Solving partial differential equations in Maple 2018.2 and Mathematica 11.3

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January 29, 2024   Compiled on January 29, 2024 at 11:03pm

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154 HNPDE, chapter 1.1.2 **1585**
154.1 Problem 1 1585
154.2 Problem 2 1586
154.3 Problem 3 1587
154.4 Problem 4 1588
154.5 Problem 5 1589

155 HNPDE, chapter 1.1.3 **1590**
155.1 Problem 1 1590
155.2 Problem 2 1591
155.3 Problem 3 1592
155.4 Problem 4 1593

156 HNPDE, chapter 1.1.4 **1594**
156.1 Problem 1 1594

1 Introduction

This report gives the result of running a number of partial differential equations in Mathematica and Maple. This is work in progress as more PDE's are being added.

The following systems are used

1. Mathematica 11.3 (64 bit).
2. Maple 2018.2.1 (64 bit) with Physics version MapleCloud 319.

The following are plain text files of the current collection of the PDE's used in this report.

Mathematica_PDE_IN_CAS_problems.txt

Maple_PDE_IN_CAS_problems.txt

10 minutes real time was used as the time limit to complete a problem. If CAS did not finish within this time limit, a failed score is given.

The PC used to run these tests is windows 10 professional 64 bit with 64 GB RAM running Intel core i7-8086K at 4 GHZ.

All possible options, assumptions and HINTS were tried to obtain a solution. The command `DSolve` was used in Mathematica and the command `pdsolve` in Maple.

It is possible I missed some option, assumption or HINT, which could help make the CAS able to solve a given PDE now marked as unsolved. Will correct such a case if found. Most of the solutions returned are not verified. If a CAS returns a solution, it is assumed to be correct and that the problem was solved by CAS.

These problems were collected from textbooks such as

1. Richard Haberman applied partial differential equations, 5th edition
2. David J Logan applied Partial differential equations.
3. Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.
4. Handbook of first order partial differential equations (HFOPDE), Volume 1, by Polyanin, Zaitsev, Moussiaux (2002).
5. Handbook of nonlinear partial differential equations (HNPDE), by Polyanin, Zaitsev (2004).

PDE's from other text books will be added with time.

Some problems were also collected from Maple and Mathematica help pages, documentation and technical forums.

Some of these problems I solved by hand. Will try to add more hand solutions in the future in order to compare with computer solution.

2 Results

The current number of partial differential equations solved is [1250].

Mathematica solved [671] and Maple solved [1099].

Mathematica	Maple
53.68%	87.92%

Table 1: Percentage solved by each CAS

Mathematica	Maple
36.27%	11.26%

Table 2: Percentage of failed due to time out among all problems that could not be solved

Mathematica	Maple
42.71 (hours)	4.18 (hours)

Table 3: Total real time used to solve all problems

Mathematica	Maple
123.019 (sec)	12.041 (sec)

Table 4: Average real time used to solve one problem

3 Lookup table of results

Table 5: Breakdown of results for each PDE. Time in seconds

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1	General first order PDE's	Linear PDE, the transport equation	✓	0.011	✓	0.078	Yes
2	General first order PDE's	Linear PDE	✓	0.003	✓	0.012	Yes
3	General first order PDE's	Linear PDE, initial value problem	✓	0.013	✓	0.116	Yes
4	General first order PDE's	Initial-boundary value problem	✓	0.044	✓	0.31	Yes
5	General first order PDE's	Linear PDE, the transport equation with initial conditions	✓	0.003	✓	0.015	Yes
6	General first order PDE's	First order wave PDE, with initial conditions (Haberman 12.2.2)	✓	0.002	✓	0.013	Yes
7	General first order PDE's	First order wave PDE, with initial and boundary conditions (Haberman 12.2.4)	✗	41.83	✓ Solution contains unresolved invlaplace calls	0.182	Yes
8	General first order PDE's	First order wave PDE, with initial conditions, non homogeneous (Haberman 12.2.5 (a))	✓	0.026	✓	0.029	Yes
9	General first order PDE's	First order wave PDE, with initial conditions, non homogeneous (Haberman 12.2.5 (d))	✓	0.018	✓	0.046	Yes
10	General first order PDE's	General solution for a quasilinear first-order PDE	✓	0.031	✓	0.032	Yes

Continued on next page

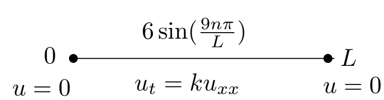
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
11	General first order PDE's	quasilinear first-order PDE, scalar conservation law	✓ Implicit solution	0.031	✓	0.011	Yes
12	General first order PDE's	quasilinear first-order PDE, scalar conservation law with initial value	✓	0.007	✓	0.015	Yes
13	General first order PDE's	nonlinear first-order PDE, the Clairaut equation	✓	0.023	✓	0.098	Yes
14	General first order PDE's	nonlinear first-order PDE, the Clairaut equation with initial value	✓	0.009	✓	0.529	No
15	General first order PDE's	Another example of nonlinear Clairaut equation	✓	0.021	✓	0.006	No
16	General first order PDE's	Recover a function from its gradient vector	✓	0.013	✓	0.062	No
17	General first order PDE's	General solution of a first order nonlinear PDE	✗	0.094	✓ Has unresolved integral in the answer	0.031	No
18	General first order PDE's	Nonlinear first order PDE	✓	0.086	✓	0.486	No
19	General first order PDE's	first order PDE of three unknowns	✗ (Timed out)	600.	✓	1.612	No
20	Heat PDE in bar (1D)	 Haberman 2.3.3 (a)	✓	0.3	✓	1.769	Yes

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Table 5 – continued from previous page

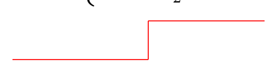
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
21	Heat PDE in bar (1D)	$3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$ 0 ● —— ● L $u=0$ $u_t = k u_{xx}$ $u=0$ Haberman 2.3.3 (b)	✓	0.78	✓	1.771	Yes
22	Heat PDE in bar (1D)	$2\cos\frac{3\pi x}{L}$ 0 ● —— ● L $u=0$ $u_t = k u_{xx}$ $u=0$ Haberman 2.3.3 (c)	✓ but $n = 3$ should be special case	0.777	✓ handled $n = 3$ case correctly.	3.359	Yes
23	Heat PDE in bar (1D)	$u(x,0) = \begin{cases} 1 & 0 < x \leq \frac{L}{2} \\ 2 & \frac{L}{2} < x \leq L \end{cases}$  0 ● —— ● L $u=0$ $u_t = k u_{xx}$ $u=0$ Haberman 2.3.3 (d)	✓	0.959	✓	1.831	Yes
24	Heat PDE in bar (1D)	$f(x)$ 0 ● —— ● L $u_x=0$ $u_t = k u_{xx}$ $u_x=0$ Haberman 2.3.7	✓	0.313	✓	0.665	Yes
25	Heat PDE in bar (1D)	$f(x)$ 0 ● —— ● L $u=0$ $u_t = k u_{xx} - \alpha u$ $u=0$ $\alpha > 0$ Haberman 2.3.8	✗	0.824	✓	0.622	Yes

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Table 5 – continued from previous page

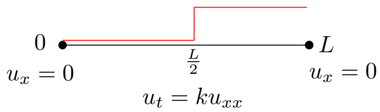
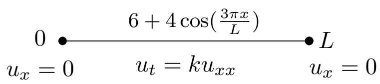
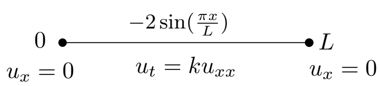
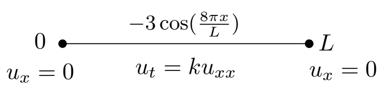
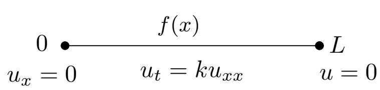
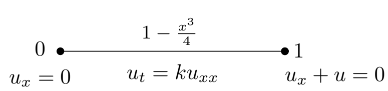
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
26	Heat PDE in bar (1D)	$u(x,0) = \begin{cases} 0 & 0 < x \le \frac{L}{2} \\ 1 & \frac{L}{2} < x \le L \end{cases}$  0, L, $\frac{L}{2}$, $u_x = 0$, $u_x = 0$, $u_t = k u_{xx}$ Haberman 2.4.1 (a)	✓	0.44	✓	1.328	No
27	Heat PDE in bar (1D)	 $6 + 4\cos(\frac{3\pi x}{L})$, 0, L, $u_x = 0$, $u_t = k u_{xx}$, $u_x = 0$ Haberman 2.4.1 (b)	✓	0.303	✓	1.766	Yes
28	Heat PDE in bar (1D)	 $-2\sin(\frac{\pi x}{L})$, 0, L, $u_x = 0$, $u_t = k u_{xx}$, $u_x = 0$ Haberman 2.4.1 (c)	✓	0.665	✓	3.277	Yes
29	Heat PDE in bar (1D)	 $-3\cos(\frac{8\pi x}{L})$, 0, L, $u_x = 0$, $u_t = k u_{xx}$, $u_x = 0$ Haberman 2.4.1 (d)	✓	0.28	✓	1.869	No
30	Heat PDE in bar (1D)	 $f(x)$, 0, L, $u_x = 0$, $u_t = k u_{xx}$, $u = 0$ Haberman 2.4.2	✓	0.214	✓	0.872	Yes
31	Heat PDE in bar (1D)	 $1 - \frac{x^3}{4}$, 0, 1, $u_x = 0$, $u_t = k u_{xx}$, $u_x + u = 0$ Convection heat loss	✗	30.405	✓	1.768	No

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Table 5 – continued from previous page

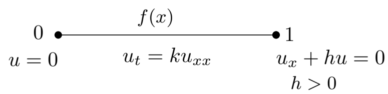
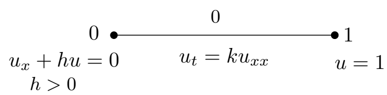
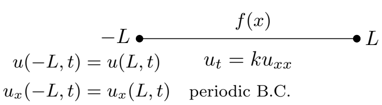
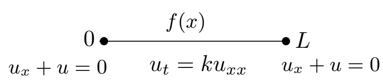
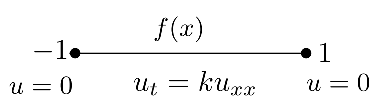
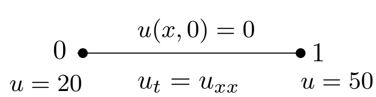
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
32	Heat PDE in bar (1D)	 convection heat loss	✗	31.243	✓	2.771	Yes
33	Heat PDE in bar (1D)	 convection heat loss	✗	8.652	✗ (Timed out)	600.	Yes
34	Heat PDE in bar (1D)	 Periodic boundary conditions	✗	0.348	✓	4.583	No
35	Heat PDE in bar (1D)	 Mixed BC	✗	31.433	✓	3.307	No
36	Heat PDE in bar (1D)	 domain -1 to +1	✗	30.528	✓	2.522	No
37	Heat PDE in bar (1D)	 non-homogeneous BC	✓	14.303	✓	1.087	No

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Table 5 – continued from previous page

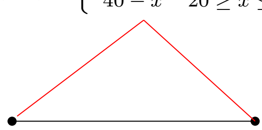
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
38	Heat PDE in bar (1D)	$f(x)$ on $0 \le x \le L$; $u = A$ at 0, $u_t = B$ at L; $u_t = k u_{xx}$ Haberman 8.2.1 (a)	✗	0.659	✓	2.095	Yes
39	Heat PDE in bar (1D)	$f(x)$ on $0 \le x \le L$; $u = A$ at 0, $u = B$ at L; $u_t = k u_{xx} + k$ Haberman 8.2.1 (d)	✗	46.008	✓	1.28	Yes
40	Heat PDE in bar (1D)	$f(x)$ on $0 \le x \le L$; $u = 0$ at 0, $u = 0$ at L; $u_t = u_{xx} - u$ Internal source	✗	0.742	✓	1.976	Yes
41	Heat PDE in bar (1D)	$\sin(2\pi x) - \sin(4\pi x)$ on $0 \le x \le 1$; $u = 0$ at 0, $u = 0$ at 1; $u_t = 100 u_{xx} - u$ Internal source	✓	0.409	✓	1.369	No
42	Heat PDE in bar (1D)	$u(x,0) = \begin{cases} x & 0 \le x < 20 \\ 40 - x & 20 \ge x \le 40 \end{cases}$  $0 \le x \le 40$; $u = 0$ at 0, $u = 0$ at 40; $u_t = u_{xx}$ IC hat function	✓	18.782	✓	3.005	No
43	Heat PDE in bar (1D)	$u(x,0) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$ $0 \le x \le 1$; $u = 0$ at 0, $u = 1$ at 1; $u_t = u_{xx}$ homogeneous BC	✓	5.378	✓	1.063	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
44	Heat PDE in bar (1D)	0 ● —— 0 —— ● π $u=t$ $\quad u_t = u_{xx} \quad$ $u=0$ BC depends on time	✓	6.514	✓	1.331	No
45	Heat PDE in bar (1D)	0 ● —— $f(x)$ —— ● π $u_x=0$ $\quad u_t = ku_{xx} + \sin(\frac{2\pi x}{L}) \quad$ $u_x=0$ Haberman 8.2.1 (f)	✗	0.38	✓	3.777	No
46	Heat PDE in bar (1D)	0 ● —— x —— ● L $u_x=0$ $\quad u_t = ku_{xx} + \cos(\omega t) \quad$ $u_x=0$ Pinchover and Rubinstein 6.25	✗	0.828	✓	7.329	No
47	Heat PDE in bar (1D)	0 ● —— $f(x)$ —— ● L $u_x=0$ $\quad u_t = ku_{xx} + e^{ct}\sin(\frac{2\pi x}{L}) \quad$ $u_x=0$ external source	✗	0.402	✓	22.746	No
48	Heat PDE in bar (1D)	0 ● —— 0 —— ● π $u=0$ $\quad u_t = u_{xx} + t(\pi - x) \quad$ $u=0$ Math 4567 Exam	✓	10.412	✓	2.304	No
49	Heat PDE in bar (1D)	0 ● —— $1+\cos(2x)$ —— ● 1 $u_x=\sin(t)$ $\quad u_t = u_{xx} + 1 + x\cos(t) \quad$ $u_x=\sin(t)$ Pinchover and Rubinstein 6.17	✗	1.442	✓	3.806	No
50	Heat PDE in bar (1D)	0 ● —— $\frac{1}{2}x^2 + x$ —— ● 1 $u_x=0$ $\quad u_t = 13u_{xx} \quad$ $u_x=1$ nonhomogeneous BC	✗	0.29	✓	7.17	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
51	Heat PDE in bar (1D)	$3\cos(42\pi x)$; left end 0: $u_x = 0$; right end 1: $u_x = 0$; $u_t = 13u_{xx} + g(x,t)$, $g(x,t) = e^{3t}\cos(17\pi x)$ Pinchover and Rubinstein 6.23	✓	0.899	✓	37.384	No
52	Heat PDE in bar (1D)	$\pi\cos(2x)$; left end 0: $u_x = 0$; right end 1: $u_x = 0$; $u_t = u_{xx} + g(x,t)$, $g(x,t) = t\cos(2001x)$ Pinchover and Rubinstein 6.21	✓	0.274	✓	4.076	No
53	Heat PDE in bar (1D)	$60 - 20x$; left end 0: $u_x = \frac{t\sin t}{5}$; right end π: $u_x = \frac{t\cos t}{10}$; $u_t = ku_{xx} + x$ nonhomogeneous BC	✗	30.606	✓	6.369	No
54	Heat PDE in bar (1D)	$f(x)$; left end 0: $u = 0$; right end 1: $u = 0$; $u_t = ku_{xx} + Q(x,t)$ With source	✗	1.536	✓	7.043	No
55	Heat PDE in bar (1D)	$u(x,0) = 0$; left end 0: $u = 1$; right end π: $u = 0$; $u_t = ku_{xx} + e^{-2t}\sin(5x)$ Haberman 8.3.6	✗	30.682	✓	20.185	Yes
56	Heat PDE in bar (1D)	$f(x)$; left end 0: $u_x = A(t)$; right end L: $u_x = B(t)$; $u_t = u_{xx} + Q(x,t)$ Haberman 8.2.2. (a)	✗	0.41	✓	7.233	Yes
57	Heat PDE in bar (1D)	$u(x,0) = 60x - 50x^2 + 10$; left end 0: $u = 10$; right end 1: $u = 20$; $u_t = \frac{1}{20}u_{xx}$ Articolo 8.4.1	✓	15.524	✓	1.203	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
58	Heat PDE in bar (1D)	0 —— 1; $\frac{-40x^2}{3}+\frac{45x}{2}+5$; $u=5$, $u_t=\frac{1}{20}u_{xx}+t$, $u_x+u=10$ Articolo 8.4.3	✗	30.722	✗ (Timed out)	600.	No
59	Heat PDE in bar (1D)	0 —— 1; $\frac{x^2}{2}+x$; $u_x=0$, $u_t=13u_{xx}$, $u_x=0$ both ends insulated	✗	14.258	✓	4.159	No
60	Heat PDE in bar (1D)	0 —— π; $\sin x$; $u_x=1$, $u_t=u_{xx}$, $u_x=-1$ both ends nonhomogeneous	✗	30.537	✓	10.458	Yes
61	Heat PDE in bar (1D)	0 —— π; 0; $u=0$, $u_t=u_{xx}$, $u_x=A$ nonhomogeneous BC	✗	5.065	✓	0.981	No
62	Heat PDE in bar (1D)	0 —— a; $rf(r)$; $u=0$, $u_t=ku_{rr}$, $u=a\phi(t)$ nonhomogeneous BC	✗	32.086	✓	7.175	Yes
63	Heat PDE in bar (1D)	1 —— b; $\ln x$; $u_x=0$, $u_t=x^2u_{xx}+xu_x$, $u_x+hu=0$ Euler-Cauchy Sturm-Liouville	✗	1.011	✓	136.857	Yes
64	Heat PDE in bar (1D)	0 —— 1; $e^{\frac{45}{10}}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x))$; $u=0$, $u_t=u_{xx}-9u_x$, $u=0$ special initial condition	✓	2.255	✗ (Timed out)	600.	Yes

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
65	Heat PDE in bar (1D)	1 ●——● b, $f(x)$; $u=0$, $u_t = x^2 u_{xx} + x u_x$, $u=0$ Euler-Cauchy Sturm-Liouville	✗	0.784	✓	1.717	No
66	Heat PDE on semi-infinite domain (1D)	0 ●——► ∞, 0; $u = f(t)$, $u_t = u_{xx}$ Logan p. 76	✓	0.263	✓	1.542	No
67	Heat PDE on semi-infinite domain (1D)	0 ●——► ∞, 0; $u = 1$, $u_t = k u_{xx}$ nonhomogeneous BC	✓	1.545	✓	0.272	No
68	Heat PDE on semi-infinite domain (1D)	$-x_0$ ●——► ∞, $u(x,t_0) = 10$; $u = 0$, $u_t = \frac{1}{4} u_{xx}$ $x > \lvert x_0 \rvert, t > \lvert t_0 \rvert$ I.C. not at zero	✗ due to IC/BC not zero	0.008	✓	1.102	No
69	Heat PDE on semi-infinite domain (1D)	0 ●——► ∞, λ; $u = \mu$, $u_t = k u_{xx}$ $x > 0, t > 0$ nonhomogeneous BC	✓	2.102	✓	0.297	No
70	Heat PDE on semi-infinite domain (1D)	0 ●——► ∞, $\cos x$; $u = 1$, $u_t = u_{xx}$ $x > 0, t > 0$ nonhomogeneous BC	✓	8.042	✓	2.295	No

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Table 5 – continued from previous page

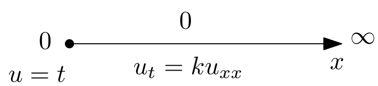
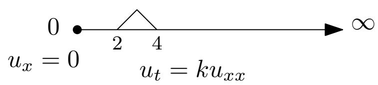
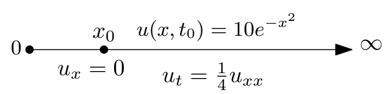
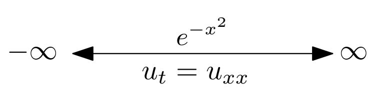
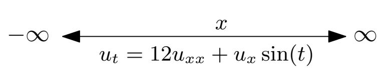
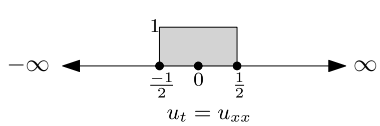
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
71	Heat PDE on semi-infinite domain (1D)	 nonhomogeneous B.C.	✓	2.294	✓	1.269	No
72	Heat PDE on semi-infinite domain (1D)	 Unit triangle I.C.	✓	26.281	✓	1.573	No
73	Heat PDE on semi-infinite domain (1D)	 I.C. not at $t = 0$	✗	0.009	✓	1.185	No
74	Heat PDE on infinite domain, 1D	 Inverse exponential I.C.	✓	1.164	✓	0.573	Yes
75	Heat PDE on infinite domain, 1D	 Advection term	✓	0.366	✓	0.512	No
76	Heat PDE on infinite domain, 1D	 UnitBox I.C.	✓	1.593	✓	0.922	No

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Table 5 – continued from previous page

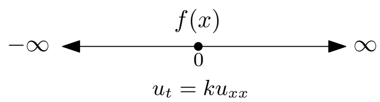
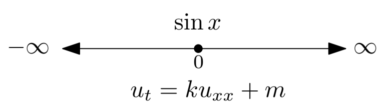
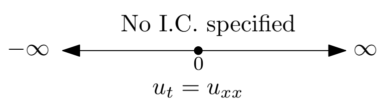
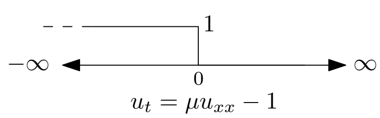
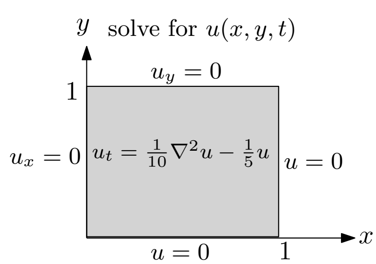
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
77	Heat PDE on infinite domain, 1D	 No source	✓	0.153	✓	1.442	No
78	Heat PDE on infinite domain, 1D	 constant as source	✓	5.562	✓	0.384	No
79	Heat PDE on infinite domain, 1D	 No intial conditions	✗	0.002	✓	0.103	Yes
80	Heat PDE on infinite domain, 1D	 piecewise I.C.	✗ due to i.c. not at zero	0.01	✓	2.676	Yes
81	Heat PDE in rectangle	No source	✗	0.004	✓	3.45	No
82	Heat PDE in rectangle	 At $t = 0, u = (1 - x^2)(1 - \frac{1}{2}y)y$ Internal source term	✗	0.003	✓	4.05	No

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Table 5 – continued from previous page

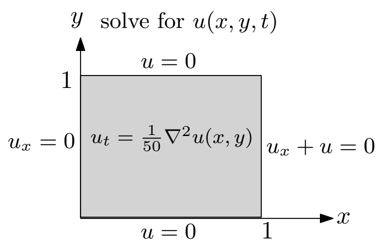
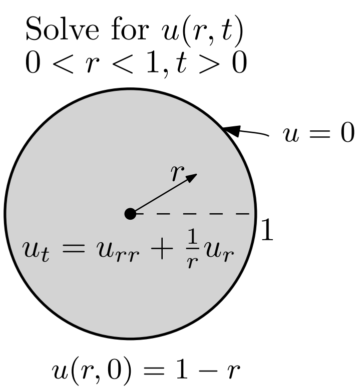
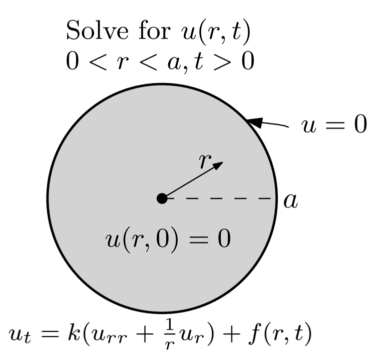
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
83	Heat PDE in rectangle	 At $t=0, u=(1-\frac{x^2}{3})y(1-y)$ Articolo 6.6.3	✗	0.003	✓	6.08	No
84	Heat PDE inside disk	 No θ dependency	✓	0.799	✓ But has unresolved inverse Laplace transforms	0.698	No
85	Heat PDE inside disk	 Haberman 8.3.5	✗	1.423	✗	1.207	Yes

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Table 5 – continued from previous page

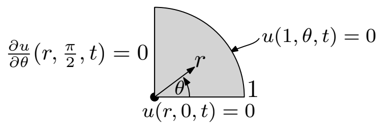
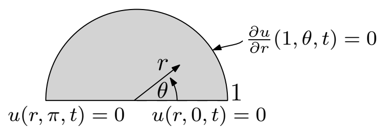
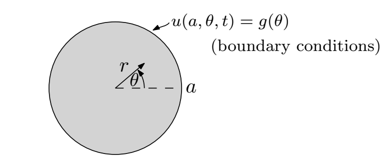
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
86	Heat PDE inside disk	Solve for $u(r,\theta,t)$ $0<r<1, 0<\theta<\frac{\pi}{2}, t>0$  I.C. $u(r,\theta,0)=(r-r^3)\sin\theta$ $u_t=\frac{1}{50}(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Articolo 6.9.1	✗	0.027	✗	0.9	No
87	Heat PDE inside disk	Solve for $u(r,\theta,t)$ $0<r<1, 0<\theta<\pi, t>0$  I.C. $u(r,\theta,0)=(r-\frac{r^3}{3})\sin\theta$ $u_t=\frac{1}{25}(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Articolo 6.9.2	✗	0.004	✓	13.496	No
88	Heat PDE inside disk	Solve for $u(r,\theta,t)$ $0<r<a, 0<\theta<2\pi, t>0$  I.C. $u(r,\theta,0)=f(r,\theta)$ $u_t=k(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$ Haberman 8.2.5	✗	0.025	✗	0.183	Yes
89	Heat PDE inside Sphere	No angle dependencies, zero intial conditions, non zero temperature at surface	✗	1.499	✓ Has un-resolved Laplace integrals	0.463	No

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Table 5 – continued from previous page

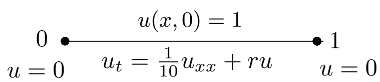
#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
90	Diffusion Reaction in 1D	 Growth form reaction term	✗	22.672	✓	1.185	Yes
91	Diffusion Reaction in 1D	using logistic form for reaction term	✗	1.172	✗	8.409	No
92	Diffusion Reaction in 1D	using Aleee form for reaction term	✗	2.196	✗	13.396	No
93	Diffusion-advection (convection) in 1D	Semi infinite domain	✗	0.454	✓	1.636	No
94	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle (Haberman 2.5.1 (a))	✓	0.179	✓	20.947	No
95	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle (Haberman 2.5.1 (b))	✓	0.161	✓	19.242	No
96	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle (Haberman 2.5.1 (c))	✗	0.278	✓	21.029	No
97	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle (Haberman 2.5.1 (d))	✗	0.281	✓	19.472	No
98	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle (Haberman 2.5.1 (e))	✗	0.554	✓	29.724	Yes
99	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, top/bottom edges non-zero	✓	2.153	✓	6.154	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
100	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, top edge at infinity, left edge nonhomogeneous constant	✗	1.543	✓	2.131	Yes
101	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, top edge at infinity, right edge nonhomogeneous constant	✗	1.54	✓	2.337	Yes
102	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, right edge at infinity, bottom edge nonhomogeneous constant	✗	1.547	✗	0.165	Yes
103	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, right edge at infinity, top edge nonhomogeneous function e^{-x}	✗	1.546	✓ I need to find out how Maple obtained the above solution. It seems to have unknown constant in it	15.123	Yes
104	Laplace PDE in Cartesian coordinates	Laplace PDE inside rectangle, right edge at infinity, top edge nonhomogeneous function $f(x)$	✗	1.458	✗ (Timed out)	600.	No
105	Laplace PDE in Cartesian coordinates	Laplace PDE in 2D Cartessian with boundary condition as Dirac function	✓	0.039	✓	0.793	No
106	Laplace PDE in Cartesian coordinates	Laplace PDE in rectangle, one side homogeneous and 3 sides are not	✓	0.527	✓	29.212	No
107	Laplace PDE in Cartesian coordinates	Laplace on all of the right half plane with $u = f(y)$ on the y axes	✓	32.327	✓	0.629	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
108	Laplace PDE in Cartesian coordinates	Laplace PDE in rectangle with infinity in the x direction with $u = \sin(y)$ on left edge.	✗	1.422	✓	8.995	No
109	Laplace PDE in Cartesian coordinates	Laplace PDE inside a disk, periodic boundary conditions	✓	0.133	✓	2.859	No
110	Laplace PDE in Cartesian coordinates	Dirichlet problem for the Laplace equation in upper half plan	✓	1.533	✓	3.012	No
111	Laplace PDE in Cartesian coordinates	Dirichlet problem for the Laplace equation in right half-plane:	✓	24.679	✓	0.153	No
112	Laplace PDE in Cartesian coordinates	Dirichlet problem for the Laplace equation in the first quadrant	✓	29.45	✗	3.667	No
113	Laplace PDE in Cartesian coordinates	Neumann problem for the Laplace equation in the upper half-plane	✓	12.711	✓ used convert(sol,Int).	0.948	No
114	Laplace PDE in Cartesian coordinates	Dirichlet problem for the Laplace equation in a rectangle	✓	1.129	✓	2.542	No
115	Laplace PDE in Cartesian coordinates	Cartessian coordinates with boundary conditions on two sides only	✗	0.199	✓	2.504	No
116	Laplace PDE in Cartesian coordinates	in Rectangle, right edge at infinity	✗	1.458	✓	9.607	No
117	Laplace PDE in Cartesian coordinates	Laplace PDE inside quarter disk, Neumann BC at edge	✗	1.485	✓	1.722	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
118	Laplace PDE in Polar coordinates	Laplace PDE inside quarter-circle (Haberman 2.5.5 (c))	✗	1.387	✓	1.114	Yes
119	Laplace PDE in Polar coordinates	Laplace PDE inside semi-circle	✗	1.653	✓	1.378	Yes
120	Laplace PDE in Polar coordinates	Laplace PDE inside circular annulus, Neumann boundary conditions using unspecified functions (Haberman 2.5.8 (b))	✗	0.01	✓ But has unresolved Invfourier and Fourier calls	7.92	No
121	Laplace PDE in Polar coordinates	Laplace PDE inside circular annulus, Dirichlet boundary conditions using specified functions	✓	1.367	✓	1.305	No
122	Laplace PDE in Polar coordinates	Laplace PDE outside a disk, periodic boundary conditions	✗	0.008	✓	2.321	No
123	Laplace PDE in Spherical coordinates	Laplace in a sphere	✗	0.021	✓	0.902	No
124	Poisson PDE in Cartesian coordinates	Poisson equation in a rectangle, all boundaries are zero	✗	0.008	✗	19.357	Yes
125	Poisson PDE in Cartesian coordinates	Dirichlet problem for the Poisson equation in a rectangle	✓	0.176	✓	0.126	No
126	Helmholtz PDE in Cartesian coordinates	Dirichlet problem for the Helmholtz equation in a rectangle	✓	2.226	✓	8.044	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
127	Helmholtz PDE in Cartesian coordinates	With no boundary conditions specified	✗ why? It solved earlier with BC?	0.004	✓	0.064	No
128	Reduced Helmholtz PDE in Cartesian coordinates	Inside square	✗	0.781	✓	30.185	No
129	Helmholtz PDE in 3D Spherical	Chain reaction PDE	✗	0.056	✗ Trivial solution	0.715	Yes
130	Wave PDE on finite length string	Both ends fixed, zero initial position, non-zero initial velocity, with extra term present	✗	28.434	✓	2.362	No
131	Wave PDE on finite length string	One end fixed, another free, both initial conduitions non zero, and source that depends on time and space	✗	0.816	✓	46.266	No
132	Wave PDE on finite length string	Both ends fixed, no initial conduitions give and no source (Logan p. 28)	✗	0.262	✓	0.602	No
133	Wave PDE on finite length string	One end fixed, other free, initial position not zero, initial velocity zero, no source (Logan p. 130)	✗	0.655	✓	6.855	No
134	Wave PDE on finite length string	Both ends fixed end, initial conditions zero, with source as generic function that depends on time and space (Logan p. 149)	✗	0.843	✓	3.249	No
135	Wave PDE on finite length string	Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space (Haberman 8.5.2 (a))	✗	0.904	✓	12.648	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
136	Wave PDE on finite length string	Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space (Haberman 8.5.2 (b))	✗	1.154	✓	55.028	Yes
137	Wave PDE on finite length string	Both ends fixed, initial conditions both not zero, No source	✓	36.935	✓	3.322	No
138	Wave PDE on finite length string	Both ends fixed end, initial conditions both not zero, and with constant source	✗	0.668	✓	10.515	No
139	Wave PDE on finite length string	Both ends fixed end, with source (Logan p. 213)	✗	0.657	✓	4.862	No
140	Wave PDE on finite length string	Telegraphy PDE, both ends fixed with damping	✗	31.266	✓ But $n = 1$ should not be included.	4.07	No
141	Wave PDE on finite length string	Both ends fixed. Initial velocity zero. Dispersion term present	✗ Due to adding dispersion term	31.329	✓	16.619	No
142	Wave PDE on finite length string	Both ends fixed, non-zero initial position	✓ But sum should not include $n = 2$	31.075	✓ Handled $n = 2$ case correctly	10.53	No
143	Wave PDE on finite length string	Both ends fixed, zero initial position, non zero initial velocity, with source that depends on time and space	✗	29.523	✓	17.288	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
144	Wave PDE on finite length string	Left end fixed, right end oscillates, initially at rest. With source that depends on time and space	✗	30.706	✓	16.13	No
145	Wave PDE on semi-infinite domain	With zero initial position and velocity, and with source term (Logan p. 115)	✓	0.157	✓	0.504	No
146	Wave PDE on semi-infinite domain	Left end having a moving boundary condition	✓	7.054	✓	0.433	No
147	Wave PDE on semi-infinite domain	Initial value with a Dirichlet condition on the half-line	✓	26.435	✓	4.447	No
148	Wave PDE on semi-infinite domain	Initial value problem with a Neumann condition on the half-line	✓	13.48	✓	0.782	No
149	Wave PDE on semi-infinite domain	With initial conditions given at $t = 1$ instead of $t = 0$	✓	24.048	✓	0.349	No
150	Wave PDE on semi-infinite domain	initial conditions at $t = 0$ but B.C. at $x = 1$	✗	9.39	✓	0.626	No
151	Wave PDE on semi-infinite domain	initial conditions at $t = 1$ with source that depends on time and space	✗	0.008	✓	3.578	No
152	Wave PDE on semi-infinite domain	Left end free with initial position and velocity given	✓	0.155	✓	0.339	No
153	Wave PDE 1D infinite domain	General solution for a second-order hyperbolic PDE on real line	✓	0.004	✓	0.179	No
154	Wave PDE 1D infinite domain	With initial conditions specified, no source	✓	0.046	✓	0.099	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
155	Wave PDE 1D infinite domain	Wave PDE on infinite domain with initial conditions specified, with source term	✓	0.009	✓	0.112	No
156	Wave PDE 1D infinite domain	non-linear wave PDE (Solitons)	✓	0.048	✓ Returning a solution that is not the most general one	0.173	No
157	Wave PDE 1D infinite domain	Hyperbolic PDE with non-rational coefficients	✓	0.105	✗	0.088	No
158	Wave PDE 1D infinite domain	Inhomogeneous hyperbolic PDE with constant coefficients	✓	0.003	✓	0.11	No
159	Wave PDE 1D infinite domain	system of 2 inhomogeneous linear hyperbolic system with constant coefficients	✓	0.274	✗	0.449	No
160	Wave PDE in 2D Cartesian coordinates	In square, given initial position but with zero initial velocity	✗	0.003	✓	13.325	No
161	Wave PDE in 2D Cartesian coordinates	In square with damping. Given zero initial position but with non-zero initial velocity	✗	0.003	✓	18.152	No
162	Wave PDE in 2D Cartesian coordinates	In rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity	✓	0.85	✓	4.921	No
163	Wave PDE in 2D Cartesian coordinates	In rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity (Haberman 8.5.5 (a))	✗	0.003	✗	0.187	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
164	Wave PDE in 2D Polar coordinates	In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given	✓	31.049	✓ Has unresolved Invlaplace calls	49.033	No
165	Wave PDE in 2D Polar coordinates	In circular disk. fixed edge of disk, with θ dependency, zero initial velocity	✗	0.004	✗	0.24	No
166	Wave PDE in 3D Spherical coordinates	No initial and no boundary conditions given	✗	0.029	✓	1.402	No
167	Wave PDE in 3D Cylindrical coordinates	No initial and no boundary conditions given	✗	0.003	✓	0.269	No
168	Schrodinger PDE	Schrodinger PDE with zero potential (Logan p. 30)	✓	0.267	✓	0.764	No
169	Schrodinger PDE	Schrodinger PDE with initial and boundary conditions. Zero potential	✓	0.011	✓	7.79	No
170	Schrodinger PDE	Initial value problem with Dirichlet boundary conditions. Zero potential	✓	0.572	✓	1.903	No
171	Schrodinger PDE	Solve a Schrodinger equation with potential over the whole real line	✓	0.005	✗ Maple does not support ∞ in boundary conditions	0.	No
172	Schrodinger PDE	Schrodinger equation, with initial conditions. Zero potential (Griffiths p. 47)	✓	35.714	✓	3.929	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
173	Schrodinger PDE	Schrodinger equation, with initial conditions. Infinite square well potential (Griffiths p. 47)	✓	0.01	✓	0.822	No
174	Schrodinger PDE	In 2 space dimensions	✗	0.658	✓	7.776	No
175	Beam PDE	Beam PDE with zero initial velocity	✓	0.19	✓	0.14	No
176	Burger's PDE	viscous fluid flow with no initial conditions	✓	0.02	✓	0.169	No
177	Burger's PDE	viscous fluid flow with initial conditions	✓	8.474	✗	0.956	No
178	Burger's PDE	viscous fluid flow with initial conditions as UnitBox	✓	11.585	✗	1.228	No
179	Black Scholes PDE	classic Black Scholes model from finance, European call version	✓	2.984	✓	1.056	No
180	Black Scholes PDE	Boundary value problem for the Black Scholes equation	✓	1.097	✓	3.356	No
181	Korteweg-deVries PDE	Korteweg-deVries (waves on shallow water surfaces) with no initial conditions	✓	0.028	✓	0.187	No
182	Tricomi PDE	Boundary value problem for the Tricomi equation	✓	5.167	✓	5.729	No
183	Cauchy Riemann PDE's	Cauchy Riemann PDE with Prescribe the values of u and v on the x axis	✓	0.003	✓	0.384	No
184	Cauchy Riemann PDE's	Cauchy Riemann PDE With extra term on right side	✗	0.001	✓	0.143	No
185	Hamilton-Jacobi PDE	Hamilton-Jacobi type PDE	✗	0.007	✓	0.408	No
186	miscellaneous PDE's	A second order PDE	✗	0.035	✓	0.081	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
187	miscellaneous PDE's	second order PDE in Polar coordinates	✗	0.011	✓	0.804	No
188	miscellaneous PDE's	Laplace like PDE with polynomial solution	✓	4.514	✓	2.437	No
189	miscellaneous PDE's	Third oder PDE	✗	0.268	✓	21.509	No
190	miscellaneous PDE's	PDE solved by Laplace transform	✗	0.181	✓	0.487	No
191	miscellaneous PDE's	Linear PDE, initial conditions at $t = 1$	✗	0.003	✓	0.633	No
192	miscellaneous PDE's	Linear PDE, initial conditions at $t = t_0$	✗	0.003	✓	0.478	No
193	miscellaneous PDE's	second order in time, Linear PDE, initial conditions at $t = t_0$	✗	0.003	✓	0.538	No
194	miscellaneous PDE's	Einstein-Weiner PDE	✗	0.003	✓	0.344	No
195	Nonlinear PDE's	Bateman-Burgers equation	✓	0.019	✓	0.178	No
196	Nonlinear PDE's	Benjamin Bona Mahony	✓	0.029	✓	0.186	No
197	Nonlinear PDE's	Benjamin Ono	✓	0.019	✓	0.172	No
198	Nonlinear PDE's	Born Infeld	✓	0.007	✓	0.376	No
199	Nonlinear PDE's	Boussinesq	✓	0.04	✓	0.192	No
200	Nonlinear PDE's	Boussinesq type PDE	✓	0.039	✓	0.204	No
201	Nonlinear PDE's	Buckmaster	✗	0.065	✓ Answer in terms of RootOf.	0.971	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
202	Nonlinear PDE's	Camassa Holm	✗	0.17	✓ Answer in terms of RootOf.	3.623	No
203	Nonlinear PDE's	Chaffee Infante equation	✗	0.09	✓	0.393	No
204	Nonlinear PDE's	Clarke's equation	✗	0.009	✗	0.038	No
205	Nonlinear PDE's	Degasperis Procesi	✗	0.167	✓ But still has unresolved ODE's in solution	0.769	No
206	Nonlinear PDE's	Dym equation	✗	0.08	✓ has RootOf	0.834	No
207	Nonlinear PDE's	Estevez Mansfield Clarkson equation	✓	0.031	✓	0.259	No
208	Nonlinear PDE's	Fisher's equation	✓	0.047	✓	0.372	No
209	Nonlinear PDE's	Hunter Saxton	✗	0.044	✓ with RootOf	0.203	No
210	Nonlinear PDE's	Kadomtsev Petviashvili	✓	0.066	✓	0.243	No
211	Nonlinear PDE's	Klein Gordon (nonlinear)	✗	0.004	✗	0.072	No
212	Nonlinear PDE's	special case Klein Gordon (nonlinear)	✗	0.208	✓	1.001	No
213	Nonlinear PDE's	Khokhlov Zabolotskaya	✗	0.056	✓	0.668	No
214	Nonlinear PDE's	Korteweg de Vries (KdV)	✓	0.013	✓	0.109	No
215	Nonlinear PDE's	Lin Tsien equation	✗	0.071	✓	0.637	No
216	Nonlinear PDE's	Liouville equation	✗	0.004	✗	0.062	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
217	Nonlinear PDE's	Plateau	✗	0.031	✓	0.416	No
218	Nonlinear PDE's	Rayleigh	✗	0.075	✓ Has RootOf	0.269	No
219	Nonlinear PDE's	Sawada Kotera	✓	0.079	✓	0.355	No
220	Nonlinear PDE's	Sine Gordon	✗	0.007	✗	0.011	No
221	Nonlinear PDE's	Sinh Gordon	✗	0.007	✗	0.011	No
222	Nonlinear PDE's	Sinh Poisson	✗	0.006	✗	0.011	No
223	Nonlinear PDE's	Thomas equation	✗	0.061	✓	0.803	No
224	Nonlinear PDE's	phi equation	✓	0.041	✓	0.352	No
225	HFOPDE, chapter 1	problem number 1	✓	0.02	✓	0.02	No
226	HFOPDE, chapter 1	problem number 2	✓	0.021	✓	0.004	No
227	HFOPDE, chapter 1	problem number 3	✓	0.02	✓	0.004	No
228	HFOPDE, chapter 1	problem number 4	✓	0.019	✓	0.003	No
229	HFOPDE, chapter 1	problem number 5	✓	0.319	✓	0.011	No
230	HFOPDE, chapter 1	problem number 6	✓	0.316	✓	0.01	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
231	HFOPDE, chapter 2.2.1	problem number 1	✓	0.004	✓ Solution missing a, b compared to book, but technically still correct.	0.002	Yes
232	HFOPDE, chapter 2.2.1	problem number 2	✓	0.005	✓	0.024	Yes
233	HFOPDE, chapter 2.2.1	problem number 3	✓	0.018	✓	0.031	Yes
234	HFOPDE, chapter 2.2.1	problem number 4	✓	0.005	✓	0.029	Yes
235	HFOPDE, chapter 2.2.1	problem number 5	✓	0.008	✓	0.111	No
236	HFOPDE, chapter 2.2.1	problem number 6	✓	0.059	✓	0.122	No
237	HFOPDE, chapter 2.2.1	problem number 7	✓	0.02	✓ But Mathematica solution is simpler, both verified correct	1.498	No
238	HFOPDE, chapter 2.2.1	problem number 8	✗	1.822	✓	2.112	No
239	HFOPDE, chapter 2.2.2	problem number 1	✓	0.005	✓	0.008	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
240	HFOPDE, chapter 2.2.2	problem number 2	✓	0.091	✓	0.164	No
241	HFOPDE, chapter 2.2.2	problem number 3	✓	0.027	✓	0.016	No
242	HFOPDE, chapter 2.2.2	problem number 4	✓	0.187	✓	0.143	No
243	HFOPDE, chapter 2.2.2	problem number 5	✓	0.073	✓	0.196	No
244	HFOPDE, chapter 2.2.2	problem number 6	✓	0.136	✓	0.151	No
245	HFOPDE, chapter 2.2.2	problem number 7	✓	0.15	✓	0.256	No
246	HFOPDE, chapter 2.2.2	problem number 8	✓	0.223	✓	0.184	No
247	HFOPDE, chapter 2.2.2	problem number 9	✓	0.071	✓	0.257	No
248	HFOPDE, chapter 2.2.2	problem number 10	✓	0.045	✓	0.137	No
249	HFOPDE, chapter 2.2.2	problem number 11	✓	0.121	✓	0.084	No
250	HFOPDE, chapter 2.2.2	problem number 12	✓	0.28	✓	0.343	No
251	HFOPDE, chapter 2.2.2	problem number 13	✓	0.009	✓	0.037	No
252	HFOPDE, chapter 2.2.2	problem number 14	✓	0.027	✓	0.526	No
253	HFOPDE, chapter 2.2.2	problem number 15	✓	0.166	✓	0.323	No
254	HFOPDE, chapter 2.2.2	problem number 16	✓	0.804	✓	0.216	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
255	HFOPDE, chapter 2.2.2	problem number 17	✓	0.519	✓	0.397	No
256	HFOPDE, chapter 2.2.2	problem number 18	✓	0.022	✓	0.086	No
257	HFOPDE, chapter 2.2.2	problem number 19	✓	0.068	✓	0.183	No
258	HFOPDE, chapter 2.2.2	problem number 20	✗	0.05	✓	0.339	No
259	HFOPDE, chapter 2.2.2	problem number 21	✓	0.05	✓	0.207	No
260	HFOPDE, chapter 2.2.2	problem number 22	✓	0.095	✓	0.639	No
261	HFOPDE, chapter 2.2.2	problem number 23	✓	0.032	✓	0.354	No
262	HFOPDE, chapter 2.2.2	problem number 24	✗	0.062	✓	0.261	No
263	HFOPDE, chapter 2.2.2	problem number 25	✗ (Timed out)	600.	✗ (Timed out)	600.	No
264	HFOPDE, chapter 2.2.2	problem number 26	✗	261.077	✗	29.987	No
265	HFOPDE, chapter 2.2.2	problem number 27	✗	101.416	✗	1.723	No
266	HFOPDE, chapter 2.2.2	problem number 28	✗	0.067	✓ solution contains RootOf	0.155	No
267	HFOPDE, chapter 2.2.2	problem number 29	✗	101.016	✗	2.472	No
268	HFOPDE, chapter 2.2.2	problem number 30	✗	0.063	✓	0.276	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
269	HFOPDE, chapter 2.2.2	problem number 31, Hesse's equation	✗	235.248	✓	8.362	No
270	HFOPDE, chapter 2.2.3	problem number 1	✓ But it can't solve it when assuming $b > 0$ which is strange.	0.066	✓ Mathematica solution is much simpler	1.557	No
271	HFOPDE, chapter 2.2.3	problem number 2	✓	0.092	✓	0.15	No
272	HFOPDE, chapter 2.2.3	problem number 3	✓	0.055	✓	1.085	No
273	HFOPDE, chapter 2.2.3	problem number 4	✗	0.087	✓	1.449	No
274	HFOPDE, chapter 2.2.3	problem number 5	✗	0.364	✓ Answer contains RootOf	0.158	No
275	HFOPDE, chapter 2.2.3	problem number 6	✗	0.07	✓ Answer contains RootOf	0.067	No
276	HFOPDE, chapter 2.2.3	problem number 7	✓	0.038	✓	0.157	No
277	HFOPDE, chapter 2.2.3	problem number 8	✓	0.017	✓	0.067	No
278	HFOPDE, chapter 2.2.3	problem number 9	✗ (Timed out)	600.	✓	0.046	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
279	HFOPDE, chapter 2.2.4	problem number 1	✓	0.016	✓	0.017	No
280	HFOPDE, chapter 2.2.4	problem number 2	✓	0.111	✓	0.312	No
281	HFOPDE, chapter 2.2.4	problem number 3	✓	0.217	✓	0.312	No
282	HFOPDE, chapter 2.2.4	problem number 4	✓	0.182	✓	1.805	No
283	HFOPDE, chapter 2.2.4	problem number 5	✗	0.186	✗ (Timed out)	600.	No
284	HFOPDE, chapter 2.2.4	problem number 6	✓	1.665	✓	0.591	No
285	HFOPDE, chapter 2.2.4	problem number 7	✗	1.085	✓	2.272	No
286	HFOPDE, chapter 2.2.5	problem number 1	✓	0.028	✓	0.15	No
287	HFOPDE, chapter 2.2.5	problem number 2	✓	0.092	✓	0.348	No
288	HFOPDE, chapter 2.2.5	problem number 3	✓	0.028	✓	0.128	No
289	HFOPDE, chapter 2.2.5	problem number 4	✗	19.143	✓	1.51	No
290	HFOPDE, chapter 2.2.5	problem number 5	✓	0.308	✓	0.431	No
291	HFOPDE, chapter 2.2.5	problem number 6	✗ (Timed out)	600.	✓	2.524	No
292	HFOPDE, chapter 2.2.5	problem number 7	✓	0.098	✓	0.187	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
293	HFOPDE, chapter 2.2.5	problem number 8	✗	22.627	✓	3.532	No
294	HFOPDE, chapter 2.2.5	problem number 9	✗	0.806	✓	6.244	No
295	HFOPDE, chapter 2.2.5	problem number 10	✗ (Timed out)	600.	✓	5.292	No
296	HFOPDE, chapter 2.2.5	problem number 11	✗	24.757	✗	6.035	No
297	HFOPDE, chapter 2.2.5	problem number 12	✗	24.269	✗	4.593	No
298	HFOPDE, chapter 2.2.5	problem number 13	✗	28.83	✗	11.215	No
299	HFOPDE, chapter 2.2.5	problem number 14	✗	1.518	✓ Solution contains RootOf	0.216	No
300	HFOPDE, chapter 2.2.5	problem number 15	✓	0.304	✓	1.568	No
301	HFOPDE, chapter 2.2.5	problem number 16	✗	75.081	✗	34.908	No
302	HFOPDE, chapter 2.2.5	problem number 17	✗	135.405	✓	1.063	No
303	HFOPDE, chapter 2.2.5	problem number 18	✗	81.147	✓	1.179	No
304	HFOPDE, chapter 2.2.5	problem number 19	✗ (Timed out)	600.	✓	1.106	No
305	HFOPDE, chapter 2.2.5	problem number 20	✓	0.052	✓	0.309	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
306	HFOPDE, chapter 2.2.5	problem number 21	✓	0.274	✓	0.252	No
307	HFOPDE, chapter 2.2.5	problem number 22	✓	0.157	✓	0.175	No
308	HFOPDE, chapter 2.2.5	problem number 23	✗	25.898	✓	0.178	No
309	HFOPDE, chapter 2.2.5	problem number 24	✓	0.086	✓	0.122	No
310	HFOPDE, chapter 2.2.5	problem number 25	✓	0.292	✓	0.089	No
311	HFOPDE, chapter 2.2.5	problem number 26	✓	0.427	✓	0.198	No
312	HFOPDE, chapter 2.2.5	problem number 27	✓	0.092	✓	0.219	No
313	HFOPDE, chapter 2.2.5	problem number 28	✗	0.1	✓ Solution contains RootOf	0.149	No
314	HFOPDE, chapter 2.2.5	problem number 29	✗	0.08	✓ Solution contains RootOf	0.116	No
315	HFOPDE, chapter 2.2.5	problem number 30	✗	0.29	✓	5.364	No
316	HFOPDE, chapter 2.2.5	problem number 31	✗	18.564	✓	1.899	No
317	HFOPDE, chapter 2.2.5	problem number 32	✗	130.256	✗	23.403	No
318	HFOPDE, chapter 2.2.5	problem number 33	✓	0.071	✓	0.428	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
319	HFOPDE, chapter 2.2.5	problem number 34	✓	0.025	✓	0.207	No
320	HFOPDE, chapter 2.2.5	problem number 35	✓	0.05	✓	0.315	No
321	HFOPDE, chapter 2.2.5	problem number 36	✓	0.163	✓	0.355	No
322	HFOPDE, chapter 2.2.5	problem number 37	✗ (Timed out)	600.	✓	0.747	No
323	HFOPDE, chapter 2.2.5	problem number 38	✓	1.367	✓	0.47	No
324	HFOPDE, chapter 2.2.5	problem number 39	✓	0.076	✓	0.328	No
325	HFOPDE, chapter 2.2.5	problem number 40	✗	140.71	✗	22.523	No
326	HFOPDE, chapter 2.2.5	problem number 41	✓	0.142	✓	0.511	No
327	HFOPDE, chapter 2.2.5	problem number 42	✗	3.386	✓	1.907	No
328	HFOPDE, chapter 2.2.5	problem number 43	✗	2.488	✓	3.795	No
329	HFOPDE, chapter 2.2.5	problem number 44	✗	144.693	✗	20.276	No
330	HFOPDE, chapter 2.2.5	problem number 45	✗	145.087	✗	30.301	No
331	HFOPDE, chapter 2.2.5	problem number 46	✗ (Timed out)	600.	✓	11.662	No
332	HFOPDE, chapter 2.2.5	problem number 47	✓	39.504	✓	1.469	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
333	HFOPDE, chapter 2.2.5	problem number 48	✗	37.35	✓	1.152	No
334	HFOPDE, chapter 2.2.5	problem number 49	✗	24.756	✗	14.524	No
335	HFOPDE, chapter 2.2.5	problem number 50	✗	114.766	✗	75.018	No
336	HFOPDE, chapter 2.2.5	problem number 51	✗	0.146	✓	2.389	No
337	HFOPDE, chapter 2.2.5	problem number 52	✗	9.411	✗	3.317	No
338	HFOPDE, chapter 2.2.5	problem number 53	✗	0.263	✓	1.75	No
339	HFOPDE, chapter 2.2.5	problem number 54	✗	0.171	✗	3.12	No
340	HFOPDE, chapter 2.2.5	problem number 55	✗	26.304	✗	9.84	No
341	HFOPDE, chapter 2.2.5	problem number 56	✗ Timed out	423.057	✓	3.039	No
342	HFOPDE, chapter 2.3.1	problem number 1	✓	0.011	✓	0.028	No
343	HFOPDE, chapter 2.3.1	problem number 2	✓	0.015	✓	0.008	No
344	HFOPDE, chapter 2.3.1	problem number 3	✓	0.035	✓	1.733	No
345	HFOPDE, chapter 2.3.1	problem number 4	✓	0.124	✓	1.608	No
346	HFOPDE, chapter 2.3.1	problem number 5	✗	24.09	✓	0.563	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
347	HFOPDE, chapter 2.3.1	problem number 6	✓	0.029	✓	0.125	No
348	HFOPDE, chapter 2.3.1	problem number 7	✓	0.097	✓	0.054	No
349	HFOPDE, chapter 2.3.1	problem number 8	✓	0.151	✓ Has RootOf	2.723	No
350	HFOPDE, chapter 2.3.1	problem number 9	✓	0.086	✓	0.515	No
351	HFOPDE, chapter 2.3.1	problem number 10	✗	0.125	✓	4.404	No
352	HFOPDE, chapter 2.3.1	problem number 11	✗	1.975	✗	3.162	No
353	HFOPDE, chapter 2.3.2	problem number 1	✓	0.429	✓	0.673	No
354	HFOPDE, chapter 2.3.2	problem number 2	✓	0.679	✓	0.259	No
355	HFOPDE, chapter 2.3.2	problem number 3	✓	0.215	✓	0.079	No
356	HFOPDE, chapter 2.3.2	problem number 4	✗	36.55	✓	0.393	No
357	HFOPDE, chapter 2.3.2	problem number 5	✓	0.116	✓	0.155	No
358	HFOPDE, chapter 2.3.2	problem number 6	✗	22.289	✗	6.686	No
359	HFOPDE, chapter 2.3.2	problem number 7	✓	0.148	✓	0.215	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
360	HFOPDE, chapter 2.3.2	problem number 8	✗ kernel error generated	0.444	✓	0.09	No
361	HFOPDE, chapter 2.3.2	problem number 9	✓	0.818	✓	0.839	No
362	HFOPDE, chapter 2.3.2	problem number 10	✓	0.8	✓	0.805	No
363	HFOPDE, chapter 2.3.2	problem number 11	✗	21.559	✓	1.039	No
364	HFOPDE, chapter 2.3.2	problem number 12	✓	1.627	✓	0.254	No
365	HFOPDE, chapter 2.3.2	problem number 13	✓	0.058	✓	0.121	No
366	HFOPDE, chapter 2.3.2	problem number 14	✗	23.365	✗	7.268	No
367	HFOPDE, chapter 2.3.2	problem number 15	✗ (Timed out)	600.	✓	2.228	No
368	HFOPDE, chapter 2.3.2	problem number 16	✗	22.075	✗	5.836	No
369	HFOPDE, chapter 2.3.2	problem number 17	✗	222.015	✓	1.382	No
370	HFOPDE, chapter 2.3.2	problem number 18	✗	22.431	✗	5.848	No
371	HFOPDE, chapter 2.3.2	problem number 19	✓	0.1	✓	0.304	No
372	HFOPDE, chapter 2.3.2	problem number 20	✗	22.241	✗	3.298	No
373	HFOPDE, chapter 2.3.2	problem number 21	✗	23.959	✗	4.204	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
374	HFOPDE, chapter 2.3.2	problem number 22	✓	1.723	✓	0.229	No
375	HFOPDE, chapter 2.3.2	problem number 23	✓	0.109	✓	0.215	No
376	HFOPDE, chapter 2.3.2	problem number 24	✗	21.281	✗	4.447	No
377	HFOPDE, chapter 2.3.2	problem number 25	✓	0.131	✓	0.169	No
378	HFOPDE, chapter 2.3.2	problem number 26	✓	0.153	✓	0.157	No
379	HFOPDE, chapter 2.3.2	problem number 27	✗	0.598	✓ Solution contains RootOf	0.263	No
380	HFOPDE, chapter 2.3.2	problem number 28	✓	7.115	✓	0.273	No
381	HFOPDE, chapter 2.3.2	problem number 29	✓	0.079	✓	0.283	No
382	HFOPDE, chapter 2.3.2	problem number 30	✓	2.549	✓	0.228	No
383	HFOPDE, chapter 2.3.2	problem number 31	✗	47.506	✓	12.006	No
384	HFOPDE, chapter 2.3.2	problem number 32	✓	0.043	✓	0.288	No
385	HFOPDE, chapter 2.3.2	problem number 33	✗	0.076	✓	0.38	No
386	HFOPDE, chapter 2.3.2	problem number 34	✗ (Timed out)	600.	✓	5.816	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
387	HFOPDE, chapter 2.3.2	problem number 35	✗ (Timed out)	600.	✓	7.093	No
388	HFOPDE, chapter 2.3.2	problem number 36	✗ (Timed out)	600.	✓	5.601	No
389	HFOPDE, chapter 2.4.1	problem number 1	✓	0.014	✓	0.051	No
390	HFOPDE, chapter 2.4.1	problem number 2	✓	0.048	✓	0.5	No
391	HFOPDE, chapter 2.4.1	problem number 3	✓	59.45	✓	3.309	No
392	HFOPDE, chapter 2.4.1	problem number 4	✗	42.013	✓	0.58	No
393	HFOPDE, chapter 2.4.1	problem number 5	✗	54.629	✓	3.659	No
394	HFOPDE, chapter 2.4.1	problem number 6	✓	0.179	✓	0.124	No
395	HFOPDE, chapter 2.4.1	problem number 7	✓	0.023	✓	0.072	No
396	HFOPDE, chapter 2.4.2	problem number 1	✓	0.011	✓	0.009	No
397	HFOPDE, chapter 2.4.2	problem number 2	✓	0.031	✓	0.489	No
398	HFOPDE, chapter 2.4.2	problem number 3	✗	58.587	✓	4.575	No
399	HFOPDE, chapter 2.4.2	problem number 4	✗	59.346	✓	3.601	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
400	HFOPDE, chapter 2.4.2	problem number 5	✗ (Timed out)	600.	✓	2.323	No
401	HFOPDE, chapter 2.4.2	problem number 6	✗ (Timed out)	600.	✓	6.564	No
402	HFOPDE, chapter 2.4.2	problem number 7	✗ (Timed out)	600.	✓	1.892	No
403	HFOPDE, chapter 2.4.2	problem number 8	✓	0.038	✓	0.106	No
404	HFOPDE, chapter 2.4.3	problem number 1	✓	0.021	✓	0.009	No
405	HFOPDE, chapter 2.4.3	problem number 2	✓	0.031	✓	0.002	No
406	HFOPDE, chapter 2.4.3	problem number 3	✓	2.293	✓	1.063	No
407	HFOPDE, chapter 2.4.3	problem number 4	✗	42.704	✓	2.809	No
408	HFOPDE, chapter 2.4.3	problem number 5	✗ (Timed out)	600.	✓	2.029	No
409	HFOPDE, chapter 2.4.3	problem number 6	✗ (Timed out)	600.	✓	5.6	No
410	HFOPDE, chapter 2.4.3	problem number 7	✗ (Timed out)	600.	✓	2.028	No
411	HFOPDE, chapter 2.4.3	problem number 8	✓	290.451	✓	0.154	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
412	HFOPDE, chapter 2.4.4	problem number 1	✓	0.021	✓	0.046	No
413	HFOPDE, chapter 2.4.4	problem number 2	✓	0.034	✓	1.662	No
414	HFOPDE, chapter 2.4.4	problem number 3	✓	2.07	✓	1.081	No
415	HFOPDE, chapter 2.4.4	problem number 4	✗	4.003	✓	0.972	No
416	HFOPDE, chapter 2.4.5	problem number 1	✓	0.066	✓	0.492	No
417	HFOPDE, chapter 2.4.5	problem number 2	✓	0.168	✓	0.113	No
418	HFOPDE, chapter 2.4.5	problem number 3	✓	1.733	✓	0.858	No
419	HFOPDE, chapter 2.4.5	problem number 4	✗	13.322	✓	5.756	No
420	HFOPDE, chapter 2.4.5	problem number 5	✓	0.035	✓	0.068	No
421	HFOPDE, chapter 2.4.5	problem number 6	✗ (Timed out)	600.	✓	5.398	No
422	HFOPDE, chapter 2.5.1	problem number 1	✓	0.033	✓	0.053	No
423	HFOPDE, chapter 2.5.1	problem number 3	✓	579.252	✓	0.045	No
424	HFOPDE, chapter 2.5.1	problem number 4	✗ (Timed out)	600.	✓	0.149	No
425	HFOPDE, chapter 2.5.2	problem number 1	✓	0.065	✓	0.361	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
426	HFOPDE, chapter 2.5.2	problem number 2	✓	0.077	✓	0.122	No
427	HFOPDE, chapter 2.5.2	problem number 3	✗ (Timed out)	600.	✓	0.508	No
428	HFOPDE, chapter 2.5.2	problem number 4	✗	22.035	✓	0.455	No
429	HFOPDE, chapter 2.5.2	problem number 5	✗	22.658	✗	2.791	No
430	HFOPDE, chapter 2.5.2	problem number 6	✗	26.279	✓	1.699	No
431	HFOPDE, chapter 2.5.2	problem number 7	✗	26.548	✗	3.14	No
432	HFOPDE, chapter 2.5.2	problem number 8	✗	23.418	✗	2.414	No
433	HFOPDE, chapter 2.5.2	problem number 9	✓	0.214	✓	0.224	No
434	HFOPDE, chapter 2.5.2	problem number 10	✗ (Timed out)	600.	✓	1.339	No
435	HFOPDE, chapter 2.5.2	problem number 11	✓	0.067	✓	0.195	No
436	HFOPDE, chapter 2.5.2	problem number 12	✗	20.671	✗	1.861	No
437	HFOPDE, chapter 2.5.2	problem number 13	✗	22.799	✗	2.928	No
438	HFOPDE, chapter 2.5.2	problem number 14	✗	21.894	✗	1.266	No
439	HFOPDE, chapter 2.5.2	problem number 15	✓	0.15	✓	0.351	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
440	HFOPDE, chapter 2.5.2	problem number 16	✓	0.057	✓	0.1	No
441	HFOPDE, chapter 2.5.2	problem number 17	✓	0.653	✓	0.304	No
442	HFOPDE, chapter 2.5.2	problem number 18	✗ (Timed out)	600.	✓	0.641	No
443	HFOPDE, chapter 2.5.2	problem number 19	✗ (Timed out)	600.	✓	3.634	No
444	HFOPDE, chapter 2.5.2	problem number 20	✗ (Timed out)	600.	✓	1.818	No
445	HFOPDE, chapter 2.5.2	problem number 21	✗	26.139	✓	0.13	No
446	HFOPDE, chapter 2.5.2	problem number 22	✗ (Timed out)	600.	✓	0.841	No
447	HFOPDE, chapter 2.5.2	problem number 23	✗ (Timed out)	600.	✓	0.372	No
448	HFOPDE, chapter 2.6.1	problem number 1	✓	0.234	✓ contains unresolved integral	0.427	No
449	HFOPDE, chapter 2.6.1	problem number 2	✓ contains unresolved integral	397.433	✓ contains unresolved integral	0.699	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
450	HFOPDE, chapter 2.6.1	problem number 3	✓	0.542	✓ contains unresolved integral	0.972	No
451	HFOPDE, chapter 2.6.1	problem number 4	✗ (Timed out)	600.	✓ contains unresolved integral	1.257	No
452	HFOPDE, chapter 2.6.1	problem number 5	✗	41.582	✓	4.007	No
453	HFOPDE, chapter 2.6.1	problem number 6	✗	44.148	✓ contains unresolved integrals	0.251	No
454	HFOPDE, chapter 2.6.1	problem number 7	✗	29.002	✓ contains unresolved integrals	2.789	No
455	HFOPDE, chapter 2.6.1	problem number 8	✗	41.969	✓	0.51	No
456	HFOPDE, chapter 2.6.1	problem number 9	✗	43.284	✓	9.662	No
457	HFOPDE, chapter 2.6.1	problem number 10	✗	43.956	✓	3.33	No
458	HFOPDE, chapter 2.6.1	problem number 11	✗ (Timed out)	600.	✗ (Timed out)	600.	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
459	HFOPDE, chapter 2.6.1	problem number 12	✗ (Timed out)	600.	✗ (Timed out)	600.	No
460	HFOPDE, chapter 2.6.1	problem number 13	✓	87.181	✓	0.555	No
461	HFOPDE, chapter 2.6.1	problem number 14	✗ (Timed out)	600.	✓	2.141	No
462	HFOPDE, chapter 2.6.2	problem number 1	✓	0.294	✓ Contains unresolved integral	0.246	No
463	HFOPDE, chapter 2.6.2	problem number 2	✓	265.927	✓	0.362	No
464	HFOPDE, chapter 2.6.2	problem number 3	✓	361.65	✓	1.932	No
465	HFOPDE, chapter 2.6.2	problem number 4	✗ (Timed out)	600.	✓	0.655	No
466	HFOPDE, chapter 2.6.2	problem number 5	✗	41.454	✓	5.51	No
467	HFOPDE, chapter 2.6.2	problem number 6	✗	41.875	✓	1.083	No
468	HFOPDE, chapter 2.6.2	problem number 7	✗	44.553	✓	3.308	No
469	HFOPDE, chapter 2.6.2	problem number 8	✗	44.042	✓	3.188	No
470	HFOPDE, chapter 2.6.2	problem number 9	✗ (Timed out)	600.	✓	2.986	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
471	HFOPDE, chapter 2.6.2	problem number 10	✗ (Timed out)	600.	✓	2.531	No
472	HFOPDE, chapter 2.6.2	problem number 11	✗ (Timed out)	600.	✓	8.403	No
473	HFOPDE, chapter 2.6.2	problem number 12	✗ (Timed out)	600.	✓	3.097	No
474	HFOPDE, chapter 2.6.3	problem number 1	✓	0.038	✓	0.01	No
475	HFOPDE, chapter 2.6.3	problem number 2	✓	0.146	✓	0.349	No
476	HFOPDE, chapter 2.6.3	problem number 3	✓	0.351	✓ Has unresolved integrals	0.543	No
477	HFOPDE, chapter 2.6.3	problem number 4	✗	42.712	✓	0.928	No
478	HFOPDE, chapter 2.6.3	problem number 5	✗	44.286	✓	0.959	No
479	HFOPDE, chapter 2.6.3	problem number 6	✗ Timed out	587.818	✓	0.885	No
480	HFOPDE, chapter 2.6.3	problem number 7	✗	228.29	✗ (Timed out)	600.	No
481	HFOPDE, chapter 2.6.3	problem number 8	✗	39.916	✗ (Timed out)	600.	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
482	HFOPDE, chapter 2.6.3	problem number 9	✗ (Timed out)	600.	✗ (Timed out)	600.	No
483	HFOPDE, chapter 2.6.3	problem number 10	✓	280.079	✓	0.593	No
484	HFOPDE, chapter 2.6.3	problem number 11	✗ (Timed out)	600.	✓	15.673	No
485	HFOPDE, chapter 2.6.3	problem number 12	✗	570.242	✓	3.002	No
486	HFOPDE, chapter 2.6.3	problem number 13	✗ (Timed out)	600.	✓	2.551	No
487	HFOPDE, chapter 2.6.3	problem number 14	✗ (Timed out)	600.	✓	4.09	No
488	HFOPDE, chapter 2.6.3	problem number 15	✗ Timed out	599.573	✓	2.529	No
489	HFOPDE, chapter 2.6.4	problem number 1	✓	0.191	✓ Has unresolved integral	0.186	No
490	HFOPDE, chapter 2.6.4	problem number 2	✓	350.572	✓	0.218	No
491	HFOPDE, chapter 2.6.4	problem number 3	✗ (Timed out)	600.	✓	0.49	No
492	HFOPDE, chapter 2.6.4	problem number 4	✗	42.808	✓	0.678	No
493	HFOPDE, chapter 2.6.4	problem number 5	✗	44.48	✓	0.614	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
494	HFOPDE, chapter 2.6.4	problem number 6	✓	0.287	✓	0.188	No
495	HFOPDE, chapter 2.6.4	problem number 7	✓	0.134	✓	3.713	No
496	HFOPDE, chapter 2.6.4	problem number 8	✓	0.118	✓	2.015	No
497	HFOPDE, chapter 2.6.4	problem number 9	✓	0.827	✓	1.06	No
498	HFOPDE, chapter 2.6.4	problem number 10	✓	1.309	✓	3.888	No
499	HFOPDE, chapter 2.6.4	problem number 11	✓	0.112	✓	1.902	No
500	HFOPDE, chapter 2.6.4	problem number 12	✗ (Timed out)	600.	✓	1.997	No
501	HFOPDE, chapter 2.6.5	problem number 1	✓	0.656	✓ Has unresolved integrals	0.494	No
502	HFOPDE, chapter 2.6.5	problem number 2	✓	1.035	✓	0.755	No
503	HFOPDE, chapter 2.6.5	problem number 3	✓	0.558	✓ Mathematica answer is simpler	1.629	No
504	HFOPDE, chapter 2.6.5	problem number 4	✗	24.028	✗ (Timed out)	600.	No
505	HFOPDE, chapter 2.6.5	problem number 5	✗	42.449	✓	0.922	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
506	HFOPDE, chapter 2.6.5	problem number 6	✗	61.038	✓	1.597	No
507	HFOPDE, chapter 2.6.5	problem number 7	✗ (Timed out)	600.	✗ (Timed out)	600.	No
508	HFOPDE, chapter 2.6.5	problem number 8	✗	24.566	✓	0.762	No
509	HFOPDE, chapter 2.6.5	problem number 9	✗	23.482	✓	0.001	No
510	HFOPDE, chapter 2.6.5	problem number 10	✗	40.018	✓	1.532	No
511	HFOPDE, chapter 2.6.5	problem number 11	✗ (Timed out)	600.	✗	233.706	No
512	HFOPDE, chapter 2.7.1	problem number 1	✓	0.095	✓	0.355	No
513	HFOPDE, chapter 2.7.1	problem number 2	✗ (Timed out)	600.	✓	0.403	No
514	HFOPDE, chapter 2.7.1	problem number 3	✗	275.58	✓	0.626	No
515	HFOPDE, chapter 2.7.1	problem number 4	✓	0.24	✓	0.398	No
516	HFOPDE, chapter 2.7.1	problem number 5	✗	248.175	✓	1.037	No
517	HFOPDE, chapter 2.7.1	problem number 6	✗	22.526	✓	1.524	No
518	HFOPDE, chapter 2.7.1	problem number 7	✗	26.795	✓	2.119	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
519	HFOPDE, chapter 2.7.1	problem number 8	✗	255.775	✓	0.844	No
520	HFOPDE, chapter 2.7.1	problem number 9	✗	277.172	✗	24.08	No
521	HFOPDE, chapter 2.7.1	problem number 10	✗	35.916	✗	29.175	No
522	HFOPDE, chapter 2.7.1	problem number 11	✗ (Timed out)	600.	✓	0.33	No
523	HFOPDE, chapter 2.7.1	problem number 12	✓	63.637	✓	0.46	No
524	HFOPDE, chapter 2.7.2	problem number 1	✓	0.093	✓	0.233	No
525	HFOPDE, chapter 2.7.2	problem number 2	✗ (Timed out)	600.	✓	0.277	No
526	HFOPDE, chapter 2.7.2	problem number 3	✗ (Timed out)	600.	✓	0.479	No
527	HFOPDE, chapter 2.7.2	problem number 4	✓	0.196	✓	0.43	No
528	HFOPDE, chapter 2.7.2	problem number 5	✗ (Timed out)	600.	✓	11.32	No
529	HFOPDE, chapter 2.7.2	problem number 6	✗	28.865	✗	8.537	No
530	HFOPDE, chapter 2.7.2	problem number 7	✗	257.983	✓	2.021	No
531	HFOPDE, chapter 2.7.2	problem number 8	✗ (Timed out)	600.	✓	12.982	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
532	HFOPDE, chapter 2.7.2	problem number 9	✗	250.287	✗	21.616	No
533	HFOPDE, chapter 2.7.2	problem number 10	✗	36.604	✗	34.822	No
534	HFOPDE, chapter 2.7.2	problem number 11	✗ (Timed out)	600.	✓	0.409	No
535	HFOPDE, chapter 2.7.2	problem number 12	✓	60.083	✓	0.532	No
536	HFOPDE, chapter 2.7.3	problem number 1	✗ (Timed out)	600.	✓	0.162	No
537	HFOPDE, chapter 2.7.3	problem number 2	✗ (Timed out)	600.	✓	0.516	No
538	HFOPDE, chapter 2.7.3	problem number 3	✗ (Timed out)	600.	✓	0.821	No
539	HFOPDE, chapter 2.7.3	problem number 4	✗ (Timed out)	600.	✓	0.319	No
540	HFOPDE, chapter 2.7.3	problem number 5	✗ (Timed out)	600.	✓	1.29	No
541	HFOPDE, chapter 2.7.3	problem number 6	✗	519.759	✓	0.816	No
542	HFOPDE, chapter 2.7.3	problem number 7	✗ (Timed out)	600.	✓	2.124	No
543	HFOPDE, chapter 2.7.3	problem number 8	✗ (Timed out)	600.	✓	0.086	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
544	HFOPDE, chapter 2.7.3	problem number 9	✗ (Timed out)	600.	✗	10.06	No
545	HFOPDE, chapter 2.7.3	problem number 10	✗	40.797	✗	551.92	No
546	HFOPDE, chapter 2.7.3	problem number 11	✗ (Timed out)	600.	✓	0.482	No
547	HFOPDE, chapter 2.7.3	problem number 12	✗ (Timed out)	600.	✓	0.6	No
548	HFOPDE, chapter 2.7.4	problem number 1	✗ (Timed out)	600.	✓	0.241	No
549	HFOPDE, chapter 2.7.4	problem number 2	✗ (Timed out)	600.	✓	0.153	No
550	HFOPDE, chapter 2.7.4	problem number 3	✗	530.186	✓	0.445	No
551	HFOPDE, chapter 2.7.4	problem number 4	✗ (Timed out)	600.	✓	0.588	No
552	HFOPDE, chapter 2.7.4	problem number 5	✗ (Timed out)	600.	✓	1.581	No
553	HFOPDE, chapter 2.7.4	problem number 6	✗ (Timed out)	600.	✓	0.798	No
554	HFOPDE, chapter 2.7.4	problem number 7	✗ (Timed out)	600.	✓	2.91	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
555	HFOPDE, chapter 2.7.4	problem number 8	✗	34.288	✗	10.886	No
556	HFOPDE, chapter 2.7.4	problem number 9	✗ (Timed out)	600.	✗	24.059	No
557	HFOPDE, chapter 2.7.4	problem number 10	✗	40.911	✗	409.914	No
558	HFOPDE, chapter 2.7.4	problem number 11	✗ (Timed out)	600.	✓	0.406	No
559	HFOPDE, chapter 2.7.4	problem number 12	✗ (Timed out)	600.	✓	0.918	No
560	HFOPDE, chapter 2.8.1	problem number 1	✓	0.643	✓	0.057	No
561	HFOPDE, chapter 2.8.1	problem number 2	✓	1.045	✓	0.247	No
562	HFOPDE, chapter 2.8.1	problem number 3	✗	124.092	✓	0.179	No
563	HFOPDE, chapter 2.8.1	problem number 4	✗	20.458	✓	0.335	No
564	HFOPDE, chapter 2.8.1	problem number 5	✗	24.544	✓	0.888	No
565	HFOPDE, chapter 2.8.1	problem number 6	✗	149.5	✓	0.197	No
566	HFOPDE, chapter 2.8.1	problem number 7	✗	23.637	✗	2.512	No
567	HFOPDE, chapter 2.8.1	problem number 8	✗	23.572	✗	2.713	No
568	HFOPDE, chapter 2.8.1	problem number 9	✗	39.714	✓	0.107	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
569	HFOPDE, chapter 2.8.1	problem number 10	✗	28.036	✗	8.516	No
570	HFOPDE, chapter 2.8.1	problem number 11	✗	27.505	✗	7.729	No
571	HFOPDE, chapter 2.8.1	problem number 12	✓	0.182	✓	0.295	No
572	HFOPDE, chapter 2.8.1	problem number 13	✗ (Timed out)	600.	✓	0.261	No
573	HFOPDE, chapter 2.8.2	problem number 1	✗	21.569	✓	1.178	No
574	HFOPDE, chapter 2.8.2	problem number 2	✗	21.565	✗	2.682	No
575	HFOPDE, chapter 2.8.2	problem number 3	✗	21.449	✗	3.266	No
576	HFOPDE, chapter 2.8.2	problem number 4	✓	0.22	✓	0.303	No
577	HFOPDE, chapter 2.8.2	problem number 5	✗	23.23	✗	3.064	No
578	HFOPDE, chapter 2.8.2	problem number 6	✗	1.441	✓	0.169	No
579	HFOPDE, chapter 2.8.2	problem number 7	✗	26.151	✗	6.43	No
580	HFOPDE, chapter 2.8.2	problem number 8	✗	27.161	✗	8.501	No
581	HFOPDE, chapter 2.8.2	problem number 9	✗	23.006	✗	4.595	No
582	HFOPDE, chapter 2.8.2	problem number 10	✓	0.232	✓	0.377	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
583	HFOPDE, chapter 2.8.2	problem number 11	✗	27.578	✓	0.408	No
584	HFOPDE, chapter 2.8.3	problem number 1	✗	26.63	✗	15.364	No
585	HFOPDE, chapter 2.8.3	problem number 2	✗	29.805	✗	28.755	No
586	HFOPDE, chapter 2.8.3	problem number 3	✗	29.649	✗	13.559	No
587	HFOPDE, chapter 2.8.4	problem number 1	✗	21.65	✓	1.262	No
588	HFOPDE, chapter 2.8.4	problem number 2	✗	21.019	✗	2.229	No
589	HFOPDE, chapter 2.8.4	problem number 3	✗	20.837	✗	1.218	No
590	HFOPDE, chapter 2.8.4	problem number 4	✓	0.168	✓	0.059	No
591	HFOPDE, chapter 2.8.5	problem number 1	✗	26.506	✓	2.769	No
592	HFOPDE, chapter 2.8.5	problem number 2	✗	25.856	✗	15.4	No
593	HFOPDE, chapter 2.8.5	problem number 3	✗	25.859	✗	9.21	No
594	HFOPDE, chapter 2.8.5	problem number 4	✗	28.685	✗	25.725	No
595	HFOPDE, chapter 2.8.5	problem number 5	✗	28.855	✗	11.957	No
596	HFOPDE, chapter 2.8.6	problem number 1	✗	20.647	✗	2.816	No
597	HFOPDE, chapter 2.8.6	problem number 2	✗	20.756	✓	0.538	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
598	HFOPDE, chapter 2.8.6	problem number 3	✓	0.092	✓	0.033	No
599	HFOPDE, chapter 2.8.6	problem number 4	✗	21.239	✓	0.307	No
600	HFOPDE, chapter 2.8.6	problem number 5	✗	21.02	✗	1.527	No
601	HFOPDE, chapter 2.8.6	problem number 6	✗	12.025	✓	0.081	No
602	HFOPDE, chapter 2.8.6	problem number 7	✗	0.153	✓	0.167	No
603	HFOPDE, chapter 2.8.6	problem number 8	✓	0.865	✓	0.237	No
604	HFOPDE, chapter 2.8.6	problem number 9	✗	0.122	✓	0.229	No
605	HFOPDE, chapter 2.8.6	problem number 10	✗	0.53	✓	0.222	No
606	HFOPDE, chapter 2.8.6	problem number 11	✓	0.239	✓	0.291	No
607	HFOPDE, chapter 2.8.6	problem number 12	✗	22.291	✓	1.062	No
608	HFOPDE, chapter 2.9.1	problem number 1	✓	0.089	✓	0.023	No
609	HFOPDE, chapter 2.9.1	problem number 2	✗	0.108	✓	0.143	No
610	HFOPDE, chapter 2.9.1	problem number 3	✗	2.718	✓	0.696	No
611	HFOPDE, chapter 2.9.1	problem number 4	✗	544.98	✗	2.433	No
612	HFOPDE, chapter 2.9.1	problem number 5	✗	2.312	✗	3.157	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
613	HFOPDE, chapter 2.9.2	problem number 1	✗	6.911	✓	0.091	No
614	HFOPDE, chapter 2.9.2	problem number 2	✗	0.074	✓	0.128	No
615	HFOPDE, chapter 2.9.2	problem number 3	✗	211.96	✓	0.413	No
616	HFOPDE, chapter 2.9.2	problem number 4	✗	455.453	✓	0.392	No
617	HFOPDE, chapter 2.9.2	problem number 5	✗	210.073	✗	7.911	No
618	HFOPDE, chapter 2.9.2	problem number 6	✗	20.316	✓	3.192	No
619	HFOPDE, chapter 2.9.2	problem number 7	✗	16.696	✓	0.349	No
620	HFOPDE, chapter 2.9.2	problem number 8	✗	208.019	✓	0.42	No
621	HFOPDE, chapter 2.9.2	problem number 9	✗ (Timed out)	600.	✓	0.417	No
622	HFOPDE, chapter 2.9.2	problem number 10	✗	31.533	✓	1.897	No
623	HFOPDE, chapter 2.9.2	problem number 11	✗	15.515	✓	0.603	No
624	HFOPDE, chapter 2.9.2	problem number 12	✗	1.358	✗	3.595	No
625	HFOPDE, chapter 2.9.2	problem number 13	✗	2.087	✗	2.786	No
626	HFOPDE, chapter 2.9.2	problem number 14	✗	24.929	✓	0.604	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
627	HFOPDE, chapter 2.9.2	problem number 15	✗	170.966	✓	0.608	No
628	HFOPDE, chapter 2.9.2	problem number 16	✗	25.345	✓	0.339	No
629	HFOPDE, chapter 2.9.3	problem number 1	✗	5.711	✓	9.172	No
630	HFOPDE, chapter 2.9.3	problem number 2	✗	181.699	✗	5.062	No
631	HFOPDE, chapter 2.9.3	problem number 3	✗	13.005	✗	8.059	No
632	HFOPDE, chapter 2.9.3	problem number 4	✗	2.543	✗	8.122	No
633	HFOPDE, chapter 2.9.3	problem number 5	✗	2.902	✗	7.008	No
634	HFOPDE, chapter 2.9.3	problem number 6	✗	25.16	✗	12.945	No
635	HFOPDE, chapter 2.9.3	problem number 7	✗	24.621	✗	10.392	No
636	HFOPDE, chapter 2.9.3	problem number 8	✗	13.313	✗	18.041	No
637	HFOPDE, chapter 2.9.3	problem number 9	✓	0.008	✓	0.053	No
638	HFOPDE, chapter 2.9.3	problem number 11	✗ (Timed out)	600.	✓	0.575	No
639	HFOPDE, chapter 2.9.3	problem number 12	✗	4.103	✓	7.826	No
640	HFOPDE, chapter 2.9.3	problem number 13	✗	11.982	✗	4.325	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
641	HFOPDE, chapter 2.9.3	problem number 14	✗	12.146	✗	3.544	No
642	HFOPDE, chapter 2.9.3	problem number 15	✗	19.498	✗	5.996	No
643	HFOPDE, chapter 2.9.3	problem number 16	✗	18.803	✗	6.034	No
644	HFOPDE, chapter 2.9.3	problem number 17	✗	4.474	✗	5.01	No
645	HFOPDE, chapter 2.9.3	problem number 18	✗	26.726	✗	7.478	No
646	HFOPDE, chapter 2.9.3	problem number 19	✗	26.007	✗	6.549	No
647	HFOPDE, chapter 2.9.3	problem number 20	✗	257.6	✗	27.366	No
648	HFOPDE, chapter 2.9.3	problem number 21	✗	77.934	✗	17.318	No
649	HFOPDE, chapter 2.9.3	problem number 22	✗	11.81	✗	8.803	No
650	HFOPDE, chapter 2.9.3	problem number 23	✗	7.111	✗	19.373	No
651	HFOPDE, chapter 3 examples	Example 1	✓	0.006	✓	0.065	No
652	HFOPDE, chapter 3 examples	Example 2	✓	0.022	✓	0.105	No
653	HFOPDE, chapter 3 examples	Example 3	✓	0.004	✓	0.008	No
654	HFOPDE, chapter 3.2.1	Problem 1	✓	0.004	✓	0.009	No
655	HFOPDE, chapter 3.2.1	Problem 2	✓	0.012	✓	0.045	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
656	HFOPDE, chapter 3.2.1	Problem 3	✓	0.009	✓	0.016	No
657	HFOPDE, chapter 3.2.1	Problem 4	✓	0.005	✓	0.008	No
658	HFOPDE, chapter 3.2.1	Problem 5	✓	0.067	✓	0.143	No
659	HFOPDE, chapter 3.2.1	Problem 6	✓	0.151	✓	0.259	No
660	HFOPDE, chapter 3.2.1	Problem 7	✓	0.026	✓	0.037	No
661	HFOPDE, chapter 3.2.1	Problem 8	✓	0.05	✓	0.051	No
662	HFOPDE, chapter 3.2.2	Problem 1	✓	0.061	✓	0.046	No
663	HFOPDE, chapter 3.2.2	Problem 2	✓	0.171	✓	0.035	No
664	HFOPDE, chapter 3.2.2	Problem 3	✓	0.063	✓	0.051	No
665	HFOPDE, chapter 3.2.2	Problem 4	✓	0.284	✓	0.11	No
666	HFOPDE, chapter 3.2.2	Problem 5	✓	0.01	✓	0.019	No
667	HFOPDE, chapter 3.2.2	Problem 6	✓	0.095	✓ Contains unresolved integral with RootOf	0.281	No
668	HFOPDE, chapter 3.2.2	Problem 7	✓	0.052	✓	0.078	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
669	HFOPDE, chapter 3.2.3	Problem 1	✓	0.011	✓	0.148	No
670	HFOPDE, chapter 3.2.3	Problem 2	✓	0.118	✓	0.067	No
671	HFOPDE, chapter 3.2.3	Problem 3	✓	0.104	✓	0.028	No
672	HFOPDE, chapter 3.2.3	Problem 4	✓	0.145	✓	0.068	No
673	HFOPDE, chapter 3.2.3	Problem 5	✓	0.006	✓	0.014	No
674	HFOPDE, chapter 3.2.3	Problem 6	✓	0.017	✓	0.167	No
675	HFOPDE, chapter 3.2.4	Problem 1	✓	0.068	✓	0.048	No
676	HFOPDE, chapter 3.2.4	Problem 2	✓	0.024	✓	0.045	No
677	HFOPDE, chapter 3.2.4	Problem 3	✓	0.012	✓	0.042	No
678	HFOPDE, chapter 3.2.4	Problem 4	✓	0.018	✓	0.071	No
679	HFOPDE, chapter 3.2.4	Problem 5	✓	0.056	✓ Result has unresolved integral	0.125	No
680	HFOPDE, chapter 3.2.4	Problem 6	✓	0.051	✓ Result has unresolved integral	0.1	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
681	HFOPDE, chapter 3.2.4	Problem 7	✓	0.65	✓ Result has unresolved integral	4.959	No
682	HFOPDE, chapter 3.2.4	Problem 8	✓	1.067	✓ Result has unresolved integral	0.686	No
683	HFOPDE, chapter 3.2.4	Problem 9	✗ (Timed out)	600.	✓	0.382	No
684	HFOPDE, chapter 3.2.4	Problem 10	✗ (Timed out)	600.	✓	7.752	No
685	HFOPDE, chapter 3.2.4	Problem 11	✓	0.404	✓	1.792	No
686	HFOPDE, chapter 3.3.1	Problem 1	✓	0.04	✓	0.144	No
687	HFOPDE, chapter 3.3.1	Problem 2	✓	0.027	✓	0.094	No
688	HFOPDE, chapter 3.3.1	Problem 3	✓	0.035	✓	0.207	No
689	HFOPDE, chapter 3.3.1	Problem 4	✓	0.046	✓	0.166	No
690	HFOPDE, chapter 3.3.1	Problem 5	✓	0.086	✓	0.419	No
691	HFOPDE, chapter 3.3.1	Problem 6	✓	0.096	✓	0.03	No
692	HFOPDE, chapter 3.3.1	Problem 7	✓	0.329	✓	0.335	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
693	HFOPDE, chapter 3.3.1	Problem 8	✗	133.129	✓	2.998	No
694	HFOPDE, chapter 3.3.1	Problem 9	✗	122.412	✓	0.736	No
695	HFOPDE, chapter 3.3.1	Problem 10	✓	0.971	✓	0.571	No
696	HFOPDE, chapter 3.3.1	Problem 11	✓	348.564	✓	0.625	No
697	HFOPDE, chapter 3.3.2	Problem 1	✓	0.112	✓	0.048	No
698	HFOPDE, chapter 3.3.2	Problem 2	✓	0.02	✓	0.144	No
699	HFOPDE, chapter 3.3.2	Problem 3	✓	0.034	✓	0.123	No
700	HFOPDE, chapter 3.3.2	Problem 4	✓	10.732	✓	0.368	No
701	HFOPDE, chapter 3.3.2	Problem 5	✓	0.332	✓	1.601	No
702	HFOPDE, chapter 3.3.2	Problem 6	✓	0.014	✓	0.073	No
703	HFOPDE, chapter 3.3.2	Problem 7	✓	0.021	✓	0.044	No
704	HFOPDE, chapter 3.3.2	Problem 8	✓	0.098	✓	0.695	No
705	HFOPDE, chapter 3.3.2	Problem 9	✓	0.248	✓	1.054	No
706	HFOPDE, chapter 3.3.2	Problem 10	✓	0.09	✓	0.118	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
707	HFOPDE, chapter 3.3.2	Problem 11	✗ (Timed out)	600.	✓	0.645	No
708	HFOPDE, chapter 3.4.1	Problem 1	✓	0.049	✓	0.024	No
709	HFOPDE, chapter 3.4.1	Problem 2	✓	0.066	✓	0.047	No
710	HFOPDE, chapter 3.4.1	Problem 3	✗ (Timed out)	600.	✓	0.022	No
711	HFOPDE, chapter 3.4.1	Problem 4	✗ (Timed out)	600.	✓	6.399	No
712	HFOPDE, chapter 3.4.1	Problem 5	✗ (Timed out)	600.	✓	1.854	No
713	HFOPDE, chapter 3.4.2	Problem 1	✓	0.046	✓	0.023	No
714	HFOPDE, chapter 3.4.2	Problem 2	✓	0.063	✓	0.021	No
715	HFOPDE, chapter 3.4.2	Problem 3	✓	0.187	✓	0.023	No
716	HFOPDE, chapter 3.4.2	Problem 4	✗ (Timed out)	600.	✓	1.135	No
717	HFOPDE, chapter 3.4.2	Problem 5	✗ (Timed out)	600.	✓	2.39	No
718	HFOPDE, chapter 3.4.3	Problem 1	✓	0.172	✓	0.023	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
719	HFOPDE, chapter 3.4.3	Problem 2	✓	0.12	✓	0.018	No
720	HFOPDE, chapter 3.4.3	Problem 3	✓	0.036	✓	0.019	No
721	HFOPDE, chapter 3.4.3	Problem 4	✗ (Timed out)	600.	✓	1.079	No
722	HFOPDE, chapter 3.4.3	Problem 5	✗ (Timed out)	600.	✓	1.013	No
723	HFOPDE, chapter 3.4.4	Problem 1	✓	0.171	✓	0.029	No
724	HFOPDE, chapter 3.4.4	Problem 2	✓	0.12	✓	0.017	No
725	HFOPDE, chapter 3.4.4	Problem 3	✓	0.035	✓	0.016	No
726	HFOPDE, chapter 3.4.4	Problem 4	✗ (Timed out)	600.	✓	0.967	No
727	HFOPDE, chapter 3.4.4	Problem 5	✗ (Timed out)	600.	✓	0.899	No
728	HFOPDE, chapter 3.4.5	Problem 1	✓	0.054	✓	0.019	No
729	HFOPDE, chapter 3.4.5	Problem 2	✓	0.155	✓	0.185	No
730	HFOPDE, chapter 3.4.5	Problem 3	✓	0.102	✓	0.059	No
731	HFOPDE, chapter 3.4.5	Problem 4	✓	0.046	✓	0.804	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
732	HFOPDE, chapter 3.4.5	Problem 5	✓	0.074	✓	0.158	No
733	HFOPDE, chapter 3.5.1	Problem 1	✓	0.03	✓	0.077	No
734	HFOPDE, chapter 3.5.1	Problem 2	✓	0.014	✓	0.048	No
735	HFOPDE, chapter 3.5.1	Problem 3	✓	0.122	✓	1.326	No
736	HFOPDE, chapter 3.5.1	Problem 4	✗ (Timed out)	600.	✓	0.569	No
737	HFOPDE, chapter 3.5.1	Problem 5	✗ (Timed out)	600.	✓	0.561	No
738	HFOPDE, chapter 3.5.1	Problem 6	✗	0.126	✓	0.297	No
739	HFOPDE, chapter 3.5.2	Problem 1	✓	0.097	✓	0.048	No
740	HFOPDE, chapter 3.5.2	Problem 2	✓	0.148	✓ Result has unresolved integrals	1.02	No
741	HFOPDE, chapter 3.5.2	Problem 3	✗ (Timed out)	600.	✓	0.536	No
742	HFOPDE, chapter 3.5.3	Problem 4	✓	0.04	✓ Result has unresolved integrals	0.133	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
743	HFOPDE, chapter 3.5.3	Problem 5	✓	0.036	✓	0.237	No
744	HFOPDE, chapter 3.5.3	Problem 6	✗ (Timed out)	600.	✓	2.775	No
745	HFOPDE, chapter 3.6.1	Problem 1	✓	0.052	✓	0.023	No
746	HFOPDE, chapter 3.6.1	Problem 2	✓	0.069	✓	0.019	No
747	HFOPDE, chapter 3.6.1	Problem 3	✓	0.025	✓	0.018	No
748	HFOPDE, chapter 3.6.1	Problem 4	✗ (Timed out)	600.	✓	2.713	No
749	HFOPDE, chapter 3.6.1	Problem 5	✓	0.424	✓ Result has unresolved integrals	4.733	No
750	HFOPDE, chapter 3.6.2	Problem 1	✓	0.05	✓	0.024	No
751	HFOPDE, chapter 3.6.2	Problem 2	✓	0.066	✓	0.023	No
752	HFOPDE, chapter 3.6.2	Problem 3	✓	0.023	✓	0.02	No
753	HFOPDE, chapter 3.6.2	Problem 4	✗ (Timed out)	600.	✓	1.329	No
754	HFOPDE, chapter 3.6.2	Problem 5	✗ (Timed out)	600.	✓	2.454	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
755	HFOPDE, chapter 3.6.3	Problem 1	✓	0.186	✓	0.022	No
756	HFOPDE, chapter 3.6.3	Problem 2	✓	0.131	✓	0.017	No
757	HFOPDE, chapter 3.6.3	Problem 3	✓	0.04	✓	0.016	No
758	HFOPDE, chapter 3.6.3	Problem 4	✗ (Timed out)	600.	✓	1.246	No
759	HFOPDE, chapter 3.6.3	Problem 5	✗ (Timed out)	600.	✓	1.76	No
760	HFOPDE, chapter 3.6.4	Problem 1	✓	0.192	✓	0.029	No
761	HFOPDE, chapter 3.6.4	Problem 2	✓	0.135	✓	0.017	No
762	HFOPDE, chapter 3.6.4	Problem 3	✓	0.041	✓	0.017	No
763	HFOPDE, chapter 3.6.4	Problem 4	✗ (Timed out)	600.	✓	1.205	No
764	HFOPDE, chapter 3.6.4	Problem 5	✗ (Timed out)	600.	✓	1.631	No
765	HFOPDE, chapter 3.6.5	Problem 1	✓	0.057	✓	0.02	No
766	HFOPDE, chapter 3.6.5	Problem 2	✓	0.085	✓	0.198	No
767	HFOPDE, chapter 3.6.5	Problem 3	✓	0.304	✓	0.033	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
768	HFOPDE, chapter 3.6.5	Problem 4	✓	0.085	✓	0.442	No
769	HFOPDE, chapter 3.6.5	Problem 5	✓	0.094	✓	2.492	No
770	HFOPDE, chapter 3.6.5	Problem 6	✓	0.112	✓	0.832	No
771	HFOPDE, chapter 3.7.1	Problem 1	✓	0.992	✓	0.104	No
772	HFOPDE, chapter 3.7.1	Problem 2	✓	0.408	✓	0.127	No
773	HFOPDE, chapter 3.7.1	Problem 3	✓	0.041	✓	0.044	No
774	HFOPDE, chapter 3.7.1	Problem 4	✗ (Timed out)	600.	✓	1.096	No
775	HFOPDE, chapter 3.7.1	Problem 5	✗ (Timed out)	600.	✓	0.814	No
776	HFOPDE, chapter 3.7.2	Problem 1	✓	0.975	✓	0.019	No
777	HFOPDE, chapter 3.7.2	Problem 2	✓	0.754	✓	0.017	No
778	HFOPDE, chapter 3.7.2	Problem 3	✓	0.042	✓	0.016	No
779	HFOPDE, chapter 3.7.2	Problem 4	✗ (Timed out)	600.	✓	0.622	No
780	HFOPDE, chapter 3.7.2	Problem 5	✗ (Timed out)	600.	✓	0.437	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
781	HFOPDE, chapter 3.7.3	Problem 1	✓	0.045	✓	0.223	No
782	HFOPDE, chapter 3.7.3	Problem 2	✓	0.075	✓	0.173	No
783	HFOPDE, chapter 3.7.3	Problem 3	✓	0.02	✓	0.116	No
784	HFOPDE, chapter 3.7.3	Problem 4	✗ (Timed out)	600.	✓	0.265	No
785	HFOPDE, chapter 3.7.3	Problem 5	✗ (Timed out)	600.	✓	0.666	No
786	HFOPDE, chapter 3.7.4	Problem 1	✓	0.038	✓	0.023	No
787	HFOPDE, chapter 3.7.4	Problem 2	✓	0.081	✓	0.019	No
788	HFOPDE, chapter 3.7.4	Problem 3	✓	0.02	✓	0.027	No
789	HFOPDE, chapter 3.7.4	Problem 4	✗ (Timed out)	600.	✓	0.326	No
790	HFOPDE, chapter 3.7.4	Problem 5	✗ (Timed out)	600.	✓	0.75	No
791	HFOPDE, chapter 3.8.1	Problem 1	✓	0.028	✓	0.011	No
792	HFOPDE, chapter 3.8.1	Problem 2	✓	0.178	✓	0.049	No
793	HFOPDE, chapter 3.8.1	Problem 3	✓	0.276	✓	0.029	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
794	HFOPDE, chapter 3.8.1	Problem 4	✓	0.147	✓	0.43	No
795	HFOPDE, chapter 3.8.1	Problem 5	✓	0.128	✓	0.08	No
796	HFOPDE, chapter 3.8.1	Problem 6	✓	0.137	✓	0.014	No
797	HFOPDE, chapter 3.8.1	Problem 7	✓	1.633	✓	0.309	No
798	HFOPDE, chapter 3.8.1	Problem 8	✓	0.128	✓	3.382	No
799	HFOPDE, chapter 3.8.1	Problem 9	✓	0.198	✓	0.116	No
800	HFOPDE, chapter 3.8.1	Problem 10	✓	0.765	✓	0.011	No
801	HFOPDE, chapter 3.8.1	Problem 11	✓	36.48	✓	0.742	No
802	HFOPDE, chapter 3.8.1	Problem 12	✓	1.861	✓	0.31	No
803	HFOPDE, chapter 3.8.1	Problem 13	✗	37.252	✓	0.548	No
804	HFOPDE, chapter 3.8.1	Problem 14	✓	1.65	✓	0.722	No
805	HFOPDE, chapter 3.8.1	Problem 15	✓	37.217	✓	0.56	No
806	HFOPDE, chapter 3.8.1	Problem 16	✓	0.274	✓	0.017	No
807	HFOPDE, chapter 3.8.2	Problem 1	✓	0.103	✓	0.048	No
808	HFOPDE, chapter 3.8.2	Problem 2	✓	0.093	✓	0.075	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
809	HFOPDE, chapter 3.8.2	Problem 3	✓	1.55	✓	0.085	No
810	HFOPDE, chapter 3.8.2	Problem 4	✗	0.078	✓	0.151	No
811	HFOPDE, chapter 3.8.2	Problem 5	✓	1.977	✓	0.326	No
812	HFOPDE, chapter 3.8.2	Problem 6	✓	0.254	✓	0.014	No
813	HFOPDE, chapter 3.8.2	Problem 7	✗	0.211	✓	0.062	No
814	HFOPDE, chapter 3.8.3	Problem 1	✓	0.098	✓	0.076	No
815	HFOPDE, chapter 3.8.3	Problem 2	✓	0.004	✓	0.01	No
816	HFOPDE, chapter 3.8.3	Problem 3	✓	0.108	✓	0.043	No
817	HFOPDE, chapter 3.8.3	Problem 4	✓	0.252	✓	0.023	No
818	HFOPDE, chapter 3.8.3	Problem 5	✓	0.114	✓	0.248	No
819	HFOPDE, chapter 3.8.3	Problem 6	✓	0.136	✓	0.195	No
820	HFOPDE, chapter 3.8.3	Problem 7	✓	0.166	✓	0.078	No
821	HFOPDE, chapter 3.8.3	Problem 8	✓	0.269	✓	3.738	No
822	HFOPDE, chapter 3.8.4	Problem 1	✓	0.044	✓	0.019	No
823	HFOPDE, chapter 3.8.4	Problem 2	✓	0.18	✓	0.022	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
824	HFOPDE, chapter 3.8.4	Problem 3	✓	1.445	✓	0.067	No
825	HFOPDE, chapter 3.8.4	Problem 4	✓	19.499	✓	0.157	No
826	HFOPDE, chapter 3.8.4	Problem 5	✓	35.412	✓	0.326	No
827	HFOPDE, chapter 3.8.4	Problem 6	✗	38.127	✓	1.043	No
828	HFOPDE, chapter 3.8.4	Problem 7	✗	0.218	✓ Contains RootOf	0.037	No
829	HFOPDE, chapter 4.1.1	Example 1	✓	0.014	✓	0.099	No
830	HFOPDE, chapter 4.1.1	Example 2	✓	0.02	✓	0.072	No
831	HFOPDE, chapter 4.1.1	Example 3	✓	0.004	✓	0.035	No
832	HFOPDE, chapter 4.2.1	Problem 1	✓	0.005	✓	0.036	No
833	HFOPDE, chapter 4.2.1	Problem 2	✓	0.013	✓	0.012	No
834	HFOPDE, chapter 4.2.1	Problem 3	✓	0.005	✓	0.008	No
835	HFOPDE, chapter 4.2.1	Problem 4	✓	0.006	✓	0.093	No
836	HFOPDE, chapter 4.2.1	Problem 5	✓	0.005	✓	0.007	No
837	HFOPDE, chapter 4.2.1	Problem 6	✓	0.009	✓	0.011	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
838	HFOPDE, chapter 4.2.1	Problem 7	✗	0.113	✗	96.38	No
839	HFOPDE, chapter 4.2.2	Problem 1	✓	0.014	✓	0.083	No
840	HFOPDE, chapter 4.2.2	Problem 2	✓	0.01	✓	0.046	No
841	HFOPDE, chapter 4.2.2	Problem 3	✓	0.031	✓	0.1	No
842	HFOPDE, chapter 4.2.2	Problem 4	✓	0.044	✓	0.119	No
843	HFOPDE, chapter 4.2.2	Problem 5	✓	0.076	✓	0.062	No
844	HFOPDE, chapter 4.2.2	Problem 6	✓	0.033	✓	0.148	No
845	HFOPDE, chapter 4.2.2	Problem 7	✗ (Timed out)	600.	✓	3.963	No
846	HFOPDE, chapter 4.2.2	Problem 8	✗	1.481	✓	2.108	No
847	HFOPDE, chapter 4.2.3	Problem 1	✓	0.014	✓	0.117	No
848	HFOPDE, chapter 4.2.3	Problem 2	✓	0.012	✓	0.021	No
849	HFOPDE, chapter 4.2.3	Problem 3	✓	0.007	✓	0.055	No
850	HFOPDE, chapter 4.2.3	Problem 4	✓	0.086	✓	0.255	No
851	HFOPDE, chapter 4.2.3	Problem 5	✓	0.033	✓	0.119	No

