{x} dx} + \int e^{\int \frac{ax^2 \ln(bx)^m - 2}{x} dx} dx}{xy + 1}\right)$$

6.2.14.5 [620] problem number 5

problem number 620

Added January 14, 2019.

Problem 2.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - abx^{n+1} y \ln(x) + b\ln(x) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^n*y^2 - a*b*x^(n + 1)*y*Log[x] + b*Log[x] + b)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*x^n*y^2- a*b*x^(n+1)*y*ln(x)  + b*ln(x) + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.6 [621] problem number 6

problem number 621

Added January 14, 2019.

Problem 2.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((n+1)x^n y^2 - a x^{n+1} (\ln x)^m y + a (\ln x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((n + 1)*x^n*y^2 - a*x^(n + 1)*Log[x]^m*y + a*Log[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-((n+1)*x^n*y^2 - a*x^(n+1)*ln(x)^m*y + a*ln(x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{n+1} e^{\int \frac{a x x^{n+1} \ln(x)^m - 2n - 2}{x} dx} + (y x^{n+1} - 1)(n+1)\left(\int \frac{x^n e^{a\left(\int x^{n+1} \ln(x)^m dx\right) - 2n \ln(x)}}{x^2} dx\right)}{y x^{n+1} - 1}\right)$$

6.2.14.7 [622] problem number 7

problem number 622

Added January 14, 2019.

Problem 2.5.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a (\ln x)^n y^2 + b m x^{m-1} - a b^2 x^{2m} (\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(2*m)*Log[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a *ln(x)^n*y^2 + b*m*x^(m-1) - a*b^2*x^(2*m)* ln(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.8 [623] problem number 8

problem number 623

Added January 14, 2019.

Problem 2.5.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a(\ln x)^n y^2 - abxy(\ln x)^{n+1} + b\ln x + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 - a*b*x*y*Log[x]^(n + 1) + b*Log[x] + b)*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*ln(x)^n*y^2 - a*b*x*y*(ln(x))^(n+1) + b*ln(x)+ b )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.9 [624] problem number 9

problem number 624

Added January 14, 2019.

Problem 2.5.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^k (y - bx^n - c)^3 + bnx^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^k*(y - b*x^n - c)^3 + b*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(\frac{2(bx^n + c - y)^2 \int_1^x a \log^k(K[1]) dK[1] + 1}{(bx^n + c - y)^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*(ln(x))^k*(y - b*x^n-c)^3 + b*n*x^(n-1) ) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(b^2x^{2n} + 2(c-y)\left(bx^n + \frac{c}{2} - \frac{y}{2}\right)\right) a\left(\int \ln(x)^k \, dx\right) + 1}{(bx^n + c - y)^2}\right)$$

6.2.14.10 [625] problem number 10

problem number 625

Added January 14, 2019.

Problem 2.5.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 + b(\ln x)^m y + bc(\ln x)^m - ac^2(\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Log[x]^n*y^2 + b*Log[x]^m*y + b*c*Log[x]^m - a*c^2*Log[x]^n)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*(ln(x))^n*y^2 + b*(ln(x))^m *y+ b*c* (ln(x))^m - a*c^2* (ln(x))^n)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(c+y)\,a\left(\int \ln(x)^n\,\mathrm{e}^{-\left(\int (2ac\ln(x)^n - b\ln(x)^m)dx\right)}dx\right) - \mathrm{e}^{-\left(\int (2ac\ln(x)^n - b\ln(x)^m)dx\right)}}{c+y}\right)$$

6.2.14.11 [626] problem number 11

problem number 626

Added January 14, 2019.

Problem 2.5.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ay + b\ln x)^2 w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*y + b*Log[x])^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ay+b\log(x)}{a\sqrt{\frac{b}{a^3}}}\right)-a^2\sqrt{\frac{b}{a^3}}\log(x)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*y+ b*ln(x))^2 *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\arctan\left(\frac{(ay+b\ln(x))a}{\sqrt{ab}}\right)-\sqrt{ab}\,\ln(x)}{\sqrt{ab}\,a}\right)$$

6.2.14.12 [627] problem number 12

problem number 627

Added January 14, 2019.

Problem 2.5.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(xy^2-A^2x(\ln\beta x)^2+A\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^2 + A)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^2 + A) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.13 [628] problem number 13

problem number 628

Added January 14, 2019.

Problem 2.5.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(xy^2 - A^2x(\ln(\beta x))^{2k} + kA(\ln(\beta x))^{k-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^(2*k) + k*A*Log[beta*x]^(k - 1))*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^(2*k) + k*A*(ln(beta*x))^(k-1))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.14 [629] problem number 14

problem number 629

Added January 14, 2019.

Problem 2.5.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(ax^n y^2 + b - ab^2 x^n (\ln x)^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*y^2 + b - a*b^2*x^n*Log[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+(a*x^n*y^2 + b - a*b^2*x^n*(ln(x))^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.14.15 [630] problem number 15

problem number 630

Added January 14, 2019.

Problem 2.5.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(a(\ln(\lambda x))^m y^2 + ky + ab^2 x^{2k} (\ln(\lambda x))^m\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Log[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Log[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\sqrt{b^2}x^k(\lambda x)^{-k}\log^{m+1}(\lambda x)(-k\log(\lambda x))^{-m-1}\,\mathrm{Gamma}(m+1,-k\log(\lambda x))+\tan^{-1}\left(\frac{yx^{-}}{\sqrt{b}}\right.\right.\right.$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*(ln(lambda*x))^m*y^2 + k*y+ a*b^2*x^(2*k)* (ln(lambda*x))^m )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(ab\left(\int x^{k-1}\ln(\lambda x)^m\,dx\right)-\arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.14.16 [631] problem number 16

problem number 631

Added January 14, 2019.

Problem 2.5.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^n(y+b\ln x)^2-b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^n*(y + b*Log[x])^2 - b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{abx^n\log(x)+ayx^n+n}{bn\log(x)+ny}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*x^n*(y + b*ln(x))^2 - b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b\ln(x)+y)\,ax^n+n}{(b\ln(x)+y)\,n}\right)$$

6.2.14.17 [632] problem number 17

problem number 632

Added January 14, 2019.

Problem 2.5.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n}\ln(x)y^2 + (bx^n\ln x - n)y + c\ln x\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*x^(2*n)*Log[x]*y^2 + (b*x^n*Log[x] - n)*y + c*Log[x])*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\left(\sqrt{b^2-4ac}+2ayx^n+b\right)e^{\frac{x^n\sqrt{b^2-4ac}(n\log(x)-1)}{n^2}}}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*x^(2*n)*ln(x)* y^2 + (b* x^n *ln(x) - n)*y + c *ln(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-2bn^2\arctan\left(\frac{2abyx^n+b^2}{\sqrt{4acb^2-b^4}}\right)+\sqrt{4acb^2-b^4}\,(n\ln(x)-1)\,x^n\right)b}{\sqrt{4acb^2-b^4}\,n^2}\right)$$

6.2.14.18 [633] problem number 18

problem number 633

Added January 14, 2019.

Problem 2.5.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^k w_x + (ay^n(\ln x)^m + by(\ln x)^s)\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y*Log[x]^s)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left((n-1)\int_1^x a\exp\left(-b(k-1)^{-s-1}(n-1)\,\mathrm{Gamma}(s+1,(k-1)\log(K[1]))\right)K[1]^{-k}\log^m\right.\right.$$

Maple ✓

```
restart;
pde := x^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m + b*y*(ln(x))^s )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int x^{-k}\ln(x)^{m}\,\mathrm{e}^{(n-1)b\left(\int x^{-k}\ln(x)^{s}dx\right)}dx\right) + y^{-n+1}\mathrm{e}^{(n-1)b\left(\int x^{-k}\ln(x)^{s}dx\right)}\right)$$

6.2.14.19 [634] problem number 19

problem number 634

Added January 14, 2019.

Problem 2.5.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left(y^2 + c(\ln x)^n y - \lambda^2 + \lambda c(\ln x)^n\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Log[x] + b)*D[w[x, y], x] + (y^2 + c*Log[x]^n*y - lambda^2 + lambda*c*Log[x]^n)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*ln(x)+b)*diff(w(x,y),x)+(y^2+ c*(ln(x))^n*y- lambda^2 + lambda*c*(ln(x))^n )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{e^{\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right)e^{\int \frac{-c\ln(x)^n+2\lambda}{a\ln(x)+b}dx+\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx} - y\left(\int \frac{e^{\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) - e^{\int \frac{c\ln(x)^n-}{a\ln(x)+}}}{\lambda e^{\int \frac{-c\ln(x)^n+2\lambda}{a\ln(x)+b}dx+\int \frac{c\ln(x)^n-2\lambda}{a\ln(x)+b}dx} + y}\right.$$

6.2.14.20 [635] problem number 20

problem number 635

Added January 14, 2019.

Problem 2.5.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left((\ln x)^n y^2 - cy - \lambda^2(\ln x)^n + c\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Log[x] + b)*D[w[x, y], x] + (Log[x]^n*y^2 - c*y - lambda^2*Log[x]^n + c*lambda)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a*ln(x)+b)*diff(w(x,y),x)+((ln(x))^n*y^2- c*y - lambda^2*(ln(x))^n + c*lambda )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{\ln(x)^n e^{\int \frac{2\lambda \ln(x)^n - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) e^{\int \frac{-2\lambda \ln(x)^n + c}{a\ln(x)+b}dx + \int \frac{2\lambda \ln(x)^n - c}{a\ln(x)+b}dx} + y\left(\int \frac{\ln(x)^n e^{\int \frac{2\lambda \ln(x)^n - c}{a\ln(x)+b}dx}}{a\ln(x)+b}dx\right) +}{\lambda e^{\int \frac{-2\lambda \ln(x)^n + c}{a\ln(x)+b}dx + \int \frac{2\lambda \ln(x)^n - c}{a\ln(x)+b}dx} - y}\right.$$

6.2.14.21 [636] problem number 21

problem number 636

Added January 14, 2019.

Problem 2.5.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 \ln(ax) w_x - \left(x^2 y^2 \ln(ax) + 1\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x^2*Log[a*x]*D[w[x, y], x] - (x^2*y^2*Log[a*x] + 1)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x^2*ln(a*x)*diff(w(x,y),x)-(x^2*y^2* ln(a*x)+ 1  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\ln(ax) - 1}{ax^2 y + xy\,\mathrm{expIntegral}(1, -\ln(ax))\ln(ax) - \mathrm{expIntegral}(1, -\ln(ax))}\right)$$

6.2.14.22 [637] problem number 22

problem number 637

Added January 14, 2019.

Problem 2.5.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n + by\ln^m x)\,w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[lambda*x]^k*D[w[x, y], x] + (a*y^n + b*y*Log[x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left((n-1)\int_1^x a\exp\left((n-1)\int_1^{K[2]} b\log^m(K[1])(\log(\lambda)+\log(K[1]))^{-k}dK[1]\right)(\log(\lambda)+\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n+ b*y* (ln(x))^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \ln(\lambda x)^{-k}\,\mathrm{e}^{(n-1)b\left(\int \ln(x)^m \ln(\lambda x)^{-k}dx\right)}dx\right) + y^{-n+1}\mathrm{e}^{(n-1)b\left(\int \ln(x)^m \ln(\lambda x)^{-k}dx\right)}\right)$$

6.2.14.23 [638] problem number 23

problem number 638

Added January 14, 2019.

Problem 2.5.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n \ln^m x + by)\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[lambda*x]^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left((n-1)\int_1^x a\exp\left(\frac{b(n-1)\,\mathrm{Gamma}(1-k,-\log(\lambda)-\log(K[1]))(-\log(\lambda)-\log(K[1])}{\lambda}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m+ b*y )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \ln(x)^m \ln(\lambda x)^{-k}\, \mathrm{e}^{(n-1)b\left(\int \ln(\lambda x)^{-k}dx\right)}dx\right) + y^{-n+1}\mathrm{e}^{(n-1)b\left(\int \ln(\lambda x)^{-k}dx\right)}\right)$$

6.2.15 6.1

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problem number 639

Added January 14, 2019.

Problem 2.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \sin^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y},Assumptions -> {Element[k,
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\sin^2(\lambda x)\right)}{k\lambda+\lambda} - bx + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming k::in
```

Unable to process Latex

6.2.15.2 [640] problem number 2

problem number 640

Added January 14, 2019.

Problem 2.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \sin^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a \sin^k(\lambda K[1]) + b} dK[1] - x\right)\right\}\right\}$$

contains unresolved integral

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\left(\sin^k(\lambda y)\right) + b} dy\right)\right)$$

contains unresolved integral

6.2.15.3 [641] problem number 3

problem number 641

Added January 14, 2019.

Problem 2.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(\lambda y)\sin^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[lambda*x]^k*Sin[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\cos^2(\mu y)}\sec(\mu y)\sin^{1-n}(\mu y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\mu y)\right)}{\mu-\mu n} - \frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(}{k\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sin(lambda*x)^k*sin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int\left(\sin^k(\lambda x)\right)dx\right) + \int\frac{\sin^{-n}(\mu y)}{a}dy\right)$$

contains unresolved integral

6.2.15.4 [642] problem number 4

problem number 642

Added January 14, 2019.

Problem 2.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(x+\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sin(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{a\lambda\left(\sin^k(\_a\lambda)\right)+1} d\_a\right)+x\right)$$

contains unresolved integral

6.2.15.5 [643] problem number 5

problem number 643

Added January 14, 2019.

Problem 2.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\sin(\lambda x) + a^2\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sin[lambda*x] + a^2*Sin[lambda*x]^2)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(y^2-a^2 + a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{2\,\mathrm{csgn}\left(\sin\left(\lambda x\right)\right)\sin\left(\lambda x\right)+2}\left(\frac{\lambda\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda},-\right.}{}\right.}{2\left(\mathrm{csgn}\left(\sin\left(\lambda x\right)\right)+\sin\left(\lambda x\right)\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda},\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\sin(\lambda x)}{2}+\frac{1}{2}\right)\cos\left(\lambda x\right.}\right.$$

6.2.15.6 [644] problem number 6

problem number 644

Added January 14, 2019.

Problem 2.6.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\sin(\beta x)y + ab\sin(\beta x) - b^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*Sin[beta*x]*y + a*b*Sin[beta*x] - b^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2 + a*sin(beta*x)* y + a*b*sin(beta*x)-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b+y)\left(\int e^{\frac{-2b\beta x-a\cos(\beta x)}{\beta}}dx\right)+e^{\frac{-2b\beta x-a\cos(\beta x)}{\beta}}}{b+y}\right)$$

contains unresolved integrals

6.2.15.7 [645] problem number 7

problem number 645

Added January 14, 2019.

Problem 2.6.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\sin^m(bx)y + a\sin^m(bx)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Sin[b*x]^m*y + a*Sin[b*x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\int_1^x \frac{\exp\left(\frac{a\sin^{m+1}(bK[1])\left(\frac{2b\cos(bK[1])\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};\sin^2(bK[1])\right)K[1]}{m+1}-2^{-m-1}\sqrt{\pi}\,\mathrm{Gamma}(m+1)\,{}_3\tilde{F}_2(1,\frac{m}{2}}{2b^2}\right.}{K[1]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2 + a*x*sin(b*x)^m*y + a*sin(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{ax^2(\sin^m(bx))-2}{x}dx}dx\right) + x\,e^{\int \frac{ax^2(\sin^m(bx))-2}{x}dx} + \int e^{\int \frac{ax^2(\sin^m(bx))-2}{x}dx}dx}{xy+1}\right)$$

6.2.15.8 [646] problem number 8

problem number 646

Added January 14, 2019.

Problem 2.6.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + \lambda \sin^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + lambda*Sin[lambda*x]^3)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(lambda*sin(lambda*x)*y^2 + lambda*sin(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sqrt{\pi}\,(y+\cos(\lambda x))}{\sqrt{\pi}\,y\,\mathrm{erfi}(\cos(\lambda x)) + \sqrt{\pi}\,\mathrm{erfi}(\cos(\lambda x))\cos(\lambda x) - 2\,e^{\cos^2(\lambda x)}}\right)$$

6.2.15.9 [647] problem number 9

problem number 647

Added January 14, 2019.

Problem 2.6.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2w_x + \left((\lambda + a - a\sin(\lambda x))y^2 + \lambda - a - a\sin(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((lambda + a - a*Sin[lambda*x])*y^2 + lambda - a - a*Sin[lambda*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+((lambda+a-a*sin(lambda*x))*y^2 +lambda -a -a*sin(lambda*x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{((\cos(\lambda x) - 1)(\cos(}{2(-a\sin(\lambda x) + (a+\lambda)\operatorname{csgn}(\sin(\lambda x)))(a\sin(\lambda x) - a - \lambda)^2(\cos(\lambda x) - 1)(\cos(\lambda x) -}\right.$$

6.2.15.10 [648] problem number 10

problem number 648

Added January 14, 2019.

Problem 2.6.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left((\lambda + a\sin^2(\lambda x))y^2 + \lambda - a + a\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((lambda + a*Sin[lambda*x]^2)*y^2 + lambda - a + a*Sin[lambda*x]^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+((lambda+a*sin(lambda*x)^2)*y^2 + lambda -a +a*sin(lambda*x)^2)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{\cos(2\lambda x)+1}\left(-(a(\sin^2(\lambda x)\right.}{-4\sqrt{\cos(2\lambda x)-1}\,(a\cos(2\lambda x)-a-2\lambda)\,\lambda\,\mathrm{e}^{\frac{a\cos(2\lambda x)}{2\lambda}}\sin(2\lambda x)+2\sqrt{\cos(2\lambda x)+1}\left(-\right.}\right.$$

6.2.15.11 [649] problem number 11

problem number 649

Added January 14, 2019.

Problem 2.6.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - a x^{k+1}(\sin x)^m y + a(\sin x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Sin[x]^m*y + a*Sin[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-((k+1)*x^k*y^2 - a*x^(k+1)*(sin(x))^m*y + a*(sin(x))^m)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-xyx^k\left(\int (k+1)\, x^k e^{\int \frac{ax^2x^k(\sin^m x)-2k-2}{x}dx}dx\right) + xx^k e^{\int \frac{ax^2x^k(\sin^m x)-2k-2}{x}dx} + \int (k+1)\, x^k e^{\int}}{xyx^k - 1}\right.$$

6.2.15.12 [650] problem number 12

problem number 650

Added January 14, 2019.

Problem 2.6.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Sin[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(a*sin(lambda*x + mu)^k * (y-b*x^n -c)^2 + y - b*x^n + b*n*x^(n-1) - c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.15.13 [651] problem number 13

problem number 651

Added January 14, 2019.

Problem 2.6.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\sin^m(\lambda x)y^2 + ky + ab^2x^{2k}\sin^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Sin[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Sin[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right) - \sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+(a*sin(lambda*x)^m*y^2 + k*y + a*b^2*x^(2*k)*sin(lambda*x)^m)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab\left(\int x^{k-1}(\sin^m(\lambda x))\,dx\right) - \arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.15.14 [652] problem number 14

problem number 652

Added January 14, 2019.

Problem 2.6.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\sin(\lambda x) + b)w_x + \left(y^2 + c\sin(\mu x)y - k^2 + ck\sin(\mu x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Sin[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Sin[mu*x]*y - k^2 + c*k*Sin[mu*x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (a *sin(lambda*x) + b)*diff(w(x,y),x)+(y^2+ c*sin(mu*x)* y - k^2 + c*k*sin(mu*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(k+y)\left(\int e^{\frac{\sqrt{-a^2+b^2}\,c\lambda\left(\int \frac{\sin(\mu x)}{a\sin(\lambda x)+b}dx\right)-4k\arctan\left(\frac{a\cos\left(\frac{\lambda x}{2}\right)+b\sin\left(\frac{\lambda x}{2}\right)}{\sqrt{-a^2+b^2}\cos\left(\frac{\lambda x}{2}\right)}\right)}{\sqrt{-a^2+b^2}\,\lambda}}{a\sin(\lambda x)+b}dx\right) + e^{\frac{\sqrt{-a^2+b^2}\,c\lambda\left(\int \frac{\sin(\mu x)}{a\sin(\lambda x)+b}d\right.}{}}}{k+y}\right.$$

6.2.16 6.2

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6.2.16.1 [653] problem number 1

problem number 653

Added January 14, 2019.

Problem 2.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cos[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{a\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cos^2(\lambda x)\right)}{k\lambda+\lambda} - bx + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*cos(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a\left(\cos^k(\lambda x)\right)dx\right)\right)$$

Contains unresolved integral

6.2.16.2 [654] problem number 2

problem number 654

Added January 14, 2019.

Problem 2.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cos[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a\cos^k(\lambda K[1]) + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(a*cos(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\left(\cos^k(\lambda y)\right) + b} dy\right)\right)$$

6.2.16.3 [655] problem number 3

problem number 655

Added January 14, 2019.

Problem 2.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cos^k(\lambda x)\cos^n(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cos[lambda*y]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cos^{-k}(\lambda K[1])\cos^{-n}(\mu K[1])dK[1]-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cos(lambda*y)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ax-\left(\int\left(\cos^{-n}\left(\mu y\right)\right)\left(\cos^{-k}\left(\lambda y\right)\right)dy\right)}{a}\right)$$

6.2.16.4 [656] problem number 4

problem number 656

Added January 14, 2019.

Problem 2.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\cos^k(x+\lambda y)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cos[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*cos(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{a\lambda\left(\cos^k(\_a\lambda)\right)+1} d\_a\right)+x\right)$$

6.2.16.5 [657] problem number 5

problem number 657

Added January 14, 2019.

Problem 2.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\cos(\lambda x) + a^2\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - a^2 + a*lambda*Cos[lambda*x] + a^2*Cos[lambda*x]^2)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( y^2-a^2 + a *lambda*cos(lambda*x) + a^2*cos(lambda*x)^2)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2\sqrt{2\cos(\lambda x)+2}\left(\frac{\lambda\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda},-\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\cos(\lambda x)}{2}+\frac{1}{2}\right)\sin(\lambda x}{2}\right.}{2\left(\cos(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4a}{\lambda},\frac{1}{2},-\frac{1}{2},-\frac{2a}{\lambda},\frac{8a+3\lambda}{8\lambda},\frac{\cos(\lambda x)}{2}+\frac{1}{2}\right)\sin(\lambda x)+\left(-4\left(\cos\right.\right.}\right.$$

6.2.16.6 [658] problem number 6

problem number 658

Added January 14, 2019.

Problem 2.6.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \cos(\lambda x) y^2 + \lambda \cos^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Cos[lambda*x]*y^2 + lambda*Cos[lambda*x]^3)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(lambda*cos(lambda*x)*y^2 + lambda*cos(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{-(y - \sin(\lambda x)) M\left(1, \frac{3}{2}, -(\sin^2(\lambda x))\right) \sin(\lambda x) - 1}{(y - \sin(\lambda x)) U\left(1, \frac{3}{2}, -(\sin^2(\lambda x))\right) \sin(\lambda x) - 2}\right)$$

6.2.16.7 [659] problem number 7

problem number 659

Added January 14, 2019.

Problem 2.6.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2w_x + \left((\lambda + a + a\cos(\lambda x))y^2 + \lambda - a + a\cos(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*D[w[x, y], x] + ((lambda + a + a*Cos[lambda*x])*y^2 + lambda - a + a*Cos[lambda*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := 2*diff(w(x,y),x)+ ((lambda+a+a*cos(lambda*x))*y^2 +lambda - a + a *cos(lambda*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y\cos(\lambda x)+y-\sin(\lambda x))\sqrt{\cos(\lambda x)+1}\sqrt{\cos(\lambda x)-1}\,\mathrm{e}^{\frac{a\cos}{\lambda}}}{\left((y\cos(\lambda x)+y-\sin(\lambda x))\sqrt{\cos(\lambda x)+1}\sqrt{\cos(\lambda x)-1}\left(\int\frac{(a\cos(\lambda x)+a+\lambda)\mathrm{e}^{-\frac{a\cos(\lambda x)}{\lambda}}\sin(\lambda}{\sqrt{\cos(\lambda x)-1}\,(\cos(\lambda x)+1)^{\frac{3}{2}}}\right.\right.}\right.$$

6.2.16.8 [660] problem number 8

problem number 660

Added January 14, 2019.

Problem 2.6.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((\lambda + a\cos^2(\lambda x))y^2 + \lambda - a + a\cos^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((lambda + a*Cos[lambda*x]^2)*y^2 + lambda - a + a*Cos[lambda*x]^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+ ((lambda+a*cos(lambda*x)^2)*y^2 + lambda - a + a*cos(lambda*x)^2)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\left(\left(a(\cos^2(\lambda x)) + \lambda\right) y(\cos^4(\lambda x)) - \frac{(\cos}{}\right.}{\left(4\left(\left(a\left(\cos^2(\lambda x)\right) + \lambda\right) y\left(\cos^4(\lambda x)\right) - \frac{(\cos(2\lambda x)+1)(a\cos(2\lambda x)+a+2\lambda)\sin(2\lambda x)}{8}\right)\sqrt{\cos(2\lambda x) - 1}\right.}\right.$$

6.2.16.9 [661] problem number 9

problem number 661

Added January 14, 2019.

Problem 2.6.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*y^m+b*x)*diff(w(x,y),x)+ cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m\left(\cos^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int\left(\cos^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int\left(\cos^{-k}(\lambda y)\right)dy\right)}\right)$$

6.2.16.10 [662] problem number 10

problem number 662

Added January 14, 2019.

Problem 2.6.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx\cos^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cos^m y)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\cos^m y)dy\right)}\right)$$

6.2.16.11 [663] problem number 11

problem number 663

Added January 14, 2019.

Problem 2.6.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx\cos^m y)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\cos^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int (\cos^m y)\left(\cos^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int (\cos^m y)\left(\cos^{-k}(\lambda y)\right.\right.}$$

6.2.16.12 [664] problem number 12

problem number 664

Added January 14, 2019.

Problem 2.6.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \cos^m y + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Cos[y]^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*cos(y)^m+b*x)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int (\cos^m y)\left(\cos^{-k}(\lambda y)\right) \mathrm{e}^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right)} dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int \left(\cos^{-k}(\lambda y)\right)dy\right)}\right)$$

6.2.17 6.3

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6.2.17.1 [665] problem number 1

problem number 665

Added January 14, 2019.

Problem 2.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a + Tan[lambda*x] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-ax-bx+\frac{\log(\cos(\lambda x))}{\lambda}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a+tan(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{((-2a-2b)\,x+2y)\,\lambda-\ln\left(\tan^2(\lambda x)+1\right)}{2\lambda}\right)$$

6.2.17.2 [666] problem number 2

problem number 666

Added January 14, 2019.

Problem 2.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\tan^k(\lambda y)+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a + Tan[lambda*y] + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-x+\frac{-i(a+b-i)\log(-\tan(\lambda y)+i)+i(a+b+i)\log(\tan(\lambda y)+i)+2\log(a+b+t}{2\lambda(a+b-i)(a+b+i)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a+tan(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(2a^2x+4abx+2b^2x+2x)\lambda+(-2a-2b)\arctan(\tan(\lambda y))+\ln(\tan^2(\lambda y)+1)-2\ln(}{2(a^2+2ab+b^2+1)\lambda}\right.$$

6.2.17.3 [667] problem number 3

problem number 667

Added January 14, 2019.

Problem 2.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tan^k(\lambda x)\tan^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tan[lambda*x]^k*Tan[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tan^{1-n}(\mu y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(\mu y)\right)}{\mu-\mu n}-\frac{a\tan^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};-\tan^2(\lambda x\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a *tan(lambda*x)^k * tan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \left(\tan^k(\lambda x)\right) dx\right) + \int \frac{\tan^{-n}(\mu y)}{a} dy\right)$$

Has unresolved integrals

6.2.17.4 [668] problem number 4

problem number 668

Added January 14, 2019.

Problem 2.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+ a *lambda + a*(lambda -a) *tan(lambda*x)^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2(\cos(2\lambda x)+1)\lambda \operatorname{LegendreP}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right) + (-4y(\cos^3(\lambda x)) - a\sin(\lambda x) - a\,\mathrm{s}}{-2(\cos(2\lambda x)+1)\lambda \operatorname{LegendreQ}\left(\frac{2a+\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right) + (4y(\cos^3(\lambda x)) + a\sin(\lambda x) + a\,\mathrm{s}}\right.$$

6.2.17.5 [669] problem number 5

problem number 669

Added January 14, 2019.

Problem 2.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + \lambda^2 + 3a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  y^2+ lambda^2 +3*a*lambda +a*(lambda-a)*tan(lambda*x)^2)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{4\lambda\,\mathrm{LegendreP}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right)(\cos^2(\lambda x)) - 2\left(\frac{\lambda(\sin^3(\lambda x))}{2} + \left(a + \frac{3\lambda}{2}\right)(\cos^2(\lambda x))\,\mathrm{s}\right.}{-4\lambda\,\mathrm{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda}, \frac{2a-\lambda}{2\lambda}, \sin(\lambda x)\right)(\cos^2(\lambda x)) + 2\left(\frac{\lambda(\sin^3(\lambda x))}{2} + \left(a + \frac{3\lambda}{2}\right)(\cos^2(\lambda x))\right.}\right.$$

6.2.17.6 [670] problem number 6

problem number 670

Added January 14, 2019.

Problem 2.6.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + ax\tan^k(bx)y + a\tan^k(bx)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*x*Tan[b*x]^k*y + a*Tan[b*x]^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\exp\left(-\int_1^x -aK[5]\tan^k(bK[5])dK[5]\right)}{x^2y+x}-\int_1^x\frac{\exp\left(-\int_1^{K[6]} -aK[5]\tan^k(bK[5])dK[5]\right.}{K[6]^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  y^2+ a*x  *tan(b*x)^k * y + a*tan(b*x)^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{xy\left(\int e^{\int\frac{ax^2\left(\tan^k(bx)\right)-2}{x}dx}dx\right)+x\,e^{\int\frac{ax^2\left(\tan^k(bx)\right)-2}{x}dx}+\int e^{\int\frac{ax^2\left(\tan^k(bx)\right)-2}{x}dx}dx}{xy+1}\right)$$

6.2.17.7 [671] problem number 7

problem number 671

Added January 14, 2019.

Problem 2.6.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x-\left((k+1)x^ky^2-ax^{k+1}(\tan x)^my+a(\tan x)^m\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Tan[x]^m*y + a*Tan[x]^m)*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2- a*x^(k+1)*tan(x)^m*y + a*tan(x)^m )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{k+1}e^{\int \frac{axx^{k+1}(\tan^m x)-2k-2}{x}dx} - \left(yx^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}e^{a\left(\int x^{k+1}\left(-\frac{i\left(e^{2ix}-1\right)}{e^{2ix}+1}\right)^m dx\right)}}{x^2}dx\right)}{yx^{k+1}-1}\right)$$

6.2.17.8 [672] problem number 8

problem number 672

Added January 20, 2019.

Problem 2.6.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^n(\lambda x)y^2 - ab^2\tan^{n+2}(\lambda x) + b\lambda\tan^2(\lambda x) + b\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Tan[lambda*x]^n*y^2 - a*b^2*Tan[lambda*x]^(n + 2) + b*lambda*Tan[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(a*tan(lambda*x)^n*y^2- a*b^2*tan(lambda*x)^(n+2) + b*lambda*tan(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.17.9 [673] problem number 9

problem number 673

Added January 20, 2019.

Problem 2.6.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Tan[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(a *tan(lambda*x+mu)^k*(y-b*x^n-c)^2 + y- b*x^n + b*n*x^(n-1)-c)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.17.10 [674] problem number 10

problem number 674

Added January 20, 2019.

Problem 2.6.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\tan^m(\lambda x)y^2 + ky + ab^2x^{2k}\tan^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (a*Tan[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Tan[lambda*x]^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right) - \sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ (a*tan(lambda*x)^m*y^2 +k*y+ a*b^2*x^(2*k)*tan(lambda*x)^m )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab\left(\int x^{k-1}(\tan^m(\lambda x))\,dx\right) - \arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.17.11 [675] problem number 11

problem number 675

Added January 20, 2019.

Problem 2.6.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\tan(\lambda x) + b)w_x + \left(y^2 + c\tan(\mu x)y - k^2 + ck\tan(\mu x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*Tan[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Tan[mu*x]*y - k^2 + c*k*Tan[mu*x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*tan(lambda*x)+b)*diff(w(x,y),x)+ (y^2+ c *tan(mu*x)*y - k^2 + c*k*tan(mu*x) )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left((k+y)\left(e^{2i\mu x}+1\right)^{\frac{ia^2c}{\left(a^2+b^2\right)(ib+a)\mu}}\left(e^{2i\mu x}+1\right)^{\frac{ib^2c}{\left(a^2+b^2\right)(ib+a)\mu}}\left(\int \frac{\left(\frac{a\sin(\lambda x)+b\cos(\lambda x)}{\cos(\lambda x)}\right)^{-\frac{2ak}{\left(a^2+b^2\right)\lambda}}\left(\frac{2}{\cos(2\lambda}\right.}{}\right.\right.}{}\right.$$

6.2.17.12 [676] problem number 12

problem number 676

Added January 20, 2019.

Problem 2.6.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*y^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*y^m + b*x)*diff(w(x,y),x)+ tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^m\left(\tan^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}\right)$$

6.2.17.13 [677] problem number 13

problem number 677

Added January 20, 2019.

Problem 2.6.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int y^{-k}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tan^m y)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int y^{-k}(\tan^m y)dy\right)}\right)$$

6.2.17.14 [678] problem number 14

problem number 678

Added January 20, 2019.

Problem 2.6.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int \left(\tan^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int(\tan^m y)\left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int(\tan^m y)\left(\tan^{-k}(\lambda y\right.\right.}\right.$$

6.2.17.15 [679] problem number 15

problem number 679

Added January 20, 2019.

Problem 2.6.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \tan^m y + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Tan[y]^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   (a*x^n*tan(y)^m+ b*x)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((n-1)\,a\left(\int (\tan^m y)\left(\tan^{-k}(\lambda y)\right)\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}dy\right) + x^{-n+1}\mathrm{e}^{(n-1)b\left(\int\left(\tan^{-k}(\lambda y)\right)dy\right)}$$

6.2.18 6.4

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6.2.18.1 [680] problem number 1

problem number 680

Added January 20, 2019.

Problem 2.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cot^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cot[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{a\cot^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};-\cot^2(\lambda x)\right)}{k\lambda+\lambda} - bx + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (a*cot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a\left(\cot^k(\lambda x)\right)dx\right)\right)$$

Has unresolved integral

6.2.18.2 [681] problem number 2

problem number 681

Added January 20, 2019.

Problem 2.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a \cot^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Cot[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\int_1^y \frac{1}{a \cot^k(\lambda K[1]) + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (a*cot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(x - \left(\int \frac{1}{a\left(\cot^k(\lambda y)\right) + b} dy\right)\right)$$

6.2.18.3 [682] problem number 3

problem number 682

Added January 20, 2019.

Problem 2.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a\cot^k(x+\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Cot[x + lambda*y]^k*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  cot(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\lambda\left(\int^{\frac{\lambda y+x}{\lambda}} \frac{1}{\lambda\left(\cot^k(\_a\lambda)\right)+1}d\_a\right)+x\right)$$

6.2.18.4 [683] problem number 4

problem number 683

Added January 20, 2019.

Problem 2.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\cot^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ ( y^2+a*lambda + a*(lambda-a)*cot(lambda*x)^2  )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-2(\cos(2\lambda x)-1)\lambda\,\mathrm{LegendreP}\left(\frac{2a+\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)+(-a\cos(\lambda x)+a\cos(3\lambda x)+3y}{2(\cos(2\lambda x)-1)\lambda\,\mathrm{LegendreQ}\left(\frac{2a+\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)+(a\cos(\lambda x)-a\cos(3\lambda x)-3y\,\mathrm{si}}\right.$$

6.2.18.5 [684] problem number 5

problem number 684

Added January 20, 2019.

Problem 2.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2+\lambda^2+3a\lambda+a(\lambda-a)\cot^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (  y^2+lambda^2 + 3*a*lambda +a*(lambda-a)*cot(lambda*x)^2   )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{4\lambda\,\mathrm{LegendreP}\left(\frac{2a+3\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)(\sin^2(\lambda x))-2\left(\frac{\lambda(\cos^3(\lambda x))}{2}+y(\cos^2(\lambda x))\sin(\lambda x)\right.}{-4\lambda\,\mathrm{LegendreQ}\left(\frac{2a+3\lambda}{2\lambda},\frac{2a-\lambda}{2\lambda},\cos(\lambda x)\right)(\sin^2(\lambda x))+2\left(\frac{\lambda(\cos^3(\lambda x))}{2}+y(\cos^2(\lambda x))\sin(\lambda x)\right.}\right.$$

6.2.18.6 [685] problem number 6

problem number 685

Added January 20, 2019.

Problem 2.6.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 - 2a\cot(ax)y + b^2 - a^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*a*Cot[a*x]*y + b^2 - a^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\tan^{-1}\left(\frac{y - a\cot(ax)}{\sqrt{b^2}}\right) - \sqrt{b^2}x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (  y^2-2*a*cot(a*x)*y + b^2-a^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{(-ia\cot(ax) + b + iy)\,\mathrm{e}^{-2ibx}}{2\,(-a\cot(ax) + ib + y)\,b}\right)$$

6.2.18.7 [686] problem number 7

problem number 686

Added January 20, 2019.

Problem 2.6.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\cot(\lambda x)w_x + a\cot(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*D[w[x, y], x] + a*Cot[mu*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2\cos(\mu y)\cos^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(lambda*x)*diff(w(x,y),x)+ a*cot(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\mu\ln\left(\frac{\cot^2(\lambda x)+1}{\cot(\lambda x)^2}\right) + \lambda\ln\left(\cos^2(\mu y)\right)}{2\lambda\mu}\right)$$

6.2.18.8 [687] problem number 8

problem number 687

Added January 20, 2019.

Problem 2.6.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\cot(\mu y)w_x + a\cot(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2\sin(\mu y)\sin^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\frac{\sqrt{(\tan^2(\mu y)+1)\left(-\frac{2}{\cos(2\lambda x)-1}\right)^{\frac{a\mu}{\lambda}}}\tan(\mu y)}{\tan^2(\mu y)+1}\right)}{a\mu}\right)$$

6.2.18.9 [688] problem number 9

problem number 688

Added January 20, 2019.

Problem 2.6.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\mu y)w_x + a\cot^2(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4\sin(\mu y)e^{\frac{a\mu(\lambda x+\cot(\lambda x))}{\lambda}}}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\lambda\ln\left(\frac{\tan^2(\mu y)}{\tan^2(\mu y)+1}\right)\sin(\lambda x)+2\left(\operatorname{arccot}\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)\sin(\lambda x)+\cos(\lambda x)-\frac{\pi\sin(\lambda x)}{2}\right)a\mu}{2a\lambda\mu\sin(\lambda x)}\right)$$

6.2.18.10 [689] problem number 10

problem number 689

Added January 20, 2019.

Problem 2.6.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(y+a)w_x+c\cot(x+b)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[y + a]*D[w[x, y], x] + c*Cot[x + b]^2*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(4\sin(a+y)e^{c(\cot(b+x)+b+x)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(y+a)*diff(w(x,y),x)+ c*cot(x+b)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(\tan(b)+\tan(x))\ln\left(\frac{1}{\sin(y)^2}\right)\tan(b)-2(\tan(b)+\tan(x))\ln\left(\frac{\cos(y)\tan(a)+\sin(y)}{\sin(y)\tan(a)}\right)\tan(b)}{2(\tan(b)+\tan(x))\tan}\right.$$

6.2.18.11 [690] problem number 11

problem number 690

Added January 20, 2019.

Problem 2.6.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x+aw_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sin(\mu y)\cos^{\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\frac{\sqrt{(\tan^2(\mu y)+1)\left(\cos^{\frac{2a\mu}{\lambda}}(\lambda x)\right)}\tan(\mu y)}{\tan^2(\mu y)+1}\right)}{a\mu}\right)$$

6.2.18.12 [691] problem number 12

problem number 691

Added January 20, 2019.

Problem 2.6.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x + a\cot(vx)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*Cot[v*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*cot(v*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln\left(\left(e^{2ivx}-1\right)^{\frac{ia\mu}{v}}\operatorname{csgn}\left(\frac{1}{\cos(\mu y)}\right)e^{a\mu x}e^{\int -\frac{\left(-2e^{2ivx}-2\right)a\mu}{\left(e^{2ivx}-1\right)\left(e^{2i\lambda x}+1\right)}dx}\sin(\mu y)\right)}{a\mu}\right)$$

6.2.19 6.5

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6.2.19.1 [692] problem number 1

problem number 692

Added January 20, 2019.

Problem 2.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(\lambda x)\cos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[lambda*x]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\sin^2(\mu y)}\csc(\mu y)\cos^{1-n}(\mu y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\mu y)\right)}{\mu(n-1)} - \frac{a\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}}{k}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ a*sin(lambda*x)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int\left(\sin^k(\lambda x)\right)dx\right) + \int\frac{\cos^{-n}(\mu y)}{a}dy\right)$$

Has unresolved integrals

6.2.19.2 [693] problem number 2

problem number 693

Added January 20, 2019.

Problem 2.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - y\tan x + a(1-a)\cot^2 x\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - y*Tan[x] + a*(1 - a)*Cot[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\left(-\sin^2(x)\right)^{\frac{1}{2}i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}}\left(i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}\cos(x)+2y\sin(x)+\cos(x)\right)}{-i\sqrt{a-1}\sqrt{a}\sqrt{\frac{1}{a-a^2}-4}\cos(x)+2y\sin(x)+\cos(x)}\right)\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-y *tan(x)+a*(1-a)*cot(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(a\cos(x)+y\sin(x))\left(\sin^{2a-1}x\right)}{-y\sin(x)+(a-1)\cos(x)}\right)$$

6.2.19.3 [694] problem number 3

problem number 694

Added January 20, 2019.

Problem 2.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2-my\tan x+b^2\cos^{2m}x\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - m*y*Tan[x] + b^2*Cos[x]^(2*m))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(x)\right)}{m+1}+\tan^{-1}\left(\frac{y\cos^{-m}(x)}{\sqrt{b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-m*y*tan(x)+b^2*cos(x)^(2*m) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{3\left(\text{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\sin^2 x\right)+\frac{(\cos(x)-1)(\cos(x)+1)(m-1)\,\text{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}+\frac{3}{2}\right],\left[\frac{5}{2}\right],s\right.}{3}\right.}{3\left(\text{hypergeom}\left(\left[\frac{1}{2},-\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\sin^2 x\right)+\frac{(\cos(x)-1)(\cos(x)+1)(m-1)\,\text{hypergeom}\left(\left[\frac{3}{2},-\frac{m}{2}+\frac{3}{2}\right],\left[\frac{5}{2}\right],si\right.}{3}\right.}\right.$$

Mathematica answer is simpler

6.2.19.4 [695] problem number 4

problem number 695

Added January 20, 2019.

Problem 2.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+my\cot x+b^2\sin^m x\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + m*y*Cot[x] + b^2*Sin[x]^m)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   diff(w(x,y),x)+ (y^2+m*y*cot(x)+b^2*sin(x)^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.19.5 [696] problem number 5

problem number 696

Added January 20, 2019.

Problem 2.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 - 2\lambda^2 \tan^2(\lambda x) - 2\lambda^2 \cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - 2*lambda^2*Tan[lambda*x]^2 - 2*lambda^2*Cot[lambda*x]^2)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (y^2-2*lambda^2*tan(lambda*x)^2-2*lambda^2*cot(lambda*x)^2)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{8\sqrt{2\cos(2\lambda x)-2}\,\lambda\cos(2\lambda x)\ln\left(\cos(\lambda x)+\sqrt{\cos^2(\lambda x)-1}\right)-4\sqrt{2\cos(2\lambda x)-2}\,y\ln}\right.$$

6.2.19.6 [697] problem number 6

problem number 697

Added January 20, 2019.

Problem 2.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) + 2ab + a(\lambda - a)\tan^2(\lambda x) + b(\lambda - b)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*(a + b) + 2*a*b + a*(lambda - a)*Tan[lambda*x]^2 + b*(l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ ( y^2+lambda*(a+b)+2*a*b+a*(lambda -a)*tan(lambda*x)^2+ b*(lambda -
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2(a(\sin^2(\lambda x)) - b(\cos^2(\lambda x)) - y\cos(\lambda x}{-4(a+b-\lambda)\lambda\,\mathrm{hypergeom}\left(\left[2, \frac{-a-b+2\lambda}{\lambda}\right], \left[-\frac{2a-5\lambda}{2\lambda}\right], \cos^2(\lambda x)\right)(\cos^2(\lambda x))(\sin^2(\lambda x)) +}\right.$$

6.2.19.7 [698] problem number 7

problem number 698

Added January 20, 2019.

Problem 2.6.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda\sin(\lambda x)y^2 + a\cos^n(\lambda x)y - a\cos^{n-1}(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Cos[lambda*x]^n*y - a*Cos[lambda*x]^(n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)* y^2 + a*cos(lambda*x)^n*y-a*cos(lambda*x)^(n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.19.8 [699] problem number 8

problem number 699

Added January 20, 2019.

Problem 2.6.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\frac{(y\cos(\lambda x) - 1)\, e^{\frac{a\cos(\lambda x)}{\lambda}}}{(y\cos(\lambda x) - 1)\, a\, \mathrm{expIntegral}\left(1, \frac{a\cos(\lambda x)}{\lambda}\right) e^{\frac{a\cos(\lambda x)}{\lambda}} - \lambda y}\right)$$

6.2.19.9 [700] problem number 9

problem number 700

Added January 20, 2019.

Problem 2.6.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(y\cos(\lambda x)-1)\,\mathrm{e}^{\frac{a\cos(\lambda x)}{\lambda}}}{(y\cos(\lambda x)-1)\,a\,\mathrm{expIntegral}\left(1,\frac{a\cos(\lambda x)}{\lambda}\right)\mathrm{e}^{\frac{a\cos(\lambda x)}{\lambda}}-\lambda y}\right)$$

6.2.19.10 [701] problem number 10

problem number 701

Added January 20, 2019.

Problem 2.6.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(Ae^{\lambda x}\cos(ay) + Be^{\mu x}\sin(ay) + Ae^{\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (A*Exp[lambda*x]*Cos[a*y] + B*Exp[mu*x]*Sin[a*y] + A*Exp[lambda*x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+ (A*exp(lambda*x)*cos(a*y) + B*exp(mu*x)*sin(a*y) + A*exp(lambda*x))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\cos(ay)+1)\,Aa\left(\int \mathrm{e}^{\frac{-Ba\,\mathrm{e}^{\mu x}+\lambda\mu x}{\mu}}dx\right) - \mathrm{e}^{-\frac{Ba\,\mathrm{e}^{\mu x}}{\mu}}\sin(ay)}{2(-\lambda+\mu)\,a\cos\left(\frac{ay}{2}\right)^2}\right)$$

6.2.19.11 [702] problem number 11

problem number 702

Added January 20, 2019.

Problem 2.6.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin^{n+1}(2x)w_x + \left(ay^2\sin^{2n}x + b\cos^{2n}x\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Sin[2*x]^(n + 1)*D[w[x, y], x] + (a*y^2*Sin[x]^(2*n) + b*Cos[x]^(2*n))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=   sin(2*x)^(n+1)*diff(w(x,y),x)+ (a*y^2*sin(x)^(2*n) + b*cos(x)^(2*n) )*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\left((-3\cos(3x)+\cos(5x))\left(\cos^{\sqrt{-ab4^{-n}+n^2}}x\right) + 2\left(\cos^{\sqrt{-ab4^{-n}+n^2}+1}x\right)\right)ay\left(\sin^{2n-\sqrt{-ab4}}\right.}{-2n\left(\sin^n(2x)\right)\sin(x)\sin(4x) + (2\cos(x) - 3\cos(3x) + \cos(5x))\,ay\,(s}\right.$$

6.2.20 7.1

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6.2.20.1 [703] problem number 1

problem number 703

Added January 20, 2019.

Problem 2.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arcsin^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcSin[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{ia\sin^{-1}(\lambda x)^k\left(\sin^{-1}(\lambda x)^2\right)^{-k}\left(\left(i\sin^{-1}(\lambda x)\right)^k \text{Gamma}\left(k+1,-i\sin^{-1}(\lambda x)\right)-\left(-i\sin^{-1}\right.\right.}{2\lambda}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-\arcsin(\lambda x)^k \arcsin(\lambda x)^{\frac{3}{2}} + \text{LommelS1}\left(k+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)\arcsin(\lambda x)\right)\sqrt{-\lambda^2 x^2 +}}{}\right.$$

6.2.20.2 [704] problem number 2

problem number 704

Added January 20, 2019.

Problem 2.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\arcsin^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcSin[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a\sin^{-1}(\lambda K[1])^k + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\arcsin(\lambda y)^k + b} dy\right)\right)$$

6.2.20.3 [705] problem number 3

problem number 705

Added January 20, 2019.

Problem 2.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\arcsin^n(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*Arcsin[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ k*arcsin(a*x + b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\arcsin(\_a+c)^n + a} d\_a\right) + x\right)$$

6.2.20.4 [706] problem number 4

problem number 706

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arcsin^k(\lambda x)\arcsin^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Arcsin[lambda*x]^k*Arcsin[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \text{Arcsin}(\mu K[1])^{-n}dK[1]-\int_1^x a\text{Arcsin}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arcsin(lambda*x)^k*arcsin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(\lambda x+1)(\lambda x-1)\left((n-1)\left(\arcsin(\lambda x)^k-\frac{\text{LommelS1}\left(k+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)\sqrt{-\lambda^2x^2+1}\,a\mu 2^k\right.}{}\right.$$

6.2.20.5 [707] problem number 5

problem number 707

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arcsin x)^n y - a^2 + a\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*Arcsin[x]^n*y - a^2 + a*lambda*Arcsin[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+ lambda*arcsin(x)^n*y -a^2 + a *lambda*arcsin(x)^n)*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int e^{-\frac{2(x-1)(x+1)\left(\frac{\left(-\arcsin(x)^n+\frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(x)\right)}{\sqrt{\arcsin(x)}}\right)\sqrt{-x^2+1}\,\lambda 2^n 2^{-n}\arcsin(x)}{2}+\left(-\frac{\lambda n 2^n 2^{-n}\,\mathrm{Lo}}{}\right.}{}}\right.}{}\right.\right.$$

6.2.20.6 [708] problem number 6

problem number 708

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arcsin x)^n y + \lambda(\arcsin y)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*Arcsin[x]^n*y + lambda*Arcsin[x]^n)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda \text{Arcsin}(K[5])^n K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda \text{Arcsin}(K[5])^n K[5]dK\right.}{K[6]^2}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+ lambda*x*arcsin(x)^n*y + lambda*arcsin(x)^n)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x}dx}dx\right) + x\, e^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x}dx} + \int e^{\int \frac{\lambda x^2 \arcsin(x)^n - 2}{x}dx}dx}{xy+1}\right)$$

6.2.20.7 [709] problem number 7

problem number 709

Added January 20, 2019.

Problem 2.7.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arcsin x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*Arcsin[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2 - lambda*arcsin(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{k+1}e^{\int \frac{\lambda x x^{k+1}\arcsin(x)^n - 2k - 2}{x}dx} - \left(yx^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k}e^{\lambda\left(\int x^{k+1}\arcsin(x)^n dx\right)}}{x^2}dx\right)}{yx^{k+1} - 1}\right)$$

6.2.20.8 [710] problem number 8

problem number 710

Added January 20, 2019.

Problem 2.7.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + ay + ab - b^2\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 + a*y + a*b - b^2*lambda*Arcsin[x]^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + a*y+ a*b -b^2 * lambda*arcsin(x)^n)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(b+y)\,\lambda\left(\int \arcsin(x)^n\, e^{\frac{(x-1)\left(-2\left(\arcsin(x)^n-\frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(x)\right)}{\sqrt{\arcsin(x)}}\right)\sqrt{-x^2+1}\,b\lambda 2^n 2^{-n}\arcsin(x)+\left(-2b\right.}{}}\right.}{}\right.$$

6.2.20.9 [711] problem number 9

problem number 711

Added January 29, 2019.

Problem 2.7.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 - b\lambda x^m(\arcsin x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 - b*lambda*x^m*ArcSin[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 - b*lambda*x^m*arcsin(x)^n*y+b*m*x^(m-1) )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.20.10 [712] problem number 10

problem number 712

Added January 29, 2019.

Problem 2.7.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m}(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcSin[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + b*m*x^(m-1) - lambda*b^2*x^(2*m)*arcsin(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.20.11 [713] problem number 11

problem number 713

Added January 29, 2019.

Problem 2.7.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcSin[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{1}{2}i\lambda\left(i\sin^{-1}(x)\right)^n \sin^{-1}(x)^n \left(\sin^{-1}(x)^2\right)^{-n} \text{Gamma}\left(n+1, -i\sin^{-1}(x)\right) + \frac{1}{2}i\lambda\left(-i\sin^{-1}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*(y - a*x^m -b)^2 + a*m*x^(m-1)  )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{-(ax^m + b - y)\,\lambda n x 2^n 2^{-n}\,\text{LommelS1}\left(n+\frac{1}{2}, \frac{3}{2}, \arcsin(x)\right)\arcsin(x) - (ax^m + b - y)}{}\right.$$

6.2.20.12 [714] problem number 12

problem number 714

Added January 29, 2019.

Problem 2.7.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arcsin x)^n y^2 + ky + \lambda b^2 x^{2k}(\arcsin x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcSin[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right) - \sqrt{b^2}\int_1^x \lambda \sin^{-1}(K[1])^n K[1]^{k-1} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 +k*y+ lambda*b^2*x^(2*k)*arcsin(x)^n  )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\arcsin(x)^n\,dx\right) - \arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.21 7.2

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6.2.21.1 [715] problem number 1

problem number 715

Added January 29, 2019.

Problem 2.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arccos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcCos[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{a\cos^{-1}(\lambda x)^k\left(-i\cos^{-1}(\lambda x)\right)^{-k}\text{Gamma}\left(k+1,-i\cos^{-1}(\lambda x)\right)+a\left(i\cos^{-1}(\lambda x)\right)^{-k}\cos^{-}}{2\lambda}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arccos(lambda*x)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-k}\,\text{LommelS1}\left(k+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(k+2)}+\frac{\sqrt{-\lambda^2x^2+1}\,2^{-k}\arccos(\lambda x)^{k+1}}{\sqrt{\pi}\,(k+2)}-\right.}{\lambda}\right.$$

6.2.21.2 [716] problem number 2

problem number 716

Added January 29, 2019.

Problem 2.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a \arccos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcCos[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a\cos^{-1}(\lambda K[1])^k + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arccos(lambda*y)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a\arccos(\lambda y)^k + b} dy\right)\right)$$

6.2.21.3 [717] problem number 3

problem number 717

Added January 29, 2019.

Problem 2.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + k \arccos^n(ax + by + c) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcCos[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k*arccos(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\arccos{(b\_a+c)}^n + a} d\_a\right) + x\right)$$

6.2.21.4 [718] problem number 4

problem number 718

Added January 29, 2019.

Problem 2.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arccos^k(\lambda x)\arccos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcCos[lambda*x]^k*ArcCos[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(\cos^{-1}(\lambda x)^2)^{-k}\left(-a(i\cos^{-1}(\lambda x))^k\cos^{-1}(\lambda x)^k\,\text{Gamma}\,(k+1,-i\cos^{-1}(\lambda x)) - a(-i\,\text{co}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arccos(lambda*x)^k*arccos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(n-2)\left(-\frac{2(k+2)2^{-k-1}\,\mathrm{LommelS1}\left(k+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)}{\sqrt{\arccos(\lambda x)}}+\left(\mathrm{LommelS1}\left(k+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{a}\right.\right.}{}\right.$$

6.2.21.5 [719] problem number 5

problem number 719

Added January 29, 2019.

Problem 2.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arccos x)^n y - a^2 + a\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcCos[x]^n*y - a^2 + a*lambda*ArcCos[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+lambda*arccos(x)^n*y- a^2 + a*lambda*arccos(x)^n )*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a+y)\left(\int -\frac{\left(-\left(-(n+2)n\,\mathrm{LommelS1}\left(n-\frac{1}{2},\frac{1}{2},\arccos(x)\right)\arccos(x)^2-\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x)^2+(n\right.\right.}{}\right.}{}\right.$$

6.2.21.6 [720] problem number 6

problem number 720

Added January 29, 2019.

Problem 2.7.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arccos x)^n y + \lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*ArcCos[x]^n*y + a*lambda*ArcCos[x]^n)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( y^2+lambda*x*arccos(x)^n*y + a*lambda*arccos(x)^n )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.21.7 [721] problem number 7

problem number 721

Added January 29, 2019.

Problem 2.7.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arccos x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCos[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2 -lambda*arccos(x)^n*(x^(k+1)*y-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{k+1}\mathrm{e}^{\int \frac{\lambda x x^{k+1} \arccos(x)^n - 2k - 2}{x} dx} - \left(y x^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k}\mathrm{e}^{\lambda\left(\int x^{k+1} \arccos(x)^n dx\right)}}{x^2} dx\right)}{y x^{k+1} - 1}\right)$$

6.2.21.8 [722] problem number 8

problem number 722

Added January 29, 2019.

Problem 2.7.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + ay + ab - b^2\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCos[x]^n)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ a*y+ a*b - b^2*lambda*arccos(x)^n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-b-y)\left(\int \frac{\left(2\left(-(n+2)n\,\mathrm{LommelS1}\left(n-\frac{1}{2},\frac{1}{2},\arccos(x)\right)\arccos(x)^2-\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x)^2+(n-\right.\right.}{}\right.}{}\right.$$

6.2.21.9 [723] problem number 9

problem number 723

Added January 29, 2019.

Problem 2.7.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 - b\lambda x^m (\arccos x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 - b*lambda*x^m*ArcCos[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2- b*lambda*x^m*arccos(x)^n*y + b*m*x^(m-1) )*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.21.10 [724] problem number 10

problem number 724

Added January 29, 2019.

Problem 2.7.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCos[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccos(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.21.11 [725] problem number 11

problem number 725

Added January 29, 2019.

Problem 2.7.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arccos x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCos[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{1}{2}\left(\lambda\left(i\cos^{-1}(x)\right)^n \cos^{-1}(x)^n \left(\cos^{-1}(x)^2\right)^{-n} \text{Gamma}\left(n+1, -i\cos^{-1}(x)\right) + \lambda\left(-i\cos^{-1}(x\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*(y- a*x^m-b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(\arccos(x)^n \arccos(x)^{\frac{3}{2}} - \text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(x)\right)\arccos(x) + (n+2)\,\text{Lomm}\right.}{}\right.$$

6.2.21.12 [726] problem number 12

problem number 726

Added January 29, 2019.

Problem 2.7.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arccos x)^n y^2 + ky + \lambda b^2 x^{2k}(\arccos x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCos[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right) - \sqrt{b^2}\int_1^x \lambda\cos^{-1}(K[1])^n K[1]^{k-1} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ k*y + lambda*b^2*x^(2*k)*arccos(x)^n )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\arccos(x)^n\,dx\right) - \arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

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6.2.22.1 [727] problem number 1

problem number 727

Added January 29, 2019.

Problem 2.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arctan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcTan[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \int_1^x \left(a \tan^{-1}(\lambda K[1])^k + b\right) dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arctan(lambda*x)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \left(\int a \arctan(\lambda x)^k \, dx\right)\right)$$

6.2.22.2 [728] problem number 2

problem number 728

Added January 29, 2019.

Problem 2.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arctan^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*ArcTan[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \frac{1}{a \tan^{-1}(\lambda K[1])^k + b} dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( a*arctan(lambda*y)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{a \arctan(\lambda y)^k + b} dy\right)\right)$$

6.2.22.3 [729] problem number 3

problem number 729

Added January 29, 2019.

Problem 2.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + k \arctan^n(ax + by + c) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcTan[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k *arctan(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk \arctan\left(b\_a + c\right)^n + a} d\_a\right) + x\right)$$

6.2.22.4 [730] problem number 4

problem number 730

Added January 29, 2019.

Problem 2.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a \arctan^k(\lambda x) \arctan^n(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcTan[lambda*x]^k*ArcTan[mu*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \tan^{-1}(\mu K[1])^{-n}dK[1]-\int_1^x a\tan^{-1}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arctan(lambda*x)^k*arctan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\left(\int \arctan\left(\lambda x\right)^k dx\right)+\int \frac{\arctan\left(\mu y\right)^{-n}}{a}dy\right)$$

6.2.22.5 [731] problem number 5

problem number 731

Added January 29, 2019.

Problem 2.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(y^2+\lambda(\arctan x)^n y-a^2+a\lambda(\arctan x)^n\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcTan[x]^n*y - a^2 + a*lambda*ArcTan[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 + lambda*arctan(x)^n*y -a^2 + a *lambda*arctan(x)^n )*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int e^{-\left(\int(-\lambda \arctan(x)^n+2a)dx\right)}dx\right) - e^{-\left(\int(-\lambda \arctan(x)^n+2a)dx\right)}}{a+y}\right)$$

6.2.22.6 [732] problem number 6

problem number 732

Added January 29, 2019.

Problem 2.7.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arctan x)^n y + \lambda(\arctan x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*x*ArcTan[x]^n*y + lambda*ArcTan[x]^n)*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda \tan^{-1}(K[5])^n K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda \tan^{-1}(K[5])^n K[5]dK\right.}{K[6]^2}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 + lambda*x*arctan(x)^n*y + lambda*arctan(x)^n )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x} dx} dx\right) + x\, e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x} dx} + \int e^{\int \frac{\lambda x^2 \arctan(x)^n - 2}{x} dx} dx}{xy+1}\right)$$

6.2.22.7 [733] problem number 7

problem number 733

Added Feb. 1, 2019.

Problem 2.7.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arctan x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcTan[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2 - lambda*arctan(x)^n*(x^(k+1)*y-1)  )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{k+1}e^{\int \frac{\lambda x x^{k+1} \arctan(x)^n - 2k - 2}{x} dx} - \left(y x^{k+1} - 1\right)(k+1)\left(\int \frac{x^{-k} e^{\lambda\left(\int x^{k+1} \arctan(x)^n dx\right)}}{x^2} dx\right)}{y x^{k+1} - 1}\right)$$

6.2.22.8 [734] problem number 8

problem number 734

Added Feb. 1, 2019.

Problem 2.7.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arctan x)^n + ay + ab - b^2\lambda(\arctan x)^n n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n + a*y + a*b - b^2*lambda*ArcTan[x]^n*n)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y e^{-ax} - \int_1^x e^{-aK[1]}\left(\left(\lambda - b^2\lambda n\right)\tan^{-1}(K[1])^n + ab\right) dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda* arctan(x)^n +a*y+ a*b - b^2*lambda*arctan(x)^n*n  )*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} + \int -\left(ab + \left(-b^2 n + 1\right)\lambda \arctan\left(x\right)^n\right)\mathrm{e}^{-ax}dx\right)$$

6.2.22.9 [735] problem number 9

problem number 735

Added Feb. 1, 2019.

Problem 2.7.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(arctanx)^n y^2 - b\lambda x^m (\arctan x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 - b*lambda*x^m*ArcTan[x]^n*y + b*m*x^(m - 1))
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 - b*lambda*x^m*arctan(x)^n*y+ b*m*x^(m-1))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.22.10 [736] problem number 10

problem number 736

Added Feb. 1, 2019.

Problem 2.7.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(arctanx)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcTan[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 +b*m*x^(m-1) - lambda*b^2*x^(2*m)*arctan(x)^n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.22.11 [737] problem number 11

problem number 737

Added Feb. 1, 2019.

Problem 2.7.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(arctanx)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcTan[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^x \lambda \tan^{-1}(K[2])^n dK[2]-\frac{1}{ax^m+b-y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*(y-a*x^m -b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(ax^m+b-y)\left(\int \lambda \arctan(x)^n\,dx\right)-1}{ax^m+b-y}\right)$$

6.2.22.12 [738] problem number 12

problem number 738

Added Feb. 1, 2019.

Problem 2.7.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(\lambda(\arctan x)^n y^2+ky+\lambda b^2x^{2k}(\arctan x)^n\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcTan[x]^n)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda \tan^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+(lambda*arctan(x)^n*y^2+k*y+lambda*b^2*x^(2*k)*arctan(x)^n  )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\arctan(x)^n\,dx\right) - \arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.23 7.4

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6.2.23.1 [739] problem number 1

problem number 739

Added Feb. 1, 2019.

Problem 2.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\,\mathrm{arccot}^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCot[lambda*x]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x\left(\lambda\cot^{-1}(\lambda K[1])^k+b\right)dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-bx+y-\left(\int\lambda\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^k dx\right)\right)$$

6.2.23.2 [740] problem number 2

problem number 740

Added Feb. 1, 2019.

Problem 2.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(a\operatorname{arccot}^k(\lambda y)+b\right)w_y=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*ArcCot[lambda*y]^k + b)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y\frac{1}{\lambda\cot^{-1}(\lambda K[1])^k+b}dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \left(\int \frac{1}{\lambda\left(-\arctan\left(\lambda y\right) + \frac{\pi}{2}\right)^{k} + b} dy\right)\right)$$

6.2.23.3 [741] problem number 3

problem number 741

Added Feb. 1, 2019.

Problem 2.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\operatorname{arccot}^n(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + k*ArcCot[a*x + b*y + c]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+k*arccot(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bk\left(-\arctan\left(\_a + c\right) + \frac{\pi}{2}\right)^{n} + a} d\_a\right) + x\right)$$

6.2.23.4 [742] problem number 4

problem number 742

Added Feb. 1, 2019.

Problem 2.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k \operatorname{arccot}^k(\lambda x) \operatorname{arccot}^n(\mu y) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*ArcCot[lambda*x]^k*ArcCot[lambda*y]^n*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(\int_1^y \cot^{-1}(\lambda K[1])^{-n} dK[1] - \int_1^x a \cot^{-1}(\lambda K[2])^k dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*arccot(lambda*x)^k*arccot(lambda*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \left(-\arctan(\lambda x) + \frac{\pi}{2}\right)^k dx\right) + \int \frac{\left(-\arctan(\lambda y) + \frac{\pi}{2}\right)^{-n}}{a} dy\right)$$

6.2.23.5 [743] problem number 5

problem number 743

Added Feb. 1, 2019.

Problem 2.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\operatorname{arccot} x)^n y - a^2 + a\lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + lambda*ArcCot[x]^n*y - a^2 + a*lambda*ArcCot[x]^n)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+lambda*arccot(x)^n*y - a^2 +a*lambda*arccot(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-a-y)\left(\int \mathrm{e}^{-\left(\int\left(-\lambda\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}+2a\right)dx\right)}dx\right) - \mathrm{e}^{-\left(\int\left(-\lambda\left(-\arctan(x)+\frac{\pi}{2}\right)^{n}+2a\right)dx\right)}}{a+y}\right)$$

6.2.23.6 [744] problem number 6

problem number 744

Added Feb. 1, 2019.

Problem 2.7.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\operatorname{arccot} x)^n y + \lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (y^2 + lambda*x*ArcCot[x]^n*y + lambda*ArcCot[x]^n)*D[w[x, y], y] == 0
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\exp\left(-\int_1^x -\lambda\cot^{-1}(K[5])^n K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -\lambda\cot^{-1}(K[5])^n K[5]dK[\right.}{K[6]^2}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2+lambda*x*arccot(x)^n*y +lambda*arccot(x)^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} dx\right) + x\, e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} + \int e^{\int \frac{\lambda x^2 \operatorname{arccot}(x)^n - 2}{x} dx} dx}{xy+1}\right)$$

6.2.23.7 [745] problem number 7

problem number 745

Added Feb. 1, 2019.

Problem 2.7.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\operatorname{arccot} x)^n (x^{k+1} y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCot[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y]
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2- lambda*arccot(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{k+1}e^{\int \frac{\lambda x x^{k+1}\left(-\arctan(x)+\frac{\pi}{2}\right)^n-2k-2}{x}dx} - \left(yx^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}e^{\lambda\left(\int x^{k+1}\left(-\arctan(x)+\frac{\pi}{2}\right)^n dx\right)}}{x^2}dx\right.}{yx^{k+1}-1}\right.$$

6.2.23.8 [746] problem number 8

problem number 746

Added Feb. 1, 2019.

Problem 2.7.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + ay + ab - b^2\lambda(\operatorname{arccot} x)^n n\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCot[x]^n*n)*D[w[x,
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2+a*y + a*b -b^2*lambda*arccot(x)^n*n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.23.9 [747] problem number 9

problem number 747

Added Feb. 1, 2019.

Problem 2.7.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 - b\lambda x^m (\operatorname{arccot} x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 - b*lambda*x^m*ArcCot[x]^n*y + b*m*x^(m - 1))*
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2- b*lambda*x^m*arccot(x)^n*y+ b*m*x^(m-1)  )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.23.10 [748] problem number 10

problem number 748

Added Feb. 1, 2019.

Problem 2.7.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\operatorname{arccot} x^n)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCot[x]
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccot(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.2.23.11 [749] problem number 11

problem number 749

Added Feb. 1, 2019.

Problem 2.7.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y] =
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x \lambda \cot^{-1}(K[2])^n dK[2] - \frac{1}{ax^m + b - y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*(y-a*x^m-b)^2+a*m*x^(m-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(ax^m+b-y)\left(\int \lambda \operatorname{arccot}(x)^n\,dx\right)-1}{ax^m+b-y}\right)$$

6.2.23.12 [750] problem number 12

problem number 750

Added Feb. 1, 2019.

Problem 2.7.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + ky + \lambda b^2 x^{2k}(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCot[x]^n)*D[w[
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda\cot^{-1}(K[1])^n K[1]^{k-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ k*y+ lambda*b^2*x^(2*k)*arccot(x)^n )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b\lambda\left(\int x^{k-1}\left(-\arctan(x)+\frac{\pi}{2}\right)^n dx\right)-\arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

6.2.24 8.1

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6.2.24.1 [751] problem number 1

problem number 751

Added Feb. 4, 2019.

Problem 2.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (f(x)y + g(x))\, w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y], x] + (f[x]*y + g[x])*D[w[x, y], y] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right)g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y+g(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\left(\int f(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f(x)dx\right)} g(x)\,dx\right)\right)$$

6.2.24.2 [752] problem number 2

problem number 752

Added Feb. 4, 2019.

Problem 2.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y + g(x)y^k\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y + g[x]*y^k)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2] + y^{1-k}\exp\left((k-1)\int_1^x f\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y+g(x)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f(x)dx\right)} g(x)\,dx\right)\right)$$

6.2.24.3 [753] problem number 3

problem number 753

Added Feb. 4, 2019.

Problem 2.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + f(x)y - a^2 - af(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + f[x]*y - a^2 - a*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+f(x)*y -a^2 -a*f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{(a - y)\left(\int e^{2ax+\int f(x)dx}dx\right) - e^{2ax+\int f(x)dx}}{a - y}\right)$$

6.2.24.4 [754] problem number 4

problem number 754

Added Feb. 4, 2019.

Problem 2.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + xf(x)y + f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 + x*f[x]*y + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\exp\left(-\int_1^x -f(K[5])K[5]dK[5]\right)}{x^2y+x} - \int_1^x \frac{\exp\left(-\int_1^{K[6]} -f(K[5])K[5]dK[5]\right)}{K[6]^2}dK[6]\right)\right\}\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( y^2+x*f(x)*y + f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\left(\int e^{\int \frac{x^2f(x)-2}{x}dx}dx\right) + x\,e^{\int \frac{x^2f(x)-2}{x}dx} + \int e^{\int \frac{x^2f(x)-2}{x}dx}dx}{xy+1}\right)$$

6.2.24.5 [755] problem number 5

problem number 755

Added Feb. 4, 2019.

Problem 2.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^ky^2 - x^{k+1}f(x)y + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - ((k + 1)*x^k*y^2 - x^(k + 1)*f[x]*y + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2-x^(k+1)*f(x)*y+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^{k+1}e^{\int \frac{xx^{k+1}f(x)-2k-2}{x}dx} + \left(yx^{k+1}-1\right)(k+1)\left(\int \frac{x^{-k}e^{\int x^{k+1}f(x)dx}}{x^2}dx\right)}{yx^{k+1}-1}\right)$$

6.2.24.6 [756] problem number 6

problem number 756

Added Feb. 4, 2019.

Problem 2.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + ay - ab - b^2 f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*y - a*b - b^2*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2+a*y-a*b- b^2*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(b-y)\left(\int e^{ax+2b\left(\int f(x)dx\right)} f(x)\,dx\right) - e^{ax+2b\left(\int f(x)dx\right)}}{b-y}\right)$$

6.2.24.7 [757] problem number 7

problem number 757

Added Feb. 4, 2019.

Problem 2.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f[x]y^2 - ax^n f[x]y + anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x^n*f[x]*y + a*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*f(x)*y+a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.24.8 [758] problem number 8

problem number 758

Added Feb. 4, 2019.

Problem 2.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + anx^{n-1} - a^2x^{2n}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*n*x^(n - 1) - a^2*x^(2*n)*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.24.9 [759] problem number 9

problem number 759

Added Feb. 4, 2019.

Problem 2.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + g(x)y - a^2f(x) - ag(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y - a^2*f[x] - a*g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)* y-a^2*f(x)-a*g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(a-y)\left(\int e^{2a\left(\int f(x)dx\right)+\int g(x)dx} f(x)\,dx\right) - e^{2a\left(\int f(x)dx\right)+\int g(x)dx}}{a-y}\right)$$

6.2.24.10 [760] problem number 10

problem number 760

Added Feb. 4, 2019.

Problem 2.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y + anx^{n-1} - ax^n g(x) - a^2x^{2n}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*n*x^(n - 1) - a*x^n*g[x] - a^2*x^(2*n)*f[x])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)*y+a*n*x^(n-1) - a*x^n*g(x)-a^2*x^(2*n)*f(x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.24.11 [761] problem number 11

problem number 761

Added Feb. 4, 2019.

Problem 2.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - ax^n g(x)y + anx^{n-1} + a^2x^{2n}(g(x) - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x^n*g*x*y + a*n*x^(n - 1) + a^2*x^(2*n)*(g*x - f*x))*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*g(x)*y+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.24.12 [762] problem number 12

problem number 762

Added Feb. 4, 2019.

Problem 2.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(f(x)y^2 + ny + ax^{2n}f(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (f[x]*y^2 + n*y + a*x^(2*n)*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{yx^{-n}}{\sqrt{a}}\right) - \sqrt{a}\int_1^x f(K[1])K[1]^{n-1}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+( f(x)*y^2+n*y+a*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int x^{n-1}f(x)\,dx\right) - \arctan\left(\frac{yx^{-n}}{\sqrt{a}}\right)\right)$$

6.2.24.13 [763] problem number 13

problem number 763

Added Feb. 4, 2019.

Problem 2.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x^{2n} f(x) y^2 + (a x^n f(x) - n) y + b f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x^(2*n)*f[x]*y^2 + (a*x^n*f[x] - n)*y + b*f[x])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1 \left(-\int_1^x \frac{b f(K[5]) \sqrt{\frac{K[5]^{2n}}{b}}}{K[5]} dK[5] - \frac{2\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\left(\sqrt{\frac{a^2}{b}} - 2y\sqrt{\frac{x^{2n}}{b}}\right)}{\sqrt{4b-a^2}}\right)}{\sqrt{4b-a^2}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x* diff(w(x,y),x)+( x^(2*n)* f(x)*y^2+(a*x^n*f(x)-n)*y+b*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{2\left(a \operatorname{arctanh}\left(\frac{(2yx^n+a)a}{\sqrt{(a^2-4b)a^2}}\right) + \frac{\sqrt{(a^2-4b)a^2}\left(\int \frac{x^n f(x)}{x} dx\right)}{2}\right) a}{\sqrt{(a^2-4b)\,a^2}}\right)$$

6.2.25 8.2

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6.2.25.1 [764] problem number 1

problem number 764

Added Feb. 4, 2019.

Problem 2.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + ae^{\lambda x}f(x)y + \lambda f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*Exp[lambda*x]*f[x]*y + lambda*f[x])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\lambda \exp\left(-\int_1^{e^{\lambda x}} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda} dK[5] - \lambda x\right)}{aye^{\lambda x} + \lambda} - \int_1^{e^{\lambda x}} \frac{\exp\left(-\int_1^{K[6]} -\frac{af\left(\frac{\log(K[5])}{\lambda}\right)}{\lambda} dK[5]\right)}{K[6]^2} d\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( a*exp(lambda*x)*y^2 + a*exp(lambda*x)*f(x)*y+lambda*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(ay\,\mathrm{e}^{\lambda x} + \lambda\right)\lambda\,\mathrm{e}^{\lambda x}}{ay\left(\int \mathrm{e}^{a\left(\int \mathrm{e}^{\lambda x} f(x)dx\right) - \lambda x} dx\right)\mathrm{e}^{2\lambda x} + \lambda\left(\int \mathrm{e}^{a\left(\int \mathrm{e}^{\lambda x} f(x)dx\right) - \lambda x} dx\right)\mathrm{e}^{\lambda x} + \mathrm{e}^{a\left(\int \mathrm{e}^{\lambda x} f(x)dx\right)}}\right)$$

6.2.25.2 [765] problem number 2

problem number 765

Added Feb. 4, 2019.

Problem 2.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x} f(x)y + a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*f[x]*y + a*lambda*Exp[lambda*x])*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*exp(lambda*x)*f(x)*y+a*lambda*exp(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.25.3 [766] problem number 3

problem number 766

Added Feb. 4, 2019.

Problem 2.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + a\lambda e^{\lambda x} - a^2 e^{2\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x]*f[x])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.2.25.4 [767] problem number 4

problem number 767

Added Feb. 4, 2019.

Problem 2.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + \lambda y + ae^{2\lambda x} f(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + lambda*y + a*Exp[2*lambda*x]*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y) \to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x}}{\sqrt{a}}\right) - \sqrt{a}\int_1^x e^{\lambda K[1]} f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+lambda*y+ a*exp(2*lambda*x)* f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int e^{\lambda x} f(x)\,dx\right) - \arctan\left(\frac{y\,e^{-\lambda x}}{\sqrt{a}}\right)\right)$$

6.2.25.5 [768] problem number 5

problem number 768

Added Feb. 4, 2019.

Problem 2.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - (ae^{\lambda x} + b)f(x)y + a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - (a*Exp[lambda*x] + b)*f[x]*y + a*lambda*Exp[lambda*x])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2-(a*exp(lambda*x)+b)*f(x)*y+a *lambda*exp(lambda*x))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.25.6 [769] problem number 6

problem number 769

Added Feb. 4, 2019.

Problem 2.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(e^{\lambda x} f(x) y^2 + (a f(x) - \lambda) y + b e^{-\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Exp[lambda*x]*f[x]*y^2 + (a*f[x] - lambda)*y + b*Exp[-(lambda*x)]*f[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  exp(lambda*x)*f(x)*y^2+(a*f(x)-lambda)*y+b*exp(-lambda*x)*f(x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \_F1\left(-\frac{2\left(a \operatorname{arctanh}\left(\frac{(2y\,\mathrm{e}^{\lambda x}+a)a}{\sqrt{(a^2-4b)a^2}}\right) + \frac{\sqrt{(a^2-4b)a^2}\,(\int f(x)dx)}{2}\right)a}{\sqrt{(a^2-4b)\,a^2}}\right)$$

6.2.25.7 [770] problem number 7

problem number 770

Added Feb. 4, 2019.

Problem 2.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x) y^2 + g(x) y + a\lambda e^{\lambda x} - a e^{\lambda x} g(x) - a^2 e^{2\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*lambda*Exp[lambda*x] - a*Exp[lambda*x]*g[x] -
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+ g(x)*y+a*lambda*exp(lambda*x) -a*exp(lambda*x)*g(x)-a^2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.25.8 [771] problem number 8

problem number 771

Added Feb. 7, 2019.

Problem 2.8.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x}g(x)y + a\lambda e^{\lambda x} + a^2e^{2\lambda x}(g(x) - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*g[x]*y + a*lambda*Exp[lambda*x] + a^2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2- a*exp(lambda*x)*g(x)*y + a*lambda*exp(lambda*x) +a^2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.25.9 [772] problem number 9

problem number 772

Added Feb. 7, 2019.

Problem 2.8.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + 2a\lambda x e^{\lambda x^2} - a^2 f(x) e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*f[x]*Exp[2*lambda*x^2]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*f(x)*exp(2*lambda*x^2))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.25.10 [773] problem number 10

problem number 773

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 + 2\lambda x y + a f(x) e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + 2*lambda*x*y + a*f[x]*Exp[2*lambda*x^2])*D[w[x, y], y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x^2}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x e^{\lambda K[1]^2}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(  f(x)*y^2+2*lambda*x*y+ a*f(x)*exp(2*lambda*x^2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y)=\_F1\left(\sqrt{a}\left(\int \mathrm{e}^{\lambda x^2}f(x)\,dx\right)-\arctan\left(\frac{y\,\mathrm{e}^{-\lambda x^2}}{\sqrt{a}}\right)\right)$$

6.2.25.11 [774] problem number 11

problem number 774

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f(x)e^{\lambda y}+g(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*Exp[lambda*y] + g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(   f(x)*exp(lambda*y) + g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int g(x)dx\right)} f(x)\,dx\right) - e^{\left(-y+\int g(x)dx\right)\lambda}}{\lambda}\right)$$

6.2.26 8.3

Local contents

6.2.26.1 [775] problem number 1 1470
6.2.26.2 [776] problem number 2 1471
6.2.26.3 [777] problem number 3 1472

6.2.26.1 [775] problem number 1

problem number 775

Added Feb. 7, 2019.

Problem 2.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda \sinh(\lambda x) - a^2 f(x)\sinh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sinh[lambda*x] - a^2*f[x]*Sinh[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a^2*f(x) + a*lambda*sinh(lambda*x) - a^2*f(x)*sinh(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.26.2 [776] problem number 2

problem number 776

Added Feb. 7, 2019.

Problem 2.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a(af(x) + \lambda)\tanh^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Tanh[lambda*x]^2 + a*lambda)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*tanh(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.26.3 [777] problem number 3

problem number 777

Added Feb. 7, 2019.

Problem 2.8.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a(af(x) + \lambda)\coth^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Coth[lambda*x]^2 + a*lambda)*D[w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*coth(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.27 8.4

Local contents

6.2.27.1 [778] problem number 1 1473
6.2.27.2 [779] problem number 2 1474
6.2.27.3 [780] problem number 3 1474
6.2.27.4 [781] problem number 4 1475

6.2.27.1 [778] problem number 1

problem number 778

Added Feb. 7, 2019.

Problem 2.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(ay^2 \ln x - axy(\ln x - 1)f(x) + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (a*y^2*Log[x] - a*x*y*(Log[x] - 1)*f[x] + f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-(   a*y^2*ln(x) -a*x*y* (ln(x)-1)*f(x)+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-(\ln(x)-1)\,x\,e^{\int \frac{ax^2 f(x)\ln(x)^2 + ax^2 f(x) + \left(-2ax^2 f(x) - 2\right)\ln(x)}{(\ln(x)-1)x}dx} + (axy\ln(x) - axy - 1)\left(\int e^{\left(\int \frac{xf(}{\ln(x)}\right.}\right.}{(axy\ln(x) - axy - 1)\,a}\right.$$

6.2.27.2 [779] problem number 2

problem number 779

Added Feb. 7, 2019.

Problem 2.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - ax(\ln x)f(x)y + a\ln x + a\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*x*Log[x]*f[x]*y + a*Log[x] + a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+(  f(x)* y^2 -a*x*ln(x)*f(x)*y+a*ln(x)+a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.27.3 [780] problem number 3

problem number 780

Added Feb. 7, 2019.

Problem 2.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(f(x)y^2 + a - a^2(\ln x)^2 f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (f[x]*y^2 + a - a^2*Log[x]^2*f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+( f(x)*y^2 +a -a^2* ln(x)^2 *f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.27.4 [781] problem number 4

problem number 781

Added Feb. 7, 2019.

Problem 2.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left((y + a\ln x)^2 f(x) - a\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + ((y + a*Log[x])^2*f[x] - a)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x \frac{f(K[2])}{K[2]}dK[2] + \frac{1}{a\log(x)+y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+( (y+a *ln(x))^2*f(x)-a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{(a\ln(x)+y)\left(\int \frac{f(x)}{x}dx\right)+1}{a\ln(x)+y}\right)$$

6.2.28 8.5

Local contents

6.2.28.1 [782] problem number 1 1476
6.2.28.2 [783] problem number 2 1477
6.2.28.3 [784] problem number 3 1478
6.2.28.4 [785] problem number 4 1478
6.2.28.5 [786] problem number 5 1479

6.2.28.1 [782] problem number 1

problem number 782

Added Feb. 7, 2019.

Problem 2.8.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda\sin(\lambda x)y^2 + f(x)\cos(\lambda x)y - f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + f[x]*Cos[lambda*x]*y - f[x])*D[w[x, y], y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+( lambda*sin(lambda*x)*y^2 + f(x)*cos(lambda*x)*y-f(x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y\,\mathrm{c}}{-y\left(\int -\lambda\,\mathrm{e}^{\int \frac{-2\lambda\left(\cos^2(\lambda x)\right)+\sqrt{-\frac{\cos(2\lambda x)}{2}+\frac{1}{2}\left(\cos^2(\lambda x)\right)f(x)+2\lambda}}{\cos(\lambda x)\sin(\lambda x)}dx}\sin(\lambda x)\,dx\right)\cos(\lambda x)+\cos(\lambda x)\,\mathrm{e}^{\int -}}\right.$$

6.2.28.2 [783] problem number 2

problem number 783

Added Feb. 7, 2019.

Problem 2.8.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda\sin(\lambda x) + a^2 f(x)\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sin[lambda*x] + a^2*f[x]*Sin[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.28.3 [784] problem number 3

problem number 784

Added Feb. 7, 2019.

Problem 2.8.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda \cos(\lambda x) + a^2 f(x) \cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Cos[lambda*x] + a^2*f[x]*Cos[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.28.4 [785] problem number 4

problem number 785

Added Feb. 7, 2019.

Problem 2.8.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda) \tan^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Tan[lambda*x]^2 + a*lambda)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*tan(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.28.5 [786] problem number 5

problem number 786

Added Feb. 7, 2019.

Problem 2.8.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda)\cot^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Cot[lambda*x]^2 + a*lambda)*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*cot(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.29 8.6

Local contents

6.2.29.1 [787] problem number 1 1480
6.2.29.2 [788] problem number 2 1481
6.2.29.3 [789] problem number 3 1482
6.2.29.4 [790] problem number 4 1483
6.2.29.5 [791] problem number 5 1484
6.2.29.6 [792] problem number 6 1484
6.2.29.7 [793] problem number 7 1485
6.2.29.8 [794] problem number 8 1486
6.2.29.9 [795] problem number 9 1487
6.2.29.10 [796] problem number 10 1488
6.2.29.11 [797] problem number 11 1489
6.2.29.12 [798] problem number 12 1490

6.2.29.1 [787] problem number 1

problem number 787

Added Feb. 7, 2019.

Problem 2.8.6.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - f(x)g(x)y + g'(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 - f[x]*g[x]*y + Derivative[1][g][x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+( f(x)*y^2 -f(x)*g(x)*y+ diff(g(x),x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.29.2 [788] problem number 2

problem number 788

Added Feb. 7, 2019.

Problem 2.8.6.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left(f'(x)y^2 - f(x)g(x)y + g(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] - (Derivative[1][f][x]*y^2 - f[x]*g[x]*y + g[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)-( diff(f(x),x)*y^2 -f(x)*g(x)*y+ g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{y\left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{\int f(x)g(x)dx}}{f(x)^2}dx\right)f(x) - e^{\int \frac{f(x)^2 g(x) - 2\frac{d}{dx}f(x)}{f(x)}dx} f(x) - \left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{\int f(x)g(x)dx}}{f(x)^2}dx\right)}{yf(x) - 1}\right)$$

6.2.29.3 [789] problem number 3

problem number 789

Added Feb. 7, 2019.

Problem 2.8.6.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(g(x)(y - f(x))^2 + f'(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (g[x]*(y - f[x])^2 + Derivative[1][f][x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^x g(K[2])dK[2] + \frac{1}{y - f(x)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(g(x)*(y-f(x))^2 + diff(f(x),x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{(y-f(x))\left(\int g(x)\,dx\right)+1}{y-f(x)}\right)$$

6.2.29.4 [790] problem number 4

problem number 790

Added Feb. 7, 2019.

Problem 2.8.6.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\frac{f'(x)}{g(x)}y^2 - \frac{g'(x)}{f(x)}\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((Derivative[1][f][x]*y^2)/g[x] - Derivative[1][g][x]/f[x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(f(x),x)/g(x)* y^2 - diff(g(x),x)/f(x)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\left(-yf(x)^2 - f(x)\,g(x)\right)\left(\int \frac{\frac{d}{dx}f(x)}{f(x)^2 g(x)}dx\right)-1}{(yf(x)+g(x))\,f(x)}\right)$$

6.2.29.5 [791] problem number 5

problem number 791

Added Feb. 7, 2019.

Problem 2.8.6.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f^2(x)w_x + \left(f'(x)y^2 - g(x)(y - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]^2*D[w[x, y], x] + (Derivative[1][f][x]*y^2 - g[x]*(y - f[x]))*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f(x)^2*diff(w(x,y),x)+(diff(f(x),x)*y^2 -g(x)*(y-f(x))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.29.6 [792] problem number 6

problem number 792

Added Feb. 7, 2019.

Problem 2.8.6.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 - \frac{f''(x)}{f(x)}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (y^2 - Derivative[2][f][x]/f[x])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(y^2 - diff(f(x),x,x)/f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\left(-yf(x)^2 - \left(\frac{d}{dx}f(x)\right)f(x)\right)\left(\int \frac{1}{f(x)^2}dx\right) - 1}{\left(yf(x) + \frac{d}{dx}f(x)\right)f(x)}\right)$$

6.2.29.7 [793] problem number 7

problem number 793

Added Feb. 7, 2019.

Problem 2.8.6.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$g(x)w_x + \left(af(x)g(x)y^3 + (bf(x)g^3(x) + g'(x))y + cf(x)g^4(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  g[x]*D[w[x, y], x] + (a*f[x]*g[x]*y^3 + (b*f[x]*g[x]^3 + Derivative[1][g][x])*y + c*f
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  g(x)*diff(w(x,y),x)+(a*f(x)*g(x)*y^3 + (b*f(x)*g(x)^3 + diff(g(x),x))*y+ c*f(x)*g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{b^3 \ln\left(\frac{-\operatorname{RootOf}\left(c^2 a\_Z^3 + \_Z b^3 - b^3\right) c g(x) - b y}{c g(x)}\right)}{3 \operatorname{RootOf}\left(c^2 a\_Z^3 + \_Z b^3 - b^3\right)^2 a c^2 + b^3} - b\left(\int f(x)\, g(x)^2\, dx\right)\right)$$

6.2.29.8 [794] problem number 8

problem number 794

Added Feb. 7, 2019.

Problem 2.8.6.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^3 + 3f(x)h(x)y^2 + (g(x) + 3f(x)h^2(x))y + f(x)h^3(x) + g(x)h(x) - h'(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^3 + 3*f[x]*h[x]*y^2 + (g[x] + 3*f[x]*h[x]^2)*y + f[x]*h[x]^3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2(h(x)+y)^2 \int_1^x \exp\left(2\int_1^{K[2]} g(K[1])dK[1]\right) f(K[2])dK[2] + \exp\left(2\int_1^x g(K[1])dK[1]\right)}{(h(x)+y)^2}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(f(x)*y^3+3*f(x)*h(x)*y^2+(g(x)+3*f(x)*h(x)^2)*y+ f(x)*h(x)^3 + g(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{2(y+h(x))^2\left(\int e^{2\left(\int g(x)dx\right)} f(x)\,dx\right) + e^{2\left(\int g(x)dx\right)}}{(y+h(x))^2}\right)$$

6.2.29.9 [795] problem number 9

problem number 795

Added Feb. 7, 2019.

Problem 2.8.6.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\frac{g'(x)}{f^2(x)(ag(x)+b)^3}y^3 + \frac{f'(x)}{f(x)}y + f(x)g'(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((Derivative[1][g][x]*y^3)/(f[x]^2*(a*g[x] + b)^3) + (Derivative[1][f
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(g(x),x)/(f(x)^2 *(a*g(x)+b)^3)*y^3 + diff(f(x),x)/f(x) * y + f(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{a^3 \ln\left(\frac{-(ag(x)+b)\,\mathrm{RootOf}\left(-a^3\_Z+\_Z^3+a^3\right)f(x)+ay}{(ag(x)+b)f(x)}\right)}{a^3-3\,\mathrm{RootOf}\left(-a^3\_Z+\_Z^3+a^3\right)^2}-\ln\left(ag(x)+b\right)\right)$$

6.2.29.10 [796] problem number 10

problem number 796

Added Feb. 7, 2019.

Problem 2.8.6.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left((y-f(x))(y-g(x))\left(y-\frac{af(x)+bg(x)}{a+b}\right)h(x)+\frac{y-g(x)}{f(x)-g(x)}f'(x)+\frac{y-f(x)}{g(x)-f(x)}g'(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + ((y - f[x])*(y - g[x])*(y - (a*f[x] + b*g[x])/(a + b))*h[x] + ((y - g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+((y-f(x))*(y-g(x))*(y- (a*f(x)+b*g(x))/(a+b))*h(x)+(y-g(x))/(f(x)-g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{(a^2+ab+b^2)\left((a+b)\,b\ln\left(\frac{9(a+b)(a^2+ab+b^2)(y-g(x))}{(f(x)-g(x))(2a+b)}\right)+\left(2\left(-\frac{\left(\int f(x)^2h(x)dx\right)}{2}-\frac{\left(\int g(x)^2h(x)d\right.}{2}\right.\right.}{}\right.$$

6.2.29.11 [797] problem number 11

problem number 797

Added Feb. 7, 2019.

Problem 2.8.6.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g'(x)y + af(x)e^{2g(x)}\right)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[x]*y^2 + Derivative[1][g][x]*y + a*f[x]*Exp[2*g[x]])*D[w[x, y], y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-g(x)}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x e^{g(K[1])}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(f(x)*y^2 + diff(g(x),x)*y+ a*f(x)*exp(2*g(x)) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y) = \_F1\left(\sqrt{a}\left(\int e^{g(x)} f(x)\,dx\right) - \arctan\left(\frac{y\,e^{-g(x)}}{\sqrt{a}}\right)\right)$$

6.2.29.12 [798] problem number 12

problem number 798

Added Feb. 7, 2019.

Problem 2.8.6.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f'(x)y^2 + ae^{\lambda x} f(x)y + ae^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Derivative[1][f][x]*y^2 + a*Exp[lambda*x]*f[x]*y + a*Exp[lambda*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y),x)+(diff(f(x),x)*y^2+ a*exp(lambda*x)* f(x)*y+a*exp(lambda*x) )*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))  ),output='rea
```

$$w(x,y) = \_F1\left(\frac{-y\left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{a\left(\int e^{\lambda x}f(x)dx\right)}}{f(x)^2}dx\right)f(x) - e^{\int \frac{a\,e^{\lambda x}f(x)^2 - 2\frac{d}{dx}f(x)}{f(x)}dx}f(x) - \left(\int \frac{\left(\frac{d}{dx}f(x)\right)e^{a\left(\int e^{\lambda x}f(x)dx\right)}}{f(x)^2}\right.}{yf(x)+1}\right.$$

6.2.30 9.1

Local contents

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6.2.30.2 [800] problem number 2 1492
6.2.30.3 [801] problem number 3 1493
6.2.30.4 [802] problem number 4 1494
6.2.30.5 [803] problem number 5 1494

6.2.30.1 [799] problem number 1

problem number 799

Added Feb. 7, 2019.

Problem 2.9.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1]-\int_1^x \frac{1}{f(K[2])}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))  ),output='rea
```

$$w(x,y)=\_F1\left(-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy\right)$$

6.2.30.2 [800] problem number 2

problem number 800

Added Feb. 7, 2019.

Problem 2.9.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(x)+g(y))w_x+f'(x)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[x] + g[y])*D[w[x, y], x] + Derivative[1][f][x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (f(x)+g(y))*diff(w(x,y),x)+diff(f(x),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(e^{-y} f(x) - \left(\int e^{-y} g(y)\, dy\right)\right)$$

6.2.30.3 [801] problem number 3

problem number 801

Added Feb. 7, 2019.

Problem 2.9.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x^n f(y) + x g(y)) w_x + h(y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (x^n*f[y] + x*g[y])*D[w[x, y], x] + h[y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := (x^n*f(y) + x*g(y))*diff(w(x,y),x)+h(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(x^{-n+1} e^{(n-1)\left(\int \frac{g(y)}{h(y)} dy\right)} + (n-1)\left(\int \frac{e^{(n-1)\left(\int \frac{g(y)}{h(y)} dy\right)} f(y)}{h(y)} dy\right)\right)$$

6.2.30.4 [802] problem number 4

problem number 802

Added Feb. 7, 2019.

Problem 2.9.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(y) + amx^n y^{m-1})w_x - (g(x) + anx^{n-1}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[y] + a*m*x^n*y^(m - 1))*D[w[x, y], x] - (g[x] + a*n*x^(n - 1)*y^m)*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(y) + a*m*x^n*y^(m-1))*diff(w(x,y),x)-(g(x)+a*n*x^(n-1)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.30.5 [803] problem number 5

problem number 803

Added Feb. 7, 2019.

Problem 2.9.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(e^{\alpha x} f(y) + c\beta)w_x - (e^{\beta y} g(x) + c\alpha)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (Exp[alpha*x]*f[y] + c*beta)*D[w[x, y], x] - (Exp[beta*y]*g[x] + c*alpha)*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (exp(alpha*x)* f(y) + c*beta)*diff(w(x,y),x)-(exp(beta*y)*g(x) + c*alpha)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.31 9.2

Local contents

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6.2.31.2 [805] problem number 2 1496
6.2.31.3 [806] problem number 3 1497
6.2.31.4 [807] problem number 4 1498
6.2.31.5 [808] problem number 5 1499
6.2.31.6 [809] problem number 6 1499
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6.2.31.1 [804] problem number 1

problem number 804

Added Feb. 7, 2019.

Problem 2.9.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + f(ax + by + c)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + f[a*x + b*y + c]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ f(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \_F1\left(-b\left(\int^{\frac{ax+by}{b}} \frac{1}{bf\,(b\_a + c) + a} d\_a\right) + x\right)$$

6.2.31.2 [805] problem number 2

problem number 805

Added Feb. 7, 2019.

Problem 2.9.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + f(\frac{y}{x})w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + f[y/x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ f(y/x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int^{\frac{y}{x}} \frac{1}{-\_a + f(\_a)} d\_a - \ln(x)\right)$$

6.2.31.3 [806] problem number 3

problem number 806

Added Feb. 7, 2019.

Problem 2.9.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(y + ax^n + b) - anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*x^n + b] - a*n*x^(n - 1))*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (f(y+a*x^n+b) - a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-x + \int_{\_b}^{y} \frac{1}{f\left(ax^n + \_a + b\right)} d\_a\right)$$

6.2.31.4 [807] problem number 4

problem number 807

Added Feb. 7, 2019.

Problem 2.9.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yf(x^n y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*f[x^n*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*f(x^n*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{m\left(\int_{\_b}^{y} \frac{1}{(mf(\_a^m x^n)+n)\_a} d\_a\right) - \ln(x)}{m}\right)$$

6.2.31.5 [808] problem number 5

problem number 808

Added Feb. 7, 2019.

Problem 2.9.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$y^{m-1}w_x + x^{n-1}f(ax^n + by^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y^(m - 1)*D[w[x, y], x] + x^(n - 1)*f[a*x^n + b*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := y^(m-1)*diff(w(x,y),x)+  x^(n-1)*f(a*x^n+b*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.31.6 [809] problem number 6

problem number 809

Added Feb. 7, 2019.

Problem 2.9.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + e^{-\lambda x}f(e^{\lambda x}y)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[-(lambda*x)]*f[Exp[lambda*x]*y]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   exp(-lambda*x)*f(exp(lambda*x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x - \left(\int^{y\,\mathrm{e}^{\lambda x}} \frac{1}{\_a\lambda + f(\_a)} d\_a\right)\right)$$

6.2.31.7 [810] problem number 7

problem number 810

Added Feb. 7, 2019.

Problem 2.9.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(e^{\lambda y} x) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*y]*f[Exp[lambda*y]*x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   exp(lambda*y)*f(exp(lambda*y)*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int^{\frac{\lambda y+\ln(x)}{\lambda}} \frac{1}{\lambda \mathrm{e}^{-\_a\lambda} f(\mathrm{e}^{-\_a\lambda})+1} d\_a\right) + \ln(x)}{\lambda}\right)$$

6.2.31.8 [811] problem number 8

problem number 811

Added Feb. 7, 2019.

Problem 2.9.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + yf(e^{\alpha x}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + y*f[Exp[alpha*x]*y^m]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+   y*f(exp(alpha*x)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{m\left(\int_{\_b}^{y} \frac{1}{(mf(\_a^m \mathrm{e}^{\alpha x})+\alpha)\_a} d\_a\right) - x}{m}\right)$$

6.2.31.9 [812] problem number 9

problem number 812

Added Feb. 7, 2019.

Problem 2.9.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + f(x^n e^{\alpha y})w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + f[x^n*Exp[alpha*y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+   f(x^n*exp(alpha*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \_F1\left(\frac{\alpha\left(\int_{\_b}^{y} \frac{1}{\alpha f(x^n e^{-a\alpha})+n} d\_a\right) - \ln(x)}{\alpha}\right)$$

6.2.31.10 [813] problem number 10

problem number 813

Added Feb. 7, 2019.

Problem 2.9.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + e^{\lambda x - \beta y} f(ae^{\lambda x} + be^{\beta y})w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*x - beta*y]*f[a*Exp[lambda*x] + b*Exp[beta*y]]*D[w[x, y],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  exp(lambda*x-beta*y)*f(a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{a\lambda^2\left(\int^{\frac{-a\,\mathrm{e}^{\lambda x}-b\,\mathrm{e}^{\beta y}}{a\lambda}} \frac{1}{b\beta f(-\_aa\lambda)+a\lambda} d\_a\right)+\mathrm{e}^{\lambda x}}{\lambda}\right)$$

6.2.31.11 [814] problem number 11

problem number 814

Added Feb. 7, 2019.

Problem 2.9.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f\left(y + ae^{\lambda x} + b\right) - a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*Exp[lambda*x] + b] - a*lambda*Exp[lambda*x])*D[w[x, y], y] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+  (f(y+a*exp(lambda*x)+b)-a * lambda*exp(lambda*x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-x + \int_{\_b}^{y} \frac{1}{f\left(a\,\mathrm{e}^{\lambda x} + \_a + b\right)} d\_a\right)$$

6.2.31.12 [815] problem number 12

problem number 815

Added Feb. 7, 2019.

Problem 2.9.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\alpha x y w_x + \left(\alpha f\left(x^n e^{\alpha y}\right) - n y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  alpha*x*y*D[w[x, y], x] + (alpha*f[x^n*Exp[alpha*y]] - n*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  alpha*x*y*diff(w(x,y),x)+  (alpha*f(x^n*exp(alpha*y)) - n*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.31.13 [816] problem number 13

problem number 816

Added Feb. 7, 2019.

Problem 2.9.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$mx(\ln y)w_x + (yf(x^n y^m) - ny\ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*x*Log[y]*D[w[x, y], x] + (y*f[x^n*y^m] - n*y*Log[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   m*x*ln(y)*diff(w(x,y),x)+  (y*f(x^n*y^m) - n*y*ln[y])*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.31.14 [817] problem number 14

problem number 817

Added Feb. 7, 2019.

Problem 2.9.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(f(y + a\tan x) - a\tan^2 x\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (f[y + a*Tan[x]] - a*Tan[x]^2)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  (f(y+a*tan(x)) - a*tan(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-x + \int^{a\tan(x)+y} \frac{1}{a + f(\_a)} d\_a\right)$$

6.2.31.15 [818] problem number 15

problem number 818

Added Feb. 7, 2019.

Problem 2.9.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$e^{\lambda x} w_x + f(\lambda x + \ln y) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Exp[lambda*x]*D[w[x, y], x] + f[lambda*x + Log[y]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=    exp(lambda*x)*diff(w(x,y),x)+  f(lambda*x+ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x - \left(\int^{y\,\mathrm{e}^{\lambda x}} \frac{1}{\_a\lambda + f\left(\ln\left(\_a\right)\right)} d\_a\right)\right)$$

6.2.31.16 [819] problem number 16

problem number 819

Added Feb. 7, 2019.

Problem 2.9.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(\lambda y + \ln x) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + Exp[lambda*y]*f[lambda*y + Log[x]]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   diff(w(x,y),x)+  exp(lambda*y)*f(lambda*y+ln(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int^{\frac{\lambda y+\ln(x)}{\lambda}} \frac{1}{\lambda\,\mathrm{e}^{-\_a\lambda} f(\_a\lambda)+1} d\_a\right) + \ln\left(x\right)}{\lambda}\right)$$

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6.2.32.1 [820] problem number 1

problem number 820

Added Feb. 7, 2019.

Problem 2.9.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x - \left(ny - xy^k f(x) g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] - (n*y - x*y^k*f[x]*g[x^n*y^m])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=   m*x*diff(w(x,y),x)- ( n*y -x*y^k*f(x)*g(x^n*y^m)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{x} \frac{-n\_a^{\frac{-kn-m+n}{m}} y^{-k+1} + \_a^{-\frac{(k-1)n}{m}} f(\_a)\, g(\_a^n y^m)}{g\left(\_a^n y^m\right)} d\_a - \left(\int \frac{m x^{-\frac{(k-1)n}{m}} y^{-k} + n\left(\int_{\_}^{x}\right.}{}\right.\right.$$

6.2.32.2 [821] problem number 2

problem number 821

Added Feb. 9, 2019.

Problem 2.9.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^n w_x - \left(a x^n + g(x) f\left(y^{n+1} + a x^{n+1}\right)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  y^n*D[w[x, y], x] - (a*x^n + g[x]*f[y^(n + 1) + a*x^(n + 1)])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  y^n*diff(w(x,y),x)- ( a*x^n + g(x)*f(y^(n+1) + a*x^(n+1))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.3 [822] problem number 3

problem number 822

Added Feb. 9, 2019.

Problem 2.9.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(\frac{y}{x})+x^{\alpha}h(\frac{y}{x})\right)w_x+\left(g(\frac{y}{x})+yx^{\alpha-1}h(\frac{y}{x})\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[y/x] + x^alpha*h[y/x])*D[w[x, y], x] + (g[y/x] + y*x^(alpha - 1)*h[y/x])*D[w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(y/x)+x^alpha * h(y/x))*diff(w(x,y),x)+ ( g(y/x)+y*x^(alpha-1)*h(y/x))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.4 [823] problem number 4

problem number 823

Added Feb. 9, 2019.

Problem 2.9.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(ax+by)+bxg(ax+by))\,w_x+(h(ax+by)-axg(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*x*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*x*g[a*x + b*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(a*x+b*y)+b*x*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*x*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.5 [824] problem number 5

problem number 824

Added Feb. 9, 2019.

Problem 2.9.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(ax+by)+byg(ax+by))\,w_x+(h(ax+by)-ayg(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*y*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*y*g[a*x + b*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (f(a*x+b*y)+b*y*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*y*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.6 [825] problem number 6

problem number 825

Added Feb. 9, 2019.

Problem 2.9.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\big(f(x^n y^m)+mx^k g(x^n y^m)\big)\,w_x+y\big(h(x^n y^m)-nx^k g(x^n y^m)\big)\,w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*x^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*x^k*g[x^n*y^m])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(f(x^n*y^m)+m*x^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*x^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.7 [826] problem number 7

problem number 826

Added Feb. 9, 2019.

Problem 2.9.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(f(x^n y^m)+m y^k g(x^n y^m)\right) w_x + y\left(h(x^n y^m)-n y^k g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*y^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*y^k*g[x^n*y^m])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(f(x^n*y^m)+m*y^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*y^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.8 [827] problem number 8

problem number 827

Added Feb. 9, 2019.

Problem 2.9.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(sf(x^n y^m)-mg(x^k y^s)\right)w_x+y\left(ng(x^k y^s)-kf(x^n y^m)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(s*f[x^n*y^m] - m*g[x^k*y^s])*D[w[x, y], x] + y*(n*g[x^k*y^s] - k*f[x^n*y^m])*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*(s*f(x^n*y^m)-m*g(x^k*y^s))*diff(w(x,y),x)+ y*(n*g(x^k*y^s)-k*f(x^n*y^m))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.9 [828] problem number 9

problem number 828

Added Feb. 9, 2019.

Problem 2.9.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$f_y * w_x - f_x w_y = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[f[x, y], y]*D[w[x, y], x] - D[f[x, y], x]*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to c_1(\text{InverseFunction}[\text{InverseFunction}[f,2,2],2,2][x,y])\}\}$$

Maple ✓

```
restart;
pde :=  diff(f(x,y),y)*diff(w(x,y),x)-diff(f(x,y),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1(-f(x,y))$$

6.2.32.10 [829] problem number 11

problem number 829

Added Feb. 9, 2019.

Problem 2.9.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$xw_x + (xf(x)g(x^n e^y) - n)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*f[x]*g[x^n*Exp[y]] - n)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+(x*f(x)*g(x^n*exp(y))-n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} \frac{1}{g\left(x^n \mathrm{e}^{-\_a}\right)} d\_a - \left(\int f(x)\, dx\right)\right)$$

6.2.32.11 [830] problem number 12

problem number 830

Added Feb. 9, 2019.

Problem 2.9.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$m w_x + \left(m y^k f(x) g(e^{\alpha x} y^m) - \alpha y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  m*D[w[x, y], x] + (m*y^k*f[x]*g[Exp[alpha*x]*y^m] - alpha*y)*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  m*diff(w(x,y),x)+(m*y^k*f(x)*g(exp(alpha*x)*y^m)- alpha*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{-m\left(\int \frac{\alpha\left(\int_{\_b}^{x} \frac{m y^{-k+m} e^{-\frac{(k-m-1)\_a\alpha}{m}} D(g)(y^m e^{-\_a\alpha})+(k-1)y^{-k} e^{-\frac{(k-1)\_a\alpha}{m}} g(y^m e^{-\_a\alpha})}{g(y^m e^{-\_a\alpha})^2} d\_a\right) g(y^m e^{\alpha x})+m y^{-k} e^{-(k}}{m g(y^m e^{\alpha x})}\right.}{m}\right.$$

6.2.32.12 [831] problem number 13

problem number 831

Added Feb. 9, 2019.

Problem 2.9.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(ax+by)+be^{\lambda y}g(ax+by)\right)w_x+\left(h(ax+by)-ae^{\lambda y}g(ax+by)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*Exp[lambda*y]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(a*x+b*y)+ b*exp(lambda*y)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)-  a*exp(lambda*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.13 [832] problem number 14

problem number 832

Added Feb. 9, 2019.

Problem 2.9.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\left(f(ax+by)+be^{\lambda x}g(ax+by)\right)w_x+\left(h(ax+by)-ae^{\lambda x}g(ax+by)\right)w_y=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[a*x + b*y] + b*Exp[lambda*x]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(a*x+b*y)+ b*exp(lambda*x)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)-  a*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.14 [833] problem number 15

problem number 833

Added Feb. 9, 2019.

Problem 2.9.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(f(x^n e^{\alpha y}) + \alpha y g(x^n e^{\alpha y}))\, w_x + (h(x^n e^{\alpha y}) - n y g(x^n e^{\alpha y})))\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*Exp[alpha*y]] + alpha*y*g[x^n*Exp[alpha*y]])*D[w[x, y], x] + (h[x^n*Exp[alph
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*exp(alpha*y))+alpha*y*g(x^n*exp(alpha*y)))*diff(w(x,y),x)+(h(x^n*exp(alpha*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.15 [834] problem number 16

problem number 834

Added Feb. 9, 2019.

Problem 2.9.3.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(e^{\alpha x} y^m) + m x g(e^{\alpha x} y^m))\, w_x + y(h(e^{\alpha x} y^m) - \alpha x g(e^{\alpha x} y^m)))\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (f[Exp[alpha*x]*y^m] + m*x*g[Exp[alpha*x]*y^m])*D[w[x, y], x] + y*(h[Exp[alpha*x]*y^m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := (f(exp(alpha*x)*y^m)+m*x*g(exp(alpha*x)*y^m))*diff(w(x,y),x)+ y*(h(exp(alpha*x)*y^m)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.16 [835] problem number 17

problem number 835

Added Feb. 9, 2019.

Problem 2.9.3.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + (xyf(x)g(x^n \ln y) - ny \ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*y*f[x]*g[x^n*Log[y]] - n*y*Log[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*diff(w(x,y),x)+ (x*y*f(x)*g(x^n*ln(y))-n*y*ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.17 [836] problem number 18

problem number 836

Added Feb. 9, 2019.

Problem 2.9.3.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(f(x^n y^m) + mg(x^n y^m) \ln y)\, w_x + y(h(x^n y^m) - ng(x^n y^m) \ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[y])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(y))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.18 [837] problem number 19

problem number 837

Added Feb. 9, 2019.

Problem 2.9.3.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(f(x^n y^m) + mg(x^n y^m) \ln x)\, w_x + y(h(x^n y^m) - ng(x^n y^m) \ln x)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[x])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(x))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.19 [838] problem number 20

problem number 838

Added Feb. 9, 2019.

Problem 2.9.3.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\cos y w_x + (f(x)g(\sin x \sin y) - \cot x \sin y)\, w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Cos[y]*D[w[x, y], x] + (f[x]*g[Sin[x]*Sin[y]] - Cot[x]*Sin[y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=cos(y)*diff(w(x,y),x)+ (f(x)* g(sin(x)*sin(y)) - cot(x)*sin(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.20 [839] problem number 21

problem number 839

Added Feb. 9, 2019.

Problem 2.9.3.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\sin 2x w_x + \left(\sin 2x \cos^2 y f(x) g(\tan x \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  Sin[2*x]*D[w[x, y], x] + (Sin[2*x]*Cos[y]^2*f[x]*g[Tan[x]*Tan[y]] - Sin[2*y])*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=sin(2*x)*diff(w(x,y),x)+ (sin(2*x)*cos(y)^2*f(x)*g(tan(x)*tan(y)) -sin(2*y))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.21 [840] problem number 22

problem number 840

Added Feb. 9, 2019.

Problem 2.9.3.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(x\cos^2 y f(x) g(x^{2n}\tan y) - n\sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + (x*Cos[y]^2*f[x]*g[x^(2*n)*Tan[y]] - n*Sin[2*y])*D[w[x, y], y] == 0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=x*diff(w(x,y),x)+ (x *cos(y)^2* f(x)* g(x^(2*n)*tan(y)) - n*sin(2*y))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.2.32.22 [841] problem number 23

problem number 841

Added Feb. 9, 2019.

Problem 2.9.3.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\cos^2 y f(x) g(e^{2x} \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (Cos[y]^2*f[x]*g[Exp[2*x]*Tan[y]] - Sin[2*y])*D[w[x, y], y] == 0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y),x)+ (cos(y)^2* f(x)* g(exp(2*x)*tan(y)) -sin(2*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

6.3 chapter 3

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6.3.1 Examples

Local contents

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6.3.1.1 [842] Example 1

problem number 842

Added Feb. 9, 2019.

Problem Chapter 3, example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\log(x)}{a} + c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\_F1\left(yx^{-\frac{b}{a}}\right) + c\ln(x)}{a}$$

6.3.1.2 [843] Example 2

problem number 843

Added Feb. 9, 2019.

Problem Chapter 3, example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ae^x w_x + bw_y = ce^{2x} y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[x]*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[2*x]*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c(ae^x y + b(-x) + b)}{a^2} + c_1\left(\frac{be^{-x}}{a} + y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(x)*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*exp(2*x)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{\left(-a\,\mathrm{e}^{-\frac{b\,\mathrm{e}^{-x}}{a}+x} + b\,\mathrm{expIntegral}\left(1, \frac{b\,\mathrm{e}^{-x}}{a}\right)\right) cy\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-x}}{a}}}{a^2} + \_F1\left(y\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-x}}{a}}\right)$$

6.3.1.3 [844] Example 3

problem number 844

Added Feb. 9, 2019.

Problem Chapter 3, example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = b$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to bx + c_1(y - ax)\}\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = bx + \_F1(-ax + y)$$

6.3.2 2.1

Local contents

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6.3.2.1 [845] Problem 1

problem number 845

Added Feb. 9, 2019.

Problem Chapter 3.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{cx}{a} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx}{a} + \_F1\left(\frac{ay - bx}{a}\right)$$

6.3.2.2 [846] Problem 2

problem number 846

Added Feb. 9, 2019.

Problem Chapter 3.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{x(a(\alpha x + 2\beta y + 2\gamma) - b\beta x)}{2a^2} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2a^2\_F1\left(\frac{ay-bx}{a}\right) + (-b\beta x + (\alpha x + 2\beta y + 2\gamma)\, a)\, x}{2a^2}$$

6.3.2.3 [847] Problem 3

problem number 847

Added Feb. 9, 2019.

Problem Chapter 3.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a\alpha x+2a\log(x)(\beta y+\gamma)-b\beta\log^2(x)}{2a^2}+c_1\left(y-\frac{b\log(x)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-b\beta\ln(x)^2+2a^2\_F1\left(\frac{ay-b\ln(x)}{a}\right)+2a\alpha x+2(\beta y+\gamma)\,a\ln(x)}{2a^2}$$

6.3.2.4 [848] Problem 4

problem number 848

Added Feb. 9, 2019.

Problem Chapter 3.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+bxw_y=c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log(x)}{a}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\_F1\left(\frac{ay-bx}{a}\right) + c\ln(x)}{a}$$

6.3.2.5 [849] Problem 5

problem number 849

Added Feb. 9, 2019.

Problem Chapter 3.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + \frac{\log(ax+b)(-a\beta d + ac\gamma - \alpha bc)}{a^2c} + \frac{\alpha x}{a} + \frac{\beta(cy+d)}{c^2}\right\}\right\}$$

Maple ✓

```
restart;
pde := (a*x+b)* diff(w(x,y),x)+(c*y+d)*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a^2c^2\_F1\left(\frac{(yc+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + (-\alpha bc + (-\beta d + \gamma c)\,a)\,c\ln(ax+b) + (\alpha c^2 x + (yc+d)\,a\beta)\,a}{a^2c^2}$$

6.3.2.6 [850] Problem 6

problem number 850

Added Feb. 9, 2019.

Problem Chapter 3.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{y^2}{2} - \frac{bx}{a}\right) - \frac{\alpha(ay^2)^{3/2}}{3\sqrt{a}b^2} + \frac{\sqrt{ay^2}(\alpha x+\gamma)}{\sqrt{a}b} + \frac{\beta x}{a}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{y^2}{2} - \frac{bx}{a}\right) + \frac{\alpha(ay^2)^{3/2}}{3\sqrt{a}b^2} - \frac{\sqrt{ay^2}(\alpha x+\gamma)}{\sqrt{a}b} + \frac{\beta x}{a}\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{6a^2b^2\_F1\left(\frac{ay^2-2bx}{a}\right) + 6ab^2\beta x - 3(a\alpha y^2 - 2(\alpha x+\gamma)\,b)\sqrt{a^2y^2}\,a + (a^2y^2)^{\frac{3}{2}}\,\alpha}{6a^2b^2}$$

6.3.2.7 [851] Problem 7

problem number 851

Added Feb. 9, 2019.

Problem Chapter 3.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to -\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}} + c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$

$$\left\{w(x,y) \to \frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}} + c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{c\ln\left(\frac{abx}{\sqrt{ab}} + \sqrt{a^2y^2}\right) + \sqrt{ab}\_F1\left(\frac{ay^2-bx^2}{a}\right)}{\sqrt{ab}}$$

6.3.2.8 [852] Problem 8

problem number 852

Added Feb. 9, 2019.

Problem Chapter 3.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cx + ky$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x + k*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{ay^2-bx^2}{2a}\right) - \frac{c\sqrt{ay^2}}{\sqrt{ab}} + \frac{kx}{a}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{ay^2-bx^2}{2a}\right) + \frac{c\sqrt{ay^2}}{\sqrt{ab}} + \frac{kx}{a}\right\}$$

Maple ✓

```
restart;
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x+k*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{kx}{a} + \frac{cy}{b} + \_F1\left(\frac{ay^2 - bx^2}{a}\right)$$

6.3.3 2.2

Local contents

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6.3.3.1 [853] Problem 1

problem number 853

Added Feb. 9, 2019.

Problem Chapter 3.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^2 + dy^2 + kxy + n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{x(a^2(2cx^2+6dy^2+3kxy+6n)-abx(6dy+kx)+2b^2dx^2)}{6a^3}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{6a^3\_F1\left(\frac{ay-bx}{a}\right)+2\left(b^2dx^2-3\left(dy+\frac{kx}{6}\right)abx+\left(cx^2+3dy^2+\frac{3}{2}kxy+3n\right)a^2\right)x}{6a^3}$$

6.3.3.2 [854] Problem 2

problem number 854

Added Feb. 9, 2019.

Problem Chapter 3.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cx^2+dy^2+kxy+n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2ab(a+b)c_1\left(yx^{-\frac{b}{a}}\right)+a^2dy^2+abcx^2+abdy^2+2abkxy+2bn(a+b)\log(x)+b^2cx^2}{2ab(a+b)}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kxy}{a+b}+\frac{cx^2}{2a}+\frac{dy^2}{2b}+\frac{n\ln(x)}{a}+\_F1\left(yx^{-\frac{b}{a}}\right)$$

6.3.3.3 [855] Problem 3

problem number 855

Added Feb. 9, 2019.

Problem Chapter 3.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x+bxw_y=cxy+d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}+\frac{cx^2}{2a}\right\}$$
$$\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}+\frac{cx^2}{2a}\right\}$$

Maple ✓

```
restart;
pde := a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x*y+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ad\ln\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2y^2}\right)+\frac{\sqrt{ab}\,cx^2}{2}+\sqrt{ab}\,a\_F1\left(\frac{ay^2-bx^2}{a}\right)}{\sqrt{ab}\,a}$$

6.3.3.4 [856] Problem 4

problem number 856

Added Feb. 9, 2019.

Problem Chapter 3.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x+by^2w_y=cx^2+dy^2+kxy+nx+my+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n*x + m*y + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{abx(ax-by)c_1\left(\frac{b}{ax}-\frac{1}{y}\right)-a^2mx^2\log\left(\frac{ax}{y}\right)+abcx^3-abdxy^2+abkx^2y\log\left(\frac{ax}{y}\right)+x\log(x}{abx(ax-by)}\right.\right.$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) =c*x^2+d*y^2+ k*x*y+ n*x+ m*y+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kxy\ln\left(\frac{ax}{y}\right)}{ax-by}-\frac{dy^2}{ax-by}+\frac{cx}{a}+\frac{n\ln(x)}{a}+\frac{m\ln(x)}{b}-\frac{m\ln\left(\frac{ax}{y}\right)}{b}+\_F1\left(\frac{ax-by}{axy}\right)-\frac{s}{ax}$$

6.3.3.5 [857] Problem 5

problem number 857

Added Feb. 9, 2019.

Problem Chapter 3.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x+axyw_y=by^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{by^2}{x-2ax}+c_1\left(yx^{-a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*diff(w(x,y),x)+a*x*y*diff(w(x,y),y) =b*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{by^2}{(2a-1)\,x}+\_F1\left(yx^{-a}\right)$$

6.3.3.6 [858] Problem 6

problem number 858

Added Feb. 9, 2019.

Problem Chapter 3.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2w_x+bx^2w_y=cx^2+d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + b*x^2*D[w[x, y], y] == c*x^2 + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to \frac{bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3}{\sqrt[3]{a}b\,(ay^3)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

$$\left\{w(x,y)\to -\frac{\sqrt[3]{-1}\left(bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3\right)}{\sqrt[3]{a}b\,(ay^3)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

$$\left\{w(x,y)\to \frac{(-1)^{2/3}\left(bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{bx^3}{bx^3-ay^3}\right)+acy^3\right)}{\sqrt[3]{a}b\,(ay^3)^{2/3}}+c_1\left(\frac{ay^3-bx^3}{3a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y^2*diff(w(x,y),x)+b*x^2*diff(w(x,y),y) =c*x^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{(\_a^2c+d)\,a}{\left(\left(\_a^3b+a\,\mathrm{RootOf}\left(ay-(a^2bx^3+a^3\_Z)^{\frac{1}{3}}\right)\right)a^2\right)^{\frac{2}{3}}}d\_a+\_F1\left(\mathrm{RootOf}\left(ay-(a^2bx^3+a\right.\right.$$

Contains unresolved integral with RootOf

6.3.3.7 [859] Problem 7

problem number 859

Added Feb. 9, 2019.

Problem Chapter 3.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2 w_x + bxyw_y = cx^2 + dy^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + b*x*y*D[w[x, y], y] == c*x^2 + d*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{ay^2-bx^2}{2a}\right) - \frac{\sqrt{a}c\sqrt{y^2-\frac{bx^2}{a}}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}\sqrt{y^2-\frac{bx^2}{a}}}\right)}{b^{3/2}} + \frac{dx}{a} + \frac{cx}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*y^2*diff(w(x,y),x)+b*x*y*diff(w(x,y),y) =c*x^2+d*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-(ay^2-bx^2)\,ac\arctan\left(\frac{bx}{\sqrt{(ay^2-bx^2)b}}\right) + \sqrt{(ay^2-bx^2)\,b}\left(ab\_F1\left(\frac{ay^2-bx^2}{a}\right) + (ac+bd)\,x\right)}{\sqrt{(ay^2-bx^2)\,b}\,ab}$$

6.3.4 2.3

Local contents

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6.3.4.2 [861] Problem 2 1546
6.3.4.3 [862] Problem 3 1547
6.3.4.4 [863] Problem 4 1548
6.3.4.5 [864] Problem 5 1549
6.3.4.6 [865] Problem 6 1549

6.3.4.1 [860] Problem 1

problem number 860

Added Feb. 9, 2019.