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Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
852	HFOPDE, chapter 4.2.3	Problem 6	✓	0.051	✓	0.069	No
853	HFOPDE, chapter 4.2.4	Problem 1	✓	0.072	✓	0.052	No
854	HFOPDE, chapter 4.2.4	Problem 2 case $n \neq -1, n \neq -2$	✓	0.023	✓	0.152	No
855	HFOPDE, chapter 4.2.4	Problem 2 case $n = -1$	✓	0.013	✓	0.147	No
856	HFOPDE, chapter 4.2.4	Problem 2 case $n = -2$	✓	0.014	✓	0.128	No
857	HFOPDE, chapter 4.2.4	Problem 3	✓	0.012	✓	0.073	No
858	HFOPDE, chapter 4.2.4	Problem 4	✓	0.02	✓	0.049	No
859	HFOPDE, chapter 4.2.4	Problem 5	✓	0.055	✓	0.045	No
860	HFOPDE, chapter 4.2.4	Problem 6	✓	0.055	✓	0.07	No
861	HFOPDE, chapter 4.2.4	Problem 7	✓	0.69	✓	2.412	No
862	HFOPDE, chapter 4.2.4	Problem 8	✓	1.138	✓	0.532	No
863	HFOPDE, chapter 4.2.4	Problem 9	✗ (Timed out)	600.	✓	2.689	No
864	HFOPDE, chapter 4.2.4	Problem 10	✗ (Timed out)	600.	✓	5.491	No
865	HFOPDE, chapter 4.2.4	Problem 11	✓	0.296	✓	1.656	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
866	HFOPDE, chapter 4.2.4	Problem 12	✗	0.07	✓	1.219	No
867	HFOPDE, chapter 4.2.4	Problem 13	✗	0.083	✗ (Timed out)	600.	No
868	HFOPDE, chapter 4.3.1	Problem 1	✓	0.041	✓	0.101	No
869	HFOPDE, chapter 4.3.1	Problem 2	✓	0.043	✓	0.053	No
870	HFOPDE, chapter 4.3.1	Problem 3	✓	0.036	✓	0.209	No
871	HFOPDE, chapter 4.3.1	Problem 4	✓	0.058	✓	0.199	No
872	HFOPDE, chapter 4.3.1	Problem 5	✓	42.318	✓	0.717	No
873	HFOPDE, chapter 4.3.1	Problem 6	✓	3.807	✓	0.308	No
874	HFOPDE, chapter 4.3.1	Problem 7	✗	137.505	✓	1.77	No
875	HFOPDE, chapter 4.3.1	Problem 8	✗	126.728	✓	1.387	No
876	HFOPDE, chapter 4.3.1	Problem 9	✓	1.035	✓	1.044	No
877	HFOPDE, chapter 4.3.1	Problem 10	✓	0.06	✓	0.042	No
878	HFOPDE, chapter 4.3.2	Problem 1	✓	0.141	✓	0.024	No
879	HFOPDE, chapter 4.3.2	Problem 2	✓	0.015	✓	0.045	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
880	HFOPDE, chapter 4.3.2	Problem 3	✓	0.021	✓	0.044	No
881	HFOPDE, chapter 4.3.2	Problem 4	✓	0.098	✓	0.732	No
882	HFOPDE, chapter 4.3.2	Problem 5	✓	0.256	✓	0.47	No
883	HFOPDE, chapter 4.3.2	Problem 6	✓	0.089	✓	0.078	No
884	HFOPDE, chapter 4.3.2	Problem 7	✗ (Timed out)	600.	✓	0.165	No
885	HFOPDE, chapter 4.4.1	Problem 1	✓	0.049	✓	0.022	No
886	HFOPDE, chapter 4.4.1	Problem 2	✓	0.066	✓	0.048	No
887	HFOPDE, chapter 4.4.1	Problem 3	✓	0.023	✓	0.017	No
888	HFOPDE, chapter 4.4.1	Problem 4	✗ (Timed out)	600.	✓	1.034	No
889	HFOPDE, chapter 4.4.1	Problem 5	✓	0.394	✓	0.966	No
890	HFOPDE, chapter 4.4.2	Problem 1	✓	0.05	✓	0.02	No
891	HFOPDE, chapter 4.4.2	Problem 2	✓	0.067	✓	0.017	No
892	HFOPDE, chapter 4.4.2	Problem 3	✓	0.023	✓	0.021	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
893	HFOPDE, chapter 4.4.2	Problem 4	✗ (Timed out)	600.	✓	0.413	No
894	HFOPDE, chapter 4.4.2	Problem 5	✗ (Timed out)	600.	✓	0.602	No
895	HFOPDE, chapter 4.4.3	Problem 1	✓	0.178	✓	0.149	No
896	HFOPDE, chapter 4.4.3	Problem 2	✓	0.121	✓	0.171	No
897	HFOPDE, chapter 4.4.3	Problem 3	✓	0.037	✓	0.102	No
898	HFOPDE, chapter 4.4.3	Problem 4	✗ (Timed out)	600.	✓	0.48	No
899	HFOPDE, chapter 4.4.3	Problem 5	✗ (Timed out)	600.	✓	0.618	No
900	HFOPDE, chapter 4.4.4	Problem 1	✓	0.181	✓	0.071	No
901	HFOPDE, chapter 4.4.4	Problem 2	✓	0.122	✓	0.054	No
902	HFOPDE, chapter 4.4.4	Problem 3	✓	0.037	✓	0.041	No
903	HFOPDE, chapter 4.4.4	Problem 4	✗ (Timed out)	600.	✓	0.469	No
904	HFOPDE, chapter 4.4.4	Problem 5	✗ (Timed out)	600.	✓	0.558	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
905	HFOPDE, chapter 4.4.5	Problem 1	✓	0.055	✓	0.019	No
906	HFOPDE, chapter 4.4.5	Problem 2	✓	0.167	✓	0.14	No
907	HFOPDE, chapter 4.4.5	Problem 3	✓	0.066	✓	0.696	No
908	HFOPDE, chapter 4.4.5	Problem 4	✓	0.068	✓	0.058	No
909	HFOPDE, chapter 4.4.5	Problem 5	✓	0.072	✓	0.424	No
910	HFOPDE, chapter 4.4.5	Problem 6	✓	0.206	✓	6.505	No
911	HFOPDE, chapter 4.5.1	Problem 1	✓	0.033	✓	0.064	No
912	HFOPDE, chapter 4.5.1	Problem 2	✓	0.018	✓	0.103	No
913	HFOPDE, chapter 4.5.1	Problem 3	✗ (Timed out)	600.	✓	0.436	No
914	HFOPDE, chapter 4.5.1	Problem 4	✗ (Timed out)	600.	✓	0.488	No
915	HFOPDE, chapter 4.5.1	Problem 5	✓	0.107	✓	1.656	No
916	HFOPDE, chapter 4.5.1	Problem 6	✗	0.142	✓	0.327	No
917	HFOPDE, chapter 4.5.2	Problem 1	✓	0.063	✓	1.302	No
918	HFOPDE, chapter 4.5.2	Problem 2	✓	0.197	✓	0.222	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
919	HFOPDE, chapter 4.5.2	Problem 3	✗ (Timed out)	600.	✓	0.066	No
920	HFOPDE, chapter 4.5.2	Problem 4	✓	0.042	✓	0.128	No
921	HFOPDE, chapter 4.5.2	Problem 5	✓	0.038	✓	0.619	No
922	HFOPDE, chapter 4.5.2	Problem 6	✗ (Timed out)	600.	✓	1.914	No
923	HFOPDE, chapter 4.6.1	Problem 1	✓	0.073	✓	0.045	No
924	HFOPDE, chapter 4.6.1	Problem 2	✓	0.054	✓	0.021	No
925	HFOPDE, chapter 4.6.1	Problem 3	✓	0.025	✓	0.014	No
926	HFOPDE, chapter 4.6.1	Problem 4	✗ (Timed out)	600.	✓	0.9	No
927	HFOPDE, chapter 4.6.1	Problem 5	✓	0.422	✓	0.798	No
928	HFOPDE, chapter 4.6.2	Problem 1	✓	0.068	✓	0.017	No
929	HFOPDE, chapter 4.6.2	Problem 2	✓	0.05	✓	0.016	No
930	HFOPDE, chapter 4.6.2	Problem 3	✓	0.025	✓	0.015	No
931	HFOPDE, chapter 4.6.2	Problem 4	✗ (Timed out)	600.	✓	0.537	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
932	HFOPDE, chapter 4.6.2	Problem 5	✗ (Timed out)	600.	✓	0.775	No
933	HFOPDE, chapter 4.6.3	Problem 1	✓	0.143	✓	0.038	No
934	HFOPDE, chapter 4.6.3	Problem 2	✓	0.201	✓	0.068	No
935	HFOPDE, chapter 4.6.3	Problem 3	✓	0.044	✓	0.031	No
936	HFOPDE, chapter 4.6.3	Problem 4	✗ (Timed out)	600.	✓	0.566	No
937	HFOPDE, chapter 4.6.3	Problem 5	✗ (Timed out)	600.	✓	0.64	No
938	HFOPDE, chapter 4.6.4	Problem 1	✓	0.14	✓	0.038	No
939	HFOPDE, chapter 4.6.4	Problem 2	✓	0.199	✓	0.051	No
940	HFOPDE, chapter 4.6.4	Problem 3	✓	0.044	✓	0.03	No
941	HFOPDE, chapter 4.6.4	Problem 4	✗ (Timed out)	600.	✓	0.859	No
942	HFOPDE, chapter 4.6.4	Problem 5	✗ (Timed out)	600.	✓	0.88	No
943	HFOPDE, chapter 4.6.5	Problem 1	✓	0.036	✓	0.024	No
944	HFOPDE, chapter 4.6.5	Problem 2	✓	0.084	✓	0.246	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
945	HFOPDE, chapter 4.6.5	Problem 3	✓	0.089	✓	0.371	No
946	HFOPDE, chapter 4.6.5	Problem 4	✓	0.073	✓	2.873	No
947	HFOPDE, chapter 4.6.5	Problem 5	✓	0.155	✓	0.087	No
948	HFOPDE, chapter 4.6.5	Problem 6	✓	0.122	✓	0.15	No
949	HFOPDE, chapter 4.7.1	Problem 1	✓	0.781	✓	0.118	No
950	HFOPDE, chapter 4.7.1	Problem 2	✓	0.415	✓	0.128	No
951	HFOPDE, chapter 4.7.1	Problem 3	✓	0.229	✓	0.065	No
952	HFOPDE, chapter 4.7.1	Problem 4	✗ (Timed out)	600.	✓	2.179	No
953	HFOPDE, chapter 4.7.1	Problem 5	✗ (Timed out)	600.	✓	1.862	No
954	HFOPDE, chapter 4.7.2	Problem 1	✓	0.824	✓	0.02	No
955	HFOPDE, chapter 4.7.2	Problem 2	✓	0.665	✓	0.019	No
956	HFOPDE, chapter 4.7.2	Problem 3	✓	0.221	✓	0.024	No
957	HFOPDE, chapter 4.7.2	Problem 4	✗ (Timed out)	600.	✓	1.176	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
958	HFOPDE, chapter 4.7.2	Problem 5	✗ (Timed out)	600.	✓	1.286	No
959	HFOPDE, chapter 4.7.3	Problem 1	✓	0.056	✓	0.305	No
960	HFOPDE, chapter 4.7.3	Problem 2	✓	0.072	✓	0.211	No
961	HFOPDE, chapter 4.7.3	Problem 3	✓	0.104	✓	0.243	No
962	HFOPDE, chapter 4.7.3	Problem 4	✗ (Timed out)	600.	✓	0.807	No
963	HFOPDE, chapter 4.7.3	Problem 5	✗ (Timed out)	600.	✓	0.753	No
964	HFOPDE, chapter 4.7.4	Problem 1	✓	0.047	✓	0.117	No
965	HFOPDE, chapter 4.7.4	Problem 2	✓	0.078	✓	0.024	No
966	HFOPDE, chapter 4.7.4	Problem 3	✓	0.109	✓	0.07	No
967	HFOPDE, chapter 4.7.4	Problem 4	✗ (Timed out)	600.	✓	0.774	No
968	HFOPDE, chapter 4.7.4	Problem 5	✗ (Timed out)	600.	✓	0.625	No
969	HFOPDE, chapter 4.8.1	Problem 1	✓	0.029	✓	0.036	No
970	HFOPDE, chapter 4.8.1	Problem 2	✓	0.182	✓	0.071	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
971	HFOPDE, chapter 4.8.1	Problem 3	✓	0.277	✓	0.013	No
972	HFOPDE, chapter 4.8.1	Problem 4	✓	0.15	✓	0.063	No
973	HFOPDE, chapter 4.8.1	Problem 5	✓	0.13	✓	0.066	No
974	HFOPDE, chapter 4.8.1	Problem 6	✓	0.142	✓	0.011	No
975	HFOPDE, chapter 4.8.1	Problem 7	✓	1.638	✓	0.076	No
976	HFOPDE, chapter 4.8.1	Problem 8	✓	0.128	✓	0.059	No
977	HFOPDE, chapter 4.8.1	Problem 9	✓	0.183	✓	0.1	No
978	HFOPDE, chapter 4.8.1	Problem 10	✓	0.764	✓	0.061	No
979	HFOPDE, chapter 4.8.1	Problem 11	✓	36.512	✓	0.148	No
980	HFOPDE, chapter 4.8.1	Problem 12	✓	1.865	✓	0.222	No
981	HFOPDE, chapter 4.8.1	Problem 13	✗	23.885	✗	11.288	No
982	HFOPDE, chapter 4.8.1	Problem 14	✓	1.613	✓	0.37	No
983	HFOPDE, chapter 4.8.1	Problem 15	✓	1.51	✓	0.121	No
984	HFOPDE, chapter 4.8.2	Problem 1	✓	0.105	✓	0.067	No
985	HFOPDE, chapter 4.8.2	Problem 2	✓	0.095	✓	0.039	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
986	HFOPDE, chapter 4.8.2	Problem 3	✓	1.546	✓	0.099	No
987	HFOPDE, chapter 4.8.2	Problem 4	✗	0.077	✓ contains RootOf	0.137	No
988	HFOPDE, chapter 4.8.2	Problem 5	✗	21.913	✗	2.278	No
989	HFOPDE, chapter 4.8.2	Problem 6	✗	0.206	✓ has RootOf	0.102	No
990	HFOPDE, chapter 4.8.2	Problem 7	✗	0.207	✓ has RootOf	0.039	No
991	HFOPDE, chapter 4.8.3	Problem 1	✓	0.101	✓	0.064	No
992	HFOPDE, chapter 4.8.3	Problem 2	✓	0.004	✓	0.041	No
993	HFOPDE, chapter 4.8.3	Problem 3	✓	0.107	✓	0.037	No
994	HFOPDE, chapter 4.8.3	Problem 4	✓	0.336	✓	0.349	No
995	HFOPDE, chapter 4.8.3	Problem 5	✓	0.307	✓	0.1	No
996	HFOPDE, chapter 4.8.3	Problem 6	✓	0.168	✓	0.068	No
997	HFOPDE, chapter 4.8.3	Problem 7	✓	2.052	✓	4.83	No
998	HFOPDE, chapter 4.8.4	Problem 1	✓	0.043	✓	0.01	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
999	HFOPDE, chapter 4.8.4	Problem 2	✓	0.18	✓	0.013	No
1000	HFOPDE, chapter 4.8.4	Problem 3	✓	1.439	✓	0.015	No
1001	HFOPDE, chapter 4.8.4	Problem 4	✓	19.257	✓	0.049	No
1002	HFOPDE, chapter 4.8.4	Problem 5	✓	34.135	✓	0.268	No
1003	HFOPDE, chapter 4.8.4	Problem 6	✗	24.08	✗	3.18	No
1004	HFOPDE, chapter 4.8.4	Problem 7	✗	0.208	✓ has RootOf	0.055	No
1005	HFOPDE, chapter 5.2.1	Problem 1	✓	0.008	✓	0.038	No
1006	HFOPDE, chapter 5.2.1	Problem 2	✓	0.01	✓	0.011	No
1007	HFOPDE, chapter 5.2.1	Problem 3	✓	0.026	✓	0.101	No
1008	HFOPDE, chapter 5.2.1	Problem 4	✓	0.023	✓	0.042	No
1009	HFOPDE, chapter 5.2.1	Problem 5	✓	0.125	✓	0.15	No
1010	HFOPDE, chapter 5.2.1	Problem 6	✓	0.04	✓	0.05	No
1011	HFOPDE, chapter 5.2.1	Problem 7	✗ (Timed out)	600.	✗ (Timed out)	600.	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1012	HFOPDE, chapter 5.2.1	Problem 8	✗ (Timed out)	600.	✓	5.948	No
1013	HFOPDE, chapter 5.2.2	Problem 1	✓	0.033	✓	0.123	No
1014	HFOPDE, chapter 5.2.2	Problem 2	✓	0.043	✓	0.052	No
1015	HFOPDE, chapter 5.2.2	Problem 3	✓	0.009	✓	0.046	No
1016	HFOPDE, chapter 5.2.2	Problem 4	✓	0.204	✓	0.08	No
1017	HFOPDE, chapter 5.2.2	Problem 5	✗ (Timed out)	600.	✓	14.365	No
1018	HFOPDE, chapter 5.2.2	Problem 6	✗ (Timed out)	600.	✓	2.185	No
1019	HFOPDE, chapter 5.2.2	Problem 7	✗	42.231	✓	2.844	No
1020	HFOPDE, chapter 5.2.2	Problem 8	✗	35.565	✓	73.979	No
1021	HFOPDE, chapter 5.2.3	Problem 1	✓	0.238	✓	0.707	No
1022	HFOPDE, chapter 5.2.3	Problem 2	✗ (Timed out)	600.	✓	0.167	No
1023	HFOPDE, chapter 5.2.3	Problem 3	✗ (Timed out)	600.	✓	0.589	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1024	HFOPDE, chapter 5.2.3	Problem 4	✗ (Timed out)	600.	✓	0.001	No
1025	HFOPDE, chapter 5.2.3	Problem 5	✓	0.176	✓	0.432	No
1026	HFOPDE, chapter 5.2.3	Problem 6	✓	0.036	✓	0.27	No
1027	HFOPDE, chapter 5.2.3	Problem 7	✗ (Timed out)	600.	✓ contains RootOf	1.327	No
1028	HFOPDE, chapter 5.2.4	Problem 1	✓	247.813	✓	0.63	No
1029	HFOPDE, chapter 5.2.4	Problem 2	✓	0.047	✗ (Timed out)	600.	No
1030	HFOPDE, chapter 5.2.4	Problem 3	✓	0.014	✓	0.485	No
1031	HFOPDE, chapter 5.2.4	Problem 4	✓	29.47	✓	0.252	No
1032	HFOPDE, chapter 5.2.4	Problem 5	✓	128.19	✓	1.27	No
1033	HFOPDE, chapter 5.2.4	Problem 6	✓	131.327	✓	1.077	No
1034	HFOPDE, chapter 5.2.4	Problem 7	✓	0.087	✓	0.239	No
1035	HFOPDE, chapter 5.2.4	Problem 8	✓	0.073	✓	0.197	No
1036	HFOPDE, chapter 5.2.4	Problem 9	✗ (Timed out)	600.	✓	7.057	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1037	HFOPDE, chapter 5.2.4	Problem 10	✗ (Timed out)	600.	✓	1.4	No
1038	HFOPDE, chapter 5.2.4	Problem 11	✗ (Timed out)	600.	✓	3.192	No
1039	HFOPDE, chapter 5.2.4	Problem 12	✓	477.189	✓	2.918	No
1040	HFOPDE, chapter 5.3.1	Problem 1	✓	38.665	✓	0.504	No
1041	HFOPDE, chapter 5.3.1	Problem 2	✓	50.115	✓	0.504	No
1042	HFOPDE, chapter 5.3.1	Problem 3	✓	387.237	✓	0.567	No
1043	HFOPDE, chapter 5.3.1	Problem 4	✗	118.14	✓	1.442	No
1044	HFOPDE, chapter 5.3.1	Problem 5	✗	118.354	✓	1.708	No
1045	HFOPDE, chapter 5.3.1	Problem 6	✗ (Timed out)	600.	✓	1.088	No
1046	HFOPDE, chapter 5.3.1	Problem 7	✓	4.909	✓	0.091	No
1047	HFOPDE, chapter 5.3.1	Problem 8	✗ (Timed out)	600.	✓	0.166	No
1048	HFOPDE, chapter 5.3.2	Problem 1	✗ (Timed out)	600.	✓	0.209	No
1049	HFOPDE, chapter 5.3.2	Problem 2	✓	3.29	✓	0.061	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1050	HFOPDE, chapter 5.3.2	Problem 3	✓	2.62	✓	0.063	No
1051	HFOPDE, chapter 5.3.2	Problem 4	✓	0.323	✓	1.493	No
1052	HFOPDE, chapter 5.3.2	Problem 5	✓	34.072	✓	0.418	No
1053	HFOPDE, chapter 5.3.2	Problem 6	✓	28.929	✓	0.307	No
1054	HFOPDE, chapter 5.3.2	Problem 7	✓	394.427	✓	0.718	No
1055	HFOPDE, chapter 5.3.2	Problem 8	✓	0.031	✓	0.05	No
1056	HFOPDE, chapter 5.3.2	Problem 9	✓	576.304	✓	1.851	No
1057	HFOPDE, chapter 5.3.2	Problem 10	✓	240.653	✓	0.396	No
1058	HFOPDE, chapter 5.4.1	Problem 1	✓	304.655	✓	10.399	No
1059	HFOPDE, chapter 5.4.1	Problem 2	✗ (Timed out)	600.	✓	0.219	No
1060	HFOPDE, chapter 5.4.1	Problem 3	✗ (Timed out)	600.	✓	5.414	No
1061	HFOPDE, chapter 5.4.1	Problem 4	✗ (Timed out)	600.	✓	8.879	No
1062	HFOPDE, chapter 5.4.1	Problem 5	✗ (Timed out)	600.	✓	1.858	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1063	HFOPDE, chapter 5.4.2	Problem 1	✓	312.306	✓	6.746	No
1064	HFOPDE, chapter 5.4.2	Problem 2	✗ (Timed out)	600.	✓	0.194	No
1065	HFOPDE, chapter 5.4.2	Problem 3	✗ (Timed out)	600.	✓	6.734	No
1066	HFOPDE, chapter 5.4.2	Problem 4	✓	0.068	✓	0.18	No
1067	HFOPDE, chapter 5.4.2	Problem 5	✗ (Timed out)	600.	✓	6.064	No
1068	HFOPDE, chapter 5.4.2	Problem 6	✗ (Timed out)	600.	✓	1.581	No
1069	HFOPDE, chapter 5.4.3	Problem 1	✓	556.053	✓	3.36	No
1070	HFOPDE, chapter 5.4.3	Problem 2	✗ (Timed out)	600.	✓	0.236	No
1071	HFOPDE, chapter 5.4.3	Problem 3	✗ (Timed out)	600.	✓	2.543	No
1072	HFOPDE, chapter 5.4.3	Problem 4	✗ (Timed out)	600.	✓	2.692	No
1073	HFOPDE, chapter 5.4.3	Problem 5	✗ (Timed out)	600.	✓	1.555	No
1074	HFOPDE, chapter 5.4.4	Problem 1	✓	532.391	✓	3.234	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1075	HFOPDE, chapter 5.4.4	Problem 2	✗ (Timed out)	600.	✓	0.129	No
1076	HFOPDE, chapter 5.4.4	Problem 3	✗ (Timed out)	600.	✓	1.551	No
1077	HFOPDE, chapter 5.4.4	Problem 4	✗ (Timed out)	600.	✓	2.108	No
1078	HFOPDE, chapter 5.4.4	Problem 5	✗ (Timed out)	600.	✓	1.29	No
1079	HFOPDE, chapter 5.4.5	Problem 1	✓	2.84	✓	6.946	No
1080	HFOPDE, chapter 5.4.5	Problem 2	✓	302.713	✓	7.021	No
1081	HFOPDE, chapter 5.4.5	Problem 3	✓	1.384	✓	0.536	No
1082	HFOPDE, chapter 5.4.5	Problem 4	✓	300.138	✓	0.505	No
1083	HFOPDE, chapter 5.4.5	Problem 5	✗ (Timed out)	600.	✓	7.285	No
1084	HFOPDE, chapter 5.4.5	Problem 6	✗ (Timed out)	600.	✓	2.78	No
1085	HFOPDE, chapter 5.5.1	Problem 1	✗ (Timed out)	600.	✓	0.512	No
1086	HFOPDE, chapter 5.5.1	Problem 2	✗ (Timed out)	600.	✓	0.067	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1087	HFOPDE, chapter 5.5.1	Problem 3	✗ (Timed out)	600.	✓	1.251	No
1088	HFOPDE, chapter 5.5.1	Problem 4	✗	0.037	✓	1.086	No
1089	HFOPDE, chapter 5.5.1	Problem 5	✗ (Timed out)	600.	✓	204.409	No
1090	HFOPDE, chapter 5.5.1	Problem 6	✗ (Timed out)	600.	✓	63.249	No
1091	HFOPDE, chapter 5.5.2	Problem 1	✓	0.728	✓	0.493	No
1092	HFOPDE, chapter 5.5.2	Problem 2	✓	0.587	✓	0.119	No
1093	HFOPDE, chapter 5.5.2	Problem 3	✓	0.24	✓	0.307	No
1094	HFOPDE, chapter 5.5.2	Problem 4	✓	0.457	✓	0.519	No
1095	HFOPDE, chapter 5.5.2	Problem 5	✓	0.196	✓	0.218	No
1096	HFOPDE, chapter 5.5.2	Problem 6	✓	1.4	✓	1.203	No
1097	HFOPDE, chapter 5.5.2	Problem 7	✓	1.208	✓	3.628	No
1098	HFOPDE, chapter 5.6.1	Problem 1	✓	0.382	✓	0.088	No
1099	HFOPDE, chapter 5.6.1	Problem 2	✓	1.036	✓	14.427	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1100	HFOPDE, chapter 5.6.1	Problem 3	✓	3.831	✓	17.877	No
1101	HFOPDE, chapter 5.6.1	Problem 4	✓	2.044	✓	0.479	No
1102	HFOPDE, chapter 5.6.1	Problem 5	✓	71.741	✓	0.637	No
1103	HFOPDE, chapter 5.6.1	Problem 6	✗ (Timed out)	600.	✓	25.934	No
1104	HFOPDE, chapter 5.6.1	Problem 7	✗ (Timed out)	600.	✓	5.616	No
1105	HFOPDE, chapter 5.6.2	Problem 1	✓	0.241	✓	0.094	No
1106	HFOPDE, chapter 5.6.2	Problem 2	✓	2.181	✓	7.437	No
1107	HFOPDE, chapter 5.6.2	Problem 3	✓	342.327	✓	7.448	No
1108	HFOPDE, chapter 5.6.2	Problem 4	✓	97.152	✓	0.3	No
1109	HFOPDE, chapter 5.6.2	Problem 5	✓	73.216	✓	0.58	No
1110	HFOPDE, chapter 5.6.2	Problem 6	✗ (Timed out)	600.	✓	9.912	No
1111	HFOPDE, chapter 5.6.3	Problem 1	✓	1.329	✓	0.244	No
1112	HFOPDE, chapter 5.6.3	Problem 2	✗ (Timed out)	600.	✓	0.673	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1113	HFOPDE, chapter 5.6.3	Problem 3	✓	467.661	✓	1.023	No
1114	HFOPDE, chapter 5.6.3	Problem 4	✓	164.461	✓	3.459	No
1115	HFOPDE, chapter 5.6.3	Problem 5	✓	193.425	✓	0.326	No
1116	HFOPDE, chapter 5.6.3	Problem 6	✗ (Timed out)	600.	✓	4.862	No
1117	HFOPDE, chapter 5.6.3	Problem 7	✗ (Timed out)	600.	✓	12.317	No
1118	HFOPDE, chapter 5.6.4	Problem 1	✓	1.96	✓	0.276	No
1119	HFOPDE, chapter 5.6.4	Problem 2	✗ (Timed out)	600.	✓	0.843	No
1120	HFOPDE, chapter 5.6.4	Problem 3	✓	348.768	✓	1.036	No
1121	HFOPDE, chapter 5.6.4	Problem 4	✓	283.629	✓	4.418	No
1122	HFOPDE, chapter 5.6.4	Problem 5	✓	196.426	✓	0.204	No
1123	HFOPDE, chapter 5.6.4	Problem 6	✗ (Timed out)	600.	✓	5.19	No
1124	HFOPDE, chapter 5.6.4	Problem 7	✗ (Timed out)	600.	✓	8.783	No
1125	HFOPDE, chapter 5.6.5	Problem 1	✓	2.124	✓	9.03	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1126	HFOPDE, chapter 5.6.5	Problem 2	✓	364.206	✓	8.324	No
1127	HFOPDE, chapter 5.6.5	Problem 3	✓	18.895	✓	0.457	No
1128	HFOPDE, chapter 5.6.5	Problem 4	✓	198.191	✓	0.091	No
1129	HFOPDE, chapter 5.6.5	Problem 5	✗ (Timed out)	600.	✓	2.755	No
1130	HFOPDE, chapter 5.6.5	Problem 6	✗ (Timed out)	600.	✓	9.767	No
1131	HFOPDE, chapter 5.6.5	Problem 7	✗ (Timed out)	600.	✓	0.987	No
1132	HFOPDE, chapter 5.7.1	Problem 1	✗ (Timed out)	600.	✓	0.189	No
1133	HFOPDE, chapter 5.7.1	Problem 2	✗ (Timed out)	600.	✓	0.033	No
1134	HFOPDE, chapter 5.7.1	Problem 3	✗ (Timed out)	600.	✓	0.557	No
1135	HFOPDE, chapter 5.7.1	Problem 4	✗ (Timed out)	600.	✓	2.384	No
1136	HFOPDE, chapter 5.7.1	Problem 5	✗ (Timed out)	600.	✓	1.951	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1137	HFOPDE, chapter 5.7.2	Problem 1	✗ (Timed out)	600.	✓	0.048	No
1138	HFOPDE, chapter 5.7.2	Problem 2	✗ (Timed out)	600.	✓	0.034	No
1139	HFOPDE, chapter 5.7.2	Problem 3	✗ (Timed out)	600.	✓	0.091	No
1140	HFOPDE, chapter 5.7.2	Problem 4	✗ (Timed out)	600.	✓	0.921	No
1141	HFOPDE, chapter 5.7.2	Problem 5	✗ (Timed out)	600.	✓	1.288	No
1142	HFOPDE, chapter 5.7.3	Problem 1	✗ (Timed out)	600.	✓	0.486	No
1143	HFOPDE, chapter 5.7.3	Problem 2	✓	529.032	✓	91.485	No
1144	HFOPDE, chapter 5.7.3	Problem 3	✗ (Timed out)	600.	✓	3.04	No
1145	HFOPDE, chapter 5.7.3	Problem 4	✗ (Timed out)	600.	✓	1.579	No
1146	HFOPDE, chapter 5.7.3	Problem 5	✗ (Timed out)	600.	✓	0.927	No
1147	HFOPDE, chapter 5.7.4	Problem 1	✗ (Timed out)	600.	✓	0.407	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1148	HFOPDE, chapter 5.7.4	Problem 2	✗ (Timed out)	600.	✓	61.295	No
1149	HFOPDE, chapter 5.7.4	Problem 3	✗ (Timed out)	600.	✓	2.405	No
1150	HFOPDE, chapter 5.7.4	Problem 4	✗ (Timed out)	600.	✓	1.546	No
1151	HFOPDE, chapter 5.7.4	Problem 5	✗ (Timed out)	600.	✓	0.822	No
1152	HFOPDE, chapter 5.8.1	Problem 1	✓	0.387	✓	0.023	No
1153	HFOPDE, chapter 5.8.1	Problem 2	✓	0.298	✓	0.166	No
1154	HFOPDE, chapter 5.8.1	Problem 3	✓	2.007	✓	0.167	No
1155	HFOPDE, chapter 5.8.1	Problem 4	✓	0.757	✓	0.052	No
1156	HFOPDE, chapter 5.8.1	Problem 5	✓	0.545	✓	0.074	No
1157	HFOPDE, chapter 5.8.1	Problem 6	✓	0.82	✓	0.023	No
1158	HFOPDE, chapter 5.8.1	Problem 7	✓	1.86	✓	0.03	No
1159	HFOPDE, chapter 5.8.1	Problem 8	✓	19.89	✓	0.5	No
1160	HFOPDE, chapter 5.8.1	Problem 9	✓	28.832	✓	0.665	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1161	HFOPDE, chapter 5.8.1	Problem 10	✗	33.908	✓	0.881	No
1162	HFOPDE, chapter 5.8.1	Problem 11	✓	16.916	✓	1.5	No
1163	HFOPDE, chapter 5.8.1	Problem 12	✓	26.893	✓	0.336	No
1164	HFOPDE, chapter 5.8.2	Problem 1	✓	0.122	✓	0.046	No
1165	HFOPDE, chapter 5.8.2	Problem 2	✓	0.373	✓	0.09	No
1166	HFOPDE, chapter 5.8.2	Problem 3	✓	0.388	✓	0.019	No
1167	HFOPDE, chapter 5.8.2	Problem 4	✓	1.912	✓	0.078	No
1168	HFOPDE, chapter 5.8.2	Problem 5	✓	0.203	✓	0.028	No
1169	HFOPDE, chapter 5.8.2	Problem 6	✗	0.068	✓	0.18	No
1170	HFOPDE, chapter 5.8.3	Problem 1	✓	0.159	✓	0.06	No
1171	HFOPDE, chapter 5.8.3	Problem 2	✓	1.714	✓	0.049	No
1172	HFOPDE, chapter 5.8.3	Problem 3	✓	16.05	✓	0.15	No
1173	HFOPDE, chapter 5.8.3	Problem 4	✓	257.546	✓	0.728	No
1174	HFOPDE, chapter 5.8.3	Problem 5	✗ (Timed out)	600.	✓	0.778	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1175	HFOPDE, chapter 5.8.3	Problem 6	✗	24.064	✗	1.849	No
1176	HFOPDE, chapter 6.2.1	Problem 1	✓	0.005	✓	0.012	No
1177	HFOPDE, chapter 6.2.1	Problem 2	✓	0.019	✓	0.018	No
1178	HFOPDE, chapter 6.2.1	Problem 3	✓	0.022	✓	0.016	No
1179	HFOPDE, chapter 6.2.1	Problem 4	✓	0.072	✓	0.251	No
1180	HFOPDE, chapter 6.2.1	Problem 5	✓	0.006	✓	0.043	No
1181	HFOPDE, chapter 6.2.1	Problem 6	✓	0.019	✓	0.077	No
1182	HFOPDE, chapter 6.2.1	Problem 7	✓	0.045	✓	0.137	No
1183	HFOPDE, chapter 6.2.1	Problem 8	✗	0.027	✓	0.024	No
1184	HFOPDE, chapter 6.2.1	Problem 9	✗	0.021	✓	2.574	No
1185	HFOPDE, chapter 6.2.1	Problem 10	✓	0.072	✓	0.101	No
1186	HFOPDE, chapter 6.2.1	Problem 11	✗	5.944	✗	14.265	No
1187	HFOPDE, chapter 6.2.1	Problem 12	✗	3.196	✗	0.15	No
1188	HFOPDE, chapter 6.2.1	Problem 13	✓	0.011	✓	0.014	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1189	HFOPDE, chapter 6.2.1	Problem 14	✗ (Timed out)	600.	✗	1.685	No
1190	HFOPDE, chapter 6.2.1	Problem 15	✗ (Timed out)	600.	✓	25.037	No
1191	HFOPDE, chapter 6.2.1	Problem 16	✗ (Timed out)	600.	✓	13.992	No
1192	HFOPDE, chapter 6.2.1	Problem 17	✗ (Timed out)	600.	✗	3.72	No
1193	HFOPDE, chapter 6.2.1	Problem 18	✗ (Timed out)	600.	✗	11.068	No
1194	HFOPDE, chapter 6.2.1	Problem 19	✗	1.44	✗	0.515	No
1195	HFOPDE, chapter 6.2.1	Problem 20	✗	231.524	✗	0.591	No
1196	HFOPDE, chapter 6.2.1	Problem 21	✗	0.195	✗	0.689	No
1197	HFOPDE, chapter 6.2.2	Problem 1	✓	0.232	✓	0.309	No
1198	HFOPDE, chapter 6.2.2	Problem 2	✓	0.115	✓	0.173	No
1199	HFOPDE, chapter 6.2.2	Problem 3	✗ (Timed out)	600.	✓	1.644	No
1200	HFOPDE, chapter 6.2.2	Problem 4	✓	0.055	✓	0.277	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1201	HFOPDE, chapter 6.2.2	Problem 5	✓	0.008	✓	0.427	No
1202	HFOPDE, chapter 6.2.2	Problem 6	✓	0.012	✓	0.048	No
1203	HFOPDE, chapter 6.2.2	Problem 7	✓	0.019	✓	0.051	No
1204	HFOPDE, chapter 6.2.2	Problem 8	✓	0.012	✓	0.018	No
1205	HFOPDE, chapter 6.2.2	Problem 9	✓	0.014	✓	0.134	No
1206	HFOPDE, chapter 6.2.2	Problem 10	✓	0.011	✓	0.018	No
1207	HFOPDE, chapter 6.2.2	Problem 11	✓	0.015	✓	0.177	No
1208	HFOPDE, chapter 6.2.2	Problem 12	✓	0.011	✓	0.271	No
1209	HFOPDE, chapter 6.2.2	Problem 13	✓	0.011	✓	0.047	No
1210	HFOPDE, chapter 6.2.2	Problem 14	✓	0.023	✓	0.044	No
1211	HFOPDE, chapter 6.2.2	Problem 15	✓	0.014	✓	0.019	No
1212	HFOPDE, chapter 6.2.2	Problem 16	✓	0.013	✓	0.127	No
1213	HFOPDE, chapter 6.2.2	Problem 17	✓	0.02	✓	0.109	No
1214	HFOPDE, chapter 6.2.2	Problem 18	✓	0.019	✓	0.039	No
1215	HFOPDE, chapter 6.2.2	Problem 19	✓	0.028	✓	0.247	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1216	HFOPDE, chapter 6.2.2	Problem 20	✓	0.019	✓	0.04	No
1217	HFOPDE, chapter 6.2.2	Problem 21	✓	0.016	✓	0.09	No
1218	HFOPDE, chapter 6.2.2	Problem 22	✓	0.029	✗	0.285	No
1219	HFOPDE, chapter 6.2.2	Problem 23	✗	19.43	✓	4.861	No
1220	HFOPDE, chapter 6.2.2	Problem 24	✓	0.056	✓	1.734	No
1221	HFOPDE, chapter 6.2.2	Problem 25	✓	0.033	✓	1.633	No
1222	HFOPDE, chapter 6.2.2	Problem 26	✗	54.235	✗	0.308	No
1223	HFOPDE, chapter 6.2.2	Problem 27	✗	0.113	✓	2.544	No
1224	HFOPDE, chapter 6.2.2	Problem 28	✓	1.258	✓	0.409	No
1225	HFOPDE, chapter 6.2.2	Problem 29	✗	0.841	✗	1.372	No
1226	HFOPDE, chapter 6.2.3	Problem 1	✗	50.123	✗	0.338	No
1227	HFOPDE, chapter 6.2.3	Problem 2	✓	0.034	✓	0.117	No
1228	HFOPDE, chapter 6.2.3	Problem 3	✓	0.024	✓	0.116	No
1229	HFOPDE, chapter 6.2.3	Problem 4	✗	0.025	✓	1.295	No
1230	HFOPDE, chapter 6.2.3	Problem 5	✗	25.608	✗	0.311	No

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#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1231	HFOPDE, chapter 6.2.3	Problem 6	✗	0.171	✓	3.454	No
1232	HFOPDE, chapter 6.2.3	Problem 7	✗ (Timed out)	600.	✓	3.548	No
1233	HFOPDE, chapter 6.2.3	Problem 8	✓	0.16	✗	0.413	No
1234	HFOPDE, chapter 6.2.3	Problem 9	✗	0.029	✓	1.278	No
1235	HFOPDE, chapter 6.2.3	Problem 10	✓	0.016	✓	0.018	No
1236	HFOPDE, chapter 6.2.3	Problem 11	✓	0.201	✓	0.616	No
1237	HFOPDE, chapter 6.2.3	Problem 12	✗	0.047	✗	2.533	No
1238	HFOPDE, chapter 6.2.3	Problem 13	✗	1.234	✗ (Timed out)	600.	No
1239	HNPDE, chapter 1.1.1	Problem 1	✗	0.276	✗	5.378	No
1240	HNPDE, chapter 1.1.1	Problem 2	✓	0.075	✓	0.702	No
1241	HNPDE, chapter 1.1.2	Problem 1	✗	0.071	✗	6.234	No
1242	HNPDE, chapter 1.1.2	Problem 2	✗	0.082	✓	0.684	No
1243	HNPDE, chapter 1.1.2	Problem 3	✗	0.162	✓	0.655	No
1244	HNPDE, chapter 1.1.2	Problem 4	✗	0.139	✓	0.899	No

Continued on next page

Table 5 – continued from previous page

#	PDE	description	Mathematica		Maple		hand solved?
			result	time	result	time	
1245	HNPDE, chapter 1.1.2	Problem 5	✗	0.797	✓	1.515	No
1246	HNPDE, chapter 1.1.3	Problem 1	✗	0.005	✗	0.093	No
1247	HNPDE, chapter 1.1.3	Problem 2	✗	0.005	✗	0.011	No
1248	HNPDE, chapter 1.1.3	Problem 3	✗	0.009	✗	0.217	No
1249	HNPDE, chapter 1.1.3	Problem 4	✗	0.009	✗	0.015	No
1250	HNPDE, chapter 1.1.4	Problem 1	✗	0.007	✗	0.011	No

4 General first order PDE's

4.1 Linear PDE, the transport equation

problem number 1

Taken from Mathematica Symbolic PDE document

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to c_1(t-x)\}\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';t:='t';
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;

cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \_F1(-x+t)$$

Hand solution

$$u_t + u_x = 0 \tag{1}$$

Let $u \equiv u(x(t), t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1) to (2) then we see that

$$\frac{du}{dt} = 0 \tag{3}$$
$$\frac{dx}{dt} = 1 \tag{4}$$

(3) says that u is constant. Since no initial conditions are given, let $u = F(x(0))$ where F is arbitrary function. To find $x(0)$ we solve (4). The solution to (4) is $x = x(0) + t$. Hence $x(0) = x - t$. Therefore

$$u(x,t) = F(x - t)$$

4.2 Linear PDE

problem number 2

Taken from Mathematica help pages

Solve for $u(x,y)$

$$3u_x + 5u_y = x$$

Mathematica ✓

```
ClearAll[u, x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[3*D[u[x, y], x] + 5*D[u[x, y], y] == x, u[x, y]
```

$$\left\{\left\{u(x,y) \to \frac{1}{6}\left(6c_1\left(\frac{1}{3}(3y - 5x)\right) + x^2\right)\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde:=3*diff(u(x, y), x) + 5*diff(u(x, y), y) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = 1/6\,x^2 + \_F1(-5/3\,x + y)$$

Hand solution

Solve

$$3u_x + 5u_y = x$$
$$u_x + \frac{5}{3}u_y = \frac{x}{3} \tag{1}$$

<u>Solution</u>

Let $u = u(y(x), x)$. Then

$$\frac{du}{dx} = \frac{\partial u}{\partial y}\frac{dy}{dx} + \frac{\partial u}{\partial x} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dx} = \frac{x}{3} \tag{3}$$
$$\frac{dy}{dx} = \frac{5}{3} \tag{4}$$

Solving (3) gives

$$u = \frac{x^2}{6} + C_1$$
$$C_1 = u - \frac{x^2}{6}$$

From (4)

$$y = \frac{5}{3}x + C_2$$
$$C_2 = y - \frac{5}{3}x$$

Let $C_1 = F(C_2)$ where F is arbutrary function. This gives

$$u - \frac{x^2}{6} = F\left(y - \frac{5}{3}x\right)$$
$$u(x,y) = F\left(y - \frac{5}{3}x\right) + \frac{x^2}{6}$$

4.3 Linear PDE, initial value problem

problem number 3

Taken from Mathematica help pages

Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -4xyu(x,y)$$

with initial value $u(x,0) = e^{-x^2}$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = x*D[u[x, y], y] + y*D[u[x, y], x] == -4*x*y*u[x, y];
 ic = u[x, 0] == Exp[-x^2];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to e^{-x^2-y^2}\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), y) + y*diff(u(x, y), x) = -4*x*y*u(x, y);
ic := u(x, 0) = exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, y))),outpu
```

$$u(x,y) = \mathrm{e}^{-x^2-y^2}$$

Hand solution

Solve

$$xu_y + yu_x = -4xyu$$

with $u(x,0) = e^{-x^2}$.

Solution

Let $u \equiv u(x(y), y)$. We've taken y as the independent variable for $x(y)$ here, since the initial conditions has $y(0)$ in it. The PDE can be written as

$$u_y + \frac{y}{x}u_x = -4yu \tag{1}$$

Then

$$\frac{du}{dy} = \frac{\partial u}{\partial x}\frac{dx}{dy} + \frac{\partial u}{\partial y} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dy} = -4yu \tag{3}$$

$$\frac{dx}{dy} = \frac{y}{x} \tag{4}$$

Solving (3) gives

$$\begin{aligned}\ln|u| &= -\frac{4y^2}{2} + C_1 \\ u &= C_1 e^{-2y^2}\end{aligned} \tag{5}$$

At $y = 0$, using initial conditions the above becomes

$$e^{-x(0)^2} = C_1$$

(5) becomes

$$\begin{aligned}u &= e^{-x(0)^2} e^{-2y^2} \\ &= e^{-x(0)^2 - 2y^2}\end{aligned} \tag{5A}$$

All what is left is to find $x(0)$ to finish the solution. From (4)

$$\frac{x^2}{2} = \frac{y^2}{2} + C_2 \tag{6}$$

At $y = 0$

$$\frac{x(0)^2}{2} = C_2$$

Hence (6) becomes

$$\begin{aligned}\frac{x^2}{2} &= \frac{y^2}{2} + \frac{x(0)^2}{2} \\ x(0)^2 &= x^2 - y^2\end{aligned}$$

Substituting the above in (5A) gives

$$\begin{aligned}u(x(y), x) &= e^{-(x^2 - y^2) - 2y^2} \\ &= e^{-x^2 - y^2}\end{aligned}$$

The following is a plot of the above solution showing the initial conditions are red line

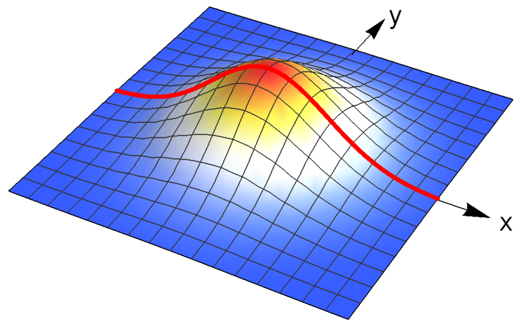


Figure 1: Solution $e^{-x^2-y^2}$

```
u[x_, y_] := Exp[-x^2 - y^2];
initialCurve = ParametricPlot3D[{x, 0, Exp[-x^2]}, {x, -2, 2},
      PlotStyle -> Red];
solution = Plot3D[u[x, y], {x, -2, 2}, {y, -2, 2},
      ColorFunction -> "TemperatureMap"];
Graphics3D[{
    First@solution,
    First@initialCurve,
    Arrow[{{0, 0, 0}, {2.6, 0, 0}}],
    Arrow[{{0, 0, 0}, {0, 2.8, 0}}],
    Text["x", {2.7, 0, 0}, {-1, 0}],
    Text["y", {0, 2.9, 0}, {-1, 0}]
  }, SphericalRegion -> True,
  Boxed → False, BaseStyle → 12,
  ImageSize → 300, PlotRange → All]
```

Figure 2: Code used for the plot

4.4 Initial-boundary value problem

problem number 4

Taken from Mathematica help pages

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

with initial value $u(x,0) = \sin x$ and boundary value $u(0,t) = 0$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] + D[u[x, t], x] == 0;
 bc = u[0, t] == 0;
 ic = u[x, 0] == Sin[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to (\theta(t-x)-1)\sin(t-x)\}\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+diff(u(x,t),x)=0;
bc:=u(0,t)=0;
ic:=u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = -\sin(-x+t)\,Heaviside(-t+x)$$

Hand solution

Since initial and boundary conditions are given, the Laplace transform method will be used to solve this PDE. Let $U(x,s)$ be the Laplace transform of $u(x,t)$. Applying Laplace transform to the PDE gives

$$sU - u(x,0) + \frac{dU}{dx} = 0$$
$$\frac{dU}{dx} + sU = \sin x$$

Integrating factor is $\mu = e^{\int s dx} = e^{sx}$. Multiplying the above by μ gives

$$\frac{d}{dx}(Ue^{sx}) = e^{sx}\sin x$$

Integrating

$$\begin{aligned}
Ue^{sx} &= \int e^{sx}\sin x dx + C \\
&= \frac{e^{sx}(s\sin x - \cos x)}{1+s^2} + C \\
U(x,s) &= \frac{s\sin x - \cos x}{1+s^2} + Ce^{-sx}
\end{aligned}$$

Applying boundary conditions $U(0,s)=0$ gives

$$0=\frac{-1}{1+s^2}+C$$
$$C=\frac{1}{1+s^2}$$

Hence

$$\begin{aligned}U(x,s)&=\frac{s\sin x-\cos x}{1+s^2}+\frac{e^{-sx}}{1+s^2}\\&=\frac{s\sin x}{1+s^2}-\frac{\cos x}{1+s^2}+\frac{e^{-sx}}{1+s^2}\end{aligned}$$

Applying inverse Laplace transform gives

$$\begin{aligned}u(x,t)&=\cos t\sin x-\cos x\sin t+\text{Heaviside}\,(t-x)\sin(t-x)\\&=-\sin(t-x)+\text{Heaviside}\,(t-x)\sin(t-x)\\&=(\text{Heaviside}\,(t-x)-1)\sin(t-x)\end{aligned}$$

4.5 Linear PDE, the transport equation with initial conditions

problem number 5

Taken from Mathematica help pages

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=0$$

With initial conditions $u(x,0)=e^{-x^2}$

Mathematica ✓

```
ClearAll[u, x, t, c];
 ic = u[x, 0] == Exp[-x^2];
 pde = D[u[x, t], {t}] + c*D[u[x, t], {x}] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{-(x-ct)^2}\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';t:='t';c:='c';
pde := diff(u(x, t), t) + c* diff(u(x, t),x) =0;
ic:=u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \mathrm{e}^{-(tc-x)^2}$$

Hand solution

Solve

$$u_t + cu_x = 0 \tag{1}$$

with initial conditions $u(x,0) = e^{-x^2}$.

Solution

Let $u = u(x(t)\,,t)$. Then

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{du}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = c \tag{4}$$

Solving (3) gives

$$\begin{aligned} u &= u(x(0)) \\ &= e^{-x(0)^2} \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= ct + x(0) \\ x(0) &= x - ct \end{aligned}$$

Then (5) becomes

$$u(x(t)\,,t) = e^{-(x-ct)^2}$$

4.6 First order wave PDE, with initial conditions (Haberman 12.2.2)

problem number 6

Added Nov 25, 2018.

Problem 12.2.2 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\frac{\partial \omega}{\partial t} - 3\frac{\partial \omega}{\partial x} = 0$$

With initial conditions $\omega(x,0) = \cos x$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll[x, t, w];
 pde = D[w[x, t], t] - 3*D[w[x, t], x] == 0;
 ic = w[x, 0] == Cos[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x, t], {x, t}], 60*10]];
```

$$\{\{w(x,t) \to \cos(3t+x)\}\}$$

Maple ✓

```
x:='x'; t:='t'; w:='w';
pde:=diff(w(x,t),t)-3*diff(w(x,t),x)=0;
ic:=w(x,0)=cos(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t) = \cos(x+3t)$$

Hand solution

Solve

$$w_t - 3w_x = 0 \tag{1}$$

With I.C. $w(x,0) = \cos x$

Solution

Let $w = w(x(t), t)$. Then

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial t} \tag{2}$$

Comparing (1),(2) shows that

$$\frac{dw}{dt} = 0 \tag{3}$$

$$\frac{dx}{dt} = -3 \tag{4}$$

Solving (3) gives

$$\begin{aligned} w &= w(x(0)) \\ &= \cos(x(0)) \end{aligned} \tag{5}$$

We need to find $x(0)$. From (4)

$$\begin{aligned} x &= -3t + x(0) \\ x(0) &= x + 3t \end{aligned}$$

Hence (5) becomes

$$w(x(t), t) = \cos(x + 3t)$$

4.7 First order wave PDE, with initial and boundary conditions (Haberman 12.2.4)

problem number 7

Added Nov 25, 2018.

Problem 12.2.4 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\omega_t + c\omega_x = 0$$

With $c > 0$. For $x > 0, t > 0$ if $\omega(x,0) = f(x)$ and $\omega(0,t) = h(t)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✗

```
ClearAll[x, t, w, f, h];
 pde = D[w[x, t], t] + c*D[w[x, t], x] == 0;
 ic = w[x, 0] == f[x];
 bc = w[0, t] == h[t];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, w[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
x:='x'; t:='t'; w:='w';f:='f';h:='h';c:='c';
pde:=diff(w(x,t),t)+c*diff(w(x,t),x)=0;
ic:=w(x,0)=f(x);
bc:=w(0,t)=h(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],w(x,t)) assumi
```

$$w(x,t)=\frac{1}{c}\left(\mathit{Heaviside}\left(\frac{tc-x}{c}\right)h\left(\frac{tc-x}{c}\right)c-\mathit{invlaplace}\left(\mathrm{e}^{-\frac{sx}{c}}\int^{0}f(\_a)\,\mathrm{e}^{\frac{s\_a}{c}}d\_a,s,t\right)+\mathit{invlaplace}\Bigg(\right.$$

Solution contains unresolved invlaplace calls

Hand solution

$$\frac{\partial w}{\partial t}+c\frac{\partial w}{\partial x}=0 \tag{1}$$

Let

$$w\equiv w(x(t)\,,t)$$

Hence

$$\frac{dw}{dt}=\frac{\partial w}{\partial t}+\frac{\partial w}{\partial x}\frac{dx}{dt} \tag{2}$$

Comparing given (1) and (2), we see that if we let $\frac{dx}{dt}=c$ in (2), then we obtain (1). Hence we conclude that $\frac{dw}{dt}=0$. Therefore, $w(x(t)\,,t)$ is constant. At $t=0$, we are given that

$$w(x(t)\,,t)=f(x(0))\qquad t=0 \tag{3}$$

We just now need to determine $x(0)$. This is found from $\frac{dx}{dt}=c$, which has the solution

$x(t) = x(0) + ct$. Hence $x(0) = x(t) - ct$. Therefore (3) becomes

$$w(x,t) = f(x - ct)$$

This is valid for $x > ct$. We now start all over again, and look at Let

$$w \equiv w(x, t(x))$$

Hence

$$\frac{dw}{dx} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\frac{dt}{dx} \tag{4}$$

Comparing (4) and (1), we see that if we let $\frac{dt}{dx} = \frac{1}{c}$ in (4), then we obtain (1). Hence we conclude that $\frac{dw}{dx} = 0$. Therefore, $w(x, t(x))$ is constant. At $x = 0$, we are given that

$$w(x, t(x)) = h(t(0)) \qquad x = 0 \tag{5}$$

We just now need to determine $t(0)$. This is found from $\frac{dt}{dx} = \frac{1}{c}$, which has the solution $t(x) = t(0) + \frac{1}{c}x$. Hence $t(0) = t(x) - \frac{1}{c}x$. Therefore (5) becomes

$$w(x,t) = h\left(t - \frac{1}{c}x\right)$$

Valid for $t > \frac{x}{c}$ or $x < ct$. Therefore, the solution is

$$w(x,t) = \begin{cases} f(x - ct) & x > ct \\ h\left(t - \frac{1}{c}x\right) & x < ct \end{cases}$$

4.8 First order wave PDE, with initial conditions, non homogeneous (Haberman 12.2.5 (a))

problem number 8

Added Nov 25, 2018.

Problem 12.2.5 (a) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\frac{\partial \omega}{\partial t} + c\frac{\partial \omega}{\partial x} = e^{2x}$$

With $\omega(x, 0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll[x, t, w, f, h];
 pde = D[w[x, t], t] + c*D[w[x, t], x] == Exp[2*x];
 ic = w[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}, Assumption
```

$$\left\{\left\{w(x,t)\to f(x-ct)+\frac{e^{2x}(1-e^{-2ct})}{2c}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; w:='w';c:='c';
pde:=diff(w(x,t),t)+c*diff(w(x,t),x)=exp(2*x);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t)) assuming
```

$$w(x,t)=1/2\,\frac{2\,f(-tc+x)\,c-\mathrm{e}^{-2\,tc+2\,x}+\mathrm{e}^{2\,x}}{c}$$

Hand solution

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds}=1 \tag{1}$$
$$\frac{dx}{ds}=c \tag{2}$$
$$\frac{du}{ds}=e^{2x} \tag{3}$$

With initial conditions at $s=0$

$$t(0)=t_1,x(0)=t_2,u(0)=t_3$$

And $u(x,0)=f(x)$ becomes

$$t_3=f(t_2)\,,t_1=0 \tag{4}$$

Equation (1) gives

$$\begin{aligned}t&=s+t_1\\&=s\end{aligned} \tag{5}$$

Equation (2) gives

$$x = cs + t_2 \tag{6}$$

From (5,6) solving for t_2 gives

$$\begin{aligned} t_2 &= x - cs \\ &= x - ct \end{aligned} \tag{7}$$

Equation (3) gives

$$\begin{aligned} du &= e^{2x} ds \\ &= e^{2(cs+t_2)} ds \end{aligned}$$

Integrating

$$u = \frac{e^{2(cs+t_2)}}{2c} + t_3$$

Using (7,4,5) in the above gives the solution

$$\begin{aligned} u(x,t) &= \frac{e^{2(ct+(x-ct))}}{2c} + f(x-ct) \\ &= \frac{1}{2c} e^{2x} + f(x-ct) \end{aligned}$$

My solution is not the same as CAS, but it was verified OK using Maple pdetest.

4.9 First order wave PDE, with initial conditions, non homogeneous (Haberman 12.2.5 (d))

problem number 9

Added Nov 25, 2018.

Problem 12.2.5 (d) from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(x,t)$

$$\frac{\partial \omega}{\partial t} + 3t \frac{\partial \omega}{\partial x} = \omega(x,t)$$

with $\omega(x,0) = f(x)$.

See my HW 12, Math 322, UW Madison.

Mathematica ✓

```
ClearAll[x, t, w, f];
 pde = D[w[x, t], t] + 3*t*D[w[x, t], x] == w[x, t];
 ic = w[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, w[x, t], {x, t}]], 60*10]];
```

$$\left\{\left\{w(x,t)\to e^{-\sqrt{t^2}}f\left(x-\frac{3t^2}{2}\right)\right\},\left\{w(x,t)\to e^{\sqrt{t^2}}f\left(x-\frac{3t^2}{2}\right)\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; w:='w';c:='c';
pde:=diff(w(x,t),t)+3*t*diff(w(x,t),x)=w(x,t);
ic:=w(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],w(x,t))),output='
```

$$w(x,t)=f\left(-3/2\,t^2+x\right)\mathrm{e}^{t}$$

Hand solution

Solve

$$\frac{\partial w}{\partial t}+3t\frac{\partial w}{\partial x}=w(x,t) \tag{1}$$

With initial conditions $w(x,0)=f(x)$

Solution

Let $w\equiv w(x(t),t)$ then

$$\frac{dw}{dt}=\frac{\partial w}{\partial x}\frac{dx}{dt}+\frac{\partial w}{\partial t} \tag{2}$$

Comparing (1,2) shows that

$$\frac{dw}{dt}=w \tag{3}$$

$$\frac{dx}{dt}=3t \tag{4}$$

Solving (3) gives

$$w = Ce^t$$

From initial conditions at $t = 0$, the above becomes $f(x(0)) = C$. Hence the above becomes

$$w(x,t) = f(x(0))\, e^t \tag{5}$$

From (4)

$$\begin{aligned} x &= \frac{3}{2}t^2 + x(0) \\ x(0) &= x - \frac{3}{2}t^2 \end{aligned}$$

Substituting the above in (5) gives

$$w(x(t)\,,t) = f\left(x - \frac{3}{2}t^2\right) e^t$$

<u>Alternative solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dt}{ds} = 1 \tag{1}$$
$$\frac{dx}{ds} = 3t \tag{2}$$
$$\frac{dw}{ds} = w \tag{3}$$

With initial conditions at $s = 0$

$$t(0) = t_1, x(0) = t_2, w(0) = t_3$$

And $w(x,0) = f(x)$ becomes

$$t_3 = f(t_2)\,, t_1 = 0 \tag{4}$$

Equation (1) gives

$$\begin{aligned} t &= s + t_1 \\ &= s \end{aligned} \tag{5}$$

Equation (2) gives, after replacing t by s from (5)

$$\begin{aligned}\frac{dx}{ds} &= 3s\\ x &= \frac{3}{2}s^2 + t_2 \qquad (6)\end{aligned}$$

Solving for t_2 gives

$$t_2 = x - \frac{3}{2}s^2 \qquad (7)$$

Equation (3) gives

$$\begin{aligned}\ln w &= s + t_3\\ w &= t_3 e^s\\ &= f(t_2)\, e^s\end{aligned}$$

Using (7,5) in the above gives the solution

$$w(x,t) = f\left(x - \frac{3}{2}t^2\right) e^t$$

4.10 General solution for a quasilinear first-order PDE

problem number 10

Taken from Mathematica help pages

Solve for $u(x,y)$

$$2\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = u^2(x,y) + 1$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = 2*D[u[x, y], x] + 5*D[u[x, y], y] == u[x, y]^2 + 1;
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \tan\left(c_1\left(y - \frac{5x}{2}\right) + \frac{x}{2}\right)\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := 2* diff(u(x, y), x) + 5*diff(u(x, y), y) = u(x, y)^2 + 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, y))),output='rea
```

$$u(x,y) = \tan\left(x/2 + 1/2\_F1(-5/2\,x + y)\right)$$

<u>Hand solution</u>

Solve for $u(x,y)$ in $2u_x + 5u_y = u^2 + 1$. Using the Lagrange-charpit method, the characteristic equations are

$$\frac{dx}{2} = \frac{dy}{5} = \frac{du}{u^2+1}$$

From the first pair of equation we obtain

$$\begin{aligned}5dx &= 2dy\\ 5x &= 2y + C_1\\ C_1 &= 5x - 2y\end{aligned}$$

Now we can pick the pair $\frac{dy}{5} = \frac{du}{u^2+1}$ or $\frac{dx}{2} = \frac{du}{u^2+1}$ to solve for u. It does not matter which. Using

$$\frac{dx}{2} = \frac{du}{u^2+1}$$

Integrating gives

$$\begin{aligned}\frac{1}{2}x &= \arctan(u) + C_2\\ C_2 &= \frac{1}{2}x - \arctan(u)\end{aligned}$$

C_1 and C_2 are always related by $C_2 = F(C_1)$ where F is arbitrary function. Hence

$$\begin{aligned}\frac{1}{2}x - \arctan(u) &= F(5x - 2y)\\ \arctan(u) &= \frac{1}{2}x - F(5x - 2y)\\ u(x,y) &= \tan\left(\frac{1}{2}x - F(5x - 2y)\right)\end{aligned}$$

4.11 quasilinear first-order PDE, scalar conservation law

problem number 11

Taken from Mathematica Symbolic PDE document

Solve for $u(x,y)$

$$\frac{\partial u}{\partial x}+u(x,y)\frac{\partial u}{\partial y}=0$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$\text{Solve}\left[u(x,y)=c_1\left(x-\frac{y}{u(x,y)}\right),u(x,y)\right]$$

Implicit solution

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
sol:=DEtools:-remove_RootOf(sol);
```

$$-y+xu(x,y)+\_F1(u(x,y))=0$$

Hand solution

Solve for $u(x,y)$ in $u_x+u\,u_y=0$. Using the Lagrange-Charpit method, the characteristic equations are

$$\frac{dx}{1}=\frac{dy}{u}=\frac{du}{0}$$

From the first pair of equation we obtain

$$u=\frac{dy}{dx}$$

But $du=0$ or $u=C_2$. Hence the above becomes

$$
\begin{aligned}
\frac{dy}{dx} &= C_2 \\
y &= xC_2 + C_1 \\
C_1 &= y - xC_2
\end{aligned}
$$

Since $C_2 = F(C_1)$ where F is arbitrary function, then

$$u(x,y) = F(y - ux)$$

4.12 quasilinear first-order PDE, scalar conservation law with initial value

problem number 12

Taken from Mathematica Symbolic PDE document

Solve for $u(x,y)$

$$u_x + u(x,y)u_y = 0$$

With $u(x,0) = \frac{1}{x+1}$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 ic = u[x, 0] == 1/(x + 1);
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic}, u[x, y], {x, y}]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \frac{y+1}{x+1}\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
ic:=u(x,0)=1/(x+1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y) = \frac{y+1}{x+1}$$

<u>Hand solution</u>

Using the method of characteristics, the systems of characteristic lines are (from the PDE itself)

$$\frac{dx}{ds} = 1 \tag{1}$$
$$\frac{dy}{ds} = u \tag{2}$$
$$\frac{du}{ds} = 0 \tag{3}$$

With initial conditions at $s = 0$

$$x(0) = t_1, y(0) = t_2, u(0) = t_3$$

We are given that $u(x,0) = \frac{1}{1+x}$. This initial condition translates to

$$t_3 = \frac{1}{1+t_1}, t_2 = 0 \tag{4}$$

Equation (1) gives

$$x = s + t_1 \tag{5}$$

Equation (2) gives

$$\begin{aligned} y &= su + t_2 \\ &= su \end{aligned} \tag{7}$$

Equation (3) gives

$$u = t_3$$

Hence the solution is

$$
\begin{aligned}
u &= t_3 \\
&= \frac{1}{1+t_1} \\
&= \frac{1}{1+(x-s)} \\
&= \frac{1}{1+\left(x-\frac{y}{u}\right)}
\end{aligned}
$$

Solving for u gives

$$
\begin{aligned}
u\left(1+\left(x-\frac{y}{u}\right)\right) &= 1 \\
u + xu - y &= 1 \\
u(1+x) &= 1+y \\
u &= \frac{1+y}{1+x}
\end{aligned}
$$

4.13 nonlinear first-order PDE, the Clairaut equation

problem number 13

Taken from Mathematica Symbolic PDE document

Solve for $u(x,y)$

$$
x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right) = 0
$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x,
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[pde, u[x, y], {x, y}]], 60*10]];
```

$$
\left\{\left\{u(x,y) \to c_1 x + c_2 y + \frac{1}{2}\left(c_1^2 + c_2^2\right)\right\}\right\}
$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = -1/2\,x^2 - 1/2\,x\sqrt{x^2+2\_c_1} - \_c_1 \ln\left(x+\sqrt{x^2+2\_c_1}\right) + \_C1 - 1/2\,y^2 - 1/2\,y\sqrt{y^2-2\_c_1} + \_$$

Hand solution

Assuming the solution is $u(x,y) = X(x) + Y(y)$. Substituting this into the PDE gives

$$\begin{aligned} xX' + yY' + \frac{1}{2}\left((X')^2 + (Y')^2\right) &= 0 \\ \frac{1}{2}(X')^2 + xX' &= -\frac{1}{2}(Y')^2 - yY' \end{aligned}$$

The above is possible when each side is equal to same constant, say C_1. This gives two ODE's

$$\frac{1}{2}(X')^2 + xX' = C_1 \tag{1}$$

$$\frac{1}{2}(Y')^2 + yY' = -C_1 \tag{2}$$

ODE (1) becomes

$$\begin{aligned} (X')^2 + 2xX' - 2C_1 &= 0 \\ X' &= \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2-4ac} \\ &= \frac{-2x}{2} \pm \frac{1}{2}\sqrt{4x^2+8C_1} \\ &= -x \pm \sqrt{x^2+2C_1} \end{aligned}$$

For the case $X' = -x + \sqrt{x^2+2C_1}$, the solution is

$$\begin{aligned} X(x) &= \int -x + \sqrt{x^2+2C_1}dx + C_2 \\ &= -\frac{x^2}{2} + \frac{x\sqrt{x^2+2C_1}}{2} + C_1 \ln\left(x+\sqrt{x^2+2C_1}\right) + C_2 \end{aligned}$$

For the case $X' = -x - \sqrt{x^2 + 2C_1}$, the solution is

$$
\begin{aligned}
X(x) &= \int -x - \sqrt{x^2 + 2C_1} dx + C_2 \\
&= -\frac{x^2}{2} - \frac{x\sqrt{x^2 + 2C_1}}{2} - C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2
\end{aligned}
$$

Combining the above two solutions to one gives

$$
X(x) = -\frac{x^2}{2} \pm \frac{x\sqrt{x^2 + 2C_1}}{2} \pm C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2 \tag{3}
$$

ODE (2) becomes

$$
\begin{aligned}
(Y')^2 + 2yY' + 2C_1 = 0 \\
Y' &= \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac} \\
&= \frac{-2y}{2} \pm \frac{1}{2}\sqrt{4y^2 - 8C_1} \\
&= -y \pm \sqrt{y^2 - 2C_1}
\end{aligned}
$$

For the case $Y' = -y + \sqrt{x^2 - 2C_1}$, the solution is

$$
\begin{aligned}
Y(y) &= \int -y + \sqrt{y^2 - 2C_1} dy + C_2 \\
&= \frac{-y^2}{2} + \frac{y\sqrt{y^2 - 2C_1}}{2} - C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3
\end{aligned}
$$

For the case $Y' = -y - \sqrt{y^2 - 2C_1}$, the solution is

$$
\begin{aligned}
Y(y) &= \int -y - \sqrt{y^2 + 2C_1} dy + C_2 \\
&= -\frac{y^2}{2} - \frac{y\sqrt{x^2 - 2C_1}}{2} + C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3
\end{aligned}
$$

Combining the above two solutions to one gives

$$
Y(x) = -\frac{y^2}{2} \pm \frac{y\sqrt{y^2 - 2C_1}}{2} \pm C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right) + C_3 \tag{4}
$$

From (3,4) the final solution is

$$
\begin{aligned}
u(x, y) &= X(x) + Y(x) \\
&= \left(-\frac{x^2}{2} \pm \frac{x\sqrt{x^2 + 2C_1}}{2} \pm C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) + C_2\right) + \left(-\frac{y^2}{2} \pm \frac{y\sqrt{y^2 - 2C_1}}{2} \pm C_1 \ln\left(y + \right.\right. \\
&= -\frac{x^2}{2} \pm \frac{x}{2}\sqrt{x^2 + 2C_1} \pm C_1 \ln\left(x + \sqrt{x^2 + 2C_1}\right) - \frac{y^2}{2} \pm \frac{y}{2}\sqrt{y^2 - 2C_1} \pm C_1 \ln\left(y + \sqrt{y^2 - 2C_1}\right)
\end{aligned}
$$

Where $C_4 = C_2 + C_3$.

4.14 nonlinear first-order PDE, the Clairaut equation with initial value

problem number 14

Taken from Mathematica Symbolic PDE document

Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)=0$$

With $u(x,0)=\frac{1}{2}(1-x^2)$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x,
 ic = u[x, 0] == (1*(1 - x^2))/2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y)\to\frac{1}{2}\left(-x^2-2y+1\right)\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y
ic:=u(x,0)=1/2*(1-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,y))),output='
```

$$u(x,y)=-1/2\,(x-y+1)\,(x-y-1)$$

4.15 Another example of nonlinear Clairaut equation

problem number 15

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u(x,y) = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \sin\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], x] + y*D[u[x, y], y] + Sin[D[u[x, y], x] + D[u[x, y], y]];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1x + c_2y + \sin(c_1 + c_2)\}\}$$

Maple ✓

```
u:='u';x:='x';y:='y';
pde:= u(x,y)= x*diff(u(x,y),x)+y*diff(u(x,y),y)+sin(diff(u(x,y),x)+diff(u(x,y),y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = x\_c_1 + y\_c_2 + \sin(\_c_1 + \_c_2)$$

4.16 Recover a function from its gradient vector

problem number 16

Taken from Mathematica DSolve help pages

Solve for $f(x,y)$

$$\frac{\partial f}{\partial x} = xy\cos(xy) + \sin(xy)$$
$$\frac{\partial f}{\partial y} = -e^{-y} + x^2\cos(xy)$$

Mathematica ✓

```
ClearAll[f, x, y];
 eq1 = D[f[x, y], x] == x*y*Cos[x*y] + Sin[x*y];
 eq2 = D[f[x, y], y] == -E^(-y) + x^2*Cos[x*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{eq1, eq2}, f[x, y], {x, y}], 60*10]];
```

$$\{\{f(x,y) \to c_1 + x\sin(xy) + e^{-y}\}\}$$

Maple ✓

```
u:='u';x:='x';y:='y';
eq1:=diff(f(x,y),x)=x*y*cos(x*y)+sin(x*y);
eq2:=diff(f(x,y),y)=-exp(-y)+x^2*cos(x*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({eq1,eq2},f(x,y))),output=
```

$$\{f(x,y) = x\sin(yx) + e^{-y} + \_C1\}$$

4.17 General solution of a first order nonlinear PDE

problem number 17

Taken from Maple pdsolve help pages

Solve for $f(x,y)$

$$x\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x} = \frac{f^2(x,y)g(x)}{h(y)}$$

Mathematica ✗

```
ClearAll[f, x, y, h, g];
 pde = x*D[f[x, y], y] - D[f[x, y], x] == (f[x, y]^2*g[x])/h[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
x:='x';y:='y';f:='f';g:='g';h:='h';
pde := x*diff(f(x,y),y)-diff(f(x,y),x)=f(x,y)^2*g(x)/h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y))),output='realt
```

$$f(x,y) = \left(\int^x \frac{g(\_a)}{h\left(-1/2\,\_a^2 + 1/2\,x^2 + y\right)} d\_a + \_F1\left(1/2\,x^2 + y\right)\right)^{-1}$$

Has unresolved integral in the answer

4.18 Nonlinear first order PDE

problem number 18

Taken from Maple pdsolve help pages, probem 5

Solve for $f(x,y,z)$

$$f_x + (f_y)^2 = f(x,y,z) + z$$

Mathematica ✓

```
ClearAll[f, x, y, z];
 pde = D[f[x, y, z], x] + D[f[x, y, z], y]^2 == f[x, y, z] + z;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, f[x, y, z], {x, y, z}], 60*10]];
```

$$\left\{\left\{f(x,y,z) \to \frac{1}{4}\left(c_1(z)^2\text{ProductLog}\left(-\frac{e^{\frac{y}{c_1(z)}+\frac{c_2(z)}{c_1(z)}+x-1}}{c_1(z)}\right)^2 + 2c_1(z)^2\text{ProductLog}\left(-\frac{e^{\frac{y}{c_1(z)}+\frac{c_2(z)}{c_1(z)}+x-1}}{c_1(z)}\right)\right.\right.\right.$$

Maple ✓

```
x:='x';y:='y';f:='f';z:='z';
pde := diff(f(x,y,z),x) + (diff(f(x,y,z),y))^2 = f(x,y,z)+z;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,f(x,y,z),'build')),out
```

$$f(x,y,z) = -\frac{e^{-x} z\_C5^2 + e^{x}\_C3^2 + \_C3\, y\_C5 + z\_C4\,\_C5 + \_C1\,\_C5}{\_C5^2 e^{-x}}$$

4.19 first order PDE of three unknowns

problem number 19

From example 3.5.4, page 212 nonlinear pde's by Lokenath Debnath, 3rd edition.

Solve for $u(x,y,z)$

$$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$$

Mathematica ✗

```
ClearAll[u, x, y, z];
 pde = (y - z)*D[u[x, y, z], x] + (z - x)*D[u[x, y, z], y] + (x - y)*D[u[x, y, z], z] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, z], {x, y, z}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
x:='x';y:='y';u:='u';z:='z';
pde:=(y-z)*diff(u(x,y,z),x)+(z-x)*diff(u(x,y,z),y)+(x-y)*diff(u(x,y,z),z)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,z),'build')),out
```

$$u(x,y,z) = e^{1/2\_C2\, x^2} e^{\_C1\, x} e^{1/2\_C2\, y^2} e^{\_C1\, y}\_C3\,\_C5\,\_C4\, e^{1/2\_C2\, z^2} e^{\_C1\, z}$$

5 Heat PDE in bar (1D)

5.1 Haberman 2.3.3 (a)

problem number 20

This is problem 2.3.3, part (a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 6\sin\left(\frac{9\pi x}{L}\right)$

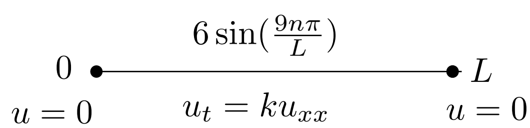


Figure 3: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L, n];
 NumericQ[L] =. ;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 6*Sin[(9*Pi*x)/L];
 NumericQ[L] = True;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t) \to 6e^{-\frac{81\pi^2 kt}{L^2}} \sin\left(\frac{9\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=6*sin(9*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=6\sin\left(9\frac{\pi x}{L}\right)e^{-81\frac{k\pi^2 t}{L^2}}$$

Hand solution

Solve $u_t = k u_{xx}$ with $0<x<L$ and initial conditions $u(x,0)=6\sin\left(\frac{9\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem **??** on page **??** as

$$u(x,t)=\sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. At $t=0$

$$6\sin\left(\frac{9\pi x}{L}\right)=\sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi}{L}x\right)$$

For $n=9$

$$6\sin\left(\frac{9\pi x}{L}\right)=B_9\sin\left(\frac{9\pi}{L}x\right)$$

Hence $B_9=6$ and all other terms are zero. Therefore the solution is

$$\begin{aligned}u(x,t)&=B_9\sin\left(\sqrt{\lambda_9}x\right)e^{-k\lambda_9 t}\\&=6\sin\left(\frac{9\pi}{L}x\right)e^{-k\left(\frac{9\pi}{L}\right)^2 t}\\&=6\sin\left(\frac{9\pi}{L}x\right)e^{-k\frac{81\pi^2}{L^2}t}\end{aligned}$$

5.2 Haberman 2.3.3 (b)

problem number 21

This is problem 2.3.3, part (b) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

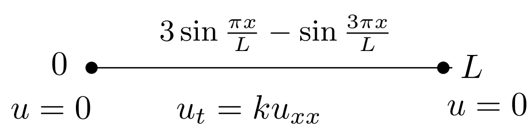


Figure 4: PDE specification

Mathematica ✓

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 3*Sin[(Pi*x)/L] - Sin[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t) \rightarrow e^{-\frac{9\pi^2 kt}{L^2}} \sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2 kt}{L^2}} - 2\cos\left(\frac{2\pi x}{L}\right) - 1\right)\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 3\sin\left(\frac{\pi x}{L}\right)\mathrm{e}^{-\frac{k\pi^2 t}{L^2}} - \sin\left(3\frac{\pi x}{L}\right)\mathrm{e}^{-9\frac{k\pi^2 t}{L^2}}$$

Hand solution

Solve $u_t = ku_{xx}$ with $0 < x < L$ and initial conditions $u(x,0) = 3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right)$. The basic solution for this type of PDE was already given in problem **??** on page **??** as

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where the eigenvalues are $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions.

Initial conditions are now applied. Setting $t = 0$, the above becomes

$$u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

As the series is unique, the terms coefficients must match for those shown only, and all other B_n terms vanish. This means that by comparing terms

$$3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right) = B_1\sin\left(\frac{\pi x}{L}\right) + B_3\sin\left(\frac{3\pi}{L}x\right)$$

Therefore

$$B_1 = 3$$
$$B_3 = -1$$

And all other $B_n = 0$. The solution is

$$u(x,t) = 3\sin\left(\frac{\pi}{L}x\right)e^{-k\left(\frac{\pi}{L}\right)^2 t} - \sin\left(\frac{3\pi}{L}x\right)e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

5.3 Haberman 2.3.3 (c)

problem number 22

This is problem 2.3.3, part (c) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 2\cos\frac{3\pi x}{L}$

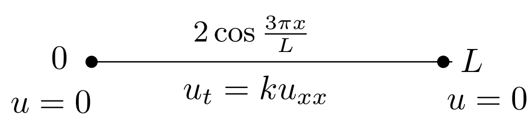


Figure 5: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 2*Cos[(3*Pi*x)/L];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}\frac{4\left(1+(-1)^{K[1]}\right)e^{-\frac{k\pi^2 tK[1]^2}{L^2}}K[1]\sin\left(\frac{\pi xK[1]}{L}\right)}{\pi\left(K[1]^2-9\right)}\right\}\right\}$$

but $n = 3$ should be special case

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=2*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 1/5\,\frac{1}{\pi}\left(5\sum_{n=4}^{\infty} 4\,\frac{n((-1)^n+1)}{\pi\,(n^2-9)}\sin\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}\pi - 16\,\sin\left(2\,\frac{\pi\,x}{L}\right)\mathrm{e}^{-4\,\frac{k\pi^2t}{L^2}}\right)$$

handled $n=3$ case correctly.
Hand solution

Solve $u_t = ku_{xx}$ with $0<x<L$ and initial conditions $u(x,0)=2\cos\left(\frac{3\pi x}{L}\right)$.

The basic solution for this type of PDE was already given in problem **??** on page **??** as

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t=0$, the above becomes

$$u(x,0) = 2\cos\left(\frac{3\pi}{L}x\right) = \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating

$$\begin{aligned}\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= \sum_{n=1}^{\infty} B_n\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\end{aligned}$$

By orthogonality of sin functions the above simplifies to

$$
\begin{aligned}
\int_0^L 2\cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx &= B_m\int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx \\
&= B_m\frac{L}{2} \\
B_m &= \frac{4}{L}\int_0^L \cos\left(\frac{3\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx
\end{aligned}
$$

For $m=3, B_3=0$. For $m\neq 3$

$$
\begin{aligned}
B_m &= \frac{4}{L}\left(\frac{1+(-1)^m}{m^2-9}\frac{nL}{\pi}\right) \\
&= \frac{4n}{\pi}\left(\frac{1+(-1)^m}{m^2-9}\right)
\end{aligned}
$$

Hence the solution becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1,n\neq 3}^{\infty}\frac{4n}{\pi}\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t} \\
&= \frac{4}{\pi}\sum_{n=1,n\neq 3}^{\infty} n\left(\frac{1+(-1)^n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}
\end{aligned}
$$

When n is odd, all terms become zero, hence the above can be also be written as

$$
u(x,t) = \frac{8}{\pi}\sum_{n=2,4,6,\cdots}^{\infty}\left(\frac{n}{n^2-9}\right)\sin\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}
$$

5.4 Haberman 2.3.3 (d)

problem number 23

This is problem 2.3.3, part (d) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$
\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}
$$

Subject to boundary conditions $u(0,t)=0$ and $u(L,t)=0$ with the temperature initially $u(x,0)=\begin{cases} 1 & 0<x\leq \frac{L}{2} \\ 2 & \frac{L}{2}<x\leq L \end{cases}$

$$u(x,0) = \begin{cases} 1 & 0 < x \le \frac{L}{2} \\ 2 & \frac{L}{2} < x \le L \end{cases}$$

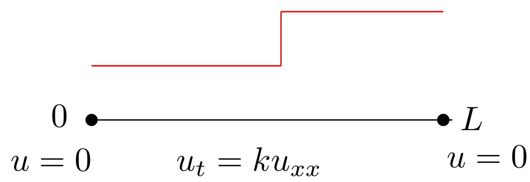


Figure 6: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == Piecewise[{{1, Inequality[0, Less, x, LessEqual, L/2]}, {2, L/2 < x < L}}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4e^{-\frac{kn^2\pi^2 t}{L^2}} \left(4\cos\left(\frac{n\pi}{2}\right) + 3\right) \sin^2\left(\frac{n\pi}{4}\right) \sin\left(\frac{n\pi x}{L}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\cos(1/2\,n\pi) + 2 + 4(-1)^{1+n}}{n\pi} \sin\left(\frac{n\pi\,x}{L}\right) e^{-\frac{k\pi^2 n^2 t}{L^2}}$$

Hand solution

The basic solution for this type of PDE was already given in problem **??** on page **??** as

$$u(x,t)=\sum_{n=1}^{\infty}B_n e^{-k\lambda_n t}\sin\left(\sqrt{\lambda_n}x\right)$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2, n=1,2,3,\cdots$ and $\sin\left(\sqrt{\lambda_n}x\right)$ are the eigenfunctions. Initial conditions are now applied. Setting $t=0$, the above becomes

$$f(x)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right) \tag{3}$$

Where

$$f(x)=\begin{cases}1 & 0<x\leq\frac{L}{2}\\ 2 & \frac{L}{2}<x<L\end{cases}$$

Multiplying both sides of (3) by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating over the domain gives

$$\int_0^L\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\int_0^L\left[\sum_{n=1}^{\infty}B_n\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)\right]dx$$

Interchanging the order of integration and summation

$$\int_0^L\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=\sum_{n=1}^{\infty}\left[B_n\left(\int_0^L\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx\right)\right]$$

But $\int_0^L\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x\right)dx=0$ for $n\neq m$, hence only one term survives

$$\int_0^L\sin\left(\frac{m\pi}{L}x\right)f(x)\,dx=B_m\int_0^L\sin^2\left(\frac{m\pi}{L}x\right)dx$$

Renaming m back to n and since $\int_0^L \sin^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$ the above becomes

$$
\begin{aligned}
\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx &= \frac{L}{2} B_n \\
B_n &= \frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx + \int_{\frac{L}{2}}^{L} \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx\right) \\
&= \frac{2}{L}\left(\int_0^{\frac{L}{2}} \sin\left(\frac{n\pi}{L}x\right) dx + 2\int_{\frac{L}{2}}^{L} \sin\left(\frac{n\pi}{L}x\right) dx\right) \\
&= \frac{2}{L}\left(\left.\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\right|_0^{\frac{L}{2}} + 2\left.\frac{-\cos\left(\frac{n\pi}{L}x\right)}{\frac{n\pi}{L}}\right|_{\frac{L}{2}}^{L}\right) \\
&= \frac{2}{n\pi}\left(\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_0^{\frac{L}{2}} + 2\left(-\cos\left(\frac{n\pi}{L}x\right)\right)_{\frac{L}{2}}^{L}\right) \\
&= \frac{2}{n\pi}\left(\left[-\cos\left(\frac{n\pi}{L}\frac{L}{2}\right) + \cos(0)\right] + 2\left[-\cos(n\pi) + \cos\left(\frac{n\pi}{2}\right)\right]\right) \\
&= \frac{2}{n\pi}\left(-\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi) + 2\cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) + 1 - 2\cos(n\pi)\right)
\end{aligned}
$$

Hence the solution is

$$
u(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}
$$

With

$$
\begin{aligned}
B_n &= \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - 2\cos(n\pi) + 1\right) \\
&= \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \\
&= \frac{2}{n\pi}\left(1 + 2(-1)^{n+1} + \cos\left(\frac{n\pi}{2}\right)\right)
\end{aligned}
$$

Therefore

$$
u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1 - 2(-1)^n + \cos\left(\frac{n\pi}{2}\right)\right) \sin\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}
$$

5.5 Haberman 2.3.7

problem number 24

This is problem 2.3.7, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

Subject to boundary conditions $u_x(0,t) = 0$, $u_x(L,t) = 0$ with initial conditions $u(x,0) = f(x)$

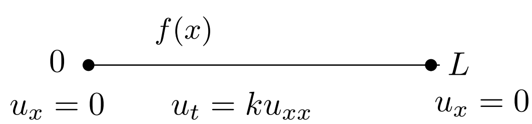


Figure 7: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L, sol, n, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{\left\{u(x,t) \to \frac{2 \sum_{n=1}^{\infty} e^{-\frac{k n^2 \pi^2 t}{L^2}} \cos\left(\frac{n\pi x}{L}\right) \int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx}{L} + \frac{\int_0^L f(x)\,dx}{L}\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\frac{1}{L}\left(\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\cos\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}\int_0^L f(x)\cos\left(\frac{n\pi\,x}{L}\right)\mathrm{d}x\right)L+\int_0^L f(x)\,\mathrm{d}x\right)$$

Hand solution

$$\begin{aligned}\frac{\partial u}{\partial t}&=k\frac{\partial^2 u}{\partial x^2}\\ u_x(0,t)&=0\\ u_x(L,t)&=0\\ u(x,0)&=f(x)\end{aligned}$$

Let $u(x,t)=T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{k}T'X=X''T$$

Dividing by $XT\neq 0$

$$\frac{1}{k}\frac{T'}{T}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

The two ODE's generated are

$$T'+k\lambda T=0 \tag{1}$$

$$X''+\lambda X=0 \tag{2}$$

Starting with the space ODE equation (2), with corresponding boundary conditions $\frac{dX}{dx}(0) = 0$, $\frac{dX}{dx}(L) = 0$. Assuming the solution is $X(x) = e^{rx}$, Then the characteristic equation is

$$\begin{aligned} r^2 + \lambda &= 0 \\ r^2 &= -\lambda \\ r &= \pm\sqrt{-\lambda} \end{aligned}$$

The following cases are considered.

<u>case $\lambda < 0$</u> In this case, $-\lambda$ and also $\sqrt{-\lambda}$ are positive. Hence the roots $\pm\sqrt{-\lambda}$ are both real. Let

$$\sqrt{-\lambda} = s$$

Where $s > 0$. This gives the solution

$$\begin{aligned} X(x) &= A\cosh(sx) + B\sinh(sx) \\ \frac{dX}{dx} &= A\sinh(sx) + B\cosh(sx) \end{aligned}$$

Applying the left B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(0) \\ &= B\cosh(0) \\ &= B \end{aligned}$$

The solution becomes $X(x) = A\cosh(sx)$ and hence $\frac{dX}{dx} = A\sinh(sx)$. Applying the right B.C. gives

$$\begin{aligned} 0 &= \frac{dX}{dx}(L) \\ &= A\sinh(sL) \end{aligned}$$

$A = 0$ result in trivial solution. Therefore assuming $\sinh(sL) = 0$ implies $sL = 0$ which is not valid since $s > 0$ and $L \neq 0$. Hence only trivial solution results from this case. <u>$\lambda < 0$ is not an eigenvalue.</u>

<u>case $\lambda = 0$</u>

The ODE becomes

$$\frac{d^2X}{dx^2} = 0$$

The solution is

$$
\begin{aligned}
X(x) &= c_1 x + c_2 \\
\frac{dX}{dx} &= c_1
\end{aligned}
$$

Applying left boundary conditions gives

$$
\begin{aligned}
0 &= \frac{dX}{dx}(0) \\
&= c_1
\end{aligned}
$$

Hence the solution becomes $X(x) = c_2$. Therefore $\frac{dX}{dx} = 0$. Applying the right B.C. provides no information.

Therefore this case leads to the solution $X(x) = c_2$. Associated with this one eigenvalue, the time equation becomes $\frac{dT_0}{dt} = 0$ hence T_0 is constant, say α. Hence the solution $u_0(x,t)$ associated with this $\lambda = 0$ is

$$
\begin{aligned}
u_0(x,t) &= X_0 T_0 \\
&= c_2 \alpha \\
&= A_0
\end{aligned}
$$

where constant $c_2\alpha$ was renamed to A_0 to indicate it is associated with $\lambda = 0$. $\lambda = 0$ is an eigenvalue.

case $\lambda > 0$

Hence $-\lambda$ is negative, and the roots are both complex.

$$
r = \pm i\sqrt{\lambda}
$$

The solution is

$$
\begin{aligned}
X(x) &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\
\frac{dX}{dx} &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right)
\end{aligned}
$$

Applying the left B.C. gives

$$
\begin{aligned}
0 &= \frac{dX}{dx}(0) \\
&= B\sqrt{\lambda}\cos(0) \\
&= B\sqrt{\lambda}
\end{aligned}
$$

Therefore $B = 0$ as $\lambda > 0$. The solution becomes $X(x) = A\cos\left(\sqrt{\lambda}x\right)$ and $\frac{dX}{dx} = -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right)$. Applying the right B.C. gives

$$\begin{aligned}0 &= \frac{dX}{dx}(L)\\ &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}L\right)\end{aligned}$$

$A = 0$ gives a trivial solution. Selecting $\sin\left(\sqrt{\lambda}L\right) = 0$ gives

$$\sqrt{\lambda}L = n\pi \qquad n = 1,2,3,\cdots$$

Or

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots$$

Therefore the space solution is

$$X_n(x) = A_n\cos\left(\frac{n\pi}{L}x\right) \qquad n = 1,2,3,\cdots$$

The time solution is found by solving

$$\frac{dT_n}{dt} + k\lambda_n T_n = 0$$

This has the solution

$$\begin{aligned}T_n(t) &= e^{-k\lambda_n t}\\ &= e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n = 1,2,3,\cdots\end{aligned}$$

For the same set of eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the A_n when combined in the following step below. Since for $\lambda = 0$ the time solution was found to be constant, and for $\lambda > 0$ the time solution is $e^{-k\left(\frac{n\pi}{L}\right)^2 t}$, then no time solution will grow with time. Time solutions always decay with time as the exponent $-k\left(\frac{n\pi}{L}\right)^2 t$ is negative quantity. The solution to the PDE for $\lambda > 0$ is

$$u_n(x,t) = T_n(t)\,X_n(x) \qquad n = 0,1,2,3,\cdots$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned}u(x,t) &= u_{\lambda=0}(x,t) + \sum_{n=1}^{\infty} u_n(x,t)\\ &= A_0 + \sum_{n=1}^{\infty} A_n\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}\end{aligned}$$

From the solution found above, setting $t=0$ gives

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides with $\cos\left(\frac{m\pi}{L}x\right)$ where in this problem $m = 0, 1, 2, \cdots$ (since there was an eigenvalue associated with $\lambda = 0$), and integrating over the domain gives

$$\begin{aligned}
\int_0^L f(x) \cos\left(\frac{m\pi}{L}x\right) dx &= \int_0^L \cos\left(\frac{m\pi}{L}x\right) \left(A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)\right) dx \\
&= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \int \cos\left(\frac{m\pi}{L}x\right) \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) dx \\
&= \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \int_0^L \sum_{n=1}^{\infty} A_n \cos\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx
\end{aligned}$$

Interchanging the order of summation and integration

$$\int_0^L f(x) \cos\left(\frac{m\pi}{L}x\right) dx = \int_0^L A_0 \cos\left(\frac{m\pi}{L}x\right) dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{m\pi}{L}x\right) \cos\left(\frac{n\pi}{L}x\right) dx \tag{1}$$

<u>case $m=0$</u>

When $m=0$ then $\cos\left(\frac{m\pi}{L}x\right) = 1$ and the above simplifies to

$$\int_0^L f(x)\, dx = \int_0^L A_0 dx + \sum_{n=1}^{\infty} A_n \int_0^L \cos\left(\frac{n\pi}{L}x\right) dx$$

But $\int_0^L \cos\left(\frac{n\pi}{L}x\right) dx = 0$ and the above becomes

$$\begin{aligned}
\int_0^L f(x)\, dx &= \int_0^L A_0 dx \\
&= A_0 L
\end{aligned}$$

Therefore

$$A_0 = \frac{1}{L} \int_0^L f(x)\, dx$$

<u>case $m>0$</u>

From (1), one term survives in the integration when only $n=m$, hence

$$\int_0^L f(x) \cos\left(\frac{m\pi}{L}x\right) dx = A_0 \int_0^L \cos\left(\frac{m\pi}{L}x\right) dx + A_m \int_0^L \cos^2\left(\frac{m\pi}{L}x\right) dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right) dx = 0$ and the above becomes

$$\int_0^L f(x) \cos\left(\frac{m\pi}{L}x\right) dx = A_m \frac{L}{2}$$

Therefore

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right) dx$$

For $n = 1, 2, 3, \cdots$

Therefore the solution is

$$\begin{aligned} u(x,t) &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \\ &= \frac{1}{L}\int_0^L f(x)\,dx + \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx\right)\cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \end{aligned}$$

In the limit as $t \to \infty$ the term $e^{-k\left(\frac{n\pi}{L}\right)^2 t} \to 0$. What is left is A_0. But $A_0 = \frac{1}{L}\int_0^L f(x)\,dx$ from above. This quantity is the average of the initial temperature.

5.6 Haberman 2.3.8

problem number 25

This is problem 2.3.8, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx} - \alpha u$$

This corresponds to a one-dimentional rod either with heat loss through the lateral sides with outside temperature zero degrees ($\alpha > 0$) or with insulated sides with a heat sink propertional to the temperature.

Suppose the boundary conditions are $u(0,t) = 0, u(L,t) = 0$, solve with the temperature initially $u(x,0) = f(x)$ if $\alpha > 0$

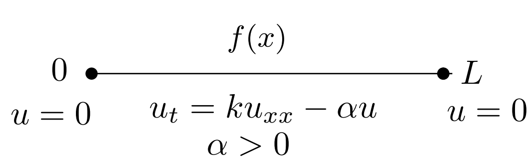


Figure 8: PDE specification

Mathematica ✗

```
ClearAll[u, t, k, x, L, a, f, alpha];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] - alpha*u[x, t];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x'; a:='a';f:='f';alpha:='alpha';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)-alpha*u(x,t);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(2\frac{1}{L}\sin\left(\frac{n\pi x}{L}\right)e^{-\frac{t\left(k\pi^2n^2+L^2\alpha\right)}{L^2}}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx\right)$$

Hand solution

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}-\alpha u$$

$$\frac{\partial u}{\partial t}+\alpha u=k\frac{\partial^2 u}{\partial x^2}$$

Assuming $u(x,t)=X(x)T(t)$ and substituting in the above gives

$$XT'+\alpha XT=kTX''$$

Dividing by $kXT\neq 0$

$$\frac{T'}{kT}+\frac{\alpha}{k}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is assumed real.

$$\frac{1}{k}\frac{T'}{T}+\frac{\alpha}{k}=\frac{X''}{X}=-\lambda$$

The two ODE's are

$$\frac{1}{k}\frac{T'}{T}+\frac{\alpha}{k}=-\lambda$$
$$\frac{X''}{X}=-\lambda$$

Or

$$T'+(\alpha+\lambda k)T=0$$
$$X''+\lambda X=0$$

The solution to the space ODE is the familiar (where $\lambda>0$ is only possible case, As found in Haberman problem 2.3.3, part d. Since it has the same B.C.)

$$X_n=B_n\sin\left(\frac{n\pi}{L}x\right)\qquad n=1,2,3,\cdots$$

Where $\lambda_n=\left(\frac{n\pi}{L}\right)^2$. The time ODE is now solved.

$$\frac{dT_n}{dt}+(\alpha+\lambda_n k)T_n=0$$

This has the solution

$$\begin{aligned}T_n(t)&=e^{-(\alpha+\lambda_n k)t}\\&=e^{-\alpha t}e^{-\left(\frac{n\pi}{L}\right)^2 kt}\end{aligned}$$

For the same eigenvalues. Notice that no need to add a constant here, since it will be absorbed in the B_n when combined in the following step below. Therefore the solution to the PDE is

$$u_n(x,t)=T_n(t)X_n(x)$$

But for linear system sum of eigenfunctions is also a solution. Hence

$$\begin{aligned}u(x,t)&=\sum_{n=1}^{\infty}u_n(x,t)\\&=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)e^{-\alpha t}e^{-\left(\frac{n\pi}{L}\right)^2 kt}\\&=e^{-\alpha t}\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)e^{-\left(\frac{n\pi}{L}\right)^2 kt}\end{aligned}$$

Where $e^{-\alpha t}$ was moved outside since it does not depend on n. From initial condition

$$u(x,0)=f(x)=\sum_{n=1}^{\infty}B_n\sin\left(\frac{n\pi}{L}x\right)$$

Applying orthogonality of sin as before to find B_n results in

$$B_n = \frac{2}{L} \int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx$$

Hence the solution becomes

$$\begin{aligned} u(x,t) &= \frac{2}{L} e^{-\alpha t} \left(\sum_{n=1}^{\infty} \left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx \right) \sin\left(\frac{n\pi}{L}x\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt} \right) \\ &= \frac{2}{L} \sum_{n=1}^{\infty} \left(\int_0^L \sin\left(\frac{n\pi}{L}x\right) f(x)\,dx \right) \sin\left(\frac{n\pi}{L}x\right) e^{-t\left(\frac{n^2\pi^2 k + \alpha L^2}{L^2}\right)} \end{aligned}$$

Hence it is clear that in the limit as t becomes large $u(x,t) \to 0$ since $\alpha > 0$ and

$$\lim_{t\to\infty} u(x,t) = 0$$

5.7 Haberman 2.4.1 (a)

problem number 26

This is problem 2.4.1 part(a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$. Initial conditions

$$u(x,0) = \begin{cases} 0 & x < \frac{L}{2} \\ 1 & x > \frac{L}{2} \end{cases}$$

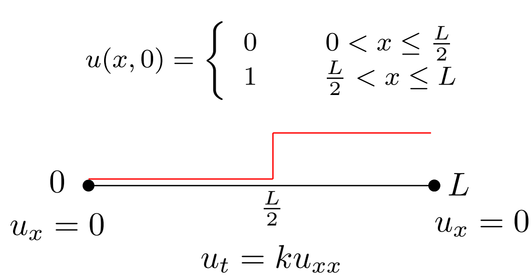


Figure 9: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,t)\to\frac{2\sum_{n=1}^{\infty}-\frac{e^{-\frac{kn^2\pi^2t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi}{2}\right)}{n\pi}}{L}+\frac{1}{2}\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0);
ic:=u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=1/2+\sum_{n=1}^{\infty}-2\,\frac{\sin\left(1/2\,n\pi\right)}{n\pi}\cos\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}$$

5.8 Haberman 2.4.1 (b)

problem number 27

This is problem 2.4.1 part(b) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t=ku_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = 6 + 4\cos\left(\frac{3\pi x}{L}\right)$

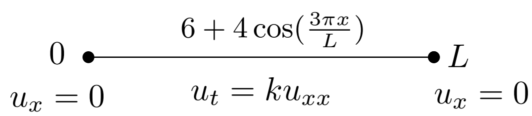


Figure 10: PDE specification

Mathematica ✓

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == 6 + 4*Cos[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 sol = sol /. {K[1] -> n};
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t)\to 4e^{-\frac{9\pi^2 kt}{L^2}}\cos\left(\frac{3\pi x}{L}\right)+6\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=6+4*cos(3*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 6 + 4\cos\left(3\frac{\pi x}{L}\right)e^{-9\frac{k\pi^2 t}{L^2}}$$

Hand solution

The general solution for this type of PDE is given in problem 5.5 on page 192 as

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \tag{1}$$

In this example $u(x,0) = f(x) = 6 + 4\cos\frac{3\pi x}{L}$. Hence at $t = 0$ the above becomes

$$\begin{aligned} f(x) &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) \\ 6 + 4\cos\frac{3\pi x}{L} &= A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Comparing terms shows that

$$\begin{aligned} A_0 &= 6 \\ A_3 &= 4 \end{aligned}$$

And all other $A_n = 0$. Hence the solution (1) is

$$u(x,t) = 6 + 4\cos\left(\frac{3\pi}{L}x\right) e^{-k\left(\frac{3\pi}{L}\right)^2 t}$$

5.9 Haberman 2.4.1 (c)

problem number 28

This is problem 2.4.1 part(c) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u_x(L,t) = 0$ with the temperature initially $u(x,0) = -2\sin\frac{\pi x}{L}$

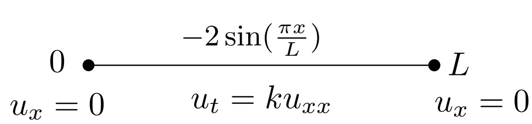


Figure 11: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -2*Sin[(Pi*x)/L];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty}\frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L}-\frac{4}{\pi}\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=-2*sin(Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\frac{1}{\pi}\left(\sum_{n=2}^{\infty}4\,\frac{(-1)^n+1}{\pi\,(n^2-1)}\cos\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\,\pi^2n^2t}{L^2}}\pi-4\right)$$

<u>Hand solution</u>

The general solution for this type of PDE is given in problem 5.5 on page 192 as

$$u(x,t)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2t} \tag{1}$$

At $t=0$ the above becomes

$$f(x)=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)$$

$$-2\sin\frac{\pi x}{L}=A_0+\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\cos\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$
\begin{aligned}
-2\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx &= \int_0^L\left(A_0\cos\left(\frac{m\pi}{L}x\right)+\cos\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty}A_n\cos\left(\frac{n\pi}{L}x\right)\right)dx\\
&= \int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\int_0^L\sum_{n=1}^{\infty}A_n\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx
\end{aligned}
$$

Interchanging the order of integration and summation

$$
\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\sum_{n=1}^{\infty}A_n\int_0^L\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx
$$

<u>Case $m=0$</u>

The above becomes

$$
-2\int_0^L\sin\left(\frac{\pi x}{L}\right)dx=\int_0^L A_0dx+\sum_{n=1}^{\infty}A_n\int_0^L\cos\left(\frac{n\pi}{L}x\right)dx
$$

But $\int_0^L\cos\left(\frac{n\pi}{L}x\right)dx=0$ hence

$$
\begin{aligned}
\int_0^L -2\sin\left(\frac{\pi x}{L}\right)dx &= \int_0^L A_0dx\\
A_0L &= -2\int_0^L\sin\left(\frac{\pi x}{L}\right)dx\\
A_0L &= -2\left(-\frac{\cos\left(\frac{\pi x}{L}\right)}{\frac{\pi}{L}}\right)_0^L\\
&= -\frac{2L}{\pi}\left(-\cos\left(\frac{\pi L}{L}\right)+\cos\left(\frac{\pi 0}{L}\right)\right)\\
&= -\frac{2L}{\pi}(-(-1)+1)\\
&= -\frac{4L}{\pi}
\end{aligned}
$$

Hence

$$
A_0=\frac{-4}{\pi}
$$

<u>Case $m>0$</u>

$$
\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \int_0^L A_0\cos\left(\frac{m\pi}{L}x\right)dx+\sum_{n=1}^{\infty}A_n\int_0^L\cos\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx
$$

One term survives the summation resulting in

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = \frac{-4}{\pi}\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx + A_m\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx$$

But $\int_0^L \cos\left(\frac{m\pi}{L}x\right)dx = 0$ and $\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx = \frac{L}{2}$, therefore

$$\int_0^L -2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx = A_m\frac{L}{2}$$

$$A_n = \frac{-4}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx$$

But

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{-L(1+\cos(n\pi))}{\pi(n^2-1)}$$

Therefore

$$\begin{aligned} A_n &= 4\frac{(1+\cos(n\pi))}{\pi(n^2-1)} \\ &= 4\frac{(-1)^n+1}{\pi(n^2-1)} \qquad n = 1,2,3,\cdots \end{aligned}$$

Hence the solution becomes

$$u(x,t) = \frac{-4}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n+1}{(n^2-1)}\cos\left(\frac{n\pi}{L}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

5.10 Haberman 2.4.1 (d)

problem number 29

This is problem 2.4.1 part(d) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ and $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = -3\cos\frac{8\pi x}{L}$

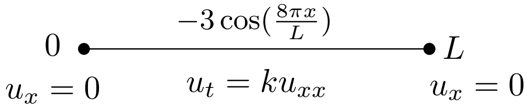


Figure 12: PDE specification

Mathematica ✓

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -3*Cos[(8*Pi*x)/L];
 NumericQ[L] = True;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 sol = sol /. {K[1] -> n};
 NumericQ[L] =. ;
```

$$\left\{\left\{u(x,t)\to -3e^{-\frac{64\pi^2 kt}{L^2}}\cos\left(\frac{8\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0 and k>0);
ic:=u(x,0)=-3*cos(8*Pi*x/L);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = -3\cos\left(8\frac{\pi x}{L}\right)e^{-64\frac{k\pi^2 t}{L^2}}$$

5.11 Haberman 2.4.2

problem number 30

This is problem 2.4.2 from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$u_t = ku_{xx}$$

The boundary conditions are $u_x(0,t) = 0$, $u(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

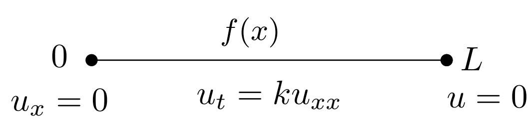


Figure 13: PDE specification

Mathematica ✓

```
ClearAll[u, t, k, x, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. {K[1] -> n, K[2] -> x};
```

$$\left\{ \left\{ u(x,t) \to \frac{2 \sum_{n=0}^{\infty} e^{-\frac{k(2n+1)^2\pi^2 t}{4L^2}} \cos\left(\frac{(2n+1)\pi x}{2L}\right) \int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right) f(x)\, dx}{L} \right\} \right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
assume(L>0);
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=0}^{\infty}\left(2\frac{1}{L}\cos\left(1/2\frac{(1+2n)\pi x}{L}\right)e^{-1/4\frac{\pi^2k(1+2n)^2t}{L^2}}\int_0^L f(x)\cos\left(1/2\frac{(1+2n)\pi x}{L}\right)dx\right)$$

Hand solution

Solve

$$\frac{\partial u}{\partial t}=\kappa\frac{\partial^2 u}{\partial x^2}$$

Let $u(x,t)=T(t)\,X(x)$, then the PDE becomes

$$\frac{1}{\kappa}T'X=X''T$$

Dividing by XT

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}$$

Since each side depends on different independent variable and both are equal, they must be both equal to same constant, say $-\lambda$. Where λ is real.

$$\frac{1}{\kappa}\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

The two ODE's are

$$T'+k\lambda T=0 \tag{1}$$
$$X''+\lambda X=0 \tag{2}$$

Per problem statement, $\lambda\geq 0$, so only two cases needs to be examined.

Case $\lambda=0$

The space equation becomes $X'' = 0$ with the solution

$$X = Ax + b$$

Hence left B.C. implies $X'(0) = 0$ or $A = 0$. Therefore the solution becomes $X = b$. The right B.C. implies $X(L) = 0$ or $b = 0$. Therefore this leads to $X = 0$ as the only solution. This results in trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$

Starting with the space ODE, the solution is

$$\begin{aligned} X(x) &= A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right) \\ X'(x) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right) \end{aligned}$$

Left B.C. gives

$$\begin{aligned} 0 &= X'(0) \\ &= B\sqrt{\lambda} \end{aligned}$$

Hence $B = 0$ since it is assumed $\lambda \neq 0$ and $\lambda > 0$. Solution becomes

$$X(x) = A\cos\left(\sqrt{\lambda}x\right)$$

Applying right B.C. gives

$$\begin{aligned} 0 &= X(L) \\ &= A\cos\left(\sqrt{\lambda}L\right) \end{aligned}$$

$A = 0$ leads to trivial solution. Therefore $\cos\left(\sqrt{\lambda}L\right) = 0$ or

$$\begin{aligned} \sqrt{\lambda} &= \frac{n\pi}{2L} \qquad n = 1,3,5,\cdots \\ &= \frac{(2n-1)\pi}{2L} \qquad n = 1,2,3\cdots \end{aligned}$$

Hence

$$\begin{aligned} \lambda_n &= \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots \\ &= \frac{(2n-1)^2\pi^2}{4L^2} \qquad n = 1,2,3\cdots \end{aligned}$$

Therefore

$$X_n(x) = A_n\cos\left(\frac{n\pi}{2L}x\right) \qquad n = 1,3,5,\cdots$$

And the corresponding time solution

$$T_n = e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \qquad n = 1, 3, 5, \cdots$$

Hence

$$\begin{aligned}
u_n(x,t) &= X_n T_n \\
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \sum_{n=1}^{\infty} A_n \cos\left(\frac{(2n-1)\pi}{2L}x\right) e^{-k\left(\frac{(2n-1)\pi}{2L}\right)^2 t}
\end{aligned}$$

From initial conditions

$$f(x) = \sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{n\pi}{2L}x\right)$$

Multiplying both sides by $\cos\left(\frac{m\pi}{2L}x\right)$ and integrating

$$\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx = \int \left(\sum_{n=1,3,5,\cdots}^{\infty} A_n \cos\left(\frac{m\pi}{2L}x\right)\cos\left(\frac{n\pi}{2L}x\right)\right) dx$$

Interchanging order of summation and integration and applying orthogonality results in

$$\begin{aligned}
\int_0^L f(x)\cos\left(\frac{m\pi}{2L}x\right) dx = A_m \frac{L}{2} \\
A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx
\end{aligned}$$

Therefore the solution is

$$\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=1,3,5,\cdots}^{\infty} \left[\int_0^L f(x)\cos\left(\frac{n\pi}{2L}x\right) dx\right] \cos\left(\frac{n\pi}{2L}x\right) e^{-k\left(\frac{n\pi}{2L}\right)^2 t} \\
&= \frac{2}{L}\sum_{n=0}^{\infty} \left(\int_0^L f(x)\cos\left(\frac{(2n+1)\pi}{2L}x\right) dx\right) \cos\left(\frac{(2n+1)\pi}{2L}x\right) e^{-k\left(\frac{(2n+1)\pi}{2L}\right)^2 t}
\end{aligned}$$

5.12 Convection heat loss

problem number 31

This problem is taken from Maple primes post

Left end insulated, right end has convection heat loss `https://www.mapleprimes.com/posts/209681-Solving-PDEs-With-Initial-And-Boundary`

Solve the heat equation

$$u_t = k u_{xx}$$

The boundary conditions are, on the left end $\frac{\partial u}{\partial x}(0,t) = 0$ and on the right end $\frac{\partial u}{\partial x}(1,t) = -u(1,t)$ with the temperature initially $u(x,0) = 1 - \frac{1}{4}x^3$

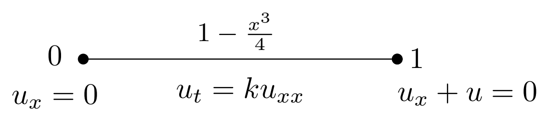


Figure 14: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 ic = u[x, 0] == 1 - (1*x^3)/4;
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == -u[1, t]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
unassign('u,t,x,k,f');
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic := u(x,0) = 1-(1/4)*x^3;
bc:= eval(diff(u(x,t), x), x = 0) = 0, eval(diff(u(x,t), x), x = 1)+u(1,t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))assumin
```

$$u(x,t) = \texttt{casesplit/ans}\left(\sum_{n=0}^{\infty} 3\,\frac{\left(\left(-{\lambda_n}^2+2\right)\cos\left(\lambda_n\right)-2+\left({\lambda_n}^3+2\,\lambda_n\right)\sin\left(\lambda_n\right)\right)\mathrm{e}^{-k{\lambda_n}^2 t}\cos\left(\lambda_n x\right)}{{\lambda_n}^3\left(\sin\left(2\,\lambda_n\right)+2\,\lambda_n\right)}, \{An$$

5.13 convection heat loss

problem number 32

Added April 28, 2019

Problem 4, section 74, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = k u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are, on the left end $u(0,t) = 0$ and on the right end $u_x(1,t) = -hu(1,t)$ with $h > 0$. Initial conditions $u(x,0) = f(x)$

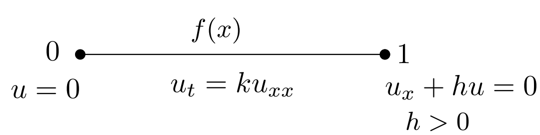


Figure 15: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, h, f];
pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
ic = u[x, 0] == f[x];
bc = {u[0, t] == 0, Derivative[1, 0][u][1, t] == -h u[1, t]};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {
```

Failed

Maple ✓

```
unassign('u,t,x,k,f,h');
pde := diff(u(x,t), t) = k*(diff(u(x,t), x, x));
ic := u(x,0) = f(x);
bc:= u(0,t)=0, eval(diff(u(x,t), x), x = 1)=-h*u(1,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \texttt{casesplit/ans}\left(\sum_{n=0}^{\infty}\left(2\, h signum(\sin(\lambda_n))\, signum(\cos(\lambda_n)) \sin(\lambda_n) \sin\left(\frac{h\sin(\lambda_n)\, x}{\cos(\lambda_n)}\right) \mathrm{e}^{-\frac{k(\sin(\lambda_n}{(\cos(\lambda}}\right.\right.$$

Hand solution

To solve the PDE, we first check the boundary conditions by writing them as

$$a_1 u(0,t) + a_2 u_x(0,t) = 0$$
$$b_1 u(1,t) + b_2 u_x(1,t) = 0$$

Then $a_1 = 0, a_2 = 0$. Hence $a_1 a_2 = 0$. And $b_1 = 1, b_2 = h$. Then since it is assumed that $h > 0$, then $b_1 b_2 \geq 0$. And since $q(x) = 0$ from the PDE itself, then we know that eigenvalues must be $\lambda \geq 0$.

Let $u = X(x)T(t)$ then the PDE becomes

$$T'X = X''T$$
$$\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

Hence the Sturm Liouville problem is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X'(1) + hX(1) &= 0 \end{aligned}$$

Where $p(x) = 1$.

<u>Case $\lambda = 0$</u>

Solution is

$$X(x) = Ax + B$$

At $x = 0$

$$0 = B$$

Hence solution becomes

$$X(x) = Ax$$

At $x = 1$ the second boundary conditions gives

$$\begin{aligned} A + hA &= 0 \\ A(1 + h) &= 0 \end{aligned}$$

For non trivial solution $1 + h = 0$ or $h = -1$. But we assumed that $h > 0$. Therefore $\lambda = 0$ is not eigenvalue.

<u>Case $\lambda > 0$</u>

Let $\lambda = \alpha^2, \alpha > 0$. Hence solution is

$$X(x) = A\cos(\alpha x) + B\sin(\alpha x)$$

At $X(0) = 0$

$$0 = A$$

The solution becomes

$$X(x) = B\sin(\alpha x)$$

At $x = 1$ the second boundary conditions gives

$$\begin{aligned} B\alpha\cos(\alpha) + hB\sin(\alpha) &= 0 \\ \alpha\cos(\alpha) + h\sin(\alpha) &= 0 \\ \tan(\alpha) &= -\frac{\alpha}{h} \end{aligned}$$

Therefore the eigenvalues are given by solution to

$$\tan\left(\alpha_n\right) = -\frac{\alpha_n}{h} \qquad n = 1,2,3,\cdots$$

And eigenfunctions are

$$X_n(x) = \sin\left(\alpha_n x\right)$$

The normalized eigenfunctions are

$$\phi_n(x) = \frac{X_n(x)}{\|X_n\left(x\right)\|}$$

But

$$\begin{aligned}
\|X_n(x)\|^2 &= \int_0^1 p(x)\, X_n^2(x)\, dx \\
&= \int_0^1 \sin^2\left(\alpha_n x\right) dx \\
&= \frac{1}{2}\int_0^1 1 - \cos\left(2\alpha_n x\right) dx \\
&= \frac{1}{2}\left(1 - \left[\frac{\sin\left(2\alpha_n x\right)}{2\alpha_n}\right]_0^1\right) \\
&= \frac{1}{2}\left(1 - \frac{1}{2\alpha_n}\left[\sin\left(2\alpha_n x\right)\right]_0^1\right) \\
&= \frac{1}{2}\left(1 - \frac{\sin\left(2\alpha_n\right)}{2\alpha_n}\right) \\
&= \frac{1}{2} - \frac{\sin\left(2\alpha_n\right)}{4\alpha_n}
\end{aligned}$$

But $\sin\left(2\alpha_n\right) = 2\sin\alpha_n\cos\alpha_n$ and $\alpha_n = -h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}$, therefore the above becomes

$$\begin{aligned}
\|X_n(x)\|^2 &= \frac{1}{2} + \frac{2\sin\alpha_n\cos\alpha_n}{4h\frac{\sin(\alpha_n)}{\cos(\alpha_n)}} \\
&= \frac{1}{2} + \frac{\cos^2\alpha_n}{2h} \\
&= \frac{h + \cos^2\alpha_n}{2h}
\end{aligned}$$

Hence

$$\begin{aligned}
\phi_n(x) &= \frac{X_n(x)}{\sqrt{\frac{h+\cos^2\alpha_n}{2h}}} \\
&= \sqrt{\frac{2h}{h + \cos^2\alpha_n}}\sin\left(\alpha_n x\right)
\end{aligned}$$

Now we use generalized Fourier series to find the solution. Let

$$u(x,t) = \sum_{n=1}^{\infty} B_n(t)\,\phi_n(x) \tag{1}$$

Substituting this back into the PDE gives

$$\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) = k\sum_{n=1}^{\infty} B_n(t)\,\phi_n''(x)$$

But $\phi_n''(x) = -\lambda_n \phi_n(x) = -\alpha_n^2 \phi_n(x)$. The above becomes

$$\begin{aligned}\sum_{n=1}^{\infty} B_n'(t)\,\phi_n(x) &= -k\sum_{n=1}^{\infty} B_n(t)\,\alpha_n^2 \phi_n(x)\\ B_n'(t) + k\alpha_n^2 B_n(t) = 0\end{aligned}$$

The solution is

$$B_n(t) = B_n(0)\,e^{-k\alpha_n^2 t}$$

Hence (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} B_n(0)\,e^{-k\alpha_n^2 t}\phi_n(x)$$

At $t = 0$ the above becomes

$$f(x) = \sum_{n=1}^{\infty} B_n(0)\,\phi_n(x)$$

Therefore

$$\begin{aligned}B_n(0) &= \langle f(x)\,,\phi_n(x)\rangle\\ &= \int_0^1 p(x)\,f(x)\,\phi_n(x)\,dx\\ &= \sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1 f(x)\sin(\alpha_n x)\,dx\end{aligned}$$

Therefore

$$\begin{aligned}B_n(t) &= B_n(0)\,e^{-k\alpha_n^2 t}\\ &= \left(\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1 f(x)\sin(\alpha_n x)\,dx\right)e^{-k\alpha_n^2 t}\end{aligned}$$

and solution (1) becomes

$$\begin{aligned}u(x,t) &= \sum_{n=1}^{\infty}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right)e^{-k\alpha_n^2 t}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin(\alpha_n x)\\ &= \frac{2h}{h+\cos^2\alpha_n}\sum_{n=1}^{\infty}\left(\int_0^1 f(x)\sin(\alpha_n x)\,dx\right)e^{-k\alpha_n^2 t}\sin(\alpha_n x)\end{aligned}$$

5.14 convection heat loss

problem number 33

Added April 28, 2019

Problem 2, section 77, Fourier series and Boundary value problem, 8th edition by Brown and Churchill.

Solve the heat equation

$$u_t = u_{xx}$$

For $0 < x < 1, t > 0$. The boundary conditions are $u_x(0,t) = hu(0,t)$ and on the right end $u(1,t) = 1$ with $h > 0$. Initial conditions $u(x,0) = 0$

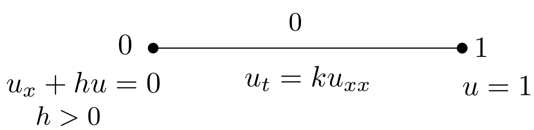


Figure 16: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k,h];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == 0;
 bc = {Derivative[1, 0][u][0, t] == h * u[0,t], u[1, t] == 1};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions->h>
```

Failed

Maple ✗

```
unassign('u,t,x,k,f,h');
pde := diff(u(x,t), t) = (diff(u(x,t), x, x));
ic := u(x,0) = 0;
bc:= eval(diff(u(x,t), x), x = 0) = h*u(0,t), u(1,t) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

Exception

Hand solution

Solve

$$u_t = u_{xx} \qquad 0 < x < 1, t > 0$$

With boundary conditions

$$\begin{aligned} u_x(0,t) - hu(0,t) &= 0 \\ u(1,t) &= 1 \end{aligned}$$

With $h > 0$. And initial conditions $u(x,0) = f(x)$.

Because the second B.C. is not zero, we need to introduce a reference function $r(x)$ which satisfies the nonhomogeneous boundary conditions.

Let $r(x) = Ax + B$. When $x = 0$ then the first BC gives

$$A - hB = 0$$

And the second BC gives

$$A + B = 1$$

From the first equation $A = hB$. Substituting in the second equation give $hB + B = 1$ or $B(1+h) = 1$ or $B = \frac{1}{1+h}$. Hence $A = \frac{h}{1+h}$. Therefore

$$\begin{aligned} r(x) &= Ax + B \\ &= \frac{h}{1+h}x + \frac{1}{1+h} \\ &= \frac{hx+1}{1+h} \end{aligned} \tag{1}$$

To verify. $r_x = \frac{h}{1+h}$. When $x = 0$ then $r(0) = \frac{1}{1+h}$. Hence $r_x(0) - hr(0) = \frac{h}{1+h} - h\frac{1}{1+h} = 0$ as expected. And when $x = 1$ then $r(1) = 1$ as expected. Now that we found $r(x)$ then we write

$$u(x,t) = v(x,t) + r(x)$$

Where $v(x,t)$ is the solution to the homogenous PDE

$$v_t = v_{xx} \qquad 0 < x < 1, t > 0$$

With boundary conditions

$$\begin{aligned} v_x(0,t) - hv(0,t) &= 0 \\ v(1,t) &= 0 \end{aligned}$$

We can now solve for $v(x,t)$ using separation of variables since boundary conditions are homogenous. Separation of variables gives

$$\begin{aligned} X'' + \lambda X &= 0 \\ X'(0) - hX(0) &= 0 \\ X(1) &= 0 \end{aligned}$$

The above is known eigenvalue problem which we found before. It has the following eigenfunctions and eigenvalues

$$\begin{aligned} \phi_n(x) &= \sqrt{\frac{2h}{h + \cos^2 \alpha_n}} \sin\left(\alpha_n (1-x)\right) \qquad n = 1, 2, \cdots \\ \tan\left(\alpha_n\right) &= \frac{-\alpha_n}{h} \end{aligned}$$

With $\alpha_n > 0$. Hence the solution $v(x,t)$ using generalized Fourier series is

$$v(x,t) = \sum_{n=1}^{\infty} B_n(t)\, \phi_n(x) \tag{2}$$

Substituting into the PDE $v_t = v_{xx}$ gives

$$\begin{aligned} \sum_{n=1}^{\infty} B_n'(t)\, \phi_n(x) &= \sum_{n=1}^{\infty} B_n(t)\, \phi_n''(x) \\ &= -\sum_{n=1}^{\infty} B_n(t)\, \alpha_n^2 \phi_n(x) \end{aligned}$$

Therefore the ODE is

$$B_n'(t) + \alpha_n^2 B_n(t) = 0$$

The solution is

$$B_n(t) = B_n(0)\, e^{-\alpha_n^2 t}$$

Hence (2) becomes

$$v(x,t) = \sum_{n=1}^{\infty} B_n(0)\, e^{-\alpha_n^2 t} \phi_n(x)$$

And since $u(x,t)=v(x,t)+r(x)$ then

$$u(x,t)=\sum_{n=1}^{\infty}B_n(0)\,e^{-\alpha_n^2 t}\phi_n(x)+\frac{hx+1}{1+h}$$

Now we find $B_n(0)$ from initial conditions. At $t=0$ the above becomes

$$\begin{aligned}0&=\sum_{n=1}^{\infty}B_n(0)\,\phi_n(x)+\frac{hx+1}{1+h}\\ -\frac{hx+1}{1+h}&=\sum_{n=1}^{\infty}B_n(0)\,\phi_n(x)\end{aligned}$$

Hence

$$\begin{aligned}B_n(0)&=\left\langle -\frac{hx+1}{1+h},\phi_n(x)\right\rangle\\ &=-\int_0^1 p(x)\,\frac{hx+1}{1+h}\phi_n(x)\,dx\\ &=-\int_0^1 \frac{hx+1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)dx\\ &=-\frac{1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\int_0^1(hx+1)\sin\left(\alpha_n(1-x)\right)dx \qquad (3)\end{aligned}$$

But

$$\begin{aligned}\int_0^1(hx+1)\sin\left(\alpha_n(1-x)\right)dx&=\int_0^1\sin\left(\alpha_n(1-x)\right)dx+h\int_0^1 x\sin\left(\alpha_n(1-x)\right)dx\\ &=\left[\frac{\cos\left(\alpha_n(1-x)\right)}{\alpha_n}\right]_0^1+h\left[\frac{\alpha_n x\cos\left(\alpha_n(1-x)\right)+\sin\left(\alpha_n(1-x)\right)}{\alpha_n^2}\right]_0^1\\ &=\frac{1-\cos\left(\alpha_n\right)}{\alpha_n}+\frac{h}{\alpha_n^2}\left[\alpha_n x\cos\left(\alpha_n(1-x)\right)+\sin\left(\alpha_n(1-x)\right)\right]_0^1\\ &=\frac{1-\cos\left(\alpha_n\right)}{\alpha_n}+\frac{h}{\alpha_n^2}\left[\alpha_n-\sin\alpha_n\right]\\ &=\frac{\alpha_n-\alpha_n\cos\left(\alpha_n\right)+h\alpha_n-h\sin\alpha_n}{\alpha_n^2}\end{aligned}$$

But $\frac{\sin(\alpha_n)}{\cos(\alpha_n)}=-\frac{\alpha_n}{h}$ or $h\sin\left(\alpha_n\right)=-\alpha_n\cos\left(\alpha_n\right)$ or $-h\sin\alpha_n=\alpha_n\cos\left(\alpha_n\right)$, hence the above simplifies to

$$\begin{aligned}\int_0^1(hx+1)\sin\left(\alpha_n(1-x)\right)dx&=\frac{\alpha_n+h\alpha_n}{\alpha_n^2}\\ &=\frac{1+h}{\alpha_n}\end{aligned}$$

Therefore (3) becomes

$$\begin{aligned}B_n(0) &= \frac{-1}{1+h}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\left(\frac{1+h}{\alpha_n}\right)\\ &= -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\end{aligned}$$

Hence final solution becomes

$$\begin{aligned}u(x,t) &= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\, e^{-\alpha_n^2 t}\phi_n(x)\\ &= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} B_n(0)\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)\\ &= \frac{hx+1}{1+h} + \sum_{n=1}^{\infty} -\frac{1}{\alpha_n}\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\exp\left(-\alpha_n^2 t\right)\sqrt{\frac{2h}{h+\cos^2\alpha_n}}\sin\left(\alpha_n(1-x)\right)\\ &= \frac{hx+1}{1+h} - 2h\sum_{n=1}^{\infty}\frac{\sin\left(\alpha_n(1-x)\right)}{\alpha_n\left(h+\cos^2\alpha_n\right)}\exp\left(-\alpha_n^2 t\right)\end{aligned}$$

5.15 Periodic boundary conditions

problem number 34

Solve the heat equation

$$u_t = k u_{xx}$$

For $-L < x < L$ and $t > 0$. The boundary conditions are periodic

$$\begin{aligned}u(-L,t) &= u(L,t)\\ \frac{\partial u}{\partial x}(-L,t) &= \frac{\partial u}{\partial x}(L,t)\end{aligned}$$

And initial conditions $u(x,0) = f(x)$

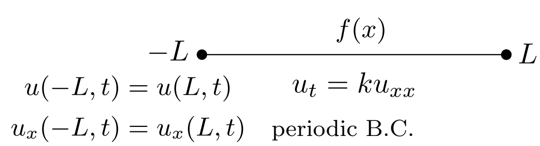


Figure 17: PDE specification

Mathematica ✗

```
ClearAll[u, t, x, L, c, f, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[-L, t] == u[L, t], Derivative[1, 0][u][-L, t] == Derivative[1, 0][u][L, t]};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(-L,t)=u(L,t),eval(diff(u(r,t),r),r=-L)=eval(diff(u(r,t),r),r=L);
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(\frac{1}{L}e^{-\frac{k\pi^2n^2t}{L^2}}\left(\sin\left(\frac{n\pi\,x}{L}\right)\int_{-L}^{L}f(x)\sin\left(\frac{n\pi\,x}{L}\right)\,dx+\cos\left(\frac{n\pi\,x}{L}\right)\int_{-L}^{L}f(x)\cos\left(\frac{n\pi\,x}{L}\right.\right.\right.\right.$$

5.16 Mixed BC

problem number 35

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) + u(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) + u(L,t) = 0$$

And initial condition $u(x,0) = f(x)$

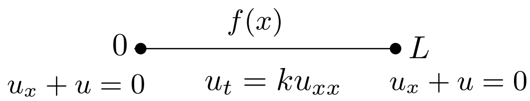


Figure 18: PDE specification

Mathematica ✗

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] + u[0, t] == 0, Derivative[1, 0][u][L, t] + u[L, t] == 0};
 ic = u[x, 0] == f[x];
 NumericQ[L] = True;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 NumericQ[L] =. ;
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2 \frac{1}{L(\pi^2 n^2 + L^2)} \left(-\pi n \cos\left(\frac{\pi n x}{L}\right) + \sin\left(\frac{\pi n x}{L}\right) L \right) e^{-\frac{k\pi^2 n^2 t}{L^2}} \int_0^L f(x) \left(-\pi n \cos\left(\frac{\pi n x}{L}\right) + \right.\right.$$

5.17 domain -1 to +1

problem number 36

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = f(x)$

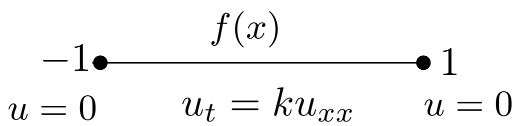


Figure 19: PDE specification

Mathematica ✗

```
ClearAll[u, t, x, f];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[-1, t] == 0, u[1, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic := u(x,0) = f(x);
bc := u(-1,t)=0,  u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t)) assu
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\sin(n\pi x) e^{-\pi^2 n^2 t} \int_{-1}^{1} f(x) \sin(n\pi x) \, dx + \cos(1/2(2n-1)\pi x) e^{-1/4\pi^2(2n-1)^2 t} \int_{-1}^{1} f(x) \cos$$

5.18 non-homogeneous BC

problem number 37

Taken from Maple PDE help pages

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 20$$
$$u(1,t) = 50$$

Initial condition is $u(x,0) = 0$

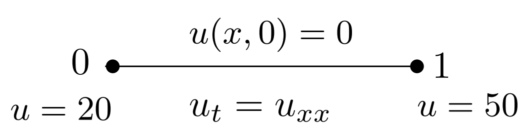


Figure 20: PDE specification

Mathematica ✓

```
ClearAll[u, x, t, n];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 20, u[1, t] == 50};
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to -\frac{2\sum_{n=1}^{\infty}\frac{(20-50(-1)^n)e^{-n^2\pi^2 t}\sin(n\pi x)}{n}}{\pi}+30x+20\right\}\right\}$$

Maple ✓

```
u:='u'; t:='t'; x:='x';
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=20, u(1,t)=50;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=20+30\,x+\sum_{n=1}^{\infty}\frac{\left(100\,(-1)^n-40\right)\sin\left(n\pi\,x\right)\mathrm{e}^{k\pi^2n^2t}}{n\pi}$$

5.19 Haberman 8.2.1 (a)

problem number 38

Added Nov 27, 2018

This is problem 8.2.1 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$\frac{\partial u}{\partial x}(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

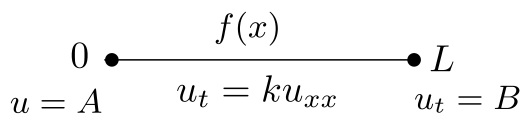


Figure 21: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, A, B, L, Q];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == A, Derivative[1, 0][u][L, t] == B};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';A:='A';B:='B';k:='k';Q:='Q';
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic  :=  u(x,0)=f(x);
bc  := u(0,t)=A, eval( diff(u(x,t),x),x=L)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} \left(-2\,\frac{1}{L} \sin\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right) \mathrm{e}^{-1/4\,\frac{k\pi^2(1+2\,n)^2 t}{L^2}} \int_0^L (Bx + A - f(x)) \sin\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right)\right.$$

Hand solution

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

We can look for $u_E(x)$ which is the steady state solution that satisfies the non-homogenous boundary conditions. In (1) $v(x,t)$ satisfies the PDE itself but with homogenous boundary conditions. The first step is to find $u_E(x)$. We use the equilibrium solution in this case. At equilibrium $\frac{\partial u_E(x,t)}{\partial t} = 0$ and hence the solution is given $\frac{d^2 u_E}{\partial x^2} = 0$ or

$$u_E(x) = c_1 x + c_2$$

At $x = 0, u_E(x) = A$, Hence

$$c_2 = A$$

And solution becomes $u_E(x) = c_1 x + A$. at $x = L$, $\frac{\partial u_E(x)}{\partial x} = c_1 = B$, Therefore

$$u_E(x) = Bx + A$$

Now we plug-in (1) into the original PDE, this gives

$$\frac{\partial v(x,t)}{\partial t} = k\left(\frac{\partial^2 v(x,t)}{\partial x} + \frac{\partial^2 u_E(x)}{\partial x}\right)$$

But $\frac{\partial^2 u_E(x)}{\partial x} = 0$, hence we need to solve

$$\frac{\partial v(x,t)}{\partial t} = k\frac{\partial^2 v(x,t)}{\partial x}$$

for $v(x,t) = u(x,t) - u_E(x)$ with homogenous boundary conditions $v(0,t) = 0$, $\frac{\partial v(L,t)}{\partial t} = 0$ and initial conditions

$$\begin{aligned} v(x,0) &= u(x,0) - u_E(x) \\ &= f(x) - (Bx + A) \end{aligned}$$

This PDE we already solved before and we know that it has the following solution

$$v(x,t) = \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right) e^{-k\lambda_n t}$$

$$\lambda_n = \left(\frac{n\pi}{2L}\right)^2 \qquad n = 1,3,5,\cdots \tag{2}$$

With b_n found from orthogonality using initial conditions $v(x,0) = f(x) - (Bx + A)$

$$v(x,0) = \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right)$$

$$\int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\lambda_m} x\right) dx = \int_0^L \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n} x\right) \sin\left(\sqrt{\lambda_m} x\right) dx$$

$$\int_0^L (f(x) - (Bx + A)) \sin\left(\sqrt{\lambda_m} x\right) dx = b_m \frac{L}{2}$$

Hence

$$b_n = \frac{2}{L}\int_0^L \left(f(x) - (Bx + A)\right)\sin\left(\sqrt{\lambda_n}x\right)dx \qquad n = 1,3,5,\cdots \tag{3}$$

Therefore, from (1) the solution is

$$\begin{aligned}
u(x,t) &= \sum_{n=1,3,5,\cdots}^{\infty} b_n \sin\left(\sqrt{\lambda_n}x\right)e^{-k\lambda_n t} + \overbrace{Bx + A}^{u_E(x)}\\
&= Bx + A + \sum_{n=1,3,5,\cdots}^{\infty}\left(\frac{2}{L}\int_0^L \left(f(x) - (Bx + A)\right)\sin\left(\sqrt{\frac{n\pi}{L}}x\right)dx\right)\sin\left(\sqrt{\frac{n\pi}{L}}x\right)e^{-k\left(\frac{n\pi}{L}\right)^2 t}
\end{aligned}$$

Or

$$u(x,t) = Bx+A+\sum_{n=0}^{\infty}\left(\frac{2}{L}\int_0^L \left(f(x) - (Bx + A)\right)\sin\left(\sqrt{\frac{(2n+1)\pi}{2L}}x\right)dx\right)\sin\left(\sqrt{\frac{(2n+1)\pi}{2L}}x\right)e^{-k\left(\frac{(2n}{}\right.}$$

5.20 Haberman 8.2.1 (d)

problem number 39

This is problem 8.2.1 part(d) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + k$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned}
u(0,t) &= A\\
u(L,t) &= B
\end{aligned}$$

Initial condition is $u(x,0) = f(x)$

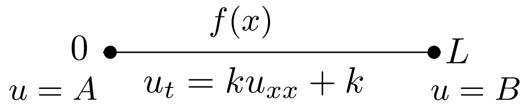


Figure 22: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, A0, B0, L0];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + k;
 bc = {u[0, t] == A0, u[L0, t] == B0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';A:='A';B:='B';
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(-\frac{1}{L^2}\sin\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}\int_0^L 2\,\sin\left(\frac{n\pi\,x}{L}\right)\left(-f(x)\,L+1/2\,L^2x+\left(-1/2\,x^2+A\right)\right.\right.\right.$$

Hand solution

Let

$$u(x,t) = v(x,t) + u_E(x) \tag{1}$$

Where $u_E(x)$ is the equilibrium solution which needs to satisfy only the nonhomogeneous B.C. And $v(x,t)$ is transient solution to heat PDE with homogeneous B.C.

At equilibrium, $u_t = ku_{xx} + Q(x)$ becomes

$$\begin{aligned} 0 &= ku_E'' + Q(x) \\ &= ku_E'' + k \\ &= k(u_E'' + 1) \end{aligned}$$

Hence

$$u_E'' = -1$$

The solution to this ODE is

$$u_E = c_1x + c_2 - \frac{1}{2}x^2$$

At $x=0$, the above gives

$$A = c_2$$

And at $x=L$

$$\begin{aligned}
B &= c_1 L + A - \frac{1}{2}L^2 \\
c_1 &= \frac{B - A + \frac{1}{2}L^2}{L} \\
&= \frac{B}{L} - \frac{A}{L} + \frac{1}{2}L
\end{aligned}$$

Hence

$$u_E = \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2$$

Hence from (1)

$$\begin{aligned}
u(x,t) &= v(x,t) + u_E \qquad (1A)\\
&= v(x,t) + \left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right)x + A - \frac{1}{2}x^2
\end{aligned}$$

Substituting this in $u_t = ku_{xx} + k$ gives

$$\begin{aligned}
v_t &= k(v_{xx} - 1) + k \\
&= kv_{xx} \qquad (2)
\end{aligned}$$

We need to solve the above for $v(x,t)$, but with homogeneous B.C. $v(0,t)=0, v(L,t)=0$. The eigenvalues for the homogeneous PDE $v_t = kv_{xx}$ with these boundary conditions is known to be $\lambda_n = \left(\frac{n\pi}{L}\right)^2$, for $n = 1, 2, \cdots$ and the corresponding eigenfunctions are $X_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$. Now, using eigenfunction expansion, let

$$v(x,t) = \sum_{n=1}^{\infty} b_n(t)\, X_n(x) \qquad (3)$$

Substituting (3) into (2) gives

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) = k\sum_{n=1}^{\infty} b_n(t)\, X_n''(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, therefore the above becomes

$$\sum_{n=1}^{\infty} b_n'(t)\, X_n(x) + k\sum_{n=1}^{\infty} \lambda_n b_n(t)\, X_n(x) = 0$$

Since the above is true for each n and since eigenfunctions can not be zero, the above simplifies to

$$b_n'(t) + k\lambda_n b_n(t) = 0 \tag{4}$$

This is linear in $b(t)$. The solution using integrating factor is

$$b_n(t) = b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

Therefore (3) becomes

$$\begin{aligned} v(x,t) &= \sum_{n=1}^{\infty} b_n(t)\, X_n(x) \\ &= \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

And from (1)

$$\begin{aligned} u(x,t) &= v(x,t) + u_E(x) \\ &= \overbrace{\left(\frac{B}{L} - \frac{A}{L} + \frac{1}{2}L\right) x + A - \frac{1}{2}x^2}^{u_E} + \sum_{n=1}^{\infty} b_0(0)\, e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned} \tag{5}$$

At $t = 0$ the above becomes

$$f(x) = \frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2 + \sum_{n=1}^{\infty} b_0(0) \sin\left(\frac{n\pi}{L}x\right)$$

For $n > 0$, and applying orthogonality

$$\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx = \int_0^L \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \sin\left(\frac{n\pi}{L}x\right) dx + \int_0^L b_0(0) \sin^2\left(\frac{n\pi}{L}x\right) dx$$

Hence

$$\int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx = \frac{L}{2} b_0(0)$$

Therefore

$$b_0(0) = \frac{2}{L} \int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx$$

Substituting the above in (5) gives

$$\begin{aligned} u(x,t) &= \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right) \\ &\quad + \sum_{n=1}^{\infty} \left(\frac{2}{L} \int_0^L \left(f(x) - \left(\frac{Bx}{L} - \frac{Ax}{L} + \frac{1}{2}Lx + A - \frac{1}{2}x^2\right)\right) \sin\left(\frac{n\pi}{L}x\right) dx\right) e^{-k\frac{n\pi}{L}t} \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

5.21 Internal source

problem number 40

Solve the heat equation

$$u_t = u_{xx} - u(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

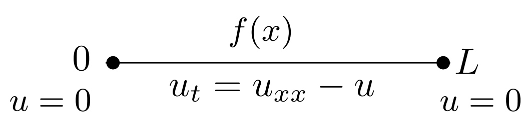


Figure 23: PDE specification

Mathematica ✗

```
ClearAll[x, t, u, f, L];
 pde = D[u[x, t], t] + u[x, t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)+u(x,t)=diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\,\frac{1}{L}\sin\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{t\left(\pi^2 n^2+L^2\right)}{L^2}}\int_0^L f(x)\sin\left(\frac{n\pi\,x}{L}\right)\mathrm{d}x\right)$$

Hand solution

$$\begin{aligned}
u_t &= u_{xx} - u \\
u(0,t) &= 0 \\
u(L,t) &= 0 \\
u(x,0) &= f(x)
\end{aligned}$$

Let $u(x,t) = v(x,t)\,e^{-t}$, hence $u_t = v_t e^{-t} - v e^{-t}$ and $u_{xx} = v_{xx} e^{-t}$. Therefore the above PDE becomes

$$\begin{aligned}
v_t e^{-t} - v e^{-t} &= v_{xx} e^{-t} - v e^{-t} \\
v_t &= v_{xx}
\end{aligned}$$

With boundary conditions

$$\begin{aligned}
v(0,t) &= 0 \\
v(L,t) &= 0
\end{aligned}$$

The solution to this PDE is known, since it has homogenouse BC and it is in standard form. The solution is

$$v(x,t) = \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)$$

Hence

$$
\begin{aligned}
u(x,t) &= e^{-t}\sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t} \sin\left(\frac{n\pi}{L}x\right) \\
&= \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n^2\pi^2}{L^2}\right)t-t} \sin\left(\frac{n\pi}{L}x\right) \\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2}{L^2}+1\right)} \sin\left(\frac{n\pi}{L}x\right) \\
&= \sum_{n=1}^{\infty} B_n e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)} \sin\left(\frac{n\pi}{L}x\right) \qquad (1)
\end{aligned}
$$

Applying initial conditions $u(x,0) = f(x)$ gives

$$
f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)
$$

Hence B_n are the Fourier sine coefficients of $f(x)$

$$
B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right) dx
$$

Therefore the solution in (1) becomes

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} \frac{2}{L}\left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right) dx\right) e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)} \sin\left(\frac{n\pi}{L}x\right) \\
&= \frac{2}{L}\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L}x\right) e^{-t\left(\frac{n^2\pi^2+L^2}{L^2}\right)} \left(\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right) dx\right)
\end{aligned}
$$

5.22 Internal source

problem number 41

Added Feb 10, 2018.

Solve the heat equation

$$
u_t + u(x,t) = 100u_{xx}
$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= 0 \\
u(1,t) &= 0
\end{aligned}
$$

Initial condition is $u(x,0) = \sin(2\pi x) - \sin(5\pi x)$

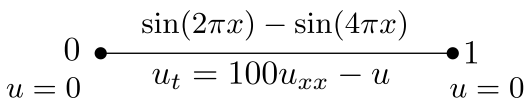


Figure 24: PDE specification

Mathematica ✓

```
ClearAll[x, t, u, f, L];
 f = Sin[2*Pi*x] - Sin[5*Pi*x];
 pde = D[u[x, t], t] == 100*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == f;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t][[1,1]], 60*10]];
```

$$u(x,t) \to e^{-400\pi^2 t} \sin(2\pi x) - e^{-2500\pi^2 t} \sin(5\pi x)$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=100*diff(u(x,t),x$2);
ic:=u(x,0)=sin(2*Pi*x)-sin(5*Pi*x);
bc:=u(0,t)=0,u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = \sin(2\pi x)\, e^{-400\pi^2 t} - \sin(5\pi x)\, e^{-2500\pi^2 t}$$

5.23 IC hat function

problem number 42

Added Feb 10, 2018.

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(40,t) = 0$$

Initial condition is

$$u(x,0) = \begin{cases} x & 0 \leq x < 20 \\ 40 - x & 20 \geq x \leq 40 \end{cases}$$

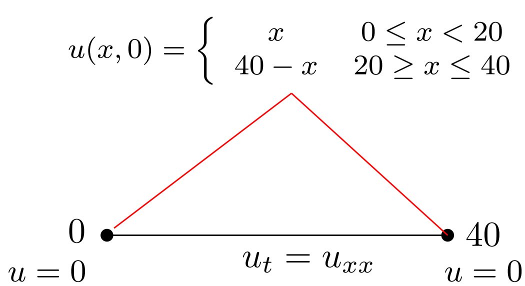


Figure 25: PDE specification

Mathematica ✓

```
ClearAll[x, t, u, f, L, n];
 f = Piecewise[{{x, Inequality[0, LessEqual, x, Less, 20]}, {40 - x, 20 <= x <= 40}}];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[40, t] == 0};
 ic = u[x, 0] == f;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{640e^{-\frac{n^2\pi^2t}{1600}}\cos\left(\frac{n\pi}{4}\right)\sin^3\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{40}\right)}{n^2\pi^2}\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
f:=piecewise(0<x and x<20,x,20<x and x<40,(40-x));
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=f;
bc:=u(0,t)=0,u(40,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=\sum_{n=1}^{\infty}160\,\frac{\sin\left(1/40\,n\pi\,x\right)\sin\left(1/2\,n\pi\right)}{\pi^2n^2}\mathrm{e}^{-\frac{\pi^2n^2t}{1600}}$$

5.24 homogeneous BC

problem number 43

Added July 2, 2018, taken from Maple 2018.1 improvement to PDE document.

Solve the heat equation

$$u_t=u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition is

$$u(x,0) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

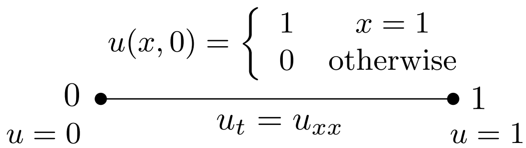


Figure 26: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == Piecewise[{{0, x == 0}, {1, True}}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
 sol = sol /. {K[1] -> n}
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} -\frac{2(-1+(-1)^n)\, e^{-n^2\pi^2 t} \sin(n\pi x)}{n\pi}\right\}\right\}$$

Maple ✓

```
pde := diff(u(x,t), t) = diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(1,t) = 1;
ic  := u(x,0) = piecewise(x = 0, 0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc])),output='r
```

$$u(x,t) = x + \sum_{n=1}^{\infty} 2\,\frac{\sin(n\pi x)\, \mathrm{e}^{-\pi^2 n^2 t}}{n\pi}$$

5.25 BC depends on time

problem number 44

added March 8, 2018. Exam problem

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = t$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

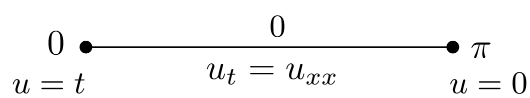


Figure 27: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == t, u[Pi, t] == 0};
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 sol = sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} -\frac{\left(2 - 2e^{-n^2 t}\right)\sin(nx)}{n^3\pi} - \frac{tx}{\pi} + t\right\}\right\}$$

Maple ✓

```
u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
bc:=u(0,t)=t,u(Pi,t)=0;
ic:=u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))),outp
```

$$u(x,t) = 1/6\,\frac{1}{\pi}\left(6\sum_{n=1}^{\infty} 2\,\frac{\sin(nx)\,\mathrm{e}^{-n^2 t}}{\pi\,n^3}\pi - 6\left(1/6\,x^2 - 1/3\,\pi\,x + t\right)(-\pi + x)\right)$$

5.26 Haberman 8.2.1 (f)

problem number 45

added March 18, 2018.

This is problem 8.2.1, part(f) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \sin\left(\frac{2\pi x}{L}\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

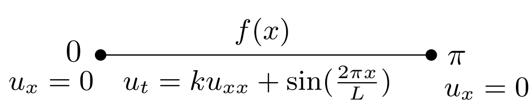


Figure 28: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f'; k:='k';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+sin(2*Pi*x/L);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = 1/4\,\frac{1}{\pi^2 kL}\left(4\sum_{n=1}^{\infty}\left(-1/2\,\frac{1}{\pi^2 kL}\int_0^L L^2\cos\left(\frac{n\pi\,x}{L}\right)\sin\left(2\,\frac{\pi\,x}{L}\right) - 2\,\pi\left(-2\,\pi\,k\_C2 + 2\,\pi\,kf(x) + L\right.\right.\right.$$

5.27 Pinchover and Rubinstein 6.25

problem number 46

Added July 2, 2018. Taken from Maple 2018.1 document, originally exercise 6.25 from Pinchover and Rubinstein.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \cos(wt)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = x$.

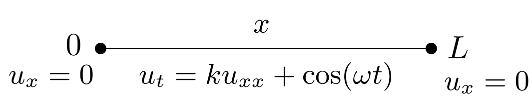


Figure 29: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Cos[w*t];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == x;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';k:='k';
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+cos(w*t);
bc := (D[1](u))(L, t) = 0, (D[1](u))(0, t) = 0;
ic:= u(x, 0) = x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = 1/2\,\frac{1}{w}\left(Lw + 2\sum_{n=1}^{\infty} 2\,\frac{L((-1)^n - 1)}{\pi^2 n^2}\cos\left(\frac{n\pi\,x}{L}\right)e^{-\frac{k\pi^2 n^2 t}{L^2}}w + 2\sin(wt)\right)$$

5.28 external source

problem number 47

Added March 18, 2018.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \left(e^{-ct}\sin\left(\frac{2\pi x}{L}\right)\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

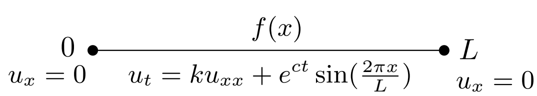


Figure 30: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, L, c];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Exp[-(c*t)]*Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';c:='c';
interface(showassumed=0);
pde :=  diff(u(x,t),t)=k*diff(u(x,t),x$2)+(exp(-c*t)*sin(2*Pi*x/L));
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = 1/3\,\frac{1}{L\pi\,(L^2c-\pi^2k)}\left(L\sum_{n=3}^{\infty}-12\,\frac{L^2((-1)^n-1)\,(L^2c-\pi^2k)}{(-\pi^2kn^2+L^2c)\,(n^2-4)}\cos\left(\frac{n\pi\,x}{L}\right)\left(\mathrm{e}^{-tc}-\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}\right)+(3\right.$$

5.29 Math 4567 Exam

problem number 48

Added April 3, 2018.

Exam question. Math 4567 UMN. Spring 2019.

Solve the heat equation

$$u_t = u_{xx} + t(\pi - x)$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

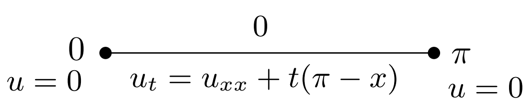


Figure 31: PDE specification

Mathematica ✓

```
ClearAll[u, x, t, k, f, L, c];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + t*(Pi-x);
 bc = {u[0,t] == 0, u[Pi,t] == 0};
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> t
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{2\left(tn^2+e^{-n^2t}-1\right)\sin(nx)}{n^5}\right\}\right\}$$

Maple ✓

```
unassign('u,t,x');
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+t*(Pi-x);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=0, u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\sum_{n=1}^{\infty}2\,\frac{\sin(nx)\left(n^2t+e^{-n^2t}-1\right)}{n^5}$$

5.30 Pinchover and Rubinstein 6.17

problem number 49

Added July 2, 2018.

Pinchover and Rubinstein's exercise 6.17. Taken from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$\frac{\partial}{\partial t}u(x,t)-\frac{\partial^2}{\partial x^2}u(x,t)=1+x\cos(t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = \sin(t)$$
$$\frac{\partial u}{\partial x}(1,t) = \sin(t)$$

Initial condition is $u(x,0) = 1 + \cos(2\pi x)$.

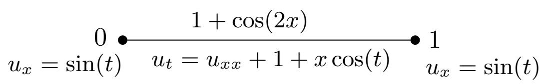


Figure 32: PDE specification

Mathematica ✗

```
pde = D[u[x, t], x] == D[u[x, t], {x, 2}] + 1 + x*Cos[t];
 bc = {Derivative[1, 0][u][0, t] == Sin[t], Derivative[1, 0][u][1, t] == Sin[t]};
 ic = u[x, 0] == 1 + Cos[2*Pi*x];
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, ic, bc}, u[x, t], x, t]], 60*10]
```

Failed

Maple ✓

```
x:='x';t:='t';
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + 1+x*cos(t);
bc := (D[1](u))(0, t) = sin(t), (D[1](u))(1, t) = sin(t);
ic := u(x, 0) = 1+cos(2*Pi*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t) = 1 + \cos(2\pi x)\,e^{-4\pi^2 t} + t + x\sin(t)$$

5.31 nonhomogeneous BC

problem number 50

Added July 2, 2018.

Second example from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$.

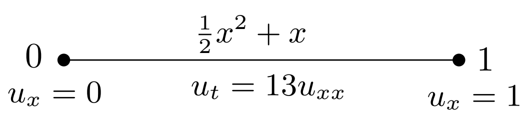


Figure 33: PDE specification

Mathematica ✗

```
pde = D[u[x, t], x] == 13*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
 ic = u[x, 0] == (1*x^2)/2 + x;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
x:='x';t:='t';
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x, x));
bc := D[1](u)(0,t)=0,D[1](u)(1,t)=1;
ic := u(x, 0) = 1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t) = 1/2 + \sum_{n=1}^{\infty} 2\,\frac{\cos(n\pi\,x)\,\mathrm{e}^{-13\,\pi^2 n^2 t}(-1+(-1)^n)}{\pi^2 n^2} + 13\,t + 1/2\,x^2$$

5.32 Pinchover and Rubinstein 6.23

problem number 51

Added July 2, 2018.

4th example from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.23 .

Solve the heat equation on bar

$$u_t = u_{xx} + g(x,t)$$

Where $g(x,t) = e^{3t}\cos(17\pi x)$ for $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = 3\cos(42\pi x)$.

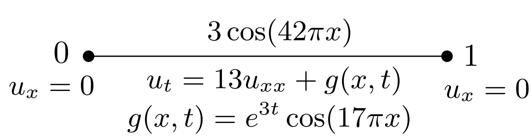


Figure 34: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, f, g];
 f[x] := 3*Cos[42*Pi*x];
 g[x, t] := Exp[3*t]*Cos[17*x*Pi];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + g[x, t];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 0};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{e^{3t}\cos(17\pi x)}{3+289\pi^2}-\frac{e^{-289\pi^2 t}\cos(17\pi x)}{3+289\pi^2}+3e^{-1764\pi^2 t}\cos(42\pi x)\right\}\right\}$$

Maple ✓

```
f:='f';g:='g';x:='x';y:='y';t:='t';
f := x->3*cos(42*x*Pi);
g :=(x,t)->exp(3*t)*cos(17*x*Pi);
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + g(x, t);
bc := (D[1](u))(0, t) = 0, (D[1](u))(1, t) = 0;
ic:= u(x, 0) = f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', simplify(pdsolve([pde, ic, bc],u(
```

$$u(x,t)=\frac{(867\,\pi^2+9)\cos(42\,\pi\,x)\,\mathrm{e}^{-1764\,\pi^2 t}+\cos(17\,\pi\,x)\left(\mathrm{e}^{3\,t}-\mathrm{e}^{-289\,\pi^2 t}\right)}{289\,\pi^2+3}$$

5.33 Pinchover and Rubinstein 6.21

problem number 52

added July 2, 2018.

Taken from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.21

Solve the heat equation on bar

$$u_t = u_{xx} + g(x,t)$$

Where $g(x,t) = t\cos(2001x)$ for $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = \pi\cos(2x)$.

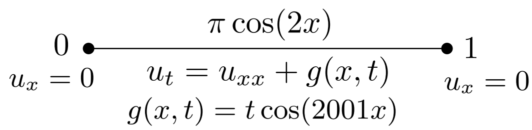


Figure 35: PDE specification

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + t*Cos[2001*x];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][Pi, t] == 0};
 ic = u[x, 0] == Pi*Cos[2*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \pi e^{-4t}\cos(2x)+\frac{e^{-4004001t}\cos(2001x)}{16032024008001}+\frac{t\cos(2001x)}{4004001}-\frac{\cos(2001x)}{16032024008001}\right\}\right\}$$

Maple ✓

```
x:='x';t:='t';
pde:= diff(u(x, t), t)= (diff(u(x, t), x, x)) + t*cos(2001*x);
bc := (D[1](u))(0, t) = 0, (D[1](u))(Pi, t) = 0;
ic:= u(x, 0) = Pi*cos(2*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t))),ou
```

$$u(x,t)=\frac{(4004001\,t+\mathrm{e}^{-4004001\,t}-1)\cos(2001\,x)}{16032024008001}+\cos(2\,x)\,\mathrm{e}^{-4\,t}\pi$$

5.34 nonhomogeneous BC

problem number 53

Added March 28, 2018. A problem from my PDE animation page.

Solve the heat equation

$$u_t = k u_{xx} + x$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = \frac{t \sin t}{5}$$
$$u(\pi,t) = \frac{t \cos t}{10}$$

Initial condition is $u(x,0) = 60 - 20x$.

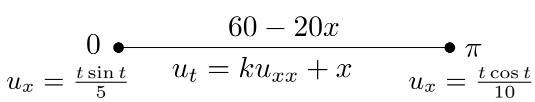


Figure 36: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, x];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + x;
 bc = {u[0, t] == (t*Sin[t])/5, u[Pi, t] == (t*Cos[t])/10};
 ic = u[x, 0] == 60 - 2*x;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {t
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';c:='c';
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+x;
ic  :=  u(x,0)=(60-2*x);
bc  :=  u(0,t)=t/5*sin(t), u(Pi,t)=t/10*cos(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = 1/10\,\frac{1}{\pi}\left(\cos\left(t\right)tx + 2\left(\pi - x\right)t\sin\left(t\right) + 10\sum_{n=1}^{\infty} 1/10\,\frac{1}{\pi\,n^3\left(n^4+1\right)^2}\left(40\left(\left(30\,n^{10} + 1/20\,n^8 + 60\right.\right.\right.\right.$$

5.35 With source

problem number 54

Taken from Maple PDE help pages

Solve the heat equation for $u(x,t)$

$$u_t = ku_{xx} + f(x,t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition is $u(x,0) = g(x)$

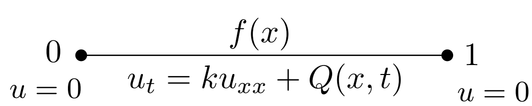


Figure 37: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, g, c];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + f[x, t];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {k
```

Failed

Maple ✓

```
u:='u'; t:='t'; x:='x';f:='f';k:='k';g:='g';
interface(showassumed=0);
pde:= diff(u(x, t), t) = k*(diff(u(x, t), x, x))+f(x, t);
bc := u(0, t) = 0, u(l, t) = 0;
ic:= u(x, 0) = g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t)) as
```

$$u(x,t) = \sum_{n=1}^{\infty}\left(2\,\frac{1}{l}\sin\left(\frac{n\pi\,x}{l}\right)\mathrm{e}^{-\frac{\pi^2 n^2 k t}{l^2}}\int_0^l g(x)\sin\left(\frac{n\pi\,x}{l}\right)\mathrm{d}x\right)+\int_0^t\sum_{n=1}^{\infty}\left(2\,\frac{1}{l}\sin\left(\frac{n\pi\,x}{l}\right)\mathrm{e}^{\frac{\pi^2 n^2 k(-t+\tau)}{l^2}}\int_0^l f(\right.$$

5.36 Haberman 8.3.6

problem number 55

Added Nov 25, 2018.

Problem 8.3.6 from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + \sin(5x)e^{-2t}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 1$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$

For hand solution see my HW9, Math 322, UW Madison.