Problem Chapter 3.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = a\sqrt{x^2 + y^2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to a\sqrt{x^2 + y^2} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*sqrt(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \sqrt{x^2 + y^2}\, a + \_F1\left(\frac{y}{x}\right)$$

6.3.4.2 [861] Problem 2

problem number 861

Added Feb. 9, 2019.

Problem Chapter 3.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = cxy^2 + dx^2y + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x*y^2 + d*x^2*y + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{a(2a^2+5ab+2b^2)\,c_1\left(yx^{-\frac{b}{a}}\right) + k(2a^2+5ab+2b^2)\log(x) + axy(2acy+adx+bcy+2bdx}{a(2a+b)(a+2b)}\right.\right.$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x*y^2+d*x^2*y+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cxy^2}{a+2b} + \frac{dx^2y}{2a+b} + \frac{k\ln(x)}{a} + \_F1\left(yx^{-\frac{b}{a}}\right)$$

6.3.4.3 [862] Problem 3

problem number 862

Added Feb. 9, 2019.

Problem Chapter 3.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cxy^2 + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y^2 + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to -\frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right) + cy^2\sqrt{ay^2}}{3\sqrt{a}b} + c_1\left(\frac{ay^2 - bx^2}{2a}\right)\right\}$$
$$\left\{w(x,y) \to \frac{3\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right) + cy^2\sqrt{ay^2}}{3\sqrt{a}b} + c_1\left(\frac{ay^2 - bx^2}{2a}\right)\right\}$$

Maple ✓

```
restart;
pde := a*y*diff(w(x,y),x) + b*x*diff(w(x,y),y) =c*x*y^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a^2bd\ln\left(\frac{abx}{\sqrt{ab}} + \sqrt{a^2y^2}\right) + \left(a^2b\_F1\left(\frac{ay^2-bx^2}{a}\right) + \left((ay^2 - bx^2)\,ay + \left(-\frac{2ay^2}{3} + bx^2\right)\sqrt{a^2y^2}\right)c\right)\sqrt{}}{\sqrt{ab}\,a^2b}$$

6.3.4.4 [863] Problem 4

problem number 863

Added Feb. 9, 2019.

Problem Chapter 3.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax + b)w_x + (cy + d)w_y = kx^3 + ny^3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == k*x^3 + n*y^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right) - \frac{\log(ax+b)\left(a^3d^3n + b^3c^3k\right)}{a^4c^3} + \frac{b^2kx}{a^3} - \frac{bkx^2}{2a^2} + \frac{kx^3}{3a} + \frac{n(-3c^2d}{}\right.\right.$$

Maple ✓

```
restart;
pde := (a*x+b)*diff(w(x,y),x) + (c*y+d)*diff(w(x,y),y) =k*x^3+n*y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{6a^4c^4\_F1\left(\frac{(yc+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + 2\left(a^2c^4kx^3 - \frac{3abc^4kx^2}{2} + 3b^2c^4kx + (yc+d)\left(c^2y^2 - \frac{5}{2}cdy + \frac{11}{2}d^2\right)a\right.}{6a^4c^4}$$

6.3.4.5 [864] Problem 5

problem number 864

Added Feb. 9, 2019.

Problem Chapter 3.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x^2 w_x + xy w_y = y^2(ax + by)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to \frac{y^2(ax + by)}{2x} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x^2*diff(w(x,y),x) + x*y*diff(w(x,y),y) =y^2*(a*x + b*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \frac{ay^2}{2} + \frac{by^3}{2x} + \_F1\left(\frac{y}{x}\right)$$

6.3.4.6 [865] Problem 6

problem number 865

Added Feb. 9, 2019.

Problem Chapter 3.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ax^3 w_x + by^3 w_y = cx + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^3*D[w[x, y], x] + b*y^3*D[w[x, y], y] == c*x + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{2cx+d}{2ax^2} + c_1\left(\frac{1}{2}\left(\frac{b}{ax^2} - \frac{1}{y^2}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^3*diff(w(x,y),x) + b*y^3*diff(w(x,y),y) =c*x+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ax^2 - by^2}{ax^2y^2}\right) - \frac{c}{ax} - \frac{d}{2ax^2}$$

6.3.5 2.4

Local contents

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6.3.5.1 [866] Problem 1

problem number 866

Added Feb. 9, 2019.

Problem Chapter 3.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n + dy^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + d*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{cx^{n+1}}{an+a} + \frac{dy^{m+1}}{bm+b}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n+d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(m+1)(n+1)\,ab\_F1\left(\frac{ay-bx}{a}\right) + (n+1)\,ady^{m+1} + (m+1)\,bcx^{n+1}}{(m+1)(n+1)\,ab}$$

6.3.5.2 [867] Problem 2

problem number 867

Added Feb. 9, 2019.

Problem Chapter 3.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n y$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{-cx^{n+1}(bx - a(n+2)y) + a^2(n^2+3n+2)\,c_1\left(y - \frac{bx}{a}\right)}{a^2(n+1)(n+2)}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(n+2)\,(n+1)\,a^2\_F1\left(\frac{ay-bx}{a}\right) + ((n+2)\,ay - bx)\,cx^{n+1}}{(n+2)\,(n+1)\,a^2}$$

6.3.5.3 [868] Problem 3

problem number 868

Added Feb. 9, 2019.

Problem Chapter 3.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2+y^2)^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\left(x^2+y^2\right)^k}{2k}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*(x^2+y^2)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\left(x^2+y^2\right)^k+2k\_F1\left(\frac{y}{x}\right)}{2k}$$

6.3.5.4 [869] Problem 4

problem number 869

Added Feb. 9, 2019.

Problem Chapter 3.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cx^ny^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{cy^mx^n}{an+bm}+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx^n y^m}{an+bm} + \_F1\left(yx^{-\frac{b}{a}}\right)$$

6.3.5.5 [870] Problem 5

problem number 870

Added Feb. 9, 2019.

Problem Chapter 3.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n + dy^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n + d*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right) + \frac{cx^n}{an} + \frac{dy^m}{bm}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n + d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c\_a^n + d\left(y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)^m}{\_aa} d\_a + \_F1\left(yx^{-\frac{b}{a}}\right)$$

Result has unresolved integral

6.3.5.6 [871] Problem 6

problem number 871

Added Feb. 9, 2019.

Problem Chapter 3.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = (ax^n + by^m)^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{(ax^n + by^m)^k}{kmn} + c_1\left(yx^{-\frac{n}{m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=m*x*diff(w(x,y),x) + n*y*diff(w(x,y),y) =(a*x^n+b*y^m)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{kmn\_F1\left(yx^{-\frac{n}{m}}\right) + (ax^n + by^m)^k}{kmn}$$

Result has unresolved integral

6.3.5.7 [872] Problem 7

problem number 872

Added Feb. 9, 2019.

Problem Chapter 3.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ax^n w_x + by^m w_y = cx^k + dy^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == c*x^k + d*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{bx^{1-n}}{a(n-1)} - \frac{y^{1-m}}{m-1}\right) + \frac{cx^{k-n+1}}{ak-an+a} - \frac{dy^{1-m}\left(\left(y^{m-1}\right)^{\frac{1}{m-1}}\right)^s}{b(m-s-1)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*y^m*diff(w(x,y),y) =c*x^k+d*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{(k-n+1)\,a^2 d a^{\frac{s}{m-1}-1} y^{-m+1}\left((n-1)\,ay^{-m+1}\right)^{-\frac{s}{m-1}}(n-1)^{\frac{s}{m-1}}\,\mathrm{e}^{-\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{n-1}\right)\operatorname{csgn}\left(iay^{-m+1}\right)^2-\operatorname{csgn}\left(i\right.\right.}{}}}{}$$

Result has unresolved integral

6.3.5.8 [873] Problem 8

problem number 873

Added Feb. 9, 2019.

Problem Chapter 3.2.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^k y^s + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^k*y^s + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{1-n}\left(\frac{d}{a-an}-\frac{cx^k y^s e^{-\frac{bsx^{m-n+1}}{am-an+a}}\left(-\frac{bsx^{m-n+1}}{am-an+a}\right)^{\frac{-k+n-1}{m-n+1}}\text{Gamma}\left(\frac{k-n+1}{m-n+1},-\frac{bsx^{m-n+1}}{am-an+a}\right)}{a(m-n+1)}\right)+c_1\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =c*x^k*y^s+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c\_a^{k-n}\left(y\,\mathrm{e}^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)n}{(m-n+1)a}}\right)^s+d\_a^{-n}}{a}d\_a+\_F1\left(y\,\mathrm{e}^{-\frac{nx^{m-n+1}}{(m-n+1)a}}\right)$$

Result has unresolved integral

6.3.5.9 [874] Problem 9

problem number 874

Added Feb. 9, 2019.

Problem Chapter 3.2.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k) w_y = sx^p y^q + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == s*x^p*y^q + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{K[1]^{-n}\left(s\left(b^{-\frac{k+1}{m-n+1}} \exp\left(-\frac{b(x^{m-n+1}-K[1]^{m-n+1})}{a(m-n+1)}\right)(a(m-n+1))^{-\frac{m}{m-n+1}}\left(b^{\frac{n}{m-n+1}} ce^{\frac{bx^{m-}}{ma-n}}\right.\right.\right.}{}\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{s\_a^{-n+p}\left(y\,\mathrm{e}^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)n}{(m-n+1)a}}\right)^q + d\_a^{-n}}{a} d\_a + \_F1\left(y\,\mathrm{e}^{-\frac{nx^{m-n+1}}{(m-n+1)a}}\right)$$

6.3.5.10 [875] Problem 10

problem number 875

Added Feb. 9, 2019.

Problem Chapter 3.2.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y^k + cx^r y) w_y = sx^p y^q + d$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y^k + c*x^r*y)*D[w[x, y], y] == s*x^p*y^q + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x) +(b*x^m*y^k + c*x^r*y)*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{s\_a^{-n+p}\left(\left(\left(\frac{-(n-r-1)(m-2n+r+2)^2 ab\_a^{m-r} y^{\frac{1}{n-r-1}} y^{\frac{r}{n-r-1}} y^{\frac{kn}{n-r-1}} \left(\frac{(k-1)c}{(n-r-1)a}\right)^{\frac{-m+n-1}{n-r-1}} \left(\frac{(k-1)c}{(n-r-1)a}\right)^{\frac{m-n+1}{n-r-1}}}{}\right.\right.\right.}{}$$

6.3.5.11 [876] Problem 11

problem number 876

Added Feb. 9, 2019.

Problem Chapter 3.2.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^m w_y = cx^m + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^m*D[w[x, y], y] == c*x^m + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(bdx\left(\frac{a(m+1)y^{k+1}}{a(m+1)y^{k+1}-b(k+1)x^{m+1}}\right)^{\frac{k}{k+1}}\ {}_2F_1\left(\frac{k}{k+1},\frac{1}{m+1};1+\frac{1}{m+1};\frac{b(k+1)x^{m+1}}{b(k+1)x^{m+1}-a(m+}\right.\right.}{ab}\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x) +b*x^n*diff(w(x,y),y) =c*x^m+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{(c\_a^m+d)\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_a^{n+1}-(k+1)bx^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}}{a}d\_a+\_F1\left(\frac{(n+1)\,ay^{k+1}-(k+1}{(n+1)\,a}\right.$$

6.3.6 3.1

Local contents

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6.3.6.1 [877] Problem 1

problem number 877

Added Feb. 9, 2019.

Problem Chapter 3.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = ce^{\lambda x} + de^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x] + d*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{ce^{\lambda x}}{a\lambda} + \frac{de^{\mu y}}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x)+d*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right) + ad\lambda\,\mathrm{e}^{\mu y} + bc\mu\,\mathrm{e}^{\lambda x}}{ab\lambda\mu}$$

6.3.6.2 [878] Problem 2

problem number 878

Added Feb. 9, 2019.

Problem Chapter 3.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\lambda x+\beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ce^{\beta y+\lambda x}}{a\lambda+b\beta} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\,\mathrm{e}^{\beta y+\lambda x}}{a\lambda+b\beta} + \_F1\left(\frac{ay-bx}{a}\right)$$

6.3.6.3 [879] Problem 3

problem number 879

Added Feb. 9, 2019.

Problem Chapter 3.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ae^{\lambda x} w_x + be^{\beta y} w_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{ce^{-\lambda x}}{a\lambda} + c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\lambda\_F1\left(-\frac{(a\lambda\, e^{\lambda x} - b\beta\, e^{\beta y})e^{-\beta y - \lambda x}}{b\beta\lambda}\right) - c\, e^{-\lambda x}}{a\lambda}$$

6.3.6.4 [880] Problem 4

problem number 880

Added Feb. 9, 2019.

Problem Chapter 3.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\beta x - \log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)\right)}{a\beta e^{\lambda y} - b\lambda e^{\beta x}} + c_1\left(\frac{e^{\lambda y}}{\lambda} - \frac{be^{\beta x}}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{\left(-\ln\left(\frac{a\beta\,\mathrm{e}^{\lambda y}}{b\lambda}\right) + \ln\left(\mathrm{e}^{\beta x}\right)\right)c + \left(a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}\right)\_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)}{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}}$$

6.3.6.5 [881] Problem 5

problem number 881

Added Feb. 9, 2019.

Problem Chapter 3.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y = ce^{\gamma x - \beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2(2\alpha^2-3\alpha\gamma+\gamma^2)\,c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta}\right)+ac(\gamma-2\alpha)e^{-\alpha x-\beta y+\gamma x}+b\beta ce^{x(\gamma-2\alpha)}}{a^2\left(2\alpha^2-3\alpha\gamma+\gamma^2\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}\right)-\frac{\left(-\frac{\mathrm{e}^{(-2\alpha+\gamma)x}}{-2\alpha+\gamma}-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y+(-2\alpha+\gamma)x}}{(-\alpha+\gamma)b\beta}\right)b\beta c}{a^2\alpha}$$

6.3.6.6 [882] Problem 6

problem number 882

Added Feb. 9, 2019.

Problem Chapter 3.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x+be^{\beta y}w_y=ce^{\gamma x-2\beta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - 2*beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-2(\alpha x+\beta y)}\left(-ce^{x(\gamma-\alpha)}\left(a^2(6\alpha^2-5\alpha\gamma+\gamma^2)\,e^{2\alpha x}-2ab\beta(3\alpha-\gamma)e^{\alpha x+\beta y}+2b^2\beta^2e^{2\beta y}\right)+a^3(-\right.}{a^3(\alpha-\gamma)(2\alpha-\gamma)(3\alpha-\gamma)}\right.$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-2*beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}\right)+\frac{\left(\frac{\mathrm{e}^{(-3\alpha+\gamma)x}}{-3\alpha+\gamma}+\frac{2\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y+(-3\alpha+\gamma)x}}{(-2\alpha+\gamma)b\beta}+\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)^2\mathrm{e}^{-}}{(-\alpha+\gamma)b^2}\right.}{a^3\alpha^2}$$

6.3.6.7 [883] Problem 7

problem number 883

Added Feb. 9, 2019.

Problem Chapter 3.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x+be^{\beta y}w_y=ce^{\gamma x}+se^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x] + s*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta}\right)-\frac{ce^{\gamma x-\alpha x}}{a\alpha-a\gamma}-\frac{s\left(e^{-\beta y}\right)^{1-\frac{\mu}{\beta}}}{b\beta-b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x) + s*exp(mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{\left((-\alpha+\gamma)\,a\_F1\left(-\frac{(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y})\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}\right)+c\,\mathrm{e}^{(-\alpha+\gamma)x}\right)(-\beta+\mu)\,\alpha b+(-\alpha+\gamma)\left(b\beta\,\mathrm{e}^{-\alpha x}+(a\right.}{(-\alpha+\gamma)\,(-\beta+\mu)\,a\alpha b}$$

6.3.6.8 [884] Problem 8

problem number 884

Added Feb. 9, 2019.

Problem Chapter 3.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x+(be^{\gamma x}+ce^{\lambda y})w_y=se^{\mu x}+ke^{\delta y}+p$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{k\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_ba\beta-b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_b}}{(\beta-\gamma)a}}d\_b\right)+a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a}\right)^{-\frac{\delta}{\lambda}}}{a}$$

6.3.6.9 [885] Problem 9

problem number 885

Added Feb. 9, 2019.

Problem Chapter 3.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x+\delta y} + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{s\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_ba\beta-b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_b}}{(\beta-\gamma)a}}d\_b\right)+a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a}\right)^{-\frac{\delta}{\lambda}}}{a}$$

6.3.6.10 [886] Problem 10

problem number 886

Added Feb. 9, 2019.

Problem Chapter 3.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = ce^{\mu x+\delta y} + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x + lambda*y]*D[w[x, y], y] == c*Exp[mu*x +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{\gamma x-\beta x}}{a\beta-a\gamma}-\frac{e^{-\lambda y}}{\lambda}\right)+\frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\delta/\lambda}e^{-\gamma x-\lambda y+\mu x}\,{}_2F_1\left(1,\frac{\mu-\gamma}{\beta-\gamma};\frac{\beta\delta-\gamma\delta-\gamma\lambda+\lambda\mu}{\beta\lambda-\gamma\lambda};1-\frac{ae^{\beta x-\gamma x-}}{b}\right.}{b(\beta(\lambda-\delta)+\delta\gamma-\lambda\mu)}\right.\right.$$

Maple ✓

```
restart;
pde :=a*exp(beta*x)*diff(w(x,y),x) +b*exp(gamma*x+lambda*y)*diff(w(x,y),y) =c*exp(mu*x+delta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{c\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_a}+(\beta-\gamma)a\,\mathrm{e}^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{(-\beta+\mu)\_a}+k\,\mathrm{e}^{-\_a\beta}}{a} d\_a + \_F1\left(-\frac{(-b\lambda\,\mathrm{e}^{\lambda y+(-}}{}\right.$$

6.3.6.11 [887] Problem 11

problem number 887

Added Feb. 9, 2019.

Problem Chapter 3.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma x} + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\lambda y}}{\lambda} - \frac{be^{\beta x}}{a\beta}\right) - \frac{d\gamma\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right) - \beta d\gamma x + c\lambda\left(e^{\lambda y}\right)^{\gamma/\lambda}}{a\beta\gamma e^{\lambda y} - b\gamma\lambda e^{\beta x}} - \frac{a\beta c\lambda\left(e^{\lambda y}\right)^{\frac{\gamma+\lambda}{\lambda}}\,{}_2F_1\left(1,\frac{\gamma+\lambda}{\lambda};\frac{\gamma}{\lambda}+\right.}{(\gamma+\lambda)\left(a\beta e^{\lambda y} - b\lambda e\right.}\right.$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c*exp(gamma*y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(c\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-(-\mathrm{e}^{-a\beta}+\mathrm{e}^{\beta x})b\lambda}{a\beta}\right)^{\frac{\gamma}{\lambda}}+d\right)\beta}{a\beta\,\mathrm{e}^{\lambda y}-(-\mathrm{e}^{-a\beta}+\mathrm{e}^{\beta x})\,b\lambda} d\_a + \_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)$$

6.3.7 3.2

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6.3.7.1 [888] Problem 1

problem number 888

Added Feb. 9, 2019.

Problem Chapter 3.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cye^{\lambda x} + kxe^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*y*Exp[lambda*x] + k*x*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{bce^{\lambda x}}{a^2\lambda^2}-\frac{ake^{\mu y}}{b^2\mu^2}+\frac{cye^{\lambda x}}{a\lambda}+\frac{kxe^{\mu y}}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*y*exp(lambda*x)+k*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a^2b^2\lambda^2\mu^2\_F1\left(\frac{ay-bx}{a}\right)-(-b\mu x+a)\,a^2k\lambda^2e^{\mu y}+(a\lambda y-b)\,b^2c\mu^2e^{\lambda x}}{a^2b^2\lambda^2\mu^2}$$

6.3.7.2 [889] Problem 2

problem number 889

Added Feb. 9, 2019.

Problem Chapter 3.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=ax^ke^{\lambda y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == a*x^k*Exp[lambda*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{x^k(-a\lambda x)^{-k}e^{\lambda(y-ax)}\,\text{Gamma}(k+1,-a\lambda x)}{\lambda}+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +a*diff(w(x,y),y) =a*x^k*exp(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(-\Gamma(k) + \Gamma(k, -a\lambda x))\, k x^k (-a\lambda x)^{-k} \mathrm{e}^{(-ax+y)\lambda} + \lambda\_F1(-ax+y) + x^k \mathrm{e}^{\lambda y}}{\lambda}$$

6.3.7.3 [890] Problem 3

problem number 890

Added Feb. 9, 2019.

Problem Chapter 3.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + be^{\lambda x}) w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*Exp[lambda*x])*D[w[x, y], y] == c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ce^{\beta x}}{\beta} + c_1\left(e^{-ax}\left(\frac{be^{\lambda x}}{a-\lambda} + y\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*y+b*exp(lambda*x))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{\beta\_F1\left(\frac{\left(b\,\mathrm{e}^{ax} + (a-\lambda) y\,\mathrm{e}^{(a-\lambda)x}\right)\mathrm{e}^{-(2a-\lambda)x}}{a-\lambda}\right) + c\,\mathrm{e}^{\beta x}}{\beta}$$

6.3.7.4 [891] Problem 4

problem number 891

Added Feb. 9, 2019.

Problem Chapter 3.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (aye^{\lambda x} + be^{\beta x}y^k)w_y = ce^{\mu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y*Exp[lambda*x] + b*Exp[beta*x]*y^k)*D[w[x, y], y] == c*Exp[mu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ce^{\mu x}}{\mu} + c_1\left((k-1)\int_1^x be^{\frac{ae^{\lambda K[1]}(k-1)}{\lambda}+\beta K[1]}dK[1] + y^{1-k}e^{\frac{a(k-1)e^{\lambda x}}{\lambda}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*y*exp(lambda*x)+b*exp(beta*x)*y^k)*diff(w(x,y),y) =c*exp(mu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\,\mathrm{e}^{\mu x} + \mu\_F1\left(\left((k-1)\,by^k\left(\int \mathrm{e}^{\frac{\beta\lambda x+(k-1)a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right)\mathrm{e}^{\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}} + y\,\mathrm{e}^{\frac{ak\,\mathrm{e}^{\lambda x}}{\lambda}}\right)y^{-k}\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)}{\mu}$$

6.3.7.5 [892] Problem 5

problem number 892

Added Feb. 9, 2019.

Problem Chapter 3.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^k + bx^n e^{\lambda y})w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*x^k + b*x^n*Exp[lambda*y])*D[w[x, y], y] == c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ce^{\beta x}}{\beta} + c_1\left(\frac{b\lambda x^{n+1}\left(-\frac{a\lambda x^{k+1}}{k+1}\right)^{-\frac{n+1}{k+1}} \text{Gamma}\left(\frac{n+1}{k+1}, -\frac{a\lambda x^{k+1}}{k+1}\right) - (k+1)e^{-\frac{\lambda\left(-ax^{k+1}+ky+y\right)}{k+1}}}{ab(k+1)\lambda^2(k-n)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +(a*x^k+b*x^n*exp(lambda*y))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{\beta\_F1\left(\frac{-\left(a\lambda x^{n+1}+(-k-n-2)x^{-k+n}\right)(k+1)^2 b\left(-\frac{a\lambda x^{k+1}}{k+1}\right)^{\frac{-k-n-2}{2k+2}} \text{WhittakerM}\left(\frac{-k+n}{2k+2}, \frac{2k+n+3}{2k+2}, -\frac{a\lambda x^{k+1}}{k+1}\right) e^{\frac{a\lambda x^{k+1}}{2k+2}} - 2\left(-\right.}{(n}\right.}{}$$

6.3.7.6 [893] Problem 6

problem number 893

Added Feb. 9, 2019.

Problem Chapter 3.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*x*exp(lambda *x+ mu* y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\,\mathrm{e}^{\lambda x+\mu y}}{\lambda + \frac{\mu y}{x}} + \_F1\left(\frac{y}{x}\right)$$

6.3.7.7 [894] Problem 7

problem number 894

Added Feb. 9, 2019.

Problem Chapter 3.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = aye^{\lambda x} + bxe^{\mu y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*y*Exp[lambda*x] + b*x*Exp[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) + \frac{aye^{\lambda x}}{\lambda x} + \frac{bxe^{\mu y}}{\mu y}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*y*exp(lambda*x)  + b*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ay\,\mathrm{e}^{\lambda x}}{\lambda x} + \frac{bx\,\mathrm{e}^{\mu y}}{\mu y} + \_F1\left(\frac{y}{x}\right)$$

6.3.7.8 [895] Problem 8

problem number 895

Added Feb. 9, 2019.

Problem Chapter 3.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + be^{\lambda y} w_y = cx^n + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == c*x^n + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{1-k}\left(\frac{cx^n}{a(-k)+an+a}+\frac{s}{a-ak}\right)+c_1\left(\frac{bx^{1-k}}{a(k-1)}-\frac{e^{-\lambda y}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^k*diff(w(x,y),x) +b*exp(lambda*y)*diff(w(x,y),y) =c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx^{-k+n+1}}{(-k+n+1)\,a}-\frac{sx^{-k+1}}{(k-1)\,a}+\_F1\left(\frac{b\lambda x^{-k+1}-(k-1)\,a\,\mathrm{e}^{-\lambda y}}{(k-1)\,b\lambda}\right)$$

6.3.7.9 [896] Problem 9

problem number 896

Added Feb. 9, 2019.

Problem Chapter 3.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x+be^{\lambda x}w_y=ce^{\mu x}+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == c*Exp[mu*x] + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\left((k+1)\mu s\,{}_2F_1\left(1,\frac{1}{k+1};\frac{k+2}{k+1};\frac{a\lambda y^{k+1}}{a\lambda y^{k+1}-be^{\lambda x}(k+1)}\right)-c\lambda e^{\mu x}\,{}_2F_1\left(1,\frac{\lambda+k\mu+\mu}{k\lambda+\lambda};\frac{\lambda}{}\right.\right.}{\mu\left(b(k+1)e^{\lambda x}-a\lambda y^{k+1}\right)}\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x) +b*exp(lambda*x)*diff(w(x,y),y) =c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{\left(c\,\mathrm{e}^{-a\mu}+s\right)\left(\left(\frac{a\lambda y^{k+1}+(k+1)b\,\mathrm{e}^{-a\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k}}{a}d\_a+\_F1\left(\frac{a\lambda y y^{k}-(k+1)\,b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)$$

6.3.7.10 [897] Problem 10

problem number 897

Added Feb. 9, 2019.

Problem Chapter 3.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x+by^kw_y=cx^n+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*x^n + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-cx^n(\lambda x)^{-n}\,\text{Gamma}(n+1,\lambda x)+a\lambda c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{y^{1-k}}{k-1}\right)-se^{-\lambda x}}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*y^k*diff(w(x,y),y) = c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx^n(\lambda x)^{-\frac{n}{2}}\,\text{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},\lambda x\right)\mathrm{e}^{-\frac{\lambda x}{2}}+(n+1)\,a\lambda\_F1\left(\frac{a\lambda y^{-k+1}-(k-1)b\,\mathrm{e}^{-\lambda x}}{a\lambda}\right)-(n+1)\,(\mathrm{e}}{(n+1)\,a\lambda}$$

6.3.7.11 [898] Problem 11

problem number 898

Added Feb. 9, 2019.

Problem Chapter 3.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+bx^kw_y=cExp[\mu x]+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == c*Exp[mu*x] + s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{(k+1)\left(e^{\mu K[1]}c+s\right)}{ae^{\lambda y}(k+1)+b\lambda\left(K[1]^{k+1}-x^{k+1}\right)}dK[1]+c_1\left(\frac{e^{\lambda y}}{\lambda}-\frac{bx^{k+1}}{ak+a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*x^k*diff(w(x,y),y) = c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{(c\,\mathrm{e}^{-a\mu}+s)\,(k+1)}{b\lambda\_a^{k+1}-b\lambda x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}d\_a+\_F1\left(\frac{-b\lambda x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}{(k+1)\,b\lambda}\right)$$

6.3.8 4.1

Local contents

6.3.8.1 [899] Problem 1 1581
6.3.8.2 [900] Problem 2 1582
6.3.8.3 [901] Problem 3 1583
6.3.8.4 [902] Problem 4 1584
6.3.8.5 [903] Problem 5 1585

6.3.8.1 [899] Problem 1

problem number 899

Added Feb. 9, 2019.

Problem Chapter 3.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\sinh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Sinh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{c\cosh(\lambda x)}{a\lambda} + \frac{k\cosh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x)+k*sinh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ab\lambda\mu\_F1\left(\frac{ay-bx}{a}\right) + ak\lambda\cosh(\mu y) + bc\mu\cosh(\lambda x)}{ab\lambda\mu}$$

6.3.8.2 [900] Problem 2

problem number 900

Added Feb. 9, 2019.

Problem Chapter 3.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\cosh(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c\cosh\left(\lambda x+\mu y\right)}{a\lambda+b\mu}+\_F1\left(\frac{ay-bx}{a}\right)$$

6.3.8.3 [901] Problem 3

problem number 901

Added Feb. 9, 2019.

Problem Chapter 3.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cx\sinh(\lambda x+\mu y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x*Sinh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*x*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-(a\sinh(\lambda x+\mu y) - (a\lambda+b\mu)\,x\cosh(\lambda x+\mu y))\,c + (a\lambda+b\mu)^2\,\_F1\left(\frac{ay-bx}{a}\right)}{(a\lambda+b\mu)^2}$$

6.3.8.4 [902] Problem 4

problem number 902

Added Feb. 9, 2019.

Problem Chapter 3.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda x)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted Kernel Exception

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(lambda*x)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\sinh^m(\_a\mu)) + s\left(\sinh^k\left(\frac{(a\lambda y + b\cosh(\_a\lambda) - b\cosh(\lambda x))\beta}{a\lambda}\right)\right)}{a} d\_a + \_F1\left(\frac{a\lambda y - b\cosh(\lambda x)}{a\lambda}\right)$$

6.3.8.5 [903] Problem 5

problem number 903

Added Feb. 9, 2019.

Problem Chapter 3.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*y]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{s\sinh^k\left(\frac{2\beta\tanh^{-1}\left(e^{\frac{b\lambda(K[1]-x)}{a}}\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda}\right) + c\sinh^m(\mu K[1])}{a} dK[1] + c_1\left(\frac{\log\left(\tanh\left(\frac{\lambda y}{2}\right)\right)}{\lambda} -\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(lambda*y)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\sinh^m(\_a\mu)) + s\left(\sinh^k\left(\frac{\beta\ln\left(\tanh\left(\frac{a\,\mathrm{arctanh}\left(e^{\lambda y}\right) + \frac{(-\_a+x)b\lambda}{2}}{a}\right)\right)}{\lambda}\right)\right)}{a} d\_a + \_F1\left(\frac{-b\lambda x - 2a\,a}{b}\right.$$

6.3.9 4.2

Local contents

6.3.9.1 [904] Problem 1 1586
6.3.9.2 [905] Problem 2 1587
6.3.9.3 [906] Problem 3 1587
6.3.9.4 [907] Problem 4 1588
6.3.9.5 [908] Problem 5 1589

6.3.9.1 [904] Problem 1

problem number 904

Added Feb. 9, 2019.

Problem Chapter 3.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\cosh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x] + k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{c\sinh(\lambda x)}{a\lambda} + \frac{k\sinh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) + ak\lambda\sinh(\mu y) + bc\mu\sinh(\lambda x)}{ab\lambda\mu}$$

6.3.9.2 [905] Problem 2

problem number 905

Added Feb. 9, 2019.

Problem Chapter 3.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\sinh(\lambda x + \mu y)}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\sinh(\lambda x + \mu y)}{a\lambda + \mu b} + \_F1\left(\frac{ya - bx}{a}\right)$$

6.3.9.3 [906] Problem 3

problem number 906

Added Feb. 9, 2019.

Problem Chapter 3.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = ax\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{a(x(a\lambda + b\mu)\sinh(\lambda x + \mu y) - a\cosh(\lambda x + \mu y))}{(a\lambda + b\mu)^2} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = a*x*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(-a\cosh(\lambda x + \mu y) + (a\lambda + \mu b)\, x\sinh(\lambda x + \mu y))\, a + (a\lambda + \mu b)^2 \_F1\left(\frac{ya - bx}{a}\right)}{(a\lambda + \mu b)^2}$$

6.3.9.4 [907] Problem 4

problem number 907

Added Feb. 9, 2019.

Problem Chapter 3.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda x)w_y = c\cosh^m(\mu x) + s\cosh^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(lambda*x)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\cosh^m(\_b\mu)) + s\left(\cosh^k\left(\frac{\left(b\left(\int(\cosh^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\cosh^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a} d\_b + \_F1\left(y - \right.$$

6.3.9.5 [908] Problem 5

problem number 908

Added Feb. 9, 2019.

Problem Chapter 3.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda y)w_y = c\cosh^m(\mu x) + s\cosh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\cosh^{-n}(\lambda K[1])\left(s\cosh^k(\beta K[1])+c\cosh^m\left(\frac{\mu\left(\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(\lambda y)\right)\sinh(\lambda y)\cosh^{1-n}(\lambda y)}{\sqrt{-\sinh^2(\lambda y)}}-b\lambda\right.}{}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(lambda*y)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\cosh^m\left(\frac{(a(\int(\cosh^{-n}(\_b\lambda))d\_b)-a(\int(\cosh^{-n}(\lambda y))dy)+bx)\mu}{b}\right)\right)+s\left(\cosh^k(\_b\beta)\right)\right)\left(\cosh^{-n}(\_b\lambda)\right)}{b}$$

6.3.10 4.3

Local contents

6.3.10.1 [909] Problem 1 1591
6.3.10.2 [910] Problem 2 1591
6.3.10.3 [911] Problem 3 1592
6.3.10.4 [912] Problem 4 1593
6.3.10.5 [913] Problem 5 1594

6.3.10.1 [909] Problem 1

problem number 909

Added Feb. 9, 2019.

Problem Chapter 3.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\tanh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x] + k*Tanh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{c\log(\cosh(\lambda x))}{a\lambda} + \frac{k\log(\cosh(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x)+ k *tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) - bc\mu\ln(\tanh(\lambda x) - 1) - bc\mu\ln(\tanh(\lambda x) + 1) - (\ln(\tanh(\mu y) - 1) + \ln}{2ab\lambda\mu}$$

6.3.10.2 [910] Problem 2

problem number 910

Added Feb. 9, 2019.

Problem Chapter 3.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\log(\cosh(\lambda x + \mu y))}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-(\ln(\tanh(\lambda x + \mu y) - 1) + \ln(\tanh(\lambda x + \mu y) + 1))\, c + (2a\lambda + 2\mu b)\_F1\left(\frac{ya - bx}{a}\right)}{2a\lambda + 2\mu b}$$

6.3.10.3 [911] Problem 3

problem number 911

Added Feb. 11, 2019.

Problem Chapter 3.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\log(\cosh(\lambda x+\mu y))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-ax\ln\left(\tanh\left(\lambda x+\mu y\right)-1\right)-ax\ln\left(\tanh\left(\lambda x+\mu y\right)+1\right)+2\lambda x\_F1\left(\frac{y}{x}\right)+2\mu y\_F1\left(\frac{y}{x}\right)}{2\lambda x+2\mu y}$$

6.3.10.4 [912] Problem 4

problem number 912

Added Feb. 11, 2019.

Problem Chapter 3.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tanh^n(\lambda x)w_y=c\tanh^m(\mu x)+s\tanh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\tanh^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\lambda x)\right)\tanh^{n+1}(\lambda x)+b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\lambda K[1])\right)\tanh^{n+1}(\lambda K[1])+a\lambda(n\right.}{a\lambda(n+1)}\right.}{a}\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*tanh(lambda*x)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\tanh^m(\_b\mu))+s\left(\frac{\sinh\left(\frac{\left(b\left(\int(\tanh^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\tanh^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}{\cosh\left(\frac{\left(b\left(\int(\tanh^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\tanh^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\right.\right.$$

6.3.10.5 [913] Problem 5

problem number 913

Added Feb. 11, 2019.

Problem Chapter 3.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tanh^n(\lambda y)w_y=c\tanh^m(\mu x)+s\tanh^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\tanh^{-n}(\lambda K[1])\left(s\tanh^k(\beta K[1])+c\tanh^m\left(\frac{-a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\tanh^2(\lambda y)\right)\tanh^{1-n}(\lambda y)+a\mu\,{}_2F_1}{b\lambda}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*tanh(lambda*y)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(-\frac{\sinh\left(\frac{\left(-a\left(\int\left(\tanh^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}{\cosh\left(\frac{\left(-a\left(\int\left(\tanh^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}\right)^m+s\left(\tanh^k(\_b\beta)\right)\right)\left(\tanh^{-n}(\_b\lambda\right.}{b}$$

6.3.11 4.4

Local contents

6.3.11.1 [914] Problem 1 1596
6.3.11.2 [915] Problem 2 1597
6.3.11.3 [916] Problem 3 1597
6.3.11.4 [917] Problem 4 1598
6.3.11.5 [918] Problem 5 1599

6.3.11.1 [914] Problem 1

problem number 914

Added Feb. 11, 2019.

Problem Chapter 3.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\coth(\lambda x) + k\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x] + k*Coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ak\lambda \log(\tanh(\mu y)) + ak\lambda \log(\cosh(\mu y)) + bc\mu \log(\sinh(\lambda x))}{ab\lambda\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x)+k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) - bc\mu \ln(\coth(\lambda x) - 1) - bc\mu \ln(\coth(\lambda x) + 1) - (\ln(\coth(\mu y) - 1) + \ln($$

6.3.11.2 [915] Problem 2

problem number 915

Added Feb. 11, 2019.

Problem Chapter 3.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c(\log(\tanh(\lambda x + \mu y)) + \log(\cosh(\lambda x + \mu y)))}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-(\ln(\coth(\lambda x + \mu y) - 1) + \ln(\coth(\lambda x + \mu y) + 1))\, c + (2a\lambda + 2\mu b)\, \_F1\left(\frac{ya - bx}{a}\right)}{2a\lambda + 2\mu b}$$

6.3.11.3 [916] Problem 3

problem number 916

Added Feb. 11, 2019.

Problem Chapter 3.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ax(\log(\tanh(\lambda x + \mu y)) + \log(\cosh(\lambda x + \mu y)))}{\lambda x + \mu y} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-ax\ln(\coth(\lambda x + \mu y) - 1) - ax\ln(\coth(\lambda x + \mu y) + 1) + 2\lambda x\_F1\left(\frac{y}{x}\right) + 2\mu y\_F1\left(\frac{y}{x}\right)}{2\lambda x + 2\mu y}$$

6.3.11.4 [917] Problem 4

problem number 917

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda x)w_y = c\coth^m(\mu x) + s\coth^k(\beta y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*coth(lambda*x)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{c(\coth^{m}(\_b\mu)) + s\left(\frac{\cosh\left(\frac{\left(b\left(\int(\coth^{n}(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\coth^{n}(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}{\sinh\left(\frac{\left(b\left(\int(\coth^{n}(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\coth^{n}(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)}\right)^{k}}{a} d\_b + \_F1\left(y - \left(\int \right.\right.$$

6.3.11.5 [918] Problem 5

problem number 918

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda y)w_y = c\coth^m(\mu x) + s\coth^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{y} \frac{\left(s\coth^{k}(\beta K[1]) + c\coth^{m}\left(\frac{-a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\coth^2(\lambda y)\right)\coth^{1-n}(\lambda y)+b\lambda\mu x-b\lambda\mu n x+a\mu\coth^{1-n}(\lambda\right.}{b\lambda-b\lambda n}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*coth(lambda*y)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{\left(c\left(-\frac{\cosh\left(\frac{\left(-a\left(\int\left(\coth^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}{\sinh\left(\frac{\left(-a\left(\int\left(\coth^{-n}(\_b\lambda)\right)d\_b\right)+a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)-bx\right)\mu}{b}\right)}\right)^{m} + s\left(\coth^{k}(\_b\beta)\right)\right)\left(\coth^{-n}(\_b\lambda)\right.}{b}$$

6.3.12 4.5

Local contents

6.3.12.1 [919] Problem 1 1600
6.3.12.2 [920] Problem 2 1601
6.3.12.3 [921] Problem 3 1602
6.3.12.4 [922] Problem 4 1603
6.3.12.5 [923] Problem 5 1603

6.3.12.1 [919] Problem 1

problem number 919

Added Feb. 11, 2019.

Problem Chapter 3.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*sinh(lambda*x)+ k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right)+ak\lambda\sinh(\mu y)+bc\mu\cosh(\lambda x)}{ab\lambda\mu}$$

6.3.12.2 [920] Problem 2

problem number 920

Added Feb. 11, 2019.

Problem Chapter 3.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=\tanh(\lambda x)+k\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Tanh[lambda*x] + k*Coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{\log(\cosh(\lambda x))}{a\lambda}+\frac{k(\log(\tanh(\mu y))+\log(\cosh(\mu y)))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  tanh(lambda*x)+ k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) + \frac{k\ln(\sinh(\mu y))}{b\mu} + \frac{\ln(\cosh(\lambda x))}{a\lambda}$$

6.3.12.3 [921] Problem 3

problem number 921

Added Feb. 11, 2019.

Problem Chapter 3.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sinh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sinh[lambda*x] + k*Tanh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y-\frac{bx}{a}\right) + \frac{\cosh(\lambda x)}{a\lambda} + \frac{k\log(\cosh(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  sinh(lambda*x)+ k*tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) + \frac{k\ln(\cosh(\mu y))}{b\mu} + \frac{\cosh(\lambda x)}{a\lambda}$$

6.3.12.4 [922] Problem 4

problem number 922

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\cosh(\mu y)w_y = \sinh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == Sinh[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{\cosh(\lambda x)}{a\lambda} + c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*cosh(mu*y)*diff(w(x,y),y) =  sinh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\lambda\_F1\left(\frac{-b\mu x+2a\arctan(e^{\mu y})}{b\mu}\right) + \cosh(\lambda x)}{a\lambda}$$

6.3.12.5 [923] Problem 5

problem number 923

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\sinh(\mu y)w_y = \cosh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[mu*y]*D[w[x, y], y] == Cosh[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{\sinh(\lambda x)}{a\lambda} + c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*sinh(mu*y)*diff(w(x,y),y) =  cosh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\lambda\_F1\left(\frac{-b\mu x - 2a\,\mathrm{arctanh}(\mathrm{e}^{\mu y})}{b\mu}\right) + \sinh(\lambda x)}{a\lambda}$$

6.3.13 5.1

Local contents

6.3.13.1 [924] Problem 1 1605
6.3.13.2 [925] Problem 2 1605
6.3.13.3 [926] Problem 3 1606
6.3.13.4 [927] Problem 4 1607
6.3.13.5 [928] Problem 5 1608
6.3.13.6 [929] Problem 6 1609

6.3.13.1 [924] Problem 1

problem number 924

Added Feb. 11, 2019.

Problem Chapter 3.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\ln(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{c(a\beta y - b\beta x)\log(a(\beta y + \lambda x))}{a(a\lambda + b\beta)} + \frac{cx\log(\beta y + \lambda x)}{a} - \frac{cx}{a}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(\ln(\beta y + \lambda x) - 1)(\beta y + \lambda x)c + (a\lambda + b\beta)\_F1\left(\frac{ya - bx}{a}\right)}{a\lambda + b\beta}$$

6.3.13.2 [925] Problem 2

problem number 925

Added Feb. 11, 2019.

Problem Chapter 3.5.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\ln(\lambda x) + k\ln(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x] + k*Log[beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{abc_1\left(y - \frac{bx}{a}\right) + aky\log(\beta y) + bcx\log(\lambda x) - bcx - bkx}{ab}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x)+k*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{aky\ln(\beta y) + bcx\ln(\lambda x) + ab\_F1\left(\frac{ya-bx}{a}\right) - aky - bcx}{ab}$$

6.3.13.3 [926] Problem 3

problem number 926

Added Feb. 11, 2019.

Problem Chapter 3.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln(\lambda x)\ln(\beta y)w_y = c\ln(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]*Log[beta*y]*D[w[x, y], y] == c*Log[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\beta\lambda c_1\left(\frac{\text{li}(\beta y)}{\beta}-\frac{bx(\log(\lambda x)-1)}{a}\right)+c\left(a\lambda\text{li}(\beta y)-b\beta\left(\log\left(e^{W\left(\frac{\lambda x(\log(\lambda x)-1)}{e}\right)+1}\right)-1\right)\text{Ei}\left(\log\left(e\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)*ln(beta*y)*diff(w(x,y),y) =  c*ln(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-\left(\text{LambertW}\left(\left(\ln\left(\lambda x\right)-1\right)\mathrm{e}^{-1}\lambda x\right)-\ln\left(\frac{(\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda x}{\text{LambertW}((\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda x)}\right)\right)\left(\ln\left(\frac{(\ln(\lambda x)-1)\mathrm{e}^{-1}x}{\text{LambertW}((\ln(\lambda x)-1)\mathrm{e}^{-1}\lambda}\right.\right.}{}$$

6.3.13.4 [927] Problem 4

problem number 927

Added Feb. 11, 2019.

Problem Chapter 3.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda x)w_y=c\ln^m(\mu x)+s\ln^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\log^k\left(\frac{\beta(-b\,\mathrm{Gamma}(n+1,-\log(\lambda x))\log^n(\lambda x)(-\log(\lambda x))^{-n}+b\,\mathrm{Gamma}(n+1,-\log(\lambda K[1]))(-\log(\lambda K[1]))^{-n}\log^n}{a\lambda}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c\ln\left(\_b\mu\right)^m+s\ln\left(\frac{\left(b\left(\int\ln(\_b\lambda)^n d\_b\right)+\left(y-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)a\right)\beta}{a}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\frac{b\ln\left(\lambda x\right)^n}{a}d\right.\right.$$

6.3.13.5 [928] Problem 5

problem number 928

Added Feb. 11, 2019.

Problem Chapter 3.5.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda y)w_y=c\ln^m(\mu x)+s\ln^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\log^{-n}(\lambda K[1])\left(s\log^k(\beta K[1])+c\log^m\left(\frac{\mu(-a\,\mathrm{Gamma}(1-n,-\log(\lambda y))(-\log(\lambda y))^n\log^{-n}(\lambda y)+a\,\mathrm{Gamm}}{b\lambda}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*ln(lambda*y)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\ln\left(-\frac{\left(-a\left(\int\ln(\_b\lambda)^{-n}d\_b\right)+a\left(\int\ln(\lambda y)^{-n}dy\right)-bx\right)\mu}{b}\right)^m+s\ln(\_b\beta)^k\right)\ln(\_b\lambda)^{-n}}{b}d\_b+\_F1\left(-\right.$$

6.3.13.6 [929] Problem 6

problem number 929

Added Feb. 11, 2019.

Problem Chapter 3.5.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x+b\ln^k(\beta y)w_y=c\ln^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[lambda*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=a*ln(lambda*x)^n*diff(w(x,y),x) + b*ln(lambda*y)^k*diff(w(x,y),y) =  c*ln(gamma*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{c(\ln(x)+\ln(\gamma))^m \ln(\lambda x)^{-n}}{a} dx + \_F1\left(-\left(\int \ln(\lambda x)^{-n} dx\right) + \int \frac{a \ln(\lambda y)^{-k}}{b} dy\right)$$

6.3.14 5.2

Local contents

6.3.14.1 [930] Problem 1 1610
6.3.14.2 [931] Problem 2 1611
6.3.14.3 [932] Problem 3 1612

6.3.14.1 [930] Problem 1

problem number 930

Added Feb. 11, 2019.

Problem Chapter 3.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n + s\ln^k(\lambda y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + s*Log[lambda*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{x(-cx^n+ns+s)}{a(n+1)}+\frac{sy\log(\lambda y)}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*x^n+s*ln(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{bcx^{n+1}+\left(b\_F1\left(\frac{ya-bx}{a}\right)+(\ln(\lambda y)-1)\,sy\right)(n+1)\,a}{(n+1)\,ab}$$

6.3.14.2 [931] Problem 2

problem number 931

Added Feb. 11, 2019.

Problem Chapter 3.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=by^2+cx^ny+s\ln^k(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*y^2 + c*x^n*y + s*Log[lambda*x]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{(-\log(\lambda x))^{-k}\left(3(n^2+3n+2)\,s\log^k(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\lambda x))+3\lambda(n^2+3n+2)\,(-\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*y^2+c*x^n*y+s*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(ac\_a^{n+1} - (ax-y)\, c\_a^{n} + s\ln(\_a\lambda)^{k} + ((-\_a+x)\, a - y)^{2}\, b\right) d\_a + \_F1(-ax+y)$$

Result has unresolved integrals

6.3.14.3 [932] Problem 3

problem number 932

Added Feb. 11, 2019.

Problem Chapter 3.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = bln^k(\lambda x)\ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{x} b\log^{k}(\lambda K[1])\log^{n}(\beta(y+a(K[1]-x)))dK[1] + c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^{x} b \ln(\_a\lambda)^{k} \ln(-((-\_a + x)\, a - y)\, \beta)^{n}\, d\_a + \_F1(-ax + y)$$

6.3.15 5.3

Local contents

6.3.15.1 [933] Problem 4 1613
6.3.15.2 [934] Problem 5 1614
6.3.15.3 [935] Problem 6 1615

6.3.15.1 [933] Problem 4

problem number 933

Added Feb. 11, 2019.

Problem Chapter 3.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^n)w_y = cln^k(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\log^{k}(\lambda x)(-\log(\lambda x))^{-k}\,\mathrm{Gamma}(k+1,-\log(\lambda x))}{\lambda} + c_1\big(ba^{-n-1}\,\mathrm{Gamma}(n+1,ax) + ye^{-a}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) +  (a*y+b*x^n)*diff(w(x,y),y) =  c*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int c \ln(\lambda x)^k \, dx + \_F1 \left(\frac{\left(-bx^n (ax)^{-\frac{n}{2}} \operatorname{WhittakerM}\left(\frac{n}{2}, \frac{n}{2}+\frac{1}{2}, ax\right) e^{\frac{ax}{2}} + (n+1)\, ay\right) e^{-ax}}{(n+1)\, a} \right)$$

Result has unresolved integrals

6.3.15.2 [934] Problem 5

problem number 934

Added Feb. 11, 2019.

Problem Chapter 3.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k(n \ln x + m \ln y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ \left\{ w(x,y) \to \frac{x^k(akm \log(y) + akn \log(x) - an - bm)}{a^2 k^2} + c_1 \left(y x^{-\frac{b}{a}} \right) \right\} \right\}$$

Maple ✓

```
restart;
pde := a*x*diff(w(x,y),x) +  b*y*diff(w(x,y),y) =  x^k*(n*ln(x)+m*ln(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2a^2k^2\_F1\left(yx^{-\frac{b}{a}}\right) + \left(2akn\ln(x) - 2bm + \left(-\left(-2\ln\left(x^{\frac{b}{a}}\right) - 2\ln\left(yx^{-\frac{b}{a}}\right) + \left(i\mathrm{csgn}(iy)^3 - i\mathrm{cs}\right.\right.\right.}{}$$

6.3.15.3 [935] Problem 6

problem number 935

Added Feb. 11, 2019.

Problem Chapter 3.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + by^n w_y = c\ln^m(\lambda x) + s\ln^l(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == c*Log[lambda*x]^m + s*Log[beta*y]^l;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{K[1]^{-k}\left(s\log^l\left(\beta\left(\frac{a(k-1)x^k y^n K[1]^k}{a(k-1)x^k y K[1]^k - b(n-1)y^n\left(xK[1]^k - x^k K[1]\right)}\right)^{\frac{1}{n-1}}\right) + c\log^m(\lambda K[1])\right)}{a} dK[1] \right.\right.$$

Maple ✓

```
restart;
pde := a*x^k*diff(w(x,y),x) +  b*y^n*diff(w(x,y),y) =  c*ln(lambda*x)+s*ln(beta*y)^l;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(c\ln\left(\_a\lambda\right) + s\ln\left(\beta\left(\frac{(k-1)ay^{-n+1}+(n-1)b\_a^{-k+1}-(n-1)bx^{-k+1}}{(k-1)a}\right)^{-\frac{1}{n-1}}\right)^{l}\right)\_a^{-k}}{a} d\_a + \_F1\left(\frac{(k}{}\right.$$

6.3.16 6.1

Local contents

6.3.16.1 [936] Problem 1 1616
6.3.16.2 [937] Problem 2 1617
6.3.16.3 [938] Problem 3 1618
6.3.16.4 [939] Problem 4 1619
6.3.16.5 [940] Problem 5 1620

6.3.16.1 [936] Problem 1

problem number 936

Added Feb. 11, 2019.

Problem Chapter 3.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\sin(\lambda x) + k\sin(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x] + k*Sin[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)-\frac{c\cos(\lambda x)}{a\lambda}-\frac{k\cos(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x)+k*sin(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right)-ak\lambda\cos(\mu y)-bc\mu\cos(\lambda x)}{ab\lambda\mu}$$

6.3.16.2 [937] Problem 2

problem number 937

Added Feb. 11, 2019.

Problem Chapter 3.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+by^nw_y=c\sin(\lambda x+\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to-\frac{c\cos(\lambda x+\mu y)}{a\lambda+b\mu}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{c\cos(\lambda x + \mu y)}{a\lambda + \mu b} + \_F1\left(\frac{ya - bx}{a}\right)$$

6.3.16.3 [938] Problem 3

problem number 938

Added Feb. 11, 2019.

Problem Chapter 3.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{ax\cos(\lambda x + \mu y)}{\lambda x + \mu y} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{a\cos(\lambda x + \mu y)}{\lambda + \frac{\mu y}{x}} + \_F1\left(\frac{y}{x}\right)$$

6.3.16.4 [939] Problem 4

problem number 939

Added Feb. 11, 2019.

Problem Chapter 3.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{s\sin^k\left(\frac{\beta\left(-b\sqrt{\cos^2(\lambda x)}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)\sec(\lambda x)\sin^{n+1}(\lambda x)+b\sqrt{\cos^2(\lambda K[1])}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda K[\right.\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*sin(lambda*x)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\sin^m(\_b\mu)) + s\left(\sin^k\left(\frac{\left(b\left(\int(\sin^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\sin^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a}d\_b+\_F1\left(y-\left(\int b\right.\right.$$

6.3.16.5 [940] Problem 5

problem number 940

Added Feb. 11, 2019.

Problem Chapter 3.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^y \frac{\sin^{-n}(\lambda K[1])\left(s\sin^k(\beta K[1]) + c\sin^m\left(\frac{-a\mu\sqrt{\cos^2(\lambda y)}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\lambda y)\right)\sec(\lambda y)\sin^{1-n}(\lambda y)+c}{}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*sin(lambda*y)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{\left(c\left(\sin^m\left(\frac{\left(a\left(\int\left(\sin^{-n}(\_b\lambda)\right)d\_b\right)-a\left(\int\left(\sin^{-n}(\lambda y)\right)dy\right)+bx\right)\mu}{b}\right)\right)+s\left(\sin^k\left(\_b\beta\right)\right)\right)\left(\sin^{-n}\left(\_b\lambda\right)\right)}{b}d\_b+\_$$

Result has unresolved integrals

6.3.17 6.2

Local contents

6.3.17.1 [941] Problem 1 1621
6.3.17.2 [942] Problem 2 1622
6.3.17.3 [943] Problem 3 1622
6.3.17.4 [944] Problem 4 1623
6.3.17.5 [945] Problem 5 1624

6.3.17.1 [941] Problem 1

problem number 941

Added Feb. 11, 2019.

Problem Chapter 3.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\cos(\lambda x) + k\cos(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x] + k*Cos[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{c\sin(\lambda x)}{a\lambda} + \frac{k\sin(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x)+k*cos(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) + ak\lambda\sin(\mu y) + bc\mu\sin(\lambda x)}{ab\lambda\mu}$$

6.3.17.2 [942] Problem 2

problem number 942

Added Feb. 11, 2019.

Problem Chapter 3.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\sin(\lambda x + \mu y)}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\sin(\lambda x + \mu y)}{a\lambda + \mu b} + \_F1\left(\frac{ya - bx}{a}\right)$$

6.3.17.3 [943] Problem 3

problem number 943

Added Feb. 11, 2019.

Problem Chapter 3.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ax\sin(\lambda x + \mu y)}{\lambda x + \mu y} + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{a\sin(\lambda x + \mu y)}{\lambda + \frac{\mu y}{x}} + \_F1\left(\frac{y}{x}\right)$$

6.3.17.4 [944] Problem 4

problem number 944

Added Feb. 11, 2019.

Problem Chapter 3.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda x)w_y = c\cos^m(\mu x) + s\cos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\cos^k\left(\frac{\beta\left(b\csc(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(\lambda x)\right)\sqrt{\sin^2(\lambda x)}\cos^{n+1}(\lambda x)+a\lambda(n+1)y-b\cos^{n+1}(\lambda K[1])\csc(\lambda K[1])\,{}_2F_1\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cos(lambda*x)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\cos^m(\_b\mu))+s\left(\cos^k\left(\frac{\left(b\left(\int(\cos^n(\_b\lambda))d\_b\right)+\left(y-\left(\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\right)a\right)\beta}{a}\right)\right)}{a}d\_b+\_F1\left(y-\left(\int\right.\right.$$

6.3.17.5 [945] Problem 5

problem number 945

Added Feb. 11, 2019.

Problem Chapter 3.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cos^n(\lambda y)w_y=c\cos^m(\mu x)+s\cos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\cos^{-n}(\lambda K[1])\left(s\cos^k(\beta K[1])+c\cos^m\left(\frac{\mu\left(a\csc(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\lambda y)\right)\sqrt{\sin^2(\lambda y)}\cos^{1-n}(\lambda y)\right.}{}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cos(lambda*y)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\cos^m\left(\frac{(a(\int(\cos^{-n}(\_b\lambda))d\_b)-a(\int(\cos^{-n}(\lambda y))dy)+bx)\mu}{b}\right)\right)+s\left(\cos^k(\_b\beta)\right)\right)(\cos^{-n}(\_b\lambda))}{b}d\_b+$$

6.3.18 6.3

Local contents

6.3.18.1 [946] Problem 1 1626
6.3.18.2 [947] Problem 2 1626
6.3.18.3 [948] Problem 3 1627
6.3.18.4 [949] Problem 4 1628
6.3.18.5 [950] Problem 5 1629

6.3.18.1 [946] Problem 1

problem number 946

Added Feb. 11, 2019.

Problem Chapter 3.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\tan(\lambda x) + k\tan(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x] + k*Tan[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) - \frac{c\log(\cos(\lambda x))}{a\lambda} - \frac{k\log(\cos(\mu y))}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x)+k*tan(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) + ak\lambda\ln\left(\tan^2(\mu y)+1\right) + bc\mu\ln\left(\tan^2(\lambda x)+1\right)}{2ab\lambda\mu}$$

6.3.18.2 [947] Problem 2

problem number 947

Added Feb. 11, 2019.

Problem Chapter 3.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{c\log(\cos(\lambda x + \mu y))}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c\ln\left(\tan^2(\lambda x + \mu y) + 1\right)}{2a\lambda + 2\mu b} + \_F1\left(\frac{ya - bx}{a}\right)$$

6.3.18.3 [948] Problem 3

problem number 948

Added Feb. 11, 2019.

Problem Chapter 3.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{ax\log(\cos(\lambda x+\mu y))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\ln\left(\tan^2\left(\lambda x+\mu y\right)+1\right)}{2\lambda+\frac{2\mu y}{x}}+\_F1\left(\frac{y}{x}\right)$$

6.3.18.4 [949] Problem 4

problem number 949

Added Feb. 11, 2019.

Problem Chapter 3.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda x)w_y=c\tan^m(\mu x)+s\tan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\tan^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda x)\right)\tan^{n+1}(\lambda x)+b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda K[1])\right)\tan^{n+1}(\lambda K[1])+a\lambda(n+1\right.}{a\lambda(n+1)}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(lambda*x)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\tan^m(\_b\mu))+s\left(\frac{\tan\left(\left(y-\left(\int\frac{b(\tan^n(\lambda x))}{a}dx\right)\right)\beta\right)+\tan\left(\frac{b\beta\left(\int(\tan^n(\_b\lambda))d\_b\right)}{a}\right)}{-\tan\left(\left(y-\left(\int\frac{b(\tan^n(\lambda x))}{a}dx\right)\right)\beta\right)\tan\left(\frac{b\beta\left(\int(\tan^n(\_b\lambda))d\_b\right)}{a}\right)+1}\right)^k}{a}d\_b+\_F1\left(y-\left(\int\right.\right.$$

6.3.18.5 [950] Problem 5

problem number 950

Added Feb. 11, 2019.

Problem Chapter 3.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda y)w_y=c\tan^m(\mu x)+s\tan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\tan^{-n}(\lambda K[1])\left(s\tan^k(\beta K[1])+c\tan^m\left(\frac{-a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(\lambda y)\right)\tan^{1-n}(\lambda y)+a\mu\,{}_2F_1\left(1,\frac{1}{2}-\right.}{b\lambda-b\lambda n}\right.\right.}{b}\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(lambda*y)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\frac{-\tan\left(\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)-\tan\left(\frac{a\mu\left(\int\left(\tan^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)}{\tan\left(\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\tan\left(\frac{a\mu\left(\int\left(\tan^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)-1}\right)^m+s\left(\tan^k(\_b\beta)\right)\right)\left(\tan^{-n}(\_\right.}{b}$$

6.3.19 6.4

Local contents

6.3.19.1 [951] Problem 1 1631
6.3.19.2 [952] Problem 2 1631
6.3.19.3 [953] Problem 3 1632
6.3.19.4 [954] Problem 4 1633
6.3.19.5 [955] Problem 5 1634

6.3.19.1 [951] Problem 1

problem number 951

Added Feb. 11, 2019.

Problem Chapter 3.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\cot(\lambda x) + k\cot(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x] + k*Cot[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{ak\lambda\log(\tan(\mu y)) + ak\lambda\log(\cos(\mu y)) + bc\mu\log(\sin(\lambda x))}{ab\lambda\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x)+k*cot(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{2ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) - ak\lambda\ln\left(\cot^2(\mu y)+1\right) - bc\mu\ln\left(\cot^2(\lambda x)+1\right)}{2ab\lambda\mu}$$

6.3.19.2 [952] Problem 2

problem number 952

Added Feb. 11, 2019.

Problem Chapter 3.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + by^n w_y = c\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c(\log(\tan(\lambda x + \mu y)) + \log(\cos(\lambda x + \mu y)))}{a\lambda + b\mu} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{c\ln\left(\cot^2(\lambda x + \mu y) + 1\right)}{2a\lambda + 2\mu b} + \_F1\left(\frac{ya - bx}{a}\right)$$

6.3.19.3 [953] Problem 3

problem number 953

Added Feb. 11, 2019.

Problem Chapter 3.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{\lambda x+\mu y}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=-\frac{a\ln\left(\cot^2\left(\lambda x+\mu y\right)+1\right)}{2\left(\lambda+\frac{\mu y}{x}\right)}+\_F1\left(\frac{y}{x}\right)$$

6.3.19.4 [954] Problem 4

problem number 954

Added Feb. 11, 2019.

Problem Chapter 3.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cot^n(\lambda x)w_y=c\cot^m(\mu x)+s\cot^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{s\cot^k\left(\frac{\beta\left(b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda x)\right)\cot^{n+1}(\lambda x)+a\lambda(n+1)y-b\cot^{n+1}(\lambda K[1])\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda K[1])\right)\right)}{a\lambda(n+1)}\right.}{a}\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cot(lambda*x)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c(\cot^m(\_b\mu))+s\left(\frac{\cot\left(\left(y-\left(\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\right)\beta\right)\cot\left(\frac{b\beta\left(\int(\cot^n(\_b\lambda))d\_b\right)}{a}\right)-1}{\cot\left(\left(y-\left(\int\frac{b(\cot^n(\lambda x))}{a}dx\right)\right)\beta\right)+\cot\left(\frac{b\beta\left(\int(\cot^n(\_b\lambda))d\_b\right)}{a}\right)}\right)^k}{a}d\_b+\_F1\left(y-\left(\int b\right.\right.$$

6.3.19.5 [955] Problem 5

problem number 955

Added Feb. 11, 2019.

Problem Chapter 3.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cot^n(\lambda y)w_y=c\cot^m(\mu x)+s\cot^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^y \frac{\left(s\cot^k(\beta K[1])+c\cot^m\left(\frac{a\mu\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\cot^2(\lambda y)\right)\cot^{1-n}(\lambda y)+b\lambda\mu x-b\lambda\mu nx-a\mu\cot^{1-n}(\lambda K[1])\,{}_2}{b\lambda-b\lambda n}\right.\right.}{b}\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*cot(lambda*y)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y \frac{\left(c\left(\frac{\cot\left(\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\cot\left(\frac{a\mu\left(\int\left(\cot^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)-1}{\cot\left(\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)+\cot\left(\frac{a\mu\left(\int\left(\cot^{-n}(\_b\lambda)\right)d\_b\right)}{b}\right)}\right)^m+s\left(\cot^k(\_b\beta)\right)\right)\left(\cot^{-n}(\_b\lambda\right.}{b}$$

6.3.20 6.5

Local contents

6.3.20.1 [956] Problem 1 1636
6.3.20.2 [957] Problem 2 1636
6.3.20.3 [958] Problem 3 1637
6.3.20.4 [959] Problem 4 1638
6.3.20.5 [960] Problem 5 1639
6.3.20.6 [961] Problem 6 1640

6.3.20.1 [956] Problem 1

problem number 956

Added Feb. 11, 2019.

Problem Chapter 3.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = \sin(\lambda x) + c\cos(\mu y) + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x] + c*Cos[mu*y] + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{kx}{a} - \frac{\cos(\lambda x)}{a\lambda} + \frac{c\sin(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)+c*cos(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{kx}{a} + \frac{ab\lambda\mu\_F1\left(\frac{ya-bx}{a}\right) + ac\lambda\sin(\mu y) - b\mu\cos(\lambda x)}{ab\lambda\mu}$$

6.3.20.2 [957] Problem 2

problem number 957

Added Feb. 11, 2019.

Problem Chapter 3.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = \tan(\lambda x) + c\sin(\mu y) + k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Tan[lambda*x] + c*Sin[mu*y] + k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) + \frac{k\lambda x - \log(\cos(\lambda x))}{a\lambda} - \frac{c\cos(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  tan(lambda*x)+c*sin(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{kx}{a} + \_F1\left(\frac{ya - bx}{a}\right) - \frac{c\cos(\mu y)}{b\mu} - \frac{\ln(\cos(\lambda x))}{a\lambda}$$

6.3.20.3 [958] Problem 3

problem number 958

Added Feb. 11, 2019.

Problem Chapter 3.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sin(\lambda x)\cos(\mu y) + c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x]*Cos[mu*y] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{(a^3\lambda^2-ab^2\mu^2)\,c_1\left(y-\frac{bx}{a}\right)+a^2c\lambda^2x-a^2\lambda\cos(\lambda x)\cos(\mu y)-ab\mu\sin(\lambda x)\sin(\mu y)-b^2c\mu^2x}{a(a\lambda-b\mu)(a\lambda+b\mu)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)*cos(mu*y)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}+\_F1\left(\frac{ya-bx}{a}\right)-\frac{(a\lambda+\mu b)\cos(\lambda x-\mu y)+(a\lambda-\mu b)\cos(\lambda x+\mu y)}{2\,(a\lambda-\mu b)\,(a\lambda+\mu b)}$$

6.3.20.4 [959] Problem 4

problem number 959

Added Feb. 11, 2019.

Problem Chapter 3.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\sin(\mu y)w_y=\cos(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == Cos[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)+c\lambda x+\sin(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*sin(mu*y)*diff(w(x,y),y) =  cos(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}\right)\right)\right)}{b\mu}\right)+c\lambda x+\sin(\lambda x)}{a\lambda}$$

6.3.20.5 [960] Problem 5

problem number 960

Added Feb. 11, 2019.

Problem Chapter 3.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan(\mu y)w_y=\sin(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Sin[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\lambda x-\cos(\lambda x)}{a\lambda}+c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  sin(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\lambda\_F1\left(\frac{-b\mu x+a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)+c\lambda x-\cos(\lambda x)}{a\lambda}$$

6.3.20.6 [961] Problem 6

problem number 961

Added Feb. 11, 2019.

Problem Chapter 3.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan(\mu y)w_y=\cot(\lambda y)+c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Cot[lambda*x] + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)+c\lambda x+\log(\sin(\lambda x))}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  cot(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{2a\lambda\_F1\left(\frac{-b\mu x+a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)+2c\lambda x-\ln\left(\cot^2(\lambda x)+1\right)}{2a\lambda}$$

6.3.21 7.1

Local contents

6.3.21.1 [962] Problem 1 1642
6.3.21.2 [963] Problem 2 1643
6.3.21.3 [964] Problem 3 1644
6.3.21.4 [965] Problem 4 1645
6.3.21.5 [966] Problem 5 1646

6.3.21.1 [962] Problem 1

problem number 962

Added Feb. 11, 2019.

Problem Chapter 3.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arcsin\frac{x}{\lambda} + k\arcsin\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[x/lambda] + k*ArcSin[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2b\beta c_1\left(y-\frac{bx}{a}\right)-\frac{bkx\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{aky\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(-\right.}{\sqrt{1-\frac{y^2}{\beta^2}}}}{a^2b\beta}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arcsin(x/lambda)+k*arcsin(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{bcx\arcsin\left(\frac{x}{\lambda}\right)+ab\_F1\left(\frac{ya-bx}{a}\right)+\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda+\left(y\arcsin\left(\frac{y}{\beta}\right)+\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,\beta\right)ak}{ab}$$

6.3.21.2 [963] Problem 2

problem number 963

Added Feb. 11, 2019.

Problem Chapter 3.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c \arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\sqrt{-\beta^2 y^2 - 2\beta\lambda xy - \lambda^2 x^2 + 1} + (\beta y + \lambda x)\sin^{-1}(\beta y + \lambda x)\right)}{a\lambda + b\beta} + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(\beta y + \lambda x)\, c \arcsin(\beta y + \lambda x) + \sqrt{-\beta^2 y^2 - 2\beta\lambda xy - \lambda^2 x^2 + 1}\, c + (a\lambda + b\beta)\, \_F1\left(\frac{ya - bx}{a}\right)}{a\lambda + b\beta}$$

6.3.21.3 [964] Problem 3

problem number 964

Added Feb. 11, 2019.

Problem Chapter 3.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax \arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to ax\left(\frac{\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}}{\beta y + \lambda x} + \sin^{-1}(\beta y + \lambda x)\right) + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{\sqrt{-\left(\frac{\beta y}{x} + \lambda\right)^2 x^2 + 1}\, ax + (\beta y + \lambda x)\left(ax \arcsin(\beta y + \lambda x) + \_F1\left(\frac{y}{x}\right)\right)}{\beta y + \lambda x}$$

6.3.21.4 [965] Problem 4

problem number 965

Added Feb. 11, 2019.

Problem Chapter 3.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda x)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \left(\frac{\sin^{-1}\left(\frac{\beta\left(ib\sin^{-1}(\lambda x)^n\left((i\sin^{-1}(\lambda x))^n \operatorname{Gamma}(n+1,-i\sin^{-1}(\lambda x))-(-i\sin^{-1}(\lambda x))^n \operatorname{Gamma}(n+1,i\sin^{-1}(\lambda x)\right.\right.\right.}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*x)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \left(\arcsin(\_a\mu)^m + \frac{\arcsin\left(\frac{\left(\sqrt{-\_a^2\lambda^2+1}\,b-\sqrt{-\lambda^2x^2+1}\,b+(\_ab\arcsin(\_a\lambda)-bx\arcsin(\lambda x)+ay)\lambda\right)\beta}{a\lambda}\right)^k}{a}\right) d\_$$

6.3.21.5 [966] Problem 5

problem number 966

Added Feb. 11, 2019.

Problem Chapter 3.7.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda y)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^y \frac{\left(\sin^{-1}(\beta K[1])^k + a\sin^{-1}\left(\frac{\mu(ia((-i\sin^{-1}(\lambda y))^n \operatorname{Gamma}(1-n,-i\sin^{-1}(\lambda y))-(i\sin^{-1}(\lambda y))^n \operatorname{Gamma}(1-n}{}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*y)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{a\left(-\arcsin\left(\frac{(-b\lambda x - a\,\mathrm{cosineIntegral}(\arcsin(\_a\lambda)) + a\,\mathrm{cosineIntegral}(\arcsin(\lambda y)))\mu}{b\lambda}\right)\right)^m + \arcsin(\_a\beta)^k}{b\arcsin(\_a\lambda)} d\_a +$$

6.3.22 7.2

Local contents

6.3.22.1 [967] Problem 1 1647
6.3.22.2 [968] Problem 2 1648
6.3.22.3 [969] Problem 3 1649
6.3.22.4 [970] Problem 4 1650
6.3.22.5 [971] Problem 5 1651

6.3.22.1 [967] Problem 1

problem number 967

Added Feb. 11, 2019.

Problem Chapter 3.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arccos\frac{x}{\lambda} + k\arccos\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[x/lambda] + k*ArcCos[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2b\beta c_1\left(y-\frac{bx}{a}\right)+\frac{bkx\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}+\frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{aky\sqrt{a^2(\beta^2-y^2)}\tan^{-1}\left(\cdots\right.}{\sqrt{1-\frac{y^2}{\beta^2}}}}{a^2b\beta}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccos(x/lambda)+k*arccos(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{bcx \arccos\left(\frac{x}{\lambda}\right) + ab\_F1\left(\frac{ay-bx}{a}\right) - \sqrt{-\frac{x^2}{\lambda^2}+1}\, bc\lambda - \left(-y \arccos\left(\frac{y}{\beta}\right) + \sqrt{\frac{\beta^2-y^2}{\beta^2}}\, \beta\right) ak}{ab}$$

6.3.22.2 [968] Problem 2

problem number 968

Added Feb. 11, 2019.

Problem Chapter 3.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \arccos(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\beta(bx-ay)\sin^{-1}(\beta y+\lambda x) + x(a\lambda+b\beta)\cos^{-1}(\beta y+\lambda x) + a\left(-\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x}\right.\right.}{a(a\lambda+b\beta)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(\beta y + \lambda x)\, c \arccos(\beta y + \lambda x) - \sqrt{-\beta^2 y^2 - 2\beta\lambda x y - \lambda^2 x^2 + 1}\, c + (a\lambda + b\beta)\, \_F1\left(\frac{ay - bx}{a}\right)}{a\lambda + b\beta}$$

6.3.22.3 [969] Problem 3

problem number 969

Added Feb. 11, 2019.

Problem Chapter 3.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + y w_y = a x \arccos(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to a x \left(\cos^{-1}(\beta y + \lambda x) - \frac{\sqrt{-\beta^2 y^2 - 2\beta\lambda x y - \lambda^2 x^2 + 1}}{\beta y + \lambda x}\right) + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-\sqrt{-\left(\frac{\beta y}{x}+\lambda\right)^2 x^2+1}\, a x + (\beta y + \lambda x)\left(a x \arccos(\beta y + \lambda x) + \_F1\left(\frac{y}{x}\right)\right)}{\beta y + \lambda x}$$

6.3.22.4 [970] Problem 4

problem number 970

Added Feb. 11, 2019.

Problem Chapter 3.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \arccos^n(\lambda x) w_y = c \arccos^m(\mu x) + s \arccos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \left(\frac{\cos^{-1}\left(\frac{\beta(\cos^{-1}(\lambda K[1])^2)^{-n}\left((\cos^{-1}(\lambda K[1])^2)^n(-b(i\cos^{-1}(\lambda x))^n \operatorname{Gamma}(n+1,-i\cos^{-1}(\lambda x))\cos^{-1}(\lambda x)^n - b(\right.\right.}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccos(lambda*x)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\arccos\left(\_a\mu\right)^{m} + \frac{\arccos\left(\frac{\left(-\sqrt{-\_a^{2}\lambda^{2}+1}\,b+\sqrt{-\lambda^{2}x^{2}+1}\,b+\left(\_ab\arccos(\_a\lambda)-bx\arccos(\lambda x)+ay\right)\lambda\right)\beta}{a\lambda}\right)^{k}}{a} \right) d$$

6.3.22.5 [971] Problem 5

problem number 971

Added Feb. 11, 2019.

Problem Chapter 3.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda y)w_y = c\arccos^m(\mu x) + s\arccos^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{ w(x,y) \to \int_{1}^{y} \frac{\left(\cos^{-1}(\beta K[1])^{k} + a\cos^{-1}\left(-\frac{a\mu\cos^{-1}(\lambda K[1])^{-n}\left(-\operatorname{Gamma}\left(1-n,-i\cos^{-1}(\lambda K[1])\right)\left(-i\cos^{-1}(\lambda K[1])\right)^{n}\right.}{}\right.\right.}{} \right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccos(lambda*y)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a \arccos\left(\frac{(b\lambda x - a\,\mathrm{sinIntegral}(\arccos(\_a\lambda)) + a\,\mathrm{sinIntegral}(\arccos(\lambda y)))\mu}{b\lambda}\right)^{m} + \arccos(\_a\beta)^{k}}{b \arccos(\_a\lambda)} d\_a + \_F1\left(\frac{b\lambda}{}\right.$$

6.3.23 7.3

Local contents

6.3.23.1 [972] Problem 1 1652
6.3.23.2 [973] Problem 2 1653
6.3.23.3 [974] Problem 3 1654
6.3.23.4 [975] Problem 4 1655
6.3.23.5 [976] Problem 5 1656

6.3.23.1 [972] Problem 1

problem number 972

Added Feb. 11, 2019.

Problem Chapter 3.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \arctan \frac{x}{\lambda} + k \arctan \frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[x/lambda] + k*ArcTan[y/beta];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{a\beta k\log\left(a^2(\beta^2+y^2)\right)-2aky\tan^{-1}\left(\frac{y}{\beta}\right)+bc\lambda\log\left(\lambda^2+x^2\right)-2bcx\tan^{-1}\left(\frac{x}{\lambda}\right)}{2ab}+c_1\Big(y-\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arctan(x/lambda)+k*arctan(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{-a\beta k\ln\left(\frac{\beta^2+y^2}{\beta^2}\right)+2aky\arctan\left(\frac{y}{\beta}\right)-bc\lambda\ln\left(\frac{x^2}{\lambda^2}+1\right)+2bcx\arctan\left(\frac{x}{\lambda}\right)+2ab\_F1\left(\frac{ay-bx}{a}\right)}{2ab}$$

6.3.23.2 [973] Problem 2

problem number 973

Added Feb. 11, 2019.

Problem Chapter 3.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\arctan(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{2(\beta y+\lambda x)\,c\arctan\left(\beta y+\lambda x\right)-c\ln\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)+(2a\lambda+2b\beta)\,\_F1\left(\frac{ay-bx}{a}\right)}{2a\lambda+2b\beta}$$

6.3.23.3 [974] Problem 3

problem number 974

Added Feb. 11, 2019.

Problem Chapter 3.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\arctan(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)-\frac{ax\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2(\beta y+\lambda x)}+ax\tan^{-1}(\beta y+\lambda x)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{-ax\ln\left(\left(\frac{\beta y}{x}+\lambda\right)^2 x^2+1\right)+2(\beta y+\lambda x)\left(ax\arctan\left(\beta y+\lambda x\right)+\_F1\left(\frac{y}{x}\right)\right)}{2\beta y+2\lambda x}$$

6.3.23.4 [975] Problem 4

problem number 975

Added Feb. 11, 2019.

Problem Chapter 3.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda x)w_y = c\arctan^m(\mu x) + s\arctan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[beta
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\left(\frac{\tan^{-1}\left(\beta\left(y-\int_1^x\frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]}\frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1]\right)\right)^k}{a}+\tan^{-1}(\mu K[2])\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arctan(lambda*x)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\left(\arctan\left(\_a\mu\right)^{m}+\frac{\arctan\left(\frac{\left(-\frac{b\ln\left(\_a^{2}\lambda^{2}+1\right)}{2}+\frac{b\ln\left(\lambda^{2}x^{2}+1\right)}{2}+\left(\_ab\arctan\left(\_a\lambda\right)-bx\arctan\left(\lambda x\right)+ay\right)\lambda\right)\beta}{a\lambda}\right)^{k}}{a}\right)$$

6.3.23.5 [976] Problem 5

problem number 976

Added Feb. 11, 2019.

Problem Chapter 3.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arctan^n(\lambda y)w_y=c\arctan^m(\mu x)+s\arctan^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_{1}^{y}\frac{\tan^{-1}(\lambda K[2])^{-n}\left(\tan^{-1}(\beta K[2])^{k}+a\tan^{-1}\left(\frac{\mu\left(bx-a\int_{1}^{y}\tan^{-1}(\lambda K[1])^{-n}dK[1]+a\int_{1}^{K[2]}\tan^{-1}(\lambda K[1\right.}{b}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arctan(lambda*y)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a \arctan\left(\frac{a\mu\left(\int \frac{1}{\arctan(\_b\lambda)} d\_b\right)}{b} + \left(x - \left(\int \frac{a}{b \arctan(\lambda y)} dy\right)\right)\mu\right)^{m} + \arctan(\_b\beta)^{k}}{b \arctan(\_b\lambda)} d\_b + \_F1\left(x\right.$$

6.3.24 7.4

Local contents

6.3.24.1 [977] Problem 1 1657
6.3.24.2 [978] Problem 2 1658
6.3.24.3 [979] Problem 3 1659
6.3.24.4 [980] Problem 4 1660
6.3.24.5 [981] Problem 5 1661

6.3.24.1 [977] Problem 1

problem number 977

Added Feb. 11, 2019.

Problem Chapter 3.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \operatorname{arccot} \frac{x}{\lambda} + k \operatorname{arccot} \frac{y}{\beta}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[x/lambda] + k*ArcCot[y/beta];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{2abc_1\left(y-\frac{bx}{a}\right)+a\beta k\log\left(a^2(\beta^2+y^2)\right)-2aky\tan^{-1}\left(\frac{y}{\beta}\right)+2bkx\tan^{-1}\left(\frac{y}{\beta}\right)+2bkx\cot^{-1}\left(\right.}{2ab}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccot(x/lambda)+k*arccot(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{a\beta k\ln\left(\frac{\beta^2+y^2}{\beta^2}\right)-2aky\arctan\left(\frac{y}{\beta}\right)+bc\lambda\ln\left(\frac{x^2}{\lambda^2}+1\right)-2bcx\arctan\left(\frac{x}{\lambda}\right)+2ab\_F1\left(\frac{ay-bx}{a}\right)+\pi(}{2ab}$$

6.3.24.2 [978] Problem 2

problem number 978

Added Feb. 11, 2019.

Problem Chapter 3.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\operatorname{arccot}(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c(a\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right)+2\beta(bx-ay)\tan^{-1}(\beta y+\lambda x)+2x(a\lambda+b\beta)\cot^{-1}}{2a(a\lambda+b\beta)}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{ac\ln\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)-2\left((\beta y+\lambda x)\,a\arctan\left(\beta y+\lambda x\right)-\frac{\pi(a\lambda+b\beta)x}{2}\right)c+(2a^2\lambda+2a}{2\left(a\lambda+b\beta\right)a}$$

6.3.24.3 [979] Problem 3

problem number 979

Added Feb. 11, 2019.

Problem Chapter 3.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\operatorname{arccot}(\lambda x+\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to ax\left(\frac{\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2\beta y+2\lambda x}+\cot^{-1}(\beta y+\lambda x)\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{ax\ln\left(\left(\frac{\beta y}{x}+\lambda\right)^2 x^2+1\right)+(\beta y+\lambda x)\left(-2ax\arctan(\beta y+\lambda x)+\pi ax+2\_F1\left(\frac{y}{x}\right)\right)}{2\beta y+2\lambda x}$$

6.3.24.4 [980] Problem 4

problem number 980

Added Feb. 11, 2019.

Problem Chapter 3.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^n(\lambda x)w_y = c\operatorname{arccot}^m(\mu x) + s\operatorname{arccot}^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\left(\frac{\cot^{-1}\left(\beta\left(y-\int_1^x\frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]}\frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]\right)\right)^k}{a}+\cot^{-1}(\mu K[2])^m\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccot(lambda*x)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\left(-\arctan\left(\_a\mu\right) + \frac{\pi}{2}\right)^{m} + \frac{\left(-\arctan\left(\frac{\left(\frac{b\ln\left(\_a^{2}\lambda^{2}+1\right)}{2} - \frac{b\ln\left(\lambda^{2}x^{2}+1\right)}{2} + \left(-\_ab\arctan(\_a\lambda) + bx\arctan(\right.\right.}{a\lambda}\right.\right.}{a}\right.$$

6.3.24.5 [981] Problem 5

problem number 981

Added Feb. 11, 2019.

Problem Chapter 3.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^n(\lambda y)w_y = c\operatorname{arccot}^m(\mu x) + s\operatorname{arccot}^k(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{y} \frac{\cot^{-1}(\lambda K[2])^{-n}\left(\cot^{-1}(\beta K[2])^{k} + a\cot^{-1}\left(\frac{\mu\left(bx - a\int_{1}^{y}\cot^{-1}(\lambda K[1])^{-n}dK[1] + a\int_{1}^{K[2]}\cot^{-1}(\lambda K[1])\right.}{b}\right.\right.}{b}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*arccot(lambda*y)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a\left(\arctan\left(\frac{2a\mu\left(\int \frac{1}{2\arctan(\_b\lambda)-\pi} d\_b\right)}{b} - \left(x - \left(\int \frac{2a}{(-2\arctan(\lambda y)+\pi)b} dy\right)\right)\mu\right) + \frac{\pi}{2}\right)^{m} + \left(-\arctan\left(\right.\right.}{b\,\mathrm{arccot}\left(\_b\lambda\right)}$$

6.3.25 8.1

Local contents

6.3.25.1 [982] Problem 1 1663
6.3.25.2 [983] Problem 2 1663
6.3.25.3 [984] Problem 3 1664
6.3.25.4 [985] Problem 4 1665
6.3.25.5 [986] Problem 5 1666
6.3.25.6 [987] Problem 6 1666
6.3.25.7 [988] Problem 7 1667
6.3.25.8 [989] Problem 8 1668
6.3.25.9 [990] Problem 9 1669
6.3.25.10 [991] Problem 10 1669
6.3.25.11 [992] Problem 11 1670
6.3.25.12 [993] Problem 12 1671
6.3.25.13 [994] Problem 13 1672
6.3.25.14 [995] Problem 14 1673
6.3.25.15 [996] Problem 15 1674
6.3.25.16 [997] Problem 16 1675

6.3.25.1 [982] Problem 1

problem number 982

Added Feb. 11, 2019.

Problem Chapter 3.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{f(K[1])}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{f(x)}{a} dx + \_F1\left(\frac{ay - bx}{a}\right)$$

6.3.25.2 [983] Problem 2

problem number 983

Added Feb. 11, 2019.

Problem Chapter 3.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = yf(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1])(y+a(K[1]-x))dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +  a*diff(w(x,y),y) =  y*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x -((-\_a+x)\,a-y)\,f(\_a)\,d\_a+\_F1\,(-ax+y)$$

6.3.25.3 [984] Problem 3

problem number 984

Added Feb. 11, 2019.

Problem Chapter 3.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=y^2f(x)+yg(x)+h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y^2*f[x] + y*g[x] + h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \left(f(K[1])(y+a(K[1]-x))^2+g(K[1])(y+a(K[1]-x))+h(K[1])\right)dK[1]+c_1(y-ax\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^2*f(x)+y*g(x)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\left(\left(-\_a + x\right) a - y\right)^{2} f(\_a) + \left(\left(\_a - x\right) a + y\right) g(\_a) + h(\_a)\right) d\_a + \_F1(-ax + y)$$

6.3.25.4 [985] Problem 4

problem number 985

Added Feb. 11, 2019.

Problem Chapter 3.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = y^k f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == y^k*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{x} f(K[1])(y + a(K[1] - x))^{k} dK[1] + c_1(y - ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^k*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \left(\left(\_a - x\right) a + y\right)^{k} f(\_a)\, d\_a + \_F1(-ax + y)$$

6.3.25.5 [986] Problem 5

problem number 986

Added Feb. 11, 2019.

Problem Chapter 3.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = e^{\lambda y} f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == Exp[lambda*y]*f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x e^{\lambda(y+a(K[1]-x))} f(K[1]) dK[1] + c_1(y - ax)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  exp(lambda*y)*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \mathrm{e}^{-((-\_a+x)a-y)\lambda} f(\_a)\, d\_a + \_F1(-ax+y)$$

6.3.25.6 [987] Problem 6

problem number 987

Added Feb. 11, 2019.

Problem Chapter 3.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (ay + f(x))w_y = g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x g(K[2])dK[2]+c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int g(x)\,dx+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right)\right)$$

6.3.25.7 [988] Problem 7

problem number 988

Added Feb. 11, 2019.

Problem Chapter 3.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+(ay+f(x))w_y=y^k g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == y^k*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x g(K[2])\left(e^{aK[2]}\left(e^{-ax}y-\int_1^x e^{-aK[1]}f(K[1])dK[1]+\int_1^{K[2]} e^{-aK[1]}f(K[1])dK[1]\right)\right)^k d\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  y^k*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int^{x} \left(\left(y\,\mathrm{e}^{-ax} + \int \mathrm{e}^{-\_ba} f(\_b)\,d\_b - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right)\right)\mathrm{e}^{-ba}\right)^{k} g(\_b)\,d\_b + \_F1\left(y\,\mathrm{e}^{-ax} - \left(\right.\right.$$

6.3.25.8 [989] Problem 8

problem number 989

Added Feb. 11, 2019.

Problem Chapter 3.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + y^k w_y = g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{g(K[2])}{f(K[2])}dK[2] + c_1\left(-\int_1^x \frac{1}{f(K[1])}dK[1] - \frac{y^{1-k}}{k-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + y^k*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \int g(x)\,dx + \_F1\left(\left((k-1)\,xy^k + y\right)y^{-k}\right)$$

6.3.25.9 [990] Problem 9

problem number 990

Added Feb. 11, 2019.

Problem Chapter 3.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+a)w_y = by + c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == b*y + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{c + b\exp\left(\int_1^{K[3]} \frac{1}{f(K[1])} dK[1]\right)\left(\exp\left(-\int_1^x \frac{1}{f(K[1])} dK[1]\right) y - \int_1^x \frac{a\exp\left(-\int_1^{K[2]} \frac{1}{f(K[1])} dK[1]\right)}{f(K[2])} d\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y+a)*diff(w(x,y),y) =  b*y+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = cx + ((-x+1)a + y)b + \_F1\left((a+y)\mathrm{e}^{-x}\right)$$

6.3.25.10 [991] Problem 10

problem number 991

Added Feb. 11, 2019.

Problem Chapter 3.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+ax)w_y = g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{g(K[3])}{f(K[3])}dK[3]+c_1\left(y\exp\left(-\int_1^x\frac{1}{f(K[1])}dK[1]\right)-\int_1^x\frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK[1]\right)}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y+a*x)*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int g(x)\,dx+\_F1\left((ax+a+y)\,\mathrm{e}^{-x}\right)$$

6.3.25.11 [992] Problem 11

problem number 992

Added Feb. 11, 2019.

Problem Chapter 3.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(yg_1(x)+g_0(x))w_y=y^2h_2(x)+yh_1(x)+h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y*g1[x] + g0[x])*D[w[x, y], y] == y^2*h2[x] + y*h1[x] + h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{\exp\left(2\int_1^{K[3]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\text{h2}(K[3])\left(\exp\left(-\int_1^x\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)y-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\text{g1}(K[1])}{f(K[1])}\right.}{f(K[2}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y*g1(x)+g0(x))*diff(w(x,y),y) =   y^2*h2(x)+y*h1(x)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\left(y^2\mathrm{e}^{2\left(\int g1(\_f)d\_f\right)}\mathrm{e}^{\int -2g1(x)dx}h2(\_f)+y\,\mathrm{e}^{-\left(\int g1(x)dx\right)}\mathrm{e}^{\int g1(\_f)d\_f}h1(\_f)+\left(\int\mathrm{e}^{-\left(\int g1(\_f)d\_f\right)}g\right.\right.$$

6.3.25.12 [993] Problem 12

problem number 993

Added Feb. 11, 2019.

Problem Chapter 3.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(yg_1(x)+y^kg_2(x))w_y=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y*g1[x] + y^k*g2[x])*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{h(K[3])}{f(K[3])}dK[3]+c_1\left((k-1)\int_1^x \frac{\exp\left((k-1)\int_1^{K[2]}\frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right)\text{g2}(K[2])}{f(K[2])}dK[2]+y^{1-}\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y),x) + (y*g1(x)+y^k*g2(x))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int h(x)\,dx+\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int g1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int g1(x)dx\right)}g2(x)\,dx\right)\right)$$

6.3.25.13 [994] Problem 13

problem number 994

Added Feb. 11, 2019.

Problem Chapter 3.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)+e^{\lambda y}g_2(x))w_y=h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x] + Exp[lambda*y])*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := f(x)*diff(w(x,y),x) +(g1(x)+exp(lambda*y))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{h(x)}{f(x)} dx + \_F1 \left(\frac{-\lambda \left(\int \frac{e^{\lambda \left(\int \frac{g1(x)}{f(x)} dx \right)}}{f(x)} dx \right) - e^{-\left(y - \left(\int \frac{g1(x)}{f(x)} dx \right) \right) \lambda}}{\lambda} \right)$$

6.3.25.14 [995] Problem 14

problem number 995

Added Feb. 11, 2019.

Problem Chapter 3.8.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^k f(x) w_x + g(x) w_y = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^k*f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ \left\{ w(x,y) \to \int_1^x \frac{h(K[2]) \left(\left(y^{k+1} - (k+1) \int_1^x \frac{g(K[1])}{f(K[1])} dK[1] + (k+1) \int_1^{K[2]} \frac{g(K[1])}{f(K[1])} dK[1] \right)^{\frac{1}{k+1}} \right)^{-k}}{f(K[2])} dK[2] + \right. \right.$$

Maple ✓

```
restart;
pde := y^k*f(x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\left(y^{k+1} + \int \frac{(k+1)g(\_b)}{f(\_b)} d\_b + \int \frac{(-k-1)g(x)}{f(x)} dx\right)^{\frac{1}{k+1}}\right)^{-k} h(\_b)}{f(\_b)} d\_b + \_F1\left(yy^{k} + (-k-1)\left(\int\right.\right.$$

6.3.25.15 [996] Problem 15

problem number 996

Added Feb. 11, 2019.

Problem Chapter 3.8.1.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^k f(x) w_x + (y^{k+1} g_1(x) + g_0(x)) w_y = y^{3k+1} h_2(x) + y^{2k+1} h_1(x) + y^k h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^k*f[x]*D[w[x, y], x] + (y^(k + 1)*g1[x] + g0[x])*D[w[x, y], y] == y^(3*k + 1)*h2[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{\mathrm{h1}(K[3])\left(\left(\exp\left(-\left((k+1)\left(\int_1^x \frac{\mathrm{g1}(K[1])}{f(K[1])} dK[1] - \int_1^{K[3]} \frac{\mathrm{g1}(K[1])}{f(K[1])} dK[1]\right)\right)\right)\left(y^{k+1} - \exp\left(\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := y^k*f(x)*diff(w(x,y),x) +(y^(k+1)* g1(x) + g0(x))*diff(w(x,y),y) =   y^(3*k +1)*h2(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\left(\left(y^{k+1}\mathrm{e}^{-(k+1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (k+1)\left(\int \frac{\mathrm{e}^{-(k+1)\left(\int \frac{g1(\_f)}{f(\_f)}d\_f\right)} g0(\_f)}{f(\_f)} d\_f\right) + (-k-1)\left(\int \mathrm{e}^{-(k+1}\right.\right.\right.}{}$$

6.3.25.16 [997] Problem 16

problem number 997

Added Feb. 11, 2019.

Problem Chapter 3.8.1.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda x}w_x + g(x)w_y = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*Exp[lambda*x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{e^{-\lambda K[2]}h(K[2])}{f(K[2])}dK[2] + c_1\left(y - \int_1^x \frac{e^{-\lambda K[1]}g(K[1])}{f(K[1])}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := f(x)*exp(lambda*x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{e^{-\lambda x} h(x)}{f(x)} dx + \_F1\left(y - \left(\int \frac{e^{-\lambda x} g(x)}{f(x)} dx\right)\right)$$

6.3.26 8.2

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6.3.26.1 [998] Problem 1

problem number 998

Added Feb. 11, 2019.

Problem Chapter 3.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x) + g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x] + g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x) +b*diff(w(x,y),y) =   f(x)+g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{f(\_a)+g\left(\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)$$

6.3.26.2 [999] Problem 2

problem number 999

Added Feb. 11, 2019.

Problem Chapter 3.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x f(K[1])g(-ax+y+aK[1])dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =   f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} f(\_a)\, g((\_a - x)\, a + y)\, d\_a + \_F1(-ax + y)$$

6.3.26.3 [1000] Problem 3

problem number 1000

Added Feb. 11, 2019.

Problem Chapter 3.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)h(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x g(K[2])h\left(e^{aK[2]}\left(e^{-ax}y - \int_1^x e^{-aK[1]}f(K[1])dK[1] + \int_1^{K[2]} e^{-aK[1]}f(K[1])dK[1]\right)\right) d\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +(a*y+f(x) )*diff(w(x,y),y) =   g(x)*h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} g(\_b)\, h\left(\left(y\, \mathrm{e}^{-ax} + \int \mathrm{e}^{-\_ba} f(\_b)\, d\_b - \left(\int \mathrm{e}^{-ax} f(x)\, dx\right)\right) \mathrm{e}^{-\_ba}\right) d\_b + \_F1\left(y\, \mathrm{e}^{-ax} - \left(\right.\right.$$

6.3.26.4 [1001] Problem 4

problem number 1001

Added Feb. 11, 2019.

Problem Chapter 3.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + g(y)w_y = h_1(x) + h_2(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == h1[x] + h2[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +g(y)*diff(w(x,y),y) =   h1(x)+h2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h1(\_f) + h2\left(\text{RootOf}\left(\int \frac{1}{f(\_f)} d\_f - \left(\int \frac{1}{f(x)} dx\right) + \int \frac{1}{g(y)} dy - \left(\int^{\_Z} \frac{1}{g(\_a)} d\_a\right)\right)\right)}{f(\_f)} d\_f + \_$$

6.3.26.5 [1002] Problem 5

problem number 1002

Added Feb. 11, 2019.

Problem Chapter 3.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f_1(x)w_x + (f_2(x)y + y^k f_3(x))w_y = g(x)h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y], x] + (y*f2[x] + y^k*f3[x])*D[w[x, y], y] == g[x]*h[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{g(K[3])h(K[3])}{\text{f1}(K[3])}dK[3]+c_1\left((k-1)\int_1^x \frac{\exp\left((k-1)\int_1^{K[2]}\frac{\text{f2}(K[1])}{\text{f1}(K[1])}dK[1]\right)\text{f3}(K[2])}{\text{f1}(K[2])}dK[2\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y),x) +(y*f2(x)+y^k*f3(x))*diff(w(x,y),y) =  g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int \frac{g(x)\,h(x)}{f1\,(x)}dx+\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{f2(x)}{f1(x)}dx\right)}+(k-1)\left(\int \frac{\mathrm{e}^{(k-1)\left(\int \frac{f2(x)}{f1(x)}dx\right)}f3(x)}{f1\,(x)}dx\right)\right)$$

6.3.26.6 [1003] Problem 6

problem number 1003

Added Feb. 11, 2019.

Problem Chapter 3.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(x)w_x+f_2(x)g_2(x)w_y=h_1(x)h_2(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*g1[x]*D[w[x, y], x] + f2[x]*g2[x]*D[w[x, y], y] == h1[x]*h2[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{\text{h1}(K[2])\text{h2}(K[2])}{\text{f1}(K[2])\text{g1}(K[2])}dK[2]+c_1\left(y-\int_1^x \frac{\text{f2}(K[1])\text{g2}(K[1])}{\text{f1}(K[1])\text{g1}(K[1])}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f1(x)*g1(x)*diff(w(x,y),x) +f2(x)*g2(x)*diff(w(x,y),y) =  h1(x)*h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int \frac{h1\,(x)\,h2\,(x)}{f1\,(x)\,g1\,(x)}dx+\_F1\left(y-\left(\int \frac{f2\,(x)\,g2\,(x)}{f1\,(x)\,g1\,(x)}dx\right)\right)$$

6.3.26.7 [1004] Problem 7

problem number 1004

Added Feb. 11, 2019.

Problem Chapter 3.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x+f_2(x)g_2(y)w_y=h_1(x)+h_2(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x] + h2[x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h1(x)+h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{h1(\_f) + h2(\_f)}{f1\,(\_f)\,g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)} d\_f - \left(\int \frac{f2(x)}{f1(x)} dx\right) + \int \frac{g1(y)}{g2(y)} dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a\right)\right)\right)} d\_f + \_$$

6.3.27 8.3

Local contents

6.3.27.1 [1005] Problem 1 1682
6.3.27.2 [1006] Problem 2 1683
6.3.27.3 [1007] Problem 3 1684
6.3.27.4 [1008] Problem 4 1685
6.3.27.5 [1009] Problem 5 1685
6.3.27.6 [1010] Problem 6 1686
6.3.27.7 [1011] Problem 7 1687
6.3.27.8 [1012] Problem 8 1688

6.3.27.1 [1005] Problem 1

problem number 1005

Added Feb. 11, 2019.

Problem Chapter 3.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(\alpha x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{f\left(\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) =  f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{f\left(\frac{-(-\_a+x)b\beta+(\_a\alpha+\beta y)a}{a}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)$$

6.3.27.2 [1006] Problem 2

problem number 1006

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=xf(\frac{y}{x})$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to xf\left(\frac{y}{x}\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = xf\left(\frac{y}{x}\right) + \_F1\left(\frac{y}{x}\right)$$

6.3.27.3 [1007] Problem 3

problem number 1007

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = f(x^2 + y^2)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{f\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]} dK[1] + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  f(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{f\left(\frac{-a^2y^2}{x^2} + \_a^2\right)}{\_a} d\_a + \_F1\left(\frac{y}{x}\right)$$

6.3.27.4 [1008] Problem 4

problem number 1008

Added Feb. 11, 2019.

Problem Chapter 3.8.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x}) + g(x^2+y^2)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x] + g[x^2 + y^2];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \left(f\left(\frac{y}{x}\right) + \frac{g\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]}\right) dK[1] + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x)+g(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{\_af\left(\frac{y}{x}\right) + g\left(\frac{-a^2y^2}{x^2} + \_a^2\right)}{\_a} d\_a + \_F1\left(\frac{y}{x}\right)$$

6.3.27.5 [1009] Problem 5

problem number 1009

Added Feb. 11, 2019.

Problem Chapter 3.8.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k f(x^n y^m)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*x^m];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{f(K[1]^{m+n})\,K[1]^{k-1}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  x^k*f(x^n*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{\_a^{k-1}f\left(\_a^n\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m\right)}{a}d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)$$

6.3.27.6 [1010] Problem 6

problem number 1010

Added Feb. 11, 2019.

Problem Chapter 3.8.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x+nyw_y=f(ax^n+by^m)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*x^m];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{f(bK[1]^m+aK[1]^n)}{mK[1]}dK[1]+c_1\left(yx^{-\frac{n}{m}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  m*x*diff(w(x,y),x) +n*y*diff(w(x,y),y) =  f(a*x^n+b*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{f\left(a\_a^{n} + b\left(y\_a^{\frac{n}{m}} x^{-\frac{n}{m}}\right)^{m}\right)}{\_am} d\_a + \_F1\left(y x^{-\frac{n}{m}}\right)$$

6.3.27.7 [1011] Problem 7

problem number 1011

Added Feb. 17, 2019.

Problem Chapter 3.8.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + x y w_y = y^k f(\alpha x + \beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{x} \frac{f\left(\left(\alpha + \frac{\beta y}{x}\right) K[1]\right) \left(\frac{y K[1]}{x}\right)^{k}}{K[1]^2} dK[1] + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x) +x*y*diff(w(x,y),y) =  y^k*f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\frac{\_ay}{x}\right)^{k} f\left(\left(\alpha + \frac{\beta y}{x}\right)\_a\right)}{\_a^{2}} d\_a + \_F1\left(\frac{y}{x}\right)$$

6.3.27.8 [1012] Problem 8

problem number 1012

Added Feb. 17, 2019.

Problem Chapter 3.8.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)} w_x + \frac{g(y)}{g'(y)} w_y = h(f(x) + g(y))$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[y]*D[w[x, y], y])/Derivative[1][g][y] ==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{x} \frac{h\left(f(K[1]) + g\left(\text{InverseFunction}\left[\text{InverseFunction}\left[g^{(-1)}, 1, 1\right], 1, 1\right]\left[\frac{f(K[1])\text{InverseFunction}}{f(x)}\right.\right.\right.}{f(K[1])}\right.\right.$$

Maple ✓

```
restart;
pde := f(x)/diff(f(x),x)*diff(w(x,y),x) +g(y)/diff(g(y),y)*diff(w(x,y),y) =  h(f(x)+g(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{\left(\frac{d}{d\_a} f(\_a)\right) h\left(\frac{(f(x)+g(y)) f(\_a)}{f(x)}\right)}{f(\_a)} d\_a + \_F1\left(\ln\left(\frac{g(y)}{f(x)}\right)\right)$$

6.3.28 8.4

Local contents

6.3.28.1 [1013] Problem 1 1689
6.3.28.2 [1014] Problem 2 1690
6.3.28.3 [1015] Problem 3 1691
6.3.28.4 [1016] Problem 4 1692
6.3.28.5 [1017] Problem 5 1693
6.3.28.6 [1018] Problem 6 1694
6.3.28.7 [1019] Problem 7 1695

6.3.28.1 [1013] Problem 1

problem number 1013

Added Feb. 17, 2019.

Problem Chapter 3.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x f(K[1], -ax + y + aK[1]) dK[1] + c_1(y - ax)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} f(\_a, (\_a - x)\, a + y)\, d\_a + \_F1(-ax + y)$$

6.3.28.2 [1014] Problem 2

problem number 1014

Added Feb. 17, 2019.

Problem Chapter 3.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_{1}^{x} \frac{f\left(K[1], x^{-\frac{b}{a}} y K[1]^{\frac{b}{a}}\right)}{aK[1]} dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{f\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_aa} d\_a + \_F1\left(yx^{-\frac{b}{a}}\right)$$

6.3.28.3 [1015] Problem 3

problem number 1015

Added Feb. 17, 2019.

Problem Chapter 3.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{h\left(K[2], \exp\left(\int_1^{K[2]} \frac{g(K[1])}{f(K[1])} dK[1] - \int_1^x \frac{g(K[1])}{f(K[1])} dK[1]\right) y\right)}{f(K[2])} dK[2] + c_1\left(y \exp\left(-\int_1^x \frac{g(K[1}{f(K[1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +g(x)*y*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h\left(\_b, y\, e^{\int \frac{g(\_b)}{f(\_b)} d\_b - \left(\int \frac{g(x)}{f(x)} dx\right)}\right)}{f(\_b)} d\_b + \_F1\left(y\, e^{-\left(\int \frac{g(x)}{f(x)} dx\right)}\right)$$

6.3.28.4 [1016] Problem 4

problem number 1016

Added Feb. 17, 2019.

Problem Chapter 3.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{h\left(K[3], \exp\left(\int_1^{K[3]} \frac{g1(K[1])}{f(K[1])} dK[1]\right)\left(\exp\left(-\int_1^x \frac{g1(K[1])}{f(K[1])} dK[1]\right) y - \int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{g1(K[1])}{f(K[1])} dK\right.}{f(K[2])}\right.\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h\left(\_f, \left(y\,\mathrm{e}^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} + \int \frac{\mathrm{e}^{-\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)} g0(\_f)}{f(\_f)} d\_f - \left(\int \frac{\mathrm{e}^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)} dx\right)\right) \mathrm{e}^{\int \frac{g1(\_f)}{f(\_f)} d\_f}\right)}{f(\_f)}$$

6.3.28.5 [1017] Problem 5

problem number 1017

Added Feb. 17, 2019.

Problem Chapter 3.8.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x \frac{h\left(K[3], \left(\exp\left(-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}dK[1] - (k-1)\int_1^{K[3]} \frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right) y^{-k}\left(\exp\left(\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}d\right.\right.\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h\left(\_f, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (-k+1)\left(\int \frac{\mathrm{e}^{(k-1)\left(\int \frac{g1(\_f)}{f(\_f)}d\_f\right)} g0(\_f)}{f(\_f)}d\_f\right) + (k-1)\left(\int \frac{\mathrm{e}^{(k-}}{}\right.\right.\right.}{f(\_f)}$$

6.3.28.6 [1018] Problem 6

problem number 1018

Added Feb. 17, 2019.

Problem Chapter 3.8.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x] + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x) +(g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{h\left(\_f, \frac{\lambda\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)+\ln\left(\frac{1}{-\lambda\left(\int \frac{e^{\lambda\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)} g0(\_f)}{f(\_f)} d\_f\right)+\lambda\left(\int \frac{e^{\lambda\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)} dx\right)+e^{-\left(y-\left(\int \frac{g1(x)}{f(x)} dx\right)\right)\lambda}}\right)}{\lambda}\right)}{f(\_f)}$$

6.3.28.7 [1019] Problem 7

problem number 1019

Added Feb. 17, 2019.

Problem Chapter 3.8.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h\left(\_f, \text{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)} d\_f - \left(\int \frac{f2(x)}{f1(x)} dx\right) + \int \frac{g1(y)}{g2(y)} dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a\right)\right)\right)}{f1\left(\_f\right) g1\left(\text{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)} d\_f - \left(\int \frac{f2(x)}{f1(x)} dx\right) + \int \frac{g1(y)}{g2(y)} dy - \left(\int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a\right)\right)\right)} d\_f + \_$$

Contains RootOf

6.4 chapter 4

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6.4.1.1 [1020] Example 1

problem number 1020

Added Feb. 17, 2019.

Chapter 4.1.1 example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + ayw_y = by^2 w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y*D[w[x, y], y] == b*y^2*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{by^2}{2a}} c_1\left(ye^{-ax}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) =  b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax}\right)\mathrm{e}^{\frac{by^2}{2a}}$$

6.4.1.2 [1021] Example 2

problem number 1021

Added Feb. 17, 2019.

Chapter 4.1.1 example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + ayw_y = be^{\lambda x} yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*y*D[w[x, y], y] == b*Exp[lambda*x]*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-ax}\right) e^{\frac{bye^{\lambda x}}{a+\lambda}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) = b*exp(lambda*x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax}\right) \mathrm{e}^{\frac{by\,\mathrm{e}^{\lambda x}}{a+\lambda}}$$

6.4.1.3 [1022] Example 3

problem number 1022

Added Feb. 17, 2019.

Chapter 4.1.1 example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = bw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to e^{bx} c_1(y-ax)\}\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{bx}$$

6.4.2 2.1

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6.4.2.1 [1023] Problem 1

problem number 1023

Added Feb. 17, 2019.

Problem Chapter 4.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{cx}{a}}$$

6.4.2.2 [1024] Problem 2

problem number 1024

Added Feb. 17, 2019.

Problem Chapter 4.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+yw_y=bw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{bx}{a}}c_1\left(ye^{-\frac{x}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +y*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{x}{a}}\right)\mathrm{e}^{\frac{bx}{a}}$$

6.4.2.3 [1025] Problem 3

problem number 1025

Added Feb. 17, 2019.

Problem Chapter 4.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = aw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^a c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^a\_F1\left(\frac{y}{x}\right)$$

6.4.2.4 [1026] Problem 4

problem number 1026

Added Feb. 17, 2019.

Problem Chapter 4.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(aw_x - bw_y) = cyw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*(D[w[x, y], x] - b*D[w[x, y], y]) == c*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x, y) \to e^{-bcx} x^{c(bx+y)} c_1(bx + y)\}\}$$

Maple ✓

```
restart;
pde :=  x*(diff(w(x,y),x) -b*diff(w(x,y),y)) = c*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = x^{(bx+y)c} \_F1(bx + y) \mathrm{e}^{-bcx}$$

6.4.2.5 [1027] Problem 5

problem number 1027

Added Feb. 17, 2019.

Problem Chapter 4.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = axw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{ax}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*x*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{ax}$$

6.4.2.6 [1028] Problem 6

problem number 1028

Added Feb. 17, 2019.

Problem Chapter 4.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)w_x+(y-b)w_y=w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\left((a-x)c_1\left(\frac{b-y}{a-x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x-a)*diff(w(x,y),x) +(y-b)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = (a-x)\_F1\left(\frac{-b+y}{a-x}\right)$$

6.4.2.7 [1029] Problem 7

problem number 1029

Added Feb. 17, 2019.

Problem Chapter 4.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y+ax)w_x + (y-ax)w_y = bw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (y + a*x)*D[w[x, y], x] + (y - a*x)*D[w[x, y], y] == b*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  (y+a*x)*diff(w(x,y),x) +(y-a*x)*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

time expired

6.4.3 2.2

Local contents

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6.4.3.1 [1030] Problem 1

problem number 1030

Added Feb. 17, 2019.

Problem Chapter 4.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (x^2 - y^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (x^2 - y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{x(a^2(x^2-3y^2)+3abxy-b^2x^2)}{3a^3}\right)c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = (x^2-y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left(x^2a^2-3y^2a^2+3xyab-b^2x^2\right)x}{3a^3}}$$

6.4.3.2 [1031] Problem 2

problem number 1031

Added Feb. 17, 2019.

Problem Chapter 4.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x^2 w_x + axyw_y = by^2 w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{by^2}{x-2ax}} c_1\left(yx^{-a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x) +a*x*y*diff(w(x,y),y) = b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(yx^{-a}\right) \mathrm{e}^{\frac{by^2}{(2a-1)x}}$$

6.4.3.3 [1032] Problem 3

problem number 1032

Added Feb. 17, 2019.

Problem Chapter 4.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ax^2 w_x + by^2 w_y = (x + cy)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == (x + c*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{1}{a}+\frac{c}{b}}\left(\frac{ax}{y}\right)^{-\frac{c}{b}}c_1\left(\frac{b}{ax}-\frac{1}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^2*diff(w(x,y),x) +b*y^2*diff(w(x,y),y) = (x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=x^{\frac{c}{b}+\frac{1}{a}}\left(\frac{ax}{y}\right)^{-\frac{c}{b}}\_F1\left(\frac{ax-by}{axy}\right)$$

6.4.3.4 [1033] Problem 4

problem number 1033

Added Feb. 17, 2019.

Problem Chapter 4.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x+ay^2w_y=(bx^2+cxy+dy^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x^2 + c*x*y + d*y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\left(\frac{x}{y}\right)^{\frac{cxy}{x-ay}}c_1\left(\frac{a}{x}-\frac{1}{y}\right)e^{\frac{bx^2-dy^2}{x-ay}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) = (b*x^2+c*x*y+d*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \left(\frac{x}{y}\right)^{-\frac{cxy}{ay-x}} \_F1\left(\frac{-ay+x}{xy}\right) \mathrm{e}^{\frac{dy^2+(ay-x)bx}{ay-x}}$$

6.4.3.5 [1034] Problem 5

problem number 1034

Added Feb. 17, 2019.

Problem Chapter 4.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^2 w_x + a x^2 w_y = (b x^2 + c y^2) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  y^2*D[w[x, y], x] + a*x^2*D[w[x, y], y] == (b*x^2 + c*y^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to c_1\left(\frac{1}{3}(y^3 - a x^3)\right) e^{\frac{b\sqrt[3]{y^3}}{a} + cx}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{1}{3}(y^3 - a x^3)\right) e^{cx - \frac{\sqrt[3]{-1}b\sqrt[3]{y^3}}{a}}\right\}$$
$$\left\{w(x,y) \to c_1\left(\frac{1}{3}(y^3 - a x^3)\right) e^{\frac{(-1)^{2/3}b\sqrt[3]{y^3}}{a} + cx}\right\}$$

Maple ✓

```
restart;
pde :=y^2*diff(w(x,y),x) +a*x^2*diff(w(x,y),y) =(b*x^2+c*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-ax^3+y^3\right)\mathrm{e}^{\frac{cax+by}{a}}$$

6.4.3.6 [1035] Problem 6

problem number 1035

Added Feb. 17, 2019.

Problem Chapter 4.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xyw_x + ay^2w_y = (bx+cy+d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*y*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x + c*y + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^c c_1\left(yx^{-a}\right)e^{-\frac{\frac{bx}{a-1}+\frac{d}{a}}{y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*y*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) =(b*x+c*y+d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^c\_F1\left(yx^{-a}\right)\mathrm{e}^{\frac{(-bx-d)a+d}{(a-1)ay}}$$

6.4.3.7 [1036] Problem 7

problem number 1036

Added Feb. 17, 2019.

Problem Chapter 4.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(ay + b)w_x + (ay^2 - bx)w_y = ayw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(a*y + b)*D[w[x, y], x] + (a*y^2 - b*x)*D[w[x, y], y] == a*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(a*y+b)*diff(w(x,y),x) +(a*y^2-b*x)*diff(w(x,y),y) =a*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay + (x+y)\,a\ln\left(\frac{-9ax+9b}{2ay+2b}\right) - (x+y)\,a\ln\left(-\frac{9(x+y)(ax-b)a}{(ay+b)x}\right) + b}{3\,(x+y)\,a}\right) \mathrm{e}^{\left(\int^x \frac{9\_aa+2b\,\mathrm{e}^{\mathrm{RootOf}\left(2\_Zax\,\mathrm{e}\right.}}{}\right.}$$

6.4.3.8 [1037] Problem 8

problem number 1037

Added Feb. 17, 2019.

Problem Chapter 4.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(ky - x + a)w_x - y(kx - y + a)w_y = b(y - x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(k*y - x + a)*D[w[x, y], x] - y*(k*x - y + a)*D[w[x, y], y] == b*(y - x)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(k*y-x+a)*diff(w(x,y),x)-y*(k*x-y+a)*diff(w(x,y),y) = b*(y-x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(-\frac{(k^2+k+1)\left(-k\ln\left(-\frac{(k+1)(k^2+k+1)(a-x)}{(k+2)(ky+a-x)}\right)+k\ln(-a+x)-\ln(x)-\ln\left(\frac{(k+1)(k^2+k+1)k}{(2k+1)(ky+a-x}\right.\right.}{3(k+1)k}\right.$$

6.4.4 2.3

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6.4.4.1 [1038] Problem 1

problem number 1038

Added Feb. 17, 2019.

Problem Chapter 4.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^3 + dy^3)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^3 + d*y^3)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{1}{4}\left(\frac{cx^4}{a}+\frac{dy^4}{b}\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^3+d*y^3)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{\left(cx^3a^3+4a^3dy^3-6a^2bdxy^2+4ab^2dx^2y-b^3dx^3\right)x}{4a^4}}$$

6.4.4.2 [1039] Problem 2

problem number 1039

Added Feb. 17, 2019.

Problem Chapter 4.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{a\sqrt{x^2+y^2}}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{\sqrt{x^2+y^2}\,a}$$

6.4.4.3 [1040] Problem 3

problem number 1040

Added Feb. 17, 2019.

Problem Chapter 4.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x+xyw_y=y^2(ax+by)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) \mathrm{e}^{\frac{ay^2}{2}+\frac{by^3}{2x}}$$

6.4.4.4 [1041] Problem 4

problem number 1041

Added Feb. 17, 2019.

Problem Chapter 4.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 y w_x + a x y^2 w_y = (b x y + c x + d y + k) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*y*D[w[x, y], x] + a*x*y^2*D[w[x, y], y] == (b*x*y + c*x + d*y + k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^b c_1\left(y x^{-a}\right) \exp\left(-\frac{a^2 d y + a c x + a d y + a k + c x}{a^2 x y + a x y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=x^2*y*diff(w(x,y),x)+a*x*y^2*diff(w(x,y),y) =(b*x*y +c*x+ d*y + k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^b \_F1\left(y x^{-a}\right) \mathrm{e}^{\frac{-a^2 d y - c x + (-c x - d y - k) a}{(a+1) a x y}}$$

6.4.4.5 [1042] Problem 5

problem number 1042

Added Feb. 17, 2019.

Problem Chapter 4.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axy^2 w_x + bx^2 y w_y = (any^2 + bmx^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*y^2*D[w[x, y], x] + b*x^2*y*D[w[x, y], y] == (a*n*y^2 + b*m*x^2)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^n \left(ay^2\right)^{m/2} c_1\left(\frac{ay^2 - bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*y^2*diff(w(x,y),x)+b*x^2*y*diff(w(x,y),y) = (a*n*y^2+ b*m*x^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = x^n \left(ay^2\right)^{\frac{m}{2}} \_F1\left(\frac{ay^2 - bx^2}{a}\right)$$

6.4.4.6 [1043] Problem 6

problem number 1043

Added Feb. 17, 2019.

Problem Chapter 4.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x^3 w_x + ay^3 w_y = x^2(bx + cy)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^3*D[w[x, y], x] + a*y^3*D[w[x, y], y] == x^2*(b*x + c*y)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(\frac{a}{x^2}-\frac{1}{y^2}\right)\right)\exp\left(bx-\frac{c\tan^{-1}\left(\frac{x\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}{\sqrt{\frac{x^2}{y^2}}}\right)}{\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}\right)\right\}$$

$$\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(\frac{a}{x^2}-\frac{1}{y^2}\right)\right)\exp\left(\frac{c\tan^{-1}\left(\frac{x\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}{\sqrt{\frac{x^2}{y^2}}}\right)}{\sqrt{\frac{a}{x^2}-\frac{1}{y^2}}}+bx\right)\right\}$$

Maple ✓

```
restart;
pde :=x^3*diff(w(x,y),x)+a*y^3*diff(w(x,y),y) =  x^2*(b*x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\left(\sqrt{\frac{-ay^2+x^2}{x^2y^2}}\,x+\sqrt{\frac{x^2}{y^2}}\right)^{\frac{c}{\sqrt{\frac{-ay^2+x^2}{x^2y^2}}}}\_F1\left(\frac{-ay^2+x^2}{x^2y^2}\right)e^{bx}$$

6.4.5 2.4

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6.4.5.1 [1044] Problem 1

problem number 1044

Added Feb. 17, 2019.

Problem Chapter 4.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + dy^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + d*y^m)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) e^{\frac{cx^{n+1}}{an+a} + \frac{dy^{m+1}}{bm+b}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n + d*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right) \mathrm{e}^{\frac{(n+1)ady^{m+1} + (m+1)bcx^{n+1}}{(n+1)(m+1)ab}}$$

6.4.5.2 [1045] Problem 2 case $n \neq -1, n \neq -2$

problem number 1045

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $neq - 1, neq - 2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
sol[[2]] = Assuming[{n != -1, n != -2}, Simplify[sol[[2]]]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{cx^{n+1}(a(n+2)y - bx)}{a^2(n+1)(n+2)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol)  assuming n<>-1, n<>-2;
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{\left((n+2)ayx^{n+1} - bx^{n+2}\right)c}{(n+2)(n+1)a^2}}$$

6.4.5.3 [1046] Problem 2 case $n = -1$

problem number 1046

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n = -1$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -1],
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c(\log(x)(ay-bx)+bx)}{a^2}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-1)
```

$$w(x,y)=x^{\frac{(ay-bx)c}{a^2}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{bcx}{a^2}}$$

6.4.5.4 [1047] Problem 2 case $n=-2$

problem number 1047

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n=-2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cx^n yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -2],
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c(-ay+bx+bx\log(x))}{a^2x}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-2)
```

$$w(x,y) = x^{\frac{bc}{a^2}} \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{-\frac{(ay-bx)c}{a^2x}}$$

6.4.5.5 [1048] Problem 3

problem number 1048

Added Feb. 17, 2019.

Problem Chapter 4.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2+y^2)^k w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) e^{\frac{a\left(x^2+y^2\right)^k}{2k}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*(x^2+y^2)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) \mathrm{e}^{\frac{a\left(x^2+y^2\right)^k}{2k}}$$

6.4.5.6 [1049] Problem 4

problem number 1049

Added Feb. 17, 2019.

Problem Chapter 4.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = cx^n y^m w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right) e^{\frac{cy^m x^n}{an+bm}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =   c*x^n*y^m*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right) \mathrm{e}^{\frac{cx^n y^m}{na+bm}}$$

6.4.5.7 [1050] Problem 5

problem number 1050

Added Feb. 17, 2019.

Problem Chapter 4.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = (cx^n + ky^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == (c*x^n + k*y^m)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cx^n}{an}+\frac{ky^m}{bm}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =  (c*x^n + k*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(yx^{-\frac{b}{a}}\right)\mathrm{e}^{\frac{akny^m+bcmx^n}{abmn}}$$

6.4.5.8 [1051] Problem 6

problem number 1051

Added Feb. 17, 2019.

Problem Chapter 4.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x+nyw_y=(ax^n+by^m)^kw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{n}{m}}\right)e^{\frac{(ax^n+by^m)^k}{kmn}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) =  (a*x^n + b*y^m)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{n}{m}}\right) \mathrm{e}^{\frac{(ax^n+by^m)^k}{kmn}}$$

6.4.5.9 [1052] Problem 7

problem number 1052

Added Feb. 17, 2019.

Problem Chapter 4.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^m w_y = (cx^k + dy^s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == (c*x^k + d*y^s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{bx^{1-n}}{a(n-1)} - \frac{y^{1-m}}{m-1}\right)\exp\left(\frac{\frac{cx^{k-n+1}}{a} + \frac{d(-k+n-1)y^{1-m}\left(\left(y^{m-1}\right)^{\frac{1}{m-1}}\right)^s}{b(m-s-1)}}{k-n+1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*y^m*diff(w(x,y),y) =  (c*x^k + d*y^s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n-1)\,ay^{-m+1} - (m-1)\,bx^{-n+1}}{(n-1)\,a}\right) \mathrm{e}^{\frac{-(k-n+1)a^2 da^{\frac{s}{m-1}-1} y^{-m+1}\left((n-1)ay^{-m+1}\right)^{-\frac{s}{m-1}} (n-1)^{\frac{s}{m-1}} \mathrm{e}^{-\frac{i\left(\mathrm{csg}\right.}{}}}{}}$$

6.4.5.10 [1053] Problem 8

problem number 1053

Added Feb. 17, 2019.