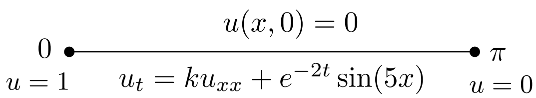


Figure 38: PDE specification

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + Sin[5*x]*Exp[-2*t];
 bc = {u[0, t] == 1, u[Pi, t] == 0};
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde:=diff(u(x,t),t)= diff(u(x,t),x$2)+ sin(5*x)*exp(-2*t);
ic:=u(x,0)=0;
bc := u(0,t) =1,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t))),outp
```

$$u(x,t) = 1/23\,\frac{1}{\pi}\left(23\sum_{n=1}^{\infty} -2\,\frac{\sin(nx)\,e^{-n^2t}}{n\pi}\pi + \pi\left(e^{-2\,t} - e^{-25\,t}\right)\sin(5\,x) - 23\,x + 23\,\pi\right)$$

Hand solution

This problem has nonhomogeneous B.C. and non-homogenous in the PDE itself (source present). First step is to use reference function to remove the nonhomogeneous B.C. then use the method of eigenfunction expansion on the resulting problem. Let

$$r(x) = c_1 x + c_2$$

At $x = 0, r(x) = 1$, hence $1 = c_2$ and at $x = \pi, r(x) = 0$, hence $0 = c_1\pi + 1$ or $c_1 = -\frac{1}{\pi}$, therefore

$$r(x) = 1 - \frac{x}{\pi}$$

Therefore

$$u(x,t) = v(x,t) + r(x)$$

Where $v(x,t)$ solution for the given PDE but with homogeneous B.C., therefore

$$\begin{aligned} v_t &= v_{xx} + e^{-2t}\sin 5x \qquad (1)\\ v(0,t) &= 0\\ v(\pi,t) &= 0\\ v(x,0) &= f(x)\\ &= u(x,0) - r(x)\\ &= 0 - \left(1 - \frac{x}{\pi}\right)\\ &= \frac{x}{\pi} - 1 \end{aligned}$$

We now solve (1). This is a PDE with homogeneous B.C. of the form $v_t = v_{xx} + Q(x,t)$. The general solution to above PDE was solved in **??** on page **??** and the solution is

$$v(x,t) = \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\frac{2}{L}\int_0^L f(s)\,\Phi_n(s)\,ds\right) + \sum_{n=1}^{\infty} e^{-k\lambda_n t}\Phi_n(x)\left(\int_0^t \frac{2}{L}e^{k\lambda_n\tau}\left(\int_0^L Q(s,\tau)\,\Phi_n(s)\,ds\right)d\tau\right) \qquad (2)$$

Where

$$\begin{aligned} \Phi_n(x) &= \sin\left(\sqrt{\lambda_n}x\right) \qquad (3)\\ \lambda_n &= \left(\frac{n\pi}{L}\right)^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

Replacing $L = \pi, f(x) = \frac{x}{\pi} - 1, Q(x,t) = e^{-2t}\sin(5x)$ into (3,2) gives

$$\begin{aligned} \Phi_n(x) &= \sin(nx) \qquad (3A)\\ \lambda_n &= n^2 \qquad n = 1,2,3,\cdots \end{aligned}$$

And

$$v(x,t) = \sum_{n=1}^{\infty} e^{-kn^2 t} \sin(nx) \left(\frac{2}{\pi} \int_0^{\pi} \left(\frac{s}{\pi} - 1 \right) \sin(ns)\, ds \right) + \sum_{n=1}^{\infty} e^{-kn^2 t} \sin(nx) \left(\int_0^t \frac{2}{\pi} e^{kn^2\tau} e^{-2\tau} \left(\int_0^{\pi} \sin(5s \tag{2A}$$

But $\int_0^{\pi} \left(\frac{s}{\pi} - 1\right) \sin(ns)\, ds = \frac{-1}{n}$ since n is integer. And $\int_0^{\pi} \sin 5s \sin(ns)\, ds = 0$ when $n \neq 5$ and for $n = 5$ it becomes $\frac{\pi}{2}$. Using these values in the above gives

$$v(x,t) = \sum_{n=1}^{\infty} e^{-kn^2 t} \sin(nx) \left(\frac{-2}{\pi n} \right) + e^{-k(5)^2 t} \sin(5x) \left(\int_0^t \frac{2}{\pi} e^{k(5)^2 \tau} e^{-2\tau} \left(\frac{\pi}{2} \right) d\tau \right) \tag{2C}$$

$$= -\frac{2}{\pi} \sum_{n=1}^{\infty} e^{-kn^2 t} \frac{\sin(nx)}{n} + e^{-25kt} \sin(5x) \left(\int_0^t e^{25k\tau} e^{-2\tau} d\tau \right) \tag{1}$$

But $\int_0^t e^{25k\tau} e^{-2\tau} d\tau = \frac{-1+e^{25kt-2t}}{25k-2}$ and the above becomes

$$\begin{aligned} v(x,t) &= -\frac{2}{\pi} \sum_{n=1}^{\infty} e^{-kn^2 t} \frac{\sin(nx)}{n} + e^{-25kt} \sin(5x) \left(\frac{-1 + e^{25kt-2t}}{25k-2} \right) \\ &= -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-kn^2 t} \sin(nx) + \sin(5x) \left(\frac{-e^{-25kt} + e^{-2t}}{25k-2} \right) \end{aligned}$$

Since $u(x,t) = v(x,t) + r(x)$ then the final solution is

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-kn^2 t} \sin(nx) \right) + \sin(5x) \left(\frac{-e^{-25kt} + e^{-2t}}{25k-2} \right)$$

Animation is below using $k = 1$, the solution becomes

$$u(x,t) = \left(1 - \frac{x}{\pi}\right) - \left(\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n^2 t} \sin(nx) \right) + \sin(5x) \left(\frac{e^{-2t} - e^{-25t}}{23} \right)$$

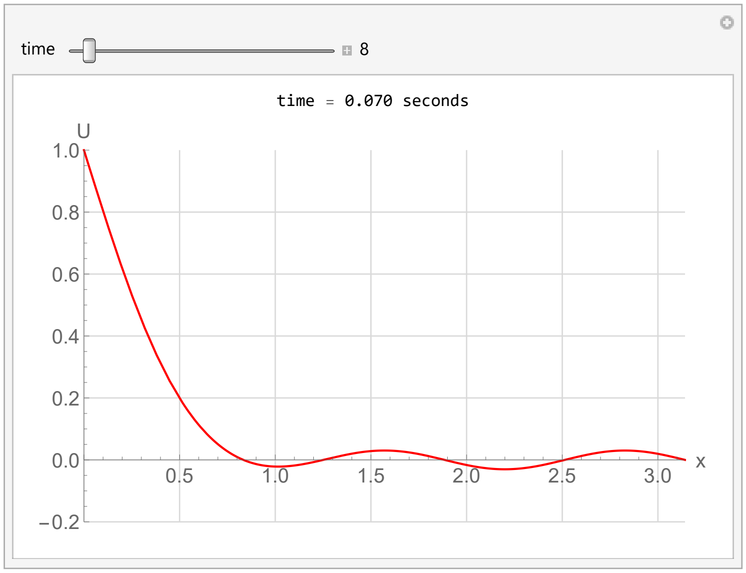


Figure 39: Initial state

Source code used for the above

```
ClearAll[x, t, n, f, A, B, s, mySol]
L = π;
k = 1;
f[x_] := 0;
Q[x_, t_] := Exp[-2 t] Sin[5 x];
ϕ[x_, n_] := Sin[n x];
λ = n^2;
numberOfTerms = 35;
padIt2[v_, f_List] := AccountingForm[v, f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
mySol[x_, t_] = N[(1 - x/π) - 2/π Sum[1/n Exp[-k λ t] ϕ[x, n] , {n, 1, numberOfTerms}]] + Sin[5 x] ((Exp[-2 t] - Exp[-25 k t])/(25 k - 2));
```

Figure 40: Source code

```
In[•]:= tab = Table[
          Grid[{
              {Row[{"time = ", padIt2[t, {4, 3}], " seconds"}]},
              {
                Quiet@Plot[Evaluate[mySol[x, t]], {x, 0, L},
                     BaseStyle → 15,
                     ImageMargins → 3,
                     PerformanceGoal → "Quality",
                     PlotRange → {{0, L}, {-.2, 1}},
                     ImageSize → 500,
                     AxesLabel → {"x", "U"},
                     GridLines → Automatic,
                     GridLinesStyle → LightGray,
                     PlotStyle → Red
                  ]

              }}],
          {t, 0, 2, .01}];

In[•]:= Manipulate[tab[[i]], {{i, 1, "time"}, 1, Length@tab, 1, Appearance → "Labeled"}]

In[•]:= Export["anim.gif", tab, "DisplayDurations" → 0.06]
```

Figure 41: Code for animation

5.37 Haberman 8.2.2. (a)

problem number 56

Added Nov 27, 2018.

Problem 8.2.2 part(a) from Richard Haberman applied partial differential equations book, 5th edition

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx} + Q(x,t)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = A(t)$$
$$\frac{\partial u}{\partial x}(L,t) = B(t)$$

Initial condition is $u(x,0) = f(x)$

For hand solution see my HW9, Math 322, UW Madison. The text does not actually asks to solve this PDE but only to reduce the problem to one with homogeneous B.C.

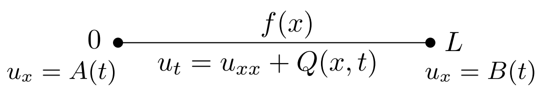


Figure 42: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, f, A, B, L, Q];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Q[x, t];
 bc = {Derivative[1, 0][u][0, t] == A[t], Derivative[1, 0][u][L, t] == B[t]};
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> L
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';A:='A';B:='B';k:='k';Q:='Q';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+Q(x,t);
ic  :=  u(x,0)=f(x);
bc  := eval( diff(u(x,t),x),x=0)=A(t), eval( diff(u(x,t),x),x=L)=B(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t) = 1/2\,\frac{1}{L^2}\left(2\,xA(t)\,L^2 - x^2A(t)\,L + x^2B(t)\,L + 2\sum_{n=1}^{\infty}\left(\frac{1}{L^2}\mathrm{e}^{\frac{k\pi^2n^2t}{L^2}}\int_0^L\left(-2\,\tau\,A(0)\,L + \tau^2A(0) - \tau^2B\right.\right.\right.$$

Hand solution

Solve

$$
\begin{aligned}
u_t &= k u_{xx} + Q(x,t) \\
u_x(0,t) &= A(t) \\
u_x(L,0) &= B(t) \\
u(x,0) &= f(x)
\end{aligned}
$$

Let

$$u(x,t) = v(x,t) + r(x,t) \tag{1}$$

Since the problem has time dependent source function $Q(x,t)$ then $r(x,t)$ is now a reference function that only needs to satisfy the non-homogenous boundary conditions which in this problem are at both ends and $v(x,t)$ has homogenous boundary conditions. The first step is to find $r(x,t)$. Let

$$r(x,t) = c_1(t)\,x + c_2(t)\,x^2$$

Then

$$\frac{\partial r(x,t)}{\partial x} = c_1(t) + 2c_2(t)\,x$$

At $x = 0$

$$A(t) = c_1(t)$$

And at $x = L$

$$
\begin{aligned}
B(t) &= c_1(t) + 2c_2(t)\,L \\
c_2(t) &= \frac{B(t) - c_1(t)}{2L}
\end{aligned}
$$

Solving for c_1, c_2 gives

$$r(x,t) = A(t)\,x + \left(\frac{B(t) - A(t)}{2L}\right) x^2 \tag{2}$$

Replacing (1) into the original PDE $u_t = k u_{xx} + Q(x,t)$ gives

$$
\begin{aligned}
\frac{\partial}{\partial t}(v(x,t) - r(x,t)) &= k\frac{\partial^2}{\partial x}(v(x,t) - r(x,t)) + Q(x,t) \\
\frac{\partial v}{\partial t} - \frac{\partial r}{\partial t} &= k\frac{\partial^2 v}{\partial x^2} - k\frac{\partial^2 r}{\partial x^2} + Q(x,t)
\end{aligned}
$$

But $r_{xx} = \frac{B(t)-A(t)}{L}$, hence the above reduces to

$$v_t = k v_{xx} + Q(x,t) - k\frac{B(t) - A(t)}{L} + r_t \tag{3}$$

Let

$$\tilde{Q}(x,t) = Q(x,t) + r_t - k\frac{B(t) - A(t)}{L}$$

Then (3) becomes

$$\begin{aligned} v_t &= kv_{xx} + \tilde{Q}(x,t) \qquad (4) \\ v_t(0,t) &= 0 \\ v_t(L,t) &= 0 \end{aligned}$$

And initial condition is

$$\begin{aligned} v(x,0) &= F(x) \\ &= u(x,0) - r(x,0) \\ &= f(x) - \left(A(0)\,x + \left(\frac{B(0) - A(0)}{2L}\right)x^2\right) \end{aligned}$$

PDE (4) with its homogenous boundary conditions is standard one, its corresponding eigenvalue boundary value ODE $X'' + \lambda X = 0$ has $\lambda = 0$ as eigenvalue with corresponding eigenfunction $\Phi_0(x) = 1$ and $\lambda_n = \left(\frac{n\pi}{L}\right)^2$ for $n = 1,2,3,\cdots$ with corresponding eigenfunctions $\Phi_n(x) = \cos\left(\sqrt{\lambda_n}x\right)$. Using these, we can write the solution to (4) using eigenfunction expansion as

$$v(x,t) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n(x) \qquad (4A)$$

Hence $v_t(x,t) = \sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x)$ and $v_{xx}(x,t) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x)$. Substituting these into (4) gives

$$\sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x) + \tilde{Q}(x,t)$$

Expanding $\tilde{Q}(x,t)$ using same eigenfunctions since they are complete, the above becomes

$$\sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} c_n(t)\,\Phi_n''(x) + \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)$$

But $\Phi_n''(x) = -\lambda_n\Phi_n(x)$ and the above becomes

$$\begin{aligned} \sum_{n=0}^{\infty} c_n'(t)\,\Phi_n(x) &= -\sum_{n=0}^{\infty} c_n(t)\,\lambda_n\Phi_n(x) + \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x) \\ c_n'(t)\,\Phi_n(x) + c_n(t)\,\lambda_n\Phi_n(x) &= b_n(t)\,\Phi_n(x) \\ c_n'(t) + c_n(t)\,\lambda_n &= b_n(t) \\ c_n'(t) + c_n(t)\,\frac{n^2\pi^2}{L^2} &= b_n(t) \qquad (5) \end{aligned}$$

To find $b_n(t)$, since $\tilde{Q}(x,t) = Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L}$ then

$$Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L} = \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)$$

Multiplying both sides by $\Phi_m(x)$ and integrating gives

$$\begin{aligned}\int_0^L \left(Q(x,t) + \frac{\partial r}{\partial t} - k\frac{B(t)-A(t)}{L}\right)\Phi_m(x)\,dx &= \int_0^L \sum_{n=0}^{\infty} b_n(t)\,\Phi_n(x)\,\Phi_m(x)\,dx\\ &= \sum_{n=0}^{\infty} b_n(t)\left(\int_0^L \Phi_n(x)\,\Phi_m(x)\,dx\right)\end{aligned}$$

By orthogonality

$$\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\Phi_m(x)\,dx = b_m(t)\int_0^L \Phi_m^2(x)\,dx$$

When $m=0, \Phi_0(x)=1$ and the above gives

$$\begin{aligned}\int_0^L Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}dx &= b_0(t)\int_0^L dx\\ b_0(t) &= \frac{1}{L}\int_0^L Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}dx\end{aligned}$$

When $m = 1,2,3,\cdots$

$$\begin{aligned}\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx &= b_m(t)\int_0^L \cos^2\left(\frac{m\pi}{L}x\right)dx\\ \int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right)dx &= b_m(t)\,\frac{L}{2}\\ b_m(t) &= \frac{2}{L}\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{m\pi}{L}x\right) d\end{aligned}$$

Therefore (5) is now solved. When $n=0$ (5) becomes

$$\begin{aligned}c_0'(t) + c_0(t)\,\frac{n^2\pi^2}{L^2} &= b_0(t)\\ c_0'(t) &= b_0(t)\\ c_0'(t) &= \frac{1}{L}\int_0^L Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}dx\end{aligned}$$

Hence

$$c_0(t) = \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0$$

For $n=1,2,3,\cdots$ (5) becomes

$$
\begin{aligned}
c_n'(t) + c_n(t)\,\frac{n^2\pi^2}{L^2} &= b_n(t)\\
&= \frac{2}{L}\int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx
\end{aligned}
$$

Integrating factor is $I = e^{\int \frac{n^2\pi^2}{L^2}dt} = e^{\frac{n^2\pi^2}{L^2}t}$ and the solution to the above becomes

$$
\begin{aligned}
\frac{d}{dt}\left(c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t}\right) &= \frac{2e^{\frac{n^2\pi^2}{L^2}t}}{L}\int_0^L \left(Q(x,t) + r_t - k\frac{B(t)-A(t)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\\
c_n(t)\,e^{\frac{n^2\pi^2}{L^2}t} &= \int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n\\
c_n(t) &= e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}
\end{aligned}
$$

Now that we found $c_n(t)$ for $n=0,1,2,3,\cdots$ the solution for $v(x,t)$ is found from 4A.

$$
\begin{aligned}
v(x,t) &= \sum_{n=0}^{\infty} c_n(t)\,\Phi_n(x)\\
&= \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0 + \sum_{n=1}^{\infty} c_n(t)\,\Phi_n(x)\\
&= \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0\\
&+ \sum_{n=1}^{\infty}\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)\cos
\end{aligned}
$$

But

$$
u(x,t) = v(x,t) + r(x,t)
$$

Hence

$$
\begin{aligned}
u(x,t) &= A(t)\,x + \left(\frac{B(t)-A(t)}{2L}\right)x^2 + \int_0^t \left(\frac{1}{L}\int_0^L Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}dx\right)dt + C_0\\
&+ \sum_{n=1}^{\infty}\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t \left(\frac{2e^{\frac{n^2\pi^2}{L^2}\tau}}{L}\int_0^L \left(Q(x,\tau) + r_\tau - k\frac{B(\tau)-A(\tau)}{L}\right)\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)\cos
\end{aligned}
$$

But

$$
\begin{aligned}
r_\tau &= \frac{d}{dt}\left(A(\tau)\,x + \left(\frac{B(\tau)-A(\tau)}{2L}\right)x^2\right)\\
&= \frac{2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2}{2L}
\end{aligned}
$$

Hence

$$
\begin{gathered}
u(x,t) = C_0 + A(t)\,x + \left(\frac{B(t)-A(t)}{2L}\right)x^2\\
+\frac{1}{2L^2}\int_0^t\left(\int_0^L 2LQ(x,\tau) + 2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2 - 2k(B(\tau)-A(\tau))\,dx\right)dt+\\
\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}x\left(e^{-\frac{n^2\pi^2}{L^2}t}\int_0^t\left(\frac{e^{\frac{n^2\pi^2}{L^2}\tau}}{L^2}\int_0^L (2LQ(x,\tau) + 2LA'(\tau)\,x + (B'(\tau)-A'(\tau))\,x^2 - 2k(B(\tau)-A(\tau)))\cos\left(\frac{n\pi}{L}x\right)dx\right)dt + C_n e^{-\frac{n^2\pi^2}{L^2}t}\right)
\end{gathered}
$$

The constants C_0, C_n are found from initial conditions $u(x,0) = f(x)$.

5.38 Articolo 8.4.1

problem number 57

Added December 20, 2018.

Example 8.4.1 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$
\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}
$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(0,t) &= 10\\
u(1,t) &= 20
\end{aligned}
$$

Initial condition is $u(x,0) = 60x - 50x^2 + 10$ and $k = \frac{1}{20}$

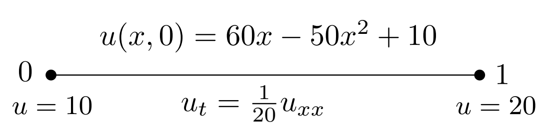


Figure 43: PDE specification

Mathematica ✓

```
ClearAll[u, x, t, k, n];
 k = 1/20;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 10, u[1, t] == 20};
 ic = u[x, 0] == 60*x - 50*x^2 + 10;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{200(-1+(-1)^n)\,e^{-\frac{1}{20}n^2\pi^2t}\sin(n\pi x)}{n^3\pi^3}+10x+10\right\}\right\}$$

Maple ✓

```
u:='u'; t:='t'; x:='x';
k := 1/20;
pde := diff(u(x,t),t)= k*diff(u(x,t),x$2);
bc := u(0, t) = 10, u(1, t) = 20;
ic := u(x, 0) = 60*x - 50*x^2 + 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

$$u(x,t)=10+10\,x+\sum_{n=1}^{\infty}-200\,\frac{(-1+(-1)^n)\sin(n\pi\,x)\,\mathrm{e}^{-1/20\,\pi^2n^2t}}{n^3\pi^3}$$

5.39 Articolo 8.4.3

problem number 58

Added December 20, 2018.

Example 8.4.3 from Partial differential equations and boundary value problems with Maple by George A. Articolo, 2nd ed.

Solve the heat equation for $u(x,t)$

$$u_t=ku_{xx}+t$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 5$$
$$u(1,t) + \frac{\partial u}{\partial x}(1,t) = 10$$

Initial condition is $u(x,0) = \frac{-40x^2}{3} + \frac{45x}{2} + 5$ and $k = \frac{1}{20}$

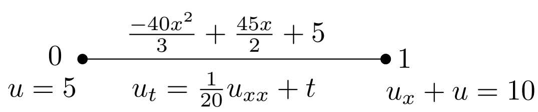


Figure 44: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, k, n];
 k = 1/20;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + t;
 bc = {u[0, t] == 5, u[1, t] + Derivative[1, 0][u][1, t] == 10};
 ic = u[x, 0] == (-40*x^2)/3 + (45*x)/2 + 5;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✗

```

u:='u'; t:='t'; x:='x';
pde := diff(u(x, t), t) = (1/20)*(diff(u(x, t), x$2))+t;
bc := u(0, t) = 5, (u(1, t)+ eval( diff(u(x,t),x),x=1)) = 10;
ic:= u(x, 0) = -40*x^2/3+45*x/2+5;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic], u(x, t)),c
```

Exception

5.40 both ends insulated

problem number 59

Added December 20, 2018.

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve the heat equation for $u(x,t)$

$$u_t = 13u_{xx}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$

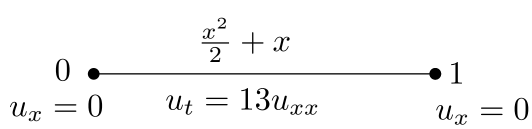


Figure 45: PDE specification

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == 13*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
 ic = u[x, 0] == (1*x^2)/2 + x;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
u:='u'; t:='t'; x:='x';
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x$2));
bc := eval( diff(u(x,t),x),x=0)=0 , eval( diff(u(x,t),x),x=1)=1;
ic := u(x,0)=1/2*x^2+x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t))),out
```

$$u(x,t) = 1/2 + \sum_{n=1}^{\infty} 2\,\frac{\cos(n\pi\,x)\,\mathrm{e}^{-13\,\pi^2 n^2 t}(-1+(-1)^n)}{\pi^2 n^2} + 13\,t + 1/2\,x^2$$

5.41 both ends nonhomogeneous

problem number 60

Added January 18, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = u_{xx}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u_x(0,t) = 1$$
$$u_x(1,t) = -1$$

Initial condition is $u(x,0) = \sin(x)$

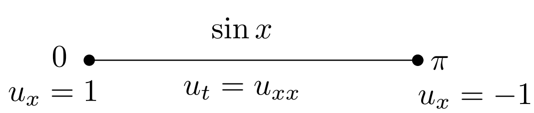


Figure 46: PDE specification

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + (x - (1*x^2)/Pi);
 ic = u[x, 0] == Sin[x];
 bc = {Derivative[1, 0][u][0, t] == 1, Derivative[1, 0][u][Pi, t] == -1};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u'; t:='t'; x:='x';
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic := u(x, 0) = sin(x):
bc := eval(diff(u(x,t),x),x=0)=1,  eval( diff(u(x,t),x),x=Pi)=-1:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

$$u(x,t) = 1/6\,\frac{1}{\pi}\left(-\pi^2 + 6\sum_{n=2}^{\infty} -2\,\frac{\cos(nx)\,\mathrm{e}^{-n^2t}((-1)^n+1)}{n^2(n^2-1)\,\pi}\pi + 6\,\pi\,x - 6\,x^2 - 12\,t + 12\right)$$

Hand solution

Since the boundary conditions are not homogeneous, we can't use separation of variables. Let the solution be

$$u = v(x,t) + r(x)$$

Where $v(x,t)$ is the solution to $v_t = v_{xx}$ and homogenous B.C. $v_x(0,t) = 0, v_x(\pi,t) = 0$ and $r(x)$ is any reference solution which only needs to satisfy the nonhomogeneous boundary conditions: $r'(0) = 1, r'(\pi) = -1$. By guessing, let $r(x) = Ax + Bx^2$. Let see if this satisfies the boundary conditions. $r' = A + 2Bx$. At $x = 0$ this implies $1 = A$. Hence $r = x + Bx^2$. Now $r' = 1 + 2Bx$. At $x = \pi$ this gives $-1 = 1 + 2B\pi$ or $B = -\frac{1}{\pi}$. Therefore

$$r(x) = x - \frac{1}{\pi}x^2$$

Substituting $u = v(x,t) + r(x)$ into the PDE $u_t = u_{xx}$ and noting that $r''(x) = -\frac{2}{\pi}$ gives

$$v_t = v_{xx} - \frac{2}{\pi} \tag{1}$$

PDE (1) is now solved using eigenfunction expansion. We need to find eigenfunctions and eigenvalues of $v_t = v_{xx}$ with $v_x(0,t) = 0, v_x(\pi,t) = 0$. This is known PDE and have eigenfunctions and eigenvalues as follows. For zero eigenvalue, the eigenfunction is an arbitrary constant. Say β. let $\beta = 1$ since scale is not important.

$$\Phi_0(x) = 1$$

And for $n = 1, 2, 3, \cdots$

$$\begin{aligned}\Phi_n(x) &= \cos\left(\sqrt{\lambda_n} x\right)\\ &= \cos(nx)\end{aligned}$$

with eigenvalues $\lambda_n = n^2$ for $n = 1, 2, 3, \cdots$. Now we can eigenfunction expansion and assume the solution to (1) is

$$v(x,t) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n(x) \tag{2}$$

Plugging this into the PDE (1) gives

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = \sum_{n=0}^{\infty} A_n(t)\,\Phi_n''(x) - \frac{2}{\pi}$$

But $\Phi_n''(x) = -\lambda_n \Phi_n(x)$ and the above simplifies to

$$\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) = -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \frac{2}{\pi}$$

Since eigenfunctions are complete, we can expand $\frac{2}{\pi}$ using them and the above becomes

$$\begin{aligned}\sum_{n=0}^{\infty} A_n'(t)\,\Phi_n(x) &= -\sum_{n=0}^{\infty} A_n(t)\,\lambda_n \Phi_n(x) - \sum_{n=0}^{\infty} C_n \Phi_n(x)\\ A_n'(t)\,\Phi_n(x) + A_n(t)\,\lambda_n \Phi_n(x) &= -C_n \Phi_n(x)\\ A_n'(t) + A_n(t)\,\lambda_n &= -C_n \qquad (3)\end{aligned}$$

To find C_n

$$\sum_{n=0}^{\infty} C_n \Phi_n(x) = \frac{2}{\pi}$$

For $n = 0$

$$C_0 \Phi_0(x) = \frac{2}{\pi}$$

But $\Phi_0(x) = 1$, hence

$$C_0 = \frac{2}{\pi}$$

All other C_m for $m > 0$ are zero. Hence (3) becomes, for $n = 0$ (since $\lambda_0 = 0$)

$$A_0'(t) = -\frac{2}{\pi}$$
$$A_0(t) = -\frac{2}{\pi}t + B_0$$

Where B_0 is integration constant. For $n > 0$ (3) becomes

$$A_n'(t) + A_n(t)\, n^2 = 0$$

This has the solution

$$A_n(t) = B_n e^{-n^2 t}$$

Where B_n is constant of integration. Hence from (2)

$$\begin{aligned} v(x,t) &= \sum_{n=0}^{\infty} A_n(t)\, \Phi_n(x) \\ &= A_0(t) + \sum_{n=1}^{\infty} A_n(t)\, \Phi_n(x) \\ &= -\frac{2}{\pi}t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t} \cos(nx) \end{aligned}$$

Since $u = v(x,t) + r(x)$ then the solution becomes

$$u(x,t) = \left(x - \frac{1}{\pi}x^2\right) - \frac{2}{\pi}t + B_0 + \sum_{n=1}^{\infty} B_n e^{-n^2 t} \cos(nx) \tag{4}$$

At $t = 0$

$$\sin(x) = \left(x - \frac{1}{\pi}x^2\right) + B_0 + \sum_{n=1}^{\infty} B_n \cos(nx) \tag{5}$$

case $n = 0$

$$\int_0^\pi \sin(x) \cos\left(\sqrt{\lambda_0}x\right) dx = \int_0^\pi \left(x - \frac{1}{\pi}x^2\right) \cos\left(\sqrt{\lambda_0}x\right) dx + \int_0^\pi B_0 \cos\left(\sqrt{\lambda_0}x\right) dx$$

But $\lambda_0 = 0$ hence

$$\begin{aligned} \int_0^\pi \sin(x)\, dx &= \int_0^\pi \left(x - \frac{1}{\pi}x^2\right) dx + \int_0^\pi B_0 dx \\ 2 &= \frac{\pi^2}{6} + B_0 \pi \\ B_0 &= \frac{2}{\pi} - \frac{\pi}{6} \end{aligned}$$

For $n>0$, Multiplying both sides of (5) by $\cos(mx)$ and integrating

$$\int_0^\pi \sin(x)\cos(mx)\,dx = \int_0^\pi \left(x-\frac{1}{\pi}x^2\right)\cos(mx)\,dx + \sum_{n=1}^\infty B_n \int_0^\pi \cos(nx)\cos(mx)\,dx$$

For $m=1$

$$\begin{aligned} 0 &= 0 + B_1\frac{\pi}{2} \\ B_1 &= 0 \end{aligned}$$

For $m>1$

$$\begin{aligned} -\frac{1+(-1)^m}{m^2(-1+m^2)} &= \frac{\pi}{2}B_m \\ B_m &= \frac{-2}{\pi}\left(\frac{1}{m^2}\frac{(-1)^m+1}{m^2-1}\right) \end{aligned}$$

Hence solution (4) becomes

$$\begin{aligned} u(x,t) &= \left(x-\frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=1}^\infty B_n e^{-n^2 t}\cos(nx) \\ u(x,t) &= \left(x-\frac{1}{\pi}x^2\right) - \frac{2}{\pi}t - \frac{\pi}{6} + \frac{2}{\pi} + \sum_{n=2}^\infty \frac{-2}{\pi}\left(\frac{1}{n^2}\frac{(-1)^n+1}{n^2-1}\right)e^{-n^2 t}\cos(nx) \end{aligned}$$

5.42 nonhomogeneous BC

problem number 61

Added March 31, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{xx}$$

For $0<x<\pi$ and $t>0$. The boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ u_x(\pi,t) &= A \end{aligned}$$

Initial condition is $u(x,0)=0$

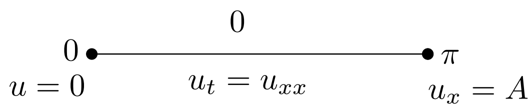


Figure 47: PDE specification

Mathematica ✗

```
ClearAll[u, x, t,A];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] ;
 ic = u[x, 0] == 0;
 bc = {u[0,t] == 0, Derivative[1, 0][u][Pi, t] == A};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t},Assumptions->A>0
```

Failed

Maple ✓

```
u:='u'; t:='t'; x:='x';A:='A';
pde := diff(u(x, t), t) = diff(u(x, t), x$2):
ic := u(x, 0) = 0:
bc := u(0,t)=0,  eval( diff(u(x,t),x),x=Pi)=A:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t) = \sum_{n=0}^{\infty} 8 \frac{\sin\left(nx + x/2\right) e^{-1/4\,(1+2\,n)^2 t} (-1)^{1+n} A}{\pi\, (1+2\,n)^2} + Ax$$

5.43 nonhomogeneous BC

problem number 62

Added April 15, 2019.

Solve the heat equation for $u(x,t)$

$$u_t = k u_{rr}$$

For $0 < r < a$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(a,t) = a\phi(t)$$

Initial condition is $u(r,0) = rf(r)$

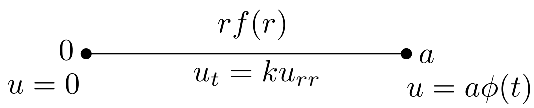


Figure 48: PDE specification

Mathematica ✗

```
ClearAll[u, x, t,A];
 pde = D[u[r, t], t] == k*D[u[r, t], {r, 2}] ;
 ic = u[r, 0] == r*f[r];
 bc = {u[0, t] == 0, u[a, t] == a*phi[t]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
```

Failed

Maple ✓

```
unassign('u,t,r,k,f,phi');
pde := diff(u(r, t), t) = k*diff(u(r, t), r$2):
ic:=u(r,0)=r*f(r);
bc:=u(0,t)=0,u(a,t)=a*phi(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(r, t))),
```

$$u(r,t) = \sum_{n=1}^{\infty}\left(-2\frac{1}{a}\sin\left(\frac{n\pi r}{a}\right)e^{-\frac{k\pi^2 n^2 t}{a^2}}\int_0^a \tau\left(\phi(0) - f(\tau)\right)\sin\left(\frac{n\pi \tau}{a}\right)d\tau\right) + \int_0^t \sum_{n=1}^{\infty} 2\frac{(-1)^n a\frac{d}{d\tau}\phi(\tau)}{n\pi}\sin\left(\right.$$

<u>Hand solution</u>

Solve

$$u_t = ku_{rr} \qquad t > 0, 0 < r < a \tag{1}$$

With boundary conditions

$$\begin{aligned}u(0,t)&=0\\u(a,t)&=a\phi(t)\end{aligned}$$

And initial conditions

$$u(r,0)=rf(r)$$

Since the boundary conditions are not homogeneous, the first step is to convert them to homogeneous. This is done using a reference function which needs to only satisfy the boundary conditions. This reference function can be seen to be $v(r,t)=r\phi(t)$. Now we write

$$u(r,t)=w(r,t)+v(r,t)$$

Where $w(r,t)$ satisfies the PDE but with homogeneous B.C. Substituting the above into (1) gives

$$\begin{aligned}w_t(r,t)+r\phi'(t)&=kw_{rr}\\w_t(r,t)&=kw_{rr}-r\phi'(t)\end{aligned}\tag{2}$$

With boundary conditions

$$\begin{aligned}w(0,t)&=0\\w(a,t)&=0\end{aligned}$$

The solution to the homogeneous PDE $w_t(r,t)=kw_{rr}$ with the above boundary conditions is easily found and known. The eigenvalues are $\lambda_n=\left(\frac{n\pi}{a}\right)^2, n=1,2,\cdots$ and eigenfunctions $\Phi_n(r)=\sin\left(\sqrt{\lambda_n}r\right)$. Let the solution to (2), using eigenfunction expansion be

$$w(r,t)=\sum_{n=1}^{\infty}C_n(t)\,\Phi_n(r)\tag{2A}$$

Substituting the above back into (2) gives

$$\sum_{n=1}^{\infty}C_n'(t)\,\Phi_n(r)=k\sum_{n=1}^{\infty}C_n(t)\,\Phi_n''(r)-\sum_{n=1}^{\infty}q_n(t)\,\Phi_n(r)\tag{3}$$

Where $q_n(t)$ are the Fourier coefficients of $r\phi'(t)$ which are found by

$$r\phi'(t)=\sum_{n=1}^{\infty}q_n(t)\,\Phi_n(r)$$

Applying orthogonality using $\Phi_n(r)$ gives

$$\begin{aligned}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr &= \int_0^a \sum_{n=1}^{\infty} q_n(t)\,\Phi_n(r)\,\Phi_m(r)\,dr\\ &= \sum_{n=1}^{\infty} q_n(t)\int_0^r \Phi_n(r)\,\Phi_m(r)\,dr\end{aligned}$$

But $\int_0^a \Phi_n(r)\,\Phi_m(r)\,dr = \int_0^a \sin\left(\frac{n\pi}{a}r\right)\sin\left(\frac{m\pi}{a}r\right)dr = \frac{a}{2}$ for $n=m$ only, and the above becomes

$$\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(s)\,dr = q_m(t)$$

Substituting the above back into (3) gives

$$\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) = k\sum_{n=1}^{\infty} C_n(t)\,\Phi_n''(r) - \sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)$$

But $\Phi_n''(r) = -\lambda_n\Phi_n(r)$ and above simplifies to

$$\begin{aligned}\sum_{n=1}^{\infty} C_n'(t)\,\Phi_n(r) + k\sum_{n=1}^{\infty} C_n(t)\,\lambda_n\Phi_n(r) &= -\sum_{n=1}^{\infty}\left(\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\right)\Phi_n(r)\\ C_n'(t) + kC_n(t)\,\lambda_n &= -\frac{2}{a}\int_0^a r\phi'(t)\,\Phi_m(r)\,dr\\ &= -\frac{2}{a}\phi'(t)\int_0^a r\sin\left(\frac{n\pi}{a}r\right)dr\\ &= -\frac{2}{a}\phi'(t)\,\frac{(-1)^{n+1}\,a^2}{n\pi}\\ &= -2a\phi'(t)\,\frac{(-1)^{n+1}}{n\pi}\end{aligned}$$

This is first order ODE in $C(t)$. The solution is

$$C_n(t) = e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau$$

From (2A)

$$w(r,t) = \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right)$$

Hence

$$\begin{aligned}u(r,t) &= w(r,t) + v(r,t)\\ &= \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}C_n(0) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\left(\frac{n\pi}{a}r\right) + r\phi(t)\end{aligned} \tag{4}$$

At $t=0$ the above becomes

$$rf(r) = \sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) + r\phi(0)$$

$$\sum_{n=1}^{\infty} C_n(0)\sin\left(\frac{n\pi}{a}r\right) = r(f(r)-\phi(0))$$

Hence $C_n(0)$ is the Fourier sine coefficients of $r(f(r)-\phi(0))$

$$\begin{aligned}\frac{a}{2}C_n(0) &= \int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\\ C_n(0) &= \frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\end{aligned}$$

Substituting this into (4) gives the final solution as

$$\begin{aligned}u(r,t) &= r\phi(t) + \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right) + 2ae^{-k\lambda_n t}\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{k\lambda_n\tau}d\tau\right)\sin\Bigg(\\ &= r\phi(t) + \sum_{n=1}^{\infty}\left(e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right) + 2a\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\right)\sin\Bigg(\frac{n\pi}{a}\\ &= r\phi(t) + \sum_{n=1}^{\infty}e^{-k\lambda_n t}\left(\frac{2}{a}\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\sin\left(\frac{n\pi}{a}r\right) + \sum_{n=1}^{\infty}2a\frac{(-1)^{n+1}}{n\pi}\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}\end{aligned}$$

Or

$$\begin{aligned}u(r,t) &= r\phi(t)\\ &\quad+\frac{2}{a}\sum_{n=1}^{\infty}e^{-k\lambda_n t}\sin\left(\frac{n\pi}{a}r\right)\left(\int_0^a r(f(r)-\phi(0))\sin\left(\frac{n\pi}{a}r\right)dr\right)\\ &\quad+\frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi}{a}r\right)\int_0^t \phi'(\tau)\,e^{-k\lambda_n(t-\tau)}d\tau\end{aligned}$$

Where $\lambda_n = \left(\frac{n\pi}{a}\right)^2$.

5.44 Euler-Cauchy Sturm-Liouville

problem number 63

Added April 20, 2019.

Solve the parabolic pde for $u(x,t)$

$$u_t = x^2u_{xx} + xu_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$u_x(1,t) = 0$$
$$hu(b,t) + u_x(b,t) = 0$$

Where $h > 0$. Initial condition is $u(x,0) = \ln x$

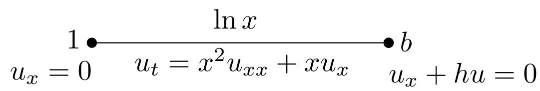


Figure 49: PDE specification

Mathematica ✗

```
ClearAll[u, x, t,h,b];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic = u[x, 0] == Log[x];
bc = {Derivative[1, 0][u][1, t] == 0, h*u[b, t] + Derivative[1, 0][u][b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

Failed

Maple ✓

```
unassign('u,t,x,h,b');
pde:=diff(u(x,t),t)=x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc:=eval( diff(u(x,t),x),x=1)=0,h*u(b,t)+eval( diff(u(x,t),x),x=b)=0;
ic:=u(x,0)=ln(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

Bad Latex generated

Hand solution

Solve

$$u_t = x^2 u_{xx} + x u_x \tag{1}$$

With $1 < x < b, t > 0$. BC

$$u_x(1,t) = 0$$
$$hu(b,t) + u_x(b,t) = 0$$

Where $h > 0$. And initial conditions

$$u(x,0) = \ln x$$

Let $u = X(x)T(x)$. Substituting into (1) gives

$$\begin{aligned} T'X &= x^2 X'' T + x X' T \\ \frac{T'}{T} &= x^2 \frac{X''}{X} + x\frac{X'}{X} = -\lambda \end{aligned}$$

Where λ is the separation constant. From the boundary conditions, we know that λ will be only positive. So we do not need to check for possibility of negative of zero eigenvalue. Letting $\lambda = \alpha^2$, then the above reduces to

$$x^2 X'' + x X' + \alpha^2 X = 0 \tag{2}$$

$$T' + \alpha^2 T = 0 \tag{3}$$

Equation (2) is Euler ODE. Assuming $X = x^r$, then $X' = r x^{r-1}, X'' = r(r-1)\,x^{r-2}$. Substituting back into (2) gives the characteristic equation

$$\begin{aligned} r(r-1)\,x^r + r x^r + \alpha^2 x^r &= 0 \\ r^2 + \alpha^2 &= 0 \\ r &= \pm i\alpha \end{aligned}$$

Hence the solution to (2) is

$$\begin{aligned} X(x) &= A x^{i\alpha} + B x^{-i\alpha} \\ &= A e^{\ln x^{i\alpha}} + B e^{\ln x^{-i\alpha}} \\ &= A e^{i\alpha \ln x} + B e^{-i\alpha \ln x} \end{aligned}$$

Which using Euler relation can be written as (using new constants, but the name of the constants kept the same for simplicity)

$$X(x) = A\cos(\alpha \ln x) + B\sin(\alpha \ln x)$$

Applying first BC. $X'(1,t) = 0$ gives

$$\begin{aligned} X'(x) &= -\frac{\alpha}{x} A\sin(\alpha \ln x) + B\frac{\alpha}{x}\cos(\alpha \ln x) \\ 0 &= -\alpha A \sin(\alpha \ln 1) + B\alpha\cos(\alpha \ln 1) \\ &= B\alpha \end{aligned}$$

Since $\alpha > 0$ then $B = 0$. Hence the solution becomes

$$X(x) = A\cos(\alpha \ln x)$$

Applying second BC. $hX(b) + X'(b) = 0$ gives

$$\begin{aligned} hA\cos(\alpha \ln b) - A\frac{\alpha}{b}\sin(\alpha \ln b) &= 0 \\ h - \frac{\alpha}{b}\tan(\alpha \ln b) &= 0 \\ \tan(\alpha \ln b) &= \frac{hb}{\alpha} \end{aligned} \tag{4}$$

There is no analytical solution to the above. The eigenvalues α_n are the solutions to the above nonlinear equation. Therefore the eigenfunctions are

$$X_n(x) = \cos(\alpha_n \ln x)$$

With eigenvalues $\alpha_n > 0$ given by solutions to (4). The solution to the time ODE is

$$\begin{aligned} T_n' + \alpha_n^2 T_n &= 0 \\ T_n(t) &= T_n(0)\, e^{-\alpha_n^2 t} \end{aligned}$$

Hence the solution to (1) is

$$u(x,t) = \sum_{n=1}^{\infty} T_n(0)\, e^{-\alpha_n^2 t} \cos(\alpha_n \ln x) \tag{5}$$

At $t = 0$

$$\ln x = \sum_{n=1}^{\infty} T_n(0) \cos(\alpha_n \ln x)$$

Applying orthogonality gives

$$\begin{aligned} \int_1^b \ln x \cos(\alpha_n \ln x)\, dx &= T_n(0) \int_1^b \cos^2(\alpha_n \ln x)\, dx \\ T_n(0) &= \frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx} \end{aligned}$$

Hence the solution (5) becomes

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{\int_1^b \ln x \cos(\alpha_n \ln x)\, dx}{\int_1^b \cos^2(\alpha_n \ln x)\, dx} \right) e^{-\alpha_n^2 t} \cos(\alpha_n \ln x)$$

5.45 special initial condition

problem number 64

Added April 28, 2019.

Taken from `https://mathematica.stackexchange.com/questions/197155/solving-a-heat-equation-problem`

Solve $u(x,t)$

$$u_t = u_{xx} - 9u_x$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition $u(x,0) = e^{\frac{45}{10}}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$

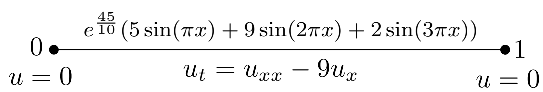


Figure 50: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - 9*D[u[x, t], x];
ic = u[x, 0] == Exp[45/10 x]*(5 Sin[Pi*x] + 9 Sin[2*Pi*x] + 2 Sin[3*Pi*x]);
bc = {u[0, t] == 0, u[1, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to 4e^{9x/2}\sin(\pi x)\left(\frac{9}{2}e^{-\frac{1}{4}(81+16\pi^2)t}\cos(\pi x) + \frac{1}{2}e^{-\frac{1}{4}(81+36\pi^2)t}(2\cos(2\pi x)+1) + \frac{5}{4}e^{-\frac{1}{4}(81+4\pi^2)t}\right)\right\}\right.$$

Maple ✗

```
unassign('u,t,x');
pde:= diff(u(x,t),t)= diff(u(x, t), x$2) - 9*diff(u(x,t),x);
bc:=u(0,t)=0,u(1,t)=0;
ic:=u(x, 0) = exp(45/10*x)*(5*sin(Pi*x) + 9*sin(2*Pi*x) + 2*sin(3*Pi*x));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t))),
```

server hangs

Hand solution

Solve

$$u_t = u_{xx} - 9u_x$$

IC

$$u(x,0) = e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x))$$

BC

$$u(0,x) = 0$$
$$u(1,x) = 0$$

Let $u = X(x)T(t)$, the PDE becomes

$$T'X = X''T - 9X'T$$
$$\frac{T'}{T} = \frac{X''}{X} - 9\frac{X'}{X} = -\lambda$$

Where λ is the seperation constant. From B.C. we know $\lambda > 0$. Hence the eigenvalue ODE is

$$X'' - 9X' + \lambda X = 0$$

The solution to the above is

$$X(x) = C_1 e^{\frac{1}{2}(9-\sqrt{81-4\lambda})x} + C_2 e^{\frac{1}{2}(9+\sqrt{81-4\lambda})x}$$
$$= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)$$

At $X(0) = 0$ this gives

$$0 = C_1 + C_2$$

And at $X(1)=0$

$$\begin{aligned}
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} + C_2 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= e^{\frac{9}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)\\
0 &= C_1 e^{\frac{9}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}}\right)
\end{aligned}$$

For nontrivial solution we want

$$\begin{aligned}
e^{-\frac{1}{2}\sqrt{81-4\lambda}} - e^{\frac{1}{2}\sqrt{81-4\lambda}} &= 0\\
e^{-\frac{1}{2}\sqrt{81-4\lambda}} &= e^{\frac{1}{2}\sqrt{81-4\lambda}} \qquad (1)
\end{aligned}$$

<u>Case $81-4\lambda>0$</u>

This means $81-4\lambda$ must be zero or

$$\lambda = \frac{81}{4}$$

But using this eigenvalue makes the eigenfunction zero as shown below

$$\begin{aligned}
X(x) &= e^{\frac{9x}{2}}\left(C_1 e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - C_1 e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}\left(e^{-\frac{1}{2}\sqrt{81-4\lambda}x} - e^{\frac{1}{2}\sqrt{81-4\lambda}x}\right)\\
&= C_1 e^{\frac{9x}{2}}(1-1)\\
&= 0
\end{aligned}$$

Therefore $\lambda=\frac{81}{4}$ can not be used as eigenfunction.

<u>Case $81-4\lambda<0$</u>

Then (1) becomes

$$\begin{aligned}
e^{-\frac{i}{2}\sqrt{4\lambda-81}} &= e^{\frac{i}{2}\sqrt{4\lambda-81}}\\
\cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) - i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= \cos\left(\frac{1}{2}\sqrt{4\lambda-81}\right) + i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right)\\
2i\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\sin\left(\frac{1}{2}\sqrt{4\lambda-81}\right) &= 0\\
\frac{1}{2}\sqrt{4\lambda-81} &= n\pi \qquad n=1,2,\cdots
\end{aligned}$$

Hence

$$\frac{1}{4}(4\lambda - 81) = n^2\pi^2$$
$$4\lambda = 81 + 4n^2\pi^2$$
$$\lambda_n = \frac{81}{4} + n^2\pi^2$$

The corresponding eigenfunctions are (and since $C_2 = -C_1$) then

$$\begin{aligned}
X_n(x) &= C_n\left(e^{\frac{1}{2}(9-\sqrt{81-4\lambda_n})x} - e^{\frac{1}{2}(9+\sqrt{81-4\lambda_n})x}\right)\\
&= C_n\left(e^{\frac{1}{2}(9-i\sqrt{4\lambda_n-81})x} - e^{\frac{1}{2}(9+i\sqrt{4\lambda_n-81})x}\right)\\
&= C_n e^{\frac{9x}{2}}\left(e^{-\frac{i}{2}\sqrt{4\lambda_n-81}x} - e^{\frac{i}{2}\sqrt{4\lambda_n-81}x}\right)\\
&= C_n e^{\frac{9x}{2}}\left(\cos\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right) - i\sin\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right) - \cos\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right) - \sin\left(\frac{1}{2}\sqrt{4\lambda_n -\right.\right.\\
&= C_n e^{\frac{9x}{2}}\left(-2i\sin\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right)\right)\\
&= A_n e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right)
\end{aligned}$$

Hence the solution is

$$u(x,t) = \sum_{n=1}^{\infty} X_n(t)\,T_n(t)$$

But $T' + \lambda_n T = 0$ has solution $T = e^{-\lambda_n t}$. Therefore the solution becomes

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-\lambda_n t} e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right)$$

At $t = 0$

$$e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x)) = \sum_{n=1}^{\infty} A_n e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_n - 81}x\right)$$

But $\lambda_n = \frac{81}{4} + n^2\pi^2$. The above becomes

$$\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x) + 9\sin(2\pi x) + 2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4} + \pi^2\right) - 81}x\right)\\
&\quad + A_2 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4} + 4\pi^2\right) - 81}x\right)\\
&\quad + A_3 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\left(\frac{81}{4} + 9\pi^2\right) - 81}x\right)\\
&\quad + \cdots
\end{aligned}$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\pi^2}x\right)\\
&+A_2 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{16\pi^2}x\right)\\
&+A_3 e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{36\pi^2}x\right)\\
&+\cdots
\end{aligned}
$$

Or

$$
\begin{aligned}
e^{\frac{45}{10}x}(5\sin(\pi x)+9\sin(2\pi x)+2\sin(3\pi x)) &= A_1 e^{\frac{9x}{2}}\sin(\pi x)\\
&+A_2 e^{\frac{9x}{2}}\sin(2\pi x)\\
&+A_3 e^{\frac{9x}{2}}\sin(3\pi x)\\
&+\cdots
\end{aligned}
$$

By comparing coefficients, we see that $A_1 e^{\frac{9x}{2}} = 5e^{\frac{45}{10}x}$ or $A_1 = e^{\left(\frac{45}{10}-\frac{9}{2}\right)x} = 5$ and $A_2 = 9$ and $A_3 = 2$ and and all other A_n for $n > 3$ are zero. Hence the solution becomes

$$
\begin{aligned}
u(x,t) &= 5e^{-\lambda_1 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_1-81}x\right)+9e^{-\lambda_2 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_2-81}x\right)+2e^{-\lambda_3 t}e^{\frac{9x}{2}}\sin\left(\frac{1}{2}\sqrt{4\lambda_3-81}x\right)\\
&= e^{-\left(\frac{81}{4}+\pi^2\right)t}e^{\frac{9x}{2}}\sin(\pi x)+e^{-\left(\frac{81}{4}+4\pi^2\right)t}e^{\frac{9x}{2}}\sin(2\pi x)+e^{-\left(\frac{81}{4}+9\pi^2\right)t}e^{\frac{9x}{2}}\sin(3\pi x)\\
&= e^{-\frac{81}{4}t+\frac{9}{2}x}\left(5e^{-\pi^2 t}\sin(\pi x)+9e^{-4\pi^2 t}\sin(2\pi x)+2e^{-9\pi^2 t}\sin(3\pi x)\right)
\end{aligned}
$$

5.46 Euler-Cauchy Sturm-Liouville

problem number 65

Added May 5, 2019.

Solve $u(x,t)$

$$u_t = x^2 u_{xx} + x u_x$$

For $1 < x < b$ and $t > 0$. The boundary conditions are

$$
\begin{aligned}
u(1,t) &= 0\\
u(b,t) &= 0
\end{aligned}
$$

Initial condition $u(x,0) = f(x)$

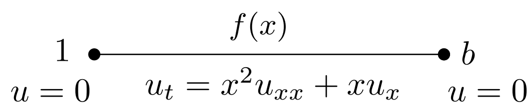


Figure 51: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, f];
pde = D[u[x, t], t] == x^2*D[u[x, t], {x, 2}] + x*D[u[x, t], x];
ic = u[x, 0] == f[x];
bc = {u[1, t] == 0, u[b, t] == 0};
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> b
```

Failed

Maple ✓

```
unassign('u,t,x,f');
pde:= diff( u(x,t),t)= x^2*diff(u(x,t),x$2)+x*diff(u(x,t),x);
bc:=u(1,t)=0,u(b,t)=0;
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve({pde, ic, bc}, u(x, t)) a
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2 \frac{1}{\ln(b)} \sin\left(\frac{n\pi \ln(x)}{\ln(b)} \right) e^{-\frac{\pi^2 n^2 t}{(\ln(b))^2}} \int_1^b \frac{f(x)}{x} \sin\left(\frac{n\pi \ln(x)}{\ln(b)} \right) dx \right)$$

6 Heat PDE on semi-infinite domain (1D)

6.1 Logan p. 76

problem number 66

This is problem at page 76 from David J Logan text book.

Solve the heat equation for $x > 0, t > 0$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $u(0,t) = f(t)$ and initial conditions $u(x,0) = 0$

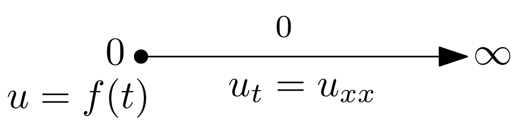


Figure 52: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, f];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = u[0, t] == f[t];
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
 sol = sol /. {K[2] -> z}
```

$$\left\{\left\{u(x,t) \to \frac{x \int_0^t \frac{f(z)e^{-\frac{x^2}{4(t-z)}}}{(t-z)^{3/2}}\, dz}{2\sqrt{\pi}}\right\}\right\}$$

Maple ✓

```
unassign('L,u,t,x');
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=0;
bc:=u(0,t)=f(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) = 1/2\,\frac{x}{\sqrt{\pi}}\int_0^t \frac{f(\zeta)}{(t-\zeta)^{3/2}}\mathrm{e}^{-\frac{x^2}{4t-4\zeta}}\,\mathrm{d}\zeta$$

6.2 nonhomogeneous BC

problem number 67

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = 1$ and And initial condition $u(x,0) = 0$

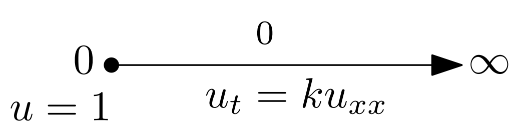


Figure 53: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == 1;
 ic = u[x, 0] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \mathrm{Erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';k:='k';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=0:
bc:=u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) = 1 - \operatorname{erf}\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right)$$

6.3 I.C. not at zero

problem number 68

Added December 20, 2018.

From `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,t_0) = 10;$$

And boundary conditions

$$u(-x_0,t) = 0$$

For $x > |x_0|$ and $t > |t_0|$.

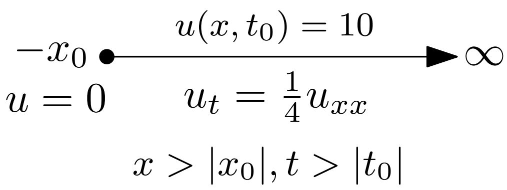


Figure 54: PDE specification

Mathematica ✗

```
ClearAll[x, t, x0, t0];
 pde = D[u[x, t], t] == (1/4)*D[u[x, t], {x, 2}];
 bc = u[-x0, t] == 0;
 ic = u[x, t0] == 10;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t, Assumptions -> {t
```

Failed

due to IC/BC not zero

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t) = (1/4)*(diff(u(x, t), x$2));
bc := u(-x0, t) = 0;
ic := u(x, t0) = 10;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, bc,ic],u(x,t)) assu
```

$$u(x,t) = 10 \operatorname{erf}\left(\frac{x + x0}{\sqrt{t - t0}}\right)$$

6.4 nonhomogeneous BC

problem number 69

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = \mu$ and And initial condition $u(x,0) = \lambda$

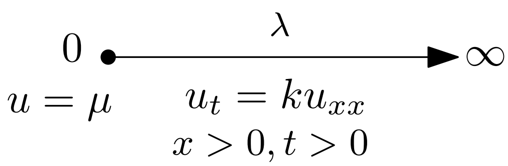


Figure 55: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, k, lambda, mu];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == lambda;
 ic = u[x, 0] == mu;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to \mu\text{Erf}\left(\frac{x}{2\sqrt{kt}}\right)+\lambda\text{Erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';k:='k';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=mu:
bc:=u(0,t)=lambda;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t) = (-\lambda + \mu)\,\mathrm{erf}\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right) + \lambda$$

6.5 nonhomogeneous BC

problem number 70

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x > 0$ and $t > 0$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0) = \cos x$$

And boundary conditions

$$u(0,t) = 1$$

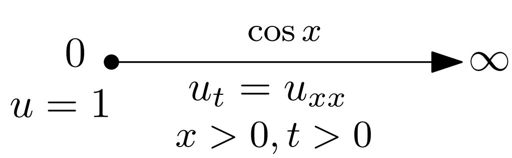


Figure 56: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == Cos[x];
 bc = u[0, t] == 1;
 sol = AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{pde, ic, bc}, u[x, t], {x, t}]],
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{ie^{-\frac{x^2}{4t}}\left(\text{DawsonF}\left(\frac{2t-ix}{2\sqrt{t}}\right)-\text{DawsonF}\left(\frac{2t+ix}{2\sqrt{t}}\right)\right)}{\sqrt{\pi}}+\text{Erfc}\left(\frac{x}{2\sqrt{t}}\right) & x>0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=cos(x);
bc:=u(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t)=1/2\,\mathrm{erf}\left(1/2\,\frac{2\,it+x}{\sqrt{t}}\right)\mathrm{e}^{-t+ix}-\mathrm{erf}\left(1/2\,\frac{x}{\sqrt{t}}\right)-1/2\,\mathrm{erf}\left(1/2\,\frac{2\,it-x}{\sqrt{t}}\right)\mathrm{e}^{-t-ix}+1$$

6.6 nonhomogeneous B.C.

problem number 71

Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$u_t = ku_{xx}$$

With initial condition

$$u(x,0)=0$$

And boundary conditions $u(0,t)=t$. Solution is bounded at infinity.

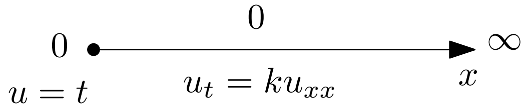


Figure 57: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 ic = u[x, 0] == 0;
 bc = u[0, t] == t;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\frac{(2kt+x^2)\,\mathrm{Erfc}\left(\frac{x}{2\sqrt{kt}}\right)-\frac{2x\sqrt{kt}e^{-\frac{x^2}{4kt}}}{\sqrt{\pi}}}{2k}\right\}\right\}$$

Maple ✓

```
x:='x'; u:='u'; t:='t'; k:='k';
interface(showassumed=0);
pde := diff(u(x, t), t)=k*diff(u(x, t), x$2);
ic:=u(x,0)=0;
bc:=u(0,t)=t;
assume(x>0);
assume(t>0);
assume(k>0);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t)=-\frac{1}{k\sqrt{\pi}}\left(\sqrt{k}\sqrt{t}\mathrm{e}^{-1/4\,\frac{x^2}{k\,t}}x+\sqrt{\pi}\left(\mathrm{erf}\left(1/2\,\frac{x}{\sqrt{k}\sqrt{t}}\right)-1\right)\left(k\,t+1/2\,x^2\right)\right)$$

6.7 Unit triangle I.C.

problem number 72

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=\text{UnitTriagle[x-3]}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(0,t)=0$$

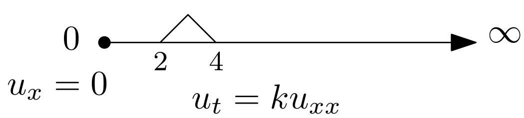


Figure 58: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitTriangle[x - 3];
 bc = Derivative[1, 0][u][0, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\ \{\quad \frac{1}{2}\left(\frac{\text{Erf}\left(\frac{|x-4|}{2\sqrt{t}}\right)(x-4)^2}{|4-x|}+(x+2)\text{Erf}\left(\frac{x+2}{2\sqrt{t}}\right)-2(x+3)\text{Erf}\left(\frac{x+3}{2\sqrt{t}}\right)+(x+4)\text{Erf}\left(\frac{x+4}{2\sqrt{t}}\right)-\frac{2(x-3}{\ }\right.\right.\right.$$

Indet

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=piecewise( x>2 and x<3,-2+x, x>3 and x<4, 4-x, 0);
bc:=(D[1](u))(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

$$u(x,t) = \frac{1}{\sqrt{\pi}\sqrt{t}}\left(t\mathrm{e}^{-1/4\,\frac{(-4+x)^2}{t}} - 2\,t\mathrm{e}^{-1/4\,\frac{(-3+x)^2}{t}} + t\mathrm{e}^{-1/4\,\frac{(-2+x)^2}{t}} + t\mathrm{e}^{-1/4\,\frac{(2+x)^2}{t}} + t\mathrm{e}^{-1/4\,\frac{(4+x)^2}{t}} - 2\,t\mathrm{e}^{-1/4\,\frac{(x+3}{t}}\right.$$

6.8 I.C. not at $t = 0$

problem number 73

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ for $t > 0, x > 0$

$$u_t = \frac{1}{4}u_{xx}$$

With initial condition

$$u(x,t_0) = 10e^{-x^2}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(x_0,t) = 0$$

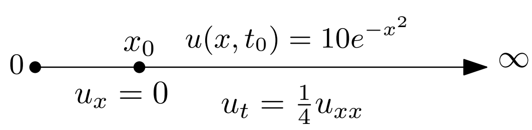


Figure 59: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, x0, t0];
 pde = D[u[x, t], t] == (1*D[u[x, t], {x, 2}])/4;
 ic = u[x, t0] == 10*Exp[-x^2];
 bc = Derivative[1, 0][u][x0, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t';x0:='x0';t0:='t0';
pde := diff(u(x, t), t) = 1/4*(diff(u(x, t), x$2));
bc := eval( diff(u(x,t),x),x=x0)=0;
ic := u(x,t0)=10*exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

$$u(x,t) = 5\frac{1}{\sqrt{t-t0+1}}\left(e^{4\frac{x0(-x+x0)}{-t+t0-1}}\operatorname{erf}\left(\frac{(t0-t+1)\,x0-x}{\sqrt{t-t0+1}\sqrt{t-t0}}\right)+e^{4\frac{x0(-x+x0)}{-t+t0-1}}+\operatorname{erf}\left(\frac{(-t+t0-1)\,x0+x}{\sqrt{t-t0+1}\sqrt{t-t0}}\right.\right.$$

7 Heat PDE on infinite domain, 1D

7.1 Inverse exponential I.C.

problem number 74

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = u_{xx}$$

With initial condition

$$u(x,0) = e^{-x^2}$$

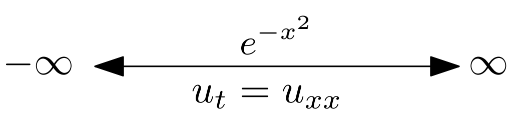


Figure 60: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == E^(-x^2);
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}\right\}\right\}$$

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=exp(-x^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = \frac{1}{\sqrt{1+4\,t}}\mathrm{e}^{-\frac{x^2}{1+4\,t}}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

On $-\infty<x<\infty, t>0$ with $u(x,0)=f(x)=e^{-x^2}$. The first step is to find Green function for the above PDE. Taking Fourier transform of both sides w.r.t. x, using $\hat{u}(k,t)$ as the Fourier transform of $u(x,t)$ gives

$$\begin{aligned}\frac{d}{dt}\hat{u}(k,t)&=(ik)^2\,\hat{u}(k,t)\\&=-k^2\hat{u}(k,t)\\\frac{d}{dt}\hat{u}(k,t)+k^2\hat{u}(k,t)&=0\end{aligned}$$

The solution to the above is

$$\hat{u}(k,t)=Ce^{-k^2t}\tag{1}$$

At $t=0$,

$$\hat{u}(k,0)=\mathcal{F}(h(x))$$

Therefore

$$C=\mathcal{F}(h(x))$$

And (1) becomes

$$\hat{u}(k,t)=\mathcal{F}(h(x))\,e^{-k^2t}$$

To find Green function, we replace $h(x)$ by $\delta(x-\xi)$ where ξ is the location of the pulse. But $\mathcal{F}(\delta(x-\xi)\,;k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta(x-\xi)\,e^{-ikx}dx=\frac{1}{\sqrt{2\pi}}e^{-i\xi k}$. Therefore the above becomes

$$\hat{G}(k,t)=\frac{1}{\sqrt{2\pi}}e^{-i\xi k}e^{-k^2t}$$

The above is the Fourier transform of the Green function. Now we invert it

$$\begin{aligned}G(x,t)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}e^{-i\xi k}e^{-k^2t}\right)e^{ikx}dk\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi k-k^2t+ikx}dk\end{aligned}\tag{2}$$

We would like to use Gaussian as the integrand, hence we want to change $-i\xi k-k^2t+ikx$ to $-\left(k\sqrt{t}-A\right)^2$. We do this by completing the square.

$$\begin{aligned}-i\xi k-k^2t+ikx&=-\left(k\sqrt{t}-A\right)^2\\&=-\left(k^2t+A^2-2Ak\sqrt{t}\right)\\&=-k^2t-A^2+2Ak\sqrt{t}\end{aligned}$$

Comparing sides then $2Ak\sqrt{t} = k(-i\xi + ix)$ or $A = \frac{-i\xi+ix}{2\sqrt{t}}$. Therefore

$$
\begin{aligned}
-i\xi k - k^2 t + ikx &= -\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2 + A^2 \\
&= -\left(k\sqrt{t} - \frac{-i\xi + ix}{2\sqrt{t}}\right)^2 + \left(\frac{-i\xi + ix}{2\sqrt{t}}\right)^2
\end{aligned}
$$

Hence

$$
\begin{aligned}
e^{-i\xi k - k^2 t + ikx} &= e^{-\left(k\sqrt{t} - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2 + \left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \\
&= e^{-\left(k\sqrt{t} - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2}
\end{aligned}
$$

Substituting the above into (2) gives

$$
\begin{aligned}
G(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} dk \\
&= \frac{1}{2\pi} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} dk
\end{aligned}
$$

To evaluate $\int_{-\infty}^{\infty} e^{-\left(k\sqrt{t} - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} dk$, let $u = k\sqrt{t}$, then $du = \sqrt{t}dk$. The above becomes

$$
\begin{aligned}
G(x,t) &= \frac{1}{2\pi} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(u - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \frac{du}{\sqrt{t}} \\
&= \frac{1}{2\pi\sqrt{t}} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \int_{-\infty}^{\infty} e^{-\left(u - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} du
\end{aligned}
$$

Now the integral is Gaussian. $\int_{-\infty}^{\infty} e^{-\left(u - \frac{-i\xi+ix}{2\sqrt{t}}\right)^2} du = \sqrt{\pi}$ and the above becomes

$$
\begin{aligned}
G(x,t) &= \frac{\sqrt{\pi}}{2\pi\sqrt{t}} e^{\left(\frac{-i\xi+ix}{2\sqrt{t}}\right)^2} \\
&= \frac{1}{2\sqrt{\pi t}} e^{\left(i\left(\frac{-\xi+x}{2\sqrt{t}}\right)\right)^2} \\
&= \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}}
\end{aligned}
$$

Now that we found the Green function for the PDE, we can find the solution as

$$
\begin{aligned}
u(x,t) &= \int_{-\infty}^{\infty} G(\xi,t)\, h(\xi)\, d\xi \\
&= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi t}} e^{-\frac{(x-\xi)^2}{4t}} h(\xi)\, d\xi \\
&= \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi
\end{aligned}
$$

But $\int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4t}} e^{-\xi^2} d\xi = \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}}$, hence the above becomes

$$
\begin{aligned}
u(x,t) &= \frac{1}{\sqrt{4\pi t}} \frac{2e^{-\frac{x^2}{1+4t}}\sqrt{\pi}}{\sqrt{\frac{1+4t}{t}}} \\
&= \frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}}
\end{aligned}
$$

7.2 Advection term

problem number 75

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t = 12u_{xx} + u_x \sin t$$

With initial condition

$$u(x,0) = x$$

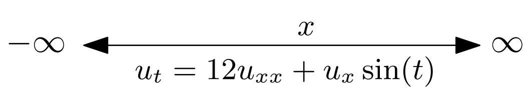


Figure 61: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == 12*D[u[x, t], {x, 2}] + Sin[t]*D[u[x, t], x];
 ic = u[x, 0] == x;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t) \to -\cos(t) + x + 1\}\}$$

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde:=diff(u(x,t),t)= 12* diff(u(x,t),x$2)+sin(t)*diff(u(x,t),x);
ic:=u(x,0)=x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t))),output=
```

$$u(x,t) = -\cos(t) + x + 1$$

7.3 UnitBox I.C.

problem number 76

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$u_t = u_{xx}$$

With initial condition

$$u(x,0) = \text{UnitBox[x]}$$

Where UnitBox is equal to 1 if $|x| \leq \frac{1}{2}$ and zero otherwise.

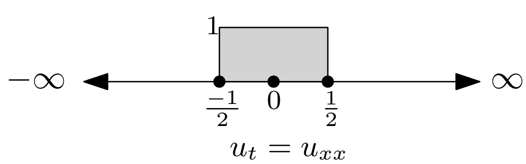


Figure 62: PDE specification

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitBox[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(\text{Erf}\left(\frac{1-2x}{4\sqrt{t}}\right) + \text{Erf}\left(\frac{2x+1}{4\sqrt{t}}\right)\right)\right\}\right\}$$

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:= u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = 1/2\ \mathrm{erf}\left(1/4\,\frac{2\,x+1}{\sqrt{t}}\right) - 1/2\ \mathrm{erf}\left(1/4\,\frac{2\,x-1}{\sqrt{t}}\right)$$

7.4 No source

problem number 77

Solve the heat equation

$$u_t = k u_{xx}$$

For $-\infty < x < \infty$ and $t > 0$, and initial condition is $u(x,0) = f(x)$

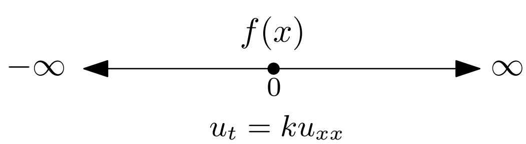


Figure 63: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, m, k, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
 sol[[2]] = sol[[2]] /. K[1] -> s;
```

$$\left\{\left\{u(x,t) \to \int_{-\infty}^{\infty} \frac{f(s) e^{-\frac{(x-s)^2}{4kt}}}{2\sqrt{\pi}\sqrt{kt}}\, ds\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';m:='m';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = 1/4\,\frac{1}{\pi^2}\int_{-\infty}^{\infty} 2\,\frac{\pi^{3/2} f(-\zeta)}{\sqrt{k}\sqrt{t}}\mathrm{e}^{-1/4\,\frac{(x+\zeta)^2}{k\,t}}\,\mathrm{d}\zeta$$

7.5 constant as source

problem number 78

Solve the heat equation

$$u_t = k u_{xx} + m$$

For $-\infty < x < \infty$ and $t > 0$. Initial condition is $u(x,0) = \sin(x)$

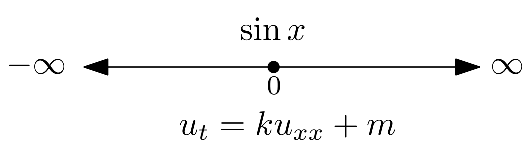


Figure 64: PDE specification

Mathematica ✓

```
ClearAll[u, t, x, m, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + m;
 ic = u[x, 0] == Sin[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to e^{-kt}\sin(x)+mt\right\}\right\}$$

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';m:='m';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic:=u(x,0)=sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t))),output='
```

$$u(x,t) = \sin(x)\, e^{-k t} + mt$$

7.6 No intial conditions

problem number 79

Solve the heat equation for $u(x,t)$

$$u_t = u_x x$$

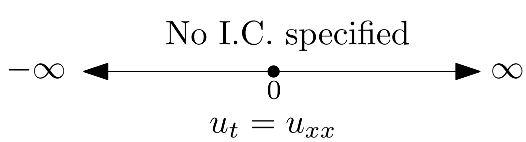


Figure 65: PDE specification

Mathematica ✗

```
ClearAll[x, y, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t) = \_C3\, \mathrm{e}^{-c_1 t} \_C1\, \mathrm{e}^{\sqrt{-c_1}\, x} + \frac{\_C3\, \mathrm{e}^{-c_1 t} \_C2}{\mathrm{e}^{\sqrt{-c_1}\, x}}$$

Hand solution

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $t > 0, -\infty < x < \infty$. Let $u = X(x)T(t)$ then we obtain

$$T'X = X''T$$

Dividing by $XT \neq 0$

$$\frac{T'}{T} = \frac{X''}{X} = -\lambda$$

(Only positive eigenvalues are possible). The two ODE's are

$$T' + \lambda T = 0 \tag{1}$$

$$X'' + \lambda X = 0 \tag{2}$$

Solution for (2) is $X(x) = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}$ and solution for (1) is $T(t) = C_3 e^{-\lambda t}$. Hence

$$\begin{aligned} u(x,t) &= C_3 e^{-\lambda t}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) \\ &= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + C_3 e^{-\lambda t} C_2 e^{-\sqrt{\lambda}x} \\ &= C_3 e^{-\lambda t} C_1 e^{\sqrt{\lambda}x} + \frac{C_3 e^{-\lambda t} C_2}{e^{\sqrt{\lambda}x}} \end{aligned}$$

7.7 piecewise I.C.

problem number 80

Added December 20, 2018.

From https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$u_t = \mu u_{xx} - 1$$

With initial condition

$$u(x,1) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$$

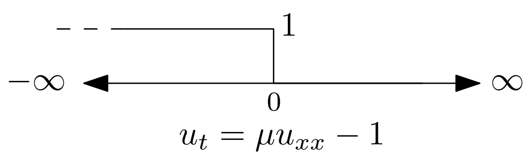


Figure 66: PDE specification

Mathematica ✗

```
ClearAll[u, x, t, mu];
 pde = D[u[x, t], t] + 1 == mu*D[u[x, t], {x, 2}];
 ic = u[x, 1] == Piecewise[{{1, x <= 0}, {0, x > 0}}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], x, t, Assumptions -> mu > 0
```

Failed

due to i.c. not at zero

Maple ✓

```
x:='x';t:='t';u:='u';mu:='mu';
pde := diff(u(x, t), t)+1 = mu* diff(u(x, t), x$2);
ic := u(x, 1) = piecewise(0 <= x, 0, x < 0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t) = 3/2 - 1/2 \operatorname{erf}\left(1/2 \frac{x}{\sqrt{\mu}\sqrt{t-1}}\right) - t$$

Hand solution

Solve

$$u_t = \mu u_{xx} - 1$$

for $t>0, -\infty<x<\infty$ with initial conditions $u(x,0)=f(x)=\begin{cases}0 & x\geq 0\\ 1 & x<0\end{cases}$

Let $v=u+t$. Hence $u=v-t$ and $u_t=v_t-1$ and $u_x=v_x$ and $u_{xx}=v_{xx}$. The above PDE becomes

$$\begin{aligned} v_t-1&=\mu v_{xx}-1\\ v_t&=\mu v_{xx} \end{aligned} \tag{1}$$

Initial conditions do not change. They are $v(x,0)=u(x,0)=\begin{cases}0 & x\geq 0\\ 1 & x<0\end{cases}$. Using Green function for 1D heat PDE on the real line, (also called heat Kernel)

$$G(x,t)=\frac{1}{\sqrt{4\pi\mu t}}e^{\frac{-x^2}{4\mu t}}$$

Then the solution to (1) is

$$\begin{aligned} v(x,t)&=\int_{-\infty}^{\infty}f(x')\,G(x-x',t)\,dx'\\ &=\int_{-\infty}^{0}\frac{1}{\sqrt{4\pi\mu t}}e^{\frac{-\left(x-x'\right)^2}{4\mu t}}dx'\\ v(x,t)&=\frac{-1}{\sqrt{4\pi\mu t}}\int_{0}^{\infty}e^{\frac{-\left(x-x'\right)^2}{4\mu t}}dx' \end{aligned}$$

But $\int_0^\infty e^{\frac{-\left(x-x'\right)^2}{4\mu t}}dx'=\sqrt{\pi\mu t}\left(1+\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$, hence

$$v(x,t)=\frac{-1}{2}\left(1+\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)$$

Since $u=v-t$ then

$$\begin{aligned} u(x,t)&=\frac{-1}{2}\left(1+\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)\right)-t\\ &=-\frac{1}{2}-\frac{1}{2}\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)-t \end{aligned}$$

8 Heat PDE in rectangle

8.1 No source

problem number 81

Taken from Maple help pages on PDE. Solve the heat equation for $u(x, y, t)$

$$u_t = \frac{1}{10}\nabla^2 u(x, y)$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$u(0, y, t) = 0$$
$$u(1, y, t) = 0$$
$$u(x, 0, t) = 0$$
$$u(x, 1, t) = 0$$

Initial condition is $u(x, y, 0) = x(1 - x)(1 - y)y$.

Mathematica ✗

```
ClearAll[x, y, t];
 pde = D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10;
 ic = u[x, y, 0] == x*(1 - x)*(1 - y)*y;
 bc = {u[0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, u[x, 1, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t'; y:='y';
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc := u(0, y, t) = 0, u(1, y, t) = 0, u(x, 0, t) = 0, u(x, 1, t) = 0;
ic:=u(x, y, 0) = x*(1-x)*(1-y)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,y,t))),out
```

$$u(x, y, t) = \sum_{n1=1}^{\infty}\left(\sum_{n=1}^{\infty} -16\,\frac{\left(-(-1)^{n1+n} + (-1)^{n} + (-1)^{n1} - 1\right)\sin(n\pi x)\sin(n1\,\pi y)\,e^{-1/10\,\pi^2 t\left(n^2+n1^2\right)}}{n^3\pi^6 n1^3}\right)$$

8.2 Internal source term

problem number 82

Taken from Maple help pages on PDE

Solve the heat equation for $u(x, y, t)$

$$\frac{\partial u}{\partial t} = 1/10\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{5}u(x, y, t);$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned}\frac{\partial u}{\partial x}u(0, y, t) &= 0\\ u(1, y, t) &= 0\\ u(x, 0, t) &= 0\\ \frac{\partial u}{\partial y}u(x, 1, t) &= 0\end{aligned}$$

Initial condition is $u(x, y, 0) = (1 - x^2)(1 - \frac{1}{2}y)y$.

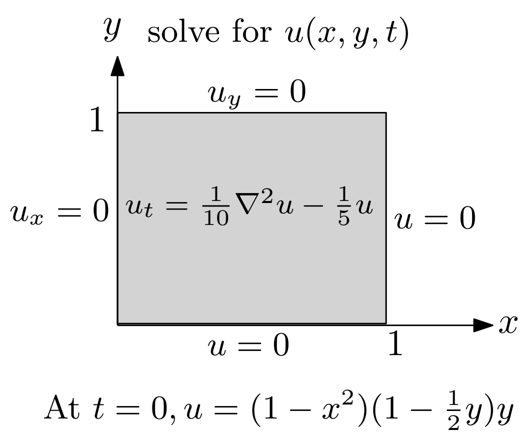


Figure 67: PDE specification

Mathematica ✗

```
ClearAll[x, y, t];
 pde = D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10 - (1*u[x,
 ic = u[x, y, 0] == (-x^2 + 1)*(1 - (1/2)*y)*y;
 bc = {Derivative[1, 0, 0][u][0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t'; y:='y';
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)
                              +diff(u(x, y, t), y$2)) - 1/5 * u(x,y,t);
ic:= u(x, y, 0) = (-x^2+1)*(1-(1/2)*y)*y;
bc := (D[1](u))(0, y, t) = 0,
       u(1, y, t) = 0,
       u(x, 0, t) = 0,
       (D[2](u))(x, 1, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
```

$$u(x,y,t) = \sum_{n1=0}^{\infty}\left(\sum_{n=0}^{\infty} 512\,\frac{(-1)^n\,e^{-1/10\,t\left(2+\left(n^2+n1^2+n+n1+1/2\right)\pi^2\right)}\sin\left(1/2\,(1+2\,n1)\,\pi\,y\right)\cos\left(1/2\,(1+2\,n)\,\pi\right.}{\pi^6\,(1+2\,n)^3\,(1+2\,n1)^3}\right.$$

8.3 Articolo 6.6.3

problem number 83

Added December 20, 2018.

Example 6.6.3 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin rectangular plate over the finite two-dimensional domain $D = (x,y)$s.t.$0 < x < 1, 0 < y < 1$. The lateral surfaces of the plate are insulated. The boundaries $y = 0$ and $y = 1$ are fixed at temperature 0, the boundary $x = 0$ is insulated, and the boundary $x = 1$ is losing heat by convection into a surrounding medium at temperature 0. The initial temperature distribution f(x, y) is

$$u(x,y,0) = \left(1 - \frac{x^2}{3}\right) y(1-y)$$

The thermal diffusivity is $k = \frac{1}{50}$. Solve for $u(x,y,t)$ the heat PDE

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With $0 < x < 1, 0 < y < 1$ and $t > 0$. Boundary conditions are

$$\frac{\partial u}{\partial x}(0,y,t) = 0$$
$$\frac{\partial u}{\partial x}(1,y,t) + u(1,y,t) = 0$$
$$u(x,0,t) = 0$$
$$u(x,1,t) = 0$$

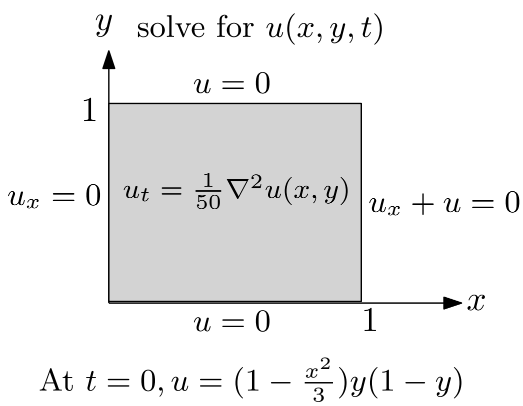


Figure 68: PDE specification

Mathematica ✗

```
ClearAll[x, y, t, k];
 k = 1/50;
 pde = D[u[x, y, t], t] == k*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]);
 bc = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] + u[1, y, t] ==
 ic = u[x, y, 0] == (1 - (1/3)*x^2)*y*(1 - y);
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; y:='y'; t:='t';k:='k';
k:=1/50;
pde := diff(u(x, y, t), t) = k*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc_left_edge:=eval( diff(u(x,y,t),x),x=0)=0;
bc_right_edge:= eval( diff(u(x,y,t),x),x=1)+u(1,y,t)=0;
bc_bottom_edge:=u(x,0,t)=0;
bc_top_edge:=u(x,1,t)=0;
bc:=bc_left_edge,bc_right_edge,bc_bottom_edge,bc_top_edge;
ic:=u(x, y, 0) = (1-(1/3)*x^2)*y*(1-y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x, y, t))
```

$$u(x,y,t) = \texttt{casesplit/ans}\left(\sum_{n=1}^{\infty}\left(\sum_{n1=0}^{\infty}\frac{32\,\mathrm{e}^{-\frac{1}{50}t\left(\pi^2 n^2+\lambda_{n1}^{\ 2}\right)}\left(-1+(-1)^n\right)\cos\left(\lambda_{n1}x\right)\sin\left(n\pi\,y\right)\left(-\lambda_{n1}^{\ 2}\sin\left(\right.\right.}{3\,\pi^3 n^3\lambda_{n1}^{\ 2}\left(\sin\left(2\,\lambda_{n1}\right)+2\,\lambda_{n1}\right)}\right.\right.$$

9 Heat PDE inside disk

9.1 No θ dependency

problem number 84

Taken from Mathematica DSolve help pages

Solve the heat equation in polar coordinates for $u(r,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

For $0 < r < 1$ and $t > 0$. The boundary conditions are

$$u(1,t) = 0$$

Initial condition is $u(r,0) = 1 - r$.

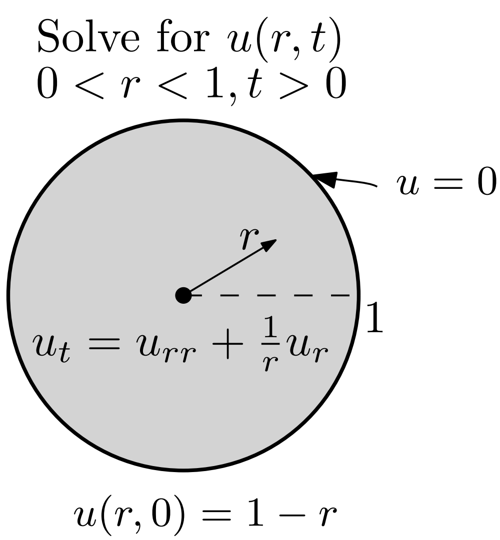


Figure 69: PDE specification

Mathematica ✓

```
ClearAll[r, t, u];
 pde = D[u[r, t], t] == D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r;
 ic = u[r, 0] == 1 - r;
 bc = u[1, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2e^{-t\text{BesselJZero}(0,n)^2}\text{BesselJ}(0, r\text{BesselJZero}(0,n))\left(\frac{\text{BesselJ}(1,\text{BesselJZero}(0,n))}{\text{BesselJZero}(0,n)} - \frac{1}{3}\text{Hypergeome}\right.}{\text{BesselJ}(0,\text{BesselJZero}(0,n))^2 + \text{BesselJ}(1,\text{BesselJZ}}\right.\right.$$

Maple ✓

```
r:='r'; u:='u'; t:='t';
pde:=diff(u(r,t),t)= diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r);
ic:=u(r,0)=1-r;
bc := u(1,t) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t) = \mathit{invlaplace}\left(\text{BesselJ}\left(0, \sqrt{-s}r\right)\_F2(s), s, t\right) - \mathit{invlaplace}\left(\frac{\text{BesselY}\left(0, \sqrt{-s}r\right)\text{BesselJ}\left(0, 1/2\ \text{Stru}\right.}{\text{BesselY}\left(0, 1/2\ \text{StruveH}\right.}\right.$$

But has unresolved inverse Laplace transforms

9.2 Haberman 8.3.5

problem number 85

Added Nov 24, 2018.

Problem 8.3.5 from Richard Haberman applied partial differential equations book, 5th edition

Solve for $u(r,t)$

$$\frac{\partial u}{\partial t} = k\nabla^2 u + f(r,t)$$

Inside the circle $(r < a)$ with $u = 0$ at $r = a$ and initially $u = 0$.

One of the problems here, is how to tell CAS the implicit condition when solving this which is that $u(0,t) < \infty$.

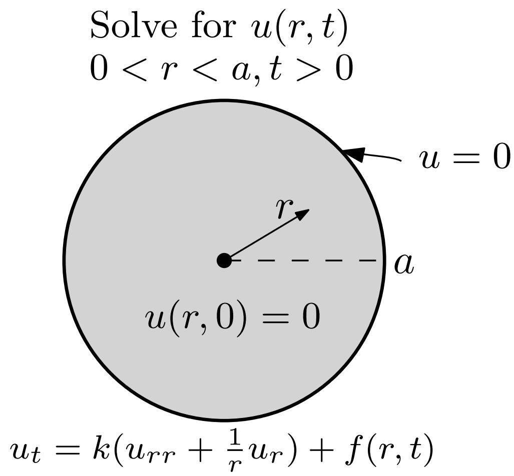


Figure 70: PDE specification

Mathematica ✗

```
ClearAll[r, t, u, k, a];
 pde = D[u[r, t], t] == k*(D[u[r, t], {r, 2}] + D[u[r, t], r]/r) + f[r, t];
 ic = u[r, 0] == 0;
 bc = u[a, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

Failed

Maple ✗

```
r:='r'; u:='u'; t:='t';a:='a';k:='k';f:='f';
pde:=diff(u(r,t),t)= k*(diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r)) + f(r,t);
ic:=u(r,0)=0;
bc := u(a,t) =0;
#do not use HINT=boundedseries below, Maple will  not solve it then
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t)) assumi
```

sol=()

Hand solution

Since this problem has homogeneous B.C. but has time dependent source (i.e. non-homogenous in the PDE itself), then we will use the method of eigenfunction expansion. In this method, we first find the eigenfunctions $\phi_n(x)$ of the associated homogenous PDE without the source being present. Then use these $\phi_n(x)$ to expand the source $f(x,t)$ as generalized Fourier series. We now switch to the associated homogenous PDE in order to find the eigenfunctions. $u \equiv u(r,t)$. There is no θ. Hence

$$
\begin{aligned}
\frac{\partial u(r,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \qquad (1) \\
u(a,t) &= 0 \\
|u(0,t)| &< \infty \\
u(r,0) &= 0
\end{aligned}
$$

We need to solve the above in order to find the eigenfunctions $\phi_n(r)$. Let $u = R(r)T(t)$. Substituting this back into (1) gives

$$
T'R = k\left(R''T + \frac{1}{r}R'T\right)
$$

Dividing by RT

$$
\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\lambda
$$

Where λ is the separation constant. The above gives

$$
T' + k\lambda T = 0
$$

And

$$
rR'' + R' + \lambda rR = 0
$$

This is a singular Sturm-Liouville ODE. Standard form is

$$(rR')' = -\lambda r R$$

Hence

$$\begin{aligned}
p &= r\\
q &= 0\\
\sigma &= r
\end{aligned}$$

The ODE $rR'' + R' + \lambda r R = 0$ is Bessel ODE whose solution is

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right) + C_2 \operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$$

Since $\operatorname{BesselY}\left(0, \sqrt{\lambda} r\right)$ blows up at $r = 0$, then $C_2 = 0$ and the solution becomes

$$R(r) = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} r\right)$$

At $r = a$ the above becomes $0 = C_1 \operatorname{BesselJ}\left(0, \sqrt{\lambda} a\right)$. Non trivial solution requires that $\sqrt{\lambda} a$ are the zeros of $\operatorname{BesselJ}(0, x)$. Let the zeros be called $\Lambda_n, n = 1, 2, 3, \cdots$. Therefore $\sqrt{\lambda_n} a = \Lambda_n$ or

$$\lambda_n = \left(\frac{\Lambda_n}{a}\right)^2 \qquad n = 1, 2, 3, \cdots$$

The corresponding eigenfunctions are $R_n(r) = \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$. Now that the eigenfunctions for the homogeneous PDE are found, eigenfunction expansion is used to find the general solution. Let

$$u(r,t) = \sum_{n=1}^{\infty} a_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) \tag{2}$$

Where $a_n(t)$ is function of time since it includes the time solution in it. Substituting the above back into the original nonhomogeneous PDE

$$\begin{aligned}
u_t &= k\nabla^2 u + f(r,t) \qquad (3)\\
&= k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + f(r,t)
\end{aligned}$$

Where $\nabla^2 u = -\lambda r u$. Substituting (2) into (3), and using $f(r,t) = \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$ gives

$$\sum_{n=1}^{\infty} a_n'(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) = k a_n(t) \left(\sum_{n=1}^{\infty} \operatorname{BesselJ}''\left(0, \frac{\Lambda_n}{a} r\right) + \frac{1}{r} \operatorname{BesselJ}'\left(0, \frac{\Lambda_n}{a} r\right)\right) + \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a}\right.$$

But $\sum_{n=1}^{\infty} \operatorname{BesselJ}''\left(0, \frac{\Lambda_n}{a} r\right) + \frac{1}{r} \operatorname{BesselJ}'\left(0, \frac{\Lambda_n}{a} r\right) = -\lambda_n \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$. The above becomes

$$\sum_{n=1}^{\infty} a_n'(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) = -k a_n(t) \sum_{n=1}^{\infty} \left(\frac{\Lambda_n}{a}\right)^2 \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) + \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0\right.$$

$$\sum_{n=1}^{\infty} \left(a_n'(t) + k \left(\frac{\Lambda_n}{a}\right)^2 a_n(t) \right) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right) = \sum_{n=1}^{\infty} b_n(t) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$$

The above simplifies to

$$a_n'(t) + k \left(\frac{\Lambda_n}{a}\right)^2 a_n(t) = b_n(t)$$

The solution is

$$a_n(t) = e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \int_0^t b_n(\tau) e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau + a_n(0) e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}$$

Hence the solution (2) becomes

$$u(r,t) = \sum_{n=1}^{\infty} \left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \left(\int_0^t b_n(\tau) e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau \right) + a_n(0) e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \right) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$$

To find $a_n(0)$, putting $t = 0$ in the above gives

$$0 = \sum_{n=1}^{\infty} a_n(0) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$$

Hence $a_n(0) = 0$. Therefore $a_n(t)$ becomes.

$$a_n(t) = e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \int_0^t b_n(\tau) e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau$$

Hence the solution from (2) now becomes

$$u(r,t) = \sum_{n=1}^{\infty} \left(e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t} \int_0^t b_n(\tau) e^{k\left(\frac{\Lambda_n}{a}\right)^2 \tau} d\tau \right) \operatorname{BesselJ}\left(0, \frac{\Lambda_n}{a} r\right)$$

And finally, to find $b_n(t)$, which is the generalized Fourier coefficient of the expansion of the source in (3) above, orthogonality is used as follows

$$\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr = b_n(t)\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr$$
$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}$$

Summary of solution

$$\begin{aligned} u(r,t) &= \sum_{n=1}^{\infty} a_n(t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) \\ &= \sum_{n=1}^{\infty}\left(\int_0^t b_n(\tau)\,e^{k\left(\frac{\Lambda_n}{a}\right)^2\tau}d\tau\right)e^{-k\left(\frac{\Lambda_n}{a}\right)^2 t}\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right) \end{aligned}$$

Where

$$b_n(t) = \frac{\int_0^a f(r,t)\,\text{BesselJ}\left(0,\frac{\Lambda_n}{a}r\right)rdr}{\int_0^a \text{BesselJ}^2\left(0,\frac{\Lambda_n}{a}r\right)rdr}$$

9.3 Articolo 6.9.1

problem number 86

Added December 20, 2018.

Example 6.9.1 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution $u(r,\theta,t)$ in a thin circular plate over the two-dimensional domain $D = \{(r,\theta)|0 < r < 1, 0 < \theta < \frac{\pi}{2}\}$.

The lateral surfaces of the plate are insulated. The edges $r = 1$ and $\theta = 0$ are at a fixed temperature of 0, and the edge $\theta = \frac{\pi}{2}$ is insulated. The initial temperature distribution $u(r,\theta,0) = f(r,\theta)$ is $u(r,\theta,0) = (r - r^3)\sin(\theta)$.

The thermal diffusivity is $k = \frac{1}{50}$. Solve for $u(r,\theta,t)$ the heat PDE

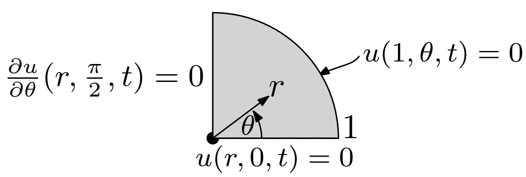
$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$
\begin{aligned}
|u(0,\theta,t)| &< \infty \\
u(1,\theta,t) &= 0 \\
u(r,0,t) &= 0 \\
\frac{\partial u}{\partial \theta}(1,\frac{\pi}{2},t) &= 0
\end{aligned}
$$

Solve for $u(r,\theta,t)$

$0 < r < 1, 0 < \theta < \frac{\pi}{2}, t > 0$



I.C. $u(r,\theta,0) = (r - r^3)\sin\theta$

$u_t = \frac{1}{50}(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta})$

Figure 71: PDE specification

Mathematica ✗

```
ClearAll[r, t, theta, u, k];
k = 1/50;
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + (1*D[u
bcOnR = u[1, theta, t] == 0;
bcOnTheta = {u[r, 0, t] == 0, Derivative[0, 1, 0][u][r, Pi/2, t] == 0};
ic = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

Failed

Maple ✗

```
unassign('r,u,t,theta,k');
k:=1/50;
pde := diff(u(r, theta, t), t) = k*( diff(u(r, theta, t), r$2) + 1/r* diff(u(r, theta, t), r
bc_on_r:= u(1,theta,t)=0;
bc_on_theta:= u(r,0,t)=0, eval(diff(u(r,theta,t),theta),theta=Pi/2)=0;
ic := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

sol=()

9.4 Articolo 6.9.2

problem number 87

Added December 20, 2018.

Example 6.9.2 from Partial differential equations and boundary value problems with Maple/George A. Articolo, 2nd ed :

We seek the temperature distribution in a thin circular plate over the two-dimensional domain $D = \{(r,\theta)|0 < r < 1, 0 < \theta < \pi\}$. The lateral surfaces of the plate are insulated. The sides $\theta = 0$ and $\theta = \pi$ are at a fixed temperature of 0, and the edge $r = 1$ is insulated. The initial temperature distribution is $u(r,\theta,0) = \left(r - \frac{r^3}{3}\right)\sin\theta$.

The thermal diffusivity is $k = \frac{1}{25}$. Solve for $u(r,\theta,t)$ the heat PDE

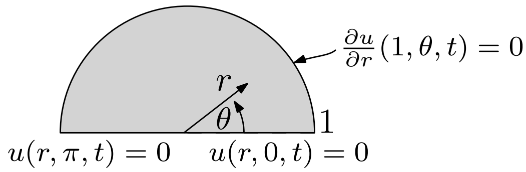
$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$$

With boundary conditions

$$|u(0,\theta,t) < \infty$$
$$u(1,\theta,t) = 0$$
$$u(r,0,t) = 0$$
$$u(r,\pi,t)0$$

Solve for $u(r,\theta,t)$

$0<r<1, 0<\theta<\pi, t>0$



I.C. $u(r,\theta,0)=(r-\frac{r^3}{3})\sin\theta$

$u_t = \frac{1}{25}(u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta})$

Figure 72: PDE specification

Mathematica ✗

```
ClearAll[r, t, theta, u, k];
 k = 1/25;
 pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + (1*D[
 bcOnR = Derivative[1, 0, 0][u][1, theta, t] == 0;
 bcOnTheta = {u[r, 0, t] == 0, u[r, Pi, t] == 0};
 ic = u[r, theta, 0] == (r - (1*r^3)/3)*Sin[theta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r
```

Failed

Maple ✓

```
r:='r'; u:='u'; t:='t';theta:='theta';k:='k';
k:=1/25;
pde := diff(u(r, theta, t), t) = k*( diff(u(r, theta, t), r$2) + 1/r* diff(u(r, theta, t), r
bc_on_r:= eval(diff(u(r,theta,t),r),r=1)=0;
bc_on_theta:= u(r,0,t)=0, u(r,Pi,t)=0;
ic := u(r,theta,0)=(r-1/3*r^3)*sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_r, bc_on_theta
```

$$u(r,\theta,t) = \texttt{casesplit/ans}\left(\sum_{n=0}^{\infty} -4/3\,\frac{\mathrm{BesselJ}\left(1,\lambda_n r\right)\sin\left(\theta\right)\mathrm{e}^{-1/25\,\lambda_n{}^2 t}\left(\mathrm{BesselJ}\left(0,\lambda_n\right)\lambda_n{}^3-\mathrm{BesselJ}\left(1,\lambda\right.\right.}{\lambda_n{}^3\left(\left(\mathrm{BesselJ}\left(0,\lambda_n\right)\right)^2\lambda_n+\left(\mathrm{BesselJ}\left(1,\lambda_n\right)\right)^2\lambda_n-\right.}$$

9.5 Haberman 8.2.5

problem number 88

Added Feb 24, 2019.

Problem 8.2.5 from from Richard Haberman applied partial differential equations book, 5th edition.

Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius a) $u_t = k\nabla^2 u$ with time-independent boundary conditions:

$$u(a,\theta,t) = g(\theta)$$

And initial conditions $u(r,\theta,0) = f(r,\theta)$. There is an implied periodic boundary conditions on θ

$$\begin{aligned} u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

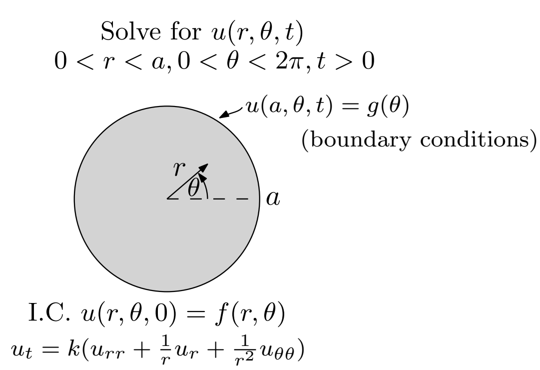


Figure 73: PDE specification

Mathematica ✗

```
ClearAll[r, t, theta, u, a, k, g, f];
pde = D[u[r, theta, t], t] == k*(D[u[r, theta, t], {r, 2}] + D[u[r, theta, t], r]/r + D[u[r,
bcOnR = u[a, theta, t] == g[theta];
bcOnTheta = {u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0,
ic = u[r, theta, 0] == f[r, theta];
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta, ic}, u[r, theta, t], {r,
```

Failed

Maple ✗

```
unassign('r,u,t,theta,g,f');
pde := diff(u(r,theta,t),t)=k*(diff(u(r,theta,t),r$2) + 1/r*diff(u(r,theta,t),r)+1/r^2*diff(
bcOnR:= u(a,theta,t)=g(theta);
bcOnTheta:= u(r,-Pi,t)=u(r,Pi,t),eval(diff(u(r,theta,t),theta),theta=-Pi)=eval(diff(u(r,thet
ic := u(r,theta,0)=f(r,theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bcOnR,bcOnTheta, ic]
```

sol=()

Hand solution

Solve

$$
\begin{aligned}
\frac{\partial u(r,\theta,t)}{\partial t} &= k\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)\\
|u(0,\theta,t)| &< \infty\\
u(a,\theta,t) &= g(\theta)\\
u(r,-\pi,t) &= u(r,\pi,t)\\
\frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

With initial conditions $u(r,\theta,0)=f(r,\theta)$.

Since the boundary conditions are not homogenous, and since there are no time dependent sources, then in this case we look for $u_E(r,\theta)$ which is solution at steady state which needs to satisfy the nonhomogeneous B.C., where $u(r,\theta,t) = \overbrace{v(r,\theta,t)}^{\text{transient}} + \overbrace{u_E(r,\theta)}^{\text{steady state}}$ and $v(r,\theta,t)$ solves the PDE but with homogenous B.C. Therefore, we need to find equilibrium (steady state) solution for Laplace PDE on disk, that only needs to satisfy the nonhomogeneous B.C.

$$
\begin{aligned}
\nabla^2 u_E &= 0\\
\frac{\partial^2 u_E}{\partial r^2} + \frac{1}{r}\frac{\partial u_E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_E}{\partial \theta^2} &= 0
\end{aligned}
$$

With boundary condition

$$
\begin{aligned}
|u_E(0,\theta)| &< \theta\\
u_E(a,\theta) &= g(\theta)\\
u_E(r,-\pi) &= u_E(r,\pi)\\
\frac{\partial u_E}{\partial \theta}(r,-\pi) &= \frac{\partial u_E}{\partial \theta}(r,\pi)
\end{aligned}
$$

Let

$$
u_E(r,\theta) = R(r)\,\Theta(\theta)
$$

Where $R(r)$ is the solution in radial dimension and $\Theta(\theta)$ is solution in angular dimension. Substituting $u_E(r,\theta)$ in the PDE gives

$$
R''\Theta + \frac{1}{r}R'\Theta + \frac{1}{r^2}\Theta'' R = 0
$$

Dividing by $R(r)\,\Phi(\theta)$

$$
\begin{aligned}
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= 0\\
r^2\frac{R''}{R} + r\frac{R'}{R} &= -\frac{\Theta''}{\Theta}
\end{aligned}
$$

Hence each side is equal to constant, say λ and we obtain

$$
\begin{aligned}
r^2\frac{R''}{R} + r\frac{R'}{R} &= \lambda\\
-\frac{\Theta''}{\Theta} &= \lambda
\end{aligned}
$$

Or

$$
r^2 R'' + rR' - \lambda R = 0 \tag{1}
$$

$$
\Theta'' + \lambda\Theta = 0 \tag{2}
$$

We start with Φ ODE. The boundary conditions on (3) are

$$\begin{aligned}\Theta(-\pi) &= \Theta(\pi)\\ \frac{\partial\Theta}{\partial\theta}(-\pi) &= \frac{\partial\Theta}{\partial\theta}(\pi)\end{aligned}$$

case $\lambda = 0$ The solution is $\Phi = c_1\theta + c_2$. Hence we obtain, from first initial conditions

$$\begin{aligned}-\pi c_1 + c_2 &= \pi c_1 + c_2\\ c_1 &= 0\end{aligned}$$

Second boundary conditions just says that $c_2 = c_2$, so any constant will do. Hence $\lambda = 0$ is an eigenvalue with constant being eigenfunction.

case $\lambda > 0$ The solution is

$$\Theta(\theta) = c_1 \cos\sqrt{\lambda}\theta + c_2 \sin\sqrt{\lambda}\theta$$

The first boundary conditions gives

$$\begin{aligned}c_1\cos\left(-\sqrt{\lambda}\pi\right) + c_2\sin\left(-\sqrt{\lambda}\pi\right) &= c_1\cos\left(\sqrt{\lambda}\pi\right) + c_2\sin\left(\sqrt{\lambda}\pi\right)\\ c_1\cos\left(\sqrt{\lambda}\pi\right) - c_2\sin\left(\sqrt{\lambda}\pi\right) &= c_1\cos\left(\sqrt{\lambda}\pi\right) + c_2\sin\left(\sqrt{\lambda}\pi\right)\\ 2c_2\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)\end{aligned}$$

From second boundary conditions we obtain

$$\Theta'(\theta) = -\sqrt{\lambda}c_1\sin\sqrt{\lambda}\theta + c_2\sqrt{\lambda}\cos\sqrt{\lambda}\theta$$

Therefore

$$\begin{aligned}-\sqrt{\lambda}c_1\sin\left(-\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\ \sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right) + c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right)\\ \sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right) &= -\sqrt{\lambda}c_1\sin\left(\sqrt{\lambda}\pi\right)\\ 2c_1\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)\end{aligned}$$

Both (3) and (4) are satisfied if

$$\begin{aligned}\sqrt{\lambda}\pi &= n\pi \qquad n = 1,2,3,\cdots\\ \lambda &= n^2 \qquad n = 1,2,3,\cdots\end{aligned}$$

Therefore

$$\Theta_n(\theta) = \overbrace{A_0}^{\lambda=0} + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

Now we go back to the R ODE (1) given by $r^2R'' + rR' - \lambda_n R = 0$ and solve it. This is Euler ODE whose solution is found by substituting $R(r) = r^\alpha$. The solution comes out to be

$$R_n(r) = c_0 + \sum_{n=1}^{\infty} c_n r^n \tag{6}$$

Combining (5,6) we now find u_E as

$$\begin{aligned} u_{E_n}(r,\theta) &= R_n(r)\,\Theta_n(\theta) \\ u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n(A_n \cos(n\theta) + B_n \sin(n\theta)) \end{aligned} \tag{7}$$

Where c_0 was combined with A_0. Now the above equilibrium solution needs to satisfy the non-homogenous B.C. $u_E(a,\theta) = g(\theta)$. Using orthogonality on (7) to find A_n, B_n gives

$$g(\theta) = A_0 + \sum_{n=1}^{\infty} a^n(A_n \cos(n\theta) + B_n \sin(n\theta))$$

For $n = 0$

$$\begin{aligned} \int_0^{2\pi} g(\theta)\,d\theta &= A_0 \int_0^{2\pi} d\theta \\ A_0 &= \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\,d\theta \end{aligned}$$

For $n > 0$, applying orthogonality using cosine to find A_n gives

$$\begin{aligned} \int_0^{2\pi} g(\theta)\cos(n\theta)\,d\theta &= A_n \int_0^{2\pi} \cos^2(n\theta)\,a^n d\theta \\ A_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\,d\theta \end{aligned}$$

Similarly, applying orthogonality using sin to find B_n gives

$$B_n = \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\,d\theta$$

Therefore, we have found $u_E(r,\theta)$ completely now. It is given by (7)

$$
\begin{aligned}
u_E(r,\theta) &= A_0 + \sum_{n=1}^{\infty} r^n \left(A_n \cos(n\theta) + B_n \sin(n\theta)\right) && \text{(7A)}\\
A_0 &= \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\, d\theta\\
A_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\, d\theta\\
B_n &= \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\, d\theta
\end{aligned}
$$

Now, since $u(r,\theta,t) = v(r,\theta,t) + u_E(r,\theta)$, then we need to solve now for $v(r,\theta,t)$ with homogeneous boundary conditions

$$
\begin{aligned}
v_t(r,\theta,t) &= k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}\right) && \text{(8)}\\
|v(0,\theta,t)| &< \theta\\
v(a,\theta,t) &= 0\\
v(r,-\pi,t) &= v(r,\pi,t)\\
\frac{\partial v}{\partial \theta}(r,-\pi,t) &= \frac{\partial v}{\partial \theta}(r,\pi,t)
\end{aligned}
$$

Let $v(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$. Substituting into (8) gives

$$
T'R\Theta = k\left(R''T\Theta + \frac{1}{r}R'T\Theta + \frac{1}{r^2}\Theta''RT\right)
$$

Dividing by $R(r)\,\Theta(\theta)\,T(t) \neq 0$ gives

$$
\frac{1}{k}\frac{T'}{T} = \frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta}
$$

Let first separation constant be $-\lambda$, hence the above becomes

$$
\begin{aligned}
\frac{1}{k}\frac{T'}{T} &= -\lambda\\
\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} &= -\lambda
\end{aligned}
$$

Or

$$
\begin{aligned}
T' + \lambda k T &= 0\\
r^2\frac{R''}{R} + r\frac{R'}{R} + r^2\lambda &= -\frac{\Theta''}{\Theta}
\end{aligned}
$$

We now separate the second equation above using μ giving

$$
\begin{aligned}
r^2\frac{R''}{R}+r\frac{R'}{R}+r^2\lambda &= \mu\\
-\frac{\Theta''}{\Theta} &= \mu
\end{aligned}
$$

Or

$$
R''+\frac{1}{r}R'+R\left(\lambda-\frac{\mu}{r^2}\right)=0 \tag{9}
$$

$$
\Theta''+\mu\Theta=0 \tag{10}
$$

Equation (9) is Sturm-Liouville ODE with boundary conditions $R(a)=0$ and bounded at $r=0$ and (10) has periodic boundary conditions as was solved above. The solution to (10) is given in (5) above, no change for this part.

$$
\Theta_n(\theta)=\overbrace{\alpha_0}^{\lambda=0}+\sum_{n=1}^{\infty}\alpha_n\cos(n\theta)+\beta_n\sin(n\theta) \tag{11}
$$

Therefore (9) becomes $R''+\frac{1}{r}R'+R\left(\lambda-\frac{n^2}{r^2}\right)=0$ with $n=0,1,2,\cdots$. We found the solution to this Sturm-Liouville before, it is given by

$$
R_{nm}(r)=J_n\left(\sqrt{\lambda_{nm}}r\right)\qquad n=0,1,2,\cdots,m=1,2,3,\cdots \tag{12}
$$

Where $\sqrt{\lambda_{nm}}=\frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n. This is found numerically. We now just need to find the time solution from $T'+\lambda_{nm}kT=0$. For This has solution

$$
T_{nm}(t)=e^{-k\lambda_{nm}t} \tag{13}
$$

Now we combine (11,12,13) to find solution for $v(r,\theta,t)$, and combining constants gives

$$
v_{nm}(r,\theta,t)=\Theta_n(\theta)\,R_{nm}(r)\,T_{nm}(t)
$$

$$
\begin{aligned}
v(r,\theta,t) &= \alpha_{0,1}J_0\left(\sqrt{\lambda_{0,1}}r\right)+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta))\\
&= \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)(\alpha_{nm}\cos(n\theta)+\beta_{nm}\sin(n\theta))
\end{aligned} \tag{14}
$$

We now need to find $\alpha_0,\alpha_{nn},\beta_{nm}$, which are found from initial conditions on $v(r,\theta,0)$ which is given by

$$
\begin{aligned}
v(r,\theta,0) &= u(r,\theta,0)-u_E(r,\theta)\\
&= f(r,\theta)-u_E(r,\theta)
\end{aligned}
$$

Hence from (14), at $t=0$

$$f(r,\theta)-u_E(r,\theta)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)\left(\alpha_{nm}\cos\left(n\theta\right)+\beta_{nm}\sin\left(n\theta\right)\right) \tag{15}$$

For each n, inside the m sum, $\cos(n\theta)$ and $\sin(n\theta)$ will be constant. So we need to apply orthogonality twice in order to remove both sums. Multiplying (15) by $\cos(n'\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos\left(n'\theta\right)d\theta &= \int_{-\pi}^{\pi}\sum_{n=0}^{\infty}\left(\sum_{m=1}^{\infty}\alpha_{nn}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\cos\left(n\theta\right)\cos\left(n'\theta\right)d\theta\\ &+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty}\beta_{nm}J_n\left(\sqrt{\lambda_{nm}}r\right)\right)\sin\left(n\theta\right)\cos\left(n'\theta\right)\end{aligned}$$

The second sum in the RHS above goes to zero due to $\int_{-\pi}^{\pi}\sin(n\theta)\cos(n'\theta)\,d\theta$ and we end up with

$$\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos\left(n\theta\right)d\theta=\alpha_{nn}\int_{-\pi}^{\pi}\cos^2\left(n\theta\right)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)d\theta$$

We now apply orthogonality again, but on Bessel functions and remembering to add the weight r, the above becomes

$$\begin{aligned}\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos\left(n\theta\right)J_n\left(\sqrt{\lambda_{nm'}}r\right)rd\theta dr &= \alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2\left(n\theta\right)\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)J_n\left(\sqrt{\lambda_{nm'}}r\right)r\\ &= \alpha_{nn}\int_0^a\int_{-\pi}^{\pi}\cos^2\left(n\theta\right)J_n^2\left(\sqrt{\lambda_{nm'}}r\right)rd\theta dr\end{aligned}$$

Therefore

$$\alpha_{nn}=\frac{\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\cos\left(n\theta\right)J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\cos^2\left(n\theta\right)J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

We will repeat the same thing to find β_{nm}. The only difference now is to use $\sin n\theta$. repeating these steps gives

$$\beta_{nm}=\frac{\int_0^a\int_{-\pi}^{\pi}\left(f(r,\theta)-u_E(r,\theta)\right)\sin\left(n\theta\right)J_n\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}{\int_0^a\int_{-\pi}^{\pi}\sin^2\left(n\theta\right)J_n^2\left(\sqrt{\lambda_{nm}}r\right)rd\theta dr}\qquad n=0,1,2,\cdots,m=1,2,3,\cdots$$

This complete the solution.

Summary of solution

$$\begin{aligned}u(r,\theta,t)&=v(r,\theta,t)+u_E(r,\theta)\\ &=u_E(r,\theta)+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)e^{-k\lambda_{nm}t}\left(\alpha_{nn}\cos\left(n\theta\right)+\beta_{nm}\sin\left(n\theta\right)\right)\end{aligned}$$

Where

$$u_E(r,\theta) = A_0 + \sum_{n=1}^{\infty} r^n (A_n \cos(n\theta) + B_n \sin(n\theta))$$
$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} g(\theta)\, d\theta$$
$$A_n = \frac{1}{\pi}\int_0^{2\pi} g(\theta)\cos(n\theta)\, d\theta$$
$$B_n = \frac{1}{\pi}\int_0^{2\pi} g(\theta)\sin(n\theta)\, d\theta$$

And

$$\alpha_{nn} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta))\cos(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \cos^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots$$

And

$$\beta_{nm} = \frac{\int_0^a \int_{-\pi}^{\pi} (f(r,\theta) - u_E(r,\theta))\sin(n\theta)\, J_n\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr}{\int_0^a \int_{-\pi}^{\pi} \sin^2(n\theta)\, J_n^2\left(\sqrt{\lambda_{nm}} r\right) r d\theta dr} \qquad n = 0,1,2,\cdots, m = 1,2,3,\cdots$$

Where $\sqrt{\lambda_{nm}} = \frac{a}{z_{nm}}$ where a is the radius of the disk and z_{nm} is the m^{th} zero of the Bessel function of order n.

10 Heat PDE inside Sphere

10.1 No angle dependencies, zero intial conditions, non zero temperature at surface

problem number 89

Added March 28, 2019.

Problem 1, section 41, Fourier series and boundary value problems 8th edition by Brown and Churchill.

Solve $u_t = \nabla u$ where $\nabla u = \frac{1}{r}(ru)_{rr}$ in Spherical coordinates with initial conditions $u(r,0) = 0$ and boundary conditions $u(1,t) = t$

Mathematica ✗

```
ClearAll[r, t, u, k];
 pde = D[u[r, t], t] == (k*D[r*u[r, t], {r, 2}])/r;
 ic = u[r, 0] == 0;
 bc = u[1, t] == t;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}, Assumptions ->
```

Failed

Maple ✓

```
r:='r'; u:='u'; t:='t';theta:='theta';k:='k';
pde:=diff(u(r,t),t)= k/r*diff(r*u(r,t),r$2);
ic:=u(r,0)=0;
bc := u(1,t) =t;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(r,t),HINT =b
```

$$u(r,t) = \frac{1}{r}\left(-invlaplace\left(\_F2(s)\sinh\left(\frac{\sqrt{s}\,r}{\sqrt{k}}\right)\cosh\left(\frac{\sqrt{s}}{\sqrt{k}}\right)\left(\sinh\left(\frac{\sqrt{s}}{\sqrt{k}}\right)\right)^{-1}, s, t\right) + invlaplace\left(\frac{1}{s^2}\,\mathrm{sinh}\right.\right.$$

Has unresolved Laplace integrals

11 Diffusion Reaction in 1D

11.1 Growth form reaction term

problem number 90

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = k u_{xx} + r u$$

with $k = \frac{1}{10}, r = 1$ and $0 < x < 1$ and $t > 0$. With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x,0) = 1$.

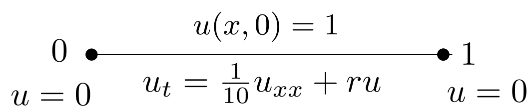


Figure 74: PDE specification

Mathematica ✗

```
ClearAll[x, t, k, r];
 k = 1/10;
 r = 1;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; u:='u'; t:='t';k:='k';r:='r';
k:=1/10; r:=1;
pde := diff(u(x,t), t) = k*diff(u(x, t), x$2) + r*u(x,t);
bc:=u(0,t)=0,u(1,t)=0;
ic:=u(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic], u(x,t))),out
```

$$u(x,t) = \sum_{n=1}^{\infty} -2 \frac{(-1+(-1)^n) \sin(n\pi x) e^{-1/10 t(\pi^2 n^2 - 10)}}{n\pi}$$

Hand solution

Solution added 4/3/2019.

$$\begin{aligned} u_t &= k u_{xx} + r u \qquad t>0, 0<x<1 \\ u(0,t) &= 0 \\ u(1,t) &= 0 \\ u(x,0) &= 1 \end{aligned}$$

Let $u = ve^{rt}$. Hence $u_t = v_t e^{rt} + vre^{rt}$ and $u_{xx} = v_{xx}e^{rt}$. Hence the PDE becomes $v_t e^{rt} + vre^{rt} = v_{xx}e^{rt} + rve^{rt}$ which simplifies to

$$\begin{aligned} v_t &= k v_{xx} \qquad t>0, 0<x<1 \\ v(0,t) &= 0 \\ v(1,t) &= 0 \end{aligned}$$

The above is now in canonical form, it is standard heat PDE with homogeneous B.C. This has the solution

$$v(x,t) = \sum_{n=1}^{\infty} B_n e^{-n^2\pi^2 t} \sin(n\pi x)$$

Therefore

$$\begin{aligned} u &= ve^{rt} \\ u &= \sum_{n=1}^{\infty} B_n e^{-t(n^2\pi^2 - r)} \sin(n\pi x) \end{aligned}$$

At $t=0$ the above becomes

$$1 = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

Hence B_n are sin Fourier coefficient of 1 which is

$$
\begin{aligned}
B_n &= 2\int_0^1 \sin(n\pi x)\,dx \\
&= 2\left(-\frac{1}{n\pi}\right)(\cos n\pi x)_0^1 \\
&= \frac{-2}{n\pi}((-1)^n - 1)
\end{aligned}
$$

Hence the solution becomes

$$
u = \sum_{n=1}^{\infty} \frac{-2}{n\pi}((-1)^n - 1)\, e^{-t(n^2\pi^2 - r)} \sin(n\pi x)
$$

But $r = \frac{1}{10}$, therefore

$$
u(x,t) = -\frac{2}{\pi}\sum_{n=1}^{\infty} \frac{((-1)^n - 1)}{n} e^{-t\left(n^2\pi^2 - \frac{1}{10}\right)} \sin(n\pi x)
$$

11.2 using logistic form for reaction term

problem number 91

Added December 29, 2018.

Solve for $u(x,t)$ in

$$
u_t = k u_{xx} + r u\left(1 - \frac{u}{\alpha}\right)
$$

with $k = \frac{1}{100}, r = \frac{1}{10}, \alpha = 10$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$
\begin{aligned}
u(0,t) &= 0 \\
u(1,t) &= 0
\end{aligned}
$$

And initial conditions $u(x,0) = 1$.

Mathematica ✗

```
ClearAll[x, t, k, r, alpha];
 k = 1/100;
 r = 1/10;
 alpha = 10;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + r*u[x, t]*(1 - u[x, t]/alpha);
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
x:='x'; u:='u'; t:='t';k:='k';r:='r';
k := 1/100; r := 1/10; alpha := 10;
pde := diff(u(x, t), t)=   k*diff(u(x, t),x$2) + r*u(x, t)*(1- u(x,t)/alpha);
bc := u(0, t) = 0, u(1, t) = 0;
ic := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc], u(x, t))),c
```

sol=()

11.3 using Aleee form for reaction term

problem number 92

Added December 29, 2018.

Solve for $u(x,t)$ in

$$u_t = k u_{xx} + \alpha u + \beta u^2 - \gamma u^3$$

with $k = \frac{1}{1000}, \alpha = \frac{1}{100}, \beta = \frac{1}{100}, \gamma = \frac{5}{1000}$ and $0 < x < 1$ and $t > 0$.

With boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 0$$

And initial conditions $u(x, 0) = 1$.

Mathematica ✗

```
ClearAll[x, t, k, alpha, beta, gamma];
 k = 1/1000;
 alpha = 1/10;
 beta = 1/100;
 gamma = 5/1000;
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + alpha*u[x, t] + beta*u[x, t]^2 - gamma*u[x, t
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
x:='x'; u:='u'; t:='t';k:='k';
k := 1/100; alpha:=1/100;beta:=1/1000; g:=5/1000;
pde := diff(u(x, t), t)= k*diff(u(x, t),x$2) +
        alpha*u(x,t)+ beta*u(x,t)^2 - g*u(x,t)^3;
bc := u(0, t) = 0, u(1, t) = 0;
ic := u(x, 0) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))),
```

sol=()

12 Diffusion-advection (convection) in 1D

12.1 Semi infinite domain

problem number 93

Added April 5, 2019.

Solve for $u(x,t)$ in

$$u_t = u_{xx} - u_x$$

For $t>0, x>0$. With boundary conditions $u(0,t)=0$ and intitial conditions $u(x,0)=f(x)$

Mathematica ✗

```
ClearAll[x, t, f];
pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - D[u[x, t], x];
ic = u[x, 0] == f[x];
bc = u[0, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {
```

Failed

Maple ✓

```
unassign('x,t,f');
pde:=diff(u(x,t),t)=diff(u(x,t),x$2)- diff(u(x,t),x);
ic:=u(x,0)=f(x);
bc:=u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc], u(x, t))as
```

$$u(x,t) = e^{x/2} invlaplace\left(\frac{1}{\sqrt{e^{x\sqrt{1+4s}}}\sqrt{1+4s}} \int^{0} \frac{e^{-a}\_C2 + f(\_a) - \_C2}{\sqrt{e^{-a\sqrt{1+4s}}}\sqrt{e^{-a}}} d\_a, s, t\right) - e^{x/2} invlaplace\left(\frac{}{\sqrt{e^{x}}}\right.$$

13 Laplace PDE in Cartesian coordinates

13.1 Laplace PDE inside rectangle (Haberman 2.5.1 (a))

problem number 94

This is problem 2.5.1 part (a) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y) &= 0\\ \frac{\partial u}{\partial x}(L,y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= f(x)\end{aligned}$$

Mathematica ✓

```
ClearAll[u, t, k, x, L, H, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H]
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
 sol = sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi x}{L}\right)\operatorname{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L\cos\left(\frac{n\pi x}{L}\right)f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L}+\frac{y\int_0^L f(x)\,dx}{HL}\right\}\right\}$$

Maple ✓

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
#these simplifications below to convert answer to one that match standard;
sol:=convert(sol,trigh);
sol:=simplify(expand(sol));
```

$$u(x,y) = \frac{1}{H\,L}\left(4\sum_{n=1}^{\infty}\left(1/2\,1\cos\left(\frac{n\pi\,x}{L}\right)\int_0^L f(x)\cos\left(\frac{n\pi\,x}{L}\right)\,\mathrm{d}x\sinh\left(\frac{n\pi\,y}{L}\right)\left(\sinh\left(\frac{n\pi\,H}{L}\right)\right)^{-1}\right)H+$$

13.2 Laplace PDE inside rectangle (Haberman 2.5.1 (b))

problem number 95

This is problem 2.5.1 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y) &= g(y)\\ \frac{\partial u}{\partial x}(L,y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= 0\end{aligned}$$

Mathematica ✓

```
ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == g[y], Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
 sol = sol /. {K[1] -> n};
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}\right\}\right\}$$

Maple ✓

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-2\,\frac{1}{n\pi}\sin\left(\frac{n\pi\,y}{H}\right)\int_0^H\sin\left(\frac{n\pi\,y}{H}\right)g(y)\,\mathrm{d}y\left(\cosh\left(\frac{n\pi\,(2\,L-x)}{H}\right)+\sinh\left(\frac{n\pi\,(2\,L-x)}{H}\right)\right.\right.$$

13.3 Laplace PDE inside rectangle (Haberman 2.5.1 (c))

problem number 96

This is problem 2.5.1 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\le x\le L, 0\le y\le H$, with following boundary conditions

$$\frac{\partial u}{\partial x}(0,y)=0$$
$$u(L,y)=g(y)$$
$$u(x,0)=0$$
$$u(x,H)=0$$

Mathematica ✗

```
ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, u[L, y] == g[y], u[x, 0] == 0, u[x, H] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

Failed

Maple ✓

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc],u(x,y)) assuming
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(4\frac{1}{H}\sin\left(\frac{n\pi\,y}{H}\right)\int_0^H\sin\left(\frac{n\pi\,y}{H}\right)g(y)\,\mathrm{d}y\left(\cosh\left(\frac{n\pi\,L}{H}\right)+\sinh\left(\frac{n\pi\,L}{H}\right)\right)\cosh\left(\frac{n\pi\,x}{H}\right)\left(\mathrm{co}\right.\right.$$

13.4 Laplace PDE inside rectangle (Haberman 2.5.1 (d))

problem number 97

This is problem 2.5.1 part (d) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} u(0, y) &= g(y) \\ u(L, y) &= 0 \\ \frac{\partial u}{\partial y} u(x, 0) &= 0 \\ u(x, H) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, Derivative[0, 1][u][x, 0] == 0, u[x, H] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

Failed

Maple ✓

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y) = \sum_{n=0}^{\infty} \left(2\,\frac{1}{H}\sin\left(1/2\,\frac{\pi\,(2\,ny+H+y)}{H}\right)\int_0^H \sin\left(1/2\,\frac{\pi\,(2\,ny+H+y)}{H}\right)g(y)\,\mathrm{d}y\left(\cosh\left(1/2\,\frac{(1+$$

13.5 Laplace PDE inside rectangle (Haberman 2.5.1 (e))

problem number 98

This is problem 2.5.1 part (e) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(L,y) &= 0 \\ u(x,0) - \frac{\partial u}{\partial y}u(x,0) &= 0 \\ u(x,H) &= f(x) \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, u[x, 0] - Derivative[0, 1][u][x, 0] == 0, u[x, H] == f[x]
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

Failed

Maple ✓

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming(
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(4\frac{1}{L}\int_{0}^{L}\sin\left(\frac{n\pi x}{L}\right)f(x)\,dx\left(\cosh\left(\frac{n\pi H}{L}\right)+\sinh\left(\frac{n\pi H}{L}\right)\right)\sin\left(\frac{n\pi x}{L}\right)\left(\pi\cosh\left(\frac{n\pi y}{L}\right)\right.\right.$$

Hand solution

Let $u(x,y)=X(x)\,Y(x)$. Substituting this into the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and simplifying gives

$$\frac{X''}{X}=-\frac{Y''}{Y}$$

Each side depends on different independent variable and they are equal, therefore they must be equal to same constant.

$$\frac{X''}{X}=-\frac{Y''}{Y}=\pm\lambda$$

Since the boundary conditions along the x direction are the homogeneous ones, $-\lambda$ is selected in the above. Two ODE's (1,2) are obtained as follows

$$X''+\lambda X=0 \tag{1}$$

With the boundary conditions

$$X(0) = 0$$
$$X(L) = 0$$

And

$$Y'' - \lambda Y = 0 \tag{2}$$

With the boundary conditions

$$Y(0) = Y'(0)$$
$$Y(H) = f(x)$$

In all these cases λ will turn out to be positive. This is shown for this problem only and not be repeated again.

Case $\lambda < 0$

The solution to (1) us

$$X = A\cosh\left(\sqrt{|\lambda|}x\right) + B\sinh\left(\sqrt{|\lambda|}x\right)$$

At $x = 0$, the above gives $0 = A$. Hence $X = B\sinh\left(\sqrt{|\lambda|}x\right)$. At $x = L$ this gives $X = B\sinh\left(\sqrt{|\lambda|}L\right)$. But $\sinh\left(\sqrt{|\lambda|}L\right) = 0$ only at 0 and $\sqrt{|\lambda|}L \neq 0$, therefore $B = 0$ and this leads to trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$

$$X = Ax + B$$

Hence at $x = 0$ this gives $0 = B$ and the solution becomes $X = B$. At $x = L$, $B = 0$. Hence the trivial solution. $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$

Solution is

$$X = A\cos\left(\sqrt{\lambda}x\right) + B\sin\left(\sqrt{\lambda}x\right)$$

At $x = 0$ this gives $0 = A$ and the solution becomes $X = B\sin\left(\sqrt{\lambda}x\right)$. At $x = L$

$$0 = B\sin\left(\sqrt{\lambda}L\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}L\right) = 0$ or $\sqrt{\lambda}L = n\pi$ where $n = 1, 2, 3, \cdots$, therefore

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \cdots$$

Eigenfunctions are

$$X_n(x) = B_n \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3, \cdots \tag{3}$$

For the Y ODE, the solution is

$$\begin{aligned} Y_n &= C_n \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ Y_n' &= C_n \frac{n\pi}{L} \sinh\left(\frac{n\pi}{L}y\right) + D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) \end{aligned}$$

Applying B.C. at $y = 0$ gives

$$\begin{aligned} Y(0) &= Y'(0) \\ C_n \cosh(0) &= D_n \frac{n\pi}{L} \cosh(0) \\ C_n &= D_n \frac{n\pi}{L} \end{aligned}$$

The eigenfunctions Y_n are

$$\begin{aligned} Y_n &= D_n \frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + D_n \sinh\left(\frac{n\pi}{L}y\right) \\ &= D_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \end{aligned}$$

Now the complete solution is produced

$$\begin{aligned} u_n(x,y) &= Y_n X_n \\ &= D_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) B_n \sin\left(\frac{n\pi}{L}x\right) \end{aligned}$$

Let $D_n B_n = B_n$ since a constant. (no need to make up a new symbol).

$$u_n(x,y) = B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Sum of eigenfunctions is the solution, hence

$$u(x,y) = \sum_{n=1}^{\infty} B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}y\right) + \sinh\left(\frac{n\pi}{L}y\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

The nonhomogeneous boundary condition is now resolved. At $y = H$

$$u(x,H) = f(x)$$

Therefore

$$f(x) = \sum_{n=1}^{\infty} B_n \left(\frac{n\pi}{L} \cosh\left(\frac{n\pi}{L}H\right) + \sinh\left(\frac{n\pi}{L}H\right)\right) \sin\left(\frac{n\pi}{L}x\right)$$

Multiplying both sides by $\sin\left(\frac{m\pi}{L}x\right)$ and integrating gives

$$\begin{aligned}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx &= \int_0^L \sin\left(\frac{m\pi}{L}x\right)\sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\sin\left(\frac{n\pi}{L}x\right)dx\\ &= \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)\int_0^L \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx\\ &= B_m\left(\frac{m\pi}{L}\cosh\left(\frac{m\pi}{L}H\right)+\sinh\left(\frac{m\pi}{L}H\right)\right)\frac{L}{2}\end{aligned}$$

Hence

$$B_n = \frac{2}{L}\frac{\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx}{\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}H\right)+\sinh\left(\frac{n\pi}{L}H\right)\right)} \tag{4}$$

This completes the solution. In summary

$$u(x,y) = \sum_{n=1}^{\infty} B_n\left(\frac{n\pi}{L}\cosh\left(\frac{n\pi}{L}y\right)+\sinh\left(\frac{n\pi}{L}y\right)\right)\sin\left(\frac{n\pi}{L}x\right)$$

With B_n given by (4).

13.6 Laplace PDE inside rectangle, top/bottom edges non-zero

problem number 99

Taken from Mathematica DSolve help pages.

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0 \leq x \leq 1, 0 \leq y \leq 2$, with following boundary conditions

$$\begin{aligned}u(0,y) &= 0\\ u(1,y) &= 0\\ u(x,0) &= \text{UnitTriagle(2 x-1)}\\ u(x,2) &= \text{UnitTriagle(2 x-1)}\end{aligned}$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 L0 = 1;
 H0 = 2;
 bc = DirichletCondition[u[x, y] == Piecewise[{{UnitTriangle[2*x - L0], y == 0 || y == H0}},
 domain = Rectangle[{0, 0}, {L0, H0}];
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], Element[{x, y}, do
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{8\operatorname{csch}(2n\pi)\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)(\sinh(n\pi(2-y))+\sinh(n\pi y))}{n^2\pi^2}\right\}\right\}$$

Maple ✓

```
u:='u'; y:='y'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x-> piecewise(x>0 and x<1/2, 2*x, x>1/2 and x<1, 2-2*x);
bc:=u(0,y)=0,u(1,y)=0,u(x,0)=f(x),u(x,2)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y)) assuming
```

$$u(x,y)=\sum_{n=1}^{\infty}8\,\frac{\sin\left(1/2\,\pi\,n\right)\mathrm{e}^{2\,\pi\,n}\sin\left(n\pi\,x\right)\left(-\mathrm{e}^{\pi\,n(-2+y)}+\mathrm{e}^{-\pi\,n(-2+y)}+\mathrm{e}^{n\pi\,y}-\mathrm{e}^{-n\pi\,y}\right)}{n^2\pi^2\left(\mathrm{e}^{4\,\pi\,n}-1\right)}$$

13.7 Laplace PDE inside rectangle, top edge at infinity, left edge nonhomogeneous constant

problem number 100

Added December 20, 2018.

Example 21, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq x \leq L, 0 \leq y \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0, y) &= A \\ u(L, y) &= 0 \\ u(x, 0) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, y, L, A];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == A, u[L, y] == 0, u[x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

Failed

Maple ✓

```
u:='u'; y:='y'; x:='x';L:='L';A:='A';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = A;
bc_right_edge:= u(L, y) = 0;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x, y) = \frac{1}{L}\left(\sum_{n=1}^{\infty} -2\frac{A}{\pi n}\sin\left(\frac{n\pi x}{L}\right)e^{-\frac{n\pi y}{L}}L + A(L - x)\right)$$

Hand solution

Let

$$u = U + v \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0) = A, v(L) = 0$. This implies

$$v(x) = A\left(1 - \frac{x}{L}\right)$$

Hence $u = U + A\left(1 - \frac{x}{L}\right)$. Substituting this back in $\nabla^2 u = 0$ gives

$$\nabla^2 U = 0$$

But with boundary condition on right edge being zero now. Let $U = X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$\begin{aligned} U(x,y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \end{aligned}$$

Using the above in (1) gives the solution

$$u(x,y) = A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}$$

At $y = 0$ the above gives

$$\begin{aligned} 0 &= A\left(1 - \frac{x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\ A\left(\frac{x}{L} - 1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $\frac{A}{L}x$

$$
\begin{aligned}
B_n &= \frac{2}{L}\int_0^L A\left(\frac{x}{L}-1\right)\sin\left(\frac{n\pi}{L}x\right)dx \\
&= \frac{2A}{L}\int_0^L \left(\frac{x}{L}-1\right)\sin\left(\frac{n\pi}{L}x\right)dx \\
&= -\frac{2A}{L}\frac{L}{n\pi} \\
&= -\frac{2A}{n\pi}
\end{aligned}
$$

Hence the solution (2) becomes

$$
u(x,y) = A\left(1-\frac{x}{L}\right) - 2\frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{L}x\right)e^{-\frac{n\pi}{L}y}
$$

13.8 Laplace PDE inside rectangle, top edge at infinity, right edge nonhomogeneous constant

problem number 101

Added March 19, 2019

Solve Laplace equation

$$
u_{xx} + u_{yy} = 0
$$

Inside a rectangle $0 \leq x \leq L, 0 \leq y \leq \infty$, with following boundary conditions

$$
\begin{aligned}
u(0,y) &= 0 \\
u(L,y) &= A \\
u(x,0) &= 0
\end{aligned}
$$

Mathematica ✗

```
ClearAll[u, x, y, L, A];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == 0, u[L, y] == A, u[x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

Failed

Maple ✓

```
u:='u'; y:='y'; x:='x';L:='L';A:='A';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_right_edge:= u(L, y) = A;
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_right_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

$$u(x,y) = \frac{1}{L}\left(Ax + \sum_{n=1}^{\infty} 2\,\frac{(-1)^n A}{\pi\, n} \sin\left(\frac{n\pi\, x}{L}\right) \mathrm{e}^{-\frac{n\pi\, y}{L}} L\right)$$

Hand solution

Let

$$u = U + v \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with right edge boundary conditions zero, and $v(x)$ satisfies the nonhomogeneous boundary conditions $v(0) = 0A, v(L) = A$. This implies

$$v(x) = A\frac{x}{L}$$

Hence $u = U + \frac{A}{L}x$. Substituting this back in $\nabla^2 u = 0$ gives

$$\nabla^2 U = 0$$

But with boundary condition on right edge being zero now. Let $U = X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ X(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $X_n(x) = \sin\left(\sqrt{\lambda_n}x\right)$. The Y ode is

$$\begin{aligned} Y_n'' - \lambda_n Y_n &= 0 \\ Y_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $Y_n(y) = c_{1_n} e^{\sqrt{\lambda_n} y} + c_{2_n} e^{-\sqrt{\lambda_n} y}$. Since $Y_n(y)$ is bounded, then $c_{1_n} = 0$ and the $Y_n(y) = c_{2_n} e^{-\sqrt{\lambda_n} y}$. Hence

$$
\begin{aligned}
U(x,y) &= \sum_{n=1}^{\infty} X_n(x)\, Y_n(y) \\
&= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} x\right) e^{-\sqrt{\lambda_n} y} \\
&= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y}
\end{aligned}
$$

Using the above in (1) gives the solution

$$
u(x,y) = \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y} \tag{2}
$$

At $y = 0$ the above gives

$$
\begin{aligned}
0 &= \frac{A}{L} x + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right) \\
-\frac{A}{L} x &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} x\right)
\end{aligned}
$$

Therefore B_n are the Fourier sine coefficients of $-\frac{A}{L} x$

$$
\begin{aligned}
B_n &= -\frac{2}{L} \int_0^L \frac{A}{L} x \sin\left(\frac{n\pi}{L} x\right) dx \\
&= -\frac{2A}{L^2} \int_0^L x \sin\left(\frac{n\pi}{L} x\right) dx \\
&= -\frac{2A}{L^2} \frac{(-1)^{n+1} L^2}{n\pi} \\
&= \frac{2A}{n\pi} (-1)^n
\end{aligned}
$$

Hence the solution (2) becomes

$$
u(x,y) = \frac{A}{L} x + \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\left(\frac{n\pi}{L} x\right) e^{-\frac{n\pi}{L} y}
$$

13.9 Laplace PDE inside rectangle, right edge at infinity, bottom edge nonhomogeneous constant

problem number 102

Added March 19, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq L, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(x,0) &= A \\ u(x,L) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, y, L, A];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == 0, u[x, 0] == A, u[x, L] == 0};
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

Failed

Maple ✗

```
u:='u'; y:='y'; x:='x';L:='L';A:='A';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = 0;
bc_bottom_edge:= u(x, 0) = A;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], HINT = boundeds
```

sol=()

Hand solution

Let

$$u(x,y) = U(x,y) + v(y) \tag{1}$$

Where U satisfies $\nabla^2 U = 0$ but with bottom edge boundary conditions zero, and $v(y)$ satisfies the nonhomogeneous boundary conditions $v(0) = A, v(L) = 0$. This implies

$$v(y) = A\left(1 - \frac{y}{L}\right)$$

Substituting (1) back in $\nabla^2 u = 0$ results in

$$\nabla^2 U = 0$$

But with boundary condition on bottom edge as $U = 0$. Now we can use separation of variables. Let $U = X(x)Y(x)$. Substituting this in the above gives

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the Y direction. Hence

$$\frac{Y''}{Y} = -\frac{X''}{X} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} Y'' + \lambda Y &= 0 \\ Y(0) &= 0 \\ Y(L) &= 0 \end{aligned}$$

This has eigenvalues $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, \cdots$ with eigenfunctions $Y_n(x) = \sin\left(\sqrt{\lambda_n} y\right)$. The X ode is

$$\begin{aligned} X_n'' - \lambda_n X_n &= 0 \\ X_n(0) &= 0 \end{aligned}$$

Since $\lambda_n > 0$ then the solution is $X_n(y) = c_{1_n} e^{\sqrt{\lambda_n} x} + c_{2_n} e^{-\sqrt{\lambda_n} x}$. Since $X_n(x)$ is bounded, then $c_{1_n} = 0$ and the $X_n(x) = c_{2_n} e^{-\sqrt{\lambda_n} x}$. Hence by superposition the solution is

$$\begin{aligned} U(x,y) &= \sum_{n=1}^{\infty} X_n(x) Y_n(y) \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\sqrt{\lambda_n} y\right) e^{-\sqrt{\lambda_n} x} \\ &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L} y\right) e^{-\frac{n\pi}{L} x} \end{aligned}$$

Substituting the above in (1) gives

$$u(x,y) = A\left(1-\frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}y\right) e^{-\frac{n\pi}{L}x} \tag{2}$$

At $x=0$ the above gives

$$\begin{aligned}
0 &= A\left(1-\frac{y}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}y\right)\\
A\left(\frac{y}{L}-1\right) &= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}y\right)
\end{aligned}$$

Therefore B_n are the Fourier sine coefficients of $A\left(\frac{y}{L}-1\right)$

$$\begin{aligned}
B_n &= \frac{2}{L}\int_0^L A\left(\frac{y}{L}-1\right)\sin\left(\frac{n\pi}{L}y\right)dy\\
&= \frac{2A}{L}\int_0^L \left(\frac{y}{L}-1\right)\sin\left(\frac{n\pi}{L}y\right)dy\\
&= -\frac{2A}{L}\frac{L}{n\pi}\\
&= -\frac{2A}{n\pi}
\end{aligned}$$

Hence the solution (2) becomes

$$u(x,y) = A\left(1-\frac{y}{L}\right) - \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{L}y\right) e^{-\frac{n\pi}{L}x}$$

13.10 Laplace PDE inside rectangle, right edge at infinity, top edge nonhomogeneous function e^{-x}

problem number 103

Added March 20, 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq L, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned}
u(0,y) &= 0\\
u(x,L) &= e^{-x}\\
u(x,0) &= 0
\end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, y, L, A];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == 0, u[x, L] == Exp[-x], u[x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

Failed

Maple ✓

```
u:='u'; y:='y'; x:='x';L:='L';A:='A';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, L) = exp(-x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y)) assuming
```

$$u(x,y) = \frac{1}{L}\left(\sum_{n=1}^{\infty} -\frac{1}{\pi n(\pi n + L)} \sin\left(\frac{n\pi y}{L}\right)\left(\_C5(n)\left(-\pi^2 n^2 + L^2\right) e^{-\frac{n\pi x}{L}} + \left((2\pi n + 4L)(-1)^{n+1} + \_\right.\right.\right.$$

I need to find out how Maple obtained the above solution. It seems to have unknown constant in it

Hand solution

Let $u = X(x)\,Y(x)$. Substituting this in $\nabla^2 u = 0$

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

We want the eigenvalue problem to be in the X direction. Hence

$$\frac{X''}{X} = -\frac{Y''}{Y} = -\lambda$$

Therefore the eigenvalue problem is

$$\begin{aligned} X'' + \lambda X &= 0 \\ X(0) &= 0 \\ |X(x)| &< \infty \end{aligned}$$

case $\lambda < 0$

Solution is $X(x) = c_1 \cosh\left(\sqrt{-\lambda} x\right) + c_2 \sinh\left(\sqrt{-\lambda} x\right)$. Since $X(0) = 0$ then $c_1 = 0$. Solution becomes $X(x) = c_2 \sinh\left(\sqrt{-\lambda} x\right)$. Since sinh is not bounded on $x > 0$ as $x \to \infty$ then $c_2 = 0$. Therefore $\lambda < 0$ is not eigenvalue.

case $\lambda = 0$

Solution is $X(x) = c_1 x + c_2$. At $x = 0$ this gives $c_2 = 0$. Hence solution is $X(x) = c_1 x$. This is bounded as $x \to \infty$ only when $c_1 = 0$. Therefore $\lambda = 0$ is not eigenvalue.

case $\lambda > 0$

Let $\lambda = \alpha^2, \alpha > 0$. Then solution is $X(x) = c_1 \cos(\alpha x) + c_2 \sin(\alpha x)$. At $x = 0$ this results in $0 = c_1$. Hence the eigenvalues are $\lambda = \alpha^2$ for all real positive real numbers and eigenfunctions are

$$X_\alpha(x) = \sin(\alpha x)$$

For the Y ode,

$$\begin{aligned} Y'' - \alpha^2 Y &= 0 \\ Y(0) &= 0 \end{aligned}$$

The solution is $Y_\alpha(y) = c_1 e^{\alpha y} + c_2 e^{-\alpha y}$. Since $Y(0) = 0$ then $c_2 = -c_1$ and the solution becomes $Y_\alpha(y) = c_1 (e^{\alpha y} - e^{-\alpha y}) = c_1 \sinh(\alpha y)$. Hence the solution is generalized linear combination of $Y(y)\, X(x)$ given by Fourier integral (since eigenvalues are continuous now and not discrete)

$$\begin{aligned} u(x,y) &= \int_0^\infty A(\alpha)\, Y_\alpha(y)\, X_\alpha(x)\, d\alpha \\ &= \int_0^\infty A(\alpha) \sinh(\alpha y) \sin(\alpha x)\, d\alpha \qquad (1) \end{aligned}$$

When $y = L$, then above becomes

$$e^{-x} = \int_0^\infty \left(A(\alpha) \sinh(\alpha L)\right) \sin(\alpha x)\, d\alpha$$

Hence the coefficient $A(\alpha)\sinh(\alpha L)$ is given by

$$\begin{aligned} A(\alpha) \sinh(\alpha L) &= \frac{2}{\pi} \int_0^\infty e^{-x} \sin(\alpha x)\, dx \\ &= \frac{2}{\pi} \frac{\alpha}{1 + \alpha^2} \end{aligned}$$

Therefore $A(\alpha) = \frac{2}{\pi \sinh(\alpha L)} \frac{\alpha}{1+\alpha}$. The solution (1) becomes

$$u(x,y) = \frac{2}{\pi} \int_0^\infty \frac{\alpha \sinh(\alpha y) \sin(\alpha x)}{(1+\alpha^2)\sinh(\alpha L)} d\alpha$$

13.11 Laplace PDE inside rectangle, right edge at infinity, top edge nonhomogeneous function $f(x)$

problem number 104

Added April 4, 2019.

Exam problem, Math 4567, UMN. Spring 2019.

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

Inside a rectangle $0 \leq y \leq 1, 0 \leq x \leq \infty$, with following boundary conditions

$$\begin{aligned} u(0, y) &= 0 \\ u(x, 1) &= f(x) \\ u(x, 0) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, y, L, A];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == 0, u[x, 1] == f[x], u[x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], {x, y}, Assumption
```

Failed

Maple ✗

```
unassign('u,y,x,f');
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge := u(0, y) = 0;
bc_top_edge:= u(x, 1) = f(x);
bc_bottom_edge:= u(x, 0) = 0;
bc:=bc_left_edge ,bc_top_edge,bc_bottom_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,t)) assuming
```

Exception

Maple can not solve it when using boundedseries(x = infinity)

13.12 Laplace PDE in 2D Cartesian with boundary condition as Dirac function

problem number 105

Added December 20, 2018

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve Laplace equation for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary condition

$$u(x,0) = \delta(x)$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = u[x, 0] == DiracDelta[x];
 sol = AbsoluteTiming[TimeConstrained[Simplify[DSolve[{pde, bc}, u[x, y], x, y]], 60*10]];
```

$$\left\{\left\{u(x,y) \to \begin{cases} \frac{y}{\pi(x^2+y^2)} & y \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
u:='u'; y:='y'; x:='x';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc := u(x, 0) = Dirac(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y),method=Fo
sol:=convert(sol,Int);
```

$$u(x,y) = 1/2\,\frac{\int_{-\infty}^{\infty} \mathrm{e}^{-sy+isx}\,\mathrm{d}s}{\pi}$$

13.13 Laplace PDE in rectangle, one side homogeneous and 3 sides are not

problem number 106

Added December 20, 2018

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation for $u(x,y)$

$$u_{xx} + u_{yy} = 0$$

With boundary condition

$$\begin{aligned} u(0,y) &= 0 \\ u(\pi,y) &= \sinh(\pi)\cos(y) \\ u(x,0) &= \sin(x) \\ u(x,\pi) &= -\sinh(x) \end{aligned}$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[0, y] == 0, u[Pi, y] == Sinh[Pi]*Cos[y], u[x, 0] == Sin[x], u[x, Pi] == -Sinh[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

$$\left\{\left\{u(x,y)\to\sum_{K[1]=1}^{\infty}\left(\frac{2\left(1+(-1)^{K[1]}\right)\operatorname{csch}(\pi K[1])K[1]\sin(yK[1])\sinh(\pi)\sinh(xK[1])}{\pi\left(K[1]^2-1\right)}+\operatorname{csch}(\pi K[1])\delta(K\right.\right.\right.$$

Maple ✓

```
u:='u'; y:='y'; x:='x';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_side:=  u(0,y)=0;
bc_right_side:= u(Pi,y)=sinh(Pi)*cos(y);
bc_bottom_side:= u(x,0)=sin(x);
bc_top_side:= u(x,Pi)=-sinh(x);
bc:= bc_left_side,bc_right_side,bc_bottom_side,bc_top_side;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = \frac{1}{e^{2\pi}-1}\left((e^{2\pi}-1)\sum_{n=1}^{\infty}\frac{(-1)^n n(e^{2\pi}-1)e^{n(\pi-y)-\pi}\sin(nx)(e^{2ny}-1)}{\pi(n^2+1)(e^{2\pi n}-1)} + (e^{2\pi}-1)\sum_{n=2}^{\infty}2\frac{\sin(ny)}{}\right.$$

13.14 Laplace on all of the right half plane with $u = f(y)$ on the y axes

problem number 107

PDE example 18 from Maple help page

see `march_20_2019_11_pm.tex` for start of solution. Not completed yet

Solve Laplace equation

$$u_{xx} + u_{yy} = 0$$

With boundary conditions

$$u(0,y) = \frac{\sin y}{y}$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = u[0, y] == Sin[y]/y;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <
```

$$\left\{\left\{u(x,y)\to\frac{(\sinh(x)-\cosh(x))(x\cos(y)-y\sin(y))+x}{x^2+y^2}\right\}\right\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(0,y)=sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\frac{\sin\left(-y+ix\right)+\_F2(y-ix)\left(y-ix\right)+\left(-y+ix\right)\_F2(y+ix)}{-y+ix}$$

13.15 Laplace PDE in rectangle with infinity in the x direction with $u=\sin(y)$ on left edge.

problem number 108

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

With boundary conditions

$$\begin{aligned}u(0,y)&=\sin y\\u(x,0)&=0\\u(x,a)&=0\\u(\infty,y)&=0\end{aligned}$$

Mathematica ✗

```
ClearAll[x, y, a];
 ode = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y], u[Infinity, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{ode, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
x:='x'; y:='y'; a:='a';
interface(showassumed=0);
ode:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(x,0)=0, u(x,a)=0, u(0,y)=sin(y), u(infinity,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve({ode, bc}, u(x,y)) assumin
```

Bad latex generated

13.16 Laplace PDE inside a disk, periodic boundary conditions

problem number 109

Solve Laplace equation in polar coordinates inside a disk

Solve for $u(r,\theta)$

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$

With $0\leq r\leq a, 0<\theta\leq 2\pi$

Boundary conditions

$$\begin{aligned} u(a,\theta)&=f(\theta)\\ |u(0,\theta)|&<\infty\\ u(r,0)&=u(r,2\pi)\\ \frac{\partial u}{\partial\theta}(r,0)&=\frac{\partial u}{\partial\theta}(r,2\pi) \end{aligned}$$

Mathematica ✓

```
ClearAll[u, theta, r, a, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bc = u[a, theta] == f[theta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(r,\theta)\to\sum_{n=1}^{\infty}r^n\left(\frac{\cos(n\theta)\left(\int_{-\pi}^{\pi}\cos(n\theta)f(\theta)\,d\theta\right)a^{-n}}{\pi}+\frac{\left(\int_{-\pi}^{\pi}f(\theta)\sin(n\theta)\,d\theta\right)\sin(n\theta)a^{-n}}{\pi}\right)+\frac{\int_{-\pi}^{\pi}f(\theta}{2\pi}\right.\right.$$

Maple ✓

```
r:='r'; theta:='theta'; a:='a'; r:='r';f:='f';
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r +(diff(u(r, theta), theta, theta))/r^2 = 0;
bc := u(a, theta) = f(theta),
      u(r, -Pi) = u(r, Pi),
      (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta)=1/2\,\frac{1}{\pi}\left(2\sum_{n=1}^{\infty}\left(\frac{\cos(n\theta)\int_{-\pi}^{\pi}f(\theta)\cos(n\theta)\,\mathrm{d}\theta+\int_{-\pi}^{\pi}f(\theta)\sin(n\theta)\,\mathrm{d}\theta\sin(n\theta)}{\pi}\left(\frac{a}{r}\right)^{-n}\right)\pi+\int_{-\pi}^{\pi}f(\theta)\right.$$

13.17 Dirichlet problem for the Laplace equation in upper half plan

problem number 110

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions $u(x,0) = 1$ for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and $x = 0$ otherwise. This is called UnitBox in Mathematica.

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = u[x, 0] == UnitBox[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \begin{cases} \dfrac{\begin{cases} \tan^{-1}\left(\frac{\frac{1}{2}-x}{y}\right) + \tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right) & y > 0 \lor x > \frac{1}{2} \lor x < -\frac{1}{2} \\ 0 & \text{True} \end{cases}}{\pi} & y \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc := u(x,0) =piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
sol:=convert(sol,Int);
```

$$u(x,y) = i\left(1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{-1/2\,s(2\,y+i)+isx}}{s}\,ds - 1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{1/2\,s(-2\,y+i)+isx}}{s}\,ds\right)$$

13.18 Dirichlet problem for the Laplace equation in right half-plane:

problem number 111

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions $u(0,y) = \mathrm{sinc}(y)$.

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = u[0, y] == Sinc[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \left\{ \begin{array}{cc} \frac{x+(x\cos(y)-y\sin(y))(\sinh(x)-\cosh(x))}{x^2+y^2} & x \geq 0 \\ \text{Indeterminate} & \text{True} \end{array} \right\}\right\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc := u(0,y) =sin(y)/y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = \frac{\sin(-y+ix) + \_F2(y-ix)(y-ix) + (-y+ix)\_F2(y+ix)}{-y+ix}$$

13.19 Dirichlet problem for the Laplace equation in the first quadrant

problem number 112

Taken from Mathematica DSolve help pages

Solve for $u(x, y)$

$$u_{xx} + y_{yy} = 0$$

Boundary conditions

$$u(x,0) = -\frac{1}{(x-2)^2+3}$$
$$u(0,y) = \frac{1}{(y-3)^2+1}$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == -((x - 2)^2 + 3)^(-1), u[0, y] == 1/((y - 3)^2 + 1)};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \left\{ \frac{2\left(\frac{3\left(y\left(3\pi(x+1)\left(x^4-4x^3+2\left(y^2+12\right)x^2-4\left(y^2+10\right)x+y^4-16y^2+100\right)+x\left(\left(6\tan^{-1}(3)-\log(10)\right)x^4+2\left(\left(6\tan^{-1}(3)-\log(10)\right)y^2+\right.\right.\right.\right.}{}\right.}{}\right.\right.\right.$$

Maple ✗

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=u(x, 0) = (-1/((x - 2)^2 + 3)), u(0, y) = 1/((y - 3)^2 + 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

sol=()

13.20 Neumann problem for the Laplace equation in the upper half-plane

problem number 113

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $\frac{u}{y}(x,0) = \text{UnitBox[x]}$ where UnitBox[x] is 1 for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and 0 otherwise. This is called UnitBox in Mathematica.

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = Derivative[0, 1][u][x, 0] == UnitBox[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{u(x,y) \to \left\{ \begin{array}{c} \frac{-4y\tan^{-1}\left(\frac{x-\frac{1}{2}}{y}\right)+4y\tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right)-2x\log\left(4x^2-4x+4y^2+1\right)+\log\left(4x^2-4x+4y^2+1\right)+2x\log\left(4x^2+4x+4y^2+1\right)+\text{lo}}{4\pi} \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=(D[2](u))(x, 0) = piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
sol:=convert(sol,Int);
```

$$u(x,y) = i\left(1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{1/2\,s(-2\,y+i)+isx}}{s^2}\,ds - 1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{-1/2\,s(2\,y+i)+isx}}{s^2}\,ds\right)$$

used convert(sol,Int).

13.21 Dirichlet problem for the Laplace equation in a rectangle

problem number 114

Taken from Mathematica DSolve help pages

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(x,0) = x^2(1-x), u(x,2) = 0, u(0,y) = 0, u(1,y) = 0$.

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == x^2*(1 - x), u[x, 2] == 0, u[0, y] == 0, u[1, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
 sol = sol /. K[1] -> n
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} -\frac{4(1+2(-1)^n)\operatorname{csch}(2n\pi)\sin(n\pi x)\sinh(n\pi(2-y))}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=u(x, 0) = x^2*(1 - x),u(x, 2) = 0, u(0, y) = 0, u(1, y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y)=\sum_{n=1}^{\infty} 4\,\frac{\sin(n\pi x)\left(2\,(-1)^{1+n}\,\mathrm{e}^{-\pi\,n(y-4)}-\mathrm{e}^{-\pi\,n(y-4)}+2\,\mathrm{e}^{n\pi\,y}(-1)^{n}+\mathrm{e}^{n\pi\,y}\right)}{n^{3}\pi^{3}\left(\mathrm{e}^{4\,\pi\,n}-1\right)}$$

13.22 Cartesian coordinates with boundary conditions on two sides only

problem number 115

Added December 20, 2018.

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions

$$u(x,0)=0$$
$$u(x,b)=h(x)$$

Mathematica ✗

```
ClearAll[u, x, y, h, b];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == 0, u[x, b] == h[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; y:='y'; u:='u';h:='h';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2)=0;
bc:=u(x,0)=0,u(x,b)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y))),output=
sol:=convert(sol,Int);
```

$$u(x,y) = -1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\int_{-\infty}^{\infty}h(x)\,\mathrm{e}^{-isx}\,\mathrm{d}x\mathrm{e}^{s(b-y)+isx}}{\mathrm{e}^{2\,sb}-1}\,\mathrm{d}s+1/2\,\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\int_{-\infty}^{\infty}h(x)\,\mathrm{e}^{-isx}\,\mathrm{d}x\mathrm{e}^{s(b+y)+isx}}{\mathrm{e}^{2\,sb}-1}\,\mathrm{d}s$$

13.23 in Rectangle, right edge at infinity

problem number 116

Added December 20, 2018.

Example 23, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions

$$\begin{aligned} u(x,0) &= 0\\ u(x,a) &= 0\\ u(0,y) &= \sin(y)\\ u(\infty,y) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, y, a];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y], u[Infinity, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> a >
```

Failed

Maple ✓

```
x:='x'; y:='y'; u:='u';a:='a';
pde := diff(u(x, y), x$2)+diff(u(x, y), y$2) = 0;
bc_left_edge:=u(0, y) = sin(y);
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,a)=0;
bc_right_edge:=u(infinity,y)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge,bc_right_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

Bad latex generated

13.24 Laplace PDE inside quarter disk, Neumann BC at edge

problem number 117

Added December 20, 2018.

Example 20, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve Laplace equation in polar coordinates inside quarter disk with $0 < r < 1$ and $0 < \theta < \frac{\pi}{2}$

Solve for $u(r, \theta)$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Boundary conditions

$$u(r,0) = 0$$
$$u(r,\frac{\pi}{2}) = 0 \quad \frac{\partial u}{\partial r}(1,\theta) = f(\theta)$$

Mathematica ✗

```
ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bcOnR = {Derivative[1, 0][u][1, theta] == f[theta]};
 bcOnTheta = {u[r, 0] == 0, u[r, Pi/2] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcOnR, bcOnTheta}, u[r, theta], {r, theta
```

Failed

Maple ✓

```
r:='r'; theta:='theta';  r:='r';f:='f';
pde := diff(u(r, theta), r$2)+1/r* diff(u(r, theta), r)+1/r^2* diff(u(r, theta), theta$2)= 0
bc_on_theta:=u(r, 0) = 0, u(r,Pi/2) = 0;
bc_on_r:= eval( diff(u(r,theta),r),r=1)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc_on_theta,bc_on_r]
```

$$u(r,\theta) = \sum_{n=1}^{\infty} \left(2\, \frac{\int_0^{\pi/2} f(\theta) \sin(2\,n\theta)\, \mathrm{d}\theta r^{2\,n} \sin(2\,n\theta)}{\pi\, n} \right)$$

14 Laplace PDE in Polar coordinates

14.1 Laplace PDE inside quarter-circle (Haberman 2.5.5 (c))

problem number 118

This is problem 2.5.5 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside quarter circle of radius 1 with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\frac{\pi}{2}) &= 0 \\ \frac{\partial u}{\partial r}(1,\theta) &= f(\theta) \end{aligned}$$

Mathematica ✗

```
ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) =
 bc = {Derivative[1, 0][u][1, theta] == f[theta], u[r, Pi/2] == 0, u[r, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+ 1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta),HINT=b
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\left(2\frac{\int_0^{\pi/2} f(\theta)\sin(2n\theta)\,d\theta r^{2n}\sin(2n\theta)}{\pi n}\right)$$

Hand solution

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0 \tag{A}$$

With boundary conditions

$$\begin{aligned} u(r,0)&=0\\ u\left(r,\frac{\pi}{2}\right)&=0\\ u(1,\theta)&=f(\theta)\end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta)=R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2R''\Theta+rR'\Theta+R\Theta''=0$$

Dividing the above by $R\Theta\neq 0$ gives

$$\begin{aligned} r^2\frac{R''}{R}+r\frac{R'}{R}+\frac{\Theta''}{\Theta}&=0\\ r^2\frac{R''}{R}+r\frac{R'}{R}&=-\frac{\Theta''}{\Theta}\end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R}+r\frac{R'}{R}=-\frac{\Theta''}{\Theta}=\lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This gives

$$\begin{aligned}\Theta'' + \lambda\Theta &= 0\\ \Theta(0) &= 0\\ \Theta\left(\frac{\pi}{2}\right) &= 0\end{aligned} \tag{1}$$

And

$$\begin{aligned}r^2R'' + rR' - \lambda R &= 0\\ |R(0)| &< \infty\end{aligned} \tag{2}$$

Starting with (1). Consider the Case $\lambda < 0$. The solution in this case will be

$$\Theta = A\cosh\left(\sqrt{\lambda}\theta\right) + B\sinh\left(\sqrt{\lambda}\theta\right)$$

Applying first B.C. gives $A = 0$. The solution becomes $\Theta = B\sinh\left(\sqrt{\lambda}\theta\right)$. Applying second B.C. gives

$$0 = B\sinh\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

But sinh is zero only when $\sqrt{\lambda}\frac{\pi}{2} = 0$ which is not the case here. Therefore $B = 0$ and hence trivial solution. Hence $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$ The ODE becomes $\Theta'' = 0$ with solution $\Theta = A\theta + B$. First B.C. gives $0 = B$. The solution becomes $\Theta = A\theta$. Second B.C. gives $0 = A\frac{\pi}{2}$, hence $A = 0$ and trivial solution. Therefore $\lambda = 0$ is not an eigenvalue.

Case $\lambda > 0$ The ODE becomes $\Theta'' + \lambda\Theta = 0$ with solution

$$\Theta = A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right)$$

The first B.C. gives $0 = A$. The solution becomes

$$\Theta = B\sin\left(\sqrt{\lambda}\theta\right)$$

And the second B.C. gives

$$0 = B\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right)$$

For non-trivial solution $\sin\left(\sqrt{\lambda}\frac{\pi}{2}\right) = 0$ or $\sqrt{\lambda}\frac{\pi}{2} = n\pi$ for $n = 1, 2, 3, \cdots$. Hence the eigenvalues are

$$\begin{aligned}\sqrt{\lambda_n} &= 2n\\ \lambda_n &= 4n^2 \qquad n = 1, 2, 3, \cdots\end{aligned}$$

And the eigenfunctions are

$$\Theta_n(\theta) = B_n \sin(2n\theta) \qquad n = 1,2,3,\cdots \tag{3}$$

Now the R ODE is solved. There is one case to consider, which is $\lambda > 0$ based on the above. The ODE is

$$\begin{aligned} r^2R'' + rR' - \lambda_n R &= 0 \\ r^2R'' + rR' - 4n^2R &= 0 \qquad n = 1,2,3,\cdots \end{aligned}$$

This is Euler ODE. Let $R(r) = r^p$. Then $R' = pr^{p-1}$ and $R'' = p(p-1)\,r^{p-2}$. This gives

$$\begin{aligned} r^2\left(p(p-1)\,r^{p-2}\right) + r\left(pr^{p-1}\right) - 4n^2r^p &= 0 \\ \left(\left(p^2 - p\right) r^p\right) + pr^p - 4n^2r^p &= 0 \\ r^p p^2 - pr^p + pr^p - 4n^2r^p &= 0 \\ p^2 - 4n^2 &= 0 \\ p &= \pm 2n \end{aligned}$$

Hence the solution is

$$R(r) = Cr^{2n} + D\frac{1}{r^{2n}}$$

Applying the condition that $|R(0)| < \infty$ implies $D = 0$, and the solution becomes

$$R_n(r) = C_n r^{2n} \qquad n = 1,2,3,\cdots \tag{4}$$

Using (3,4) the solution $u_n(r,\theta)$ is

$$\begin{aligned} u_n(r,\theta) &= R_n\Theta_n \\ &= C_n r^{2n} B_n \sin(2n\theta) \\ &= B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Where C_nB_n was combined into one constant B_n. (No need to introduce new symbol). The final solution is

$$\begin{aligned} u(r,\theta) &= \sum_{n=1}^{\infty} u_n(r,\theta) \\ &= \sum_{n=1}^{\infty} B_n r^{2n} \sin(2n\theta) \end{aligned}$$

Now the nonhomogeneous condition is applied to find B_n.

$$\frac{\partial}{\partial r}u(r,\theta) = \sum_{n=1}^{\infty} B_n (2n)\, r^{2n-1} \sin(2n\theta)$$

Hence $\frac{\partial}{\partial r}u(1,\theta)=f(\theta)$ becomes

$$f(\theta)=\sum_{n=1}^{\infty}2B_n n\sin(2n\theta)$$

Multiplying by $\sin(2m\theta)$ and integrating gives

$$\begin{aligned}\int_0^{\frac{\pi}{2}}f(\theta)\sin(2m\theta)\,d\theta &= \int_0^{\frac{\pi}{2}}\sin(2m\theta)\sum_{n=1}^{\infty}2B_n n\sin(2n\theta)\,d\theta\\ &=\sum_{n=1}^{\infty}2nB_n\int_0^{\frac{\pi}{2}}\sin(2m\theta)\sin(2n\theta)\,d\theta \qquad (5)\end{aligned}$$

When $n=m$ then

$$\begin{aligned}\int_0^{\frac{\pi}{2}}\sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}}\sin^2(2n\theta)\,d\theta\\ &=\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}-\frac{1}{2}\cos 4n\theta\right)d\theta\\ &=\frac{1}{2}[\theta]_0^{\frac{\pi}{2}}-\frac{1}{2}\left[\frac{\sin 4n\theta}{4n}\right]_0^{\frac{\pi}{2}}\\ &=\frac{\pi}{4}-\left(\frac{1}{8n}\left(\sin\frac{4n}{2}\pi\right)-\sin(0)\right)\end{aligned}$$

And since n is integer, then $\sin\frac{4n}{2}\pi=\sin 2n\pi=0$ and the above becomes $\frac{\pi}{4}$.

Now for the case when $n\neq m$ using $\sin A\sin B=\frac{1}{2}(\cos(A-B)-\cos(A+B))$ then

$$\begin{aligned}\int_0^{\frac{\pi}{2}}\sin(2m\theta)\sin(2n\theta)\,d\theta &= \int_0^{\frac{\pi}{2}}\frac{1}{2}(\cos(2m\theta-2n\theta)-\cos(2m\theta+2n\theta))\,d\theta\\ &=\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(2m\theta-2n\theta)\,d\theta-\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos(2m\theta+2n\theta)\,d\theta\\ &=\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos((2m-2n)\theta)\,d\theta-\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos((2m+2n)\theta)\,d\theta\\ &=\frac{1}{2}\left[\frac{\sin((2m-2n)\theta)}{(2m-2n)}\right]_0^{\frac{\pi}{2}}-\frac{1}{2}\left[\frac{\sin((2m+2n)\theta)}{(2m+2n)}\right]_0^{\frac{\pi}{2}}\\ &=\frac{1}{4(m-n)}[\sin((2m-2n)\theta)]_0^{\frac{\pi}{2}}-\frac{1}{4(m+n)}[\sin((2m+2n)\theta)]_0^{\frac{\pi}{2}}\\ &=\frac{1}{4(m-n)}\left[\sin\left((2m-2n)\frac{\pi}{2}\right)-0\right]-\frac{1}{4(m+n)}\left[\sin\left((2m+2n)\frac{\pi}{2}\right)-0\right]\end{aligned}$$

Since $2m-2n\frac{\pi}{2}=\pi(m-n)$ which is integer multiple of π and also $(2m+2n)\frac{\pi}{2}$ is integer multiple of π then the whole term above becomes zero. Therefore (5) becomes

$$\int_0^{\frac{\pi}{2}}f(\theta)\sin(2m\theta)\,d\theta=2mB_m\frac{\pi}{4}$$

Hence

$$B_n = \frac{2}{\pi n} \int_0^{\frac{\pi}{2}} f(\theta) \sin(2n\theta)\, d\theta$$

Summary: the final solution is

$$u(r,\theta) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left[\int_0^{\frac{\pi}{2}} f(\theta) \sin(2n\theta)\, d\theta \right] \left(r^{2n} \sin(2n\theta) \right)$$

14.2 Laplace PDE inside semi-circle

problem number 119

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside semi-circle of radius 1 with $0 \leq \theta \leq \pi$ and $0 \leq r \leq 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi) &= 0 \\ u(0,\theta) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned}$$

Mathematica ✗

```
ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) =
 bc = {u[r, 0] == 0, u[r, Pi] == 0, u[0, theta] == 0, u[1, theta] == f[theta]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
L:='L'; u:='u';f:='f';theta:='theta';
pde:=diff(u(r,theta),r$2)
        +1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi)=0,u(0,theta)=0,u(1,theta)=f(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta))),outp
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\left(2\frac{\sin(n\theta)\left(r^n\int_0^{\pi}\sin(n\theta)\,f(\theta)\,\mathrm{d}\theta-1/2\_C5(n)\,\pi\left(r^n-r^{-n}\right)\right)}{\pi}\right)$$

Hand solution

The Laplace PDE in polar coordinates is

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0 \tag{A}$$

With

$$\begin{aligned}\frac{\partial u}{\partial r}(a,\theta)&=0\\ u(b,\theta)&=g(\theta)\end{aligned} \tag{B}$$

Assuming the solution can be written as

$$u(r,\theta)=R(r)\,\Theta(\theta)$$

And substituting this assumed solution back into the (A) gives

$$r^2R''\Theta+rR'\Theta+R\Theta''=0$$

Dividing the above by $R\Theta$ gives

$$\begin{aligned}r^2\frac{R''}{R}+r\frac{R'}{R}+\frac{\Theta''}{\Theta}&=0\\ r^2\frac{R''}{R}+r\frac{R'}{R}&=-\frac{\Theta''}{\Theta}\end{aligned}$$

Since each side depends on different independent variable and they are equal, they must be equal to same constant. say λ.

$$r^2\frac{R''}{R}+r\frac{R'}{R}=-\frac{\Theta''}{\Theta}=\lambda$$

This results in the following two ODE's. The boundaries conditions in (B) are also transferred to each ODE. This results in

$$\begin{aligned}\Theta'' + \lambda\Theta &= 0 \\ \Theta(-\pi) &= \Theta(\pi) \\ \Theta'(-\pi) &= \Theta'(\pi)\end{aligned} \tag{1}$$

And

$$\begin{aligned}r^2 R'' + rR' - \lambda R &= 0 \\ R'(a) &= 0\end{aligned} \tag{2}$$

Starting with (1)

Case $\lambda < 0$ The solution is

$$\Theta(\theta) = A\cosh\left(\sqrt{|\lambda|}\theta\right) + B\sinh\left(\sqrt{|\lambda|}\theta\right)$$

First B.C. gives

$$\begin{aligned}\Theta(-\pi) &= \Theta(\pi) \\ A\cosh\left(-\sqrt{|\lambda|}\pi\right) + B\sinh\left(-\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\ A\cosh\left(\sqrt{|\lambda|}\pi\right) - B\sinh\left(\sqrt{|\lambda|}\pi\right) &= A\cosh\left(\sqrt{|\lambda|}\pi\right) + B\sinh\left(\sqrt{|\lambda|}\pi\right) \\ 2B\sinh\left(\sqrt{|\lambda|}\pi\right) &= 0\end{aligned}$$

But $\sinh = 0$ only at zero and $\lambda \neq 0$, hence $B = 0$ and the solution becomes

$$\begin{aligned}\Theta(\theta) &= A\cosh\left(\sqrt{|\lambda|}\theta\right) \\ \Theta'(\theta) &= A\sqrt{\lambda}\cosh\left(\sqrt{|\lambda|}\theta\right)\end{aligned}$$

Applying the second B.C. gives

$$\begin{aligned}\Theta'(-\pi) &= \Theta'(\pi) \\ A\sqrt{|\lambda|}\cosh\left(-\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) \\ A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) \\ 2A\sqrt{|\lambda|}\cosh\left(\sqrt{|\lambda|}\pi\right) &= 0\end{aligned}$$

But cosh is never zero, hence $A = 0$. Therefore trivial solution and $\lambda < 0$ is not an eigenvalue.

Case $\lambda = 0$ The solution is $\Theta = A\theta + B$. Applying the first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
-A\pi + B &= \pi A + B \\
2\pi A &= 0 \\
A &= 0
\end{aligned}$$

And the solution becomes $\Theta = B_0$. A constant. Hence $\lambda = 0$ is an eigenvalue.

Case $\lambda > 0$

The solution becomes

$$\begin{aligned}
\Theta &= A\cos\left(\sqrt{\lambda}\theta\right) + B\sin\left(\sqrt{\lambda}\theta\right) \\
\Theta' &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\theta\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\theta\right)
\end{aligned}$$

Applying first B.C. gives

$$\begin{aligned}
\Theta(-\pi) &= \Theta(\pi) \\
A\cos\left(-\sqrt{\lambda}\pi\right) + B\sin\left(-\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
A\cos\left(\sqrt{\lambda}\pi\right) - B\sin\left(\sqrt{\lambda}\pi\right) &= A\cos\left(\sqrt{\lambda}\pi\right) + B\sin\left(\sqrt{\lambda}\pi\right) \\
2B\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (3)
\end{aligned}$$

Applying second B.C. gives

$$\begin{aligned}
\Theta'(-\pi) &= \Theta'(\pi) \\
-A\sqrt{\lambda}\sin\left(-\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(-\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) + B\sqrt{\lambda}\cos\left(\sqrt{\lambda}\pi\right) \\
A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) &= -A\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi\right) \\
2A\sin\left(\sqrt{\lambda}\pi\right) &= 0 \qquad (4)
\end{aligned}$$

Equations (3,4) can be both zero only if $A = B = 0$ which gives trivial solution, or when $\sin\left(\sqrt{\lambda}\pi\right) = 0$. Therefore taking $\sin\left(\sqrt{\lambda}\pi\right) = 0$ gives a non-trivial solution. Hence

$$\begin{aligned}
\sqrt{\lambda}\pi &= n\pi \qquad n = 1, 2, 3, \cdots \\
\lambda_n &= n^2 \qquad n = 1, 2, 3, \cdots
\end{aligned}$$

Hence the solution for Θ is

$$\Theta = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta) + B_n \sin(n\theta) \tag{5}$$

Now the R equation is solved

The case for $\lambda = 0$ gives

$$\begin{aligned} r^2 R'' + rR' &= 0 \\ R'' + \frac{1}{r} R' &= 0 \qquad r \neq 0 \end{aligned}$$

As was done in last problem, the solution to this is

$$R(r) = A \ln |r| + C$$

Since $r > 0$ no need to keep worrying about $|r|$ and is removed for simplicity. Applying the B.C. gives

$$R' = A\frac{1}{r}$$

Evaluating at $r = a$ gives

$$0 = A\frac{1}{a}$$

Hence $A = 0$, and the solution becomes

$$R(r) = C_0$$

Which is a constant.

Case $\lambda > 0$ The ODE in this case is

$$r^2 R'' + rR' - n^2 R = 0 \qquad n = 1, 2, 3, \cdots$$

Let $R = r^p$, the above becomes

$$\begin{aligned} r^2 p(p-1) r^{p-2} + rpr^{p-1} - n^2 r^p &= 0 \\ p(p-1) r^p + pr^p - n^2 r^p &= 0 \\ p(p-1) + p - n^2 &= 0 \\ p^2 &= n^2 \\ p &= \pm n \end{aligned}$$

Hence the solution is

$$R_n(r) = Cr^n + D\frac{1}{r^n} \qquad n = 1, 2, 3, \cdots$$

Applying the boundary condition $R'(a)=0$ gives

$$\begin{aligned}
R_n'(r) &= nC_n r^{n-1} - nD_n \frac{1}{r^{n+1}}\\
0 &= R_n'(a)\\
&= nC_n a^{n-1} - nD_n \frac{1}{a^{n+1}}\\
&= nC_n a^{2n} - nD_n\\
&= C_n a^{2n} - D_n\\
D_n &= C_n a^{2n}
\end{aligned}$$

The solution becomes

$$\begin{aligned}
R_n(r) &= C_n r^n + C_n a^{2n}\frac{1}{r^n} \qquad n=1,2,3,\cdots\\
&= C_n\left(r^n + \frac{a^{2n}}{r^n}\right)
\end{aligned}$$

Hence the complete solution for $R(r)$ is

$$R(r) = C_0 + \sum_{n=1}^{\infty} C_n\left(r^n + \frac{a^{2n}}{r^n}\right) \tag{6}$$

Using (5),(6) gives

$$\begin{aligned}
u_n(r,\theta) &= R_n\Theta_n\\
u(r,\theta) &= \left[C_0 + \sum_{n=1}^{\infty} C_n\left(r^n + \frac{a^{2n}}{r^n}\right)\right]\left[A_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta) + B_n\sin(n\theta)\right]\\
&= D_0 + \sum_{n=1}^{\infty} A_n\cos(n\theta)\, C_n\left(r^n + \frac{a^{2n}}{r^n}\right) + \sum_{n=1}^{\infty} B_n\sin(n\theta)\, C_n\left(r^n + \frac{a^{2n}}{r^n}\right)
\end{aligned}$$

Where $D_0 = C_0A_0$. To simplify more, A_nC_n is combined to A_n and B_nC_n is combined to B_n. The full solution is

$$u(r,\theta) = D_0 + \sum_{n=1}^{\infty} A_n\left(r^n + \frac{a^{2n}}{r^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n\left(r^n + \frac{a^{2n}}{r^n}\right)\sin(n\theta)$$

The final nonhomogeneous B.C. is applied.

$$\begin{aligned}
u(b,\theta) &= g(\theta)\\
g(\theta) &= D_0 + \sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\sin(n\theta)
\end{aligned}$$

For $n = 0$, integrating both sides give

$$\int_{-\pi}^{\pi} g(\theta)\,d\theta = \int_{-\pi}^{\pi} D_0 d\theta$$
$$D_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\theta)\,d\theta$$

For $n > 0$, multiplying both sides by $\cos(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta = &\int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta \\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\cos(m\theta)\cos(n\theta)\,d\theta \\ &+ \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\cos(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\cos(m\theta)\,d\theta = &\int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta \\ &+ \sum_{n=1}^{\infty} A_n\left(b^n + \frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta \\ &+ \sum_{n=1}^{\infty} B_n\left(b^n + \frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta \qquad (7)\end{aligned}$$

But

$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = \pi \qquad n = m \neq 0$$
$$\int_{-\pi}^{\pi}\cos(m\theta)\cos(n\theta)\,d\theta = 0 \qquad n \neq m$$

And

$$\int_{-\pi}^{\pi}\cos(m\theta)\sin(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\cos(m\theta)\,d\theta = 0$$

Then (7) becomes

$$\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta = \pi A_n\left(b^n + \frac{a^{2n}}{b^n}\right)$$
$$A_n = \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}} \qquad (8)$$

Again, multiplying both sides by $\sin(m\theta)$ and integrating gives

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0 \sin(m\theta)\,d\theta \\ &\quad + \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\sin(m\theta)\cos(n\theta)\,d\theta \\ &\quad + \int_{-\pi}^{\pi}\sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\sin(m\theta)\sin(n\theta)\,d\theta\end{aligned}$$

Hence

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(m\theta)\,d\theta &= \int_{-\pi}^{\pi} D_0 \sin(m\theta)\,d\theta \\ &\quad + \sum_{n=1}^{\infty} A_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta \\ &\quad + \sum_{n=1}^{\infty} B_n\left(b^n+\frac{a^{2n}}{b^n}\right)\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta \qquad (9)\end{aligned}$$

But

$$\begin{aligned}\int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= \pi \qquad n=m\neq 0 \\ \int_{-\pi}^{\pi}\sin(m\theta)\sin(n\theta)\,d\theta &= 0 \qquad n\neq m\end{aligned}$$

And

$$\int_{-\pi}^{\pi}\sin(m\theta)\cos(n\theta)\,d\theta = 0$$

And

$$\int_{-\pi}^{\pi} D_0\sin(m\theta)\,d\theta = 0$$

Then (9) becomes

$$\begin{aligned}\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta &= \pi B_n\left(b^n+\frac{a^{2n}}{b^n}\right) \\ B_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta}{b^n+\frac{a^{2n}}{b^n}}\end{aligned}$$

This complete the solution. Summary

$$
\begin{aligned}
u(r,\theta) &= D_0 + \sum_{n=1}^{\infty} A_n\left(r^n + \frac{a^{2n}}{r^n}\right)\cos(n\theta) + \sum_{n=1}^{\infty} B_n\left(r^n + \frac{a^{2n}}{r^n}\right)\sin(n\theta) \\
D_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\theta)\,d\theta \\
A_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\cos(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}} \\
B_n &= \frac{1}{\pi}\frac{\int_{-\pi}^{\pi} g(\theta)\sin(n\theta)\,d\theta}{b^n + \frac{a^{2n}}{b^n}}
\end{aligned}
$$

14.3 Laplace PDE inside circular annulus, Neumann boundary conditions using unspecified functions (Haberman 2.5.8 (b))

problem number 120

This is problem 2.5.8 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$
\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0
$$

Inside circular annulus $a < r < b$ subject to the following boundary conditions

$$
\begin{aligned}
\frac{\partial u}{\partial r}(a,\theta) &= 0 \\
u(b,0) &= g(\theta)
\end{aligned}
$$

Mathematica ✗

```
ClearAll[u, a, theta, r, g];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bc = {Derivative[1, 0][u][a, theta] == 0, u[b, theta] == g[theta]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
a:='a'; u:='u'; r:='r'; theta:='theta';g:='g';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta)) assum
```

$$u(r,\theta) = \mathit{invfourier}\left(\frac{\mathit{fourier}(g(\theta)\,,\theta,s)\,\mathrm{e}^{s(2\ln(a)-\ln(r)-\ln(b))}}{\mathrm{e}^{2\,(\ln(a)-\ln(b))s}+1},s,\theta\right)+\mathit{invfourier}\left(\frac{\mathit{fourier}(g(\theta)\,,\theta,s)\,\mathrm{e}^{s(\ln(r)-\ln}}{\mathrm{e}^{2\,(\ln(a)-\ln(b))s}+1}\right.$$

But has unresolved Invfourier and Fourier calls

14.4 Laplace PDE inside circular annulus, Dirichlet boundary conditions using specified functions

problem number 121

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$$

Inside circular annulus $1<r<2$ subject to the following boundary conditions

$$u(1,\theta)=0$$
$$u(2,\theta)=\sin\theta$$

Mathematica ✓

```
ClearAll[u, r, theta];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bc = {u[1, theta] == 0, u[2, theta] == Sin[theta]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

$$\left\{\left\{u(r,\theta)\to\ \left\{\begin{array}{cc}\frac{2(r^2-1)\sin(\theta)}{3r} & 1\leq r\leq 2\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

Maple ✓

```
u:='u'; r:='r'; theta:='theta';
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(1,theta)=0,u(2,theta)=sin(theta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(r,theta))),outp
```

$$u(r,\theta) = 2/3\,\frac{\sin(\theta)\,(r^2-1)}{r}$$

14.5 Laplace PDE outside a disk, periodic boundary conditions

problem number 122

Solve Laplace equation in polar coordinates outside a disk

Solve for $u(r,\theta)$

$$\begin{aligned}\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} &= 0\\ a &\leq r\\ 0 &< \theta \leq 2\pi\end{aligned}$$

Boundary conditions

$$\begin{aligned}u(a,\theta) &= f(\theta)\\ |u(0,\theta)| &< \infty\\ u(r,0) &= u(r,2\pi)\\ \frac{\partial u}{\partial \theta}(r,0) &= \frac{\partial u}{\partial \theta}(r,2\pi)\end{aligned}$$

Mathematica ✗

```
ClearAll[u, theta, r, a, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2
 bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Deriva
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions
```

Failed

Maple ✓

```
r:='r'; theta:='theta'; a:='a'; r:='r';f:='f';
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r+(diff(u(r, theta), theta, theta))/r^2 = 0;
bc := u(a, theta) = f(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r, theta), HI
```

$$u(r,\theta) = 1/2\,\frac{1}{\pi}\left(2\sum_{n=1}^{\infty}\left(\frac{\int_{-\pi}^{\pi}\sin(n\theta)\,f(\theta)\,\mathrm{d}\theta\sin(n\theta)+\int_{-\pi}^{\pi}f(\theta)\cos(n\theta)\,\mathrm{d}\theta\cos(n\theta)}{\pi}\left(\frac{r}{a}\right)^{-n}\right)\pi+\int_{-\pi}^{\pi}f(\theta)$$

15 Laplace PDE in Spherical coordinates

15.1 Laplace in a sphere

problem number 123

Taken from Maple pdsolve help pages

Solve for $u(r,\theta,\phi)$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2}=0$$

Mathematica ✗

```
ClearAll[u, \[Theta], \[Phi], r];
 ClearAll[u, r, \[Theta], \[Phi]];
 lap = Laplacian[f[r, \[Theta], \[Phi]], {r, \[Theta], \[Phi]}, "Spherical"];
 sol = AbsoluteTiming[TimeConstrained[DSolve[lap == 0, f[r, \[Theta], \[Phi]], {r, \[Theta],
```

Failed

Maple ✓

```
r:='r'; theta:='theta'; phi:='phi'; r:='r';
PDE := Diff(r^2*diff(F(r,theta,phi),r),r)
          + 1/sin(theta)*Diff(sin(theta)*diff(F(r,theta,phi),theta),theta)
          + 1/sin(theta)^2*diff(F(r,theta,phi),phi,phi) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(PDE,F(r,theta,phi),'build'
sol:=simplify(sol,size);
```

$$F(r,\theta,\phi)=\frac{(\sin(\theta))^{\sqrt{\_c_2}}\sqrt{2}(-1)^{1/2\sqrt{\_c_2}}\left(\_C5\ \sin\left(\sqrt{\_c_2}\phi\right)+\_C6\ \cos\left(\sqrt{\_c_2}\phi\right)\right)\left(\_C1\ r^{1/2\sqrt{1+4\_c_1}}+\right.}{}$$

16 Poisson PDE in Cartesian coordinates

16.1 Poisson equation in a rectangle, all boundaries are zero

problem number 124

Added March 13, 2019.

Solve for $u(x,y)$

$$\frac{u_{xx}}{A}+\frac{u_{xx}}{B}=-2\theta$$

Where A,B,θ are constants, and the boundary conditions are

$$\begin{aligned} u(x,-b)&=0\\ u(x,b)&=0\\ u(-a,y)&=0\\ u(a,y)&=0 \end{aligned}$$

Mathematica ✗

```
ClearAll[a, b, A, B, theta, x, y, u];
 pde = D[u[x, y], {x, 2}]/A + D[u[x, y], {y, 2}]/B == -2*theta;
 bc = {u[x, -b] == 0, u[x, b] == 0, u[-a, y] == 0, u[a, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
x:='x'; y:='y'; u:='u';A:='A';B:='B';theta:='theta';a:='a';b:='b';
pde:=diff(u(x,y),x$2)/A+diff(u(x,y),y$2)/B=-2*theta;
bc:=u(x,-b)=0,
    u(x,b)=0,
    u(-a,y)=0,
    u(a,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

sol=()

Hand solution

solve

$$\frac{u_{xx}}{A}+\frac{u_{yy}}{B}=-2\theta$$
$$Bu_{xx}+Au_{yy}=-2\theta AB$$
$$=C$$

Where $C=-2\theta AB$ is a new constant. With boundary conditions

$$u(x,-b)=0$$
$$u(x,b)=0$$
$$u(-a,y)=0$$
$$u(a,y)=0$$

To simplify solution, shift the rectangle so its lower left corner on the origin. Let $\tilde{x}=x+a$, and $\tilde{y}=y+b$. The boundary conditions becomes

$$u(\tilde{x},0)=0$$
$$u(\tilde{x},2b)=0$$
$$u(0,\tilde{y})=0$$
$$u(2a,\tilde{y})=0$$

And the pde becomes $Bu_{\tilde{x}\tilde{x}}+Au_{\tilde{y}\tilde{y}}=C$. Instead of keep writing $\tilde{x},\tilde{y}$, will use x,y, but remember that these are shifted version. At the end, we shift back.

Hence the PDE to solve is $Bu_{xx} + Au_{yy} = C$ with BC

$$\begin{aligned} u(x,0) &= 0 \\ u(x,2b) &= 0 \\ u(0,y) &= 0 \\ u(2a,y) &= 0 \end{aligned}$$

Using eigenfunction expansion method. Let

$$u(x,y) = \sum_{n=1}^{\infty} b_n(y)\, X_n(x) \tag{1}$$

Where $X_n(x)$ is eigenfunctions for $X''(x) + \lambda_n X(x) = 0$ with boundary conditions $X(0) = X(2a) = 0$. This has eigenfunctions as $X_n(x) = \sin\left(\sqrt{\lambda_n} x\right)$ with eigenvalues $\lambda_n = \left(\frac{n\pi}{2a}\right)^2$ for $n = 1, 2, \cdots$.

Substituting (1) into the PDE $Bu_{xx} + Au_{yy} = C$ gives

$$B\sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = C$$

Expanding C (a constant) as Fourier sine series the above becomes

$$B\sum_{n=1}^{\infty} b_n(y)\, X_n''(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = \sum_{n=1}^{\infty} q_n X_n(x)$$

But $X_n''(x) = -\lambda_n X_n(x)$, hence the above becomes

$$-B\sum_{n=1}^{\infty} \lambda_n b_n(y)\, X_n(x) + A\sum_{n=1}^{\infty} b_n''(y)\, X_n(x) = \sum_{n=1}^{\infty} q_n X_n(x)$$

$$Ab_n''(y) - B\lambda_n b_n(y) = q_n \tag{1A}$$

But

$$\begin{aligned} C &= \sum_{n=1}^{\infty} q_n X_n(x) \\ \int_0^{2a} C X_n(x)\, dx &= q_n \int_0^{2a} X_n^2(x)\, dx \\ \int_0^{2a} C \sin\left(\sqrt{\lambda_n} x\right) dx &= q_n \int_0^{2a} \sin^2\left(\sqrt{\lambda_n} x\right) dx \\ \frac{-C}{\sqrt{\lambda_n}}\left((-1)^n - 1\right) &= q_n a \\ q_n &= \frac{-C}{a\sqrt{\lambda_n}}\left((-1)^n - 1\right) \end{aligned}$$

Hence (1A) becomes

$$Ab_n''(y) - B\lambda_n b_n(y) = \frac{-C}{a\sqrt{\lambda_n}}((-1)^n - 1)$$

This is standard second order linear ODE. The solution is

$$b_n(y) = D_n e^{\sqrt{\frac{B}{A}\lambda_n} y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1)$$

Using the above in (1) gives the solution

$$u(x,y) = \sum_{n=1}^{\infty} \left(D_n e^{\sqrt{\frac{B}{A}\lambda_n} y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} y} + \frac{C}{aB\lambda_n^{\frac{3}{2}}}((-1)^n - 1) \right) X_n(x) \tag{1A}$$

We now need to find D_n, E_n.

<u>Case n even</u>

When n is even $((-1)^n - 1) = 0$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty} \left(D_n e^{\sqrt{\frac{B}{A}\lambda_n} y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} y} \right) X_n(x)$$

At $y = 0$ the above gives

$$0 = \sum_{n=1}^{\infty} (D_n + E_n) \sin\left(\sqrt{\lambda_n} x\right)$$

Therefore

$$D_n + E_n = 0 \tag{2}$$

And at $y = 2b$

$$0 = \sum_{n=1}^{\infty} \left(D_n e^{\sqrt{\frac{B}{A}\lambda_n} 2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} 2b} \right) \sin\left(\sqrt{\lambda_n} x\right)$$

Therefore

$$D_n e^{\sqrt{\frac{B}{A}\lambda_n} 2b} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} 2b} = 0 \tag{3}$$

From (2,3) we see that $D_n = E_n = 0$, Hence $u(x,y) = 0$ when n even.

<u>Case n odd</u>

When n is odd $((-1)^n - 1) = -2$ and the solution (1A) becomes

$$u(x,y) = \sum_{n=1}^{\infty} \left(D_n e^{\sqrt{\frac{B}{A}\lambda_n} y} + E_n e^{-\sqrt{\frac{B}{A}\lambda_n} y} - \frac{2C}{aB\lambda_n^{\frac{3}{2}}} \right) X_n(x)$$

At $y=0$ the above gives

$$0=\sum_{n=1}^{\infty}\left(D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n+E_n-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{4}$$

And at $y=2b$

$$0=\sum_{n=1}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda_n}x\right)$$

Therefore

$$D_n e^{\sqrt{\frac{B}{A}\lambda_n}2b}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}2b}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}=0 \tag{5}$$

Solving (4,5) for D_n, E_n gives

$$D_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$

$$E_n=\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}$$

Therefore the final solution from (1A) becomes

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(D_n e^{\sqrt{\frac{B}{A}\lambda_n}y}+E_n e^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)X_n(x)$$

$$=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}y}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}y}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)\sin\left(\sqrt{\lambda}\right.$$

Where $\lambda_n=\left(\frac{n\pi}{2a}\right)^2$. Switching back to original coordinates using $\tilde{x}=x+a$, and $\tilde{y}=y+b$, then the above is

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\left(-\sqrt{\frac{B}{A}\lambda_n}y+b\right)}-\frac{2C}{aB\lambda_n^{\frac{3}{2}}}\right)$$

Where $C=-2\theta AB$, hence

$$u(x,y)=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta AB}{aB\lambda_n^{\frac{3}{2}}}\right.$$

$$=\sum_{n=1,3,5,\cdots}^{\infty}\left(\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{1}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\left(\frac{-4\theta A}{a\lambda_n^{\frac{3}{2}}}\frac{e^{\sqrt{\frac{B}{A}\lambda_n}2b}}{1+e^{\sqrt{\frac{B}{A}\lambda_n}2b}}\right)e^{-\sqrt{\frac{B}{A}\lambda_n}(y+b)}+\frac{4\theta A}{a\lambda_n^{\frac{3}{2}}}\right)\sin$$

16.2 Dirichlet problem for the Poisson equation in a rectangle

problem number 125

Taken from Mathematica DSolve help pages.

Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6y$$

Boundary conditions

$$u(x,0) = 1 + 11x + x^3$$
$$u(x,2) = -7 + 11x + x^3$$
$$u(0,y) = 1 - y^3$$
$$u(4,y) = 109 - y^3$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 6*x - 6*y;
 bc = {u[x, 0] == 1 + 11*x + x^3, u[x, 2] == -7 + 11*x + x^3, u[0, y] == 1 - y^3, u[4, y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to x^3 + 11x - y^3 + 1\}\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=6*x-6*y;
bc:=u(x,0)=1+11*x+x^3,
    u(x,2)=-7+11*x+x^3,
    u(0,y)=1-y^3,
    u(4,y)=109-y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = x^3 - y^3 + 11\,x + 1$$

17 Helmholtz PDE in Cartesian coordinates

17.1 Dirichlet problem for the Helmholtz equation in a rectangle

problem number 126

Taken from Mathematica DSolve help pages.

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 5u(x,y) = 0$$

Boundary conditions

$$u(x,0) = \text{UnitTriangle[x-2]}$$
$$u(x,2) = 0$$
$$u(0,y) = 0$$
$$u(4,y) = 0$$

Mathematica ✓

```
ClearAll[x, y, n, u];
 pde = {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
 bc = {u[x, 0] == Piecewise[{{-1 + x, x > 1 && x < 2}, {3 - x, x > 2 && x < 3}}], u[x, 2] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
 sol = sol /. K[1] -> n
```

$$\left\{\left\{u(x,y) \to \frac{1}{2}\sum_{n=1}^{\infty} \frac{128\left(\cos\left(\frac{n\pi}{8}\right) + \cos\left(\frac{3n\pi}{8}\right)\right) \operatorname{csch}\left(\frac{1}{2}\sqrt{n^2\pi^2-80}\right) \sin^3\left(\frac{n\pi}{8}\right) \sin\left(\frac{n\pi x}{4}\right) \sinh\left(\frac{1}{4}\sqrt{n^2\pi^2-80}\right.}{n^2\pi^2}\right.\right.$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
bc:=u(x,0)=piecewise( x>1 and x<2,
                      -1+x,x>2 and x<3 ,
                      3-x),
    u(x,2)=0,
    u(0,y)=0,
    u(4,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = \sum_{n=1}^{\infty} 32 \frac{\sin(1/4\,n\pi\,x)\left(1/2\left(\sin(1/2\,\pi\,n) - 1/2\,\sin(1/4\,\pi\,n) - 1/2\,\sin(3/4\,\pi\,n)\right)\sin\left(1/2\sqrt{-\pi^2 n^{}}\right.\right.}{}$$

17.2 With no boundary conditions specified

problem number 127

Added December 27, 2018.