Problem Chapter 4.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = (cx^k y^s + d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == (c*x^k*y^s + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-\frac{bx^{m-n+1}}{am-an+a}}\right) \exp\left(\frac{x^{1-n}\left(\frac{d}{1-n} - \frac{cx^k y^s e^{-\frac{bsx^{m-n+1}}{am-an+a}}\left(-\frac{bsx^{m-n+1}}{am-an+a}\right)^{\frac{-k+n-1}{m-n+1}} \mathrm{Gamma}\left(\frac{k-n+1}{m-n+1}, -\frac{bsx^{m-n+}}{am-an+}\right.}{m-n+1}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y*diff(w(x,y),y) =  (c*x^k*y^s + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\frac{bx^{m-n+1}}{(m-n+1)a}}\right)\mathrm{e}^{\int_{}^{x}\frac{\left(c\_a^{k}\left(y\,\mathrm{e}^{-\frac{\left(-\_a^{m-n+1}+x^{m-n+1}\right)b}{(m-n+1)a}}\right)^{s}+d\right)\_a^{-n}}{a}d\_a}$$

6.4.5.11 [1054] Problem 9

problem number 1054

Added Feb. 17, 2019.

Problem Chapter 4.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k)w_y = (sx^p y^q + d)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == (s*x^p*y^q + d)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(c(a(m-n+1))^{\frac{k-m}{m-n+1}}b^{\frac{-k+n-1}{m-n+1}}\,\mathrm{Gamma}\left(\frac{k-n+1}{m-n+1},\frac{bx^{m-n+1}}{am-an+a}\right)+ye^{-\frac{bx^{m-n+1}}{am-an+a}}\right)\exp\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+(b*x^m*y+c*x^k)*diff(w(x,y),y) =   (s*x^p*y^q + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(m-n+1)(k+m-2n+2)^2 acx^{k-m}\left(\frac{bx^{m-n+1}}{(m-n+1)a}\right)^{\frac{-k-m+2n-2}{2m-2n+2}} \text{WhittakerM}\left(\frac{k+m-2n+2}{2m-2n+2},\right.}{}\right.$$

6.4.5.12 [1055] Problem 10

problem number 1055

Added Feb. 17, 2019.

Problem Chapter 4.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y^k w_y = (cx^p y^q + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == (c*x^p*y^q + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\frac{y^{1-k}(-m+n-1)}{k-1} - \frac{bx^{m-n+1}}{a}}{m-n+1}\right)\exp\left(\frac{x^{1-n}\left(-\frac{cx^p\left(\left(y^{k-1}\right)^{\frac{1}{k-1}}\right)^q\left(\frac{ay(m-n+1)x^n}{ay(m-n+1)x^n+b(k-1)y^k x^{m+1}}\right)^{\frac{q}{k-1}} {}_2F_1\left(\right.}{n-p-}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y^k*diff(w(x,y),y) =    (c*x^p*y^q + s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(m-n+1)\,ay^{-k+1} + (k-1)\,bx^{m-n+1}}{(m-n+1)\,a}\right) e^{\int^x \frac{\left(c\_a^p\left(\left(\frac{(m-n+1)ay^{-k+1}-(k-1)b\_a^{m-n+1}+(k-1)bx^{m-n+1}}{(m-n+1)a}\right.\right.}{a}}$$

6.4.5.13 [1056] Problem 11

problem number 1056

Added Feb. 17, 2019.

Problem Chapter 4.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = (cx^m + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == (c*x^m + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y^{k+1}}{k+1} - \frac{bx^{n+1}}{an+a}\right) \exp\left(\frac{x\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\left(cx^m\,{}_2F_1\left(\frac{k}{k+1},\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=a*y^k*diff(w(x,y),x)+b*x^n*diff(w(x,y),y) =   (c*x^m+ s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(n+1)\,ay^{k+1} - (k+1)\,bx^{n+1}}{(n+1)\,a}\right) \mathrm{e}^{\int^{x} \frac{(c\_a^{m}+s)\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_a^{n+1}-(k+1)bx^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}}{a} d\_a}$$

6.4.5.14 [1057] Problem 12

problem number 1057

Added Feb. 17, 2019.

Problem Chapter 4.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(x^n + (an-1)y^n)w_x + y(y^n + (an-1)x^n)w_y = kn(x^n + y^n)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(x^n + (a*n - 1)*y^n)*D[w[x, y], x] + y*(y^n + (a*n - 1)*x^n)*D[w[x, y], y] == k*n*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*(x^n + (a*n -1)*y^n)*diff(w(x,y),x)+y*(y^n + (a*n -1)*x^n)*diff(w(x,y),y) =   k*n*(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(x^{-\frac{1}{a}}y^{n} - x^{\frac{na-1}{a}}\right)(y^{n})^{-\frac{1}{an}}\right) \mathrm{e}^{\int^{x} \frac{\left(\_a^{n}+\mathrm{RootOf}\left(-\_a^{\frac{1}{a}}x^{-\frac{1}{a}}y^{n}(\_Z^{n}\right.\right.}{\left(an\,\mathrm{RootOf}\left(-\_a^{\frac{1}{a}}x^{-\frac{1}{a}}y^{n}(\_Z^{n})^{\frac{1}{an}}(y^{n})^{-\frac{1}{an}}+\_a^{\frac{1}{a}}x^{\frac{na-1}{a}}(\_Z^{n})^{\frac{1}{an}}(y^{n})^{-\frac{1}{an}}+\right.\right.}}$$

6.4.5.15 [1058] Problem 13

problem number 1058

Added Feb. 17, 2019.

Problem Chapter 4.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x((n-2)y^n-2x^n)w_x+y(2y^n-(n-2)x^n)w_y=((a(n-2)+2b)y^n-(2a+b(n-2))x^n))\,w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = x*((n - 2)*y^n - 2*x^n)*D[w[x, y], x] + y*(2*y^n - (n - 2)*x^n)*D[w[x, y], y] == ((a*(
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*((n-2)*y^n -2*x^n )*diff(w(x,y),x)+y*(2*y^n - (n-2)*x^n)*diff(w(x,y),y) = ((a*(n-2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(x^n+y^n)\,(y^n)^{-\frac{2}{n}}}{x^2}\right)\mathrm{e}^{\int^x \frac{2\left(\frac{1}{2}bn+a-b\right)\_a^n+(-na+2a-2b)\,\mathrm{RootOf}\left(\_a^2x^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}\right.}{\left(-n\,\mathrm{RootOf}\left(\_a^2x^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}+\_a^2y^n(\_Z^n)^{\frac{2}{n}}(y^n)^{-\frac{2}{n}}-x^2\_Z^n-x^2\_a^n\right)\right)^n+2\_a^n+2\,\mathrm{Roo}}}$$

6.4.6 3.1

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6.4.6.1 [1059] Problem 1

problem number 1059

Added Feb. 23, 2019.

Problem Chapter 4.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\alpha x+\beta y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x + beta*y]*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{ce^{\alpha x+\beta y}}{a\alpha+b\beta}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*exp(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\alpha x+\beta y}}{a\alpha+b\beta}}$$

6.4.6.2 [1060] Problem 2

problem number 1060

Added Feb. 23, 2019.

Problem Chapter 4.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (ce^{\lambda x} + ke^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x] + k*Exp[mu*y])*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) e^{\frac{ce^{\lambda x}}{a\lambda} + \frac{ke^{\mu y}}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*exp(lambda*x)+k*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right) \mathrm{e}^{\frac{ak\lambda\, \mathrm{e}^{\mu y} + bc\mu\, \mathrm{e}^{\lambda x}}{ab\lambda\mu}}$$

6.4.6.3 [1061] Problem 3

problem number 1061

Added Feb. 23, 2019.

Problem Chapter 4.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*w[x, y];
sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ce^{-\lambda x}}{a\lambda}}c_1\left(\frac{be^{-\lambda x}}{a\lambda}-\frac{e^{-\beta y}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\left(-a\lambda\,\mathrm{e}^{\lambda x}+b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)\mathrm{e}^{-\frac{c\,\mathrm{e}^{-\lambda x}}{a\lambda}}$$

6.4.6.4 [1062] Problem 4

problem number 1062

Added Feb. 23, 2019.

Problem Chapter 4.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+be^{\beta x}w_y=cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\lambda y}}{\lambda}-\frac{be^{\beta x}}{a\beta}\right)\exp\left(\frac{c\left(\beta x-\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)\right)}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{a\beta\,\mathrm{e}^{\lambda y}}{b\lambda}\right)^{-\frac{c}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}} \left(\mathrm{e}^{\beta x}\right)^{\frac{c}{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}} \_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y}-b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right)$$

6.4.6.5 [1063] Problem 5

problem number 1063

Added Feb. 23, 2019.

Problem Chapter 4.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)}+y\right)\exp\left(\int_1^x \frac{c\exp\left(y\gamma-\frac{b\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K[1]}\right)\gamma}{a(\beta-\lambda)}-\lambda K[1]\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*exp(gamma*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\beta-\lambda)\,ay - b\,\mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\,a}\right)\mathrm{e}^{\int^{x} \frac{c\,\mathrm{e}^{\frac{\gamma b\,\mathrm{e}^{(\beta-\lambda)\_a} - \gamma b\,\mathrm{e}^{(\beta-\lambda)x} + (\beta-\lambda)(-\_a\lambda+\gamma y)a}{(\beta-\lambda)a}}}{a}\,d\_a}$$

6.4.6.6 [1064] Problem 6

problem number 1064

Added Feb. 23, 2019.

Problem Chapter 4.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = (ce^{\gamma y} + se^{\delta y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == (c*Exp[gamma*y] + s*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{e^{-\beta y}}{\beta}\right)\exp\left(-\frac{e^{-\lambda x}\left(e^{-\beta y}\right)^{-\frac{\delta+\gamma}{\beta}}\left((\beta-\delta)\left(c\gamma\left(e^{-\beta y}\right)^{\frac{\delta}{\beta}}\left(\frac{a\lambda e^{\lambda x-\beta y}}{b\beta}\right)^{\frac{\gamma}{\beta}}\,{}_2F_1\left(\frac{\beta+\gamma}{\beta},\frac{\gamma}{\beta}\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  (c*exp(gamma*y)+s*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-a\lambda\,\mathrm{e}^{\lambda x}+b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y-\lambda x}}{b\beta\lambda}\right)\mathrm{e}^{-\frac{\left((\beta-\delta)c\left(\frac{a\lambda}{b\beta\,\mathrm{e}^{-\lambda x}+\left(a\lambda\,\mathrm{e}^{\lambda x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\beta y-\lambda x}}\right)^{\frac{\gamma}{\beta}}+(\beta-\gamma)s\left(\frac{a\lambda}{b\beta\,\mathrm{e}^{-\lambda x}+\left(a\lambda\,\mathrm{e}^{\lambda x}-b\right.}\right.\right.}{(\beta-\gamma)(\beta-\delta)ab\lambda}}$$

6.4.6.7 [1065] Problem 7

problem number 1065

Added Feb. 23, 2019.

Problem Chapter 4.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = (se^{\mu x} + ke^{\delta y} + p)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right) - \mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{\lambda}\right)\mathrm{e}^{\int x \frac{k\left(a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}\right.}{}}$$

6.4.6.8 [1066] Problem 8

problem number 1066

Added Feb. 23, 2019.

Problem Chapter 4.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = (se^{\mu x+\delta y} + k)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right) - \mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{\lambda}\right)\mathrm{e}^{\int^{x}\frac{s\left(a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}\right.}{}\right.}$$

6.4.6.9 [1067] Problem 9

problem number 1067

Added Feb. 23, 2019.

Problem Chapter 4.3.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = (ce^{\mu x+\delta y}+k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x + lambda*y])*D[w[x, y], y] == (c*Exp[mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{\gamma x-\beta x}}{a\beta-a\gamma}-\frac{e^{-\lambda y}}{\lambda}\right)\exp\left(\frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\delta/\lambda}e^{-\gamma x-\lambda y+\mu x}\,{}_2F_1\left(1,\frac{\mu-\gamma}{\beta-\gamma};\frac{\beta\delta-\gamma\delta-\gamma\lambda+\lambda\mu}{\beta\lambda-\gamma\lambda};1-\frac{ae^{\beta x}}{}\right.}{b(\beta(\lambda-\delta)+\delta\gamma-\lambda\mu)}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x+lambda*y))*diff(w(x,y),y) =  (c*exp(mu*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(-b\lambda\,\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+(\beta-\gamma)\,a\right)\mathrm{e}^{-\lambda y}}{(\beta-\gamma)\,b\lambda}\right)\mathrm{e}^{\int^{x}\frac{\left(c\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_a}+(\beta-\gamma)a\,\mathrm{e}^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{\_a}}{a}}$$

6.4.6.10 [1068] Problem 10

problem number 1068

Added Feb. 23, 2019.

Problem Chapter 4.3.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = (ce^{\mu x}+k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == (c*Exp[mu*x] + k)*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{\frac{ce^{x(\mu-\lambda)}}{\lambda-\mu}+\frac{ke^{-\lambda x}}{\lambda}}{a}}c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  (c*exp(mu*x) + k)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\beta-\lambda)\,ay - b\,\mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\,a}\right)\mathrm{e}^{\frac{c\lambda\,\mathrm{e}^{(-\lambda+\mu)x}-(-\lambda+\mu)k\,\mathrm{e}^{-\lambda x}}{(-\lambda+\mu)a\lambda}}$$

6.4.7 3.2

Local contents

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6.4.7.1 [1069] Problem 1

problem number 1069

Added Feb. 23, 2019.

Problem Chapter 4.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (cye^{\lambda x} + kxe^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y*Exp[lambda*x] + k*x*Exp[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(-\frac{bce^{\lambda x}}{a^2\lambda^2} - \frac{ake^{\mu y}}{b^2\mu^2} + \frac{cye^{\lambda x}}{a\lambda} + \frac{kxe^{\mu y}}{b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*y*exp(lambda*x) + k*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{-(-b\mu x+a)a^2k\lambda^2\mathrm{e}^{\mu y}+(a\lambda y-b)b^2c\mu^2\mathrm{e}^{\lambda x}}{a^2b^2\lambda^2\mu^2}}$$

6.4.7.2 [1070] Problem 2

problem number 1070

Added Feb. 23, 2019.

Problem Chapter 4.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) e^{\frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*exp(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) \mathrm{e}^{\frac{a\,\mathrm{e}^{\lambda x+\mu y}}{\lambda+\frac{\mu y}{x}}}$$

6.4.7.3 [1071] Problem 3

problem number 1071

Added Feb. 23, 2019.

Problem Chapter 4.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = (aye^{\lambda x} + bxe^{\mu y})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x] + b*x*Exp[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =    (a*y*exp(lambda*x)+ b*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) \mathrm{e}^{\frac{\left(\frac{a\mu y^2 \mathrm{e}^{\lambda x}}{x^2}+b\lambda\,\mathrm{e}^{\mu y}\right)x}{\lambda\mu y}}$$

6.4.7.4 [1072] Problem 4

problem number 1072

Added Feb. 23, 2019.

Problem Chapter 4.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ax^k w_x + be^{\lambda y} w_y = (cx^n + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == (c*x^n + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x^{1-k}\left(\frac{cx^n}{-k+n+1}+\frac{s}{1-k}\right)}{a}}c_1\left(\frac{bx^{1-k}}{a(k-1)}-\frac{e^{-\lambda y}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*x^k*diff(w(x,y),x)+b*exp(lambda*y)*diff(w(x,y),y) =    (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{b\lambda x^{-k+1}-(k-1)\,a\,\mathrm{e}^{-\lambda y}}{(k-1)\,b\lambda}\right)\mathrm{e}^{-\frac{((k-1)cx^n+(k-n-1)s)x^{-k+1}}{(k-n-1)(k-1)a}}$$

6.4.7.5 [1073] Problem 5

problem number 1073

Added Feb. 23, 2019.

Problem Chapter 4.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x+be^{\lambda x}w_y=(ce^{\mu x}+s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\frac{be^{\lambda x}}{a\lambda}\right)\exp\left(-\frac{y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\left(c\lambda e^{\mu x}\,{}_2F_1\left(1,\frac{\lambda+k\mu+\mu}{k\lambda+\lambda};\frac{\lambda+\mu}{\lambda};\frac{be^{\lambda x}(k+1)}{be^{\lambda x}(k+1)-a\lambda y^{k+1}}\right.\right.}{\mu\left(b(k+1)e^{\lambda x}-a\lambda\right.}\right.\right.$$

Maple ✓

```
restart;
pde := a*y^k*diff(w(x,y),x)+b*exp(lambda*x)*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\lambda y y^k - (k+1)\, b\, \mathrm{e}^{\lambda x}}{a\lambda}\right) \mathrm{e}^{\int^x \frac{(c\,\mathrm{e}^{-a\mu}+s)\left(\left(\frac{a\lambda y^{k+1}+(k+1)b\,\mathrm{e}^{-a\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k}}{a} d\_a}$$

6.4.7.6 [1074] Problem 6

problem number 1074

Added Feb. 23, 2019.

Problem Chapter 4.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x} w_x + by^k w_y = (cx^n + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == (c*x^n + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{be^{-\lambda x}}{a\lambda} - \frac{y^{1-k}}{k-1}\right) \exp\left(-\frac{cx^n(\lambda x)^{-n}\,\mathrm{Gamma}(n+1,\lambda x) + se^{-\lambda x}}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*y^k*diff(w(x,y),y) =   (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a\lambda y^{-k+1} - (k-1)\,b\,\mathrm{e}^{-\lambda x}}{a\lambda}\right)\mathrm{e}^{\frac{c x^{n} (\lambda x)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},\lambda x\right)\mathrm{e}^{-\frac{\lambda x}{2}} - \left(\mathrm{e}^{-\lambda x}-1\right)(n+1)s}{(n+1)a\lambda}}$$

6.4.7.7 [1075] Problem 7

problem number 1075

Added Feb. 23, 2019.

Problem Chapter 4.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^k w_y = (ce^{\mu x} + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\lambda y}}{\lambda} - \frac{bx^{k+1}}{ak+a}\right)\exp\left(\int_1^x \frac{(k+1)\left(e^{\mu K[1]}c+s\right)}{ae^{\lambda y}(k+1) + b\lambda\left(K[1]^{k+1} - x^{k+1}\right)}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*x^k*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-b\lambda x^{k+1} + (k+1)\,a\,\mathrm{e}^{\lambda y}}{(k+1)\,b\lambda}\right)\mathrm{e}^{\int^{x} \frac{(c\,\mathrm{e}^{-a\mu}+s)(k+1)}{b\lambda\_a^{k+1} - b\lambda x^{k+1} + (k+1)a\,\mathrm{e}^{\lambda y}}d\_a}$$

6.4.8 4.1

Local contents

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6.4.8.1 [1076] Problem 1

problem number 1076

Added Feb. 23, 2019.

Problem Chapter 4.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\sinh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Sinh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x)}{a\lambda} + \frac{k\cosh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*sinh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\cosh(\mu y) + bc\mu\cosh(\lambda x)}{ab\lambda\mu}}$$

6.4.8.2 [1077] Problem 2

problem number 1077

Added Feb. 23, 2019.

Problem Chapter 4.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{c\cosh(\lambda x+\mu y)}{a\lambda+\mu b}}$$

6.4.8.3 [1078] Problem 3

problem number 1078

Added Feb. 23, 2019.

Problem Chapter 4.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sinh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ax\cosh(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\cosh(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

6.4.8.4 [1079] Problem 4

problem number 1079

Added Feb. 23, 2019.

Problem Chapter 4.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\sinh^n(\lambda x)w_y=(c\sinh^m(\mu x)+s\sinh^k(\beta y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*sinh(lambda*x)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\sinh^n(\lambda x))}{a} dx\right)\right) e^{\int^x \frac{c(\sinh^m(\_b\mu)) + s\left(-\sinh\left(\left(-y - \left(\int \frac{b(\sinh^n(\_b\lambda))}{a} d\_b\right) + \int \frac{b(\sinh^n(\lambda x))}{a} dx\right)\beta\right)\right)^k}{a} d}$$

6.4.8.5 [1080] Problem 5

problem number 1080

Added Feb. 23, 2019.

Problem Chapter 4.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = (c\sinh^m(\mu x) + s\sinh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*y]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\cosh^2(\lambda y)}\operatorname{sech}(\lambda y)\sinh^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\sinh}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*sinh(lambda*y)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\left(\int \left(\sinh^{-n}(\lambda y)\right) dy\right) + bx}{b}\right) e^{\int y \frac{\left(c\left(-\sinh\left(\frac{\left(a\left(\int\left(\sinh^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int \frac{a\left(\sinh^{-n}(\_b\lambda)\right)}{b} d\_b\right)\right)\mu}{b}\right)\right)^m +}{b}}$$

6.4.9 4.2

Local contents

6.4.9.1 [1081] Problem 1 1749
6.4.9.2 [1082] Problem 2 1750
6.4.9.3 [1083] Problem 3 1751
6.4.9.4 [1084] Problem 4 1752
6.4.9.5 [1085] Problem 5 1752

6.4.9.1 [1081] Problem 1

problem number 1081

Added Feb. 23, 2019.

Problem Chapter 4.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cosh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cosh[lambda*x] + k*Cosh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sinh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*cosh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\sinh(\mu y)+bc\mu\sinh(\lambda x)}{ab\lambda\mu}}$$

6.4.9.2 [1082] Problem 2

problem number 1082

Added Feb. 23, 2019.

Problem Chapter 4.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\cosh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sinh(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{c\sinh(\lambda x+\mu y)}{a\lambda+\mu b}}$$

6.4.9.3 [1083] Problem 3

problem number 1083

Added Feb. 23, 2019.

Problem Chapter 4.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cosh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ax\sinh(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\sinh(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

6.4.9.4 [1084] Problem 4

problem number 1084

Added Feb. 23, 2019.

Problem Chapter 4.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\cosh^n(\lambda x)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*cosh(lambda*x)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\cosh^n(\lambda x))}{a}dx\right)\right) e^{\int^x \frac{c(\cosh^m(\_b\mu))+s\left(\cosh^k\left(\left(-y-\left(\int \frac{b(\cosh^n(\_b\lambda))}{a}d\_b\right)+\int \frac{b(\cosh^n(\lambda x))}{a}dx\right)\beta\right)\right)}{a}d\_}$$

6.4.9.5 [1085] Problem 5

problem number 1085

Added Feb. 23, 2019.

Problem Chapter 4.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\cosh^n(\lambda y)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{-\sinh^2(\lambda y)}\operatorname{csch}(\lambda y)\cosh^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(\lambda y)\right)}{\lambda-\lambda n}-\frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cosh}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*cosh(lambda*y)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\left(\int\left(\cosh^{-n}(\lambda y)\right)dy\right)+bx}{b}\right)\mathrm{e}^{\int^y \frac{\left(c\left(\cosh^m\left(\frac{\left(a\left(\int\left(\cosh^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\cosh^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)+}{b}}$$

6.4.10 4.3

Local contents

6.4.10.1 [1086] Problem 1 1754
6.4.10.2 [1087] Problem 2 1754
6.4.10.3 [1088] Problem 3 1755
6.4.10.4 [1089] Problem 4 1756
6.4.10.5 [1090] Problem 5 1757

6.4.10.1 [1086] Problem 1

problem number 1086

Added Feb. 23, 2019.

Problem Chapter 4.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\tanh(\lambda x) + k\tanh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tanh[lambda*x] + k*Tanh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cosh^{\frac{c}{a\lambda}}(\lambda x)\cosh^{\frac{k}{b\mu}}(\mu y)c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*tanh(lambda*x) + k*tanh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x) - 1)^{-\frac{c}{2a\lambda}}(\tanh(\lambda x) + 1)^{-\frac{c}{2a\lambda}}(\tanh(\mu y) - 1)^{-\frac{k}{2b\mu}}(\tanh(\mu y) + 1)^{-\frac{k}{2b\mu}}\_F1\left(\frac{ay - bx}{a}\right.$$

6.4.10.2 [1087] Problem 2

problem number 1087

Added Feb. 23, 2019.

Problem Chapter 4.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cosh^{\frac{c}{a\lambda+b\mu}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=(\tanh(\lambda x+\mu y)-1)^{-\frac{c}{2a\lambda+2\mu b}}(\tanh(\lambda x+\mu y)+1)^{-\frac{c}{2a\lambda+2\mu b}}\_F1\left(\frac{ay-bx}{a}\right)$$

6.4.10.3 [1088] Problem 3

problem number 1088

Added Feb. 23, 2019.

Problem Chapter 4.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=ax\tanh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cosh^{\frac{ax}{\lambda x+\mu y}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x + \mu y) - 1)^{-\frac{a}{2\left(\lambda + \frac{\mu y}{x}\right)}} (\tanh(\lambda x + \mu y) + 1)^{-\frac{a}{2\left(\lambda + \frac{\mu y}{x}\right)}} \_F1\left(\frac{y}{x}\right)$$

6.4.10.4 [1089] Problem 4

problem number 1089

Added Feb. 23, 2019.

Problem Chapter 4.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda x)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{b\tanh^{n+1}(\lambda x)\,{}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \tanh^2(\lambda x)\right)}{a\lambda n + a\lambda}\right)\exp\left(\int_1^x \frac{s\tanh^k\left(\frac{\beta(-b\,{}_2F_1(1,\frac{n+1}{2};\frac{n+3}{2};\text{ta}}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*tanh(lambda*x)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\tanh^n(\lambda x))}{a} dx\right)\right) e^{\int^x \frac{c(\tanh^m(\_b\mu)) + s\left(-\tanh\left(\left(-y - \left(\int \frac{b(\tanh^n(\_b\lambda))}{a} d\_b\right) + \int \frac{b(\tanh^n(\lambda x))}{a} dx\right)\beta\right)\right)}{a}}$$

6.4.10.5 [1090] Problem 5

problem number 1090

Added Feb. 23, 2019.

Problem Chapter 4.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda y)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\tanh^{1-n}(\lambda y)\,{}_2F_1\left(1, \frac{1}{2} - \frac{n}{2}; \frac{3}{2} - \frac{n}{2}; \tanh^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right) \exp\left(\int_1^y \frac{\tanh^{-n}(\lambda K[1])\left(s\tanh\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*tanh(lambda*y)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)}{b}+x\right)e^{\int^{y}\frac{\left(c\left(-\tanh\left(-\mu\left(\int\frac{a\left(\tanh^{-n}(\_b\lambda)\right)}{b}d\_b\right)-\left(-\frac{a\left(\int\left(\tanh^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)}{b}}$$

6.4.11 4.4

Local contents

6.4.11.1 [1091] Problem 1 1758
6.4.11.2 [1092] Problem 2 1759
6.4.11.3 [1093] Problem 3 1760
6.4.11.4 [1094] Problem 4 1760
6.4.11.5 [1095] Problem 5 1761

6.4.11.1 [1091] Problem 1

problem number 1091

Added Feb. 23, 2019.

Problem Chapter 4.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\coth(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Coth[lambda*x] + k*Coth[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sinh^{\frac{c}{a\lambda}}(\lambda x)c_1\left(y-\frac{bx}{a}\right)e^{\frac{k(\log(\tanh(\mu y))+\log(\cosh(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*coth(lambda*x) + k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\coth(\lambda x) - 1)^{-\frac{c}{2a\lambda}} (\coth(\lambda x) + 1)^{-\frac{c}{2a\lambda}} (\coth(\mu y) - 1)^{-\frac{k}{2b\mu}} (\coth(\mu y) + 1)^{-\frac{k}{2b\mu}} \_F1\left(\frac{ay - bx}{a}\right)$$

6.4.11.2 [1092] Problem 2

problem number 1092

Added Feb. 23, 2019.

Problem Chapter 4.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{c(\log(\tanh(\lambda x + \mu y)) + \log(\cosh(\lambda x + \mu y)))}{a\lambda + b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\coth(\lambda x + \mu y) - 1)^{-\frac{c}{2a\lambda + 2\mu b}} (\coth(\lambda x + \mu y) + 1)^{-\frac{c}{2a\lambda + 2\mu b}} \_F1\left(\frac{ay - bx}{a}\right)$$

6.4.11.3 [1093] Problem 3

problem number 1093

Added Feb. 23, 2019.

Problem Chapter 4.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax \coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right)\exp\left(\frac{ax(\log(\tanh(\lambda x+\mu y))+\log(\cosh(\lambda x+\mu y)))}{\lambda x+\mu y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\coth(\lambda x+\mu y)-1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}}(\coth(\lambda x+\mu y)+1)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}}\_F1\left(\frac{y}{x}\right)$$

6.4.11.4 [1094] Problem 4

problem number 1094

Added Feb. 23, 2019.

Problem Chapter 4.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\coth^n(\lambda x)w_y = (c\coth^m(\mu x) + s\coth^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\coth^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x \frac{s\coth^k\left(\frac{\beta(-b\,{}_2F_1(1,\frac{n+1}{2};\frac{n+3}{2};\text{cot}}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*coth(lambda*x)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b(\coth^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c(\coth^m(\_b\mu))+s\left(-\coth\left(\left(-y-\left(\int\frac{b(\coth^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\coth^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}}$$

6.4.11.5 [1095] Problem 5

problem number 1095

Added Feb. 23, 2019.

Problem Chapter 4.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\coth^n(\lambda y)w_y=(c\coth^m(\mu x)+s\coth^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\coth^{1-n}(\lambda y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\coth^2(\lambda y)\right)}{\lambda-\lambda n}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\left(s\coth^k(\beta K[1])+c\cot\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*coth(lambda*y)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mathrm{e}^{\int^y\frac{\left(c\left(-\coth\left(-\mu\left(\int\frac{a\left(\coth^{-n}(\_b\lambda)\right)}{b}d\_b\right)-\left(-\frac{a\left(\int\left(\coth^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)^{n}}{b}}$$

6.4.12 4.5

Local contents

6.4.12.1 [1096] Problem 1 1763
6.4.12.2 [1097] Problem 2 1763
6.4.12.3 [1098] Problem 3 1764
6.4.12.4 [1099] Problem 4 1765
6.4.12.5 [1100] Problem 5 1766
6.4.12.6 [1101] Problem 6 1766

6.4.12.1 [1096] Problem 1

problem number 1096

Added Feb. 23, 2019.

Problem Chapter 4.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Cosh[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x)}{a\lambda} + \frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\sinh(\mu y) + bc\mu\cosh(\lambda x)}{ab\lambda\mu}}$$

6.4.12.2 [1097] Problem 2

problem number 1097

Added Feb. 23, 2019.

Problem Chapter 4.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (\tanh(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (Tanh[lambda*x] + k*Coth[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\cosh(\lambda x)}c_1\left(y-\frac{bx}{a}\right)e^{\frac{k(\log(\tanh(\mu y))+\log(\cosh(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (tanh(lambda*x)+k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\cosh^{\frac{1}{a\lambda}}(\lambda x)\right)\left(\sinh^{\frac{k}{b\mu}}(\mu y)\right)\_F1\left(\frac{ay-bx}{a}\right)$$

6.4.12.3 [1098] Problem 3

problem number 1098

Added Feb. 23, 2019.

Problem Chapter 4.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\sinh(\mu y)w_y=b\cosh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Cosh[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sinh(\lambda x)}{\lambda}}c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*cosh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\mu x - 2\operatorname{arctanh}\left(e^{\mu y}\right)}{a\mu}\right) e^{\frac{b\sinh(\lambda x)}{\lambda}}$$

6.4.12.4 [1099] Problem 4

problem number 1099

Added Feb. 23, 2019.

Problem Chapter 4.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = b\tanh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Tanh[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cosh^{\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu} - ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*tanh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = (\tanh(\lambda x) - 1)^{-\frac{b}{2\lambda}}(\tanh(\lambda x) + 1)^{-\frac{b}{2\lambda}}\_F1\left(\frac{-a\mu x - 2\operatorname{arctanh}\left(e^{\mu y}\right)}{a\mu}\right)$$

6.4.12.5 [1100] Problem 5

problem number 1100

Added Feb. 23, 2019.

Problem Chapter 4.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh(\lambda x)w_x + b\cosh(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\tanh\left(\frac{\lambda x}{2}\right)}c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{b\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*sinh(lambda*x)*diff(w(x,y),x)+b*cosh(mu*y)^n*diff(w(x,y),y) =  w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda\left(\int \frac{a(\cosh^{-n}(\mu y))}{b}dy\right)+2\operatorname{arctanh}\left(e^{\lambda x}\right)}{\lambda}\right)e^{-\frac{2\operatorname{arctanh}\left(e^{\lambda x}\right)}{a\lambda}}$$

6.4.12.6 [1101] Problem 6

problem number 1101

Added Feb. 23, 2019.

Problem Chapter 4.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh(\lambda x)w_x + b\coth(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y], x] + b*Coth[mu*y]*D[w[x, y], y] == w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\sinh(\lambda x)}c_1\left(-\frac{2a\cosh(\mu y)\sinh^{-\frac{b\mu}{a\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*tanh(lambda*x)*diff(w(x,y),x)+b*coth(mu*y)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y),'build')),outpu
```

$$w(x,y)=c_1c_2\left(\frac{-\cosh(\lambda x)+\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-\frac{-c_1}{2\lambda}}\left(\frac{\cosh(\lambda x)+\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-\frac{-c_1}{2\lambda}}\left(\frac{\cosh(\mu y)}{\sinh(\mu y)}\right)^{\frac{1}{b\mu}}\left(\frac{\cosh(\mu y)}{\sinh(\mu y)}\right.$$

6.4.13 5.1

Local contents

6.4.13.1 [1102] Problem 1 1767
6.4.13.2 [1103] Problem 2 1768
6.4.13.3 [1104] Problem 3 1769
6.4.13.4 [1105] Problem 4 1770
6.4.13.5 [1106] Problem 5 1771
6.4.13.6 [1107] Problem 6 1772

6.4.13.1 [1102] Problem 1

problem number 1102

Added Feb. 25, 2019.

Problem Chapter 4.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\ln(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\frac{(a\beta y-b\beta x)\log(a(\beta y+\lambda x))}{a\lambda+b\beta}+x\log(\beta y+\lambda x)-x\right)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*ln(lambda*x + beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=(\beta y+\lambda x)^{\frac{(\beta y+\lambda x)c}{a\lambda+b\beta}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{-\frac{(\beta y+\lambda x)c}{a\lambda+b\beta}}$$

6.4.13.2 [1103] Problem 2

problem number 1103

Added Feb. 25, 2019.

Problem Chapter 4.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c\ln(\lambda x)+k\ln(\beta y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Log[lambda*x] + k*Log[beta*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{x(c+k)}{a}}(\lambda x)^{\frac{cx}{a}}(\beta y)^{\frac{ky}{b}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*ln(lambda*x)+k*ln(beta*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=(\beta y)^{\frac{ky}{b}}(\lambda x)^{\frac{cx}{a}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{-aky-bcx}{ab}}$$

6.4.13.3 [1104] Problem 3

problem number 1104

Added Feb. 25, 2019.

Problem Chapter 4.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda x)w_y=\left(c\ln^m(\mu x)+s\ln^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b(-\log(\lambda x))^{-n}\log^n(\lambda x)\,\mathrm{Gamma}(n+1,-\log(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x \frac{s\log^k\left(\frac{\beta(-b\,\mathrm{Gamma}(}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*ln(lambda*x)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c\ln(\_b\lambda)^m+s\ln\left(\left(y+\int\frac{b\ln(\_b\lambda)^n}{a}d\_b-\left(\int\frac{b\ln(\lambda x)^n}{a}dx\right)\right)\beta\right)^k}{a}d\_b}$$

6.4.13.4 [1105] Problem 4

problem number 1105

Added Feb. 25, 2019.

Problem Chapter 4.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\ln^n(\lambda y)w_y=\left(c\ln^m(\mu x)+s\ln^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(-\log(\lambda y))^n\log^{-n}(\lambda y)\,\mathrm{Gamma}(1-n,-\log(\lambda y))}{\lambda}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\log^{-n}(\lambda K[1])\left(s\,\mathrm{lo}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*ln(lambda*y)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\ln(\lambda y)^{-n}\,dy\right)}{b}+x\right)\mathrm{e}^{\int^y\frac{\left(c\ln\left(\left(-\frac{a\left(\int\ln(\lambda y)^{-n}dy\right)}{b}+x+\int\frac{a\ln(\_b\lambda)^{-n}}{b}d\_b\right)\lambda\right)^m+s\ln(\_b\beta)^k\right)\ln(\_b\lambda)^{-n}}{b}d\_b}$$

6.4.13.5 [1106] Problem 5

problem number 1106

Added Feb. 25, 2019.

Problem Chapter 4.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln(\beta y)w_x+a\ln(\lambda x)w_y=bw\ln(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Log[beta*y]*D[w[x, y], x] + a*Log[lambda*x]*D[w[x, y], y] == b*w[x, y]*Log[beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{bx}c_1\left(y\left(\log\left(\beta y e^{\frac{ax}{y}}x^{-\frac{ax}{y}}\lambda^{-\frac{ax}{y}}\right)-1\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := ln(beta*y)*diff(w(x,y),x)+a*ln(lambda*x)*diff(w(x,y),y) =  b*w(x,y)*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-ax\ln(\lambda x)+ax+y\ln(\beta y)-y}{a}\right)\mathrm{e}^{bx}$$

6.4.13.6 [1107] Problem 6

problem number 1107

Added Feb. 25, 2019.

Problem Chapter 4.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln(\lambda x)^n w_x+b\ln(\beta y)^k w_y=c\ln(\gamma x)^m w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[beta*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := a*ln(lambda*x)^n*diff(w(x,y),x)+b*ln(beta*y)^k*diff(w(x,y),y) = c*log(gamma*x)^m*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \ln(\lambda x)^{-n}\,dx\right) + \int \frac{a\ln(\beta y)^{-k}}{b}dy\right) e^{\int \frac{c(\ln(x)+\ln(\gamma))^m \ln(\lambda x)^{-n}}{a}dx}$$

6.4.14 5.2

Local contents

6.4.14.1 [1108] Problem 1 1773
6.4.14.2 [1109] Problem 2 1774
6.4.14.3 [1110] Problem 3 1775
6.4.14.4 [1111] Problem 4 1776
6.4.14.5 [1112] Problem 5 1777
6.4.14.6 [1113] Problem 6 1778

6.4.14.1 [1108] Problem 1

problem number 1108

Added Feb. 25, 2019.

Problem Chapter 4.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + s\ln^k(\lambda y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + s*Log[gamma*y]^k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{s\log^k(\gamma y)(-\log(\gamma y))^{-k}\,\text{Gamma}(k+1,-\log(\gamma y))}{b\gamma}+\frac{cx^{n+1}}{an+a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n+s*ln(gamma*y)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\int^x\frac{\_c\_a^n+s\left(\ln\left(\frac{ay-(-\_a+x)b}{a}\right)+\ln(\gamma)\right)^k}{a}d\_a}$$

6.4.14.2 [1109] Problem 2

problem number 1109

Added Feb. 25, 2019.

Problem Chapter 4.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=(by^2+cx^ny+s\ln^k(\lambda x))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*y^2 + c*x^n*y + s*Log[lambda*x]^k)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\frac{s\log^k(\lambda x)(-\log(\lambda x))^{-k}\,\mathrm{Gamma}(k+1,-\log(\lambda x))}{\lambda}+\frac{1}{3}a^2bx^3+abx^2(y-a\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = (b*y^2+c*x^n*y+ s*ln(lambda*x)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(-ax+y)\,\mathrm{e}^{\int^x\left(a\_c\_a^{n+1}-(ax-y)\_c\_a^n+s\ln(\_a\lambda)^k+((-\_a+x)a-y)^2b\right)d\_a}$$

6.4.14.3 [1110] Problem 3

problem number 1110

Added March 9, 2019.

Problem Chapter 4.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=b\ln^k(\lambda x)\ln^n(\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x b\log^k(\lambda K[1])\log^n(\beta(y+a(K[1]-x)))dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b*ln(lambda*x)^k*ln(beta*y)^n*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x b\ln(\_a\lambda)^k \ln(-((-\_a+x)a-y)\beta)^n d\_a}$$

6.4.14.4 [1111] Problem 4

problem number 1111

Added March 9, 2019.

Problem Chapter 4.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^n)w_y = c\ln^k(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ba^{-n-1}\,\mathrm{Gamma}(n+1,ax) + ye^{-ax}\right)\exp\left(\frac{c(-\log(\lambda x))^{-k}\log^k(\lambda x)\,\mathrm{Gamma}(k+1,-\log(}{\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a*y+b*x^n)*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-bx^n(ax)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},ax\right)\mathrm{e}^{\frac{ax}{2}}+(n+1)\,ay\right)\mathrm{e}^{-ax}}{(n+1)\,a}\right)\mathrm{e}^{\int c\ln(\lambda x)^k dx}$$

6.4.14.5 [1112] Problem 5

problem number 1112

Added March 9, 2019.

Problem Chapter 4.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k(n\ln x + m\ln y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\frac{x^k(akm\log(y)+akn\log(x)-an-bm)}{a^2k^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = x^k*(n*ln(x)+m*ln(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = x^{\frac{nx^k}{ak}} \left(x^{\frac{b}{a}}\right)^{\frac{mx^k}{ak}} \left(yx^{-\frac{b}{a}}\right)^{\frac{mx^k}{ak}} \_F1\left(yx^{-\frac{b}{a}}\right) \mathrm{e}^{-\frac{\left(i\pi akm\operatorname{csgn}(iy)^3 - i\pi akm\operatorname{csgn}(iy)^2\operatorname{csgn}\left(ix^{\frac{b}{a}}\right) - i\pi\left(\operatorname{csgn}(iy) - \operatorname{csgn}\left(ix^{\frac{b}{a}}\right)\right)akm}{2a^2k^2}}$$

6.4.14.6 [1113] Problem 6

problem number 1113

Added March 9, 2019.

Problem Chapter 4.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + by^n w_y = (c\ln^m(\lambda x) + s\ln^t(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*y]^t)*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{bx^{1-k}}{a(k-1)} - \frac{y^{1-n}}{n-1}\right)\exp\left(\int_1^x \frac{K[1]^{-k}\left(c\log^m(\lambda K[1]) + s\log^t\left(\beta\left(\frac{a(k-1)x^k y^n}{a(k-1)x^k yK[1]^k - b(n-1)y}\right.\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^k*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = (c*ln(lambda*x)^m+s*ln(beta*y)^t)*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(k-1)\,ay^{-n+1} - (n-1)\,bx^{-k+1}}{(k-1)\,a}\right) \mathrm{e}^{\int^x \frac{\left(c\ln(\_a\lambda)^m + s\ln\left(\beta\left(\frac{(k-1)ay^{-n+1}+(n-1)b\_a^{-k+1}-(n-1)bx^{-k+1}}{(k-1)a}\right)^{-}\right.\right.}{a}}$$

6.4.15 6.1

Local contents

6.4.15.1 [1114] Problem 1 1779
6.4.15.2 [1115] Problem 2 1780
6.4.15.3 [1116] Problem 3 1781
6.4.15.4 [1117] Problem 4 1782
6.4.15.5 [1118] Problem 5 1783

6.4.15.1 [1114] Problem 1

problem number 1114

Added March 9, 2019.

Problem Chapter 4.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sin(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{-\frac{c\cos(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{-\frac{c\cos(\lambda x+\mu y)}{a\lambda+\mu b}}$$

6.4.15.2 [1115] Problem 2

problem number 1115

Added March 9, 2019.

Problem Chapter 4.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c\sin(\lambda x)+k\sin(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sin[lambda*x] + k*Sin[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{-\frac{c\cos(\lambda x)}{a\lambda}-\frac{k\cos(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*sin(lambda*x)+k*sin(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{-ak\lambda\cos(\mu y)-bc\mu\cos(\lambda x)}{ab\lambda\mu}}$$

6.4.15.3 [1116] Problem 3

problem number 1116

Added March 9, 2019.

Problem Chapter 4.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{-\frac{ax\cos(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{-\frac{a\cos(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

6.4.15.4 [1117] Problem 4

problem number 1117

Added March 9, 2019.

Problem Chapter 4.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{b\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)}{a\lambda n + a\lambda}\right)\exp\left(\int_1^x \frac{s\sin^k\left(\frac{\beta(-b}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*x)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\sin^n(\lambda x))}{a}dx\right)\right)e^{\int^x \frac{c(\sin^m(\_b\mu)) + s\left(-\sin\left(\left(-y - \left(\int \frac{b(\sin^n(\_b\lambda))}{a}d\_b\right) + \int \frac{b(\sin^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}d\_b}$$

6.4.15.5 [1118] Problem 5

problem number 1118

Added March 9, 2019.

Problem Chapter 4.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\sqrt{\cos^2(\lambda y)}\sec(\lambda y)\sin^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(\lambda y)\right)}{\lambda-\lambda n} - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\sin^{-n}(\lambda K[1]}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*y)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\left(\int(\sin^{-n}(\lambda y))\,dy\right)+bx}{b}\right)\mathrm{e}^{\int^y \frac{\left(c\left(-\sin\left(\frac{\left(a\left(\int\left(\sin^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\sin^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)^m+s\left(\sin^k\right.}{b}}$$

6.4.16 6.2

Local contents

6.4.16.1 [1119] Problem 1 1784
6.4.16.2 [1120] Problem 2 1785
6.4.16.3 [1121] Problem 3 1785
6.4.16.4 [1122] Problem 4 1786
6.4.16.5 [1123] Problem 5 1787

6.4.16.1 [1119] Problem 1

problem number 1119

Added March 9, 2019.

Problem Chapter 4.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\cos(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{c\sin(\lambda x+\mu y)}{a\lambda+b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{c\sin(\lambda x+\mu y)}{a\lambda+\mu b}}$$

6.4.16.2 [1120] Problem 2

problem number 1120

Added March 9, 2019.

Problem Chapter 4.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\cos(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cos[lambda*x] + k*Cos[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)e^{\frac{c\sin(\lambda x)}{a\lambda} + \frac{k\sin(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cos(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{ak\lambda\sin(\mu y) + bc\mu\sin(\lambda x)}{ab\lambda\mu}}$$

6.4.16.3 [1121] Problem 3

problem number 1121

Added March 9, 2019.

Problem Chapter 4.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right)e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{\frac{a\sin(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

6.4.16.4 [1122] Problem 4

problem number 1122

Added March 9, 2019.

Problem Chapter 4.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda x)w_y = (c\cos^m(\mu x) + s\cos^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{b\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(\lambda x)\right)}{a\lambda n+a\lambda}+y\right)\exp\left(\int_1^x \frac{s\cos^k\left(\frac{\beta\left(b\,c\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*x)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x\frac{c(\cos^m(\_b\mu))+s\left(\cos^k\left(\left(-y-\left(\int\frac{b(\cos^n(\_b\lambda))}{a}d\_b\right)+\int\frac{b(\cos^n(\lambda x))}{a}dx\right)\beta\right)\right)}{a}d\_b}$$

6.4.16.5 [1123] Problem 5

problem number 1123

Added March 9, 2019.

Problem Chapter 4.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\cos^n(\lambda y)w_y=(c\cos^m(\mu x)+s\cos^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{\sin^2(\lambda y)}\csc(\lambda y)\cos^{1-n}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\lambda y)\right)}{\lambda(n-1)}-\frac{bx}{a}\right)\exp\left(\int_1^y\frac{\cos^{-n}(\lambda K[1}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*y)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-a\left(\int\left(\cos^{-n}(\lambda y)\right)dy\right)+bx}{b}\right)\mathrm{e}^{\int^y\frac{\left(c\left(\cos^m\left(\frac{\left(a\left(\int\left(\cos^{-n}(\lambda y)\right)dy\right)-bx-b\left(\int\frac{a\left(\cos^{-n}(\_b\lambda)\right)}{b}d\_b\right)\right)\mu}{b}\right)\right)+s\left(\cos^k\right.}{b}}$$

6.4.17 6.3

Local contents

6.4.17.1 [1124] Problem 1 1789
6.4.17.2 [1125] Problem 2 1789
6.4.17.3 [1126] Problem 3 1790
6.4.17.4 [1127] Problem 4 1791
6.4.17.5 [1128] Problem 5 1792

6.4.17.1 [1124] Problem 1

problem number 1124

Added March 9, 2019.

Problem Chapter 4.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\cos^{-\frac{c}{a\lambda+b\mu}}(\lambda x + \mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\tan^2(\lambda x + \mu y) + 1\right)^{\frac{c}{2a\lambda+2\mu b}} \_F1\left(\frac{ay - bx}{a}\right)$$

6.4.17.2 [1125] Problem 2

problem number 1125

Added March 9, 2019.

Problem Chapter 4.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (c\tan(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tan[lambda*x] + k*Tan[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cos^{-\frac{c}{a\lambda}}(\lambda x)\cos^{-\frac{k}{b\mu}}(\mu y)c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*tan(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\tan^2(\lambda x) + 1\right)^{\frac{c}{2a\lambda}}\left(\tan^2(\mu y) + 1\right)^{\frac{k}{2b\mu}} \_F1\left(\frac{ay - bx}{a}\right)$$

6.4.17.3 [1126] Problem 3

problem number 1126

Added March 9, 2019.

Problem Chapter 4.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cos^{-\frac{ax}{\lambda x+\mu y}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\tan^2(\lambda x+\mu y)+1\right)^{\frac{a}{2\lambda+\frac{2\mu y}{x}}}\_F1\left(\frac{y}{x}\right)$$

6.4.17.4 [1127] Problem 4

problem number 1127

Added March 9, 2019.

Problem Chapter 4.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\tan^n(\lambda x)w_y=(c\tan^m(\mu x)+s\tan^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{b\tan^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(\lambda x)\right)}{a\lambda n+a\lambda}\right)\exp\left(\int_1^x\frac{s\tan^k\left(\frac{\beta\left(-b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*x)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\tan^n(\lambda x))}{a} dx\right)\right) e^{\int^x \frac{c(\tan^m(\_b\mu)) + s\left(-\tan\left(\left(-y - \left(\int \frac{b(\tan^n(\_b\lambda))}{a} d\_b\right) + \int \frac{b(\tan^n(\lambda x))}{a} dx\right)\beta\right)\right)^k}{a} d\_b}$$

6.4.17.5 [1128] Problem 5

problem number 1128

Added March 9, 2019.

Problem Chapter 4.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan^n(\lambda y)w_y = (c\tan^m(\mu x) + s\tan^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\tan^{1-n}(\lambda y)\,{}_2F_1\left(1, \frac{1}{2} - \frac{n}{2}; \frac{3}{2} - \frac{n}{2}; -\tan^2(\lambda y)\right)}{\lambda - \lambda n} - \frac{bx}{a}\right) \exp\left(\int_1^y \frac{\tan^{-n}(\lambda K[1])\left(s\tan^k(\beta\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*y)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)e^{\int^{y}\frac{\left(c\left(-\tan\left(-\mu\left(\int\frac{a\left(\tan^{-n}(\_b\lambda)\right)}{b}d\_b\right)-\left(-\frac{a\left(\int\left(\tan^{-n}(\lambda y)\right)dy\right)}{b}+x\right)\mu\right)\right)^{m}+s(}{b}}$$

6.4.18 6.4

Local contents

6.4.18.1 [1129] Problem 1 1793
6.4.18.2 [1130] Problem 2 1794
6.4.18.3 [1131] Problem 3 1795
6.4.18.4 [1132] Problem 4 1796
6.4.18.5 [1133] Problem 5 1797

6.4.18.1 [1129] Problem 1

problem number 1129

Added March 9, 2019.

Problem Chapter 4.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cot(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\cot^2(\lambda x+\mu y)+1\right)^{-\frac{c}{2a\lambda+2\mu b}}\_F1\left(\frac{ay-bx}{a}\right)$$

6.4.18.2 [1130] Problem 2

problem number 1130

Added March 9, 2019.

Problem Chapter 4.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c\cot(\lambda x)+k\cot(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cot[lambda*x] + k*Cot[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sin^{\frac{c}{a\lambda}}(\lambda x)c_1\left(y-\frac{bx}{a}\right)e^{\frac{k(\log(\tan(\mu y))+\log(\cos(\mu y)))}{b\mu}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cot(lambda*x)+k*cot(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cot^2(\lambda x)+1\right)^{-\frac{c}{2a\lambda}} \left(\cot^2(\mu y)+1\right)^{-\frac{k}{2b\mu}} \_F1\left(\frac{ay-bx}{a}\right)$$

6.4.18.3 [1131] Problem 3

problem number 1131

Added March 9, 2019.

Problem Chapter 4.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cot(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\frac{ax(\log(\tan(\lambda x+\mu y))+\log(\cos(\lambda x+\mu y)))}{\lambda x+\mu y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cot^2(\lambda x+\mu y)+1\right)^{-\frac{a}{2\left(\lambda+\frac{\mu y}{x}\right)}} \_F1\left(\frac{y}{x}\right)$$

6.4.18.4 [1132] Problem 4

problem number 1132

Added March 9, 2019.

Problem Chapter 4.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\cot^n(\lambda x)w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{b\cot^{n+1}(\lambda x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\lambda x)\right)}{a\lambda n + a\lambda} + y\right)\exp\left(\int_1^x \frac{s\cot^k\left(\frac{\beta\left(b\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(\right.\right.}{}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*x)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b(\cot^n(\lambda x))}{a}dx\right)\right)\mathrm{e}^{\int^x \frac{c(\cot^m(\_b\mu)) + s\left(-\cot\left(\left(-y-\left(\int \frac{b(\cot^n(\_b\lambda))}{a}d\_b\right)+\int \frac{b(\cot^n(\lambda x))}{a}dx\right)\beta\right)\right)^k}{a}d\_b}$$

6.4.18.5 [1133] Problem 5

problem number 1133

Added March 9, 2019.

Problem Chapter 4.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot^n(\lambda y)w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\cot^{1-n}(\lambda y)\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\cot^2(\lambda y)\right)}{\lambda(n-1)} - \frac{bx}{a}\right)\exp\left(\int_1^y \frac{\left(s\cot^k(\beta K[1]) + c\cot^m\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*y)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b} + x\right)e^{\int^y \frac{\left(c\left(-\cot\left(-\mu\left(\int \frac{a\left(\cot^{-n}(\_b\lambda)\right)}{b}d\_b\right) - \left(-\frac{a\left(\int\left(\cot^{-n}(\lambda y)\right)dy\right)}{b} + x\right)\mu\right)\right)^m + s\left(c\right.}{b}}$$

6.4.19 6.5

Local contents

6.4.19.1 [1134] Problem 1 1798
6.4.19.2 [1135] Problem 2 1799
6.4.19.3 [1136] Problem 3 1799
6.4.19.4 [1137] Problem 4 1800
6.4.19.5 [1138] Problem 5 1801
6.4.19.6 [1139] Problem 6 1802

6.4.19.1 [1134] Problem 1

problem number 1134

Added March 9, 2019.

Problem Chapter 4.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = (b\sin(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Cos[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)e^{\frac{k\sin(\mu y)}{a\mu}-\frac{b\cos(\lambda x)}{\lambda}}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\frac{-ab\mu\cos(\lambda x)+k\lambda\sin(\mu y)}{a\lambda\mu}}$$

6.4.19.2 [1135] Problem 2

problem number 1135

Added March 9, 2019.

Problem Chapter 4.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a\sin(\mu y)w_y = (b\sin(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Tan[mu*y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{b\cos(\lambda x)}{\lambda}} c_1(y-ax)\cos^{-\frac{k}{a\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cos^{-\frac{k}{a\mu}}(\mu y)\right) \_F1(-ax+y)\, \mathrm{e}^{-\frac{b\cos(\lambda x)}{\lambda}}$$

6.4.19.3 [1136] Problem 3

problem number 1136

Added March 9, 2019.

Problem Chapter 4.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a\sin(\mu y)w_y = b\tan(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Sin[mu*y]*D[w[x, y], y] == b*Tan[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*sin(mu*y)*diff(w(x,y),y) = b*tan(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\tan^2(\lambda x)+1\right)^{\frac{b}{2\lambda}}\_F1\left(\frac{\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{a\mu x}}{\_Z^2\mathrm{e}^{2a\mu x}+1},-\frac{\_Z^2\mathrm{e}^{2a\mu x}-1}{\_Z^2\mathrm{e}^{2a\mu x}+1}\right)\right)\right)}{a\mu}\right)$$

6.4.19.4 [1137] Problem 4

problem number 1137

Added March 9, 2019.

Problem Chapter 4.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+a\tan(\mu y)w_y=b\sin(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*Tan[mu*y]*D[w[x, y], y] == b*Sin[lambda*x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{b\cos(\lambda x)}{\lambda}}c_1\left(\frac{\log(\sin(\mu y))}{\mu}-ax\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*tan(mu*y)*diff(w(x,y),y) = b*sin(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\ln\left(\operatorname{csgn}\left(\frac{1}{\cos(\mu y)}\right)\mathrm{e}^{-a\mu x}\sin(\mu y)\right)}{a\mu}\right)\mathrm{e}^{-\frac{b\cos(\lambda x)}{\lambda}}$$

6.4.19.5 [1138] Problem 5

problem number 1138

Added March 9, 2019.

Problem Chapter 4.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin(\lambda x)w_x+aw_y=b\cos(\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Sin[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Cos[mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sin(\mu y)}{a\mu}}c_1\left(\frac{-a\log\left(\sin\left(\frac{\lambda x}{2}\right)\right)+a\log\left(\cos\left(\frac{\lambda x}{2}\right)\right)+\lambda y}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  sin(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*cos(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-a\ln\left(-\cot\left(\lambda x\right)+\csc\left(\lambda x\right)\right)+\lambda y}{\lambda}\right)\mathrm{e}^{\frac{b\sin(\mu y)}{a\mu}}$$

6.4.19.6 [1139] Problem 6

problem number 1139

Added March 9, 2019.

Problem Chapter 4.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)w_x + aw_y = b\tan(\mu y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Cot[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Tan[mu*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\cos^{-\frac{b}{a\mu}}(\mu y)c_1\left(\frac{a\log(\cos(\lambda x))}{\lambda}+y\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  cot(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*tan(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\cos^{-\frac{b}{a\mu}}(\mu y)\right) \_F1\left(\frac{-a\ln\left(\cot^2(\lambda x)+1\right)+2a\ln\left(\cot(\lambda x)\right)+2\lambda y}{2\lambda}\right)$$

6.4.20 7.1

Local contents

6.4.20.1 [1140] Problem 1 1803
6.4.20.2 [1141] Problem 2 1804
6.4.20.3 [1142] Problem 3 1805
6.4.20.4 [1143] Problem 4 1806
6.4.20.5 [1144] Problem 5 1807

6.4.20.1 [1140] Problem 1

problem number 1140

Added March 9, 2019.

Problem Chapter 4.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arcsin(\frac{x}{\lambda} + k\arcsin(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcSin[x/lambda] + k*ArcSin[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{\frac{k\left(\sqrt{a^2(\beta^2-y^2)}(ay-bx)\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)+a^2(\beta^2-y^2)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}}+akx\sin^{-1}\left(\frac{y}{\beta}\right)+ac\lambda\right.}{a^2}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arcsin(x/lambda)+k*arcsin(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{aky\arcsin\left(\frac{y}{\beta}\right)+bcx\arcsin\left(\frac{x}{\lambda}\right)+\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,a\beta k+\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda}{ab}}$$

6.4.20.2 [1141] Problem 2

problem number 1141

Added March 9, 2019.

Problem Chapter 4.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\arcsin(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}+(\beta y+\lambda x)\sin^{-1}(\beta y+\lambda x)\right)}{a\lambda+b\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left((\beta y+\lambda x)\arcsin(\beta y+\lambda x)+\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)c}{a\lambda+b\beta}}$$

6.4.20.3 [1142] Problem 3

problem number 1142

Added March 9, 2019.

Problem Chapter 4.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ax\arcsin(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{a\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}(-3a\beta y+a\lambda x+4b\beta x)+\sin^{-1}(\beta y+\lambda\right.}{4(a\lambda+b\beta)^2}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{\left(\left(2(\beta y+\lambda x)b\beta x+\left(-\beta^2y^2+\lambda^2x^2-\frac{1}{2}\right)a\right)\arcsin(\beta y+\lambda x)+\left(2b\beta x+\left(-\frac{3\beta y}{2}+\frac{\lambda x}{2}\right)a\right)\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)a}{2(a\lambda+b\beta)^2}}$$

6.4.20.4 [1143] Problem 4

problem number 1143

Added March 9, 2019.

Problem Chapter 4.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^n(\lambda x)w_y=\left(c\arcsin^m(\mu x)+s\arcsin^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{b\sin^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x \frac{s\sin^{-1}\left(\beta\left(y-\int_1^x \frac{b\sin^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2]}\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*x)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-\left(-\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)\arcsin(\lambda x)+\arcsin(\lambda x)^{n+\frac{3}{2}}\right)\sqrt{-\lambda^2x^2+1}\,b+\left(\right.}{}\right.$$

6.4.20.5 [1144] Problem 5

problem number 1144

Added March 9, 2019.

Problem Chapter 4.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^n(\lambda y)w_y=\left(c\arcsin^m(\mu x)+s\arcsin^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{bx}{a}-\frac{i\sin^{-1}(\lambda y)^{-n}\left((-i\sin^{-1}(\lambda y))^n\,\text{Gamma}\left(1-n,-i\sin^{-1}(\lambda y)\right)-(i\sin^{-1}(\lambda y))^n\,\text{G}\right.}{2\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*y)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\left(-\text{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda y)\right)\arcsin(\lambda y)+\arcsin(\lambda y)^{-n+\frac{3}{2}}\right)\sqrt{-\lambda^2y^2+1}\,a-}{}\right.$$

6.4.21 7.2

Local contents

6.4.21.1 [1145] Problem 1 1809
6.4.21.2 [1146] Problem 2 1810
6.4.21.3 [1147] Problem 3 1811
6.4.21.4 [1148] Problem 4 1812
6.4.21.5 [1149] Problem 5 1813

6.4.21.1 [1145] Problem 1

problem number 1145

Added March 9, 2019.

Problem Chapter 4.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arccos(\frac{x}{\lambda} + k\arccos(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCos[x/lambda] + k*ArcCos[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{-\frac{k\left(\sqrt{a^2(\beta^2-y^2)}(ay-bx)\tan^{-1}\left(\frac{ay}{\sqrt{a^2(\beta^2-y^2)}}\right)+a^2(\beta^2-y^2)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}} + akx\cos^{-1}\left(\frac{y}{\beta}\right) - ac}{a^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccos(x/lambda)+k*arccos(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{aky\arccos\left(\frac{y}{\beta}\right)+bcx\arccos\left(\frac{x}{\lambda}\right)-\sqrt{\frac{\beta^2-y^2}{\beta^2}}\,a\beta k-\sqrt{-\frac{x^2}{\lambda^2}+1}\,bc\lambda}{ab}}$$

6.4.21.2 [1146] Problem 2

problem number 1146

Added March 9, 2019.

Problem Chapter 4.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c \arccos(\lambda x + \beta y) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{c\left(\beta(bx - ay)\sin^{-1}(\beta y + \lambda x) + x(a\lambda + b\beta)\cos^{-1}(\beta y + \lambda x) + a\left(-\sqrt{-\beta^2}\right.\right.}{a(a\lambda + b\beta)}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right) e^{\frac{\left((\beta y + \lambda x)\arccos(\beta y + \lambda x) - \sqrt{-\beta^2 y^2 - 2\beta\lambda xy - \lambda^2 x^2 + 1}\right)c}{a\lambda + b\beta}}$$

6.4.21.3 [1147] Problem 3

problem number 1147

Added March 9, 2019.

Problem Chapter 4.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = ax \arccos(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{(a^2 + 2\beta^2(bx - ay)^2)\sin^{-1}(\beta y + \lambda x) - a\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}(-}{4(a\lambda + b\beta)^2}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)\mathrm{e}^{\frac{\left(\frac{a\arcsin(\beta y+\lambda x)}{2} + (\beta y+\lambda x)(2b\beta x + (-\beta y+\lambda x)a)\arccos(\beta y+\lambda x) + \left(-2b\beta x + \left(\frac{3\beta y}{2} - \frac{\lambda x}{2}\right)a\right)\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}\right)}{2(a\lambda+b\beta)^2}}$$

6.4.21.4 [1148] Problem 4

problem number 1148

Added March 9, 2019.

Problem Chapter 4.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda x)w_y = \left(c\arccos^m(\mu x) + s\arccos^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{b\cos^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x \frac{s\cos^{-1}\left(\beta\left(y-\int_1^x \frac{b\cos^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[}\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*x)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(-\text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}+\arccos(\lambda x)^{n+1}+\frac{(n+2)\,\text{LommelS1}(n+}{\sqrt{\arccos(}}\right.}{}\right.$$

6.4.21.5 [1149] Problem 5

problem number 1149

Added March 9, 2019.

Problem Chapter 4.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda y)w_y = \left(c\arccos^m(\mu x) + s\arccos^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cos^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cos^{-1}(\lambda K[2])^{-n}\left(s\cos^{-1}(\beta K[2])^k+c\cos^{-1}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*y)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(\text{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)}-\arccos(\lambda y)^{-n+1}+\frac{(n-2)\,\text{LommelS1}(-n}{\sqrt{\arccos}}\right.}{}\right.$$

6.4.22 7.3

Local contents

6.4.22.1 [1150] Problem 1 1814
6.4.22.2 [1151] Problem 2 1815
6.4.22.3 [1152] Problem 3 1816
6.4.22.4 [1153] Problem 4 1817
6.4.22.5 [1154] Problem 5 1818

6.4.22.1 [1150] Problem 1

problem number 1150

Added March 9, 2019.

Problem Chapter 4.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arctan(\frac{x}{\lambda} + k\arctan(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcTan[x/lambda] + k*ArcTan[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \left(\lambda^2+x^2\right)^{-\frac{c\lambda}{2a}} c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{k\left(2y\tan^{-1}\left(\frac{y}{\beta}\right)-\beta\log\left(a^2(\beta^2+y^2)\right)\right)}{2b}+\frac{cx\tan^{-1}\left(\frac{x}{\lambda}\right)}{a}\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arctan(x/lambda)+k*arctan(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{\beta^2+y^2}{\beta^2}\right)^{-\frac{\beta k}{2b}} \left(\frac{x^2}{\lambda^2}+1\right)^{-\frac{c\lambda}{2a}} \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{aky\arctan\left(\frac{y}{\beta}\right)+bcx\arctan\left(\frac{x}{\lambda}\right)}{ab}}$$

6.4.22.2 [1151] Problem 2

problem number 1151

Added March 9, 2019.

Problem Chapter 4.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arctan(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}\right)\right\}\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\beta^2 y^2 + 2\beta\lambda xy + \lambda^2 x^2 + 1\right)^{-\frac{c}{2a\lambda+2b\beta}} \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{(\beta y+\lambda x)c\arctan(\beta y+\lambda x)}{a\lambda+b\beta}}$$

6.4.22.3 [1152] Problem 3

problem number 1152

Added March 9, 2019.

Problem Chapter 4.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\arctan(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{1}{4}\left(2x^2\tan^{-1}(\beta y+\lambda x) + \frac{i(a+i\beta(bx-ay))^2\log(a(\beta y+\lambda x+i)) + i(b\beta}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\beta^2 y^2 + 2\beta\lambda xy + \lambda^2 x^2 + 1\right)^{\frac{(ay-bx)a\beta}{2(a\lambda+b\beta)^2}} \_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{\left(-(\beta y+\lambda x)a+\left(2(\beta y+\lambda x)b\beta x+\left(-\beta^2 y^2+\lambda^2 x^2+1\right)a\right)\arctan(\beta y}{2(a\lambda+b\beta)^2}}$$

6.4.22.4 [1153] Problem 4

problem number 1153

Added March 9, 2019.

Problem Chapter 4.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda x)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a} dK[1]\right) \exp\left(\int_1^x \frac{s\tan^{-1}\left(\beta\left(y - \int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a} dK[1] + \int_1^K\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*x)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y - \left(\int \frac{b\arctan(\lambda x)^n}{a} dx\right)\right) e^{\int^x \frac{c\arctan(\_b\mu)^m + s\arctan\left(\left(y+\int \frac{b\arctan(\_b\lambda)^n}{a} d\_b - \left(\int \frac{b\arctan(\lambda x)^n}{a} dx\right)\right)\beta\right)^k}{a} d\_}$$

6.4.22.5 [1154] Problem 5

problem number 1154

Added March 9, 2019.

Problem Chapter 4.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda y)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\int_1^y \tan^{-1}(\lambda K[1])^{-n} dK[1] - \frac{bx}{a}\right) \exp\left(\int_1^y \frac{\tan^{-1}(\lambda K[2])^{-n}\left(s\tan^{-1}(\beta K[2])^k + c\tan^{-}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*y)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\left(\int \arctan\left(\lambda y\right)^{-n} dy\right)}{b} + x\right) e^{\int^{y} \frac{\left(c \arctan\left(\left(-\frac{a\left(\int \arctan(\lambda y)^{-n} dy\right)}{b} + x + \int \frac{a \arctan(\_b\lambda)^{-n}}{b} d\_b\right)\mu\right)^{m} + s \arctan}{b}}$$

6.4.23 7.4

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6.4.23.1 [1155] Problem 1

problem number 1155

Added March 9, 2019.

Problem Chapter 4.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c \operatorname{arccot}(\frac{x}{\lambda} + k \operatorname{arccot}(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCot[x/lambda] + k*ArcCot[y/beta])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \left(\lambda^2+x^2\right)^{\frac{c\lambda}{2a}} c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{k\left(a\beta\log\left(a^2(\beta^2+y^2)\right)+2\tan^{-1}\left(\frac{y}{\beta}\right)(bx-ay)+2bx\cot^{-}\right.}{2ab}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccot(x/lambda)+k*arccot(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\frac{\beta^2+y^2}{\beta^2}\right)^{\frac{\beta k}{2b}}\left(\frac{x^2}{\lambda^2}+1\right)^{\frac{c\lambda}{2a}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{-2aky\arctan\left(\frac{y}{\beta}\right)-2\left(c\arctan\left(\frac{x}{\lambda}\right)-\frac{\pi(c+k)}{2}\right)bx}{2ab}}$$

6.4.23.2 [1156] Problem 2

problem number 1156

Added March 9, 2019.

Problem Chapter 4.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\operatorname{arccot}(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(a\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right)+2\beta(bx-ay)\tan^{-1}(\beta y+\lambda x)+}{2a(a\lambda+b\beta)}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)^{\frac{c}{2a\lambda+2b\beta}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{(-2(\beta y+\lambda x)a\arctan(\beta y+\lambda x)+\pi(a\lambda+b\beta)x)c}{2(a\lambda+b\beta)a}}$$

6.4.23.3 [1157] Problem 3

problem number 1157

Added March 9, 2019.

Problem Chapter 4.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ax\operatorname{arccot}(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{1}{4}\left(2x^2\cot^{-1}(\beta y+\lambda x)+\frac{i(ia\beta y+a-ib\beta x)^2\log(-a(\beta y+\lambda x-i))+i(}{(}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)^{-\frac{(ay-bx)a\beta}{2(a\lambda+b\beta)^2}}\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{2\pi ab\beta\lambda x^2+\pi b^2\beta^2x^2+\left(\pi\lambda^2x^2+2\beta y+2\lambda x\right)a^2-2\left(2(\beta y+\lambda x)b\beta\right.}{4(a\lambda+b\beta)^2}}$$

6.4.23.4 [1158] Problem 4

problem number 1158

Added March 9, 2019.

Problem Chapter 4.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\operatorname{arccot}^n(\lambda x)w_y=\left(c\operatorname{arccot}^m(\mu x)+s\operatorname{arccot}^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]\right)\exp\left(\int_1^x \frac{s\cot^{-1}\left(\beta\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1]+\int_1^{K[2}\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*x)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y-\left(\int \frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx\right)\right)\mathrm{e}^{\int^x \frac{c\left(-\arctan(\_b\mu)+\frac{\pi}{2}\right)^m+s\left(-\arctan\left(\left(y+\int \frac{b\left(-\arctan(\_b\lambda)+\frac{\pi}{2}\right)^n}{a}d\right.\right.\right.}{a}}$$

6.4.23.5 [1159] Problem 5

problem number 1159

Added March 9, 2019.

Problem Chapter 4.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\operatorname{arccot}^n(\lambda y)w_y=\left(c\operatorname{arccot}^m(\mu x)+s\operatorname{arccot}^k(\beta y)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cot^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a}\right)\exp\left(\int_1^y \frac{\cot^{-1}(\lambda K[2])^{-n}\left(s\cot^{-1}(\beta K[2])^k+c\cot^{-1}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*y)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{a\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x\right)\mathrm{e}^{\int^y \frac{\left(c\left(-\arctan\left(\left(-\frac{a\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x+\int\frac{a\left(-\arctan\right.}{}\right.\right.\right.}{}}$$

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6.4.24.1 [1160] Problem 1

problem number 1160

Added March 10, 2019.

Problem Chapter 4.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = f(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =f(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ay - bx}{a}\right)e^{\int \frac{f(x)}{a}dx}$$

6.4.24.2 [1161] Problem 2

problem number 1161

Added March 10, 2019.

Problem Chapter 4.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = f(x)yw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)\exp\left(\int_1^x f(K[1])(y+a(K[1]-x))dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x -((-\_a+x)a-y)f(\_a)d\_a}$$

6.4.24.3 [1162] Problem 3

problem number 1162

Added March 10, 2019.

Problem Chapter 4.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + aw_y = (f(x)y^2 + g(x)y + h(x))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == (f[x]*y^2 + g[x]*y + h[x])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x \left(f(K[1])(y+a(K[1]-x))^2+g(K[1])(y+a(K[1]-x))+h(K[1])\right)dK\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =(f(x)*y^2+g(x)*y+h(x))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(-ax+y)\,\mathrm{e}^{\int^x\left(((-\_a+x)a-y)^2f(\_a)+((\_a-x)a+y)g(\_a)+h(\_a)\right)d\_a}$$

6.4.24.4 [1163] Problem 4

problem number 1163

Added March 10, 2019.

Problem Chapter 4.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)y^kw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])(y+a(K[1]-x))^kdK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x ((\_a-x)a+y)^k f(\_a)d\_a}$$

6.4.24.5 [1164] Problem 5

problem number 1164

Added March 10, 2019.

Problem Chapter 4.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x)e^{\lambda y}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*Exp[lambda*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)\exp\left(\int_1^x e^{\lambda(y+a(K[1]-x))}f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*exp(lambda*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x \mathrm{e}^{-((-\_a+x)a-y)\lambda} f(\_a)d\_a}$$

6.4.24.6 [1165] Problem 6

problem number 1165

Added March 10, 2019.

Problem Chapter 4.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (ay + f(x))w_y = g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x g(K[2])dK[2]\right) c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]} f(K[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right)\right)\mathrm{e}^{\int g(x)dx}$$

6.4.24.7 [1166] Problem 7

problem number 1166

Added March 10, 2019.

Problem Chapter 4.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (ay + f(x))w_y = g(x)y^k w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*y^k*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]} f(K[1])dK[1]\right)\exp\left(\int_1^x g(K[2])\left(e^{aK[2]}\left(e^{-ax}y - \int_1^x e^{-aK[1]} f(K[1\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right)\right)\mathrm{e}^{\int^x \left(\left(y\,\mathrm{e}^{-ax} + \int \mathrm{e}^{-\_ba} f(\_b)d\_b - \left(\int \mathrm{e}^{-ax} f(x)dx\right)\right)\mathrm{e}^{-\_ba}\right)^k g(\_b)d\_b}$$

6.4.24.8 [1167] Problem 8

problem number 1167

Added March 10, 2019.

Problem Chapter 4.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + y^k w_y = g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g(K[2])}{f(K[2])}dK[2]\right)c_1\left(-\int_1^x \frac{1}{f(K[1])}dK[1]-\frac{y^{1-k}}{k-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\left((k-1)\,y^k\left(\int \frac{1}{f(x)}dx\right)+y\right)y^{-k}\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}$$

6.4.24.9 [1168] Problem 9

problem number 1168

Added March 10, 2019.

Problem Chapter 4.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(y+a)w_y=(by+c)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == (b*y + c)*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x\frac{1}{f(K[1])}dK[1]\right)-\int_1^x\frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK[1]\right)}{f(K[2])}dK[2]\right)\exp\left(\int_1^x\frac{c+}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (y+a)*diff(w(x,y),y) =(b*y+c)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left((a+y)\,\mathrm{e}^{-\left(\int\frac{1}{f(x)}dx\right)}\right)\mathrm{e}^{(a+y)b+(-ba+c)\left(\int\frac{1}{f(x)}dx\right)}$$

6.4.24.10 [1169] Problem 10

problem number 1169

Added March 10, 2019.

Problem Chapter 4.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(y+ax)w_y=g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g(K[3])}{f(K[3])}dK[3]\right)c_1\left(y\exp\left(-\int_1^x \frac{1}{f(K[1])}dK[1]\right)-\int_1^x \frac{a\exp\left(-\int_1^{K[2]}\frac{1}{f(K[1])}dK\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (y+a*x)*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-a\left(\int \frac{x\,\mathrm{e}^{-\left(\int \frac{1}{f(x)}dx\right)}}{f(x)}dx\right)+y\,\mathrm{e}^{-\left(\int \frac{1}{f(x)}dx\right)}\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}$$

6.4.24.11 [1170] Problem 11

problem number 1170

Added March 10, 2019.

Problem Chapter 4.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_0(x))w_y=\left(h_2(x)y^2+h_1(x)y+h_0(x)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == (h2[x]*y^2 + h1[x]*y + h0[x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y\exp\left(-\int_1^x\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\mathrm{g0}(K[2])}{f(K[2])}dK[2]\right)\exp\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) =(h2(x)*y^2+h1(x)*y+h0(x))*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y\,\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}-\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int^x}\frac{\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}g0(\_f)}{f(\_f)}d\_f\right)^2\mathrm{e}^{2\left(\int\frac{g1(\_f)}{f(\_f)}d\_f\right)}h2(}{}$$

6.4.24.12 [1171] Problem 12

problem number 1171

Added March 10, 2019.

Problem Chapter 4.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_2(x)y^k)w_y=h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*y^k)*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h(K[3])}{f(K[3])}dK[3]\right)c_1\left((k-1)\int_1^x \frac{\exp\left((k-1)\int_1^{K[2]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\mathrm{g2}(K[2])}{f(K[2])}dK[2]\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*y^k)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol := simplify(sol);
```

$$w(x,y)=\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int\frac{g1(x)}{f(x)}dx\right)}+(k-1)\left(\int\frac{\mathrm{e}^{(k-1)\left(\int\frac{g1(x)}{f(x)}dx\right)}g2(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int\frac{h(x)}{f(x)}dx}$$

6.4.24.13 [1172] Problem 13

problem number 1172

Added March 10, 2019.

Problem Chapter 4.8.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)+g_2(x)e^{\lambda y})w_y=h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*Exp[lambda*y])*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*exp(lambda*y))*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.4.24.14 [1173] Problem 14

problem number 1173

Added March 10, 2019.

Problem Chapter 4.8.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)y^k w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*y^k*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]\right)\exp\left(\int_1^x\frac{h(K[2])\left(\left(y^{k+1}-(k+1)\int_1^x\frac{g(K[1])}{f(K[1])}dK[1]+(k\right.\right.}{f(K[2])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*y^k*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yy^k + (-k-1)\left(\int \frac{g(x)}{f(x)}dx\right)\right) e^{\int^x \frac{\left(\left(y^{k+1} + \int \frac{(k+1)g(\_b)}{f(\_b)} d\_b + \int \frac{(-k-1)g(x)}{f(x)} dx\right)^{\frac{1}{k+1}}\right)^{-k} h(\_b)}{f(\_b)} d\_b}$$

6.4.24.15 [1174] Problem 15

problem number 1174

Added March 10, 2019.

Problem Chapter 4.8.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda y}w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*Exp[lambda*y]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{\lambda y}}{\lambda} - \int_1^x \frac{g(K[1])}{f(K[1])}dK[1]\right) \exp\left(\int_1^x \frac{h(K[2])}{f(K[2])\left(-\lambda \int_1^x \frac{g(K[1])}{f(K[1])}dK[1] + e^{\lambda y} + \lambda \int_1^{K[2]} \frac{g(K[1]}{f(K[1]}\right.}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*exp(lambda*y)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\lambda\left(\int \frac{g(x)}{f(x)}dx\right) + e^{\lambda y}}{\lambda}\right) e^{\int^{x} \frac{h(\_b)}{\left(\lambda\left(\int \frac{g(\_b)}{f(\_b)}d\_b\right) - \lambda\left(\int \frac{g(x)}{f(x)}dx\right) + e^{\lambda y}\right) f(\_b)} d\_b}$$

6.4.25 8.2

Local contents

6.4.25.1 [1175] Problem 1 1838
6.4.25.2 [1176] Problem 2 1839
6.4.25.3 [1177] Problem 3 1840
6.4.25.4 [1178] Problem 4 1841
6.4.25.5 [1179] Problem 5 1842
6.4.25.6 [1180] Problem 6 1842
6.4.25.7 [1181] Problem 7 1843

6.4.25.1 [1175] Problem 1

problem number 1175

Added March 10, 2019.

Problem Chapter 4.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (f(x) + g(y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x] + g[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) =(f(x)+g(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\int^x\frac{f(\_a)+g\left(\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a}$$

6.4.25.2 [1176] Problem 2

problem number 1176

Added March 10, 2019.

Problem Chapter 4.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+aw_y=f(x)g(y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])g(-ax+y+aK[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x)*g(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a)g((\_a-x)a+y)d\_a}$$

6.4.25.3 [1177] Problem 3

problem number 1177

Added March 10, 2019.

Problem Chapter 4.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)h(y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1]\right)\exp\left(\int_1^x g(K[2])h\left(e^{aK[2]}\left(e^{-ax}y-\int_1^x e^{-aK[1]}f(K\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+(a*y+f(x))*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-ax} - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right)\right) \mathrm{e}^{\int^{x} g(\_b)h\left(\left(y\,\mathrm{e}^{-ax}+\int \mathrm{e}^{-\_ba} f(\_b)d\_b-\left(\int \mathrm{e}^{-ax} f(x)dx\right)\right)\mathrm{e}^{-\_ba}\right)d\_b}$$

6.4.25.4 [1178] Problem 4

problem number 1178

Added March 10, 2019.

Problem Chapter 4.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy\right) \mathrm{e}^{\int^{x} \frac{h1(\_f)+h2\left(\mathrm{RootOf}\left(\int \frac{1}{f(\_f)}d\_f-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy-\left(\int_{\_Z} \frac{1}{g(\_a)}d\_a\right)\right)\right)}{f(\_f)}}$$

contains RootOf

6.4.25.5 [1179] Problem 5

problem number 1179

Added March 10, 2019.

Problem Chapter 4.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f_1(x)w_x + (f_2(x) + f_3(x)y^k)w_y = g(x)h(y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x, y], x] + (f2[x] + f3[x]*y^k)*D[w[x, y], y] == g[x]*h[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f1(x)*diff(w(x,y),x)+(f2(x)+f3(x)*y^k)*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.4.25.6 [1180] Problem 6

problem number 1180

Added March 10, 2019.

Problem Chapter 4.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h_1(x)h_2(y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x]*h2[y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h1(x)*h2(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy\right) e^{\int^x \frac{h1(\_f)h2\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int \_Z \frac{g1(\_a)}{g2(\_a)}d\_\right.\right.}{f1(\_f)g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int \_Z \frac{g1(\_a)}{g2(\_a)}d\_\right.\right.}}$$

has RootOf

6.4.25.7 [1181] Problem 7

problem number 1181

Added March 10, 2019.

Problem Chapter 4.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy\right) \mathrm{e}^{\int^x \frac{h1(\_f)+h2\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f-\left(\int \frac{f2(x)}{f1(x)}dx\right)+\int \frac{g1(y)}{g2(y)}dy-\left(\int\_Z \frac{g1(\_a)}{g2(\_a)}d\right.\right.}{f1(\_f)g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f-\left(\int \frac{f2(x)}{f1(x)}dx\right)+\int \frac{g1(y)}{g2(y)}dy-\left(\int\_Z \frac{g1(\_a)}{g2(\_a)}d\_\right.\right.}}$$

has RootOf

6.4.26 8.3

Local contents

6.4.26.1 [1182] Problem 1 1844
6.4.26.2 [1183] Problem 2 1845
6.4.26.3 [1184] Problem 3 1846
6.4.26.4 [1185] Problem 4 1847
6.4.26.5 [1186] Problem 5 1848
6.4.26.6 [1187] Problem 6 1849
6.4.26.7 [1188] Problem 7 1850

6.4.26.1 [1182] Problem 1

problem number 1182

Added March 10, 2019.

Problem Chapter 4.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(\alpha x + \beta y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{f\left(\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ay-bx}{a}\right)e^{\int^x\frac{f\left(\frac{-(-\_a+x)b\beta+(\_a\alpha+\beta y)a}{a}\right)}{a}d\_a}$$

6.4.26.2 [1183] Problem 2

problem number 1183

Added March 10, 2019.

Problem Chapter 4.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=xf(\frac{y}{x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{xf\left(\frac{y}{x}\right)}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = x*f(y/x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{y}{x}\right)\mathrm{e}^{xf\left(\frac{y}{x}\right)}$$

6.4.26.3 [1184] Problem 3

problem number 1184

Added March 10, 2019.

Problem Chapter 4.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y=f(x^2+y^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\int_1^x\frac{f\left(\frac{(x^2+y^2)K[1]^2}{x^2}\right)}{K[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = f(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\int^x \frac{f\left(\frac{-\_a^2 y^2}{x^2}+\_a^2\right)}{\_a} d\_a}$$

6.4.26.4 [1185] Problem 4

problem number 1185

Added March 10, 2019.

Problem Chapter 4.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k f(x^n * y^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*y^m]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right) \exp\left(\int_1^x \frac{f\left(K[1]^n \left(x^{-\frac{b}{a}} yK[1]^{\frac{b}{a}}\right)^m\right) K[1]^{k-1}}{a} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = x^k*f(x^n+y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right) e^{\int x \frac{\_a^{k-1} f\left(\_a^n + \left(y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)^m\right)}{a} d\_a}$$

6.4.26.5 [1186] Problem 5

problem number 1186

Added March 10, 2019.

Problem Chapter 4.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = f(ax^n + by^m)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*y^m]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{n}{m}}\right) \exp\left(\int_1^x \frac{f\left(b\left(x^{-\frac{n}{m}} yK[1]^{\frac{n}{m}}\right)^m + aK[1]^n\right)}{mK[1]} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) = f(a*x^n+b*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{n}{m}}\right) e^{\int x \frac{f\left(a\_a^n + b\left(y\_a^{\frac{n}{m}} x^{-\frac{n}{m}}\right)^m\right)}{\_am} d\_a}$$

6.4.26.6 [1187] Problem 6

problem number 1187

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xy w_y = y^k f(\alpha x^n + \beta y^m) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) \exp\left(\int_1^x \frac{f\left(\left(\alpha + \frac{\beta y}{x}\right) K[1]\right) \left(\frac{y K[1]}{x}\right)^k}{K[1]^2} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^k*f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\int^x \frac{\left(\frac{\_a y}{x}\right)^k f\left(\left(\alpha + \frac{\beta y}{x}\right) \_a\right)}{\_a^2} d\_a}$$

6.4.26.7 [1188] Problem 7

problem number 1188

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)}w_x + \frac{g(x)}{g'(x)}w_y = h(f(x)+g(y))w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[x]*D[w[x, y], y])/Derivative[1][g][x] =
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])}dK[1]\right)\exp\left(\int_1^x \frac{h\left(f(K[2])+g\left(y-\int_1^x \frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])}dK[1]+\right.\right.}{f(K[2])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)/diff(f(x),x)*diff(w(x,y),x)+g(x)/diff(g(x),x)*diff(w(x,y),y) = h(f(x)+g(y))*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y-\left(\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{\left(\frac{d}{dx}g(x)\right)f(x)}dx\right)\right)\mathrm{e}^{\int^x \frac{\left(\frac{d}{d\_b}f(\_b)\right)h\left(f(\_b)+g\left(y+\int \frac{\left(\frac{d}{d\_b}f(\_b)\right)g(\_b)}{\left(\frac{d}{d\_b}g(\_b)\right)f(\_b)}d\_b-\left(\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{\left(\frac{d}{dx}g(x)\right)f(x)}dx\right)\right)\right)}{f(\_b)}}$$

6.4.27 8.4

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6.4.27.1 [1189] Problem 1

problem number 1189

Added March 10, 2019.

Problem Chapter 4.8.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + a*D[w[x, y], y] == f[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)\exp\left(\int_1^x f(K[1],-ax+y+aK[1])dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a,(\_a-x)a+y)d\_a}$$

6.4.27.2 [1190] Problem 2

problem number 1190

Added March 10, 2019.

Problem Chapter 4.8.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\int_1^x \frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)e^{\int_x \frac{f\left(\_a,y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_aa}d\_a}$$

6.4.27.3 [1191] Problem 3

problem number 1191

Added March 10, 2019.

Problem Chapter 4.8.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{h\left(K[2], \exp\left(\int_1^{K[2]} \frac{g(K[1])}{f(K[1])} dK[1] - \int_1^x \frac{g(K[1])}{f(K[1])} dK[1]\right) y\right)}{f(K[2])} dK[2]\right) c_1\left(y \exp\left(-\int_1^x \right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+g(x)*y*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\, e^{-\left(\int \frac{g(x)}{f(x)} dx\right)}\right) e^{\int^x \frac{h\left(\_b, y\, e^{\int \frac{g(\_b)}{f(\_b)} d\_b - \left(\int \frac{g(x)}{f(x)} dx\right)}\right)}{f(\_b)} d\_b}$$

6.4.27.4 [1192] Problem 4

problem number 1192

Added March 10, 2019.

Problem Chapter 4.8.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{h\left(K[3], \exp\left(\int_1^{K[3]} \frac{\text{g1}(K[1])}{f(K[1])} dK[1]\right)\left(\exp\left(-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])} dK[1]\right) y - \int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{\text{g1}}{f(}\right.}{f}\right.\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y\,\mathrm{e}^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} - \left(\int \frac{\mathrm{e}^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)} dx\right)\right) \mathrm{e}^{\int^x \frac{h\left(\_f, \left(y\,\mathrm{e}^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} + \int \frac{\mathrm{e}^{-\left(\int \frac{g1(\_f)}{f(\_f)} d\_f\right)} g0(\_f)}{f(\_f)} d\_\right.\right.}{f(\_f)}}$$

6.4.27.5 [1193] Problem 5

problem number 1193

Added March 10, 2019.

Problem Chapter 4.8.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{h\left(K[3], \left(\exp\left(-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}dK[1] - (k-1)\int_1^{K[3]} \frac{\text{g1}(K[1])}{f(K[1])}dK[1]\right) y^{-k}\left(\exp\left(\int_1^x \frac{\text{g1}}{f}\right.\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} + (k-1)\left(\int \frac{\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}dx\right)} g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int^x \frac{h\left(\_f, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int \frac{g1(x)}{f(x)}\right.}\right.\right.}{}}$$

6.4.27.6 [1194] Problem 6

problem number 1194

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x, y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y]*w[x, y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

6.4.27.7 [1195] Problem 7

problem number 1195

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x, y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y]*w[x, y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy\right) e^{\int^x \frac{h\left(\_f,\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int \_Z \frac{g1(\_a)}{g2(\_a)}d\_a\right)\right.}{f1(\_f)g1\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}d\_f - \left(\int \frac{f2(x)}{f1(x)}dx\right) + \int \frac{g1(y)}{g2(y)}dy - \left(\int \_Z \frac{g1(\_a)}{g2(\_a)}d\_\right.\right.}}$$

has RootOf

6.5 chapter 5

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6.5.1.1 [1196] Problem 1

problem number 1196

Added March 10, 2019.

Problem Chapter 5.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cw + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{d}{c} + e^{\frac{cx}{a}} c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = c*w(x,y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c\_F1\left(\frac{ay-bx}{a}\right) \mathrm{e}^{\frac{cx}{a}} - d}{c}$$

6.5.1.2 [1197] Problem 2

problem number 1197

Added March 10, 2019.

Problem Chapter 5.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(x-a)w_x + (y-b)w_y = w - c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y] - c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c + (x-a)c_1\left(\frac{b-y}{a-x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x-a)*diff(w(x,y),x)+(y-b)*diff(w(x,y),y) = w(x,y)-c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = c + (a-x)\_F1\left(\frac{-b+y}{a-x}\right)$$

6.5.1.3 [1198] Problem 3

problem number 1198

Added March 10, 2019.

Problem Chapter 5.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax+b)w_x+(cx+d)w_y=\alpha w+\beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*x + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(bc-ad)\log(ax+b)+a(ay-cx)}{a^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{a^2y-acx-ad\ln(ax+b)+bc\ln(ax+b)}{a^2}\right)-\beta}{\alpha}$$

6.5.1.4 [1199] Problem 4

problem number 1199

Added March 10, 2019.

Problem Chapter 5.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x+(cy+d)w_y=\alpha w+\beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-\beta}{\alpha}$$

6.5.1.5 [1200] Problem 5

problem number 1200

Added March 10, 2019.

Problem Chapter 5.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x+(cy+d)w_y=\alpha w+\beta y+\gamma x$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta*y + gamma*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\alpha(a-\alpha)(\alpha-c)(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-a\beta(\alpha y+d)+\alpha^2\beta y+\alpha^2\gamma x+\alpha b\gamma+\alpha\beta a}{\alpha(\alpha-a)(\alpha-c)}\right.\right.$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta*y+gamma*x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{a\beta d+(-\alpha+c)(a-\alpha)\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)+(-\beta y-\gamma x)\alpha^2+\gamma bc+(a\beta y-\beta d}{(a-\alpha)(-\alpha+c)\alpha}$$

6.5.1.6 [1201] Problem 6

problem number 1201

Added March 10, 2019.

Problem Chapter 5.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x+(cx+dy)w_y=\alpha w+\beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a*x + b)*D[w[x, y], x] + (c*x + d*y)*D[w[x, y], y] == alpha*w[x, y] + beta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{\beta}{\alpha}+(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(ax+b)^{-\frac{d}{a}}(-d(-ay+cx+dy)-bc)}{d(a-d)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d*y)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\alpha(ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{(dya-cxd-d^2y-bc)(ax+b)^{-\frac{d}{a}}}{(a-d)d}\right)-\beta}{\alpha}$$

6.5.1.7 [1202] Problem 7

problem number 1202

Added March 10, 2019.

Problem Chapter 5.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x+a_0)w_x+(b_2y+b_1x+b_0)w_y=(c_2y+c_1x+c_0)w+k_2y+k_1x+k_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (a1*x + a0)*D[w[x, y], x] + (b2*y + b1*x + b0)*D[w[x, y], y] == (c2*y + c1*x + c0)*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to (\mathrm{a0}+\mathrm{a1}x)^{\frac{\mathrm{a0b1c2}-\mathrm{a0b2c1}-\mathrm{a1b0c2}+\mathrm{a1b2c0}}{\mathrm{a1}^2\mathrm{b2}}}\exp\left(\frac{-\mathrm{a1}(\mathrm{a0b1c2}+\mathrm{b2}(\mathrm{b0c2}+\mathrm{b1c2}x-\mathrm{b2c1}x+\mathrm{b2c2}y))}{\mathrm{a1b2}^2(\mathrm{a1}-\mathrm{b}}\right.\right.$$

Maple ✗

```
restart;
pde :=  (a1*x+a0)*diff(w(x,y),x)+ (b2*y+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,y)+k2*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

time expired

6.5.1.8 [1203] Problem 8

problem number 1203

Added March 10, 2019.

Problem Chapter 5.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x+(b_1x+b_0)w_y=(c_1x+c_0)w+s_1x+s0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*y*D[w[x, y], x] + (b1*x + b0)*D[w[x, y], y] == (c1*x + c0)*w[x, y] + s1*x + s0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ w(x,y) \to \exp\left(\frac{(\mathrm{b0c1} - \mathrm{b1c0}) \tanh^{-1}\left(\frac{\mathrm{b0}+\mathrm{b1}x}{\sqrt{\mathrm{b1}}\sqrt{ay^2}}\right) - \sqrt{\mathrm{b1}}\mathrm{c1}\sqrt{ay^2}}{\sqrt{a}\mathrm{b1}^{3/2}} \right) \left(\int_1^x - \frac{\exp\left(\frac{\sqrt{-\mathrm{b1}x^2 - 2\mathrm{b0}x + ay^2 + \mathrm{b1}K[1]^2 +}}{\mathrm{b1}} \right.}{\sqrt{a}} \right. \right.$$

$$\left\{ w(x,y) \to \exp\left(\frac{(\mathrm{b1c0} - \mathrm{b0c1}) \tanh^{-1}\left(\frac{\mathrm{b0}+\mathrm{b1}x}{\sqrt{\mathrm{b1}}\sqrt{ay^2}}\right) + \sqrt{\mathrm{b1}}\mathrm{c1}\sqrt{ay^2}}{\sqrt{a}\mathrm{b1}^{3/2}} \right) \left(\int_1^x \frac{\exp\left(- \frac{\sqrt{-\mathrm{b1}x^2 - 2\mathrm{b0}x + ay^2 + \mathrm{b1}K[2]^2 +}}{\mathrm{b1}} \right.}{\sqrt{a}} \right. \right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b1*x+b0)*diff(w(x,y),y) =  (c1*x+c0)*w(x,y)+s1*x+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{(\_as1 + s0) \left(\frac{(\_ab1 + b0)a + \sqrt{(ay^2 + (2\_a - 2x)b0 + (\_a^2 - x^2)b1)a}\sqrt{ab1}}{\sqrt{ab1}} \right)^{\frac{b0\,c1 - b1\,c0}{\sqrt{ab1}\,b1}} \mathrm{e}^{-\frac{\sqrt{\left(ay^2 + (2\_a - 2x)b0 + \left(\_a^2\right.\right.}}{ab1}}}{\sqrt{(ay^2 + (2\_a - 2x)\,b0 + (\_a^2 - x^2)\,b1)\,a}} \right.$$

6.5.2 2.2

Local contents

6.5.2.1 [1204] Problem 1 1867
6.5.2.2 [1205] Problem 2 1868
6.5.2.3 [1206] Problem 3 1869
6.5.2.4 [1207] Problem 4 1870
6.5.2.5 [1208] Problem 5 1871
6.5.2.6 [1209] Problem 6 1871
6.5.2.7 [1210] Problem 7 1872
6.5.2.8 [1211] Problem 8 1873

6.5.2.1 [1204] Problem 1

problem number 1204

Added March 10, 2019.

Problem Chapter 5.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cw + \beta xy + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + beta*x*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{a\beta(2b+cy)+c(b\beta x+\beta cxy+c\gamma)}{c^3} + e^{\frac{cx}{a}} c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c^3 \_F1\left(\frac{ay-bx}{a}\right) e^{\frac{cx}{a}} - 2ab\beta - (ay+bx)\beta c + (-\beta xy - \gamma) c^2}{c^3}$$

6.5.2.2 [1205] Problem 2

problem number 1205

Added March 10, 2019.

Problem Chapter 5.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{c^3\left(-e^{\frac{cx}{a}}\right) c_1\left(y - \frac{bx}{a}\right) + 2a^2\beta + a(2b\gamma + 2\beta cx + c\gamma y) + c(b\gamma x + c(\beta x^2 + \delta + \gamma xy))}{c^3}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c^3\_F1\left(\frac{ay-bx}{a}\right)\mathrm{e}^{\frac{cx}{a}} - 2a^2\beta - 2\gamma ab + \left(-\beta x^2 - \gamma xy - \delta\right)c^2 + \left(-2a\beta x + \left(-ay - bx\right)\gamma\right)c}{c^3}$$

6.5.2.3 [1206] Problem 3

problem number 1206

Added March 10, 2019.

Problem Chapter 5.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = w + ax^2 + by^2 + c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == w[x, y] + a*x^2 + b*y^2 + c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to xc_1\left(\frac{y}{x}\right) + ax^2 + by^2 - c\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =  w(x,y)+a*x^2+b*y^2+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = ax^2 + by^2 + x\_F1\left(\frac{y}{x}\right) - c$$

6.5.2.4 [1207] Problem 4

problem number 1207

Added March 10, 2019.

Problem Chapter 5.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{c(2a-c)(a+b-c)x^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right) - 2a^2\delta - 2ab\delta + ac(x(\beta x + 2\gamma y) + 3\delta) + bc(\beta x^2 + \delta) -}{c(c-2a)(-a-b+c)}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\beta x^2}{2a-c} + \frac{\gamma y x^{-\frac{b}{a}+\frac{a+b}{a}}}{a+b-c} + x^{\frac{c}{a}}\_F1\left(yx^{-\frac{b}{a}}\right) - \frac{\delta}{c}$$

6.5.2.5 [1208] Problem 5

problem number 1208

Added March 10, 2019.

Problem Chapter 5.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ayw_x + (b_2x^2 + b_1x + b_0)w_y = (c_2x^2 + c_1x + c_0)w + s_2x^2 + s_1x + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + (b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*x^2 + c1*x + c0)*w[x, y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*x^2+c1*x+c0)*w(x,y)+s2*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

6.5.2.6 [1209] Problem 6

problem number 1209

Added March 10, 2019.

Problem Chapter 5.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$ay^2w_x + (b_1x^2 + b_0)w_y = (c_1x^2 + c_0)w + s_1x^2 + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y^2*D[w[x, y], x] + (b1*x^2 + b0)*D[w[x, y], y] == (c1*x^2 + c0)*w[x, y] + s1*x^2 +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ (b1*x^2+b0)*diff(w(x,y),y) =  (c1*x^2+c0)*w(x,y)+s1*x^2+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

6.5.2.7 [1210] Problem 7

problem number 1210

Added March 10, 2019.

Problem Chapter 5.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(a_1x^2 + a_0)w_x + (y + b_2x^2 + b_1x + b_0)w_y = (c_2y + c_1x + c_0)w + k_{22}y^2 + k12xy + k_{11}x^2 + k_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (y + b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*y + c1
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (y+b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(y\,\mathrm{e}^{-\frac{\arctan\left(\frac{a1\,x}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}} + \int \frac{(\_f^{2} b2 + \_f b1 + b0)\mathrm{e}^{-\frac{\arctan\left(\frac{\_f a1}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}}}{\_f^{2} a1 + a0} d\_f - \left(\int \frac{(b2x^{2} + b1x + b0)\mathrm{e}^{-\frac{\arctan\left(\frac{a1\,x}{\sqrt{a0\,a1}}\right)}{\sqrt{a0\,a1}}}}{a1x^{2} + a0} \right.}{} \right.$$

6.5.2.8 [1211] Problem 8

problem number 1211

Added March 10, 2019.

Problem Chapter 5.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2 + a_0)w_x + (b_2y^2 + b_1xy)w_y = (c_2y^2 + c_1x^2)w + s_{22}y^2 + s_{12}xy + s_{11}x^2 + s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (b2*y^2 + b1*x^2)*D[w[x, y], y] == (c2*y^2 + c1*x^2)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (b2*y^2+b1*x^2)*diff(w(x,y),y) =  (c2*y^2+c1*x^2)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Expression too large to display

6.5.3 2.3

Local contents

6.5.3.1 [1212] Problem 1 1874
6.5.3.2 [1213] Problem 2 1875
6.5.3.3 [1214] Problem 3 1876
6.5.3.4 [1215] Problem 4 1878
6.5.3.5 [1216] Problem 5 1879
6.5.3.6 [1217] Problem 6 1880
6.5.3.7 [1218] Problem 7 1881

6.5.3.1 [1212] Problem 1

problem number 1212

Added March 12, 2019.

Problem Chapter 5.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = \alpha yw + \beta\sqrt{xy} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == alpha*y*w[x, y] + beta*Sqrt[x*y] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{\frac{\alpha y}{b}}\left(-\beta\sqrt{xy}\left(\frac{\alpha y}{b}\right)^{-\frac{a+b}{2b}}\text{Gamma}\left(\frac{a+b}{2b},\frac{\alpha y}{b}\right)+bc_1\left(yx^{-\frac{b}{a}}\right)+\gamma\text{Ei}\left(-\frac{\alpha y}{b}\right)\right)}{b}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = alpha*y*w(x,y)+ beta*sqrt(x*y)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=-\frac{\left(-4\sqrt{xy}\,(2y\alpha+a+3b)\,ab^3\beta x^{-\frac{a+b}{2a}}x^{\frac{b}{2a}+\frac{1}{2}}\left(\frac{\alpha y}{b}\right)^{-\frac{a+3b}{4b}}\left(\frac{\alpha yx^{-\frac{b}{a}}}{b}\right)^{-\frac{a}{2b}-\frac{1}{2}}\left(\frac{\alpha yx^{-\frac{b}{a}}}{b}\right)^{\frac{a}{2b}+\frac{1}{2}}\text{Whittaker}\right.}{}$$

6.5.3.2 [1213] Problem 2

problem number 1213

Added March 12, 2019.

Problem Chapter 5.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=\lambda\sqrt{xy}w+\beta xy+\gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == lambda*Sqrt[x*y]*w[x, y] + beta*x*y + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{2\lambda\sqrt{xy}}{a+b}}\left(\int_1^x \frac{e^{-\frac{2\lambda\sqrt{x^{-\frac{b}{a}}yK[1]^{\frac{a+b}{a}}}}{a+b}}\left(\beta yK[1]^{\frac{a+b}{a}}x^{-\frac{b}{a}}+\gamma\right)}{aK[1]}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = lambda*sqrt(x*y)*w(x,y)+ beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=-\frac{\left((a+b)\left(a+b+2\sqrt{xy}\,\lambda\right)\beta\,\mathrm{e}^{-\frac{2\sqrt{xy}\,\lambda}{a+b}}-2\left(-2\gamma\,\mathrm{expIntegral}\left(1,\frac{2\sqrt{xy}\,\lambda}{a+b}\right)+(a+b)\,\_F1\left(yx^{-\frac{b}{a}}\right)\right.\right.}{2\,(a+b)\,\lambda^2}$$

6.5.3.3 [1214] Problem 3

problem number 1214

Added March 12, 2019.

Problem Chapter 5.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x+bxw_y=\alpha w+\beta\sqrt{x}+\gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ w(x,y) \to e^{-\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}} \left(\int_1^x -\frac{\exp\left(\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}K[1]}{\sqrt{-bx^2+ay^2+bK[1]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[1]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[1]^2-x^2\right)}} dK[1] + c_1 \left(\frac{ay^2-b}{2a} \right. \right. \right.$$

$$\left\{ w(x,y) \to e^{\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}} \left(\int_1^x \frac{\exp\left(-\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}K[2]}{\sqrt{-bx^2+ay^2+bK[2]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[2]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[2]^2-x^2\right)}} dK[2] + c_1 \left(\frac{ay^2-bx}{2a} \right. \right. \right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(\sqrt{\_a}\,\beta+\gamma\right)\left(\frac{\_aab+\sqrt{(ay^2+(\_a^2-x^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}}}{\sqrt{\left(ay^2+\left(\_a^2-x^2\right)b\right)a}} d\_a + \_F1\left(\frac{ay^2-bx^2}{a}\right) \right) \left(\frac{abx}{\sqrt{ab}} + \sqrt{a^2 g} \right.$$

6.5.3.4 [1215] Problem 4

problem number 1215

Added March 12, 2019.

Problem Chapter 5.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ w(x,y) \to e^{-\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}} \left(\int_1^x -\frac{\exp\left(\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}K[1]}{\sqrt{-bx^2+ay^2+bK[1]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[1]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[1]^2-x^2\right)}} dK[1] + c_1\left(\frac{ay^2-b}{2a}\right.\right.\right.$$

$$\left\{ w(x,y) \to e^{\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}} \left(\int_1^x \frac{\exp\left(-\frac{\alpha \tanh^{-1}\left(\frac{\sqrt{b}K[2]}{\sqrt{-bx^2+ay^2+bK[2]^2}}\right)}{\sqrt{a}\sqrt{b}}\right)\left(\sqrt{K[2]}\beta+\gamma\right)}{\sqrt{a}\sqrt{ay^2+b\left(K[2]^2-x^2\right)}} dK[2] + c_1\left(\frac{ay^2-bx}{2a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\sqrt{\_a}\,\beta+\gamma\right)\left(\frac{\_aab+\sqrt{(ay^2+(\_a^2-x^2)b)a}\,\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}}}{\sqrt{(ay^2+(\_a^2-x^2)\,b)\,a}}d\_a+\_F1\left(\frac{ay^2-bx^2}{a}\right)\right)\left(\frac{abx}{\sqrt{ab}}+\sqrt{a^2}\right.$$

6.5.3.5 [1216] Problem 5

problem number 1216

Added March 12, 2019.

Problem Chapter 5.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x+b\sqrt{y}w_y=\alpha w+\beta x+\gamma y+\delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*x + gamma*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to-\frac{-2\alpha^3e^{\frac{2\alpha\sqrt{x}}{a}}c_1\left(2\sqrt{y}-\frac{2b\sqrt{x}}{a}\right)+a^2\beta+2a\alpha\beta\sqrt{x}+2\alpha^2\beta x+2\alpha^2\delta+2\alpha^2\gamma y+2\alpha b\gamma\sqrt{y}+b^2}{2\alpha^3}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*x+gamma*y+de
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(-2\alpha^3\_F1\left(\frac{-a\sqrt{y}+b\sqrt{x}}{b}\right) + \left(2a\alpha\beta\sqrt{x} + a^2\beta + 2\gamma\alpha b\sqrt{y} + (2\beta x + 2\delta + 2\gamma y)\alpha^2 + \gamma b^2\right) e^{-\frac{2\alpha\sqrt{y}}{b}}\right)}{2\alpha^3}$$

6.5.3.6 [1217] Problem 6

problem number 1217

Added March 12, 2019.

Problem Chapter 5.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x + b\sqrt{y}w_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{a\beta + 2\alpha\left(\beta\sqrt{x} + \gamma\right)}{2\alpha^2} + e^{\frac{2\alpha\sqrt{x}}{a}} c_1\left(2\sqrt{y} - \frac{2b\sqrt{x}}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{y} \frac{\left(\gamma\_a + \sqrt{\frac{(\sqrt{\_a}\,a - a\sqrt{y} + b\sqrt{x})^2}{b^2}}\,\beta + \delta\right) \mathrm{e}^{-\frac{2\sqrt{\_a}\,\alpha}{b}}}{\sqrt{\_a}\,b} d\_a + \_F1\left(\frac{-a\sqrt{y} + b\sqrt{x}}{b}\right)\right) \mathrm{e}^{\frac{2\alpha\sqrt{y}}{b}}$$

6.5.3.7 [1218] Problem 7

problem number 1218

Added March 12, 2019.

Problem Chapter 5.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{y}w_x + b\sqrt{x}w_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sqrt[y]*D[w[x, y], x] + b*Sqrt[x]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\alpha x \sqrt[3]{\frac{ay^{3/2}}{ay^{3/2}-bx^{3/2}}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{bx^{3/2}}{bx^{3/2}-ay^{3/2}}\right)}{a\sqrt[3]{y^{3/2}}}\right)\left(\int_1^x \frac{\exp\left(-\frac{\alpha\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{bK[1]^{3/2}}{bx^{3/2}-ay^{3/2}}\right)K[1]\sqrt[3]{1-}}{a\sqrt[3]{y^{3/2}+\frac{b\left(K[1]^{3/2}-x^{3/2}\right.}{a}}}\right.}{a\sqrt[3]{y^{3/2}+\frac{b(K[1]}{}}}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sqrt(y)*diff(w(x,y),x)+ b*sqrt(x)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{y} \frac{\left(\gamma\_b + \sqrt{\frac{\left(\left(\_b^{\frac{3}{2}} a + b\,\mathrm{RootOf}\left(b^2 x - \left(a b^2 y^{\frac{3}{2}} + \_Z b^3\right)^{\frac{2}{3}}\right)\right) b^2\right)^{\frac{2}{3}}}{b^2}}\,\beta + \delta\right) \mathrm{e}^{-\frac{\alpha \left(\int \frac{1}{\sqrt{\frac{\left(\left(\_b^{\frac{3}{2}} a + b\,\mathrm{RootOf}\left(b^2 x - \left(a b\right.\right.\right.\right.}{b^2}}}}{b}}}{\sqrt{\frac{\left(\left(\_b^{\frac{3}{2}} a + b\,\mathrm{RootOf}\left(b^2 x - \left(a b^2 y^{\frac{3}{2}} + \_Z b^3\right)^{\frac{2}{3}}\right)\right) b^2\right)^{\frac{2}{3}}}{b^2}}\, b} \right.$$

contains RootOf

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6.5.4.1 [1219] Problem 1

problem number 1219

Added March 12, 2019.

Problem Chapter 5.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + kx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + k*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}kK[1]^n\left(y+\frac{b(K[1]-x)}{a}\right)^m}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\_a^n\left(\frac{ay-(-\_a+x)b}{a}\right)^m \mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.4.2 [1220] Problem 2

problem number 1220

Added March 12, 2019.

Problem Chapter 5.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + yw_y = bw + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x(b-m)}{a}}\left(-\frac{cy^m x^n \left(\frac{x(b-m)}{a}\right)^{-n} \text{Gamma}\left(n+1,\frac{x(b-m)}{a}\right)}{b-m} + e^{\frac{mx}{a}} c_1\left(ye^{-\frac{x}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ y*diff(w(x,y),y) = b*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{c\_a^n \left(y\,\mathrm{e}^{-\frac{a-x}{a}}\right)^m \mathrm{e}^{-\frac{ab}{a}}}{a} d\_a + \_F1\left(y\,\mathrm{e}^{-\frac{x}{a}}\right)\right) \mathrm{e}^{\frac{bx}{a}}$$

6.5.4.3 [1221] Problem 3

problem number 1221

Added April 1, 2019.

Problem Chapter 5.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = axw + bx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y] + b*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{ax}\left(-by^m x^n (ax)^{-m-n}\,\text{Gamma}(m+n, ax) + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{bx^n y^m (ax)^{-\frac{m}{2}-\frac{n}{2}}\,\text{WhittakerM}\left(\frac{m}{2}+\frac{n}{2}, \frac{m}{2}+\frac{n}{2}+\frac{1}{2}, ax\right) e^{\frac{ax}{2}}}{(m+n+1)(m+n)} + \frac{bx^{n-1} y^m (ax)^{-\frac{m}{2}-\frac{n}{2}}\,\text{WhittakerM}\left(\frac{m}{2}\right.}{(m+n)}$$

6.5.4.4 [1222] Problem 4

problem number 1222

Added April 1, 2019.

Problem Chapter 5.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}w + bx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2+y^2]*w[x,y] + b*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{a\sqrt{x^2+y^2}}\left(\int_1^x be^{-a\sqrt{\left(\frac{y^2}{x^2}+1\right)K[1]^2}}K[1]^{n-1}\left(\frac{yK[1]}{x}\right)^m dK[1] + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left((m+n+1)\,bx^{-m}x^{m+n}y^m\left(\sqrt{x^2+y^2}\,a\right)^{-\frac{m}{2}-\frac{n}{2}}\left(\frac{\sqrt{x^2+y^2}\,a}{x}\right)^{-m-n}\left(\frac{\sqrt{x^2+y^2}\,a}{x}\right)^{m+n}\text{WhittakerM}\left(\frac{m}{2}\right.\right.}{}$$

6.5.4.5 [1223] Problem 5

problem number 1223

Added April 1, 2019.

Problem Chapter 5.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m w + px^k y^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x,y] + p*x^k*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cy^m x^n}{an+bm}}\left(\int_1^x \frac{\exp\left(-\frac{cK[1]^n\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^m}{bm+an}\right)pK[1]^{k-1}\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^s}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*x^n*y^m*w(x,y)+ p*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left((an+bm)^2\left(cx^ny^m+(k+n)a+(m+s)b\right)px^{\frac{bs+(k-n)a}{a}}x^{-\frac{bs}{a}}y^sy^{-m}\left(\frac{cx^ny^m}{an+bm}\right)^{\frac{(-k-n)a-(m+s)b}{2an+2bm}}\left(\frac{cx^{-\frac{b}{}}}{an+}\right.\right.}{}$$

6.5.4.6 [1224] Problem 6

problem number 1224

Added April 1, 2019.

Problem Chapter 5.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=(cx^n+py^m)w+qx^ky^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*(x^n+p*y^m)*w[x,y] + q*x^k*y^s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^n}{an}+\frac{cpy^m}{bm}}\left(\int_1^x \frac{\exp\left(-\frac{c\left(\frac{ap\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^m}{bm}+\frac{K[1]^n}{n}\right)}{a}\right)qK[1]^{k-1}\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)^s}{a}dK[1]+c_1\left(yx\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*(x^n+y^m)*w(x,y)+ q*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{q\_b^{k-1}\left(y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^s \mathrm{e}^{-\frac{c\left(\int \frac{\_b^n+\left(y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m}{\_b}d\_b\right)}{a}}}{a}d\_b+\_F1\left(yx^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int^x \frac{\left(\_a^n+\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m\right)}{\_aa}}$$

6.5.4.7 [1225] Problem 7

problem number 1225

Added April 1, 2019.

Problem Chapter 5.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axy w_y = by^2 w + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x,y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{by^2}{x-2ax}}\left(-\frac{cy^m x^{n-1}\left(-\frac{by^2}{x-2ax}\right)^{-\frac{am+n-1}{2a-1}}\text{Gamma}\left(\frac{am+n-1}{2a-1},-\frac{by^2}{x-2ax}\right)}{2a-1}+c_1\left(yx^{-a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+ a*x*y*diff(w(x,y),y) = b*y^2*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{4\left(\frac{\left(a-\frac{1}{2}\right)\left((m+2)a+n-2\right)^2 cxx^{-am}x^{am+n}y^m\left(\frac{by^2}{(2a-1)x}\right)^{\frac{(-m-2)a-n+2}{4a-2}}\left(\frac{by^2x^{-2a}}{2a-1}\right)^{\frac{-am-n+1}{2a-1}}\left(\frac{by^2x^{-2a}}{2a-1}\right)^{\frac{am+n-1}{2a-1}}\text{WhittakerM}}{2}\right.}{}$$

6.5.4.8 [1226] Problem 8

problem number 1226

Added April 1, 2019.

Problem Chapter 5.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xy w_y = y^2(ax+by)w + cx^n y^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x+b*y)*w[x,y] + c*x^n*y^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -c2^{\frac{1}{2}(m+n-3)} y^m x^{n-1} e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)} \left(y^2\left(a+\frac{by}{x}\right)\right)^{\frac{1}{2}(-m-n+1)} \text{Gamma}\left(\frac{1}{2}(m+n-1), \frac{1}{2}y^2\left(a+\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  x^2*diff(w(x,y),x)+ x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left((m+n+3)\left(by^3+\left(ay^2+m+n+1\right)x\right)cx2^{-\frac{m}{4}-\frac{n}{4}+\frac{3}{4}}2^{\frac{m}{2}+\frac{n}{2}+\frac{1}{2}}x^{-m-2}x^{m+n-1}y^{m+2}\left(\left(a+\frac{by}{x}\right)y^2\right.\right.}{}$$

6.5.4.9 [1227] Problem 9

problem number 1227

Added April 1, 2019.

Problem Chapter 5.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^p y^q w + sx^\gamma y^\delta + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^p*y^q*w[x,y] + s*x^gamma*y^delta+d
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(-\frac{cy^q x^{-n+p+1} e^{-\frac{bqx^{m-n+1}}{am-an+a}}\left(-\frac{bqx^{m-n+1}}{am-an+a}\right)^{\frac{n-p-1}{m-n+1}}\text{Gamma}\left(\frac{-n+p+1}{m-n+1},-\frac{bqx^{m-n+1}}{am-an+a}\right)}{a(m-n+1)}\right)\left(\int_1^x \text{ex}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y*diff(w(x,y),y) = c*x^p*y^q*w(x,y)+ s*x^gamma*y^delta+d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(s\_b^{-n+\gamma} \left(y\, \mathrm{e}^{-\frac{\left(-\_b^{m-n+1}+x^{m-n+1}\right) b}{(m-n+1)a}} \right)^{\delta} + d\_b^{-n} \right) \mathrm{e}^{-\frac{c\left(\int \_b^{-n+p} \left(y\, \mathrm{e}^{-\frac{\left(-\_b^{m-n+1}+x^{m-n+1}\right) b}{(m-n+1)a}} \right)^{q} d\_b \right.}{a}}}{a} \right.$$

6.5.4.10 [1228] Problem 10

problem number 1228

Added April 1, 2019.

Problem Chapter 5.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k) w_y = sx^p y^q w + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + (b*x^m*y+x*x^k)*D[w[x, y], y] == s*x^p*y^q*w[x,y] + d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{s\left(b^{-\frac{k+2}{m-n+1}}\exp\left(-\frac{b(x^{m-n+1}-K[1]^{m-n+1})}{a(m-n+1)}\right)(a(m-n+1))^{-\frac{m}{m-n+1}}\left(b^{\frac{n}{m-n+1}}e^{\frac{bx^{m-n+1}}{ma-na+a}}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ (b*x^m*y+c*x^k)*diff(w(x,y),y) = s*x^p*y^q*w(x,y)+ d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{d\_b^{-n}\mathrm{e}^{-\frac{s\left(\int\_b^{-n+p}\left(\frac{-(m-n+1)(k+m-2n+2)^2acx^{k-m}\left(\frac{bx^{m-n+1}}{(m-n+1)a}\right)^{\frac{-k-m+2n-2}{2m-2n+2}}\mathrm{WhittakerM}\left(\frac{k+m-2n+2}{2m-2n+2},\frac{k+2m-3n+3}{2m-2n+2}\right.}{}\right.\right.}{}}{}\right.$$

6.5.4.11 [1229] Problem 11

problem number 1229

Added April 1, 2019.

Problem Chapter 5.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y^k w_y = cw + sx^p y^q + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == c*w[x,y] + s*x^p*y^q+d;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx^{1-n}}{a-an}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-n}}{a(n-1)}} K[1]^{-n}\left(s\left(\left(-\frac{a(m-n+1)x^n y^k K[1]^n}{a(-m+n-1)x^n y K[1]^n - b(k-1)y^k(x^{m+1}K[1]^n - x^n K[1]^{m+1})}\right)^{\frac{1}{k-1}}\right)^q K\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y^k*diff(w(x,y),y) = c*w(x,y)+ s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s\_a^{-n+p}\left(\left(\frac{(m-n+1)ay^{-k+1} - (k-1)b\_a^{m-n+1} + (k-1)bx^{m-n+1}}{(m-n+1)a}\right)^{-\frac{1}{k-1}}\right)^q + d\_a^{-n}\right) e^{\frac{c\_a^{-n+1}}{(n-1)a}}}{a} d\_a +$$

6.5.4.12 [1230] Problem 12

problem number 1230

Added April 1, 2019.

Problem Chapter 5.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y] + s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\,{}_2F_1\left(\frac{k}{k+1},\frac{1}{n+1};1+\frac{1}{n+1};\frac{b(k+1)x^{n+}}{b(k+1)x^{n+1}-a(n-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{x}\frac{s\_b^{m}\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_b^{n+1}-(k+1)bx^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}\mathrm{e}^{-\frac{c\left(\int\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_b^{n+1}-(k+1)bx^{n+1}}{(n+1)a}\right)^{\frac{1}{k+}}\right.}{a}}}{a}\right.$$

6.5.5 3.1

Local contents

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6.5.5.6 [1236] Problem 6 1901
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6.5.5.1 [1231] Problem 1

problem number 1231

Added April 1, 2019.

Problem Chapter 5.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (ce^{\lambda x} + se^{\mu y})w + ke^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x]+s*Exp[mu*y])*w[x,y] + k*Exp[nu*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\lambda x}}{a\lambda}+\frac{se^{\mu y}}{b\mu}}\left(\int_1^x \frac{\exp\left(-\frac{e^{\lambda K[1]}c}{a\lambda}-\frac{e^{\mu\left(y+\frac{b(K[1]-x)}{a}\right)}s}{b\mu}+\nu K[1]\right)k}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*exp(lambda*x)+s*exp(mu*y))*w(x,y)+ k*exp(nu*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\,\mathrm{e}^{\frac{-a\lambda s\,\mathrm{e}^{\frac{(ay-(-\_a+x)b)\mu}{a}}+\left(\_a a\lambda\nu-c\,\mathrm{e}^{-\_a\lambda}\right)b\mu}{ab\lambda\mu}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{a\lambda s\,\mathrm{e}^{\mu y}+bc\mu\,\mathrm{e}^{\lambda x}}{ab\lambda\mu}}$$

6.5.5.2 [1232] Problem 2

problem number 1232

Added April 1, 2019.

Problem Chapter 5.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\alpha x+\beta y}w + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x+beta*y]*w[x,y] + k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\alpha x+\beta y}}{a\alpha+b\beta}}\left(\int_1^x \frac{\exp\left(\gamma K[1]-\frac{ce^{\beta y+\alpha K[1]+\frac{b\beta(K[1]-x)}{a}}}{a\alpha+b\beta}\right)k}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*exp(alpha*x+beta*y)*w(x,y)+ k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\,\mathrm{e}^{\frac{-c\,\mathrm{e}^{\frac{-(-\_a+x)b\beta+(\alpha\_a+\beta y)a}{a}}}{a\alpha+b\beta}+\gamma(a\alpha+b\beta)\_a}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\alpha x+\beta y}}{a\alpha+b\beta}}$$

6.5.5.3 [1233] Problem 3

problem number 1233

Added April 1, 2019.

Problem Chapter 5.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma y}w + se^{\mu x+\delta y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\exp\left(y\gamma-\frac{b\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K[1]}\right)\gamma}{a(\beta-\lambda)}-\lambda K[1]\right)}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\frac{b\delta\left(e^{(\beta-\lambda)x}-e^{(\beta-\lambda)K}\right.}{a(\beta-\lambda)}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*exp(gamma*y)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{s\,\mathrm{e}^{\frac{b\delta\,\mathrm{e}^{(\beta-\lambda)x}-b\delta\,\mathrm{e}^{(\beta-\lambda)\_b}-(-\beta+\lambda)c\left(\int \mathrm{e}^{\frac{-\gamma b\,\mathrm{e}^{(\beta-\lambda)x}+\gamma b\,\mathrm{e}^{(\beta-\lambda)\_b}+(\beta-\lambda)(-\_b\lambda+\gamma y)a}{(\beta-\lambda)a}}d\_b\right)-(-\beta+\lambda)(\_b\lambda-\_b\mu-\delta y)a}{(-\beta+\lambda)a}}}{a}d\_b\right.$$

6.5.5.4 [1234] Problem 4

problem number 1234

Added April 1, 2019.

Problem Chapter 5.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = sw + ke^{\mu x+\delta y}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x-b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right)-c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_ba\beta-b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_b}}{(\beta-\gamma)a}}d\_b\right)+a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay+b\,\mathrm{e}^{(-\beta+\gamma)x}\right)\lambda}{(\beta-\gamma)a}}}{a}\right)^{-}}{a} \right.$$

6.5.5.5 [1235] Problem 5

problem number 1235

Added April 1, 2019.

Problem Chapter 5.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x+\delta y}w + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\,\mathrm{e}^{\frac{-\_f a\beta - s\left(\int\left(\frac{a\lambda\left(\int \frac{c\,\mathrm{e}^{\frac{-(\beta-\gamma)a\beta x - b\lambda\,\mathrm{e}^{(-\beta+\gamma)x}}{(\beta-\gamma)a}}}{a}dx\right) - c\lambda\left(\int \mathrm{e}^{\frac{-(\beta-\gamma)\_f a\beta - b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_f}}{(\beta-\gamma)a}}d\_f\right) + a\,\mathrm{e}^{-\frac{\left((\beta-\gamma)ay + b\,\mathrm{e}^{(-\beta+\gamma)x}\right.}{(\beta-\gamma)a}}}{a}\right.}{a}}}{a} \right.$$

6.5.5.6 [1236] Problem 6

problem number 1236

Added April 1, 2019.

Problem Chapter 5.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = ce^{\sigma y}w + ke^{\mu x+deltay} + d$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x+lambda*y]*D[w[x, y], y] == c*Exp[sigma*y]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c(\gamma-\beta)\left(e^{\lambda y}\right)^{\frac{\sigma}{\lambda}}e^{-\gamma x-\lambda y}\,{}_2F_1\left(1,-\frac{\gamma}{\beta-\gamma};\frac{\beta\sigma-\gamma(\lambda+\sigma)}{(\beta-\gamma)\lambda};1-\frac{ae^{\beta x-\gamma x-\lambda y}(\beta-\gamma)}{b\lambda}\right)}{b(\beta(\lambda-\sigma)+\gamma\sigma)}\right)\left(\int_1^x \frac{\exp\left(\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ b*exp(gamma*x+lambda*y)*diff(w(x,y),y) = c*exp(sigma*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{k\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_b}+(\beta-\gamma)a\,\mathrm{e}^{-\lambda y}}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{\frac{(-\beta+\mu)\_ba-c\left(\int\left(\frac{(\beta-\gamma)a}{-b\lambda\,\mathrm{e}^{-\lambda y}\mathrm{e}^{\lambda y+(-\beta+\gamma)x}+b\lambda\,\mathrm{e}^{(-\beta+\gamma)\_b}+(\beta}\right.\right.}{a}}}{a}\right.$$

6.5.5.7 [1237] Problem 7

problem number 1237

Added April 1, 2019.

Problem Chapter 5.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = cw + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x,y]+s*Exp[gamma*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{ce^{-\lambda x}}{a\lambda}}\left(\int_1^x \frac{e^{\frac{e^{-\lambda K[1]}c}{a\lambda}+(\gamma-\lambda)K[1]}s}{a}dK[1] + c_1\left(\frac{be^{x(\beta-\lambda)}}{a(\lambda-\beta)} + y\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*w(x,y)+s*exp(gamma*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s\,\mathrm{e}^{\frac{(-\lambda+\gamma)a\lambda x + c\,\mathrm{e}^{-\lambda x}}{a\lambda}}}{a}dx + \_F1\left(\frac{(\beta-\lambda)\,ay - b\,\mathrm{e}^{(\beta-\lambda)x}}{(\beta-\lambda)\,a}\right)\right)\mathrm{e}^{-\frac{c\,\mathrm{e}^{-\lambda x}}{a\lambda}}$$

6.5.5.8 [1238] Problem 8

problem number 1238

Added April 1, 2019.

Problem Chapter 5.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^{\beta x}w_y = ce^{\gamma x}w + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*x^(beta*x)*D[w[x, y], y] == c*Exp[gamma*x]*w[x,y]+s
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{x(\gamma-\lambda)}}{a(\gamma-\lambda)}}\left(\int_1^x \frac{\exp\left(-\frac{e^{(\gamma-\lambda)K[2]}c}{a(\gamma-\lambda)}-\lambda K[2]\right)s}{a}dK[2]+c_1\left(y-\int_1^x \frac{be^{-\lambda K[1]}K[1]^{\beta K[1]}}{a}dK[1]\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*x^(beta*x)*diff(w(x,y),y) = c*exp(gamma*x)*w(x,y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s\,\mathrm{e}^{\frac{-(-\lambda+\gamma)a\lambda x-c\,\mathrm{e}^{(-\lambda+\gamma)x}}{(-\lambda+\gamma)a}}}{a}dx+\_F1\left(\frac{ay-b\left(\int x^{\beta x}\mathrm{e}^{-\lambda x}dx\right)}{a}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{(-\lambda+\gamma)x}}{(-\lambda+\gamma)a}}$$

6.5.6 3.2

Local contents

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6.5.6.3 [1241] Problem 3 1906
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6.5.6.6 [1244] Problem 6 1909
6.5.6.7 [1245] Problem 7 1910
6.5.6.8 [1246] Problem 8 1911
6.5.6.9 [1247] Problem 9 1912
6.5.6.10 [1248] Problem 10 1913

6.5.6.1 [1239] Problem 1

problem number 1239

Added April 2, 2019.

Problem Chapter 5.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + bx^n)w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*x^n)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}}-\int_1^x be^{-\frac{ae^{\lambda K[1]}}{\lambda}}K[1]^n dK[1]\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*x^n)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{k\,\mathrm{e}^{(-c+\gamma)x}}{-c+\gamma} + \_F1\left(-b\left(\int x^n \mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\,dx\right) + y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)\mathrm{e}^{cx}$$

6.5.6.2 [1240] Problem 2

problem number 1240

Added April 2, 2019.

Problem Chapter 5.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + be^{\beta x})w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*Exp[gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{e^{cx}\left(ke^{x(\gamma-c)} + (\gamma-c)c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}} - \int_1^x be^{\beta K[1]-\frac{ae^{\lambda K[1]}}{\lambda}}\,dK[1]\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*exp(ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(k\,\mathrm{e}^{(-c+\gamma)x} + (-c+\gamma)\,\_F1\left(-b\left(\int \mathrm{e}^{\frac{\beta\lambda x - a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right) + y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)\mathrm{e}^{cx}}{c-\gamma}$$

6.5.6.3 [1241] Problem 3

problem number 1241

Added April 2, 2019.

Problem Chapter 5.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + be^{\beta x})w_y = cw + kx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{cx}\left(-\frac{kx^n(cx)^{-n}\,\mathrm{Gamma}(n+1,cx)}{c} + c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}} - \int_1^x be^{\beta K[1] - \frac{ae^{\lambda K[1]}}{\lambda}}dK[1]\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left(kx^n(cx)^{-\frac{n}{2}}\,\mathrm{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},cx\right)\mathrm{e}^{-\frac{cx}{2}} + (n+1)\,c\_F1\left(-b\left(\int \mathrm{e}^{\frac{\beta\lambda x - a\,\mathrm{e}^{\lambda x}}{\lambda}}dx\right) + y\,\mathrm{e}^{-\frac{a\,\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)}{(n+1)\,c}$$

6.5.6.4 [1242] Problem 4

problem number 1242

Added April 2, 2019.

Problem Chapter 5.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda y} + bx^k)w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y], x] + (a*Exp[lambda*y]+b*x^k)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{e^{cx}\left(ke^{x(\gamma-c)} + (\gamma-c)c_1\left(\frac{a\lambda x\left(-\frac{b\lambda x^{k+1}}{k+1}\right)^{-\frac{1}{k+1}}\mathrm{Gamma}\left(\frac{1}{k+1},-\frac{b\lambda x^{k+1}}{k+1}\right)-(k+1)e^{-\frac{\lambda\left(-bx^{k+1}+ky+y\right)}{k+1}}}{abk(k+1)\lambda^2}\right)\right)}{c-\gamma}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y),x)+ (a*exp(lambda*y)+b*x^k)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\left(k\,\mathrm{e}^{(-c+\gamma)x} + (-c+\gamma)\_F1\left(\frac{(k+1)(k+2)^2 a x^{-k}\left(-\frac{b\lambda x^{k+1}}{k+1}\right)^{\frac{-k-2}{2k+2}} \mathrm{WhittakerM}\left(\frac{k+2}{2k+2},\frac{2k+3}{2k+2},-\frac{b\lambda x^{k+1}}{k+1}\right)\mathrm{e}^{\frac{b\lambda x^{k+1}}{2k+2}} -}{}\right.\right.}{}$$

6.5.6.5 [1243] Problem 5

problem number 1243

Added April 2, 2019.

Problem Chapter 5.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w + be^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x+mu*y]*w[x,y]+b*Exp[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}}\left(\int_1^x \frac{b\exp\left(\nu K[1] - \frac{ae^{\left(\lambda+\frac{\mu y}{x}\right)K[1]}x}{\lambda x+\mu y}\right)}{K[1]}dK[1] + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) =  a*x*exp(lambda*x+mu*y)*w(x,y)+k*exp(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\,\mathrm{e}^{-\frac{a x\,\mathrm{e}^{-a\lambda+\frac{-a\mu y}{x}}-(\lambda x+\mu y)\_a\nu}{\lambda x+\mu y}}}{\_a} d\_a + \_F1\left(\frac{y}{x}\right)\right) \mathrm{e}^{\frac{a\,\mathrm{e}^{\lambda x+\mu y}}{\lambda+\frac{\mu y}{x}}}$$

6.5.6.6 [1244] Problem 6

problem number 1244

Added April 2, 2019.

Problem Chapter 5.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = (aye^{\lambda x} + bxe^{\mu y})w + ce^{\nu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x]+b*x*Exp[mu*y])*w[x,y]+c*Exp[n
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\left(\int_1^x \frac{c\exp\left(-\frac{be^{\frac{\mu y K[1]}{x}}x}{\mu y}+\nu K[1]-\frac{ae^{\lambda K[1]}y}{\lambda x}\right)}{K[1]} dK[1] + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) = (a*y*exp(lambda*x)+b*x*exp(mu*y))*w(x,y)+c*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{c\,\mathrm{e}^{-\frac{\left(-\frac{a\lambda\mu\nu y}{x}+\frac{a\mu y^{2}\mathrm{e}^{-a\lambda}}{x^{2}}+b\lambda\,\mathrm{e}^{\frac{a\mu y}{x}}\right)x}{\lambda\mu y}}}{\_a}\,d\_a + \_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{\left(\frac{a\mu y^{2}\mathrm{e}^{\lambda x}}{x^{2}}+b\lambda\,\mathrm{e}^{\mu y}\right)x}{\lambda\mu y}}$$

6.5.6.7 [1245] Problem 7

problem number 1245

Added April 2, 2019.

Problem Chapter 5.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^{k}w_{x} + be^{\lambda x}w_{y} = w + ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(-\frac{(k+1)y^{k+1}\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\,{}_2F_1\left(1,\frac{1}{k+1};\frac{k+2}{k+1};\frac{a\lambda y^{k+1}}{a\lambda y^{k+1}-be^{\lambda x}(k+1)}\right)}{a\lambda y^{k+1}-b(k+1)e^{\lambda x}}\right)\left(\int_{1}^{x}\frac{c\exp\left(\frac{(k+1)(}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k* diff(w(x,y),x)+ b*exp(lambda*x)*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{c\left(\left(\frac{a\lambda y^{k+1}+(k+1)b\,\mathrm{e}^{-b\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k} \mathrm{e}^{\frac{-ba\beta-\left(\int\left(\left(\frac{a\lambda y^{k+1}+(k+1)b\,\mathrm{e}^{-b\lambda}-(k+1)b\,\mathrm{e}^{\lambda x}}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k} d\_b\right)}{a}}{a} d\_b \right.$$

6.5.6.8 [1246] Problem 8

problem number 1246

Added April 2, 2019.

Problem Chapter 5.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + byw_y = w + ce^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*x]*D[w[x, y], x] + b*y*D[w[x, y], y] == w[x,y]+c*Exp[lambda*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{e^{-\frac{e^{-\lambda x}}{a\lambda}}\left(-c\mathrm{Ei}\left(\frac{e^{-\lambda x}}{a\lambda}\right)+a\lambda c_1\left(ye^{\frac{be^{-\lambda x}}{a\lambda}}\right)\right)}{a\lambda}\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*x)* diff(w(x,y),x)+ b*y*diff(w(x,y),y) = w(x,y)+c*exp(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left(a\lambda\_F1\left(y\,\mathrm{e}^{\frac{b\,\mathrm{e}^{-\lambda x}}{a\lambda}}\right) + c\,\mathrm{expIntegral}\left(1, -\frac{\mathrm{e}^{-\lambda x}}{a\lambda}\right)\right)\mathrm{e}^{-\frac{\mathrm{e}^{-\lambda x}}{a\lambda}}}{a\lambda}$$

6.5.6.9 [1247] Problem 9

problem number 1247

Added April 2, 2019.

Problem Chapter 5.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^k w_y = w + ce^{\beta x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{(k+1)x\,{}_2F_1\left(1, \frac{1}{k+1}; 1+\frac{1}{k+1}; \frac{b\lambda x^{k+1}}{b\lambda x^{k+1} - ae^{\lambda y}(k+1)}\right)}{a(k+1)e^{\lambda y} - b\lambda x^{k+1}}\right)\left(\int_1^x \frac{c\exp\left(\left(\beta - \frac{(k+1)\,{}_2F_1\left(1, \frac{1}{k+1}; 1+\right.}{ae^{\lambda y}(k+}\right.\right.}{ae^{\lambda y}(k+1)+}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*x^k*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{(k+1)\,c\,\mathrm{e}^{\frac{\_bb\beta\lambda+(-k-1)\left(\int \frac{b\lambda}{b\lambda\_b^{k+1}-b\lambda x^{k+1}+(k+1)a\,\mathrm{e}^{\lambda y}}d\_b\right)}{b\lambda}}}{b\lambda\_b^{k+1}-b\lambda x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}d\_b + \_F1\left(\frac{-b\lambda x^{k+1}+(k+1)\,a\,\mathrm{e}^{\lambda y}}{(k+1)\,b\lambda}\right)\right)$$

6.5.6.10 [1248] Problem 10

problem number 1248

Added April 2, 2019.

Problem Chapter 5.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = w + cx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == w[x,y]+c*x^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\beta x - \log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)}{a\beta e^{\lambda y} - b\lambda e^{\beta x}}\right)\left(\int_1^x \frac{\beta c \exp\left(\frac{\log\left(\frac{ae^{\lambda y}\beta}{\lambda}+b\left(-e^{\beta x}+e^{\beta K[1]}\right)\right)-\beta K[1]}{a\beta e^{\lambda y}-be^{\beta x}\lambda}\right)K[1]^k}{ae^{\lambda y}\beta + b\left(-e^{\beta x}+e^{\beta K[1]}\right)\lambda}dK[1] + c_1\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = w(x,y)+c*x^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\beta c\_a^{k} \left(\mathrm{e}^{-a\beta} - \frac{-a\beta\,\mathrm{e}^{\lambda y} + b\lambda\,\mathrm{e}^{\beta x}}{b\lambda}\right)^{\frac{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x} - 1}{-a\beta\,\mathrm{e}^{\lambda y} + b\lambda\,\mathrm{e}^{\beta x}}} \left(\mathrm{e}^{-a\beta}\right)^{\frac{1}{-a\beta\,\mathrm{e}^{\lambda y} + b\lambda\,\mathrm{e}^{\beta x}}}}{b\lambda} d\_a + \_F1\left(\frac{a\beta\,\mathrm{e}^{\lambda y} - b\lambda\,\mathrm{e}^{\beta x}}{b\beta\lambda}\right.$$

6.5.7 4.1

Local contents

6.5.7.1 [1249] Problem 1 1914
6.5.7.2 [1250] Problem 2 1915
6.5.7.3 [1251] Problem 3 1916
6.5.7.4 [1252] Problem 4 1917
6.5.7.5 [1253] Problem 5 1918

6.5.7.1 [1249] Problem 1

problem number 1249

Added April 3, 2019.

Problem Chapter 5.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sinh^k(\lambda x)\sinh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sinh[lambda*x]^k*Sinh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sinh^k(\lambda K[1])\sinh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*sinh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\sinh^k(\_a\lambda)\right)\left(\sinh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.7.2 [1250] Problem 2

problem number 1250

Added April 3, 2019.

Problem Chapter 5.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=c\sinh^k(\lambda x)w+s\sinh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x]^k*w[x,y]+ s*Sinh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\sqrt{\cosh^2(\lambda x)}\operatorname{sech}(\lambda x)\sinh^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};-\sinh^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(-\frac{c\sqrt{c}}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sinh(lambda*x)^k*w(x,y)+s*sinh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{s(\sinh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\sinh^k(\lambda x)\right)dx\right)}{a}}}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int\frac{c\left(\sinh^k(\lambda x)\right)}{a}dx}$$

6.5.7.3 [1251] Problem 3

problem number 1251

Added April 3, 2019.

Problem Chapter 5.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c_1\sinh^{n_1}(\lambda_1 x)+c_2\sinh^{n_2}(\lambda_2 y))\,w+s_1\sinh^{k_1}(\beta_1 x)+s_2\sinh^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Sinh[lambda1*x]^n1 + c2*Sinh[lambda2*y]^n2)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{\mathrm{c1}\sqrt{\cosh^2(\mathrm{lambda1}x)}\operatorname{sech}(\mathrm{lambda1}x)\sinh^{\mathrm{n1}+1}(\mathrm{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{\mathrm{n1}+1}{2};\frac{\mathrm{n1}+3}{2};-\sinh^2\right.}{a\mathrm{lambda1n1}+a\mathrm{lambda1}}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*sinh(lambda1*x)^n1 + c2*sinh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(s1\left(\sinh^{k1}\left(\_b\beta 1\right)\right)+s2\left(\sinh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right)\mathrm{e}^{-\frac{\int\left(c1\left(\sinh^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\sinh^{n2}\left(\frac{(ay-(-\_b+x}{a}\right.\right.}{a}}}{a}\right.$$

6.5.7.4 [1252] Problem 4

problem number 1252

Added April 3, 2019.

Problem Chapter 5.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^n(\lambda x)w_x+b\sinh^m(\mu x)w_y=c\sinh^k(\nu x)w+p\sinh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c2^{n-1}e^{-\nu x}\left(e^{\lambda x}-e^{-\lambda x}\right)^{-n}\left(1-e^{2\lambda x}\right)^{n}\left((\lambda n+\nu)\,{}_2F_1\left(n,\frac{\lambda n-\nu}{2\lambda};\frac{1}{2}\left(n-\frac{\nu}{\lambda}+2\right);e^{2\lambda x}\right)+e\right.}{a(\nu-\lambda n)(\lambda n+\nu)}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^{x}\frac{\left(cw_{x,y}\sinh\left(\_b\nu\right)+p\left(\sinh^{s}\left(\frac{(ay+b(\int(\sinh^{-n}(\_b\lambda))(\sinh^{m}(\_b\mu))d\_b)-b(\int(\sinh^{-n}(\lambda x))(\sinh^{m}(\mu x))dx))\beta}{a}\right)\right.\right.}{a}$$

6.5.7.5 [1253] Problem 5

problem number 1253

Added April 3, 2019.

Problem Chapter 5.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^{n}(\lambda x)w_x+b\sinh^{m}(\mu x)w_y=c\sinh^{k}(\nu y)w+p\sinh^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sinh^{-n}(\lambda K[2])\sinh\left(\nu\left(y-\int_1^x \frac{b\sinh^{-n}(\lambda K[1])\sinh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\sinh^{-n}(\lambda K[1}{}\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*y)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(cw_{x,y}\sinh\left(\frac{(ay+b(\int(\sinh^{-n}(\_b\lambda))(\sinh^m(\_b\mu))d\_b)-b(\int(\sinh^{-n}(\lambda x))(\sinh^m(\mu x))dx))\nu}{a}\right)+p(\sinh^s(\_b\beta)\right.}{a}$$

6.5.8 4.2

Local contents

6.5.8.1 [1254] Problem 1 1920
6.5.8.2 [1255] Problem 2 1921
6.5.8.3 [1256] Problem 3 1922
6.5.8.4 [1257] Problem 4 1923
6.5.8.5 [1258] Problem 5 1923
6.5.8.6 [1259] Problem 6 1924

6.5.8.1 [1254] Problem 1

problem number 1254

Added April 3, 2019.

Problem Chapter 5.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cosh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Cosh[lambda*x]^k*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cosh^k(\lambda K[1])\cosh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+cosh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\cosh^k(\_a\lambda)\right)\left(\cosh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.8.2 [1255] Problem 2

problem number 1255

Added April 3, 2019.

Problem Chapter 5.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh^k(\lambda x)w + s\cosh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x]^k*w[x,y]+ s*Cosh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{c\sqrt{-\sinh^2(\lambda x)}\operatorname{csch}(\lambda x)\cosh^{k+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cosh^2(\lambda x)\right)}{ak\lambda + a\lambda}\right)\left(\int_1^x \frac{\exp\left(\frac{c\cosh^k}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cosh(lambda*x)^k*w(x,y)+s*cosh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s(\cosh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\cosh^k(\lambda x)\right)dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{c\left(\cosh^k(\lambda x)\right)}{a}dx}$$

6.5.8.3 [1256] Problem 3

problem number 1256

Added April 3, 2019.

Problem Chapter 5.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \cosh^{n_1}(\lambda_1 x) + c_2 \cosh^{n_2}(\lambda_2 y))\, w + s_1 \cosh^{k_1}(\beta_1 x) + s_2 \cosh^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Cosh[lambda1*x]^n1 + c2*Cosh[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\mathrm{c1}\sqrt{-\sinh^2(\mathrm{lambda1}x)}\,\mathrm{csch}(\mathrm{lambda1}x)\cosh^{\mathrm{n1}+1}(\mathrm{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{\mathrm{n1}+1}{2};\frac{\mathrm{n1}+3}{2};\cosh^{2}\right.}{a\mathrm{lambda1n1}+a\mathrm{lambda1}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*cosh(lambda1*x)^n1 + c2*cosh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(s1\left(\cosh^{k1}\left(\_b\beta 1\right)\right) + s2\left(\cosh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right) \mathrm{e}^{-\frac{\int\left(c1\left(\cosh^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\cosh^{n2}\left(\frac{(ay-(-\_b+}{a}\right.\right.}{a}}}{a}\right.$$

6.5.8.4 [1257] Problem 4

problem number 1257

Added April 3, 2019.

Problem Chapter 5.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + yw_y = ax\cosh(\lambda x + \mu y)w + b\cosh(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x+my*y]+b*Cosh[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) + \frac{ax\sinh(\lambda x + \text{my}y)}{\lambda x + \text{my}y} + b\text{Chi}(\nu x)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cosh(lambda*x+my*y)+b*cosh(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{a\sinh(\lambda x + myy)}{\lambda + \frac{myy}{x}} + b\,\text{hyperbolicCosineIntegral}(\nu x) + \_F1\left(\frac{y}{x}\right)$$

6.5.8.5 [1258] Problem 5

problem number 1258

Added April 3, 2019.

Problem Chapter 5.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a\cosh^n(\lambda x)w_x + b\cosh^m(\mu x)w_y = c\cosh^k(\nu x)w + p\cosh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k+p*C
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x\frac{\cosh^{-n}(\lambda K[2])\left(c\cosh^k(\nu K[2])+p\cosh^s\left(\beta\left(y-\int_1^x\frac{b\cosh^{-n}(\lambda K[1])\cosh^m(\mu K[1])}{a}dK[1]+\int\right.\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x\frac{\left(c\left(\cosh^k(\_b\nu)\right)+p\left(\cosh^s\left(\frac{(ay+b(\int(\cosh^{-n}(\_b\lambda))(\cosh^m(\_b\mu))d\_b)-b(\int(\cosh^{-n}(\lambda x))(\cosh^m(\mu x))dx))\beta}{a}\right)\right.\right.}{a}$$

6.5.8.6 [1259] Problem 6

problem number 1259

Added April 3, 2019.

Problem Chapter 5.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cosh^n(\lambda x)w_x+b\cosh^m(\mu x)w_y=c\cosh^k(\nu y)w+p\cosh^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*y]^k+p*C
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{\cosh^{-n}(\lambda K[2])\left(c\cosh^k\left(\nu\left(y-\int_1^x \frac{b\cosh^{-n}(\lambda K[1])\cosh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\cosh^{-n}(\lambda K[1])\,c}{a}\right.\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*y)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(c\left(\cosh^k\left(\frac{(ay+b(\int(\cosh^{-n}(\_b\lambda))(\cosh^m(\_b\mu))d\_b)-b(\int(\cosh^{-n}(\lambda x))(\cosh^m(\mu x))dx))\nu}{a}\right)\right)+p(\cosh^s(\_b\beta)}{a}$$

6.5.9 4.3

Local contents

6.5.9.1 [1260] Problem 1 1926
6.5.9.2 [1261] Problem 2 1927
6.5.9.3 [1262] Problem 3 1928
6.5.9.4 [1263] Problem 4 1928
6.5.9.5 [1264] Problem 5 1929

6.5.9.1 [1260] Problem 1

problem number 1260

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \tanh^k(\lambda x)\tanh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Tanh[lambda*x]^k*Tanh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tanh^k(\lambda K[1])\tanh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+tanh(lambda*x)^k*tanh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\tanh^k(\_a\lambda)\right)\left(\tanh^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.9.2 [1261] Problem 2

problem number 1261

Added April 3, 2019.

Problem Chapter 5.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh^k(\lambda x)w + s\tanh^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x]^k*w[x,y]+ s*Tanh[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\tanh^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\tanh^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(-\frac{c\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\tanh^2(\lambda K[1])\right)\text{ta}}{a\lambda+ak\lambda}\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tanh(lambda*x)^k*w(x,y)+s*tanh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s(\tanh^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\tanh^k(\lambda x)\right)dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{c\left(\tanh^k(\lambda x)\right)}{a}dx}$$

6.5.9.3 [1262] Problem 3

problem number 1262

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \tanh^{n_1}(\lambda_1 x) + c_2 \tanh^{n_2}(\lambda_2 y))\, w + s_1 \tanh^{k_1}(\beta_1 x) + s_2 \tanh^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Tanh[lambda1*x]^n1 + c2*Tanh[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*tanh(lambda1*x)^n1 + c2*tanh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1\left(\tanh^{k1}\left(\_b\beta 1\right)\right) + s2\left(\tanh^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right) \mathrm{e}^{-\frac{\int\left(c1\left(\tanh^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\tanh^{n2}\left(\frac{(ay-(-\_b-}{a}\right.\right.}{a}}}{a}\right.$$

6.5.9.4 [1263] Problem 4

problem number 1263

Added April 3, 2019.

Problem Chapter 5.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a \tanh^n(\lambda x) w_x + b \tanh^m(\mu x) w_y = c \tanh^k(\nu x) w + p \tanh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tanh^{-n}(\lambda K[2])\tanh(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\tanh^{-n}(\lambda K[2])\tanh(\nu K[2])}{a}d\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int^x \frac{\left(cw_{x,y}\tanh(\_b\nu)+p\left(\frac{\sinh\left(\frac{\left(ay-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\frac{\sinh(\mu x)}{\cosh(\mu x)}\right)^{m}dx\right)+b\left(\int\left(\tanh^{-n}(\_b\lambda)\right)(\tanh^{m}(\_b\mu))d\_b\right)\right)\beta}{a}\right)}{\cosh\left(\frac{\left(ay-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\frac{\sinh(\mu x)}{\cosh(\mu x)}\right)^{m}dx\right)+b\left(\int\left(\tanh^{-n}(\_b\lambda)\right)(\tanh^{m}(\_b\mu))d\_b\right)\right)\beta}{a}\right)}\right)\right)}{a}$$

6.5.9.5 [1264] Problem 5

problem number 1264

Added April 3, 2019.

Problem Chapter 5.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^n(\lambda x)w_x+b\tanh^m(\mu x)w_y=c\tanh^k(\nu y)w+p\tanh^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tanh^{-n}(\lambda K[2])\tanh\left(\nu\left(y-\int_1^x \frac{b\tanh^{-n}(\lambda K[1])\tanh^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\tanh^{-n}(\lambda K}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}\left(\frac{\sinh(\_f\beta)}{\cosh(\_f\beta)}\right)^s \mathrm{e}^{-\frac{c\left(\int \frac{\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\sinh(\_f\mu)}{\cosh(\_f\mu)}\right)^m\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sinh(}{\cosh(}\right.\right.}{a}\right.}{\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\sinh(\_f\mu)}{\cosh(\_f\mu)}\right)^m\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\right.}{a}\right.}\right.}{a}}}{a}\right.$$

6.5.10 4.4

Local contents

6.5.10.1 [1265] Problem 1 1931
6.5.10.2 [1266] Problem 2 1932
6.5.10.3 [1267] Problem 3 1933
6.5.10.4 [1268] Problem 4 1933
6.5.10.5 [1269] Problem 5 1934

6.5.10.1 [1265] Problem 1

problem number 1265

Added April 3, 2019.

Problem Chapter 5.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \coth^k(\lambda x)\coth^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Coth[lambda*x]^k*Coth[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\coth^k(\lambda K[1])\coth^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+coth(lambda*x)^k*coth(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\coth^k(\_a\lambda)\right)\left(\coth^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)e^{-\frac{-ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

6.5.10.2 [1266] Problem 2

problem number 1266

Added April 3, 2019.

Problem Chapter 5.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth^k(\lambda x)w + s\coth^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x]^k*w[x,y]+ s*Coth[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{c\coth^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\coth^2(\lambda x)\right)}{ak\lambda+a\lambda}\right)\left(\int_1^x \frac{\exp\left(-\frac{c\coth^{k+1}(\lambda K[1])\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\mathrm{co}\right.}{a\lambda+ak\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*coth(lambda*x)^k*w(x,y)+s*coth(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s(\coth^n(\beta x))\,\mathrm{e}^{-\frac{c\left(\int\left(\coth^k(\lambda x)\right)dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{c\left(\coth^k(\lambda x)\right)}{a}dx}$$

6.5.10.3 [1267] Problem 3

problem number 1267

Added April 3, 2019.

Problem Chapter 5.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \coth^{n_1}(\lambda_1 x) + c_2 \coth^{n_2}(\lambda_2 y))\, w + s_1 \coth^{k_1}(\beta_1 x) + s_2 \coth^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Coth[lambda1*x]^n1 + c2*Coth[lambda2*y]^n2)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*coth(lambda1*x)^n1 + c2*coth(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(s1\left(\coth^{k1}\left(\_b\beta 1\right)\right) + s2\left(\coth^{k2}\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right)\right)\right) e^{-\frac{\int\left(c1\left(\coth^{n1}\left(\_b\lambda 1\right)\right)+c2\left(\coth^{n2}\left(\frac{(ay-(-\_b+}{a}\right.\right.}}{a}}{a}\right.$$

6.5.10.4 [1268] Problem 4

problem number 1268

Added April 3, 2019.

Problem Chapter 5.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a \coth^{n}(\lambda x) w_x + b \coth^{m}(\mu x) w_y = c \coth^{k}(\nu x) w + p \coth^{s}(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*x]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\coth^{-n}(\lambda K[2])\coth(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\coth^{-n}(\lambda K[2])\coth(\nu K[2])}{a}dK\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*x)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}\left(\frac{\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)}{\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^m dx\right)\right)\beta}{a}\right)}\right)^s}{a}\right.$$

6.5.10.5 [1269] Problem 5

problem number 1269

Added April 3, 2019.

Problem Chapter 5.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\coth^n(\lambda x)w_x+b\coth^m(\mu x)w_y=c\coth^k(\nu y)w+p\coth^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*y]*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\coth^{-n}(\lambda K[2])\coth\left(\nu\left(y-\int_1^x \frac{b\coth^{-n}(\lambda K[1])\coth^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\coth^{-n}(\lambda K}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cosh(\_f\beta)}{\sinh(\_f\beta)}\right)^s\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n} e^{-\frac{c\left(\int \frac{\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(}{\sinh(}\right.\right.}{a}\right.}{\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^m\left(\frac{\cosh(\_f\lambda)}{\sinh(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n}\right.}{a}\right.}\right.}{a}}}{a}\right.$$

6.5.11 4.5

Local contents

6.5.11.1 [1270] Problem 1 1936
6.5.11.2 [1271] Problem 2 1937
6.5.11.3 [1272] Problem 3 1938
6.5.11.4 [1273] Problem 4 1939
6.5.11.5 [1274] Problem 5 1940
6.5.11.6 [1275] Problem 6 1941

6.5.11.1 [1270] Problem 1

problem number 1270

Added April 4, 2019.

Problem Chapter 5.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sinh^k(\lambda x) + c_2 \cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sinh[lambda*x]^k+c2*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} c_1\left(y - \frac{bx}{a}\right) + \frac{\mathrm{c1}\left(e^{2\lambda x} - 1\right)\sinh^k(\lambda x)\,{}_2F_1\left(1, \frac{1}{2}\left(k - \frac{1}{a\lambda} + 2\right); \frac{1}{2}\left(-k - \frac{1}{a\lambda} + 2\right); e^{2\lambda x}\right)}{ak\lambda + 1} - \right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sinh(lambda*x)^k+c2*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\sinh^{k}\left(\_a\lambda\right)\right) + c2\left(\cosh^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.11.2 [1271] Problem 2

problem number 1271

Added April 4, 2019.

Problem Chapter 5.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sinh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sinh[lambda*x]^k*Cosh[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cosh^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\sinh^k(\lambda K[1])}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\cosh^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\left(\sinh^{k}(\_a\lambda)\right)\mathrm{e}^{-\frac{ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.11.3 [1272] Problem 3

problem number 1272

Added April 4, 2019.

Problem Chapter 5.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tanh(\lambda x) + s\coth(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Tanh[lambda*x]+s*coth[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}}\left(\int_{1}^{x} \frac{e^{-\frac{cK[1]}{a}}\left(s\coth\left(\mu\left(y+\frac{b(K[1]-x)}{a}\right)\right)+k\tanh(\lambda K[1])\right)}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*tanh(lambda*x)+s*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} -\frac{\left((k-s)\cosh\left(\frac{-(-\_a+x)b\mu+(-\_a\lambda+\mu y)a}{a}\right)-(k+s)\cosh\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{\left(\sinh\left(\frac{-(-\_a+x)b\mu+(-\_a\lambda+\mu y)a}{a}\right)+\sinh\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\right)a} d\_\right.$$

6.5.11.4 [1273] Problem 4

problem number 1273

Added April 4, 2019.

Problem Chapter 5.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh(\lambda x)w_y = cw + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*w[x,y]+ k*Cosh[mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}k\cosh\left(\frac{\mu(a\lambda y-b\cosh(\lambda x)+b\cosh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\cosh(\lambda x)}{a\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sinh(lambda*x)*diff(w(x,y),y) = c*w(x,y)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\cosh\left(\frac{(a\lambda y+b\cosh(\_a\lambda)-b\cosh(\lambda x))\mu}{a\lambda}\right)\mathrm{e}^{-\frac{ac}{a}}}{a}d\_a + \_F1\left(\frac{a\lambda y - b\cosh(\lambda x)}{a\lambda}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.11.5 [1274] Problem 5

problem number 1274

Added April 4, 2019.

Problem Chapter 5.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^n(\lambda x)w_x + b\cosh^m(\mu x)w_y = c\cosh^k(\nu x)w + p\sinh^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cosh^k(\nu K[2])\sinh^{-n}(\lambda K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\cosh^k(\nu K[2])\sinh^{-n}(\lambda K[2])}{a}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\left(\sinh^{-n}(\_f\lambda)\right)\left(\sinh^s\left(\frac{(ay+b(\int(\cosh^m(\_f\mu))(\sinh^{-n}(\_f\lambda))d\_f)-b(\int(\cosh^m(\mu x))(\sinh^{-n}(\lambda x))dx))\beta}{a}\right)\right)e}{a}\right.$$

6.5.11.6 [1275] Problem 6

problem number 1275

Added April 4, 2019.

Problem Chapter 5.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^n(\lambda x)w_x + b\coth^m(\mu x)w_y = c\tanh^k(\nu y)w + p\coth^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]^k*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c\tanh^{-n}(\lambda K[2])\tanh^k\left(\nu\left(y - \int_1^x \frac{b\coth^m(\mu K[1])\tanh^{-n}(\lambda K[1])}{a}dK[1] + \int_1^{K[2]} \frac{b\coth^m(\mu K}{}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p\left(\frac{\cosh(\_f\beta)}{\sinh(\_f\beta)}\right)^{s} \left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} \mathrm{e}^{-\frac{c\left(\int\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n}\left(\sinh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^{m}\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\cosh}{\sinh}\right.\right.}{a}}{\cosh\left(\frac{\left(ay+b\left(\int\left(\frac{\cosh(\_f\mu)}{\sinh(\_f\mu)}\right)^{m}\left(\frac{\sinh(\_f\lambda)}{\cosh(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\cosh}{\sinh}\right.\right.}{a}}\right.}{a}}}{a} \right.$$

6.5.12 5.1

Local contents

6.5.12.1 [1276] Problem 1 1943
6.5.12.2 [1277] Problem 2 1944
6.5.12.3 [1278] Problem 3 1945
6.5.12.4 [1279] Problem 4 1946
6.5.12.5 [1280] Problem 5 1946
6.5.12.6 [1281] Problem 6 1947

6.5.12.1 [1276] Problem 1

problem number 1276

Added April 5, 2019.

Problem Chapter 5.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \ln^k(\lambda x)\ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Log[lambda*x]^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\log^k(\lambda K[1])\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\ln(\_a\lambda)^k\ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n e^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

6.5.12.2 [1277] Problem 2

problem number 1277

Added April 5, 2019.

Problem Chapter 5.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln^k(\lambda x)w + s\ln^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x]^k*w[x,y]+s*Log[beta*x]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^k(\lambda K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{c\log^k(\lambda K[1])}{a}dK[1]\right)s\log^n(\beta K[2])}{a}dK[2]+\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y)+s*ln(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{s\ln(\beta x)^n e^{-\frac{c\left(\int \ln(\lambda x)^k dx\right)}{a}}}{a}dx + \_F1\left(\frac{ay-bx}{a}\right)\right)e^{\int \frac{c\ln(\lambda x)^k}{a}dx}$$

6.5.12.3 [1278] Problem 3

problem number 1278

Added April 5, 2019.

Problem Chapter 5.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \ln^{n_1}(\lambda_1 x) + c_2 \ln^{n_2}(\lambda_2 y))\, w + s_1 \ln^{k_1}(\beta_1 x) + s_2 \ln^{k_2}(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*Log[lambda1*x]^n1 +c2*Log[lambda2*y]^n2)*w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{\mathrm{c1}\log^{\mathrm{n1}}(\mathrm{lambda1}x)(-\log(\mathrm{lambda1}x))^{-\mathrm{n1}}\,\mathrm{Gamma}(\mathrm{n1}+1,-\log(\mathrm{lambda1}x))}{a\mathrm{lambda1}} + \frac{\mathrm{c2}(-}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*ln(lambda1*x)^n1 +c2*ln(lambda2*y)^n2)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(k2\,s2\ln\left(\frac{(ay-(-\_b+x)b)\beta 2}{a}\right) + s1\ln(\_b\beta 1)^{k1}\right)\mathrm{e}^{-\frac{\int\left(c1\ln(\_b\lambda 1)^{n1}+c2\ln\left(\frac{(ay-(-\_b+x)b)\lambda 2}{a}\right)^{n2}\right)d\_b}{a}}}{a}d\_b\right.$$

6.5.12.4 [1279] Problem 4

problem number 1279

Added April 5, 2019.

Problem Chapter 5.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a \ln(\lambda x) w_x + b \ln(\mu y) w_y = cw + k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]*D[w[x, y], x] + b*Log[mu*y]*D[w[x, y], y] == c*w[x,y]+k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)*diff(w(x,y),x)+ b*ln(mu*y)*diff(w(x,y),y) =c*w(x,y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{c\_F1\left(\frac{-a\lambda \operatorname{expIntegral}(1,-\ln(\mu y))+b\mu \operatorname{expIntegral}(1,-\ln(\lambda x))}{b\lambda\mu}\right) \mathrm{e}^{-\frac{c \operatorname{expIntegral}(1,-\ln(\lambda x))}{a\lambda}} - k}{c}$$

6.5.12.5 [1280] Problem 5

problem number 1280

Added April 5, 2019.

Problem Chapter 5.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a \ln^n(\lambda x) w_x + b \ln^m(\mu x) w_y = c \ln^k(\nu x) w + p \ln^s(\beta y) + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^{-n}(\lambda K[2])\log^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\log^{-n}(\lambda K[2])\log^k(\nu K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*x)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{e^{-\frac{c\left(\int \ln(\lambda\_f)^{-n}\ln(\nu\_f)^k d\_f\right)}{a}}\left(p\ln\left(\frac{\beta\left(b\left(\int \ln(\mu\_f)^m \ln(\lambda\_f)^{-n} d\_f\right)+\left(-\frac{b\left(\int \ln(\mu x)^m \ln(\lambda x)^{-n} dx\right)}{a}+y\right)a\right)}{a}\right)^s+}{a}\right.$$

6.5.12.6 [1281] Problem 6

problem number 1281

Added April 5, 2019.

Problem Chapter 5.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x+b\ln^m(\mu x)w_y=c\ln^k(\nu y)w+p\ln^s(\beta x)+q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\log^{-n}(\lambda K[2])\log^k\left(\nu\left(y-\int_1^x \frac{b\log^{-n}(\lambda K[1])\log^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\log^{-n}(\lambda K[1])\log}{a}\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*y)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{e^{-\frac{c\left(\int \ln\left(\frac{\nu\left(b\left(\int \ln(\mu\_f)^m \ln(\lambda\_f)^{-n} d\_f\right)+\left(-\frac{b\left(\int \ln(\mu x)^m \ln(\lambda x)^{-n} dx\right)}{a}+y\right)a\right)}{a}\right)^k \ln(\lambda\_f)^{-n} d\_f\right)}{a}(p\ln(\beta\_f)^s+q)\ln}{a}\right.$$

6.5.13 5.2

Local contents

6.5.13.1 [1282] Problem 1 1949
6.5.13.2 [1283] Problem 2 1950
6.5.13.3 [1284] Problem 3 1951
6.5.13.4 [1285] Problem 4 1952
6.5.13.5 [1286] Problem 5 1953
6.5.13.6 [1287] Problem 6 1954

6.5.13.7 [1288] Problem 7 1955

6.5.13.1 [1282] Problem 1

problem number 1282

Added April 8, 2019.

Problem Chapter 5.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 x^k + c_2 \ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*x^k+c2*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}K[1]^k+\text{c2}\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*x^k+c2*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\_a^k+c2\ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.13.2 [1283] Problem 2

problem number 1283

Added April 8, 2019.

Problem Chapter 5.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + x^k \ln^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x^k*Log[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}} \left(\int_1^x \frac{e^{-\frac{cK[1]}{a}} K[1]^k \log^n\left(\beta\left(y + \frac{b(K[1]-x)}{a}\right)\right)}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\_a^k \ln\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n e^{-\frac{\_ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\frac{cx}{a}}$$

6.5.13.3 [1284] Problem 3

problem number 1284

Added April 8, 2019.

Problem Chapter 5.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-k}}{a-ak}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-k}}{a(k-1)}}sK[1]^{-k}\log^m(\beta K[1])}{a}dK[1]+c_1\left(y-\frac{bx^{-k+n+1}}{a(-k)+an+a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{sx^{-k}\ln(\beta x)^m\,\mathrm{e}^{\frac{cx^{-k+1}}{(k-1)a}}}{a}dx+\_F1\left(\frac{(k-n-1)\,ay+bx^{-k+n+1}}{(k-n-1)\,a}\right)\right)\mathrm{e}^{-\frac{cx^{-k+1}}{(k-1)a}}$$

6.5.13.4 [1285] Problem 4

problem number 1285

Added April 8, 2019.

Problem Chapter 5.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^k w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-n}}{a(n-1)}}sK[1]^{-n}\log^m(\beta K[1])}{a}dK[1]+c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-k}}{k-1}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*y^k*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{sx^{-n}\ln(\beta x)^m e^{\frac{cx^{-n+1}}{(n-1)a}}}{a}dx+\_F1\left(\frac{(n-1)ay^{-k+1}-(k-1)bx^{-n+1}}{(n-1)a}\right)\right)e^{-\frac{cx^{-n+1}}{(n-1)a}}$$

6.5.13.5 [1286] Problem 5

problem number 1286

Added April 8, 2019.

Problem Chapter 5.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + b\ln^n(\lambda x) w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^n*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\frac{sx^{m-n+1}\left(\frac{cx^{1-n}}{a-an}\right)^{\frac{m-n+1}{n-1}}\text{Gamma}\left(\frac{-m+n-1}{n-1},\frac{cx^{1-n}}{a-an}\right)}{a(n-1)}+c_1\left(\frac{(n-1)^{-n-1}\left(b\lambda^n\,\text{Gamma}(\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{\left(-(n-1)\left(m-2n+2\right)^2 asx^m\left(-\frac{c}{(n-1)a}\right)^{\frac{-m+n-1}{n-1}}\left(-\frac{c}{(n-1)a}\right)^{\frac{m-n+1}{n-1}}\left(-\frac{cx^{-n+1}}{(n-1)a}\right)^{\frac{m-2n+2}{2n-2}}\text{Whittaker}\right.}{}$$

6.5.13.6 [1287] Problem 6

problem number 1287

Added April 8, 2019.

Problem Chapter 5.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\,{}_2F_1\left(\frac{k}{k+1},\frac{1}{n+1};1+\frac{1}{n+1};\frac{b(k+1)x^{n+}}{b(k+1)x^{n+1}-a(n-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{s\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_b^{n+1}-(k+1)bx^{n+1}}{(n+1)a}\right)^{\frac{1}{k+1}}\right)^{-k}\ln\left(\_b\beta\right)^m \mathrm{e}^{-\frac{c\left(\int\left(\left(\frac{(n+1)ay^{k+1}+(k+1)b\_b^{n+1}-(k+1)bx^{n}}{(n+1)a}\right.\right.}{a}}}{a}\right.$$

6.5.13.7 [1288] Problem 7

problem number 1288

Added April 8, 2019.

Problem Chapter 5.5.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + b\ln^n(\lambda x) w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*y^k*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c\left(\left(y^{k+1} - (k+1)\int_1^x \frac{b\log^n(\lambda K[1])}{a} dK[1] + (k+1)\int_1^{K[2]} \frac{b\log^n(\lambda K[1])}{a} dK[1]\right)^{\frac{1}{k+1}}\right)^{-k}}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*y^k*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{s\_f^m\left(\left(\frac{ay^{k+1} + \int (k+1) b\ln(\_f\lambda)^n d\_f + \int (-k-1) b\ln(\lambda x)^n dx}{a}\right)^{\frac{1}{k+1}}\right)^{-k} e^{-\frac{\left((k+1)\_fb + \left(ayy^k - (k+1)b\left(\int \ln(\lambda x)^n dx\right)\right) \ln}{}}}{a}\right.$$

6.5.14 6.1

Local contents

6.5.14.1 [1289] Problem 1 1956
6.5.14.2 [1290] Problem 2 1957
6.5.14.3 [1291] Problem 3 1958
6.5.14.4 [1292] Problem 4 1959
6.5.14.5 [1293] Problem 5 1960
6.5.14.6 [1294] Problem 6 1961
6.5.14.7 [1295] Problem 7 1962

6.5.14.1 [1289] Problem 1

problem number 1289

Added April 8, 2019.

Problem Chapter 5.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{k((a\lambda+b\mu)\cos(\lambda x+\mu y)+c\sin(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2} + e^{\frac{cx}{a}} c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left(-(c\sin(\lambda x+\mu y)+(a\lambda+\mu b)\cos(\lambda x+\mu y))\,k\,\mathrm{e}^{-\frac{cx}{a}} + (a^2\lambda^2+2ab\lambda\mu+b^2\mu^2+c^2)\_F1\left(\frac{ay-bx}{a}\right.\right.}{a^2\lambda^2+2ab\lambda\mu+b^2\mu^2+c^2}$$

6.5.14.2 [1290] Problem 2

problem number 1290

Added April 8, 2019.

Problem Chapter 5.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \sin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sin[lambda*x]^k+c2*Sin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} c_1\left(y - \frac{bx}{a}\right) - i\left(\frac{\mathrm{c1}\left(-1+e^{2i\lambda x}\right)\sin^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k+\frac{i}{a\lambda}+2\right);\frac{1}{2}\left(-k+\frac{i}{a\lambda}+2\right);e^{2i\lambda x}\right)}{ak\lambda - i}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sin(lambda*x)^k+c2*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(c1\left(\sin^k(\_a\lambda)\right) + c2\left(\sin^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.14.3 [1291] Problem 3

problem number 1291

Added April 8, 2019.

Problem Chapter 5.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sin^k(\lambda x)\sin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sin[lambda*x]^k*Sin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^k(\lambda K[1])\sin^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sin(lambda*x)^k*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\sin^k(\_a\lambda)\right)\left(\sin^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.14.4 [1292] Problem 4

problem number 1292

Added April 8, 2019.

Problem Chapter 5.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+beta*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\sin\left(\beta yK[1]^{\frac{b}{a}}x^{-\frac{b}{a}} + \lambda K[1]\right)}{a}dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\_a^{\frac{-a-c}{a}}\sin\left(\beta y\_a^{\frac{b}{a}}x^{-\frac{b}{a}} + \_a\lambda\right)}{a}d\_a + \_F1\left(yx^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

6.5.14.5 [1293] Problem 5

problem number 1293

Added April 8, 2019.

Problem Chapter 5.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)w + b\sin(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x+beta*y]*w[x,y]+ b*Sin[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ax\cos(\beta y+\lambda x)}{\beta y+\lambda x}}\left(\int_1^x \frac{be^{\frac{ax\cos\left(\left(\lambda+\frac{\beta y}{x}\right)K[1]\right)}{\lambda x+\beta y}}\sin(\nu K[1])}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*sin(lambda*x+beta*y)*w(x,y)+ b*sin(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{b\,\mathrm{e}^{-a\left(\int \sin(b\beta\ln(\_b)+\_b\lambda+(-b\ln(x)+y)\beta)d\_b\right)}\sin(\_b\nu)}{\_b}d\_b+\_F1(-b\ln(x)+y)\right)\mathrm{e}^{\int^x a\sin(b\beta\ln(\_}$$

6.5.14.6 [1294] Problem 6

problem number 1294

Added April 8, 2019.

Problem Chapter 5.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sin^{-n}(\lambda K[2])\sin^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\sin^{-n}(\lambda K[2])\sin^k(\nu K[2])}{a}dK[2]\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*x)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p(\sin^{-n}(\_f\lambda))\left(\sin^s\left(\frac{(ay+b(\int(\sin^m(\_f\mu))(\sin^{-n}(\_f\lambda))d\_f)-b(\int(\sin^{-n}(\lambda x))(\sin^m(\mu x))dx))\beta}{a}\right)\right)\mathrm{e}^{-\frac{c(\int(\sin}{}}}{a}\right.$$

6.5.14.7 [1295] Problem 7

problem number 1295

Added April 8, 2019.

Problem Chapter 5.6.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu y)w + p\sin^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sin^{-n}(\lambda K[2])\sin^k\left(\nu\left(y-\int_1^x \frac{b\sin^{-n}(\lambda K[1])\sin^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\sin^{-n}(\lambda K[1])\sin^n}{a}\right.\right.\right.}{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*y)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p(\sin^s(\_f\beta))(\sin^{-n}(\_f\lambda))\,\mathrm{e}^{-\frac{c\left(\int(\sin^{-n}(\_f\lambda))\left(\sin^k\left(\frac{\frac{\left(ay+b\left(\int(\sin^m(\_f\mu))\left(\sin^{-n}(\_f\lambda)\right)d\_f\right)-b\left(\int\left(\sin^{-n}(\lambda x)\right)(\sin\right.}{a}}{a}\right.\right.\right.}{a}}}{a}\right.$$

6.5.15 6.2

Local contents

6.5.15.1 [1296] Problem 1 1963
6.5.15.2 [1297] Problem 2 1964
6.5.15.3 [1298] Problem 3 1965
6.5.15.4 [1299] Problem 4 1966
6.5.15.5 [1300] Problem 5 1967
6.5.15.6 [1301] Problem 6 1968

6.5.15.1 [1296] Problem 1

problem number 1296

Added April 11, 2019.

Problem Chapter 5.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{k((a\lambda+b\mu)\sin(\lambda x+\mu y) - c\cos(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2} + e^{\frac{cx}{a}} c_1\left(y - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\left(\left(-c\cos\left(\lambda x+\mu y\right)+\left(a\lambda+\mu b\right)\sin\left(\lambda x+\mu y\right)\right)k\,\mathrm{e}^{-\frac{cx}{a}}+\left(a^2\lambda^2+2ab\lambda\mu+b^2\mu^2+c^2\right)\_F1\left(\frac{ay-bx}{a}\right.\right.}{a^2\lambda^2+2ab\lambda\mu+b^2\mu^2+c^2}$$

6.5.15.2 [1297] Problem 2

problem number 1297

Added April 11, 2019.

Problem Chapter 5.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \cos^k(\lambda x) + c_2 \cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cos[lambda*x]^k + c2*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{e^{\frac{x}{a}}(ak\lambda - i)(b\beta n - i)c_1\left(y - \frac{bx}{a}\right) + \mathrm{c1}\left(1 + e^{2i\lambda x}\right)(1 + ib\beta n)\cos^k(\lambda x)\,{}_2F_1\left(1, \frac{1}{2}\left(k + \frac{i}{a\lambda} + 2\right)\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cos(lambda*x)^k + c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(c1\left(\cos^k(\_a\lambda)\right) + c2\left(\cos^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay - bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.15.3 [1298] Problem 3

problem number 1298

Added April 11, 2019.

Problem Chapter 5.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cos^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cos[lambda*x]^k * Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^k(\lambda K[1])\cos^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cos(lambda*x)^k *cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(\cos^k(\_a\lambda)\right)\left(\cos^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.15.4 [1299] Problem 4

problem number 1299

Added April 11, 2019.

Problem Chapter 5.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^{\frac{c}{a}}\left(\int_1^x \frac{k\cos\left(\mu y K[1]^{\frac{b}{a}} x^{-\frac{b}{a}} + \lambda K[1]\right) K[1]^{-\frac{a+c}{a}}}{a} dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{k\_a^{\frac{-a-c}{a}} \cos\left(\mu y\_a^{\frac{b}{a}} x^{-\frac{b}{a}} + \_a\lambda\right)}{a} d\_a + \_F1\left(yx^{-\frac{b}{a}}\right)\right) x^{\frac{c}{a}}$$

6.5.15.5 [1300] Problem 5

problem number 1300

Added April 11, 2019.

Problem Chapter 5.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)w + b\cos(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x+mu*y]*w[x,y]+b*Cos[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\left(\int_1^x \frac{b\exp\left(-\frac{ax\sin\left(\left(\lambda+\frac{\mu y}{x}\right)K[1]\right)}{\lambda x+\mu y}\right)\cos(\nu K[1])}{K[1]}dK[1] + c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =a*x*cos(lambda*x+mu*y)*w(x,y)+b*cos(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{b\cos(\_a\nu)\,\mathrm{e}^{-\frac{a\sin\left(\_a\lambda+\frac{a\mu y}{x}\right)}{\lambda+\frac{\mu y}{x}}}}{\_a}d\_a + \_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{a\sin(\lambda x+\mu y)}{\lambda+\frac{\mu y}{x}}}$$

6.5.15.6 [1301] Problem 6

problem number 1301

Added April 11, 2019.

Problem Chapter 5.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cos^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\cos^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cos[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cos^{-n}(\lambda K[2])\cos^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\cos^{-n}(\lambda K[2])\cos^k(\nu K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cos(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =c*cos(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p(\cos^{-n}(\_f\lambda))\left(\cos^s\left(\frac{(ay+b(\int(\cos^m(\_f\mu))(\cos^{-n}(\_f\lambda))d\_f)-b(\int(\cos^{-n}(\lambda x))(\cos^m(\mu x))dx))\beta}{a}\right)\right)\mathrm{e}^{-\frac{c(\int(c}{}}}{a}\right.$$

6.5.16 6.3

Local contents

6.5.16.1 [1302] Problem 1 1969
6.5.16.2 [1303] Problem 2 1970
6.5.16.3 [1304] Problem 3 1971
6.5.16.4 [1305] Problem 4 1972
6.5.16.5 [1306] Problem 5 1973
6.5.16.6 [1307] Problem 6 1974
6.5.16.7 [1308] Problem 7 1975

6.5.16.1 [1302] Problem 1

problem number 1302

Added April 11, 2019.

Problem Chapter 5.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)-\frac{ik\left(-2\,{}_2F_1\left(1,-\frac{ic}{2a\lambda+2b\mu};\frac{-ic+2a\lambda+2b\mu}{2(a\lambda+b\mu)};-e^{-2i(\lambda x+\mu y)}\right)+2e^{\frac{2i\mu(ay-bx)}{a}}\,{}_2F_1\left(1,\right.\right.}{c\left(1+e^{\frac{2i\mu(ay-bx)}{a}}\right)}\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\,\mathrm{e}^{-\frac{\_ac}{a}} \tan\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right) \mathrm{e}^{\frac{cx}{a}}$$

6.5.16.2 [1303] Problem 2

problem number 1303

Added April 11, 2019.

Problem Chapter 5.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \tan^k(\lambda x) + c_2 \tan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Tan[lambda*x]^k + c2*Tan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\mathrm{c1}\tan^k(\lambda K[1]) + \mathrm{c2}\tan^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*tan(lambda*x)^k + c2*tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\tan^{k}\left(\_a\lambda\right)\right) + c2\left(\tan^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{-\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.16.3 [1304] Problem 3

problem number 1304

Added April 11, 2019.

Problem Chapter 5.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \tan^k(\lambda x)\tan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Tan[lambda*x]^k * Tan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tan^k(\lambda K[1])\tan^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ tan(lambda*x)^k *tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\tan^{k}\left(\_a\lambda\right)\right)\left(\tan^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.16.4 [1305] Problem 4

problem number 1305

Added April 11, 2019.

Problem Chapter 5.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = c\tan(\lambda x)w + k\tan(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Tan[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cos^{-\frac{c}{a\lambda}}(\lambda x)\left(\int_{1}^{x} \frac{k\cos^{\frac{c}{a\lambda}}(\lambda K[1])\tan(\nu K[1])}{a} dK[1] + c_1\left(\frac{\log(\sin(\mu y))}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) = c*tan(lambda*x)*w(x,y)+ k*tan(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{k\left(\frac{2}{\cos(2\lambda x)+1}\right)^{-\frac{c}{2a\lambda}} \sin(\nu x)}{a\cos(\nu x)} dx + \_F1\left(\frac{-b\mu x + a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)\right)\left(\tan^2(\lambda x)+1\right)^{\frac{c}{2a\lambda}}$$

6.5.16.5 [1306] Problem 5

problem number 1306

Added April 11, 2019.

Problem Chapter 5.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\tan(\lambda x + \nu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+nu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}} \tan\left(\nu y K[1]^{\frac{b}{a}} x^{-\frac{b}{a}} + \lambda K[1]\right)}{a} dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\_a^{\frac{-a-c}{a}} \tan\left(\nu y\_a^{\frac{b}{a}} x^{-\frac{b}{a}} + \_a\lambda\right)}{a} d\_a + \_F1\left(y x^{-\frac{b}{a}}\right)\right) x^{\frac{c}{a}}$$

6.5.16.6 [1307] Problem 6

problem number 1307

Added April 11, 2019.

Problem Chapter 5.6.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\tan^m(\mu x)w_y = c\tan^k(\nu x)w + p\tan^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c\tan^{-n}(\lambda K[2])\tan^k(\nu K[2])}{a} dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\tan^{-n}(\lambda K[2])\tan^k(\nu K[2])}{a} dK[\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n} \left(\frac{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^{m}\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\left(\frac{\sin(\mu x)}{\cos(\mu x)}\right)^{m} dx\right)\right)\beta}{a}\right)}{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^{m}\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\left(\frac{\sin(\mu x)}{\cos(\mu x)}\right)^{m} dx\right)\right)\beta}{a}\right)} \right)^{s} \mathrm{e}^{-\frac{c\left(\int\left(\right.\right.}{}}}{a}$$

6.5.16.7 [1308] Problem 7

problem number 1308

Added April 11, 2019.

Problem Chapter 5.6.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^{n}(\lambda x)w_x + b\tan^{m}(\mu x)w_y = c\tan^{k}(\nu y)w + p\tan^{s}(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-n}(\lambda K[2])\tan^k\left(\nu\left(y-\int_1^x \frac{b\tan^{-n}(\lambda K[1])\tan^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\tan^{-n}(\lambda K[1])\,\mathrm{t}}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\sin(\_f\beta)}{\cos(\_f\beta)}\right)^s\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}\mathrm{e}^{-\frac{c\left(\int\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}\left(\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\right.\right.}{a}\right.\right.}{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\sin(\_f\mu)}{\cos(\_f\mu)}\right)^m\left(\frac{\sin(\_f\lambda)}{\cos(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\right.\right.}{a}\right.\right.}\right.}{a}}}{a}\right.$$

6.5.17 6.4

Local contents

6.5.17.1 [1309] Problem 1 1977
6.5.17.2 [1310] Problem 2 1978
6.5.17.3 [1311] Problem 3 1979
6.5.17.4 [1312] Problem 4 1980
6.5.17.5 [1313] Problem 5 1981
6.5.17.6 [1314] Problem 6 1982
6.5.17.7 [1315] Problem 7 1983

6.5.17.1 [1309] Problem 1

problem number 1309

Added April 11, 2019.

Problem Chapter 5.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cw + k\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+mu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{c(c-2i(a\lambda+b\mu))e^{\frac{x(c-2ib\mu)}{a}}\left(e^{\frac{2ib\mu x}{a}}-e^{2i\mu y}\right)c_1\left(y-\frac{bx}{a}\right)+k\left(-2(2a\lambda+2b\mu+ic)e^{\frac{2i\mu(ay-bx)}{a}}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\cot\left(\frac{-(-\_a+x)b\mu+(\_a\lambda+\mu y)a}{a}\right)\mathrm{e}^{-\frac{ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.17.2 [1310] Problem 2

problem number 1310

Added April 11, 2019.

Problem Chapter 5.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1\cot^k(\lambda x) + c_2\cot^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cot[lambda*x]^k + c2*Cot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\mathrm{c1}\cot^k(\lambda K[1])+\mathrm{c2}\cot^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cot(lambda*x)^k + c2*cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\cot^{k}\left(\_a\lambda\right)\right) + c2\left(\cot^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.17.3 [1311] Problem 3

problem number 1311

Added April 11, 2019.

Problem Chapter 5.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cot^k(\lambda x)\cot^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cot[lambda*x]^k * Cot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cot^k(\lambda K[1])\cot^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cot(lambda*x)^k *cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(\cot^k(\_a\lambda)\right)\left(\cot^n\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.17.4 [1312] Problem 4

problem number 1312

Added April 11, 2019.

Problem Chapter 5.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot(\mu y)w_y = c\cot(\lambda x)w + k\cot(\nu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Cot[mu*y]*D[w[x, y], y] == c*Cot[lambda*x]*w[x,y]+k*Cot[nu*x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{w(x,y) \to \sin^{\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{k\cot(\nu K[1])\sin^{-\frac{c}{a\lambda}}(\lambda K[1])}{a} dK[1] + c_1\left(\frac{\log(\sec(\mu y))}{\mu} - \frac{bx}{a}\right)\right)\right\}$$
$$\left\{w(x,y) \to \sin^{\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{k\cot(\nu K[2])\sin^{-\frac{c}{a\lambda}}(\lambda K[2])}{a} dK[2] + c_1\left(\frac{\log(\sec(\mu y))}{\mu} - \frac{bx}{a}\right)\right)\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*cot(mu*y)*diff(w(x,y),y) = c*cot(lambda*x)*w(x,y)+ k*cot(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(-\left(\int^{y} \frac{\left(\sin\left(\frac{2a\nu\ln\left(\frac{\cos(\_a\mu)}{\sin(\_a\mu)}\right)-a\nu\ln\left(-\frac{2}{\cos(2\_a\mu)-1}\right)+a\nu\ln(\cot^2(\mu y)+1)-2a\nu\ln(\cot(\mu y))-2(-\_a\mu+\nu x)b\mu}{2b\mu}\right) - \sin\right.}{\left(\sin\left(\frac{2a\nu\ln\left(\frac{\cos(\_a\mu)}{\sin(\_a\mu)}\right)-a\nu\ln\left(-\frac{2}{\cos(2\_a\mu)-1}\right)+a\nu\ln(\cot^2(}{2b\mu}\right.\right.}\right.\right.$$

6.5.17.5 [1313] Problem 5

problem number 1313

Added April 11, 2019.

Problem Chapter 5.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\cot(\lambda x + \nu y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+nu*y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^{\frac{c}{a}}\left(\int_1^x \frac{k\cot\left(\nu y K[1]^{\frac{b}{a}} x^{-\frac{b}{a}} + \lambda K[1]\right) K[1]^{-\frac{a+c}{a}}}{a} dK[1] + c_1\left(y x^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{k\_a^{\frac{-a-c}{a}} \cot\left(\nu y\_a^{\frac{b}{a}} x^{-\frac{b}{a}} + \_a\lambda\right)}{a} d\_a + \_F1\left(y x^{-\frac{b}{a}}\right)\right) x^{\frac{c}{a}}$$

6.5.17.6 [1314] Problem 6

problem number 1314

Added April 11, 2019.

Problem Chapter 5.6.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\cot^k(\nu x)w + p\cot^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c\cot^{-n}(\lambda K[2])\cot^k(\nu K[2])}{a} dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\cot^{-n}(\lambda K[2])\cot^k(\nu K[2])}{a} dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n} \left(\frac{\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^{m}\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m} dx\right)\right)\beta}{a}\right)}{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^{m}\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n} d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m} dx\right)\right)\beta}{a}\right)} \right)^{s} \mathrm{e}^{-\frac{c\left(\int\left(\right.\right.}{}} \right.$$

6.5.17.7 [1315] Problem 7

problem number 1315

Added April 11, 2019.

Problem Chapter 5.6.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\cot^k(\nu y)w + p\cot^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*y]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cot^{-n}(\lambda K[2])\cot^k\left(\nu\left(y-\int_1^x \frac{b\cot^{-n}(\lambda K[1])\cot^m(\mu K[1])}{a}dK[1]+\int_1^{K[2]} \frac{b\cot^{-n}(\lambda K[1])\cot}{a}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(\frac{\cos(\_f\beta)}{\sin(\_f\beta)}\right)^s\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}\mathrm{e}^{-\frac{c\left(\int\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}\left(\cos\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^m\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\right.\right.}{a}\right.\right.}{\sin\left(\frac{\left(ay+b\left(\int\left(\frac{\cos(\_f\mu)}{\sin(\_f\mu)}\right)^m\left(\frac{\cos(\_f\lambda)}{\sin(\_f\lambda)}\right)^{-n}d\_f\right)-b\left(\int\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\right.\right.}{a}\right.\right.}\right.}{a}}}{a}\right.$$

6.5.18 6.5

Local contents

6.5.18.1 [1316] Problem 1 1985
6.5.18.2 [1317] Problem 2 1986
6.5.18.3 [1318] Problem 3 1987
6.5.18.4 [1319] Problem 4 1988
6.5.18.5 [1320] Problem 5 1989
6.5.18.6 [1321] Problem 6 1990
6.5.18.7 [1322] Problem 7 1991

6.5.18.1 [1316] Problem 1

problem number 1316

Added April 11, 2019.

Problem Chapter 5.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*Sin[lambda*x]^k+c2*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} c_1 \left(y - \frac{bx}{a}\right) - \frac{ic1\left(-1+e^{2i\lambda x}\right)\sin^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k+\frac{i}{a\lambda}+2\right);\frac{1}{2}\left(-k+\frac{i}{a\lambda}+2\right);e^{2i\lambda x}\right)}{ak\lambda - i} +\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*sin(lambda*x)^k+c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(c1\left(\sin^{k}\left(\_a\lambda\right)\right) + c2\left(\cos^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\right)\mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.18.2 [1317] Problem 2

problem number 1317

Added April 11, 2019.

Problem Chapter 5.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sin^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sin[lambda*x]^k*Cos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{cx}{a}}\left(\int_{1}^{x} \frac{e^{-\frac{cK[1]}{a}}\cos^{n}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\sin^{k}(\lambda K[1])}{a} dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+sin(lambda*x)^k*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\cos^{n}\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)\left(\sin^{k}\left(\_a\lambda\right)\right)\mathrm{e}^{-\frac{\_ac}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.18.3 [1318] Problem 3

problem number 1318

Added April 11, 2019.

Problem Chapter 5.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = c\sin(\lambda x)w + k\cos(\nu x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Cos[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_{1}^{x} \frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(s+k\cos(\nu K[1]))}{a} dK[1] + c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*cos(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\cos(\nu x)+s)\,\mathrm{e}^{\frac{c\cos(\lambda x)}{a\lambda}}}{a}dx + \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y - \arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}, -\frac{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}\right.\right.\right.}{b\mu}\right.\right.$$

6.5.18.4 [1319] Problem 4

problem number 1319

Added April 11, 2019.

Problem Chapter 5.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = c\sin(\lambda x)w + k\tan(\nu x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Tan[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_1^x \frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(s+k\tan(\nu K[1]))}{a}dK[1] + c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*tan(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\sin(\nu x) + s\cos(\nu x))\,\mathrm{e}^{\frac{c\cos(\lambda x)}{a\lambda}}}{a\cos(\nu x)}dx + \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\mu y - \arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\mu x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\mu x}{a}}+1}, -\frac{\_Z}{\_Z}\right.\right.\right.}{b\mu}\right.\right.$$

6.5.18.5 [1320] Problem 5

problem number 1320

Added April 11, 2019.

Problem Chapter 5.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = c\tan(\lambda x)w + k\cot(\nu x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Cot[nu*x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \cos^{-\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{\cos^{\frac{c}{a\lambda}}(\lambda K[1])(s + k\cot(\nu K[1]))}{a}dK[1] + c_1\left(\frac{\log(\sin(\mu y))}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) =  c*tan(lambda*x)*w(x,y)+k*cot(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{(k\cos(\nu x) + s\sin(\nu x))\left(\cos^{\frac{c}{a\lambda}}(\lambda x)\right)}{a\sin(\nu x)}dx + \_F1\left(\frac{-b\mu x + a\ln\left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}}\right)}{b\mu}\right)\right)\left(\cos^{-\frac{c}{a\lambda}}(\right.$$

6.5.18.6 [1321] Problem 6

problem number 1321

Added April 11, 2019.

Problem Chapter 5.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sin[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c\cos^k(\nu K[2])\sin^{-n}(\lambda K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{c\cos^k(\nu K[2])\sin^{-n}(\lambda K[2])}{a}dK[2\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =  c*cos(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p(\sin^{-n}(\_f\lambda))\left(\sin^s\left(\frac{(ay+b(\int(\cos^m(\_f\mu))(\sin^{-n}(\_f\lambda))d\_f)-b(\int(\cos^m(\mu x))(\sin^{-n}(\lambda x))dx))\beta}{a}\right)\right)\mathrm{e}^{-\frac{c(\int(\mathrm{co}}{}}}{a}\right.$$

6.5.18.7 [1322] Problem 7

problem number 1322

Added April 11, 2019.

Problem Chapter 5.6.5.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\tan^k(\nu x)w + p\cot^s(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Tan[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-n}(\lambda K[2])\tan^k(\nu K[2])}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\tan^{-n}(\lambda K[2])\tan^k(\nu K[2])}{a}dK[\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =  c*tan(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{p\left(\frac{\cos(\beta x)}{\sin(\beta x)}\right)^{s}\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n} \mathrm{e}^{-\frac{c\left(\int\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\left(\frac{\sin(\nu x)}{\cos(\nu x)}\right)^{k} dx\right)}{a}}}{a} dx + \_F1\left(\frac{ay - b\left(\int\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^{m}\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right.\right.}{a}\right.\right.$$

6.5.19 7.1

Local contents

6.5.19.1 [1323] Problem 1 1992
6.5.19.2 [1324] Problem 2 1993
6.5.19.3 [1325] Problem 3 1994
6.5.19.4 [1326] Problem 4 1995
6.5.19.5 [1327] Problem 5 1996

6.5.19.1 [1323] Problem 1

problem number 1323

Added April 13, 2019.

Problem Chapter 5.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arcsin^k(\lambda x) + c_2 \arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcSin[lambda*x]^k+c2*ArcSin[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\sin^{-1}(\lambda K[1])^k+\text{c2}\sin^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arcsin(lambda*x)^k+c2*arcsin(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\arcsin\left(\_a\lambda\right)^k+c2\arcsin\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.19.2 [1324] Problem 2

problem number 1324

Added April 13, 2019.

Problem Chapter 5.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\arcsin^k(\lambda x)\arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcSin[lambda*x]^k*ArcSin[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^{-1}(\lambda K[1])^k\sin^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arcsin(lambda*x)^k*arcsin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\arcsin\left(\_a\lambda\right)^k\arcsin\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n \mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.19.3 [1325] Problem 3

problem number 1325

Added April 13, 2019.

Problem Chapter 5.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c_1\arcsin(\lambda_1 x)+c_2\arcsin(\lambda_2 y))\,w+s_1\arcsin^n(\beta_1 x)+s_2\arcsin^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcSin[lambda1*x] + c2*ArcSin[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{\text{c1}\sqrt{1-\text{lambda1}^2x^2}}{a\text{lambda1}}+\frac{\text{c1}x\sin^{-1}(\text{lambda1}x)}{a}+\frac{\text{c2}\sqrt{1-\text{lambda2}^2y^2}}{b\text{lambda2}}+\frac{\text{c2}y\sin^{-1}(\text{la}}{b}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arcsin(lambda1*x) + c2*arcsin(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(s1\arcsin(\_a\beta 1)^n+s2\arcsin\left(\frac{(ay-(-\_a+x)b)\beta 2}{a}\right)^k\right)\mathrm{e}^{\frac{-\sqrt{-\frac{(-(-\_a+x)b\lambda 2+(\lambda 2y-1)a)(-(-\_a+x)b\lambda 2+(\lambda 2}{a^2}}}{}}}{a}\right.$$

6.5.19.4 [1326] Problem 4

problem number 1326

Added April 13, 2019.

Problem Chapter 5.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^m(\mu x)w_y=c\arcsin^k(\nu x)w+p\arcsin^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*x]^k*w[x,y]+p*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{ic\sin^{-1}(\nu x)^k\left(\sin^{-1}(\nu x)^2\right)^{-k}\left(\left(-i\sin^{-1}(\nu x)\right)^k\text{Gamma}\left(k+1,i\sin^{-1}(\nu x)\right)-\left(i\sin^{-1}\right.\right.}{2a\nu}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*x)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(-\arcsin\left(\frac{\left(-\left(\arcsin(\_f\mu)^m-\frac{\text{LommelS1}\left(m+\frac{3}{2},\frac{1}{2},\arcsin(\_f\mu)\right)}{\sqrt{\arcsin(\_f\mu)}}\right)\sqrt{-\_f^2\mu^2+1}\,b2^m2^{-m}\arcsin(\_f\mu)+\left(-\_fbm2^m\right.}{}\right.\right.}{}\right.\right.$$

6.5.19.5 [1327] Problem 5

problem number 1327

Added April 13, 2019.

Problem Chapter 5.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arcsin^m(\mu x)w_y=c\arcsin^k(\nu y)w+p\arcsin^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*y]^k*w[x,y]+p*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\sin^{-1}\left(\nu\left(y-\int_1^x \frac{b\sin^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]} \frac{b\sin^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\left(\int\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*y)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\arcsin(\_f\beta)^n\,\mathrm{e}^{-\frac{c\left(\int\left(-\arcsin\left(\frac{\left(-\left(\arcsin(\_f\mu)^m-\frac{\mathrm{LommelS1}\left(m+\frac{3}{2},\frac{1}{2},\arcsin(\_f\mu)\right)}{\sqrt{\arcsin(\_f\mu)}}\right)\sqrt{-\_f^2\mu^2+1}\,b2^m2^{-m}\arcsin(\_f\mu)\right.}{}\right.\right.\right.}}{}\right.$$

6.5.20 7.2

Local contents

6.5.20.1 [1328] Problem 1 1998
6.5.20.2 [1329] Problem 2 1999
6.5.20.3 [1330] Problem 3 2000
6.5.20.4 [1331] Problem 4 2001
6.5.20.5 [1332] Problem 5 2002

6.5.20.1 [1328] Problem 1

problem number 1328

Added April 13, 2019.

Problem Chapter 5.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arccos^k(\lambda x) + c_2 \arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCos[lambda*x]^k+c2*ArcCos[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} \left(\int_1^x \frac{e^{-\frac{K[1]}{a}} \left(\text{c1} \cos^{-1}(\lambda K[1])^k + \text{c2} \cos^{-1}\left(\beta\left(y + \frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccos(lambda*x)^k+c2*arccos(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(c1 \arccos(\_a\lambda)^k + c2 \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right) \mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right) \mathrm{e}^{\frac{x}{a}}$$

6.5.20.2 [1329] Problem 2

problem number 1329

Added April 13, 2019.

Problem Chapter 5.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \arccos^k(\lambda x)\arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCos[lambda*x]^k*ArcCos[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^{-1}(\lambda K[1])^k\cos^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccos(lambda*x)^k*arccos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\arccos(\_a\lambda)^k\arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n e^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

6.5.20.3 [1330] Problem 3

problem number 1330

Added April 13, 2019.

Problem Chapter 5.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \arccos(\lambda_1 x) + c_2 \arccos(\lambda_2 y))\, w + s_1 \arccos^n(\beta_1 x) + s_2 \arccos^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCos[lambda1*x] + c2*ArcCos[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(-\frac{\mathrm{c1}\sqrt{1-\mathrm{lambda1}^2 x^2}}{a\mathrm{lambda1}} + \frac{\mathrm{c1}x\cos^{-1}(\mathrm{lambda1}x)}{a} + \frac{\mathrm{c2}x\sin^{-1}(\mathrm{lambda2}y)}{a} + \frac{\mathrm{c2}x\cos^{-1}}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccos(lambda1*x) + c2*arccos(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1 \arccos(\_a\beta 1)^n + s2 \arccos\left(\frac{(ay-(-\_a+x)b)\beta 2}{a}\right)^k\right) \mathrm{e}^{\frac{\sqrt{-\frac{(-(-\_a+x)b\lambda 2+(\lambda 2y-1)a)(-(-\_a+x)b\lambda 2+(\lambda 2}{a^2}}}{}}}{a}\right.$$

6.5.20.4 [1331] Problem 4

problem number 1331

Added April 13, 2019.

Problem Chapter 5.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^m(\mu x)w_y = c\arccos^k(\nu x)w + p\arccos^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*x]^k*w[x,y]+p*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\frac{c\cos^{-1}(\nu x)^k\left(\cos^{-1}(\nu x)^2\right)^{-k}\left(\left(-i\cos^{-1}(\nu x)\right)^k \text{Gamma}\left(k+1, i\cos^{-1}(\nu x)\right) + \left(i\cos^{-1}\right.\right.}{2a\nu}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*x)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{p\arccos\left(\frac{\left(\frac{\left(-\text{LommelS1}\left(m+\frac{3}{2},\frac{3}{2},\arccos(\mu x)\right)\arccos(\mu x)+\arccos(\mu x)^{m+\frac{3}{2}}+(m+2)\,\text{LommelS1}\left(m+\frac{1}{2},\frac{1}{2},\arccos(\mu x)\right)\right)\sqrt{-\mu^2x^2+}}{\sqrt{\arccos(\mu x)}}}{}\right.}{}\right.\right.$$

6.5.20.5 [1332] Problem 5

problem number 1332

Added April 13, 2019.

Problem Chapter 5.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^m(\mu x)w_y = c\arccos^k(\nu y)w + p\arccos^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*y]^k*w[x,y]+p*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cos^{-1}\left(\nu\left(y-\int_1^x \frac{b\cos^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]} \frac{b\cos^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*y)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{p \arccos(\_b\beta)^{n} \mathrm{e}^{-\frac{c \left(\int \arccos \left(\frac{\left(-\mathrm{LommelS1}\left(m+\frac{3}{2},\frac{3}{2},\arccos(\mu x)\right) \arccos(\mu x)+\arccos(\mu x)^{m+\frac{3}{2}}+(m+2)\,\mathrm{LommelS1}\left(m+\frac{1}{2},\frac{1}{2}\right.\right.}{\sqrt{\arccos(\mu x)}} \right.\right.}{}}{} \right.$$

6.5.21 7.3

Local contents

6.5.21.1 [1333] Problem 1 2003
6.5.21.2 [1334] Problem 2 2004
6.5.21.3 [1335] Problem 3 2005
6.5.21.4 [1336] Problem 4 2006
6.5.21.5 [1337] Problem 5 2007

6.5.21.1 [1333] Problem 1

problem number 1333

Added April 13, 2019.

Problem Chapter 5.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arctan^k(\lambda x) + c_2 \arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcTan[lambda*x]^k+c2*ArcTan[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\text{c1}\tan^{-1}(\lambda K[1])^k+\text{c2}\tan^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arctan(lambda*x)^k+c2*arctan(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(c1\arctan\left(\_a\lambda\right)^k+c2\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

6.5.21.2 [1334] Problem 2

problem number 1334

Added April 13, 2019.

Problem Chapter 5.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+\arctan^k(\lambda x)\arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcTan[lambda*x]^k*ArcTan[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tan^{-1}(\lambda K[1])^k\tan^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arctan(lambda*x)^k*arctan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\arctan(\lambda\_a)^k\arctan\left(\frac{\beta(\_ab+ay-bx)}{a}\right)^n \mathrm{e}^{-\frac{c\_a}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.21.3 [1335] Problem 3

problem number 1335

Added April 13, 2019.

Problem Chapter 5.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(c_1\arctan(\lambda_1 x)+c_2\arctan(\lambda_2 y))\,w+s_1\arctan^n(\beta_1 x)+s_2\arctan^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcTan[lambda1*x] + c2*ArcTan[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \left(\text{lambda1}^2x^2+1\right)^{-\frac{\text{c1}}{2a\text{lambda1}}}\exp\left(\frac{\text{c2}\left(2\text{lambda2}y\tan^{-1}(\text{lambda2}y)-\log\left(a^2\left(\text{lambda2}^2y^2+\right.\right.\right.}{2b\text{lambda2}}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arctan(lambda1*x) + c2*arctan(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x\frac{\left(s1\arctan\left(\_a\beta1\right)^n+s2\arctan\left(\frac{(ay-(-\_a+x)b)\beta2}{a}\right)^k\right)\left(\frac{a^2+(ay-(-\_a+x)b)^2\lambda2^2}{a^2}\right)^{\frac{c2}{2b\lambda2}}\left(\_a^2\lambda1^2\right.}{a}\right.$$

6.5.21.4 [1336] Problem 4

problem number 1336

Added April 13, 2019.

Problem Chapter 5.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arctan^m(\mu x)w_y=c\arctan^k(\nu x)w+p\arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*x]^k*w[x,y]+p*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-1}(\nu K[2])^k}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\tan^{-1}(\nu K[2])^k}{a}dK[2]\right)p\tan^{-1}\left(\beta\left(y-\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*x)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\arctan\left(\frac{\left(b\left(\int \arctan(\_f\mu)^m d\_f\right)+\left(y-\left(\int \frac{b\arctan(\mu x)^m}{a}dx\right)\right)a\right)\beta}{a}\right)^n e^{-\frac{c\left(\int \arctan(\_f\nu)^k d\_f\right)}{a}}}{a}d\_f+\_F1\left(y\right.\right.$$

6.5.21.5 [1337] Problem 5

problem number 1337

Added April 13, 2019.

Problem Chapter 5.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+b\arctan^m(\mu x)w_y=c\arctan^k(\nu y)w+p\arctan^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*y]^k*w[x,y]+p*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\tan^{-1}\left(\nu\left(y-\int_1^x \frac{b\tan^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]} \frac{b\tan^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*y)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\arctan\left(\_f\beta\right)^n \mathrm{e}^{-\frac{c\left(\int \arctan\left(\frac{\left(b\left(\int \arctan(\_f\mu)^m d\_f\right)+\left(y-\left(\int \frac{b\arctan(\mu x)^m}{a}dx\right)\right)a\right)\nu}{a}\right)^k d\_f\right)}{a}}}{a}d\_f+\_F1\left(y-\right.\right.$$

6.5.22 7.4

Local contents

6.5.22.1 [1338] Problem 1 2009
6.5.22.2 [1339] Problem 2 2010
6.5.22.3 [1340] Problem 3 2011
6.5.22.4 [1341] Problem 4 2012
6.5.22.5 [1342] Problem 5 2013

6.5.22.1 [1338] Problem 1

problem number 1338

Added April 13, 2019.

Problem Chapter 5.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \operatorname{arccot}^k(\lambda x) + c_2 \operatorname{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCot[lambda*x]^k+c2*ArcCot[beta*y]^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x}{a}} \left(\int_1^x \frac{e^{-\frac{K[1]}{a}} \left(\text{c1} \cot^{-1}(\lambda K[1])^k + \text{c2} \cot^{-1}\left(\beta\left(y + \frac{b(K[1]-x)}{a}\right)\right)^n\right)}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccot(lambda*x)^k+c2*arccot(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(c1\left(-\arctan\left(\_a\lambda\right) + \frac{\pi}{2}\right)^k + c2\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right) + \frac{\pi}{2}\right)^n\right) \mathrm{e}^{-\frac{\_a}{a}}}{a} d\_a + \_F1\left(\frac{a}{}\right.\right.$$

6.5.22.2 [1339] Problem 2

problem number 1339

Added April 13, 2019.

Problem Chapter 5.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \operatorname{arccot}^k(\lambda x)\operatorname{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCot[lambda*x]^k*ArcCot[beta*y]^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cot^{-1}(\lambda K[1])^k\cot^{-1}\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccot(lambda*x)^k*arccot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(-\arctan\left(\_a\lambda\right)+\frac{\pi}{2}\right)^k\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)+\frac{\pi}{2}\right)^n \mathrm{e}^{-\frac{ac}{a}}}{a}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)$$

6.5.22.3 [1340] Problem 3

problem number 1340

Added April 13, 2019.

Problem Chapter 5.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \operatorname{arccot}(\lambda_1 x) + c_2 \operatorname{arccot}(\lambda_2 y))\, w + s_1 \operatorname{arccot}^n(\beta_1 x) + s_2 \operatorname{arccot}^k(\beta_2 y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCot[lambda1*x] + c2*ArcCot[lambda2*y])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \left(\text{lambda1}^2 x^2+1\right)^{\frac{\text{c1}}{2a\text{lambda1}}} \exp\left(\frac{\frac{\text{c2}\left(a\log\left(a^2\left(\text{lambda2}^2 y^2+1\right)\right)+2\text{lambda2}\tan^{-1}(\text{lambda2}y)(bx-ay)+2b\text{lambda}\right.}{b\text{lambda2}}}{2a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccot(lambda1*x) + c2*arccot(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\left(s1\left(-\arctan\left(\_a\beta 1\right)+\frac{\pi}{2}\right)^n + s2\left(-\arctan\left(\frac{(ay-(-\_a+x)b)\beta 2}{a}\right)+\frac{\pi}{2}\right)^k\right)\left(\frac{a^2+(ay-(-\_a+x)b)^2}{a^2}\right.}{a}\right.$$

6.5.22.4 [1341] Problem 4

problem number 1341

Added April 13, 2019.

Problem Chapter 5.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^m(\mu x)w_y = c\operatorname{arccot}^k(\nu x)w + p\operatorname{arccot}^n(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*x]^k*w[x,y]+p*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cot^{-1}(\nu K[2])^k}{a}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]}\frac{c\cot^{-1}(\nu K[2])^k}{a}dK[2]\right)p\cot^{-1}\left(\beta\left(y-\int\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*x)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(-\arctan\left(\frac{\left(b\left(\int\left(-\arctan(\_f\mu)+\frac{\pi}{2}\right)^m d\_f\right)+\left(y-\left(\int\frac{b\left(-\arctan(\mu x)+\frac{\pi}{2}\right)^m}{a}dx\right)\right)a\right)\beta}{a}\right)+\frac{\pi}{2}\right)^n e^{-\frac{c\left(\int\left(-\mathrm{arc}\right.\right.}{}}}{a}\right.$$

6.5.22.5 [1342] Problem 5

problem number 1342

Added April 13, 2019.

Problem Chapter 5.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^m(\mu x)w_y = c\operatorname{arccot}^k(\nu y)w + p\operatorname{arccot}^n(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*y]^k*w[x,y]+p*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\cot^{-1}\left(\nu\left(y-\int_1^x \frac{b\cot^{-1}(\mu K[1])^m}{a}dK[1]+\int_1^{K[2]} \frac{b\cot^{-1}(\mu K[1])^m}{a}dK[1]\right)\right)^k}{a}dK[2]\right)\left(\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*y)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{p\left(-\arctan\left(\_f\beta\right)+\frac{\pi}{2}\right)^n \mathrm{e}^{-\frac{c\left(\int\left(-\arctan\left(\frac{\left(b\left(\int\left(-\arctan(\_f\mu)+\frac{\pi}{2}\right)^m d\_f\right)+\left(y-\left(\int \frac{b\left(-\arctan(\mu x)+\frac{\pi}{2}\right)^m}{a}dx\right)\right)a\right)\nu}{a}\right.\right.\right.}{a}}}{a}\right.$$

6.5.23 8.1

Local contents

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6.5.23.1 [1343] Problem 1

problem number 1343

Added April 13, 2019.

Problem Chapter 5.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)w + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])}{a}dK[1]\right)g(K[2])}{a}dK[2]+c_1\left(y-\frac{bx}{a}\right)\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\frac{\int f(x)dx}{a}} g(x)}{a} dx + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\int \frac{f(x)}{a} dx}$$

6.5.23.2 [1344] Problem 2

problem number 1344

Added April 13, 2019.

Problem Chapter 5.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cy+k)w + f(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y+k)*w[x,y]+f[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x(2a(cy+k)-bcx)}{2a^2}} \left(\int_1^x \frac{\exp\left(-\frac{K[1](2a(k+cy)+bc(K[1]-2x))}{2a^2}\right) f(K[1])}{a} dK[1] + c_1\left(y - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*y+k)*w(x,y)+f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\frac{\left(-\left(-\frac{a}{2}+x\right)bc+(cy+k)a\right)\_a}{a^2}} f(\_a)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\frac{\left(-\frac{bcx}{2}+(cy+k)a\right)x}{a^2}}$$

6.5.23.3 [1345] Problem 3

problem number 1345

Added April 13, 2019.

Problem Chapter 5.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)yw + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*y*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{f(K[1])(ay+b(K[1]-x))}{a^2} dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{f(K[1])(ay+b(K[1]-x))}{a^2} dK[1]\right)}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*y*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\frac{\int (ay+(\_b-x)b)f(\_b)d\_b}{a^2}} g(\_b)}{a} d\_b + \_F1\left(\frac{ay-bx}{a}\right)\right) e^{\int^{x} \frac{(ay-(-\_a+x)b)f(\_a)}{a^2} d\_a}$$

6.5.23.4 [1346] Problem 4

problem number 1346

Added April 13, 2019.

Problem Chapter 5.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x)w + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x]*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{f(K[1])}{aK[1]} dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{f(K[1])}{aK[1]} dK[1]\right) g(K[2])}{aK[2]} dK[2] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\frac{\int \frac{f(x)}{x}dx}{a}} g(x)}{ax} dx + \_F1\left(y x^{-\frac{b}{a}}\right)\right) e^{\int \frac{f(x)}{ax}dx}$$

6.5.23.5 [1347] Problem 5

problem number 1347

Added April 13, 2019.

Problem Chapter 5.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (ay+b)w_y = cw + g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (a+y+b)*D[w[x, y], y] == c*w[x,y]+g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{c}{f(K[3])} dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{c}{f(K[3])} dK[3]\right) g(K[4])}{f(K[4])} dK[4] + c_1\left(y \exp\left(-\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (a*y+b)*diff(w(x,y),y) = c*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-c\left(\int \frac{1}{f(x)}dx\right)}g(x)}{f(x)}dx + \_F1\left(\frac{(ay+b)\,e^{-a\left(\int \frac{1}{f(x)}dx\right)}}{a}\right)\right)e^{\int \frac{c}{f(x)}dx}$$

6.5.23.6 [1348] Problem 6

problem number 1348

Added April 13, 2019.

Problem Chapter 5.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)w_y = h(x)w + p(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x,y]+p[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{h(K[2])}{f(K[2])}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{h(K[2])}{f(K[2])}dK[2]\right)p(K[3])}{f(K[3])}dK[3] + c_1\left(y - \int_1^x \frac{g(}{f(}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x)*w(x,y)+p(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\left(\int \frac{h(x)}{f(x)}dx\right)}p(x)}{f(x)}dx + \_F1\left(y - \left(\int \frac{g(x)}{f(x)}dx\right)\right)\right) e^{\int \frac{h(x)}{f(x)}dx}$$

6.5.23.7 [1349] Problem 7

problem number 1349

Added April 13, 2019.

Problem Chapter 5.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_1(x)w + h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h1[x]*w[x,y]+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\text{h1}(K[3])}{f(K[3])}dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\text{h1}(K[3])}{f(K[3])}dK[3]\right)\text{h0}(K[4])}{f(K[4])}dK[4] + c_1\left(y\exp\left(\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h1(x)*w(x,y)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int \frac{e^{-\left(\int \frac{h1(x)}{f(x)}dx\right)} h0(x)}{f(x)}dx + \_F1\left(y\, e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} - \left(\int \frac{e^{-\left(\int \frac{g1(x)}{f(x)}dx\right)} g0(x)}{f(x)}dx\right)\right)\right) e^{\int \frac{h1(x)}{f(x)}dx}$$

6.5.23.8 [1350] Problem 8

problem number 1350

Added April 13, 2019.

Problem Chapter 5.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_2(x)w + h_1(x)y + h0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\text{h2}(K[3])}{f(K[3])}dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\text{h2}(K[3])}{f(K[3])}dK[3]\right)\left(\text{h0}(K[4]) + \exp\left(\int_1^{K[4]} \frac{\text{g1}(K[}{f(K[}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\left(\int \frac{h2(\_g)}{f(\_g)} d\_g\right)} h0(\_g) + \left(y\, e^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} + \int \frac{e^{-\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right)} g0(\_g)}{f(\_g)} d\_g - \left(\int \frac{e^{-\left(\int \frac{g1(x)}{f(x)} dx\right)} g0(x)}{f(x)}\right.\right.}{f(\_g)}\right.$$

6.5.23.9 [1351] Problem 9

problem number 1351

Added April 13, 2019.

Problem Chapter 5.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h_2(x)w + h_1(x)y^n + h0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y^n+h0[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{\mathrm{h2}(K[3])}{f(K[3])} dK[3]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[4]} \frac{\mathrm{h2}(K[3])}{f(K[3])} dK[3]\right)\left(\mathrm{h1}(K[4])\left(\left(\exp\left(-\int_1^x \frac{\mathrm{g1}(K}{f(K}\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y^n+h0(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\left(\left(y^{-k+1} e^{(k-1)\left(\int \frac{g1(x)}{f(x)} dx\right)} + (-k+1)\left(\int \frac{e^{(k-1)\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right)} g0(\_g)}{f(\_g)} d\_g\right) + (k-1)\left(\int e^{(k-1}\right.\right.\right.}{f(\_g)}\right.$$

6.5.23.10 [1352] Problem 10

problem number 1352

Added April 13, 2019.

Problem Chapter 5.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h_2(x)w + h_1(x)e^{\beta y} + h0(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]+g0[x]*Exp[lambda*y])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\frac{1}{\int \frac{\lambda e^{\int \frac{\lambda g1(x)}{f(x)} dx} g0(x)}{f(x)} dx + \int -\frac{\lambda e^{\int \frac{\lambda g1(\_g)}{f(\_g)} d\_g} g0(\_g)}{f(\_g)} d\_g + e^{\left(-y + \int \frac{g1(x)}{f(x)} dx\right)\lambda}} \right)^{\frac{\beta}{\lambda}} e^{\beta\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right) - \left(\int \frac{h2(\_g)}{f(\_g)} d\_g\right)} h}{f(\_g)} \right.$$

6.5.23.11 [1353] Problem 11

problem number 1353

Added April 13, 2019.

Problem Chapter 5.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)y^k w_x + f_2(x)w_y = g(x)w + h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f1[x]*y^k*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{ \left\{ w(x,y) \to \exp\left(\int_1^x \frac{g(K[2]) \left(\left(y^{k+1} - (k+1) \int_1^x \frac{\text{f2}(K[1])}{\text{f1}(K[1])} dK[1] + (k+1) \int_1^{K[2]} \frac{\text{f2}(K[1])}{\text{f1}(K[1])} dK[1] \right)^{\frac{1}{k+1}} \right)^{-k}}{\text{f1}(K[2])} d \right. \right.$$

Maple ✓

```
restart;
pde :=  f1(x)*y^k*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(\left(y^{k+1} + \int \frac{(-k-1)f2(x)}{f1(x)} dx + \int \frac{(k+1)f2(\_f)}{f1(\_f)} d\_f\right)^{\frac{1}{k+1}}\right)^{-k} \mathrm{e}^{-\left(\int \frac{\left(\left(y^{k+1} + \int \frac{(-k-1)f2(x)}{f1(x)} dx + \int \frac{(k+1)f2(\_f)}{f1(\_f)} d}{f1(\_f)}\right.}{}\right.}}{f1\left(\_f\right)}\right.$$

6.5.23.12 [1354] Problem 12

problem number 1354

Added April 13, 2019.

Problem Chapter 5.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)e^{\lambda y}w_x + f_2(x)w_y = g(x)w + h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f1[x]*Exp[lambda*y]*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \exp\left(\int_1^x \frac{g(K[2])}{\mathrm{f1}(K[2])\left(-\lambda \int_1^x \frac{\mathrm{f2}(K[1])}{\mathrm{f1}(K[1])} dK[1] + e^{\lambda y} + \lambda \int_1^{K[2]} \frac{\mathrm{f2}(K[1])}{\mathrm{f1}(K[1])} dK[1]\right)} dK[2]\right)\left(\int_1^x \frac{\exp\left(\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  f1(x)*exp(lambda*y)*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{e^{-\frac{\int \frac{g(\_f)}{\left(\int \frac{f2(\_f)}{f1(\_f)} d\_f + \frac{-\lambda\left(\int \frac{f2(x)}{f1(x)} dx\right) + e^{\lambda y}}{\lambda}\right) f1(\_f)} d\_f}{\lambda}} h(\_f)}{\left(\lambda\left(\int \frac{f2(\_f)}{f1(\_f)} d\_f\right) - \lambda\left(\int \frac{f2(x)}{f1(x)} dx\right) + e^{\lambda y}\right) f1(\_f)} d\_f + \_F1\left(\frac{-\lambda\left(\int \frac{f2(x)}{f1(x)} dx\right) + e^{\lambda y}}{\lambda}\right) \right)$$

6.5.24 8.2

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6.5.24.1 [1355] Problem 1

problem number 1355

Added April 13, 2019.

Problem Chapter 5.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + f(x)g(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+f[x]*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}f(K[1])g(K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+f(x)*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{\mathrm{e}^{-\frac{cx}{a}}f(x)\,g(x)}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.24.2 [1356] Problem 2

problem number 1356

Added April 13, 2019.

Problem Chapter 5.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+xf(x)+yg(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x*f[x]+y*g[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}(af(K[1])K[1]+g(K[1])(-bx+ay+bK[1]))}{a^2}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x*f(x)+y*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{(\_aaf(\_a)+(ay-(-\_a+x)\,b)\,g(\_a))\,\mathrm{e}^{-\frac{ac}{a}}}{a^2}d\_a+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

6.5.24.3 [1357] Problem 3

problem number 1357

Added April 13, 2019.

Problem Chapter 5.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=f(x)w+g(x)h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x]*h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])}{a}dK[1]\right)g(K[2])h(K[2])}{a}dK[2]+c_1\left(y-\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int \frac{\mathrm{e}^{-\frac{\int f(x)dx}{a}}g(x)\,h(x)}{a}dx+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int \frac{f(x)}{a}dx}$$

6.5.24.4 [1358] Problem 4

problem number 1358

Added April 13, 2019.

Problem Chapter 5.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=(f(x)+g(y))w+p(x)+q(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x]+g[y])*w[x,y]+p[x]+q[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f(K[1])+g\left(y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (f(x)+g(y))*w(x,y)+p(x)+q(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\left(p(\_b)+q\left(\frac{ay-(-\_b+x)b}{a}\right)\right)\mathrm{e}^{-\frac{\int\left(f(\_b)+g\left(\frac{ay-(-\_b+x)b}{a}\right)\right)d\_b}{a}}}{a}d\_b+\_F1\left(\frac{ay-bx}{a}\right)\right)\mathrm{e}^{\int^x \frac{f(\_a)+g}{}}$$

6.5.24.5 [1359] Problem 5

problem number 1359

Added April 13, 2019.

Problem Chapter 5.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=cw+f(x)g(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+f[x]*g[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{f(K[1])g\left(x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)K[1]^{-\frac{a+c}{a}}}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\_a^{\frac{-a-c}{a}}f(\_a)\,g\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{a}d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

6.5.24.6 [1360] Problem 6

problem number 1360

Added April 13, 2019.

Problem Chapter 5.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x+f_2(y)w_y=aw+g_1(x)+g_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x, y], x] + f2[y]*D[w[x, y], y] == a*w[x,y]+g1[x]+g2[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y),x)+ f2(y)*diff(w(x,y),y) = a*w(x,y)+g1(x)+g2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\left(g1(\_f) + g2\left(\text{RootOf}\left(\int \frac{1}{f1(\_f)} d\_f - \left(\int \frac{1}{f1(x)} dx\right) + \int \frac{1}{f2(y)} dy - \left(\int^{-Z} \frac{1}{f2(\_a)} d\_a\right)\right)\right)\right) e}{f1\,(\_f)}\right.$$

6.5.25 8.3

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6.5.25.1 [1361] Problem 1

problem number 1361

Added April 13, 2019.

Problem Chapter 5.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x})w + g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x,y]+g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{xf\left(\frac{y}{x}\right)}\left(\int_1^x \frac{e^{-f\left(\frac{y}{x}\right)K[1]}g\left(K[1],\frac{yK[1]}{x}\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = x*f(y/x)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\mathrm{e}^{-\_a f\left(\frac{y}{x}\right)}g\left(\_a,\frac{-\_a y}{x}\right)}{\_a}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{xf\left(\frac{y}{x}\right)}$$

6.5.25.2 [1362] Problem 2

problem number 1362

Added April 13, 2019.

Problem Chapter 5.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x+byw_y=f(x,y)w+g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x,y]*w[x,y]+g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[2]}\frac{f\left(K[1],x^{-\frac{b}{a}}yK[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)g\left(K[2\right.}{aK[2]}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x,y)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{e^{-\frac{\int \frac{f\left(\_b,y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_b}d\_b}{a}}g\left(\_b,y\_b^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_ba}d\_b+\_F1\left(yx^{-\frac{b}{a}}\right)\right)e^{\int^x \frac{f\left(\_a,y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)}{\_aa}d\_a}$$

6.5.25.3 [1363] Problem 3

problem number 1363

Added April 13, 2019.

Problem Chapter 5.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+g(x)w_y=h(x,y)w+F(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[2],y-\int_1^x \frac{g(K[1])}{f(K[1])}dK[1]+\int_1^{K[2]} \frac{g(K[1])}{f(K[1])}dK[1]\right)}{f(K[2])}dK[2]\right)\left(\int_1^x \frac{\exp\left(-\int_1^{K[3]} \frac{h(}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{F\left(\_f,y+\int \frac{g(\_f)}{f(\_f)}d\_f-\left(\int \frac{g(x)}{f(x)}dx\right)\right)\mathrm{e}^{-\left(\int \frac{h\left(\_f,y+\int \frac{g(\_f)}{f(\_f)}d\_f-\left(\int \frac{g(x)}{f(x)}dx\right)\right)}{f(\_f)}d\_f\right)}}{f(\_f)}d\_f+\_F1\left(y-\right.\right.$$

6.5.25.4 [1364] Problem 4

problem number 1364

Added April 13, 2019.

Problem Chapter 5.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x+(g_1(x)y+g_0(x))w_y=h(x,y)w+F(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[3],\exp\left(\int_1^{K[3]}\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)\left(\exp\left(-\int_1^x\frac{\mathrm{g1}(K[1])}{f(K[1])}dK[1]\right)y-\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{\mathrm{g1}}{f(}\right.}{f}\right.\right.}{f(K[3])}\right.\right.$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{F\left(\_g,\left(y\,\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}+\int\frac{\mathrm{e}^{-\left(\int\frac{g1(\_g)}{f(\_g)}d\_g\right)}g0(\_g)}{f(\_g)}d\_g-\left(\int\frac{\mathrm{e}^{-\left(\int\frac{g1(x)}{f(x)}dx\right)}g0(x)}{f(x)}dx\right)\right)\mathrm{e}^{\int\frac{g1(\_g)}{f(\_g)}d\_}}{f(\_}\right.\right.$$

6.5.25.5 [1365] Problem 5

problem number 1365

Added April 13, 2019.

Problem Chapter 5.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{F\left(\_g, \left(y^{-k+1} e^{(k-1)\left(\int \frac{g1(x)}{f(x)} dx\right)} + (-k+1)\left(\int \frac{e^{(k-1)\left(\int \frac{g1(\_g)}{f(\_g)} d\_g\right)} g0(\_g)}{f(\_g)} d\_g\right) + (k-1)\left(\int \right.\right.\right.}{}\right.$$

6.5.25.6 [1366] Problem 6

problem number 1366

Added April 13, 2019.

Problem Chapter 5.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)e^{\lambda y})w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*Exp[lambda*y])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

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6.6.1.1 [1367] Problem 1

problem number 1367

Added April 13, 2019.

Problem Chapter 6.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y) + c*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{za-cx}{a}\right)$$

Hand solution

Solve

$$aw_x + bw_y + cw_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c} = \frac{dw}{0}$$

From first two pair of equations, integrating gives $\frac{b}{a}x - y = C_1$ and from $\frac{dx}{a} = \frac{dz}{c}$ by Integrating gives $\frac{c}{a}x - z = C_2$. Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(\frac{b}{a}x - y, \frac{c}{a}x - z\right)$$

6.6.1.2 [1368] Problem 2

problem number 1368

Added April 13, 2019.

Problem Chapter 6.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + axw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ax^2}{2},\frac{1}{3}abx^3-bxy+z\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*x*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{ax^2}{2}+y,\frac{(ax^2-3y)\,bx}{3}+z\right)$$

<u>Hand solution</u>

Solve

$$w_x+axw_y+byw_z=0 \tag{1}$$

Using Lagrange-charpit

$$dx=\frac{dy}{ax}=\frac{dz}{by}=\frac{dw}{0}$$

From first two pair of equations, integrating gives

$$a\frac{x^2}{2}-y=C_1 \tag{1}$$

And from $dx=\frac{dz}{by}$ we obtain

$$bydx=dz$$

But from (1) $y=\frac{ax^2}{2}-C_1$ and the above becomes

$$b\left(\frac{ax^2}{2}-C_1\right)dx=dz$$

Now we can integrate and the result is

$$b\left(\frac{ax^3}{6}-C_1x\right)-z=C_2$$

Using (1) again the above becomes

$$
\begin{aligned}
b\left(\frac{ax^3}{6}-\left(a\frac{x^2}{2}-y\right)x\right)-z&=C_2\\
b\left(\frac{ax^3}{6}-a\frac{x^3}{2}+yx\right)-z&=C_2\\
-\frac{b}{3}ax^3+byx-z&=C_2
\end{aligned}
$$

Since $dw=0$ then $w=C_3$. Where C_1,C_2,C_3 are constants. But $C_3=F(C_1,C_2)$ where F is arbitrary function. Hence

$$
\begin{aligned}
u(x,y,z)&=F\left(a\frac{x^2}{2}-y,-\frac{b}{3}ax^3+byx-z\right)\\
&=F\left(y-a\frac{x^2}{2},\frac{1}{3}abx^3-byx+z\right)
\end{aligned}
$$

6.6.1.3 [1369] Problem 3

problem number 1369

Added April 13, 2019.

Problem Chapter 6.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+byw_y+czw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y) + c*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

Hand solution

Solve

$$aw_x + byw_y + czw_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{by} = \frac{dz}{cz} = \frac{dw}{0}$$

From first two pair of equations, integrating gives

$$\begin{aligned} abx + C_1 &= \ln y \\ y &= C_1 e^{abx} \\ C_1 &= ye^{-abx} \end{aligned} \tag{1}$$

And from $\frac{dx}{a} = \frac{dz}{cz}$ we obtain

$$\begin{aligned} \frac{c}{a}x + C_2 &= \ln z \\ z &= C_2 e^{\frac{c}{a}x} \\ C_2 &= ze^{-\frac{c}{a}x} \end{aligned}$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(ye^{-abx}, ze^{-\frac{c}{a}x}\right)$$

6.6.1.4 [1370] Problem 4

problem number 1370

Added April 13, 2019.

Problem Chapter 6.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}}, \frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{z^2a - by^2}{a}, -\frac{-\sqrt{ab}\,x + \ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)$$

<u>Hand solution</u>

Solve

$$w_x + azw_y + byw_z = 0$$

Using Lagrange-charpit

$$dx = \frac{dy}{az} = \frac{dz}{by} = \frac{dw}{0}$$

Starting with $\frac{dy}{az} = \frac{dz}{by}$ or $\frac{b}{a}ydy = zdz$ and integrating gives

$$\begin{aligned}\frac{b}{a}\frac{y^2}{2} &= \frac{z^2}{2} + C_1\\ \frac{b}{a}y^2 &= z^2 + C_1\\ C_1 &= \frac{b}{a}y^2 - z^2 \qquad (1)\end{aligned}$$

Now we can either consider $dx = \frac{dz}{by}$ or $dx = \frac{dy}{az}$. Both will give valid solutions. Lets try both to see.

<u>case $dx = \frac{dz}{by}$</u>

From (1) solving for y in terms of z gives

$$\sqrt{\frac{a}{b}C_1 + \frac{a}{b}z^2} = y$$

Hence $dx = \frac{dz}{by}$ becomes $bdx = \frac{dz}{\sqrt{\frac{a}{b}C_1 + \frac{a}{b}z^2}}$. Integrating gives

$$bx = \sqrt{\frac{b}{a}}\ln\left(z + \sqrt{z^2 + C_1}\right) + C_2$$

Using (1) in the above gives

$$\begin{aligned}bx &= \sqrt{\frac{b}{a}}\ln\left(z + \sqrt{z^2 + \left(\frac{b}{a}y^2 - z^2\right)}\right) + C_2\\ &= \sqrt{\frac{b}{a}}\ln\left(z + \sqrt{\frac{b}{a}y^2}\right) + C_2\\ C_2 &= bx - \sqrt{\frac{b}{a}}\ln\left(z + \sqrt{\frac{b}{a}y^2}\right)\end{aligned}$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x, y, z) = F\left(\frac{b}{a}y^2 - z^2, bx - \sqrt{\frac{b}{a}}\ln\left(\sqrt{\frac{b}{a}}y + z\right)\right) \qquad (2)$$

<u>case $dx = \frac{dy}{az}$</u>

From (1) we solve for z in terms of y. This gives $z^2 = \frac{b}{a}y^2 - C_1$ or $z = \sqrt{\frac{b}{a}y^2 - C_1}$, taking the positive root only. Hence $dx = \frac{dy}{az}$ becomes $adx = \frac{dy}{\sqrt{\frac{b}{a}y^2 - C_1}}$ or $adx = \sqrt{\frac{a}{b}}\frac{dy}{\sqrt{y^2 - \frac{a}{b}C_1}}$

Integrating give

$$ax = \sqrt{\frac{a}{b}} \ln \left(y + \sqrt{y^2 - \frac{a}{b} C_1} \right) + C_2$$

Using (1) in the above gives

$$ax = \sqrt{\frac{a}{b}} \ln \left(y + \sqrt{y^2 - \frac{a}{b} \left(\frac{b}{a} y^2 - z^2 \right)} \right) + C_2$$

$$ax = \sqrt{\frac{a}{b}} \ln \left(y + \frac{a}{b} z \right) + C_2$$

Hence

$$C_2 = ax - \sqrt{\frac{a}{b}} \ln \left(y + \sqrt{\frac{a}{b}} z \right)$$

Since $dw = 0$ then $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x, y, z) = F\left(\frac{b}{a} y^2 - z^2, ax - \sqrt{\frac{a}{b}} \ln \left(y + \frac{a}{b} z \right) \right) \tag{3}$$

Both (2,3) are valid solutions.

6.6.1.5 [1371] Problem 5

problem number 1371

Added April 13, 2019.

Problem Chapter 6.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$x w_x + a y w_y + b z w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*y*D[w[x, y,z], y] +b*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\{\{w(x, y, z) \to c_1(y x^{-a}, z x^{-b})\}\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ a*y*diff(w(x,y,z),y) + b*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-a}, zx^{-b}\right)$$

6.6.1.6 [1372] Problem 6

problem number 1372

Added April 13, 2019.

Problem Chapter 6.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{\sqrt{a}z\,s}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{z^2a - by^2}{a}, x\left(az + \sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)$$

6.6.1.7 [1373] Problem 7

problem number 1373

Added April 13, 2019.

Problem Chapter 6.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + (ax+by)w_y + (\alpha x + \beta y + \gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(alpha*x+beta*y+gamma*z)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{x^{-b}(ax+(b-1)y)}{b-1}, \frac{x^{-\gamma}(-a\beta x + \alpha x(b-\gamma) - (\gamma-1)(-bz+\beta y+\gamma z))}{(\gamma-1)(b-\gamma)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (alpha*x+beta*y+gamma*z)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(ax+(b-1)y)x^{-b}}{b-1}, \frac{-(-b^2z+(\beta y+z)b+(ax-y)\beta+(b-1)\gamma z)(-1+\gamma)x^{-\gamma}-(}{(-1+\gamma)(b-1)(b-\gamma)}\right.$$

6.6.1.8 [1374] Problem 8

problem number 1374

Added April 13, 2019.

Problem Chapter 6.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + (ay+bz)(bw_y - aw_z) = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + (a*y+b*z)*(b*D[w[x, y,z], y] -a*D[w[x,y,z],z])==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*b*x*diff(w(x,y,z),x)+ (a*y+b*z)*(b*diff(w(x,y,z),y) - a*diff(w(x,y,z),z))= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay+bz}{b}, x\,\mathrm{e}^{-\frac{ay}{ay+bz}}\right)$$

6.6.1.9 [1375] Problem 9

problem number 1375

Added April 13, 2019.

Problem Chapter 6.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$abxw_x + b(ay + bz)w_y + a(ay - bz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*b*x*diff(w(x,y,z),x)+ b*(a*y+b*z)*diff(w(x,y,z),y) + a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{\sqrt{-a^2y^2+2abyz+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2abyz+b^2z^2} + \left(\frac{ay}{\sqrt{-a^2y^2+2abyz+b^2z^2}} + \frac{bz}{\sqrt{-a^2y^2+2abyz+b^2}}\right.}{\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}\right.\right.$$

6.6.1.10 [1376] Problem 10

problem number 1376

Added April 13, 2019.

Problem Chapter 6.2.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b^2cyw_x + a^2cxw_y - ab(ax+by)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b^2*c*y*D[w[x, y,z], x] + a^2*c*x*D[w[x, y,z], y] -a*b*(a*x+b*y)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{1}{2}\left(y^2 - \frac{a^2x^2}{b^2}\right), \frac{ax+by+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  b^2*c*y*diff(w(x,y,z),x)+ a^2*c*x*diff(w(x,y,z),y) - a*b*(a*x+b*y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+b^2y^2}{b^2}, \frac{ax+by+cz}{c}\right)$$

6.6.1.11 [1377] Problem 11

problem number 1377

Added April 13, 2019.

Problem Chapter 6.2.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$czw_x + (ax+by)w_y + (ax+by+cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  c*z*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(a*x+b*y+c*z)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  c*z*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (a*x+b*y+c*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.12 [1378] Problem 12

problem number 1378

Added April 13, 2019.

Problem Chapter 6.2.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b^2czw_x - a^2cxw_y + ab^2yw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b^2*c*z*D[w[x, y,z], x] - a^2*c*x*D[w[x, y,z], y] +a*b^2*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  b^2*c*z*diff(w(x,y,z),x)-a^2*c*x*diff(w(x,y,z),y) + a*b^2*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.13 [1379] Problem 13

problem number 1379

Added April 13, 2019.

Problem Chapter 6.2.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(x+a)w_x + (y+b)xw_y + (z+c)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (x+a)*D[w[x, y,z], x] + (y+b)*D[w[x, y,z], y] +(z+c)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b+y}{a+x}, \frac{c+z}{a+x}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (x+a)*diff(w(x,y,z),x)+(y+b)*diff(w(x,y,z),y) + (z+c)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{b+y}{a+x}, \frac{c+z}{a+x}\right)$$

6.6.1.14 [1380] Problem 14

problem number 1380

Added April 13, 2019.

Problem Chapter 6.2.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2bc(ax-by)w_x - ac(ax-by-cz)w_y - ab(ax-by-3cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*b*c*(a*x-b*y)*D[w[x, y,z], x] -a*c*(a*x-b*y-c*z)*D[w[x, y,z], y] - a*b*(a*x -b*y-3*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b*c*(a*x-b*y)*diff(w(x,y,z),x)-a*c*(a*x-b*y-c*z)*diff(w(x,y,z),y)- a*b*(a*x -b*y-3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.15 [1381] Problem 15

problem number 1381

Added April 13, 2019.

Problem Chapter 6.2.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(y-z)w_x + ac(z-x)w_y + ab(x-y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*(y-z)*D[w[x, y,z], x] +a*c*(z-x)*D[w[x, y,z], y] + a*b*(x -y)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  b*c*(y-z)*diff(w(x,y,z),x)+a*c*(z-x)*diff(w(x,y,z),y)+ a*b*(x -y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3 c_4 c_5 e^{c_1 x} e^{\frac{c_1 by}{a}} e^{\frac{c_1 cz}{a}} e^{\frac{c_2 x^2}{2}} e^{\frac{c_2 by^2}{2a}} e^{\frac{c_2 cz^2}{2a}}$$

6.6.1.16 [1382] Problem 16

problem number 1382

Added April 13, 2019.

Problem Chapter 6.2.1.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(by-2cz)w_x + ac(3cz-ax)w_y + ab(2ax-3by)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*(b*y-2*c*z)*D[w[x, y,z], x] +a*c*(3*c*z-a*x)*D[w[x, y,z], y] + a*b*(2*a*x -3*b*y)
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  b*c*(b*y-2*c*z)*diff(w(x,y,z),x)+a*c*(3*c*z-a*x)*diff(w(x,y,z),y)+ a*b*(2*a*x-3*b*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3 c_4 c_5 e^{c_1 x} e^{\frac{c_2 x^2}{2}} e^{\frac{2c_1 by}{3a}} e^{\frac{c_2 b^2 y^2}{2a^2}} e^{\frac{c_1 cz}{3a}} e^{\frac{c_2 c^2 z^2}{2a^2}}$$

6.6.1.17 [1383] Problem 17

problem number 1383

Added April 13, 2019.

Problem Chapter 6.2.1.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2bc(by - cz)w_x - ac(4ax - 3by - cz)w_y + 3ab(4ax - by - 3cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  2*b*c*(b*y-c*z)*D[w[x, y,z], x] -a*c*(4*a*x-3*b*y-c*z)*D[w[x, y,z], y] + 3*a*b*(4*a*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b*c*(b*y-c*z)*diff(w(x,y,z),x)-a*c*(4*a*x-3*b*y-c*z)*diff(w(x,y,z),y)+ 3*a*b*(4*a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.18 [1384] Problem 18

problem number 1384

Added April 13, 2019.

Problem Chapter 6.2.1.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(ax + y - z)w_x - (x + ay - z)w_y + (a - 1)(y - x)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x+y-z)*D[w[x, y,z], x] -(x+a*y-z)*D[w[x, y,z], y] + (a-1)*(y-x)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  (a*x+y-z)*diff(w(x,y,z),x)-(x+a*y-z)*diff(w(x,y,z),y)+ (a-1)*(y-x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.19 [1385] Problem 19

problem number 1385

Added April 13, 2019.

Problem Chapter 6.2.1.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2bc(3ax - 2by + cz)w_x - 2ac(2ax - 5by + 3cz)w_y + ab(2ax - 6by + 11cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   2*b*c*(3*a*x-2*b*y+c*z)*D[w[x, y,z], x] -2*a*c(2*a*x-5*b*y+3*c*z)*D[w[x, y,z], y] +
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   2*b*c*(3*a*x-2*b*y+c*z)*diff(w(x,y,z),x)-2*a*c*(2*a*x-5*b*y+3*c*z)*diff(w(x,y,z),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.20 [1386] Problem 20

problem number 1386

Added April 13, 2019.

Problem Chapter 6.2.1.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(Ax + cy + bz)w_x + (cx + By + az)w_y + (bx + ay + Cz)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (A*x+c*y+b*z)*D[w[x, y,z], x] +(c*x+B*y+a*z)*D[w[x, y,z], y] +(b*x+a*y+C1*z)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   (A*x+c*y+b*z)*diff(w(x,y,z),x)+(c*x+B*y+a*z)*diff(w(x,y,z),y)+ (b*x+a*y+C1*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.1.21 [1387] Problem 21

problem number 1387

Added April 13, 2019.

Problem Chapter 6.2.1.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(a_1x + b_1y + c_1z + d_1)w_x + (a_2x + b_2y + c_2z + d_2)w_y + (a_3x + b_3y + c_3z + d_3)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   (a1*x+b1*y+c1*z+d1)*D[w[x, y,z], x] +(a2*x+b2*y+c2*z+d2)*D[w[x, y,z], y] +(a3*x+b3*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=   (a1*x+b1*y+c1*z+d1)*diff(w(x,y,z),x)+(a2*x+b2*y+c2*z+d2)*diff(w(x,y,z),y)+ (a3*x+b3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.2 2.2

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6.6.2.1 [1388] Problem 1

problem number 1388

Added April 14, 2019.

Problem Chapter 6.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 xy + b_1 x^2 + c_1 x) w_y + (a_2 xy + b_2 x^2 + c_2 x) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*y+b2*x^2+c2*x)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-2\text{a1b2}x^3-3\text{a1c2}x^2+6\text{a1}z+2\text{a2b1}x^3+3\text{a2c1}x^2-6\text{a2}y}{6\text{a1}},\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*y+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-\frac{\pi\sqrt{2}\,b1\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}+\sqrt{\pi}\left(a1^{\frac{3}{2}}y+(b1x+c1)\sqrt{a1}\right)\mathrm{e}^{-\frac{a1x^2}{2}}}{\sqrt{\pi}\,a1^{\frac{3}{2}}},-\frac{-\frac{3\sqrt{2}\left(\frac{\sqrt{\pi}\sqrt{\frac{a1}{\pi}}}{\sqrt{a1}}-1\right)a2\,b1\,\mathrm{erf}}{2}}{}\right.$$

6.6.2.2 [1389] Problem 2

problem number 1389

Added April 14, 2019.

Problem Chapter 6.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+b_1x^2+c_1x)w_y+(a_2xz+b_2x^2+c_2x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*z+b2*x^2+c2*x)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}-\frac{\sqrt{\frac{\pi}{2}}\text{b1}\text{erf}\left(\frac{\sqrt{\text{a1}}x}{\sqrt{2}}\right)}{\text{a1}^{3/2}},\frac{e^{-\frac{\text{a2}x^2}{2}}(\text{a2}z+\text{b2}x+\text{c2})}{\text{a2}}-\frac{\sqrt{\frac{\pi}{2}}\text{b2}\text{erf}\left(\frac{\sqrt{}}{}\right.}{\text{a2}^{3/2}}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*z+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-\frac{\pi\sqrt{2}\,b1\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}+\sqrt{\pi}\left(a1^{\frac{3}{2}}y+(b1x+c1)\sqrt{a1}\right)\mathrm{e}^{-\frac{a1x^2}{2}}}{\sqrt{\pi}\,a1^{\frac{3}{2}}},\frac{-\frac{\pi\sqrt{2}\,b2\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a2}\,x}{2}\right)}{2}+\sqrt{\pi}}{}\right.$$

6.6.2.3 [1390] Problem 3

problem number 1390

Added April 14, 2019.

Problem Chapter 6.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+b_1x^2+c_1x)w_y+(a_2yz+b_2y^2+c_2y)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*y*z+b2*y^2+c2*y)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*y*z+b2*y^2+c2*y)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\frac{\pi\sqrt{2}\,b1\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2} + \sqrt{\pi}\left(a1^{\frac{3}{2}}y + (b1x + c1)\sqrt{a1}\right)\mathrm{e}^{-\frac{a1x^2}{2}}}{\sqrt{\pi}\,a1^{\frac{3}{2}}}, z\,\mathrm{e}^{-\frac{\int^x 2\left(-\frac{\sqrt{2}\,\pi\left(-\mathrm{erf}\left(\frac{\sqrt{2}-b\sqrt{a1}}{2}\right.\right.}{}\right.}{}}\right.$$

6.6.2.4 [1391] Problem 4

problem number 1391

Added April 14, 2019.

Problem Chapter 6.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1xy + b_1y^2)w_y + (a_2xz + b_2z^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   D[w[x, y,z], x] +(a1*x+b1*y^2)*D[w[x, y,z], y] +(a2*x*z+b2*z^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{2\left(\text{b1}xyJ_{\frac{1}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}J_{-\frac{2}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/}\right.\right.}{(2\text{b1}xy+1)J_{-\frac{1}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}J_{-\frac{4}{3}}\left(\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\right)-\sqrt{\text{a1}}\sqrt{\text{b1}}}\right.\right.$$

Maple ✓

```
restart;
pde :=   diff(w(x,y,z),x)+(a1*x+b1*y^2)*diff(w(x,y,z),y)+ (a2*x*z+b2*z^2)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{b1\,y\,\text{AiryBi}\left(-(a1\,b1)^{\frac{1}{3}}\,x\right)-(a1\,b1)^{\frac{1}{3}}\,\text{AiryBi}\left(1,-(a1\,b1)^{\frac{1}{3}}\,x\right)}{-b1\,y\,\text{AiryAi}\left(-(a1\,b1)^{\frac{1}{3}}\,x\right)+(a1\,b1)^{\frac{1}{3}}\,\text{AiryAi}\left(1,-(a1\,b1)^{\frac{1}{3}}\,x\right)},\frac{\sqrt{\pi}\,b2\,z\,\text{erf}\left(\frac{\sqrt{-2a2}}{2}\right.}{\sqrt{-2}}\right.$$

6.6.2.5 [1392] Problem 5

problem number 1392

Added April 14, 2019.