Solve for $u(x,y)$

$$u_{xx} + u_{yy} + 5u(x,y) = 0$$

Mathematica ✗

```
ClearAll[x, y, n, u];
 pde = {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

why? It solved earlier with BC?

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = \_C1\, e^{\sqrt{\_c_1}x}\_C3\, \sin\left(\sqrt{\_c_1+5}y\right)+\_C1\, e^{\sqrt{\_c_1}x}\_C4\, \cos\left(\sqrt{\_c_1+5}y\right)+\frac{\_C2\,\_C3\, \sin\left(\sqrt{\_c_1+\right.}}{e^{\sqrt{\_c_1}x}}$$

18 Reduced Helmholtz PDE in Cartesian coordinates

18.1 Inside square

problem number 128

Added December 20, 2018.

Example 24, taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - ku(x,y) = 0$$

With $k > 0$. It is called reduced Helmholtz, because of the minus sign above. Otherwise, standard Helmholtz has a positive sign.

Boundary conditions

$$\begin{aligned} u(x,0) &= 0 \\ u(x,\pi) &= 0 \\ u(0,y) &= 1 \\ u(\pi,y) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[x, y, n, u, k];
 pde = Laplacian[u[x, y], {x, y}] - k*u[x, y] == 0;
 bc = {u[x, 0] == 0, u[x, Pi] == 0, u[0, y] == 1, u[Pi, y] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> k >
```

Failed

Maple ✓

```
x:='x'; y:='y'; u:='u';k:='k';
pde:= diff(u(x, y), x$2)+diff(u(x, y), y$2)-k*u(x, y) = 0;
bc_left_edge:=u(0, y) = 1;
bc_lower_edge:=u(x, 0) = 0;
bc_top_edge:=u(x,Pi)=0;
bc_right_edge:=u(Pi,y)=0;
bc:=bc_left_edge,bc_lower_edge,bc_top_edge,bc_right_edge;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc ], u(x, y)) assumi
```

$$u(x,y)=\sum_{n=1}^{\infty}-2\,\frac{\sin(ny)\left(-1+(-1)^{n}\right)\left(\mathrm{e}^{-(x-2\,\pi)\sqrt{n^2+k}}-\mathrm{e}^{\sqrt{n^2+k}\,x}\right)}{\left(\mathrm{e}^{2\,\sqrt{n^2+k}\,\pi}-1\right)\pi\,n}$$

19 Helmholtz PDE in 3D Spherical

19.1 Chain reaction PDE

problem number 129

Added May 7, 2019.

Assume ϕ independence. Solve for $u(r,\theta,t)$

$$u_t = k(\lambda u + \nabla^2(u))$$

Where $\nabla^2(u) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta u_\theta)$ with $k > 0$.

Boundary conditions $u(R,\theta,t) = 0$.

Mathematica ✗

```
ClearAll[r,theta,U,k,lambda,t];
U = u[r, theta, t];
pde = D[U, t] == k*(lambda*U + (1/r^2)*D[r^2*D[U, r], r] + (1/(r^2*Sin[theta]))*D[Sin[theta]
bc = u[R, theta, t] == 0;
sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, U, {r, theta, t}, Assumptions -> {k >
```

Failed

Maple ✗

```
unassign('r,theta,t,U,k,lambda');
U:=u(r,theta,t);
pde:= diff(U,t) = k*(lambda*U +1/r^2* diff(r^2*diff(U,r),r)+ 1/(r^2*sin(theta))*diff(sin(the
bc:=u(R,theta,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc], U) assuming k>0,
```

$$u(r,\theta,t) = 0$$

Trivial solution

Hand solution

Solve for $u(r,\theta,t)$ in spherical coordinates (assuming ϕ independence) the chain reaction equation $\frac{1}{k}u_t = \lambda u + \nabla^2 u$ with boundary conditions $u(R,\theta,t) = 0$.

$$\begin{aligned}
\frac{1}{k}u_t &= \lambda u + \nabla^2 u \\
&= \lambda u + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)\right) \\
&= \lambda u + \frac{1}{r^2}(2ru_r + r^2 u_{rr}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)
\end{aligned}$$

Let $u = R(r)\,\Theta(\theta)\,T(t)$. Substituting into the above gives

$$\begin{aligned}
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{1}{r^2}(2rR'\Theta T + r^2 R''\Theta T) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta(\Theta' RT)) \\
\frac{1}{k}T'R\Theta &= \lambda TR\Theta + \frac{2}{r}R'\Theta T + R''\Theta T + \frac{RT}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

Dividing by $TR\Theta$ gives

$$\frac{1}{k}\frac{T'}{T} = \lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)$$

The left side depends on t only and the right depends on r,θ only. Let the separation variable be $-n$. This gives the following 2 equations

$$\frac{1}{k}\frac{T'}{T} = -n \tag{1}$$

$$\lambda + \frac{2}{r}\frac{R'}{R} + \frac{R''}{R} + \frac{1}{\Theta r^2\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = -n \tag{2}$$

Now we consider (2). Multiplying both sides of (2) by r^2 gives

$$\begin{aligned}
\lambda r^2 + 2r\frac{R'}{R} + r^2\frac{R''}{R} + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) &= -nr^2 \\
2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda + n) &= -\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta)
\end{aligned}$$

The left side depends on r and the right side depends on θ. Let the separation variable be $l(l+1)$ where l is integer. Hence we obtain the following two equations

$$-\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}(\Theta'\sin\theta) = l(l+1) \tag{4}$$

$$2r\frac{R'}{R} + r^2\frac{R''}{R} + r^2(\lambda + n) = l(l+1) \tag{5}$$

Starting with (4)

$$\begin{aligned}
\frac{\partial}{\partial\theta}(\Theta'\sin\theta) + l(l+1)\,\Theta\sin\theta &= 0 \\
\Theta''\sin\theta + \Theta'\cos\theta + l(l+1)\,\Theta\sin\theta &= 0
\end{aligned}$$

Using the substitution $z = \cos\theta$ the above becomes

$$\left(1 - z^2\right)\Theta'' - 2z\Theta' + l(l+1)\,\Theta = 0$$

This Legendre ODE. Solution is $P_l(\theta)$. The other solution to the above ODE is ignored as not bounded. Now back to solving (5). Writing it as

$$\begin{aligned} 2rR' + r^2R'' + r^2(\lambda + n)\,R &= l(l+1)\,R \\ r^2R'' + 2rR' + \left(r^2(\lambda + n) - l(l+1)\right)R &= 0 \end{aligned} \tag{6}$$

This can be converted to Bessel ODE using substitution. First let $v = r\sqrt{(\lambda + n)}$. Then $R'(r) = \sqrt{(\lambda + n)}R'(v)\,, R''(r) = (\lambda + n)\,R''(v)$ and (6) becomes

$$\begin{aligned} (\lambda + n)\,r^2R''(v) + 2r\sqrt{(\lambda + n)}R'(v) + \left(r^2(\lambda + n) - l(l+1)\right)R &= 0 \\ v^2R''(v) + 2vR'(v) + \left(v^2 - l(l+1)\right)R &= 0 \end{aligned} \tag{7}$$

Now, we apply second transformation $R(v) = \frac{Z(v)}{\sqrt{v}}$ Then

$$\begin{aligned} R'(v) &= \frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}} \\ R''(v) &= \frac{Z''(v)}{\sqrt{v}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}Z'(v)\,\frac{1}{v^{\frac{3}{2}}} - \frac{1}{2}\left(-\frac{3}{2}\right)Z(v)\,\frac{1}{v^{\frac{5}{2}}} \\ &= \frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}} \end{aligned}$$

Hence (7) becomes

$$v^2\left(\frac{Z''(v)}{\sqrt{v}} - Z'(v)\,\frac{1}{r^{\frac{3}{2}}} + \frac{3}{4}Z(v)\,\frac{1}{v^{\frac{5}{2}}}\right) + 2v\left(\frac{Z'(v)}{\sqrt{v}} - \frac{1}{2}Z(v)\,\frac{1}{v^{\frac{3}{2}}}\right) + \left(v^2 - l(l+1)\right)\frac{Z(v)}{\sqrt{v}} = 0$$

Multiplying by $\sqrt{v}$ gives

$$\begin{aligned} v^2\left(Z''(v) - Z'(v)\,\frac{1}{v} + \frac{3}{4}Z(v)\,\frac{1}{v^2}\right) + 2v\left(Z'(v) - \frac{1}{2}Z(v)\,\frac{1}{v}\right) + \left(v^2 - l(l+1)\right)Z(v) &= 0 \\ \left(v^2Z''(v) - vZ'(v) + \frac{3}{4}Z(v)\right) + \left(2vZ'(v) - Z(v)\right) + \left(v^2 - l(l+1)\right)Z(v) &= 0 \\ v^2Z''(v) + vZ'(v) + \frac{1}{4}Z(v) + \left(v^2 - l(l+1)\right)Z(v) &= 0 \\ v^2Z''(v) + vZ'(v) + \left(v^2 - l(l+1) + \frac{1}{4}\right)Z(v) &= 0 \\ v^2Z''(v) + vZ'(v) + \left(v^2 - l^2 + l + \frac{1}{4}\right)Z(v) &= 0 \\ v^2Z''(v) + vZ'(v) + \left(v^2 - \left(l + \frac{1}{2}\right)^2\right)Z(v) &= 0 \end{aligned}$$

This is now in standard Bessel ODE form. Comparing it to $v^2 Z''(v)+vZ'(v)+(v^2-d^2)\,Z(v)=0$ shows the order is $d=l+\frac{1}{2}$. The solutions are

$$Z(v)=c_1 J_{l+\frac{1}{2}}(v)+c_2 Y_{l+\frac{1}{2}}(v)$$

But $R(v)=\frac{Z(v)}{\sqrt{v}}$, hence

$$R(v)=c_1\frac{J_{l+\frac{1}{2}}(v)}{\sqrt{v}}+c_2\frac{Y_{l+\frac{1}{2}}(v)}{\sqrt{v}}$$

But $v=r\sqrt{\lambda+n}$ then above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}+c_2\frac{Y_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

The solution assumed bounded at $r=0$ hence $c_2=0$ and the above becomes

$$R(r)=c_1\frac{J_{l+\frac{1}{2}}\left(r\sqrt{(\lambda+n)}\right)}{\sqrt{r\sqrt{(\lambda+n)}}}$$

Let $m^2=(\lambda+n)$, hence

$$\begin{aligned}R(r)&=c_1\frac{J_{l+\frac{1}{2}}(mr)}{\sqrt{mr}}\\&=j_l(mr)\end{aligned}$$

where $j_l(mr)$ are the spherical Bessel functions. Boundary conditions at $r=R$ gives

$$j_l(mR)=0$$

Hence mR or $\sqrt{\lambda+n}R$ are the zeros of spherical Bessel functions $j_l(mR)$. There are infinite zeros for each l. Let the v^{th} zero of j_l be called $Z_{l,v}$. Then $mR_{l,v}=Z_{l,v}$ or $\sqrt{\lambda+n}_{l,v}=\frac{Z_{l,v}}{R}$. or

$$n=\left(\frac{Z_{l,v}}{R}\right)^2-\lambda$$

The solution to the time ODE is therefore

$$\begin{aligned}T_{l,v}&=A_{l,v}e^{-nkt}\\&=A_{l,v}e^{-\left(\left(\frac{Z_{l,v}}{R}\right)^2-\lambda\right)kt}\end{aligned}$$

Hence the complete solution is

$$
\begin{aligned}
u(r,\theta,t) &= e^{-i\alpha kt} P_l(\theta)\, j_l(mr) \\
&= \sum_{l=1}^{\infty}\sum_{v=0}^{\infty} A_{l,v} e^{-nkt} P_l(\theta)\, j_l\left(\frac{Z_{l,v}}{R} r\right)
\end{aligned}
$$

$A_{l.v}$ constants still need to be found from initial conditions. For each l, we have infinite sum over all $v's$ zeros of j_l.

20 Wave PDE on finite length string

20.1 Both ends fixed, zero initial position, non-zero initial velocity, with extra term present

problem number 130

Added Feb 25, 2019. Exam 1 problem, MATH 4567 Applied Fourier Analysis, University of Minnesota, Twin Cities.

Solve for $u(x,t)$

$$u_{tt} = u_{xx} - u$$

With boundary condition

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$u_t(x,0) = 1$$

Mathematica ✗

```
ClearAll[u, x, t, k, L];
 pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] - u[x, t];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 1};
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde:=diff(u(x,t),t$2)=diff(u(x,t),x$2)-u(x,t);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = \sum_{n=1}^{\infty} -2 \frac{(-1+(-1)^n)\sin(nx)\sin\left(\sqrt{n^2+1}t\right)}{\pi\sqrt{n^2+1}n}$$

20.2 One end fixed, another free, both initial conduitions non zero, and source that depends on time and space

problem number 131

Added July 2, 2018. Taken from Maple 2018.1 improvement to PDE document.

Solve

$$-\frac{\partial^2 u}{\partial t^2} + u(x,t) = \frac{\partial^2 u}{\partial x^2} + 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)$$

With boundary condition

$$\begin{aligned} u(0,t) &= 0 \\ \frac{\partial u(1,t)}{\partial x} &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^2 - 2x \\ u(x,1) &= u(x,\frac{1}{2}) + e^{-1}\left(\frac{1}{2}x^2 - x\right) \end{aligned}$$

Mathematica ✗

```
ClearAll[u, x, t, k, L];
 pde = -D[u[x, t], {t, 2}] + u[x, t] == D[u[x, t], {x, 2}] + 2*Exp[-t]*(x - (1/2)*x^2 + (1/2
 bc = {u[0, t] == 0, Derivative[1, 0][u][1, t] == 0};
 ic = {u[x, 0] == x^2 - 2*x, u[x, 1] == u[x, 1/2] + ((1/2)*x^2 - x)*Exp[-1] - ((3*x^2)/4 - (
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := -diff(u(x, t), t, t) + u(x, t) = diff(u(x, t), x, x)+ 2*exp(-t)*(x-(1/2)*x^2+(1/2)*t-
ic:= u(x, 0) = x^2-2*x,
     u(x, 1) = u(x, 1/2)+((1/2)*x^2-x)*exp(-1)-(3/4*(x^2)-3/2*x)*exp(-1/2);
bc:= u(0, t) = 0, eval(diff(u(x, t), x), {x = 1}) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic, bc],u(x,t))),out
```

$$u(x,t) = -1/2\, \mathrm{e}^{-t} x(-2+x)\,(t-2)$$

20.3 Both ends fixed, no initial conduitions give and no source (Logan p. 28)

problem number 132

This is problem at page 28, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary condition

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Mathematica ✗

```
ClearAll[u, t, x, L, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, t], {x, t}, Assumptions -> {L >
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,t)) assuming
```

$$u(x,t)=\sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{L}\right)\left(\sin\left(\frac{cn\pi t}{L}\right)\_C1(n)+\cos\left(\frac{cn\pi t}{L}\right)\_C5(n)\right)$$

20.4 One end fixed, other free, initial position not zero, initial velocity zero, no source (Logan p. 130)

problem number 133

This is problem at page 130, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$\frac{\partial u}{\partial x}(L,0)=0$$
$$u(0,t)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

Mathematica ✗

```
ClearAll[u, t, x, L, c, f];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic:=D[2](u)(x,0)=0,u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=0}^{\infty} \left(2\,\frac{1}{L} \sin \left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L} \right) \cos \left(1/2\,\frac{c(1+2\,n)\,\pi\,t}{L} \right) \int_0^L \sin \left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L} \right) f(x)\,\mathrm{d}x \right)$$

20.5 Both ends fixed end, initial conditions zero, with source as generic function that depends on time and space (Logan p. 149)

problem number 134

This is problem at page 149, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + p(x,t)$$

With boundary conditions

$$u(\pi,0)=0$$
$$u(0,t)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=0$$

Mathematica ✗

```
ClearAll[u, t, x, c, p];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + p[x, t];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';p:='p';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t)=\int_0^t \sum_{n=1}^{\infty}\left(2\,\frac{\int_0^{\pi}\sin(nx)\,p(x,\tau)\,\mathrm{d}x\sin(nx)\sin(cn(t-\tau))}{\pi\,nc}\right)\mathrm{d}\tau$$

20.6 Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space (Haberman 8.5.2 (a))

problem number 135

Added Nov 25, 2018.

This is problem 8.5.2 (a), Richard Haberman applied partial differential equations book, 5th edition

Consider a vibrating string with time-dependent forcing:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Q(x,t)$$

With boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

Solve the initial value problem.

my hand solution in file `feb_24_2019_4_24_pm.tex`, but I need to go over my solution again to make sure it is correct.

Mathematica ✗

```
ClearAll[u, t, x, c, Q, L, f];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + Q[x, t];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';Q:='Q';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+Q(x,t);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=f(x), eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\frac{1}{L}\int_0^L \sin\left(\frac{\pi n\tau}{L}\right) f(\tau)\, d\tau \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{\pi nct}{L}\right)\right) + \int_0^t \sum_{n=1}^{\infty} \left(2\frac{1}{\pi nc}\int_0^L \sin\left(\frac{n\pi x}{L}\right) Q(x$$

20.7 Both ends fixed end, initial position given, zero initial velocity, with source that depends on time and space (Haberman 8.5.2 (b))

problem number 136

Added Nov 25, 2018.

This is problem 8.5.2 (b), Richard Haberman applied partial differential equations book, 5th edition

Consider a vibrating string with time-dependent forcing:

$$u_{tt} = c^2 u_{xx} + g(x)\cos(\omega t)$$

With boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

With initial conditions

$$u_t(x,0) = 0$$
$$u(x,0) = f(x)$$

Solve the initial value problem.

See my solution at HW 9, Math 322. UW Madison.

Mathematica ✗

```
ClearAll[u, t, x, c, Q, L, f, g, w];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + g[x]*Cos[w*t];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';g:='Q';w:='w';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+ g(x)*cos(w*t);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=0, eval( diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t)) assumi
```

Bad latex generated

Hand solution

Let

$$u(x,t)=\sum_{n=1}^{\infty}A_n(t)\,\phi_n(x)$$

Where we used $=$ instead of $\sim$ above, since the PDE given has homogeneous B.C. We know that $\phi_n(x)=\sin\left(\sqrt{\lambda_n}x\right)$ for $n=1,2,3,\cdots$ where $\lambda_n=\left(\frac{n\pi}{L}\right)^2$. Substituting the above in the given PDE gives

$$\sum_{n=1}^{\infty}A_n''(t)\,\phi_n(x)=c^2\sum_{n=1}^{\infty}A_n(t)\,\frac{d^2\phi_n(x)}{dx^2}+Q(x,t)$$

But $Q(x,t)=\sum_{n=1}^{\infty}q_n(t)\,\phi_n(x)$, hence the above becomes

$$\sum_{n=1}^{\infty}A_n''(t)\,\phi_n(x)=c^2\sum_{n=1}^{\infty}A_n(t)\,\frac{d^2\phi_n(x)}{dx^2}+\sum_{n=1}^{\infty}g_n(t)\,\phi_n(x)$$

But $\frac{d^2\phi_n(x)}{dx^2} = -\lambda_n\phi_n(x)$, hence

$$\sum_{n=1}^{\infty} A_n''(t)\,\phi_n(x) = -c^2\sum_{n=1}^{\infty}\lambda_n A_n(t)\,\phi_n(x) + \sum_{n=1}^{\infty} g_n(t)\,\phi_n(x)$$

Multiplying both sides by $\phi_m(x)$ and integrating gives

$$\int_0^L \sum_{n=1}^{\infty} A_n''(t)\,\phi_m(x)\,\phi_n(x)\,dx = -c^2\int_0^L \sum_{n=1}^{\infty}\lambda_n A_n(t)\,\phi_m(x)\,\phi_n(x)\,dx + \int_0^L \sum_{n=1}^{\infty} g_n(t)\,\phi_m(x)\,\phi_n(x)\,dx$$

$$A_n''(t)\int_0^L \phi_n^2(x)\,dx = -c^2\lambda_n A_n(t)\int_0^L \phi_n^2(x)\,dx + g_n(t)\int_0^L \phi_n^2(x)\,dx$$

Hence

$$A_n''(t) + c^2\lambda_n A_n(t) = g_n(t)$$

Now we solve the above ODE. Let solution be

$$A_n(t) = A_n^h(t) + A_n^p(t)$$

Which is the sum of the homogenous and particular solutions. The homogenous solution is

$$A_n^h(t) = c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right)$$

And the particular solution depends on $q_n(t)$. Once we find $q_n(t)$, we plug-in everything back into $u(x,t) = \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x)$ and then use initial conditions to find c_{1_n}, c_{2_n}, the two constant of integrations. Now we are given that $Q(x,t) = g(x)\cos(\omega t)$. Hence

$$g_n(t) = \frac{\int_0^L Q(x,t)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \frac{\cos(\omega t)\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx} = \cos(\omega t)\,\gamma_n$$

Where

$$\gamma_n = \frac{\int_0^L g(x)\,\phi_n(x)\,dx}{\int_0^L \phi_n^2(x)\,dx}$$

is constant that depends on n. Now we use the above in result found in part (a)

$$A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n\cos(\omega t) \tag{1}$$

We know the homogenous solution from part (a).

$$A_n^h(t) = c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right)$$

We now need to find the particular solution. Will solve using method of undetermined coefficients.

<u>Case 1</u> $\omega \neq c\sqrt{\lambda_n}$ (no resonance)

We can now guess

$$A_n^p(t) = z_1 \cos(\omega t) + z_2 \sin(\omega t)$$

Plugging this back into (1) gives

$$\begin{aligned}
(z_1 \cos(\omega t) + z_2 \sin(\omega t))'' + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t) \\
(-\omega z_1 \sin(\omega t) + \omega z_2 \cos(\omega t))' + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t) \\
-\omega^2 z_1 \cos(\omega t) - \omega^2 z_2 \sin(\omega t) + c^2\lambda_n(z_1 \cos(\omega t) + z_2 \sin(\omega t)) &= \gamma_n \cos(\omega t)
\end{aligned}$$

Collecting terms

$$\cos(\omega t)\left(-\omega^2 z_1 + c^2\lambda_n z_1\right) + \sin(\omega t)\left(-\omega^2 z_2 + c^2\lambda_n z_2\right) = \gamma_n \cos(\omega t)$$

Therefore we obtain two equations in two unknowns

$$\begin{aligned}
-\omega^2 z_1 + c^2\lambda_n z_1 &= \gamma_n \\
-\omega^2 z_2 + c^2\lambda_n z_2 &= 0
\end{aligned}$$

From the second equation, $z_2 = 0$ and from the first equation

$$\begin{aligned}
z_1\left(c^2\lambda_n - \omega^2\right) &= \gamma_n \\
z_1 &= \frac{\gamma_n}{c^2\lambda_n - \omega^2}
\end{aligned}$$

Hence

$$\begin{aligned}
A_n^p(t) &= z_1 \cos(\omega t) + z_2 \sin(\omega t) \\
&= \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Therefore

$$\begin{aligned}
A_n(t) &= A_n^h(t) + A_n^p(t) \\
&= c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)
\end{aligned}$$

Now we need to find c_{1_n}, c_{2_n}. Since

$$\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\, \phi_n(x) \\
&= \sum_{n=1}^{\infty} \left(c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + c_{2_n} \sin\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{c^2\lambda_n - \omega^2} \cos(\omega t)\right) \sin\left(\frac{n\pi}{L} x\right)
\end{aligned}$$

At $t=0$ the above becomes

$$\begin{aligned}
f(x) &= \sum_{n=1}^{\infty}\left(c_{1_n}+\frac{\gamma_n}{c^2\lambda_n-\omega^2}\right)\sin\left(\frac{n\pi}{L}x\right)\\
&= \sum_{n=1}^{\infty}c_{1_n}\sin\left(\frac{n\pi}{L}x\right)+\sum_{n=1}^{\infty}\frac{\gamma_n}{c^2\lambda_n-\omega^2}\sin\left(\frac{n\pi}{L}x\right)
\end{aligned}$$

Applying orthogonality

$$\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx = \int_0^L\sum_{n=1}^{\infty}c_{1_n}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx+\int_0^L\sum_{n=1}^{\infty}\frac{\gamma_n}{c^2\lambda_n-\omega^2}\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx$$

$$\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx = c_{1_n}\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx+\frac{\gamma_n}{c^2\lambda_n-\omega^2}\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx$$

Rearranging

$$\begin{aligned}
\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx-\frac{\gamma_n}{c^2\lambda_n-\omega^2}\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx &= c_{1_n}\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx\\
c_{1_n} &= \frac{\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx}{\int_0^L\sin^2\left(\frac{n\pi}{L}x\right)dx}-\frac{\gamma_n}{c^2\lambda_n-\omega^2}\\
&= \frac{2}{L}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx-\frac{\gamma_n}{c^2\lambda_n-\omega^2}
\end{aligned}$$

We now need to find c_{2_n}. For this we need to differentiate the solution once.

$$\frac{\partial u(x,t)}{\partial t}=\sum_{n=1}^{\infty}\left(-c\sqrt{\lambda_n}c_{1_n}\sin\left(c\sqrt{\lambda_n}t\right)+c\sqrt{\lambda_n}c_{2_n}\cos\left(c\sqrt{\lambda_n}t\right)-\frac{\gamma_n}{c^2\lambda_n-\omega^2}\omega\sin(\omega t)\right)\sin\left(\frac{n\pi}{L}x\right)$$

Applying initial conditions $\frac{\partial u(x,0)}{\partial t}=0$ gives

$$0=\sum_{n=1}^{\infty}c\sqrt{\lambda_n}c_{2_n}\sin\left(\frac{n\pi}{L}x\right)$$

Hence

$$c_{2_n}=0$$

Therefore the final solution is

$$A_n(t)=c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right)+\frac{\gamma_n}{c^2\lambda_n-\omega^2}\cos(\omega t)$$

And

$$u(x,t)=\sum_{n=1}^{\infty}A_n(t)\sin\left(\frac{n\pi}{L}x\right)$$

Where

$$c_{1_n} = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{m\pi}{L}x\right)dx - \frac{\gamma_n}{c^2\lambda_n - \omega^2}$$

<u>Case 2</u> $\omega = c\sqrt{\lambda_n}$ <u>Resonance case</u>. Now we can't guess $A_n^p(t) = z_1\cos(\omega t) + z_2\sin(\omega t)$ so we have to use

$$A_n^p(t) = z_1 t\cos(\omega t) + z_2 t\sin(\omega t)$$

Substituting this in $A_n''(t) + c^2\lambda_n A_n(t) = \gamma_n\cos(\omega t)$ gives

$$(z_1 t\cos(\omega t) + z_2 t\sin(\omega t))'' + c^2\lambda_n(z_1 t\cos(\omega t) + z_2 t\sin(\omega t)) = \gamma_n\cos(\omega t) \tag{2}$$

But

$$\begin{aligned}(z_1 t\cos(\omega t) + z_2 t\sin(\omega t))'' &= (z_1\cos(\omega t) - z_1\omega t\sin(\omega t) + z_2\sin(\omega t) + z_2\omega t\cos(\omega t))' \\ &= -z_1\omega\sin(\omega t) - \left(z_1\omega\sin(\omega t) + z_1\omega^2 t\cos(\omega t)\right) \\ &\quad + z_2\omega\cos(\omega t) + \left(z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t)\right) \\ &= -2z_1\omega\sin(\omega t) - z_1\omega^2 t\cos(\omega t) + 2z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t)\end{aligned}$$

Hence (2) becomes

$$-2z_1\omega\sin(\omega t) - z_1\omega^2 t\cos(\omega t) + 2z_2\omega\cos(\omega t) - z_2\omega^2 t\sin(\omega t) + c^2\lambda_n(z_1 t\cos(\omega t) + z_2 t\sin(\omega t)) = \gamma_n\cos(\omega t$$

Comparing coefficients we see that $2z_2\omega = \gamma_n$ or

$$z_2 = \frac{\gamma_n}{2\omega}$$

And $z_1 = 0$. Therefore

$$A_n^p(t) = \frac{\gamma_n}{2\omega}t\sin(\omega t)$$

Therefore

$$\begin{aligned}A_n(t) &= A_n^h(t) + A_n^p(t) \\ &= c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}}t\sin(\omega t)\end{aligned}$$

We now can find c_{1_n}, c_{2_n} from initial conditions.

$$\begin{aligned}u(x,t) &= \sum_{n=1}^{\infty} A_n(t)\,\phi_n(x) \\ &= \sum_{n=1}^{\infty}\left(c_{1_n}\cos\left(c\sqrt{\lambda_n}t\right) + c_{2_n}\sin\left(c\sqrt{\lambda_n}t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}}t\sin(\omega t)\right)\sin\left(\frac{n\pi}{L}x\right) \quad (4)\end{aligned}$$

At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} c_{1_n} \sin\left(\frac{n\pi}{L} x\right)$$

$$c_{1n} = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

Taking time derivative of (4) and setting it to zero will give c_{2n}. Since initial speed is zero then $c_{2_n} = 0$. Hence

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)$$

This completes the solution.

<u>Summary of solution</u>

The solution is given by

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\, \phi_n(x)$$

<u>Case $\omega \neq c\sqrt{\lambda_n}$</u>

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{c^2 \lambda_n - \omega^2} \cos(\omega t)$$

And

$$c_{1_n} = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{m\pi}{L} x\right) dx - \frac{\gamma_n}{c^2 \lambda_n - \omega^2}$$

And

$$\gamma_n = \frac{\int_0^L g(x)\, \phi_n(x)\, dx}{\int_0^L \phi_n^2(x)\, dx}$$

And $\lambda_n = \left(\frac{n\pi}{L}\right)^2, n = 1, 2, 3,$

<u>Case $\omega = c\sqrt{\lambda_n}$</u> (resonance)

$$A_n(t) = c_{1_n} \cos\left(c\sqrt{\lambda_n} t\right) + \frac{\gamma_n}{2c\sqrt{\lambda_n}} t \sin(\omega t)$$

And

$$c_{1_n} = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

20.8 Both ends fixed, initial conditions both not zero, No source

problem number 137

Added July 2, 2018.

Taken from Maple 2018.1 improvements to PDE's document.

Solve

$$\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial x^2}$$

For $t > 0$ and $0 < x < 1$. With boundary conditions

$$v(0,t) = 0$$
$$v(1,0) = 0$$

With initial conditions

$$v(x,0) = f(x)$$
$$\frac{\partial v}{\partial t}(x,0) = g(x)$$

Where $f(x) = -\frac{e^{2}x - e^{x+1} - x + e^{1-x}}{e^{2}-1}$ and $g(x) = 1 + \frac{e^{2}x - e^{x+1} - x + e^{1-x}}{e^{2}-1}$

Mathematica ✓

```
ClearAll[v, t, t];
 pde = D[v[x, t], {t, 2}] == D[v[x, t], {x, 2}];
 bc = {v[0, t] == 0, v[1, t] == 0};
 ic = {v[x, 0] == -((Exp[2]*x - Exp[x + 1] - x + Exp[1 - x])/(Exp[2] - 1)), Derivative[0, 1]
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, v[x, t], {x, t}], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{v(x,t) \to \sum_{n=1}^{\infty} \left(\frac{2(-1)^n \cos(n\pi t)}{\pi^3 n^3 + \pi n} + \frac{(-2(-1+(-1)^n)\,\pi^2 n^2 - 4(-1)^n + 2)\sin(n\pi t)}{\pi^4 n^4 + \pi^2 n^2}\right)\sin(n\pi x)\right\}\right\}$$

Maple ✓

```
v:='v';x:='x';t:='t';
pde := diff(v(x, t), t, t)=(diff(v(x, t), x, x));
bc := v(0, t) = 0, v(1, t) = 0;
ic:= v(x, 0) = -(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1),
               (D[2](v))(x, 0) = 1+(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],v(x,t))),outp
```

$$v(x,t) = \sum_{n=1}^{\infty} -2 \frac{\sin(n\pi x)\left((\pi^2(-1)^n n^2 - \pi^2 n^2 + 2(-1)^n - 1)\sin(n\pi t) - (-1)^n \cos(n\pi t)\pi n\right)}{\pi^2 n^2 (\pi^2 n^2 + 1)}$$

20.9 Both ends fixed end, initial conditions both not zero, and with constant source

problem number 138

Added July 2, 2018.

Third example, from Maple 2018.1 improvements to PDE's document.

Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + 1$$

For $t > 0$ and $0 < x < L$. With boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

Mathematica ✗

```
ClearAll[u, t, x, c, L, f, g];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + 1;
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
interface(showassumed=0);
x:='x';t:='t';a:='a';f='f';L:='L';g:='g';
pde :=diff(u(x, t), t, t) = c^2* diff(u(x, t), x, x) + 1 ;
bc := u(0, t) = 0, u(L, t) = 0;
ic:= u(x, 0) = f(x), (D[2](u))(x, 0) = g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde, ic, bc],u(x,t)) ass
```

$$u(x,t) = 1/2\,\frac{1}{c^2}\left(2\sum_{n=1}^{\infty}\left(\frac{1}{\pi\,nc^2L}\sin\left(\frac{n\pi\,x}{L}\right)\left(2\,L\sin\left(\frac{\pi\,nct}{L}\right)\int_0^L\sin\left(\frac{n\pi\,x}{L}\right)g(x)\,\mathrm{d}xc-\pi\,\cos\left(\frac{\pi\,nct}{L}\right)\right.\right.\right.$$

20.10 Both ends fixed end, with source (Logan p. 213)

problem number 139

This is problem at page 213, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} + Ax$$

With boundary conditions

$$u(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica ✗

```
ClearAll[u, t, x, c, A, L];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + A*x;
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L';c:='c';u:='u';A:='A';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,bc,ic],u(x,t))  assu
```

$$u(x,t) = 1/6\,\frac{1}{c^2}\left(AL^2x - Ax^3 + 6\sum_{n=1}^{\infty} 2\,\frac{L^3(-1)^n A}{n^3\pi^3c^2}\sin\left(\frac{n\pi\,x}{L}\right)\cos\left(\frac{\pi\,nct}{L}\right)c^2\right)$$

20.11 Telegraphy PDE, both ends fixed with damping

problem number 140

Solve

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t)=0$$
$$u(\pi,0)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=f(x)$$

Mathematica ✗

```
pde = D[u[x, t], {t, 2}] + 2*D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], x, t], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+2*diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=f(x);
bc:=u(0,t)=0,u(Pi,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol', pdsolve([pde,ic,bc],u(x,t)) assum
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{\int_0^\pi \sin(nx)\,f(x)\,dx \sin(nx)\left(\left(-1+\sqrt{-n^2+1}\right)\mathrm{e}^{-\left(\sqrt{-n^2+1}+1\right)t}+\mathrm{e}^{\left(-1+\sqrt{-n^2+1}\right)t}\left(\sqrt{-n^2+\right.\right.}{\sqrt{-n^2+1}\pi}\right.$$

But $n=1$ should not be included.

20.12 Both ends fixed. Initial velocity zero. Dispersion term present

problem number 141

Solve

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} + \gamma^2 u(x,t) = c^2 \frac{\partial^2 u}{\partial x^2}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time.

With $0 < x < \pi$ and $t > 0$ and with boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

Mathematica ✗

```
ClearAll[a, u, x, t, gamma];
 pde = (1*D[u[x, t], {t, 2}])/a^2 + gamma^2*u[x, t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Due to adding dispersion term

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';a:='a';g:='g';
interface(showassumed=0);
pde:=1/a^2*diff(u(x,t),t$2)+g^2*u(x,t)=diff(u(x,t),x$2);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=sin(x)^2,(D[2](u))(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t))),outpu
```

$$u(x,t) = 1/3\,\frac{1}{\pi}\left(3\sum_{n=3}^{\infty} 4\,\frac{\sin(nx)\cos\left(a\sqrt{g^2+n^2}t\right)\left(-1+(-1)^n\right)}{\pi\, n\,(n^2-4)}\pi + 8\,\sin(x)\cos\left(a\sqrt{g^2+1}t\right)\right)$$

20.13 Both ends fixed, non-zero initial position

problem number 142

Added March 9, 2018.

Solve

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

Mathematica ✓

```
ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}];
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4(\cos(n\pi)-1)\cos(2nt)\sin(nx)}{(n^3-4n)\,\pi}\right\}\right\}$$

But sum should not include $n = 2$

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)= 4*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=sin(x)^2,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,t))),outpu
```

$$u(x,t) = 1/3\,\frac{1}{\pi}\left(3\sum_{n=3}^{\infty} 4\,\frac{\sin(nx)\cos(2\,nt)\left(-1+(-1)^n\right)}{\pi\,n\,(n^2-4)}\pi + 8\,\sin(x)\cos(2\,t)\right)$$

Handled $n = 2$ case correctly

20.14 Both ends fixed, zero initial position, non zero initial velocity, with source that depends on time and space

problem number 143

Added December 20, 2018.

Example 18, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,t)$ with $0<x<1$ and $t>0$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x e^{-t}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(1,0) = 0$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

Mathematica ✗

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + x*Exp[-t];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 bc = {u[0, t] == 0, u[1, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2)+x*exp(-t);
bc := u(0,t)=0,u(1,t)=0;
ic := u(x,0)=0,eval(diff(u(x,t),t),t=0)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{\left(-\pi^2(-1)^n n^2 + \pi^2 n^2 + 2\left(-1\right)^{1+n} + 1\right)\cos\left(\pi n(-x+t)\right) - \pi\left(-1\right)^n n \sin\left(\pi n(-x+t)\right) + (\pi}{\pi}$$

20.15 Left end fixed, right end oscillates, initially at rest. With source that depends on time and space

problem number 144

Added December 20, 2018.

Example 19, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $0 < x < \pi$ and $t > 0$

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} + (1+t)x$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = \sin(t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica ✗

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}] + (1 + t)*x;
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 bc = {u[0, t] == 0, u[Pi, t] == Sin[t]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t$2) = 4*diff(u(x, t), x$2)+(1+t)*x;
bc := u(0,t)=0,u(Pi,t)=sin(t);
ic := u(x,0)=0,eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t))),outp
```

$$u(x,t) = \frac{1}{\pi}\left(x\sin(t) + \sum_{n=1}^{\infty} -2\,\frac{(1/2\,\cos(nx-t)\,n^3 - 1/2\,\cos(nx+t)\,n^3 + \sin(nx)\,((-2\,n^4 - 1/2\,\pi\,n^2 +}{\pi\,n^4\,(4\,n^2}\right.$$

21 Wave PDE on semi-infinite domain

21.1 With zero initial position and velocity, and with source term (Logan p. 115)

problem number 145

This is problem at page 115, David J Logan textbook, applied PDE textbook.

Falling cable lying on a table that is suddenly removed.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - g$$

With boundary condition

$$u(0,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica ✓

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] - g;
 bc = u[0, t] == 0;
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to \frac{1}{2} g \left(\left(t - \frac{x}{c}\right)^2 \theta\left(t - \frac{x}{c}\right) - t^2\right) - c_1 \delta\left(t - \frac{x}{c}\right)\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; g:='g';c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t),HINT = bou
```

$$u(x,t) = 1/2\,\frac{g}{c^2}\left(\mathit{Heaviside}\left(t - \frac{x}{c}\right)(tc-x)^2 - c^2t^2\right)$$

21.2 Left end having a moving boundary condition

problem number 146

Solve for $u(x,t)$ with $t>0$ and $x>0$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = g(t)$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica ✓

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = u[0, t] == g[t];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t)\to\ \left\{\ \begin{array}{cc} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\leq ct \\ \text{Indeterminate} & \text{True} \end{array}\right.\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';g:='g';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic:=u(x,0)=0,D[2](u)(x,0)=0;
bc:=u(0,t)=g(t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT =
```

$$u(x,t)=-\mathit{invlaplace}\left(\mathrm{e}^{\frac{sx}{c}}\_F2(s),s,t\right)+\mathit{invlaplace}\left(\mathrm{e}^{\frac{sx}{c}}\mathit{laplace}(g(t),t,s),s,t\right)+\mathit{invlaplace}\left(\_F2(s)\,\mathrm{e}^{-\frac{sx}{c}},s$$

21.3 Initial value with a Dirichlet condition on the half-line

problem number 147

Taken from Mathematica DSolve help pages.

Solve for $u(x,t)$ initial value wave PDE on infinite domain with $t>0$ and $x>0$.

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \sin^2(x) \qquad \pi < x < 2\pi$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary conditions $u(0,t) = 0$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == Piecewise[{{Sin[x]^2, Pi < x < 2*Pi}}], Derivative[0, 1][u][x, 0] == 0};
 bc = u[0, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \left\{ \begin{array}{l} \frac{1}{2}\left(\left(\left(\begin{cases} \sin^2\left(\sqrt{c^2}t - x\right) & \pi < x - \sqrt{c^2}t < 2\pi \\ 0 & \text{True} \end{cases} \right) + \left(\begin{cases} \sin^2\left(\sqrt{c^2}t + x\right) & \pi < \end{cases} \right.\right.\right. \\ \frac{1}{2}\left(\left(\left(\begin{cases} \sin^2\left(\sqrt{c^2}t + x\right) & \pi < \sqrt{c^2}t + x < 2\pi \\ 0 & \text{True} \end{cases} \right) - \left(\begin{cases} \sin^2\left(\sqrt{c^2}t - x\right) & \pi < \end{cases} \right.\right.\right. \\ \text{Indeterminate} \end{array}\right.\right.\right.$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde:=diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2) ;
ic:=u(x,0)= piecewise(Pi<x and x<2*Pi,sin(x)^2),(D[2](u))(x,0)=0;
bc:=u(0,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

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21.4 Initial value problem with a Neumann condition on the half-line

problem number 148

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \sin^3(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 1 - e^{-\frac{x}{10}}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == Sin[x]^3, Derivative[0, 1][u][x, 0] == 1 - E^(-(x/10))};
 bc = Derivative[1, 0][u][0, t] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t}], 60*10]];
```

$$\begin{cases} \frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t + x\right) - \sin^3\left(\sqrt{c^2}t - x\right)\right) + \frac{2\sqrt{c^2}t - 20e^{-x/10}\sinh\left(\frac{\sqrt{c^2}t}{10}\right)}{2\sqrt{c^2}} & x > \sqrt{\ldots} \\ \frac{10e^{\frac{1}{10}\left(-\sqrt{c^2}t - x\right)} + 10e^{\frac{1}{10}\left(x - \sqrt{c^2}t\right)} + 2\sqrt{c^2}t - 20}{2\sqrt{c^2}} - \sqrt{c^2}\left(t - \frac{x}{\sqrt{c^2}}\right) + \frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t - x\right) + \sin^3\left(\sqrt{c^2}t + x\right)\right) & 0 \le x \ldots \end{cases}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde:=diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2) ;
ic:=u(x,0)= sin(x)^3,(D[2](u))(x,0)=1-exp(-x/10);
bc:=(D[1](u))(0,t)=1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t)) assumi
```

Bad latex generated

21.5 With initial conditions given at $t = 1$ instead of $t = 0$

problem number 149

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$\begin{aligned} u(x,1) &= e^{-(x-6)^2} + e^{-(x+6)^2} \\ \frac{\partial u}{\partial t}(x,1) &= \frac{1}{2} \end{aligned}$$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == Sin[x]^3, Derivative[0, 1][u][x, 0] == 1 - E^(-(x/10))};
 bc = Derivative[1, 0][u][0, t] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolveValue[{pde, ic, bc}, u[x, t], {x, t}, Assumption
```

$$\begin{cases} \frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t+x\right)-\sin^3\left(\sqrt{c^2}t-x\right)\right)+\frac{2\sqrt{c^2}t-20e^{-x/10}\sinh\left(\frac{\sqrt{c^2}t}{10}\right)}{2\sqrt{c^2}} & x>\sqrt{} \\ \frac{10e^{\frac{1}{10}\left(-\sqrt{c^2}t-x\right)}+10e^{\frac{1}{10}\left(x-\sqrt{c^2}t\right)}+2\sqrt{c^2}t-20}{2\sqrt{c^2}}-\sqrt{c^2}\left(t-\frac{x}{\sqrt{c^2}}\right)+\frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t-x\right)+\sin^3\left(\sqrt{c^2}t+x\right)\right) & 0\leq x \end{cases}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t$2) = diff(u(x, t), x$2);
ic := u(x, 1) = exp(-(x-6)^2)+exp(-(x+6)^2), eval(diff(u(x,t),t),t=1)= 1/2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

$$u(x,t)=1/2\,e^{-(-x+t+5)^2}+1/2\,e^{-(-x+t-7)^2}+1/2\,e^{-(x+t-7)^2}+1/2\,e^{-(x+t+5)^2}+t/2-1/2$$

21.6 initial conditions at $t=0$ but B.C. at $x=1$

problem number 150

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And Boundary conditions $\frac{\partial u}{\partial x}(1,t) = 0$

Mathematica ✗

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == (1*D[u[x, t], {x, 2}])/4;
 ic = {u[x, 0] == Exp[-x^2], Derivative[0, 1][u][x, 0] == 0};
 bc = Derivative[1, 0][u][1, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t$2)=(1/4)*(diff(u(x, t), x$2)) ;
bc:= eval(diff(u(x,t),x),x=1)=0;
ic:=  u(x, 0) = exp(-x^2), eval(diff(u(x,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],u(x,t)) assum
```

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21.7 initial conditions at $t = 1$ with source that depends on time and space

problem number 151

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + f(x,t)$$

With initial conditions

$$u(x,1) = g(x)$$
$$\frac{\partial u}{\partial t}(x,1) = h(x)$$

Mathematica ✗

```
ClearAll[u, t, x, c, h, f, g];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 1] == g[x], Derivative[0, 1][u][x, 1] == h[x]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> {t >
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';h:='h';f:='f';c:='c';g:='g';
pde := diff(u(x, t), t$2) = c^2*(diff(u(x, t), x$2)) + f(x, t);
ic := u(x, 1) = g(x),eval(diff(u(x,t),t),t=1)=h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t)) assumi
```

$$u(x,t) = 1/2\,\frac{\int_0^{t-1}\int_{(-t+\tau+1)c+x}^{x+c(t-1-\tau)} c^2\left(\frac{d^2}{d\zeta^2}h(\zeta)\right)\tau + c^2\frac{d^2}{d\zeta^2}g(\zeta) + f(\zeta,\tau+1)\,d\zeta\,d\tau + (2\,t-2)\,ch(x) + 2\,g(x)\,c}{c}$$

21.8 Left end free with initial position and velocity given

problem number 152

Added December 20, 2018.

Example 17, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,t)$ with $x>0, t>0$

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = x^3$$

And boundary condition $\frac{\partial u}{\partial x}(0,t) = 0$.

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == 9*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^3};
 bc = Derivative[1, 0][u][0, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions ->
```

$$\left\{\left\{u(x,t) \to 3c_1\delta(3t-x) + \frac{1}{12}(x-3t)^4\theta\left(t-\frac{x}{3}\right) + 9t^3x + tx^3\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t$2) = 9*(diff(u(x, t), x$2));
bc := eval( diff(u(x,t),x),x=0)=0;
ic := u(x,0)=0,eval(diff(u(x,t),t),t=0)=x^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc,ic],u(x,t)) assum
```

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22 Wave PDE 1D infinite domain

22.1 General solution for a second-order hyperbolic PDE on real line

problem number 153

From Mathematica DSolve help pages (slightly modified)

Solve for $u(x,t)$ with $t>0$ on real line

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2 u}{\partial t \partial x}=c^2\frac{\partial^2 u}{\partial x^2}$$

Mathematica ✓

```
ClearAll[u, t, x, c];
 ode = D[u[x, t], {t, 2}] + D[u[x, t], x, t] == c^2*D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{\left(\sqrt{4c^2+1}-1\right)x}{2c^2}\right)+c_2\left(t-\frac{\left(-\sqrt{4c^2+1}-1\right)x}{2c^2}\right)\right\}\right\}$$

Maple ✓

```

x:='x'; t:='t';c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+diff(u(x,t),t,x)=c^2*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t)) assuming t>0,x
```

$$u(x,t)=\_F1\left(1/2\,\frac{2\,c^2t+x\sqrt{4\,c^2+1}+x}{c^2}\right)+\_F2\left(1/2\,\frac{2\,c^2t-x\sqrt{4\,c^2+1}+x}{c^2}\right)$$

22.2 With initial conditions specified, no source

problem number 154

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
 ic = {u[x, 0] == E^(-x^2), Derivative[0, 1][u][x, 0] == 1};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(e^{-(x-t)^2} + e^{-(t+x)^2}\right) + t\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x,t), t$2) = diff(u(x,t), x$2);
ic:= u(x, 0) = exp(-x^2), (D[2](u))(x,0) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = 1/2\,\mathrm{e}^{-(-x+t)^2} + t + 1/2\,\mathrm{e}^{-(x+t)^2}$$

22.3 Wave PDE on infinite domain with initial conditions specified, with source term

problem number 155

Taken from Mathematica DSolve help pages.

Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + m$$

With initial conditions

$$u(x,0) = \sin x - \frac{\cos 3x}{e^{\frac{abs(x)}{6}}}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = {D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + m};
 ic = {u[x, 0] == Sin[x] - Cos[3*x]/E^(Abs[x]/6), Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(-e^{-\frac{|x-t|}{6}}\cos(3(x-t)) - e^{-\frac{|t+x|}{6}}\cos(3(t+x)) - \sin(t-x) + \sin(t+x)\right) + \frac{mt^2}{2}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde:= diff(u(x, t), t$2) = diff(u(x, t), x$2) + m;
ic := u(x, 0) = sin(x) - cos(3*x)/exp(abs(x)/6), (D[2](u))(x, 0) =0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic], u(x, t))),outpu
```

$$u(x,t) = 1/2\,\mathrm{e}^{-1/6\,|-x+t|-1/6\,|x+t|}\left(\left(mt^2 - \sin\left(-x+t\right) + \sin\left(x+t\right)\right)\mathrm{e}^{1/6\,|-x+t|+1/6\,|x+t|} - \mathrm{e}^{1/6\,|-x+t|}\cos\left(3\,x\right.\right.$$

22.4 non-linear wave PDE (Solitons)

problem number 156

This was first solved analytically by (Krvskal, Zabrsky 1965).

Solve

$$\frac{\partial u}{\partial t} + 6u(x,t)\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

Mathematica ✓

```
ClearAll[u, t, x];
 pde = D[u[x, t], t] + 6*u[x, t]*D[u[x, t], x] + D[u[x, t], {x, 3}] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -\frac{12c_1^3\tanh^2\left(c_2t + c_1x + c_3\right) - 8c_1^3 + c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde :=  diff(u(x,t),t)+6*u(x,t)*diff(u(x,t),x)+diff(u(x,t),x$3)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t)) assuming t>0,x
```

$$u(x,t) = -2\_C2^2(\tanh(\_C2\,x + \_C3\,t + \_C1))^2 + 1/6\,\frac{8\_C2^3 - \_C3}{\_C2}$$

Returning a solution that is not the most general one

22.5 Hyperbolic PDE with non-rational coefficients

problem number 157

From Mathematica DSolve help pages

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} - 2\sin x\frac{\partial^2 u}{\partial x\partial y} - \cos^2 x\frac{\partial^2 u}{\partial y^2} - \cos x\frac{\partial u}{\partial y} = 0$$

Mathematica ✓

```
ClearAll[u, x, y];
 ode = D[u[x, y], {x, 2}] - 2*Sin[x]*D[u[x, y], x, y] - Cos[x]^2*D[u[x, y], {y, 2}] - Cos[x]
 sol = AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to c_1(x - \cos(x) + y) + c_2(-x - \cos(x) + y)\}\}$$

Maple ✗

```
x:='x'; t:='t';c:='c';u:='u';
interface(showassumed=0);
ode := diff(u(x, y), x$2) - 2*sin(x)*diff(u(x, y),x,y)-cos(x)^2*diff(u(x, y), y$2) - cos(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(ode, u(x, y))),output='rea
```

sol=()

22.6 Inhomogeneous hyperbolic PDE with constant coefficients

problem number 158

From Mathematica DSolve help pages

Solve for $u(x,t)$

$$3\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x \partial t} = 1$$

Mathematica ✓

```
ClearAll[u, x, t];
 ode = 3*D[u[x, t], {x, 2}] - D[u[x, t], {t, 2}] + D[u[x, t], x, t] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[ode, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to c_1\left(t - \frac{1}{6}\left(1+\sqrt{13}\right)x\right) + c_2\left(t - \frac{1}{6}\left(1-\sqrt{13}\right)x\right) + \frac{x^2}{6}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t';y:='y';u:='u';
ode := 3*diff(u(x, t), x$2) - diff(u(x, t),t$2)+diff(u(x, t), x,t) =1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(ode, u(x, t))),output='rea
```

$$u(x,t) = \_F2\left(1/6\left(-1+\sqrt{13}\right)x+t\right) + \_F1\left(1/2\left(1/13\sqrt{13}+1\right)x - 3/13\,t\sqrt{13}\right) + 1/13\sqrt{13}\left(1/6\left(-1\right.\right.$$

22.7 system of 2 inhomogeneous linear hyperbolic system with constant coefficients

problem number 159

From Mathematica DSolve help pages

Solve for $u(x,t), v(x,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial x} + 1$$
$$\frac{\partial v}{\partial t} = -\frac{\partial u}{\partial x} - 1$$

With initial conditions

$$u(x,0) = \cos^2 x$$
$$v(x,0) = \sin x$$

Mathematica ✓

```
ClearAll[u, v, x, t];
 eqns = {D[u[x, t], t] == D[v[x, t], x] + 1, D[v[x, t], t] == -D[u[x, t], x] - 1};
 ic = {u[x, 0] == Cos[x]^2, v[x, 0] == Sin[x]};
 sol = AbsoluteTiming[TimeConstrained[FullSimplify[DSolve[{eqns, ic}, {u[x, t], v[x, t]}, {x
```

$$\left\{\left\{u(x,t) \to \sinh(t)\cos(x) + \frac{1}{2}\cosh(2t)\cos(2x) + t + \frac{1}{2}, v(x,t) \to \cosh(t)\sin(x)(2\sinh(t)\cos(x)+1) - \right.\right.$$

Maple ✗

```
x:='x'; t:='t';v:='v';u:='u';
pde1 := diff(u(x, t), t) = diff(v(x, t), x) + 1;
pde2 := diff(v(x, t), t) = -diff(u(x, t), x) - 1;
ic := u(x, 0) = cos(x)^2, v(x, 0) = sin(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2, ic], {u(x, t),
```

$$\text{sol}=()$$

23 Wave PDE in 2D Cartesian coordinates

23.1 In square, given initial position but with zero initial velocity

problem number 160

Taken from Maple PDE help pages. This wave PDE inside square with free to move on left edge and right edge, and top and bottom edges are fixed. It has zero initial velocity, but given a non-zero initial position. Where $0 < x < \pi$ and $0 < y < \pi$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x} u(0, y, t) &= 0\\ \frac{\partial u}{\partial x} u(\pi, y, t) &= 0\\ u(x, 0, t) &= 0\\ u(x, \pi, 0) &= 0\end{aligned}$$

With initial conditions

$$\begin{aligned}\frac{\partial u}{\partial t}(x, y, 0) &= 0\\ u(x, 0) &= xy(\pi - y)\end{aligned}$$

Mathematica ✗

```
ClearAll[u, t, y, x];
 pde = D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4;
 ic = {Derivative[0, 0, 1][u][x, y, 0] == 0, u[x, y, 0] == x*y*(Pi - y)};
 bc = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][Pi, y, t] == 0, u[x, 0,
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t, t) = (1/4)*(diff(u(x, y, t), x, x))+(1/4)*(diff(u(x, y, t), y, y)
bc := (D[1](u))(0, y, t) = 0,
      (D[1](u))(Pi, y, t) = 0,
      u(x, 0, t) = 0,
      u(x, Pi, t) = 0;
ic:= u(x, y, 0) = x*y*(Pi-y),(D[3](u))(x, y, 0) = 0;
sol:=pdsolve([pde,bc,ic],u(x,y,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t))),out
sol:=subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{n=1}^{\infty} -2\,\frac{(-1+(-1)^n)\sin(ny)\cos(1/2\,nt)}{n^3} + \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} -8\,\frac{((-1)^{n+m}-(-1)^n-(-1)^m+1)\cos}{\pi^2 m}\right.$$

23.2 In square with damping. Given zero initial position but with non-zero initial velocity

problem number 161

Taken from Maple PDE help pages. This wave PDE inside square with damping present.

Membrane is free to move on the right edge and also on top edge. But fixed at left edge and bottom edge.

It has zero initial position, but given a non-zero initial velocity. Where $0 < x < 1$ and $0 < y < 1$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{10}\frac{\partial u}{\partial t}$$

With boundary conditions

$$
\begin{aligned}
u(0,y,t) &= 0 \\
\frac{\partial u}{\partial x}u(1,y,t) &= 0 \\
u(x,0,t) &= 0 \\
\frac{\partial u}{\partial y}u(x,1,t) &= 0
\end{aligned}
$$

With initial conditions

$$
\begin{aligned}
u(x,y,0) &= 0 \\
\frac{\partial u}{\partial t}(x,y,0) &= x(1-\frac{1}{2}x)(1-\frac{1}{2}y)y
\end{aligned}
$$

Mathematica ✗

```
ClearAll[u, t, y, x];
 pde = D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4 - (1*D
 ic = {u[x, y, 0] == 0, Derivative[0, 0, 1][u][x, y, 0] == x*(1 - (1/2)*x)*(1 - (1/2)*y)*y};
 bc = {u[0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] == 0, u[x, 0, t] == 0, Derivative[0,
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t$2) = 1/4*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2))-(1/10)*(dif
bc := u(0, y, t) = 0,
      (D[1](u))(1, y, t) = 0,
      u(x, 0, t) = 0,
      (D[2](u))(x, 1, t) = 0;
ic:= u(x, y, 0) = 0, (D[3](u))(x, y, 0) = x*(1-(1/2)*x)*(1-(1/2)*y)*y;
sol:=pdsolve([pde, ic,bc], u(x, y, t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol:=subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty} 5120 \frac{e^{-t/20}\sin\left(1/2\left(1+2m\right)\pi y\right)\sin\left(1/2\left(1+2n\right)\pi x\right)\sin\left(1/20\, t\sqrt{-1+\left(100\, m^2\right.}\right.}{\sqrt{-1+\left(100\, m^2+100\, n^2+100\, m+100\, n+50\right)\pi^2}\pi^6\left(1+\right.}\right.$$

23.3 In rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity

problem number 162

Taken from Mathematica helps pages on DSolve

Solve for $u(x,y,t)$ with $0 < x < 1$ and $0 < y < 2$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

With boundary conditions

$$\begin{aligned} u(x,0,t) &= 0 \\ u(0,y,t) &= 0 \\ u(1,y,t) &= 0 \\ u(x,2,t) &= 0 \end{aligned}$$

With initial conditions

$$u(x,y,0) = \frac{1}{10}(x-x^2)(2y-y^2)$$
$$\frac{\partial u}{\partial t}(x,y,0) = 0$$

Mathematica ✓

```
ClearAll[u, t, y, x, n, m];
 pde = D[u[x, y, t], {t, 2}] == Laplacian[u[x, y, t], {x, y}];
 ic = {u[x, y, 0] == (1/10)*(x - x^2)*(2*y - y^2), Derivative[0, 0, 1][u][x, y, 0] == 0};
 bc = {u[x, 0, t] == 0, u[0, y, t] == 0, u[1, y, t] == 0, u[x, 2, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}], 60*10]];
 sol = sol /. {K[1] -> n, K[2] -> m};
 sol = Assuming[Element[{n, m}, Integers], FullSimplify[sol]];
```

Bad latex generated

Maple ✓

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t$2) = diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2);
ic:=u(x,y,0)=(1/10)*(x-x^2)*(2*y-y^2),(D[3](u))(x,y,0)=0;
bc:=u(x,0,t)=0,u(0,y,t)=0,u(1,y,t)=0,u(x,2,t)=0;
sol:=pdsolve([pde, ic,bc], u(x, y, t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(x, y, t)))
sol:=subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty} -\frac{32\sin(n\pi x)\sin(1/2\,m\pi\,y)\cos\left(1/2\,\pi\sqrt{m^2+4\,n^2}t\right)\left(-(-1)^{m+n}+(-1)^m+(-1)^n\right.}{5\,n^3\pi^6m^3}\right.$$

23.4 In rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity (Haberman 8.5.5 (a))

problem number 163

Added Nov 27, 2018.

This is problem 8.5.5 part(a) from Richard Haberman applied partial differential equations 5th edition.

Solve the initial value problem for membrane with time-dependent forcing and fixed boundaries $u = 0$.

$$\frac{\partial^2 u(x,y,t)}{\partial t^2} = c^2 \nabla(u) + Q(x,y,t)$$

If the memberane is rectangle $(0 < x < L, 0 < y < H)$.

With initial conditions

$$\begin{aligned} u(x,y,0) &= f(x,y) \\ \frac{\partial u}{\partial t}(x,y,0) &= 0 \end{aligned}$$

See my HW9, Math 322, UW Madison.

Mathematica ✗

```
ClearAll[u, t, y, x, n, m, L, H, Q, f];
 pde = D[u[x, y, t], {t, 2}] == c^2*Laplacian[u[x, y, t], {x, y}] + Q[x, y, t];
 ic = {u[x, y, 0] == f[x, y], Derivative[0, 0, 1][u][x, y, 0] == 0};
 bc = {u[0, y, t] == 0, u[L, y, t] == 0, u[x, 0, t] == 0, u[x, H, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}, Assumptio
```

Failed

Maple ✗

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';Q:='Q';
interface(showassumed=0);
pde:=diff(u(x,y,t),t$2)=c^2*(diff(u(x,y,t),x$2)+diff(u(x,y,t),y$2))+Q(x,y,t);
bc:=u(0,y,t)=0,u(L,y,t)=0,u(x,0,t)=0,u(x,H,t)=0;
ic:=u(x,y,0)=f(x,y), eval( diff(u(x,y,t),t),t=0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],u(x,y,t)) assu
```

sol=()

24 Wave PDE in 2D Polar coordinates

24.1 In circular disk. fixed edge of disk, no θ dependency, with initial position and velocity given

problem number 164

Taken from Mathematica helps pages on DSolve

Solve for $u(r,t)$ with $0 < r < 1$ and $t > 0$.

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

With boundary conditions

$$u(1,t) = 0$$

With initial conditions

$$u(r,0) = 1$$
$$\frac{\partial u}{\partial t}(r,0) = \frac{r}{3}$$

Mathematica ✓

```
ClearAll[u, t, r, n];
 pde = D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r);
 ic = {u[r, 0] == 1, Derivative[0, 1][u][r, 0] == r/3};
 bc = u[1, t] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, t], {r, t}], 60*10]];
 sol = sol /. K[1] -> n;
 sol = FullSimplify[sol];
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2\text{BesselJ}(0, r\text{BesselJZero}(0,n))\left(9\sqrt{c^2}\text{BesselJ}(1, \text{BesselJZero}(0,n))\cos(ct\text{BesselJZero}(0\right.}{9\sqrt{c^2}\left(\text{BesselJ}(0, \text{BesselJZero}(0,n))^2 + \right.}\right.\right.$$

Maple ✓

```
x:='x'; t:='t'; y:='y'; u:='u';c:='c';
pde := diff(u(r, t), t$2) = c^2*( diff(u(r,t), r$2)+ (1/r)* diff(u(r,t),r)  ) ;
ic:=u(r,0)=1, eval( diff(u(r,t),t),t=0)=r/3;
bc:=u(1,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, t)) ass
```

$$u(r,t) = -invlaplace\left(\frac{1}{s}\operatorname{BesselI}\left(0,\frac{sr}{c}\right)\left(\operatorname{BesselI}\left(0,\frac{s}{c}\right)\right)^{-1},s,t\right) - 1/3\, invlaplace\left(\frac{1}{s^2}\operatorname{BesselI}\left(0,\frac{sr}{c}\right)\left(\operatorname{Be}\right.\right.$$

Has unresolved Invlaplace calls

24.2 In circular disk. fixed edge of disk, with θ dependency, zero initial velocity

problem number 165

Solve for $u(r,\theta,t)$ with $0 < r < a$ and $t > 0$ and $-\pi < \theta < \pi$

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t) &= 0 \\ |u(0,\theta,t)| &< \infty \\ u(r,-\pi,t) &= u(r,\pi,t) \\ \frac{\partial u}{\partial \theta}(r,-\pi,t) &= \frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$u(r,\theta,0) = f(r,\theta)$$
$$\frac{\partial u}{\partial t}(r,\theta,0) = 0$$

Mathematica ✗

```
ClearAll[u, t, r, n, theta, a, f];
 pde = D[u[r, theta, t], {t, 2}] == c^2*(D[u[r, theta, t], {r, 2}] + (1*D[u[r, theta, t], r]
 ic = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
 bc = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, A
```

Failed

Maple ✗

```
x:='x'; t:='t'; y:='y'; u:='u';theta:='theta';
pde := diff(u(r, theta, t), t$2) = c^2*(diff(u(r, theta, t), r$2)  + 1/r*diff(u(r, theta, t)
ic := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc := u(a, theta, t) = 0,
      u(r, -Pi, t) = u(r, Pi, t),
      (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc], u(r, theta ,
```

sol=()

25 Wave PDE in 3D Spherical coordinates

25.1 No initial and no boundary conditions given

problem number 166

Added Jan 10, 2019.

Solve for $u(r,\theta,\phi,t)$ the wave PDE in 3D

$$u_{tt} = c^2\nabla^2 u$$

Using the Physics convention for Spherical coordinates system.

Mathematica ✗

```
ClearAll[u, t, r, theta, phi];
 lap = Laplacian[u[r, theta, phi, t], {r, theta, phi}, "Spherical"];
 pde = D[u[r, theta, phi, t], {t, 2}] == c^2*lap;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, theta, phi, t], {r, theta, phi, t}, A
```

Failed

Maple ✓

```
u:='u';t:='t'; theta:='theta';phi:='phi';r:='r';
lap:=VectorCalculus:-Laplacian( u(r,theta,phi,t), 'spherical'[r,theta,phi] );
pde:= diff(u(r,theta,phi,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,theta,phi,t),'buil
sol := simplify(sol);
```

$$u(r,\theta,\phi,t) = \frac{\sqrt{2}e^{1/2\,(-\pi-2\,\phi)\sqrt{\_c_3}-\sqrt{\_c_4}t}(\sin(\theta))^{i\sqrt{\_c_3}}\left(e^{2\sqrt{\_c_4}t}\_C7+\_C8\right)\left(e^{2\sqrt{\_c_3}\phi}\_C5+\_C6\right)}{\sqrt{r}}\Bigg(\_C2$$

26 Wave PDE in 3D Cylindrical coordinates

26.1 No initial and no boundary conditions given

problem number 167

Added Jan 10, 2019.

Solve for $u(r,\phi,z,t)$ the wave PDE in 3D

$$u_{tt} = c^2\nabla^2 u$$

Mathematica ✗

```
ClearAll[u, t, r, z, phi];
 lap = Laplacian[u[r, phi, z, t], {r, phi, z}, "Cylindrical"];
 pde = D[u[r, phi, z, t], {t, 2}] == c^2*lap;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[r, phi, z, t], {r, phi, z, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';t:='t'; phi:='phi';r:='r';z:='z';
lap:=VectorCalculus:-Laplacian( u(r,phi,z,t), 'cylindrical'[r,phi,z] );
pde:= diff(u(r,phi,z,t),t$2)= c^2* lap;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(r,phi,z,t),'build'))
sol:=simplify(sol);
```

$$u(r,\phi,z,t) = \mathrm{e}^{-\sqrt{\_c_2}\phi-\sqrt{\_c_3}z-\sqrt{\_c_4}t}\left(\_C1\ \mathrm{BesselJ}\left(\sqrt{-\_c_2}, \frac{\sqrt{\_c_3c^2-\_c_4}r}{c}\right)+\_C2\ \mathrm{BesselY}\left(\sqrt{-\_c_2}\right.\right.$$

27 Schrodinger PDE

27.1 Schrodinger PDE with zero potential (Logan p. 30)

problem number 168

From page 30, David J Logan textbook, applied PDE textbook.

Solve

$$Ih\frac{\partial f}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 f}{\partial x^2}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,0) = 0$$

Mathematica ✓

```
ClearAll[f, t, x, L, m, h];
 pde = I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
 bc = {f[0, t] == 0, f[L, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L >
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t)\to\sum_{n=1}^{\infty}e^{-\frac{ihn^2\pi^2t}{2L^2m}}c_n\sin\left(\frac{n\pi x}{L}\right)\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';f:='f';
interface(showassumed=0);
pde:=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t)) assuming
```

$$f(x,t)=\sum_{n=1}^{\infty}\_C1(n)\sin\left(\frac{n\pi x}{L}\right)\mathrm{e}^{\frac{-i/2h\pi^2n^2t}{mL^2}}$$

27.2 Schrodinger PDE with initial and boundary conditions. Zero potential

problem number 169

Solve for $f(x, y, t)$

$$I\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned} f(0, y, t) &= 0 \\ f(1, y, t) &= 0 \\ f(x, 1, t) &= 0 \\ f(x, 0, t) &= 0 \end{aligned}$$

And initial conditions $f(x, y, 0) = \sqrt{2}(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y))$

Mathematica ✓

```
ClearAll[f, t, x, y];
 pde = I*D[f[x, y, t], {t}] == -((hBar^2*Laplacian[f[x, y, t], {x, y}])/(2*m));
 initSum = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[2*Pi*y]);
 bcs = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bcs, initSum}, f[x, y, t], {x, y, t}], 60
```

$$\left\{\left\{f(x, y, t) \to \sqrt{2}e^{-\frac{5i\pi^2 \text{hBar}^2 t}{2m}}(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y))\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; y:='y'; hbar:='hbar';f:='f';
interface(showassumed=0);
pde:=  I* diff(f(x,y,t),t) = -hBar^2/(2*m) * (diff(f(x,y,t),x$2) +  diff(f(x,y,t),y$2));
ic := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y) + sin(Pi*x)*sin(2*Pi*y));
bc := f(0, y, t) = 0,
      f(1, y, t) = 0,
      f(x, 1, t) = 0,
      f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],f(x,y,t))),out
```

$$f(x,y,t)=\sqrt{2}\sin(\pi x)\,\mathrm{e}^{\frac{-5/2\,ihBar^2t\pi^2}{m}}(2\cos(\pi x)\sin(\pi y)+\sin(2\pi y))$$

27.3 Initial value problem with Dirichlet boundary conditions. Zero potential

problem number 170

Taken from Mathematica DSolve help pages

Solve for $f(x,t)$

$$I\frac{\partial f}{\partial t}=-2\frac{\partial^2 f}{\partial x^2}$$

With boundary conditions

$$f(5,t)=0$$
$$f(10,t)=0$$

And initial conditions $f(x,2)=f(x)$ where $f(x)=-350+155x-22x^2+x^3$

Mathematica ✓

```
ClearAll[g, f, t, x];
 pde = I*D[f[x, t], t] == -2*D[f[x, t], {x, 2}];
 g[x_] := -350 + 155*x - 22*x^2 + x^3;
 ic = f[x, 2] == g[x];
 bc = {f[5, t] == 0, f[10, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc, ic}, f[x, t], {x, t}], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t)\to \sum_{n=1}^{\infty}\frac{100(7+8(-1)^n)\,e^{-\frac{2}{25}in^2\pi^2(t-2)}\sin\left(\frac{1}{5}n\pi(x-5)\right)}{n^3\pi^3}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; y:='y'; f:='f';g:='g';
pde:=I*diff(f(x,t),t)=-2*diff(f(x,t),x$2);
bc:=f(5,t)=0,f(10,t)=0;
g:=x->-350+155*x-22*x^2+x^3;
ic:=f(x,2)=g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc,ic],f(x,t))),outpu
```

$$f(x,t)=\sum_{n=1}^{\infty}\frac{\left(800+700\,(-1)^n\right)\sin\left(1/5\,n\pi\,x\right)\mathrm{e}^{-\frac{2}{25}\,i\pi^2n^2(t-2)}}{n^3\pi^3}$$

27.4 Solve a Schrodinger equation with potential over the whole real line

problem number 171

Taken from Mathematica DSolve help pages

Solve for $f(x,t)$

$$I\frac{\partial f}{\partial t}=-\frac{\partial^2 f}{\partial x^2}+2x^2f(x,t)$$

With boundary conditions

$$f(-\infty, t) = 0$$
$$f(\infty, t) = 0$$

Mathematica ✓

```
ClearAll[f, t, x];
 pde = I*D[f[x, t], t] == -D[f[x, t], {x, 2}] + 2*x^2*f[x, t];
 bc = {f[-Infinity, t] == 0, f[Infinity, t] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, f[x, t], {x, t}], 60*10]];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{f(x,t)\to\sum_{n=0}^{\infty}e^{-\frac{x^2}{\sqrt{2}}-2i\sqrt{2}\left(n+\frac{1}{2}\right)t}c_n\text{HermiteH}\left(n,\sqrt[4]{2}x\right)\right\}\right\}$$

Maple ✗

```
x:='x'; t:='t'; y:='y'; f:='f';g:='g';
pde:=I*diff(f(x,t),t)=-diff(f(x,t),x$2)+2*x^2*f(x,t);
bc:=f(-infinity ,t)=0,f(infinity,t)=0;
try
  cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],f(x,t))),output
catch:
  sol:=();
  cpu_time :=0;
end try;
```

sol=()

Maple does not support ∞ in boundary conditions

27.5 Schrodinger equation, with initial conditions. Zero potential (Griffiths p. 47)

problem number 172

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47.

Solve for $f(x,t)$

$$Ih\frac{\partial f}{\partial t}=-\frac{h^2}{2m}\frac{\partial^2 f}{\partial x^2}$$

With initial conditions $f(x,0)=Ax(a-x)$ for $0\leq x\leq a$ and zero otherwise.

Mathematica ✓

```
ClearAll[x, t, f, a, A, m, h];
 ic = Piecewise[{{A*x*(a - x), 0 <= x <= a}, {0, True}}];
 pde = I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, f[x, 0] == ic}, f[x, t], {x, t}, Assumpti
```

$$\left\{\left\{f(x,t)\to\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}A\sqrt{h}\sqrt{t}\left(-i\sqrt{\pi}mx^2\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{m}(a-x)}{\sqrt{h}\sqrt{t}}\right)+i\sqrt{\pi}amx\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{m}(a-x)}{\sqrt{h}\sqrt{t}}\right)+i\right.}{}\right.\right.$$

Maple ✓

```
x:='x'; t:='t'; f:='f'; a:='a';A:='A';h:='h';m:='m';
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde:=I*h*diff(f(x,t),t) = -h^2/(2*m)*diff(f(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',dsolve([pde,ic],f(x,t)) assuming a
sol:=convert(sol,Int);
```

$$f(x,t)=-2\,A\left(\frac{i/2}{\pi}\int_{-\infty}^{\infty}\frac{1}{s^3}\mathrm{e}^{\frac{-i/2hs^2t}{m}+isx}\,\mathrm{d}s-\frac{i/2}{\pi}\int_{-\infty}^{\infty}\frac{1}{s^3}\mathrm{e}^{\frac{-i/2s(hst+2\,am)}{m}+isx}\,\mathrm{d}s+1/4\,\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{1}{s^2}\mathrm{e}^{\frac{-i/2hs^2t}{m}+isx}\,\mathrm{d}s\right.$$

27.6 Schrodinger equation, with initial conditions. Infinite square well potential (Griffiths p. 47)

problem number 173

Taken from Introduction to Quantum mechanics, second edition, by David Griffiths, page 47. This is the same as the above problem but has an extra $V(x)f(x,t)$ terms where $V(x)$ is the infinite square well potential defined by $V(x)=0$ if $0\leq x\leq a$ and $V(x)=\infty$ otherwise.

Solve for $f(x,t)$

$$I\hbar\frac{\partial f}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2 f}{\partial x^2}+V(x)f(x,t)$$

With initial conditions $f(x,0)=Ax(a-x)$ for $0\leq x\leq a$ and zero otherwise.

Mathematica ✓

```
ClearAll[x, y, t, f, m];
 ic = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[3*Pi*y]);
 bc = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
 pde = I*h*D[f[x, y, t], t] == -((h^2*(D[f[x, y, t], {x, 2}] + D[f[x, y, t], {y, 2}]))/(2*m)
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, f[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{f(x,y,t)\rightarrow\sqrt{2}e^{-\frac{5i\pi^2ht}{m}}\left(\sin(\pi x)\sin(3\pi y)+\sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2ht}{2m}}\right)\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; f:='f'; a:='a';A:='A';h:='h';m:='m';
V:=x->piecewise(0<=x and x<=a,0,infinity);
ic:=f(x,0)=piecewise(0<=x and x<=a,A*x*(a-x),0);
pde:=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2) +V(x)*f(x,t);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],f(x,t)) assuming
```

Bad latex generated

27.7 In 2 space dimensions

problem number 174

Added December 20, 2018.

Example 28, taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $f(x,y,t)$

$$I\hbar \frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

With initial conditions $f(x,y,0) = \sqrt{2}(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(3\pi y))$ and boundary conditions

$$\begin{aligned} f(0,y,t) &= 0 \\ f(1,y,t) &= 0 \\ f(x,1,t) &= 0 \\ f(x,0,t) &= 0 \end{aligned}$$

Mathematica ✗

```
ClearAll[x, t, f, a, A, m, h];
 v[x_] = Piecewise[{{0, 0 <= x <= a}}, Infinity];
 ic = Piecewise[{{A*x*(a - x), 0 <= x <= a}, {0, True}}];
 ode = I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m)) + v[x]*f[x, t];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{ode, f[x, 0] == ic}, f[x, t], {x, t}, Assumpti
```

Failed

Maple ✓

```
x:='x'; t:='t'; f:='f'; y:='y';m:='m';
pde := I*hbar* diff(f(x, y, t), t) = - hbar^2/(2*m)*  (diff(f(x, y, t), x$2)+diff(f(x, y, t)
ic := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y)+sin(Pi*x)*sin(3*Pi*y));
bc:= f(0, y, t) = 0, f(1, y, t) = 0, f(x, 1, t) = 0, f(x, 0, t) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic,bc],f(x,y,t))),ou
```

$$f(x,y,t) = \sqrt{2}\sin(\pi x)\left(2\,\mathrm{e}^{\frac{-5/2\, ihbar\, t\pi^2}{m}}\cos(\pi x)\sin(\pi y) + \sin(3\pi y)\,\mathrm{e}^{\frac{-5\, ihbar\, t\pi^2}{m}}\right)$$

28 Beam PDE

28.1 Beam PDE with zero initial velocity

problem number 175

Added January 20, 2018.

Solve

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= -12t^2 \\ f(1,t) &= 1 - 12t^2 \\ \frac{\partial^2 u}{\partial x^2} u(0,t) &= 0 \\ \frac{\partial^2 u}{\partial x^2} u(1,t) &= 12 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^4 \\ \frac{\partial u}{\partial t} u(x,0) &= 0 \end{aligned}$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t, 2}] + D[u[x, t], {x, 4}] == 0;
 bc = {u[0, t] == -12*t^2, u[1, t] == 1 - 12*t^2, Derivative[2, 0][u][0, t] == 0, Derivative
 ic = {u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic, bc}, u[x, t], x, t], 60*10]];
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Maple ✓

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+diff(u(x,t),x$4)=0;
bc:=u(0,t)=-12*t^2,
    u(1,t)=1-12*t^2,D[1,1](u)(0,t)=0,
    D[1,1](u)(1,t)=12;
ic:=u(x,0)=x^4,D[2](u)(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic,bc],u(x,t),HINT=`+
```

$$u(x,t) = x^4 - 12\,t^2$$

29 Burger's PDE

29.1 viscous fluid flow with no initial conditions

problem number 176

From Mathematica symbolic PDE document.

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + u(x,t)\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

Mathematica ✓

```
ClearAll[u, x, t, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -\frac{2c_1^2\mu\tanh\left(c_2t + c_1x + c_3\right) + c_2}{c_1}\right\}\right\}$$

Maple ✓

```
x:='x'; t:='t'; y:='y'; mu:='mu';
interface(showassumed=0);
pde := diff(u(x, t), t) + u(x, t)*diff(u(x, t), x) = mu* diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde, u(x, t))),output='rea
```

$$u(x,t) = -2\,\mu\,\_C2\,\tanh\left(\_C2\,x + \_C3\,t + \_C1\right) - \frac{\_C3}{\_C2}$$

29.2 viscous fluid flow with initial conditions

problem number 177

From Mathematica symbolic PDE document.

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + u(x,t)\frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

Mathematica ✓

```
ClearAll[u, x, y, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
 ic = u[x, 0] == Piecewise[{{1, x < 0}, {0, x >= 1}}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> mu >
```

$$\left\{\left\{u(x,t) \to \frac{1}{\frac{e^{-\frac{t-2x}{4\mu}}\left(\operatorname{Erf}\left(\frac{x}{2\sqrt{\mu}\sqrt{t}}\right)+1\right)}{\operatorname{Erf}\left(\frac{t-x}{2\sqrt{\mu}\sqrt{t}}\right)+1} + 1}\right\}\right\}$$

Maple ✗

```

x:='x'; y:='y'; mu:='mu';u:='u';
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x))  = mu*(diff(u(x, t), x$2));
ic := u(x, 0) =  piecewise(x>=0,0,x<0,1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

29.3 viscous fluid flow with initial conditions as UnitBox

problem number 178

From Mathematica DSolve help pages.

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+u(x,t)\frac{\partial u}{\partial x}=\mu\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0)=\begin{cases}1 & |x|\leq\frac{1}{2}\\ 0 & \text{otherwise}\end{cases}$$

Mathematica ✓

```
ClearAll[u, x, y, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitBox[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to\frac{e^{\frac{t+1}{4\mu}}\left(\text{Erf}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)-\text{Erf}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)\right)}{e^{\frac{x}{2\mu}}\text{Erf}\left(\frac{1-2x}{4\sqrt{\mu t}}\right)+e^{\frac{t+1}{4\mu}}\text{Erf}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)-e^{\frac{t+1}{4\mu}}\text{Erf}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)+e^{\frac{x+1}{2\mu}}\text{Erf}\left(\frac{2x+1}{4\sqrt{\mu t}}\right)-e^{\frac{x}{2\mu}}-e^{\frac{x+1}{2\mu}}}\right\}\right\}$$

Maple ✗

```
x:='x'; y:='y'; mu:='mu';u:='u';
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic:= u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t)) assuming
```

sol=()

30 Black Scholes PDE

30.1 classic Black Scholes model from finance, European call version

problem number 179

From Mathematica symbolic PDE document.

Solve for $V(S,t)$ where V is the price of the option as a function of stock price S and time t. r is the risk-free interest rate, and σ is the volatility of the stock.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV - rS\frac{\partial V}{\partial S}$$

With boundary condition $V(S,T) = \max\{S-k, 0\}$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation` See the European call version at bottom of the page.

Mathematica ✓

```
ClearAll[u, t, x, sigma, k];
 pde = D[u[x, t], t] == (1*sigma^2*D[u[x, t], {x, 2}])/2;
 ic = u[x, 0] == k*Exp[x - 1]*HeavisideTheta[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> k >
```

$$\left\{\left\{u(x,t) \to \frac{1}{2} k e^{\frac{\sigma^2 t}{2}+x-1}\left(\operatorname{Erf}\left(\frac{\sigma^2 t + x}{\sqrt{2}\sqrt{t}\,|\sigma|}\right)+1\right)\right\}\right\}$$

Maple ✓

```
x:='x';t:='t';u:='u';sigma:='sigma';k:='k';
pde := diff(u(x,t),t) = 1/2*sigma^2*diff(u(x,t),x$2);
ic := u(x, 0) = k*exp(x - 1)*Heaviside(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],u(x,t)) assuming
```

$$u(x,t) = -k e^{-1}\left(i\,invfourier\left(\frac{e^{-1/2\,s^2\sigma^2 t}}{s+i}, s, x\right) - invfourier\left(e^{-1/2\,s^2\sigma^2 t} fourier(e^x, x, s), s, x\right)\right)$$

30.2 Boundary value problem for the Black Scholes equation

problem number 180

From Mathematica DSolve help pages.

Solve for $V(t,s)$

$$\frac{\partial v}{\partial t}+\frac{1}{2}\sigma^2 s^2\frac{\partial^2 v}{\partial s^2}+(r-q)s\frac{\partial v}{\partial s}-rv(t,s)=0$$

With boundary condition $v(T,s)=\psi(s)$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation`

Mathematica ✓

```
ClearAll[t, s, v, sigma, psi];
 pde = D[v[t, s], t] + (1*sigma^2*s^2*D[v[t, s], {s, 2}])/2 + (r - q)*s*D[v[t, s], s] - r*v[
 bc = v[T, s] == psi[s];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, v[t, s], {t, s}], 60*10]];
```

$$\left\{\left\{v(t,s)\to\frac{e^{r(t-T)}\int_{-\infty}^{\infty}\psi\left(e^{K[1]}\right)\exp\left(-\frac{\left(-K[1]+(T-t)\left(-q+r-\frac{\sigma^2}{2}\right)+\log(s)\right)^2}{2\sigma^2(T-t)}\right)dK[1]}{\sqrt{2\pi}\sqrt{\sigma^2(T-t)}}\right\}\right\}$$

Maple ✓

```
t:='t'; s:='s'; sigma:='sigma';v:='v';psi:='psi';
interface(showassumed=0);
pde:=diff(v(t, s), t) +s^2*(diff(v(t, s), s, s))/(2*sigma^2)+(r-q)*s*(diff(v(t, s), s))-r*v(
ic:=v(T, s) = psi(s);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,ic],v(t,s))),output='
```

$$v(t,s)=\psi(s)+\sum_{n=1}^{\infty}\frac{(t-T)^n\left(U\mapsto rU^{(n)}\right)(\psi(s))}{n!}$$

31 Korteweg-deVries PDE

31.1 Korteweg-deVries (waves on shallow water surfaces) with no initial conditions

problem number 181

From Mathematica symbolic PDE document.

Solve for $u(x,t)$

$$\frac{\partial^3 u}{\partial x^3} + \frac{\partial u}{\partial t} - 6u(x,t)\frac{\partial u}{\partial x} = 0$$

Reference `https://en.wikipedia.org/wiki/Korteweg%E2%80%93de_Vries_equation`

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], {x, 3}] + D[u[x, t], {t}] - 6*u[x, t]*D[u[x, t], {x}] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{12c_1^3\tanh^2\left(c_2t + c_1x + c_3\right) - 8c_1^3 + c_2}{6c_1}\right\}\right\}$$

Maple ✓

```
x:='x'; y:='y';u:='u';
pde:= diff(u(x,t),x$3)+ diff(u(x,t),t)-6*u(x,t)* diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 2\_C2^2(\tanh\left(\_C2\,x + \_C3\,t + \_C1\right))^2 - 1/6\,\frac{8\_C2^3 - \_C3}{\_C2}$$

32 Tricomi PDE

32.1 Boundary value problem for the Tricomi equation

problem number 182

From Mathematica DSolve helps pages.

Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -y(y - x^2)\}\}$$

Maple ✓

```
x:='x'; y:='y';u:='u';
pde:= diff(u(x,y),x$2)+ y*diff(u(x,y),y$2)=0;
bc:=u(x,0)=0,(D[2](u))(x,0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde,bc],u(x,y))),output='
```

$$u(x,y) = y(x^2 - y)$$

33 Cauchy Riemann PDE's

33.1 Cauchy Riemann PDE with Prescribe the values of u and v on the x axis

problem number 183

From Mathematica DSolve helps pages.

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

With boundary conditions

$$u(x,0) = x^3$$
$$v(x,0) = 0$$

Mathematica ✓

```
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x];
 bc = {u[x, 0] == x^3, v[x, 0] == 0};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2, bc}, {u[x, y], v[x, y]}, {x, y}],
```

$$\{\{u(x,y) \to x^3 - 3xy^2, v(x,y) \to 3x^2y - y^3\}\}$$

Maple ✓

```
x:='x'; y:='y';u:='u';
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y);
bc:=u(x,0)=x^3,v(x,0)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2,bc],[u(x,y),v(x
```

$$\{u(x,y) = x^3 - 3\,y^2x, v(x,y) = -3\,yx^2 + y^3\}$$

33.2 Cauchy Riemann PDE With extra term on right side

problem number 184

Solve for $u(x,y), v(x,y$

$$
\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x} + y
\end{aligned}
$$

Mathematica ✗

```
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x] + y;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde1, pde2}, {u[x, y], v[x, y]}, {x, y}], 60*1
```

Failed

Maple ✓

```
x:='x'; y:='y';u:='u';
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y)+y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde1,pde2],[u(x,y),v(x,y)
```

$$\{u(x,y) = -i\_F1(y-ix) + i\_F2(y+ix) + yx + \_C1, v(x,y) = \_F1(y-ix) + \_F2(y+ix) + 1/2\,x^2$$

34 Hamilton-Jacobi PDE

34.1 Hamilton-Jacobi type PDE

problem number 185

Taken from Maple pdsolve help pages, which is taken from Landau, L.D. and Lifshitz, E.M. Translated by Sykes, J.B. and Bell, J.S. Mechanics. Oxford: Pergamon Press, 1969

Solve for $S(t,\xi,\eta,\phi)$

$$-\frac{\partial}{\partial t}S(t,\xi,\eta,\phi) = 1/2\,\frac{\left(\frac{\partial}{\partial \xi}S(t,\xi,\eta,\phi)\right)^2(\xi^2-1)}{\sigma^2 m\,(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial \eta}S(t,\xi,\eta,\phi)\right)^2(-\eta^2+1)}{\sigma^2 m\,(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial \phi}S(t,\xi,\eta\right.}{\sigma^2 m\,(\xi^2-1)\,(}$$

Mathematica ✗

```
ClearAll[t, \[Zeta], \[Eta], \[Phi], a, b, s];
 pde = -D[s[t, \[Zeta], \[Eta], \[Phi]], t] == ((\[Zeta]^2 - 1)*D[s[t, \[Zeta], \[Eta], \[Ph
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, s[t, \[Zeta], \[Eta], \[Phi]], {t, \[Zeta]
```

Failed

Maple ✓

```
S:='S';t:='t'; xi:='xi';eta:='eta';phi:='phi';
pde := -diff(S(t,xi,eta,phi),t) =1/2*diff(S(t,xi,eta,phi),xi)^2*(xi^2-1)/sigma^2/m/(xi^2-eta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,'build')),output='real
```

$$S(t,\xi,\eta,\phi) = \_c_4\phi + \_c_1 t + \_C1 + \_C2 + \_C3 + \_C4 - \int \frac{\sqrt{-2\,\eta^4 m\sigma^2\_c_1 + 2\,b(\eta)\,\eta^2 m\sigma^2 + 2\,\eta^2\_c_1\sigma^2 m}}{\eta^2}$$

35 miscellaneous PDE's

35.1 A second order PDE

problem number 186

Taken from Maple pdsolve help pages, problem 4.

Solve for $S(x,y)$

$$S(x,y)\left(\frac{\partial^2 S}{\partial x\partial y}\right)+\frac{\partial S}{\partial x}\frac{\partial S}{\partial y}=1$$

Mathematica ✗

```
ClearAll[s, x, y];
 pde = s[x, y]*D[s[x, y], x, y] + D[s[x, y], x]*D[s[x, y], y] == 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, s[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
S:='S';x:='x'; y:='y';
pde := S(x,y)*diff(S(x,y),y,x) + diff(S(x,y),x)*diff(S(x,y),y) = 1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,S(x,y),'build')),outpu
```

$$S(x,y)=\frac{\sqrt{2\_c_1x+\_C1}\sqrt{\_C2\,\_c_1{}^2+\_c_1y}}{\_c_1}$$

35.2 second order PDE in Polar coordinates

problem number 187

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(r,\theta)$

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

With boundary conditions

$$u(2,\theta) = 3\sin(2\theta) + 1$$

Mathematica ✗

```
ClearAll[u, r, theta];
 pde = D[u[r, theta], {r, 2}] + D[u[r, theta], {theta, 2}] == 0;
 bc = u[2, theta] == 3*Sin[2*theta] + 1;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[r, theta], {r, theta}], 60*10]];
```

Failed

Maple ✓

```
r:='r'; theta:='theta'; t:='t';
pde := diff(u(r, theta), r$2)+diff(u(r, theta), theta$2) = 0;
bc := u(2, theta) = 3*sin(2*theta)+1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(r,theta),meth
```

$$u(r,\theta) = -3/2\, i e^{-2\,r+4+2\,i\theta} + 3/2\, i e^{2\,r-4-2\,i\theta} + 1$$

35.3 Laplace like PDE with polynomial solution

problem number 188

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica ✓

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], {x, y}], 60*10]];
```

$$\{\{u(x,y) \to -y(y-x^2)\}\}$$

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde := diff(u(x, y), x$2)+y*(diff(u(x, y), y$2)) = 0;
bc:=u(x,0)=0, eval(diff(u(x,y),y),y=0)=x^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc], u(x, y))),outpu
```

$$u(x,y) = y(x^2 - y)$$

35.4 Third oder PDE

problem number 189

Added December 20, 2018.

Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $u(x,y)$

$$\frac{\partial u}{\partial t} = -\frac{\partial^3 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = f(x)$$

Mathematica ✗

```
ClearAll[u, x, t, f];
 pde = D[u[x, t], t] == -D[u[x, t], {x, 3}];
 ic = u[x, 0] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x, t), t)=- diff(u(x, t), x$3);
ic:=u(x,0)=f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],u(x,t))),output=
```

$$u(x,t) = 1/4\,\frac{1}{\pi^2}\int_{-\infty}^{\infty} 4/3\,\frac{\pi\, f(-\zeta)}{\sqrt[3]{-t}}\sqrt{-\frac{x+\zeta}{\sqrt[3]{-t}}}\,\mathrm{BesselK}\left(1/3, -2/9\,\frac{\sqrt{3}(x+\zeta)}{\sqrt[3]{-t}}\sqrt{-\frac{x+\zeta}{\sqrt[3]{-t}}}\right)d\zeta$$

35.5 PDE solved by Laplace transform

problem number 190

Added December 20, 2018.

Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $u(x,y)$

$$\frac{\partial u^2}{\partial xy} = \sin(x)\sin(y)$$

With boundary conditions

$$u(x,0) = 1 + \cos(x)$$
$$\frac{\partial u}{\partial y}(0,y) = -2\sin y$$

Mathematica ✗

```
ClearAll[u, x, y];
 pde = D[u[x, y], y, x] == Sin[x]*Sin[y];
 bc = {u[x, 0] == 1 + Cos[x], Derivative[0, 1][u][0, y] == -2*Sin[y]};
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, bc}, u[x, y], x, y], 60*10]];
```

Failed

Maple ✓

```
x:='x'; y:='y'; u:='u';
pde := diff(u(x, y), y,x)=sin(x)*sin(y);
bc:=u(x,0)=1+cos(x),eval( diff(u(x,y),y),x=0)=-2*sin(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, bc],u(x,y))),output=
```

$$u(x,y) = 1/2\,\cos(x-y) + 1/2\,\cos(x+y) + \cos(y)$$

35.6 Linear PDE, initial conditions at $t = 1$

problem number 191

Added December 20, 2018.

Example 25, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w}{\partial t} = \frac{\partial w^2}{\partial x_1^2} + \frac{\partial w^2}{\partial x_2^2} + \frac{\partial w^2}{\partial x_3^2}$$

With initial condition $w(x_1, x_2, x_3, 1) = e^a x_1^2 + x_2 x_3$

Mathematica ✗

```
ClearAll[w, x1, x2, x3, t, a];
 pde = D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], {x1, 2}] + D[w[x1, x2, x3, t], {x2, 2}]
 ic = w[x1, x2, x3, 1] == Exp[a]*x1^2 + x2*x3;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

Failed

Maple ✓

```
w:='w';x1:='x1';x2:='x2';x3:='x3';t:='t';a:='a';
pde := diff(w(x1, x2, x3, t), t) = diff(w(x1, x2, x3, t), x1$2)+diff(w(x1, x2, x3, t), x2$2)
ic:= w(x1, x2, x3, 1) = exp(a)*x1^2+x2*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = \left(x1^2 + 2t - 2\right) e^a + x2\, x3$$

35.7 Linear PDE, initial conditions at $t = t_0$

problem number 192

Added December 20, 2018.

Example 26, Taken from https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w}{\partial t} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} + \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition $w(x_1, x_2, x_3, t_0) = e^{x_1} + x_2 - 3x_3$

Mathematica ✗

```
ClearAll[w, x1, x2, x3, t, t0];
 pde = D[w[x1, x2, x3, t], t] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1, x3] +
 ic = w[x1, x2, x3, t0] == Exp[x1] + x2 - 3*x3;
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

Failed

Maple ✓

```
w:='w';x1:='x1';x2:='x2';x3:='x3';t:='t';t0:='t0';
pde := diff(w(x1, x2, x3, t), t)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff(w
ic:= w(x1, x2, x3, t0) = exp(x1)+x2-3*x3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = e^{x1} + x2 - 3\,x3$$

35.8 second order in time, Linear PDE, initial conditions at $t = t_0$

problem number 193

Added December 20, 2018.

Example 27, Taken from `https://www.mapleprimes.com/posts/209970-Exact-Solutions-For-PDE-And-Boundary--Initial-Conditions-2018`

Solve for $w(x_1, x_2, x_3, t)$

$$\frac{\partial w^2}{\partial t^2} = \frac{\partial w^2}{\partial x_1 x_2} + \frac{\partial w^2}{\partial x_1 x_3} + \frac{\partial w^2}{\partial x_3^2} - \frac{\partial w^2}{\partial x_2 x_3}$$

With initial condition

$$w(x_1, x_2, x_3, t_0) = x_1^3 x_2^2 + x_3$$
$$\frac{\partial w}{\partial t}(x_1, x_2, x_3, t_0) = -x_2 x_3 + x_1$$

Mathematica ✗

```
ClearAll[w, x1, x2, x3, t, t0];
 pde = D[w[x1, x2, x3, t], {t, 2}] == D[w[x1, x2, x3, t], x1, x2] + D[w[x1, x2, x3, t], x1,
 ic = {w[x1, x2, x3, t0] == x1^3*x2^2 + x3, Derivative[0, 0, 0, 1][w][x1, x2, x3, t0] == -(x
 sol = AbsoluteTiming[TimeConstrained[DSolve[{pde, ic}, w[x1, x2, x3, t], {x1, x2, x3, t}],
```

Failed

Maple ✓

```
w:='w';x1:='x1';x2:='x2';x3:='x3';t:='t';t0:='t0';
pde := diff(w(x1, x2, x3, t), t$2)= diff(w(x1,x2,x3,t),x1,x2)+diff(w(x1,x2,x3,t),x1,x3)+diff
ic:= w(x1, x2, x3, t0) = x1^3*x2^2+x3, eval( diff( w(x1,x2,x3,t),t),t=t0)=-x2*x3+x1;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve([pde, ic],w(x1,x2,x3,t))),
```

$$w(x1, x2, x3, t) = 1/2\, t0^4 x1 + 1/6\,(-12\, x1\, t - 1)\, t0^3 + 1/6\,(18\, x1\, t^2 + 18\, x1^2 x2 + 3\, t)\, t0^2 + 1/6\,(-36\, t x1^2 x$$

35.9 Einstein-Weiner PDE

problem number 194

Added January 2, 2018.

Solve for $u(x,t)$ with $x > 0, t > 0$

$$u_t = -\beta u_x + D u_{xx}$$

Assuming $\beta > 0, D > 0$

Mathematica ✗

```
ClearAll[u, x, t, beta, d];
 pde = D[u[x, t], t] == beta*D[u[x, t], x] + d*D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}, Assumptions -> {beta > 0,
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';beta:='beta';d:='d';
pde:=diff(u(x,t),t)=-beta*diff(u(x,t),x)+d*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build') assumi
```

$$u(x,t)=\frac{\_C3\,\_C1}{\mathrm{e}^{-c_1 t}}\sqrt{\mathrm{e}^{\frac{x\beta}{d}}}\,\mathrm{e}^{1/2\,\frac{x\sqrt{\beta^2-4\,d\,\_c_1}}{d}}+\frac{\_C3\,\_C2}{\mathrm{e}^{-c_1 t}}\sqrt{\mathrm{e}^{\frac{x\beta}{d}}}\,\mathrm{e}^{-1/2\,\frac{x\sqrt{\beta^2-4\,d\,\_c_1}}{d}}$$

36 Nonlinear PDE's

36.1 Bateman-Burgers equation

problem number 195

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t + uu_x = \nu u_{xx}$$

Mathematica ✓

```
ClearAll[u, x, t, v];
 pde = D[u[x, t], t] + u[x, t]*D[u[x, t], x] == v*D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{2c_1^2 v\tanh\left(c_2 t + c_1 x + c_3\right) + c_2}{c_1}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+u(x,t)*diff(u(x,t),x)=v*diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = -2\,v\_C2\,\tanh\left(\_C2\,x + \_C3\,t + \_C1\right) - \frac{\_C3}{\_C2}$$

36.2 Benjamin Bona Mahony

problem number 196

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t + u_x + uu + x - u_{xxt} = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] + D[u[x, t], x] + u[x, t]*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{12c_2c_1^2\tanh^2\left(c_2t+c_1x+c_3\right)-8c_2c_1^2-c_1-c_2}{c_1}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+diff(u(x,t),x)+u(x,t)*diff(u(x,t),x)-diff(u(x,t),x,x,t)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 12\_C2\,\_C3\left(\tanh\left(\_C2\,x+\_C3\,t+\_C1\right)\right)^2-\frac{8\_C2^2\_C3+\_C2+\_C3}{\_C2}$$

36.3 Benjamin Ono

problem number 197

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_t + Hu_{xx} + uu_x = 0$$

Important note. H above is meant to be Hilbert transform. https://en.wikipedia.org/wiki/Benjamin%E2%80%93Ono_equation However, here in the code below it is taken as just a scalar.