Problem Chapter 6.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+xzw_y-xyw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*D[w[x, y,z], x] +x*z*D[w[x, y,z], y] -x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\sin\left(\frac{x^2}{2a}\right)+z\cos\left(\frac{x^2}{2a}\right),y\cos\left(\frac{x^2}{2a}\right)-z\sin\left(\frac{x^2}{2a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*diff(w(x,y,z),x)+x*z*diff(w(x,y,z),y)- x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y^2+z^2,-2a\arctan\left(\frac{y}{z}\right)+x^2\right)$$

6.6.2.6 [1393] Problem 6

problem number 1393

Added April 14, 2019.

Problem Chapter 6.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxw_x+cyw_y+(ax^2+by^2)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*D[w[x, y,z], x] +c*y*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},-\frac{ax^2+by^2-2cz}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   c*x*diff(w(x,y,z),x)+c*y*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-ax^2 - by^2 + 2cz}{2c}\right)$$

6.6.2.7 [1394] Problem 7

problem number 1394

Added April 14, 2019.

Problem Chapter 6.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$czw_x - a(2ax-b)yw_y + a(2ax-b)zw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*z*D[w[x, y,z], x] -a*(2*a*x-b)*y*D[w[x, y,z], y] +a*(2*a*x-b)*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(cyz, \frac{-a^2x^2 + abx + cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   c*z*diff(w(x,y,z),x)-a*(2*a*x-b)*diff(w(x,y,z),y)+a*(2*a*x-b)*z*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2 + abx + cz}{c}, y + \ln(cz)\right)$$

6.6.2.8 [1395] Problem 8

problem number 1395

Added April 14, 2019.

Problem Chapter 6.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$acx^2 w_x - acxyw_y - b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*c*x^2*D[w[x, y,z], x] -a*c*x*y*D[w[x, y,z], y] -b^2*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to c_1\left(xy, z - \frac{b^2y^2}{3acx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*c*x^2*diff(w(x,y,z),x) -a*c*x*y*diff(w(x,y,z),y)-b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(xy, \frac{3zacx^3 - b^2x^2y^2}{3acx^3}\right)$$

6.6.2.9 [1396] Problem 9

problem number 1396

Added April 14, 2019.

Problem Chapter 6.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*x^2*D[w[x, y,z], x] +b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b}{ax} - \frac{1}{y}, \frac{c}{ax} - \frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*x^2*diff(w(x,y,z),x) +b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax - by}{axy}, \frac{ax - cz}{axz}\right)$$

6.6.2.10 [1397] Problem 10

problem number 1397

Added April 14, 2019.

Problem Chapter 6.2.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abx^2 w_x + cz^2 w_y + 2abxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*b*x^2*D[w[x, y,z], x] +c*z^2*D[w[x, y,z], y] +2*a*b*x*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{z}{x^2},y-\frac{cz^2}{3abx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=    a*b*x^2*diff(w(x,y,z),x) +c*z^2*diff(w(x,y,z),y)+2*a*b*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{z}{x^2},\frac{3yxab-cz^2}{3abx}\right)$$

6.6.2.11 [1398] Problem 11

problem number 1398

Added April 14, 2019.

Problem Chapter 6.2.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxyw_x+a^2cx^2w_y-by(2ax+cz)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*x*y*D[w[x, y,z], x] +a^2*c*x^2*D[w[x, y,z], y] -b*y*(2*a*x+c*z)*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-a^2x^2}{2b},\frac{x(ax+cz)}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=     b*c*x*y*diff(w(x,y,z),x) +a^2*c*x^2*diff(w(x,y,z),y)-b*y*(2*a*x+c*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+by^2}{b}, \frac{(ax+cz)\,x}{c}\right)$$

6.6.2.12 [1399] Problem 12

problem number 1399

Added April 14, 2019.

Problem Chapter 6.2.2.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxyw_x + c^2yzw_y + b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*x*y*D[w[x, y,z], x] +c^2*y*z*D[w[x, y,z], y] +b^2*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{by+cz}{2bx}, \frac{x(by-cz)}{2b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y*diff(w(x,y,z),x) +c^2*y*z*diff(w(x,y,z),y)+b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b^2y^2+c^2z^2}{c^2}, x(by\,\mathrm{csgn}(b)+cz)^{-\mathrm{csgn}(b)}\right)$$

6.6.2.13 [1400] Problem 13

problem number 1400

Added April 14, 2019.

Problem Chapter 6.2.2.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xyw_x + y(y-a)w_y + z(y-a)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*y*D[w[x, y,z], x] +y*(y-a)*D[w[x, y,z], y] +z*(y-a)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{y-a}{x}, \frac{z}{y}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*y*diff(w(x,y,z),x) +y*(y-a)*diff(w(x,y,z),y)+z*(y-a)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a+y}{x}, \frac{z}{y}\right)$$

6.6.2.14 [1401] Problem 14

problem number 1401

Added April 14, 2019.

Problem Chapter 6.2.2.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$by^2 w_x - axyw_y + cxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*y^2*D[w[x, y,z], x] -a*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{ax^2+by^2}{2b}, z\left(-by^2\right)^{\frac{c}{2a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*y^2*diff(w(x,y,z),x) -a*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax^2+by^2}{b}, z\left(-by^2\right)^{\frac{c}{2a}}\right)$$

6.6.2.15 [1402] Problem 15

problem number 1402

Added April 14, 2019.

Problem Chapter 6.2.2.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x + 2axyw_y - (2ax+cz)zw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] +2*a*x*y*D[w[x, y,z], y] -(2*a*x+c*z)*z*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-cxyz,x\left(\frac{ax}{c}+z\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x) +2*a*x*y*diff(w(x,y,z),y)-(2*a*x+c*z)*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(ax+cz)\,x}{c},-cxyz\right)$$

6.6.2.16 [1403] Problem 16

problem number 1403

Added April 14, 2019.

Problem Chapter 6.2.2.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x+cyzw_y+abxyw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] +c*y*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{cz^2-abxy}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x) +c*y*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-yxab + cz^2}{c}\right)$$

6.6.2.17 [1404] Problem 17

problem number 1404

Added April 14, 2019.

Problem Chapter 6.2.2.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$cxzw_x - cyzw_y + (by^2 - ax)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(b*y^2-a*x)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(xy, \frac{2ax + by^2 + cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(b*y^2-a*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(xy, \frac{bx^2y^2 + z^2cx^2 + 2ax^3}{cx^2}\right)$$

6.6.2.18 [1405] Problem 18

problem number 1405

Added April 14, 2019.

Problem Chapter 6.2.2.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$cxzw_x - cyzw_y + (ax^2 + by^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to c_1\left(xy, \frac{-ax^2 + by^2 + cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(xy, \frac{-ax^2 + by^2 + cz^2}{c}\right)$$

6.6.2.19 [1406] Problem 19

problem number 1406

Added April 14, 2019.

Problem Chapter 6.2.2.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xzw_x + yzw_y + (ax^2 + ay^2 + bz^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*z*D[w[x, y,z], x] +y*z*D[w[x, y,z], y] +(a*x^2+a*y^2+b*z^2)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \frac{x^{-2b}(a(x^2+y^2)+(b-1)z^2)}{b-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*z*diff(w(x,y,z),x)+y*z*diff(w(x,y,z),y)+(a*x^2+a*y^2+b*z^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{((b-1)\,z^2 + (x^2+y^2)\,a)\,x^{-2b}}{b-1}\right)$$

6.6.2.20 [1407] Problem 20

problem number 1407

Added April 14, 2019.

Problem Chapter 6.2.2.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$2cxzw_x + 2cyzw_y + (cz^2 - ax^2 - by^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   2*c*x*z*D[w[x, y,z], x] +2*c*y*z*D[w[x, y,z], y] +(c*z^2-a*x^2-b*y^2)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{ax^2+by^2+cz^2}{cx}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := 2*c*x*z*diff(w(x,y,z),x)+2*c*y*z*diff(w(x,y,z),y)+(c*z^2-a*x^2-b*y^2)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{ax^2+by^2+cz^2}{cx}\right)$$

6.6.2.21 [1408] Problem 21

problem number 1408

Added April 14, 2019.

Problem Chapter 6.2.2.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcyzw_x+acxzw_y+abxyw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*y*z*D[w[x, y,z], x] +a*c*x*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-ax^2}{2b},\frac{cz^2-ax^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*y*z*diff(w(x,y,z),x)+a*c*x*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-ax^2+by^2}{b}, \frac{-ax^2+cz^2}{c}\right)$$

6.6.2.22 [1409] Problem 22

problem number 1409

Added April 14, 2019.

Problem Chapter 6.2.2.22, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(x^2-a^2)w_x + c(bxy+acz)w_y + b(cxz+aby)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   b*c*(x^2-a^2)*D[w[x, y,z], x] +c*(b*x*y+a*c*z)*D[w[x, y,z], y] +b*(c*x*z + a*b*y)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{acz+bxy}{a^2b-bx^2}, \frac{a(aby+cxz)}{b\left(a^2-x^2\right)}\right)\right\}\right\}$$

Maple ✗

```
restart;
pde :=  b*c*(x^2-a^2)*diff(w(x,y,z),x)+c*(b*x*y+a*c*z)*diff(w(x,y,z),y)+b*(c*x*z + a*b*y)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.2.23 [1410] Problem 23

problem number 1410

Added April 14, 2019.

Problem Chapter 6.2.2.23, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$bx(by + c)w_x + (b^2y^2 - acx)w_y + b^2yzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   b*x*(b*y+c)*D[w[x, y,z], x] + (b^2*y^2-a*c*x )*D[w[x, y,z], y] + b^2*y*z*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  b*x*(b*y+c)*diff(w(x,y,z),x)+(b^2*y^2-a*c*x )*diff(w(x,y,z),y)+b^2*y*z*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{by + c + (ax + by)\ln\left(\frac{-9ax+9c}{2by+2c}\right) + (-ax - by)\ln\left(-\frac{9(ax+by)(ax-c)}{(by+c)x}\right)}{3ax + 3by}, z\,\mathrm{e}^{-\left(\int^x \frac{9\_aa+2c\,\mathrm{e}^{\mathrm{RootO}}}{}\right.}\right.$$

6.6.2.24 [1411] Problem 24

problem number 1411

Added April 14, 2019.

Problem Chapter 6.2.2.24, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$x(by - cz)w_x + y(cz - ax)w_y + z(ax - by)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   x*(b*y -c*z)*D[w[x, y,z], x] + y*(c*z-a*x)*D[w[x, y,z], y] + z*(a*x - b*y)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to c_1\left(-\frac{cxyz}{b}, \frac{ax + by + cz}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   x*(b*y -c*z)*diff(w(x,y,z),x)+ y*(c*z-a*x)*diff(w(x,y,z),y)+z*(a*x - b*y)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x, y, z) = \frac{c_4 c_5 x^{c_2} y^{c_2} z^{c_2} e^{c_2} e^{-c_1 x} e^{-\frac{c_1 by}{a}} e^{-\frac{c_1 cz}{a}}}{c_3}$$

6.6.2.25 [1412] Problem 25

problem number 1412

Added April 14, 2019.

Problem Chapter 6.2.2.25, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a(y+\beta)(z+\gamma)w_x - b(x+\alpha)(z+\gamma)w_y - c(x+\alpha)(y+\beta)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =   a*(y+beta)*(z+gamma)*D[w[x, y,z], x] -b*(x+alpha)*(z+gamma)*D[w[x, y,z], y] - c*(x+a
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{ay(2\beta+y)+2\alpha bx+bx^2}{2a}, \frac{az(2\gamma+z)+2\alpha cx+cx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=   a*(y+beta)*(z+gamma)*diff(w(x,y,z),x)-b*(x+alpha)*(z+gamma)*diff(w(x,y,z),y)- c*(x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_2c_4c_5e^{c_1\beta y}e^{c_3\alpha x}e^{\frac{c_3\gamma az}{c}}e^{\frac{c_1y^2}{2}}e^{\frac{c_3x^2}{2}}e^{-\frac{c_1bz^2}{2c}}e^{\frac{c_3az^2}{2c}}e^{-\frac{c_1\gamma bz}{c}}$$

6.6.2.26 [1413] Problem 26

problem number 1413

Added April 14, 2019.

Problem Chapter 6.2.2.26, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bc(acxz+b^2y^2)w_x + ac(bcyz-2a^2x^2)w_y - ab(2abxy+c^2z^2)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   b*c*(a*c*x*z + b^2*y^2)*D[w[x, y,z], x] +a*c*(b*c*y*z-2*a^2*x^2)*D[w[x, y,z], y] - a
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=   b*c*(a*c*x*z + b^2*y^2)*diff(w(x,y,z),x)+a*c*(b*c*y*z-2*a^2*x^2)*diff(w(x,y,z),y)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.2.27 [1414] Problem 27

problem number 1414

Added April 14, 2019.

Problem Chapter 6.2.2.27, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a(y^2 + z^2)w_x + x(bz - ay)w_y - x(by + az)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   a*(y^2+z^2)*D[w[x, y,z], x] +x*(b*z-a*y)*D[w[x, y,z], y] -x*(b*y + a*z)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*(y^2+z^2)*diff(w(x,y,z),x)+x*(b*z-a*y)*diff(w(x,y,z),y)-x*(b*y + a*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1\_F5\left(-\frac{-2a\arctan\left(\frac{z}{y}\right) + b\ln\left(y^2+z^2\right)}{2b}\right)\mathrm{e}^{\frac{x^2\_c_1}{2}}\mathrm{e}^{-a\_c_1\left(\int^y -\frac{}{a\cos\left(2\,\mathrm{RootOf}\left(2\_Za-2a\arctan\left(\frac{z}{y}\right)-b\ln\left(\frac{}{c}\right.\right.\right.}\right.}$$

6.6.2.28 [1415] Problem 28

problem number 1415

Added April 14, 2019.

Problem Chapter 6.2.2.28, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b(by + cz)^2 w_x - ax(by + 2cz)w_y + abxzw_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b*(b*y + c*z)^2*D[w[x, y,z], x] - a*x*(b*y + 2*c*z)*D[w[x, y,z], y] +a*b*x*z*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z) \to c_1\left(\frac{2(ax^2 + c^2z^2)}{b}, \log(z(by + cz))\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{2(ax^2 + c^2z^2)}{b}, \log(z(cz - by))\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{2(ax^2 + (by - cz)^2)}{b}, \log(z(cz - by))\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{2(ax^2 + (by + cz)^2)}{b}, \log(z(by + cz))\right)\right\}$$

Maple ✓

```
restart;
pde :=  b*(b*y + c*z)^2*diff(w(x,y,z),x)- a*x*(b*y + 2*c*z)*diff(w(x,y,z),y)+a*b*x*z*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1\_F5\left(\frac{(by + cz)\,z}{b}\right) e^{\frac{x^2\_c_1}{2}} e^{\frac{b^2y^2\_c_1}{2a}} e^{\frac{bcyz\_c_1}{2a}}$$

6.6.2.29 [1416] Problem 29

problem number 1416

Added April 14, 2019.

Problem Chapter 6.2.2.29, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(f_0 x - f_1) w_x + (f_0 y - f_2) w_y + (f_0 z - f_3) w_z = 0$$

Where

$$f_n = a_n + b_n x + c_n y + d_n z$$

Mathematica ✗

```
ClearAll["Global`*"];
f[n_]:= a[n] + b[n]*x + c[n]*y+ d[n]*z;
pde = (f[0]*x - f[1])*D[w[x, y,z], x] +(f[0]*y-f[2])*D[w[x, y,z], y] +(f[0]*z -f[3])*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
f:= n ->  a[n] + b[n]*x + c[n]*y+ d[n]*z;
pde :=  (f(0)*x - f(1))*diff(w(x,y,z),x)+(f(0)*y-f(2))*diff(w(x,y,z),y)+(f(0)*z -f(3))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.3 2.3

Local contents

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6.6.3.1 [1417] Problem 1

problem number 1417

Added April 15, 2019.

Problem Chapter 6.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$2b^2xzw_x + by(b^2z^2 + 1)w_y + axy(bz + 1)^2w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = 2*b^2*x*z*D[w[x, y,z], x] + b*y*(b^2*z^2 +1)*D[w[x, y,z], y] + a*x*y*(b*z +1)^2*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
pde :=  2*b^2*x*z*diff(w(x,y,z),x)+b*y*(b^2*z^2 +1)*diff(w(x,y,z),y)+a*x*y*(b*z +1)^2*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.3.2 [1418] Problem 2

problem number 1418

Added April 15, 2019.

Problem Chapter 6.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$bcxy^2 w_x + 2bcy^3 w_y + 2(cyz - ax^2)^2 w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = b*c*x*y^2*D[w[x, y,z], x] + 2*b*c*y^3*D[w[x, y,z], y] + 2*(c*y*z-a*x^2)^2*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to c_1\left(\frac{y}{x^2}, \frac{x^4(\log(x)\,(2cyz - 2ax^2) + by)}{bcy^2\,(ax^2 - cyz)}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y^2*diff(w(x,y,z),x)+2*b*c*y^3*diff(w(x,y,z),y)+2*(c*y*z-a*x^2)^2*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{y}{x^2}, \frac{by + (-2ax^2 + 2cyz)\ln(x)}{2ax^2 - 2cyz}\right)$$

6.6.3.3 [1419] Problem 3

problem number 1419

Added April 15, 2019.

Problem Chapter 6.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$bc^2y^2zw_x + ac^2xz^2w_y - abxy^2w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b*c*x*y^2*D[w[x, y,z], x] + a*c^2*x*z^2*D[w[x, y,z], y] - a*b*x*y^2*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{ax}{c} + z, \frac{by^3 + c^2z^3}{3b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := b*c*x*y^2*diff(w(x,y,z),x)+a*c^2*x*z^2*diff(w(x,y,z),y)- a*b*x*y^2*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax + cz}{c}, \frac{-a^3x^3 - 3a^2cx^2z - 3ac^2xz^2 + bcy^3}{bc}\right)$$

6.6.3.4 [1420] Problem 4

problem number 1420

Added April 15, 2019.

Problem Chapter 6.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(by^2 - cz^2)w_x + y(cz^2 - ax^2)w_y + z(ax^2 - by^2)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(b*y^2-c*z^2)*D[w[x, y,z], x] + y*(c*z^2-a*x^2)*D[w[x, y,z], y] + z*(a*x^2-b*y^2)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=x*(b*y^2-c*z^2)*diff(w(x,y,z),x)+ y*(c*z^2-a*x^2)*diff(w(x,y,z),y) + z*(a*x^2-b*y^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3 c_4 c_5 x^{\frac{c_2}{2}} y^{\frac{c_2}{2}} z^{\frac{c_2}{2}} \mathrm{e}^{\frac{c_2}{4}} \mathrm{e}^{-\frac{c_1 x^2}{4}} \mathrm{e}^{-\frac{c_1 b y^2}{4a}} \mathrm{e}^{-\frac{c_1 c z^2}{4a}}$$

6.6.3.5 [1421] Problem 5

problem number 1421

Added April 15, 2019.

Problem Chapter 6.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$by(3ax^2 + by^2 + cz^2)w_x - 2ax(ax^2 + cz^2)w_y + 2abxyzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*y*(3*a*x^2+ b*y^2+c*z^2)*D[w[x, y,z], x] - 2*a*x*(a*x^2+c*z^2)*D[w[x, y,z], y] + 2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := b*y*(3*a*x^2+ b*y^2+c*z^2)*diff(w(x,y,z),x)- 2*a*x*(a*x^2+c*z^2)*diff(w(x,y,z),y) + 2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.3.6 [1422] Problem 6

problem number 1422

Added April 15, 2019.

Problem Chapter 6.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$b(a(a^2x^2 + b^2y^2 - 1)x + by)w_x + a(b(a^2x^2 + b^2y^2 - 1)y - ax)w_y + 2abzw_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*D[w[x, y,z], x] +a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*diff(w(x,y,z),x)+a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a^2x^2+b^2y^2-1)\,\mathrm{e}^{2\arctan\left(\frac{by}{ax}\right)}}{a^2x^2+b^2y^2}, z\,\mathrm{e}^{2\left(\int^x \frac{}{\left(-\_a^2a^2+\cos^2\left(\mathrm{RootOf}\left(2\_Z-\ln\left(-\frac{\left(a^2x^2+b^2y^2-1\right)\_a^2a^2\mathrm{e}^{2\arctan\left(\right.}}{\left(a^2x^2+b^2y^2\right)\left(-\_a^2a^2+\cos^2\_\right.}\right.\right.\right.\right.}\right.}\right.$$

6.6.3.7 [1423] Problem 7

problem number 1423

Added April 15, 2019.

Problem Chapter 6.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$x(b^3y^3 - 2a^3x^3)w_x + y(2b^3y^3 - a^3x^3)w_y + 9z(a^3x^3 - b^3y^3)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(b^3*y^3 - 2*a^3*x^3)*D[w[x, y,z], x] +y*(2*b^3*y^3 -a^3*x^3)*D[w[x, y,z], y] +9*z*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde := x*(b^3*y^3 - 2*a^3*x^3)*diff(w(x,y,z),x)+y*(2*b^3*y^3 -a^3*x^3)*diff(w(x,y,z),y) + 9*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.6.3.8 [1424] Problem 8

problem number 1424

Added April 15, 2019.

Problem Chapter 6.2.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2(abxy - c^2z^2)w_x + axy(abxy - c^2z^2)w_y + byz(bcyz + 2a^2x^2)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*(a*b*x*y-c^2*z^2)*D[w[x, y,z], x] +a*x*y*(a*b*x*y-c^2*z^2)*D[w[x, y,z], y] +b*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{\log\left(\frac{xz}{a^2bx^2y+ac^2xz^2+b^2cy^2z}\right)}{a}\right)\right\}\right\}$$

Maple ✗

```
restart;
pde := a*x^2*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),x)+a*x*y*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),y) + b
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.3.9 [1425] Problem 9

problem number 1425

Added April 15, 2019.

Problem Chapter 6.2.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(cz^4-by^4)w_x+y(ax^4-2cz^4)w_y+z(2by^4-ax^4)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(c*z^4 - b*y^4)*D[w[x, y,z], x] +y*(a*x^4-2*c*z^4)*D[w[x, y,z], y] +z*(2*b*y^4-a*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := x*(c*z^4 - b*y^4)*diff(w(x,y,z),x)+y*(a*x^4-2*c*z^4)*diff(w(x,y,z),y) + z*(2*b*y^4-a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_3 c_4 c_5 x^{\frac{c_2}{4}} y^{\frac{c_2}{8}} z^{\frac{c_2}{8}} \mathrm{e}^{\frac{c_2}{16}} \mathrm{e}^{-\frac{c_1 x^4}{16}} \mathrm{e}^{-\frac{c_1 b y^4}{16a}} \mathrm{e}^{-\frac{c_1 c z^4}{16a}}$$

6.6.3.10 [1426] Problem 10

problem number 1426

Added April 15, 2019.

Problem Chapter 6.2.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x w_x + y w_y + a\sqrt{x^2+y^2} w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, z - a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + a*sqrt(x^2+y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, -\sqrt{x^2+y^2}\, a + z\right)$$

6.6.3.11 [1427] Problem 11

problem number 1427

Added April 15, 2019.

Problem Chapter 6.2.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + (z - a\sqrt{x^2+y^2+z^2})w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right) - 2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)} + x^{2a+2}}{x^2+y^2}}\right)\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \frac{1}{2}\log\left(\frac{x^{2a}(y^2+2z^2) - 2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)} + x^{2a+2}}{x^2+y^2}\right)\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right) + 2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)} + x^{2a+2}}{x^2+y^2}}\right)\right)\right\}$$
$$\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \frac{1}{2}\log\left(\frac{x^{2a}(y^2+2z^2) + 2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)} + x^{2a+2}}{x^2+y^2}\right)\right)\right\}$$

Maple ✓

```
restart;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + (z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \left(z + \sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)$$

6.6.3.12 [1428] Problem 12

problem number 1428

Added April 15, 2019.

Problem Chapter 6.2.3.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$z\sqrt{y^2+z^2}w_x + az\sqrt{x^2+z^2}w_y - (x\sqrt{y^2+z^2} + ay\sqrt{x^2+z^2})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  z*Sqrt[y^2+z^2]*D[w[x, y,z], x] +a*z*Sqrt[x^2+z^2]*D[w[x, y,z], y] -(x*Sqrt[y^2+z^2]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := z*sqrt(y^2+z^2)*diff(w(x,y,z),x)+a*z*sqrt(x^2+z^2)*diff(w(x,y,z),y) -(x*sqrt(y^2+z^2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.3.13 [1429] Problem 13

problem number 1429

Added April 15, 2019.

Problem Chapter 6.2.3.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(y-z)\sqrt{f(x)}w_x + (z-x)\sqrt{f(y)}w_y + (x-y)\sqrt{f(z)}w_z = 0$$

Where

$$f(t) = a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0$$

Mathematica ✗

```
ClearAll["Global`*"];
f[t_]:= a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
pde =  (y-z)*Sqrt[f[x]]*D[w[x, y,z], x] +(z-x)*Sqrt[f[y]]*D[w[x, y,z], y] +(x-y)*Sqrt[f[z]]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
f := t-> a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
pde :=  (y-z)*sqrt(f(x))*diff(w(x,y,z),x)+(z-x)*sqrt(f(y))*diff(w(x,y,z),y)+(x-y)*sqrt(f(z))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.6.4 2.4

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6.6.4.1 [1430] Problem 1

problem number 1430

Added May 18, 2019.

Problem Chapter 6.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +a*x^n*y^m*D[w[x, y,z], y] +b*x^nu *y^mu* z^lambda *D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{ax^{n+1}}{n+1}-(m-1)^{\frac{1}{m-1}}y\left(\frac{(m-1)^{\frac{1}{1-m}}}{y}\right)^m,-\frac{b(m-1)^{\frac{\mu}{1-m}}x^{\nu+1}\left((m-1)^{\frac{1}{m-1}}y\left(\frac{(m-1)}{y}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(m-1)\,ax^{n+1}+(n+1)\,y^{-m+1}}{n+1},(\lambda-1)\,b\left(\int^x \_a^{\nu}\left(\left(\frac{(-\_a^{n+1}+x^{n+1})\,(m-1)\,a+(}{n+1}\right.\right.\right.\right.$$

6.6.4.2 [1431] Problem 2

problem number 1431

Added May 18, 2019.

Problem Chapter 6.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}y+b_2x^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m1)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},-\int_1^x\frac{\left((-1)^{-\frac{\mathrm{n2}}{\mathrm{n1}}}\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(a1x^{m1+1}+(m1+n1+2)x^{m1-n1}\right)b1\left(\frac{a1x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\frac{n}{2}\right.}{}\right.$$

6.6.4.3 [1432] Problem 3

problem number 1432

Added May 18, 2019.

Problem Chapter 6.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2x^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m1)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},\mathrm{b2}(\mathrm{n2}+1)^{\frac{\mathrm{m1}-\mathrm{n2}}{\mathrm{n2}+1}}\mathrm{a}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(a1x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)b1\left(\frac{a1x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\frac{n}{2}\right.}{}\right.$$

6.6.4.4 [1433] Problem 4

problem number 1433

Added May 18, 2019.

Problem Chapter 6.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2y^{m_1})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*y^m1)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},ze^{-\frac{\mathrm{a2}x^{\mathrm{n2}+1}}{\mathrm{n2}+1}}-\int_1^x\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*y^m1)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(a1x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)b1\left(\frac{a1x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1-2}{2n1+2}}\,\mathrm{WhittakerM}\left(\frac{n}{2}\right.}{}\right.$$

6.6.4.5 [1434] Problem 5

problem number 1434

Added May 18, 2019.

Problem Chapter 6.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1}y^k1)w_y+(a_2x^{n_2}z+b_2x^{m_2}z^{k_2})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(a1*x^n1*y+b1*x^m1*y^k1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2*z^k1)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(-1)^{\frac{\mathrm{n1}-\mathrm{m1}}{\mathrm{n1}+1}}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}(\mathrm{k1}-1)^{\frac{\mathrm{n1}-\mathrm{m1}}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},-\frac{\mathrm{a1}(\mathrm{k1}-1)x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1*y^k1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2*z^k1)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n1+1)^2\left(-(k1-1)\,a1x^{m1+1}+(m1+n1+2)\,x^{m1-n1}\right)b1\left(-\frac{(k1-1)a1x^{n1+1}}{n1+1}\right)^{\frac{-m1-n1}{2n1+2}}}{}\right.$$

6.6.4.6 [1435] Problem 6

problem number 1435

Added May 18, 2019.

Problem Chapter 6.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^n w_x+by^m w_y+cz^l w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = a*x^n*D[w[x, y,z], x] +b*y^m*D[w[x, y,z], y] +c*z^L*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{\left(\frac{1}{y}\right)^{m-1}}{m-1},\frac{cx^{1-n}}{a(n-1)}-\frac{\left(\frac{1}{z}\right)^{L-1}}{L-1}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x^n*diff(w(x,y,z),x)+b*y^m*diff(w(x,y,z),y)+c*z^L*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(n-1)\,ay^{-m+1}-(m-1)\,bx^{-n+1}}{(n-1)\,a},\frac{(n-1)\,az^{-L+1}-(L-1)\,cx^{-n+1}}{(n-1)\,a}\right)$$

6.6.4.7 [1436] Problem 7

problem number 1436

Added May 18, 2019.

Problem Chapter 6.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ay^m w_x+bx^n w_y+cz^l w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y^m*D[w[x, y,z], x] +b*x^n*D[w[x, y,z], y] +c*z^L*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y^m*diff(w(x,y,z),x)+b*x^n*diff(w(x,y,z),y)+c*z^L*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(n+1)\,ay^{m+1}-(m+1)\,bx^{n+1}}{(n+1)\,a}, \frac{az^{-L+1}+(L-1)\,c\left(\int^{x}\left(\left(\frac{(n+1)ay^{m+1}+(m+1)b\_a^{n+1}-(m+}{(n+1)a}\right.\right.\right.}{a}\right.$$

6.6.4.8 [1437] Problem 8

problem number 1437

Added May 18, 2019.

Problem Chapter 6.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$x(y^n-z^n)w_x+y(z^n-x^n)w_y+z(x^n-y^n)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*(y^n-z^n)*D[w[x, y,z], x] +y*(z^n-x^n)*D[w[x, y,z], y] +z*(x^n-y^n)*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  x*(y^n-z^n)*diff(w(x,y,z),x)+y*(z^n-x^n)*diff(w(x,y,z),y)+z*(x^n-y^n)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_4c_5x^{\frac{c_1}{n}}(y^n)^{\frac{c_1}{n^2}}(z^n)^{\frac{c_1}{n^2}}\,\mathrm{e}^{\frac{c_1}{n^2}}\mathrm{e}^{-\frac{c_2}{n}}\mathrm{e}^{-\frac{c_3x^n}{n^2}}\mathrm{e}^{-\frac{c_3y^n}{n^2}}\mathrm{e}^{-\frac{c_3z^n}{n^2}}$$

6.6.5 3.1

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6.6.5.1 [1438] Problem 1

problem number 1438

Added May 18, 2019.

Problem Chapter 6.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + be^{\alpha x} w_y + ce^{\beta y} w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[beta*y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{be^{\alpha x}}{a\alpha}, z - \frac{c\text{Ei}\left(\frac{b\beta e^{\alpha x}}{a\alpha}\right) e^{\beta\left(y - \frac{be^{\alpha x}}{a\alpha}\right)}}{a\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(beta*y)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha y - b\,\mathrm{e}^{\alpha x}}{a\alpha}, \frac{a\alpha z + c\,\mathrm{expIntegral}\left(1, -\frac{b\beta\,\mathrm{e}^{\alpha x}}{a\alpha}\right)\mathrm{e}^{\frac{(a\alpha y - b\,\mathrm{e}^{\alpha x})\beta}{a\alpha}}}{a\alpha}\right)$$

6.6.5.2 [1439] Problem 2

problem number 1439

Added May 18, 2019.

Problem Chapter 6.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} - \frac{e^{-\gamma z}}{\gamma}, y - \frac{be^{\alpha x}}{a\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha y - b\,\mathrm{e}^{\alpha x}}{a\alpha}, \frac{-a\,\mathrm{e}^{-\gamma z} - \gamma c x}{\gamma c}\right)$$

6.6.5.3 [1440] Problem 3

problem number 1440

Added May 18, 2019.

Problem Chapter 6.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + be^{\beta y} w_y + ce^{\gamma z} w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] +b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{bx}{a} - \frac{e^{-\beta y}}{\beta}, -\frac{cx}{a} - \frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x - a\,\mathrm{e}^{-\beta y}}{b\beta}, \frac{-a\,\mathrm{e}^{-\gamma z} - \gamma c x}{\gamma c}\right)$$

6.6.5.4 [1441] Problem 4

problem number 1441

Added May 18, 2019.

Problem Chapter 6.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (A_1 e^{\alpha_1 x} + B_1 e^{\nu_1 x + \lambda y}) w_y + (A_2 e^{\alpha_2 x} + B_2 e^{\nu_2 x + \beta y}) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +(A1*Exp[alpha1*x]+B1*Exp[nu1*x+lambda*y])*D[w[x, y,z], y] +(A2*Exp[al
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+(A1*exp(alpha1*x)+B1*exp(nu1*x+lambda*y))*diff(w(x,y,z),y)+(A2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-B1\lambda\left(\int e^{\frac{A1\lambda e^{\alpha 1 x}}{\alpha 1}+\nu 1 x}dx\right) - e^{\frac{\left(A1\, e^{\alpha 1 x}-\alpha 1 y\right)\lambda}{\alpha 1}}}{\lambda}, z - \left(\int^{x}\left(B2\left(-B1\lambda\left(\int e^{\frac{A1\lambda e^{-b\alpha 1}+\_b\alpha 1\nu 1}{\alpha 1}}\right.\right.\right.\right.\right.$$

6.6.5.5 [1442] Problem 5

problem number 1442

Added May 18, 2019.

Problem Chapter 6.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a e^{\alpha x} w_x + b e^{\beta y} w_y + c e^{\gamma z} w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] +b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{be^{-\alpha x}}{a\alpha}-\frac{e^{-\beta y}}{\beta},\frac{ce^{-\alpha x}}{a\alpha}-\frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta},-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x}-\gamma c\,\mathrm{e}^{\gamma z}\right)\mathrm{e}^{-\alpha x-\gamma z}}{\gamma\alpha c}\right)$$

6.6.5.6 [1443] Problem 6

problem number 1443

Added May 18, 2019.

Problem Chapter 6.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\beta y}w_x+be^{\alpha x}w_y+ce^{\gamma z}w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*y]*D[w[x, y,z], x] +b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{\beta y}}{\beta}-\frac{be^{\alpha x}}{a\alpha},-\frac{c\gamma\log\left(\frac{a\alpha e^{\beta y}}{\beta}\right)-a\alpha e^{\beta y-\gamma z}+b\beta e^{\alpha x-\gamma z}-\alpha c\gamma x}{b\beta\gamma e^{\alpha x}-a\alpha\gamma e^{\beta y}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*exp(beta*y)*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\alpha\,\mathrm{e}^{\beta y}-b\beta\,\mathrm{e}^{\alpha x}}{\alpha b\beta},-\frac{\left(\gamma\alpha cx-\gamma c\ln\left(\frac{a\alpha\,\mathrm{e}^{\beta y}}{b\beta}\right)+\left(a\alpha\,\mathrm{e}^{\beta y}-b\beta\,\mathrm{e}^{\alpha x}\right)\mathrm{e}^{-\gamma z}\right)b\beta}{\gamma\left(a\alpha\,\mathrm{e}^{\beta y}-b\beta\,\mathrm{e}^{\alpha x}\right)\alpha c}\right)$$

6.6.5.7 [1444] Problem 7

problem number 1444

Added May 18, 2019.

Problem Chapter 6.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1+a_2e^{\alpha x})w_x+(b_1+b_2e^{\beta y}w_y+(c_1+c_2e^{\gamma z})w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] +(b1+b2*Exp[beta*y])*D[w[x, y,z], y] +(c1+c2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\frac{e^{\beta y}(\text{a1}+\text{a2}e^{\alpha x})^{\frac{\text{b1}\beta}{\text{a1}\alpha}}}{\text{b1}+\text{b2}e^{\beta y}}\right)}{\text{b1}\beta}-\frac{x}{\text{a1}},\frac{\log\left(\frac{e^{\gamma z}(\text{a1}+\text{a2}e^{\alpha x})^{\frac{\text{c1}\gamma}{\text{a1}\alpha}}}{\text{c1}+\text{c2}e^{\gamma z}}\right)}{\text{c1}\gamma}-\frac{x}{\text{a1}}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+(b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a1\,\alpha\,\mathrm{RootOf}\left(a1\,\alpha\beta y-a1\,\alpha\ln\left(\frac{(-b1+\mathrm{e}^{\_Z})(a2\,\mathrm{e}^{\alpha x}+a1)^{\frac{b1\beta}{a1\alpha}}}{b2}\right)+b1\,\beta\ln\left(a2\,\mathrm{e}^{\alpha x}+a1\right)\right)-}{a1\,\alpha\,b1\,\beta}\right.$$

6.6.5.8 [1445] Problem 8

problem number 1445

Added May 18, 2019.

Problem Chapter 6.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$e^{\beta y}(a_1+a_2e^{\alpha x})w_x+e^{\alpha x}(b_1+b_2e^{\beta y}w_y+ce^{\beta y+\gamma z}w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] +Exp[alpha*x]*(b1+b2*Exp[beta*y])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\log\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)}{\mathrm{a1}\alpha}-\frac{cx}{\mathrm{a1}}-\frac{e^{-\gamma z}}{\gamma},\frac{\log\left(\mathrm{b1}+\mathrm{b2}e^{\beta y}\right)}{\mathrm{b2}\beta}-\frac{\log\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)}{\mathrm{a2}\alpha}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+exp(alpha*x)*(b1+b2*exp(beta*y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a2\alpha\beta y+a2\alpha\,\mathrm{RootOf}\left(a2\alpha\beta y-a2\alpha\ln\left(\frac{b1\left(a2\,\mathrm{e}^{\alpha x}+a1\right)^{-\frac{b2\beta}{a2\alpha}}}{-b2+\mathrm{e}^{-Z}}\right)-b2\beta\ln\left(a2\,\mathrm{e}^{\alpha x}+a1\right)\right)}{a2\alpha b2\beta}\right.$$

6.6.6 3.2

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6.6.6.1 [1446] Problem 1

problem number 1446

Added May 18, 2019.

Problem Chapter 6.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aye^{\alpha x} w_x + be^{\beta y} w_y + ce^{\gamma z} w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left((\beta y+1)\,a\alpha\,\mathrm{e}^{\alpha x-\beta y}-b\beta^2\right)\mathrm{e}^{-\alpha x}}{\alpha b\beta^2}, \frac{\left(\gamma\alpha b\beta^2 c\left(\int^x -\frac{}{((\beta y+1)a\alpha\,\mathrm{e}^{-(-\_a+x)\alpha}\mathrm{e}^{\alpha x-\beta y}-(\mathrm{e}^{-(-\_a+x)\alpha}}\right.\right.}{}\right.$$

6.6.6.2 [1447] Problem 2

problem number 1447

Added May 18, 2019.

Problem Chapter 6.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aye^{\alpha x} w_x + be^{\beta y} w_y + cze^{\gamma z} w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*y*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*z*Exp[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*y*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*z*exp(gamma*z)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left((\beta y+1)\,a\alpha\,\mathrm{e}^{\alpha x-\beta y}-b\beta^2\right)\mathrm{e}^{-\alpha x}}{\alpha b\beta^2}, \frac{b\beta\,\mathrm{expIntegral}\left(1,\gamma z\right)\mathrm{LambertW}\left(-(\beta y+1)\,\mathrm{e}^{-\alpha x-}\right.}{\alpha b\beta^2 c\,\mathrm{LambertW}\left(-\left(\beta y+1\right)\mathrm{e}\right.}\right.$$

6.6.6.3 [1448] Problem 3

problem number 1448

Added May 18, 2019.

Problem Chapter 6.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[y^2 + ae^{\alpha x}(\alpha - ae^{\alpha x})\right] w_y + \left[z^2 + bz + ce^{\beta x}(\beta - b - ce^{\beta x})\right] w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ a*Exp[alpha*x]*(alpha-a*Exp[alpha*x]))*D[w[x, y,z], y] +(z^2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\mathrm{Ei}\left(\frac{2ae^{\alpha x}}{\alpha}\right)\left(y-ae^{\alpha x}\right)+\alpha e^{\frac{2ae^{\alpha x}}{\alpha}}}{ae^{\alpha x}-y},\frac{2^{b/\beta}\beta^{-\frac{b}{\beta}}e^{bx}c^{b/\beta}\left(\left(b-ce^{\beta x}+z\right)\mathrm{LaguerreL}\left(-\frac{b}{\beta},\frac{b}{\beta},\frac{2c}{}\right.\right.}{ce^{\beta x}-z}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ a*exp(alpha*x)*(alpha-a*exp(alpha*x)))*diff(w(x,y,z),y)+(z^2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\alpha x}+y}{\alpha\,\mathrm{e}^{\frac{2a\,\mathrm{e}^{\alpha x}}{\alpha}}+\left(a\,\mathrm{e}^{\alpha x}-y\right)\mathrm{expIntegral}\left(1,-\frac{2a\,\mathrm{e}^{\alpha x}}{\alpha}\right)},\frac{\left(-c\,\mathrm{e}^{\beta x}+z\right)\left(\int\mathrm{e}^{\frac{b\beta x+2c\,\mathrm{e}^{\beta x}}{\beta}}dx\right)+\mathrm{e}^{\frac{b\beta x+2}{\beta}}}{c\,\mathrm{e}^{\beta x}-z}\right.$$

6.6.6.4 [1449] Problem 4

problem number 1449

Added May 18, 2019.

Problem Chapter 6.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left[y^2+by+ae^{\alpha x}(\alpha-b-ae^{\alpha x})\right]w_y+\left[z^2+ce^{\beta x}(z-k)-k^2\right]w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ b*y + a*Exp[alpha*x]*(alpha-b-a*Exp[alpha*x]))*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(2k(-1)^{-\frac{k}{\beta}}\left(-\frac{\text{Gamma}\left(\frac{2k}{\beta},0,-\frac{ce^{\beta x}}{\beta}\right)}{\beta}+\frac{\beta^{-\frac{2k}{\beta}}c^{\frac{2k}{\beta}}e^{\frac{ce^{\beta x}+2\beta kx+2i\pi k}{\beta}}}{k-z}\right),\frac{2^{\frac{b}{\alpha}}\alpha^{-\frac{b}{\alpha}}e^{bx}a^{\frac{b}{\alpha}}\left((a(-\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ b*y+a*exp(alpha*x)*(alpha-b-a*exp(alpha*x)))*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(-a\,\mathrm{e}^{\alpha x}+y)\left(\int \mathrm{e}^{\frac{\alpha bx+2a\,\mathrm{e}^{\alpha x}}{\alpha}}dx\right)+\mathrm{e}^{\frac{\alpha bx+2a\,\mathrm{e}^{\alpha x}}{\alpha}}}{a\,\mathrm{e}^{\alpha x}-y},\frac{(k-z)\left(\int \mathrm{e}^{\frac{2\beta kx+c\,\mathrm{e}^{\beta x}}{\beta}}dx\right)-\mathrm{e}^{\frac{2\beta kx+c\,\mathrm{e}^{\beta x}}{\beta}}}{k-z}\right)$$

6.6.6.5 [1450] Problem 5

problem number 1450

Added May 18, 2019.

Problem Chapter 6.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left[y^2+by+ae^{\alpha x}(y-b)-b^2\right]w_y+\left[z^2+c(xz-1)e^{\beta x}\right]w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+ b*y + a*Exp[alpha*x]*(y-b)-b^2)*D[w[x, y,z], y] +(z^2 +c*(x*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2+ b*y+a*exp(alpha*x)*(y-b)-b^2)*diff(w(x,y,z),y)+(z^2 +c*(x*z-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.6 [1451] Problem 6

problem number 1451

Added May 18, 2019.

Problem Chapter 6.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[y^2 - ae^{\alpha x}(xy - 1)\right] w_y + \left(ce^{\beta x} z^2 + be^{-\beta x}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2- a*Exp[alpha*x]*(x*y-1))*D[w[x, y,z], y] +(c*Exp[beta*x]*z^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( y^2- a*exp(alpha*x)*(x*y-1))*diff(w(x,y,z),y)+(c*exp(beta*x)*z^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.7 [1452] Problem 7

problem number 1452

Added May 18, 2019.

Problem Chapter 6.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(ay^2 e^{\alpha x} + be^{-\alpha x}\right) w_y + \left[de^{\beta x} z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x})\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a*y^2- a*Exp[alpha*x]+ b * Exp[-alpha*x])*D[w[x, y,z], y] +(d*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+( a*y^2- a*exp(alpha*x)+ b * exp(-alpha*x))*diff(w(x,y,z),y)+(d*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.8 [1453] Problem 8

problem number 1453

Added May 18, 2019.

Problem Chapter 6.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left[be^{\alpha x}y^2 + ae^{\beta x}(\beta - abe^{(\alpha+\beta)x})\right] w_y + \left(cz^2e^{\gamma x} + dz + ke^{-\gamma x}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (b*Exp[alpha*x]*y^2 + a*Exp[beta*x]*(beta- a*b*Exp[(alpha+beta)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (b*exp(alpha*x)*y^2 + a*exp(beta*x)*(beta- a*b*exp((alpha+beta)*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.9 [1454] Problem 9

problem number 1454

Added May 18, 2019.

Problem Chapter 6.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(ae^{\alpha x}y^2 + by + ce^{\alpha x}\right) w_y + \left(e^{\beta x}z^2 + de^{\gamma x}(z + \beta e^{-\beta x})\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[alpha*x]*y^2 + b*y +  c*Exp[alpha*x])*D[w[x, y,z], y] +(Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*exp(alpha*x)*y^2 + b*y +  c*exp(alpha*x))*diff(w(x,y,z),y)+(exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\sqrt{a}\,y\,\mathrm{BesselJ}\left(-\frac{\alpha+b}{2\alpha},\frac{\sqrt{a}\sqrt{c}\,\mathrm{e}^{\alpha x}}{\alpha}\right)+\sqrt{c}\,\mathrm{BesselJ}\left(\frac{\alpha-b}{2\alpha},\frac{\sqrt{a}\sqrt{c}\,\mathrm{e}^{\alpha x}}{\alpha}\right)}{\sqrt{a}\,y\,\mathrm{BesselY}\left(-\frac{\alpha+b}{2\alpha},\frac{\sqrt{a}\sqrt{c}\,\mathrm{e}^{\alpha x}}{\alpha}\right)-\sqrt{c}\,\mathrm{BesselY}\left(\frac{\alpha-b}{2\alpha},\frac{\sqrt{a}\sqrt{c}\,\mathrm{e}^{\alpha x}}{\alpha}\right)},-\frac{}{\beta d\,\mathrm{hypergeom}\left(\left[\frac{-\beta}{}\right.\right.}\right.$$

6.6.6.10 [1455] Problem 10

problem number 1455

Added May 18, 2019.

Problem Chapter 6.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left[e^{\alpha x}y^2 + aye^{\beta x} + a\alpha e^{(\beta-\alpha)x}\right]w_y + \left[\gamma e^{\gamma x}z^2 + be^{\delta x}(z+e^{-\gamma x})\right]w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( Exp[alpha*x]*y^2 + a*y*Exp[beta*x] + a*alpha*Exp[(beta-alpha)*x])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( exp(alpha*x)*y^2 + a*y*exp(beta*x) + a*alpha*exp((beta-alpha)*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(-\alpha+\beta)\left(y\,\mathrm{e}^{\alpha x}+\alpha\right)\mathrm{e}^{\alpha x}}{a\alpha\,\mathrm{hypergeom}\left(\left[\frac{-\alpha+\beta}{\beta}\right],\left[\frac{-\alpha+2\beta}{\beta}\right],\frac{a\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{\beta x}-(-\alpha+\beta)\,y\,\mathrm{hypergeom}\left(\left[-\frac{\alpha}{\beta}\right],\left[\frac{-\alpha+\beta}{\beta}\right]\right.}\right.$$

6.6.6.11 [1456] Problem 11

problem number 1456

Added May 18, 2019.

Problem Chapter 6.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left[\alpha e^{\alpha x}y^2 + ae^{\beta x}(y+e^{-\alpha x})\right]w_y + \left[e^{\gamma x}(z-be^{\delta x})^2 + b\delta e^{\delta x}\right]w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( alpha*Exp[alpha*x]*y^2 + a*Exp[beta*x]*(y+Exp[-alpha*x]))*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( alpha*exp(alpha*x)*y^2 + a*exp(beta*x)*(y+exp(-alpha*x)))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(-\alpha+\beta)\,(y\,\mathrm{e}^{\alpha x}+1)\,\mathrm{e}^{\alpha x}}{a\,\mathrm{hypergeom}\left(\left[\frac{-\alpha+\beta}{\beta}\right],\left[\frac{-\alpha+2\beta}{\beta}\right],\frac{a\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{\beta x}-(-\alpha+\beta)\,y\,\mathrm{hypergeom}\left(\left[-\frac{\alpha}{\beta}\right],\left[\frac{-\alpha+\beta}{\beta}\right],\right.}\right.$$

6.6.6.12 [1457] Problem 12

problem number 1457

Added May 18, 2019.

Problem Chapter 6.3.2.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + \left(a_1 e^{\alpha x} y^2 + \beta y + a_1 b_2^2 x^{2\beta} e^{\alpha x}\right) w_y + \left[a_2 x^{2n} z^2 e^{\lambda x} + (b_2 x^n e^{\lambda x} - n) z + c e^{\lambda x}\right] w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + ( a1*Exp[alpha*x]*y^2 + beta*y+ a1*b2^2*x^(2*beta)*Exp[alpha*x])*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  x*diff(w(x,y,z),x)+ ( a1*exp(alpha*x)*y^2 + beta*y+ a1*b2^2*x^(2*beta)*exp(alpha*x))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.13 [1458] Problem 13

problem number 1458

Added May 18, 2019.

Problem Chapter 6.3.2.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k\right) w_y + \left(a_2 e^{\lambda_2 x} z + b_2 e^{\beta_1 x} z^m\right) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*Exp[lambda1*x]*y + b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +(a2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left((k-1)\int_1^x \mathrm{b1}e^{\frac{\mathrm{a1}e^{\mathrm{lambda1}K[1]}(k-1)}{\mathrm{lambda1}}+\mathrm{beta1}K[1]}dK[1] + y^{1-k}e^{\frac{\mathrm{a1}(k-1)e^{\mathrm{lambda1}x}}{\mathrm{lambda1}}}, (m-1)\int_1^x \mathrm{b2}e^{\frac{\mathrm{a2}}{}}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a1*exp(lambda1*x)*y + b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+(a2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\left((k-1)\,b1\,y^k\left(\int \mathrm{e}^{\frac{\beta 1\lambda 1x+(k-1)a1\,\mathrm{e}^{\lambda 1x}}{\lambda 1}}dx\right)\mathrm{e}^{\frac{a1\,\mathrm{e}^{\lambda 1x}}{\lambda 1}} + y\,\mathrm{e}^{\frac{a1\,k\,\mathrm{e}^{\lambda 1x}}{\lambda 1}}\right)y^{-k}\mathrm{e}^{-\frac{a1\,\mathrm{e}^{\lambda 1x}}{\lambda 1}}, \left((m-1)\,b2z\right.\right.$$

6.6.6.14 [1459] Problem 14

problem number 1459

Added May 18, 2019.

Problem Chapter 6.3.2.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(a_1 e^{\beta_1 x} y + b_1 e^{\gamma_1 x} y^k\right) w_y + \left(a_2 e^{\beta_2 x} + b_2 e^{\gamma_1 x + \lambda z}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*Exp[beta1*x]*y + b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +(a2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a1*exp(beta1*x)*y + b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+(a2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\left((k-1)\, b1 y^k \left(\int e^{\frac{\beta 1 \gamma 1 x + (k-1) a1\, e^{\beta 1 x}}{\beta 1}} dx\right) e^{\frac{a1\, e^{\beta 1 x}}{\beta 1}} + y\, e^{\frac{a1\, k\, e^{\beta 1 x}}{\beta 1}}\right) y^{-k} e^{-\frac{a1\, e^{\beta 1 x}}{\beta 1}}, \frac{-b2\lambda\left(\int e^{\frac{a2}{}}\right.}{}\right.$$

6.6.6.15 [1460] Problem 15

problem number 1460

Added May 18, 2019.

Problem Chapter 6.3.2.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(a_1 x^n + b_1 x^m e^{\lambda y}\right) w_y + \left(a_2 x^k + b_2 x^s e^{\beta z}\right) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] +( a1*x^n+b1*x^m*Exp[lambda*y])*D[w[x, y,z], y] +(a2*x^k+b2*x^2*Exp[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\mathrm{b2}\beta x^3\left(-\frac{\mathrm{a2}\beta x^{k+1}}{k+1}\right)^{-\frac{3}{k+1}}\mathrm{Gamma}\left(\frac{3}{k+1},-\frac{\mathrm{a2}\beta x^{k+1}}{k+1}\right)-(k+1)e^{-\frac{\beta\left(-\mathrm{a2}x^{k+1}+kz+z\right)}{k+1}}}{\mathrm{a2b2}\beta^2\left(k^2-k-2\right)},\frac{(n+1)e}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ ( a1*x^n+b1*x^m*exp(lambda*y))*diff(w(x,y,z),y)+(a2*x^k+b2*x^2*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(n+1)^2\left(-a1\lambda x^{m+1}+(m+n+2)x^{m-n}\right)b1\left(-\frac{a1\lambda x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}}\mathrm{WhittakerM}\left(\frac{m-n}{2n+2},\right.}{}\right.$$

6.6.6.16 [1461] Problem 16

problem number 1461

Added May 18, 2019.

Problem Chapter 6.3.2.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(ax^n e^{\lambda y} + bxy^m)w_x + e^{\mu y} w_y + \left(cy^l z^k + dy^p z\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  (a*x^n*Exp[lambda*y] + b*x*y^m)*D[w[x, y,z], x] +Exp[mu*y]*D[w[x, y,z], y] +(c*y^L*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  (a*x^n*exp(lambda*y) + b*x*y^m)*diff(w(x,y,z),x)+ exp(mu*y)*diff(w(x,y,z),y)+(c*y^L*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x^{\frac{1}{m+1}} x^{\frac{m}{m+1}} x^{-\frac{n}{m+1}} x^{-\frac{mn}{m+1}} \mathrm{e}^{\frac{bny^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)\mathrm{e}^{-\frac{\mu y}{2}}}{(m+1)\mu}} \mathrm{e}^{-\frac{by^m(\mu y)^{-\frac{m}{2}}\,\mathrm{WhittakerM}\left(\frac{m}{2},\frac{m}{2}+\frac{1}{2},\mu y\right)}{(m+1)\mu}}\right.$$

6.6.6.17 [1462] Problem 17

problem number 1462

Added May 18, 2019.

Problem Chapter 6.3.2.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 + 2a\alpha e^{\alpha x^2} - a^2 e^{2\alpha x^2}) w_y + \left(ce^{-2\beta x^2} z^2 + 2\beta xz + b^2 c\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2 + 2*a*alpha*Exp[alpha*x^2]-a^2*Exp[2*alpha*x^2] )*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (y^2 + 2*a*alpha*exp(alpha*x^2)-a^2*exp(2*alpha*x^2) )*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.6.18 [1463] Problem 18

problem number 1463

Added May 18, 2019.

Problem Chapter 6.3.2.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ae^{2\alpha x^2}y^2 + 2\alpha xy + ab^2)w_y + \left(cx^\beta z^2 + 2\gamma xz + cd^2x^\beta e^{2\gamma x^2}\right) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[2*alpha*x^2]*y^2 + 2*alpha*x*y + a*b^2 )*D[w[x, y,z], y] +(
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*exp(2*alpha*x^2)*y^2 + 2*alpha*x*y + a*b^2 )*diff(w(x,y,z),y)+(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.6.7 4.1

Local contents

6.6.7.1 [1464] Problem 1 2127
6.6.7.2 [1465] Problem 2 2128
6.6.7.3 [1466] Problem 3 2128
6.6.7.4 [1467] Problem 4 2129
6.6.7.5 [1468] Problem 5 2130
6.6.7.6 [1469] Problem 6 2131

6.6.7.1 [1464] Problem 1

problem number 1464

Added May 19, 2019.

Problem Chapter 6.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Sinh[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, z - \frac{c\cosh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*sinh(lambda*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \frac{a\lambda z - c\cosh(\lambda x)}{a\lambda}\right)$$

6.6.7.2 [1465] Problem 2

problem number 1465

Added May 19, 2019.

Problem Chapter 6.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(z - \frac{c\cosh(\lambda x)}{a\lambda}, \frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x - 2a\operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta}, \frac{a\lambda z - c\cosh(\lambda x)}{a\lambda}\right)$$

6.6.7.3 [1466] Problem 3

problem number 1466

Added May 19, 2019.

Problem Chapter 6.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{bx}{a}, \frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x - 2a\,\mathrm{arctanh}\left(\mathrm{e}^{\beta y}\right)}{b\beta}, \frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{\gamma z}\right) - \gamma c x}{\gamma c}\right)$$

6.6.7.4 [1467] Problem 4

problem number 1467

Added May 19, 2019.

Problem Chapter 6.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta},\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-2a\lambda\operatorname{arctanh}\left(e^{\beta y}\right)+2b\beta\operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{-2a\lambda\operatorname{arctanh}\left(e^{\gamma z}\right)+2\gamma c\operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

6.6.7.5 [1468] Problem 5

problem number 1468

Added May 19, 2019.

Problem Chapter 6.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x+b\sinh(\lambda x)w_y+c\sinh(\gamma z)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh(\beta y) - b\beta\cosh(\lambda x)}{b\beta\lambda}, \frac{-2a\operatorname{arctanh}(e^{\gamma z}) - \gamma c\left(\int^{x} \frac{}{\sqrt{\frac{b\beta\cosh(\_a\lambda)-b\beta\cosh(\lambda x)+(\cosh(\beta y)-1)}{a\lambda}}}\right.}{\gamma c}\right.$$

6.6.7.6 [1469] Problem 6

problem number 1469

Added May 19, 2019.

Problem Chapter 6.4.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x + b\sinh(\lambda x)w_y + c\sinh(\lambda x)\sinh(\beta y)\sinh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[lambda*x]*S
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(lambda*x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh(\beta y) - b\beta\cosh(\lambda x)}{b\beta\lambda}, -\frac{4a\lambda\operatorname{arctanh}(e^{\gamma z}) + \gamma c\, e^{\lambda x} + \gamma c\, e^{-\lambda x}}{2\gamma c\lambda}\right)$$

6.6.8 4.2

Local contents

6.6.8.1 [1470] Problem 1 2132
6.6.8.2 [1471] Problem 2 2133
6.6.8.3 [1472] Problem 3 2134
6.6.8.4 [1473] Problem 4 2135
6.6.8.5 [1474] Problem 5 2136
6.6.8.6 [1475] Problem 6 2137

6.6.8.1 [1470] Problem 1

problem number 1470

Added May 19, 2019.

Problem Chapter 6.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cosh(\beta x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cosh[beta*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sinh(\beta x)}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cosh(beta*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\beta z-c\sinh(\beta x)}{a\beta}\right)$$

6.6.8.2 [1471] Problem 2

problem number 1471

Added May 19, 2019.

Problem Chapter 6.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta x)w_y+c\cosh(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*x]*D[w[x, y,z], y] +c*Cosh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{b\sinh(\beta x)}{a\beta},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*x)*diff(w(x,y,z),y)+c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\beta y - b\sinh(\beta x)}{a\beta}, \frac{a\lambda z - c\sinh(\lambda x)}{a\lambda}\right)$$

6.6.8.3 [1472] Problem 3

problem number 1472

Added May 19, 2019.

Problem Chapter 6.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cosh(\beta y)w_y + c\cosh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{bx}{a}, z - \frac{c\sinh(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + 2a\arctan\left(e^{\beta y}\right)}{b\beta}, \frac{a\lambda z - c\sinh(\lambda x)}{a\lambda}\right)$$

6.6.8.4 [1473] Problem 4

problem number 1473

Added May 19, 2019.

Problem Chapter 6.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\cosh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{bx}{a}, \frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + 2a\arctan\left(e^{\beta y}\right)}{b\beta}, \frac{2a\arctan\left(e^{\gamma z}\right) - \gamma c x}{\gamma c}\right)$$

6.6.8.5 [1474] Problem 5

problem number 1474

Added May 19, 2019.

Problem Chapter 6.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a\cosh(\lambda x)w_x + b\cosh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Cosh[lambda*x]*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{2b\tan^{-1}\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}, \frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{2c\tan^{-1}\left(\tanh\left(\right.\right.}{a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*cosh(lambda*x)*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\lambda\arctan\left(e^{\beta y}\right) - 2b\beta\arctan\left(e^{\lambda x}\right)}{b\beta\lambda}, \frac{2a\lambda\arctan\left(e^{\gamma z}\right) - 2\gamma c\arctan\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

6.6.8.6 [1475] Problem 6

problem number 1475

Added May 19, 2019.

Problem Chapter 6.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a\cosh(\beta y)w_x + b\cosh(\lambda x)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Cosh[beta*y]*D[w[x, y,z], x] + b*Cosh[lambda*x]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*cosh(beta*y)*diff(w(x,y,z),x)+ b*cosh(lambda*x)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\sinh(\beta y) - b\beta\sinh(\lambda x)}{b\beta\lambda}, -\frac{4\left(-\frac{\left((a\lambda-b\beta)\sqrt{-(\sinh(\lambda x)-i)(\sinh(\lambda x)+i)(\sinh(\beta y)-i)(\sinh(\beta y)+i)}+\right.}{}\right.}{}\right.$$

6.6.9 4.3

Local contents

6.6.9.1 [1476] Problem 1 2138
6.6.9.2 [1477] Problem 2 2139
6.6.9.3 [1478] Problem 3 2140
6.6.9.4 [1479] Problem 4 2141
6.6.9.5 [1480] Problem 5 2142
6.6.9.6 [1481] Problem 6 2143

6.6.9.1 [1476] Problem 1

problem number 1476

Added May 19, 2019.

Problem Chapter 6.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\sinh(\gamma z))}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*tanh(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \operatorname{RootOf}\left(\gamma z + \operatorname{arctanh}\left(\frac{\sqrt{\left(e^{\frac{2\gamma(\_Z+x)c}{a}}-1\right)e^{-\frac{2\gamma(\_Z+x)c}{a}}}\,e^{\frac{2\gamma(\_Z+x)c}{a}}}{e^{\frac{2\gamma(\_Z+x)c}{a}}-1}\right)\right)\right)$$

6.6.9.2 [1477] Problem 2

problem number 1477

Added May 19, 2019.

Problem Chapter 6.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta x)w_y + c\tanh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*x]*D[w[x, y,z], y] +c*Tanh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{b\log(\cosh(\beta x))}{a\beta}, z - \frac{c\log(\cosh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*x)*diff(w(x,y,z),y)+c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\beta y + b\ln(\tanh(\beta x)-1) + b\ln(\tanh(\beta x)+1)}{2a\beta}, \frac{2a\lambda z + c\ln(\tanh(\lambda x)-1) + c\ln(}{2a\lambda}\right.$$

6.6.9.3 [1478] Problem 3

problem number 1478

Added May 19, 2019.

Problem Chapter 6.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(z - \frac{c\log(\cosh(\lambda x))}{a\lambda}, \frac{\log(\sinh(\beta y))}{\beta} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta}, \frac{a\lambda z - c\ln(\cosh}{a\lambda}\right.$$

6.6.9.4 [1479] Problem 4

problem number 1479

Added May 19, 2019.

Problem Chapter 6.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sinh(\beta y))}{\beta} - \frac{bx}{a}, \frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\sinh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 c_2 c_3 (\tanh(\beta y) - 1)^{-\frac{-c_2}{2\beta}} (\tanh(\beta y) + 1)^{-\frac{-c_2}{2\beta}} (\tanh(\gamma z) - 1)^{\frac{a\_c_1}{2\gamma c}} (\tanh(\gamma z) - 1)^{\frac{b\_c_2}{2\gamma c}} (\tanh($$

6.6.9.5 [1480] Problem 5

problem number 1480

Added May 19, 2019.

Problem Chapter 6.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\tanh(\lambda x)w_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*tanh(lambda*x)*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 c_2 c_3 (\tanh(\beta y) - 1)^{-\frac{-c_2}{2\beta}} (\tanh(\beta y) + 1)^{-\frac{-c_2}{2\beta}} (\tanh(\gamma z) - 1)^{\frac{a\_c_1}{2\gamma c}} (\tanh(\gamma z) - 1)^{\frac{b\_c_2}{2\gamma c}} (\tanh($$

6.6.9.6 [1481] Problem 6

problem number 1481

Added May 19, 2019.

Problem Chapter 6.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \tanh(\beta y) w_x + b \tanh(\lambda x) w_y + c \tanh(\gamma z) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*tanh(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 (\tanh(\gamma z) - 1)^{-\frac{-c_3}{2\gamma}} (\tanh(\gamma z) + 1)^{-\frac{-c_3}{2\gamma}} \left(\tanh^{\frac{-c_3}{\gamma}}(\gamma z)\right) \_F5 \left(\frac{a \ln \left(\text{RootOf}\left(\beta y - \text{arccosh}\right.\right.}{} \right.$$

6.6.10 4.4

Local contents

6.6.10.1 [1482] Problem 1 2144
6.6.10.2 [1483] Problem 2 2145
6.6.10.3 [1484] Problem 3 2146
6.6.10.4 [1485] Problem 4 2147
6.6.10.5 [1486] Problem 5 2148
6.6.10.6 [1487] Problem 6 2149

6.6.10.1 [1482] Problem 1

problem number 1482

Added May 19, 2019.

Problem Chapter 6.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Coth[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\cosh(\gamma z))}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*coth(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\frac{-a\ln\left(\frac{(\mathrm{RootOf}(\gamma z-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\gamma z-\mathrm{arccoth}(\_Z-1))-2}\right)+a\ln\left(\mathrm{RootOf}\left(\gamma z-\mathrm{arccoth}(\_Z-1)\right)\right.}{2\gamma c}\right.$$

6.6.10.2 [1483] Problem 2

problem number 1483

Added May 19, 2019.

Problem Chapter 6.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta x)w_y + c\coth(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*x]*D[w[x, y,z], y] +c*Coth[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{b\log(\sinh(\beta x))}{a\beta}, z-\frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*x)*diff(w(x,y,z),y)+c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\beta y + b\ln(\coth(\beta x)-1) + b\ln(\coth(\beta x)+1)}{2a\beta}, \frac{2a\lambda z + c\ln(\coth(\lambda x)-1) + c\ln(c}{2a\lambda}\right.$$

6.6.10.3 [1484] Problem 3

problem number 1484

Added May 19, 2019.

Problem Chapter 6.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[lambda*x]*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\cosh(\beta y))}{\beta} - \frac{bx}{a}, z - \frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-2}\right) - a\ln\left(\mathrm{RootOf}\left(\beta y - \mathrm{arccoth}(\_Z-1)\right)\right)}{2b\beta},\right.$$

6.6.10.4 [1485] Problem 4

problem number 1485

Added May 19, 2019.

Problem Chapter 6.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\cosh(\beta y))}{\beta} - \frac{bx}{a}, \frac{b\log\left(\cosh^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\cosh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-2}\right) - a\ln\left(\mathrm{RootOf}\left(\beta y - \mathrm{arccoth}(\_Z-1)\right)\right)}{2b\beta},\right.$$

6.6.10.5 [1486] Problem 5

problem number 1486

Added May 19, 2019.

Problem Chapter 6.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\coth(\lambda x)w_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[lambda*x]*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(lambda*x)*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1c_2c_3(\coth(\beta y)-1)^{-\frac{-c_2}{2\beta}}(\coth(\beta y)+1)^{-\frac{-c_2}{2\beta}}(\coth(\gamma z)-1)^{\frac{a\_c_1}{2\gamma c}}(\coth(\gamma z)-1)^{\frac{b\_c_2}{2\gamma c}}(\coth(\gamma$$

6.6.10.6 [1487] Problem 6

problem number 1487

Added May 19, 2019.

Problem Chapter 6.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \coth(\beta y) w_x + b \coth(\lambda x) w_y + c \coth(\gamma z) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Coth[lambda*x]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*coth(lambda*x)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 (\coth(\gamma z) - 1)^{-\frac{c_3}{2\gamma}} (\coth(\gamma z) + 1)^{-\frac{c_3}{2\gamma}} \left(\coth^{\frac{c_3}{\gamma}}(\gamma z)\right) \_F5 \left(\frac{a \ln\left(\text{RootOf}\left(\beta y - \text{arcsinh}\right.\right.}{} \right.$$

6.6.11 4.5

Local contents

6.6.11.1 [1488] Problem 1 2150
6.6.11.2 [1489] Problem 2 2151
6.6.11.3 [1490] Problem 3 2152
6.6.11.4 [1491] Problem 4 2153
6.6.11.5 [1492] Problem 5 2154
6.6.11.6 [1493] Problem 6 2155

6.6.11.1 [1488] Problem 1

problem number 1488

Added May 19, 2019.

Problem Chapter 6.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \sinh(\lambda x) w_x + b \sinh(\beta y) w_y + c \cosh(\gamma z) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1 \left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{c\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}, \frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2a\lambda \operatorname{arctanh}\left(e^{\beta y}\right) + 2b\beta \operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda}, \frac{2a\lambda \arctan\left(e^{\gamma z}\right) + 2\gamma c \operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

6.6.11.2 [1489] Problem 2

problem number 1489

Added May 19, 2019.

Problem Chapter 6.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\cosh(\beta y)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{b\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}, \frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{c\log\left(\tanh\left(\frac{\lambda x}{2}\right)\right)}{a\lambda}\right)\right\}\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\lambda\arctan\left(e^{\beta y}\right)+2b\beta\operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda},\frac{2a\lambda\arctan\left(e^{\gamma z}\right)+2\gamma c\operatorname{arctanh}\left(e^{\lambda x}\right)}{\gamma c\lambda}\right)$$

6.6.11.3 [1490] Problem 3

problem number 1490

Added May 19, 2019.

Problem Chapter 6.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x + b\sinh(\lambda x)w_y + c\sinh(\lambda x)\sinh(\beta y)\cosh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[lambda*x]*D[w[x, y,z], y] +c*Sinh[lambda*x]*S
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+ b*sinh(lambda*x)*diff(w(x,y,z),y)+c*sinh(lambda*x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\lambda\cosh(\beta y)-b\beta\cosh(\lambda x)}{b\beta\lambda},\frac{2a\lambda\arctan\left(e^{\gamma z}\right)-\gamma c\cosh(\lambda x)}{\gamma c\lambda}\right)$$

6.6.11.4 [1491] Problem 4

problem number 1491

Added May 19, 2019.

Problem Chapter 6.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a\cosh(\beta y)w_x + b\tanh(\lambda x)w_y + c\cosh(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Cosh[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Cosh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*cosh(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*cosh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2a\lambda\sinh(\beta y) + b\beta\ln(\tanh(\lambda x) - 1) + b\beta\ln(\tanh(\lambda x) + 1)}{2b\beta\lambda}, \frac{a\arctan(e^{\gamma z}) - \gamma c\left(\int^{x}\right.}{}\right.$$

6.6.11.5 [1492] Problem 5

problem number 1492

Added May 19, 2019.

Problem Chapter 6.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \coth(\beta y) w_x + b \tanh(\lambda x) w_y + c \tanh(\gamma z) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Tanh[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 (\tanh(\gamma z) - 1)^{-\frac{-c_3}{2\gamma}} (\tanh(\gamma z) + 1)^{-\frac{-c_3}{2\gamma}} \left(\tanh^{\frac{-c_3}{\gamma}}(\gamma z)\right) \_F5 \left(\frac{a \ln \left(\text{RootOf}\left(\beta y - \text{arcsinh}\right.\right.}{} \right.$$

6.6.11.6 [1493] Problem 6

problem number 1493

Added May 19, 2019.

Problem Chapter 6.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \coth(\beta y) w_x + b \tanh(\lambda x) w_y + c \coth(\gamma z) w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Coth[beta*y]*D[w[x, y,z], x] + b*Tanh[lambda*x]*D[w[x, y,z], y] +c*Coth[gamma*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  a*coth(beta*y)*diff(w(x,y,z),x)+ b*tanh(lambda*x)*diff(w(x,y,z),y)+c*coth(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 (\coth(\gamma z) - 1)^{-\frac{c_3}{2\gamma}} (\coth(\gamma z) + 1)^{-\frac{c_3}{2\gamma}} \left(\coth^{\frac{c_3}{\gamma}}(\gamma z)\right) \_F5 \left(\frac{a \ln \left(\text{RootOf}\left(\beta y - \text{arcsinh}\right.\right.}{} \right.$$

6.6.12 5.1

Local contents

6.6.12.1 [1494] Problem 1 2156
6.6.12.2 [1495] Problem 2 2157
6.6.12.3 [1496] Problem 3 2157
6.6.12.4 [1497] Problem 4 2158

6.6.12.1 [1494] Problem 1

problem number 1494

Added May 26, 2019.

Problem Chapter 6.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\ln(\beta y)\ln(\lambda z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Log[beta*y]*Log[lambda*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\text{LogIntegral}(\lambda z)}{\lambda} + \frac{cx}{a} - \frac{cy\log(\beta y)}{b}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*ln(beta*y)*ln(lambda*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-ay + bx}{b}, \frac{-(\ln(\beta y) - 1)\, c\lambda y - b\,\text{expIntegral}(1, -\ln(\lambda z))}{c\lambda}\right)$$

6.6.12.2 [1495] Problem 2

problem number 1495

Added May 26, 2019.

Problem Chapter 6.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\ln(\beta x)w_y + c\ln(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Log[beta*x]*D[w[x, y,z], y] +c*Log[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{ay - bx\log(\beta x) + bx}{a}, \frac{az - cx\log(\lambda x) + cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*ln(beta*x)*diff(w(x,y,z),y)+c*ln(lambda*x)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-bx\ln(\beta x) + ay + bx}{a}, \frac{-cx\ln(\lambda x) + az + cx}{a}\right)$$

6.6.12.3 [1496] Problem 3

problem number 1496

Added May 26, 2019.

Problem Chapter 6.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\ln(\beta x)\ln(\lambda y)w_y + c\ln(\mu x)\ln(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Log[beta*x]*Log[lambda*y]*D[w[x, y,z], y] +c*Log[mu*x]*Log[gamm
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\text{LogIntegral}(\lambda y)}{\lambda} - \frac{bx(\log(\beta x)-1)}{a}, \frac{\text{LogIntegral}(\gamma z)}{\gamma} - \frac{cx(\log(\mu x)-1)}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*ln(beta*x)*ln(lambda*y)*diff(w(x,y,z),y)+c*ln(mu*x)*ln(gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(\ln(\beta x)-1)\,b\lambda x + a\,\text{expIntegral}(1,-\ln(\lambda y))}{a\lambda}, -\frac{\left(-\gamma\left(-\ln\left(\frac{(\ln(\beta x)-1)\beta x}{\text{LambertW}((\ln(\beta x)-1)e^{-1}\beta x)}\right)\right.\right.}{}\right.$$

6.6.12.4 [1497] Problem 4

problem number 1497

Added May 26, 2019.

Problem Chapter 6.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln(\beta x)w_x + b\ln(\lambda y)w_y + c\ln(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Log[beta*x]*D[w[x, y,z], x] + b*Log[lambda*y]*D[w[x, y,z], y] +c*Log[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\text{LogIntegral}(\lambda y)}{\lambda}-\frac{b\,\text{LogIntegral}(\beta x)}{a\beta},\frac{\text{LogIntegral}(\gamma z)}{\gamma}-\frac{c\,\text{LogIntegral}(\beta x)}{a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*ln(beta*x)*diff(w(x,y,z),x)+ b*ln(lambda*y)*diff(w(x,y,z),y)+c*ln(gamma*z)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\beta\,\text{expIntegral}\left(1,-\ln\left(\lambda y\right)\right)+b\lambda\,\text{expIntegral}\left(1,-\ln\left(\beta x\right)\right)}{b\beta\lambda},\frac{-a\beta\,\text{expIntegral}\left(1,-\ln\right.}{}\right.$$

6.6.13 5.2

Local contents

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6.6.13.2 [1499] Problem 2 2160
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6.6.13.1 [1498] Problem 1

problem number 1498

Added May 26, 2019.

Problem Chapter 6.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + b\ln^k(\lambda x) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +b*Log[lambda*x]^k*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, z - \frac{b(-\log(\lambda x))^{-k}\log^k(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+b*ln(lambda*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-axx^n+(n+1)\,y}{n+1}, -bx\ln(\lambda x) + bx + z\right)$$

6.6.13.2 [1499] Problem 2

problem number 1499

Added May 26, 2019.

Problem Chapter 6.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + c\ln^k(\lambda x))w_y + (bz + s\ln^n(\lambda x))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+c*Log[lambda*x]^k)*D[w[x, y,z], y] +(b*z+s*Log[lambda*x]^n)*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-ax}-\int_1^x ce^{-aK[1]}(\log(\lambda)+\log(K[1]))^k dK[1], ze^{-bx}-\int_1^x e^{-bK[2]}s(\log(\lambda)+\log(K[\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*y+c*ln(lambda*x)^k)*diff(w(x,y,z),y)+(b*z+s*log(lambda*x)^n)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-c\left(\int \ln(\lambda x)^k e^{-ax}dx\right)+y\,e^{-ax},-s\left(\int \ln(\lambda x)^n e^{-bx}dx\right)+z\,e^{-bx}\right)$$

6.6.13.3 [1500] Problem 3

problem number 1500

Added May 26, 2019.

Problem Chapter 6.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+(c\ln^n(\lambda x)+s\ln^k(\beta y))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +(c*Log[lambda*x]^n+s*Log[beta*y]^k)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},-\frac{c\log^{n+1}(\lambda x)}{an+a}-\frac{s\log^{k+1}(\beta y)}{bk+b}+z\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+(c*ln(lambda*x)^n+s*ln(beta*y)^k)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(yx^{-\frac{b}{a}},\frac{(n+1)\left(i\pi\,\mathrm{csgn}(i\beta)\,\mathrm{csgn}(iy)\,\mathrm{csgn}(i\beta y)+i\pi\mathrm{csgn}(iy)^3-i\pi\mathrm{csgn}(iy)^2\,\mathrm{csgn}\left(ix^{\frac{b}{a}}\right)\right.}{}\right.$$

6.6.13.4 [1501] Problem 4

problem number 1501

Added May 26, 2019.

Problem Chapter 6.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln(\lambda x)w_x+by\ln(\beta y)w_y+cz\ln(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[lambda*x]*D[w[x, y,z], x] + b*y*Log[beta*y]*D[w[x, y,z], y] +c*Log[gamma*z]*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\log\left((\log(\beta)+\log(y))(\log(\lambda)+\log(x))^{-\frac{b}{a}}\right),\frac{\operatorname{li}(\gamma z)}{\gamma}-\frac{c\log(\log(\lambda x))}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*ln(lambda*x)*diff(w(x,y,z),x)+ b*y*ln(beta*y)*diff(w(x,y,z),y)+c*ln(gamma*z)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\ln(\ln(\beta y))+b\ln(\ln(\lambda x))}{a},\frac{-b\,\mathrm{expIntegral}(1,-\ln(z)-\ln(\gamma))-\gamma c\ln(\ln(\beta y))}{\gamma c}\right)$$

6.6.13.5 [1502] Problem 5

problem number 1502

Added May 26, 2019.

Problem Chapter 6.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln(\lambda x)w_x+by\ln(\beta y)w_y+cz\ln(\gamma x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[lambda*x]*D[w[x, y,z], x] + b*y*Log[beta*y]*D[w[x, y,z], y] +c*Log[gamma*x]*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{az-c\log(\gamma x)\log(\log(\lambda x))+c\log(\lambda x)\log(\log(\lambda x))-c\log(x)}{a},\log\left((\log(\beta)+\log(y)\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*ln(lambda*x)*diff(w(x,y,z),x)+ b*y*ln(beta*y)*diff(w(x,y,z),y)+c*ln(gamma*x)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln(\ln(\beta y))-b\ln(\ln(\lambda x))}{b},\frac{2az-2c\ln(x)-\left(i\pi\,\mathrm{csgn}(i\lambda)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(i\lambda x)+i\pi\mathrm{cs}\right.}{}\right.$$

6.6.13.6 [1503] Problem 6

problem number 1503

Added May 26, 2019.

Problem Chapter 6.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln^n(x)w_x+by\ln^m(y)w_y+cz\ln^k(z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +c*z*Log[z]^k*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\log^{1-n}(x)}{a(n-1)}-(m-1)^{\frac{1}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m,\frac{c\log^{1-n}(x)}{a(n-1)}-(k-1)^{\frac{1}{k-1}}\log(z)\right.\right.$$

Maple ✓

```
restart;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+ b*y*ln(y)^m*diff(w(x,y,z),y)+c*z*ln(z)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-(n-1)\,a\ln(y)^{-m+1}+(m-1)\,b\ln(x)^{-n+1}}{(n-1)\,(m-1)\,b},\frac{-(n-1)\,a\ln(z)^{-k+1}+(k-1)\,c\ln(x)^{-}}{(n-1)\,(k-1)\,c}\right.$$

6.6.14 6.1

Local contents

6.6.14.1 [1504] Problem 1 2165
6.6.14.2 [1505] Problem 2 2166
6.6.14.3 [1506] Problem 3 2167
6.6.14.4 [1507] Problem 4 2168
6.6.14.5 [1508] Problem 5 2169

6.6.14.1 [1504] Problem 1

problem number 1504

Added May 26, 2019.

Problem Chapter 6.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sin(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Sin[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*sin(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z - \arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{\gamma cx}{a}}}{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}+1}, -\frac{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2\gamma cx}{a}}+1}\right)\right)\right)}{\gamma c}\right)$$

6.6.14.2 [1505] Problem 2

problem number 1505

Added May 26, 2019.

Problem Chapter 6.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sin(\beta y)w_y + c\sin(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Sin[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\cos(\lambda x)}{a\lambda}+z,\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*sin(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\lambda z+c\cos(\lambda x)}{a\lambda}\right)$$

6.6.14.3 [1506] Problem 3

problem number 1506

Added May 26, 2019.

Problem Chapter 6.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\beta y)w_y+c\sin(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Sin[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*sin(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\ln\left(\mathrm{RootOf}\left(\gamma z-\mathrm{arct}\right.\right.}{}\right.$$

6.6.14.4 [1507] Problem 4

problem number 1507

Added May 26, 2019.

Problem Chapter 6.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\lambda x)\sin(\beta y)w_y+cw_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[lambda*x]*Sin[beta*y]*D[w[x, y,z], y] +c*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{cx}{a},\frac{a\log\left(\tan^2\left(\frac{\beta y}{2}\right)\right)}{\beta}+\frac{2b\cos(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(lambda*x)*sin(beta*y)*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\lambda\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z}{\_Z^2+1},\frac{-\_Z^2-1}{\_Z^2+1}\right)\right)\right)+b\beta\cos(\lambda x)}{b\beta\lambda},\frac{az-cx}{a}\right)$$

6.6.14.5 [1508] Problem 5

problem number 1508

Added May 26, 2019.

Problem Chapter 6.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin^n(\lambda x)\sin^m(\beta y)w_y+c\sin^k(\mu x)\sin^r(\gamma*z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[lambda*x]^n*Sin[beta*y]^m*D[w[x, y,z], y] +c*Sin[mu*x]^k*Si
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\sqrt{\cos^2(\beta y)}\sec(\beta y)\sin^{1-m}(\beta y)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\sin^2(\beta y)\right)}{\beta-\beta m}-\frac{b\sqrt{\cos}}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*sin(lambda*x)^n*sin(beta*y)^m*diff(w(x,y,z),y)+c*sin(mu*x)^k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\left(\int(\sin^n(\lambda x))\,dx\right)+\int\frac{a(\sin^{-m}(\beta y))}{b}dy,-\left(\int(\sin^k(\mu x))\,dx\right)+\int\frac{a(\sin^{-r}(\gamma z))}{c}\right.$$

6.6.15 6.2

Local contents

6.6.15.1 [1509] Problem 1 2170
6.6.15.2 [1510] Problem 2 2172
6.6.15.3 [1511] Problem 3 2174
6.6.15.4 [1512] Problem 4 2175
6.6.15.5 [1513] Problem 5 2176

6.6.15.1 [1509] Problem 1

problem number 1509

Added May 26, 2019.

Problem Chapter 6.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{ w(x,y,z) \to c_1\left(y - \frac{bx}{a}, -\frac{\cosh^{-1}\left(-\frac{\sec(\gamma z)\left(2\left(2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}+\sinh^3(\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right.} \right.}{\gamma} \right.\right.$$

$$\left\{ w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\cosh^{-1}\left(-\frac{\sec(\gamma z)\left(2\left(2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}+\sinh^3\left(\frac{c\gamma}{a}\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)-} \right.}{\gamma} \right.\right.$$

$$\left\{ w(x,y,z) \to c_1\left(y - \frac{bx}{a}, -\frac{\cosh^{-1}\left(\frac{\sec(\gamma z)\left(-2\left(-2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}+\sinh^3\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)} \right.}{\gamma} \right.\right.$$

$$\left\{ w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\cosh^{-1}\left(\frac{\sec(\gamma z)\left(-2\left(-2\sec(\gamma z)\sqrt{\sin^2(\gamma z)\cos^2(\gamma z)\sinh^2\left(\frac{c\gamma x}{a}\right)\left(\cosh\left(\frac{4c\gamma x}{a}\right)-\sinh\left(\frac{4c\gamma x}{a}\right)\right)}+\sinh^3\left(\frac{c}{}\right.\right.}{4\cosh\left(\frac{2c\gamma x}{a}\right)-} \right.}{\gamma} \right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z - \arctan\left(\frac{\_Z^2 e^{\frac{2\gamma c x}{a}}-1}{\_Z^2 e^{\frac{2\gamma c x}{a}}+1}, \frac{2\_Z e^{\frac{\gamma c x}{a}}}{\_Z^2 e^{\frac{2\gamma c x}{a}}+1}\right)\right)\right)}{\gamma c}\right)$$

6.6.15.2 [1510] Problem 2

problem number 1510

Added May 26, 2019.

Problem Chapter 6.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sin\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right)}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sinh\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh(}\right)}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(\frac{\sec(\beta y)\left(-2\left(-2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+s\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right)}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\frac{\cosh^{-1}\left(\frac{\sec(\beta y)\left(-2\left(-2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\sinh\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh(}\right)}{\beta}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y - \arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\lambda z - c\sin(\lambda x)}{a\lambda}\right)$$

6.6.15.3 [1511] Problem 3

problem number 1511

Added May 26, 2019.

Problem Chapter 6.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y - \arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z - \arctan\right.\right.}{}\right.$$

6.6.15.4 [1512] Problem 4

problem number 1512

Added May 26, 2019.

Problem Chapter 6.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\gamma z)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +c*Cos[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*y)*diff(w(x,y,z),y)+c*cos(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\text{RootOf}\left(\beta y - \arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, \frac{a\ln\left(\text{RootOf}\left(\gamma z - \arctan\right.\right.}{}\right.$$

6.6.15.5 [1513] Problem 5

problem number 1513

Added May 26, 2019.

Problem Chapter 6.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos^n(\lambda x)\cos^m(\beta y)w_y + c\cos^k(\mu x)\cos^r(\gamma * z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[lambda*x]^n*Cos[beta*y]^m*D[w[x, y,z], y] +c*Cos[mu*x]^k*Co
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{n+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda x)\right)}{a\lambda n + a\lambda} + \frac{\sqrt{\sin^2}}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*cos(lambda*x)^n*cos(beta*y)^m*diff(w(x,y,z),y)+c*cos(mu*x)^k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int (\cos^n(\lambda x))\,dx\right) + \int \frac{a(\cos^{-m}(\beta y))}{b}dy, -\left(\int (\cos^k(\mu x))\,dx\right) + \int \frac{a(\cos^{-r}(\gamma z)}{c}$$

6.6.16 6.3

Local contents

6.6.16.1 [1514] Problem 1 2177
6.6.16.2 [1515] Problem 2 2178
6.6.16.3 [1516] Problem 3 2180
6.6.16.4 [1517] Problem 4 2181
6.6.16.5 [1518] Problem 5 2182

6.6.16.1 [1514] Problem 1

problem number 1514

Added May 26, 2019.

Problem Chapter 6.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\sin(\gamma z))}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right)-\gamma c x}{\gamma c}\right)$$

Hand solution

Solve

$$aw_x + bw_y + c\tan(\gamma z)\, w_z = 0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c\tan(\gamma z)} = \frac{dw}{0}$$

From first two pair of equation we obtain $\frac{b}{a}x - y = C_1$ and from $\frac{dx}{a} = \frac{dz}{c\tan(\gamma z)}$ we obtain $\frac{c}{a}x - \frac{1}{\gamma}\ln(\sin(\gamma z)) = C_2$. Since $dw = 0$ and $w = C_3$. Where C_1, C_2, C_3 are constants. But $C_3 = F(C_1, C_2)$ where F is arbitrary function. Hence

$$u(x,y,z) = F\left(\frac{b}{a}x - y, \frac{c}{a}x - \frac{1}{\gamma}\ln(\sin(\gamma z))\right)$$

6.6.16.2 [1515] Problem 2

problem number 1515

Added May 26, 2019.

Problem Chapter 6.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{c\log(\cos(\lambda x))}{a\lambda}+z,\frac{\log(\sin(\beta y))}{\beta}-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x+a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta},-\frac{-2a\lambda z+c\ln\left(\tan^2(\lambda x)+1\right)}{2a\lambda}\right)$$

Hand solution

Solve

$$aw_x+b\tan(\beta y)\,w_y+c\tan(\lambda x)\,w_z=0 \tag{1}$$

Using Lagrange-charpit

$$\frac{dx}{a}=\frac{dy}{b\tan(\beta y)}=\frac{dz}{c\tan(\lambda x)}=\frac{dw}{0}$$

From first two pair of equations, integrating gives $\frac{b}{a}x-\frac{1}{\beta}\ln(\sin(\beta y))=C_1$ and from $\frac{dx}{a}=\frac{dz}{c\tan(\lambda x)}$ we obtain $\frac{c}{a}\tan(\lambda x)\,dx=dz$. Integrating gives $-\frac{c}{a\lambda}\ln(\cos(\lambda x))-z=C_2$. Since $dw=0$ then $w=C_3$. Where C_1,C_2,C_3 are constants. But $C_3=F(C_1,C_2)$ where F is arbitrary function. Hence

$$u(x,y,z)=F\left(\frac{b}{a}x-\frac{1}{\beta}\ln(\sin(\beta y)),-\frac{c}{a\lambda}\ln(\cos(\lambda x))-z\right)$$

6.6.16.3 [1516] Problem 3

problem number 1516

Added May 26, 2019.

Problem Chapter 6.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}, \frac{b\log\left(\sin^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\sin(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right) - \gamma c x}{\gamma c}\right)$$

6.6.16.4 [1517] Problem 4

problem number 1517

Added May 26, 2019.

Problem Chapter 6.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Tan[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}, \frac{b\log\left(\sin^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\sin(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*tan(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{a\ln\left(\frac{\tan(\gamma z)}{\sqrt{\tan^2(\gamma z)+1}}\right) - \gamma c x}{\gamma c}\right)$$

6.6.16.5 [1518] Problem 5

problem number 1518

Added May 26, 2019.

Problem Chapter 6.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$\mu\nu \tan(\lambda x) w_x + \lambda\nu \tan(\mu y) w_y + \lambda\mu \tan(\nu z) w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  mu*nu*Tan[lambda*x]*D[w[x, y,z], x] + lambda*nu*Tan[mu*y]*D[w[x, y,z], y] +lambda*mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\{\{w(x, y, z) \to c_1 (\log (\csc^2(\mu y) \sin^2(\nu z)), -4 \sin(\lambda x) \csc(\mu y))\}\}$$

Maple ✓

```
restart;
pde :=  mu*nu*tan(lambda*x)*diff(w(x,y,z),x)+ lambda*nu*tan(mu*y)*diff(w(x,y,z),y)+lambda*mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1 \left(\frac{\ln \left(\frac{\tan(\mu y)}{\sqrt{\tan^2(\mu y)+1}\, \sin(\lambda x)} \right)}{\lambda}, \frac{\ln \left(\frac{\tan(\nu z)}{\sqrt{\tan^2(\nu z)+1}\, \sin(\lambda x)} \right)}{\lambda} \right)$$

6.6.17 6.4

Local contents

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6.6.17.1 [1519] Problem 1

problem number 1519

Added May 26, 2019.

Problem Chapter 6.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\cot(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\sec(\gamma z))}{\gamma} - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-ay + bx}{b}, \frac{b\ln\left(\cot^2(\gamma z) + 1\right) - 2b\ln\left(\cot(\gamma z)\right) - 2\gamma cy}{2\gamma c}\right)$$

6.6.17.2 [1520] Problem 2

problem number 1520

Added May 26, 2019.

Problem Chapter 6.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sec(\beta y))}{\beta} - \frac{bx}{a}, z - \frac{c\log(\sin(\lambda x))}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x - a\ln\left(\cot^2(\beta y)+1\right) + 2a\ln\left(\cot(\beta y)\right)}{2b\beta}, \frac{-2b\beta c\ln\left(\left(\left(-\left(\cot^2(\beta y)+1\right)^{\frac{ia\lambda}{b\beta}}\left(e^{2i\beta y}\right.\right.\right.\right.}{}\right.$$

6.6.17.3 [1521] Problem 3

problem number 1521

Added May 26, 2019.

Problem Chapter 6.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx}{a}+\frac{\log(\cos(\beta y))}{\beta},\frac{2c\log(\cos(\beta y))}{\beta}-\frac{b\log\left(\cos^2(\gamma z)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2b\beta x-a\ln\left(\cot^2(\beta y)+1\right)+2a\ln\left(\cot(\beta y)\right)}{2b\beta},\frac{-2b\beta\ln\left(\frac{\cos(\gamma z)}{\sin(\gamma z)}\right)+b\beta\ln\left(\frac{\left(\frac{\cos(2\beta y)}{2}+\frac{1}{2}\right)^{\frac{\gamma}{b\beta}}}{}\right.}{}\right.$$

6.6.17.4 [1522] Problem 4

problem number 1522

Added May 26, 2019.

Problem Chapter 6.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cot(\beta y)w_y+c\cot(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +c*Cot[gamma*z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{bx}{a}+\frac{\log(\cos(\beta y))}{\beta},\frac{2c\log(\cos(\beta y))}{\beta}-\frac{b\log\left(\cos^2(\gamma z)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*y)*diff(w(x,y,z),y)+c*cot(gamma*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{2b\beta x-a\ln\left(\cot^2(\beta y)+1\right)+2a\ln\left(\cot(\beta y)\right)}{2b\beta},\frac{-2b\beta\ln\left(\frac{\cos(\gamma z)}{\sin(\gamma z)}\right)+b\beta\ln\left(\frac{\left(\frac{\cos(2\beta y)}{2}+\frac{1}{2}\right)^{\frac{\gamma}{b\beta}}}{}\right.}{}\right.$$

6.6.17.5 [1523] Problem 5

problem number 1523

Added May 26, 2019.

Problem Chapter 6.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$\mu\nu\cot(\lambda x)w_x+\lambda\nu\cot(\mu y)w_y+\lambda\mu\cot(\nu z)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  mu*nu*Cot[lambda*x]*D[w[x, y,z], x] + lambda*nu*Cot[mu*y]*D[w[x, y,z], y] +lambda*mu*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  mu*nu*cot(lambda*x)*diff(w(x,y,z),x)+ lambda*nu*cot(mu*y)*diff(w(x,y,z),y)+lambda*mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{\ln\left(\sqrt{\tan^2(\mu y)+1}\,\cos(\lambda x)\right)}{\lambda}, \frac{\ln\left(\sqrt{\tan^2(\nu z)+1}\,\cos(\lambda x)\right)}{\lambda}\right)$$

6.6.18 6.5

Local contents

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6.6.18.2 [1525] Problem 2 2188
6.6.18.3 [1526] Problem 3 2189
6.6.18.4 [1527] Problem 4 2190
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6.6.18.1 [1524] Problem 1

problem number 1524

Added May 31, 2019.

Problem Chapter 6.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + (c\sin^n(\lambda x) + s\cos^k(\beta y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +(c*Sin[lambda*x]^n+s*Cos[beta*y]^k)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(\lambda x)\right)}{a\lambda n+a\lambda}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+(c*sin(lambda*x)^n+s*cos(beta*y)^k)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c(\sin^n(\_a\lambda))+s\left(\cos^k\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)\right)}{a}d\_a\right)\right)$$

6.6.18.2 [1525] Problem 2

problem number 1525

Added May 31, 2019.

Problem Chapter 6.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sin(\beta y)w_y+c\cos(\lambda x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +c*Cos[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sin(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*y)*diff(w(x,y,z),y)+c*cos(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\mathrm{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{b\beta x}{a}}}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},\frac{a\lambda z-c\sin(\lambda x)}{a\lambda}\right)$$

6.6.18.3 [1526] Problem 3

problem number 1526

Added May 31, 2019.

Problem Chapter 6.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\cos^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +b*Cos[beta*x]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{k+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(\beta x)\right)}{\beta k+\beta}+z,y-\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+b*cos(beta*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sin^n(\lambda x))\,dx\right),z-\left(\int b(\cos^k(\beta x))\,dx\right)\right)$$

6.6.18.4 [1527] Problem 4

problem number 1527

Added May 31, 2019.

Problem Chapter 6.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\sin^k(\beta y)w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +b*Sin[beta*y]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+b*sin(beta*y)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sin^k\left(\left(a\left(\int (\sin^n(\_b\lambda))\,d\_b\right) + y - \left(\int a(s\right.\right.\right.\right.$$

6.6.18.5 [1528] Problem 5

problem number 1528

Added May 31, 2019.

Problem Chapter 6.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\cot(\lambda x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +c*Cot[lambda*x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(z - \frac{c\log(\sin(\lambda x))}{a\lambda}, \frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*tan(beta*y)*diff(w(x,y,z),y)+c*cot(lambda*x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, \frac{2a\lambda z + c\ln\left(\cot^2(\lambda x)+1\right)}{2a\lambda}\right)$$

6.6.18.6 [1529] Problem 6

problem number 1529

Added May 31, 2019.

Problem Chapter 6.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\lambda x)w_y + b\tan^k(\beta y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[lambda*x]^n*D[w[x, y,z], y] +b*Tan[beta*y]^k*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ a*cot(lambda*x)^n*diff(w(x,y,z),y)+b*tan(beta*y)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cot^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\frac{-\tan\left(\left(y - \left(\int a(\cot^n(\lambda x))\,dx\right)\right)\beta\right) - \tan\left(a\right.}{\tan\left(\left(y - \left(\int a\left(\cot^n(\lambda x)\right)dx\right)\right)\beta\right)\tan\left(a\beta\left(\int\right.\right.}\right.\right.\right.$$

6.6.19 7.1

Local contents

6.6.19.1 [1530] Problem 1 2193
6.6.19.2 [1531] Problem 2 2194
6.6.19.3 [1532] Problem 3 2195
6.6.19.4 [1533] Problem 4 2196
6.6.19.5 [1534] Problem 5 2197

6.6.19.1 [1530] Problem 1

problem number 1530

Added May 31, 2019.

Problem Chapter 6.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\int_1^x \frac{c\sin^{-1}(\lambda K[1])^n}{a}dK[1]-\frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^k\,\text{Gamma}\,(1-\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda x 2^n 2^{-n}}{2} + \frac{(\lambda x-1)(\lambda x+1)(k}{}\right.}{}\right.$$

6.6.19.2 [1531] Problem 2

problem number 1531

Added May 31, 2019.

Problem Chapter 6.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},-\int_1^x \frac{c\sin^{-1}(\lambda K[1])^n\left(\left(\frac{a\sin^{-1}(\lambda K[1])^{-n}\text{InverseFunction[Inactive[Integrate],1,2]}\left[\int_1^x \frac{c\sin^{-}}{}\right.}{c}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\left(\int^x \arcsin(\_a\lambda)^n \arcsin\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right)^m d\_a\right)-\frac{\left(-\gamma 1kz2^k\,\mathrm{I}\right.}{}\right.$$

6.6.19.3 [1532] Problem 3

problem number 1532

Added May 31, 2019.

Problem Chapter 6.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arcsin^n(\lambda x)w_y+c\arcsin^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] +c*ArcSin[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z+\frac{ic\sin^{-1}(\beta x)^k\left(\sin^{-1}(\beta x)^2\right)^{-k}\left(\left(i\sin^{-1}(\beta x)\right)^k\operatorname{Gamma}\left(k+1,-i\sin^{-1}(\beta x)\right)-(-\right.}{2a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+c*arcsin(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{\left(-\arcsin(\lambda x)^n\arcsin(\lambda x)^{\frac{3}{2}}+\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)\arcsin(\lambda x)\right)\sqrt{-\lambda^2x^2}}{}\right.$$

6.6.19.4 [1533] Problem 4

problem number 1533

Added May 31, 2019.

Problem Chapter 6.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arcsin^n(\lambda x)w_y+c\arcsin^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] +c*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^k\,\text{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-(i\sin^{-1}(\beta z))\right.}{2\beta}\right.\right.\right.$$

Maple ✗

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+c*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.6.19.5 [1534] Problem 5

problem number 1534

Added May 31, 2019.

Problem Chapter 6.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arcsin^n(\lambda y)w_y+c\arcsin^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*y]^n*D[w[x, y,z], y] +c*ArcSin[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^k\,\mathrm{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-(i\sin^{-1}(\beta z))\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+c*arcsin(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{\left(-\arcsin(\lambda y)^{-n}\arcsin(\lambda y)^{\frac{3}{2}}+\mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda y)\right)\arcsin(\lambda y)\right)\sqrt{-}}{}\right.$$

6.6.20 7.2

Local contents

6.6.20.1 [1535] Problem 1 2199
6.6.20.2 [1536] Problem 2 2200
6.6.20.3 [1537] Problem 3 2201
6.6.20.4 [1538] Problem 4 2202
6.6.20.5 [1539] Problem 5 2203

6.6.20.1 [1535] Problem 1

problem number 1535

Added May 31, 2019.

Problem Chapter 6.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, -\int_1^x \frac{c\cos^{-1}(\lambda K[1])^n}{a} dK[1] + \frac{\cos^{-1}(\beta z)^{-k}\left((-i\cos^{-1}(\beta z))^k \text{Gamma}(1-k\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\text{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^n}{\sqrt{\pi}\,(n+2)}\right.}{\lambda}\right.$$

6.6.20.2 [1536] Problem 2

problem number 1536

Added May 31, 2019.

Problem Chapter 6.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^m(\beta y)\arccos^k(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCos[lambda*x]^n*ArcCos[beta*y]^m*ArcCos[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\ \cos^{-1}(\gamma z)^{-k}\left(-2\gamma\cos^{-1}(\gamma z)^k\int_1^x \frac{c\cos^{-1}(\lambda K[1])^n\left(\left(\frac{a\cos^{-1}(\lambda K[1])^{-n}\text{InverseFunction[Inacti}}{}\right.\right.}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccos(lambda*x)^n*arccos(beta*y)^m*arccos(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{-(k-2)\,c\gamma 1\left(\int^{x} \arccos\left(\_a\lambda\right)^{n} \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^{m} d\_a\right) + \frac{\left((k-2)\gamma 1 z\,\mathrm{Lo}\right.}{}}{}\right.$$

6.6.20.3 [1537] Problem 3

problem number 1537

Added May 31, 2019.

Problem Chapter 6.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] +c*ArcCos[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\left(\cos^{-1}(\beta x)^2\right)^{-k}\left(-c(i\cos^{-1}(\beta x))^{k}\cos^{-1}(\beta x)^{k}\,\mathrm{Gamma}\left(k+1,-i\cos^{-1}(\beta x)\right) - c(-i\right.}{2a\beta}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+c*arccos(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y + \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^{n+1}}{\sqrt{\pi}\,(n+2)} - \right.}{a\lambda}\right.$$

6.6.20.4 [1538] Problem 4

problem number 1538

Added May 31, 2019.

Problem Chapter 6.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] +c*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} + \frac{\cos^{-1}(\beta z)^{-k}\left(\left(-i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right) + \left(i\cos^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+c*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{\left(-2(n+2)2^{-n-1}\,\mathrm{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)+\left(\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\arccos(\lambda x)-\arccos(\lambda x)^{n+1}\sqrt{\mathrm{arcc}}\right.\right.}{\sqrt{\arccos(\lambda x)}}\right.$$

6.6.20.5 [1539] Problem 5

problem number 1539

Added May 31, 2019.

Problem Chapter 6.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda y)w_y + c\arccos^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*y]^n*D[w[x, y,z], y] +c*ArcCos[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}+\frac{\cos^{-1}(\beta z)^{-k}\left(\left(-i\cos^{-1}(\beta z)\right)^k\mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right)+\left(i\cos^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*y)^n*diff(w(x,y,z),y)+c*arccos(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x + \frac{\sqrt{\pi}\left(\frac{\sqrt{-\lambda^2y^2+1}\,2^n\,\mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)}}{\sqrt{\pi}\,(n-2)} - \frac{\sqrt{-\lambda^2y^2+1}\,2^n\arccos(\lambda y)^{-n+1}}{\sqrt{\pi}\,(n-2)} + \frac{3}{}\right.}{b\lambda}\right.$$

6.6.21 7.3

Local contents

6.6.21.1 [1540] Problem 1 2204
6.6.21.2 [1541] Problem 2 2205
6.6.21.3 [1542] Problem 3 2206
6.6.21.4 [1543] Problem 4 2207
6.6.21.5 [1544] Problem 5 2208

6.6.21.1 [1540] Problem 1

problem number 1540

Added May 31, 2019.

Problem Chapter 6.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arctan^n(\lambda x)\arctan^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z \tan^{-1}(\beta K[1])^{-k}dK[1]-\int_1^x \frac{c\tan^{-1}(\lambda K[2])^n}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},-\left(\int \arctan(\lambda x)^n\,dx\right)+\int \frac{a\arctan(\beta z)^{-k}}{c}dz\right)$$

6.6.21.2 [1541] Problem 2

problem number 1541

Added May 31, 2019.

Problem Chapter 6.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^m(\beta y)\arctan^k(\gamma z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*ArcTan[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z \tan^{-1}(\gamma K[1])^{-k}dK[1]-\int_1^x \frac{c\tan^{-1}(\lambda K[2])^n\left(\left(\frac{a\tan^{-1}(\lambda K[2])^{-n}\text{InverseFunct}}{}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arctan(lambda*x)^n*arctan(beta*y)^m*arctan(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-xb}{a},\int\frac{a\arctan(\gamma 1z)^{-k}}{c}dz-\left(\int^x \arctan(\_a\lambda)^n\arctan\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right.\right.\right.$$

6.6.21.3 [1542] Problem 3

problem number 1542

Added May 31, 2019.

Problem Chapter 6.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda x)w_y+c\arctan^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] +c*ArcTan[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1],z-\int_1^x \frac{c\tan^{-1}(\beta K[2])^k}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+c*arctan(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int \frac{b\arctan(\lambda x)^n}{a}dx\right),z-\left(\int \frac{c\arctan(\beta x)^k}{a}dx\right)\right)$$

6.6.21.4 [1543] Problem 4

problem number 1543

Added May 31, 2019.

Problem Chapter 6.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda x)w_y+c\arctan^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] +c*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\tan^{-1}(\lambda K[1])^n}{a}dK[1],\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+c*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int\frac{b\arctan(\lambda x)^n}{a}dx,\int\frac{b\arctan(\beta z)^{-k}}{c}dz-\left(\int^y\arctan\left(\lambda\,\mathrm{RootOf}\left(\_b-y+\right.\right.\right.\right.$$

6.6.21.5 [1544] Problem 5

problem number 1544

Added May 31, 2019.

Problem Chapter 6.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda y)w_y+c\arctan^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*y]^n*D[w[x, y,z], y] +c*ArcTan[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \tan^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a},\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*y)^n*diff(w(x,y,z),y)+c*arctan(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{a\left(\int \arctan(\lambda y)^{-n}\,dy\right)}{b}+x,-\left(\int \arctan(\lambda y)^{-n}\,dy\right)+\int \frac{b\arctan(\beta z)^{-k}}{c}dz\right)$$

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6.6.22.1 [1545] Problem 1

problem number 1545

Added May 31, 2019.

Problem Chapter 6.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\operatorname{arccot}^n(\lambda x)\operatorname{arccot}^k(\beta z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \int_1^z \cot^{-1}(\beta K[1])^{-k} dK[1] - \int_1^x \frac{c\cot^{-1}(\lambda K[2])^n}{a} dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\left(\int \left(-\arctan(\lambda x) + \frac{\pi}{2}\right)^n dx\right) + \int \frac{a\left(-\arctan(\beta z) + \frac{\pi}{2}\right)^{-k}}{c} dz\right)$$

6.6.22.2 [1546] Problem 2

problem number 1546

Added May 31, 2019.

Problem Chapter 6.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\operatorname{arccot}^n(\lambda x)\operatorname{arccot}^m(\beta y)\operatorname{arccot}^k(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},\int_1^z \cot^{-1}(\gamma K[1])^{-k}dK[1]-\int_1^x \frac{c\cot^{-1}(\lambda K[2])^n\left(\left(\frac{a\cot^{-1}(\lambda K[2])^{-n}\text{InverseFunctio}}{}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\int\frac{a\left(-\arctan(\gamma 1z)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\int^x\left(-\arctan(\_a\lambda)+\frac{\pi}{2}\right)^n\left(-\arctan\left(\right.\right.\right.\right.$$

6.6.22.3 [1547] Problem 3

problem number 1547

Added May 31, 2019.

Problem Chapter 6.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\operatorname{arccot}^n(\lambda x)w_y+c\operatorname{arccot}^k(\beta x)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*x]^n*D[w[x, y,z], y] +c*ArcCot[beta*x]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1],z-\int_1^x \frac{c\cot^{-1}(\beta K[2])^k}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+c*arccot(beta*x)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int \frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx\right),z-\left(\int \frac{c\left(-\arctan(\beta x)+\frac{\pi}{2}\right)^k}{a}dx\right)\right)$$

6.6.22.4 [1548] Problem 4

problem number 1548

Added May 31, 2019.

Problem Chapter 6.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\operatorname{arccot}^n(\lambda x)w_y+c\operatorname{arccot}^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*x]^n*D[w[x, y,z], y] +c*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x \frac{b\cot^{-1}(\lambda K[1])^n}{a}dK[1],\int_1^z \cot^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+c*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int\frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx,\int\frac{b\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\int^y\left(-\arctan\left(\right.\right.\right.$$

6.6.22.5 [1549] Problem 5

problem number 1549

Added May 31, 2019.

Problem Chapter 6.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\operatorname{arccot}^n(\lambda y)w_y+c\operatorname{arccot}^k(\beta z)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCot[lambda*y]^n*D[w[x, y,z], y] +c*ArcCot[beta*z]^k*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \cot^{-1}(\lambda K[1])^{-n}dK[1]-\frac{bx}{a},\int_1^z \cot^{-1}(\beta K[2])^{-k}dK[2]-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*y)^n*diff(w(x,y,z),y)+c*arccot(beta*z)^k*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{a\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)}{b}+x,-\left(\int\left(-\arctan(\lambda y)+\frac{\pi}{2}\right)^{-n}dy\right)+\int\frac{b(-\mathrm{ar}}{}\right.$$

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6.6.23.1 [1550] Problem 1

problem number 1550

Added May 31, 2019.

Problem Chapter 6.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + f(x)w_y + g(x)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*D[w[x, y,z], y] +g[x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \int_1^x f(K[1])dK[1], z - \int_1^x g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+g(x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int f(x)\,dx\right), z - \left(\int g(x)\,dx\right)\right)$$

6.6.23.2 [1551] Problem 2

problem number 1551

Added May 31, 2019.

Problem Chapter 6.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + f(x)(y+a)w_y + g(x)(z+b)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*(y+a)*D[w[x, y,z], y] +g[x]*(z+b)*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right) - \int_1^x a\exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right) f(K[2])dK[2], z\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+g(x)*(z+b)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((a+y)\,\mathrm{e}^{-(\int f(x)dx)}, (b+z)\,\mathrm{e}^{-(\int g(x)dx)}\right)$$

6.6.23.3 [1552] Problem 3

problem number 1552

Added May 31, 2019.

Problem Chapter 6.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + f(x))w_y + (bz + g(x))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+f[x])*D[w[x, y,z], y] +(b*z+g[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1],ze^{-bx}-\int_1^x e^{-bK[2]}g(K[2])dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+(b*z+g(x))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right),z\,\mathrm{e}^{-bx}-\left(\int \mathrm{e}^{-bx}g(x)\,dx\right)\right)$$

6.6.23.4 [1553] Problem 4

problem number 1553

Added May 31, 2019.

Problem Chapter 6.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)y+g_2(x))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[x]*y+g2[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[3]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[3])dK[3],-\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(x)*y+g2(x))*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-(\int f1(x)dx)}-\left(\int \mathrm{e}^{-(\int f1(x)dx)}f2(x)\,dx\right),z-\left(\int^x\left(y\,\mathrm{e}^{-(\int f1(x)dx)}\mathrm{e}^{\int f1(\_f)d\_f}g1(\_f)\right.\right.\right.$$

6.6.23.5 [1554] Problem 5

problem number 1554

Added May 31, 2019.

Problem Chapter 6.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)z+g_2(x))w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[x]*z+g2[x])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2],z\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(x)*z+g2(x))*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-(\int f1(x)dx)}-\left(\int \mathrm{e}^{-(\int f1(x)dx)}f2(x)\,dx\right),z\,\mathrm{e}^{-(\int g1(x)dx)}-\left(\int \mathrm{e}^{-(\int g1(x)dx)}g2(x)\,dx\right)\right)$$

6.6.23.6 [1555] Problem 6

problem number 1555

Added May 31, 2019.

Problem Chapter 6.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_2(x)y+f_1(x)z+f_0(x))w_y+(g_2(x)y+g_1(x)z+g_0(x))w_z=0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (f2[x]*y+f1[x]*z+f0[x])*D[w[x, y,z], y] +(g2[x]*y+g1[x]*z+g0[x])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,y,z),x)+ (f2(x)*y+f1(x)*z+f0(x))*diff(w(x,y,z),y)+(g2(x)*y+g1(x)*z+g0(x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.23.7 [1556] Problem 7

problem number 1556

Added May 31, 2019.

Problem Chapter 6.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x))w_y + f(x) \sinh(\gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \int_1^x f(K[2])dK[2], \frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}} - \int_1^{e^{\lambda x}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right)\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{H}\right.}{-(-\sinh(\lambda x)+i)\left(\sinh^2(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda}{2}\right.}\right.$$

6.6.23.8 [1557] Problem 8

problem number 1557

Added May 31, 2019.

Problem Chapter 6.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(x)z^m)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g1[x]*z+g2[x]*z^m)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]}\mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2]+y^{1-k}\exp\left((k-1)\int\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g1(x)*z+g2(x)*z^m)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int g1(x)dx\right)} + ($$

6.6.23.9 [1558] Problem 9

problem number 1558

Added May 31, 2019.

Problem Chapter 6.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x) + g_2(x)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g1[x]+g2[x]*Exp[lambda*z])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g1(x)+g2(x)*exp(lambda*z))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g1(x)dx\right)} g2(x)\,d}{\lambda}\right.$$

6.6.23.10 [1559] Problem 10

problem number 1559

Added May 31, 2019.

Problem Chapter 6.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x) + g_2(x)e^{\beta z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g1[x]+g2[x]*Exp[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g1(x)+g2(x)*exp(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda(\int f1(x)dx)} f2(x)\,dx\right) - e^{-(y-(\int f1(x)dx))\lambda}}{\lambda}, \frac{-\beta\left(\int e^{\beta(\int g1(x)dx)} g2(x)\,dx\right) - e^{-(z-(\int}}{\beta}\right.$$

6.6.24 8.2

Local contents

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6.6.24.1 [1560] Problem 1

problem number 1560

Added May 31, 2019.

Problem Chapter 6.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x)g(y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z+f[x]*g[y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \frac{z}{x} - \int_1^x \frac{f(K[1])g\left(\frac{yK[1]}{x}\right)}{K[1]^2} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+(z+f(x)*g(y))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-x\left(\int^x \frac{f(\_a)g\left(\frac{-ay}{x}\right)}{\_a^2} d\_a\right) + z}{x}\right)$$

6.6.24.2 [1561] Problem 2

problem number 1561

Added May 31, 2019.

Problem Chapter 6.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g_1(y)z + g_2(y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g1[y]*z+g2[y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right) - \int_1^x \exp\left(-\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2], z\,\mathrm{e}\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+  (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g1(y)*z+g2(y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-(\int f1(x)dx)} - \left(\int \mathrm{e}^{-(\int f1(x)dx)} f2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int^x g1\left(\left(y\,\mathrm{e}^{-(\int f1(x)dx)} + \int \mathrm{e}^{-(\int f1(\_f)d\_f)} f2(\_f)d\_f\right.\right.\right.}$$

6.6.24.3 [1562] Problem 3

problem number 1562

Added May 31, 2019.

Problem Chapter 6.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x))w_y + f(x)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\int_1^z \frac{1}{g(K[2])}dK[2] - \int_1^x f(K[3])dK[3], \frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}} - \int_1^{e^{\lambda x}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}dK[1]\right.\right.\right.$$

Maple ✓

```
restart;
pde :=  diff(w(x,y,z),x)+  (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda \mathrm{H}\right.}{-(-\sinh(\lambda x)+i)\left(\sinh^2(\lambda x)+1\right)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda}{2}\right.}\right.$$

6.6.24.4 [1563] Problem 4

problem number 1563

Added May 31, 2019.

Problem Chapter 6.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f(x)w_x + z^k w_y + g(y)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + z^k*D[w[x, y,z], y] +g[y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y,z),x)+  z^k*diff(w(x,y,z),y)+g(y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(zz^k + (-k-1)\left(\int g(y)\,dy\right), \int \frac{1}{f(x)}dx - \left(\int^y \left(\left(z^{k+1} + \int (-k-1)\,g(y)\,dy + \int\right.\right.\right.$$

6.6.24.5 [1564] Problem 5

problem number 1564

Added May 31, 2019.

Problem Chapter 6.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + g[y]*D[w[x, y,z], y] +h[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1] - \int_1^x \frac{1}{f(K[2])}dK[2], \int_1^z \frac{1}{h(K[3])}dK[3] - \int_1^x \frac{1}{f(K[4])}dK[4]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f(x)*diff(w(x,y,z),x)+  g(y)*diff(w(x,y,z),y)+h(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy, -\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{h(z)}dz\right)$$

6.6.24.6 [1565] Problem 6

problem number 1565

Added May 31, 2019.

Problem Chapter 6.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y,z], x] + f2[x]*g[y]*D[w[x, y,z], y] +f3[x]*g[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\int_1^y \frac{1}{g(K[1])}dK[1]-\int_1^x \frac{\text{f2}(K[2])}{\text{f1}(K[2])}dK[2],\int_1^z \frac{1}{g(K[3])}dK[3]-\int_1^x \frac{\text{f3}(K[4])}{\text{f1}(K[4])}dK[4]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  f1(x)*diff(w(x,y,z),x)+  f2(x)*g(y)*diff(w(x,y,z),y)+f3(x)*h(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\int \frac{1}{g(y)}dy-\left(\int \frac{f2(x)}{f1(x)}dx\right),\int \frac{1}{h(z)}dz-\left(\int \frac{f3(x)}{f1(x)}dx\right)\right)$$

6.6.24.7 [1566] Problem 7

problem number 1566

Added May 31, 2019.

Problem Chapter 6.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\beta y)w_x+b\sinh(\gamma z)w_y+f_1(x)f_2(z)\sinh(\beta y)w_z=0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[beta*y]*D[w[x, y,z], x] + b*Sinh[gamma*z]*D[w[x, y,z], y] +f1[x]*f2[z]*Sinh[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(\int_1^z \frac{1}{\text{f2}(K[1])}dK[1]-\int_1^x \frac{\text{f1}(K[2])}{a}dK[2],-\frac{\beta\int_1^x \frac{b\sinh\left(\gamma\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{\text{f2}(K[1])}dK[1]\&\right]\left[-\int_1\right.\right.}{}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(\int_1^z \frac{1}{\text{f2}(K[1])}dK[1]-\int_1^x \frac{\text{f1}(K[2])}{a}dK[2],\frac{\cosh(\beta y)}{\beta}-\int_1^x \frac{b\sinh\left(\gamma\text{InverseFunction}\left[\int_1^{\#}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
pde :=  a*sinh(beta*y)*diff(w(x,y,z),x)+  b*sinh(gamma1*z)*diff(w(x,y,z),y)+f2(x)*f2(z)*sinh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\int \frac{a}{f2\,(z)}dz-\left(\int f2(x)\,dx\right),\frac{-b\beta\left(\int^x \sinh\left(\gamma 1\,\text{RootOf}\left(\int \frac{a}{f2(z)}dz+\int f2(\_f)\,d\_f-\right.\right.\right.}{b}\right.$$

6.6.25 8.3

Local contents

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6.6.25.1 [1567] Problem 1

problem number 1567

Added May 31, 2019.

Problem Chapter 6.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + f(x, y)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +f[x,y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\int_1^x\frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+f(x,y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{f\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a\right)\right)$$

6.6.25.2 [1568] Problem 2

problem number 1568

Added May 31, 2019.

Problem Chapter 6.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + f(x, y)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +f[x,y]*g[z]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \int_1^z \frac{1}{g(K[1])}dK[1] - \int_1^x \frac{f\left(K[2], y + \frac{b(K[2]-x)}{a}\right)}{a}dK[2]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+f(x,y)*g(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \int \frac{a}{g(z)}dz - \left(\int^x f\left(\_a, \frac{ay - (-\_a + x)b}{a}\right)d\_a\right)\right)$$

6.6.25.3 [1569] Problem 3

problem number 1569

Added May 31, 2019.

Problem Chapter 6.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x, y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z+f[x,y])*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{y}{x}, \frac{z}{x} - \int_1^x \frac{f\left(K[1], \frac{yK[1]}{x}\right)}{K[1]^2} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  x*diff(w(x,y,z),x)+  y*diff(w(x,y,z),y)+(z+f(x,y))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, \frac{-x\left(\int^x \frac{f\left(\_a, \frac{-ay}{x}\right)}{\_a^2} d\_a\right) + z}{x}\right)$$

6.6.25.4 [1570] Problem 4

problem number 1570

Added May 31, 2019.

Problem Chapter 6.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + f(x,y)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +f[x,y]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-\frac{b}{a}}, z - \int_1^x \frac{f\left(K[1], x^{-\frac{b}{a}} yK[1]^{\frac{b}{a}}\right)}{aK[1]} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+f(x,y)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-\frac{b}{a}}, z - \left(\int^x \frac{f\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_aa} d\_a\right)\right)$$

6.6.25.5 [1571] Problem 5

problem number 1571

Added May 31, 2019.

Problem Chapter 6.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + f(x,y)g(z)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +f[x,y]*g[x]*D[w[x,y,z],z]==0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-\frac{b}{a}}, z - \int_1^x \frac{f\left(K[1], x^{-\frac{b}{a}} yK[1]^{\frac{b}{a}}\right) g(K[1])}{aK[1]} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
pde :=  a*x*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+f(x,y)*g(z)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-\frac{b}{a}}, \int \frac{a}{g(z)} dz - \left(\int^x \frac{f\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_a} d\_a\right)\right)$$

6.6.25.6 [1572] Problem 6

problem number 1572

Added May 31, 2019.

Problem Chapter 6.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g(x, y)z + h(x, y))w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] +(g[x,y]*z+h[x,y])*D[w[x,y,z],z]==0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right) - \int_1^x \exp\left(-\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2], z\,\mathrm{e}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-(\int f1(x)dx)} - \left(\int \mathrm{e}^{-(\int f1(x)dx)} f2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int^x g\left(\_f, \left(y\,\mathrm{e}^{-(\int f1(x)dx)} + \int \mathrm{e}^{-(\int f1(\_f)d\_f)} f2(\_f)d\_\right.\right.\right.}$$

6.6.25.7 [1573] Problem 7

problem number 1573

Added May 31, 2019.

Problem Chapter 6.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g(x,y)z + h(x,y)z^m)w_z = 0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*z^m)*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2] + y^{1-k}\exp\left((k-1)\right.\right.\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int^x g\left(\_f,\left(y^{-k+}\right.\right.\right.}\right.$$

6.6.25.8 [1574] Problem 8

problem number 1574

Added May 31, 2019.

Problem Chapter 6.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g(x, y) + h(x, y)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*Exp[lambda*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*exp(lambda*z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.6.25.9 [1575] Problem 9

problem number 1575

Added May 31, 2019.

Problem Chapter 6.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g(x, y)z + h(x, y)z^k)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g[x,y]*z+h[x,y]*z^k)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g(x,y)*z+h(x,y)*z^k)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda(\int f1(x)dx)} f2(x)\,dx\right) - e^{-(y-(\int f1(x)dx))\lambda}}{\lambda}, z^{-k+1} e^{(k-1)\left(\int^{x} g\left(\_a, \frac{\lambda(\int f1(\_a)d\_a)+\ln\left(\frac{}{(-(\int}\right.}{}\right.\right.}\right.$$

6.6.25.10 [1576] Problem 10

problem number 1576

Added May 31, 2019.

Problem Chapter 6.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g(x,y) + h(x,y)e^{\beta z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] +(g[x,y]+h[x,y]*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+(g(x,y)+h(x,y)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int f1(x)dx\right)} f2(x)\,dx\right) - e^{-\left(y-\left(\int f1(x)dx\right)\right)\lambda}}{\lambda}, \frac{-\beta\left(\int^{x} e^{\beta\left(\int g\left(\_a, \frac{\lambda\left(\int f1(\_a)d\_a\right)+\ln\left(\frac{}{\left(-\left(\int e^{\lambda}\right.\right.}\right.}\right.}\right.}{}\right.$$

6.6.25.11 [1577] Problem 11

problem number 1577

Added May 31, 2019.

Problem Chapter 6.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x,y) + h_2(x,y)z^m)w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] +(h1[x,y]+h2[x,y]*z^m)*D[w[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+(h1(x,y)+h2(x,y)*z^m)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.6.25.12 [1578] Problem 12

problem number 1578

Added May 31, 2019.

Problem Chapter 6.8.3.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x, y) + h_2(x, y)e^{\lambda z})w_z = 0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] +(h1[x,y]+h2[x,y]*Exp[lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde := f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+(h1(x,y)+h2(x,y)*exp(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

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6.7.1.1 [1579] Problem 1

problem number 1579

Added May 31, 2019.

Problem Chapter 7.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==alpha*x+beta*y+gamma*z+delta;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{x(a(\alpha x + 2\beta y + 2\delta + 2\gamma z) - x(b\beta + c\gamma))}{2a^2} + c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)= alpha*x+beta*y+gamma*z+del
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \frac{az - cx}{a}\right) + \frac{(\beta y + \gamma z + \delta)\,x}{a} + \frac{(\alpha a - b\beta - \gamma c)\,x^2}{2a^2}$$

6.7.1.2 [1580] Problem 2

problem number 1580

Added May 31, 2019.

Problem Chapter 7.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + azw_y + byw_z = cx + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c*x+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}}, \frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.$$

Maple ✓

```
restart;
pde := diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)= c*x+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{\left(\ln\left(\frac{\_aab+\sqrt{ab}\sqrt{(az^2+(\_a^2-y^2)b)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)\right)c+\sqrt{ab}\,(cx+s)}{\sqrt{ab}\sqrt{(az^2+(\_a^2-y^2)b)a}} d\_a + \_F1\left(\frac{az^2}{}\right.$$

6.7.1.3 [1581] Problem 3

problem number 1581

Added May 31, 2019.

Problem Chapter 7.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x + a_0) w_y + (b_1 x + b_0) w_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x+a0)*D[w[x, y,z], y] +(b1*x+b0)*D[w[x,y,z],z]==alpha*x+beta*y+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{6}x\left(-3\text{a0}\beta x - 2\text{a1}\beta x^2 + 3\alpha x - 3\text{b0}\gamma x - 2\text{b1}\gamma x^2 + 6\beta y + 6\delta + 6\gamma z\right) + c_1\left(-\text{a0}x - \frac{\text{a1}x^2}{2}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a1*x+a0)*diff(w(x,y,z),y)+(b1*x+b0)*diff(w(x,y,z),z)=alpha*x+beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(-2a1\beta - 2\gamma b1)\,x^3}{6} + \frac{(-3\beta a0 - 3\gamma b0 + 3\alpha)\,x^2}{6} + \frac{(6\beta y + 6\gamma z + 6\delta)\,x}{6} + \_F1\left(-\frac{1}{2}a1x^2 - a0x\right.$$

6.7.1.4 [1582] Problem 4

problem number 1582

Added May 31, 2019.

Problem Chapter 7.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_2 y + a_1 x + a_0) w_y + (b_2 y + b_1 x + b_0) w_z = c_2 y + c_1 z + c_0 x + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*y+a1*x+a0)*D[w[x, y,z], y] +(b2*y+b1*x+b0)*D[w[x,y,z],z]==c2*y+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{6\mathrm{a}2^4 c_1 \left(\frac{e^{-\mathrm{a}2x}(\mathrm{a}2(\mathrm{a}0+\mathrm{a}2y)+\mathrm{a}1\mathrm{a}2x+\mathrm{a}1)}{\mathrm{a}2^2}, \frac{e^{-\mathrm{a}2x}(\mathrm{a}2(2\mathrm{a}0\mathrm{b}2(\mathrm{a}2xe^{\mathrm{a}2x}+1)-\mathrm{a}2(\mathrm{a}2e^{\mathrm{a}2x}(2\mathrm{b}0x+\mathrm{b}1x^2-2z)+2\mathrm{b}2y(e^{\mathrm{a}2x}-1)}{2\mathrm{a}2^3}\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a2*y+a1*x+a0)*diff(w(x,y,z),y)+(b2*y+b1*x+b0)*diff(w(x,y,z),z)=c2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{6\left(-\frac{b1\,c1\,x^2}{3} + c1\,z + s + \left(-\frac{b0\,c1}{2} + \frac{c0}{2}\right)x\right) a2^4 x + 6a2^4\_F1\left(\frac{(a2^2 y + a1 + (a1x + a0)a2)\mathrm{e}^{-a2x}}{a2^2}, \frac{-2a0\,a2\,b}{}\right.}{}$$

6.7.1.5 [1583] Problem 5

problem number 1583

Added May 31, 2019.

Problem Chapter 7.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ay + k_1 x + k_0) w_y + (bz + s_1 x + s_0) w_z = c_1 x + c_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x+k0)*D[w[x, y,z], y] +(b*z+s1*x+s0)*D[w[x,y,z],z]==c1*x+c0
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1 \left(\frac{e^{-ax}(a^2 y + a(\mathrm{k0} + \mathrm{k1} x) + \mathrm{k1})}{a^2}, \frac{e^{-bx}(b^2 z + b(\mathrm{s0} + \mathrm{s1} x) + \mathrm{s1})}{b^2}\right) + \mathrm{c0} x + \frac{\mathrm{c1} x^2}{2}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a*y+k1*x+k0)*diff(w(x,y,z),y)+(b*z+s1*x+s0)*diff(w(x,y,z),z)=c1*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c1 x^2}{2} + c0 x + \_F1\left(\frac{(a^2 y + (k1 x + k0) a + k1) e^{-ax}}{a^2}, \frac{(b^2 z + (s1 x + s0) b + s1) e^{-bx}}{b^2}\right)$$

6.7.1.6 [1584] Problem 6

problem number 1584

Added May 31, 2019.

Problem Chapter 7.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + byw_y + czw_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==alpha*x+beta*y+gamma*z+de
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right) + \frac{x(\alpha x + 2\delta)}{2a} + \frac{\beta y}{b} + \frac{\gamma z}{c}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+  b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x+beta*y+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\alpha x^2}{2a} + \frac{\delta x}{a} + \frac{\beta y}{b} + \frac{\gamma z}{c} + \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

Hand solution

Solve

$$aw_x + byw_y + czw_z = \alpha x + \beta y + \gamma z + \delta$$

The parametrization invariant Lagrange-Charpit equations are

$$\frac{dx}{a} = \frac{dy}{by} = \frac{dz}{cz} = \frac{dw}{\alpha x + \beta y + \gamma z + \delta}$$

Solving $\frac{dx}{a} = \frac{dy}{by}$ gives

$$\begin{aligned}
\frac{b}{a}dx &= \frac{dy}{y} \\
\frac{b}{a}x &= \ln y + C_1 \qquad (1) \\
\ln y &= \frac{b}{a}x - C_1 \\
y &= C_1 e^{\frac{b}{a}x} \\
C_1 &= y e^{-\frac{b}{a}x}
\end{aligned}$$

Equation $\frac{dx}{a} = \frac{dz}{cz}$ gives

$$\begin{aligned}
\frac{c}{a}dx &= \frac{dz}{z} \\
\frac{c}{a}x &= \ln z + C_2 \\
\ln z &= \frac{c}{a}x - C_2 \\
z &= C_2 e^{\frac{c}{a}x} \\
C_2 &= z e^{-\frac{c}{a}x} \qquad (2)
\end{aligned}$$

And $\frac{dx}{a} = \frac{dw}{\alpha x+\beta y+\gamma z+\delta}$ gives

$$\begin{aligned}
\frac{\alpha x + \beta y + \gamma z + \delta}{a} dx &= dw \\
\left(\frac{\alpha}{a}x + \beta\frac{y}{a} + \gamma\frac{z}{a} + \frac{\delta}{a}\right) dx &= dw
\end{aligned}$$

But from (1) $y = C_1 e^{\frac{b}{a}x}$ and from (2) $z = C_2 e^{\frac{c}{a}x}$. Hence the above becomes

$$\left(\frac{\alpha}{a}x + \frac{\beta}{a}C_1 e^{\frac{b}{a}x} + \frac{\gamma}{a}C_2 e^{\frac{c}{a}x} + \frac{\delta}{a}\right) dx = dw$$

Integrating

$$\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}C_1 e^{\frac{b}{a}x} + \frac{\gamma}{c}C_2 e^{\frac{c}{a}x} + \frac{\delta}{a}x = w + C_3$$

But $C_2 = z e^{-\frac{c}{a}x}$ and $C_1 = y e^{-\frac{b}{a}x}$, hence the above becomes

$$\begin{aligned}
\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y e^{-\frac{b}{a}x} e^{\frac{b}{a}x} + \frac{\gamma}{c}z e^{-\frac{c}{a}x} e^{\frac{c}{a}x} + \frac{\delta}{a}x &= w + C_3 \\
\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x &= w + C_3 \\
C_3 &= \left(\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x\right) - w
\end{aligned}$$

Since $C_3 = F(C_1, C_2)$ then the solution is

$$\left(\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x\right) - w = F\left(ye^{-\frac{b}{a}x}, ze^{-\frac{c}{a}x}\right)$$

$$w(x,y,z) = F\left(ye^{-\frac{b}{a}x}, ze^{-\frac{c}{a}x}\right) + \left(\frac{\alpha}{a}\frac{x^2}{2} + \frac{\beta}{b}y + \frac{\gamma}{c}z + \frac{\delta}{a}x\right)$$

(sign change on F does not matter, since arbitrary function, can be renamed).

6.7.1.7 [1585] Problem 7

problem number 1585

Added June 1, 2019.

Problem Chapter 7.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = c$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c\log(x) + c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+  a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c\ln\left(\frac{aby}{\sqrt{ab}} + \sqrt{a^2z^2}\right) + \sqrt{ab}\_F1\left(\frac{az^2-by^2}{a}, x\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)}{\sqrt{ab}}$$

Hand solution

Solve

$$xw_x + azw_y + byw_z = c$$

The parametrization invariant Lagrange-Charpit equations are

$$\frac{dx}{x} = \frac{dy}{az} = \frac{dz}{by} = \frac{dw}{c}$$

Solving $\frac{dy}{az} = \frac{dz}{by}$ gives

$$\begin{aligned} \frac{b}{a}ydy &= zdz \\ \frac{b}{a}y^2 &= z^2 + C_1 \qquad (1) \\ C_1 &= \frac{b}{a}y^2 - z^2 \\ &= \frac{by^2 - az^2}{a} \end{aligned}$$

Equation $\frac{dx}{x} = \frac{dy}{az}$ gives

$$a\frac{dx}{x} = \frac{dy}{z}$$

But from (1) $z = \sqrt{\frac{b}{a}y^2 - C_1}$, hence the above becomes

$$a\frac{dx}{x} = \frac{dy}{\sqrt{\frac{b}{a}y^2 - C_1}}$$

Integrating gives

$$a\ln x=\sqrt{\frac{a}{b}}\ln\left(\sqrt{\frac{b}{a}}y+\sqrt{\frac{b}{a}y^2-C_1}\right)+C_2$$
$$\ln x=\sqrt{\frac{1}{ab}}\ln\left(\sqrt{\frac{b}{a}}y+\sqrt{\frac{b}{a}y^2-C_1}\right)+\frac{C_2}{a}$$

Let $\frac{C_2}{a}=C_3$ and the above becomes

$$x=C_3\left(\sqrt{\frac{b}{a}}y+\sqrt{\frac{b}{a}y^2-C_1}\right)^{\sqrt{\frac{1}{ab}}}$$
$$C_3=x\left(\sqrt{\frac{b}{a}}y+\sqrt{\frac{b}{a}y^2-C_1}\right)^{-\sqrt{\frac{1}{ab}}}$$

But from (1)

$$C_1=\frac{b}{a}y^2-z^2 \tag{2}$$

Hence C_3 simplifies to

$$\begin{aligned}C_3&=x\left(\sqrt{\frac{b}{a}}y+\sqrt{\frac{b}{a}y^2-\left(\frac{b}{a}y^2-z^2\right)}\right)^{-\sqrt{\frac{1}{ab}}}\\&=x\left(\sqrt{\frac{b}{a}}y+z\right)^{-\sqrt{\frac{1}{ab}}}\end{aligned} \tag{4}$$

And $\frac{dx}{x}=\frac{dw}{c}$ gives

$$\ln x=\frac{1}{c}w+C_4$$

But $C_4=F(C_1,C_3)$. Hence

$$\ln x-\frac{1}{c}w=F\left(\frac{b}{a}y^2-z^2,x\left(\sqrt{\frac{b}{a}}y+z\right)^{-\sqrt{\frac{1}{ab}}}\right)$$
$$-\frac{1}{c}w=F\left(\frac{b}{a}y^2-z^2,x\left(\sqrt{\frac{b}{a}}y+z\right)^{-\sqrt{\frac{1}{ab}}}\right)-\ln x$$
$$w(x,y,z)=cF\left(\frac{b}{a}y^2-z^2,x\left(\sqrt{\frac{b}{a}}y+z\right)^{-\sqrt{\frac{1}{ab}}}\right)+c\ln x$$

Verified OK under the assumptions that $a>0,b>0,z>0$.

6.7.1.8 [1586] Problem 8

problem number 1586

Added June 1, 2019.

Problem Chapter 7.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$abxw_x + b(ay + bz)w_y + a(ay - bz)w_z = c$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]==c;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*b*x*diff(w(x,y,z),x)+  b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{2\sqrt{-a^2y^2+2abyz+b^2z^2}\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}\,b\_F1\left(-\frac{1}{\sqrt{-a^2y^2+2abyz+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2abyz+b^2z^2}+\left(\right.}{}\right.\right.}{}$$

6.7.1.9 [1587] Problem 9

problem number 1587

Added June 1, 2019.

Problem Chapter 7.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$(a_1x + a_0)w_x + (b_1y + b_0)w_y + (c_1z + c_0)w_z = \alpha x + \beta y + \gamma z + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x+a0)*D[w[x, y,z], x] + (b1*y+b0)*D[w[x, y,z], y] +(c1*z+c0)*D[w[x,y,z],z]==alpha
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{(\mathrm{b0}+\mathrm{b1}y)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{b1}}{\mathrm{a1}}}}{\mathrm{b1}}, \frac{(\mathrm{c0}+\mathrm{c1}z)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{c1}}{\mathrm{a1}}}}{\mathrm{c1}}\right) - \frac{\mathrm{a0}\alpha\log(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{a1}^2} + \frac{\log(\mathrm{a0}}{}\right.$$

Maple ✓

```
restart;
local gamma;
pde := (a1*x+a0)*diff(w(x,y,z),x)+  (b1*y+b0)*diff(w(x,y,z),y)+(c1*z+c0)*diff(w(x,y,z),z)=al
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a1^2 b1^2 c1^2 \_F1\left(\frac{(b1y+b0)(a1x+a0)^{-\frac{b1}{a1}}}{b1}, \frac{(c1z+c0)(a1x+a0)^{-\frac{c1}{a1}}}{c1}\right) - (a0\alpha b1c1 + (b0\beta c1 + (\gamma c0 - \delta}{a1^2}$$

6.7.2 2.2

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6.7.2.1 [1588] Problem 1

problem number 1588

Added June 1, 2019.

Problem Chapter 7.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = \alpha x^2 + \beta y^2 + \gamma z^2 + \delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==alpha*x^2+beta*y^2+gamma*z^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{3}\left(3c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right) + \frac{\alpha x^3 + 3\delta x}{a} + \frac{\beta y^3}{b} + \frac{\gamma z^3}{c}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+  b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=alpha*x^2+beta*y^2+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{3a^3\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)+3\left(\left(\frac{\alpha x^2}{3}+\beta y^2+\gamma z^2+\delta\right)a^2-(b\beta y+c\gamma z)\,ax+\frac{(b^2\beta+\gamma c^2)x^2}{3}\right)x}{3a^3}$$

6.7.2.2 [1589] Problem 2

problem number 1589

Added June 1, 2019.

Problem Chapter 7.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^2+a_0)w_y+(b_1x^2+b_0)w_z=\alpha x+\beta y+\gamma z+\delta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^2+a0)*D[w[x, y,z], y] +(b1*x^2+b0)*D[w[x,y,z],z]==alpha*x+bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to-\frac{1}{4}x\left(2\text{a0}\beta x+\text{a1}\beta x^3-2\alpha x+2\text{b0}\gamma x+\text{b1}\gamma x^3-4\beta y-4\delta-4\gamma z\right)+c_1\left(-\text{a0}x-\frac{\text{a1}x^3}{3}+\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a1*x^2+a0)*diff(w(x,y,z),y)+(b1*x^2+b0)*diff(w(x,y,z),z)=alpha*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(-a1\beta - \gamma b1)\,x^4}{4} + \frac{(-2\beta a0 - 2\gamma b0 + 2\alpha)\,x^2}{4} + \frac{(4\beta y + 4\gamma z + 4\delta)\,x}{4} + \_F1\left(-\frac{1}{3}a1x^3 - a0x +\right.$$

6.7.2.3 [1590] Problem 3

problem number 1590

Added June 1, 2019.

Problem Chapter 7.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + s_1x^2 + s_0)w_z = c_1x^2 + c_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x^2+k0)*D[w[x, y,z], y] +(b*z+s1*x^2+s0)*D[w[x,y,z],z]==c1*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{e^{-ax}(a^2(\mathrm{k0} + \mathrm{k1}x^2) + a^3y + 2a\mathrm{k1}x + 2\mathrm{k1})}{a^3}, \frac{e^{-bx}(b^2(\mathrm{s0} + \mathrm{s1}x^2) + b^3z + 2b\mathrm{s1}x + 2\mathrm{s1})}{b^3}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+  (a*y+k1*x^2+k0)*diff(w(x,y,z),y)+(b*z+s1*x^2+s0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c1x^3}{3} + c0x + \_F1\left(\frac{(a^3y + 2ak1x + (k1x^2 + k0)a^2 + 2k1)\,\mathrm{e}^{-ax}}{a^3}, \frac{(b^3z + 2bs1x + (s1x^2 + s0)}{b^3}\right.$$

6.7.2.4 [1591] Problem 4

problem number 1591

Added June 1, 2019.

Problem Chapter 7.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2xy + a_1x^2 + a_0)w_y + (b_2xy + b_1x^2 + b_0)w_z = c_2y + c_1z + c_0x + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*x+a1*x^2+a0)*D[w[x, y,z], y] +(b2*x*y+b1*x^2+b0)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{180}x\left(15\text{a0b2c1}x^3 - 90\text{a0c2}x + 10\text{a1b2c1}x^5 - 45\text{a1c2}x^3 + 12\text{a2b2c1}x^4 - 60\text{a2c2}x^2 - 90\text{b}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a2*x+a1*x^2+a0)*diff(w(x,y,z),y)+(b2*x*y+b1*x^2+b0)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a1\,b2\,c1\,x^6}{18} + \frac{a2\,b2\,c1\,x^5}{15} + \frac{(-45a1\,c2 + (15b2\,a0 - 45b1)\,c1)\,x^4}{180} + \frac{(-60b2\,c1\,y - 60a2\,c2)\,x^3}{180} +$$

6.7.2.5 [1592] Problem 5

problem number 1592

Added June 1, 2019.

Problem Chapter 7.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = x(\alpha x + \beta y + \gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==x*(alpha*x+beta*y+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{x(a^2(\alpha x + 2\beta y + 2\gamma z) + a\alpha x(b+c) + 2a(b\gamma z + \beta c y) + \alpha b c x)}{2a(a+b)(a+c)} + c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=x*(alpha*x+beta*y+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha x^2}{2a}+\left(\frac{\beta y}{\left(\frac{b}{a}+1\right)a}+\frac{\gamma z}{\left(\frac{c}{a}+1\right)a}\right)x+\_F1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)$$

6.7.2.6 [1593] Problem 6

problem number 1593

Added June 1, 2019.

Problem Chapter 7.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+bxyw_y+cxzw_z=\alpha x+\beta y+\gamma z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]==alpha*x+beta*y+ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)+\frac{\alpha\log(x)}{a}+\frac{-a(\beta y+\gamma z)+b\gamma z+\beta cy}{x(a-b)(a-c)}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)=alpha*x+beta*y+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\alpha \ln(x)}{a} + \_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) + \frac{-\frac{\beta y}{a-b} - \frac{\gamma z}{a-c}}{x}$$

6.7.2.7 [1594] Problem 7

problem number 1594

Added June 1, 2019.

Problem Chapter 7.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + bxyw_y + cz^2 w_z = ky^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{ky^2}{ax-2bx} + c_1\left(yx^{-\frac{b}{a}}, \frac{c}{ax} - \frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*x*y*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{-\frac{ky^2}{x} + (a-2b)\_F1\left(yx^{-\frac{b}{a}}, \frac{ax-cz}{axz}\right)}{a-2b}$$

6.7.2.8 [1595] Problem 8

problem number 1595

Added June 1, 2019.

Problem Chapter 7.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = kxy$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*x*y;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{kxy\log\left(\frac{ax}{y}\right)}{ax-by} + c_1\left(\frac{b}{ax}-\frac{1}{y}, \frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*x*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{kxy\ln\left(\frac{ax}{y}\right)}{-ax+by} + \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

6.7.2.9 [1596] Problem 9

problem number 1596

Added June 1, 2019.

Problem Chapter 7.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = \alpha x^2 + \beta y^2 + \gamma z^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==alpha*x^2+beta*y^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a(ax-by)(ax-cz)c_1\left(\frac{b}{ax}-\frac{1}{y}, \frac{c}{ax}-\frac{1}{z}\right) + a^2x(\alpha x^2-\beta y^2-\gamma z^2) - a\alpha x^2(by+cz) + ayz}{a(ax-by)(ax-cz)}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=alpha*x^2+beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{\beta y^2}{ax-by} - \frac{\gamma z^2}{ax-cz} + \frac{\alpha x}{a} + \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

6.7.3 2.3

Local contents

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6.7.3.1 [1597] Problem 1

problem number 1597

Added June 1, 2019.

Problem Chapter 7.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = xyz$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +b*D[w[x,y,z],z]==x*y*z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{1}{12}x^2(ax(bx-2z)-2bxy+6yz)+c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+b*diff(w(x,y,z),z)=x*y*z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{abx^4}{12} + \frac{x^2yz}{2} + \frac{(-2az-2by)\,x^3}{12} + \_F1(-ax+y,-bx+z)$$

6.7.3.2 [1598] Problem 2

problem number 1598

Added June 1, 2019.

Problem Chapter 7.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = kx^3 + sy^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==k*x^3+s*y^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{x(3a^2(kx^3+4sy^2)-12absxy+4b^2sx^2)}{12a^3} + c_1\left(y-\frac{bx}{a}, z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=k*x^3+s*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{kx^4}{4a} + \frac{sxy^2}{a} - \frac{bsx^2y}{a^2} + \frac{b^2sx^3}{3a^3} + \_F1\left(\frac{ay-bx}{a}, \frac{az-cx}{a}\right)$$

6.7.3.3 [1599] Problem 3

problem number 1599

Added June 1, 2019.

Problem Chapter 7.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = kx + s\sqrt{x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==k*x+s*Sqrt[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{6ac_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right) + 3kx^2 + 4sx^{3/2}}{6a}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=k*x+s*sqrt(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{kx^2}{2a} + \frac{2sx^{\frac{3}{2}}}{3a} + \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right)$$

6.7.3.4 [1600] Problem 4

problem number 1600

Added June 1, 2019.

Problem Chapter 7.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + azw_y + byw_z = c\sqrt{x} + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==c*Sqrt[x]+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}}, \frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*sqrt(x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{\sqrt{\frac{\frac{\sqrt{ab}\,x+\ln\left(\frac{-aab+\sqrt{ab}\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}c+s}}{\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}} d\_a + \_F1\left(\frac{az^2-by^2}{a}, -\frac{-\sqrt{a}}{}\right.$$

6.7.3.5 [1601] Problem 5

problem number 1601

Added June 1, 2019.

Problem Chapter 7.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = kxyz$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==k*x*y*z;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{kxyz\left(by(ax-cz)\log\left(\frac{ax}{y}\right)+cz(by-ax)\log\left(\frac{ax}{z}\right)\right)}{(ax-by)(ax-cz)(by-cz)} + c_1\left(\frac{b}{ax}-\frac{1}{y}, \frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=k*x*y*z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left((ax-cz)\,by\ln\left(\frac{ax}{y}\right)-(ax-by)\,cz\ln\left(\frac{ax}{z}\right)\right)kxyz+(by-cz)\,(ax-by)\,(ax-cz)\,\_F1\left(\frac{ax-}{ax}\right.}{(by-cz)\,(ax-by)\,(ax-cz)}$$

6.7.4 2.4

Local contents

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6.7.4.1 [1602] Problem 1

problem number 1602

Added June 10, 2019.

Problem Chapter 7.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = \alpha x^n + \beta y^m + \gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gamma*z^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)+\frac{\alpha x^{n+1}}{an+a}+\frac{\beta y^{m+1}}{bm+b}+\frac{\gamma z^{k+1}}{ck+c}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gamma*z^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{(m+1)(k+1)(n+1)\,abc\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)+(m+1)(k+1)\,\alpha bcx^{n+1}+(n+1)\left((m+1)\right.}{(m+1)(k+1)(n+1)\,abc}$$

6.7.4.2 [1603] Problem 2

problem number 1603

Added June 10, 2019.

Problem Chapter 7.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+byw_y+czw_z=\alpha x^n+\beta y^m+\gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)+\frac{\alpha x^{n+1}}{an+a}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gamma
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\alpha x^{n+1}}{(n+1)\,a}+\_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}},z\,\mathrm{e}^{-\frac{cx}{a}}\right)+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}$$

6.7.4.3 [1604] Problem 3

problem number 1604

Added June 10, 2019.

Problem Chapter 7.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=cx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^{n+1}}{n+1}+c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}\frac{c\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_aab+\sqrt{ab}\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)^n}{\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}}d\_a+\_F1\left(\frac{az^2-by^2}{a},-\frac{-\sqrt{ab}}{}\right.$$

6.7.4.4 [1605] Problem 4

problem number 1605

Added June 10, 2019.

Problem Chapter 7.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+czw_z=\alpha x^n+\beta y^m+\gamma z^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== alpha*x^n+beta*y^m+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)+\frac{\alpha x^n}{an}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=alpha*x^n+beta*y^m+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{\alpha\_a^n+\beta\left(y\_a^{\frac{b}{a}}x^{-\frac{b}{a}}\right)^m+\gamma\left(z\_a^{\frac{c}{a}}x^{-\frac{c}{a}}\right)^k}{\_aa}d\_a+\_F1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)$$

6.7.4.5 [1606] Problem 5

problem number 1606

Added June 10, 2019.

Problem Chapter 7.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+azw_y+byw_z=cx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^n}{n}+c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}},y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=c*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^y \frac{c\left(x\left(\frac{\_aab+\sqrt{ab}\sqrt{(az^2+(\_a^2-y^2)b)a}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^n}{\sqrt{(az^2+(\_a^2-y^2)\,b)\,a}}d\_a+\_F1\left(\frac{az^2-by^2}{a},x\left(az+\right.\right.$$

6.7.4.6 [1607] Problem 6

problem number 1607

Added June 10, 2019.

Problem Chapter 7.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x+b(ay+bz)w_y+a(ay-bz)w_z=cx^n$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]== c*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*b*x*diff(w(x,y,z),x)+b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=c*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\left(\int^{y} -\frac{c\left(x\left(\frac{\frac{\sqrt{2}\,\_aa^2}{-a^2y^2+2abyz+b^2z^2}+\sqrt{\frac{2\,\_a^2a^2}{-a^2y^2+2abyz+b^2z^2}+1}\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}{\sqrt{\frac{a^2}{-a^2y^2+2abyz+b^2z^2}}}\right)^{\frac{\sqrt{2}\,a}{2\sqrt{-a^2y^2+2abyz+b^2z^2}\sqrt{\frac{a^2}{-a^2y^2+2a}}}}\right.}{\sqrt{-a^2y^2}}\right.$$

6.7.4.7 [1608] Problem 7

problem number 1608

Added June 10, 2019.

Problem Chapter 7.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = cx^k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*y^m*D[w[x, y,z], y] +b*x^nu*y^mu*z^lambda*D[w[x,y,z],z]== c*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)=c*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{cx^{k+1} + (k+1)\,\_F1\left(\frac{(m-1)ax^{n+1}+(n+1)y^{-m+1}}{n+1}, (\lambda-1)\,b\left(\int^{x} \_a^{\nu}\left(\left(\frac{(-\_a^{n+1}+x^{n+1})(m-1)a+(n+1)y^{-}}{n+1}\right.\right.\right.}{k+1}$$

6.7.4.8 [1609] Problem 8

problem number 1609

Added June 10, 2019.

Problem Chapter 7.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{n_1} y + b_1 x^{m_1}) w_y + (a_2 x^{n_2} y + b_2 x^{m_2}) w_z = c_2 x^{k_2} y + c_1 x^{k_1} z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n1*y+ b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}+e^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}}y,(-1)^{-\frac{\mathrm{n2}+1}{\mathrm{n1}+1}}\mathrm{a2}e^{-\mathrm{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.7.4.9 [1610] Problem 9

problem number 1610

Added June 10, 2019.

Problem Chapter 7.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1x^{m_1})w_y+(a_2x^{n_2}z+b_2x^{m_2})w_z=c_2x^{k_2}y+c_1x^{k_1}z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+ b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\frac{\mathrm{a2}^{-\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}\mathrm{c1}e^{\frac{\mathrm{a2}\left(K[1]^{\mathrm{n2}+1}-x^{\mathrm{n2}+1}\right)}{\mathrm{n2}+1}}\left((\mathrm{n2}+1)z\mathrm{a2}^{\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}+\mathrm{b2}e^{\frac{\mathrm{a2}x^{\mathrm{n2}+1}}{\mathrm{n2}+1}}(\mathrm{n2}+1)^{\frac{\mathrm{m2}+1}{\mathrm{n2}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m}}{\mathrm{n}}\right.\right.}{\mathrm{n2}+1}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.7.4.10 [1611] Problem 10

problem number 1611

Added June 10, 2019.

Problem Chapter 7.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1y^k)w_y+(a_2x^{n_2}z+b_2z^m)w_z=cx^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+ b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{cx^{s+1}}{s+1}+c_1\left(\mathrm{b1}(-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}}(\mathrm{n1}+1)^{-\frac{\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{1}{\mathrm{n1}+1}}(k-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{1}{\mathrm{n1}+1},-\frac{\mathrm{a1}(k-1)x^{\mathrm{n}}}{\mathrm{n1}+1}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{cx^{s+1}+(s+1)\_F1\left(\frac{-(n1+1)(n1+2)^2b1x^{-n1}\left(-\frac{(k-1)a1x^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\frac{n1+2}{2n1+2},\frac{2n1+3}{2n1+2},-\frac{(k-1)a1x^{n1+}}{n1+1}\right.}{}\right.}{}$$

6.7.4.11 [1612] Problem 11

problem number 1612

Added June 10, 2019.

Problem Chapter 7.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x^{n_1}y+b_1y^k)w_y+(a_2x^{n_2}z+b_2z^m)w_z=c_1x^{s_1}+c_2y^{s_2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n1*y+ b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\mathrm{c1}K[1]^{\mathrm{s1}}+\mathrm{c2}\left(\left(\frac{(-1)^{-\frac{1}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{1}{\mathrm{n1}+1}}\exp\left(-\frac{\mathrm{a1}\left(x^{\mathrm{n1}+1}+(k-1)K[1]^{\mathrm{n1}+1}\right)}{\mathrm{n1}+1}\right)(k-1)^{-\frac{1}{\mathrm{n1}+1}}y^{-k}}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+(a1*x^n1*y+ b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c1\_a^{s1}+c2\left(\left(\frac{(n1+1)(n1+2)^2\,b1\_a^{-n1}\left(-\frac{(k-1)a1\_a^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}}\mathrm{WhittakerM}\left(\frac{n1+}{2n1}\right.}{}\right.\right.\right.$$

6.7.4.12 [1613] Problem 12

problem number 1613

Added June 10, 2019.

Problem Chapter 7.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+yw_y+a\sqrt{x^2+y^2}w_z=bx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]== b*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{bx^n}{n}+c_1\left(\frac{y}{x},z-a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+a*sqrt(x^2+y^2)*diff(w(x,y,z),z)=  b*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{bx^n+n\_F1\left(\frac{y}{x},-\sqrt{x^2+y^2}\,a+z\right)}{n}$$

6.7.4.13 [1614] Problem 13

problem number 1614

Added June 10, 2019.

Problem Chapter 7.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x+yw_y+(z-a\sqrt{x^2+y^2+z^2}w_z=bx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]== b*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{bx^n}{n}+c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+(z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)=  b*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{bx^n+n\_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)}{n}$$

6.7.5 3.1

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6.7.5.1 [1615] Problem 1

problem number 1615

Added June 11, 2019.

Problem Chapter 7.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ae^{\lambda x} w_y + be^{\beta x} w_z = ce^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to \frac{ce^{\gamma x}}{\gamma} + c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, z - \frac{be^{\beta x}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \frac{c\,\mathrm{e}^{\gamma x} + \gamma\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x} + \lambda y}{\lambda}, \frac{-b\,\mathrm{e}^{\beta x} + \beta z}{\beta}\right)}{\gamma}$$

6.7.5.2 [1616] Problem 2

problem number 1616

Added June 11, 2019.

Problem Chapter 7.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x} w_y + be^{\beta y} w_z = ce^{\gamma y} + se^{\mu z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(e^{\gamma\left(\frac{a\left(-e^{\lambda x}+e^{\lambda K[1]}\right)}{\lambda}+y\right)} c + \exp\left(\frac{\mu\left(\lambda z - be^{\beta\left(y-\frac{ae^{\lambda x}}{\lambda}\right)} \mathrm{Ei}\left(\frac{a\beta e^{\lambda x}}{\lambda}\right) + be^{\beta\left(y-\frac{ae^{\lambda x}}{\lambda}\right)} \mathrm{Ei}\left(\frac{a\beta e^{\lambda}}{\lambda}\right.\right.}{\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\,\mathrm{e}^{-\frac{\left(-a\,\mathrm{e}^{-a\lambda}+a\,\mathrm{e}^{\lambda x}-\lambda y\right)\gamma}{\lambda}} + s\,\mathrm{e}^{\frac{\left(\left(-\mathrm{expIntegral}\left(1,-\frac{a\beta\,\mathrm{e}^{-a\lambda}}{\lambda}\right)+\mathrm{expIntegral}\left(1,-\frac{a\beta\,\mathrm{e}^{\lambda x}}{\lambda}\right)\right) b\,\mathrm{e}^{\frac{\left(-a\,\mathrm{e}^{\lambda x}+\lambda y\right)\beta}{\lambda}}+\lambda z\right)\mu}{\lambda}}\right) d$$

6.7.5.3 [1617] Problem 3

problem number 1617

Added June 11, 2019.

Problem Chapter 7.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ae^{\lambda y} w_y + be^{\beta y} w_z = ce^{\gamma x} + se^{\mu z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*y]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(e^{\gamma K[1]} c + \exp\left(-\frac{\mu\left(b\lambda(x-K[1])\left(a\lambda(x-K[1])+e^{-\lambda y}\right)^{-\frac{\beta}{\lambda}} + (\beta-\lambda)z + \frac{be^{-\lambda y}\left((a\right.}{}\right.}{\lambda-\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(lambda*y)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)=  c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\,\mathrm{e}^{-a\gamma} + s\,\mathrm{e}^{\frac{\left(-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y} + (\beta-\lambda)az + \left((-\_a+x)a\lambda+\mathrm{e}^{-\lambda y}\right)b\left(\frac{1}{(-\_a+x)a\lambda+\mathrm{e}^{-\lambda y}}\right)^{\frac{\beta}{\lambda}}\right)\mu}{(\beta-\lambda)a}}\right) d\_a + \_F1\left(\frac{-a\lambda x}{a}\right.$$

6.7.5.4 [1618] Problem 4

problem number 1618

Added June 11, 2019.

Problem Chapter 7.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (A_1 e^{\alpha_1 x} + B_1 e^{\nu_1 x + \lambda y}) w_y + (A_2 e^{\alpha_2 x} + B_2 e^{\nu_2 x + \beta y}) w_z = k e^{\gamma z}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (A1*Exp[alpha1*x] +B1*Exp[nu1*x+lambda*y] )*D[w[x, y,z], y] +(A2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (A1*exp(alpha1*x) +B1*exp(nu1*x+lambda*y) )*diff(w(x,y,z),y)+(A2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x k\,\mathrm{e}^{\left(z+\int\left(B2\left(-B1\lambda\left(\int \mathrm{e}^{\frac{A1\lambda\,\mathrm{e}^{\_f\alpha1}+\_f\alpha1\nu1}{\alpha1}}d\_f\right)+B1\lambda\left(\int \mathrm{e}^{\frac{A1\lambda\,\mathrm{e}^{\alpha1x}}{\alpha1}+\nu1x}dx\right)+\mathrm{e}^{\frac{\left(A1\,\mathrm{e}^{\alpha1x}-\alpha1y\right)\lambda}{\alpha1}}\right)^{-\frac{\beta}{\lambda}}\mathrm{e}^{\frac{A1\beta\,\mathrm{e}^{\_f\alpha1}+\_}{\alpha1}}\right.\right.}$$

6.7.5.5 [1619] Problem 5

problem number 1619

Added June 11, 2019.

Problem Chapter 7.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y + ce^{\gamma z}w_z = ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{ke^{x(\lambda-\alpha)}}{a(\alpha-\lambda)} + c_1\left(\frac{be^{-\alpha x}}{a\alpha} - \frac{e^{-\beta y}}{\beta}, \frac{ce^{-\alpha x}}{a\alpha} - \frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*exp(alpha*x)*diff(w(x,y,z),x)+  b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{(-\alpha+\lambda)\,a\_F1\left(-\frac{(a\alpha\,\mathrm{e}^{\alpha x} - b\beta\,\mathrm{e}^{\beta y})\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}, -\frac{(a\alpha\,\mathrm{e}^{\alpha x} - c\gamma\,\mathrm{e}^{\gamma z})\mathrm{e}^{-\alpha x-\gamma z}}{\alpha c\gamma}\right) + k\,\mathrm{e}^{-(\alpha-\lambda)x}}{(-\alpha+\lambda)\,a}$$

6.7.5.6 [1620] Problem 6

problem number 1620

Added June 11, 2019.

Problem Chapter 7.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ae^{\beta y}w_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[beta*x]*D[w[x, y,z], x] + b*Exp[alpha*x]*D[w[x, y,z], y] +c*Exp[gamma*z]*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{ke^{x(\lambda-\beta)}}{a(\beta-\lambda)} + c_1\left(\frac{ce^{-\beta x}}{a\beta} - \frac{e^{-\gamma z}}{\gamma}, y - \frac{be^{\alpha x-\beta x}}{a\alpha - a\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*exp(beta*y)*diff(w(x,y,z),x)+  b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\alpha k\,\mathrm{e}^{-a\lambda}}{a\alpha\,\mathrm{e}^{\beta y} - (-\mathrm{e}^{-a\alpha} + \mathrm{e}^{\alpha x})\,b\beta} d\_a + \_F1\left(\frac{a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha x}}{\alpha b\beta}, -\frac{\left(\alpha c\gamma x - c\gamma \ln\left(\frac{a\alpha\,\mathrm{e}^{\beta y}}{b\beta}\right) + (a\alpha\,\mathrm{e}\right.}{(a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha}}\right.$$

6.7.5.7 [1621] Problem 7

problem number 1621

Added June 11, 2019.

Problem Chapter 7.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1+a_2e^{\alpha x})w_x+(b_1+b_2e^{\beta y})w_y+(c_1+c_2e^{\gamma z})w_z=k_1+k_2e^{\alpha x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + (b1+b2*Exp[beta*y])*D[w[x, y,z], y] +(c1+c2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{(\text{a1k2}-\text{a2k1})\log\left(\text{a1}+\text{a2}e^{\alpha x}\right)+\text{a2}\alpha\text{k1}x}{\text{a1a2}\alpha}+c_1\left(\frac{\log\left(\frac{e^{\beta y}(\text{a1}+\text{a2}e^{\alpha x})^{\frac{\text{b1}\beta}{\text{a1}\alpha}}}{\text{b1}+\text{b2}e^{\beta y}}\right)}{\text{b1}\beta}-\frac{x}{\text{a1}},\frac{\log\left(\frac{e^{\gamma z}(\text{a1}}{\text{c}}\right.}{\text{c}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+ (b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1\,a2\alpha\_F1\left(\frac{-a1\,\alpha\,\text{RootOf}\left(a1\,\alpha\beta y-a1\,\alpha\ln\left(\frac{\left(-b1+\text{e}^{\_Z}\right)(a2\,\text{e}^{\alpha x}+a1)^{\frac{b1\beta}{a1\alpha}}}{b2}\right)+b1\,\beta\ln(a2\,\text{e}^{\alpha x}+a1)\right)-(-b1\ln(a2\,\text{e}^{\alpha x}+a1}{a1\,\alpha\,b1\,\beta}\right.}{}$$

6.7.5.8 [1622] Problem 8

problem number 1622

Added June 11, 2019.

Problem Chapter 7.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$e^{\beta y}(a_1 + a_2 e^{\alpha x})w_x + e^{\alpha x}(b_1 + b_2 e^{\beta y})w_y + ce^{\beta y+\gamma z}w_z = k_3 e^{\beta y}(k_1 + k_2 e^{\alpha x})$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + Exp[alpha*x]*(b1+b2*Exp[beta*y])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{\mathrm{k3}((\mathrm{a1k2} - \mathrm{a2k1})\log(\mathrm{a1} + \mathrm{a2}e^{\alpha x}) + \mathrm{a2}\alpha\mathrm{k1}x)}{\mathrm{a1a2}\alpha} + c_1\left(\frac{c\log(\mathrm{a1} + \mathrm{a2}e^{\alpha x})}{\mathrm{a1}\alpha} - \frac{cx}{\mathrm{a1}} - \frac{e^{-\gamma z}}{\gamma}, \log\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+  exp(alpha*x)*(b1+b2*exp(beta*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(k1\,k3x + a1\_F1\left(\frac{a2\alpha\beta y + a2\alpha\,\mathrm{RootOf}\left(a2\alpha\beta y - a2\alpha\ln\left(\frac{b1\left(a2\,\mathrm{e}^{\alpha x} + a1\right)^{-\frac{b2\beta}{a2\alpha}}}{-b2 + \mathrm{e}\_Z}\right) - b2\beta\ln(a2\,\mathrm{e}^{\alpha x} + a1)\right) - b2\beta\ln(a2}{a2\alpha b2\beta}\right.\right.}{a1\,a2\alpha}$$

6.7.6 3.2

Local contents

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6.7.6.1 [1623] Problem 1

problem number 1623

Added June 11, 2019.

Problem Chapter 7.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + ax^n w_y + bx^m w_z = ce^{\lambda x} y + ke^{\beta x} z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +b*x^m*D[w[x,y,z],z]== c*Exp[lambda*x]*y+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, \frac{-bx^{m+1}+mz+z}{m+1}\right) - \frac{acx^n(-\lambda x)^{-n}\,\text{Gamma}(n+2,-\lambda x)}{\lambda^2(n+1)} - \frac{bkx^{\prime}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*x^n*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)=c*exp(lambda*x)*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{-(n+1)(m+1)(-\Gamma(m)+\Gamma(m,-\beta x))\, b\gamma k\lambda^2 m x^m (-\beta)^m (-\beta)^{-m} (-\beta x)^{-m} - (m+1)\left((n\right.}{}$$

6.7.6.2 [1624] Problem 2

problem number 1624

Added June 11, 2019.

Problem Chapter 7.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x} w_y + bx^m w_z = cx^n y + ke^{\beta x} z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*x^m*D[w[x,y,z],z]== c*x^n*y+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, \frac{-bx^{m+1}+mz+z}{m+1}\right) + \frac{acx^n(-\lambda x)^{-n}\,\text{Gamma}(n+1,-\lambda x)}{\lambda^2} - \frac{bkx^m(-\beta x)^{-}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*exp(lambda*x)*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)=c*x^n*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{bk\lambda\_a^{m+1}\mathrm{e}^{-a\beta} - bk\lambda x^{m+1}\mathrm{e}^{-a\beta} - (m+1)\left(-ac\_a^{n}\mathrm{e}^{-a\lambda} - k\lambda z\,\mathrm{e}^{-a\beta} - \lambda s\,\mathrm{e}^{-a\gamma} + \left(a\,\mathrm{e}^{\lambda x} -\right.\right.}{(m+1)\,\lambda}$$

6.7.6.3 [1625] Problem 3

problem number 1625

Added June 11, 2019.

Problem Chapter 7.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x}w_y + byw_z = ke^{\beta x}z + se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== k*Exp[beta*x]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, \frac{abe^{\lambda x}(\lambda x - 1)}{\lambda^2} - bxy + z\right) + \frac{abke^{x(\beta+\lambda)}}{\beta^2(\beta+\lambda)} - \frac{bkye^{\beta x}}{\beta^2} + \frac{kze^{\beta x}}{\beta} + \frac{se^{\gamma x}}{\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   a*exp(lambda*x)*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=k*exp(beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{ab\beta^2\gamma k\,\mathrm{e}^{(\beta+\lambda)x} + (\beta+\lambda)\,\beta^2\gamma\lambda^2\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda}, \frac{(\lambda x-1)ab\,\mathrm{e}^{\lambda x}-(bxy-z)\lambda^2}{\lambda^2}\right) + \left(\beta^2\lambda^2 s\,\mathrm{e}^{\gamma x} + \left((-\beta+\right.\right.}{(\beta+\lambda)\,\beta^2\gamma\lambda^2}$$

6.7.6.4 [1626] Problem 4

problem number 1626

Added June 11, 2019.

Problem Chapter 7.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ay^n w_y + bz^m w_z = ce^{\lambda x} + ke^{\beta y} + se^{\gamma z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*y^n*D[w[x, y,z], y] +b*z^m*D[w[x,y,z],z]== c*Exp[lambda*x]+k*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-ax - \frac{\left(\frac{1}{y}\right)^{n-1}}{n-1}, -bx - \frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right) + \frac{k\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{n}{n-1}}\left(-\beta\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{1}{1-n}}\right)^{n}\text{Gamma}}{a\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+    a*y^n*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)=c*exp(lambda*x)+k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(c\,\mathrm{e}^{-a\lambda} + k\,\mathrm{e}^{\beta\left(\left(\left(-\_a+x\right)(n-1)ay^{n}+y\right)y^{-n}\right)^{-\frac{1}{n-1}}} + s\,\mathrm{e}^{\gamma\left(\left(\left(-\_a+x\right)(m-1)bz^{m}+z\right)z^{-m}\right)^{-\frac{1}{m-1}}} \right) d\_a + \_F1$$

6.7.6.5 [1627] Problem 5

problem number 1627

Added June 11, 2019.

Problem Chapter 7.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\beta y} w_y + bz^m w_z = ce^{\lambda x} + ky^n + se^{\gamma z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[beta*y]*D[w[x, y,z], y] +b*z^m*D[w[x,y,z],z]== x*Exp[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{a\beta x + e^{-\beta y}}{\beta}, -bx - \frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right) + \frac{k\log^{n-1}\left(e^{-\beta y}\right)\left(-\frac{\log\left(e^{-\beta y}\right)}{\beta}\right)^{n+1}\left(-\log^{2}\left(e^{-\beta y}\right)\right)^{-n}}{a}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+    a*exp(beta*y)*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)=x*exp(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\left(\_a\,\mathrm{e}^{-a\lambda}+k\left(\frac{\ln\left(\frac{1}{(-\_a+x)a\beta+\mathrm{e}^{-\beta y}}\right)}{\beta}\right)^{n}+s\,\mathrm{e}^{\gamma\left(\left(\left(-\_a+x\right)(m-1)bz^{m}+z\right)z^{-m}\right)^{-\frac{1}{m-1}}}\right)d\_a+\_F1\left(\right.$$

6.7.6.6 [1628] Problem 6

problem number 1628

Added June 11, 2019.

Problem Chapter 7.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(y^2+by+ae^{\alpha x}(y-b)-b^2\right)w_y+\left(z^2+c(xz-1)e^{\beta x}\right)w_z=ke^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+b*y+a*Exp[alpha*x]*(y-b)-b^2 )*D[w[x, y,z], y] +( z^2+c*(x*z-
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   ( y^2+b*y+a*exp(alpha*x)*(y-b)-b^2 )*diff(w(x,y,z),y)+( z^2+c*(x*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.6.7 [1629] Problem 7

problem number 1629

Added June 11, 2019.

Problem Chapter 7.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(y^2 + ae^{\alpha x}(x+1)\right) w_y + \left(ce^{\beta x} z^2 + be^{-\beta x}\right) w_z = ke^{\lambda x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( y^2+a*Exp[alpha*x]*(x+1))*D[w[x, y,z], y] +( c*Exp[beta*x]*z^2+b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+   ( y^2+a*exp(alpha*x)*(x+1))*diff(w(x,y,z),y)+( c*exp(beta*x)*z^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.6.8 [1630] Problem 8

problem number 1630

Added June 11, 2019.

Problem Chapter 7.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(ae^{\alpha x} y^2 + be^{-\alpha x}\right) w_y + \left(de^{\beta x} z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x})\right) w_z = ke^{\lambda x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a*Exp[alpha*x]*y^2+b*Exp[-alpha*x])*D[w[x, y,z], y] + ( d*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a*exp(alpha*x)*y^2+b*exp(-alpha*x))*diff(w(x,y,z),y)+ ( d*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.7.6.9 [1631] Problem 9

problem number 1631

Added June 11, 2019.

Problem Chapter 7.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k\right) w_y + \left(a_2 e^{\lambda_2 x} z + b_2 e^{\beta_2 x} z^m\right) w_z = c x^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a1*Exp[lambda1*x]*y+ b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +  ( a2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{c x^{s+1}}{s+1} + c_1\left((k-1)\int_1^x \mathrm{b1} e^{\frac{\mathrm{a1} e^{\mathrm{lambda1} K[1]}(k-1)}{\mathrm{lambda1}} + \mathrm{beta1} K[1]} dK[1] + y^{1-k} e^{\frac{\mathrm{a1}(k-1) e^{\mathrm{lambda1} x}}{\mathrm{lambda1}}}, (m-1)\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a1*exp(lambda1*x)*y+ b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+ ( a2*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{cx^{s+1} + (s+1)\_F1\left(\left((k-1)\,b1\,y^k\left(\int e^{\frac{\beta 1\lambda 1x+(k-1)a1\,e^{\lambda 1x}}{\lambda 1}}dx\right)e^{\frac{a1\,e^{\lambda 1x}}{\lambda 1}} + y\,e^{\frac{a1\,k\,e^{\lambda 1x}}{\lambda 1}}\right)y^{-k}e^{-\frac{a1\,e^{\lambda 1x}}{\lambda 1}}\right.}{s+1}$$

6.7.6.10 [1632] Problem 10

problem number 1632

Added June 11, 2019.

Problem Chapter 7.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(a_1 e^{\beta_1 x} y + b_1 e^{\gamma_1 x} y^k\right) w_y + \left(a_2 e^{\beta_2 x} z + b_2 e^{\gamma_1 x + \lambda z}\right) w_z = c x^s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + ( a1*Exp[beta1*x]*y+ b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +  ( a2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ ( a1*exp(beta1*x)*y+ b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+ ( a2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.6.11 [1633] Problem 11

problem number 1633

Added June 11, 2019.

Problem Chapter 7.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^n + b_1 x^m e^{\lambda y}) w_y + \left(a_2 x^k + b_2 x^L e^{\beta z}\right) w_z = c x^s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde = D[w[x, y,z], x] + (a1*x^n+b1*x^m*Exp[lambda*y] )*D[w[x, y,z], y] +  ( a2*x^k+b2*x^L*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{c x^{s+1}}{s+1} + c_1 \left(\frac{\mathrm{b2}\beta x^{L+1}\left(-\frac{\mathrm{a2}\beta x^{k+1}}{k+1}\right)^{-\frac{L+1}{k+1}} \mathrm{Gamma}\left(\frac{L+1}{k+1}, -\frac{\mathrm{a2}\beta x^{k+1}}{k+1}\right) - (k+1)e^{-\frac{\beta\left(-\mathrm{a2}x^{k+1}+kz+\right.}{k+1}}}{\mathrm{a2b2}\beta^2(k+1)(k-L)}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a1*x^n+b1*x^m*exp(lambda*y) )*diff(w(x,y,z),y)+ ( a2*x^k+b2*x^L*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c x^{s+1} + (s+1)\_F1\left(\frac{(n+1)^2\left(-a1\lambda x^{m+1}+(m+n+2)x^{m-n}\right)b1\left(-\frac{a1\lambda x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}} \mathrm{WhittakerM}\left(\frac{m-n}{2n+2}, \frac{m+2n+3}{2n+2},\right.}{}\right.}{}$$

6.7.7 4.1

Local contents

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6.7.7.1 [1634] Problem 1

problem number 1634

Added June 19, 2019.

Problem Chapter 7.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\sinh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sinh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-bx) + \frac{c\sqrt{\cosh^2(\lambda x)}\operatorname{sech}(\lambda x)\sinh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{k+1}{2}, \frac{k+3}{2}\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\sinh^k(\lambda x)\right) dx + \_F1(-ax+y, -bx+z)$$

6.7.7.2 [1635] Problem 2

problem number 1635

Added June 19, 2019.

Problem Chapter 7.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = k\sinh(\beta y) + s\sinh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]== k*Sinh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{s\sinh\left(\frac{\gamma(a\lambda z - c\cosh(\lambda x) + c\cosh(\lambda K[1]))}{a\lambda}\right) + k\sinh\left(\beta\left(y + \frac{b(K[1]-x)}{a}\right)\right)}{a} dK[1] + c_1\left(y - \frac{bx}{a},\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(lambda*x)*diff(w(x,y,z),z)=k*sinh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{k\sinh\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right) + s\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}, \frac{a\lambda z - c}{}\right.$$

6.7.7.3 [1636] Problem 3

problem number 1636

Added June 19, 2019.

Problem Chapter 7.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sinh^n(\beta x)w_y + c\sinh^k(\lambda x)w_z = c\sinh^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[beta*x]^n*D[w[x, y,z], y] +  c*Sinh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{a\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{n+3}{2}, -\sinh^2(\beta x\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*sinh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\sinh^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\sinh^n(\beta x))\,dx\right), z - \left(\int c(\sinh^k(\lambda x))\,dx\right)\right)$$

6.7.7.4 [1637] Problem 4

problem number 1637

Added June 19, 2019.

Problem Chapter 7.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sinh(\beta y)w_y + c\sinh(\lambda x)w_z = k\sinh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{k\sinh\left(\frac{\gamma(a\lambda z - c\cosh(\lambda x) + c\cosh(\lambda K[1]))}{a\lambda}\right)}{a} dK[1] + c_1\left(z - \frac{c\cosh(\lambda x)}{a\lambda}, \frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta} - \frac{b}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{k \sinh\left(\frac{(a\lambda z + c\cosh(\_a\lambda) - c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a} d\_a + \_F1\left(\frac{-b\beta x - 2a\operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta}, \frac{a\lambda z - c\cosh(\lambda$$

6.7.7.5 [1638] Problem 5

problem number 1638

Added June 19, 2019.

Problem Chapter 7.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sinh^{n_1}(\lambda_1 x) w_x + b_1 \sinh^{m_1}(\beta_1 y) w_y + c_1 \sinh^{k_1}(\gamma_1 z) w_z = a_2 \sinh^{n_2}(\lambda_2 x) + b_2 \sinh^{m_2}(\beta_2 y) w_y + c_2 \sinh^{k_2}(\gamma_2 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Sinh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Sinh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Sin
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{\sinh^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\sinh^{\mathrm{k2}}(\mathrm{gamma2}K[3]) + \mathrm{b2}\sinh^{\mathrm{m2}}(\mathrm{beta2}K[3]) + \mathrm{a2}\sinh^{\mathrm{n2}}\right.}{\mathrm{a1}}\right.$$

Maple ✗

```
restart;
local gamma;
pde := a1*sinh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*sinh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*sinh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.7.8 4.2

Local contents

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6.7.8.1 [1639] Problem 1

problem number 1639

Added June 19, 2019.

Problem Chapter 7.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\cosh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cosh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\sqrt{-\sinh^2(\lambda x)}\operatorname{csch}(\lambda x)\cosh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k}{}\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*cosh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\cosh^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

6.7.8.2 [1640] Problem 2

problem number 1640

Added June 19, 2019.

Problem Chapter 7.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cosh(\lambda x)w_z=k\cosh(\beta y)+s\cosh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]== k*Cosh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{k\cosh\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a},\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cosh(lambda*x)*diff(w(x,y,z),z)=k*cosh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{k\cosh\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)+s\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-}{}\right.$$

6.7.8.3 [1641] Problem 3

problem number 1641

Added June 19, 2019.

Problem Chapter 7.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\beta x)w_y+b\cosh^k(\lambda x)w_z=c\cosh^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[beta*x]^n*D[w[x, y,z], y] +  b*Cosh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{a\sinh(\beta x)\cosh^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cosh^2(\beta x)\right)}{(\beta n+\beta)\sqrt{-\sinh^2(\beta x)}}+y,\frac{b\sinh(\lambda x)}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*cosh(beta*x)^n*diff(w(x,y,z),y)+ b*cosh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\cosh^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\cosh^n(\beta x))\,dx\right),z-\left(\int b(\cosh^k(\lambda x))\,dx\right)\right)$$

6.7.8.4 [1642] Problem 4

problem number 1642

Added June 19, 2019.

Problem Chapter 7.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\lambda x)w_z=k\cosh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{k\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sinh}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+ c*cosh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{k\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\sinh(\lambda x)}{a\lambda}\right.$$

6.7.8.5 [1643] Problem 5

problem number 1643

Added June 19, 2019.

Problem Chapter 7.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\gamma z)w_z=p\cosh(\lambda x)+q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{p\sinh(\lambda x)+\lambda q x}{a\lambda}+c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},\frac{2\tan^{-1}\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*cosh(beta*y)*diff(w(x,y,z),y)+ c*cosh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a\lambda\_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{-c\gamma x+2a\arctan\left(e^{\gamma z}\right)}{c\gamma}\right)+\lambda q x+p\sinh(\lambda x)}{a\lambda}$$

6.7.8.6 [1644] Problem 6

problem number 1644

Added June 19, 2019.

Problem Chapter 7.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cosh^{n_1}(\lambda_1 x)w_x+b_1\cosh^{m_1}(\beta_1 y)w_y+c_1\cosh^{k_1}(\gamma_1 z)w_z=a_2\cosh^{n_2}(\lambda_2 x)+b_2\cosh^{m_2}(\beta_2 y)w_y+c_2\cosh^{k_2}(\gamma_2 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Cosh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cosh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Cos
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{\cosh^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\cosh^{\mathrm{k2}}(\mathrm{gamma2}K[3])+\mathrm{b2}\cosh^{\mathrm{m2}}(\mathrm{beta2}K[3])+\mathrm{a2}\cosh^{\mathrm{n}}\right.}{\mathrm{a1}}\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde := a1*cosh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cosh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cosh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.7.9 4.3

Local contents

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6.7.9.1 [1645] Problem 1

problem number 1645

Added June 20, 2019.

Problem Chapter 7.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tanh[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax,z-bx) + \frac{c\tanh^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\tanh^2(\lambda x)\right)}{k\lambda+\lambda} + sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\tanh^k(\lambda x)\right)dx + \_F1(-ax+y,-bx+z)$$

6.7.9.2 [1646] Problem 2

problem number 1646

Added June 20, 2019.

Problem Chapter 7.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\tanh(\lambda x)w_z = k\tanh(\beta y) + s\tanh(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]== k*Tanh[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{k\tanh\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\tanh\left(\frac{\gamma(a\lambda z-c\log(\cosh(\lambda x))+c\log(\cosh(\lambda K[1])))}{a\lambda}\right)}{a}dK[1]+c_1\Bigg(\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tanh(lambda*x)*diff(w(x,y,z),z)=k*tanh(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{-(k+s)\sinh\left(\frac{c\gamma\ln(\tanh(\_a\lambda)-1)+c\gamma\ln(\tanh(\_a\lambda)+1)-c\gamma\ln(\tanh(\lambda x)-1)-c\gamma\ln(\tanh(\lambda x)+1)-2(a\gamma z+(ay-(-}{2a\lambda}\right.}{\left(\cosh\left(\frac{c\gamma\ln(\tanh(\_a\lambda)-1)+c\gamma\ln(\tanh(\_a\lambda)+1)-c\gamma\ln(\tanh(\lambda x)-1)-c\gamma\ln(\tanh(\lambda x)+1)-2(a\gamma z+(ay-(-\_}{2a\lambda}\right.\right.}$$

6.7.9.3 [1647] Problem 3

problem number 1647

Added June 19, 2019.

Problem Chapter 7.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\beta x)w_y+c\tanh^k(\lambda x)w_z=c\tanh^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[beta*x]^n*D[w[x, y,z], y] +  b*Tanh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{a\tanh^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\tanh^2(\beta x)\right)}{\beta n+\beta},z-\frac{b\tanh^{k+1}(\lambda x)\,\text{H}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*tanh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\tanh^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\tanh^k(\lambda x))\,dx\right)\right)$$

6.7.9.4 [1648] Problem 4

problem number 1648

Added June 19, 2019.

Problem Chapter 7.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\tanh(\beta y)w_y+c\tanh(\lambda x)w_z=k\tanh(\gamma z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{y}\frac{k\sinh\left(\frac{\left(a\lambda z-c\ln(\cosh(\lambda x))+c\ln\left(\cosh\left(\frac{(-2b\beta x+a\ln(\tanh(\_a\beta)-1)+a\ln(\tanh(\_a\beta)+1)-a\ln(\tanh(\beta y)-1)-a\ln(\tanh}{2b\beta}\right.\right.\right.}{a\lambda}\right.}{b\cosh\left(\frac{\left(a\lambda z-c\ln(\cosh(\lambda x))+c\ln\left(\cosh\left(\frac{(-2b\beta x+a\ln(\tanh(\_a\beta)-1)+a\ln(\tanh(\_a\beta)+1)-a\ln(\tanh(\beta y)-1)-a\ln(\tanh(\beta y)+1)}{2b\beta}\right.\right.\right.}{a\lambda}\right.}$$

6.7.9.5 [1649] Problem 5

problem number 1649

Added June 19, 2019.

Problem Chapter 7.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\tanh(\beta y)w_y+c\tanh(\gamma z)w_z=k\tanh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{k\log(\cosh(\lambda x))}{a\lambda}+c_1\left(\frac{1}{2}\left(\frac{\log(\sinh(\beta y))}{\beta}-\frac{bx}{a}\right),\frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma}-\frac{2c\log(\sinh(\beta y))}{\beta}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 c_2 c_3 \left(\frac{-\cosh(\beta y) + \sinh(\beta y)}{\cosh(\beta y)}\right)^{-\frac{c_2}{2\beta}} \left(\frac{\cosh(\beta y) + \sinh(\beta y)}{\cosh(\beta y)}\right)^{-\frac{c_2}{2\beta}} \left(\frac{-\cosh(\gamma z) + \sinh(\gamma z}{\cosh(\gamma z)}\right.$$

6.7.9.6 [1650] Problem 6

problem number 1650

Added June 19, 2019.

Problem Chapter 7.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\tanh(\lambda x)w_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Tanh[lambda*x]*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*tanh(lambda*x)*diff(w(x,y,z),x)+ b*tanh(beta*y)*diff(w(x,y,z),y)+ c*tanh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 c_2 c_3 \left(\frac{-\cosh(\beta y) + \sinh(\beta y)}{\cosh(\beta y)}\right)^{-\frac{c_2}{2\beta}} \left(\frac{\cosh(\beta y) + \sinh(\beta y)}{\cosh(\beta y)}\right)^{-\frac{c_2}{2\beta}} \left(\frac{-\cosh(\gamma z) + \sinh(\gamma z}{\cosh(\gamma z)}\right.$$

6.7.9.7 [1651] Problem 7

problem number 1651

Added June 19, 2019.

Problem Chapter 7.4.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \tanh^{n_1}(\lambda_1 x) w_x + b_1 \tanh^{m_1}(\beta_1 y) w_y + c_1 \tanh^{k_1}(\gamma_1 z) w_z = a_2 \tanh^{n_2}(\lambda_2 x) + b_2 \tanh^{m_2}(\beta_2 y) w_y + c_2 \tanh^{k_2}(\gamma$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Tanh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Tanh[beta1*x]^m1*D[w[x, y,z], y] +  c1*Tan
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{\tanh^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\tanh^{\mathrm{k2}}(\mathrm{gamma2}K[3]) + \mathrm{b2}\tanh^{\mathrm{m2}}(\mathrm{beta2}K[3]) + \mathrm{a2}\tanh\right.}{\mathrm{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a1*tanh(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*tanh(beta1*x)^m1*diff(w(x,y,z),y)+ c1*tanh
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{b2\left(\tanh^{m2}(\beta 2x)\right)\left(\tanh^{-n1}(\lambda 1x)\right) + c2\left(\tanh^{k2}(\gamma 2x)\right)\left(\tanh^{-n1}(\lambda 1x)\right) + a2\left(\tanh^{-n1+n}\right.}{a1}$$

6.7.10 4.4

Local contents

6.7.10.1 [1652] Problem 1 2319
6.7.10.2 [1653] Problem 2 2320
6.7.10.3 [1654] Problem 3 2321
6.7.10.4 [1655] Problem 4 2322
6.7.10.5 [1656] Problem 5 2323
6.7.10.6 [1657] Problem 6 2324

6.7.10.1 [1652] Problem 1

problem number 1652

Added June 20, 2019.

Problem Chapter 7.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Coth[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)+\frac{c\coth^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\lambda x)\right)}{k\lambda+\lambda}+sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*coth(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(\coth^k(\lambda x)\right)dx+\_F1(-ax+y,-bx+z)$$

6.7.10.2 [1653] Problem 2

problem number 1653

Added June 20, 2019.

Problem Chapter 7.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\coth(\lambda x)w_z=k\coth(\beta y)+s\coth(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]== k*Coth[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{k\coth\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)+s\coth\left(\frac{\gamma(a\lambda z-c\log(\sinh(\lambda x))+c\log(\sinh(\lambda K[1])))}{a\lambda}\right)}{a}dK[1]+c_1\Big(y\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*coth(lambda*x)*diff(w(x,y,z),z)=k*coth(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{-(k+s)\sinh\left(\frac{c\gamma\ln(\coth(\_a\lambda)-1)+c\gamma\ln(\coth(\_a\lambda)+1)-c\gamma\ln(\coth(\lambda x)-1)-c\gamma\ln(\coth(\lambda x)+1)-2(a\gamma z+(ay-(-\_}{2a\lambda}\right.}{\left(\cosh\left(\frac{c\gamma\ln(\coth(\_a\lambda)-1)+c\gamma\ln(\coth(\_a\lambda)+1)-c\gamma\ln(\coth(\lambda x)-1)-c\gamma\ln(\coth(\lambda x)+1)-2(a\gamma z+(ay-(-\_}{2a\lambda}\right.\right.}$$

6.7.10.3 [1654] Problem 3

problem number 1654

Added June 20, 2019.

Problem Chapter 7.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\coth^n(\beta x)w_y+b\coth^k(\lambda x)w_z=c\coth^m(\gamma x)+s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Coth[beta*x]^n*D[w[x, y,z], y] +  b*Coth[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{b\coth^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\lambda x)\right)}{k\lambda+\lambda},y-\frac{a\coth^{n+1}(\beta x)\,\text{H}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*coth(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c(\coth^m(\gamma x))\,dx+\_F1\left(y-\left(\int a(\coth^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)$$

6.7.10.4 [1655] Problem 4

problem number 1655

Added June 20, 2019.

Problem Chapter 7.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta y)w_y+c\coth(\lambda x)w_z=k\coth(\gamma z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+ c*coth(lambda*x)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\left(\int^{x} \frac{k\cosh\left(\frac{(-2a\lambda z+c\ln(\coth(\_a\lambda)-1)+c\ln(\coth(\_a\lambda)+1)-c\ln(\coth(\lambda x)-1)-c\ln(\coth(\lambda x)+1))\gamma}{2a\lambda}\right)}{a\sinh\left(\frac{(-2a\lambda z+c\ln(\coth(\_a\lambda)-1)+c\ln(\coth(\_a\lambda)+1)-c\ln(\coth(\lambda x)-1)-c\ln(\coth(\lambda x)+1))\gamma}{2a\lambda}\right)} d\_a\right) + \_$$

6.7.10.5 [1656] Problem 5

problem number 1656

Added June 20, 2019.

Problem Chapter 7.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = p\coth(\lambda x) + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a\lambda q\log(\cosh(\beta y)) + b\beta p\log(-\tanh(\lambda x)) + b\beta p\log(\cosh(\lambda x))}{ab\beta\lambda} + c_1\left(\frac{a\log(\operatorname{sech}(\beta y)) + b}{2a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*coth(beta*y)*diff(w(x,y,z),y)+ c*coth(gamma*z)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{2a\lambda\_F1\left(\frac{-2b\beta x+a\ln\left(\frac{(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1))-2}\right)-a\ln(\mathrm{RootOf}(\beta y-\mathrm{arccoth}(\_Z-1)))}{2b\beta}, \frac{-2c\gamma x+a\ln\left(\frac{(\mathrm{RootOf}(\gamma z-}{\mathrm{RootOf}(\gamma z-}\right.}{}\right.}{}$$

6.7.10.6 [1657] Problem 6

problem number 1657

Added June 20, 2019.

Problem Chapter 7.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\coth^{n_1}(\lambda_1 x)w_x+b_1\coth^{m_1}(\beta_1 y)w_y+c_1\coth^{k_1}(\gamma_1 z)w_z = a_2\coth^{n_2}(\lambda_2 x)+b_2\coth^{m_2}(\beta_2 y)w_y+c_2\coth^{k_2}(\gamma_2 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*Coth[lambda1*x]^n1*D[w[x, y,z], x] + b1*Coth[beta1*x]^m1*D[w[x, y,z], y] +  c1*Cot
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{\coth^{-\mathrm{n1}}(\mathrm{lambda1}K[3])\left(\mathrm{c2}\coth^{\mathrm{k2}}(\mathrm{gamma2}K[3])+\mathrm{b2}\coth^{\mathrm{m2}}(\mathrm{beta2}K[3])+\mathrm{a2}\coth^{\mathrm{n}}\right.}{\mathrm{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a1*coth(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*coth(beta1*x)^m1*diff(w(x,y,z),y)+ c1*coth
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{b2\left(\coth^{m2}\left(\beta 2x\right)\right)\left(\coth^{-n1}\left(\lambda 1x\right)\right) + c2\left(\coth^{k2}\left(\gamma 2x\right)\right)\left(\coth^{-n1}\left(\lambda 1x\right)\right) + a2\left(\coth^{-n1+n2}\right.}{a1}$$

6.7.11 4.5

Local contents

6.7.11.1 [1658] Problem 1 2325
6.7.11.2 [1659] Problem 2 2326
6.7.11.3 [1660] Problem 3 2327
6.7.11.4 [1661] Problem 4 2328
6.7.11.5 [1662] Problem 5 2329

6.7.11.1 [1658] Problem 1

problem number 1658

Added June 20, 2019.

Problem Chapter 7.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh^n(\lambda y)w_z = s\cosh^m(\beta x) + k\sinh^r(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*y]^n*D[w[x,y,z],z]== s*Cosh[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sqrt{\cosh^2(\lambda y)}\operatorname{sech}(\lambda y)\sinh^{n+1}(\lambda y)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\text{si}\right.}{b\lambda n+b\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(lambda*y)^n*diff(w(x,y,z),z)=s*cosh(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\frac{k\left(\sinh^r\left(\frac{(ay-(-\_a+x)b)\gamma}{a}\right)\right)+s(\cosh^m(\_a\beta))}{a}d\_a+\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c\left(\sinh^n\left(\frac{(a}{}\right.\right.}{}\right.\right.$$

6.7.11.2 [1659] Problem 2

problem number 1659

Added June 20, 2019.

Problem Chapter 7.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\lambda x)w_y+b\cosh^m(\beta x)w_z=s\cosh^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[lambda*x]^n*D[w[x, y,z], y] +  b*Cosh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{s\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{k+1}(\gamma x)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cosh^2(\gamma x)\right)}{\gamma k+\gamma}+c_1\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sinh(lambda*x)^n*diff(w(x,y,z),y)+ b*cosh(beta*x)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int s\left(\cosh^k(\gamma x)\right)dx+\_F1\left(y-\left(\int a(\sinh^n(\lambda x))\,dx\right),z-\left(\int b(\cosh^m(\beta x))\,dx\right)\right)$$

6.7.11.3 [1660] Problem 3

problem number 1660

Added June 20, 2019.

Problem Chapter 7.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\lambda x)w_y+b\sinh^m(\beta y)w_z=s\sinh^k(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[lambda*x]^n*D[w[x, y,z], y] +  b*Sinh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x s\sinh^k\left(\frac{\gamma\left(-b\sqrt{\cosh^2(\beta x)}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\sinh^2(\beta x)\right)\operatorname{sech}(\beta x)\sinh^{m+1}(\beta x)+\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cosh(lambda*x)^n*diff(w(x,y,z),y)+ b*sinh(beta*y)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x s\left(\sinh^k\left(\left(b\left(\int\left(\sinh^m\left(\left(a\left(\int(\cosh^n(\_f\lambda))\,d\_f\right)+y-\left(\int a(\cosh^n(\lambda x))\,dx\right)\right)\beta\right.\right.\right.\right.\right.$$

6.7.11.4 [1661] Problem 4

problem number 1661

Added June 20, 2019.

Problem Chapter 7.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\lambda x)w_y+b\coth^m(\beta x)w_z=s\coth^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[lambda*x]^n*D[w[x, y,z], y] +  b*Coth[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{s\coth^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},\coth^2(\gamma x)\right)}{\gamma k+\gamma}+c_1\left(z-\frac{b\coth^{m+1}(\beta x)\,\text{Hyp}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tanh(lambda*x)^n*diff(w(x,y,z),y)+ b*coth(beta*x)^m*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int s\left(\coth^k(\gamma x)\right)dx+\_F1\left(y-\left(\int a(\tanh^n(\lambda x))\,dx\right),z-\left(\int b(\coth^m(\beta x))\,dx\right)\right)$$

6.7.11.5 [1662] Problem 5

problem number 1662

Added June 20, 2019.

Problem Chapter 7.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x+b\sinh(\beta y)w_y+c\sinh(\gamma z)w_z=k\cosh(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{k\log(\sinh(\lambda x))}{a\lambda}+c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta},\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+ b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(gamma*z)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a\lambda\_F1\left(\frac{-2a\lambda\operatorname{arctanh}(e^{\beta y})+2b\beta\operatorname{arctanh}(e^{\lambda x})}{b\beta\lambda},\frac{-2a\lambda\operatorname{arctanh}(e^{\gamma z})+2c\gamma\operatorname{arctanh}(e^{\lambda x})}{c\gamma\lambda}\right)+k\ln(\sinh(\lambda x))}{a\lambda}$$

6.7.12 5.1

Local contents

6.7.12.1 [1663] Problem 1 2331
6.7.12.2 [1664] Problem 2 2332
6.7.12.3 [1665] Problem 3 2332
6.7.12.4 [1666] Problem 4 2333
6.7.12.5 [1667] Problem 5 2334

6.7.12.1 [1663] Problem 1

problem number 1663

Added June 26, 2019.

Problem Chapter 7.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\ln^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Log[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-bx) + \frac{c\log^k(\lambda x)(-\log(\lambda x))^{-k}\,\text{Gamma}(k+1, -\log(\lambda x))}{\lambda} + sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*ln(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\ln(\lambda x)^k\,dx + \_F1(-ax+y, -bx+z)$$

Contains unresolve integral because maple can not integrate $\ln^n(x)$

6.7.12.2 [1664] Problem 2

problem number 1664

Added June 26, 2019.

Problem Chapter 7.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\ln(\beta y)\ln(\gamma z)w_z = k\ln(\alpha x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Log[beta*y]*Log[gamma*z]*D[w[x,y,z],z]== k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*ln(beta*y)*ln(gamma*z)*diff(w(x,y,z),z)=k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a\_F1\left(\frac{-ay+bx}{b}, \frac{-(\ln(\beta y)-1)c\gamma y - b\,\mathrm{expIntegral}(1,-\ln(\gamma z))}{c\gamma}\right) + (\ln(\alpha x) - 1)\,kx}{a}$$

6.7.12.3 [1665] Problem 3

problem number 1665

Added June 26, 2019.

Problem Chapter 7.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = c\ln^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to x(c\log(\gamma x) - c + s) + c_1\left(y - \frac{a(-\log(\beta x))^{-n}\log^n(\beta x)\,\mathrm{Gamma}(n+1,-\log(\beta x))}{\beta}, z - \right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = cx\ln(\gamma x) + (-c+s)\,x + \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int b\ln(\lambda x)^k\,dx\right)\right)$$

Contains unresolve integral because maple can not integrate $\ln^n(x)$

6.7.12.4 [1666] Problem 4

problem number 1666

Added June 26, 2019.

Problem Chapter 7.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\lambda x)w_y + b\ln^m(\beta y)w_z = c\ln^k(\gamma y) + s\ln^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*Log[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*ln(beta*y)^m*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\left(c\ln\left(\left(a\left(\int\ln\left(\_f\lambda\right)^{n}d\_f\right)+y-\left(\int a\ln\left(\lambda x\right)^{n}dx\right)\right)\gamma\right)^{k}+s\ln\left(\left(b\left(\int\ln\left(\left(a\left(\right.\right.\right.\right.\right.$$

6.7.12.5 [1667] Problem 5

problem number 1667

Added June 26, 2019.

Problem Chapter 7.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\ln^{n_1}(\lambda_1 x)w_x+b_1\ln^{m_1}(\beta_1 y)w_y+c_1\ln^{k_1}(\gamma_1 z)w_z=a_2\ln^{n_2}(\lambda_2 x)+b_2\ln^{m_2}(\beta_2 y)+c_2\ln^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Log[lambda1*x]^n1*D[w[x, y,z], x] + b1*Log[beta1*y]^m1*D[w[x, y,z], y] +  c1*Log[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*ln(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*ln(beta1*y)^m1*diff(w(x,y,z),y)+ c1*ln(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{\left(a2 \ln\left(\_f\lambda 2\right)^{n2} + b2 \ln\left(\beta 2 \operatorname{RootOf}\left(\int \ln\left(\_f\lambda 1\right)^{-n1} d\_f - \left(\int \ln\left(\lambda 1 x\right)^{-n1} dx\right) + \int \frac{a1 \ln}{}\right.\right.\right.}{}$$

Contains RootOf and unresolved integrals $\ln^n(x)$

6.7.13 5.2

Local contents

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6.7.13.2 [1669] Problem 2 2336
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6.7.13.4 [1671] Problem 4 2338
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6.7.13.1 [1668] Problem 1

problem number 1668

Added June 26, 2019.

Problem Chapter 7.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cx^n \ln^k(\lambda y) w_z = sy^m \ln^r(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*x^n*Log[lambda*y]*D[w[x,y,z],z]== s*y^m*Lo
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x \frac{s\left(y+\frac{b(K[1]-x)}{a}\right)^m \log^r(\beta K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{bcx^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{bx}{bx-ay}\right.}{a(n+1)(n+2)(ay-bx)}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*x^n*ln(lambda*y)^k*diff(w(x,y,z),z)=s*y^m*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{s\left(\frac{ay-(-\_a+x)b}{a}\right)^m \ln(\_a\beta)^m}{a}d\_a+\_F1\left(\frac{ay-bx}{a},z-\left(\int^x \frac{c\_a^n \ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^k}{a}d\_\right.\right.$$

Answer has unresolved integrals

6.7.13.2 [1669] Problem 2

problem number 1669

Added June 26, 2019.

Problem Chapter 7.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ax^n w_y+bx^m w_z=cy\ln^k(\lambda x)+sz\ln^r(\beta x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +  b*x^m*D[w[x,y,z],z]== c*y*Log[lambda*x]^k+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{\frac{(m+2)(-\log(\beta x))^{-r}(-\log(\lambda x))^{-k}(-\beta c(m+1)(ax^{n+1}-(n+1)y)(-\log(\beta x))^{r}\log^{k}(\lambda x)\,\mathrm{Gamma}(k+1,-\log(\lambda x))-\lambda(n+1}{\beta\lambda}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)= c*y*ln(lambda*x)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^{x}\frac{(a\_a^{n+1}-ax^{n+1}+ny+y)\,(m+1)\,c\ln(\_a\lambda)^{k}+(b\_a^{m+1}-bx^{m+1}+mz+z)\,(n+1)\,s}{(n+1)\,(m+1)}$$

Answer has unresolved integrals

6.7.13.3 [1670] Problem 3

problem number 1670

Added June 26, 2019.

Problem Chapter 7.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\ln^n(\lambda x)w_y+by^m w_z=c\ln^k(\beta x)+s\ln^r(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*y^m*D[w[x,y,z],z]== c*Log[be
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(c\log^k(\beta K[3])+s\log^r\left(\gamma\left(z-\int_1^x b\left(y-\int_1^x a\log^n(\lambda K[1])dK[1]+\int_1^{K[2]} a\log^n(\lambda\right.\right.\right.\right.\right.$$

Generated internal errors from solve : inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*y^m*diff(w(x,y,z),z)= c*ln(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(c\ln\left(\_b\beta\right)^k+s\ln\left(-\frac{\left(-(m+1)\,az-\left(\_ba+y\ln\left(\_a\lambda\right)^{-n}-\ln\left(\_a\lambda\right)^{-n}\left(\int a\ln\left(\lambda x\right)^n\right.\right.\right.}{}\right.\right.$$

Answer has unresolved integrals and RootOf

6.7.13.4 [1671] Problem 4

problem number 1671

Added June 26, 2019.

Problem Chapter 7.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln^n(\lambda x)w_x+zw_y+b\ln^k(\beta y)w_z=cx^m+s\ln(\gamma y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y,z], x] + z*D[w[x, y,z], y] +  b*Log[beta*y]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*ln(lambda*x)^n*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ b*ln(beta*y)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{c\,\mathrm{RootOf}\left(b\ln\left(\_a\beta\right)^{k}\left(\int \frac{\ln(\lambda x)^{-n}}{a}dx\right) - b\ln\left(\_a\beta\right)^{k}\left(\int^{\_Z} \frac{\ln(\_a\lambda)^{-n}}{a}d\_a\right) - \sqrt{2by\ln\left(\_a\beta\right.}\right.}{\sqrt{2b\left(\int \ln\left(\_f\beta\right)^{k}d\_f\right)}}$$

6.7.13.5 [1672] Problem 5

problem number 1672

Added June 26, 2019.

Problem Chapter 7.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax\ln^{n}(x)w_{x} + by\ln^{m}(y)w_{y} + cz\ln(z)^{r}w_{z} = k\ln^{s}(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +  c*z*Log[z]^r*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{k(m-1)^{\frac{1}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^{m}\log^{s}\left(\exp\left(\left((m-1)^{\frac{m}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^{m}\right)^{\frac{1}{1-}}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+ b*y*ln(y)^m*diff(w(x,y,z),y)+ c*z*ln(z)^r*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{(n-s-1)\,a\_F1\left(\frac{-(n-1)a\ln(y)^{-m+1}+(m-1)b\ln(x)^{-n+1}}{(n-1)(m-1)b},\frac{-(n-1)a\ln(z)^{-r+1}+(r-1)c\ln(x)^{-n+1}}{(r-1)(n-1)c}\right)-k\ln(x)^{-}}{(n-s-1)\,a}$$

6.7.14 6.1

Local contents

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6.7.14.1 [1673] Problem 1

problem number 1673

Added June 26, 2019.

Problem Chapter 7.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\sin^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Sin[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-cx) + \frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{k+1}{2}, \frac{k+3}{2}, \sin\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sin(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\sin^k(\lambda x)\right)dx + \_F1(-ax+y, -bx+z)$$

6.7.14.2 [1674] Problem 2

problem number 1674

Added June 26, 2019.

Problem Chapter 7.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\sin(\gamma z)w_z = k\sin(\alpha x) + s\sin(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sin[gamma*z]*D[w[x,y,z],z]== k*Sin[alpha*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log\left(\tan\left(\frac{\gamma z}{2}\right)\right)}{\gamma} - \frac{cx}{a}\right) - \frac{k\cos(\alpha x)}{a\alpha} - \frac{s\cos(\beta y)}{b\beta}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sin(gamma*z)*diff(w(x,y,z),z)= k*sin(alpha
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a\alpha b\beta\_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\mathrm{RootOf}\left(\gamma z - \arctan\left(\frac{2\_Z e^{\frac{c\gamma x}{a}}}{\_Z^2 e^{\frac{2c\gamma x}{a}}+1}, -\frac{\_Z^2 e^{\frac{2c\gamma x}{a}}-1}{\_Z^2 e^{\frac{2c\gamma x}{a}}+1}\right)\right)\right)}{c\gamma}\right) - a\alpha s\cos(\beta y) - b\beta k\cos(}{a\alpha b\beta}$$

6.7.14.3 [1675] Problem 3

problem number 1675

Added June 26, 2019.

Problem Chapter 7.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta x)w_z = c\sin^k(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{c\sqrt{\cos^2(\gamma x)}\sec(\gamma x)\sin^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(\gamma x)\right)}{\gamma k+\gamma} + c_1\left(z - \frac{b\sqrt{}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int c\left(\sin^k(\gamma x)\right)dx + \_F1\left(y - \left(\int a\left(\sin^n(\lambda x)\right)dx\right), z - \left(\int b\left(\sin^m(\beta x)\right)dx\right)\right)$$

6.7.14.4 [1676] Problem 4

problem number 1676

Added June 26, 2019.

Problem Chapter 7.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta y)w_z = c\sin^k(\gamma y) + s\sin^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(c\sin^k\left(\frac{\gamma\left(-a\sqrt{\cos^2(\lambda x)}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)\sec(\lambda x)\sin^{n+1}(\lambda x) + a\sqrt{\cos^2(\right.}{\lambda(n}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\sin^k\left(\left(a\left(\int (\sin^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\sin^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\sin^r\left(\left(b\left(\right.\right.\right.\right.$$

6.7.14.5 [1677] Problem 5

problem number 1677

Added June 26, 2019.

Problem Chapter 7.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\sin(\beta x)w_y + c\sin(\lambda x)w_z = k\sin(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*x]*D[w[x, y,z], y] +  c*Sin[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{k\sin\left(\frac{\gamma(a\lambda z+c\cos(\lambda x)-c\cos(\lambda K[1]))}{a\lambda}\right)}{a}dK[1] + c_1\left(\frac{b\cos(\beta x)}{a\beta}+y, \frac{c\cos(\lambda x)}{a\lambda}+z\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*sin(beta*x)*diff(w(x,y,z),y)+ c*sin(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\left(\int^x -\frac{k\sin\left(\frac{(a\lambda z-c\cos(\_a\lambda)+c\cos(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a\right) + \_F1\left(\frac{a\beta y+b\cos(\beta x)}{a\beta}, \frac{a\lambda z+c\cos(\lambda x)}{a\lambda}\right)$$

6.7.14.6 [1678] Problem 6

problem number 1678

Added June 26, 2019.

Problem Chapter 7.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sin^{n_1}(\lambda_1 x) w_x + b_1 \sin^{m_1}(\beta_1 y) w_y + c_1 \sin^{k_1}(\gamma_1 z) w_z = a_2 \sin^{n_2}(\lambda_2 x) + b_2 \sin^{m_2}(\beta_2 y) + c_2 \sin^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Sin[beta1*y]^m1*D[w[x, y,z], y] +  c1*Sin[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*sin(beta1*y)^m1*diff(w(x,y,z),y)+ c1*sin(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2(\sin^{n2}(\_f\lambda 2)) + b2\left(\sin^{m2}\left(\beta 2\,\mathrm{RootOf}\left(\int(\sin^{-n1}(\_f\lambda 1))\,d\_f - \left(\int(\sin^{-n1}(\lambda 1 x))\right.\right.\right.\right.}{}$$

6.7.15 6.2

Local contents

6.7.15.1 [1679] Problem 1 2347
6.7.15.2 [1680] Problem 2 2348
6.7.15.3 [1681] Problem 3 2349
6.7.15.4 [1682] Problem 4 2350

6.7.15.5 [1683] Problem 5 2351
6.7.15.6 [1684] Problem 6 2352

6.7.15.1 [1679] Problem 1

problem number 1679

Added June 26, 2019.

Problem Chapter 7.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\cos^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Cos[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-cx) - \frac{c\sqrt{\sin^2(\lambda x)}\csc(\lambda x)\cos^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{k+1}{2}, \frac{k+3}{2}, \cos\right.}{k\lambda+\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cos(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\cos^k(\lambda x)\right) dx + \_F1(-ax+y, -bx+z)$$

6.7.15.2 [1680] Problem 2

problem number 1680

Added June 26, 2019.

Problem Chapter 7.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\cos(\beta z)w_z = k\cos(\lambda x) + s\cos(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cos[beta*z]*D[w[x,y,z],z]== k*Cos[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, -\frac{\cosh^{-1}\left(-\frac{\sec(\beta z)\left(2\left(2\sec(\beta z)\sqrt{\sin^2(\beta z)\cos^2(\beta z)\sinh^2\left(\frac{\beta cx}{a}\right)\left(\cosh\left(\frac{4\beta cx}{a}\right)-\sinh\left(\frac{4\beta cx}{a}\right)\right)}+\sin\right.\right.}{4\cosh(}\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cos(beta*z)*diff(w(x,y,z),z)= k*cos(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{ab\gamma\lambda\_F1\left(\frac{ay-bx}{a}, \frac{a\ln\left(\mathrm{RootOf}\left(\beta z - \arctan\left(\frac{\_Z^2 e^{\frac{2\beta cx}{a}}-1}{\_Z^2 e^{\frac{2\beta cx}{a}}+1}, \frac{2\_Z e^{\frac{\beta cx}{a}}}{\_Z^2 e^{\frac{2\beta cx}{a}}+1}\right)\right)\right)}{\beta c}\right) + a\lambda s\sin(\gamma y) + b\gamma k\sin(\lambda x}{ab\gamma\lambda}$$

6.7.15.3 [1681] Problem 3

problem number 1681

Added June 26, 2019.

Problem Chapter 7.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\beta x)w_y + b\cos^k(\lambda x)w_z = c\cos^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{a\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\beta x)\right)}{\beta n+\beta} + y, \frac{b\sqrt{}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\cos^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\cos^n(\beta x))\,dx\right), z - \left(\int b(\cos^k(\lambda x))\,dx\right)\right)$$

6.7.15.4 [1682] Problem 4

problem number 1682

Added June 26, 2019.

Problem Chapter 7.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\cos^m(\beta y)w_z = c\cos^k(\gamma y) + s\cos^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(c\cos^k\left(\frac{\gamma\left(a\csc(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(\lambda x)\right)\sqrt{\sin^2(\lambda x)}\cos^{n+1}\right.}{}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int(\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\cos^r\left(\left(b\right.\right.\right.\right.$$

6.7.15.5 [1683] Problem 5

problem number 1683

Added June 26, 2019.

Problem Chapter 7.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\cos(\beta x)w_y + c\cos(\lambda x)w_z = k\cos(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*x]*D[w[x, y,z], y] +  c*Cos[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \frac{k\cos\left(\frac{\gamma(a\lambda z-c\sin(\lambda x)+c\sin(\lambda K[1]))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\sin(\beta x)}{a\beta},z-\frac{c\sin(\lambda x)}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*cos(beta*x)*diff(w(x,y,z),y)+ c*cos(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \frac{k\cos\left(\frac{(a\lambda z+c\sin(\_a\lambda)-c\sin(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a+\_F1\left(\frac{a\beta y-b\sin(\beta x)}{a\beta},\frac{a\lambda z-c\sin(\lambda x)}{a\lambda}\right)$$

6.7.15.6 [1684] Problem 6

problem number 1684

Added June 26, 2019.

Problem Chapter 7.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \cos^{n_1}(\lambda_1 x) w_x + b_1 \cos^{m_1}(\beta_1 y) w_y + c_1 \cos^{k_1}(\gamma_1 z) w_z = a_2 \cos^{n_2}(\lambda_2 x) + b_2 \cos^{m_2}(\beta_2 y) + c_2 \cos^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cos[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cos[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cos(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2(\cos^{n2}(\_f\lambda 2)) + b2\left(\cos^{m2}\left(\beta 2\,\mathrm{RootOf}\left(\int(\cos^{-n1}(\_f\lambda 1))\,d\_f - \left(\int(\cos^{-n1}(\lambda 1 x)\right.\right.\right.\right.}{}$$

6.7.16 6.3

Local contents

6.7.16.1 [1685] Problem 1 2353
6.7.16.2 [1686] Problem 2 2354
6.7.16.3 [1687] Problem 3 2355
6.7.16.4 [1688] Problem 4 2356
6.7.16.5 [1689] Problem 5 2357

6.7.16.1 [1685] Problem 1

problem number 1685

Added June 26, 2019.

Problem Chapter 7.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\tan^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Tan[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-cx) + \frac{c\tan^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1, \frac{k+1}{2}, \frac{k+3}{2}, -\tan^2(\lambda x)\right)}{k\lambda+\lambda} + sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tan(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\tan^k(\lambda x)\right) dx + \_F1(-ax+y, -bx+z)$$

6.7.16.2 [1686] Problem 2

problem number 1686

Added June 26, 2019.

Problem Chapter 7.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\tan(\beta z)w_z = k\tan(\lambda x) + s\tan(\gamma y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tan[beta*z]*D[w[x,y,z],z]== k*Tan[lambda*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, \frac{\log(\sin(\beta z))}{\beta} - \frac{cx}{a}\right) - \frac{k\log(\cos(\lambda x))}{a\lambda} - \frac{s\log(\cos(\gamma y))}{b\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tan(beta*z)*diff(w(x,y,z),z)= k*tan(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{2ab\gamma\lambda\_F1\left(\frac{ay-bx}{a}, \frac{-\beta cx + a\ln\left(\frac{\tan(\beta z)}{\sqrt{\tan^2(\beta z)+1}}\right)}{\beta c}\right) + a\lambda s\ln\left(\tan^2(\gamma y)+1\right) + b\gamma k\ln\left(\tan^2(\lambda x)+1\right)}{2ab\gamma\lambda}$$

6.7.16.3 [1687] Problem 3

problem number 1687

Added June 26, 2019.

Problem Chapter 7.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\tan^n(\beta x)w_y + b\tan^k(\lambda x)w_z = c\tan^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[beta*x]^n*D[w[x, y,z], y] +  b*Tan[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{a\tan^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1, \frac{n+1}{2}, \frac{n+3}{2}, -\tan^2(\beta x)\right)}{\beta n + \beta}, z - \frac{b\tan^{k+1}(\lambda x)\,\text{Hy}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tan(beta*x)^n*diff(w(x,y,z),y)+ b*tan(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\tan^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\tan^n(\beta x))\,dx\right), z - \left(\int b(\tan^k(\lambda x))\,dx\right)\right)$$

6.7.16.4 [1688] Problem 4

problem number 1688

Added June 26, 2019.

Problem Chapter 7.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tan^n(\lambda x)w_y + b\tan^m(\beta y)w_z = c\tan^k(\gamma y) + s\tan^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[lambda*x]^n*D[w[x, y,z], y] +  b*Tan[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(c\tan^k\left(\frac{\gamma\left(-a\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(\lambda x)\right)\tan^{n+1}(\lambda x) + a\,\text{Hyperg}\right.}{\lambda(n+}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*tan(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\frac{\tan\left(\left(y - \left(\int a(\tan^n(\lambda x))\,dx\right)\right)\gamma\right) + \tan\left(a\gamma\left(\int(\tan^n(\_f\lambda))\,d\_f\right)\right)}{-\tan\left(\left(y - \left(\int a(\tan^n(\lambda x))\,dx\right)\right)\gamma\right)\tan\left(a\gamma\left(\int(\tan^n(\_f\lambda))\,d\_f\right)\right) + 1}\right)^k + s\left(\frac{t}{-t}\right.\right.$$

6.7.16.5 [1689] Problem 5

problem number 1689

Added June 26, 2019.

Problem Chapter 7.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \tan^{n_1}(\lambda_1 x) w_x + b_1 \tan^{m_1}(\beta_1 y) w_y + c_1 \tan^{k_1}(\gamma_1 z) w_z = a_2 \tan^{n_2}(\lambda_2 x) + b_2 \tan^{m_2}(\beta_2 y) + c_2 \tan^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Tan[beta1*y]^m1*D[w[x, y,z], y] +  c1*Tan[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*tan(beta1*y)^m1*diff(w(x,y,z),y)+ c1*tan(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2(\tan^{n2}(\_f\lambda 2)) + b2\left(\tan^{m2}\left(\beta 2\,\mathrm{RootOf}\left(\int(\tan^{-n1}(\_f\lambda 1))\,d\_f - \left(\int(\tan^{-n1}(\lambda 1 x\right.\right.\right.\right.}{}$$

6.7.17 6.4

Local contents

6.7.17.1 [1690] Problem 1 2358
6.7.17.2 [1691] Problem 2 2359
6.7.17.3 [1692] Problem 3 2360
6.7.17.4 [1693] Problem 4 2361
6.7.17.5 [1694] Problem 5 2362

6.7.17.1 [1690] Problem 1

problem number 1690

Added June 26, 2019.

Problem Chapter 7.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c\cot^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  c*D[w[x,y,z],z]== c*Cot[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-cx) - \frac{c\cot^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1, \frac{k+1}{2}, \frac{k+3}{2}, -\cot^2(\lambda x)\right)}{k\lambda+\lambda} + sx\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cot(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\left(\cot^k(\lambda x)\right) dx + \_F1(-ax+y, -bx+z)$$

6.7.17.2 [1691] Problem 2

problem number 1691

Added June 26, 2019.

Problem Chapter 7.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\cot(\gamma z)w_z = k\cot(\lambda x) + s\cot(\beta y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cot[gamma*z]*D[w[x,y,z],z]== k*Cot[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a\lambda s\log(\tan(\beta y)) + a\lambda s\log(\cos(\beta y)) + b\beta k\log(\sin(\lambda x))}{ab\beta\lambda} + c_1\left(y - \frac{bx}{a}, \frac{\log(\sec(\gamma z))}{\gamma} - \frac{cx}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cot(gamma*z)*diff(w(x,y,z),z)= k*cot(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{2ab\beta\lambda\_F1\left(\frac{-ay+bx}{b}, \frac{-2c\gamma y + b\ln(\cot^2(\gamma z)+1) - 2b\ln(\cot(\gamma z))}{2c\gamma}\right) - a\lambda s\ln\left(\cot^2(\beta y)+1\right) - b\beta k\ln\left(\cot^2(\right.}{2ab\beta\lambda}$$

6.7.17.3 [1692] Problem 3

problem number 1692

Added June 26, 2019.

Problem Chapter 7.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\beta x)w_y + b\cot^k(\lambda x)w_z = c\cot^m(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[beta*x]^n*D[w[x, y,z], y] +  b*Cot[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b\cot^{k+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\cot^2(\lambda x)\right)}{k\lambda+\lambda} + z, \frac{a\cot^{n+1}(\beta x)\,\text{Hyperge}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cot(beta*x)^n*diff(w(x,y,z),y)+ b*cot(lambda*x)^k*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\cot^m(\gamma x))\,dx + \_F1\left(y - \left(\int a(\cot^n(\beta x))\,dx\right), z - \left(\int b(\cot^k(\lambda x))\,dx\right)\right)$$

6.7.17.4 [1693] Problem 4

problem number 1693

Added June 26, 2019.

Problem Chapter 7.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\lambda x)w_y + b\cot^m(\beta y)w_z = c\cot^k(\gamma y) + s\cot^r(\mu z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[lambda*x]^n*D[w[x, y,z], y] +  b*Cot[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(c\cot^k\left(\frac{\gamma\left(a\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(\lambda x)\right)\cot^{n+1}(\lambda x)+\lambda(n+1)y\right.}{\lambda(n+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cot(lambda*x)^n*diff(w(x,y,z),y)+ b*cot(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\frac{\cot\left(\left(y-\left(\int a(\cot^n(\lambda x))\,dx\right)\right)\gamma\right)\cot\left(a\gamma\left(\int(\cot^n(\_f\lambda))\,d\_f\right)\right)-1}{\cot\left(\left(y-\left(\int a(\cot^n(\lambda x))\,dx\right)\right)\gamma\right)+\cot\left(a\gamma\left(\int(\cot^n(\_f\lambda))\,d\_f\right)\right)}\right)^k + s\left(\frac{\cot((z}{\cot((z}\right.\right.$$

6.7.17.5 [1694] Problem 5

problem number 1694

Added June 26, 2019.

Problem Chapter 7.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\cot(\beta x)w_y + c\cot(\lambda x)w_z = k\cot(\gamma z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*x]*D[w[x, y,z], y] +  c*Cot[lambda*x]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{k\cot\left(\frac{\gamma(a\lambda z - c\log(\sin(\lambda x)) + c\log(\sin(\lambda K[1])))}{a\lambda}\right)}{a} dK[1] + c_1\left(y - \frac{b\log(\sin(\beta x))}{a\beta}, z - \frac{c\log(\sin(}{a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*cot(beta*x)*diff(w(x,y,z),y)+ c*cot(lambda*x)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(\cot\left(\frac{(2a\lambda z + c\ln(\cot^2(\lambda x)+1))\gamma}{2a\lambda}\right)\cot\left(\frac{c\gamma\ln(\cot^2(\_a\lambda)+1)}{2a\lambda}\right) + 1\right)k}{\left(-\cot\left(\frac{(2a\lambda z + c\ln(\cot^2(\lambda x)+1))\gamma}{2a\lambda}\right) + \cot\left(\frac{c\gamma\ln(\cot^2(\_a\lambda)+1)}{2a\lambda}\right)\right)a} d\_a + \_F1\left(\frac{2a\beta y + b\ln(\cot^2(}{2a\beta}\right.$$

6.7.18 6.2

Local contents

6.7.18.1 [1695] Problem 6 2363

6.7.18.1 [1695] Problem 6

problem number 1695

Added June 26, 2019.

Problem Chapter 7.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 \cot^{n_1}(\lambda_1 x) w_x + b_1 \cot^{m_1}(\beta_1 y) w_y + c_1 \cot^{k_1}(\gamma_1 z) w_z = a_2 \cot^{n_2}(\lambda_2 x) + b_2 \cot^{m_2}(\beta_2 y) + c_2 \cot^{k_2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cot[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cot[g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cot(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \int^x \frac{\left(a2(\cot^{n2}(\_f\lambda 2)) + b2\left(\cot^{m2}\left(\beta 2\,\mathrm{RootOf}\left(\int(\cot^{-n1}(\_f\lambda 1))\,d\_f - \left(\int(\cot^{-n1}(\lambda 1 x)\right.\right.\right.\right.}{}$$

6.7.19 6.5

Local contents

6.7.19.1 [1696] Problem 1 2364
6.7.19.2 [1697] Problem 2 2365
6.7.19.3 [1698] Problem 3 2366
6.7.19.4 [1699] Problem 4 2367
6.7.19.5 [1700] Problem 5 2367

6.7.19.1 [1696] Problem 1

problem number 1696

Added June 26, 2019.

Problem Chapter 7.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\sin^n(\lambda x)w_y + b\cos^m(\beta x)w_z = c\sin^k(\gamma x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{m+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(\beta x)\right)}{\beta m+\beta} + z, y -\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c(\sin^k(\gamma x))\,dx + \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int b(\cos^m(\beta x))\,dx\right)\right)$$

6.7.19.2 [1697] Problem 2

problem number 1697

Added June 26, 2019.

Problem Chapter 7.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\sin^m(\beta y)w_z = c\cos^k(\gamma y) + s\sin^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] + b*Sin[beta*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int (\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\sin^r\left(\left(b\left(\right.\right.\right.\right.$$

6.7.19.3 [1698] Problem 3

problem number 1698

Added June 26, 2019.

Problem Chapter 7.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\cos^n(\lambda x)w_y + b\tan^m(\beta y)w_z = c\cos^k(\gamma y) + s\tan^r(\mu z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] + b*Tan[beta*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(c\left(\cos^k\left(\left(a\left(\int (\cos^n(\_f\lambda))\,d\_f\right) + y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\gamma\right)\right) + s\left(\frac{\sin\left(\left(z +\right.\right.}{\cos\left(\left(z +\right.\right.}\right.$$

6.7.19.4 [1699] Problem 4

problem number 1699

Added June 26, 2019.

Problem Chapter 7.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a1 \sin^{n1}(\lambda_1 x) w_x + b1 \cos^{m1}(\beta_1 y) w_y + c1 \cos^{k1}(\gamma_1 z) w_z = a2 \cos^{n2}(\lambda_2 x) + b2 \sin^{m2}(\beta_2 y) + c2 \cos^{k2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \int^x \frac{\left(a2(\cos^{n2}(\_f\lambda 2)) + b2(\sin^{m2}(\_f\beta 2)) + c2\left(\cos^{k2}\left(\gamma 2\,\mathrm{RootOf}\left(\int (\sin^{-n1}(\_f\lambda 1))\, d\_f\right.\right.\right.\right.}{}$$

6.7.19.5 [1700] Problem 5

problem number 1700

Added June 26, 2019.

Problem Chapter 7.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a1\tan^{n1}(\lambda_1 x)w_x+b1\cot^{m1}(\beta_1 y)w_y+c1\cot^{k1}(\gamma_1 z)w_z = a2\cot^{n2}(\lambda_2 x)+b2\tan^{m2}(\beta_2 y)+c2\cot^{k2}(\gamma_2 z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*x)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{\left(a2\left(\frac{\cos(\_f\lambda 2)}{\sin(\_f\lambda 2)}\right)^{n2} + b2\left(\frac{\sin(\_f\beta 2)}{\cos(\_f\beta 2)}\right)^{m2} + c2\left(\frac{\cos\left(\gamma 2\,\mathrm{RootOf}\left(\int\left(\frac{\sin(\lambda 1 x)}{\cos(\lambda 1 x)}\right)^{-n1}dx-\left(\int(\tan^{-n1}(\_f\lambda 1))d\_\right.\right.\right.}{\sin\left(\gamma 2\,\mathrm{RootOf}\left(\int\left(\frac{\sin(\lambda 1 x)}{\cos(\lambda 1 x)}\right)^{-n1}dx-\left(\int(\tan^{-n1}(\_f\lambda 1))d\_\right.\right.\right.}\right.\right.}{a1}$$

6.7.20 7.1

Local contents

6.7.20.1 [1701] Problem 1 2369
6.7.20.2 [1702] Problem 2 2370
6.7.20.3 [1703] Problem 3 2371
6.7.20.4 [1704] Problem 4 2372
6.7.20.5 [1705] Problem 5 2372
6.7.20.6 [1706] Problem 6 2373

6.7.20.1 [1701] Problem 1

problem number 1701

Added June 26, 2019.

Problem Chapter 7.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + aw_y + bw_z = c \arcsin^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcSin[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax, z-bx) + \frac{(\sin^{-1}(\lambda x)^2)^{-k}\left(-ic(i\sin^{-1}(\lambda x))^k \sin^{-1}(\lambda x)^k \operatorname{Gamma}(k+1, -i\sin\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arcsin(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c \arcsin(\lambda x)^k \, dx + \_F1(-ax+y, -bx+z)$$

6.7.20.2 [1702] Problem 2

problem number 1702

Added June 26, 2019.

Problem Chapter 7.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 w_x + a_2 w_y + a_3 w_z = b_1 \arcsin(\lambda_1 x) + b_2 \arcsin(\lambda_2 y) + b_3 \arcsin(\lambda_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcSin[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{\text{a2}x}{\text{a1}}, z - \frac{\text{a3}x}{\text{a1}}\right) + \frac{\text{b1}\sqrt{1 - \text{lambda1}^2 x^2}}{\text{a1lambda1}} + \frac{\text{b1}x \sin^{-1}(\text{lambda1}x)}{\text{a1}} + \frac{\text{b2}\sqrt{1 - \text{laml}}}{\text{a2lambd}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arcsin(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a1\, a2\, a3\, \lambda 1 \lambda 2 \lambda 3\_F1\left(\frac{a1y - a2x}{a1}, \frac{za1 - a3x}{a1}\right) + \sqrt{-\lambda 1^2 x^2 + 1}\, a2\, a3\, b1\, \lambda 2 \lambda 3 + \left(\sqrt{-\lambda 2^2 y^2 + 1}\, a1\, a3\right.}{}$$

6.7.20.3 [1703] Problem 3

problem number 1703

Added June 26, 2019.

Problem Chapter 7.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s\arcsin(\gamma x)^m}{a}dx + \_F1\left(\frac{ya-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\text{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda}{2}\right.}{}\right.$$

6.7.20.4 [1704] Problem 4

problem number 1704

Added June 26, 2019.

Problem Chapter 7.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{sx}{a} + \_F1\left(\frac{ay-bx}{a}, -\left(\int^x \arcsin(\_a\lambda)^n \arcsin\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right)^m d\_a\right) - \frac{\left(-\gamma kz\right.}{}$$

6.7.20.5 [1705] Problem 5

problem number 1705

Added June 26, 2019.

Problem Chapter 7.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\sin^{-1}\left(\frac{\gamma\left(ia(-i\sin^{-1}(\beta z))^k \operatorname{Gamma}(1-k,-i\sin^{-1}(\beta z))\sin^{-1}(\beta z)^{-k} - ia(i\sin^{-1}(\beta z))^k \operatorname{Gamma}(1-k,i\sin^{-1}(\right.}{}\right.}{}\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.7.20.6 [1706] Problem 6

problem number 1706

Added June 26, 2019.

Problem Chapter 7.7.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*x]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{cx}{a}-\frac{i\sin^{-1}(\beta z)^{-k}\left(\left(-i\sin^{-1}(\beta z)\right)^k\text{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right)-\left(i\sin^{-1}(\beta z)\right)\right.}{2\beta}\right.\right.\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.7.21 7.2

Local contents

6.7.21.1 [1707] Problem 1 2374
6.7.21.2 [1708] Problem 2 2375
6.7.21.3 [1709] Problem 3 2376
6.7.21.4 [1710] Problem 4 2377
6.7.21.5 [1711] Problem 5 2378

6.7.21.1 [1707] Problem 1

problem number 1707

Added June 26, 2019.

Problem Chapter 7.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arccos^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcCos[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax,z-bx) + \frac{(\cos^{-1}(\lambda x)^2)^{-k}\left(c(i\cos^{-1}(\lambda x))^k \cos^{-1}(\lambda x)^k \operatorname{Gamma}(k+1,-i\cos^{-}\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccos(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = sx + \int c\arccos(\lambda x)^k\,dx + \_F1(-ax+y,-bx+z)$$

Answer contains unresolved integrals

6.7.21.2 [1708] Problem 2

problem number 1708

Added June 26, 2019.

Problem Chapter 7.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x + a_2w_y + a_3w_z = b_1\arccos(\lambda_1 x) + b_2\arccos(\lambda_2 y) + b_3\arccos(\lambda_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcCos[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)-\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\cos^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}x\sin^{-1}(\text{lar}}{\text{a1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arccos(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a1\,a2\,a3\,\lambda1\lambda2\lambda3\_F1\left(\frac{a1\,y-a2\,x}{a1},\frac{z\,a1-a3\,x}{a1}\right)-\sqrt{-\lambda1^2x^2+1}\,a2\,a3\,b1\,\lambda2\lambda3-\left(\sqrt{-\lambda2^2y^2+1}\,a1\,a3}{}$$

6.7.21.3 [1709] Problem 3

problem number 1709

Added June 26, 2019.

Problem Chapter 7.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arccos^n(\lambda x)\arccos^k(\beta z)w_z=s\arccos^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s \arccos(\gamma x)^m}{a} dx + \_F1\left(\frac{ay-bx}{a}, \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2 x^2+1}\, 2^{-n}\, \mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \right.}{}\right.$$

6.7.21.4 [1710] Problem 4

problem number 1710

Added June 26, 2019.

Problem Chapter 7.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = s\arccos^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcCos[lambda*x]^n*D[w[x, y,z], y] + c*ArcCos[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\cos^{-1}\left(\frac{\gamma\left(-a\left(-i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k,-i\cos^{-1}(\beta z)\right)\cos^{-1}(\beta z)^{-k} - a\left(i\cos^{-1}(\beta z)\right)^k \mathrm{Gamma}\left(1-k,i\cos^{-}\right.\right.}{}\right.}{}\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s \arccos\left(\gamma \operatorname{RootOf}\left(\_Zb\lambda n \operatorname{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos\left(\_Z\lambda\right)\right)\arccos\left(\_Z\lambda\right)+2\_Zb\lambda\right.\right.}{}$$

6.7.21.5 [1711] Problem 5

problem number 1711

Added June 26, 2019.

Problem Chapter 7.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda y)w_y + c\arcsin^k(\beta z)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcSin[lambda*y]^n*D[w[x, y,z], y] + c*ArcSin[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s \arcsin(\lambda y)^{-n}}{b} dy + \_F1 \left(- \frac{\left(-\arcsin(\lambda y)^{-n} \arcsin(\lambda y)^{\frac{3}{2}} + \text{LommelS1}\left(-n+\frac{3}{2}, \frac{1}{2}, \arcsin\right.\right.}{} \right.$$

6.7.22 7.3

Local contents

6.7.22.1 [1712] Problem 1 2379
6.7.22.2 [1713] Problem 2 2380
6.7.22.3 [1714] Problem 3 2381
6.7.22.4 [1715] Problem 4 2382
6.7.22.5 [1716] Problem 5 2383

6.7.22.1 [1712] Problem 1

problem number 1712

Added June 26, 2019.

Problem Chapter 7.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c \arctan^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcTan[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \left(c\tan^{-1}(\lambda K[1])^k+s\right)dK[1]+c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arctan(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\arctan\left(\lambda x\right)^k dx+\_F1(-ax+y,-bx+z)$$

6.7.22.2 [1713] Problem 2

problem number 1713

Added June 26, 2019.

Problem Chapter 7.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=b_1\arctan(\lambda_1x)+b_2\arctan(\lambda_2y)+b_3\arctan(\lambda_3z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcTan[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)-\frac{\text{b2}\log\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)}{\text{2a2lambda2}}-\frac{\text{b3}\log\left(\text{a1}^2\left(\text{lambda3}^2z^2+1\right)\right.}{\text{2a3lambda3}}\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arctan(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{b1\,x\arctan(\lambda 1x)}{a1}+\frac{b2\,y\arctan(\lambda 2y)}{a2}+\frac{b3\,z\arctan(\lambda 3z)}{a3}+\frac{2a1\,a2\,a3\,\lambda 1\lambda 2\lambda 3\_F1\left(\frac{a1y-a2x}{a1},\frac{za1-}{a}\right.}{}$$

6.7.22.3 [1714] Problem 3

problem number 1714

Added June 26, 2019.

Problem Chapter 7.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^k(\beta z)w_z=s\arctan^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s \arctan(\gamma x)^m}{a} dx + \_F1\left(\frac{ay-bx}{a}, -\left(\int \arctan(\lambda x)^n dx\right) + \int \frac{a \arctan(\beta z)^{-k}}{c} dz\right)$$

6.7.22.4 [1715] Problem 4

problem number 1715

Added June 26, 2019.

Problem Chapter 7.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arctan^n(\lambda x)\arctan^m(\beta y)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*y)^m*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{sx}{a}+\_F1\left(\frac{ay-bx}{a},-\left(\int^{x}\frac{c\arctan\left(\lambda\_a\right)^{n}\arctan\left(\frac{\beta(\_ab+ay-bx)}{a}\right)^{m}}{a}d\_a\right)+z\right)$$

6.7.22.5 [1716] Problem 5

problem number 1716

Added June 26, 2019.

Problem Chapter 7.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\arctan^n(\lambda x)w_y+c\arctan^k(\beta z)w_z=s\arctan^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] + c*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^z\frac{s\tan^{-1}(\beta K[3])^{-k}\tan^{-1}\left(\frac{\gamma\left(cx-a\int_1^z\tan^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\tan^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)^{m}}{c}dK[3\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s \arctan\left(\gamma \operatorname{RootOf}\left(\_b - y + \int \frac{b \arctan(\lambda x)^n}{a} dx - \left(\int^{\_Z} \frac{b \arctan(\_a \lambda)^n}{a} d\_a\right)\right)\right)^m \arctan\left(\lambda \mathrm{R}\right.}{b}$$

6.7.23 7.4

Local contents

6.7.23.1 [1717] Problem 1 2384
6.7.23.2 [1718] Problem 2 2385
6.7.23.3 [1719] Problem 3 2386
6.7.23.4 [1720] Problem 4 2387
6.7.23.5 [1721] Problem 5 2388

6.7.23.1 [1717] Problem 1

problem number 1717

Added June 26, 2019.

Problem Chapter 7.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a w_y + b w_z = c \operatorname{arccot}^k(\lambda x) + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] + b*D[w[x,y,z],z]==c*ArcCot[lambda*x]^k+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \left(c\cot^{-1}(\lambda K[1])^k+s\right)dK[1]+c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccot(lambda*x)^k+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=sx+\int c\left(-\arctan\left(\lambda x\right)+\frac{\pi}{2}\right)^k dx+\_F1\left(-ax+y,-bx+z\right)$$

6.7.23.2 [1718] Problem 2

problem number 1718

Added June 26, 2019.

Problem Chapter 7.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=b_1\operatorname{arccot}(\lambda_1x)+b_2\operatorname{arccot}(\lambda_2y)+b_3\operatorname{arccot}(\lambda_3z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x, y,z], x] + a2*D[w[x, y,z], y] + a3*D[w[x,y,z],z]== b1*ArcCot[lambda1*x]+b2*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)+\frac{1}{2}\left(\frac{\text{b2}\log\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)}{\text{a2lambda2}}+\frac{\text{b3}\log\left(\text{a1}^2\left(\text{lambda3}^2z^2+\right.\right.}{\text{a3lambda3}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a1*diff(w(x,y,z),x)+ a2*diff(w(x,y,z),y)+ a3*diff(w(x,y,z),z)= b1*arccot(lambda1*x)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{2a1\,a2\,a3\,\lambda1\lambda2\lambda3\_F1\left(\frac{a1y-a2x}{a1},\frac{za1-a3x}{a1}\right)+a2\,a3\,b1\,\lambda2\lambda3\ln\left(\lambda1^2x^2+1\right)+\left(a1\,a3\,b2\,\lambda3\ln\left(\lambda2^2\right.\right.}{}$$

6.7.23.3 [1719] Problem 3

problem number 1719

Added June 26, 2019.

Problem Chapter 7.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\operatorname{arccot}^n(\lambda x)\operatorname{arccot}^k(\beta z)w_z=s\operatorname{arccot}^m(\gamma x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{s\left(-\arctan(\gamma x)+\frac{\pi}{2}\right)^m}{a}dx + \_F1\left(\frac{ay-bx}{a}, -\left(\int \left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n dx\right) + \int \frac{a(-\mathrm{ar}}{}\right.$$

6.7.23.4 [1720] Problem 4

problem number 1720

Added June 26, 2019.

Problem Chapter 7.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\operatorname{arccot}^n(\lambda x)\operatorname{arccot}^m(\beta y)\operatorname{arccot}^k(\gamma z)w_z = s$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{sx}{a} + \_F1\left(\frac{ay-bx}{a}, \int \frac{a\left(-\arctan(\gamma z)+\frac{\pi}{2}\right)^{-k}}{c} dz - \left(\int^{x}\left(-\arctan\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right.\right.\right.\right.$$

6.7.23.5 [1721] Problem 5

problem number 1721

Added June 26, 2019.

Problem Chapter 7.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\operatorname{arccot}^n(\lambda x)w_y + c\operatorname{arccot}^k(\beta z)w_z = s\operatorname{arccot}^m(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*ArcTan[lambda*x]^n*D[w[x, y,z], y] + c*ArcTan[beta*z]^k*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^z \frac{s\cot^{-1}\left(\frac{\gamma\left(cx-a\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\tan^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)^m \tan^{-1}(\beta K[3])^{-k}}{c} dK[3\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{s\left(-\arctan\left(\gamma\,\mathrm{RootOf}\left(\_b - y + \int \frac{b\arctan(\lambda x)^n}{a}dx - \left(\int^{\_Z} \frac{b\arctan(\_a\lambda)^n}{a}d\_a\right)\right)\right) + \frac{\pi}{2}\right)^m \mathrm{ar}}{b}$$

6.7.24 8.1

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6.7.24.1 [1722] Problem 1

problem number 1722

Added June 27, 2019.

Problem Chapter 7.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + f(x)w_y + g(x)w_z = h_2(x)y + h_1(x) + h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*D[w[x, y,z], y] + g[x]*D[w[x,y,z],z]== h2[x]*y+h1[x]*z+h0[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\int_1^x\left(\mathrm{h0}(K[3])+\mathrm{h2}(K[3])\left(y-\int_1^x f(K[1])dK[1]+\int_1^{K[3]} f(K[1])dK[1]\right)+\mathrm{h1}(K[3])\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+ g(x)*diff(w(x,y,z),z)= h2(x)*y+h1(x)*z+h0(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x\left(\left(\int f(\_f)\,d\_f\right)h2(\_f)+\left(\int g(\_f)\,d\_f\right)h1(\_f)+h0(\_f)+\left(z-\left(\int g(x)\,dx\right)\right)\right.$$

6.7.24.2 [1723] Problem 2

problem number 1723

Added June 27, 2019.

Problem Chapter 7.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+f(x)(y+a)w_y+g(x)(z+b)w_z=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + f[x]*(y+a)*D[w[x, y,z], y] + g[x]*(z+b)*D[w[x,y,z],z]== h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x h(K[5])dK[5]+c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right)-\int_1^x a\exp\left(-\int_1^{K[2]} f(K[1])dK[\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+ g(x)*(z+b)*diff(w(x,y,z),z)= h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int h(x)\,dx+\_F1\left((a+y)\,\mathrm{e}^{-\left(\int f(x)dx\right)},(b+z)\,\mathrm{e}^{-\left(\int g(x)dx\right)}\right)$$

6.7.24.3 [1724] Problem 3

problem number 1724

Added June 27, 2019.

Problem Chapter 7.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+f(x))w_y+(bz+g(x))w_z=h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+f[x])*D[w[x, y,z], y] + (b*z+g[x])*D[w[x,y,z],z]== h[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x h(K[3])dK[3]+c_1\left(ye^{-ax}-\int_1^x e^{-aK[1]}f(K[1])dK[1],ze^{-bx}-\int_1^x e^{-bK[2]}g(K[2])dK[\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+ (b*z+g(x))*diff(w(x,y,z),z)= h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int h(x)\,dx+\_F1\left(y\,\mathrm{e}^{-ax}-\left(\int \mathrm{e}^{-ax}f(x)\,dx\right),z\,\mathrm{e}^{-bx}-\left(\int \mathrm{e}^{-bx}g(x)\,dx\right)\right)$$

6.7.24.4 [1725] Problem 4

problem number 1725

Added June 27, 2019.

Problem Chapter 7.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)y+g_2(x))w_z=h_2(x)y+h_1(x)z+h_0(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x]*y+g2[x])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right)\left(\exp\left(\int_1^x \text{f1}(K[1])dK[1]\right)\text{h0}(K[5])+\exp\left(\int_1^{K[5]} \text{f1}(K\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\int^x \left(y\,\mathrm{e}^{-(\int f1(x)dx)}\mathrm{e}^{\int f1(\_g)d\_g}h2(\_g)+\left(\int \mathrm{e}^{-(\int f1(\_g)d\_g)}f2(\_g)\,d\_g\right)\mathrm{e}^{\int f1(\_g)d\_g}h2(\_g)-\right.$$

6.7.24.5 [1726] Problem 5

problem number 1726

Added June 27, 2019.

Problem Chapter 7.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))w_y+(g_1(x)z+g_2(x))w_z=h_2(x)y+h_1(x)z+h_0(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*z+f2[x])*D[w[x, y,z], y] + (g1[x]*y+g2[x])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*z+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.24.6 [1727] Problem 6

problem number 1727

Added June 27, 2019.

Problem Chapter 7.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x))w_y + f(x) \sinh(\gamma z)w_z = g(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2-a^2 + a *lambda*Sin[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x, y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2 + a *lambda*sin(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 - \frac{2\_c_3 \operatorname{arctanh}\left(\mathrm{e}^{\gamma z}\right)}{\gamma} + \_F1(x) + \_F2(y) \textbf{ where } \left[\left\{-\frac{a^2 \cosh\left(2\lambda x\right)\left(\frac{d}{dy}\_F2(y)\right)}{2} + a\lambda\left(\frac{d}{dy}\right.\right.\right.$$

Gives Warning: Incomplete separation

6.7.24.7 [1728] Problem 7

problem number 1728

Added June 27, 2019.