Mathematica ✓

```
ClearAll[u, x, t, h];
 pde = D[u[x, t], t] + h*D[u[x, t], {x, 2}] + u[x, t]*D[u[x, t], x] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{2c_1^2 h \tanh\left(c_2 t + c_1 x + c_3\right) - c_2}{c_1}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+H*diff(u(x,t),x$2)+u(x,t)*diff(u(x,t),x)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 2\,H\,\_C2\,\tanh\left(\_C2\,x + \_C3\,t + \_C1\right) - \frac{\_C3}{\_C2}$$

36.4 Born Infeld

problem number 198

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$(1-u_t^2)u_{xx}+2u_xu_tu_{xt}-(1+u_x^2)u_{tt}=0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = (1 - D[u[x, t], t]^2)*D[u[x, t], {x, 2}] + 2*D[u[x, t], x]*D[u[x, t], t]*D[D[u[x, t],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\{\{u(x,t)\to c_1(t+x)+c_2(t-x)\}\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=(1-diff(u(x,t),t)^2)*diff(u(x,t),x$2)+2*diff(u(x,t),x)*diff(u(x,t),t)*diff(u(x,t),x,t)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=\_C7\,(\tanh(-\_C2\,t+\_C2\,x+\_C1))^3+\_C5\,\tanh(-\_C2\,t+\_C2\,x+\_C1)+\_C4$$

36.5 Boussinesq

problem number 199

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_{tt} - u_{xx} - u_{xxxx} - 3(u^2)_{xx} = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to -\frac{12c_1^4 \tanh^2\left(c_2 t + c_1 x + c_3\right) - 8c_1^4 + c_1^2 - c_2^2}{6c_1^2}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t$2)-diff(u(x,t),x$2)-diff(u(x,t),x$4)- 3 * diff( u(x,t)^2, x$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = -2\_C2^2(\tanh\left(\_C2\,x + \_C3\,t + \_C1\right))^2 + 1/6\,\frac{8\_C2^4 - \_C2^2 + \_C3^2}{\_C2^2}$$

36.6 Boussinesq type PDE

problem number 200

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_{tt} - u_{xx} - 2\alpha(uu_x)_x - \beta u_{xxtt} = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] - D[u[x, t], {x, 4}] - 3*D[u[x, t]^2, {x, 2}]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{12c_1^4\tanh^2\left(c_2t+c_1x+c_3\right)-8c_1^4+c_1^2-c_2^2}{6c_1^2}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';alpha:='alpha';beta:='beta';
pde:=diff(u(x,t),t$2)-diff(u(x,t),x$2)-2*alpha*diff( (u(x,t)*diff(u(x,t),x)) ,x) - beta*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t)=-6\,\frac{\_C3^2\beta\left(\tanh\left(\_C2\,x+\_C3\,t+\_C1\right)\right)^2}{\alpha}+1/2\,\frac{8\,\_C2^2\,\_C3^2\beta-\_C2^2+\_C3^2}{\alpha\,\_C2^2}$$

36.7 Buckmaster

problem number 201

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t=(u^4)_{xx}+(u^3)_x$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t]^4, {x, 2}] + D[u[x, t]^3, x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)= diff(u(x,t)^4,x$2)+diff(u(x,t)^3,x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \mathrm{RootOf}\left(\_C1\,x + \_C2\,t + \_C3 + \int^{\_Z} 4\,\frac{\_C1^2\,\_f^3}{\_C1\,\_f^3 + 4\,\_C3\,\_C1^2 - \_C2\,\_f} d\_f + \_C4\right)$$

Answer in terms of RootOf.

36.8 Camassa Holm

problem number 202

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_diff erential_equations`

Solve for $u(x,t)$

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_{xxx}$$

Mathematica ✗

```
ClearAll[u, x, t, k];
 pde = D[u[x, t], t] + 2*k*D[u[x, t], x] - D[D[u[x, t], {x, 2}], t] + 3*u[x, t]*D[u[x, t], x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+2*k*diff(u(x,t),x)- diff(u(x,t),x,x,t)+3*u(x,t)*diff(u(x,t),x)=2*diff(u(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = \frac{1}{\_C1}\left(\left(\mathrm{RootOf}\left(-i\_C1\,x - i\_C2\,t - i\_C3 + \int^{-\frac{-\_Z^2+\_C2}{\_C1}} \frac{}{\sqrt{\_C1^3\_C3\_f + \_C1^2\_C2\_}}\right.\right.\right.$$

Answer in terms of RootOf.

36.9 Chaffee Infante equation

problem number 203

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t = u_{xx} + \lambda(u^3 - u) = 0$$

Mathematica ✗

```
ClearAll[u, x, t, lambda];
 pde = D[u[x, t], t] - D[u[x, t], {x, 2}] + lambda*(u[x, t]^3 - u[x, t]) == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)-diff(u(x,t),x$2)+lambda*(u(x,t)^3-u(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = 1/2\tanh\left(-3/4\,\lambda\,t + 1/4\,\sqrt{2}\sqrt{\lambda}x + \_C1\right) - 1/2$$

36.10 Clarke's equation

problem number 204

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $\theta(x,t)$

$$\left(\theta_t - \gamma e^{\theta}\right)_{tt} = \left(\theta_t - e^{\theta}\right)_{xx}$$

Mathematica ✗

```
ClearAll[theta, x, t, gamma];
 pde = D[D[theta[x, t], t] - gamma*Exp[theta[x, t]], {t, 2}] == D[D[theta[x, t], t] - Exp[th
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, theta[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
theta:='theta';x:='x';t:='t';g:='g';
pde := diff(diff(theta(x,t),t)-g*exp(theta(x,t)),t$2) = diff( diff(theta(x,t),t)-exp(theta(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,theta(x,t))),output='r
```

sol=()

36.11 Degasperis Procesi

problem number 205

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t - u_{xxt} + 4uu_x = 3u_xu_xx + uu_{xxx}$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] - D[D[u[x, t], {x, 2}], t] + 4*u[x, t]*D[u[x, t], x] == 3*D[u[x, t], x]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:= diff(u(x,t),t)-diff(u(x,t),x,x,t)+4*u(x,t)*diff(u(x,t),x)=3*diff(u(x,t),x)*diff(u(x,t)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$PDESolStruc\left(u(x,t) = \frac{\_F1(x)}{-\_c_2t + \_C2}, [\left\{\left\{\_F1(x) = ODESolStruc\left(\_a, [\left\{\left(\frac{d^2}{d\_a^2}\_b(\_a)\right)(\_b(\_a\right.\right.\right.\right.\right.$$

But still has unresolved ODE's in solution

36.12 Dym equation

problem number 206

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_t = u^3 u_{xxx}$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == u[x, t]^3*D[u[x, t], {x, 3}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:= diff(u(x,t),t)=u(x,t)^3 * diff(u(x,t),x$3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = \frac{1}{\sqrt[3]{-3\_c_1 t + \_C4}} \operatorname{RootOf}\left(-\int^{\_Z}\left(\operatorname{RootOf}\left(-\ln\left(\_f\right) + 2\int^{\_Z} \frac{}{2\sqrt[3]{2}\sqrt[3]{-\_c_1{}^2}\operatorname{RootOf}\left(\operatorname{Airy}\right.}\right.\right.\right.$$

has RootOf

36.13 Estevez Mansfield Clarkson equation

problem number 207

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x, y, t)$

$$u_{tyyy} + \beta u_y u_{yt} + \beta u_{yy} u_t + u_{tt} = 0$$

Mathematica ✓

```
ClearAll[u, x, t, y, beta];
 pde = D[D[u[x, y, t], t], {y, 3}] + beta*D[u[x, y, t], y]*D[D[u[x, y, t], y], t] + beta*D[u
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t) \to \frac{\beta c_4(x) + 6c_1(x)\tanh\left(-4tc_1(x)^3 + yc_1(x) + c_3(x)\right)}{\beta}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';y:='y';beta='beta';
pde:= diff(u(x,y,t),t,y,y,y)+ beta*diff(u(x,y,t),y)*diff(u(x,y,t),y,t) +  beta*diff(u(x,y,t)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = 6\,\frac{\_C3\ \tanh\left(-4\,\_C3^3 t + \_C2\,x + \_C3\,y + \_C1\right)}{\beta} + \_C5$$

36.14 Fisher's equation

problem number 208

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_t = u(1-u) + u_{xx}$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == u[x, t]*(1 - u[x, t]) + D[u[x, t], {x, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to \frac{1}{4}\left(\tanh\left(-c_3+\frac{5t}{12}-\frac{x}{2\sqrt{6}}\right)+1\right)^2\right\},\left\{u(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(-c_3+\frac{5t}{12}-\frac{ix}{2\sqrt{6}}\right)\right)\right.\right.$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:= diff(u(x,t),t)= u(x,t)*(1-u(x,t))+ diff(u(x,t),x$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',{PDEtools:-TWSolutions(pde,u(x,t))
```

$$\left\{\{u(x,t)=1\},\left\{u(x,t)=1/4\left(\tanh\left(-\frac{5\,t}{12}-1/12\,\sqrt{6}x+\_C1\right)\right)^2-1/2\tanh\left(-\frac{5\,t}{12}-1/12\,\sqrt{6}x+\_\right.\right.\right.$$

36.15 Hunter Saxton

problem number 209

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$(u_t + uu_x))_x = \frac{1}{2}(u_x)^2$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[D[u[x, t], t] + u[x, t]*D[u[x, t], x], x] == (1*D[u[x, t], x]^2)/2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';
pde:= diff( (diff(u(x,t),t)+ u(x,t)* diff(u(x,t),x)) , x) = 1/2* (diff(u(x,t),x))^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = 2\,\frac{\mathrm{RootOf}\left(-\_C2\,\_c_1{}^3 - x\_c_1{}^3 - 2\,\_C1\,\sqrt{\_Z}\,\_c_1 + 2\,\_C1^2\ln\left(\sqrt{\_Z}\,\_c_1 + \_C1\right) + \_Z\,\_c_1{}^2}{\_c_1 t + 2\,\_C3}$$

with RootOf

36.16 Kadomtsev Petviashvili

problem number 210

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x, y, t)$

$$(u_t + uu_x + \epsilon^2 u_{xxx})_x + \lambda u_{yy} = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[D[u[x, y, t], t] + u[x, y, t]*D[u[x, y, t], x] + eps^2*D[u[x, y, t], {x, 3}], t] +
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

$$\left\{\left\{u(x,y,t)\to -\frac{12c_3c_1^3\text{eps}^2\tanh^2\left(c_3t+c_1x+c_2y+c_4\right)-8c_3c_1^3\text{eps}^2+c_2^2\lambda+c_3^2}{c_1c_3}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';y:='y';lambda:='lambda';epsilon:='epsilon';
pde:= diff( diff(u(x,y,t),t)+u(x,y,t)*diff(u(x,y,t),x)+epsilon^2* diff(u(x,y,t),x$3),x)+ lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = -12\,\epsilon^2\_C2^2(\tanh\left(\_C2\,x+\_C3\,y+\_C4\,t+\_C1\right))^2+\frac{8\_C2^4\epsilon^2-\_C3^2\lambda-\_C4\_C2}{\_C2^2}$$

36.17 Klein Gordon (nonlinear)

problem number 211

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x, y)$

$$u_{xx} + u_{yy} + \lambda u^p = 0$$

Mathematica ✗

```
ClearAll[u, x, y, lambda];
 pde = Laplacian[u[x, y], {x, y}] + lambda*u[x, y]^p == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
u:='u';x:='x';y:='y';lambda:='lambda';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+lambda*u(x,y)^p=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

sol=()

36.18 special case Klein Gordon (nonlinear)

problem number 212

Added December 27, 2018.

Solve for $u(x, y)$

$$u_{xx} + u_{yy} + u^2 = 0$$

Mathematica ✗

```
ClearAll[u, x, y, lambda];
 pde = Laplacian[u[x, y], {x, y}] + u[x, y]^2 == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';y:='y';lambda:='lambda';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+u(x,y)^2=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = -6\, \mathit{WeierstrassP}(\_C1\, x + \_C2\, y + 2\, \_C3, 0, \_C4)\, (\_C1^2 + \_C2^2)$$

36.19 Khokhlov Zabolotskaya

problem number 213

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,y,t)$

$$u_{xt} - (uu_x)_x = u_{yy}$$

Mathematica ✗

```
ClearAll[u, x, y, t];
 pde = D[D[u[x, y, t], x], t] - D[u[x, y, t]*D[u[x, y, t], x], x] == D[u[x, y, t], {y, 2}];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';y:='y';t:='t';
pde:=diff(u(x,y,t),x,t)- diff( (u(x,y,t)* diff(u(x,y,t),x)) ,x ) = diff(u(x,y,t),y$2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = \frac{\_C3\,\_C1 - \_C2^2 + \sqrt{2\,(\_C1\,x + \_C2\,y + \_C3\,t + \_C4)\,\_C1^2\,\_C4 + \_C1^2\,\_C3^2 - 2\,\_}}{\_C1^2}$$

36.20 Korteweg de Vries (KdV)

problem number 214

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t + (u_x)^3 + 6uu_x = 0$$

Mathematica ✓

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] + D[u[x, t], x]^3 + 6*u[x, t]*D[u[x, t], x] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t) \to \frac{-18c_1tx - 18c_2t - 9c_1^2x^2 - 18c_1c_2x - 9c_2^2 - c_1 - 9t^2}{6c_1^2}\right\}\right\}$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+ diff( u(x,t),x )^3 + 6 * u(x,t)* diff(u(x,t),x) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

$$u(x,t) = -3/2\_C1^2 + 3\_C1\left(\_c_2t + x\right) - 3/2\left(\_c_2t + x\right)^2 - 1/6\_c_2$$

36.21 Lin Tsien equation

problem number 215

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,y,t)$

$$2u_{tx} + u_x u_{xx} - u_{yy} = 0$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = 2*D[u[x, y, t], t, x] + D[u[x, y, t], x]*D[u[x, y, t], {x, 2}] - D[u[x, y, t], {y, 2}
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y, t], {x, y, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';y:='y';
pde:=2*diff(u(x,y,t),t,x)+ diff(u(x,y,t),x)* diff(u(x,y,t),x$2) - diff(u(x,y,t),y$2) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y,t))),output='rea
```

$$u(x,y,t) = \_C4 + \_C5\left(\_C1\,x + \_C2\,y + \_C3\,t + \_C4\right)$$

36.22 Liouville equation

problem number 216

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,y)$

$$u_{xx} + u_{yy} + e^{\lambda u} = 0$$

Mathematica ✗

```
ClearAll[u, x, lam, y];
 pde = Laplacian[u[x, y], {x, y}] + Exp[lam*u[x, y]] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
u:='u';x:='x';y:='y';lambda:='lambda';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)+exp(lambda*u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

36.23 Plateau

problem number 217

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,y)$

$$(1+u_y^2)u_{xx} - 2u_x u_y y_{xy} + (1+u_x^2)u_{yy} = 0$$

Mathematica ✗

```
ClearAll[u, x, y];
 pde = (1 + D[u[x, y], y]^2)*D[u[x, y], {x, 2}] - 2*D[u[x, y], x]*D[u[x, y], y]*D[u[x, y], x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';y:='y';
pde:=(1+diff(u(x,y),y)^2)* diff(u(x,y),x$2)-2* diff(u(x,y),x)*
       diff(u(x,y),y)*diff(u(x,y),x,y)+
       (1+diff(u(x,y),x)^2)*diff(u(x,y),y$2)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

$$u(x,y) = \_C7\,(\tanh(\_C2\,x - i\_C2\,y + \_C1))^3 + \_C5\,\tanh(\_C2\,x - i\_C2\,y + \_C1) + \_C4$$

36.24 Rayleigh

problem number 218

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_{tt} - u_{xx} = \epsilon(u_t - u_t^3)$$

Mathematica ✗

```
ClearAll[u, x, t, epsilon];
 pde = D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] == epsilon*(D[u[x, t], t] - D[u[x, t], t]^3);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';t:='t';epsilon:='epsilon';
pde:=diff(u(x,t),t$2)-diff(u(x,t),x$2)=epsilon*(diff(u(x,t),t)-diff(u(x,t),t)^3);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t),'build')),outpu
```

$$u(x,t) = 1/2\_c_1x^2 + \_C1\,x + \_C2 + \int \mathrm{RootOf}\left(t + \int^{\_Z} \left(\_f^3\epsilon - \_f\,\epsilon - \_c_1\right)^{-1} d\_f + \_C3\right) \mathrm{d}t + \_C4$$

Has RootOf

36.25 Sawada Kotera

problem number 219

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_t + 45u^2u_x + 15u_xu_{xx} + 15uu_{xxx} + u_{xxxxx} = 0$$

Mathematica ✓

```
ClearAll[phi, x, t];
 pde = D[u[x, t], t] + 45*u[x, t]^2*D[u[x, t], x] + 15*D[u[x, t], x]*D[u[x, t], {x, 2}] + 15
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{u(x,t)\to -\frac{4}{3}c_1^2\left(3\tanh^2\left(-16c_1^5t+c_1x+c_3\right)-2\right)\right\},\left\{u(x,t)\to \frac{-30c_1^{5/2}\tanh^2\left(c_2t+c_1x+c_3\right)+20c}{15\sqrt{c_1}}\right.\right.$$

Maple ✓

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+45* u(x,t)^2* diff(u(x,t),x)+ 15* diff(u(x,t),x)*
      diff(u(x,t),x$2)+15*u(x,t)*diff(u(x,t),x$3)+diff(u(x,t),x$5);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',{PDEtools:-TWSolutions(pde,u(x,t))
```

$$\left\{\{u(x,t)=\_C4\},\left\{u(x,t)=-4\_C2^2\left(\tanh\left(-16\_C2^5t+\_C2\,x+\_C1\right)\right)^2+8/3\_C2^2\right\},\left\{u(x,t)=\right.\right.$$

36.26 Sine Gordon

problem number 220

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$\phi_{tt}-\phi_{xx}+\sin\phi=0$$

Mathematica ✗

```
ClearAll[phi, x, t];
 pde = D[phi[x, t], {t, 2}] - D[phi[x, t], {x, 2}] + Sin[phi[x, t]] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```

phi:='phi';x:='x';t:='t';
pde:=diff(phi(x,t),t$2)-diff(phi(x,t),x$2)+sin(phi(x,t))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,phi(x,t))),output='rea
```

sol=()

36.27 Sinh Gordon

problem number 221

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_{xt} = \sinh u$$

Mathematica ✗

```
ClearAll[u, x, t];
 pde = D[u[x, t], x, t] == Sinh[u[x, t]];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),x,t)=sinh(u(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,t))),output='realt
```

sol=()

36.28 Sinh Poisson

problem number 222

Added December 27, 2018.

Taken from `https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations`

Solve for $u(x,t)$

$$u_{xx} + u_{yy} + \sinh u = 0$$

Mathematica ✗

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] + Sinh[u[x, y]] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
u:='u';x:='x';y:='y';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+ sinh(u(x,y))=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y))),output='realt
```

sol=()

36.29 Thomas equation

problem number 223

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$u_{xy} + \alpha u_x + \beta u_y + \nu u_x u_y = 0$$

Mathematica ✗

```
ClearAll[u, x, y, alpha, beta, nu];
 pde = D[u[x, y], x, y] + alpha*D[u[x, y], x] + beta*D[u[x, y], y] + nu*D[u[x, y], x]*D[u[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, u[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
u:='u';x:='x';y:='y';beta:='beta';alpha:='alpha';nu:='nu';
pde:=diff(u(x,y),x,y)+alpha*diff(u(x,y),x)+beta*diff(u(x,y),y)
      +nu* diff(u(x,y),x)*diff(u(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,u(x,y),'build')),outpu
```

$$u(x,y) = -1/2\,\frac{\sqrt{\alpha^2 - 2\,\alpha\,\beta + \beta^2 - 4\_c_1\nu}x}{\nu} + 1/2\,\frac{\sqrt{\alpha^2 - 2\,\alpha\,\beta + \beta^2 - 4\_c_1\nu}y}{\nu} - 1/2\,\frac{\sqrt{\alpha^2 + 2\,\alpha\,\beta + \beta^2 -}}{\nu}$$

36.30 phi equation

problem number 224

Added December 27, 2018.

Taken from https://en.wikipedia.org/wiki/List_of_nonlinear_partial_differential_equations

Solve for $u(x,t)$

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0$$

Mathematica ✓

```
ClearAll[u, x, y, alpha, beta, nu];
 pde = D[phi[x, t], t, t] - D[phi[x, t], x, x] - phi[x, t] + phi[x, t]^3 == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, phi[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{\phi(x,t) \to -\tanh\left(c_2t - \frac{\sqrt{2c_2^2+1}x}{\sqrt{2}} + c_3\right)\right\}, \left\{\phi(x,t) \to \tanh\left(c_2t - \frac{\sqrt{2c_2^2+1}x}{\sqrt{2}} + c_3\right)\right\}, \left\{\phi(x,t)\right.\right.$$

Maple ✓

```
phi:='phi';x:='x';t:='t';
pde:=diff(phi(x,t),t$2)-diff(phi(x,t),x$2) - phi(x,t) + phi(x,t)^3=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',{PDEtools:-TWSolutions(pde,phi(x,t
```

$$\left\{\{\phi(x,t) = -1\}, \{\phi(x,t) = 1\}, \left\{\phi(x,t) = -\tanh\left(-1/2\sqrt{4\_C2^2 - 2}t + \_C2\,x + \_C1\right)\right\}, \left\{\phi(x,t) =\right.\right.$$

37 HFOPDE, chapter 1

37.1 problem number 1

problem number 225

Added January 2, 2019.

Problem 1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x = f(x, y)$$

Mathematica ✓

```
ClearAll[w, x, y, f];
 pde = D[w[x, y], x] == f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x f(K[1],y)\,dK[1]+c_1(y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';
pde:=diff(w(x,y),x)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int f(x,y)\,\mathrm{d}x+\_F1(y)$$

37.2 problem number 2

problem number 226

Added January 2, 2019.

Problem 1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = f(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, f];
 pde = D[w[x, y], y] == f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^y f(x,K[1])\,dK[1]+c_1(x)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';
pde:=diff(w(x,y),y)=f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\int f(x,y)\,\mathrm{d}y+\_F1(x)$$

37.3 problem number 3

problem number 227

Added January 2, 2019.

Problem 1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x = wf(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, f];
 pde = D[w[x, y], x] == w[x, y]*f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y)e^{\int_1^x f(K[1],y)\,dK[1]}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';
pde:=diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(y)\,\mathrm{e}^{\int f(x,y)\,\mathrm{d}x}$$

37.4 problem number 4

problem number 228

Added January 2, 2019.

Problem 1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = wf(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, f];
 pde = D[w[x, y], y] == w[x, y]*f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(x)e^{\int_1^y f(x,K[1])\,dK[1]}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';
pde:=diff(w(x,y),x)=w(x,y)*f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(y)\,\mathrm{e}^{\int f(x,y)\,\mathrm{d}x}$$

37.5 problem number 5

problem number 229

Added January 2, 2019.

Problem 1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x = wf(x,y) + g(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, f, g];
 pde = D[w[x, y], x] == w[x, y]*f[x, y] + g[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y)e^{\int_1^x f(K[1],y)\,dK[1]}+e^{\int_1^x f(K[1],y)\,dK[1]}\int_1^x g(K[2],y)e^{-\int_1^{K[2]} f(K[1],y)\,dK[1]}\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';g:='g';
pde:=diff(w(x,y),x)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int g(x,y)\,\mathrm{e}^{-\int f(x,y)\,\mathrm{d}x}\,\mathrm{d}x+\_F1(y)\right)\mathrm{e}^{\int f(x,y)\,\mathrm{d}x}$$

37.6 problem number 6

problem number 230

Added January 2, 2019.

Problem 1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_y = wf(x,y)+g(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, f, g];
 pde = D[w[x, y], y] == w[x, y]*f[x, y] + g[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(x)e^{\int_1^y f(x,K[1])\,dK[1]}+e^{\int_1^y f(x,K[1])\,dK[1]}\int_1^y g(x,K[2])e^{-\int_1^{K[2]} f(x,K[1])\,dK[1]}\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';f:='f';g:='g';
pde:=diff(w(x,y),y)=w(x,y)*f(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int g(x,y)\,e^{-\int f(x,y)\,dy}\,dy+\_F1(x)\right)e^{\int f(x,y)\,dy}$$

38 HFOPDE, chapter 2.2.1

38.1 problem number 1

problem number 231

Added January 2, 2019.

Problem 2.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde:=a*diff(w(x,y),y)+b*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(x)$$

Solution missing a, b compared to book, but technically still correct.
Hand solution

$$aw_x + bw_y = 0$$

The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dw}{0}$$

The first pair of equations results in $bdx = ady$ or $bx = ay + C_1$. Hence

$$C_1 = bx - ay$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$ where F is arbitrary function, therefore the solution is

$$w(x, y) = F(bx - ay)$$

38.2 problem number 2

problem number 232

Added January 2, 2019.

Problem 2.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + (bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = a*D[w[x, y], x] + (b*x + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2ay - bx^2 - 2cx}{2a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=a*diff(w(x,y),x)+(b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{-bx^2+2\,ya-2\,cx}{a}\right)$$

Hand solution

Solve $aw_x + (bx+c)\,w_y = 0$. The Lagrange-charpit equations are

$$\frac{dx}{a} = \frac{dy}{(bx+c)} = \frac{dw}{0}$$

The first pair of equations gives $\frac{(bx+c)}{a}dx = dy$. Integrating results in

$$\frac{1}{a}\left(\frac{bx^2}{2}+cx\right) = y + C_1$$

$$C_1 = \frac{1}{a}\left(\frac{bx^2}{2}+cx\right) - y$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Where F is arbitray function. Therefore

$$w(x,y) = F\left(\frac{bx^2}{2a}+\frac{c}{a}x-y\right)$$

38.3 problem number 3

problem number 233

Added January 2, 2019.

Problem 2.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax+by+c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = D[w[x, y], x] + (a*x + b*y + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-bx}(abx+a+b^2y+bc)}{b^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=diff(w(x,y),x)+(a*x+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{(axb+yb^2+bc+a)\,\mathrm{e}^{-bx}}{b^2}\right)$$

Hand solution

Solve

$$w_x+(ax+by+c)\,w_y=0 \tag{1}$$

The Lagrange-charpit equations are

$$dx=\frac{dy}{(ax+by+c)}=\frac{dw}{0}$$

The first pair of equations gives

$$\frac{dy}{dx}=ax+by+c$$
$$\frac{dy}{dx}-by=ax+c$$

This is linear. Integrating factor is $I = e^{-bx}$. Hence the above becomes

$$
\begin{aligned}
\frac{d}{dx}\left(ye^{-bx}\right) &= (ax+c)\,e^{-bx} \\
ye^{-bx} &= a\int xe^{-bx} + c\int e^{-bx} + C_1 \\
ye^{-bx} &= a\left(-\frac{(1+bx)\,e^{-bx}}{b^2}\right) - c\frac{e^{-bx}}{b} + C_1 \\
y &= -a\frac{(1+bx)}{b^2} - \frac{c}{b} - C_1 e^{bx} \\
C_1 &= -\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}
\end{aligned}
$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$
\begin{aligned}
w(x,y) &= F\left(-\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}\right) \\
&= F\left(\left(y + \frac{a}{b^2}(1+bx) + \frac{c}{b}\right) e^{-bx}\right)
\end{aligned}
$$

38.4 problem number 4

problem number 234

Added January 2, 2019.

Problem 2.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$
axw_x + byw_y = 0
$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$
\left\{\left\{w(x,y) \to c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}
$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)$$

Hand solution

Solve

$$axw_x + byw_y = 0 \tag{1}$$

The Lagrange-charpit equations are

$$\frac{dx}{ax} = \frac{dy}{by} = \frac{dw}{0}$$

The first pair of equations gives

$$\begin{aligned}
\frac{b}{a}\frac{dx}{x} &= \frac{dy}{y} \\
\frac{b}{a}\ln x &= \ln y + C_1 \\
x^{\frac{b}{a}} &= C_1 y \\
C_1 &= \frac{x^{\frac{b}{a}}}{y}
\end{aligned}$$

Since $dw = 0$ then $w = C_2$. But $C_2 = F(C_1)$. Therefore

$$w(x,y) = F\left(\frac{x^{\frac{b}{a}}}{y}\right)$$

38.5 problem number 5

problem number 235

Added January 2, 2019.

Problem 2.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde:=a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y^2a-bx^2}{a}\right)$$

38.6 problem number 6

problem number 236

Added January 2, 2019.

Problem 2.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + (y+a)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = y*D[w[x, y], x] + (y + a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\left(-\log\left(a\left(\frac{ye^{-\frac{y}{a}-1}}{a}+e^{-\frac{y}{a}-1}\right)\right)\right)-a-x\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde:=y*diff(w(x,y),x)+(y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1(-a\ln(y+a)+y-x)$$

38.7 problem number 7

problem number 237

Added January 2, 2019.

Problem 2.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay+bx+c)w_x-(by+kx+s)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c, s];
 pde = (a*y + b*x + c)*D[w[x, y], x] - (b*y + k*x + s)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2+2bxy+2cy+kx^2+2sx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';s:='s';
pde:=(a*y+b*x+c)*diff(w(x,y),x)-(b*y+k*x+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{}{\sqrt{a^3k^2y^2 - 2\,a^2b^2ky^2 + 2\,a^2bk^2xy + a^2k^3x^2 + ab^4y^2 - 4\,ab^3kxy - 2\,ab^2k^2x^2 + 2\,b^5xy + b^4}}\right.$$

But Mathematica solution is simpler, both verified correct

38.8 problem number 8

problem number 238

Added January 2, 2019.

Problem 2.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x + b_1y + c_1)w_x + (a_2x + b_2y + c_2)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a1, a2, b1, b2, c1, c2];
 pde = (a1*x + b1*y + c1)*D[w[x, y], x] + (a1*x + b2*y + c2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a1:='a1';b1:='b1';c1:='c1';a2:='a2';b2:='b2';c2:='c2';
pde:=(a1*x+b1*y+c1)*diff(w(x,y),x)+(a2*x+b2*y+c2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{\sqrt{-a1^2+2\,a1\,b2-4\,a2\,b1-b2^2}}\left(\sqrt{-a1^2+2\,a1\,b2-4\,a2\,b1-b2^2}\ln\left(y a1^3 b2\right.\right.\right.$$

39 HFOPDE, chapter 2.2.2

39.1 problem number 1

problem number 239

Added January 2, 2019.

Problem 2.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^2 + bx + c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = D[w[x, y], x] + (a*x^2 + b*x + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{1}{6}(-2ax^3 - 3bx^2 - 6cx + 6y)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=diff(w(x,y),x)+(a*x^2+b*x+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/3\,ax^3 - 1/2\,bx^2 - cx + y\right)$$

39.2 problem number 2

problem number 240

Added January 2, 2019.

Problem 2.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay^2 + by + c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = D[w[x, y], x] + (a*y^2 + b*y + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sqrt{4ac-b^2}\tan^{-1}\left(\frac{2ay\sqrt{4ac-b^2}+b\sqrt{4ac-b^2}}{4ac-b^2}\right)-4acx+b^2x}{4ac-b^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=diff(w(x,y),x)+(a*y^2+b*y+c)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{4\,ca-b^2}}\left(-x\sqrt{4\,ca-b^2}+2\,\arctan\left(\frac{2\,ya+b}{\sqrt{4\,ca-b^2}}\right)\right)\right)$$

39.3 problem number 3

problem number 241

Added January 2, 2019.

Problem 2.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^2 + cx)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = D[w[x, y], x] + (a*y + b*x^2 + c*x)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-ax}(a^3y + a^2bx^2 + a^2cx + 2abx + ac + 2b)}{a^3}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde:=diff(w(x,y),x)+(a*y+b*x^2+c*x)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(bx^2a^2 + ya^3 + a^2cx + 2\,axb + ca + 2\,b)\,\mathrm{e}^{-ax}}{a^3}\right)$$

39.4 problem number 4

problem number 242

Added January 2, 2019.

Problem 2.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (axy + bx^2 + cx + ky + s)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c, k, s];
 pde = D[w[x, y], x] + (a*x*y + b*x^2 + c*x + k*y + s)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{ax^2}{2}-kx}\left(2a^{3/2}bx+2a^{3/2}c+2a^{5/2}y-\sqrt{2\pi}a^2se^{\frac{(ax+k)^2}{2a}}\operatorname{Erf}\left(\frac{k}{\sqrt{2}\sqrt{a}}+\frac{\sqrt{a}x}{\sqrt{2}}\right)-\sqrt{2\pi}bk^2e^{\frac{(ax\cdots}{2\cdots}}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';k:='k';s:='s';
pde:=diff(w(x,y),x)+(a*x*y+b*x^2+c*x+k*y+s)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-1/2\,\frac{e^{-x(ax+2k)}}{a^{5/2}}\left(\sqrt{2}e^{1/2\,\frac{2a^2x^2+4akx+k^2}{a}}\operatorname{erf}\left(1/2\,\frac{\sqrt{2}(ax+k)}{\sqrt{a}}\right)a^2s\sqrt{\pi}-\sqrt{2}e^{1/2\,\frac{2a^2x^2+4akx+\cdots}{a}}\right.\right.$$

39.5 problem number 5

problem number 243

Added January 2, 2019.

Problem 2.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (y^2 - a^2x^2 + 3a)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (y^2 - a^2*x^2 + 3*a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-y\text{ParabolicCylinderD}\left(-2,i\sqrt{2}\sqrt{a}x\right)+ax\text{ParabolicCylinderD}\left(-2,i\sqrt{2}\sqrt{a}x\right)+i\sqrt{2}\sqrt{}}{y\text{ParabolicCylinderD}\left(1,\sqrt{2}\sqrt{a}x\right)+ax\text{ParabolicCylinderD}\left(1,\sqrt{2}\sqrt{a}x\right)-\sqrt{2}\sqrt{a}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde:=diff(w(x,y),x)+(y^2-a^2*x^2+3*a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{-ax^2+yx+1}{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-a}\,x\right)(-a)^{3/2}x^2+\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-a}\,x\right)\sqrt{-a}\,yx-\mathrm{e}^{ax^2}ax+\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-a}\,x\right)\sqrt{-a}}\right.$$

39.6 problem number 6

problem number 244

Added January 2, 2019.

Problem 2.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 - a^2x^2 + a)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (y^2 - a^2*x^2 + a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{-\sqrt{\pi}y\text{Erfi}\left(\sqrt{a}x\right) + \sqrt{\pi}ax\text{Erfi}\left(\sqrt{a}x\right) - 2\sqrt{a}e^{ax^2}}{2\sqrt{a}(ax-y)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde:=diff(w(x,y),x)+(y^2-a^2*x^2+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-ax+y)\sqrt{\pi}}{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-a}x\right)ax - \sqrt{\pi}\operatorname{erf}\left(\sqrt{-a}x\right)y - 2\sqrt{-a}e^{ax^2}}\right)$$

39.7 problem number 7

problem number 245

Added January 2, 2019.

Problem 2.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + axy + a)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (y^2 + a*x*y + a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{2\pi}\sqrt{a}xy\mathrm{Erfi}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)+\sqrt{2\pi}\sqrt{a}\mathrm{Erfi}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)-2ye^{\frac{ax^2}{2}}}{2\sqrt{2}\sqrt{a}(xy+1)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde:=diff(w(x,y),x)+(y^2+a*x*y+a)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{yx+1}\left(-\operatorname{erf}\left(1/2\sqrt{-2\,a}x\right)yax+e^{1/2\,ax^2}\sqrt{-2\,\frac{a}{\pi}}y-a\operatorname{erf}\left(1/2\sqrt{-2\,a}x\right)\right)\frac{1}{\sqrt{-2\,\frac{a}{\pi}}}\right)$$

39.8 problem number 8

problem number 246

Added January 2, 2019.

Problem 2.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + axy - abx - b^2)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = D[w[x, y], x] + (y^2 + a*x*y - a*b*x - b^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{2b^2}{a}}\left(2\sqrt{a}e^{\frac{2b^2}{a}+\frac{ax^2}{2}+2bx}+\sqrt{2\pi}y\text{Erfi}\left(\frac{\sqrt{2}b}{\sqrt{a}}+\frac{\sqrt{a}x}{\sqrt{2}}\right)-\sqrt{2\pi}b\text{Erfi}\left(\frac{\sqrt{2}b}{\sqrt{a}}+\frac{\sqrt{a}x}{\sqrt{2}}\right)\right)}{2\sqrt{a}(b-y)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde:=diff(w(x,y),x)+(y^2+a*x*y-a*b*x-b^2)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{\sqrt{2}}{\sqrt{-a}\,(b-y)}\left(e^{1/2\,\frac{(ax+2\,b)^2}{a}}\sqrt{2}\sqrt{-a}+\sqrt{\pi}\,\text{erf}\left(1/2\,\frac{(ax+2\,b)\sqrt{2}}{\sqrt{-a}}\right)b-\sqrt{\pi}\,\text{erf}\left(1/2\right.\right.\right.$$

39.9 problem number 9

problem number 247

Added January 2, 2019.

Problem 2.2.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + k(ax + by + c)^2 w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c, k];
 pde = D[w[x, y], x] + k*(a*x + a*y + c)^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-2ia\sqrt{k}x}\left(ia\sqrt{k}x+ia\sqrt{k}y+ic\sqrt{k}+1\right)}{2a\sqrt{k}\left(a\sqrt{k}x+a\sqrt{k}y+c\sqrt{k}+i\right)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';k:='k';
pde:=diff(w(x,y),x)+k*(a*x+a*y+c)^2*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xa\sqrt{k}-\arctan\left(ya\sqrt{k}+xa\sqrt{k}+\sqrt{k}c\right)}{a\sqrt{k}}\right)$$

39.10 problem number 10

problem number 248

Added January 2, 2019.

Problem 2.2.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ay^2 + cx^2 + y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, c];
 pde = x*D[w[x, y], x] + (a*y^2 + c*x^2 + y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}y}{\sqrt{c}x}\right)}{\sqrt{a}}-cx}{c}\right)\right\}\right\}$$

Maple ✓

```

w:='w';x:='x';y:='y';A:='A'; C:='C';
pde:=x*diff(w(x,y),x)+(a*y^2+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{ca}}\left(-x\sqrt{ca}+\arctan\left(\frac{ya}{x\sqrt{ca}}\right)\right)\right)$$

39.11 problem number 11

problem number 249

Added January 2, 2019.

Problem 2.2.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ay^2 + bxy + cx^2 + y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, c, b];
 pde = x*D[w[x, y], x] + (a*y^2 + b*x*y + c*x^2 + y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sqrt{4ac-b^2}\tan^{-1}\left(\frac{bx\sqrt{4ac-b^2}+2ay\sqrt{4ac-b^2}}{4acx-b^2x}\right)-4acx+b^2x}{4ac-b^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';A:='A'; C:='C';b:='b';
pde:=x*diff(w(x,y),x)+(a*y^2+b*x*y+c*x^2+y)*diff(w(x,y),y)=0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{4\,ca-b^2}}\left(-x\sqrt{4\,ca-b^2}+2\arctan\left(\frac{2\,ya+bx}{x\sqrt{4\,ca-b^2}}\right)\right)\right)$$

39.12 problem number 12

problem number 250

Added January 2, 2019.

Problem 2.2.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+c)w_x + \left(\alpha(ay+bx)^2 + \beta(ay+bx) - bx + \gamma\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, c, b, alpha, gamma, beta];
 pde = (a*x + c)*D[w[x, y], x] + (alpha*(a*y + b*x)^2 + beta*(a*y + b*x) - b*x + gamma)*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(2\tan^{-1}\left(\frac{2a^3\alpha^2y\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}+2a^2\alpha^2bx\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}+a^2\alpha\beta\sqrt{\frac{4a\alpha\gamma-a\beta^2+4\alpha bc}{a^3\alpha^2}}\right.}{4a\alpha\gamma-a\beta^2+4\alpha bc}\right.\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';A:='A'; C:='C';b:='b';alpha:='alpha';g:='g';beta:='beta';
pde := (a*x + c)*diff(w(x,y),x)+(alpha*(a*y+b*x)^2+beta*(a*y+b*x)-b*x+g)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{a^3\left(4\,ga\alpha-a\beta^2+4\,\alpha\,bc\right)}}\left(-2\,\arctan\left(\frac{a^2(2\,ya\alpha+2\,\alpha\,bx+\beta)}{\sqrt{4\,a^4\alpha\,g-a^4\beta^2+4\,a^3\alpha\,bc}}\right)a^2+\ln\left(ax+c\right)\right.\right.$$

39.13 problem number 13

problem number 251

Added January 2, 2019.

Problem 2.2.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x + by^2w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{by-ax}{axy}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{-ax+by}{axy}\right)$$

39.14 problem number 14

problem number 252

Added January 2, 2019.

Problem 2.2.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2+b)w_x - \left(y^2 - 2xy + (1-a)x^2 - b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = (a*x^2 + b)*D[w[x, y], x] - (y^2 - 2*x*y + (1 - a)*x^2 - b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right) - x\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right) - \sqrt{a}\sqrt{b}}{\sqrt{a}\sqrt{b}x - \sqrt{a}\sqrt{b}y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde := (a*x^2+b)*diff(w(x,y),x)-(y^2-2*x*y+(1-a)*x^2-b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{ab}\,(x-y)}\left(\arctan\left(\frac{ax}{\sqrt{ab}}\right)y - \arctan\left(\frac{ax}{\sqrt{ab}}\right)x - \sqrt{ab}\right)\right)$$

39.15 problem number 15

problem number 253

Added January 2, 2019.

Problem 2.2.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2 + b_1x + x_1)w_x + (a_2y^2 + b_2y + c_2)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a1, b1, c1, a2, b2, c2];
 pde = (a1*x^2 + b1*x + c1)*D[w[x, y], x] + (a2*y^2 + b2*y + c2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\left(\frac{4\text{a2c2}\sqrt{4\text{a1c1}-\text{b1}^2}\tan^{-1}\left(\frac{2\text{a1}x+\text{b1}}{\sqrt{4\text{a1c1}-\text{b1}^2}}\right)}{\text{b1}^2-4\text{a1c1}}-\frac{\text{b2}^2\sqrt{4\text{a1c1}-\text{b1}^2}\tan^{-1}\left(\frac{2\text{a1}x+\text{b1}}{\sqrt{4\text{a1c1}-\text{b1}^2}}\right)}{\text{b1}^2-4\text{a1c1}}+\sqrt{4\text{a2c2}-\text{b2}^2}\tan\right.}{4\text{a2c2}-\text{b2}^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a1:='a1';b1:='b1';c1:='c1';a2:='a2';b2:='b2';c2:='c2';
pde := (a1*x^2+b1*x+c1)*diff(w(x,y),x)+ (a2*y^2+b2*y+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-2\frac{1}{\sqrt{4\,c1\,a1-b1^2}\sqrt{4\,c2\,a2-b2^2}}\left(\sqrt{4\,c2\,a2-b2^2}\arctan\left(\frac{2\,a1\,x+b1}{\sqrt{4\,c1\,a1-b1^2}}\right)-\text{arc}\right.\right.$$

39.16 problem number 16

problem number 254

Added January 2, 2019.

Problem 2.2.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)(x-b)w_x - \left(y^2 + k(y+x-a)(y+x-b)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = (x - a)*(x - b)*D[w[x, y], x] - (y^2 + k*(y + x - a)*(y + x - b))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{1}{2}\left(2\tan^{-1}\left(\frac{-\frac{2k^2x\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}{(a-b)^2} - \frac{2kx\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}{(a-b)^2} - \frac{2k^2y\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}{(a-b)^2} - \frac{4ky\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}}{(a-b)^2} - \frac{2y\sqrt{-}}{(a-}}{k^2}\right.\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde := (x-a)*(x-b)*diff(w(x,y),x)- (y^2+k*(y+x-a)*(y+x-b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(b-x)^k(-kb+kx+ky+y)(a-x)^{-k}}{-ak+kx+ky+y}\right)$$

39.17 problem number 17

problem number 255

Added January 2, 2019.

Problem 2.2.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1y^2+b_1y+c_1)w_x+(a_2x^2+b_2x+c_2)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, a1, b1, c1, a2, b2, c2];
 pde = (a1*y^2 + b1*y + c1)*D[w[x, y], x] + (a2*x^2 + b2*x + c2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{6}\left(2\text{a1}y^3-2\text{a2}x^3+3\text{b1}y^2-3\text{b2}x^2+6\text{c1}y-6\text{c2}x\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a1:='a1';b1:='b1';c1:='c1';a2:='a2';b2:='b2';c2:='c2';
pde := (a1*y^2+b1*y+c1)*diff(w(x,y),x)+ (a2*x^2+b2*x+c2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(1/3\,a1\,y^3-1/3\,a2\,x^3+1/2\,b1\,y^2-1/2\,x^2b2+c1\,y-c2\,x\right)$$

39.18 problem number 18

problem number 256

Added January 2, 2019.

Problem 2.2.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y(ax+b)w_x + (ay^2 - cx)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = y*(a*x + b)*D[w[x, y], x] + (a*y^2 - c*x)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a^2y^2-2acx-bc}{a^2(ax+b)^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  y*(a*x+b)*diff(w(x,y),x)+ (a*y^2-c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{a^2y^2-2\,acx-bc}{a^2\,(a^2x^2+2\,axb+b^2)}\right)$$

39.19 problem number 19

problem number 257

Added January 2, 2019.

Problem 2.2.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx)w_x - (cx^2+by)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = (a*y^2 + b*x)*D[w[x, y], x] - (x*x^2 + b*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{12}\left(4ay^3+12bxy+3x^4\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  (a*y^2+b*x)*diff(w(x,y),x)- (x*x^2+b*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/3\,ay^3 - 1/4\,x^4 - bxy\right)$$

39.20 problem number 20

problem number 258

Added January 2, 2019.

Problem 2.2.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2+bx^2)w_x + 2bxw_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b];
 pde = (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y^2a + bx^2 + 2\,ya + 2\,a)\,\mathrm{e}^{-y}}{b}\right)$$

39.21 problem number 21

problem number 259

Added January 2, 2019.

Problem 2.2.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2 + bx^2)w_x + 2bxyw_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = (a*y^2 + b*x^2)*D[w[x, y], x] + 2*b*x*y*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\log\left(\frac{bx^2 - ay^2}{y}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  (a*y^2+b*x^2)*diff(w(x,y),x)+ 2*b*x*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{y^2a - bx^2}\right)$$

39.22 problem number 22

problem number 260

Added January 2, 2019.

Problem 2.2.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2 + x^2)w_x + (bx^2 + c - 2xy)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = (a*y^2 + x^2)*D[w[x, y], x] + (b*x^2 + c - 2*x*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{1}{3}(ay^3 - bx^3 - 3cx + 3x^2y)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  (a*y^2+x^2)*diff(w(x,y),x)+(b*x^2+c-2*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/3\,ay^3 + 1/3\,bx^3 - yx^2 + cx\right)$$

39.23 problem number 23

problem number 261

Added January 2, 2019.

Problem 2.2.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + Bx^2 - a^2B)w_x + (Cy^2 + 2Bxy)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, A, B, C0];
 pde = (A*y^2 + B*x^2 - a^2*B)*D[w[x, y], x] + (C0*y^2 + 2*B*x*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a^2(-B) - Ay^2 + Bx^2 + \mathrm{C0}xy}{y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';A:='A';B:='B';C:='C';
pde :=  (A*y^2+B*x^2-a^2*B)*diff(w(x,y),x)+(C*y^2+2*B*x*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{Ay^2 + a^2B - Bx^2 - Cyx}{y}\right)$$

39.24 problem number 24

problem number 262

Added January 2, 2019.

Problem 2.2.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^2 + bx^2 + cy)w_x + 2bxw_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b, c];
 pde = (a*y^2 + b*x^2 + c*y)*D[w[x, y], x] + 2*b*x*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  (a*y^2+b*x^2+c*y)*diff(w(x,y),x)+2*b*x*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(y^2a + bx^2 + 2\,ya + cy + 2\,a + c)\,\mathrm{e}^{-y}}{b}\right)$$

39.25 problem number 25

problem number 263

Added January 2, 2019.

Problem 2.2.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Axy + Bx^2 + kx)w_x + (Dy^2 + Exy + Fx^2 + ky)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, A, B, D0, E0, F, k];
 pde = (A*x*y + B*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F*x^2 + k*y)*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';A:='A';B:='B';D0:='D0';E0:='E0';k:='k';F0:='F0';
pde :=  (A*x*y+B*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

39.26 problem number 26

problem number 264

Added January 2, 2019.

Problem 2.2.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Axy + Aky + Bx^2 + Bkx)w_x + (Cy^2 + Dxy + k(D - B)y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, A, B, D0, E0, C0, k];
 pde = (A*x*y + A*k*y + B*x^2 + B*k*x)*D[w[x, y], x] + (C0*y^2 + D0*x*y + k*(D0 - B)*y)*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';A:='A';B:='B';D0:='D0';E0:='E0';k:='k';C0:='C0';
pde :=  (A*x*y+A*k*y+B*x^2+B*k*x)*diff(w(x,y),x)+(C0*y^2+D0*x*y+k*(D0-B)*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

39.27 problem number 27

problem number 265

Added January 2, 2019.

Problem 2.2.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + Bxy + Cx^2 + kx)w_x + (Dy^2 + Exy + Fx^2 + ky)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, A, B, D0, E0, C0, k, F0];
 pde = (A*y^2 + B*x*y + C0*x^2 + k*x)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2 + k*y)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';A:='A';B:='B';D0:='D0';E0:='E0';k:='k';C0:='C0';
pde :=  (A*y^2+B*x*y+C0*x^2+k*x)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2+k*y)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

39.28 problem number 28

problem number 266

Added January 2, 2019.

Problem 2.2.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + Bxy + Cx^2)w_x + (Dy^2 + Exy + Fx^2)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, A, B, D0, E0, C0, F0];
 pde = (A*y^2 + B*x*y + C0*x^2)*D[w[x, y], x] + (D0*y^2 + E0*x*y + F0*x^2)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';A:='A';B:='B';D0:='D0';E0:='E0';C0:='C0';
pde :=  (A*y^2+B*x*y+C0*x^2)*diff(w(x,y),x)+(D0*y^2+E0*x*y+F0*x^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\sum_{\_R=\mathrm{RootOf}\left(A\_Z^3+(B-D0)\_Z^2+(C0-E0)\_Z-F0\right)} \frac{A\_R^2 + B\_R + C0}{3\,A\_R^2 + 2\,B\_R - 2\,D0\,\_R + C0 - E0} \ln\right.$$

solution contains RootOf

39.29 problem number 29

problem number 267

Added January 2, 2019.

Problem 2.2.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ay^2 + 2Bxy + Dx^2 + a)w_x - (Dy^2 + 2Dxy - Ex^2 - b)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, A, B, D0, E0, b, a];
 pde = (A*y^2 + 2*B*x*y + D0*x^2 + a)*D[w[x, y], x] - (D0*y^2 + 2*D0*x*y - E0*x^2 - b)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';A:='A';B:='B';D0:='D0';E0:='E0';a:='a';b:='b';
pde :=  (A*y^2+2*B*x*y+D0*x^2+a)*diff(w(x,y),x)-(D0*y^2+2*D0*x*y-E0*x^2-b)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

39.30 problem number 30

problem number 268

Added January 2, 2019.

Problem 2.2.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y^2 - 2xy + x^2 + ay)w_x + ayw_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a];
 pde = (y^2 - 2*x*y + x^2 + a*y)*D[w[x, y], x] + a*y*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde :=  (y^2-2*x*y+x^2+a*y)*diff(w(x,y),x)+a*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\ln(y)\,x - \ln(y)\,y + a}{x-y}\right)$$

39.31 problem number 31, Hesse's equation

problem number 269

Added January 2, 2019.

Problem 2.2.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke (1965).

Solve for $w(x,y)$

$$(xf_1 - f_2)w_x + (yf_1 - f_3)w_y = 0$$

Where $f_n = a_n + b_n x + c_n y$.

Mathematica ✗

```
ClearAll[w, x, y, a1, a2, a3, b1, b2, b3, c1, c2, c3];
 pde = (x*(a1 + b1*x + c1*y) - (a2 + b2*x + c2*y))*D[w[x, y], x] + (y*(a1 + b1*x + c1*y) - (
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a1:='a1';a2:='a2';a3:='a3';b1:='b1';b2:='b2';b3:='b3';c1:='c1';c2:='c2'
pde :=  (x *(a1+b1*x+c1*y)-(a2+b2*x+c2*y))*diff(w(x,y),x)+(y*(a1+b1*x+c1*y)-(a3+b3*x+c3*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Too large to display

40 HFOPDE, chapter 2.2.3

40.1 problem number 1

problem number 270

Added January 2, 2019.

Problem 2.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + bx^2y - a^2 - abx^2)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = D[w[x, y], x] + (y^2 + b*x^2*y - a^2 - a*b*x)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^3+2xy+2}{bx^2+2y}\right)\right\}\right\}$$

But it can't solve it when assuming $b > 0$ which is strange.

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  diff(w(x,y),x)+ (y^2+b*x^2*y-a^2-a*b*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1\left(-3\,csgn(b)\,HeunT\left(-\frac{a^2 3^{2/3}}{\sqrt[3]{b^2}}, -3\,\frac{(a-1)\sqrt{b^2}}{b}, 0, 1/3\,3^{2/3}\sqrt[6]{b^2}x\right)bx^2 + 2\,3^{2/3}\sqrt[6]{b^2}\,Heun\right.\right.$$

Mathematica solution is much simpler

40.2 problem number 2

problem number 271

Added January 2, 2019.

Problem 2.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^2y + bx^3 + c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = D[w[x, y], x] + (a*x^2*y + b*x^3 + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{ax^3}{3}}\left(\sqrt[3]{3}be^{\frac{ax^3}{3}}\text{Gamma}\left(\frac{1}{3},\frac{ax^3}{3}\right)+\sqrt[3]{3}ace^{\frac{ax^3}{3}}\text{Gamma}\left(\frac{1}{3},\frac{ax^3}{3}\right)+3a^{4/3}y+3\sqrt[3]{a}bx\right)}{3a^{4/3}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*x^3+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/4\frac{\left(3\sqrt[6]{3}\,\text{WhittakerM}\left(1/6,2/3,1/3\,ax^3\right)e^{1/6\,ax^3}acx+3\sqrt[6]{3}\,\text{WhittakerM}\left(1/6,2/3,1/\right.\right.}{a\sqrt[6]{ax^3}}\right.$$

40.3 problem number 3

problem number 272

Added January 2, 2019.

Problem 2.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^2y + by^3)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b];
 pde = D[w[x, y], x] + (a*x^2*y + b*y^3)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{(-1)^{2/3}\left(-2^{2/3}\sqrt[3]{3}by^2\text{Gamma}\left(\frac{1}{3},-\frac{2ax^3}{3}\right)+3\sqrt[3]{a}e^{\frac{2ax^3}{3}+\frac{i\pi}{3}}\right)}{3\sqrt[3]{a}y^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  diff(w(x,y),x)+ (a*x^2*y+b*y^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/9\,\frac{2\,2^{2/3}3^{5/6}bxOy^2 - 3\,2^{2/3}\sqrt[3]{3}bx\Gamma(1/3,-2/3\,ax^3)\,\Gamma(2/3)\,y^2 + 9\,O\Gamma(2/3)\,e^{2/3\,ax^3}}{O\Gamma\left(2/3\right)y^2}\right)$$

40.4 problem number 4

problem number 273

Added January 2, 2019.

Problem 2.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (axy + b)y^2 w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b];
 pde = D[w[x, y], x] + (a*x*y + b)*y^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  diff(w(x,y),x)+ (a*x*y+b)*y^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{-b^2+4\,a}\left(2\,\sqrt{b^2-4\,a}b\,\mathrm{arctanh}\left(\frac{\sqrt{b^2-4\,a}(2\,axy+b)}{-b^2+4\,a}\right)+\ln\left(x^2\left(ax^2y^2+bxy+\right.\right.\right.\right.$$

40.5 problem number 5

problem number 274

Added January 2, 2019.

Problem 2.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + A(ax+by+c)^3 y^2 w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b, A];
 pde = D[w[x, y], x] + A*(a*x + b*y + c)^3*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';A:='A';
pde :=  diff(w(x,y),x)+  A*(a*x+b*y+c)^3*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/3\,\frac{1}{Ab}\sum_{\_R=\mathrm{RootOf}\left(Ab^4\_Z^3+3\,Ab^3c\_Z^2+3\,Ab^2c^2\_Z+Abc^3+a\right)}\frac{1}{b^2\_R^2+2\,bc\_R+c^2}\ln\left(\frac{-\_R\,b+\right.\right.$$

Answer contains RootOf

40.6 problem number 6

problem number 275

Added January 2, 2019.

Problem 2.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ax^4y^3 + (bx^2 - 1)y + cx)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b, c];
 pde = x*D[w[x, y], x] + (a*x^4*y^3 + (b*x^2 - 1)*y + c*x)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  x*diff(w(x,y),x)+  (a*x^4*y^3+(b*x^2-1)*y+c*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,bx^2 + b^3 \sum_{\_R=\mathrm{RootOf}(c^2a\_Z^3+b^3\_Z-b^3)} \frac{1}{3\_R^2ac^2 + b^3} \ln\left(-\frac{bxy + \_R\,c}{c}\right)\right)$$

Answer contains RootOf

40.7 problem number 7

problem number 276

Added January 2, 2019.

Problem 2.2.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + (ax^2y^2 + bxy + c)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = x^2*D[w[x, y], x] + (a*x^2*y^2 + b*x*y + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2ayx^{\sqrt{-4ac+b^2+2b+1}+1}+bx^{\sqrt{-4ac+b^2+2b+1}}+\sqrt{-4ac+b^2+2b+1}x^{\sqrt{-4ac+b^2+2b+1}}+x^{\sqrt{-4ac}}}{\sqrt{-4ac+b^2+2b+1}-2axy-b-1}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  x^2*diff(w(x,y),x)+  (a*x^2*y^2+b*x*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{4\,ca-b^2-2\,b-1}}\left(\ln(x)\sqrt{4\,ca-b^2-2\,b-1}-2\arctan\left(\frac{2\,axy+b+1}{\sqrt{4\,ca-b^2-2\,b-1}}\right)\right)\right)$$

40.8 problem number 8

problem number 277

Added January 2, 2019.

Problem 2.2.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^2y+b)w_x-(axy^2+c)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = (a*x^2*y + b)*D[w[x, y], x] - (a*x*y^2 + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ax^2y^2+2by+2cx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  (a*x^2*y+b)*diff(w(x,y),x)- (a*x*y^2+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-1/2\,ax^2y^2-by-cx\right)$$

40.9 problem number 9

problem number 278

Added January 2, 2019.

Problem 2.2.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax + by^3)w_x - (cx^3 + ay)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b, c];
 pde = (a*x + b*y^3)*D[w[x, y], x] - (c*x^3 + a*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde :=  (a*x+b*y^3)*diff(w(x,y),x)- (c*x^3+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-1/4\, by^4 - 1/4\, x^4 c - axy\right)$$

41 HFOPDE, chapter 2.2.4

41.1 problem number 1

problem number 279

Added January 2, 2019.

Problem 2.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a\sqrt{x}y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (a*Sqrt[x]*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\frac{2}{3}ax^{3/2}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ye^{-2/3\,x^{3/2}a}\right)$$

41.2 problem number 2

problem number 280

Added January 2, 2019.

Problem 2.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a\sqrt{x}y + b\sqrt{y})w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (a*Sqrt[x]*y + b*Sqrt[y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{1}{3}ax^{3/2}}\left(3^{2/3}be^{\frac{1}{3}ax^{3/2}}\text{Gamma}\left(\frac{2}{3},\frac{1}{3}ax^{3/2}\right)-3a^{2/3}\sqrt{y}\right)}{3a^{2/3}}\right)\right\},\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{1}{3}ax^{3/2}}\left(3\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/10\,\frac{\left(3\sqrt[3]{3}\,\text{WhittakerM}\left(1/3,5/6,1/3\,x^{3/2}a\right)\text{e}^{1/6\,x^{3/2}a}bx+5\,bx\sqrt[3]{x^{3/2}a}-10\,\sqrt{y}\sqrt[3]{x^{3/2}a}\right.}{\sqrt[3]{x^{3/2}a}}\right.$$

41.3 problem number 3

problem number 281

Added January 2, 2019.

Problem 2.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a\sqrt{x}y + bx\sqrt{y})w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a];
 pde = D[w[x, y], x] + (a*Sqrt[x]*y + b*x*Sqrt[y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-\frac{1}{3}ax^{3/2}}\left(\sqrt[3]{3}be^{\frac{1}{3}ax^{3/2}}\text{Gamma}\left(\frac{1}{3},\frac{1}{3}ax^{3/2}\right) - 3a^{4/3}\sqrt{y} + 3\sqrt[3]{a}b\sqrt{x}\right)}{3a^{4/3}}\right)\right\},\left\{w(x,y) \to c_1\left(\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';
pde :=  diff(w(x,y),x)+ (a*sqrt(x)*y+b*x*sqrt(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/4\,\frac{e^{-1/3\,x^{3/2}a}\left(3\,\sqrt[6]{3}\,\text{WhittakerM}\left(1/6,2/3,1/3\,x^{3/2}a\right)\sqrt{x}e^{1/6\,x^{3/2}a}b - 4\,\sqrt{y}a\sqrt[6]{x^{3/2}a}\right)}{\sqrt[6]{x^{3/2}a}a}\right)$$

41.4 problem number 4

problem number 282

Added January 2, 2019.

Problem 2.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + A\sqrt{ax + by + c}w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c, A];
 pde = D[w[x, y], x] + A*Sqrt[a*x + b*y + c]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2a^2\log\left(-ae^{-\frac{\sqrt{aA^2b^2x+A^2b^3y+A^2b^2c}}{a}}\left(-\frac{\sqrt{aA^2b^2x+A^2b^3y+A^2b^2c}}{a}-1\right)\right)+aA^2b^2x+A^2b^2c}{aA^2b^2}\right)\right\}\right\},$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';A:='A'; C:='C';b:='b';
pde :=  diff(w(x,y),x)+ A*sqrt(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xA^2b^2 - 2\,A\sqrt{ax+by+c}b + a\ln\left(A^2ab^2x + A^2b^3y + A^2b^2c - a^2\right) - a\ln\left(A\sqrt{ax+by+c}\right.}{A^2b^2}\right.$$

41.5 problem number 5

problem number 283

Added January 2, 2019.

Problem 2.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay + b\sqrt{y^2 + cx^2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, a, b];
 pde = x*D[w[x, y], x] + (a*y + b*Sqrt[y^2 + c*x^2])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';
pde :=  x*diff(w(x,y),x)+ ( a*y + b *sqrt(y^2+c*x^2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

41.6 problem number 6

problem number 284

Added January 2, 2019.

Problem 2.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax + b\sqrt{y})\, w_x - (c\sqrt{x} + ay)\, w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, a, b, c];
 pde = (a*x + b*Sqrt[y])*D[w[x, y], x] - (c*Sqrt[x] + a*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{9a^3x^3 + 16b^2cx^{3/2}}{24b^2}\right)\right\}, \left\{w(x,y) \to c_1\left(\frac{1}{3}(3axy - 2by^{3/2} + 2cx^{3/2})\right)\right\}, \left\{w(x,y) \to c_1\left(\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';c:='c';
pde := (a*x+b*sqrt(y))* diff(w(x,y),x)- (c*sqrt(x)+a*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\mathrm{RootOf}\left(8\,y^{5/2}abc^2 + 3\,y^4a^4 - 2\sqrt[3]{-4\,y^{3/2}bc^2 - y^3a^3 - 6\,c^2\_Z + 2\sqrt{4\,y^3b^2c^2 + 2\,y^{9/2}a^3b}}\right.\right.$$

41.7 problem number 7

problem number 285

Added January 2, 2019.

Problem 2.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sqrt{f(x)}w_x + \sqrt{f(y)}w_y = 0$$

Where $f(t) = \sum_{n=0}^{4} a_n t^n$

Mathematica ✗

```
ClearAll[w, x, y, t, n, a];
 f[t_] := Sum[a[n]*t^n, {n, 1, 4}];
 pde = Sqrt[f[x]]*D[w[x, y], x] + Sqrt[f[y]]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';n:='n';t:='t';
f:=t->sum(a[n]*t^n,n=1..4);
pde := sqrt(f(x))* diff(w(x,y),x)+ sqrt(f(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-2\,1\left(1/12\,\frac{\sqrt[3]{12\sqrt{3}\sqrt{27\,a_1{}^2a_4{}^2 - 18\,a_1a_2a_3a_4 + 4\,a_1a_3{}^3 + 4\,a_2{}^3a_4 - a_2{}^2a_3{}^2a_4 - 108\,a_1a_4{}^2}}}{a_4}\right.\right.$$

$$8\,a_1a_4{}^2+36\,a_2a_3a_4-8\,a_3{}^3\Big)^{2/3} - 2\,a_3\sqrt[3]{12\sqrt{3}\sqrt{27\,a_1{}^2a_4{}^2-18\,a_1a_2a_3a_4+4\,a_1a_3{}^3+4\,a_2{}^3a_4-a_2{}^2a_3{}^2a_4-108\,a_1a_4{}^2+36\,a_2a_3a_4-8\,a_3{}^3}-12\,a_2a_4+4\,a_3{}^2}\Big)\Big(i\Big(12\sqrt{3}\sqrt{27\,a_1{}^2a$$

42 HFOPDE, chapter 2.2.5

42.1 problem number 1

problem number 286

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay + bx^k\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, k];
 pde = D[w[x, y], x] + (a*y + b*x^k)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(a^{-k-1}e^{-ax}\left(be^{ax}\text{Gamma}(k+1,ax) + ya^{k+1}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';k:='k';b:='b';
pde := diff(w(x,y),x)+ (a*y+b*x^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{e^{-ax}\left(x^k e^{1/2\,ax}(ax)^{-k/2}\,\text{WhittakerM}\left(k/2, k/2+1/2, ax\right)b - aky - ya\right)}{a\,(k+1)}\right)$$

42.2 problem number 2

problem number 287

Added January 2, 2019.

Problem 2.2.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^k y + bx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, k];
 pde = D[w[x, y], x] + (a*x^k*y + b*x^n)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a^{-\frac{n}{k+1}-\frac{1}{k+1}}e^{-\frac{ax^{k+1}}{k+1}}\left(b(k+1)^{\frac{n}{k+1}+\frac{1}{k+1}}e^{\frac{ax^{k+1}}{k+1}}\text{Gamma}\left(\frac{n}{k+1}+\frac{1}{k+1},\frac{ax^{k+1}}{k+1}\right)+kya^{\frac{n}{k+1}+\frac{1}{k+1}}+\right.}{k+1}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';k:='k';b:='b';n:='n';
pde := diff(w(x,y),x)+ (a*x^k*y+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{a\left(2\,k^2n+3\,kn^2+n^3+2\,k^2+10\,kn+6\,n^2+7\,k+11\,n+6\right)}\left(-6\,\mathrm{e}^{-\frac{ax^{k+1}}{k+1}}ya+\mathrm{e}^{-1/2\,\frac{ax}{k}}\right.\right.$$

42.3 problem number 3

problem number 288

Added January 2, 2019.

Problem 2.2.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay^2 + bx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b];
 pde = D[w[x, y], x] + (a*y^2 + b*x^n)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\left(-axy\text{BesselJ}\left(\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n+2}{2}}}{n+2}\right)-\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\text{Bes}\right.}{-\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\text{BesselJ}\left(\frac{n+1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}}{n+2}\right)+2axy\text{BesselJ}\left(-\frac{1}{n+2},\frac{2\sqrt{a}\sqrt{b}x^{\frac{n+2}{2}}}{n+2}\right)+\text{BesselJ}\left(\right.}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';
pde := diff(w(x,y),x)+ (a*y^2+b*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1\left(-\text{BesselY}\left(\frac{n+3}{n+2},2\,\frac{\sqrt{ab}x^{n/2}x}{n+2}\right)\sqrt{ab}x^{n/2}x+\text{BesselY}\left((n+2)^{-1},2\,\frac{\sqrt{ab}x^{n/2}x}{n+2}\right)yax\right.\right.$$

42.4 problem number 4

problem number 289

Added January 2, 2019.

Problem 2.2.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + anx^{n-1} - a^2x^{2n}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a];
 pde = D[w[x, y], x] + (y^2 + a*n*x^(n - 1) - a^2*x^(2*n))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';
pde := diff(w(x,y),x)+ (y^2+a*n*x^(n-1)-a^2*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(2\left(x^{5/2\,n+2}a - x^{3/2\,n+2}y\right)e^{-\frac{x^{n+1}a}{n+1}}\left(-4\,yx\,\mathrm{WhittakerM}\left(1/2\,\frac{n+2}{n+1}, 1/2\,\frac{2\,n+3}{n+1}, -2\,\frac{x^{n+1}a}{n+1}\right)\right.\right.$$

42.5 problem number 5

problem number 290

Added January 2, 2019.

Problem 2.2.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y + ax^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a];
 pde = D[w[x, y], x] + (y^2 + a*x^n*y + a*x^(n - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{(n+1)^{-\frac{1}{n+1}-1}\left((-1)^{\frac{1}{n+1}+1} x a^{\frac{1}{n+1}} \text{Gamma}\left(\frac{1}{-n-1}, -\frac{ax^{n+1}}{n+1}\right) + (-1)^{\frac{1}{n+1}+1} x^2 y a^{\frac{1}{n+1}} \text{Gamm}\right.}{x(xy+1)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';
pde := diff(w(x,y),x)+ (y^2+a*x^n*y+a*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{anx\left(2\,nyx + yx + 2\,n + 1\right)}\left(\mathrm{e}^{1/2\,\frac{x^{n+1}a}{n+1}} x^{-n} y \left(-\frac{x^{n+1}a}{n+1}\right)^{-1/2\,\frac{n}{n+1}} \text{WhittakerM}\left(-1/2\,\frac{n+}{n+}\right.\right.\right.$$

42.6 problem number 6

problem number 291

Added January 2, 2019.

Problem 2.2.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax^n y - abx^n - b^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b];
 pde = D[w[x, y], x] + (y^2 + a*x^n*y - a*b*x^n - b^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';
pde := diff(w(x,y),x)+ (y^2+a*x^n*y-a*b*x^n-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{-b+y}\left(\int \mathrm{e}^{\frac{x(x^n a+2\,bn+2\,b)}{n+1}}\,\mathrm{d}xy - \int \mathrm{e}^{\frac{x(x^n a+2\,bn+2\,b)}{n+1}}\,\mathrm{d}xb + \mathrm{e}^{\frac{x(x^n a+2\,bn+2\,b)}{n+1}}\right)\right)$$

42.7 problem number 7

problem number 292

Added January 2, 2019.

Problem 2.2.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b];
 pde = D[w[x, y], x] + (a*x^n*y^2 + b*x^(-n - 2))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-2ayx^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(\frac{\sqrt{-4ab+n^2+2n+1}}{\sqrt{a}\sqrt{b}}-\frac{-n-1}{\sqrt{a}\sqrt{b}}\right)+n+1}-nx^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(\frac{\sqrt{-4ab+n^2+2n+1}}{\sqrt{a}\sqrt{b}}-\frac{-n-1}{\sqrt{a}\sqrt{b}}\right)}-\sqrt{-4ab+n}}{2ayx^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(-\frac{\sqrt{-4ab+n^2+2n+1}}{\sqrt{a}\sqrt{b}}-\frac{-n-1}{\sqrt{a}\sqrt{b}}\right)+n+1}+nx^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(-\frac{\sqrt{-4ab+n^2+2n+1}}{\sqrt{a}\sqrt{b}}-\frac{-n-1}{\sqrt{a}\sqrt{b}}\right)}-\sqrt{-4ab+n}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{4\,ab-n^2-2\,n-1}}\left(\ln\left(x\right)\sqrt{4\,ab-n^2-2\,n-1}-2\,\arctan\left(\frac{2\,ax^nyx+n+1}{\sqrt{4\,ab-n^2-2\,n-1}}\right)\right)\right)$$

42.8 problem number 8

problem number 293

Added January 2, 2019.

Problem 2.2.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + bmx^{m-1} - ab^2 x^{n+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m];
 pde = D[w[x, y], x] + (a*x^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(n + 2*m))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*m*x^(m-1) -a*b^2*x^(n+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-2\,a\left(x^{5/2\,m+2\,n+2}b - x^{3/2\,m+2\,n+2}y\right)e^{-\frac{x^{m+n+1}ab}{m+n+1}}\left(2\,ayn^2x^{n+1}\,\mathrm{WhittakerM}\left(-1/2\,\frac{m}{m+n+}\right.\right.\right.$$

42.9 problem number 9

problem number 294

Added January 2, 2019.

Problem 2.2.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((n+1)x^n y^2 - a x^{n+m+1} y + a x^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m];
 pde = D[w[x, y], x] + ((n + 1)*x^n*y^2 - a*x^(n + m + 1)*y + a*x^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';
pde := diff(w(x,y),x)+ ((n+1)*x^n*y^2 - a*x^(n+m+1)* y + a*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(-2\,{}_1F_1\left(\frac{-n+m}{m+n+2};\ \frac{m+1}{m+n+2};\ \frac{x^2x^nx^ma}{m+n+2}\right)x^m amnx + {}_1F_1\left(2\,\frac{m+1}{m+n+2};\ \frac{2\,m+3}{m+n+}\right.\right.\right.$$

42.10 problem number 10

problem number 295

Added January 2, 2019.

Problem 2.2.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a x^n y^2 + b x^m y + b c x^m - a c^2 x^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c];
 pde = D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + b*c*x^m - a*c^2*x^n)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
pde := diff(w(x,y),x)+ (a*x^n*y^2 + b*x^m*y+ b*c*x^m -a*c^2*x^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{c+y}\left(\left(\int e^{\frac{x(-2\,c a x^n m + x^m b n - 2\,x^n a c + x^m b)}{(m+1)(n+1)}} x^n a\, dx y + \int e^{\frac{x(-2\,c a x^n m + x^m b n - 2\,x^n a c + x^m b)}{(m+1)(n+1)}} x^n a\, dx c + e^{-$$

42.11 problem number 11

problem number 296

Added January 2, 2019.

Problem 2.2.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 - ax^n(bx^m + c)y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c];
 pde = D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*x^m + c)*y + b*m*x^(m - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
pde := diff(w(x,y),x)+ (a*x^n*y^2-a*x^n*(b*x^m +c)*y+ b*m*x^(m-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.12 problem number 12

problem number 297

Added January 2, 2019.

Problem 2.2.5.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left(anx^{n-1}y^2 - cx^m(ax^n + b) + cx^m\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c];
 pde = D[w[x, y], x] - (a*n*x^(n - 1)*y^2 - c*x^m*(a*x^n + b) + c*x^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
pde := diff(w(x,y),x)- (a*n*x^(n-1)*y^2 - c*x^m*(a*x^n+b) + c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.13 problem number 13

problem number 298

Added January 2, 2019.

Problem 2.2.5.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^2 + bx^my + ckx^{k-1} - bcx^{m+k} - ac^2x^{n+2k}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^n*y^2 + b*x^m*y + c*k*x^(k - 1) - b*c*x^(m + k) - a*c^2*x^(n + 2
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^n*y^2+b*x^m*y+ c*k*x^(k-1)-b*c*x^(m+k)-a*c^2*x^(n+2*k))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.14 problem number 14

problem number 299

Added January 2, 2019.

Problem 2.2.5.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^{2n+1}y^3 + bx^{-n-2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + b*x^(-n - 2))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^(2*n+1)*y^3 + b*x^(-n-2))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\ln(x) - \sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+\_Z(n+1)+b\right)} \frac{\ln\left(yx^n x - \_R\right)}{3\_R^2 a + n + 1}\right)$$

Solution contains RootOf

42.15 problem number 15

problem number 300

Added January 2, 2019.

Problem 2.2.5.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^3 + 3abx^{n+m}y^2 - bmx^{m-1} - 2ab^3x^{n+3m}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 - b*m*x^(m - 1) - 2*a*b^3*x^(n + 3*m
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{6^{-\frac{n}{2m+n+1}-\frac{1}{2m+n+1}}(2m+n+1)^{-\frac{2m}{2m+n+1}} b^{-\frac{2n}{2m+n+1}-\frac{2}{2m+n+1}} e^{-\frac{6ab^2x^{2m+n+1}}{2m+n+1}}\left(-2y^2 a^{\frac{2m}{2m+n+1}} e^{\frac{6ab^2}{2}}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2 - b*m*x^(m-1) - 2*a*b^3*x^(n+3*m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

42.16 problem number 16

problem number 301

Added January 2, 2019.

Problem 2.2.5.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^3 + 3abx^{n+m}y^2 + cx^k y - 2ab^3x^{n+3m} + bcx^{m+l} - bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^n*y^3 + 3*a*b*x^(n + m)*y^2 + c*x^k*y - 2*a*b^3*x^(n + 3*m) + b*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^n*y^3 + 3*a*b*x^(n+m)*y^2+ c*x^k*y-2*a*b^3*x^(n+3*m) + b*c*x^(m+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.17 problem number 17

problem number 302

Added January 2, 2019.

Problem 2.2.5.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ay^n + bx^{\frac{n}{1-n}}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*y^n + b*x^(n/(1 - n)))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*y^n+b*x^(n/(1-n)))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int_{\_b}^{y} 1x^{\frac{n}{n-1}}\left(\_a^n x^{\frac{n}{n-1}} anx - a\_a^n x^{\frac{n}{n-1}} x + bxn + x^{\frac{n}{n-1}}\_a - bx\right)^{-1} d\_an + \int_{\_b}^{y} 1x^{\frac{n}{n-1}}\left(\_\right.\right.$$

42.18 problem number 18

problem number 303

Added January 2, 2019.

Problem 2.2.5.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a x^{m-n-(mn)} y^n + b x^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^(m - n - m*n)*y^n + b*x^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^(m-n-(m*n))*y^n + b*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\int^{y}_{\_b} -\frac{x^{mn}x^n}{x^m b x x^{mn} x^n - x^{mn} x^n \_a\, m + \_a^n a x^m x - x^{mn} x^n \_a} \mathrm{d}\_a + \ln(x)\right)$$

42.19 problem number 19

problem number 304

Added January 2, 2019.

Problem 2.2.5.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ax^n y^k + bx^m y)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = D[w[x, y], x] + (a*x^n*y^k + b*x^m*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := diff(w(x,y),x)+ (a*x^n*y^k + b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{b\,(2\,m^2n + 3\,mn^2 + n^3 + 2\,m^2 + 10\,mn + 6\,n^2 + 7\,m + 11\,n + 6)}\left(10\,\mathrm{e}^{\frac{x^{m+1}b(k-1)}{m+1}}y^{1-k}bmn\right.\right.$$

42.20 problem number 20

problem number 305

Added January 2, 2019.

Problem 2.2.5.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^2 + by + cx^{2b}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*y^2 + b*y + c*x^(2*b))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-\sqrt{a}y\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)-\sqrt{c}x^b\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}{\sqrt{a}y\cos\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)-\sqrt{c}x^b\sin\left(\frac{\sqrt{a}\sqrt{c}x^b}{b}\right)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*y^2 + b*y+ c*x^(2*b))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{b}\left(\sqrt{c}\sqrt{a}x^b - \arctan\left(\frac{\sqrt{a}x^{-b}y}{\sqrt{c}}\right)b\right)\right)$$

42.21 problem number 21

problem number 306

Added January 2, 2019.

Problem 2.2.5.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^2 + (n + bx^n)y + cx^{2n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*y^2 + (n + b*x^n)*y + c*x^(2*n))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\exp\left(\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}+\frac{b}{\sqrt{a}\sqrt{c}}\right)}{2n}-\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{b}{\sqrt{a}\sqrt{c}}-\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}\right)}{2n}\right)\left(x^n\sqrt{b^2-4ac}+2ay+bx^n\right)}{x^n\sqrt{b^2-4ac}-2ay-bx^n}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*y^2+(n+b*x^n)*y + c*x^(2*n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b}{\sqrt{b^2\left(4\,ca-b^2\right)}n}\left(2\,bn\arctan\left(\frac{b(b+2\,x^{-n}ya)}{\sqrt{b^2\left(4\,ca-b^2\right)}}\right)-\sqrt{b^2\left(4\,ca-b^2\right)}x^n\right)\right)$$

42.22 problem number 22

problem number 307

Added January 2, 2019.

Problem 2.2.5.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + by + cx^{-n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^n*y^2 + b*y + c/x^n)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-2ayx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(\frac{\sqrt{-4ac+b^2+2bn+n^2}}{\sqrt{a}\sqrt{c}}-\frac{-b-n}{\sqrt{a}\sqrt{c}}\right)+n}-bx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(\frac{\sqrt{-4ac+b^2+2bn+n^2}}{\sqrt{a}\sqrt{c}}-\frac{-b-n}{\sqrt{a}\sqrt{c}}\right)}-nx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(\right.}}{2ayx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(-\frac{\sqrt{-4ac+b^2+2bn+n^2}}{\sqrt{a}\sqrt{c}}-\frac{-b-n}{\sqrt{a}\sqrt{c}}\right)+n}+bx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(-\frac{\sqrt{-4ac+b^2+2bn+n^2}}{\sqrt{a}\sqrt{c}}-\frac{-b-n}{\sqrt{a}\sqrt{c}}\right)}+nx^{\frac{1}{2}\sqrt{a}\sqrt{c}\left(-\right.}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*x^n*y^2+b*y+c*x^(-n))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{4\,ca-b^2-2\,bn-n^2}}\left(\ln\left(x\right)\sqrt{4\,ca-b^2-2\,bn-n^2}-2\,\arctan\left(\frac{2\,ax^ny+b+n}{\sqrt{4\,ca-b^2-2\,bn-}}\right.\right.\right.$$

42.23 problem number 23

problem number 308

Added January 2, 2019.

Problem 2.2.5.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + my - ab^2 x^{x+2m}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^n*y^2 + m*y - a*b^2*x^(x + 2*m))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+  (a*x^n*y^2+ m*y- a*b^2*x^(x+2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(2 \operatorname{BesselI}\left(-\frac{n+m}{n+x+2\,m}, 2\,\frac{abx^{n/2}x^{x/2}x^m}{n+x+2\,m}\right) x^n y a + 2\,x \frac{\partial}{\partial x} \operatorname{BesselI}\left(-\frac{n+m}{n+x+2\,m},\right.\right.\right.$$

42.24 problem number 24

problem number 309

Added January 2, 2019.

Problem 2.2.5.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x^{2n}y^2 + (m-n)y + x^{2m}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (x^(2*n)*y^2 + (m - n)*y + x^(2*m))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{m\tan^{-1}\left(yx^{n-m}\right)+n\tan^{-1}\left(yx^{n-m}\right)-x^{m+n}}{m+n}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+  (x^(2*n)*y^2+(m-n)*y+ x^(2*m))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\arctan\left(x^{n-m}y\right)m - \arctan\left(x^{n-m}y\right)n + x^{n+m}}{n+m}\right)$$

42.25 problem number 25

problem number 310

Added January 2, 2019.

Problem 2.2.5.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n}y^2 + (bx^n - n)y + c\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^(2*n)*y^2 + (b*x^n - n)*y + c)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\exp\left(\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}+\frac{b}{\sqrt{a}\sqrt{c}}\right)}{2n}-\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{b}{\sqrt{a}\sqrt{c}}-\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}\right)}{2n}\right)\left(\sqrt{b^2-4ac}+2ayx^n+b\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*x^(2*n)*y^2+ (b*x^n -n)*y + c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b}{\sqrt{b^2\,(4\,ca-b^2)}n}\left(2\,bn\arctan\left(\frac{b(2\,ax^ny+b)}{\sqrt{4\,acb^2-b^4}}\right)-\sqrt{b^2\,(4\,ca-b^2)}x^n\right)\right)$$

42.26 problem number 26

problem number 311

Added January 2, 2019.

Problem 2.2.5.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n+m}y^2 + (bx^{n+m} - n)y + cx^m\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^(2*n + m)*y^2 + (b*x^(n + m) - n)*y + c*x^m)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\left(\sqrt{b^2-4ac}+2ayx^n+b\right)\exp\left(\frac{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}+\frac{b}{\sqrt{a}\sqrt{c}}\right)x^{m+n}}{2(m+n)}-\frac{\sqrt{a}\sqrt{c}\left(\frac{b}{\sqrt{a}\sqrt{c}}-\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}\right)x^{m+n}}{2(m+n)}\right)}{\sqrt{b^2-4ac}-2ayx^n-b}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*x^(2*n + m)*y^2 +(b*x^(n+m)-n)*y+ c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b}{\sqrt{b^2\left(4\,ca-b^2\right)}\,(n+m)}\left(2\,bm\arctan\left(\frac{b(2\,ax^ny+b)}{\sqrt{4\,acb^2-b^4}}\right)+2\,bn\arctan\left(\frac{b(2\,ax^ny+b)}{\sqrt{4\,acb^2-b^4}}\right)-\right.\right.$$

42.27 problem number 27

problem number 312

Added January 2, 2019.

Problem 2.2.5.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ay^3 + 3abx^n y^2 - bnx^n - 2ab^3 x^{3n}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*y^3 + 3*a*b*x^n*y^2 - b*n*x^n - 2*a*b^3*x^(3*n))*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-\frac{3ab^2x^{2n}}{n}}\left(ay^2 e^{\frac{3ab^2x^{2n}}{n}}\text{ExpIntegralEi}\left(-\frac{3ab^2x^{2n}}{n}\right) + ab^2x^{2n}e^{\frac{3ab^2x^{2n}}{n}}\text{ExpIntegralEi}\left(-\frac{3ab^2x^2}{n}\right.\right.}{n\left(bx^n+y\right)^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*y^3+3*a*b*x^n*y^2 - b*n*x^n -2*a*b^3*x^(3*n) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{n\left(x^{2n}b^2 + 2x^n by + y^2\right)}\left(\text{expIntegral}\left(1, 3\frac{x^{2n}ab^2}{n}\right)x^{2n}ab^2 + 2\,\text{expIntegral}\left(1, 3\frac{x^{2n}a}{n}\right.\right.\right.$$

42.28 problem number 28

problem number 313

Added January 2, 2019.

Problem 2.2.5.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{2n+1}y^3 + (bx-n)y + cx^{1-n}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^(2*n + 1)*y^3 + (b*x - n)*y + c*x^(1 - n))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*x^(2*n +1)*y^3 + (b*x-n)*y + c*x^(1-n) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(b^3 \sum_{\_R=\mathrm{RootOf}\left(\_Z^3ac^2+\_Z\,b^3-b^3\right)} \frac{1}{3\_R^2ac^2+b^3}\ln\left(-\frac{x^nby+\_R\,c}{c}\right) - bx\right)$$

Solution contains RootOf

42.29 problem number 29

problem number 314

Added January 2, 2019.

Problem 2.2.5.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^{n+2}y^3 + (bx^n - 1)y + cx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (a*x^(n + 2)*y^3 + (b*x^n - 1)*y + c*x^(n - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ (a*x^(n+2)*y^3+ (b*x^n-1)*y + c*x^(n-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{n}\left(b^3 \sum_{\_R=\mathrm{RootOf}\left(\_Z^3ac^2+\_Z\,b^3-b^3\right)} \frac{1}{3\_R^2ac^2+b^3}\ln\left(-\frac{bxy+\_R\,c}{c}\right)n - bx^n\right)\right)$$

Solution contains RootOf

42.30 problem number 30

problem number 315

Added January 2, 2019.

Problem 2.2.5.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(y + ax^{n-m}y^m + bx^{n-k}y^k\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*D[w[x, y], x] + (y + a*x^(n - m)*y^m + b*x^(n - k)*y^k)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*diff(w(x,y),x)+ ( y+a*x^(n - m)*y^m+b*x^(n-k)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{(n-1)\,x}\left(\int_{\_b}^{y} -\frac{x^m x^k}{x\,(ax^k\_a^m + \_a^k x^m b)}\,\mathrm{d}\_axn - \int_{\_b}^{y} -\frac{x^m x^k}{x\,(ax^k\_a^m + \_a^k x^m b)}\,\mathrm{d}\_ax + x\right.\right.$$

42.31 problem number 31

problem number 316

Added January 2, 2019.

Problem 2.2.5.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + \left(x^{n-1}((1+2n)x+an)y - nx^{2n}(x+a)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = y*D[w[x, y], x] + (x^(n - 1)*((1 + 2*n)*x + a*n)*y - n*x^(2*n)*(x + a))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := y*diff(w(x,y),x)+ ( x^(n-1)*((1+2*n)*x+a*n)*y-n*x^(2*n)*(x+a) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{x}\left(\sqrt{-n^2}\int^{-2\,\frac{1}{\sqrt{-n^2}}\arctan\left(\frac{n\left(2\,x^n a+x^{n+1}-y\right)}{\sqrt{-n^2}\left(x^{n+1}-y\right)}\right)}\tan\left(1/2\,\_a\sqrt{-n^2}\right)\mathrm{e}^{-\_a}d\_ax - 2\,\mathrm{e}^{2\,\frac{1}{\sqrt{-n}}}\right.\right.$$

42.32 problem number 32

problem number 317

Added January 2, 2019.

Problem 2.2.5.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x + \left((a(2n+k)x^k + b)x^{n-1}y - (a^2nx^{2k} + abx^k - c)x^{2n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = y*D[w[x, y], x] + ((a*(2*n + k)*x^k + b)*x^(n - 1)*y - (a^2*n*x^(2*k) + a*b*x^k - c)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := y*diff(w(x,y),x)+ ( (a*(2*n+k)*x^k+b)*x^(n-1)*y -(a^2*n*x^(2*k)+ a*b*x^k-c)*x^(2*n-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.33 problem number 33

problem number 318

Added January 2, 2019.

Problem 2.2.5.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2axy+b)w_x - \left(a(m+3)xy^2 + b(m+2)y - cx^m\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x*(2*a*x*y + b)*D[w[x, y], x] - (a*(m + 3)*x*y^2 + b*(m + 2)*y - c*x^m)*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{x^{m+2}(-2amxy^2-2axy^2-2bmy-2by+cx^m)}{2a(m+1)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x*(2*a*x*y+b)*diff(w(x,y),x)- ( a*(m+3)*x*y^2+b*(m+2)*y-c*x^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{(2\,axy^2m+2\,axy^2+2\,bym+2\,by-cx^m)\,x^2x^m}{m+1}\right)$$

42.34 problem number 34

problem number 319

Added January 2, 2019.

Problem 2.2.5.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2(2axy+b)w_x - \left(4ax^2y^2+3bxy-cx^2-k\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = x^2*(2*a*x*y + b)*D[w[x, y], x] - (4*a*x^2*y^2 + 3*b*x*y - c*x^2 - k)*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4ax^4y^2+4bx^3y-cx^4-2kx^2}{4a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde := x^2*(2*a*x*y+b)*diff(w(x,y),x)- ( 4*a*x^2*y^2 + 3*b*x*y-c*x^2 - k )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-ax^4y^2 - bx^3y + 1/4\,x^4c + 1/2\,kx^2\right)$$

42.35 problem number 35

problem number 320

Added January 2, 2019.

Problem 2.2.5.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^m w_x + by^n w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = a*x^m*D[w[x, y], x] + b*y^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{x^{-m}y^{-n}(-ayx^m+amyx^m+bxy^n-bnxy^n)}{a(m-1)(n-1)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde :=a*x^m*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{x^{1-m}bn - y^{-n+1}am - x^{1-m}b + y^{-n+1}a}{a\,(m-1)}\right)$$

42.36 problem number 36

problem number 321

Added January 2, 2019.

Problem 2.2.5.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (by + cx^m)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = a*x^n*D[w[x, y], x] + (b*y + c*x^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{b^{-\frac{n}{n-1}}(a-an)^{-\frac{m}{n-1}-\frac{1}{n-1}}e^{-\frac{bx^{1-n}}{a(1-n)}}\left(-c(a-an)^{\frac{n}{n-1}}b^{\frac{m}{n-1}+\frac{1}{n-1}}e^{\frac{bx^{1-n}}{a(1-n)}}\text{Gamma}\left(\frac{-m+n-1}{n-1},\frac{bx^{1-}}{a-an}\right.\right.}{a(n-1)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde :=a*x^n*diff(w(x,y),x)+ (b*y+c*x^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{ab\,(m^3-6\,m^2n+11\,mn^2-6\,n^3+6\,m^2-22\,mn+18\,n^2+11\,m-18\,n+6)}\left(-e^{1/2\,\frac{x^{-n-}}{a(n-}}\right.\right.$$

42.37 problem number 37

problem number 322

Added January 2, 2019.

Problem 2.2.5.37 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + (y^n + bx^m y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k];
 pde = a*x^k*D[w[x, y], x] + (y^n + b*x^m*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> {n != 1}],
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';
pde :=a*x^k*diff(w(x,y),x)+ (y^n+b*x^m*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n<>1)
```

$$w(x,y) = \_F1\left(\frac{1}{a}\left(\frac{a}{y^{(k-m-1)^{-1}}}y^{\frac{mn}{k-m-1}}y^{\frac{n}{k-m-1}}y^{\frac{k}{k-m-1}}\mathrm{e}^{\frac{bx^{-k+m+1}}{(k-m-1)a}}\left(y^{\frac{kn}{k-m-1}}\right)^{-1}\left(\mathrm{e}^{\frac{bx^{-k+m+1}n}{(k-m-1)a}}\right)^{-1}\left(y^{\frac{m}{k-m-1}}\right)^{-1}+\right.\right.$$

42.38 problem number 38

problem number 323

Added January 2, 2019.