Problem Chapter 7.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(x)z^m)w_z = h(x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[x]*z^m)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x h(K[5])dK[5] + c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2] + \right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(x)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int h(x)\,dx + \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)(\int f1(x)dx)} + (k-1)\left(\int \mathrm{e}^{(k-1)(\int f1(x)dx)} f2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)(}$$

6.7.24.8 [1729] Problem 8

problem number 1729

Added June 27, 2019.

Problem Chapter 7.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x) + g_2(x)e^{\lambda z})w_z = h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]+g2[x]*Exp[lambda*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(lambda*z))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int h(x)\,dx + \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g1\right.}\right.}{}\right.$$

6.7.24.9 [1730] Problem 9

problem number 1730

Added June 27, 2019.

Problem Chapter 7.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x) + g_2(x)e^{\beta z})w_z = h(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]+g2[x]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int h(x)\,dx + \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int f1(x)dx\right)} f2(x)\,dx\right) - e^{\left(-y+\int f1(x)dx\right)\lambda}}{\lambda}, \frac{-\beta\left(\int e^{\beta\left(\int g1(x)dx\right)} g2(x)\,dx\right)}{\beta}\right.$$

6.7.25 8.2

Local contents

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6.7.25.1 [1731] Problem 1

problem number 1731

Added June 27, 2019.

Problem Chapter 7.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = \Phi(x) + \Psi(x) + \chi(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f[x]*D[w[x, y,z], x] + g[y]*D[w[x, y,z], y] + h[z]*D[w[x,y,z],z]== phi[x]+psi[x]+chi[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ g(y)*diff(w(x,y,z),y)+ h(z)*diff(w(x,y,z),z)= phi(x)+psi(x)+c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{\chi(x)+\phi(x)+\psi(x)}{f(x)}dx + \_F1\left(-\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{g(y)}dy, -\left(\int \frac{1}{f(x)}dx\right)+\int \frac{1}{h(z)}d$$

6.7.25.2 [1732] Problem 2

problem number 1732

Added June 27, 2019.

Problem Chapter 7.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + zw_y + g(y)w_z = h_2(x) + h_1(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x, y,z], x] + z*D[w[x, y,z], y] + g[y]*D[w[x,y,z],z]== h2[x]+h1[y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ g(y)*diff(w(x,y,z),z)= h2(x)+h1(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{y} \frac{h1(\_g) + h2\left(\text{RootOf}\left(\int \frac{1}{\sqrt{z^2+2(\int g(\_g)d\_g)-2(\int g(y)dy)}}d\_g + \int \frac{1}{f(x)}dx - \left(\int^{y} \frac{}{\sqrt{z^2+2(\int g(\_b)}}\right.\right.\right.}{\sqrt{z^2 + 2\left(\int g\left(\_g\right) d\_g\right) - 2\left(\int g\left(y\right) dy\right)}}$$

6.7.25.3 [1733] Problem 3

problem number 1733

Added June 27, 2019.

Problem Chapter 7.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = f_4(x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = f1[x]*D[w[x, y,z], x] + f2[x]*g[y]*D[w[x, y,z], y] + f3[x]*h[z]*D[w[x,y,z],z]== f4[x];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f1(x)*diff(w(x,y,z),x)+ f2(x)*g(y)*diff(w(x,y,z),y)+ f3(x)*h(z)*diff(w(x,y,z),z)= f4
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int \frac{f4(x)}{f1(x)}dx + \_F1\left(\int \frac{1}{g(y)}dy - \left(\int \frac{f2(x)}{f1(x)}dx\right), \int \frac{1}{h(z)}dz - \left(\int \frac{f3(x)}{f1(x)}dx\right)\right)$$

6.7.25.4 [1734] Problem 4

problem number 1734

Added June 27, 2019.

Problem Chapter 7.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g_1(x)z + g_2(y))w_z = h_1(x) + h_2(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x]*z+g2[y])*D[w[x,y,z],z]== h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(\text{h1}(K[5]) + \text{h2}\left(\exp\left(\int_1^{K[5]} \text{f1}(K[1])dK[1]\right)\right)\left(\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right)y - \int_1^x\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*z+g2(y))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(h1(\_f) + h2\left(\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)} + \int \mathrm{e}^{-\left(\int f1(\_f)d\_f\right)} f2(\_f)\,d\_f - \left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)} f2(x)\,dx\right.\right.\right.\right.$$

6.7.25.5 [1735] Problem 5

problem number 1735

Added June 27, 2019.

Problem Chapter 7.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(y)z^m)w_z = h_1(x) + h_2(y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[y]*z^m)*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \left(\mathrm{h1}(K[5]) + \mathrm{h2}\left(\left(\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1] - (k-1)\int_1^{K[5]} \mathrm{f1}(K[1])dK[1]\right) y^{-k}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*z^m)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(h1(\_f) + h2\left(\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right) + (-k+1\right.\right.\right.$$

6.7.25.6 [1736] Problem 6

problem number 1736

Added June 27, 2019.

Problem Chapter 7.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x)z + g_2(y)e^{\lambda z})w_z = h_1(x) + h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x]*z+g2[y]*Exp[lambda*z])
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*exp(lambda*z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.25.7 [1737] Problem 7

problem number 1737

Added June 27, 2019.

Problem Chapter 7.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x)z + g_2(y)z^k)w_z = h_1(x) + h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]*z+g2[y]*z^k)*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)*z+g2(y)*z^k)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \left(h1(\_f) + h2 \left(\frac{\lambda \left(\int f1(\_f)\, d\_f \right) + \ln \left(\frac{1}{\left(-\left(\int e^{\lambda \left(\int f1(\_f) d\_f \right)} f2(\_f) d\_f \right) + \int e^{\lambda \left(\int f1(x) dx \right)} f2(x) dx \right) \lambda + e^{(-}} \right.}{\lambda} \right.\right.$$

6.7.25.8 [1738] Problem 8

problem number 1738

Added June 27, 2019.

Problem Chapter 7.8.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x) + g_2(y)e^{\beta z})w_z = h_1(x) + h_2(y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x]+g2[y]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(y)*exp(bet
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \left(h1(\_f) + h2 \left(\frac{\lambda \left(\int f1(\_f)\, d\_f \right) + \ln \left(\frac{1}{\left(-\left(\int e^{\lambda (\int f1(\_f) d\_f)} f2(\_f) d\_f \right) + \int e^{\lambda (\int f1(x) dx)} f2(x) dx \right) \lambda + e^{(-}} \right.}{\lambda} \right.$$

6.7.26 8.3

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6.7.26.1 [1739] Problem 1

problem number 1739

Added June 27, 2019.

Problem Chapter 7.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)w_z = g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + f[x,y]*D[w[x,y,z],z]== g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \int_1^x \frac{g\left(K[2],y+\frac{b(K[2]-x)}{a}\right)}{a}dK[2]+c_1\left(y-\frac{bx}{a},z-\int_1^x \frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[1]\right)\right\}\right\}$$

Kernel message inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*diff(w(x,y,z),z)=  g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{g\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}, z - \left(\int^{x} \frac{f\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a} d\_a\right)\right)$$

6.7.26.2 [1740] Problem 2

problem number 1740

Added June 27, 2019.

Problem Chapter 7.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)g(z)w_z = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + f[x,y]*g[z]*D[w[x,y,z],z]== h[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x \frac{h\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a} d\_a + \_F1\left(\frac{ay-bx}{a}, \int \frac{a}{g(z)} dz - \left(\int^x f\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)\right.\right.$$

6.7.26.3 [1741] Problem 3

problem number 1741

Added June 27, 2019.

Problem Chapter 7.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + yw_y + (z + f(x,y))w_z = g(x,y)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] + (z+f[x,y])*D[w[x,y,z],z]== g[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \int_1^x \frac{g\left(K[2], \frac{yK[2]}{x}\right)}{K[2]} dK[2] + c_1\left(\frac{y}{x}, \frac{z}{x} - \int_1^x \frac{f\left(K[1], \frac{yK[1]}{x}\right)}{K[1]^2} dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+ (z+f(x,y))*diff(w(x,y,z),z)=  g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{g\left(\_a, \frac{-ay}{x}\right)}{\_a} d\_a + \_F1\left(\frac{y}{x}, \frac{-x\left(\int^{x} \frac{f\left(\_a, \frac{-ay}{x}\right)}{\_a^2} d\_a\right) + z}{x}\right)$$

6.7.26.4 [1742] Problem 4

problem number 1742

Added June 27, 2019.

Problem Chapter 7.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + f(x,y)g(z)w_z = h(x,y)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] + f[x,y]*g[z]*D[w[x,y,z],z]== h[x,y];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} \frac{h\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_aa} d\_a + \_F1\left(y x^{-\frac{b}{a}}, \int \frac{a}{g(z)} dz - \left(\int^{x} \frac{f\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_a} d\_a\right)\right)$$

6.7.26.5 [1743] Problem 5

problem number 1743

Added June 27, 2019.

Problem Chapter 7.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))w_y + (g_1(x,y)z + g_2(x,y))w_z = h(x,y,z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x])*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y])*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y))*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \left(y\,\mathrm{e}^{-(\int f1(x)dx)} + \int \mathrm{e}^{-(\int f1(\_h)d\_h)} f2(\_h)\,d\_h - \left(\int \mathrm{e}^{-(\int f1(x)dx)} f2(x)\,dx\right)\right)\mathrm{e}^{\int f1(}$$

6.7.26.6 [1744] Problem 6

problem number 1744

Added June 27, 2019.

Problem Chapter 7.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x,y)z + g_2(x,y)z^m)w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y]*z^m)*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
\
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y)*z^m)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^x h\left(\_h, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right) + (-k+1)\left(\int \mathrm{e}^{(k-}\right.\right.\right.$$

6.7.26.7 [1745] Problem 7

problem number 1745

Added June 27, 2019.

Problem Chapter 7.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x)y^k)w_y + (g_1(x,y) + g_2(x,y)e^{\lambda z})w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]*y+f2[x]*y^k)*D[w[x, y,z], y] + (g1[x,y]+g2[x,y]*Exp[lambda*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x,y)+g2(x,y)*exp(lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right) + (-k+1)\left(\int \mathrm{e}^{(k-}\right.\right.\right.$$

6.7.26.8 [1746] Problem 8

problem number 1746

Added June 27, 2019.

Problem Chapter 7.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x,y)z + g_2(x,y)z^k)w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x,y]*z+g2[x,y]*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x,y)*z+g2(x,y)*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \frac{\lambda\left(\int f1(\_h)\,d\_h\right) + \ln\left(\frac{1}{\left(-\left(\int e^{\lambda\left(\int f1(\_h)d\_h\right)} f2(\_h)d\_h\right) + \int e^{\lambda\left(\int f1(x)dx\right)} f2(x)dx\right)\lambda + e^{\left(-y+\int f1(x)dx\right.}}\right.}{\lambda}\right.$$

6.7.26.9 [1747] Problem 9

problem number 1747

Added June 27, 2019.

Problem Chapter 7.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x) + f_2(x)e^{\lambda y})w_y + (g_1(x,y) + g_2(x,y)e^{\beta z})w_z = h(x,y,z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (f1[x]+f2[x]*Exp[lambda*y])*D[w[x, y,z], y] + (g1[x,y]+g2[x,y]*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x,y)+g2(x,y)*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \int^{x} h\left(\_h, \frac{\lambda\left(\int f1(\_h)\,d\_h\right) + \ln\left(\frac{1}{\left(-\left(\int e^{\lambda\left(\int f1(\_h)d\_h\right)} f2(\_h)d\_h\right) + \int e^{\lambda\left(\int f1(x)dx\right)} f2(x)dx\right)\lambda + e^{\left(-y+\int f1(x)dx\right.}}\right.}{\lambda}\right.$$

6.7.26.10 [1748] Problem 10

problem number 1748

Added June 27, 2019.

Problem Chapter 7.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x, y) + h_2(x, y)z^m)w_z = h_3(x, y, z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] + (h1[x,y]+h2[x,y]*z^m)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+ (h1(x,y)+h2(x,y)*z^m)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.7.26.11 [1749] Problem 11

problem number 1749

Added June 27, 2019.

Problem Chapter 7.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x, y) + h_2(x, y)e^{\lambda z})w_z = h_3(x, y, z)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x, y,z], x] + f2[x]*g2[y]*D[w[x, y,z], y] + (h1[x,y]+h2[x,y]*Exp[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f1(x)*g1(y)*diff(w(x,y,z),x)+ f2(x)*g2(y)*diff(w(x,y,z),y)+ (h1(x,y)+h2(x,y)*exp(lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8 chapter 8

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6.8.1.1 [1750] Problem 1

problem number 1750

Added June 27, 2019.

Problem Chapter 8.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] + c*D[w[x,y,z],z]== (alpha*x+beta*y+gamma*z+del
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right) \exp\left(\frac{x(a(\alpha x + 2\beta y + 2\delta + 2\gamma z) - x(b\beta + c\gamma))}{2a^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=  (alpha*x+beta*y+gamma*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \frac{za - cx}{a}\right) \mathrm{e}^{\frac{\left(\left(\frac{\alpha x}{2} + \beta y + \gamma z + \delta\right)a - \frac{(b\beta + \gamma c)x}{2}\right)x}{a^2}}$$

6.8.1.2 [1751] Problem 2

problem number 1751

Added June 27, 2019.

Problem Chapter 8.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + azw_y + byw_z = (cx + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== (c*x+s)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{cx^2}{2}+sx} c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}}, \frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}\right.\right.}{2}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=  (c*x+s)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{az^2-by^2}{a}, -\frac{-\sqrt{ab}\,x+\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right) e^{\int^y \frac{\left(\ln\left(\frac{-aab+\sqrt{\left(az^2+\left(-a^2-y^2\right)b\right)a}\sqrt{ab}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2}}{\sqrt{ab}}\right.\right.}{\sqrt{\left(az^2+\left(-a^2-y^2\right)b\right)a}\sqrt{ab}}}$$

6.8.1.3 [1752] Problem 3

problem number 1752

Added June 27, 2019.

Problem Chapter 8.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x + a_0) w_y + (b_1 x + b_0) w_z = (\alpha x + \beta y + \gamma z + \delta) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x+a0)*D[w[x, y,z], y] + (b1*x+b0)*D[w[x,y,z],z]== (alpha*x+beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\mathrm{a0}x - \frac{\mathrm{a1}x^2}{2} + y, -\mathrm{b0}x - \frac{\mathrm{b1}x^2}{2} + z\right) \exp\left(\frac{1}{6}x\left(-3\mathrm{a0}\beta x - 2\mathrm{a1}\beta x^2 + 3\alpha x - 3\mathrm{b0}\gamma x -\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a1*x+a0)*diff(w(x,y,z),y)+ (b1*x+b0)*diff(w(x,y,z),z)=  (alpha*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{2}a1x^2 - a0x + y, -\frac{1}{2}b1x^2 - b0x + z\right) \mathrm{e}^{-\frac{\left(-\frac{3\alpha x}{2} + \left(a1x^2 + \frac{3}{2}a0x - 3y\right)\beta - 3\delta + \left(b1x^2 + \frac{3}{2}b0x - 3z\right)\gamma\right)x}{3}}$$

6.8.1.4 [1753] Problem 4

problem number 1753

Added June 27, 2019.

Problem Chapter 8.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_2 y + a_1 x + a_0) w_y + (b_2 y + b_1 x + b_0) w_z = (c_2 y + c_1 z + c_0 x + s) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a2*y+a1*x+a0)*D[w[x, y,z], y] + (b2*y+b1*x+b0)*D[w[x,y,z],z]== (c2
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{e^{-\mathrm{a2}x}(\mathrm{a2}(\mathrm{a0}+\mathrm{a2}y)+\mathrm{a1a2}x+\mathrm{a1})}{\mathrm{a2}^2}, \frac{e^{-\mathrm{a2}x}(\mathrm{a2}(2\mathrm{a0b2}(\mathrm{a2}xe^{\mathrm{a2}x}+1)-\mathrm{a2}(\mathrm{a2}e^{\mathrm{a2}x}(2\mathrm{b0}x+\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a2*y+a1*x+a0)*diff(w(x,y,z),y)+ (b2*y+b1*x+b0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a2^2 y + a1 + (a1x + a0)\,a2)\,\mathrm{e}^{-a2x}}{a2^2}, \frac{-2a0\,a2\,b2 + (-b1x^2 - 2b0x + 2z)\,a2^3 + (a1x^2}{2a2^3}\right.$$

6.8.1.5 [1754] Problem 5

problem number 1754

Added June 27, 2019.

Problem Chapter 8.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ay + k_1 x + k_0) w_y + (bz + s_1 x + s_0) w_z = (c_1 x + c_0) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x+k0)*D[w[x, y,z], y] + (b*z+s1*x+s0)*D[w[x,y,z],z]== (c1*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\mathrm{c0}x+\frac{\mathrm{c1}x^2}{2}} c_1\left(\frac{e^{-ax}(a^2 y + a(\mathrm{k0}+\mathrm{k1}x)+\mathrm{k1})}{a^2}, \frac{e^{-bx}(b^2 z + b(\mathrm{s0}+\mathrm{s1}x)+\mathrm{s1})}{b^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a*y+k1*x+k0)*diff(w(x,y,z),y)+(b*z+s1*x+s0)*diff(w(x,y,z),z)=  (c1
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a^2 y + (k1\,x + k0)\,a + k1)\,\mathrm{e}^{-ax}}{a^2}, \frac{(b^2 z + (s1\,x + s0)\,b + s1)\,\mathrm{e}^{-bx}}{b^2}\right)\mathrm{e}^{\frac{(c1\,x+2\,c0)\,x}{2}}$$

6.8.1.6 [1755] Problem 6

problem number 1755

Added June 27, 2019.

Problem Chapter 8.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$axw_x + byw_y + czw_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] + c*z*D[w[x,y,z],z]== (alpha*x+beta*y+gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to x^{\frac{\beta}{a}} c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\frac{\alpha x}{a}+\frac{\beta y}{b}+\frac{\gamma z}{c}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (alpha*x+beta*y+ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = x^{\frac{\beta}{a}} \_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) \mathrm{e}^{\frac{ab\gamma z+(a\beta y+\alpha bx)c}{abc}}$$

6.8.1.7 [1756] Problem 7

problem number 1756

Added June 27, 2019.

Problem Chapter 8.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + azw_y + byw_z = cw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== c*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to x^c c_1 \left(iy \sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{\sqrt{a}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*y*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{az^2-2by}{a}, x\,\mathrm{e}^{\frac{2\operatorname{arctanh}\left(\frac{z}{\sqrt{\frac{az^2-2by}{a}}}\right)}{\sqrt{\frac{az^2-2by}{a}}\,a}}\right)\mathrm{e}^{-\frac{2c\operatorname{arctanh}\left(\frac{\sqrt{a^2z^2}}{\sqrt{\frac{az^2-2by}{a}}\,a}\right)}{\sqrt{\frac{az^2-2by}{a}}\,a}}$$

6.8.1.8 [1757] Problem 8

problem number 1757

Added June 27, 2019.

Problem Chapter 8.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + b(ay+bz)w_y + a(ay-bz)w_z = cw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] + a*(a*y-b*z)*D[w[x,y,z],z]== c*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a*b*x*diff(w(x,y,z),x)+ b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\frac{\sqrt{2}\, a^2 y}{(-a^2y^2 + 2zaby + b^2z^2)\sqrt{\frac{a^2}{-a^2y^2+2zaby+b^2z^2}}} + \frac{ay}{\sqrt{-a^2y^2 + 2zaby + b^2z^2}} + \frac{bz}{\sqrt{-a^2y^2 + 2zaby}} \right.$$

6.8.1.9 [1758] Problem 9

problem number 1758

Added June 27, 2019.

Problem Chapter 8.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1x + a_0)w_x + (b_1y + b_0)w_y + (c_1z + c_0)w_z = (\alpha x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1*x+a0)*D[w[x, y,z], x] + (b1*y+b0)*D[w[x, y,z], y] +(c1*z+c0)*D[w[x,y,z],z]== (alp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{a0}\alpha\mathrm{b1c1}+\mathrm{a1b0}\beta\mathrm{c1}+\mathrm{a1b1c0}\gamma-\mathrm{a1b1c1}\delta}{\mathrm{a1}^2\mathrm{b1c1}}}c_1\left(\frac{(\mathrm{b0}+\mathrm{b1}y)(\mathrm{a0}+\mathrm{a1}x)^{-\frac{\mathrm{b1}}{\mathrm{a1}}}}{\mathrm{b1}},\frac{(\mathrm{c0}+\mathrm{c1}z)(\mathrm{a0}+\mathrm{a1}x)}{\mathrm{c1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   (a1*x+a0)*diff(w(x,y,z),x)+(b1*y+b0)*diff(w(x,y,z),y)+(c1*z+c0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=(a1x+a0)^{-\frac{a0\alpha}{a1^2}-\frac{b0\beta}{a1b1}-\frac{c0\gamma}{a1c1}+\frac{\delta}{a1}}\_F1\left(\frac{(b1y+b0)(a1x+a0)^{-\frac{b1}{a1}}}{b1},\frac{(c1z+c0)(a1x+a0)^{-\frac{c1}{a1}}}{c1}\right)$$

6.8.2 2.2

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6.8.2.1 [1759] Problem 1

problem number 1759

Added June 28, 2019.

Problem Chapter 8.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\lambda x^2 + \beta y^2 + \gamma z^2 + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (alpha*x^2+beta*y^2+gamma*z^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{bx}{a}, z - \frac{cx}{a}\right) \exp\left(\frac{1}{3}\left(\frac{\alpha x^3 + 3\delta x}{a} + \frac{\beta y^3}{b} + \frac{\gamma z^3}{c}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (alpha*x^2+beta*y^2+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay - bx}{a}, \frac{za - cx}{a}\right) \mathrm{e}^{\frac{\left(\left(\frac{\alpha x^2}{3} + \beta y^2 + \gamma z^2 + \delta\right) a^2 - (b\beta y + c\gamma z) a x + \frac{\left(b^2\beta + \gamma c^2\right) x^2}{3}\right) x}{a^3}}$$

6.8.2.2 [1760] Problem 2

problem number 1760

Added June 28, 2019.

Problem Chapter 8.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^2 + a_0)w_y + (b_1x^2 + b_0)w_z = (\lambda x + \beta y + \gamma z + \delta)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^2+a0)*D[w[x, y,z], y] +(b1*x^2+b0)*D[w[x,y,z],z]== (alpha*x+b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\mathrm{a0}x - \frac{\mathrm{a1}x^3}{3} + y, -\mathrm{b0}x - \frac{\mathrm{b1}x^3}{3} + z\right)\exp\left(-\frac{1}{4}x\left(2\mathrm{a0}\beta x + \mathrm{a1}\beta x^3 - 2\alpha x + 2\mathrm{b0}\gamma x +\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a1*x^2+a0)*diff(w(x,y,z),y)+(b1*x^2+b0)*diff(w(x,y,z),z)=  (alpha
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{3}a1x^3 - a0x + y, -\frac{1}{3}b1x^3 - b0x + z\right)\mathrm{e}^{-\frac{\left(-2\alpha x + \left(a1x^3 + 2a0x - 4y\right)\beta - 4\delta + \left(b1x^3 + 2b0x - 4z\right)\gamma\right)x}{4}}$$

6.8.2.3 [1761] Problem 3

problem number 1761

Added June 28, 2019.

Problem Chapter 8.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + s_1x^2 + s_0)w_z = (c_1x^2 + c_0)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*y+k1*x^2+k0)*D[w[x, y,z], y] +(b*z+s1*x^2+s0)*D[w[x,y,z],z]== (c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\mathrm{c0}x+\frac{\mathrm{c1}x^3}{3}} c_1\left(\frac{e^{-ax}(a^2(\mathrm{k0}+\mathrm{k1}x^2)+a^3y+2a\mathrm{k1}x+2\mathrm{k1})}{a^3}, \frac{e^{-bx}(b^2(\mathrm{s0}+\mathrm{s1}x^2)+b^3z+2b\mathrm{s1}x\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a*y+k1*x^2+k0)*diff(w(x,y,z),y)+(b*z+s1*x^2+s0)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(a^3y+2ak1x+(k1x^2+k0)a^2+2k1)\,\mathrm{e}^{-ax}}{a^3}, \frac{(b^3z+2bs1x+(s1x^2+s0)b^2+2s1)\,\mathrm{e}^{-b}}{b^3}\right.$$

6.8.2.4 [1762] Problem 4

problem number 1762

Added June 28, 2019.

Problem Chapter 8.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_2xy + a_1x^2 + a_0)w_y + (b_2xy + b_1x^2 + b_0)w_z = (c_2y + c_1z + c_0x + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x*y+a1*x^2+a0)*D[w[x, y,z], y] +(b2*x*y+b1*x^2+b0)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\text{a0b2}x}{\text{a1}}-\frac{\text{b2}y}{\text{a1}}-\text{b0}x-\frac{\text{b1}x^3}{3}+\frac{\text{b2}x^3}{3}+z,e^{-\frac{\text{a1}x^2}{2}}(x+y)-\frac{\sqrt{\frac{\pi}{2}}(\text{a0}+1)\text{erf}\left(\frac{\sqrt{\text{a1}}x}{\sqrt{2}}\right)}{\sqrt{\text{a1}}}\right)e\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+(a1*x*y+a1*x^2+a0)*diff(w(x,y,z),y)+(b2*x*y+b1*x^2+b0)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{(x+y)\sqrt{a1}\,e^{-\frac{a1x^2}{2}}-\frac{\sqrt{2}\sqrt{\pi}\,(a0+1)\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}}{\sqrt{a1}},-\frac{3\left(-\frac{\sqrt{2}\,(a0+1)\left(\frac{\sqrt{\pi}\sqrt{\frac{a1}{\pi}}}{\sqrt{a1}}-1\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a1}\,x}{2}\right)}{2}+\right.}{}\right.$$

6.8.2.5 [1763] Problem 5

problem number 1763

Added June 28, 2019.

Problem Chapter 8.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$axw_x + byw_y + czw_z = x(\lambda x + \beta y + \gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== x*(lambda*x+beta*y+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\frac{\beta xy}{a+b}+\frac{\text{gamaxz}}{a+c}+\frac{\lambda x^2}{2a}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  x*(lambda*x+beta*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) \mathrm{e}^{\frac{\left(bc\lambda x+(2\beta y+2gamaz+\lambda x)a^2+(2bgamaz+2\beta cy+(b+c)\lambda x)a\right)x}{2(a+b)(a+c)a}}$$

6.8.2.6 [1764] Problem 6

problem number 1764

Added June 28, 2019.

Problem Chapter 8.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$ax^2w_x + bxyw_y + cxzw_z = (\lambda x + \beta y + \gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]== (lambda*x+beta*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to x^{\frac{\lambda}{a}} e^{-\frac{\frac{\beta y}{a-b}+\frac{\text{gamaz}}{a-c}}{x}} c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)=  (lambda*x+be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = x^{\frac{\lambda}{a}} \_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) \mathrm{e}^{\frac{-(a-c)\beta y-(a-b)gamaz}{(a-b)(a-c)x}}$$

6.8.2.7 [1765] Problem 7

problem number 1765

Added June 28, 2019.

Problem Chapter 8.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x + bxyw_y + cz^2w_z = ky^2w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*x*y*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*y^2*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{ky^2}{ax-2bx}}c_1\left(yx^{-\frac{b}{a}},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*x*y*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*y^2*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(yx^{-\frac{b}{a}},\frac{ax-cz}{axz}\right)\mathrm{e}^{-\frac{ky^2}{(a-2b)x}}$$

6.8.2.8 [1766] Problem 8

problem number 1766

Added June 28, 2019.

Problem Chapter 8.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2w_x+by^2w_y+cz^2w_z=kxyw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*x*y*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\left(\frac{ax}{y}\right)^{\frac{kxy}{ax-by}}c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*x*y*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\frac{ax}{y}\right)^{\frac{kxy}{ax-by}} \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)$$

6.8.2.9 [1767] Problem 9

problem number 1767

Added June 28, 2019.

Problem Chapter 8.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = (\lambda x^2 + \beta y^2 + \gamma z^2) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== (lambda*x^2+beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{b}{ax} - \frac{1}{y}, \frac{c}{ax} - \frac{1}{z}\right) \exp\left(\frac{\beta y^2}{by - ax} + \frac{\gamma z^2}{cz - ax} + \frac{\lambda x}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  (lambda*x^2+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right) \mathrm{e}^{\frac{bc\lambda xyz+\left(-\beta y^2-\gamma z^2+\lambda x^2\right)a^2x+\left(\beta cy^2z-c\lambda x^2z-\left(-\gamma z^2+\lambda x^2\right)by\right)a}{(ax-cz)(ax-by)a}}$$

6.8.3 2.3

Local contents

6.8.3.1 [1768] Problem 1 2437
6.8.3.2 [1769] Problem 2 2438
6.8.3.3 [1770] Problem 3 2439
6.8.3.4 [1771] Problem 4 2440
6.8.3.5 [1772] Problem 5 2441

6.8.3.1 [1768] Problem 1

problem number 1768

Added July 1, 2019.

Problem Chapter 8.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = xyzw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +b*D[w[x,y,z],z]== x*y*z*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{1}{12}x^2(ax(bx-2z)-2bxy+6yz)\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+b*diff(w(x,y,z),z)=  x*y*z*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\frac{\left(abx^2+6yz+(-2za-2by)x\right)x^2}{12}}$$

6.8.3.2 [1769] Problem 2

problem number 1769

Added July 1, 2019.

Problem Chapter 8.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = (kx^3 + sy^2)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (k*x^3+s*y^2)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{x(3a^2(kx^3+4sy^2)-12absxy+4b^2sx^2)}{12a^3}\right)c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (k*x^3+s*y^2)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{za-cx}{a}\right) \mathrm{e}^{\frac{\left(a^2kx^3+4a^2sy^2-4absxy+\frac{4}{3}b^2sx^2\right)x}{4a^3}}$$

6.8.3.3 [1770] Problem 3

problem number 1770

Added July 1, 2019.

Problem Chapter 8.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = (kx + s\sqrt{x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (k*x+s*Sqrt[x])*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{3kx^2+4sx^{3/2}}{6a}} c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (k*x+s*sqrt(x))*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right) \mathrm{e}^{\frac{3kx^2+4sx^{\frac{3}{2}}}{6a}}$$

6.8.3.4 [1771] Problem 4

problem number 1771

Added July 1, 2019.

Problem Chapter 8.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + azw_y + byw_z = (c\sqrt{x} + s)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + b*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== (c*Sqrt[x]+s)*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{2}{3}cx^{3/2}+sx}c_1\left(\frac{1}{2}e^{-bx}\left(y\left(-e^{2bx}\right)+ze^{2bx}+y+z\right),\frac{1}{2}e^{-bx}\left(ye^{2bx}-ze^{2bx}+y+z\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+b*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  (c*sqrt(x)+s)*w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y^2+z^2, \frac{bx-\ln(y+z)}{b}\right) e^{\int^y \frac{\sqrt{\frac{bx+\ln\left(\_a+\sqrt{\_a^2-y^2+z^2}\right)-\ln(y+z)}{b}}c+s}{\sqrt{\_a^2-y^2+z^2}\,b} d\_a}$$

6.8.3.5 [1772] Problem 5

problem number 1772

Added July 1, 2019.

Problem Chapter 8.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = kxyzw$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x, y,z], x] + b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]== k*x*y*z*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\exp\left(\frac{kxyz\left(by(ax-cz)\log\left(\frac{ax}{y}\right)+cz(by-ax)\log\left(\frac{ax}{z}\right)\right)}{(ax-by)(ax-cz)(by-cz)}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x^2*diff(w(x,y,z),x)+b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)=  k*x*y*z*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\frac{ax}{y}\right)^{\frac{bkxy^2z}{(by-cz)(ax-by)}}\left(\frac{ax}{z}\right)^{-\frac{ckxyz^2}{(by-cz)(ax-cz)}}\_F1\left(\frac{ax-by}{axy},\frac{ax-cz}{axz}\right)$$

6.8.4 2.4

Local contents

6.8.4.1 [1773] Problem 1 2442
6.8.4.2 [1774] Problem 2 2443
6.8.4.3 [1775] Problem 3 2444
6.8.4.4 [1776] Problem 4 2445
6.8.4.5 [1777] Problem 5 2446
6.8.4.6 [1778] Problem 6 2447
6.8.4.7 [1779] Problem 7 2448
6.8.4.8 [1780] Problem 8 2449
6.8.4.9 [1781] Problem 9 2450
6.8.4.10 [1782] Problem 10 2451
6.8.4.11 [1783] Problem 11 2452
6.8.4.12 [1784] Problem 12 2453
6.8.4.13 [1785] Problem 13 2453

6.8.4.1 [1773] Problem 1

problem number 1773

Added July 1, 2019.

Problem Chapter 8.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+gamma*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\exp\left(\frac{\lambda x^{n+1}}{an+a}+\frac{\beta y^{m+1}}{bm+b}+\frac{\gamma z^{k+1}}{ck+c}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+c*diff(w(x,y,z),z)=  (lambda*x^n+beta*y^m+gam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{za-cx}{a}\right) \mathrm{e}^{\frac{(n+1)(k+1)a\beta c y^{m+1}+\left((n+1)a\gamma z^{k+1}+(k+1)c\lambda x^{n+1}\right)(m+1)b}{(m+1)(k+1)(n+1)abc}}$$

6.8.4.2 [1774] Problem 2

problem number 1774

Added July 1, 2019.

Problem Chapter 8.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + byw_y + czw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(ye^{-\frac{bx}{a}}, ze^{-\frac{cx}{a}}\right) \exp\left(\frac{\lambda x^{n+1}}{an+a} + \frac{\beta y^m}{bm} + \frac{\gamma z^k}{ck}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (lambda*x^n+beta*y^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\frac{bx}{a}}, z\,\mathrm{e}^{-\frac{cx}{a}}\right)\mathrm{e}^{\frac{bck\lambda m x^{n+1}+(n+1)\left(b\gamma m z^{k}+\beta c k y^{m}\right)a}{(n+1)abckm}}$$

6.8.4.3 [1775] Problem 3

problem number 1775

Added July 1, 2019.

Problem Chapter 8.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + azw_y + byw_z = cx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{n+1}}{n+1}}c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+\right.\right.}{2}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*x^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{az^2-by^2}{a}, -\frac{-\sqrt{ab}\,x+\ln\left(\frac{aby+\sqrt{a^2z^2}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right) \mathrm{e}^{\int^y \frac{c\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_aab+\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}\,\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}} - \ln\left(\frac{a}{}\right.\right.}{\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}}}$$

6.8.4.4 [1776] Problem 4

problem number 1776

Added July 1, 2019.

Problem Chapter 8.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = (\lambda x^n + \beta y^m + \gamma z^k)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]== (lambda*x^n+beta*y^m+g
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\frac{\lambda x^n}{an}+\frac{\beta y^m}{bm}+\frac{\gamma z^k}{ck}}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   a*x*diff(w(x,y,z),x)+b*y*diff(w(x,y,z),y)+c*z*diff(w(x,y,z),z)=  (lambda*x^n+beta*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\int^x \frac{\beta\left(y\_a\frac{b}{a}x^{-\frac{b}{a}}\right)^m + \gamma\left(z\_a\frac{c}{a}x^{-\frac{c}{a}}\right)^k + \lambda\_a^n}{\_aa} d\_a}$$

6.8.4.5 [1777] Problem 5

problem number 1777

Added July 1, 2019.

Problem Chapter 8.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = cx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]== c*x^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{cx^n}{n}} c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}}, y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right) - \right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   x*diff(w(x,y,z),x)+a*z*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)=  c*x^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{az^2-by^2}{a}, x\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right) e^{\int^y \frac{c\left(x\left(\frac{\_aab+\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}\sqrt{ab}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^n}{\sqrt{\left(az^2+\left(\_a^2-y^2\right)b\right)a}} d\_a}$$

6.8.4.6 [1778] Problem 6

problem number 1778

Added July 1, 2019.

Problem Chapter 8.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$abxw_x + b(ay+bz)w_y + a(ay-bz)w_z = cx^n w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]== c*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a*b*x*diff(w(x,y,z),x)+b*(a*y+b*z)*diff(w(x,y,z),y)+a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{1}{\sqrt{-a^2y^2+2zaby+b^2z^2}}, x\left(\frac{\frac{\sqrt{2}\,a^2y}{-a^2y^2+2zaby+b^2z^2}+\left(\frac{ay}{\sqrt{-a^2y^2+2zaby+b^2z^2}}+\frac{bz}{\sqrt{-a^2y^2+2zaby+b^2}}\right.}{\sqrt{\frac{a^2}{-a^2y^2+2zaby+b^2z^2}}}\right.\right.$$

6.8.4.7 [1779] Problem 7

problem number 1779

Added July 1, 2019.

Problem Chapter 8.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n y^m w_y + bx^\nu y^\mu z^\lambda w_z = cx^k w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*y^m*D[w[x, y,z], y] +b*x^nu*y^mu*z^lambda*D[w[x,y,z],z]== c*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*y^m*diff(w(x,y,z),y)+b*x^nu*y^mu*z^lambda*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(m-1)\,a x^{n+1} + (n+1)\,y^{-m+1}}{n+1}, (\lambda-1)\,b\left(\int^{x} \_a^{\nu}\left(\left(\frac{(-\_a^{n+1} + x^{n+1})\,(m-1)\,a + (}{n+1}\right.\right.\right.$$

6.8.4.8 [1780] Problem 8

problem number 1780

Added July 1, 2019.

Problem Chapter 8.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{n_1} y + b_1 x^{m_1}) w_y + (a_2 x^{n_2} y + b_2 x^{m_2}) w_z = (c_1 x^{k_2} y + c_1 x^{k_1} z) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*y+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\int_1^x \frac{(-1)^{-\frac{n2+1}{n1+1}a1-\frac{m1+n2+2}{n1+1}c1 e^{-\frac{a1 x^{n1+1}}{n1+1}}\left((-1)^{\frac{n2+1}{n1+1}a1\frac{m1+n2+2}{n1+1}e^{\frac{a1 x^{n1+1}}{n1+1}}zK[2]^{k1}+(-1)^{\frac{n2+1}{n1+1}a1\frac{m1+n2+2}{n1+1}e^{\frac{a1 x^{n1+1}}{n1+1}}n1^2 zK}\right.}{}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*y+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.8.4.9 [1781] Problem 9

problem number 1781

Added July 1, 2019.

Problem Chapter 8.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^{n_1} y + b_1 x^{m_1}) w_y + (a_2 x^{n_2} z + b_2 x^{m_2}) w_z = (c_1 x^{k_2} y + c_1 x^{k_1} z) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*x^m1)*D[w[x, y,z], y] +(a2*x^n2*z+b2*x^m2)*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\mathrm{b1}(\mathrm{n1}+1)^{\frac{\mathrm{m1}-\mathrm{n1}}{\mathrm{n1}+1}}\mathrm{a1}^{-\frac{\mathrm{m1}+1}{\mathrm{n1}+1}}\,\mathrm{Gamma}\left(\frac{\mathrm{m1}+1}{\mathrm{n1}+1},\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right)+ye^{-\frac{\mathrm{a1}x^{\mathrm{n1}+1}}{\mathrm{n1}+1}},\mathrm{b2}(\mathrm{n2}+1)^{\frac{\mathrm{m2}-\mathrm{n2}}{\mathrm{n2}+1}}\mathrm{a}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*x^m1)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*x^m2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.8.4.10 [1782] Problem 10

problem number 1782

Added July 1, 2019.

Problem Chapter 8.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^{n_1} y + b_1 y^k) w_y + (a_2 x^{n_2} z + b_2 z^m) w_z = c x^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*y^k)*D[w[x, y,z], y] +(a2*x^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{cx^{s+1}}{s+1}} c_1 \left(\mathrm{b1}(-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}} (\mathrm{n1}+1)^{-\frac{\mathrm{n1}}{\mathrm{n1}+1}} \mathrm{a1}^{-\frac{1}{\mathrm{n1}+1}} (k-1)^{\frac{\mathrm{n1}}{\mathrm{n1}+1}} \operatorname{Gamma}\left(\frac{1}{\mathrm{n1}+1}, -\frac{\mathrm{a1}(k-1)x^{\mathrm{n1}+1}}{\mathrm{n1}+1}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*y^k)*diff(w(x,y,z),y)+(a2*x^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-(n1+1)(n1+2)^2 b1 x^{-n1}\left(-\frac{(k-1)a1x^{n1+1}}{n1+1}\right)^{\frac{-n1-2}{2n1+2}} \text{WhittakerM}\left(\frac{n1+2}{2n1+2}, \frac{2n1+3}{2n1+2}, -\frac{(k-1)a}{n1}\right.}{}\right.$$

6.8.4.11 [1783] Problem 11

problem number 1783

Added July 1, 2019.

Problem Chapter 8.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{n_1} y + b_1 y^k) w_y + (a_2 y^{n_2} z + b_2 z^m) w_z = (c_1 x^{s_1} + c_2 y^{s_2} + c_3 z^{s_3}) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n1*y+b1*y^k)*D[w[x, y,z], y] +(a2*y^n2*z+b2*z^m)*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n1*y+b1*y^k)*diff(w(x,y,z),y)+(a2*y^n2*z+b2*z^m)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.8.4.12 [1784] Problem 12

problem number 1784

Added July 1, 2019.

Problem Chapter 8.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + a\sqrt{x^2 + y^2}w_z = bx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]== b*x^n*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x, y, z) \to e^{\frac{bx^n}{n}} c_1\left(\frac{y}{x}, z - a\sqrt{x^2 + y^2}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+a*sqrt(x^2+y^2)*diff(w(x,y,z),z)= b*x^n*w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{y}{x}, -\sqrt{x^2 + y^2}\, a + z\right) \mathrm{e}^{\frac{bx^n}{n}}$$

6.8.4.13 [1785] Problem 13

problem number 1785

Added July 1, 2019.

Problem Chapter 8.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z - a\sqrt{x^2 + y^2 + z^2}w_z = bx^n w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x, y,z], x] + y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]== b*x^n*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{bx^n}{n}}c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}+x^{2a+2}}{x^2+y^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y)+(z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)= b*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)\mathrm{e}^{\frac{bx^n}{n}}$$

6.8.5 3.1

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6.8.5.1 [1786] Problem 1

problem number 1786

Added July 1, 2019.

Problem Chapter 8.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x} w_y + be^{\beta x} w_z = ce^{\gamma x} w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== c*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{ce^{\gamma x}}{\gamma}} c_1\left(y - \frac{ae^{\lambda x}}{\lambda}, z - \frac{be^{\beta x}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)= c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-a\,\mathrm{e}^{\lambda x} + \lambda y}{\lambda}, \frac{-b\,\mathrm{e}^{\beta x} + \beta z}{\beta}\right) \mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

6.8.5.2 [1787] Problem 2

problem number 1787

Added July 1, 2019.

Problem Chapter 8.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\lambda x} w_y + be^{\beta x} w_z = (ce^{\gamma y} + se^{\mu z}) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] +b*Exp[beta*x]*D[w[x,y,z],z]== (c*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},z-\frac{be^{\beta x}}{\beta}\right)\exp\left(\frac{c\operatorname{Ei}\left(\frac{ae^{\lambda x}\gamma}{\lambda}\right)e^{\gamma\left(y-\frac{ae^{\lambda x}}{\lambda}\right)}}{\lambda}+\frac{s\operatorname{Ei}\left(\frac{be^{\beta x}\mu}{\beta}\right)e^{\mu\left(z-\frac{be^{\beta x}}{\beta}\right)}}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*exp(beta*x)*diff(w(x,y,z),z)= (c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{-b\,\mathrm{e}^{\beta x}+\beta z}{\beta}\right)\mathrm{e}^{\frac{-\beta c\,\mathrm{expIntegral}\left(1,-\frac{a\gamma\,\mathrm{e}^{\lambda x}}{\lambda}\right)\mathrm{e}^{\frac{\left(-a\,\mathrm{e}^{\lambda x}+\lambda y\right)\gamma}{\lambda}}-\lambda s\,\mathrm{expIntegral}\left(1,-\frac{b\mu\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{\frac{\left(-b\,\mathrm{e}^{\beta x}\right.}{\beta}}}{\beta\lambda}}$$

6.8.5.3 [1788] Problem 3

problem number 1788

Added July 2, 2019.

Problem Chapter 8.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\lambda y}w_y+be^{\beta y}w_z=(ce^{\gamma x}+se^{\mu z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*y]*D[w[x, y,z], y] +b*Exp[beta*y]*D[w[x,y,z],z]== (c*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{a\lambda x+e^{-\lambda y}}{\lambda},\frac{b\left(e^{-\lambda y}\right)^{1-\frac{\beta}{\lambda}}}{a(\lambda-\beta)}+z\right)\exp\left(\int_1^x\left(e^{\gamma K[1]}c+\exp\left(-\frac{\mu\left(b\lambda(x-K[1])\left(a\lambda(\right.\right.}{}\right.\right.\right.\right.$$

Generates Solve::incnst: Inconsistent or redundant transcendental equation

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*y)*diff(w(x,y,z),y)+b*exp(beta*y)*diff(w(x,y,z),z)= (c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\lambda x-\mathrm{e}^{-\lambda y}}{a\lambda},\frac{-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y}+(\beta-\lambda)\,az}{(\beta-\lambda)\,a}\right)\mathrm{e}^{\int^x\left(c\,\mathrm{e}^{-a\gamma}+s\,\mathrm{e}^{\frac{\left(-b\left(\mathrm{e}^{\lambda y}\right)^{\frac{\beta}{\lambda}}\mathrm{e}^{-\lambda y}+(\beta-\lambda)az+\left((-\_a+x)a\lambda\right.\right.}{(\beta-\lambda)}}\right.}$$

6.8.5.4 [1789] Problem 4

problem number 1789

Added July 2, 2019.

Problem Chapter 8.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(A_1e^{\alpha_1x}+B_1e^{\nu_1x+\lambda y})w_y+(A_2e^{\alpha_2x}+B_2e^{\nu_2x+\beta z})w_z=ke^{\gamma z}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (A1*Exp[alpha1*x] + B1*Exp[nu1*x+lambda*y])*D[w[x, y,z], y] + (A2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(A1*exp(alpha1*x) + B1*exp(nu1*x+lambda*y))*diff(w(x,y,z),y)+(A2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-B1\lambda\left(\int e^{\frac{A1\lambda e^{\alpha 1x}}{\alpha 1}+\nu 1x}dx\right) - e^{\frac{\left(A1\, e^{\alpha 1x}-\alpha 1y\right)\lambda}{\alpha 1}}}{\lambda}, \frac{-B2\beta\left(\int e^{\frac{A2\beta e^{\alpha 2x}}{\alpha 2}+\nu 2x}dx\right) - e^{\frac{\left(A2\, e^{\alpha 2x}-\alpha 2z\right)\beta}{\alpha 2}}}{\beta}\right.$$

6.8.5.5 [1790] Problem 5

problem number 1790

Added July 2, 2019.

Problem Chapter 8.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y + ce^{\gamma z}w_z = ke^{\lambda x}w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Exp[alpha*x]*D[w[x, y,z], x] + b*Exp[beta*y]*D[w[x, y,z], y] + c*Exp[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{ke^{x(\lambda-\alpha)}}{a(\lambda-\alpha)}}c_1\left(\frac{be^{-\alpha x}}{a\alpha} - \frac{e^{-\beta y}}{\beta}, \frac{ce^{-\alpha x}}{a\alpha} - \frac{e^{-\gamma z}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*exp(alpha*x)*diff(w(x,y,z),x)+b*exp(beta*y)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left(a\alpha\,\mathrm{e}^{\alpha x} - b\beta\,\mathrm{e}^{\beta y}\right)\mathrm{e}^{-\alpha x-\beta y}}{\alpha b\beta}, -\frac{\left(a\alpha\,\mathrm{e}^{\alpha x} - c\gamma\,\mathrm{e}^{\gamma z}\right)\mathrm{e}^{-\alpha x-\gamma z}}{\alpha c\gamma}\right)\mathrm{e}^{-\frac{k\,\mathrm{e}^{-(\alpha-\lambda)x}}{(\alpha-\lambda)a}}$$

6.8.5.6 [1791] Problem 6

problem number 1791

Added July 2, 2019.

Problem Chapter 8.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ae^{\beta y}w_x + be^{\alpha x}w_y + ce^{\gamma z}w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Exp[beta*y]*D[w[x, y,z], x] + b*Exp[alpha*x]*D[w[x, y,z], y] + c*Exp[gamma*z]*D[w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*exp(beta*y)*diff(w(x,y,z),x)+b*exp(alpha*x)*diff(w(x,y,z),y)+c*exp(gamma*z)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha x}}{\alpha b\beta}, -\frac{\left(a\alpha\,\mathrm{e}^{\beta y}\mathrm{e}^{-\gamma z} + \alpha c\gamma x - b\beta\,\mathrm{e}^{\alpha x}\mathrm{e}^{-\gamma z} - c\gamma\ln\left(\frac{a\alpha\,\mathrm{e}^{\beta y}}{b\beta}\right)\right)b\beta}{\left(a\alpha\,\mathrm{e}^{\beta y} - b\beta\,\mathrm{e}^{\alpha x}\right)\alpha c\gamma}\right)\mathrm{e}^{\int^{x}\frac{\alpha b}{a\alpha\,\mathrm{e}^{\beta y}-(-}}$$

6.8.5.7 [1792] Problem 7

problem number 1792

Added July 2, 2019.

Problem Chapter 8.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$(a_1 + a_2 e^{\alpha x})w_x + (b_1 + b_2 e^{\beta y})w_y + (c_1 + c_2 e^{\gamma z})w_z = (k_1 + k_2 e^{\alpha x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  (a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + (b1+b2*Exp[beta*y])*D[w[x, y,z], y] + (c1+c2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{\mathrm{k1}x}{\mathrm{a1}}}\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)^{\frac{\mathrm{a1k2}-\mathrm{a2k1}}{\mathrm{a1a2}\alpha}} c_1\left(\frac{\log\left(\frac{e^{\beta y}\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)^{\frac{\mathrm{b1}\beta}{\mathrm{a1}\alpha}}}{\mathrm{b1}+\mathrm{b2}e^{\beta y}}\right)}{\mathrm{b1}\beta} - \frac{x}{\mathrm{a1}}, \frac{\log\left(\frac{e^{\gamma z}\left(\mathrm{a1}+\mathrm{a2}e^{\alpha x}\right)^{\frac{\mathrm{c1}\gamma}{\mathrm{a1}\alpha}}}{\mathrm{c1}+\mathrm{c2}e^{\gamma z}}\right)}{\mathrm{c1}\gamma} - \frac{x}{\mathrm{a1}}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  (a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+(b1+b2*exp(beta*y))*diff(w(x,y,z),y)+(c1+c2*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = (a2\,\mathrm{e}^{\alpha x} + a1)^{-\frac{k1}{a1\alpha}+\frac{k2}{a2\alpha}} (\mathrm{e}^{\alpha x})^{\frac{k1}{a1\alpha}} \_F1\left(\frac{-a1\alpha\,\mathrm{RootOf}\left(a1\alpha\beta y - a1\alpha\ln\left(\frac{(-b1+\mathrm{e}^{-Z})(a2\,\mathrm{e}^{\alpha x}+a1)}{b2}\right.\right.}{}\right.$$

6.8.5.8 [1793] Problem 8

problem number 1793

Added July 2, 2019.

Problem Chapter 8.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$e^{\beta y}(a_1 + a_2 e^{\alpha x})w_x + e^{\alpha x}(b_1 + b_2 e^{\beta y})w_y + ce^{\beta y+\gamma z}w_z = k_3 e^{\beta y}(k_1 + k_2 e^{\alpha x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  Exp[beta*y]*(a1+a2*Exp[alpha*x])*D[w[x, y,z], x] + Exp[alpha*x]*(b1+b2*Exp[beta*y])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{\mathrm{k3}((\mathrm{a1k2}-\mathrm{a2k1})\log(\mathrm{a1}+\mathrm{a2}e^{\alpha x})+\mathrm{a2}\alpha\mathrm{k1}x)}{\mathrm{a1a2}\alpha}\right)c_1\left(\frac{c\log(\mathrm{a1}+\mathrm{a2}e^{\alpha x})}{\mathrm{a1}\alpha} - \frac{cx}{\mathrm{a1}} - \frac{e^{-\gamma z}}{\gamma}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  exp(beta*y)*(a1+a2*exp(alpha*x))*diff(w(x,y,z),x)+exp(alpha*x)*(b1+b2*exp(beta*y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = (a2\,\mathrm{e}^{\alpha x} + a1)^{-\frac{k1\,k3}{a1\,\alpha}+\frac{k2\,k3}{a2\,\alpha}} \left(\mathrm{e}^{\alpha x}\right)^{\frac{k1\,k3}{a1\,\alpha}} \_F1\left(\frac{a2\alpha\beta y + a2\alpha\,\mathrm{RootOf}\left(a2\alpha\beta y - a2\alpha\ln\left(\frac{b1(a2\,\mathrm{e}^{\alpha x}+a}{-b2+\mathrm{e}}\right.\right.}{a2}\right.$$

6.8.6 3.2

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6.8.6.1 [1794] Problem 1

problem number 1794

Added July 2, 2019.

Problem Chapter 8.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + bx^m w_z = (ce^{\lambda x}y + ke^{\beta x}z + se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] + b*x^m*D[w[x,y,z],z]== (c*Exp[lambda*x]*y+k*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1},\frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(-\frac{acx^n(-\lambda x)^{-n}\,\mathrm{Gamma}(n+2,-\lambda x)}{\lambda^2(n+1)}-\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*diff(w(x,y,z),y)+c*x^m*diff(w(x,y,z),z)= (c*exp(lambda*x)*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-ax^{n+1}+(n+1)\,y}{n+1},\frac{-cx^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\frac{(n+1)(m+1)(\Gamma(n)-\Gamma(n,-\lambda x))a\beta^2c\gamma nx^n(-\lambda x)^{-n}+(n+1)(m+}{}}$$

6.8.6.2 [1795] Problem 2

problem number 1795

Added July 2, 2019.

Problem Chapter 8.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\lambda x}w_y+bx^mw_z=(cx^ny+ke^{\beta x}z+se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] + b*x^m*D[w[x,y,z],z]== (c*x^n*y+k*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},\frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(\frac{acx^n(-\lambda x)^{-n}\,\mathrm{Gamma}(n+1,-\lambda x)}{\lambda^2}-\frac{bkx^m(-\beta\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*x^m*diff(w(x,y,z),z)= (c*x^n*y+k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{-bx^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\int^x\frac{bk\lambda\_a^{m+1}\mathrm{e}^{-a\beta}-bk\lambda x^{m+1}\mathrm{e}^{-a\beta}-\left(-ac\_a^n\mathrm{e}^{-a\lambda}-k\lambda z\,\mathrm{e}^{-a\beta}-\lambda s\,\mathrm{e}^{-a}\right.}{(m+1)\lambda}}$$

6.8.6.3 [1796] Problem 3

problem number 1796

Added July 2, 2019.

Problem Chapter 8.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\lambda x}w_y+byw_z=(ke^{\beta x}z+se^{\gamma x})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[lambda*x]*D[w[x, y,z], y] + b*y*D[w[x,y,z],z]== (k*Exp[beta*x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ae^{\lambda x}}{\lambda},\frac{abe^{\lambda x}(\lambda x-1)}{\lambda^2}-bxy+z\right)\exp\left(\frac{abke^{x(\beta+\lambda)}}{\beta^2(\beta+\lambda)}+\frac{ke^{\beta x}(\beta z-by)}{\beta^2}+\frac{se^{\gamma x}}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(lambda*x)*diff(w(x,y,z),y)+b*y*diff(w(x,y,z),z)= (k*exp(beta*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\,\mathrm{e}^{\lambda x}+\lambda y}{\lambda},\frac{(\lambda x-1)\,ab\,\mathrm{e}^{\lambda x}-(bxy-z)\,\lambda^2}{\lambda^2}\right)\mathrm{e}^{\frac{ab\beta^2\gamma k\,\mathrm{e}^{(\beta+\lambda)x}+(\beta+\lambda)\left(\beta^2\lambda^2 s\,\mathrm{e}^{\gamma x}+\left((-\beta+\lambda)ab\,\mathrm{e}^{\lambda x}-(by-\right.\right.}{(\beta+\lambda)\beta^2\gamma\lambda^2}}$$

6.8.6.4 [1797] Problem 4

problem number 1797

Added July 2, 2019.

Problem Chapter 8.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ay^nw_y+bz^mw_z=(ce^{\lambda x}+ke^{\beta y}+se^{\gamma z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*y^n*D[w[x, y,z], y] + b*z^m*D[w[x,y,z],z]== (c*Exp[lambda*x]+k*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-ax-\frac{\left(\frac{1}{y}\right)^{n-1}}{n-1},-bx-\frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right)\exp\left(\frac{k\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{n}{n-1}}\left(-\beta\left(\left(\frac{1}{y}\right)^{n-1}\right)^{\frac{1}{1-n}}\right)^{n}\text{Gam}}{a\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*y^n*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)= (c*exp(lambda*x)+k*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\left((n-1)\,axy^n+y\right)y^{-n},\left((m-1)\,bxz^m+z\right)z^{-m}\right)\mathrm{e}^{\int^x\left(c\,\mathrm{e}^{-a\lambda}+k\,\mathrm{e}^{\beta\left(\left((-\_a+x)(n-1)ay^n+y\right)y^{-n}\right)^{-\frac{1}{n-1}}}\right.}$$

6.8.6.5 [1798] Problem 5

problem number 1798

Added July 2, 2019.

Problem Chapter 8.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\beta y}w_y+bz^m w_z=(ce^{\lambda x}+ky^n+se^{\gamma z})w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Exp[beta*y]*D[w[x, y,z], y] + b*z^m*D[w[x,y,z],z]== (c*Exp[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{a\beta x+e^{-\beta y}}{\beta},-bx-\frac{\left(\frac{1}{z}\right)^{m-1}}{m-1}\right)\exp\left(-\frac{k\left(-\log\left(e^{-\beta y}\right)\right)^{-n}\left(-\frac{\log\left(e^{-\beta y}\right)}{\beta}\right)^{n}\text{Gamma}\left(n\right.}{a\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*exp(beta*y)*diff(w(x,y,z),y)+b*z^m*diff(w(x,y,z),z)= (c*exp(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-a\beta x-\mathrm{e}^{-\beta y}}{a\beta},\left(\left(m-1\right)bxz^{m}+z\right)z^{-m}\right)\mathrm{e}^{\int^{x}\left(c\,\mathrm{e}^{-a\lambda}+k\left(\frac{\ln\left(\frac{1}{(-\_a+x)a\beta+\mathrm{e}^{-\beta y}}\right)}{\beta}\right)^{n}+s\,\mathrm{e}^{\gamma\left(\left(\left(-\_a+x\right)(m\right.\right.}\right.}$$

6.8.6.6 [1799] Problem 6

problem number 1799

Added July 2, 2019.

Problem Chapter 8.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(y^2+by+ae^{\alpha y}(y-b)-b^2)w_y+(z^2+c(xz-1)e^{\beta x})w_z=ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2+ b*y+ a*Exp[alpha*y]*(y-b)-b^2)*D[w[x, y,z], y] +  (z^2+c*(x*z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(y^2+ b*y+ a*exp(alpha*y)*(y-b)-b^2)*diff(w(x,y,z),y)+(z^2+c*(x*z-1
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
```

$$w(x,y,z) = c_1 \_F1(x) \_F3(z) \, e^{-c_2 \left(\int \frac{1}{(b-y)a\, e^{\alpha y}+b^2-by-y^2} dy\right)} \textbf{where} \left[\left\{(zx-1)\, c\_F1(x) \left(\frac{d}{dz}\_F3(z)\right) e^{\beta x} - \right.\right.$$

6.8.6.7 [1800] Problem 7

problem number 1800

Added July 2, 2019.

Problem Chapter 8.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (y^2 + ae^{\alpha x}(x+1))w_y + (ce^{\beta x}z^2 + be^{-\beta x})w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (y^2+ a*Exp[alpha*x]*(x+1))*D[w[x, y,z], y] + (c*Exp[beta*x]*z^2+b*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(y^2+ a*exp(alpha*x)*(x+1))*diff(w(x,y,z),y)+(c*exp(beta*x)*z^2+b*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.6.8 [1801] Problem 8

problem number 1801

Added July 2, 2019.

Problem Chapter 8.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (ae^{\alpha x}y^2 + be^{-\alpha x})w_y + (de^{\beta x}z^2 + ce^{\gamma x}(\gamma - cde^{(\beta+\gamma)x}))w_z = ke^{\lambda x}w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a*Exp[alpha*x]*y^2+b*Exp[-alpha*x])*D[w[x, y,z], y] + (d*Exp[beta*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a*exp(alpha*x)*y^2+b*exp(-alpha*x))*diff(w(x,y,z),y)+(d*exp(beta*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.8.6.9 [1802] Problem 9

problem number 1802

Added July 2, 2019.

Problem Chapter 8.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\lambda_1 x} y + b_1 e^{\beta_1 x} y^k) w_y + (a_2 e^{\lambda_2 x} z + b_2 e^{\beta_2 x} z^m) w_z = c x^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*Exp[lambda1*x]*y+b1*Exp[beta1*x]*y^k)*D[w[x, y,z], y] +  (a2*Ex
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{c x^{s+1}}{s+1}} c_1 \left((k-1)\int_1^x \mathrm{b1} e^{\frac{\mathrm{a1} e^{\mathrm{lambda1} K[1]}(k-1)}{\mathrm{lambda1}} + \mathrm{beta1} K[1]} dK[1] + y^{1-k} e^{\frac{\mathrm{a1}(k-1) e^{\mathrm{lambda1} x}}{\mathrm{lambda1}}}, (m-1)\int_1^x \right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*exp(lambda1*x)*y+b1*exp(beta1*x)*y^k)*diff(w(x,y,z),y)+(a2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\left((k-1)\, b1\, y^k \left(\int \mathrm{e}^{\frac{\beta 1 \lambda 1 x + (k-1) a1\, \mathrm{e}^{\lambda 1 x}}{\lambda 1}} dx\right) \mathrm{e}^{\frac{a1\, \mathrm{e}^{\lambda 1 x}}{\lambda 1}} + y\, \mathrm{e}^{\frac{a1\, k\, \mathrm{e}^{\lambda 1 x}}{\lambda 1}}\right) y^{-k} \mathrm{e}^{-\frac{a1\, \mathrm{e}^{\lambda 1 x}}{\lambda 1}}, \left((m-1)\, b2 z\right.\right.$$

6.8.6.10 [1803] Problem 10

problem number 1803

Added July 2, 2019.

Problem Chapter 8.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\beta_1 x} y + b_1 e^{\gamma_1 x} y^k) w_y + (a_2 e^{\beta_2 x} + b_2 e^{\gamma_2 x + \lambda_2 z}) w_z = c x^s w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*Exp[beta1*x]*y+b1*Exp[gamma1*x]*y^k)*D[w[x, y,z], y] +  (a2*Exp
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*exp(beta1*x)*y+b1*exp(gamma1*x)*y^k)*diff(w(x,y,z),y)+ (a2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\left((k-1)\, b1 y^k \left(\int e^{\frac{\beta 1 \gamma 1 x + (k-1) a1\, e^{\beta 1 x}}{\beta 1}} dx\right) e^{\frac{a1\, e^{\beta 1 x}}{\beta 1}} + y\, e^{\frac{a1\, k\, e^{\beta 1 x}}{\beta 1}}\right) y^{-k} e^{-\frac{a1\, e^{\beta 1 x}}{\beta 1}}, \frac{-b2\lambda 2\left(\int e^{\frac{a}{}}\right.}{}\right.$$

6.8.6.11 [1804] Problem 11

problem number 1804

Added July 2, 2019.

Problem Chapter 8.3.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^n + b_1 x^m e^{\lambda y}) w_y + (a_2 x^k + b_2 x^r e^{\beta z}) w_z = c x^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + (a1*x^n+b1*x^m*Exp[lambda*y] )*D[w[x, y,z], y] +   (a2*x^k+b2*x^r*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{c x^{s+1}}{s+1}} c_1 \left(\frac{(n+1) e^{-\frac{\lambda\left(-\mathrm{a1} x^{n+1}+n y+y\right)}{n+1}} - \mathrm{b1}\lambda x^{m+1}\left(-\frac{\mathrm{a1}\lambda x^{n+1}}{n+1}\right)^{-\frac{m+1}{n+1}} \mathrm{Gamma}\left(\frac{m+1}{n+1}, -\frac{\mathrm{a1}\lambda x^{n+1}}{n+1}\right)}{\mathrm{a1b1}\lambda^2 (n+1)(m-n)}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+(a1*x^n+b1*x^m*exp(lambda*y) )*diff(w(x,y,z),y)+ (a2*x^k+b2*x^r*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{(n+1)^2 \left(-a1\lambda x^{m+1} + (m+n+2) x^{m-n}\right) b1 \left(-\frac{a1\lambda x^{n+1}}{n+1}\right)^{\frac{-m-n-2}{2n+2}} \mathrm{WhittakerM}\left(\frac{m-n}{2n+2}, \right.}{}\right.$$

6.8.7 4.1

Local contents

6.8.7.1 [1805] Problem 1 2473
6.8.7.2 [1806] Problem 2 2473
6.8.7.3 [1807] Problem 3 2474
6.8.7.4 [1808] Problem 4 2475
6.8.7.5 [1809] Problem 5 2476

6.8.7.1 [1805] Problem 1

problem number 1805

Added Oct 10, 2019.

Problem Chapter 8.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sinh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sinh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

6.8.7.2 [1806] Problem 2

problem number 1806

Added Oct 10, 2019.

Problem Chapter 8.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\lambda x)w_z = (k\sinh(\beta x) + s\sinh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]== (k*Sinh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\cosh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{s\sinh\left(\frac{\gamma(a\lambda z-c\cosh(\lambda x)+c\cosh(\lambda K[1]))}{a\lambda}\right)+k\sinh(\beta K[1}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*sinh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c\cosh(\lambda x)}{a\lambda}\right)e^{\int^x\frac{k\sinh(\_a\beta)+s\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

6.8.7.3 [1807] Problem 3

problem number 1807

Added Oct 10, 2019.

Problem Chapter 8.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\beta x)w_y+b\sinh^k(\lambda x)w_z=c\sinh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[beta*x]^n*D[w[x, y,z], y] +  b*Sinh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{\cosh^2(\gamma x)}\operatorname{sech}(\gamma x)\sinh^{m+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\sinh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(y-\frac{a\sqrt{c}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*sinh(beta*x)^n*diff(w(x, y,z), y) +  b*sinh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\beta x))\,dx\right),z-\left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\sinh^m(\gamma x))dx}$$

6.8.7.4 [1808] Problem 4

problem number 1808

Added Oct 10, 2019.

Problem Chapter 8.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\sinh(\beta y)w_y+c\sinh(\lambda x)w_z=k\sinh(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(z-\frac{c\cosh(\lambda x)}{a\lambda},\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a}\right)\exp\left(\int_1^x\frac{k\sinh\left(\frac{\gamma(a\lambda z-c\cosh(\lambda x)+c\cosh(\lambda K[1]))}{a\lambda}\right)}{a}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*sinh(beta*y)*diff(w(x, y,z), y) +  c*sinh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x-2a\operatorname{arctanh}\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\cosh\left(\lambda x\right)}{a\lambda}\right)e^{\int^x\frac{k\sinh\left(\frac{(a\lambda z+c\cosh(\_a\lambda)-c\cosh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

6.8.7.5 [1809] Problem 5

problem number 1809

Added Oct 10, 2019.

Problem Chapter 8.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\sinh^{n_1}(\lambda_1 x)w_x+b_1\sinh^{m_1}(\beta_1 y)w_y+c_1\sinh^{k_1}(\gamma_1 z)w_z=\left(a_2\sinh^{n_2}(\lambda_2 x)w_x+b_2\sinh^{m_2}(\beta_2 y)w_y+c_2\sinh^{k_2}\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sinh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Sinh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Sin
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sinh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*sinh(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\sinh^{-n1}(\lambda 1x)\right) dx\right) + \int \frac{a1\left(\sinh^{-m1}(\beta 1y)\right)}{b1} dy, \frac{a1\,z - c1\left(\int \left(\sinh^{k1}(\gamma 1x)\right)\left(\mathrm{si}\right.\right.}{a1}\right.$$

6.8.8 4.2

Local contents

6.8.8.1 [1810] Problem 1 2477
6.8.8.2 [1811] Problem 2 2478
6.8.8.3 [1812] Problem 3 2479
6.8.8.4 [1813] Problem 4 2480
6.8.8.5 [1814] Problem 5 2481

6.8.8.1 [1810] Problem 1

problem number 1810

Added Oct 10, 2019.

Problem Chapter 8.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cosh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cosh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{-\sinh^2(\beta x)}\operatorname{csch}(\beta x)\cosh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(\beta x\right.}{\beta n+\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cosh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cosh^n(\beta x))dx}$$

6.8.8.2 [1811] Problem 2

problem number 1811

Added Oct 10, 2019.

Problem Chapter 8.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cosh(\lambda x)w_z=(k\cosh(\beta x)+s\cosh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]== (k*Cosh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{k\cosh(\beta K[1])+s\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K[1])}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*cosh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\lambda z-c\sinh(\lambda x)}{a\lambda}\right)e^{\int^x\frac{k\cosh(\_a\beta)+s\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

6.8.8.3 [1812] Problem 3

problem number 1812

Added Oct 10, 2019.

Problem Chapter 8.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\beta x)w_y+b\cosh^k(\lambda x)w_z=c\cosh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[beta*x]^n*D[w[x, y,z], y] +  b*Cosh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{m+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cosh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(\frac{a\sinh(\beta x}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*sinh(beta*x)^n*diff(w(x, y,z), y) +  b*sinh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\beta x))\,dx\right),z-\left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\sinh^m(\gamma x))dx}$$

6.8.8.4 [1813] Problem 4

problem number 1813

Added Oct 10, 2019.

Problem Chapter 8.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\cosh(\beta y)w_y+c\cosh(\lambda x)w_z=k\cosh(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cosh[beta*y]*D[w[x, y,z], y] +  c*Cosh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sinh(\lambda x)}{a\lambda}\right)\exp\left(\int_1^x\frac{k\cosh\left(\frac{\gamma(a\lambda z-c\sinh(\lambda x)+c\sinh(\lambda K}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*cosh(beta*y)*diff(w(x, y,z), y) +  c*cosh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{-b\beta x+2a\arctan\left(e^{\beta y}\right)}{b\beta},\frac{a\lambda z-c\sinh(\lambda x)}{a\lambda}\right)e^{\int_x\frac{k\cosh\left(\frac{(a\lambda z+c\sinh(\_a\lambda)-c\sinh(\lambda x))\gamma}{a\lambda}\right)}{a}d\_a}$$

6.8.8.5 [1814] Problem 5

problem number 1814

Added Oct 10, 2019.

Problem Chapter 8.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cosh^{n_1}(\lambda_1 x)w_x+b_1\cosh^{m_1}(\beta_1 y)w_y+c_1\cosh^{k_1}(\gamma_1 z)w_z=\left(a_2\cosh^{n_2}(\lambda_2 x)w_x+b_2\cosh^{m_2}(\beta_2 y)w_y+c_2\cosh\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cosh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Cosh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cos
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cosh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*cosh(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cosh^{-n1}(\lambda 1x)\right) dx\right) + \int \frac{a1\left(\cosh^{-m1}(\beta 1y)\right)}{b1} dy, \frac{a1\,z - c1\left(\int \left(\cosh^{k1}(\gamma 1x)\right)\right.}{a1}\right.$$

6.8.9 4.3

Local contents

6.8.9.1 [1815] Problem 1 2482
6.8.9.2 [1816] Problem 2 2483
6.8.9.3 [1817] Problem 3 2484
6.8.9.4 [1818] Problem 4 2485
6.8.9.5 [1819] Problem 5 2486
6.8.9.6 [1820] Problem 6 2487

6.8.9.1 [1815] Problem 1

problem number 1815

Added Oct 10, 2019.

Problem Chapter 8.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tanh[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tanh(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\tanh^n(\beta x))dx}$$

6.8.9.2 [1816] Problem 2

problem number 1816

Added Oct 10, 2019.

Problem Chapter 8.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\tanh(\lambda x)w_z=(k\tanh(\beta x)+s\tanh(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]== (k*Tanh[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\log(\cosh(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x\frac{k\tanh(\beta K[1])+s\tanh\left(\frac{\gamma(a\lambda z-c\log(\cosh(\lambda x))+c}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*tanh(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{2a\lambda z+c\ln\left(\tanh\left(\lambda x\right)-1\right)+c\ln\left(\tanh\left(\lambda x\right)+1\right)}{2a\lambda}\right)\mathrm{e}^{\int^x\frac{k\tanh(\_a\beta)-s\tanh\left(\frac{(-2a\lambda z+c}{}\right.}{}}$$

6.8.9.3 [1817] Problem 3

problem number 1817

Added Oct 10, 2019.

Problem Chapter 8.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\tanh^n(\beta x)w_y+b\tanh^k(\lambda x)w_z=c\tanh^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[beta*x]^n*D[w[x, y,z], y] +  b*Tanh[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{m+1}(\gamma x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\tanh^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(y-\frac{a\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+}{2}\right.}{\beta n+\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*tanh(beta*x)^n*diff(w(x, y,z), y) +  b*tanh(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\tanh^k(\lambda x))\,dx\right)\right)e^{\int c(\tanh^m(\gamma x))dx}$$

6.8.9.4 [1818] Problem 4

problem number 1818

Added Oct 10, 2019.

Problem Chapter 8.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\tanh(\beta y)w_y+c\tanh(\lambda x)w_z=k\tanh(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*tanh(beta*y)*diff(w(x, y,z), y) +  c*tanh(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x + a\ln(\tanh(\beta y)-1) + a\ln(\tanh(\beta y)+1) - 2a\ln(\tanh(\beta y))}{2b\beta}, \frac{a\lambda z - c\ln(\cosh}{a\lambda}\right.$$

6.8.9.5 [1819] Problem 5

problem number 1819

Added Oct 10, 2019.

Problem Chapter 8.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tanh(\beta y)w_y + c\tanh(\gamma z)w_z = k\tanh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tanh[beta*y]*D[w[x, y,z], y] +  c*Tanh[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \cosh^{\frac{k}{a\lambda}}(\lambda x)c_1\left(\frac{1}{2}\left(\frac{\log(\sinh(\beta y))}{\beta} - \frac{bx}{a}\right), \frac{b\log\left(\sinh^2(\gamma z)\right)}{\gamma} - \frac{2c\log(\sinh(\beta y))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*tanh(beta*y)*diff(w(x, y,z), y) +  c*tanh(gamma*z)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(x)\,\_F2(y)\,\_F3(z) \textbf{ where} \left[\left\{\frac{d}{dx}\_F1(x) = \frac{(k\tanh(\lambda x) - \_c_1)\,\_F1(x)}{a}, \frac{d}{dy}\_F2(y) =\right.\right.$$

6.8.9.6 [1820] Problem 6

problem number 1820

Added Oct 10, 2019.

Problem Chapter 8.4.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\tanh^{n_1}(\lambda_1 x)w_x + b_1\tanh^{m_1}(\beta_1 y)w_y + c_1\tanh^{k_1}(\gamma_1 z)w_z = \left(a_2\tanh^{n_2}(\lambda_2 x)w_x + b_2\tanh^{m_2}(\beta_2 y)w_y + c_2\tan\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tanh[lambda1*x]^n1*D[w[x, y,z], x] + b1*Tanh[beta1*y]^m1*D[w[x, y,z], y] +  c1*Tan
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tanh(lambda1*x)^n1*diff(w(x, y,z), x) + b1*tanh(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\tanh^{-n1}(\lambda 1x)\right) dx\right) + \int \frac{a1\left(\tanh^{-m1}(\beta 1y)\right)}{b1} dy, \frac{a1\,z - c1\left(\int \left(\frac{\sinh(\gamma 1x)}{\cosh(\gamma 1x)}\right)^{k1} \left(\frac{s}{c}\right.\right.}{a1}\right.$$

6.8.10 4.4

Local contents

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6.8.10.1 [1821] Problem 1

problem number 1821

Added Oct 10, 2019.

Problem Chapter 8.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Coth[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\beta x)\right)}{\beta n+\beta}\right)c_1(y-ax,z-bx)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*coth(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\coth^n(\beta x))dx}$$

6.8.10.2 [1822] Problem 2

problem number 1822

Added Oct 10, 2019.

Problem Chapter 8.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\coth(\lambda x)w_z=(k\coth(\beta x)+s\coth(\gamma z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]== (k*Coth[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\log(\sinh(\lambda x))}{a\lambda}\right)\exp\left(\int_1^x\frac{k\coth(\beta K[1])+s\coth\left(\frac{\gamma(a\lambda z-c\log(\sinh(\lambda x))+c\,l}{a\lambda}\right.}{a}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*diff(w(x, y,z), y) +  c*coth(lambda*x)*diff(w(x,y,z),z)= (k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{2a\lambda z+c\ln(\coth(\lambda x)-1)+c\ln(\coth(\lambda x)+1)}{2a\lambda}\right)\mathrm{e}^{-\left(\int^x\frac{-k\coth(\_a\beta)+s\coth\left(\frac{(-2a\lambda}{}\right.}{}\right.}$$

6.8.10.3 [1823] Problem 3

problem number 1823

Added Oct 10, 2019.

Problem Chapter 8.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\coth^n(\beta x)w_y+b\coth^k(\lambda x)w_z=c\coth^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Coth[beta*x]^n*D[w[x, y,z], y] +  b*Coth[lambda*x]^k*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{m+1}(\gamma x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\coth^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\left(z-\frac{b\coth^{k+1}(\lambda x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2}\right.}{k\lambda+\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x, y,z), x) + a*coth(beta*x)^n*diff(w(x, y,z), y) +  b*coth(lambda*x)^k*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\coth^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)\mathrm{e}^{\int c(\coth^m(\gamma x))dx}$$

6.8.10.4 [1824] Problem 4

problem number 1824

Added Oct 10, 2019.

Problem Chapter 8.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+b\coth(\beta y)w_y+c\coth(\lambda x)w_z=k\coth(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[lambda*x]*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*coth(beta*y)*diff(w(x, y,z), y) +  c*coth(lambda*x)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-2}\right) - a\ln\left(\mathrm{RootOf}\left(\beta y - \mathrm{arccoth}(\_Z-1)\right)\right)}{2b\beta},\right.$$

6.8.10.5 [1825] Problem 5

problem number 1825

Added Oct 10, 2019.

Problem Chapter 8.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\coth(\beta y)w_y + c\coth(\gamma z)w_z = k\coth(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Coth[beta*y]*D[w[x, y,z], y] +  c*Coth[gamma*z]*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{k(\log(-\tanh(\lambda x))+\log(\cosh(\lambda x)))}{a\lambda}} c_1\left(\frac{a\log(\mathrm{sech}(\beta y)) + b\beta x}{2a\beta}, \frac{2c\log(\mathrm{sech}(\beta y))}{\beta} - \frac{b\log\left(\mathrm{sech}^2(\gamma z)\right)}{\gamma}\right)\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x, y,z), x) + b*coth(beta*y)*diff(w(x, y,z), y) +  c*coth(gamma*z)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = (\coth(\lambda x) - 1)^{-\frac{k}{2a\lambda}} (\coth(\lambda x) + 1)^{-\frac{k}{2a\lambda}} \_F1\left(\frac{-2b\beta x + a\ln\left(\frac{(\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-1)^2}{\mathrm{RootOf}(\beta y - \mathrm{arccoth}(\_Z-1))-2}\right) -}{2b\beta}\right.$$

6.8.10.6 [1826] Problem 6

problem number 1826

Added Oct 10, 2019.

Problem Chapter 8.4.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \coth^{n_1}(\lambda_1 x) w_x + b_1 \coth^{m_1}(\beta_1 y) w_y + c_1 \coth^{k_1}(\gamma_1 z) w_z = \left(a_2 \coth^{n_2}(\lambda_2 x) w_x + b_2 \coth^{m_2}(\beta_2 y) w_y + c_2 \coth\right.$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Coth[lambda1*x]^n1*D[w[x, y,z], x] + b1*Coth[beta1*y]^m1*D[w[x, y,z], y] +  c1*Cot
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*coth(lambda1*x)^n1*diff(w(x, y,z), x) + b1*coth(beta1*y)^m1*diff(w(x, y,z), y) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\coth^{-n1}(\lambda 1x)\right) dx\right) + \int \frac{a1\left(\coth^{-m1}(\beta 1y)\right)}{b1} dy, \frac{a1\,z - c1\left(\int \left(\frac{\cosh(\gamma 1x)}{\sinh(\gamma 1x)}\right)^{k1} \left(\frac{\mathrm{co}}{\mathrm{si}}\right.\right.}{a1}\right.$$

6.8.11 4.5

Local contents

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6.8.11.3 [1829] Problem 3 2496
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6.8.11.1 [1827] Problem 1

problem number 1827

Added Oct 10, 2019.

Problem Chapter 8.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh^n(\lambda y)w_z = (s\cosh^m(\beta x) + k\sinh^r(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sinh[lambda*y]^n*D[w[x,y,z],z]== (s*Cosh[b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{c\sqrt{\cosh^2(\lambda y)}\operatorname{sech}(\lambda y)\sinh^{n+1}(\lambda y)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\lambda y)\right)}{b\lambda n+b\lambda}\right)\exp\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*sinh(lambda*y)^n*diff(w(x,y,z),z)= (s*cosh(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},z-\left(\int^x\frac{c\left(\sinh^n\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)\right)}{a}d\_a\right)\right)\mathrm{e}^{\int^x\frac{k\left(\sinh^r\left(\frac{(ay-(-\_a+x)b)\gamma}{a}\right)\right)+s(\cosh^m(}{a}}$$

6.8.11.2 [1828] Problem 2

problem number 1828

Added Oct 10, 2019.

Problem Chapter 8.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sinh^n(\lambda x)w_y+b\cosh^m(\beta x)w_z=s\cosh^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sinh[lambda*x]^n*D[w[x, y,z], y] +  b*Cosh[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{s\sqrt{-\sinh^2(\gamma x)}\operatorname{csch}(\gamma x)\cosh^{k+1}(\gamma x)\,{}_2F_1\left(\frac{1}{2},\frac{k+1}{2};\frac{k+3}{2};\cosh^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(\frac{b\sinh(\beta x)}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sinh(lambda*x)^n*diff(w(x,y,z),y)+ b*cosh(beta*x)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sinh^n(\lambda x))\,dx\right),z-\left(\int b(\cosh^m(\beta x))\,dx\right)\right)\mathrm{e}^{\int s(\cosh^k(\gamma x))dx}$$

6.8.11.3 [1829] Problem 3

problem number 1829

Added Oct 10, 2019.

Problem Chapter 8.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cosh^n(\lambda x)w_y+b\sinh^m(\beta y)w_z=s\sinh^k(\gamma z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cosh[lambda*x]^n*D[w[x, y,z], y] +  b*Sinh[beta*y]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cosh(lambda*x)^n*diff(w(x,y,z),y)+ b*sinh(beta*y)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cosh^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sinh^m\left(\left(a\left(\int(\cosh^n(\_b\lambda))\,d\_b\right) + y - \left(\right.\right.\right.\right.\right.$$

6.8.11.4 [1830] Problem 4

problem number 1830

Added Oct 10, 2019.

Problem Chapter 8.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tanh^n(\lambda x)w_y + b\coth^m(\beta x)w_z = s\coth^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tanh[lambda*x]^n*D[w[x, y,z], y] +  b*Coth[beta*x]^m*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{s\coth^{k+1}(\gamma x)\,{}_2F_1\left(1,\frac{k+1}{2};\frac{k+3}{2};\coth^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(z - \frac{b\coth^{m+1}(\beta x)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2}\right.}{\beta m+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*tanh(lambda*x)^n*diff(w(x,y,z),y)+ b*coth(beta*x)^m*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\tanh^n(\lambda x))\,dx\right), z - \left(\int b(\coth^m(\beta x))\,dx\right)\right) e^{\int s(\coth^k(\gamma x))dx}$$

6.8.11.5 [1831] Problem 5

problem number 1831

Added Oct 10, 2019.

Problem Chapter 8.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\sinh(\lambda x)w_x + b\sinh(\beta y)w_y + c\sinh(\gamma z)w_z = k\cosh(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*Sinh[lambda*x]*D[w[x, y,z], x] + b*Sinh[beta*y]*D[w[x, y,z], y] +  c*Sinh[gamma*z]*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \sinh^{\frac{k}{a\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tanh\left(\frac{\beta y}{2}\right)\tanh^{-\frac{b\beta}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\beta}, \frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\tanh^{-\frac{c\gamma}{a\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\gamma}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*sinh(lambda*x)*diff(w(x,y,z),x)+b*sinh(beta*y)*diff(w(x,y,z),y)+ c*sinh(gamma*z)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\sinh^{\frac{k}{a\lambda}}(\lambda x)\right) \_F1\left(\frac{-2a\lambda \operatorname{arctanh}\left(e^{\beta y}\right) + 2b\beta \operatorname{arctanh}\left(e^{\lambda x}\right)}{b\beta\lambda}, \frac{-2a\lambda \operatorname{arctanh}\left(e^{\gamma z}\right) + 2c\gamma \operatorname{arct}}{c\gamma\lambda}\right.$$

6.8.12 5.1

Local contents

6.8.12.1 [1832] Problem 1 2499
6.8.12.2 [1833] Problem 2 2500
6.8.12.3 [1834] Problem 3 2501
6.8.12.4 [1835] Problem 4 2502
6.8.12.5 [1836] Problem 5 2503

6.8.12.1 [1832] Problem 1

problem number 1832

Added Oct 17, 2019.

Problem Chapter 8.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\ln^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Log[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1,-\log(\beta x))}{\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*ln(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\ln(\beta x)^n dx}$$

6.8.12.2 [1833] Problem 2

problem number 1833

Added Oct 17, 2019.

Problem Chapter 8.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\ln^n(\beta x)w_z=s\ln^m(\lambda y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Log[beta*x]^n*D[w[x,y,z],z]== s*Log[lambda
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{s(-\log(\lambda y))^{-m}\log^m(\lambda y)\,\text{Gamma}(m+1,-\log(\lambda y))}{b\lambda}\right)c_1\left(y-\frac{bx}{a},z-\frac{c(-\log(\beta x)}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*ln(beta*x)^n*diff(w(x,y,z),z)= s*ln(lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z - \left(\int \frac{c\ln(\beta x)^n}{a}dx\right)\right) e^{\int^x \frac{s\ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^m}{a} d\_a}$$

6.8.12.3 [1834] Problem 3

problem number 1834

Added Oct 17, 2019.

Problem Chapter 8.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = c\ln^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c(-\log(\gamma x))^{-m}\log^m(\gamma x)\,\mathrm{Gamma}(m+1,-\log(\gamma x))}{\gamma}\right)c_1\left(y - \frac{a(-\log(\beta x))^{-n}\log^n}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int b\ln(\lambda x)^k\,dx\right)\right)\mathrm{e}^{\int c\ln(\gamma x)^m dx}$$

6.8.12.4 [1835] Problem 4

problem number 1835

Added Oct 17, 2019.

Problem Chapter 8.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda y)w_z = c\ln^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[beta*x]^n*D[w[x, y,z], y] +  b*Log[lambda*y]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*y)^k*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a\ln(\beta x)^n\,dx\right), z - \left(\int^x b\ln\left(\left(a\left(\int \ln(\_b\beta)^n\,d\_b\right) + y - \left(\int a\ln(\beta x)^n\,d\right.\right.\right.\right.$$

6.8.12.5 [1836] Problem 5

problem number 1836

Added Oct 17, 2019.

Problem Chapter 8.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \ln^{n_1}(\lambda_1 x) w_x + b_1 \ln^{m_1}(\beta_1 y) w_y + c_1 \ln^{k_1}(\gamma_1 z) w_z = \left(a_2 \ln^{n_2}(\lambda_2 x) + b_2 \ln^{m_2}(\beta_2 y) + c_2 \ln^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =   a1*Log[lambda1*x]^n1*D[w[x, y,z], x] + b1*Log[beta1*y]^m1*D[w[x, y,z], x]*D[w[x, y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=   a1*ln(lambda1*x)^n1*diff(w(x,y,z),x)+ b1*ln(beta1*y)^m1*diff(w(x,y,z),y)+  c1*ln(ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \ln(\lambda 1 x)^{-n1}\,dx\right) + \int \frac{a1 \ln(\beta 1 y)^{-m1}}{b1} dy, -\left(\int \ln(\lambda 1 x)^{-n1}\,dx\right) + \int \frac{a1 \ln(\gamma 1 z}{c1}\right.$$

6.8.13 5.2

Local contents

6.8.13.1 [1837] Problem 1 2504
6.8.13.2 [1838] Problem 2 2505
6.8.13.3 [1839] Problem 3 2506
6.8.13.4 [1840] Problem 4 2506
6.8.13.5 [1841] Problem 5 2507

6.8.13.1 [1837] Problem 1

problem number 1837

Added Oct 17, 2019.

Problem Chapter 8.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cx^n \ln^k(\lambda y)w_z = sy^m \ln^r(\beta x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*x^n*Log[lambda*y]^k*D[w[x,y,z],z]== s*y^m*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*x^n*ln(lambda*y)^k*diff(w(x,y,z),z)= s*y^m*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z - \left(\int^x \frac{c\_a^n \ln\left(\frac{(ay-(-\_a+x)b)\lambda}{a}\right)^k}{a} d\_a\right)\right) e^{\int^x \frac{s\left(\frac{ay-(-\_a+x)b}{a}\right)^m \ln(\_a\beta)^r}{a} d\_a}$$

6.8.13.2 [1838] Problem 2

problem number 1838

Added Oct 17, 2019.

Problem Chapter 8.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + bx^m w_z = \left(cy\ln^k(\lambda x) + sz\ln^r(\beta x)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*x^n*D[w[x, y,z], y] +  b*x^m*D[w[x,y,z],z]== (c*y*Log[lambda*x]^k
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, \frac{-bx^{m+1}+mz+z}{m+1}\right)\exp\left(\frac{(n^2+3n+2)\left(-\log(\beta x)\right)^{-r}(-\log(\lambda x)}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)= (c*y*ln(lambda*x)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-ax^{n+1}+(n+1)\,y}{n+1}, \frac{-bx^{m+1}+(m+1)\,z}{m+1}\right)\mathrm{e}^{\int^x \frac{(m+1)\left(a\_a^{n+1}-ax^{n+1}+ny+y\right)c\ln(\_a\lambda)^k+(n+1)\left(b\_a^m\right.}{(n+1)(m+1)}}$$

6.8.13.3 [1839] Problem 3

problem number 1839

Added Oct 17, 2019.

Problem Chapter 8.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a \ln^n(\lambda x) w_y + b y^m w_z = \left(c \ln^k(\beta x) + s \ln^r(\gamma z)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Log[lambda*x]^n*D[w[x, y,z], y] +  b*y^m*D[w[x,y,z],z]== (c*Log[b
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(y - \frac{a(-\log(\lambda x))^{-n} \log^n(\lambda x) \operatorname{Gamma}(n+1, -\log(\lambda x))}{\lambda}, z - \int_1^x b\left(\frac{-a \operatorname{Gamma}(n+}{}\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*ln(lambda*x)^n*diff(w(x,y,z),y)+ b*y^m*diff(w(x,y,z),z)= (c*ln(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

server hangs

6.8.13.4 [1840] Problem 4

problem number 1840

Added Oct 17, 2019.

Problem Chapter 8.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a \ln^n(\lambda x) w_x + z w_y + b \ln^k(\beta y) w_z = \left(c x^m + s \ln(\gamma y)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[lambda*x]^n*D[w[x, y,z], x] + z*D[w[x, y,z], y] +  b*Log[lambda*y]^k*D[w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*ln(lambda*x)^n*diff(w(x,y,z),x)+z*diff(w(x,y,z),y)+ b*ln(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-2b\left(\int \ln(\lambda y)^k\,dy\right) + z^2, \frac{b\left(\int \frac{\ln(\lambda x)^{-n}}{a}dx\right) - \sqrt{2by\ln(\_a\lambda)^k - 2b\left(\int \ln(\lambda y)^k\,dy\right) +}}{b}\right.$$

6.8.13.5 [1841] Problem 5

problem number 1841

Added Oct 17, 2019.

Problem Chapter 8.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax(\ln x)^n w_x + by(\ln y)^m w_y + cz(\ln z)^r w_z = k(\ln x)^s w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*Log[x]^n*D[w[x, y,z], x] + b*y*Log[y]^m*D[w[x, y,z], y] +  c*z*Log[z]^r*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{k\log^{-n+s+1}(x)}{a(-n)+as+a}}c_1\left(\frac{b\log^{1-n}(x)}{a(n-1)}-(m-1)^{\frac{1}{m-1}}\log(y)\left(\frac{(m-1)^{\frac{1}{1-m}}}{\log(y)}\right)^m,\frac{c\log^{1-n}(x)}{a(n-1)}-(r-1\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*ln(x)^n*diff(w(x,y,z),x)+b*y*ln(y)^m*diff(w(x,y,z),y)+ c*z*ln(z)^r*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=x^{-\frac{k\ln(x)^{-n+s}}{(n-s-1)a}}\_F1\left(\frac{-(n-1)\,a\ln(y)^{-m+1}+(m-1)\,b\ln(x)^{-n+1}}{(n-1)\,(m-1)\,b},\frac{-(n-1)\,a\ln(z)^{-r+1}+(r-}{(r-1)\,(n-1)}\right.$$

6.8.14 6.1

Local contents

6.8.14.1 [1842] Problem 1 2509
6.8.14.2 [1843] Problem 2 2509
6.8.14.3 [1844] Problem 3 2510
6.8.14.4 [1845] Problem 4 2511
6.8.14.5 [1846] Problem 5 2512
6.8.14.6 [1847] Problem 6 2513

6.8.14.1 [1842] Problem 1

problem number 1842

Added Oct 18, 2019.

Problem Chapter 8.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sin^n(\lambda x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Sin[lambda*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax,z-bx)\exp\left(\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{n+1}(\lambda x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(\lambda x)\right)}{\lambda n+\lambda}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*sin(lambda*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\sin^n(\lambda x))dx}$$

6.8.14.2 [1843] Problem 2

problem number 1843

Added Oct 18, 2019.

Problem Chapter 8.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sin(\lambda z)w_z = (k\sin(\gamma x) + s\sin(\beta y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Sin[lambda*z]*D[w[x,y,z],z]== (k*Sin[gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{k\cos(\gamma x)}{a\gamma}-\frac{s\cos(\beta y)}{b\beta}}c_1\left(y-\frac{bx}{a},\frac{\log\left(\tan\left(\frac{\lambda z}{2}\right)\right)}{\lambda}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*sin(lambda*z)*diff(w(x,y,z),z)= (k*sin(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\text{RootOf}\left(\lambda z-\arctan\left(\frac{2\_Z\,\mathrm{e}^{\frac{c\lambda x}{a}}}{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}+1},-\frac{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}-1}{\_Z^2\mathrm{e}^{\frac{2c\lambda x}{a}}+1}\right)\right)\right)}{c\lambda}\right)\mathrm{e}^{\frac{-a\gamma s\cos(\beta y)-b\beta k\cos(\gamma}{ab\beta\gamma}}$$

6.8.14.3 [1844] Problem 3

problem number 1844

Added Oct 18, 2019.

Problem Chapter 8.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\sin^n(\lambda x)w_y+b\sin^m(\beta x)w_z=c\sin^k(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*z]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*z)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y + \int a(\sin^n(\lambda x))\,dx, \int \frac{a(\sin^{-m}(\beta z))}{b}dz - \left(\int^y \left(\sin^{-n}\left(\lambda\,\mathrm{RootOf}\left(\_b - y + \int\right.\right.\right.\right.$$

6.8.14.4 [1845] Problem 4

problem number 1845

Added Oct 18, 2019.

Problem Chapter 8.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\sin^m(\beta y)w_z = \left(c\sin^k(\gamma y) + s\sin^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*sin(beta*y)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\sin^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sin^m\left(\left(a\left(\int (\sin^n(\_b\lambda))\,d\_b\right) + y - \left(\int a(\right.\right.\right.\right.\right.$$

6.8.14.5 [1846] Problem 5

problem number 1846

Added Oct 18, 2019.

Problem Chapter 8.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\sin(\beta y)w_y + c\sin(\lambda x)w_z = k\sin(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Sin[beta*y]*D[w[x, y,z], y] +  c*Sin[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{\log\left(\tan\left(\frac{\beta y}{2}\right)\right)}{\beta}-\frac{bx}{a},z-\frac{c\sqrt{\cos^2(\lambda x)}\sec(\lambda x)\sin^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+}{2}\right.}{a\lambda m+a\lambda}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*sin(beta*y)*diff(w(x,y,z),y)+ c*sin(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{a\ln\left(\text{RootOf}\left(\beta y-\arctan\left(\frac{2\_Z\,e^{\frac{b\beta x}{a}}}{\_Z^2e^{\frac{2b\beta x}{a}}+1},-\frac{\_Z^2e^{\frac{2b\beta x}{a}}-1}{\_Z^2e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta},z-\left(\int\frac{c(\sin^m(\lambda x))}{a}dx\right)\right.$$

6.8.14.6 [1847] Problem 6

problem number 1847

Added Oct 18, 2019.

Problem Chapter 8.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sin^{n_1}(\lambda_1 x) w_x + b_1 \sin^{m_1}(\beta_1 y) w_y + c_1 \sin^{k_1}(\gamma_1 z) w_z = \left(a_2 \sin^{n_2}(\lambda_2 x) + b_2 \sin^{m_2}(\beta_2 y) + c_2 \sin^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Sin[beta1*y]^m1*D[w[x, y,z], y] + c1*Sin[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*sin(beta1*y)^m1*diff(w(x,y,z),y)+ c1*sin(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\sin^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\sin^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\sin^{-m1}(\_f\beta 1)\right)\left(\sin\right.}{}\right.\right.$$

6.8.15 6.2

Local contents

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6.8.15.2 [1849] Problem 2 2514
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6.8.15.1 [1848] Problem 1

problem number 1848

Added Oct 18, 2019.

Problem Chapter 8.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cos^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cos[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1(y-ax,z-bx)\exp\left(-\frac{c\sqrt{\sin^2(\beta x)}\csc(\beta x)\cos^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n}{}\right.}{\beta n+\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cos(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cos^n(\beta x))dx}$$

6.8.15.2 [1849] Problem 2

problem number 1849

Added Oct 18, 2019.

Problem Chapter 8.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cos(\beta z)w_z = (k\cos(\lambda x) + s\cos(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cos[beta*z]*D[w[x,y,z],z]== (k*Cos[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{k\sin(\lambda x)}{a\lambda}+\frac{s\sin(\gamma y)}{b\gamma}}c_1\left(y-\frac{bx}{a},-\frac{\cosh^{-1}\left(-\frac{\sec(\beta z)\left(2\left(2\sec(\beta z)\sqrt{\sin^2(\beta z)\cos^2(\beta z)\sinh^2\left(\frac{\beta cx}{a}\right)\left(\cosh\left(\frac{4\beta cx}{a}\right.\right.\right.}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*cos(beta*z)*diff(w(x,y,z),z)= (k*cos(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ay-bx}{a},\frac{a\ln\left(\mathrm{RootOf}\left(\beta z-\arctan\left(\frac{\_Z^2e^{\frac{2\beta cx}{a}}-1}{\_Z^2e^{\frac{2\beta cx}{a}}+1},\frac{2\_Z\,e^{\frac{\beta cx}{a}}}{\_Z^2e^{\frac{2\beta cx}{a}}+1}\right)\right)\right)}{\beta c}\right)e^{\frac{a\lambda s\sin(\gamma y)+b\gamma k\sin(\lambda x}{ab\gamma\lambda}}$$

6.8.15.3 [1850] Problem 3

problem number 1850

Added Oct 18, 2019.

Problem Chapter 8.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cos^n(\beta x)w_y+b\cos^k(\lambda x)w_z=c\cos^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(-\frac{c\sqrt{\sin^2(\gamma x)}\csc(\gamma x)\cos^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(\gamma x)\right)}{\gamma m+\gamma}\right)c_1\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\cos^n(\beta x))\,dx\right),z-\left(\int b(\cos^k(\lambda x))\,dx\right)\right)e^{\int c(\cos^m(\gamma x))dx}$$

6.8.15.4 [1851] Problem 4

problem number 1851

Added Oct 18, 2019.

Problem Chapter 8.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cos^n(\beta x)w_y+b\cos^m(\gamma y)w_z=\left(c\cos^k(\gamma y)+s\cos^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[beta*x]^n*D[w[x, y,z], y] +  b*Cos[gamma*y]^m*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(beta*x)^n*diff(w(x,y,z),y)+ b*cos(gamma*y)^m*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cos^n(\beta x))\,dx\right), z - \left(\int^x b\left(\cos^m\left(\left(a\left(\int(\cos^n(\_b\beta))\,d\_b\right) + y - \left(\int a\right.\right.\right.\right.\right.$$

6.8.15.5 [1852] Problem 5

problem number 1852

Added Oct 18, 2019.

Problem Chapter 8.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cos(\beta y)w_y + c\cos(\lambda x)w_z = k\cos(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cos[beta*y]*D[w[x, y,z], y] +  c*Cos[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{w(x,y,z)\to c_1\left(-\frac{\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.}{\beta}\right.\right.$$

$$\left\{\left(\cosh^{-1}\left(-\frac{\sec(\beta y)\left(2\left(2\sec(\beta y)\sqrt{\sin^2(\beta y)\cos^2(\beta y)\sinh^2\left(\frac{b\beta x}{a}\right)\left(\cosh\left(\frac{4b\beta x}{a}\right)-\sinh\left(\frac{4b\beta x}{a}\right)\right)}+\sinh^3\left(\frac{b\beta x}{a}\right)+\text{si}\right.\right.}{4\cosh\left(\frac{2b\beta x}{a}\right)-4\sinh}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*cos(beta*y)*diff(w(x,y,z),y)+ c*cos(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{a\ln\left(\text{RootOf}\left(\beta y - \arctan\left(\frac{\_Z^2 e^{\frac{2b\beta x}{a}}-1}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}, \frac{2\_Z e^{\frac{b\beta x}{a}}}{\_Z^2 e^{\frac{2b\beta x}{a}}+1}\right)\right)\right)}{b\beta}, z - \left(\int \frac{c(\cos^m(\lambda x))}{a} dx\right)\right)$$

6.8.15.6 [1853] Problem 6

problem number 1853

Added Oct 18, 2019.

Problem Chapter 8.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \cos^{n_1}(\lambda_1 x) w_x + b_1 \cos^{m_1}(\beta_1 y) w_y + c_1 \cos^{k_1}(\gamma_1 z) w_z = \left(a_2 \cos^{n_2}(\lambda_2 x) + b_2 \cos^{m_2}(\beta_2 y) + c_2 \cos^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cos[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cos[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cos(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cos(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cos^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\cos^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\cos^{-m1}(\_f\beta 1)\right)\left(c\right.}{}\right.\right.$$

6.8.16 6.3

Local contents

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6.8.16.1 [1854] Problem 1

problem number 1854

Added Oct 18, 2019.

Problem Chapter 8.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tan^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Tan[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\tan^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(\beta x)\right)}{\beta n+\beta}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*tan(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\tan^n(\beta x))dx}$$

6.8.16.2 [1855] Problem 2

problem number 1855

Added Oct 18, 2019.

Problem Chapter 8.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\tan(\beta z)w_z=(k\tan(\lambda x)+s\tan(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Tan[beta*z]*D[w[x,y,z],z]== (k*Tan[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\cos^{-\frac{k}{a\lambda}}(\lambda x)\cos^{-\frac{s}{b\gamma}}(\gamma y)c_1\left(y-\frac{bx}{a},\frac{\log(\sin(\beta z))}{\beta}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*tan(beta*z)*diff(w(x,y,z),z)= (k*tan(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\tan^2(\gamma y)+1\right)^{\frac{s}{2b\gamma}} \left(\tan^2(\lambda x)+1\right)^{\frac{k}{2a\lambda}} \_F1\left(\frac{ay-bx}{a}, \frac{-\beta c x + a\ln\left(\frac{\tan(\beta z)}{\sqrt{\tan^2(\beta z)+1}}\right)}{\beta c}\right)$$

6.8.16.3 [1856] Problem 3

problem number 1856

Added Oct 18, 2019.

Problem Chapter 8.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tan^n(\beta x) w_y + b\tan^k(\lambda x) w_z = c\tan^m(\gamma x) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Tan[beta*x]^n*D[w[x, y,z], y] +  b*Tan[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\frac{c\tan^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(\gamma x)\right)}{\gamma m+\gamma}\right) c_1\left(y - \frac{a\tan^{n+1}(\beta x}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*tan(beta*x)^n*diff(w(x,y,z),y)+ b*tan(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\tan^n(\beta x))\,dx\right), z - \left(\int b(\tan^k(\lambda x))\,dx\right)\right) e^{\int c(\tan^m(\gamma x))dx}$$

6.8.16.4 [1857] Problem 4

problem number 1857

Added Oct 18, 2019.

Problem Chapter 8.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\tan(\beta y)w_y + c\tan(\lambda x)w_z = k\tan(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Tan[beta*y]*D[w[x, y,z], y] +  c*Tan[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{\log(\sin(\beta y))}{\beta} - \frac{bx}{a}, \frac{-c\tan^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\tan^2(\lambda x)\right) +}{a\lambda m + a\lambda}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*tan(beta*y)*diff(w(x,y,z),y)+ c*tan(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-b\beta x + a\ln\left(\frac{\tan(\beta y)}{\sqrt{\tan^2(\beta y)+1}}\right)}{b\beta}, z - \left(\int \frac{c(\tan^m(\lambda x))}{a}dx\right)\right) e^{\int^x -\frac{k\tan\left(\left(-z-\left(\int \frac{c(\tan^m(\_b\lambda))}{a}d\_b\right)+\right.\right.}{a}}$$

6.8.16.5 [1858] Problem 5

problem number 1858

Added Oct 18, 2019.

Problem Chapter 8.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\tan^{n_1}(\lambda_1 x)w_x + b_1\tan^{m_1}(\beta_1 y)w_y + c_1\tan^{k_1}(\gamma_1 z)w_z = \left(a_2\tan^{n_2}(\lambda_2 x) + b_2\tan^{m_2}(\beta_2 y) + c_2\tan^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Tan[beta1*y]^m1*D[w[x, y,z], y] + c1*Tan[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*tan(beta1*y)^m1*diff(w(x,y,z),y)+ c1*tan(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\tan^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\tan^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\tan^{-m1}(\_f\beta 1)\right)\left(t\right.}{}\right.\right.$$

6.8.17 6.4

Local contents

6.8.17.1 [1859] Problem 1 2525
6.8.17.2 [1860] Problem 2 2526
6.8.17.3 [1861] Problem 3 2527
6.8.17.4 [1862] Problem 4 2528
6.8.17.5 [1863] Problem 5 2529

6.8.17.1 [1859] Problem 1

problem number 1859

Added Oct 18, 2019.

Problem Chapter 8.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cot^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*D[w[x, y,z], y] +  b*D[w[x,y,z],z]== c*Cot[beta*x]^n*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(-\frac{c\cot^{n+1}(\beta x)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(\beta x)\right)}{\beta n+\beta}\right)\right\}\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*cot(beta*x)^n*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c(\cot^n(\beta x))dx}$$

6.8.17.2 [1860] Problem 2

problem number 1860

Added Oct 18, 2019.

Problem Chapter 8.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\cot(\beta z)w_z=(k\cot(\lambda x)+s\cot(\gamma y))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +  c*Cot[beta*z]*D[w[x,y,z],z]== (k*Cot[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \sin^{\frac{k}{a\lambda}}(\lambda x)e^{\frac{s(\log(\tan(\gamma y))+\log(\cos(\gamma y)))}{b\gamma}}c_1\left(y-\frac{bx}{a},\frac{\log(\sec(\beta z))}{\beta}-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*diff(w(x,y,z),y)+ c*cot(beta*z)*diff(w(x,y,z),z)= (k*cot(lambda
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\cot^2(\gamma y)+1\right)^{-\frac{s}{2b\gamma}} \left(\cot^2(\lambda x)+1\right)^{-\frac{k}{2a\lambda}} \_F1\left(\frac{-ay+bx}{b}, \frac{-2\beta cy + b\ln\left(\cot^2(\beta z)+1\right) - 2b\ln}{2\beta c}\right.$$

6.8.17.3 [1861] Problem 3

problem number 1861

Added Oct 18, 2019.

Problem Chapter 8.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cot^n(\beta x)w_y + b\cot^k(\lambda x)w_z = c\cot^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cot[beta*x]^n*D[w[x, y,z], y] +  b*Cot[lambda*x]^k*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(-\frac{c\cot^{m+1}(\gamma x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\gamma x)\right)}{\gamma m+\gamma}\right) c_1\left(\frac{b\cot^{k+1}(\lambda x)\,\text{H}}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cot(beta*x)^n*diff(w(x,y,z),y)+ b*cot(lambda*x)^k*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cot^n(\beta x))\,dx\right), z - \left(\int b(\cot^k(\lambda x))\,dx\right)\right) e^{\int c(\cot^m(\gamma x))dx}$$

6.8.17.4 [1862] Problem 4

problem number 1862

Added Oct 18, 2019.

Problem Chapter 8.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\cot(\beta y)w_y + c\cot(\lambda x)w_z = k\cot(\gamma z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x, y,z], x] + b*Cot[beta*y]*D[w[x, y,z], y] +  c*Cot[lambda*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z) \to c_1\left(\frac{c\cot^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\lambda x)\right) + a\lambda mz + a\lambda z}{a\lambda m + a\lambda}, \frac{\log(\sec(\beta$$

$$\left\{w(x,y,z) \to c_1\left(\frac{c\cot^{m+1}(\lambda x)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\cot^2(\lambda x)\right) + a\lambda mz + a\lambda z}{a\lambda m + a\lambda}, \frac{\log(\sec(\beta$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+b*cot(beta*y)*diff(w(x,y,z),y)+ c*cot(lambda*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{2b\beta x - a\ln\left(\cot^2(\beta y)+1\right) + 2a\ln\left(\cot(\beta y)\right)}{2b\beta}, z - \left(\int^y \frac{c\left(\frac{\cot\left(\frac{a\lambda\ln(\cot(\_a\beta))}{b\beta}\right)\cot\left(\frac{(2b\beta x - a\ln}{}\right.}{\cot\left(\frac{a\lambda\ln(\cot(\_}{b\beta}\right.}\right.}{}\right.\right.$$

6.8.17.5 [1863] Problem 5

problem number 1863

Added Oct 18, 2019.

Problem Chapter 8.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1\cot^{n_1}(\lambda_1 x)w_x + b_1\cot^{m_1}(\beta_1 y)w_y + c_1\cot^{k_1}(\gamma_1 z)w_z = \left(a_2\cot^{n_2}(\lambda_2 x) + b_2\cot^{m_2}(\beta_2 y) + c_2\cot^{k_2}(\gamma_2 z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Cot[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*cot(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cot^{-m1}(\beta 1y)\right) dy\right) + \int \frac{b1\left(\cot^{-k1}(\gamma 1z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\cot^{-m1}(\_f\beta 1)\right)\left(c\right.}{}\right.\right.$$

6.8.18 6.5

Local contents

6.8.18.1 [1864] Problem 1 2530
6.8.18.2 [1865] Problem 2 2531
6.8.18.3 [1866] Problem 3 2532
6.8.18.4 [1867] Problem 4 2533
6.8.18.5 [1868] Problem 5 2534

6.8.18.1 [1864] Problem 1

problem number 1864

Added Oct 18, 2019.

Problem Chapter 8.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sin^n(\lambda x)w_y + b\cos^m(\beta x)w_z = c\sin^k(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Sin[lambda*x]^n*D[w[x, y,z], y] +  b*Cos[beta*x]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{\cos^2(\gamma x)}\sec(\gamma x)\sin^{k+1}(\gamma x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\sin^2(\gamma x)\right)}{\gamma k+\gamma}\right)c_1\left(\frac{b}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*sin(lambda*x)^n*diff(w(x,y,z),y)+ b*cos(beta*x)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y-\left(\int a(\sin^n(\lambda x))\,dx\right),z-\left(\int b(\cos^m(\beta x))\,dx\right)\right)e^{\int c(\sin^k(\gamma x))\,dx}$$

6.8.18.2 [1865] Problem 2

problem number 1865

Added Oct 18, 2019.

Problem Chapter 8.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+a\cos^n(\lambda x)w_y+b\sin^m(\beta y)w_z=\left(c\cos^k(\gamma y)+s\sin^r(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Sin[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+b*cos(lambda*x)^n*diff(w(x,y,z),y)+  b*sin(beta*y)^m*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int b(\cos^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\sin^m\left(\left(b\left(\int(\cos^n(\_b\lambda))\,d\_b\right) + y - \left(\int b(\right.\right.\right.\right.$$

6.8.18.3 [1866] Problem 3

problem number 1866

Added Oct 18, 2019.

Problem Chapter 8.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cos^n(\lambda x)w_y + b\tan^m(\beta y)w_z = \left(c\cos^k(\gamma y) + s\tan^k(\mu z)\right)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, y,z], x] + a*Cos[lambda*x]^n*D[w[x, y,z], y] +  b*Tan[beta*y]^m*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+a*cos(lambda*x)^n*diff(w(x,y,z),y)+ b*tan(beta*y)^m*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y - \left(\int a(\cos^n(\lambda x))\,dx\right), z - \left(\int^x b\left(\frac{-\tan\left(\left(y - \left(\int a(\cos^n(\lambda x))\,dx\right)\right)\beta\right) - \tan\left(a\beta\right.}{\tan\left(\left(y - \left(\int a\left(\cos^n(\lambda x)\right)dx\right)\right)\beta\right)\tan\left(a\beta\left(\int\right.\right.}\right.\right.$$

6.8.18.4 [1867] Problem 4

problem number 1867

Added Oct 18, 2019.

Problem Chapter 8.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \sin^{n_1}(\lambda_1 x) w_x + b_1 \cot^{m_1}(\beta_1 y) w_y + c_1 \cos^{k_1}(\gamma_1 z) w_z = \left(a_2 \cos^{n_2}(\lambda_2 x) + b_2 \sin^{m_2}(\beta_2 y) + c_2 \cos^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Sin[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cos[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*sin(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cos(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\frac{\cos(\beta 1 y)}{\sin(\beta 1 y)}\right)^{-m1} dy\right) + \int \frac{b1\left(\cos^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\frac{\cos(\_f\beta 1)}{\sin(\_f\beta 1)}\right)^{-m1}\left(s\right.}{}\right.\right.$$

6.8.18.5 [1868] Problem 5

problem number 1868

Added Oct 18, 2019.

Problem Chapter 8.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 \tan^{n_1}(\lambda_1 x) w_x + b_1 \cot^{m_1}(\beta_1 y) w_y + c_1 \cot^{k_1}(\gamma_1 z) w_z = \left(a_2 \cot^{n_2}(\lambda_2 x) + b_2 \tan^{m_2}(\beta_2 y) + c_2 \cot^{k_2}(\gamma_2 z)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Tan[lambda1*z]^n1*D[w[x, y,z], x] + b1*Cot[beta1*y]^m1*D[w[x, y,z], y] + c1*Cot[ga
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a1*tan(lambda1*z)^n1*diff(w(x,y,z),x)+ b1*cot(beta1*y)^m1*diff(w(x,y,z),y)+ c1*cot(g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \left(\cot^{-m1}(\beta 1 y)\right) dy\right) + \int \frac{b1\left(\cot^{-k1}(\gamma 1 z)\right)}{c1} dz, x - \left(\int^{y} \frac{a1\left(\frac{\cos(\_f\beta 1)}{\sin(\_f\beta 1)}\right)^{-m1}\left(\sin\right.}{\left.\cos\right.}\right.\right.$$

6.8.19 7.1

Local contents

6.8.19.1 [1869] Problem 1 2535
6.8.19.2 [1870] Problem 2 2536
6.8.19.3 [1871] Problem 3 2537
6.8.19.4 [1872] Problem 4 2538
6.8.19.5 [1873] Problem 5 2539
6.8.19.6 [1874] Problem 6 2540

6.8.19.1 [1869] Problem 1

problem number 1869

Added Nov 30, 2019.

Problem Chapter 8.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arcsin^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcSin[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{ic\sin^{-1}(\beta x)^n\left(\sin^{-1}(\beta x)^2\right)^{-n}\left(\left(-i\sin^{-1}(\beta x)\right)^n\text{Gamma}\left(n+1,i\,s\right.\right.}{2\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arcsin(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\arcsin(\beta x)^n dx}$$

6.8.19.2 [1870] Problem 2

problem number 1870

Added Nov 30, 2019.

Problem Chapter 8.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=(b_1\arcsin(\lambda_1x)+b_2\arcsin(\lambda_2y)+b_3\arcsin(\lambda_3z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcSin[lambda1*x]+b2*ArcSin[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)\exp\left(\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\sin^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}\sqrt{1-\text{l}}}{\text{a2lar}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arcsin(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right)\mathrm{e}^{\frac{\sqrt{-y^2\lambda_2^2+1}\,a_1a_3b_2\lambda_1\lambda_3+\left(\sqrt{-z^2\lambda_3^2+1}\,a_1a_2b_3\lambda_1+\left(\sqrt{-\lambda_1^2x^2+1}\,a_2a_3b_1+(a_2a_3b_1x\arcsin(x\lambda}{a_1a_2a_3\lambda_1\lambda_2\lambda_3}}$$

6.8.19.3 [1871] Problem 3

problem number 1871

Added Nov 30, 2019.

Problem Chapter 8.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arcsin^n(\lambda x)\arcsin^k(\beta z)w_z=s\arcsin^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcSin[lambda*x]^n*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\frac{2\left(\frac{(k-1)\left(\arcsin(\lambda x)^n - \frac{\text{LommelS1}\left(n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda x)\right)}{\sqrt{\arcsin(\lambda x)}}\right)(-\lambda^2x^2+1)\beta c\lambda x 2^n 2^{-n}}{2} + \frac{(\lambda x-1)(\lambda x+1)(k}{}\right.}{}\right.$$

6.8.19.4 [1872] Problem 4

problem number 1872

Added Nov 30, 2019.

Problem Chapter 8.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arcsin^n(\lambda x)\arcsin^m(\beta y)\arcsin^k(\gamma z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcSin[lambda*x]^n*ArcSin[beta*y]^m*ArcSin[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arcsin(lambda*x)^n*arcsin(beta*y)^m*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, -\left(\int^{x} \arcsin(\_a\lambda)^{n} \arcsin\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right)^{m} d\_a\right) - \frac{\left(-\gamma k z 2^{k}\,\mathrm{Lo}\right.}{}$$

6.8.19.5 [1873] Problem 5

problem number 1873

Added Nov 30, 2019.

Problem Chapter 8.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arcsin^n(\lambda x)w_y + c\arcsin^k(\beta z)w_z = s\arcsin^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcSin[lambda*x]^n*D[w[x,y,z],y]+c*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} - \frac{i\sin^{-1}(\beta z)^{-k}\left((-i\sin^{-1}(\beta z))^{k}\,\mathrm{Gamma}\left(1-k,-i\sin^{-1}(\beta z)\right) - (i\sin^{-1}(\beta z))\right.}{2\beta}\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*x)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.8.19.6 [1874] Problem 6

problem number 1874

Added Nov 30, 2019.

Problem Chapter 8.7.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + b\arcsin^n(\lambda y)w_y + c\arcsin^k(\beta z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcSin[lambda*y]^n*D[w[x,y,z],y]+c*ArcSin[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arcsin(lambda*y)^n*diff(w(x,y,z),y)+ c*arcsin(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\frac{\left(-\arcsin(\lambda y)^{-n}\arcsin(\lambda y)^{\frac{3}{2}} + \text{LommelS1}\left(-n+\frac{3}{2},\frac{1}{2},\arcsin(\lambda y)\right)\arcsin(\lambda y)\right)\sqrt{-}}{}\right.$$

6.8.20 7.2

Local contents

6.8.20.1 [1875] Problem 1 2541
6.8.20.2 [1876] Problem 2 2542
6.8.20.3 [1877] Problem 3 2543
6.8.20.4 [1878] Problem 4 2544
6.8.20.5 [1879] Problem 5 2545
6.8.20.6 [1880] Problem 6 2546

6.8.20.1 [1875] Problem 1

problem number 1875

Added Nov 30, 2019.

Problem Chapter 8.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arccos^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcCos[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\frac{c\cos^{-1}(\beta x)^n\left(\cos^{-1}(\beta x)^2\right)^{-n}\left((-i\cos^{-1}(\beta x))^n\,\text{Gamma}\left(n+1,i\,c\right.\right.}{2\beta}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccos(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-ax+y,-bx+z\right)\mathrm{e}^{\int c\arccos(\beta x)^n dx}$$

6.8.20.2 [1876] Problem 2

problem number 1876

Added Nov 30, 2019.

Problem Chapter 8.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=\left(b_1\arccos(\lambda_1x)+b_2\arccos(\lambda_2y)+b_3\arccos(\lambda_3z)\right)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcCos[lambda1*x]+b2*ArcCos[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{\text{a2}x}{\text{a1}},z-\frac{\text{a3}x}{\text{a1}}\right)\exp\left(-\frac{\text{b1}\sqrt{1-\text{lambda1}^2x^2}}{\text{a1lambda1}}+\frac{\text{b1}x\cos^{-1}(\text{lambda1}x)}{\text{a1}}+\frac{\text{b2}x\sin^{-}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arccos(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right)\mathrm{e}^{\frac{-\sqrt{-y^2\lambda_2^2+1}\,a_1a_3b_2\lambda_1\lambda_3+\left(-\sqrt{-z^2\lambda_3^2+1}\,a_1a_2b_3\lambda_1+\left(-\sqrt{-\lambda_1^2x^2+1}\,a_2a_3b_1+(a_2a_3b_1x\,\mathrm{arcc}\right.\right.}{a_1a_2a_3\lambda_1\lambda_2\lambda_3}}$$

6.8.20.3 [1877] Problem 3

problem number 1877

Added Nov 30, 2019.

Problem Chapter 8.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arccos^n(\lambda x)\arccos^k(\beta z)w_z=s\arccos^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCos[lambda*x]^n*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{\sqrt{\pi}\left(-\frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\sqrt{\arccos(\lambda x)}}{\sqrt{\pi}\,(n+2)} + \frac{\sqrt{-\lambda^2x^2+1}\,2^{-n}\arccos(\lambda x)^n}{\sqrt{\pi}\,(n+2)}\right.}{\lambda}\right.$$

6.8.20.4 [1878] Problem 4

problem number 1878

Added Nov 30, 2019.

Problem Chapter 8.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\arccos^n(\lambda x)\arccos^m(\beta y)\arccos^k(\gamma z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCos[lambda*x]^n*ArcCos[beta*y]^m*ArcCos[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccos(lambda*x)^n*arccos(beta*y)^m*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \frac{-(k-2)\,c\gamma\left(\int^{x} \arccos(\_a\lambda)^{n} \arccos\left(\frac{(ay-(-\_a+x)b)\beta}{a}\right)^{m} d\_a\right) + \frac{\left((k-2)\gamma z\,\mathrm{Lom}\right.}{}}{}\right.$$

6.8.20.5 [1879] Problem 5

problem number 1879

Added Nov 30, 2019.

Problem Chapter 8.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda x)w_y + c\arccos^k(\beta z)w_z = s\arccos^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCos[lambda*x]^n*D[w[x,y,z],y]+c*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cx}{a} + \frac{\cos^{-1}(\beta z)^{-k}\left((-i\cos^{-1}(\beta z))^{k}\,\mathrm{Gamma}\left(1-k, -i\cos^{-1}(\beta z)\right) + (i\cos^{-1}(\beta z))^{k}\right.}{2\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*x)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\frac{\left(-\operatorname{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(\lambda x)\right)\arccos(\lambda x)+\arccos(\lambda x)^{n+\frac{3}{2}}+(n+2)\operatorname{LommelS1}\left(n+\frac{1}{2},\frac{1}{2},\arccos(\lambda x)\right)\right)\sqrt{-\lambda^2x^2+}}{\sqrt{\arccos(\lambda x)}}}{(n+}\right.$$

6.8.20.6 [1880] Problem 6

problem number 1880

Added Nov 30, 2019.

Problem Chapter 8.7.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arccos^n(\lambda y)w_y + c\arccos^k(\beta z)w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCos[lambda*y]^n*D[w[x,y,z],y]+c*ArcCos[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccos(lambda*y)^n*diff(w(x,y,z),y)+ c*arccos(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(x + \frac{\sqrt{\pi}\left(\frac{\sqrt{-\lambda^2y^2+1}\,2^n\,\mathrm{LommelS1}\left(-n+\frac{3}{2},\frac{3}{2},\arccos(\lambda y)\right)\sqrt{\arccos(\lambda y)}}{\sqrt{\pi}\,(n-2)} - \frac{\sqrt{-\lambda^2y^2+1}\,2^n\arccos(\lambda y)^{-n+1}}{\sqrt{\pi}\,(n-2)} + \frac{3}{}\right.}{b\lambda}\right.$$

6.8.21 7.3

Local contents

6.8.21.1 [1881] Problem 1 2547
6.8.21.2 [1882] Problem 2 2548
6.8.21.3 [1883] Problem 3 2549
6.8.21.4 [1884] Problem 4 2550
6.8.21.5 [1885] Problem 5 2551

6.8.21.1 [1881] Problem 1

problem number 1881

Added Nov 30, 2019.

Problem Chapter 8.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\arctan^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcTan[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\int_1^x c\tan^{-1}(\beta K[1])^n dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arctan(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\arctan(\beta x)^n dx}$$

6.8.21.2 [1882] Problem 2

problem number 1882

Added Nov 30, 2019.

Problem Chapter 8.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1w_x+a_2w_y+a_3w_z=(b_1\arctan(\lambda_1x)+b_2\arctan(\lambda_2y)+b_3\arctan(\lambda_3z))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcTan[lambda1*x]+b2*ArcTan[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\left(\text{lambda1}^2x^2+1\right)^{-\frac{\text{b1}}{\text{2a1lambda1}}}\left(\text{a1}^2\left(\text{lambda2}^2y^2+1\right)\right)^{-\frac{\text{b2}}{\text{2a2lambda2}}}\left(\text{a1}^2\left(\text{lambda3}^2z^2+1\right)\right)^{-\frac{}{\text{2a3}}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arctan(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\lambda_1^2 x^2+1\right)^{-\frac{b_1}{2a_1\lambda_1}} \left(y^2\lambda_2^2+1\right)^{-\frac{b_2}{2a_2\lambda_2}} \left(z^2\lambda_3^2+1\right)^{-\frac{b_3}{2a_3\lambda_3}} \_F1\left(\frac{ya_1-xa_2}{a_1},\frac{za_1-a_3x}{a_1}\right) \mathrm{e}^{\frac{a_1a_3b_2y\arctan}{}}$$

6.8.21.3 [1883] Problem 3

problem number 1883

Added Nov 30, 2019.

Problem Chapter 8.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+bw_y+c\arctan^n(\lambda x)\arctan^k(\beta z)w_z = s\arctan^m(\gamma x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcTan[lambda*x]^n*ArcTan[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\gamma^2x^2+1\right)^{-\frac{s}{2a\gamma}} \_F1\left(\frac{ay-bx}{a},-\left(\int \arctan(\lambda x)^n\,dx\right)+\int \frac{a\arctan(\beta z)^{-k}}{c}dz\right) \mathrm{e}^{\frac{sx\arctan(\gamma x)}{a}}$$

6.8.21.4 [1884] Problem 4

problem number 1884

Added Nov 30, 2019.

Problem Chapter 8.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c \arctan^n(\lambda x) \arctan^m(\beta y) \arctan^k(\gamma z) w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcTan[lambda*x]^n*ArcTan[beta*y]^m*ArcTan[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arctan(lambda*x)^n*arctan(beta*y)^m*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \_F1\left(\frac{ay - bx}{a}, -\left(\int^x \arctan(\lambda\_a)^n \arctan\left(\frac{\beta(\_ab + ay - bx)}{a}\right)^m d\_a\right) + \int \frac{\arctan(\gamma z)}{c}$$

6.8.21.5 [1885] Problem 5

problem number 1885

Added Nov 30, 2019.

Problem Chapter 8.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\arctan^n(\lambda x)w_y + c\arctan^k(\beta z)w_z = s\arctan^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcTan[lambda*x]^n*D[w[x,y,z],y]+c*ArcTan[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^z \frac{s\tan^{-1}(\beta K[3])^{-k}\tan^{-1}\left(\frac{\gamma\left(cx-a\int_1^z \tan^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\tan^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)}{c}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arctan(lambda*x)^n*diff(w(x,y,z),y)+ c*arctan(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-y+\int \frac{b\arctan(\lambda x)^n}{a}dx, \int \frac{b\arctan(\beta z)^{-k}}{c}dz - \left(\int^y \arctan\left(\lambda\,\mathrm{RootOf}\left(\_b - y + \right.\right.\right.\right.$$

6.8.22 7.4

Local contents

6.8.22.1 [1886] Problem 1 2552
6.8.22.2 [1887] Problem 2 2553
6.8.22.3 [1888] Problem 3 2554
6.8.22.4 [1889] Problem 4 2554
6.8.22.5 [1890] Problem 5 2555

6.8.22.1 [1886] Problem 1

problem number 1886

Added Nov 30, 2019.

Problem Chapter 8.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\operatorname{arccot}^n(\beta x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*ArcCot[beta*x]^n * w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1(y-ax,z-bx)\exp\left(\int_1^x c\cot^{-1}(\beta K[1])^n dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)= c*arccot(beta*x)^n*w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1(-ax+y,-bx+z)\,\mathrm{e}^{\int c\left(-\arctan(\beta x)+\frac{\pi}{2}\right)^n dx}$$

6.8.22.2 [1887] Problem 2

problem number 1887

Added Nov 30, 2019.

Problem Chapter 8.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1 w_x + a_2 w_y + a_3 w_z = (b_1 \operatorname{arccot}(\lambda_1 x) + b_2 \operatorname{arccot}(\lambda_2 y) + b_3 \operatorname{arccot}(\lambda_3 z))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*D[w[x,y,z],x]+a2*D[w[x,y,z],y]+a3*D[w[x,y,z],z]== (b1*ArcCot[lambda1*x]+b2*ArcCot[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \left(\text{lambda1}^2 x^2+1\right)^{\frac{\text{b1}}{\text{2a1lambda1}}} \left(\text{a1}^2\left(\text{lambda2}^2 y^2+1\right)\right)^{\frac{\text{b2}}{\text{2a2lambda2}}} \left(\text{a1}^2\left(\text{lambda3}^2 z^2+1\right)\right)^{\frac{\text{b3}}{\text{2a3lambd}}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a__1*diff(w(x,y,z),x)+ a__2*diff(w(x,y,z),y)+ a__3*diff(w(x,y,z),z)= (b__1*arccot(la
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\lambda_1^2 x^2+1\right)^{\frac{b_1}{2a_1\lambda_1}} \left(y^2\lambda_2^2+1\right)^{\frac{b_2}{2a_2\lambda_2}} \left(z^2\lambda_3^2+1\right)^{\frac{b_3}{2a_3\lambda_3}} \_F1\left(\frac{ya_1-xa_2}{a_1}, \frac{za_1-a_3x}{a_1}\right) \mathrm{e}^{\frac{-2a_1a_3b_2y\arctan(y\lambda}{}}$$

6.8.22.3 [1888] Problem 3

problem number 1888

Added Nov 30, 2019.

Problem Chapter 8.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c \operatorname{arccot}^n(\lambda x) \operatorname{arccot}^k(\beta z) w_z = s \operatorname{arccot}^m(\gamma x) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCot[lambda*x]^n*ArcCot[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x, y, z) = \left(\gamma^2 x^2 + 1\right)^{\frac{s}{2a\gamma}} \_F1\left(\frac{ay - bx}{a}, -\left(\int \left(-\arctan(\lambda x) + \frac{\pi}{2}\right)^n dx\right) + \int \frac{a\left(-\arctan(\beta z) + \frac{\pi}{2}\right)^{-}}{c}\right.$$

6.8.22.4 [1889] Problem 4

problem number 1889

Added Nov 30, 2019.

Problem Chapter 8.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c \operatorname{arccot}^n(\lambda x) \operatorname{arccot}^m(\beta y) \operatorname{arccot}^k(\gamma z) w_z = sw$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*ArcCot[lambda*x]^n*ArcCot[beta*y]^m*ArcCot[gamma*z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*arccot(lambda*x)^n*arccot(beta*y)^m*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \int \frac{a\left(-\arctan(\gamma z)+\frac{\pi}{2}\right)^{-k}}{c}dz - \left(\int^{x}\left(-\arctan\left(\frac{(ay-(-\_a+x)\,b)\,\beta}{a}\right)+\right.\right.$$

6.8.22.5 [1890] Problem 5

problem number 1890

Added Nov 30, 2019.

Problem Chapter 8.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + b\operatorname{arccot}^n(\lambda x)w_y + c\operatorname{arccot}^k(\beta z)w_z = s\operatorname{arccot}^m(\gamma x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*ArcCot[lambda*x]^n*D[w[x,y,z],y]+c*ArcCot[beta*z]^k*D[w[x,y,z],z]==
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^z \frac{s\cot^{-1}(\beta K[3])^{-k}\cot^{-1}\left(\frac{\gamma\left(cx-a\int_1^z \cot^{-1}(\beta K[2])^{-k}dK[2]+a\int_1^{K[3]}\cot^{-1}(\beta K[2])^{-k}dK[2]\right)}{c}\right)m}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*arccot(lambda*x)^n*diff(w(x,y,z),y)+ c*arccot(beta*z)^k*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-y+\int\frac{b\left(-\arctan(\lambda x)+\frac{\pi}{2}\right)^n}{a}dx,\int\frac{b\left(-\arctan(\beta z)+\frac{\pi}{2}\right)^{-k}}{c}dz-\left(\int^y\left(-\arctan\left(\right.\right.\right.\right.$$

6.8.23 8.1

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6.8.23.1 [1891] Problem 1

problem number 1891

Added December 1, 2019.

Problem Chapter 8.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + f(x)w_y + g(x)w_z = (h_2(x)y + h_1(x)z + h_0(x))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+f[x]*D[w[x,y,z],y]+g[x]*D[w[x,y,z],z]==(h2[x]*y+h1[x]*z+h0[x])*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\int_1^x f(K[1])dK[1],z-\int_1^x g(K[2])dK[2]\right)\exp\left(\int_1^x\left(\text{h0}(K[3])+\text{h2}(K[3])\left(y-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*diff(w(x,y,z),y)+ g(x)*diff(w(x,y,z),z)= (h2(x)*y+h1(x)*z+h0(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y-\left(\int f(x)\,dx\right), z-\left(\int g(x)\,dx\right)\right)\mathrm{e}^{\int^x((\int f(\_f)d\_f)h2(\_f)+(\int g(\_f)d\_f)h1(\_f)+h0(\_f)+(z-}$$

6.8.23.2 [1892] Problem 2

problem number 1892

Added December 1, 2019.

Problem Chapter 8.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + f(x)(y + a)w_y + g(x)(z + b)w_z = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+f[x]*(y+a)*D[w[x,y,z],y]+g[x]*(z+b)*D[w[x,y,z],z]==h[x]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x h(K[5])dK[5]\right) c_1\left(y\exp\left(-\int_1^x f(K[1])dK[1]\right) - \int_1^x a\exp\left(-\int_1^{K[2]} f(K[1]\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ f(x)*(y+a)*diff(w(x,y,z),y)+ g(x)*(z+b)*diff(w(x,y,z),z)= h(x)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left((a+y)\,\mathrm{e}^{-(\int f(x)dx)}, (b+z)\,\mathrm{e}^{-(\int g(x)dx)}\right)\mathrm{e}^{\int h(x)dx}$$

6.8.23.3 [1893] Problem 3

problem number 1893

Added December 1, 2019.

Problem Chapter 8.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + f(x))\, w_y + (bz + g(x))\, w_z = h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+f[x])*D[w[x,y,z],y]+(b*z+g[x])*D[w[x,y,z],z]==h[x]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x h(K[3])dK[3]\right) c_1\left(ye^{-ax} - \int_1^x e^{-aK[1]} f(K[1])dK[1], ze^{-bx} - \int_1^x e^{-bK[2]} g(K[2]\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+f(x))*diff(w(x,y,z),y)+ (b*z+g(x))*diff(w(x,y,z),z)= h(x)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-ax} - \left(\int \mathrm{e}^{-ax} f(x)\,dx\right), z\,\mathrm{e}^{-bx} - \left(\int \mathrm{e}^{-bx} g(x)\,dx\right)\right) \mathrm{e}^{\int h(x)dx}$$

6.8.23.4 [1894] Problem 4

problem number 1894

Added December 1, 2019.

Problem Chapter 8.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))\, w_y + (g_1(x)y + g_2(x))\, w_z = (h_2(x)y + h_1(x)z + h_0(x))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*y+g2[x])*D[w[x,y,z],z]==(h2[x]*y+h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[3]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[3])dK[3],-\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*y+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-(\int f1(x)dx)}-\left(\int \mathrm{e}^{-(\int f1(x)dx)}f2(x)\,dx\right),z-\left(\int^x\left(y\,\mathrm{e}^{-(\int f1(x)dx)}\mathrm{e}^{\int f1(\_f)d\_f}g1(\_f)\right.\right.\right.$$

6.8.23.5 [1895] Problem 5

problem number 1895

Added December 1, 2019.

Problem Chapter 8.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(f_1(x)y+f_2(x))\,w_y+(g_1(x)z+g_2(x))\,w_z=(h_2(x)y+h_1(x)z+h_0(x))\,w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*z+g2[x])*D[w[x,y,z],z]==(h2[x]*y+h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1]\right)-\int_1^x \exp\left(-\int_1^{K[2]} \mathrm{f1}(K[1])dK[1]\right)\mathrm{f2}(K[2])dK[2],z\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f1(x)*y+f2(x))*diff(w(x,y,z),y)+ (g1(x)*z+g2(x))*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y\,\mathrm{e}^{-\left(\int f1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int f1(x)dx\right)}f2(x)\,dx\right),z\,\mathrm{e}^{-\left(\int g1(x)dx\right)}-\left(\int \mathrm{e}^{-\left(\int g1(x)dx\right)}g2(x)\,dx\right)\right)$$

6.8.23.6 [1896] Problem 6

problem number 1896

Added December 1, 2019.

Problem Chapter 8.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(y^2-a^2+a\lambda\sinh(\lambda x)-a^2\sinh^2(\lambda x)\right)w_y+f(x)\sinh(\gamma z)w_z=g(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(y^2-a^2+a*lambda*Sinh[lambda*x]-a^2*Sinh[lambda*x]^2)*D[w[x,y,z],y]+f[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x g(K[3])dK[3]\right)c_1\left(\frac{\log\left(\tanh\left(\frac{\gamma z}{2}\right)\right)}{\gamma}-\int_1^x f(K[2])dK[2],\frac{2\lambda e^{\frac{ae^{-\lambda x}\left(e^{2\lambda x}-1\right)}{\lambda}+\lambda x}}{ae^{2\lambda x}+a-2ye^{\lambda x}}-\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (y^2-a^2+a*lambda*sinh(lambda*x)-a^2*sinh(lambda*x)^2)*diff(w(x,y,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(-\frac{2\sqrt{\sinh(\lambda x)+i}\left(-\left(-\frac{(\sinh^2(\lambda x))}{2}+i\sinh(\lambda x)+\frac{1}{2}\right)\lambda\,\mathrm{H}\right.}{-\left(\sinh^2(\lambda x)+1\right)(-\sinh(\lambda x)+i)\lambda\,\mathrm{HeunCPrime}\left(\frac{4ia}{\lambda},\frac{1}{2},-\frac{1}{2},\frac{2ia}{\lambda},-\frac{8ia-3\lambda}{8\lambda},-\frac{i\sinh(\lambda}{2}\right.}\right.$$

6.8.23.7 [1897] Problem 7

problem number 1897

Added December 1, 2019.

Problem Chapter 8.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(f_1(x)y+f_2(x)y^k\right)w_y+\left(g_1(x)z+g_2(x)z^m\right)w_z=h(x)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]*z+g2[x]*z^m)*D[w[x,y,z],z]==h
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x h(K[5])dK[5]\right)c_1\left((k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} \text{f1}(K[1])dK[1]\right)\text{f2}(K[2])dK\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)*z+g2(x)*z^m)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}+(k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)}f2(x)\,dx\right),z^{-m+1}\mathrm{e}^{(m-1)\left(\int g1(x)dx\right)}+(\right.$$

6.8.23.8 [1898] Problem 8

problem number 1898

Added December 1, 2019.

Problem Chapter 8.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+\left(f_1(x)y+f_2(x)y^k\right)w_y+\left(g_1(x)+g_2(x)e^{\lambda z}\right)w_z=h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[lambda*z])*D[w[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)*y+f2(x)*y^k)*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(lambda*z))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f1(x)dx\right)} f2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g1(x)dx\right)} g2(x)\,d}{\lambda}\right.$$

6.8.23.9 [1899] Problem 9

problem number 1899

Added December 1, 2019.

Problem Chapter 8.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x) + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x) + g_2(x)e^{\beta z}\right) w_z = h(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+( f1[x]+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[beta*z])*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ ( f1(x)+f2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g1(x)+g2(x)*exp(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{-\lambda\left(\int e^{\lambda\left(\int f1(x)dx\right)} f2(x)\,dx\right) - e^{\left(-y+\int f1(x)dx\right)\lambda}}{\lambda}, \frac{-\beta\left(\int e^{\beta\left(\int g1(x)dx\right)} g2(x)\,dx\right) - e^{\left(-z+\int g1\right.}}{\beta}\right.$$

6.8.24 8.2

Local contents

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6.8.24.1 [1900] Problem 1

problem number 1900

Added December 1, 2019.

Problem Chapter 8.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + g(y)w_y + h(z)w_z = (\varphi(z) + \psi(y) + \chi(z))\,w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x,y,z],x]+g[y]*D[w[x,y,z],y]+h[z]*D[w[x,y,z],z]==(varphi[z]+psi[y]+chi[z])*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ g(y)*diff(w(x,y,z),y)+ h(x)*diff(w(x,y,z),z)=(varphi(z)+psi(y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{1}{f(x)}dx\right) + \int \frac{1}{g(y)}dy, z - \left(\int \frac{h(x)}{f(x)}dx\right)\right) e^{\int^x \frac{\chi\left(z+\int \frac{h(\_g)}{f(\_g)}d\_g - \left(\int \frac{h(x)}{f(x)}dx\right)\right)+\psi\left(\mathrm{RootOf}\left(\right.\right.}{}}$$

6.8.24.2 [1901] Problem 2

problem number 1901

Added December 1, 2019.

Problem Chapter 8.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f(x)w_x + zw_y + g(y)w_z = (h_2(x) + h_1(y))\, w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f[x]*D[w[x,y,z],x]+z*D[w[x,y,z],y]+g[y]*D[w[x,y,z],z]==(h2[x]+h1[y])*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f(x)*diff(w(x,y,z),x)+ z*diff(w(x,y,z),y)+ g(y)*diff(w(x,y,z),z)=(h__2(x)+h__1(y))*w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(z^2 - 2\left(\int g(y)\,dy\right), \int \frac{1}{f(x)}dx - \left(\int^y \frac{1}{\sqrt{z^2 + 2\left(\int g(\_b)\,d\_b\right) - 2\left(\int g(y)\,dy\right)}}d\_b\right.\right.$$

6.8.24.3 [1902] Problem 3

problem number 1902

Added December 1, 2019.

Problem Chapter 8.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)w_x + f_2(x)g(y)w_y + f_3(x)h(z)w_z = f_4(x)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*D[w[x,y,z],x]+f2[x]*g[y]*D[w[x,y,z],y]+f3[x]*h[z]*D[w[x,y,z],z]==f4[x]*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f__1(x)*diff(w(x,y,z),x)+ f__2(x)*g(y)*diff(w(x,y,z),y)+ f__3(x)*h(z)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\int \frac{1}{g(y)}dy - \left(\int \frac{f_2(x)}{f_1(x)}dx\right), \int \frac{1}{h(z)}dz - \left(\int \frac{f_3(x)}{f_1(x)}dx\right)\right) e^{\int \frac{f_4(x)}{f_1(x)}dx}$$

6.8.24.4 [1903] Problem 4

problem number 1903

Added December 1, 2019.

Problem Chapter 8.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))\, w_y + (g_1(x)z + g_2(y))\, w_z = (h_1(x) + h_2(y))\, w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x]*z+g2[x])*D[w[x,y,z],z]==(h1[x]+h2[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x \left(\text{h1}(K[5]) + \text{h2}\left(\exp\left(\int_1^{K[5]} \text{f1}(K[1])dK[1]\right)\left(\exp\left(-\int_1^x \text{f1}(K[1])dK[1]\right) y\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x))*diff(w(x,y,z),y)+ (g__1(x)*z+g__2(x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int g_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int g_1(x)dx\right)} g_2(x)\,dx\right)\right)$$

6.8.24.5 [1904] Problem 5

problem number 1904

Added December 1, 2019.

Problem Chapter 8.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(y)z + g_2(x)z^m\right) w_z = \left(h_1(x) + h_2(y)\right) w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[y]*z+g2[x])*D[w[x,y,z],z]==(h1[x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \exp\left(\int_1^x \left(\mathrm{h1}(K[5]) + \mathrm{h2}\left(\left(\exp\left(-\int_1^x \mathrm{f1}(K[1])dK[1] - (k-1)\int_1^{K[5]} \mathrm{f1}(K[1])dK[1]\right)\right.\right.\right.\right.\right.$$

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(y)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(y)*z+g__2(x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.24.6 [1905] Problem 6

problem number 1905

Added December 1, 2019.

Problem Chapter 8.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x) + g_2(y)e^{\lambda z}\right) w_z = \left(h_1(x) + h_2(y)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x]+g2[x]*Exp[lambda*z])*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x)+g__2(x)*exp(lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), \frac{-\lambda\left(\int \mathrm{e}^{\lambda\left(\int g_1(x)dx\right)} g_2(x)\,d\right.}{\lambda}$$

6.8.24.7 [1906] Problem 7

problem number 1906

Added December 1, 2019.

Problem Chapter 8.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(y)z + g_2(x)z^k\right) w_z = \left(h_1(x) + h_2(y)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[y]*z+g2[x]*z^k)*D[w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(y)*z+g__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.24.8 [1907] Problem 8

problem number 1907

Added December 1, 2019.

Problem Chapter 8.8.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x) + g_2(x)e^{\beta z}\right) w_z = \left(h_1(x) + h_2(y)\right) w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x]+g2[y]*Exp[beta*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x)+g__2(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.25 8.3

Local contents

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6.8.25.1 [1908] Problem 1

problem number 1908

Added Jan 1, 2020.

Problem Chapter 8.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + f(x,y)w_z = g(x,y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+f[x,y]*D[w[x,y,z],z]==g[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x \frac{g\left(K[2],y+\frac{b(K[2]-x)}{a}\right)}{a}dK[2]\right)c_1\left(y-\frac{bx}{a},z-\int_1^x \frac{f\left(K[1],y+\frac{b(K[1]-x)}{a}\right)}{a}dK[\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*diff(w(x,y,z),z)=g(x,y)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, z-\left(\int^x \frac{f\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a\right)\right)e^{\int^x \frac{g\left(\_a,\frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a}$$

6.8.25.2 [1909] Problem 2

problem number 1909

Added Jan 1, 2020.

Problem Chapter 8.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + f(x, y)g(z)w_z = h(x, y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+f[x,y]*g[z]*D[w[x,y,z],z]==h[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=h(x,y)*w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(\frac{ay-bx}{a}, \int \frac{a}{g(z)}dz - \left(\int^x f\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)d\_a\right)\right) e^{\int^x \frac{h\left(\_a, \frac{ay-(-\_a+x)b}{a}\right)}{a}d\_a}$$

6.8.25.3 [1910] Problem 3

problem number 1910

Added Jan 1, 2020.

Problem Chapter 8.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$xw_x + yw_y + (z + f(x, y))w_z = g(x, y)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+y*D[w[x,y,z],y]+(z+f[x,y])*D[w[x,y,z],z]==g[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\int_1^x \frac{g\left(K[2],\frac{yK[2]}{x}\right)}{K[2]}dK[2]\right)c_1\left(\frac{y}{x},\frac{z}{x}-\int_1^x \frac{f\left(K[1],\frac{yK[1]}{x}\right)}{K[1]^2}dK[1]\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ y*diff(w(x,y,z),y)+ (z+f(x,y))*diff(w(x,y,z),z)=g(x,y)*w(x,y,z);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{-x\left(\int^x \frac{f\left(\_a,\frac{-ay}{x}\right)}{\_a^2}d\_a\right)+z}{x}\right)e^{\int^x \frac{g\left(\_a,\frac{-ay}{x}\right)}{\_a}d\_a}$$

6.8.25.4 [1911] Problem 4

problem number 1911

Added Jan 1, 2020.

Problem Chapter 8.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+f(x,y)g(z)w_z=h(x,y)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+f[x,y]*g[z]*D[w[x,y,z],z]==h[x,y]*w[x,y,z];
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ f(x,y)*g(z)*diff(w(x,y,z),z)=h(x,y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(yx^{-\frac{b}{a}}, \int \frac{a}{g(z)}dz - \left(\int^{x} \frac{f\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_a} d\_a\right)\right) e^{\int^{x} \frac{h\left(\_a, y\_a^{\frac{b}{a}} x^{-\frac{b}{a}}\right)}{\_a a} d\_a}$$

6.8.25.5 [1912] Problem 5

problem number 1912

Added Jan 1, 2020.

Problem Chapter 8.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (f_1(x)y + f_2(x))\, w_y + (g_1(x,y)z + g_2(x,y))\, w_z = h(x,y,z)w$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y])*D[w[x,y,z],z]==h[x,y,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)} - \left(\int \mathrm{e}^{-\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z\,\mathrm{e}^{-\left(\int^x g_1\left(\_f,\left(y\,\mathrm{e}^{-\left(\int f_1(x)dx\right)}+\int \mathrm{e}^{-\left(\int f_1(\_f)d\_f\right)} f_2(\_f)\right.\right.\right.}$$

6.8.25.6 [1913] Problem 6

problem number 1913

Added Jan 1, 2020.

Problem Chapter 8.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x,y)z + g_2(x,y)z^m\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*z^m)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y)*z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(y^{-k+1}\mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} + (k-1)\left(\int \mathrm{e}^{(k-1)\left(\int f_1(x)dx\right)} f_2(x)\,dx\right), z^{-m+1}\mathrm{e}^{(m-1)\left(\int^x g_1\left(\_f,\left(y^{-k}\right.\right.\right.}\right.$$

6.8.25.7 [1914] Problem 7

problem number 1914

Added Jan 1, 2020.

Problem Chapter 8.8.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + \left(f_1(x)y + f_2(x)y^k\right) w_y + \left(g_1(x,y)z + g_2(x,y)e^{\lambda z}\right) w_z = h(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*y^k)*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*Exp[lambda*z])*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*y^k)*diff(w(x,y,z),y)+ (g__1(x,y)*z+g__2(x,y)*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.8.25.8 [1915] Problem 8

problem number 1915

Added Jan 1, 2020.

Problem Chapter 8.8.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x, y)z + g_2(x, y)z^k\right) w_z = h(x, y, z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*z^k)*D[w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.25.9 [1916] Problem 9

problem number 1916

Added Jan 1, 2020.

Problem Chapter 8.8.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + \left(f_1(x)y + f_2(x)e^{\lambda y}\right) w_y + \left(g_1(x, y)z + g_2(x, y)e^{\beta z}\right) w_z = h(x, y, z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(f1[x]*y+f2[x]*Exp[lambda*y])*D[w[x,y,z],y]+(g1[x,y]*z+g2[x,y]*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (f__1(x)*y+f__2(x)*exp(lambda*y))*diff(w(x,y,z),y)+ (g__1(x,y)*z+g
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.8.25.10 [1917] Problem 10

problem number 1917

Added Jan 1, 2020.

Problem Chapter 8.8.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + (h_1(x, y) + h_2(x, y)z^m)\, w_z = h_3(x, y, z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x,y,z],x]+f2[x]*g2[y]*D[w[x,y,z],y]+(h1[x,y]+h2[x,y]*z^m)*D[w[x,y,z],
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  f__1(x)*g__1(y)*diff(w(x,y,z),x)+ f__2(x)*g__2(y)*diff(w(x,y,z),y)+ (h__1(x,y)*z+h__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \_F1\left(-\left(\int \frac{f_2(x)}{f_1(x)}dx\right) + \int \frac{g_1(y)}{g_2(y)}dy, c_4\right) \mathrm{e}^{\int^x \frac{h_3\left(\_h, \mathrm{RootOf}\left(\int \frac{f_2(\_h)}{f_1(\_h)} d\_h - \left(\int \frac{f_2(x)}{f_1(x)} dx\right) + \int \frac{g_1(y)}{g_2(y)} dy - \left(\int -Z \frac{g_1(\_}{g_2(\_}\right.\right.}{}}$$

6.8.25.11 [1918] Problem 11

problem number 1918

Added Jan 1, 2020.

Problem Chapter 8.8.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y + \left(h_1(x,y) + h_2(x,y)e^{\lambda z}\right) w_z = h_3(x,y,z)w$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  f1[x]*g1[y]*D[w[x,y,z],x]+f2[x]*g2[y]*D[w[x,y,z],y]+(h1[x,y]+h2[x,y]*Exp[lambda*z])*D
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✗

```
restart;
local gamma;
pde :=  f__1(x)*g__1(y)*diff(w(x,y,z),x)+ f__2(x)*g__2(y)*diff(w(x,y,z),y)+ (h__1(x,y)*z+h__
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

sol=()

6.9 chapter 9

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6.9.1.1 [1919] Problem 1

problem number 1919

Added Jan 6, 2020.

Problem Chapter 9.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + cw_z = (\alpha x + \beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{x(\alpha x+2\beta)}{2a}}c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\alpha x+\beta)^2}{2a\alpha}}(\alpha q-\beta p)\operatorname{erf}\left(\frac{\alpha x+\beta}{\sqrt{2}\sqrt{a}\sqrt{\alpha}}\right)}{\sqrt{a}\alpha^{3/2}}-\frac{p}{\alpha}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,y,z)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(-\frac{\sqrt{2}\sqrt{\pi}\,(-\alpha q+\beta p)\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\frac{\alpha}{a}}\,x}{2}+\frac{\sqrt{2}\,\beta}{2\sqrt{\frac{\alpha}{a}}\,a}\right)\mathrm{e}^{\frac{\beta^2}{2a\alpha}}}{2}+\sqrt{\frac{\alpha}{a}}\left(\alpha\_F1\left(\frac{ay-bx}{a},\frac{az-cx}{a}\right)-p\,\mathrm{e}^{-\frac{(\alpha x+2\beta)x}{2a}}\right)a\right)\mathrm{e}^{\frac{(\alpha x+}{2a}}}{\sqrt{\frac{\alpha}{a}}\,a\alpha}$$

6.9.1.2 [1920] Problem 2

problem number 1920

Added Jan 6, 2020.

Problem Chapter 9.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=(cx+k)w+px+q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==(c*x+k)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(cx+2k)}c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=(c*x+k)*w(x,y,z)+p*x+q;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^{y}\frac{\left(\left(\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{(az^2+(\_b^2-y^2)b)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)\right)p+\sqrt{ab}\,(px+q)\right)\mathrm{e}^{-\int\frac{\left(\ln\left(\frac{\_ba}{}\right.\right.}{}}}{\sqrt{ab}\sqrt{(az^2+(\_b^2-y^2)b)a}}\right.$$

6.9.1.3 [1921] Problem 3

problem number 1921

Added Jan 6, 2020.

Problem Chapter 9.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1x+a_0)w_y+(b_1x+b_0)w_z=(c_1x+c_0)w+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x+a0)*D[w[x,y,z],y]+(b1*x+b0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,y,z]+s1*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(2\mathrm{c0}+\mathrm{c1}x)}c_1\left(-\mathrm{a0}x-\frac{\mathrm{a1}x^2}{2}+y,-\mathrm{b0}x-\frac{\mathrm{b1}x^2}{2}+z\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\mathrm{c0}+\mathrm{c1}x)^2}{2\mathrm{c1}}}\operatorname{erf}\left(\frac{\mathrm{c0}+\mathrm{c1}x}{\sqrt{2}\sqrt{\mathrm{c1}}}\right)(\mathrm{c1s0}-}{\mathrm{c1}^{3/2}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x+a__0)*diff(w(x,y,z),y)+ (b__1*x+b__0)*diff(w(x,y,z),z)=(c_
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(2c_1^{\frac{5}{2}}\_F1\left(-\frac{1}{2}a_1x^2-a_0x+y,-\frac{1}{2}b_1x^2-b_0x+z\right)-2c_1^{\frac{3}{2}}s_1\,\mathrm{e}^{-\frac{1}{2}c_1x^2-c_0x}+\sqrt{2}\sqrt{\pi}\left(-c_0s_1+c_1s_0\right.\right.}{2c_1^{\frac{5}{2}}}$$

6.9.1.4 [1922] Problem 4

problem number 1922

Added Jan 6, 2020.

Problem Chapter 9.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(b_1x+b_0)w_y+(c_1y+c_0)w_z=aw+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x+b0)*D[w[x,y,z],y]+(c1*y+c0)*D[w[x,y,z],z]==a*w[x,y,z]+s1*x+s0;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{a^2(-e^{ax})\,c_1\left(-\text{b0}x-\frac{\text{b1}x^2}{2}+y,\frac{1}{2}\text{b0c1}x^2+\frac{1}{3}\text{b1c1}x^3-\text{c0}x-\text{c1}xy+z\right)+a\text{s0}+a\text{s1}x+}{a^2}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x+b__0)*diff(w(x,y,z),y)+ (c__1*x+c__0)*diff(w(x,y,z),z)=a*w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a^2\_F1\left(-\frac{1}{2}b_1x^2-b_0x+y,-\frac{1}{2}c_1x^2-c_0x+z\right)\mathrm{e}^{ax}+(-s_1x-s_0)\,a-s_1}{a^2}$$

6.9.1.5 [1923] Problem 5

problem number 1923

Added Jan 6, 2020.

Problem Chapter 9.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+k_1x+k_0)w_y+(bz+n_1x+n_0)w_z=(c_1x+c_0)w+s_1x+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k1*x+k0)*D[w[x,y,z],y]+(b*z+n1*x+n0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{1}{2}x(2\mathrm{c0}+\mathrm{c1}x)}c_1\left(\frac{e^{-ax}(a^2y+a(\mathrm{k0}+\mathrm{k1}x)+\mathrm{k1})}{a^2},\frac{e^{-bx}(b^2z+b(\mathrm{n0}+\mathrm{n1}x)+\mathrm{n1})}{b^2}\right)+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\mathrm{c0}}{}}}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k__1*x+k__0)*diff(w(x,y,z),y)+ (b*z+n__1*x+n__0)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(2c_1^{\frac{5}{2}}\_F1\left(\frac{(a^2y+(k_1x+k_0)a+k_1)\mathrm{e}^{-ax}}{a^2},\frac{(b^2z+(n_1x+n_0)b+n_1)\mathrm{e}^{-bx}}{b^2}\right)-2c_1^{\frac{3}{2}}s_1\,\mathrm{e}^{-\frac{1}{2}c_1x^2-c_0x}+\sqrt{2}\,\sqrt{\pi}\,(-c_0s_1\right.}{2c_1^{\frac{5}{2}}}$$

6.9.1.6 [1924] Problem 6

problem number 1924

Added Jan 6, 2020.

Problem Chapter 9.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_2y+a_1x+a_0)w_y+(b_3z+b_2y+b_1x+b_0)w_z=(c_3z+c_2y+c_1x+c_0)w+s_3z+s_2y+s_1x+s_0$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a2*y+a1*x+a0)*D[w[x,y,z],y]+(b3*z+b2*y+b1*x+b0)*D[w[x,y,z],z]==(c3*z+c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✗

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__2*y+a__1*x+a__0)*diff(w(x,y,z),y)+ (b__3*z+b__2*y+b__1*x+b__0)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

time expired

6.9.1.7 [1925] Problem 7

problem number 1925

Added Jan 6, 2020.

Problem Chapter 9.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$axw_x + bxw_y + czw_z = (\alpha x + \beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*x*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{e^{\frac{\alpha x}{a}}\left(-\left(\frac{\alpha x}{a}\right)^{\frac{\beta}{a}}\left(ap\,\mathrm{Gamma}\left(1-\frac{\beta}{a},\frac{\alpha x}{a}\right)+\alpha q\,\mathrm{Gamma}\left(-\frac{\beta}{a},\frac{\alpha x}{a}\right)\right)+a\alpha x^{\frac{\beta}{a}}c_1\left(y-\frac{bx}{a},zx^{-\frac{c}{a}}\right)\right)}{a\alpha}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*x*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{3\left(\left(a-\frac{\beta}{3}\right)(\alpha x+a-\beta)\,a^2 q x^{-\frac{\beta}{a}-1}\left(\frac{\alpha}{a}\right)^{\frac{\beta}{a}}\left(\frac{\alpha}{a}\right)^{-\frac{\beta}{a}}\left(\frac{\alpha x}{a}\right)^{-\frac{a-\beta}{2a}}\text{WhittakerM}\left(\frac{-a-\beta}{2a},\frac{2a-\beta}{2a},\frac{\alpha x}{a}\right)\mathrm{e}^{-\frac{\alpha x}{2a}}\right.}{}$$

6.9.1.8 [1926] Problem 8

problem number 1926

Added Jan 6, 2020.

Problem Chapter 9.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + byw_y + czw_z = (\alpha x+\beta)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==(alpha*x+beta)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{e^{\frac{\alpha x}{a}}\left(-\left(\frac{\alpha x}{a}\right)^{\frac{\beta}{a}}\left(ap\,\text{Gamma}\left(1-\frac{\beta}{a},\frac{\alpha x}{a}\right)+\alpha q\,\text{Gamma}\left(-\frac{\beta}{a},\frac{\alpha x}{a}\right)\right)+a\alpha x^{\frac{\beta}{a}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right)}{a\alpha}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=(alpha*x+beta)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{3\left(\left(a-\frac{\beta}{3}\right)(\alpha x+a-\beta)\,a^2 q x^{-\frac{\beta}{a}-1}\left(\frac{\alpha}{a}\right)^{\frac{\beta}{a}}\left(\frac{\alpha}{a}\right)^{-\frac{\beta}{a}}\left(\frac{\alpha x}{a}\right)^{-\frac{a-\beta}{2a}}\text{WhittakerM}\left(\frac{-a-\beta}{2a},\frac{2a-\beta}{2a},\frac{\alpha x}{a}\right)\mathrm{e}^{-\frac{\alpha}{2}}\right.}{}$$

6.9.1.9 [1927] Problem 9

problem number 1927

Added Jan 6, 2020.

Problem Chapter 9.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = (cx+k)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==(c*x+k)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\frac{e^{cx}\left(-(cx)^k(p\,\text{Gamma}(1-k,cx)+cq\,\text{Gamma}(-k,cx))+cx^k c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)\right.\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=(c*x+k)*w(x,y,z)+p*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{y} \frac{\left(px\left(\frac{\_bab+\sqrt{ab}\sqrt{(az^2+(\_b^2-y^2)b)a}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}+q\right)\mathrm{e}^{-\left(\int \frac{cx\left(\frac{\_bab+\sqrt{ab}\sqrt{(az^2+(\_b^2}}{\sqrt{ab}}\right.}{\sqrt{(az^2}}\right.}}{\sqrt{\left(az^2+\left(\_b^2-y^2\right)b\right)a}} \right.$$

6.9.2 2.2

Local contents

6.9.2.1 [1928] Problem 1 2594
6.9.2.2 [1929] Problem 2 2595
6.9.2.3 [1930] Problem 3 2596
6.9.2.4 [1931] Problem 4 2597
6.9.2.5 [1932] Problem 5 2597
6.9.2.6 [1933] Problem 6 2598
6.9.2.7 [1934] Problem 7 2599

6.9.2.1 [1928] Problem 1

problem number 1928

Added Jan 6, 2020.

Problem Chapter 9.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x^2 + a_0)w_y + (b_1x^2 + b_0)w_z = (c_1x + c_0)w + s_1x^2 + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^2+a0)*D[w[x,y,z],y]+(b1*x^2+b0)*D[w[x,y,z],z]==(c1*x+c0)*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{1}{2}x(2\text{c0}+\text{c1}x)}c_1\left(-\text{a0}x - \frac{\text{a1}x^3}{3} + y, -\text{b0}x - \frac{\text{b1}x^3}{3} + z\right) + \frac{\sqrt{\frac{\pi}{2}}e^{\frac{(\text{c0}+\text{c1}x)^2}{2\text{c1}}}\operatorname{erf}\left(\frac{\text{c0}+\text{c1}x}{\sqrt{2}\sqrt{\text{c1}}}\right)(\text{c0}^2\text{s1} -}{\text{c1}^{5/2}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^2+a__0)*diff(w(x,y,z),y)+ (b__1*x^2+b__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(c_1^{\frac{9}{2}}\_F1\left(-\frac{1}{3}a_1x^3 - a_0x + y, -\frac{1}{3}b_1x^3 - b_0x + z\right) + \frac{\sqrt{\pi}\sqrt{2}\left(c_0^2s_1 + c_1^2s\_0 + c_1s_1\right)c_1^2\operatorname{erf}\left(\frac{\sqrt{2}\left(\sqrt{c_1}\,x + \frac{c_0}{\sqrt{c_1}}\right)}{2}\right)\mathrm{e}}{2}\right.}{c_1^{\frac{9}{2}}}$$

6.9.2.2 [1929] Problem 2

problem number 1929

Added Jan 6, 2020.

Problem Chapter 9.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (b_1x^2 + b_0)w_y + (c_1y^2 + c_0)w_z = aw + s_1x^2 + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x^2+b0)*D[w[x,y,z],y]+(c1*y^2+c0)*D[w[x,y,z],z]==a*w[x,y,z]+s1*x^2+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{a^3e^{ax}c_1\left(-\text{b0}x - \frac{\text{b1}x^3}{3} + y, -\frac{1}{3}\text{b0}^2\text{c1}x^3 - \frac{3}{10}\text{b0b1c1}x^5 + \text{b0c1}x^2y - \frac{1}{14}\text{b1}^2\text{c1}x^7 + \frac{1}{2}\text{b1c1}x^4\right.}{a^3}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x^2+b__0)*diff(w(x,y,z),y)+ (c__1*y^2+c__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{a^3\_F1\left(-\frac{1}{3}b_1x^3 - b_0x + y, -c_0x - \frac{\left(b_1^2x^6 + \frac{21}{5}b_0b_1x^4 - 7b_1x^3y + \frac{14}{3}b_0^2x^2 - 14b_0xy + 14y^2\right)c_1x}{14} + z\right)e^{ax} - 2as_1x}{a^3}$$

6.9.2.3 [1930] Problem 3

problem number 1930

Added Jan 6, 2020.

Problem Chapter 9.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + k_1x^2 + k_0)w_y + (bz + n_1x^2 + n_0)w_z = (c_1x + c_0)w + s_1x + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k1*x^2+k0)*D[w[x,y,z],y]+(b*z+n1*x^2+n0)*D[w[x,y,z],z]==(c1*x+c0)*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{1}{2}x(2\text{c0}+\text{c1}x)}c_1\left(\frac{e^{-ax}(a^2(\text{k0}+\text{k1}x^2)+a^3y+2a\text{k1}x+2\text{k1})}{a^3}, \frac{e^{-bx}(b^2(\text{n0}+\text{n1}x^2)+b^3z+2b}{b^3}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k__1*x^2+k__0)*diff(w(x,y,z),y)+ (b*z+n__1*x^2+n__0)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = -\frac{\left(-2c_1^{\frac{5}{2}}\_F1\left(\frac{(a^3y+2ak_1x+(k_1x^2+k_0)a^2+2k_1)\mathrm{e}^{-ax}}{a^3}, \frac{(b^3z+2bn_1x+(n_1x^2+n_0)b^2+2n_1)\mathrm{e}^{-bx}}{b^3}\right) + 2c_1^{\frac{3}{2}}s_1\,\mathrm{e}^{-\frac{1}{2}c_1x^2-}\right.}{2c_1^{\frac{5}{2}}}$$

6.9.2.4 [1931] Problem 4

problem number 1931

Added Jan 6, 2020.

Problem Chapter 9.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_2xy+a_1x+a_0)w_y+(b_3yz+b_2y^2+b_1x^2+b_0)w_z = (c_3z+c_2y+c_1x+c_0)w+s_1xy+s_2xz$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a2*x*y+a1*x+a0)*D[w[x,y,z],y]+(b3*y*z+b2*y^2+b1*x^2+b0)*D[w[x,y,z],z]=
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__2*x*y+a__1*x+a__0)*diff(w(x,y,z),y)+ (b__3*y*z+b__2*y^2+b__1*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

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6.9.2.5 [1932] Problem 5

problem number 1932

Added Jan 6, 2020.

Problem Chapter 9.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x + bxw_y + czw_z = kxw + sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*x*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x*w[x,y,z]+s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(a+kx)}{k^2}+e^{\frac{kx}{a}}c_1\left(y-\frac{bx}{a},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*x*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x*w(x,y,z)+s*x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{k^2\_F1\left(\frac{ay-bx}{a},zx^{-\frac{c}{a}}\right)\mathrm{e}^{\frac{kx}{a}}-(xk+a)\,s}{k^2}$$

6.9.2.6 [1933] Problem 6

problem number 1933

Added Jan 6, 2020.

Problem Chapter 9.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$axw_x+byw_y+czw_z=kxw+sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x*w[x,y,z]+s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(a+kx)}{k^2}+e^{\frac{kx}{a}}c_1\left(yx^{-\frac{b}{a}},zx^{-\frac{c}{a}}\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x*w(x,y,z)+s*x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{k^2\_F1\left(yx^{-\frac{b}{a}}, zx^{-\frac{c}{a}}\right) e^{\frac{kx}{a}} - (xk+a)\,s}{k^2}$$

6.9.2.7 [1934] Problem 7

problem number 1934

Added Jan 6, 2020.

Problem Chapter 9.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^2 w_x + by^2 w_y + cz^2 w_z = (kx+s)w + px + q$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*x^2*D[w[x,y,z],x]+b*y^2*D[w[x,y,z],y]+c*z^2*D[w[x,y,z],z]==(k*x+s)*w[x,y,z]+p*x+q;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to \frac{e^{-\frac{s}{ax}}\left(-\frac{s}{ax}\right)^{-\frac{k}{a}}\left(asx^{\frac{k}{a}}\left(-\frac{s}{ax}\right)^{\frac{k}{a}} c_1\left(\frac{b}{ax}-\frac{1}{y}, \frac{c}{ax}-\frac{1}{z}\right) + ps\,\text{Gamma}\left(\frac{k}{a}, -\frac{s}{ax}\right) - aq\,\text{Gamma}\left(\frac{a}{}\right.\right.}{as}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  a*x^2*diff(w(x,y,z),x)+ b*y^2*diff(w(x,y,z),y)+ c*z^2*diff(w(x,y,z),z)=(k*x+s)*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int \frac{(px+q)\, x^{\frac{-2a-k}{a}} \mathrm{e}^{\frac{s}{ax}}}{a} dx + \_F1\left(\frac{ax-by}{axy}, \frac{ax-cz}{axz}\right)\right) x^{\frac{k}{a}} \mathrm{e}^{-\frac{s}{ax}}$$

6.9.3 2.3

Local contents

6.9.3.1 [1935] Problem 1 2600
6.9.3.2 [1936] Problem 2 2601
6.9.3.3 [1937] Problem 3 2602
6.9.3.4 [1938] Problem 4 2603
6.9.3.5 [1939] Problem 5 2604

6.9.3.1 [1935] Problem 1

problem number 1935

Added Jan 16, 2020.

Problem Chapter 9.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1\sqrt{x} + a_0)w_y + (b_1\sqrt{x} + b_0)w_z = cw + s_1\sqrt{x} + s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*Sqrt[x]+a0)*D[w[x,y,z],y]+(b1*Sqrt[x]+b0)*D[w[x,y,z],z]==c*w[x,y,z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{cx}c_1\left(-\text{a0}x-\frac{2}{3}\text{a1}x^{3/2}+y,-\text{b0}x-\frac{2}{3}\text{b1}x^{3/2}+z\right)+\frac{\sqrt{\pi}\text{s1}e^{cx}\text{erf}\left(\sqrt{c}\sqrt{x}\right)}{2c^{3/2}}-\frac{\text{s0}+\text{s1}\sqrt{x}}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*sqrt(x)+a__0)*diff(w(x,y,z),y)+ (b__1*sqrt(x)+b__0)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(\sqrt{\frac{c}{\pi}}\,c\_F1\left(-\frac{2a_1x^{\frac{3}{2}}}{3}-a_0x+y,-\frac{2b_1x^{\frac{3}{2}}}{3}-b_0x+z\right)+\frac{s_1\,\text{erf}\left(\sqrt{c}\sqrt{x}\right)}{2}\right)\mathrm{e}^{cx}-\left(s_1\sqrt{x}+s_0\right)\sqrt{\frac{c}{\pi}}}{\sqrt{\frac{c}{\pi}}\,c}$$

6.9.3.2 [1936] Problem 2

problem number 1936

Added Jan 16, 2020.

Problem Chapter 9.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(b_1x^2+b_0)w_y+(c_1y^3+c_0)w_z=aw+s_1x^3+s_0$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(b1*x^2+b0)*D[w[x,y,z],y]+(c1*y^3+c0)*D[w[x,y,z],z]==a*w[x,y,z]+ s1*x^3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{a^4(-e^{ax})\,c_1\left(-\mathrm{b0}x-\frac{\mathrm{b1}x^3}{3}+y,\mathrm{b0}^2\left(\frac{19}{60}\mathrm{b1c1}x^6-\mathrm{c1}x^3y\right)+\frac{1}{4}\mathrm{b0}^3\mathrm{c1}x^4+\frac{3}{280}\mathrm{b0c1}x^2\left(13\mathrm{b1}^2\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (b__1*x^2+b__0)*diff(w(x,y,z),y)+ (c__1*y^3+c__0)*diff(w(x,y,z),z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{a^4\_F1\left(-\frac{1}{3}b_1x^3-b_0x+y,-c_0x+\frac{3\left(b_1^3x^9+\frac{13b_0b_1^2x^7}{2}-10b_1^2x^6y+\frac{133b_0^2b_1x^5}{9}-42b_0b_1x^4y-\frac{140b_0^2x^2y}{3}+70b_0xy^2+\left(\right.\right.}{140}}{a^4}$$

6.9.3.3 [1937] Problem 3

problem number 1937

Added Jan 16, 2020.

Problem Chapter 9.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(ay+kx^3)w_y+(bz+nx^3)w_z=cw+sx^2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+k*x^3)*D[w[x,y,z],y]+(b*z+n*x^3)*D[w[x,y,z],z]==c*w[x,y,z]+ s*x^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s(c^2x^2+2cx+2)}{c^3}+e^{cx}c_1\left(\frac{e^{-ax}(k(a^3x^3+3a^2x^2+6ax+6)+a^4y)}{a^4},\frac{e^{-bx}(n(b^3x^3+3b^2\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+k*x^3)*diff(w(x,y,z),y)+ (b*z+n*x^3)*diff(w(x,y,z),z)=c*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{c^3\_F1\left(\frac{(x^3ka^3+a^4y+3x^2ka^2+6kax+6k)\mathrm{e}^{-ax}}{a^4},\frac{(nx^3b^3+b^4z+3nx^2b^2+6nxb+6n)\mathrm{e}^{-bx}}{b^4}\right)\mathrm{e}^{cx}-(x^2c^2+2cx+2)\,s}{c^3}$$

6.9.3.4 [1938] Problem 4

problem number 1938

Added Jan 16, 2020.

Problem Chapter 9.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1xy+a_2x^3)w_y+(b_1yz+b_2y^3)w_z=(c_1z+c_2y)w+s_1x^2y+s_2xz^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x*y+a2*x^3)*D[w[x,y,z],y]+(b1*y*z+b2*y^3)*D[w[x,y,z],z]==(c1*z+c2*y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x*y+a__2*x^3)*diff(w(x,y,z),y)+ (b__1*y*z+b__2*y^3)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.9.3.5 [1939] Problem 5

problem number 1939

Added Jan 16, 2020.

Problem Chapter 9.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$ax^3 w_x + by^3 w_y + cz^3 w_z = xw + kx + s$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*y^3*D[w[x,y,z],y]+c*z^3*D[w[x,y,z],z]==x*w[x,y,z]+ k*x+s;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{x^2}{2a}}c_1\left(-\frac{bx}{a}-\frac{1}{2y^2},-\frac{cx}{a}-\frac{1}{2z^2}\right)+\frac{\sqrt{\frac{\pi}{2}}se^{\frac{x^2}{2a}}\operatorname{erf}\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-k\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*y^3*diff(w(x,y,z),y)+ c*z^3*diff(w(x,y,z),z)=x*w(x,y,z)+ k*x+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\sqrt{2}\, s \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2\sqrt{a}}\right) e^{\frac{x^2}{2a}} + 2\sqrt{\frac{a}{\pi}}\_F1\left(\frac{2bxy^2+a}{ay^2}, \frac{2cxz^2+a}{az^2}\right) e^{\frac{x^2}{2a}} - 2\sqrt{\frac{a}{\pi}}\, k}{2\sqrt{\frac{a}{\pi}}}$$

6.9.4 2.4

Local contents

6.9.4.1 [1940] Problem 1 2605
6.9.4.2 [1941] Problem 2 2606
6.9.4.3 [1942] Problem 3 2607
6.9.4.4 [1943] Problem 4 2609
6.9.4.5 [1944] Problem 5 2610
6.9.4.6 [1945] Problem 6 2611
6.9.4.7 [1946] Problem 7 2612
6.9.4.8 [1947] Problem 8 2614
6.9.4.9 [1948] Problem 9 2615
6.9.4.10 [1949] Problem 10 2616
6.9.4.11 [1950] Problem 11 2617
6.9.4.12 [1951] Problem 12 2618
6.9.4.13 [1952] Problem 13 2619
6.9.4.14 [1953] Problem 14 2620

6.9.4.1 [1940] Problem 1

problem number 1940

Added Jan 16, 2020.

Problem Chapter 9.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + cw_z = kx^n w + sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*D[w[x,y,z],y]+c*D[w[x,y,z],z]==k*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{kx^{n+1}}{an+a}}\left(-\frac{sx^{m+1}\left(\frac{kx^{n+1}}{an+a}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{kx^{n+1}}{an+a}\right)}{a(n+1)}+c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*diff(w(x,y,z),z)=k*x^n*w(x,y,z)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(\left(kx^{n+1}+(m+n+2)\,a\right)(n+1)^2\,sx^{m-n}\left(\frac{k}{(n+1)a}\right)^{\frac{-m-1}{n+1}}\left(\frac{k}{(n+1)a}\right)^{\frac{m+1}{n+1}}\left(\frac{kx^{n+1}}{(n+1)a}\right)^{\frac{-m-n-2}{2n+2}}\text{Whittal}\right.}{}$$

6.9.4.2 [1941] Problem 2

problem number 1941

Added Jan 16, 2020.

Problem Chapter 9.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x+byw_y+czw_z=kx^nw+sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+b*y*D[w[x,y,z],y]+c*z*D[w[x,y,z],z]==k*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{kx^{n+1}}{an+a}}\left(-\frac{sx^{m+1}\left(\frac{kx^{n+1}}{an+a}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{kx^{n+1}}{an+a}\right)}{a(n+1)}+c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y)+ c*z*diff(w(x,y,z),z)=k*x^n*w(x,y,z)+ s*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(\left(kx^{n+1}+(m+n+2)\,a\right)(n+1)^2\,sx^{m-n}\left(\frac{k}{(n+1)a}\right)^{\frac{-m-1}{n+1}}\left(\frac{k}{(n+1)a}\right)^{\frac{m+1}{n+1}}\left(\frac{kx^{n+1}}{(n+1)a}\right)^{\frac{-m-n-2}{2n+2}}\text{Whittal}}{}$$

6.9.4.3 [1942] Problem 3

problem number 1942

Added Jan 16, 2020.

Problem Chapter 9.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+azw_y+byw_z=cx^nw+sx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ s*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{n+1}}{n+1}}\left(-\frac{sx^{m+1}\left(\frac{cx^{n+1}}{n+1}\right)^{-\frac{m+1}{n+1}}\text{Gamma}\left(\frac{m+1}{n+1},\frac{cx^{n+1}}{n+1}\right)}{n+1}+c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)\right.}{2\sqrt{b}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^{y}\frac{s\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{\left(az^2+\left(\_b^2-y^2\right)b\right)a}}{\sqrt{ab}}\right)-\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)^{m}\mathrm{e}^{-c\left(\int\frac{\left(\frac{\sqrt{ab}\,x+\ln\left(\frac{\_bab+\sqrt{ab}\sqrt{\left(az^2+\left(\_\right.\right.}}{\sqrt{ab}}\right.}{}\right.}{\sqrt{\left(az^2+\right.}}\right.}}{\sqrt{\left(az^2+\left(\_b^2-y^2\right)b\right)a}}\right.$$

6.9.4.4 [1943] Problem 4

problem number 1943

Added Jan 16, 2020.

Problem Chapter 9.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + bx^m w_z = cx^k w + sx^r$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+a*x^n*D[w[x,y,z],y]+b*x^m*D[w[x,y,z],z]==c*x^k*w[x,y,z]+ s*x^r;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^{k+1}}{k+1}}\left(-\frac{sx^{r+1}\left(\frac{cx^{k+1}}{k+1}\right)^{-\frac{r+1}{k+1}}\text{Gamma}\left(\frac{r+1}{k+1},\frac{cx^{k+1}}{k+1}\right)}{k+1}+c_1\left(\frac{-ax^{n+1}+ny+y}{n+1},\frac{-bx^{m+1}+m}{m+1}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*x^m*diff(w(x,y,z),z)=c*x^k*w(x,y,z)+ s*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(\left((k+1)\left(cx^{k+1}+k+r+2\right)\text{WhittakerM}\left(\frac{-k+r}{2k+2},\frac{2k+r+3}{2k+2},\frac{cx^{k+1}}{k+1}\right)+(k+r+2)^2\text{WhittakerM}\right.\right.}{}$$

6.9.4.5 [1944] Problem 5

problem number 1944

Added Jan 16, 2020.

Problem Chapter 9.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + bx^n w_y + cy^m w_z = aw + sx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+b*x^n*D[w[x,y,z],y]+c*x^m*D[w[x,y,z],z]==a*w[x,y,z]+ s*x^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{ax}\left(-\frac{sx^k(ax)^{-k}\,\text{Gamma}(k+1,ax)}{a} + c_1\left(\frac{-bx^{n+1}+ny+y}{n+1}, \frac{-cx^{m+1}+mz+z}{m+1}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ b*x^n*diff(w(x,y,z),y)+ c*x^m*diff(w(x,y,z),z)=a*w(x,y,z)+ s*x^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(sx^k(ax)^{-\frac{k}{2}}\,\text{WhittakerM}\left(\frac{k}{2},\frac{k}{2}+\frac{1}{2},ax\right)\mathrm{e}^{-\frac{ax}{2}} + (k+1)\,a\_F1\left(\frac{-bxx^n+(n+1)y}{n+1}, \frac{-cxx^m+(m+1)z}{m+1}\right)\right)\mathrm{e}}{(k+1)\,a}$$

6.9.4.6 [1945] Problem 6

problem number 1945

Added Jan 16, 2020.

Problem Chapter 9.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (ay + \beta x^n) w_y + (bz + \gamma x^m) w_z = c x^k w + s x^r$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a*y+beta*x^n)*D[w[x,y,z],y]+(b*z+gamma*x^m)*D[w[x,y,z],z]==c*x^k*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{cx^{k+1}}{k+1}} \left(-\frac{s x^{r+1} \left(\frac{cx^{k+1}}{k+1}\right)^{-\frac{r+1}{k+1}} \text{Gamma}\left(\frac{r+1}{k+1}, \frac{cx^{k+1}}{k+1}\right)}{k+1} + c_1 \left(\gamma b^{-m-1} \text{Gamma}(m+1, bx) + ze\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a*y+beta*x^n)*diff(w(x,y,z),y)+ (b*z+gamma*x^m)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(\left((k+1)\left(cx^{k+1}+k+r+2\right) \text{WhittakerM}\left(\frac{-k+r}{2k+2}, \frac{2k+r+3}{2k+2}, \frac{cx^{k+1}}{k+1}\right) + (k+r+2)^2 \text{WhittakerM}\right.\right.}{}$$

6.9.4.7 [1946] Problem 7

problem number 1946

Added Jan 16, 2020.

Problem Chapter 9.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^{n_1} y + a_2 x^{n_2}) w_y + (b_1 y^{m_1} z + b_2 y^{m_2}) w_z = cw + s_1 x y^{k_1} + s_2 x^{k_2} z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^n1*y + a2*x^n2)*D[w[x,y,z],y]+(b1*y^m1*z + b2*y^m2)*D[w[x,y,z],z]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(n__1)*y + a__2*x^(n__2))*diff(w(x,y,z),y)+ (b__1*y^(m__1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.9.4.8 [1947] Problem 8

problem number 1947

Added Jan 16, 2020.

Problem Chapter 9.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + (a_1 x^{\lambda_1} y + a_2 x^{\lambda_2} y^k) w_y + (b_1 x^{\beta_1} z + b_2 x^{\beta_2} z^m) w_z = c_1 x^{\gamma_1} w + c_2 y^{\gamma_2}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^lambda1*y + a2*x^lambda2*y^k)*D[w[x,y,z],y]+(b1*x^beta1*z + b2*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{\text{c1}x^{\text{gamma1}+1}}{\text{gamma1}+1}} \left(\int_1^x \text{c2}e^{-\frac{\text{c1}K[1]^{\text{gamma1}+1}}{\text{gamma1}+1}} \left(\left(\frac{(-1)^{-\frac{\text{lambda2}+1}{\text{lambda1}+1}} \text{a1}^{-\frac{\text{lambda2}+1}{\text{lambda1}+1}} \exp\left(-\frac{\text{a1}\left(x^{\text{lambda1}+1}+(k-1\right.}{\text{lambda1}}\right.}{}\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(lambda__1)*y + a__2*x^(lambda__2)*y^k)*diff(w(x,y,z),y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{x} c_2 \left(\left(\frac{-\left((k-1)\,a_1\_a^{\lambda_2+1} + (-\lambda_1-\lambda_2-2)\,\_a^{-\lambda_1+\lambda_2}\right)(\lambda_1+1)^2\,a_2\left(-\frac{(k-1)a_1\_a^{\lambda_1+1}}{\lambda_1+1}\right)^{-\lambda}}{}\right.\right.\right.$$

6.9.4.9 [1948] Problem 9

problem number 1948

Added Jan 16, 2020.

Problem Chapter 9.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 x^{\lambda_1} y + a_2 x^{\lambda_2} y^k) w_y + (b_1 y^{\beta_1} z + b_2 y^{\beta_2} z^m) w_z = c_1 x^{\gamma_1} w + c_2 z^{\gamma_2}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+(a1*x^lambda1*y + a2*x^lambda2*y^k)*D[w[x,y,z],y]+(b1*y^beta1*z + b2*y^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  diff(w(x,y,z),x)+ (a__1*x^(lambda__1)*y + a__2*x^(lambda__2)*y^k)*diff(w(x,y,z),y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.9.4.10 [1949] Problem 10

problem number 1949

Added Jan 16, 2020.

Problem Chapter 9.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + ayw_y + bzw_z = cx^n w + kx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*y*D[w[x,y,z],y]+b*z*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ k*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^n}{n}}\left(-\frac{kx^m\left(\frac{cx^n}{n}\right)^{-\frac{m}{n}}\text{Gamma}\left(\frac{m}{n},\frac{cx^n}{n}\right)}{n}+c_1\left(yx^{-a},zx^{-b}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*y*diff(w(x,y,z),y)+ b*z*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ k*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left((cx^n+m+n)\,kn^2x^{m-n}\left(\frac{c}{n}\right)^{\frac{m}{n}}\left(\frac{c}{n}\right)^{-\frac{m}{n}}\left(\frac{cx^n}{n}\right)^{-\frac{m+n}{2n}}\text{WhittakerM}\left(\frac{m-n}{2n},\frac{m+2n}{2n},\frac{cx^n}{n}\right)\mathrm{e}^{-\frac{cx^n}{2n}}+\left((m\right.\right.}{}$$

6.9.4.11 [1950] Problem 11

problem number 1950

Added Jan 16, 2020.

Problem Chapter 9.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$xw_x + azw_y + byw_z = cx^n w + kx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  x*D[w[x,y,z],x]+a*z*D[w[x,y,z],y]+b*y*D[w[x,y,z],z]==c*x^n*w[x,y,z]+ k*x^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{cx^n}{n}}\left(-\frac{kx^m\left(\frac{cx^n}{n}\right)^{-\frac{m}{n}}\text{Gamma}\left(\frac{m}{n},\frac{cx^n}{n}\right)}{n}+c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{\right.}}{\sqrt{b}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y)+ b*y*diff(w(x,y,z),z)=c*x^n*w(x,y,z)+ k*x^m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{y} \frac{k\left(x\left(\frac{\_bab+\sqrt{(az^2+(\_b^2-y^2)b)a}\sqrt{ab}}{\sqrt{ab}}\right)^{\frac{1}{\sqrt{ab}}}\left(az+\sqrt{ab}\,y\right)^{-\frac{\sqrt{ab}}{ab}}\right)^{m} \mathrm{e}^{-c\left(\int \frac{\left(x\left(\frac{\_bab+\sqrt{(az^2+(\_b^2-y^2)b}}{\sqrt{ab}}\right.\right.}{\sqrt{(az^2+}}\right.}}{\sqrt{(az^2+(\_b^2-y^2)\,b)\,a}} \right.$$

6.9.4.12 [1951] Problem 12

problem number 1951

Added Jan 16, 2020.

Problem Chapter 9.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$bcxw_x + c(by+cz)w_y + b(by-cz)w_z = kx^n w + sx^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  b*c*x*D[w[x,y,z],x]+ c*(b*y + c*z)*D[w[x,y,z],y]+b*(b*y - c*z)*D[w[x,y,z],z]==k*x^n*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde :=  b*c*x*diff(w(x,y,z),x)+ c*(b*y + c*z)*diff(w(x,y,z),y)+ b*(b*y - c*z)*diff(w(x,y,z),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(- \left(\int^{y} - \frac{s\left(x \left(\frac{\frac{\sqrt{2}\_bb^2}{-b^2y^2+2bzcy+c^2z^2} + \sqrt{\frac{2\_b^2b^2}{-b^2y^2+2bzcy+c^2z^2}+1}\sqrt{\frac{b^2}{-b^2y^2+2bzcy+c^2z^2}}}{\sqrt{\frac{b^2}{-b^2y^2+2bzcy+c^2z^2}}} \right)^{\frac{\sqrt{2}\,b}{2\sqrt{-b^2y^2+2bzcy+c^2z^2}\sqrt{\frac{}{-b^2y^2+2}}}} \right.}{} \right.\right.$$

6.9.4.13 [1952] Problem 13

problem number 1952

Added Jan 16, 2020.

Problem Chapter 9.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1x^{n_1}w_x + b_2y^{n_2}w_y + b_3z^{n_3}w_z = aw + c_1x^{k_1} + c_2y^{k_2} + c_3x^{k_3}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*x^n1*D[w[x,y,z],x]+ b2*y^n2*D[w[x,y,z],y]+b3*z^n3*D[w[x,y,z],z]==a*w[x,y,z]+ c1*x^
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ax^{1-\text{n1}}}{\text{b1}-\text{b1n1}}}\left(\int_1^x \frac{e^{\frac{aK[1]^{1-\text{n1}}}{\text{b1}(\text{n1}-1)}}K[1]^{-\text{n1}}\left(\text{c1}K[1]^{\text{k1}}+\text{c3}K[1]^{\text{k3}}+\text{c2}\left(\left(\frac{\text{b2}(\text{n2}-1)x^{-\text{n1}}(x^{\text{n1}}K[1]-xK[1]^{\text{n1}})}{\text{b1}(\text{n1}-1)}\right.\right.\right.}{\text{b1}}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   b__1*x^(n__1)*diff(w(x,y,z),x)+ b__2*y^(n__2)*diff(w(x,y,z),y)+ b__3*z^(n__3)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x \frac{\left(c_2\_\_a^{-n_1}\left(\left(\frac{(n_1-1)b_1y^{-n_2+1}+(n_2-1)b_2\_\_a^{-n_1+1}-(n_2-1)b_2x^{-n_1+1}}{(n_1-1)b_1}\right)^{-\frac{1}{n_2-1}}\right)^{k_2}+c_1\_\_a^{k_1-n_1}+c_3\_\_a\right.}{b_1}\right.$$

6.9.4.14 [1953] Problem 14

problem number 1953

Added Jan 16, 2020.

Problem Chapter 9.2.4.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1x^{n_1}w_x+a_2y^{n_2}w_y+a_3z^{n_3}w_z=bx^kw+cx^m$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a1*x^n1*D[w[x,y,z],x]+ a2*y^n2*D[w[x,y,z],y]+a3*z^n3*D[w[x,y,z],z]==b*x^k*w[x,y,z]+ c
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{bx^{k-\text{n1}+1}}{\text{a1}k-\text{a1n1}+\text{a1}}}\left(-\frac{cx^{m-\text{n1}+1}\left(\frac{bx^{k-\text{n1}+1}}{\text{a1}k-\text{a1n1}+\text{a1}}\right)^{\frac{-m+\text{n1}-1}{k-\text{n1}+1}}\text{Gamma}\left(\frac{m-\text{n1}+1}{k-\text{n1}+1},\frac{bx^{k-\text{n1}+1}}{\text{a1}k-\text{a1n1}+\text{a1}}\right)}{\text{a1}(k-\text{n1}+1)}+c_1\left(\frac{\text{a2}}{\text{a1}(\text{n}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde :=   a__1*x^(n__1)*diff(w(x,y,z),x)+ a__2*y^(n__2)*diff(w(x,y,z),y)+ a__3*z^(n__3)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{6\left(-\frac{(-k+n_1-1)\left(-4\left(-\frac{k}{2}-\frac{m}{2}+n_1-\frac{3}{2}\right)^2a_1\,\text{WhittakerM}\left(\frac{k+m-2n_1+3}{2k-2n_1+2},\frac{2k+m-3n_1+4}{2k-2n_1+2},\frac{bx^{k-n_1+1}}{(k-n_1+1)a_1}\right)+(-k+n_1-1)\left(bx^{k-n_1+1}\right.\right.}{}\right.}{}$$

6.9.5 3.1

Local contents

6.9.5.1 [1954] Problem 1 2622
6.9.5.2 [1955] Problem 2 2622
6.9.5.3 [1956] Problem 3 2623
6.9.5.4 [1957] Problem 4 2624
6.9.5.5 [1958] Problem 5 2625
6.9.5.6 [1959] Problem 6 2626
6.9.5.7 [1960] Problem 7 2627

6.9.5.1 [1954] Problem 1

problem number 1954

Added Jan 18, 2020.

Problem Chapter 9.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = ce^{\beta x}w + ke^{\lambda x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Exp[beta*x]*w[x,y,z]+ k*Exp[lambda*
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\beta x}}{\beta}}\left(\int_1^x e^{\lambda K[1]-\frac{ce^{\beta K[1]}}{\beta}}kdK[1]+c_1(y-ax,z-bx)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde :=   diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*exp(beta*x)*w(x,y,z)+ k*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k\,\mathrm{e}^{\frac{\beta\lambda x-c\,\mathrm{e}^{\beta x}}{\beta}}dx+\_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\beta x}}{\beta}}$$

6.9.5.2 [1955] Problem 2

problem number 1955

Added Jan 18, 2020.

Problem Chapter 9.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ae^{\beta x}w_y + be^{\lambda x}w_z = ce^{\gamma x}w + se^{\mu x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Exp[beta*x]*D[w[x,y,z],y]+b*Exp[lambda*x]*D[w[x,y,z],z]==c*Exp[gamma
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\gamma x}}{\gamma}}\left(\int_1^x e^{\mu K[1]-\frac{ce^{\gamma K[1]}}{\gamma}}sdK[1]+c_1\left(y-\frac{ae^{\beta x}}{\beta},z-\frac{be^{\lambda x}}{\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(beta*x)*diff(w(x,y,z),y)+ b*exp(lambda*x)*diff(w(x,y,z),z)=c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s\,\mathrm{e}^{\frac{\gamma\mu x-c\,\mathrm{e}^{\gamma x}}{\gamma}}dx+\_F1\left(\frac{-a\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{-b\,\mathrm{e}^{\lambda x}+\lambda z}{\lambda}\right)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

6.9.5.3 [1956] Problem 3

problem number 1956

Added Jan 18, 2020.

Problem Chapter 9.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+be^{\beta x}w_y+ce^{\lambda y}w_z=aw+se^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Exp[beta*x]*D[w[x,y,z],y]+c*Exp[lambda*y]*D[w[x,y,z],z]==a*w[x,y,z]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{e^{ax}\left(se^{x(\gamma-a)}+(\gamma-a)c_1\left(y-\frac{be^{\beta x}}{\beta},z-\frac{c\operatorname{Ei}\left(\frac{be^{\beta x}\lambda}{\beta}\right)e^{\lambda\left(y-\frac{be^{\beta x}}{\beta}\right)}}{\beta}\right)\right)}{a-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*exp(beta*x)*diff(w(x,y,z),y)+ c*exp(lambda*y)*diff(w(x,y,z),z)=a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\frac{\left(-s\,\mathrm{e}^{-(a-\gamma)x}+(a-\gamma)\_F1\left(\frac{-b\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{c\,\mathrm{expIntegral}\left(1,-\frac{b\lambda\,\mathrm{e}^{\beta x}}{\beta}\right)\mathrm{e}^{-\frac{\left(b\,\mathrm{e}^{\beta x}-\beta y\right)\lambda}{\beta}}}{\beta}+\beta z\right)\right)\mathrm{e}^{ax}}{a-\gamma}$$

6.9.5.4 [1957] Problem 4

problem number 1957

Added Jan 18, 2020.

Problem Chapter 9.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+ae^{\beta x}w_y+be^{\lambda z}w_z=cw+ke^{\gamma x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Exp[beta*x]*D[w[x,y,z],y]+b*Exp[lambda*z]*D[w[x,y,z],z]==c*w[x,y,z]+
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{e^{cx}\left(ke^{x(\gamma-c)}+(\gamma-c)c_1\left(-\frac{b\lambda x+e^{-\lambda z}}{\lambda},y-\frac{ae^{\beta x}}{\beta}\right)\right)}{c-\gamma}\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*exp(beta*x)*diff(w(x,y,z),y)+ b*exp(lambda*z)*diff(w(x,y,z),z)=c*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(-\frac{k\,\mathrm{e}^{-(c-\gamma)x}}{c-\gamma}+\_F1\left(\frac{-a\,\mathrm{e}^{\beta x}+\beta y}{\beta},\frac{-b\lambda x-\mathrm{e}^{-\lambda z}}{b\lambda}\right)\right)\mathrm{e}^{cx}$$

6.9.5.5 [1958] Problem 5

problem number 1958

Added Jan 18, 2020.

Problem Chapter 9.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\sigma x}+a_2e^{\lambda y})w_y+(b_1e^{\mu y}+b_2e^{\beta z})w_z=c_1w+c_2e^{\nu x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[sigma*x]+ a2*Exp[lambda*y] )*D[w[x,y,z],y]+(b1*Exp[mu*y]+ b2*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(sigma*x)+ a__2*exp(lambda*y) )*diff(w(x,y,z),y)+ (b__1*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(-c_2\,\mathrm{e}^{(-c_1+\nu)x} + (c_1-\nu)\,\_F1\left(\frac{a_2\lambda\,\mathrm{expIntegral}\left(1,-\frac{a_1\lambda\,\mathrm{e}^{\sigma x}}{\sigma}\right)-\sigma\,\mathrm{e}^{\frac{(a_1\,\mathrm{e}^{\sigma x}-\sigma y)\lambda}{\sigma}}}{\lambda\sigma}, \frac{-b_2\beta\left(\int^x \mathrm{e}^{b_1\beta\left(\int\left(\frac{-\left(-\mathrm{expInte}\right.}{}\right.\right.}\right.}{}\right.\right.$$

6.9.5.6 [1959] Problem 6

problem number 1959

Added Jan 18, 2020.

Problem Chapter 9.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1 e^{\lambda_1 x} w_x + b_2 e^{\lambda_2 y} w_y + b_3 e^{\lambda_3 z} w_z = a w + c_1 e^{\beta_1 x} + c_2 e^{\beta_2 y} + c_3 e^{\beta_3 z}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Exp[lambda1*x]*D[w[x,y,z],x]+ b2*Exp[lambda2*y]*D[w[x,y,z],y]+b3*Exp[lambda3*z]*D[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{-\frac{ae^{-\text{lambda1}x}}{\text{b1lambda1}}}\left(\int_1^x \frac{e^{\frac{ae^{-\text{lambda1}K[1]}}{\text{b1lambda1}}-\text{lambda1}K[1]}\left(\text{c2}\left(\frac{\text{b2}\left(-e^{-\text{lambda1}x}+e^{-\text{lambda1}K[1]}\right)\text{lambda2}}{\text{b1lambda1}}+e^{-\text{lamb}}\right.\right.}{}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*exp(lambda__1*x)*diff(w(x,y,z),x)+ b__2*exp(lambda__2*y)*diff(w(x,y,z),y)+ b__3*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x \frac{c_2\left(\frac{b_1\lambda_1}{b_2\lambda_2\,\mathrm{e}^{-\_a\lambda_1}+\left(b_1\lambda_1\,\mathrm{e}^{x\lambda_1}-b_2\lambda_2\,\mathrm{e}^{y\lambda_2}\right)\mathrm{e}^{-x\lambda_1-y\lambda_2}}\right)^{\frac{\beta_2}{\lambda_2}}\mathrm{e}^{\frac{-\_ab_1\lambda_1^2+a\,\mathrm{e}^{-\_a\lambda_1}}{b_1\lambda_1}}+c_3\left(\frac{b_1\lambda_1}{b_3\lambda_3\,\mathrm{e}^{-\_a\lambda_1}+\left(b_1\lambda_1\,\mathrm{e}^{x\lambda_1}-b_3\right.}\right.}{b_1}\right.$$

6.9.5.7 [1960] Problem 7

problem number 1960

Added Jan 18, 2020.

Problem Chapter 9.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a_1e^{\sigma_1x+\beta_1y}w_x+a_2e^{\sigma_2y+\beta_2y}w_y+\left(b_1e^{\nu_1x+\mu_1y}+b_2e^{\nu_2x+\mu_2y+\lambda z}\right)w_z=c_1w+c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Exp[sigma1*x+beta1*y]*D[w[x,y,z],x]+ a2*Exp[sigma2*y+beta2*y]*D[w[x,y,z],y]+( b1*E
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde := a__1*exp(sigma__1*x+beta__1*y)*diff(w(x,y,z),x)+ a__2*exp(sigma__2*y+beta__2*y)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int^{x} \frac{c_2 \left(\frac{a_1 \sigma_1 \mathrm{e}^{-x\sigma_1} \mathrm{e}^{x\sigma_1 + (\beta_1 - \beta_2 - \sigma_2) y} + (\beta_1 - \beta_2 - \sigma_2)(-\mathrm{e}^{-\_a\sigma_1} + \mathrm{e}^{-x\sigma_1}) a_2}{a_1 \sigma_1} \right)^{-\frac{\beta_1}{\beta_1 - \beta_2 - \sigma_2}} \mathrm{e}^{\frac{-(\beta_2 + \sigma_2)\_a a_2 \sigma_1 + c_1 \left(\frac{a_1 \sigma_1 \mathrm{e}^{-}}{}}{}} }{a_1} \right.$$

6.9.6 3.2

Local contents

6.9.6.1 [1961] Problem 1 2629
6.9.6.2 [1962] Problem 2 2630
6.9.6.3 [1963] Problem 3 2630
6.9.6.4 [1964] Problem 4 2631
6.9.6.5 [1965] Problem 5 2632
6.9.6.6 [1966] Problem 6 2633
6.9.6.7 [1967] Problem 7 2634
6.9.6.8 [1968] Problem 8 2635
6.9.6.9 [1969] Problem 9 2636

6.9.6.10 [1970] Problem 10 2637

6.9.6.1 [1961] Problem 1

problem number 1961

Added Jan 19, 2020.

Problem Chapter 9.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = ce^{\beta x}w + kx^n$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Exp[beta*x]*w[x,y,z]+ k*x^n;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{ce^{\beta x}}{\beta}}\left(\int_1^x e^{-\frac{ce^{\beta K[1]}}{\beta}}kK[1]^n dK[1]+c_1(y-ax,z-bx)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*exp(beta*x)*w(x,y,z)+ k*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int kx^n \mathrm{e}^{-\frac{c\,\mathrm{e}^{\beta x}}{\beta}}dx+\_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\frac{c\,\mathrm{e}^{\beta x}}{\beta}}$$

6.9.6.2 [1962] Problem 2

problem number 1962

Added Jan 19, 2020.

Problem Chapter 9.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + ax^n w_y + be^{\lambda x} w_z = ce^{\gamma x} w + sx^k$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*x^n*D[w[x,y,z],y]+b*Exp[lambda*x]*D[w[x,y,z],z]==c*Exp[gamma*x]*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{ce^{\gamma x}}{\gamma}} \left(\int_1^x e^{-\frac{ce^{\gamma K[1]}}{\gamma}} sK[1]^k dK[1] + c_1\left(\frac{-ax^{n+1}+ny+y}{n+1}, z - \frac{be^{\lambda x}}{\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*x^n*diff(w(x,y,z),y)+ b*exp(lambda*x)*diff(w(x,y,z),z)=c*exp(gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int sx^k \mathrm{e}^{-\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}} dx + \_F1\left(\frac{-axx^n + (n+1)\,y}{n+1}, \frac{-b\,\mathrm{e}^{\lambda x} + \lambda z}{\lambda}\right)\right) \mathrm{e}^{\frac{c\,\mathrm{e}^{\gamma x}}{\gamma}}$$

6.9.6.3 [1963] Problem 3

problem number 1963

Added Jan 19, 2020.

Problem Chapter 9.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + be^{\beta x} w_y + cy^n w_z = aw + se^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Exp[beta*x]*D[w[x,y,z],y]+c*y^n*D[w[x,y,z],z]==a*w[x,y,z]+ s*Exp[gam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*exp(beta*x)*diff(w(x,y,z),y)+ c*y^n*diff(w(x,y,z),z)=a*w(x,y,z)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(-s\,\mathrm{e}^{-(a-\gamma)x} + (a-\gamma)\_F1\left(\frac{-b\,\mathrm{e}^{\beta x}+\beta y}{\beta}, z - \left(\int^x c\left(\frac{b\,\mathrm{e}^{-a\beta}-b\,\mathrm{e}^{\beta x}+\beta y}{\beta}\right)^n d\_a\right)\right)\right)\mathrm{e}^{ax}}{a-\gamma}$$

6.9.6.4 [1964] Problem 4

problem number 1964

Added Jan 19, 2020.

Problem Chapter 9.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 y + a_2 x y^k) w_y + (b_1 x + b_2 e^{\beta y + \lambda z}) w_z = c_1 w + c_2 e^{\gamma x}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*y+a2*x*y^k)*D[w[x,y,z],y]+(b1*x+b2*Exp[beta*y+lambda*z])*D[w[x,y,z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*y+a__2*x*y^k)*diff(w(x,y,z),y)+ (b__1*x+b__2*exp(beta*y+lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{\left(-c_2\,\mathrm{e}^{-(c_1-\gamma)x} + (c_1-\gamma)\,\_F1\left(\frac{((k-1)a_1^2y+((k-1)a_1x-1)a_2y^k)y^{-k}\mathrm{e}^{(k-1)a_1x}}{(k-1)a_1^2}, \frac{-b_2\lambda\left(\int^x \mathrm{e}^{\frac{-a^2b_1\lambda}{2}+\beta\left(\frac{(-((k-1}{}\right.}}{}\right.\right.}{c_1-\gamma}$$

6.9.6.5 [1965] Problem 5

problem number 1965

Added Jan 19, 2020.

Problem Chapter 9.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1x + a_2e^{\lambda y})w_y + (b_1z + b_2e^{\beta y}z^k)w_z = c_1w + c_2$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*x+a2*Exp[lambda*y])*D[w[x,y,z],y]+(b1*z+b2*Exp[beta*y]*z^k)*D[w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to -\frac{\mathrm{c2}}{\mathrm{c1}} + e^{\mathrm{c1}x}c_1\left(-\frac{\frac{\sqrt{2\pi}\mathrm{a2}\sqrt{\lambda}\,\mathrm{Erfi}\left(\frac{\sqrt{\mathrm{a1}}\sqrt{\lambda}x}{\sqrt{2}}\right)}{\mathrm{a1}^{3/2}} + \frac{2e^{\frac{1}{2}\mathrm{a1}\lambda x^2-\lambda y}}{\mathrm{a1}}}{2\mathrm{a2}\lambda^2}, (k-1)\int_1^x \mathrm{b2}\exp\left(\frac{1}{2}\mathrm{a1}\beta K[1]^2 + \mathrm{b1}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*x+a__2*exp(lambda*y))*diff(w(x,y,z),y)+ (b__1*z+b__2*exp(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1 \_F1\left(\frac{\sqrt{2}\sqrt{\pi}\sqrt{-a_1\lambda}\,a_2 \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{-a_1\lambda}\,x}{2}\right)\sqrt{e^{-\left(a_1x^2-2y\right)\lambda}}+2a_1}{2a_1\lambda\sqrt{e^{-\left(a_1x^2-2y\right)\lambda}}}, (k-1)\,b_2 2^{\frac{\beta}{2\lambda}}\pi^{-\frac{\beta}{2\lambda}}\left(\int^x \pi^{\frac{\beta}{\lambda}}\left(-\frac{}{\left(2\pi a_1 a_2 \lambda \operatorname{er}\right.}\right.\right.\right.}{}$$

6.9.6.6 [1966] Problem 6

problem number 1966

Added Jan 19, 2020.

Problem Chapter 9.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\mu x} + a_2 e^{\lambda y}) w_y + (b_1 e^{\nu y} + b_2 e^{\beta z}) w_z = c_1 w + c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[mu*x]+a2*Exp[lambda*y])*D[w[x,y,z],y]+(b1*Exp[nu*y]+b2*Exp[bet
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(mu*x)+a__2*exp(lambda*y))*diff(w(x,y,z),y)+ (b__1*exp(nu*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1\_F1\left(\frac{a_2\lambda\,\mathrm{expIntegral}\left(1,-\frac{a_1\lambda\,\mathrm{e}^{\mu x}}{\mu}\right)-\mu\,\mathrm{e}^{-\frac{(-a_1\,\mathrm{e}^{\mu x}+\mu y)\lambda}{\mu}}}{\lambda\mu}, \frac{-b_2\beta\left(\int^x \mathrm{e}^{b_1\beta\left(\int\left(\frac{-\left(-\mathrm{expIntegral}\left(1,-\frac{a_1\lambda\,\mathrm{e}^{-b\mu}}{\mu}\right)+\mathrm{expIntegra}\right.}{\mu}\right.\right.}\right.}{}\right.}{}$$

6.9.6.7 [1967] Problem 7

problem number 1967

Added Jan 19, 2020.

Problem Chapter 9.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + (a_1 e^{\lambda_1 x} y + a_2 e^{\lambda_2 x}) w_y + (b_1 e^{\beta_1 x} z + b_2 e^{\beta_2 x}) w_z = c_1 e^{\gamma_1 x} w + c_2 e^{\gamma_2 x}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x])*D[w[x,y,z],y]+(b1*Exp[beta1*x]
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{\text{c1}e^{\text{gamma1}x}}{\text{gamma1}}}\left(\int_1^x \text{c2}e^{\text{gamma2}K[3]-\frac{\text{c1}e^{\text{gamma1}K[3]}}{\text{gamma1}}}dK[3]+c_1\left(ye^{-\frac{\text{a1}e^{\text{lambda1}x}}{\text{lambda1}}}-\int_1^x \text{a2}e^{\text{lambda2}K[1]-\frac{\text{a1}}{}}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x))*diff(w(x,y,z),y)+ (
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int c_2\,\mathrm{e}^{\frac{x\gamma_1\gamma_2-c_1\,\mathrm{e}^{x\gamma_1}}{\gamma_1}}dx+\_F1\left(-a_2\left(\int \mathrm{e}^{\frac{x\lambda_1\lambda_2-a_1\,\mathrm{e}^{x\lambda_1}}{\lambda_1}}dx\right)+y\,\mathrm{e}^{-\frac{a_1\,\mathrm{e}^{x\lambda_1}}{\lambda_1}},\frac{-b_1\beta_2\,\mathrm{e}^{\beta_1 x}-b_2\beta_1\,\mathrm{e}^{\beta_2 x}}{\beta_1\beta_2}\right.\right.$$

6.9.6.8 [1968] Problem 8

problem number 1968

Added Jan 19, 2020.

Problem Chapter 9.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\lambda_1 x}y+a_2e^{\lambda_2 x}y^k)w_y+(b_1e^{\beta_1 x}z+b_2e^{\beta_2 x}z^m)w_z=c_1e^{\gamma_1 x}w+c_2e^{\gamma_2 y}$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x]*y^k)*D[w[x,y,z],y]+(b1*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{\text{c1}e^{\text{gamma1}x}}{\text{gamma1}}}\left(\int_1^x \text{c2}\exp\left(\text{gamma2}\left(e^{-\frac{\text{a1}\left(e^{\text{lambda1}K[3]}(k-1)+e^{\text{lambda1}x}\right)}{\text{lambda1}}}y^{-k}\left(e^{\frac{\text{a1}e^{\text{lambda1}x}}{\text{lambda1}}}(k-1)\int_1^x\right.\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x)*y^k)*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int^x c_2\,\mathrm{e}^{\frac{\gamma_1\gamma_2\left(\left(-(k-1)a_2y^k\left(\int \mathrm{e}^{\frac{-b\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{-b\lambda_1}}{\lambda_1}}d\_b\right)\mathrm{e}^{\frac{a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}+(k-1)a_2y^k\left(\int \mathrm{e}^{\frac{x\lambda_1\lambda_2+(k-1)a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}dx\right)\mathrm{e}^{\frac{a_1\mathrm{e}^{x\lambda_1}}{\lambda_1}}+y\,\mathrm{e}^{\cdots}\right.\right.}{\gamma_1}}\right.$$

6.9.6.9 [1969] Problem 9

problem number 1969

Added Jan 19, 2020.

Problem Chapter 9.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x+(a_1e^{\lambda_1x}y+a_2e^{\lambda_2x}y^k)w_y+(b_1e^{\beta_1y}z+b_2e^{\beta_2y}z^m)w_z=c_1e^{\gamma_1x}w+c_2e^{\gamma_2z}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ (a1*Exp[lambda1*x]*y+a2*Exp[lambda2*x]*y^k)*D[w[x,y,z],y]+(b1*Exp[beta
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$Aborted

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ (a__1*exp(lambda__1*x)*y+a__2*exp(lambda__2*x)*y^k)*diff(w(x,y,z),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

Expression too large to display

6.9.6.10 [1970] Problem 10

problem number 1970

Added Jan 19, 2020.

Problem Chapter 9.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$a_1 e^{\beta y} w_x + a_2 e^{sigmax} w_y + (b_1 x^n e^{\mu y} + b_2 y^m e^{\nu x + \lambda z}) w_z = c_1 w + c_2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a1*Exp[beta*y]*D[w[x,y,z],x]+ a2*Exp[sigma*x]*D[w[x,y,z],y]+(b1*x^n*Exp[mu*y]+b2*Exp[
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a__1*exp(beta*y)*diff(w(x,y,z),x)+ a__1*exp(sigma*x)*diff(w(x,y,z),y)+ (b__1*x^n*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \frac{c_1 \left(\frac{\sigma\,\mathrm{e}^{\beta y}}{\beta}\right)^{\frac{c_1}{\left(\beta\,\mathrm{e}^{\sigma x}-\sigma\,\mathrm{e}^{\beta y}\right)a_1}} _F1\left(\frac{-\beta\,\mathrm{e}^{\sigma x}+\sigma\,\mathrm{e}^{\beta y}}{\beta\sigma}, \frac{-b_2\lambda\sigma\left(\int^x \frac{\beta\,\mathrm{e}^{\frac{_b a_1\beta\nu+b_1\lambda\sigma\left(\int \frac{_b^n\left(-\frac{-\sigma\,\mathrm{e}^{\beta y}+\left(-\mathrm{e}^{_b\sigma}+\mathrm{e}^{\sigma x}\right)\beta}{\sigma}\right)^{\frac{\mu}{\beta}}}{\mathrm{e}^{_b\sigma}-\frac{\beta\,\mathrm{e}^{\sigma x}-\sigma\,\mathrm{e}^{\beta y}}{\beta}} d_\cdots}{a_1\beta}}}{\beta\,\mathrm{e}^{_b\sigma}-\beta\,\mathrm{e}^{\sigma x}+\sigma\,\mathrm{e}^{\beta y}} \cdots\right)}{\cdots}\right)}{\cdots}$$

6.9.7 4.1

Local contents

6.9.7.1 [1971] Problem 1 2639
6.9.7.2 [1972] Problem 2 2640
6.9.7.3 [1973] Problem 3 2641
6.9.7.4 [1974] Problem 4 2642
6.9.7.5 [1975] Problem 5 2642

6.9.7.1 [1971] Problem 1

problem number 1971

Added Jan 19, 2020.

Problem Chapter 9.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w + k\sinh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Sinh[beta*x]^n*w[x,y,z]+ k*Sinh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\sinh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\sinh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

6.9.7.2 [1972] Problem 2

problem number 1972

Added Jan 19, 2020.

Problem Chapter 9.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\sinh(\beta z)w_z = (p\sinh(\lambda x) + q)\,w + k\sinh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Sinh[beta*z]*D[w[x,y,z],z]==(p*Sinh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{p\cosh(\lambda x)+\lambda q x}{a\lambda}}\left(\int_1^x \frac{e^{-\frac{p\cosh(\lambda K[1])+\lambda q K[1]}{a\lambda}}k\sinh(\gamma K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{\log\left(\tanh\left(\frac{\beta z}{2}\right)\right)}{\beta}-\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*sinh(beta*z)*diff(w(x,y,z),z)=(p*sinh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{k\,\mathrm{e}^{\frac{-\lambda q x-p\cosh(\lambda x)}{a\lambda}}\sinh(\gamma x)}{a}dx+\_F1\left(\frac{ya-xb}{a},\frac{-\beta c x-2a\,\mathrm{arctanh}\left(\mathrm{e}^{\beta z}\right)}{\beta c}\right)\right)\mathrm{e}^{\frac{\lambda q x+p\cosh(\lambda x)}{a\lambda}}$$

6.9.7.3 [1973] Problem 3

problem number 1973

Added Jan 19, 2020.

Problem Chapter 9.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\sinh^n(\beta x)w_y + b\sinh^k(\lambda x)w_z = cw + s\sinh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Sinh[beta*x]^n*D[w[x,y,z],y]+b*Sinh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \frac{k(e^{2\mu x}-1)\sinh^m(\mu x)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{c}{\mu}+m+2\right),-\frac{c+(m-2)\mu}{2\mu},e^{2\mu x}\right)}{c+m\mu}+e^{cx}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*sinh(beta*x)^n*diff(w(x,y,z),y)+ b*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k(\sinh^m(\mu x))\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a(\sinh^n(\beta x))\,dx\right),z-\left(\int b(\sinh^k(\lambda x))\,dx\right)\right)\right)$$

6.9.7.4 [1974] Problem 4

problem number 1974

Added Jan 19, 2020.

Problem Chapter 9.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\sinh^n(\beta x)w_y + c\sinh^k(\lambda y)w_z = aw + s\sinh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Sinh[beta*x]^n*D[w[x,y,z],y]+c*Sinh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*sinh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s(\sinh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\sinh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\sinh^k\left(\left(b\left(\int (\mathrm{si}\right.\right.\right.\right.\right.$$

6.9.7.5 [1975] Problem 5

problem number 1975

Added Jan 19, 2020.

Problem Chapter 9.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\sinh^{n_1}(\lambda_1 x)w_x + b_2\sinh^{n_2}(\lambda_2 y)w_y + b_3\sinh^{n_3}(\lambda_3 z)w_z = aw + c_1\sinh^{k_1}(\beta_1 x) + c_2\sinh^{k_2}(\beta_2 y) + c_3\sinh^{k_3}(\beta_3 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Sinh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Sinh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Sinh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\sqrt{\cosh^2(\text{lambda1}x)}\text{sech}(\text{lambda1}x)\sinh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};-\sinh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*sinh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*sinh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\sinh^{k_1}(\beta_1 x)\right)+c_2\left(\sinh^{k_2}(\beta_2 x)\right)+c_3\left(\sinh^{k_3}(\beta_3 x)\right)\right)\left(\sinh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\sinh^{-n_1}(x\lambda_1)\right.}{b_1}}}{b_1}\right.$$

6.9.8 4.2

Local contents

6.9.8.1 [1976] Problem 1 2644
6.9.8.2 [1977] Problem 2 2645
6.9.8.3 [1978] Problem 3 2646
6.9.8.4 [1979] Problem 4 2647
6.9.8.5 [1980] Problem 5 2647

6.9.8.1 [1976] Problem 1

problem number 1976

Added Jan 19, 2020.

Problem Chapter 9.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\cosh^n(\beta x)w + k\cosh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Cosh[beta*x]^n*w[x,y,z]+ k*Cosh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{-\sinh^2(\beta x)}\operatorname{csch}(\beta x)\cosh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(\frac{c}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*cosh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\cosh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c(\cosh^n(\beta x))dx}$$

6.9.8.2 [1977] Problem 2

problem number 1977

Added Jan 19, 2020.

Problem Chapter 9.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\cosh(\beta z)w_z = (p\cosh(\lambda x) + q)\,w + k\cosh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Cosh[beta*z]*D[w[x,y,z],z]==(p*Cosh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{\frac{p\sinh(\lambda x)+\lambda qx}{a\lambda}}\left(\int_1^x \frac{e^{-\frac{\lambda qK[1]+p\sinh(\lambda K[1])}{a\lambda}}k\cosh(\gamma K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{2\tan^{-1}\left(\tanh\left(\frac{\beta z}{2}\right)\right.}{\beta}\right.\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*cosh(beta*z)*diff(w(x,y,z),z)=(p*cosh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{k\cosh(\gamma x)\,\mathrm{e}^{\frac{-\lambda qx-p\sinh(\lambda x)}{a\lambda}}}{a}dx+\_F1\left(\frac{ya-xb}{a},\frac{-\beta cx+2a\arctan\left(\mathrm{e}^{\beta z}\right)}{\beta c}\right)\right)\mathrm{e}^{\frac{\lambda qx+p\sinh(\lambda x)}{a\lambda}}$$

6.9.8.3 [1978] Problem 3

problem number 1978

Added Jan 19, 2020.

Problem Chapter 9.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\cosh^n(\beta x)w_y + b\cosh^k(\lambda x)w_z = cw + s\cosh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Cosh[beta*x]^n*D[w[x,y,z],y]+b*Cosh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{k(e^{2\mu x}+1)\cosh^m(\mu x)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{c}{\mu}+m+2\right),-\frac{c+(m-2)\mu}{2\mu},-e^{2\mu x}\right)}{c+m\mu}+\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*cosh(beta*x)^n*diff(w(x,y,z),y)+ b*cosh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\mu x))\,\mathrm{e}^{-cx}dx + \_F1\left(y-\left(\int a(\cosh^n(\beta x))\,dx\right), z-\left(\int b(\cosh^k(\lambda x))\,dx\right)\right)\right.$$

6.9.8.4 [1979] Problem 4

problem number 1979

Added Jan 19, 2020.

Problem Chapter 9.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\cosh^n(\beta x)w_y + c\cosh^k(\lambda y)w_z = aw + s\cosh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Cosh[beta*x]^n*D[w[x,y,z],y]+c*Cosh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*cosh(beta*x)^n*diff(w(x,y,z),y)+ c*cosh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s(\cosh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\cosh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\cosh^k\left(\left(b\left(\int (c\right.\right.\right.\right.\right.$$

6.9.8.5 [1980] Problem 5

problem number 1980

Added Jan 19, 2020.

Problem Chapter 9.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\cosh^{n_1}(\lambda_1 x)w_x + b_2\cosh^{n_2}(\lambda_2 y)w_y + b_3\cosh^{n_3}(\lambda_3 z)w_z = aw + c_1\cosh^{k_1}(\beta_1 x) + c_2\cosh^{k_2}(\beta_2 y) + c_3\cosh^{k_3}(\beta_3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Cosh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Cosh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Cosh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\sqrt{-\sinh^2(\text{lambda1}x)}\text{csch}(\text{lambda1}x)\cosh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};\cosh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*cosh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*cosh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\cosh^{k_1}(\beta_1 x)\right)+c_2\left(\cosh^{k_2}(\beta_2 x)\right)+c_3\left(\cosh^{k_3}(\beta_3 x)\right)\right)\left(\cosh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\cosh^{-n_1}(x\lambda\right.\right.}{b_1}}}{b_1}\right.$$

6.9.9 4.3

Local contents

6.9.9.1 [1981] Problem 1 2649
6.9.9.2 [1982] Problem 2 2650
6.9.9.3 [1983] Problem 3 2651
6.9.9.4 [1984] Problem 4 2652
6.9.9.5 [1985] Problem 5 2652

6.9.9.1 [1981] Problem 1

problem number 1981

Added Jan 19, 2020.

Problem Chapter 9.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^n(\beta x)w + k\tanh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Tanh[beta*x]^n*w[x,y,z]+ k*Tanh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\right.}{n\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\tanh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\tanh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c(\tanh^n(\beta x))dx}$$

6.9.9.2 [1982] Problem 2

problem number 1982

Added Jan 19, 2020.

Problem Chapter 9.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$aw_x + bw_y + c\tanh(\beta z)w_z = (p\tanh(\lambda x) + q)\, w + k\tanh(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Tanh[beta*z]*D[w[x,y,z],z]==(p*Tanh[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z) \to e^{\frac{qx}{a}} \cosh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[1]}{a}} k \cosh^{-\frac{p}{a\lambda}}(\lambda K[1]) \tanh(\gamma K[1])}{a} dK[1] + c_1\left(y - \frac{bx}{a}, \frac{\log(\sinh(}{\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*tanh(beta*z)*diff(w(x,y,z),z)=(p*tanh(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(-\left(\int^z \frac{k\left(\tanh\left(\frac{(-2\beta cx + a\ln(\tanh(\_a\beta)-1) + a\ln(\tanh(\_a\beta)+1) - a\ln(\tanh(\beta z)-1) - a\ln(\tanh(\beta z)+1) - 2a\ln(\tanh}{2\beta c}\right.\right.}{}\right.\right.$$

6.9.9.3 [1983] Problem 3

problem number 1983

Added Jan 19, 2020.

Problem Chapter 9.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\tanh^n(\beta x)w_y + b\tanh^k(\lambda x)w_z = cw + s\tanh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Tanh[beta*x]^n*D[w[x,y,z],y]+b*Tanh[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{k(e^{-2\mu x}-1)^m\left(e^{-2\mu x}+1\right)^m\left(-e^{-4\mu x}(e^{2\mu x}-1)^2\right)^{-m}\tanh^m(\mu x)F_1\left(\frac{c}{2\mu};m,-m;\frac{c}{2\mu}+1;\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*tanh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\tanh^m(\mu x))\,\mathrm{e}^{-cx}dx + \_F1\left(y - \left(\int a(\tanh^n(\beta x))\,dx\right), z - \left(\int b(\tanh^k(\lambda x))\,dx\right)\right)\right.$$

6.9.9.4 [1984] Problem 4

problem number 1984

Added Jan 19, 2020.

Problem Chapter 9.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\tanh^n(\beta x)w_y + c\tanh^k(\lambda y)w_z = aw + s\tanh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Tanh[beta*x]^n*D[w[x,y,z],y]+c*Tanh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*tanh(beta*x)^n*diff(w(x,y,z),y)+ c*tanh(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s(\tanh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\tanh^n(\beta x))\,dx\right), z - \left(\int^x c\left(\frac{\sinh\left(\left(b\left(\int(\tan\right.\right.\right.}{\cosh\left(\left(b\left(\int(\tan\right.\right.\right.}\right.\right.\right.$$

6.9.9.5 [1985] Problem 5

problem number 1985

Added Jan 19, 2020.

Problem Chapter 9.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\tanh^{n_1}(\lambda_1 x)w_x + b_2\tanh^{n_2}(\lambda_2 y)w_y + b_3\tanh^{n_3}(\lambda_3 z)w_z = aw + c_1\tanh^{k_1}(\beta_1 x) + c_2\tanh^{k_2}(\beta_2 y) + c_3\tanh^{k_3}(\beta$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Tanh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Tanh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Tanh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\tanh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(1,\frac{1}{2}-\frac{\text{n1}}{2};\frac{3}{2}-\frac{\text{n1}}{2};\tanh^2(\text{lambda1}x)\right)}{\text{b1lambda1}-\text{b1lambda1n1}}\right)\left(\int_1^x \frac{\exp\left(\frac{a\,{}_2F_1(}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*tanh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*tanh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{\left(c_1\left(\tanh^{k_1}(\beta_1 x)\right)+c_2\left(\tanh^{k_2}(\beta_2 x)\right)+c_3\left(\tanh^{k_3}(\beta_3 x)\right)\right)\left(\tanh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\tanh^{-n_1}(x\right.}{b_1}}}{b_1}\right.$$

6.9.10 4.4

Local contents

6.9.10.1 [1986] Problem 1 2654
6.9.10.2 [1987] Problem 2 2655
6.9.10.3 [1988] Problem 3 2656
6.9.10.4 [1989] Problem 4 2657
6.9.10.5 [1990] Problem 5 2657

6.9.10.1 [1986] Problem 1

problem number 1986

Added Jan 19, 2020.

Problem Chapter 9.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\coth^n(\beta x)w + k\coth^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Coth[beta*x]^n*w[x,y,z]+ k*Coth[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\coth^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\coth^{n+1}(\beta K[1])\,{}_2F_1\left(1,\right.}{n\beta +}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*coth(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\coth^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\coth^n(\beta x))dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c(\coth^n(\beta x))dx}$$

6.9.10.2 [1987] Problem 2

problem number 1987

Added Jan 19, 2020.

Problem Chapter 9.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$aw_x + bw_y + c\coth(\beta z)w_z = (p\coth(\lambda x) + q)\,w + k\coth(\gamma x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  a*D[w[x,y,z],x]+ b*D[w[x,y,z],y]+c*Coth[beta*z]*D[w[x,y,z],z]==(p*Coth[lambda*x]+q)*w
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{w(x,y,z)\to e^{\frac{qx}{a}}\sinh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[1]}{a}}k\coth(\gamma K[1])\sinh^{-\frac{p}{a\lambda}}(\lambda K[1])}{a}dK[1]+c_1\left(y-\frac{bx}{a},\frac{\log(\cosh(\beta z}{\beta}\right.\right.\right.$$

$$\left\{w(x,y,z)\to e^{\frac{qx}{a}}\sinh^{\frac{p}{a\lambda}}(\lambda x)\left(\int_1^x \frac{e^{-\frac{qK[2]}{a}}k\coth(\gamma K[2])\sinh^{-\frac{p}{a\lambda}}(\lambda K[2])}{a}dK[2]+c_1\left(y-\frac{bx}{a},\frac{\log(\cosh(\beta z}{\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y)+ c*coth(beta*z)*diff(w(x,y,z),z)=(p*coth(lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{k(\coth(\lambda x)-1)^{\frac{p}{2a\lambda}}(\coth(\lambda x)+1)^{\frac{p}{2a\lambda}}\coth(\gamma x)\,\mathrm{e}^{-\frac{qx}{a}}}{a}dx+\_F1\left(\frac{ya-xb}{a},\frac{-2\beta cx+a\ln}{}\right.\right.$$

6.9.10.3 [1988] Problem 3

problem number 1988

Added Jan 19, 2020.

Problem Chapter 9.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\coth^n(\beta x)w_y + b\coth^k(\lambda x)w_z = cw + s\coth^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Coth[beta*x]^n*D[w[x,y,z],y]+b*Coth[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{k(e^{-2\mu x}-1)^{-m}\left(e^{-2\mu x}+1\right)^{-m}\left(-e^{-4\mu x}(e^{2\mu x}-1)^2\right)^m\coth^m(\mu x)F_1\left(\frac{c}{2\mu};-m,m;\frac{c}{2\mu}+1\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*coth(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\coth^m(\mu x))\,\mathrm{e}^{-cx}dx + \_F1\left(y - \left(\int a(\coth^n(\beta x))\,dx\right), z - \left(\int b(\coth^k(\lambda x))\,dx\right)\right)\right.$$

6.9.10.4 [1989] Problem 4

problem number 1989

Added Jan 19, 2020.

Problem Chapter 9.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\coth^n(\beta x)w_y + c\coth^k(\lambda y)w_z = aw + s\coth^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Coth[beta*x]^n*D[w[x,y,z],y]+c*Coth[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*coth(beta*x)^n*diff(w(x,y,z),y)+ c*coth(lambda*y)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s(\coth^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\coth^n(\beta x))\,dx\right), z - \left(\int^x c\left(\frac{\cosh\left(\left(b\left(\int(\cot\right.\right.\right.}{\sinh\left(\left(b\left(\int(\cot\right.\right.}\right.\right.\right.$$

6.9.10.5 [1990] Problem 5

problem number 1990

Added Jan 19, 2020.

Problem Chapter 9.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\coth^{n_1}(\lambda_1 x)w_x + b_2\coth^{n_2}(\lambda_2 y)w_y + b_3\coth^{n_3}(\lambda_3 z)w_z = aw + c_1\coth^{k_1}(\beta_1 x) + c_2\coth^{k_2}(\beta_2 y) + c_3\coth^{k_3}(\beta_3$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Coth[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Coth[lambda2*x]^n2*D[w[x,y,z],y]+b3*Coth[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\coth^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(1,\frac{1}{2}-\frac{\text{n1}}{2};\frac{3}{2}-\frac{\text{n1}}{2};\coth^2(\text{lambda1}x)\right)}{\text{b1lambda1}-\text{b1lambda1n1}}\right)\left(\int_1^x \frac{\exp\left(\frac{a\coth^{1}}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*coth(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*coth(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{\left(c_1\left(\coth^{k_1}(\beta_1 x)\right)+c_2\left(\coth^{k_2}(\beta_2 x)\right)+c_3\left(\coth^{k_3}(\beta_3 x)\right)\right)\left(\coth^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\coth^{-n_1}(x\lambda_1\right.\right.}{b_1}}}{b_1}\right.$$

6.9.11 4.5

Local contents

6.9.11.1 [1991] Problem 1 2659
6.9.11.2 [1992] Problem 2 2660
6.9.11.3 [1993] Problem 3 2661
6.9.11.4 [1994] Problem 4 2661
6.9.11.5 [1995] Problem 5 2662

6.9.11.1 [1991] Problem 1

problem number 1991

Added Jan 19, 2020.

Problem Chapter 9.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\sinh^n(\beta x)w + k\cosh^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Sinh[beta*x]^n*w[x,y,z]+ k*Cosh[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\sqrt{\cosh^2(\beta x)}\operatorname{sech}(\beta x)\sinh^{n+1}(\beta x)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*sinh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\lambda x))\,\mathrm{e}^{-c\left(\int(\sinh^n(\beta x))dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c(\sinh^n(\beta x))dx}$$

6.9.11.2 [1992] Problem 2

problem number 1992

Added Jan 19, 2020.

Problem Chapter 9.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\tanh^n(\beta x)w + k\coth^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Tanh[beta*x]^n*w[x,y,z]+ k*Coth[lam
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c\tanh^{n+1}(\beta x)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\beta x)\right)}{\beta n+\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(\right.}{n\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*tanh(beta*x)^n*w(x,y,z)+ k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int k\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^m \mathrm{e}^{-c\left(\int\left(\frac{\sinh(\beta x)}{\cosh(\beta x)}\right)^n dx\right)}dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c\left(\frac{\sinh(\beta x)}{\cosh(\beta x)}\right)^n dx}$$

6.9.11.3 [1993] Problem 3

problem number 1993

Added Jan 19, 2020.

Problem Chapter 9.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + b\cosh^n(\beta x)w_y + c\sinh^k(\lambda y)w_z = aw + s\cosh^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Cosh[beta*x]^n*D[w[x,y,z],y]+c*Sinh[lambda*y]^k*D[w[x,y,z],z]==a*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ b*cosh(beta*x)^n*diff(w(x,y,z),y)+ c*sinh(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k(\cosh^m(\mu x))\,\mathrm{e}^{-ax}dx + \_F1\left(y - \left(\int b(\cosh^n(\beta x))\,dx\right), z - \left(\int c(\sinh^k(\lambda x))\,dx\right)\right)\right)$$

6.9.11.4 [1994] Problem 4

problem number 1994

Added Jan 19, 2020.

Problem Chapter 9.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y, z)$

$$w_x + a\tanh^n(\beta x)w_y + b\coth^k(\lambda x)w_z = cw + s\tanh^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Tanh[beta*x]^n*D[w[x,y,z],y]+b*Coth[lambda*x]^k*D[w[x,y,z],z]==c*w[x
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to -\frac{s\left(e^{-2\mu x}-1\right)^m\left(e^{-2\mu x}+1\right)^m\left(-e^{-4\mu x}\left(e^{2\mu x}-1\right)^2\right)^{-m}\tanh^m(\mu x)F_1\left(\frac{c}{2\mu};m,-m;\frac{c}{2\mu}+1;\right.}{c}\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+ a*tanh(beta*x)^n*diff(w(x,y,z),y)+ b*coth(lambda*x)^k*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s(\tanh^m(\mu x))\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a(\tanh^n(\beta x))\,dx\right),z-\left(\int b(\coth^k(\lambda x))\,dx\right)\right)\right.$$

6.9.11.5 [1995] Problem 5

problem number 1995

Added Jan 19, 2020.

Problem Chapter 9.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\sinh^{n_1}(\lambda_1 x)w_x+b_2\cosh^{n_2}(\lambda_2 y)w_y+b_3\sinh^{n_3}(\lambda_3 z)w_z=aw+c_1\cosh^{k_1}(\beta_1 x)+c_2\sinh^{k_2}(\beta_2 y)+c_3\sinh^{k_3}(\beta_3 z$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Sinh[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Cosh[lambda2*x]^n2*D[w[x,y,z],y]+b3*Sinh[lamb
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{a\sqrt{\cosh^2(\text{lambda1}x)}\text{sech}(\text{lambda1}x)\sinh^{1-\text{n1}}(\text{lambda1}x)\,{}_2F_1\left(\frac{1}{2},\frac{1-\text{n1}}{2};\frac{3-\text{n1}}{2};-\sinh\right.}{\text{b1lambda1}-\text{b1lambda1n1}}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*sinh(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*cosh(lambda__2*x)^(n__2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int\frac{\left(c_1\left(\cosh^{k_1}(\beta_1 x)\right)+c_2\left(\sinh^{k_2}(\beta_2 x)\right)+c_3\left(\sinh^{k_3}(\beta_3 x)\right)\right)\left(\sinh^{-n_1}(x\lambda_1)\right)\mathrm{e}^{-\frac{a\left(\int\left(\sinh^{-n_1}(x\lambda_1)\right.}{b_1}}}{b_1}\right.$$

6.9.12 5.1

Local contents

6.9.12.1 [1996] Problem 1 2664
6.9.12.2 [1997] Problem 2 2665
6.9.12.3 [1998] Problem 3 2666
6.9.12.4 [1999] Problem 4 2666
6.9.12.5 [2000] Problem 5 2667

6.9.12.1 [1996] Problem 1

problem number 1996

Added Jan 19, 2020.

Problem Chapter 9.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + aw_y + bw_z = c\ln^n(\beta x)w + k\ln^m(\lambda x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*D[w[x,y,z],y]+b*D[w[x,y,z],z]==c*Log[beta*x]^n*w[x,y,z]+ k*Log[lambd
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to \exp\left(\frac{c(-\log(\beta x))^{-n}\log^n(\beta x)\,\text{Gamma}(n+1,-\log(\beta x))}{\beta}\right)\left(\int_1^x \exp\left(-\frac{c\,\text{Gamma}(n+1}{}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*diff(w(x,y,z),y)+ b*diff(w(x,y,z),z)=c*ln(beta*x)^n*w(x,y,z)+ k*ln
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int k\ln(\lambda x)^m \mathrm{e}^{-c\left(\int \ln(\beta x)^n dx\right)} dx + \_F1(-ax+y,-xb+z)\right)\mathrm{e}^{\int c\ln(\beta x)^n dx}$$

6.9.12.2 [1997] Problem 2

problem number 1997

Added Jan 19, 2020.

Problem Chapter 9.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + a\ln^n(\beta x)w_y + b\ln^k(\lambda x)w_z = cw + s\ln^m(\mu x)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ a*Log[beta*x]^n*D[w[x,y,z],y]+b*Log[lambda*x]^k*D[w[x,y,z],z]==c*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to e^{cx}\left(\int_1^x e^{-cK[1]}s\log^m(\mu K[1])dK[1]+c_1\left(y-\frac{a(-\log(\beta x))^{-n}\log^n(\beta x)\,\mathrm{Gamma}(n+1,-}{\beta}\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+a*ln(beta*x)^n*diff(w(x,y,z),y)+ b*ln(lambda*x)^k*diff(w(x,y,z),z)=c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int s\ln(\mu x)^m\,\mathrm{e}^{-cx}dx+\_F1\left(y-\left(\int a\ln(\beta x)^n\,dx\right),z-\left(\int b\ln(\lambda x)^k\,dx\right)\right)\right)\mathrm{e}^{cx}$$

6.9.12.3 [1998] Problem 3

problem number 1998

Added Jan 19, 2020.

Problem Chapter 9.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$w_x + b\ln^n(\beta x)w_y + c\ln^k(\lambda y)w_z = aw + s\ln^m(\mu x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x,y,z],x]+ b*Log[beta*x]^n*D[w[x,y,z],y]+c*Log[lambda*y]^k*D[w[x,y,z],z]==a*w[x,y
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := diff(w(x,y,z),x)+b*ln(beta*x)^n*diff(w(x,y,z),y)+ c*ln(lambda*y)^k*diff(w(x,y,z),z)=a
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int s\ln(\mu x)^m e^{-ax}dx + \_F1\left(y - \left(\int b\ln(\beta x)^n dx\right), z - \left(\int^x c\ln\left(\left(b\left(\int \ln(\_b\beta)^n d\_b\right.\right.\right.\right.\right.$$

6.9.12.4 [1999] Problem 4

problem number 1999

Added Jan 19, 2020.

Problem Chapter 9.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$a\ln(\alpha x)w_x + b\ln(\beta y)w_y + c\ln(\gamma z)w_z = pw + q\ln(\lambda x)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  a*Log[alpha*x]*D[w[x,y,z],x]+ b*Log[beta*y]*D[w[x,y,z],y]+c*Log[gamma*z]*D[w[x,y,z],z
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

Failed

Maple ✓

```
restart;
local gamma;
pde := a*ln(alpha*x)*diff(w(x,y,z),x)+b*ln(beta*y)*diff(w(x,y,z),y)+ c*ln(gamma*z)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z)=\left(\int \frac{q\,\mathrm{e}^{\frac{p\,\mathrm{expIntegral}(1,-\ln(\alpha x))}{a\alpha}}\ln(\lambda x)}{a\ln(\alpha x)}dx+\_F1\left(\frac{-a\alpha\,\mathrm{expIntegral}(1,-\ln(\beta y))+b\beta\,\mathrm{expIntegral}(\right.\right.$$

6.9.12.5 [2000] Problem 5

problem number 2000

Added Jan 19, 2020.

Problem Chapter 9.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y,z)$

$$b_1\ln^{n_1}(\lambda_1 x)w_x+b_2\ln^{n_2}(\lambda_2 y)w_y+b_3\ln^{n_3}(\lambda_3 z)w_z=aw+c_1\ln^{k_1}(\beta_1 x)+c_2\ln^{k_2}(\beta_2 y)+c_3\ln^{k_3}(\beta_3 z)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  b1*Log[lambda1*x]^n1*D[w[x,y,z],x]+ b2*Log[lambda2*x]^n2*D[w[x,y,z],y]+b3*Log[lambda3
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x,y,z}], 60*10]];
```

$$\left\{\left\{w(x,y,z)\to\exp\left(\frac{a(-\log(\mathrm{lambda1}x))^{\mathrm{n1}}\log^{-\mathrm{n1}}(\mathrm{lambda1}x)\,\mathrm{Gamma}(1-\mathrm{n1},-\log(\mathrm{lambda1}x))}{\mathrm{b1lambda1}}\right)\left(\int\right.\right.\right.$$

Maple ✓

```
restart;
local gamma;
pde := b__1*ln(lambda__1*x)^(n__1)*diff(w(x,y,z),x)+b__2*ln(lambda__2*x)^(n__2)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
```

$$w(x,y,z) = \left(\int \frac{e^{-\frac{a\left(\int \ln(\lambda_1 x)^{-n_1} dx\right)}{b_1}} \left(c_1 \ln(\beta_1 x)^{k_1} + c_2 \ln(\beta_2 x)^{k_2} + c_3 \ln(\beta_3 x)^{k_3}\right) \ln(\lambda_1 x)^{-n_1}}{b_1} dx + \_F1 \left(- \frac{b}{}\right.\right.$$

CHAPTER 7

Handbook of nonlinear partial differential equations

Local contents

7.1 chapter 1 2670

7.1 chapter 1

Local contents

7.1.1 1.1 2670
7.1.2 1.2 2672
7.1.3 1.3 2676
7.1.4 1.4 2679

7.1.1 1.1

Local contents

7.1.1.1 [2001] Problem 1 2670
7.1.1.2 [2002] Problem 2 2671

7.1.1.1 [2001] Problem 1

problem number 2001

Added March 23, 2019.

Problem Chapter 1.1.1.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + bw^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) + b*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.1.2 [2002] Problem 2

problem number 2002

Added March 23, 2019.

Problem Chapter 1.1.1.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw(1-w)$$

Mathematica ✓

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]*(1 - w[x, t]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5at-\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5at-i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5at-i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\right.$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{1}{12}\left(5at+i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\left(1+\tanh\left(\frac{1}{12}\left(5at+i\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)\right.$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{1}{12}\left(5at+\sqrt{6}\sqrt{a}x-12c_3\right)\right)\right)^2\right\}$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{5at}{12}-\frac{\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{5at}{12}-\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5at}{12}-\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{w(x,t) \to -\frac{1}{4}\left(-3+\tanh\left(\frac{5at}{12}+\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\left(1+\tanh\left(\frac{5at}{12}+\frac{i\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)\right\}$$
$$\left\{w(x,t) \to \frac{1}{4}\left(1+\tanh\left(\frac{5at}{12}+\frac{\sqrt{a}x}{2\sqrt{6}}+c_3\right)\right)^2\right\}$$

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) + a*w(x,t)*(1-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = -\frac{\left(\tanh^2\left(-\frac{5at}{12} + c_1 + \frac{\sqrt{-6a}\,x}{12}\right)\right)}{4} - \frac{\tanh\left(-\frac{5at}{12} + c_1 + \frac{\sqrt{-6a}\,x}{12}\right)}{2} + \frac{3}{4}$$

7.1.2 1.2

Local contents

7.1.2.1 [2003] Problem 1 2672
7.1.2.2 [2004] Problem 2 2673
7.1.2.3 [2005] Problem 3 2674
7.1.2.4 [2006] Problem 4 2674
7.1.2.5 [2007] Problem 5 2675

7.1.2.1 [2003] Problem 1

problem number 2003

Added March 23, 2019.

Problem Chapter 1.1.2.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} - bw^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.2.2 [2004] Problem 2

problem number 2004

Added March 23, 2019.

Problem Chapter 1.1.2.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw - bw^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] - b*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)- b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{\sqrt{ab}\left(\tanh\left(-\frac{3at}{4} + \frac{\sqrt{2}\sqrt{a}\,x}{4} + c_1\right) - 1\right)}{2b}$$

7.1.2.3 [2005] Problem 3

problem number 2005

Added March 23, 2019.

Problem Chapter 1.1.2.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} - bw^3 - cw^2$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3 - c*w[x, t]^2;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3- c*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{\left(\tanh\left(-\frac{c^2 t}{4b} + \frac{\sqrt{2}\,cx}{4\sqrt{ab}} + c_1\right) - 1\right) c}{2b}$$

7.1.2.4 [2006] Problem 4

problem number 2006

Added March 23, 2019.

Problem Chapter 1.1.2.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} - w(1-w)(a-w)$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] - w[x, t]*(1 - w[x, t])*(a - w[x, t]);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) - w(x,t)*(1-w(x,t))*(a-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{\tanh\left(c_1 + \frac{(-2a+1)t}{4} + \frac{\sqrt{2}\,x}{4}\right)}{2} + \frac{1}{2}$$

7.1.2.5 [2007] Problem 5

problem number 2007

Added March 23, 2019.

Problem Chapter 1.1.2.5, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + b_0 + b_1 w + b_2 w^2 + b_3 w^3$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b0 + b1*w[x, t] + b2*w[x, t]^2 + b3*w[x, t]^3;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b0+b1*w(x,t)+b2*w(x,t)^2+b3*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = \frac{-2304\Big(ab3\,\mathrm{RootOf}\left(512\_Z^6a^3b3^2 - 27b0^2b3^3 + 18b0\,b1\,b2\,b3^2 - 4b0\,b2^3b3 - 4b1^3b3^2 + b1^2b\right.}{}$$

7.1.3 1.3

Local contents

7.1.3.1 [2008] Problem 1 2676
7.1.3.2 [2009] Problem 2 2677
7.1.3.3 [2010] Problem 3 2678
7.1.3.4 [2011] Problem 4 2678

7.1.3.1 [2008] Problem 1

problem number 2008

Added March 23, 2019.

Problem Chapter 1.1.3.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + bw^k$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^k;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b*w(x,t)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.3.2 [2009] Problem 2

problem number 2009

Added March 23, 2019.

Problem Chapter 1.1.3.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.3.3 [2010] Problem 3

problem number 2010

Added March 23, 2019.

Problem Chapter 1.1.3.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m + cw^{2m-1}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m + c*w[x, t]^(2*m - 1);
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m+c*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.3.4 [2011] Problem 4

problem number 2011

Added March 23, 2019.

Problem Chapter 1.1.3.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw^{m-1} + bmw^m - mb^2w^{2m-1}$$

Mathematica ✗

```
ClearAll["Global`*"];
pde =  D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]^(m - 1) + b*m*w[x, t]^m - m*b^2*w[x,
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)^(m-1)+b*m*w(x,t)^m-m*b^2*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

7.1.4 1.4

Local contents

7.1.4.1 [2012] Problem 1 2679

7.1.4.1 [2012] Problem 1

problem number 2012

Added March 23, 2019.

Problem Chapter 1.1.4.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + s_1(bx+ct)^k + s_2 w^m$$

Mathematica ✗

```
ClearAll["Global`*"];
pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + s1*(b*x + c*t)^k + s2*w[x, t]^m;
sol =  AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
restart;
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +s1*(b*x+c*t)^k+s2*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

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