Problem 2.2.5.38 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^k+b)w_x + \left(\alpha x^n y^2 + (\beta - anx^k)y + \gamma x^{-n}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma];
 pde = x*(a*x^k + b)*D[w[x, y], x] + (alpha*x^n*y^2 + (beta - a*n*x^k)*y + gamma/x^n)*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-\beta\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}(k\log(x)-\log(ax^k+b))\left(\sqrt{\frac{(-bn-\beta)^2}{\alpha\gamma}-4}-\frac{-bn-\beta}{\sqrt{\alpha}\sqrt{\gamma}}\right)}{2bk}\right)-bn\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}(k\log(x)-\log(ax\right.}{}\right.}{\beta\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}(k\log(x)-\log(ax^k+b))\left(-\frac{-bn-\beta}{\sqrt{\alpha}\sqrt{\gamma}}-\sqrt{\frac{(-bn-\beta)^2}{\alpha\gamma}-4}\right)}{2bk}\right)+bn\exp\left(\frac{\sqrt{\alpha}\sqrt{\gamma}(k\log(x)-\log(ax^k+\right.}{}\right.}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=x*(a*x^k+b)*diff(w(x,y),x)+ (alpha*x^n*y^2+(beta-a*n*x^k)*y+g*x^(-n))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\sqrt{-\left(bn+\beta\right)^2\left(-b^2n^2-2\,b\beta\,n+4\,\alpha\,g-\beta^2\right)bk}}\left(-2\,b^3kn^2\operatorname{arctanh}\left(\frac{2\,x^ny\alpha\,bn+2\,x^ny\alpha}{\sqrt{-\left(bn+\beta\right)^2\left(-b^2\right.}}\right.\right.\right.$$

42.39 problem number 39

problem number 324

Added January 2, 2019.

Problem 2.2.5.39 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y+Ax^n+a)w_x-\left(nAx^{n-1}y+kx^m+b\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A];
 pde = (y + A*x^n + a)*D[w[x, y], x] - (n*A*x^(n - 1)*y + k*x^m + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2amy+2ay+2Amyx^n+2Ayx^n+2bmx+2bx+2kx^{m+1}+my^2+y^2}{m+1}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(y+ A*x^n + a)*diff(w(x,y),x)- ( n*A*x^(n-1)*y + k*x^m + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{2\,Ax^{n}ym + 2\,Ax^{n}y + y^{2}m + 2\,aym + 2\,kx^{m}x + 2\,bxm + y^{2} + 2\,ya + 2\,bx}{m+1}\right)$$

42.40 problem number 40

problem number 325

Added January 2, 2019.

Problem 2.2.5.40 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y + ax^{n+1} + bx^{n})w_x + \left(anx^{n} + cx^{n-1}\right)yw_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A];
 pde = (y + a*x^(n + 1) + b*x^n)*D[w[x, y], x] + (a*n*x^n + c*x^(n - 1))*y*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(y+ a*x^(n+1)+b*x^n)*diff(w(x,y),x)+ ( a*n*x^n + c*x^(n-1))*y*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.41 problem number 41

problem number 326

Added January 2, 2019.

Problem 2.2.5.41 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(2ax^ny+b)w_x - \left(a(3n+m)x^ny^2 + b(2n+m)y - Ax^m - Cx^{-n}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0];
 pde = x*(2*a*x^n*y + b)*D[w[x, y], x] - (a*(3*n + m)*x^n*y^2 + b*(2*n + m)*y - A*x^m - C0/x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{x^{m+n}(2amy^2x^{2n}+2any^2x^{2n}-Ax^{m+n}+2bmyx^n+2bnyx^n-2\text{C0})}{2a(m+n)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=x*(2*a*x^n*y+b)*diff(w(x,y),x)- ( a*(3*n+m)*x^n*y^2+b*(2*n+m)*y-A*x^m -C*x^(-n))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{-2\,x^{3\,n+m}y^2am - 2\,x^{3\,n+m}y^2an - 2\,x^{m+2\,n}ybm - 2\,x^{m+2\,n}ybn + x^{2\,n+2\,m}A + 2\,x^{n+m}C}{n+m}\right.$$

42.42 problem number 42

problem number 327

Added January 2, 2019.

Problem 2.2.5.42 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^2 + xy)w_x + \left(cx^n + bxy + y^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0];
 pde = (a*x^n + b*x^2 + x*y)*D[w[x, y], x] + (c*x^n + b*x*y + y^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*x^n+b*x^2+ x*y)*diff(w(x,y),x)+ ( c*x^n + b*x*y+ y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/3\,\frac{1}{n-2}\left(\ln\left(abx^2+a^2x^n+cx^2\right)n^2-2\,\ln\left(x\right)n^2+\frac{n}{(n-2)\,(n-1)}\left(-n^3\ln\left(9\,\frac{abx^2n^2}{\right.\right.\right.\right.$$

42.43 problem number 43

problem number 328

Added January 2, 2019.

Problem 2.2.5.43 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^n + bx^2 + cxy)w_x + \left(ky^n + bxy + cy^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0];
 pde = (a*y^n + b*x^2 + c*x*y)*D[w[x, y], x] + (k*y^n + b*x*y + c*y^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*y^n+b*x^2+c*x*y)*diff(w(x,y),x)+ ( k*y^n+ b*x*y+c*y^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/3\,\frac{1}{n-2}\left(2\,\ln(y)\,n^2 - \ln\left(aby^2 + cky^2 + y^n k^2\right)n^2 - 6\,n\ln(y) - \frac{n}{(n-2)\,(n-1)}\left(-n\right.\right.\right.$$

42.44 problem number 44

problem number 329

Added January 2, 2019.

Problem 2.2.5.44 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left(cy^2 - bx^{m-1}y + ax^{n-2}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0];
 pde = (a*x^n + b*x^m + c)*D[w[x, y], x] + (c*y^2 - b*x^(m - 1)*y + a*x^(n - 2))*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( c*y^2-b*x^(m-1)*y+ a*x^(n-2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.45 problem number 45

problem number 330

Added January 2, 2019.

Problem 2.2.5.45 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left(ax^{n-2}y^2 + bx^{m-1}y + c\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0];
 pde = (a*x^n + b*x^m + c)*D[w[x, y], x] + (a*x^(n - 2)*y^2 + b*x^(m - 1)*y + c)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ( a*x^(n-2)*y^2 + b*x^(m-1)*y + c)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.46 problem number 46

problem number 331

Added January 2, 2019.

Problem 2.2.5.46 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left(\alpha x^k y^2 + \beta x^s y - \alpha\lambda^2 x^k + \beta\lambda x^s\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda];
 pde = (a*x^n + b*x^m + c)*D[w[x, y], x] + (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ (alpha*x^k*y^2 + beta*x^s*y - alpha*lambda^2*x^k +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\int \frac{x^k\alpha}{x^n a + x^m b + c} e^{\int \frac{-2x^k\alpha\lambda + x^s\beta}{x^n a + x^m b + c} dx} dx \lambda\, e^{\int \frac{-2x^k\alpha\lambda + x^s\beta}{x^n a + x^m b + c} dx + \int -\frac{-2x^k\alpha\lambda + x^s\beta}{x^n a + x^m b + c} dx} + y \int \frac{x^k\alpha}{x^n a + x^m}\right.\right.$$

42.47 problem number 47

problem number 332

Added January 2, 2019.

Problem 2.2.5.47 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n + bx^m + c)w_x - \left(sx^k y^2 - (ax^n + bx^m + c)y - s\lambda x^{k+2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda];
 pde = x*(a*x^n + b*x^m + c)*D[w[x, y], x] - (s*x^k*y^2 - (a*x^n + b*x^m + c)*y - s*lambda*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\tanh^{-1}\left(\frac{y}{\sqrt{\lambda}x}\right)-\sqrt{\lambda}\int_1^x\frac{sK[1]^k}{aK[1]^n+bK[1]^m+c}dK[1]}{\sqrt{\lambda}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=x*(a*x^n + b*x^m + c)*diff(w(x,y),x)- (s*x^k*y^2 -(a*x^n + b*x^m+c)*y - s*lambda*x^(k+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{1}{s\sqrt{\lambda}}\left(-\int\frac{x^k}{x^n a+x^m b+c}\,\mathrm{d}x s\sqrt{\lambda}+\operatorname{arctanh}\left(\frac{y}{x\sqrt{\lambda}}\right)\right)\right)$$

42.48 problem number 48

problem number 333

Added January 2, 2019.

Problem 2.2.5.48 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx^m + c)w_x + \left((ax^n + bx^m + c)y^2 - an(n-1)x^{n-2} - bm(m-1)x^{m-2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda];
 pde = (a*x^n + b*x^m + c)*D[w[x, y], x] + ((a*x^n + b*x^m + c)*y^2 - a*n*(n - 1)*x^(n - 2)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=(a*x^n + b*x^m + c)*diff(w(x,y),x)+ ((a*x^n+b*x^m + c)*y^2-a*n*(n-1)*x^(n-2)-b*m*(m-1)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{x^n acn + x^m bcm + x^{1+2\,n}ya^2 + yx^{2\,m+1}b^2 + a^2x^{2\,n}n + x^{2\,m}b^2m + c^2xy + 2\,ax^{m+n+1}yb +}\right.$$

42.49 problem number 49

problem number 334

Added January 2, 2019.

Problem 2.2.5.49 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + by^n + x)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + y\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda];
 pde = (a*x^n + b*y^n + x)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n - m) + y)*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n + b*y^n + x)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k) +beta*x^m*y^(n-m) + y )*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.50 problem number 50

problem number 335

Added January 2, 2019.

Problem 2.2.5.50 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + by^n + Ax^2 + Bxy)w_x + \left(\alpha x^k y^{n-k} + \beta x^m y^{n-m} + Axy + By^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B];
 pde = (a*x^n + b*y^n + A*x^2 + B*x*y)*D[w[x, y], x] + (alpha*x^k*y^(n - k) + beta*x^m*y^(n
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n + b*y^n + A*x^2 + B*x*y)*diff(w(x,y),x)+ (alpha*x^k*y^(n-k)+beta*x^m*y^(n-m) +
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.51 problem number 51

problem number 336

Added January 2, 2019.

Problem 2.2.5.51 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ay^m + bx^n + s)w_x - \left(\alpha x^k + bnx^{n-1}y + \beta\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*y^m + b*x^n + s)*D[w[x, y], x] - (alpha*x^k + b*n*x^(n - 1)*y + beta)*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*y^m + b*x^n + s)*diff(w(x,y),x)- (alpha*x^k + b*n*x^(n-1)* y + beta)*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{x^nbkmy + x^kx\alpha\, m + x^nbky + bymx^n + \beta\, kmx + kmsy + ay^myk + x^kx\alpha + x^nby + \beta\, k}{km + k + m + 1}\right.$$

42.52 problem number 52

problem number 337

Added January 2, 2019.

Problem 2.2.5.52 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^ny^m + x)w_x + \left(bx^ky^{n+m-k} + y\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*x^n*y^m + x)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n*y^m +x)*diff(w(x,y),x)+  (b*x^k*y^(n+m-k) + y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.53 problem number 53

problem number 338

Added January 2, 2019.

Problem 2.2.5.53 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ax^n y^m + \alpha)w_x - y(bx^n y^m + \beta)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = x*(a*x^n*y^m + alpha)*D[w[x, y], x] - y*(b*x^n*y^m + beta)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  x*(a*x^n*y^m +alpha)*diff(w(x,y),x)-  y*( b*x^n*y^m + beta )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((y^m)^{\alpha\,(an-bm)}\,x^{\beta\,m(an-bm)}(x^n y^m an - bx^n y^m m + \alpha\,n - \beta\,m)^{-m(a\beta-\alpha\,b)}\right)$$

42.54 problem number 54

problem number 339

Added January 2, 2019.

Problem 2.2.5.54 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(anx^k y^{n+k} + s)w_x - y\left(bmx^{m+k}y^k + s\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = x*(a*n*x^k*y^(n + k) + s)*D[w[x, y], x] - y*(b*m*x^(m + k)*y^k + s)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  x*(a*n*x^k*y^(n+k) + s)*diff(w(x,y),x)- y*( b*m*x^(m+k)*y^k + s )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.55 problem number 55

problem number 340

Added January 2, 2019.

Problem 2.2.5.55 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + Ax^2 + Bxy)w_x + \left(bx^k y^{n+m-k} + Axy + By^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*x^n*y^m + A*x^2 + B*x*y)*D[w[x, y], x] + (b*x^k*y^(n + m - k) + A*x*y + B*y^2)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n*y^m + A*x^2 + B*x*y)*diff(w(x,y),x)+ (b*x^k*y^(n+m-k) + A*x*y+ B*y^2)*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

42.56 problem number 56

problem number 341

Added January 2, 2019.

Problem 2.2.5.56 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxy^k)w_x + (\alpha y^s + \beta)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + (alpha*y^s + beta)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n*y^m + b*x*y^k)*diff(w(x,y),x)+ (alpha*y^s + beta)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b(n-1)\int\frac{y^k}{\alpha\,y^s+\beta}\,\mathrm{d}y} + an\int\frac{y^m}{\alpha\,y^s+\beta}\mathrm{e}^{b(n-1)\int\frac{y^k}{\alpha\,y^s+\beta}\,\mathrm{d}y}\,\mathrm{d}y - a\int\frac{y^m}{\alpha\,y^s+\beta}\mathrm{e}^{b(n-1)\int\frac{y^k}{\alpha\,y^s+\beta}\,\mathrm{d}y}\,\mathrm{c}\right.$$

43 HFOPDE, chapter 2.3.1

43.1 problem number 1

problem number 342

Added January 2, 2019.

Problem 2.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ae^{\lambda x} w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = D[w[x, y], x] + a*Exp[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - ae^{\lambda x}}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ a*exp(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a e^{\lambda x} - y\lambda}{\lambda}\right)$$

43.2 problem number 2

problem number 343

Added January 7, 2019.

Problem 2.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = D[w[x, y], x] + (a*Exp[lambda*x] + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{a\left(-e^{\lambda x}\right) - b\lambda x + \lambda y}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ (a*exp(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{bx\lambda + a e^{\lambda x} - y\lambda}{\lambda}\right)$$

43.3 problem number 3

problem number 344

Added January 7, 2019.

Problem 2.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = D[w[x, y], x] + (a*Exp[lambda*y] + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\log\left(\frac{e^{\lambda y}}{ae^{\lambda y}+b}\right) - b\lambda x}{b\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ (a*exp(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\ln\left(b\mathrm{e}^{bx\lambda - y\lambda} + \mathrm{e}^{bx\lambda}a\right)}{\lambda\, b}\right)$$

43.4 problem number 4

problem number 345

Added January 7, 2019.

Problem 2.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y+\beta x} + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(a\lambda e^{b\lambda x+\beta x}+\beta e^{\lambda(bx-y)}+b\lambda e^{\lambda(bx-y)}\right)}{b\lambda+\beta}\right)\right\}\right\}$$

Maple ✓

```

w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{-bx\lambda + y\lambda + \ln\left(\left(a\lambda\,\mathrm{e}^{\beta\,x+y\lambda}+\lambda\,b+\beta\right)^{-1}\right)}{\lambda\,b+\beta}\right)$$

43.5 problem number 5

problem number 346

Added January 7, 2019.

Problem 2.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda y+\beta x} + be^{\gamma x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = D[w[x, y], x] + (a*Exp[lambda*y + beta*x] + b*Exp[gamma*x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ (a*exp(lambda*y+beta*x)+b*exp(g*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{\lambda}\left(\mathrm{e}^{\frac{\lambda\,(b\mathrm{e}^{gx}-gy)}{g}} + a\int \mathrm{e}^{\beta\,x+\frac{\lambda\,b\mathrm{e}^{gx}}{g}}\,\mathrm{d}x\lambda\right)\right)$$

43.6 problem number 6

problem number 347

Added January 7, 2019.

Problem 2.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\beta y-\lambda x}\left(b\beta e^{\beta y}-a\lambda e^{\lambda x}\right)}{a\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(e^{\beta y}b\beta - a\lambda\, e^{\lambda x}\right)e^{-\beta y-\lambda x}}{b\beta\,\lambda}\right)$$

43.7 problem number 7

problem number 348

Added January 7, 2019.

Problem 2.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x}+b\right)w_x + \left(ce^{\beta x}+d\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-\lambda e^{\beta x}\text{Hypergeometric2F1}\left(1,\frac{\beta}{\lambda},\frac{\beta}{\lambda}+1,-\frac{ae^{\lambda x}}{b}\right)+\beta c\log\left(ae^{\lambda x}+b\right)+\beta d\log\left(ae^{\lambda x}+b\right)}{b\beta\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\int\frac{c+\mathrm{e}^{\beta x}+d}{a\mathrm{e}^{\lambda x}+b}\,\mathrm{d}x+y\right)$$

43.8 problem number 8

problem number 349

Added January 7, 2019.

Problem 2.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x}+b\right)w_x+\left(ce^{\beta y}+d\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*Exp[lambda*x] + b)*D[w[x, y], x] + (c + Exp[beta*y] + d)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\log\left(\left(ce^{\frac{\beta cx}{b}+\frac{\beta dx}{b}-\beta y}+de^{\frac{\beta cx}{b}+\frac{\beta dx}{b}-\beta y}+e^{\frac{\beta x(c+d)}{b}}\right)\left(ae^{\lambda x}+b\right)^{-\frac{\beta c}{b\lambda}-\frac{\beta d}{b\lambda}}\right)}{\beta(c+d)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*exp(lambda*x)+b)*diff(w(x,y),x)+ (c+exp(beta*y)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{1}{\lambda\,b\beta\,(c+d)}\left(yb\beta\,\lambda-\lambda\,x\beta\,c-\lambda\,x\beta\,d+\ln\left(a\mathrm{e}^{\lambda\,x}+b\right)\beta\,c+\ln\left(a\mathrm{e}^{\lambda\,x}+b\right)\beta\,d-\mathrm{RootOf}\right.\right.$$

Has RootOf

43.9 problem number 9

problem number 350

Added January 7, 2019.

Problem 2.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda y}+b\right)w_x+\left(ce^{\beta x}+d\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*Exp[lambda*y] + b)*D[w[x, y], x] + (c + Exp[beta*x] + d)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\beta e^{\lambda y}+b\beta\lambda y-\beta c\lambda x-\beta d\lambda x-\lambda e^{\beta x}}{\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*exp(lambda*y)+b)*diff(w(x,y),x)+ (c+exp(beta*x)+d)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{-yb\beta\,\lambda+\lambda\,x\beta\,c+\lambda\,x\beta\,d-a\mathrm{e}^{y\lambda}\beta+\mathrm{e}^{\beta\,x}\lambda}{\beta\,\lambda}\right)$$

43.10 problem number 10

problem number 351

Added January 7, 2019.

Problem 2.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x}+be^{\beta y}\right)w_x+a\lambda e^{\lambda x}w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s];
 pde = (a*Exp[lambda*x] + b*Exp[beta*y])*D[w[x, y], x] + a*lambda*Exp[lambda*x]*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),x)+ a*lambda*exp(lambda*x)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda x + \ln\left(a\beta - b\mathrm{e}^{\beta y - \lambda x} - a\right) - y}{\beta - 1}\right)$$

43.11 problem number 11

problem number 352

Added January 7, 2019.

Problem 2.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(ae^{\lambda x+\beta y} + c\mu\right) w_x - \left(be^{\gamma x+muy} + c\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = (a*Exp[lambda*x + beta*y] + c*mu)*D[w[x, y], x] - (b*Exp[gamma*x + mu*y] + c*lambda)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*exp(lambda*x+beta*y)+c*mu)*diff(w(x,y),x)- (b*exp(g*x+ mu*y)+c*lambda)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44 HFOPDE, chapter 2.3.2

44.1 problem number 1

problem number 353

Added January 7, 2019.

Problem 2.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y\text{ExpIntegralEi}\left(\frac{2ae^{\lambda x}}{\lambda}\right)-ae^{\lambda x}\text{ExpIntegralEi}\left(\frac{2ae^{\lambda x}}{\lambda}\right)+\lambda e^{\frac{2ae^{\lambda x}}{\lambda}}}{ae^{\lambda x}-y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (y^2+a*lambda*exp(lambda*x)- a^2*exp(2*lambda *x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-(a\mathrm{e}^{\lambda x}-y)\left(\text{expIntegral}\left(1,-2\,\frac{a\mathrm{e}^{\lambda x}}{\lambda}\right)\mathrm{e}^{\lambda x}a+\mathrm{e}^{2\,\frac{a\mathrm{e}^{\lambda x}}{\lambda}}\lambda-\text{expIntegral}\left(1,-2\,\frac{a\mathrm{e}^{\lambda x}}{\lambda}\right)y\right)\right.$$

44.2 problem number 2

problem number 354

Added January 7, 2019.

Problem 2.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + by + a(\lambda - b)e^{\lambda x} - a^2 e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (y^2 + b*y + a*(lambda - b)*Exp[lambda*x] - a^2*Exp[2*lambda*x])*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{2^{b/\lambda}\lambda^{-\frac{b}{\lambda}}e^{bx}a^{b/\lambda}\left(-y\text{LaguerreL}\left(-\frac{b}{\lambda},\frac{b}{\lambda},\frac{2ae^{\lambda x}}{\lambda}\right)+2ae^{\lambda x}\text{LaguerreL}\left(-\frac{b}{\lambda}-1,\frac{b}{\lambda}+1,\frac{2ae^{\lambda x}}{\lambda}\right.\right.}{ae^{\lambda x}-y}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (y^2+b*y+ a*(lambda-b)*exp(lambda*x) - a^2*exp(2*lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{a\mathrm{e}^{\lambda x}-y}\left(\mathrm{e}^{\frac{bx\lambda+2\,a\mathrm{e}^{\lambda x}}{\lambda}} - a\mathrm{e}^{\lambda x}\int \mathrm{e}^{\frac{bx\lambda+2\,a\mathrm{e}^{\lambda x}}{\lambda}}\,\mathrm{d}x + y\int \mathrm{e}^{\frac{bx\lambda+2\,a\mathrm{e}^{\lambda x}}{\lambda}}\,\mathrm{d}x\right)\right)$$

44.3 problem number 3

problem number 355

Added January 7, 2019.

Problem 2.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ae^{\lambda x}y - abe^{\lambda x} - b^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (y^2 + a*Exp[lambda*x]*y - a*b*Exp[lambda*x] - b^2)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{2b(-1)^{-\frac{b}{\lambda}}\lambda^{-\frac{2b}{\lambda}-1}\left(y\lambda^{\frac{2b}{\lambda}}\text{Gamma}\left(\frac{2b}{\lambda},0,-\frac{ae^{\lambda x}}{\lambda}\right) - b\lambda^{\frac{2b}{\lambda}}\text{Gamma}\left(\frac{2b}{\lambda},0,-\frac{ae^{\lambda x}}{\lambda}\right) + \lambda(-1)^{\frac{2b}{\lambda}}a\right.}{b-y}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (y^2+a*exp(lambda*x)*y-a*b*exp(lambda*x)- b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{-b+y}\left(\mathrm{e}^{\frac{2\,bx\lambda+a\mathrm{e}^{\lambda\,x}}{\lambda}} + y\int \mathrm{e}^{\frac{2\,bx\lambda+a\mathrm{e}^{\lambda\,x}}{\lambda}}\,\mathrm{d}x - b\int \mathrm{e}^{\frac{2\,bx\lambda+a\mathrm{e}^{\lambda\,x}}{\lambda}}\,\mathrm{d}x\right)\right)$$

44.4 problem number 4

problem number 356

Added January 7, 2019.

Problem 2.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(y^2 - axe^{\lambda x}y + ae^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] - (y^2 - a*x*Exp[lambda*x]*y + a*Exp[lambda*x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)-  (y^2-a*x*exp(lambda*x)*y + a*exp(lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\lambda^2 x (yx-1)}\left(yx^2\int \frac{1}{x^2}\mathrm{e}^{\frac{a\mathrm{e}^{\lambda x}(\lambda x-1)}{\lambda^2}}\,\mathrm{d}x - \mathrm{e}^{\frac{a\mathrm{e}^{\lambda x}(\lambda x-1)}{\lambda^2}} - x\int \frac{1}{x^2}\mathrm{e}^{\frac{a\mathrm{e}^{\lambda x}(\lambda x-1)}{\lambda^2}}\,\mathrm{d}x\right)\right)$$

44.5 problem number 5

problem number 357

Added January 7, 2019.

Problem 2.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + be^{-\lambda x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{x\left(-\sqrt{\lambda^2-4ab}\right)}\left(\sqrt{\lambda^2-4ab}-2aye^{\lambda x}-\lambda\right)}{2aye^{\lambda x}\sqrt{\lambda^2-4ab}+\lambda\sqrt{\lambda^2-4ab}-4ab+\lambda^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\lambda}{\sqrt{\lambda^2\left(4ab-\lambda^2\right)}}\left(2\lambda\arctan\left(\frac{\lambda\left(2\,e^{\lambda x}ay+\lambda\right)}{\sqrt{4\lambda^2ab-\lambda^4}}\right)-\sqrt{\lambda^2\left(4ab-\lambda^2\right)}x\right)\right)$$

44.6 problem number 6

problem number 358

Added January 7, 2019.

Problem 2.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ae^{\lambda x}y^2 + b\mu e^{\mu x} - ab^2e^{(\lambda+2\mu)x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*mu*Exp[mu*x] - a*b^2*Exp[(lambda + 2*mu)*x])
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*mu*exp(mu*x) - a*b^2*exp((lambda + 2*mu)*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.7 problem number 7

problem number 359

Added January 7, 2019.

Problem 2.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ae^{\lambda x}y^2 + by + ce^{-\lambda x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*y + c*Exp[-(lambda*x)])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{}{2ay\sqrt{-4ac+b^2+2b\lambda+\lambda^2}e^{x\sqrt{-4ac+b^2+2b\lambda+\lambda^2}+\lambda x} + b^2e^{x\sqrt{-4ac+b^2+2b\lambda+\lambda^2}} + 2b\lambda e^{x\sqrt{-4ac+b^2}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + b*y +c*exp(-lambda*x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{\sqrt{(b+\lambda)^2(4\,ca-b^2-2\,\lambda\,b-\lambda^2)}}\left(\sqrt{(b+\lambda)^2(4\,ca-b^2-2\,\lambda\,b-\lambda^2)}bx + \sqrt{(b+\lambda)^2}\right.\right.$$

44.8 problem number 8

problem number 360

Added January 7, 2019.

Problem 2.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + \mu y - ab^2e^{(\lambda+2\mu)x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + mu*y - a*b^2*Exp[(lambda + 2*mu)*x])*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

kernel error generated

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2 + mu*y - a*b^2*exp((lambda+2*mu)*x))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(e^{\lambda x}\sinh\left(\frac{abe^{x(\lambda+\mu)}}{\lambda+\mu}\right)y + e^{x(\lambda+\mu)}\cosh\left(\frac{abe^{x(\lambda+\mu)}}{\lambda+\mu}\right)b\right)\left(e^{\lambda x}\cosh\left(\frac{abe^{x(\lambda+\mu)}}{\lambda+\mu}\right)y + e\right.\right.$$

44.9 problem number 9

problem number 361

Added January 7, 2019.

Problem 2.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}y^2 + ae^{\mu x}y + a\lambda e^{(\mu-lambda)x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda)*x])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\mu^{-\frac{\lambda}{\mu}}e^{-\lambda x}\left(y(-1)^{\lambda/\mu}e^{2\lambda x}a^{\lambda/\mu}\text{Gamma}\left(-\frac{\lambda}{\mu},-\frac{ae^{\mu x}}{\mu}\right)+\lambda(-1)^{\lambda/\mu}e^{\lambda x}a^{\lambda/\mu}\text{Gamma}\left(-\frac{\lambda}{\mu},-\frac{ae}{\mu}\right.\right.}{ye^{\lambda x}+\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (exp(lambda*x)*y^2  + a*exp(mu*x)*y+a*lambda*exp((mu-lambda)*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-e^{\lambda x}\left(e^{\lambda x}y\lambda-e^{\lambda x}y\mu+\lambda^2-\lambda\mu\right)\left(e^{\lambda x}y_1F_1(-\frac{\lambda}{\mu};-\frac{\lambda-\mu}{\mu};\frac{ae^{\mu x}}{\mu})\lambda-e^{\lambda x}y_1F_1(-\frac{\lambda}{\mu};-\frac{\lambda}{\mu}\right.\right.$$

44.10 problem number 10

problem number 362

Added January 7, 2019.

Problem 2.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x-\left(\lambda e^{\lambda x}y^2-ae^{\mu x}y+ae^{(\mu-lambda)x}\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] - (lambda*Exp[lambda*x]*y^2 - a*Exp[mu*x]*y + a*lambda*Exp[(mu - lambda
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\mu\left(\lambda y e^{\lambda x}\text{LaguerreL}\left(\frac{\lambda^2}{\mu}-\frac{\lambda}{\mu},\frac{\lambda}{\mu},\frac{ae^{\mu x}}{\mu}\right)+ae\right.}{\lambda\left(-\mu y e^{\lambda x}\text{HypergeometricU}\left(\frac{\lambda}{\mu}-\frac{\lambda^2}{\mu},\frac{\lambda}{\mu}+1,\frac{ae^{\mu x}}{\mu}\right)+\mu\text{HypergeometricU}\left(\frac{\lambda}{\mu}-\frac{\lambda^2}{\mu},\frac{\lambda}{\mu}+\right.\right.}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)-  (lambda*exp(lambda*x)*y^2  - a*exp(mu*x)*y + a*lambda*exp((mu-lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-1\left(\mathrm{e}^{\lambda x}yM\left(-\frac{\lambda(\lambda-1)}{\mu},\frac{\lambda+\mu}{\mu},\frac{a\mathrm{e}^{\mu x}}{\mu}\right)\lambda-\mathrm{e}^{\mu x}M\left(-\frac{\lambda(\lambda-1)}{\mu},\frac{\lambda+\mu}{\mu},\frac{a\mathrm{e}^{\mu x}}{\mu}\right)a+M\left(\right.\right.\right.$$

44.11 problem number 11

problem number 363

Added January 7, 2019.

Problem 2.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}y^2+abe^{(\lambda+\mu)x}y-b\mu e^{\mu x}\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*b*Exp[(lambda + mu)*x]*y - b*mu*Exp[mu*x])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp(lambda*x)*y^2+ a*b*exp((lambda +mu)*x)*y - b*mu*exp(mu*x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-a\left(ye^{\lambda x} + e^{x(\lambda+\mu)}b\right) e^{\frac{abe^{x(\lambda+\mu)}}{\lambda+\mu}} \left(\text{WhittakerM}\left(-1/2\,\frac{\mu}{\lambda+\mu}, 1/2\,\frac{3\,\lambda+2\,\mu}{\lambda+\mu}, \frac{abe^{x(\lambda+\mu)}}{\lambda+\mu}\right) ya^2\right.\right.$$

44.12 problem number 12

problem number 364

Added January 7, 2019.

Problem 2.3.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{(2\lambda+\mu)x}y^2 + \left(be^{(\lambda+\mu)x} - \lambda\right)y + ce^{\mu x}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[(2*lambda + mu)*x]*y^2 + (b*Exp[(lambda + mu)*x] - lambda)*y +
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{i\pi e^{-\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}}\left(\sqrt{b^2-4ac}e^{x(\lambda+\mu)}-2aye^{x(2\lambda+\mu)}+b\left(-e^{x(\lambda+\mu)}\right)\right.}{2\left(2aye^{x(2\lambda+\mu)}\cosh\left(\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}\right)+\sqrt{b^2-4ac}e^{x(\lambda+\mu)}\sinh\left(\frac{\sqrt{b^2-4ac}e^{x(\lambda+\mu)}}{2(\lambda+\mu)}\right)+be^{x(\lambda+\mu}\right.}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  (a*exp((2*lambda +mu)*x)*y^2+ (b*exp((lambda +mu)*x) -lambda)*y + c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{b}{\sqrt{b^2\left(4\,ca-b^2\right)}\left(\lambda+\mu\right)}\left(2\,b\lambda\,\arctan\left(\frac{b\left(2\,e^{\lambda\,x}ay+b\right)}{\sqrt{b^2\left(4\,ca-b^2\right)}}\right)+2\,b\mu\,\arctan\left(\frac{b\left(2\,e^{\lambda\,x}ay+b\right)}{\sqrt{b^2\left(4\,ca-b^2\right)}}\right.\right.\right.$$

44.13 problem number 13

problem number 365

Added January 7, 2019.

Problem 2.3.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(e^{\lambda x}(y-be^{\mu x})^2+b\mu e^{\mu x}\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (Exp[lambda*x]*(y - b*Exp[mu*x])^2 + b*mu*Exp[mu*x])*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{be^{\lambda x+\mu x}-ye^{\lambda x}-\lambda}{\lambda\left(be^{\mu x}-y\right)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( exp(lambda*x) *(y- b*exp(mu*x))^2 + b*mu*exp(mu*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\mathrm{b}e^{\lambda x+\mu x}-y\mathrm{e}^{\lambda x}-\lambda}{\lambda\left(b\mathrm{e}^{\mu x}-y\right)}\right)$$

44.14 problem number 14

problem number 366

Added January 7, 2019.

Problem 2.3.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{\lambda x}y^2+bnx^{n-1}-ab^2e^{\lambda x}x^{2n}\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + b*n*x^(n - 1) - a*b^2*Exp[lambda*x]*x^(2*n))*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^2+ b*n*x^(n-1) - a*b^2*exp(lambda*x)*x^(2*n))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.15 problem number 15

problem number 367

Added January 7, 2019.

Problem 2.3.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x}y^2 + ax^n y + a\lambda x^n e^{-\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (Exp[lambda*x]*y^2 + a*x^n*y + a*lambda*x^n*Exp[-(lambda*x)])*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( exp(lambda*x)*y^2+ a*x^n*y + a*lambda*x^n*exp(-lambda*x))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{y\mathrm{e}^{\lambda x}+\lambda}\left(\mathrm{e}^{\lambda x}y\int \mathrm{e}^{\frac{x(x^n a-\lambda n-\lambda)}{n+1}}dx+\int \mathrm{e}^{\frac{x(x^n a-\lambda n-\lambda)}{n+1}}dx\lambda+\mathrm{e}^{\frac{x(x^n a-\lambda n-\lambda)}{n+1}}\right)\right)$$

44.16 problem number 16

problem number 368

Added January 7, 2019.

Problem 2.3.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda e^{\lambda x}y^2 + ax^n e^{\lambda x}y - ax^n e^{2\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (lambda*Exp[lambda*x]*y^2 + a*x^n*Exp[lambda*x]*y - a*x^n*Exp[2*lambd
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  ( lambda*exp(lambda*x)*y^2+ a*x^n*exp(lambda*x)*y - a*x^n*exp(2*lam
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.17 problem number 17

problem number 369

Added January 7, 2019.

Problem 2.3.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 - abx^n e^{\lambda x}y + bnx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 - a*b*x^n*Exp[lambda*x]*y + b*n*x^(n - 1))*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  ( a*exp(lambda*x)*y^2- a*b*x^n*exp(lambda*x)*y + b*n*x^(n-1))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(a(bx^n - y)\left(-\int \lambda\, \mathrm{e}^{-\frac{ab(-\lambda)^{-n}\left(x^n(-\lambda)^n n\Gamma(n)(-\lambda x)^{-n} - x^n(-\lambda)^n \mathrm{e}^{\lambda x} - x^n(-\lambda)^n n(-\lambda x)^{-n}\Gamma(n,-\lambda x)\right)}{\lambda} + \lambda x}\, \mathrm{d}x x^n a$$

44.18 problem number 18

problem number 370

Added January 7, 2019.

Problem 2.3.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + b\lambda e^{\lambda x} - ab^2 x^n e^{2\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*y^2 + b*lambda*Exp[lambda*x] - a*b^2*x^n*Exp[2*lambda*x])*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 + b*lambda*exp(lambda*x) - a*b^2*x^n*exp(2*lambda*x))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.19 problem number 19

problem number 371

Added January 7, 2019.

Problem 2.3.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 + \lambda y - ab^2 x^n e^{2\lambda x}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*y^2 + lambda*y - a*b^2*x^n*Exp[2*lambda*x])*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-i\left(ab(-1)^{-n}\lambda^{-n-1}\text{Gamma}(n+1,-\lambda x)+\tanh^{-1}\left(\frac{ye^{-\lambda x}}{b}\right)\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+  ( a*x^n*y^2 + lambda*y - a*b^2*x^n*exp(2*lambda*x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{i}{\lambda}\left(a\Gamma(n)\,x^n(-\lambda\,x)^{-n}\,bn - \Gamma(n,-\lambda\,x)\,ax^n(-\lambda\,x)^{-n}\,bn - e^{\lambda\,x}x^nab - \operatorname{arctanh}\left(\frac{e^{-\lambda\,x}y}{b}\right)\lambda\right.\right.$$

44.20 problem number 20

problem number 372

Added January 7, 2019.

Problem 2.3.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^2 - abx^ne^{\lambda x}y + b\lambda e^{\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*y^2 - a*b*x^n*Exp[lambda*x]*y + b*lambda*Exp[lambda*x])*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*b*x^n*exp(lambda*x)*y + b*lambda*exp(lambda*x) )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.21 problem number 21

problem number 373

Added January 7, 2019.

Problem 2.3.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(ax^n y^2 - ax^n \left(be^{\lambda x} + c\right) y + b\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*y^2 - a*x^n*(b*Exp[lambda*x] + c)*y + b*lambda*Exp[lambda*x])*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( a*x^n*y^2 - a*x^n*(b*exp(lambda*x) + c )*y + b*lambda*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.22 problem number 22

problem number 374

Added January 7, 2019.

Problem 2.3.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n e^{2\lambda x} y^2 + \left(bx^n e^{\lambda x} - \lambda\right) y + cx^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*Exp[2*lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - lambda)*y + c*x^n
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-4a^{3/2}c^{3/2}(-1)^{-n}\lambda^{-n-1}\text{Gamma}(n+1,-\lambda x)+\sqrt{a}b^2\sqrt{c}(-1)^{-n}\lambda^{-n-1}\text{Gamma}(n+1,-\right.\right.\right.}{4ac-b^2}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (a*x^n*exp(2*lambda*x)*y^2 + (b*x^n*exp(lambda*x) - lambda)*y + c*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{b}{\sqrt{b^2(4ca-b^2)}\lambda}\left(-e^{\lambda x}\sqrt{b^2(4ca-b^2)}x^n - (-\lambda x)^{-n}\Gamma(n,-\lambda x)\sqrt{b^2(4ca-b^2)}x^n n + 2\right.\right.$$

44.23 problem number 23

problem number 375

Added January 10, 2019.

Problem 2.3.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}(y - bx^n - c)^2 + bnx^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*(y - b*x^n - c)^2 + b*n*x^(n - 1))*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{abe^{\lambda x}x^n + ace^{\lambda x} - aye^{\lambda x} - \lambda}{\lambda\left(bx^n + c - y\right)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (  a*exp(lambda*x)*(y- b*x^n - c)^2 +b*n*x^(n-1))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{e^{\lambda x}x^n ab - e^{\lambda x}ay + e^{\lambda x}ac - \lambda}{\lambda\left(bx^n + c - y\right)}\right)$$

44.24 problem number 24

problem number 376

Added January 10, 2019.

Problem 2.3.2.24 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + 2a\lambda x e^{\lambda x^2} - a^2 e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*Exp[2*lambda*x^2])*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (    y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*exp(2*lambda*x^2))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

44.25 problem number 25

problem number 377

Added January 10, 2019.

Problem 2.3.2.25 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{-\lambda x^2}y^2 + \lambda xy + ab^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*Exp[-(lambda*x^2)]*y^2 + lambda*x*y + a*b^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{2}\left(2\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right)-\frac{\sqrt{2\pi}ab\text{Erf}\left(\frac{\sqrt{\lambda}x}{\sqrt{2}}\right)}{\sqrt{\lambda}}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (     a*exp(-lambda*x^2)*y^2 + lambda*x*y + a*b^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{1}{\sqrt{\lambda}}\left(ab\sqrt{\pi}\sqrt{2}\,\text{erf}\left(1/2\,\sqrt{2}\sqrt{\lambda}x\right)-2\,\arctan\left(\frac{e^{-1/2\,\lambda\,x^2}y}{b}\right)\sqrt{\lambda}\right)\right)$$

44.26 problem number 26

problem number 378

Added January 10, 2019.

Problem 2.3.2.26 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^ny^2 + \lambda xy + ab^2x^ne^{\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu];
 pde = D[w[x, y], x] + (a*x^n*y^2 + lambda*x*y + a*b^2*x^n*Exp[lambda*x^2])*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{1}{2}i\left(abi^{-n}2^{\frac{n}{2}+\frac{1}{2}}\lambda^{-\frac{n}{2}-\frac{1}{2}}\text{Gamma}\left(\frac{n}{2}+\frac{1}{2},-\frac{\lambda x^2}{2}\right)+2i\tan^{-1}\left(\frac{ye^{-\frac{\lambda x^2}{2}}}{b}\right)\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (   a*x^n*y^2 + lambda*x*y + a*b^2*x^n*exp(lambda*x^2) )*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(2^{n/2-1/2}x^{n+1}ab\left(-\lambda\,x^2\right)^{-1/2-n/2}\Gamma(n/2+1/2)-2^{n/2-1/2}x^{n+1}ab\left(-\lambda\,x^2\right)^{-1/2-n/2}\Gamma\left(n/2+\right.\right.$$

44.27 problem number 27

problem number 379

Added January 10, 2019.

Problem 2.3.2.27 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(ae^{2\lambda x}y^3+be^{\lambda x}y^2+cy+de^{-\lambda x}\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Exp[2*lambda*x]*y^3 + b*Exp[lambda*x]*y^2 + c*y + d*Exp[-(lambda*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ (  a*exp(2*lambda*x)*y^3 + b*exp(lambda*x)*y^2 + c*y+ d*exp(-lambda*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x - \sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\_Z^2+(c+\lambda)\_Z+d\right)} \frac{\ln\left(y\mathrm{e}^{\lambda x} - \_R\right)}{3\_R^2 a + 2\_R b + c + \lambda}\right)$$

Solution contains RootOf

44.28 problem number 28

problem number 380

Added January 10, 2019.

Problem 2.3.2.28 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^3 + 3abe^{\lambda x}y^2 + cy - 2ab^3e^{\lambda x} + bc\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^3 + 3*a*b*Exp[lambda*x]*y^2 + c*y - 2*a*b^3*Exp[la
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\frac{6ab^2e^{\lambda x}}{\lambda}}\left(2y^2e^{\frac{6ab^2e^{\lambda x}}{\lambda}}\int_1^x a\exp\left(-\frac{6ab^2e^{\lambda K[1]}}{\lambda}+2cK[1]+\lambda K[1]\right)dK[1]+4bye^{\frac{6ab^2e^{\lambda x}}{\lambda}}\left(\int_1^x\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+ ( a*exp(lambda*x)*y^3 + 3*a*b*exp(lambda*x)*y^2 + c*y- 2*a*b^3*exp(l
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{1}{(b+y)^2}\left(2\,ab^2\int\mathrm{e}^{-\frac{6\,ae^{\lambda\,x}b^2-2\,c\lambda\,x-\lambda^2x}{\lambda}}\,\mathrm{d}x+4\,yab\int\mathrm{e}^{-\frac{6\,ae^{\lambda\,x}b^2-2\,c\lambda\,x-\lambda^2x}{\lambda}}\,\mathrm{d}x+2\,y^2a\int\mathrm{e}^{-\frac{6\,ae^{\lambda\,x}b^2}{}}\right.\right.$$

44.29 problem number 29

problem number 381

Added January 10, 2019.

Problem 2.3.2.29 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ae^{\lambda x}y^2+ky+ab^2x^{2k}e^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + k*y + a*b^2*x^(2*k)*Exp[lambda*x])*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(a\sqrt{b^2}x^k(-\lambda x)^{-k}\text{Gamma}(k,-\lambda x)+\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  x*diff(w(x,y),x)+ ( a*exp(lambda*x)* y^2 + k*y + a*b^2*x^(2*k)*exp(lambda*x) )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(a\Gamma(k)\,x^k(-\lambda\,x)^{-k}\,b-\Gamma(k,-\lambda\,x)\,ax^k(-\lambda\,x)^{-k}\,b-\arctan\left(\frac{yx^{-k}}{b}\right)\right)$$

44.30 problem number 30

problem number 382

Added January 10, 2019.

Problem 2.3.2.30 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n}e^{\lambda x}y^2+(bx^ne^{\lambda x}-n)y+ce^{\lambda x}\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*x^(2*n)*Exp[lambda*x]*y^2 + (b*x^n*Exp[lambda*x] - n)*y + c*Exp[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-b^2c(-\lambda x)^{-n}\sqrt{\frac{ax^{2n}}{c}}\text{Gamma}(n,-\lambda x)+4ac^2(-\lambda x)^{-n}\sqrt{\frac{ax^{2n}}{c}}\text{Gamma}(n,-\lambda x)-2\sqrt{a}\sqrt{}}{4ac-b^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  x*diff(w(x,y),x)+ (  a*x^(2*n)*exp(lambda*x)*y^2 + (b*x^n*exp(lambda*x) - n)*y + c*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{b}{\sqrt{b^2\left(4\,ca-b^2\right)}}\left(-\Gamma(n)\left(-\lambda\,x\right)^{-n}\sqrt{b^2\left(4\,ca-b^2\right)}x^n+\Gamma(n,-\lambda\,x)\left(-\lambda\,x\right)^{-n}\sqrt{b^2\left(4\,ca-\right.}\right.\right.$$

44.31 problem number 31

problem number 383

Added January 10, 2019.

Problem 2.3.2.31 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$yw_x+e^{\lambda x}\left((2a\lambda x+a+b)y-e^{\lambda x}(a^2\lambda x^2+abx-c)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = y*D[w[x, y], x] + Exp[lambda*x]*((2*a*lambda*x + a + b)*y - Exp[lambda*x]*(a^2*lambda
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  y*diff(w(x,y),x)+ exp(lambda*x)* ( (2*a*lambda*x+a + b)*y - exp(lambda*x)*(a^2*lambd
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{a}\left(2\,ax\lambda\,\mathrm{e}^{2\,1\arctan\left(\frac{2\,y\lambda\,\mathrm{e}^{-\lambda\,x}-2\,ax\lambda-b}{a}\frac{1}{\sqrt{-\frac{b^2+4\,\lambda\,c}{a^2}}}\right)\frac{1}{\sqrt{-\frac{b^2+4\,\lambda\,c}{a^2}}}} + \sqrt{-\frac{b^2+4\,\lambda\,c}{a^2}}\int^{-2\,1\arctan\left(\right.}\right.\right.$$

44.32 problem number 32

problem number 384

Added January 10, 2019.

Problem 2.3.2.32 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^m w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*y^m*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{e^{-\lambda x}y^{-m}\left(a\lambda ye^{\lambda x}+by^m-bmy^m\right)}{a\lambda(m-1)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   a*exp(lambda*x)*diff(w(x,y),x)+  b*y^m*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{be^{-\lambda x}m-y^{1-m}a\lambda-be^{-\lambda x}}{a\lambda}\right)$$

44.33 problem number 33

problem number 385

Added January 10, 2019.

Problem 2.3.2.33 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ae^y+bx)w_x+w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*Exp[y] + b*x)*D[w[x, y], x] + D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*exp(y)+b*x)*diff(w(x,y),x)+ diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{\left(x\mathrm{e}^{y(b-1)}b - x\mathrm{e}^{y(b-1)} + a\mathrm{e}^{by}\right)\mathrm{e}^{-y(2b-1)}}{b-1}\right)$$

44.34 problem number 34

problem number 386

Added January 10, 2019.

Problem 2.3.2.34 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n e^{\lambda y} + bxy^m)w_x + e^{\mu y}w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*Exp[lambda*y] + b*x*y^m)*D[w[x, y], x] + Exp[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*x^n*exp(lambda*y)+ b*x*y^m)*diff(w(x,y),x)+ exp(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{(m+1)^{-1}} e^{\frac{bne^{-1/2\,\mu\,y}y^m(\mu\,y)^{-m/2}\,\mathrm{WhittakerM}(m/2,m/2+1/2,\mu\,y)}{\mu\,(m+1)}} x^{\frac{m}{m+1}}\left(x^{\frac{mn}{m+1}}\right)^{-1}\left(x^{\frac{n}{m+1}}\right)^{-1}\left(e^{\frac{be^{-1/2\,\mu\,y}y^m(\mu}{}}\right.\right.$$

44.35 problem number 35

problem number 387

Added January 10, 2019.

Problem 2.3.2.35 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxe^{\lambda y})w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*y^m + b*x*Exp[lambda*y])*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n*y^m+ b *x*exp(lambda*y))*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x}{x^n}\mathrm{e}^{\frac{y^{-k}\mathrm{e}^{y\lambda}bn}{\lambda}}\mathrm{e}^{\frac{(-y\lambda)^k\Gamma(-k,-y\lambda)bky^{-k}}{\lambda}}\mathrm{e}^{\frac{y^{-k}(-y\lambda)^k b\Gamma(1-k)}{\lambda}}\left(\mathrm{e}^{\frac{(-y\lambda)^k\Gamma(-k,-y\lambda)bkny^{-k}}{\lambda}}\right)^{-1}\left(\mathrm{e}^{\frac{y^{-k}(-y\lambda)^k b\Gamma(1-k)n}{\lambda}}\right.\right.$$

44.36 problem number 36

problem number 388

Added January 10, 2019.

Problem 2.3.2.36 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bxy^k)w_x + e^{\lambda y}w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*y^m + b*x*y^k)*D[w[x, y], x] + Exp[lambda*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n*y^m+ b *x*y^k)*diff(w(x,y),x)+ exp(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{(k+1)^{-1}} e^{\frac{bn e^{-1/2\,y\lambda} y^{k} (y\lambda)^{-k/2} \mathrm{WhittakerM}(k/2,k/2+1/2,y\lambda)}{\lambda\,(k+1)}} x^{\frac{k}{k+1}} \left(x^{\frac{kn}{k+1}}\right)^{-1} \left(x^{\frac{n}{k+1}}\right)^{-1} \left(e^{\frac{b e^{-1/2\,y\lambda} y^{k} (y\lambda)^{-k/2}}{\lambda}}\right.\right.$$

45 HFOPDE, chapter 2.4.1

45.1 problem number 1

problem number 389

Added January 10, 2019.

Problem 2.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - a\cosh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\cosh(\lambda x)\,a - y\lambda}{\lambda}\right)$$

45.2 problem number 2

problem number 390

Added January 10, 2019.

Problem 2.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)-a\mu x}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{x\mu\, a + 2\,\mathrm{arctanh}\left(\mathrm{e}^{\mu\, y}\right)}{\mu\, a}\right)$$

45.3 problem number 3

problem number 391

Added January 10, 2019.

Problem 2.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda \sinh(\lambda x) - a^2 \sinh^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sinh[lambda*x] - a^2*Sinh[lambda*x]^2)*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{e^{-\frac{ae^{-\lambda x}}{\lambda}}\left(-2ye^{\frac{ae^{-\lambda x}}{\lambda}+\lambda x}\int_1^{e^{\lambda x}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}\,dK[1] + ae^{\frac{ae^{-\lambda x}}{\lambda}}\int_1^{e^{\lambda x}} \frac{e^{\frac{a\left(K[1]^2-1\right)}{\lambda K[1]}}}{K[1]}\,dK[1] + ae^{\frac{ae^{-}}{\lambda}}\right.}{ae^{2\lambda x} + a - 2ye^{\lambda x}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ (y^2-a^2 + a*lambda*sinh(lambda*x) - a^2*sinh(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\sqrt{\sinh(\lambda x) + i}\left(-2\,i(\sinh(\lambda x))^2 \cosh(\lambda x)\,\mathrm{HeunC}\left(\frac{4\,ia}{\lambda}, -1/2, -1/2, \frac{2\,ia}{\lambda}, -1/8\,\frac{8\,ia -}{\lambda}\right.\right.\right.$$

45.4 problem number 4

problem number 392

Added January 10, 2019.

Problem 2.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \lambda\big(\sinh(\lambda x)y^2 - \sinh^3(\lambda x)\big)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + lambda*(Sinh[lambda*x]*y^2 - Sinh[lambda*x]^3)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ lambda*(sinh(lambda*x)*y^2 - sinh(lambda*x)^3)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-\cosh(\lambda x) + y)\sqrt{\pi}}{\sqrt{\pi}\cosh(\lambda x)\operatorname{erfi}(\cosh(\lambda x)) - \sqrt{\pi}\operatorname{erfi}(\cosh(\lambda x))\, y - 2\,\mathrm{e}^{(\cosh(\lambda x))^2}}\right)$$

45.5 problem number 5

problem number 393

Added January 10, 2019.

Problem 2.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((a\sinh^2(\lambda x) - \lambda)y^2 - a\sinh^2(\lambda x) + \lambda - a\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + ((a*Sinh[lambda*x]^2 - lambda)*y^2 - a*Sinh[lambda*x]^2 + lambda - a)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ ((a*sinh(lambda*x)^2-lambda)*y^2 - a*sinh(lambda*x)^2 + lambda - a)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\left(-2\left(\cosh\left(2\lambda x\right)\right)^2 \left(\sinh\left(\lambda x\right)\right)^2 ya + 4\cosh\left(2\lambda x\right)\left(\sinh\left(\lambda x\right)\right)^2 ya + \left(\cosh\left(2\lambda x\right)\right)^2 \sinh\right.\right.$$

45.6 problem number 6

problem number 394

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\lambda x)w_x + a(\sinh(\mu y))\,w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Sinh[lambda*x]*D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\sinh^{-\frac{a\mu}{\lambda}}\left(\frac{\lambda x}{2}\right)\cosh^{\frac{a\mu}{\lambda}}\left(\frac{\lambda x}{2}\right)\right)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := sinh(lambda*x)*diff(w(x,y),x)+ a*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-2\,\frac{-\operatorname{arctanh}\left(e^{\lambda x}\right)\mu\,a + \operatorname{arctanh}\left(e^{\mu y}\right)\lambda}{\lambda\,\mu\,a}\right)$$

45.7 problem number 7

problem number 395

Added January 10, 2019.

Problem 2.4.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\mu y)w_x + a(\sinh(\lambda x))\, w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Sinh[mu*yx]*D[w[x, y], x] + a*Sinh[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - a\cosh(\lambda x)\text{csch}(\mu\text{yx})}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := sinh(mu*y)*diff(w(x,y),x)+ a*sinh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\cosh\left(\lambda\, x\right)\mu\, a - \cosh\left(\mu\, y\right)\lambda}{\lambda\,\mu\, a}\right)$$

46 HFOPDE, chapter 2.4.2

46.1 problem number 1

problem number 396

Added January 10, 2019.

Problem 2.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a(\cosh(\lambda x))\, w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - a\sinh(\lambda x)}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\sinh(\lambda x) - y\lambda}{\lambda}\right)$$

46.2 problem number 2

problem number 397

Added January 10, 2019.

Problem 2.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a(\cosh(\lambda x))\, w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Cosh[lambda*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\tan^{-1}\left(\tanh\left(\frac{\lambda y}{2}\right)\right)-a\lambda x}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ a*cosh(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-ax\lambda + 2\arctan\left(\mathrm{e}^{y\lambda}\right)}{a\lambda}\right)$$

46.3 problem number 3

problem number 398

Added January 10, 2019.

Problem 2.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((a\cosh^2(\lambda x) - \lambda)y^2 - a\cosh^2(\lambda x) + \lambda + a\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + ((a*Cosh[lambda*x]^2 - lambda)*y^2 - a*Cosh[lambda*x]^2 + lambda + a)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ ( (a *cosh(lambda*x)^2-lambda)*y^2 - a*cosh(lambda*x)^2+ lambda + a)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\sqrt{-1+\cosh(2\lambda x)}\left(-8(\cosh(\lambda x))^6 ya + 8(\cosh(\lambda x))^4 y\lambda + (\cosh(2\lambda x))^2 \sinh(2\lambda\right.\right.$$

46.4 problem number 4

problem number 399

Added January 10, 2019.

Problem 2.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2w_x + \left((a - \lambda + a\cosh(\lambda x))y^2 + a + \lambda - a\cosh(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = 2*D[w[x, y], x] + ((a - lambda + a*Cosh[lambda*x])*y^2 + a + lambda - a*Cosh[lambda*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := 2*diff(w(x,y),x)+ ( (a - lambda + a*cosh(lambda*x))*y^2 + a+ lambda- a *cosh(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\sqrt{\cosh(\lambda x) - 1}(\cosh(\lambda x) + 1)^{3/2}(-y\cosh(\lambda x) + \sinh(\lambda x) - y)\left(\sqrt{\cosh(\lambda x) - 1}(\text{co}\right.\right.$$

46.5 problem number 5

problem number 400

Added January 10, 2019.

Problem 2.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx\cosh^m(y))\, w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(x^{-n+1} e^{b\int(\cosh(y))^m y^{-k}\, dy(n-1)} + an\int e^{b\int(\cosh(y))^m y^{-k}\, dy(n-1)} y^{-k}\, dy - a\int e^{b\int(\cosh(y))^m y^{-k}\, dy(n-\right.$$

46.6 problem number 6

problem number 401

Added January 10, 2019.

Problem 2.4.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cosh^m(y))\,w_x + \cosh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n + b*x*Cosh[y]^m)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n+ b*x*cosh(y)^m)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\cosh(y))^m(\cosh(y\lambda))^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\cosh(y))^m(\cosh(y\lambda))^{-k}\,\mathrm{d}y(n-1)}(\cosh(y\lambda))^{-k}\,\mathrm{d}y -\right.$$

46.7 problem number 7

problem number 402

Added January 10, 2019.

Problem 2.4.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n y^m + bx) w_x + \cosh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*y^m + b*x)*D[w[x, y], x] + Cosh[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n*y^m+ b*x)*diff(w(x,y),x)+cosh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = _F1\left(x^{-n+1}\mathrm{e}^{b\int(\cosh(y\lambda))^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\cosh(y\lambda))^{-k}\,\mathrm{d}y(n-1)} y^m (\cosh(y\lambda))^{-k}\,\mathrm{d}y - a\int \mathrm{e}^{b\int(\cosh(y

46.8 problem number 8

problem number 403

Added January 10, 2019.

Problem 2.4.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(\cosh(\mu y))\, w_x + a\cosh(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Cosh[mu*y]*D[w[x, y], x] + a*Cosh[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda\sinh(\mu y)-a\mu\sinh(\lambda x)}{\lambda\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := cosh(mu*y)*diff(w(x,y),x)+a*cosh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sinh(\lambda x)\,\mu\, a-\sinh(\mu y)\,\lambda}{\lambda\,\mu\, a}\right)$$

47 HFOPDE, chapter 2.4.3

47.1 problem number 1

problem number 404

Added January 10, 2019.

Problem 2.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tanh(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Tanh[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - a\log(\cosh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{\ln(\tanh(\lambda\,x)-1)\,a+\ln(\tanh(\lambda\,x)+1)\,a+2\,y\lambda}{\lambda}\right)$$

47.2 problem number 2

problem number 405

Added January 10, 2019.

Problem 2.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tanh(\lambda y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Tanh[lambda*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log(\sinh(\lambda y))-a\lambda x}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*tanh(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(1/2\,\frac{\ln\left(\tanh\left(\lambda\,x\right)-1\right)a+\ln\left(\tanh\left(\lambda\,x\right)+1\right)a+2\,y\lambda}{\lambda}\right)$$

47.3 problem number 3

problem number 406

Added January 10, 2019.

Problem 2.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda - a(a+\lambda)\tanh^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Tanh[lambda*x]^2)*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\lambda\left(-ye^{2\lambda x}\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda}, -\frac{a}{\lambda}, 1-\frac{a}{\lambda}, -e^{2\lambda x}\right) - y\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+( y^2+a*lambda - a*(a+lambda)*tanh(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\tanh(\lambda x)\,\text{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x)\right)a + \tanh(\lambda x)\,\text{LegendreP}\left(\frac{a}{\lambda}, \frac{a}{\lambda}, \tanh(\lambda x\right.\right.\right.$$

47.4 problem number 4

problem number 407

Added January 10, 2019.

Problem 2.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + 3a\lambda - \lambda^2 - a(a+\lambda)\tanh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Tanh[lambda*x])*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+( y^2+3*a*lambda - lambda^2 -a*(a+lambda)*tanh(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

47.5 problem number 5

problem number 408

Added January 10, 2019.

Problem 2.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tanh^m(y))\, w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\tanh(y))^m y^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\tanh(y))^m y^{-k}\,\mathrm{d}y(n-1)} y^{-k}\,\mathrm{d}y - a\int \mathrm{e}^{b\int(\tanh(y))^m y^{-k}\,\mathrm{d}y(n-\right.$$

47.6 problem number 6

problem number 409

Added January 10, 2019.

Problem 2.4.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tanh^m(y))\,w_x + \tanh^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n + b*x*Tanh[y]^m)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := ( a*x^n + b*x*tanh(y)^m)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\tanh(y))^m(\tanh(y\lambda))^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\tanh(y))^m(\tanh(y\lambda))^{-k}\,\mathrm{d}y(n-1)}(\tanh(y\lambda))^{-k}\,\mathrm{d}y\right.$$

47.7 problem number 7

problem number 410

Added January 10, 2019.

Problem 2.4.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)\, w_x + \tanh^k(\lambda y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*y^m + b*x)*D[w[x, y], x] + Tanh[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := ( a*x^n*y^m + b*x)*diff(w(x,y),x)+tanh(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1} e^{b\int(\tanh(y\lambda))^{-k}\,dy(n-1)} + an\int e^{b\int(\tanh(y\lambda))^{-k}\,dy(n-1)} y^m (\tanh(y\lambda))^{-k}\,dy - a\int e^{b\int(\tanh(y$$

47.8 problem number 8

problem number 411

Added January 10, 2019.

Problem 2.4.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n\tanh^m y + bx)\, w_x + y^k w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*Tanh[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{an\int_1^y K[1]^{-k}\tanh^m(K[1])\,dK[1]-a\int_1^y K[1]^{-k}\tanh^m(K[1])\,dK[1]+x^{1-n}}{an-a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := ( a*x^n*tanh(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1} + an\int (\tanh(y))^m\, y^{-k}\, \mathrm{d}y - a\int (\tanh(y))^m\, y^{-k}\, \mathrm{d}y\right)$$

48 HFOPDE, chapter 2.4.4

48.1 problem number 1

problem number 412

Added January 10, 2019.

Problem 2.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\coth(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Coth[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda y - a\log(\sinh(\lambda x))}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*coth(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{a\ln(\coth(\lambda\,x)-1)+a\ln(\coth(\lambda\,x)+1)+2\,y\lambda}{\lambda}\right)$$

48.2 problem number 2

problem number 413

Added January 10, 2019.

Problem 2.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\coth(\lambda y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Coth[lambda*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\log(\cosh(\lambda y))-a\lambda x}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*coth(lambda*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{2\,ax\lambda+\ln\left(\coth(y\lambda)-1\right)+\ln\left(\coth(y\lambda)+1\right)-2\,\ln\left(\coth(y\lambda)\right)}{a\lambda}\right)$$

48.3 problem number 3

problem number 414

Added January 10, 2019.

Problem 2.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda - a(a+\lambda)\coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*lambda - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{\lambda\left(-ye^{2\lambda x}\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda},1-\frac{a}{\lambda},e^{2\lambda x}\right)+y\text{Hypergeometric2F1}\left(-\frac{2a}{\lambda},-\frac{a}{\lambda}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2 + a*lambda - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\coth(\lambda x)\operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)a + \coth(\lambda x)\operatorname{LegendreP}\left(\frac{a}{\lambda},\frac{a}{\lambda},\coth(\lambda x)\right)\right.\right.$$

48.4 problem number 4

problem number 415

Added January 10, 2019.

Problem 2.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + 3a\lambda - \lambda^2 - a(a+\lambda)\coth^2(\lambda x))\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + 3*a*lambda - lambda^2 - a*(a + lambda)*Coth[lambda*x]^2)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2 + a*lambda  -lambda^2 - a*(a+lambda)*coth(lambda*x)^2)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\coth(\lambda x)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{\sqrt{a^2+\lambda^2}}{\lambda}, \coth(\lambda x)\right) a + \coth(\lambda x)\,\mathrm{LegendreP}\left(\frac{a}{\lambda}, \frac{\sqrt{a^2}}{\lambda}\right.\right.\right.$$

49 HFOPDE, chapter 2.4.5

49.1 problem number 1

problem number 416

Added January 10, 2019.

Problem 2.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\lambda x)\cosh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Sinh[lambda*x]*Cosh[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{2\left(a\mu\cosh(\lambda x)-2\lambda\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)\right)}{\lambda\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*sinh(lambda*x)*cosh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\cosh(\lambda x)\,\mu\,a-2\arctan\left(e^{\mu y}\right)\lambda}{\lambda\,\mu\,a}\right)$$

49.2 problem number 2

problem number 417

Added January 10, 2019.

Problem 2.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cosh(\lambda x)\sinh(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Cosh[lambda*x]*Sinh[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\lambda\log\left(\tanh^2\left(\frac{\mu y}{2}\right)\right)-2a\mu\sinh(\lambda x)}{\lambda\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*cosh(lambda*x)*sinh(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-1/2\,\frac{e^{\lambda\,x}a\mu-\mu\,ae^{-\lambda\,x}+4\,\mathrm{arctanh}\left(e^{\mu\,y}\right)\lambda}{\lambda\,\mu\,a}\right)$$

49.3 problem number 3

problem number 418

Added January 10, 2019.

Problem 2.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - 2\lambda^2 \tanh^2(\lambda x) - 2\lambda^2 \coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 - 2*lambda^2*Tanh[lambda*x]^2 - 2*lambda^2*Coth[lambda*x]^2)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{e^{-4\lambda x}\left(16\lambda^2 x e^{4\lambda x} + 16\lambda^2 x e^{8\lambda x} + 8\lambda x y e^{4\lambda x} - 8\lambda x y e^{8\lambda x} - y e^{4\lambda x} - y e^{8\lambda x} + y e^{12\lambda x} + 14\lambda e^{4}\right.}{2\left(-y e^{4\lambda x} + 2\lambda e^{4\lambda x} + 2\lambda + y\right)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2 -2 *lambda^2*tanh(lambda*x)^2 - 2*lambda^2*coth(lambda*x)^2)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-4\left(\sinh(\lambda x)(\coth(\lambda x))^2 \lambda + \sinh(\lambda x)\coth(\lambda x)\, y - 2\cosh(\lambda x)\coth(\lambda x)\,\lambda - \sinh(\lambda\right.\right.$$

49.4 problem number 4

problem number 419

Added January 10, 2019.

Problem 2.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) - 2ab - a(a+\lambda)\tanh^2(\lambda x) - b(b+\lambda)\coth^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + lambda*(a + b) - 2*a*b - a*(a + lambda)*Tanh[lambda*x]^2 - b*(
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2 +lambda*(a+b)-2*a*b -a*(a+lambda)*tanh(lambda*x)^2 - b*(b+lambda)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((-2a-3\lambda)\left(\sinh(\lambda x)\cosh(\lambda x)y - a(\cosh(\lambda x))^2 - (\cosh(\lambda x))^2 b + a\right)(\sinh(\lambda x))^2 \left(\right.\right.$$

49.5 problem number 5

problem number 420

Added January 10, 2019.

Problem 2.4.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sinh(\lambda y)w_x + a\cosh(\beta x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Sinh[lambda*y]*D[w[x, y], x] + a*Cosh[beta*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\beta\cosh(\lambda y)-a\lambda\sinh(\beta x)}{\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := sinh(lambda*y)*diff(w(x,y),x)+a*cosh(beta*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\sinh(\beta x)\,a\lambda+\cosh(y\lambda)\,\beta}{a\beta\,\lambda}\right)$$

49.6 problem number 6

problem number 421

Added January 10, 2019.

Problem 2.4.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n \cosh^m(\lambda y) + bx)\, w_x + \sinh^k(\beta y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*Cosh[lambda*y]^m + b*x)*D[w[x, y], x] + Sinh[beta*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*x^n*cosh(lambda*y)^m+b*x)*diff(w(x,y),x)+sinh(beta*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(x^{-n+1} e^{b \int (\sinh(\beta y))^{-k} \, dy (n-1)} + an \int e^{b \int (\sinh(\beta y))^{-k} \, dy (n-1)} (\cosh(y\lambda))^m (\sinh(\beta y))^{-k} \, dy - a\right.$$

50 HFOPDE, chapter 2.5.1

50.1 problem number 1

problem number 422

Added January 14, 2019.

Problem 2.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\ln^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{(-\log(\lambda x))^{-k}\left(a\log^k(\lambda x)\text{Gamma}(k+1,-\log(\lambda x)) + b\lambda x(-\log(\lambda x))^k - \lambda y(-\log(\lambda\right.}{\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*ln(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx - \int a(\ln(\lambda x))^k \,dx + y\right)$$

50.2 problem number 3

problem number 423

Added January 14, 2019.

Problem 2.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\ln^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\log^k(\lambda K[1])+b}\,dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*ln(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\ln(y\lambda))^k + b\right)^{-1} \mathrm{d}y + x\right)$$

50.3 problem number 4

problem number 424

Added January 14, 2019.

Problem 2.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\ln^k(x+\lambda y)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Log[x + lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*ln(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{y\lambda+x}{\lambda}} \left(1 + a(\ln(\lambda\_a))^k \lambda\right)^{-1} d\_a\lambda + x\right)$$

51 HFOPDE, chapter 2.5.2

51.1 problem number 1

problem number 425

Added January 14, 2019.

Problem 2.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ax^n \ln^k(\lambda y) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*x^n*Log[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\frac{n(-\log(\lambda y))^k\log^{-k}(\lambda y)\text{Gamma}(1-k,-\log(\lambda y))}{\lambda}+\frac{(-\log(\lambda y))^k\log^{-k}(\lambda y)\text{Gamma}(1-k,-\log(\lambda y))}{\lambda}-ax^{n+1}}{n+1}\right)\right\}\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*x^n*ln(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{n+1}a - n\int(\ln(y\lambda))^{-k}\,dy - \int(\ln(y\lambda))^{-k}\,dy}{a}\right)$$

51.2 problem number 2

problem number 426

Added January 14, 2019.

Problem 2.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ay^n \ln^k(\lambda x) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*y^n*Log[lambda*x]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{y^{-n}(-\log(\lambda x))^{-k}\left(-ay^n \log^k(\lambda x)\text{Gamma}(k+1, -\log(\lambda x)) + any^n \log^k(\lambda x)\text{Gamma}\right.}{\lambda(n-1)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*y^n*ln(lambda*x)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{y^n} + an\int (\ln(\lambda x))^k \, dx - a\int (\ln(\lambda x))^k \, dx\right)$$

51.3 problem number 3

problem number 427

Added January 14, 2019.

Problem 2.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + a\ln(\beta x)y - ab\ln(\beta x) - b^2\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*Log[beta*x]*y - a*b*Log[beta*x] - b^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2+ a*ln(beta*x)* y - a*b*ln(beta*x) - b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{(\beta x)^{ax} e^{-(a-2b)x} + y\int(\beta x)^{ax} e^{-(a-2b)x} dx - b\int(\beta x)^{ax} e^{-(a-2b)x} dx}{-b+y}\right)$$

51.4 problem number 4

problem number 428

Added January 14, 2019.

Problem 2.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\ln^m(bx)y + a\ln^m(bx)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*x*Log[b*x]^m*y + a*Log[b*x]^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2+ a*x*ln(b*x)^m * y + a *ln(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int \frac{a(\ln(bx))^m x^2-2}{x}\,dx}\,dx + e^{\int \frac{a(\ln(bx))^m x^2-2}{x}\,dx}x + \int e^{\int \frac{a(\ln(bx))^m x^2-2}{x}\,dx}\,dx\right)\right)$$

51.5 problem number 5

problem number 429

Added January 14, 2019.

Problem 2.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ax^n y^2 - abx^{n+1} y \ln(x) + b\ln(x) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*x^n*y^2 - a*b*x^(n + 1)*y*Log[x] + b*Log[x] + b)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*x^n*y^2- a*b*x^(n+1)*y*ln(x)  + b*ln(x) + b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.6 problem number 6

problem number 430

Added January 14, 2019.

Problem 2.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((n+1)x^n y^2 - ax^{n+1}(\ln x)^m y + a(\ln x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] - ((n + 1)*x^n*y^2 - a*x^(n + 1)*Log[x]^m*y + a*Log[x]^m)*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)-((n+1)*x^n*y^2 - a*x^(n+1)*ln(x)^m*y + a*ln(x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx^{n+1}-1}\left(yx^{n+1}\int \frac{e^{a\int x^{n+1}(\ln(x))^m\,dx - 2n\ln(x)}x^n}{x^2}\,dxn + yx^{n+1}\int \frac{e^{a\int x^{n+1}(\ln(x))^m\,dx-2n\ln(x)}x^{n}}{x^2}\right.\right.$$

51.7 problem number 7

problem number 431

Added January 14, 2019.

Problem 2.5.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 + bmx^{m-1} - ab^2x^{2m}(\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[x]^n*y^2 + b*m*x^(m - 1) - a*b^2*x^(2*m)*Log[x]^n)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a *ln(x)^n*y^2 + b*m*x^(m-1) - a*b^2*x^(2*m)* ln(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.8 problem number 8

problem number 432

Added January 14, 2019.

Problem 2.5.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a(\ln x)^n y^2 - abxy(\ln x)^{n+1} + b\ln x + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[x]^n*y^2 - a*b*x*y*Log[x]^(n + 1) + b*Log[x] + b)*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*ln(x)^n*y^2 - a*b*x*y*(ln(x))^(n+1) + b*ln(x)+ b )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.9 problem number 9

problem number 433

Added January 14, 2019.

Problem 2.5.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a(\ln x)^k(y - bx^n - c)^3 + bnx^{n-1}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[x]^k*(y - b*x^n - c)^3 + b*n*x^(n - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{(-\log(x))^{-k}\left(2ab^2x^{2n}\log^k(x)\text{Gamma}(k+1,-\log(x)) + 4abcx^n\log^k(x)\text{Gamma}(k+1,\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*(ln(x))^k*(y - b*x^n-c)^3 + b*n*x^(n-1) ) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2\,x^{2\,n}ab^{2}\int(\ln(x))^{k}\,\mathrm{d}x - 4\,x^{n}yab\int(\ln(x))^{k}\,\mathrm{d}x + 4\,x^{n}abc\int(\ln(x))^{k}\,\mathrm{d}x + 2\,y^{2}a\int(\ln(x))^{k}}{(bx^{n}+c-y)^{2}}\right.$$

51.10 problem number 10

problem number 434

Added January 14, 2019.

Problem 2.5.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a(\ln x)^n y^2 + b(\ln x)^m y + bc(\ln x)^m - ac^2(\ln x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Log[x]^n*y^2 + b*Log[x]^m*y + b*c*Log[x]^m - a*c^2*Log[x]^n)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*(ln(x))^n*y^2 + b*(ln(x))^m *y+ b*c* (ln(x))^m - a*c^2* (ln(x))^n)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{ya\int(\ln(x))^n \mathrm{e}^{-2\,ca\int(\ln(x))^n\,\mathrm{d}x+b\int(\ln(x))^m\,\mathrm{d}x}\,\mathrm{d}x + a\int(\ln(x))^n \mathrm{e}^{-2\,ca\int(\ln(x))^n\,\mathrm{d}x+b\int(\ln(x))^m\,\mathrm{d}x}\,\mathrm{d}}{c+y}\right.$$

51.11 problem number 11

problem number 435

Added January 14, 2019.

Problem 2.5.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ay + b\ln x)^2 w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*y + b*Log[x])^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{a^3y\sqrt{\frac{b}{a^3}}+a^2b\sqrt{\frac{b}{a^3}}\log(x)}{b}\right)-a^2\sqrt{\frac{b}{a^3}}\log(x)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*y+ b*ln(x))^2 *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{a\sqrt{ab}}\left(-\ln(x)\sqrt{ab} + \arctan\left(\frac{a(ya + b\ln(x))}{\sqrt{ab}}\right)\right)\right)$$

51.12 problem number 12

problem number 436

Added January 14, 2019.

Problem 2.5.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(xy^2 - A^2x(\ln\beta x)^2 + A\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^2 + A)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^2 + A) *diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.13 problem number 13

problem number 437

Added January 14, 2019.

Problem 2.5.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(xy^2 - A^2x(\ln(\beta x))^{2k} + kA(\ln(\beta x))^{k-1}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (x*y^2 - A^2*x*Log[beta*x]^(2*k) + k*A*Log[beta*x]^(k - 1))*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(x*y^2 - A^2*x*(ln(beta*x))^(2*k) + k*A*(ln(beta*x))^(k-1))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.14 problem number 14

problem number 438

Added January 14, 2019.

Problem 2.5.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(ax^n y^2 + b - ab^2 x^n (\ln x)^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*x^n*y^2 + b - a*b^2*x^n*Log[x]^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*x^n*y^2 + b - a*b^2*x^n*(ln(x))^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

51.15 problem number 15

problem number 439

Added January 14, 2019.

Problem 2.5.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left(a(\ln(\lambda x))^m y^2 + ky + ab^2 x^{2k} (\ln(\lambda x))^m\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*Log[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Log[lambda*x]^m)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\frac{a\sqrt{b^2}x^k(\lambda x)^{-k}\log^m(\lambda x)(-k\log(\lambda x))^{-m}\text{Gamma}(m+1,-k\log(\lambda x))}{k}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*(ln(lambda*x))^m*y^2 + k*y+ a*b^2*x^(2*k)* (ln(lambda*x))^m )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(ab\int(\ln(\lambda x))^m x^{k-1}\,dx-\arctan\left(\frac{x^{-k}y}{b}\right)\right)$$

51.16 problem number 16

problem number 440

Added January 14, 2019.

Problem 2.5.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^n(y+b\ln x)^2-b\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*x^n*(y + b*Log[x])^2 - b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{abx^n\log(x)+ayx^n+n}{n(b\log(x)+y)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*x^n*(y + b*ln(x))^2 - b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ab\ln(x)\,x^n+ax^ny+n}{n\,(y+b\ln(x))}\right)$$

51.17 problem number 17

problem number 441

Added January 14, 2019.

Problem 2.5.2.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+\left(ax^{2n}\ln(x)y^2+(bx^n\ln x-n)y+c\ln x\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*x^(2*n)*Log[x]*y^2 + (b*x^n*Log[x] - n)*y + c*Log[x])*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\left(\sqrt{b^2-4ac}+2ayx^n+b\right)\exp\left(\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}+\frac{b}{\sqrt{a}\sqrt{c}}\right)(n\log(x)-1)}{2n^2}-\frac{\sqrt{a}\sqrt{c}x^n\left(\frac{b}{\sqrt{a}\sqrt{c}}-\frac{\sqrt{b^2-4ac}}{\sqrt{a}\sqrt{c}}\right.}{2n^2}\right.}{\sqrt{b^2-4ac}-2ayx^n-b}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*x^(2*n)*ln(x)* y^2 + (b* x^n *ln(x) - n)*y + c *ln(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{b}{\sqrt{b^2\left(4ca-b^2\right)}n^2}\left(-\ln(x)\,x^n\sqrt{b^2\left(4ca-b^2\right)}n+2\,bn^2\arctan\left(\frac{b(2\,ax^ny+b)}{\sqrt{4\,acb^2-b^4}}\right)+\sqrt{b^2\left(\right.}\right.\right.$$

51.18 problem number 18

problem number 442

Added January 14, 2019.

Problem 2.5.2.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^k w_x + (ay^n(\ln x)^m + by(\ln x)^s)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y*Log[x]^s)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m + b*y*(ln(x))^s )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-n+1}e^{b\int x^{-k}(\ln(x))^s\,dx(n-1)} + an\int e^{b\int x^{-k}(\ln(x))^s\,dx(n-1)}x^{-k}(\ln(x))^m\,dx - a\int e^{b\int x^{-k}(\ln(x))^s\,dx}\right.$$

51.19 problem number 19

problem number 443

Added January 14, 2019.

Problem 2.5.2.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left(y^2 + c(\ln x)^n y - \lambda^2 + \lambda c(\ln x)^n\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*Log[x] + b)*D[w[x, y], x] + (y^2 + c*Log[x]^n*y - lambda^2 + lambda*c*Log[x]^n)*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*ln(x)+b)*diff(w(x,y),x)+(y^2+ c*(ln(x))^n*y- lambda^2 + lambda*c*(ln(x))^n )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\int \frac{1}{\ln(x)\,a+b} e^{\int \frac{c(\ln(x))^n-2\lambda}{\ln(x)a+b}\,dx}\,dx\lambda\, e^{\int \frac{c(\ln(x))^n-2\lambda}{\ln(x)a+b}\,dx+\int -\frac{c(\ln(x))^n-2\lambda}{\ln(x)a+b}\,dx} + y\int \frac{1}{\ln(x)\,a+b} e^{\int \frac{c}{}}\right.\right.$$

51.20 problem number 20

problem number 444

Added January 14, 2019.

Problem 2.5.2.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\ln x + b)w_x + \left((\ln x)^n y^2 - cy - \lambda^2(\ln x)^n + c\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*Log[x] + b)*D[w[x, y], x] + (Log[x]^n*y^2 - c*y - lambda^2*Log[x]^n + c*lambda)*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a*ln(x)+b)*diff(w(x,y),x)+((ln(x))^n*y^2- c*y - lambda^2*(ln(x))^n + c*lambda )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\int \frac{(\ln(x))^n}{\ln(x)\,a+b} e^{\int \frac{2(\ln(x))^n\lambda-c}{\ln(x)a+b}\,dx}\, dx \lambda\, e^{\int \frac{2(\ln(x))^n\lambda-c}{\ln(x)a+b}\,dx + \int -\frac{2(\ln(x))^n\lambda-c}{\ln(x)a+b}\,dx} - y \int \frac{(\ln(x))^n}{\ln(x)\,a+b} e^{\int 2\cdots}\right.\right.$$

51.21 problem number 21

problem number 445

Added January 14, 2019.

Problem 2.5.2.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 \ln(ax) w_x - \left(x^2y^2 \ln(ax) + 1\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x^2*Log[a*x]*D[w[x, y], x] - (x^2*y^2*Log[a*x] + 1)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x^2*ln(a*x)*diff(w(x,y),x)-(x^2*y^2* ln(a*x)+ 1  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{xy\ln(ax) - 1}{\ln(ax)\operatorname{expIntegral}(1, -\ln(ax))\,yx + ax^2y - \operatorname{expIntegral}(1, -\ln(ax))}\right)$$

51.22 problem number 22

problem number 446

Added January 14, 2019.

Problem 2.5.2.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n + by\ln^m x)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Log[lambda*x]^k*D[w[x, y], x] + (a*y^n + b*y*Log[x]^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n+ b*y* (ln(x))^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-n+1}\mathrm{e}^{b\int(\ln(x))^m(\ln(\lambda x))^{-k}\,\mathrm{d}x(n-1)} + an\int \mathrm{e}^{b\int(\ln(x))^m(\ln(\lambda x))^{-k}\,\mathrm{d}x(n-1)}(\ln(\lambda x))^{-k}\,\mathrm{d}x - a\int \mathrm{e}^{b\int(\right.$$

51.23 problem number 23

problem number 447

Added January 14, 2019.

Problem 2.5.2.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\ln^k(\lambda x)w_x + (ay^n\ln^m x + by)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = Log[lambda*x]^k*D[w[x, y], x] + (a*y^n*Log[x]^m + b*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (ln(lambda*x))^k*diff(w(x,y),x)+(a*y^n*(ln(x))^m+ b*y )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{-n+1}\mathrm{e}^{b\int(\ln(\lambda x))^{-k}\,\mathrm{d}x(n-1)} + an\int \mathrm{e}^{b\int(\ln(\lambda x))^{-k}\,\mathrm{d}x(n-1)}(\ln(\lambda x))^{-k}(\ln(x))^{m}\,\mathrm{d}x - a\int \mathrm{e}^{b\int(\ln(\lambda$$

52 HFOPDE, chapter 2.6.1

52.1 problem number 1

problem number 448

Added January 14, 2019.

Problem 2.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda x) + b\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Sin[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{\sin^2(\lambda x)^{-\frac{k}{2}-\frac{1}{2}}\left(-a\cos(\lambda x)\sin^{k+1}(\lambda x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-k}{2},\frac{3}{2},\cos^2(\lambda x)\right)+b\lambda x\right.}{\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*sin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx - \int a(\sin(\lambda x))^k \, dx + y\right)$$

contains unresolved integral

52.2 problem number 2

problem number 449

Added January 14, 2019.

Problem 2.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Sin[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\sin^k(\lambda K[1])+b}\,dK[1]-x\right)\right\}\right\}$$

contains unresolved integral

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*sin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\sin(y\lambda))^k + b\right)^{-1} \mathrm{d}y + x\right)$$

contains unresolved integral

52.3 problem number 3

problem number 450

Added January 14, 2019.

Problem 2.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + a\sin^k(\lambda y)\sin^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Sin[lambda*x]^k*Sin[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x, y) \to c_1()\}\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*sin(lambda*x)^k*sin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\int (\sin(\lambda x))^k \, dx + \int \frac{(\sin(\mu y))^{-n}}{a} \, dy\right)$$

contains unresolved integral

52.4 problem number 4

problem number 451

Added January 14, 2019.

Problem 2.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(x+\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Sin[x + lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*sin(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{y\lambda+x}{\lambda}} \left(1+a(\sin(\lambda\_a))^k\lambda\right)^{-1} d\_a\lambda + x\right)$$

contains unresolved integral

52.5 problem number 5

problem number 452

Added January 14, 2019.

Problem 2.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\sin(\lambda x) + a^2\sin^2(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 - a^2 + a*lambda*Sin[lambda*x] + a^2*Sin[lambda*x]^2)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(y^2-a^2 + a*lambda*sin(lambda*x)+a^2*sin(lambda*x)^2)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\sqrt{2\,csgn\left(\sin\left(\lambda\,x\right)\right)\sin\left(\lambda\,x\right)+2}\left(2\,\cos\left(\lambda\,x\right)\left(csgn(\sin\left(\lambda\,x\right))\right)^2 \mathrm{HeunC}\left(4\,\frac{a}{\lambda}, -1/2, \right.\right.\right.$$

52.6 problem number 6

problem number 453

Added January 14, 2019.

Problem 2.6.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\sin(\beta x)y + ab\sin(\beta x) - b^2\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*Sin[beta*x]*y + a*b*Sin[beta*x] - b^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+( y^2 + a*sin(beta*x)* y + a*b*sin(beta*x)-b^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{b+y}\left(b\int \mathrm{e}^{-\frac{2bx\beta+\cos(\beta x)a}{\beta}}\,\mathrm{d}x + y\int \mathrm{e}^{-\frac{2bx\beta+\cos(\beta x)a}{\beta}}\,\mathrm{d}x + \mathrm{e}^{-\frac{2bx\beta+\cos(\beta x)a}{\beta}}\right)\right)$$

contains unresolved integrals

52.7 problem number 7

problem number 454

Added January 14, 2019.

Problem 2.6.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\sin^m(bx)y + a\sin^m(bx)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 + a*x*Sin[b*x]^m*y + a*Sin[b*x]^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+( y^2 + a*x*sin(b*x)^m*y + a*sin(b*x)^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int \frac{a(\sin(bx))^m x^2-2}{x}\,dx}\,dx + e^{\int \frac{a(\sin(bx))^m x^2-2}{x}\,dx}x + \int e^{\int \frac{a(\sin(bx))^m x^2-2}{x}\,dx}\,dx\right)\right)$$

contains unresolved integrals

52.8 problem number 8

problem number 455

Added January 14, 2019.

Problem 2.6.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + \lambda \sin^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + lambda*Sin[lambda*x]^3)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(lambda*sin(lambda*x)*y^2 + lambda*sin(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\sqrt{\pi}(\cos(\lambda x) + y)}{\sqrt{\pi}\cos(\lambda x)\,\mathrm{erfi}(\cos(\lambda x)) + \sqrt{\pi}\,\mathrm{erfi}(\cos(\lambda x))\, y - 2\,\mathrm{e}^{(\cos(\lambda x))^2}}\right)$$

52.9 problem number 9

problem number 456

Added January 14, 2019.

Problem 2.6.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2w_x + \left((\lambda + a - a\sin(\lambda x))y^2 + \lambda - a - a\sin(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = 2*D[w[x, y], x] + ((lambda + a - a*Sin[lambda*x])*y^2 + lambda - a - a*Sin[lambda*x])
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := 2*diff(w(x,y),x)+((lambda+a-a*sin(lambda*x))*y^2 +lambda -a -a*sin(lambda*x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((\sin(\lambda x) - 1)^{3/2}\sqrt{\sin(\lambda x) + 1}\left((\cos(\lambda x))^3 (\mathit{csgn}(\sin(\lambda x)))^2 \lambda^3 - \cos(\lambda x)(\mathit{csgn}(\sin(\lambda\right.\right.$$

52.10 problem number 10

problem number 457

Added January 14, 2019.

Problem 2.6.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((\lambda + a\sin^2(\lambda x))y^2 + \lambda - a + a\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + ((lambda + a*Sin[lambda*x]^2)*y^2 + lambda - a + a*Sin[lambda*x]^2)*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+((lambda+a*sin(lambda*x)^2)*y^2 + lambda -a +a*sin(lambda*x)^2)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\sqrt{\cos(2\lambda x)+1}\left(2(\sin(\lambda x))^2(\cos(2\lambda x))^2 ya - 4(\sin(\lambda x))^2\cos(2\lambda x)\, ya + \sin(2\lambda x)\right.\right.$$

52.11 problem number 11

problem number 458

Added January 14, 2019.

Problem 2.6.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left((k+1)x^k y^2 - a x^{k+1}(\sin x)^m y + a(\sin x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Sin[x]^m*y + a*Sin[x]^m)*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)-((k+1)*x^k*y^2 - a*x^(k+1)*(sin(x))^m*y + a*(sin(x))^m)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

server hangs

(server hangs)

52.12 problem number 12

problem number 459

Added January 14, 2019.

Problem 2.6.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\sin^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Sin[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*sin(lambda*x + mu)^k * (y-b*x^n -c)^2 + y - b*x^n + b*n*x^(n-1) - c
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

52.13 problem number 13

problem number 460

Added January 14, 2019.

Problem 2.6.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\sin^m(\lambda x)y^2 + ky + ab^2x^{2k}\sin^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = x*D[w[x, y], x] + (a*Sin[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Sin[lambda*x]^m)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x aK[1]^{k-1}\sin^m(\lambda K[1])\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := x*diff(w(x,y),x)+(a*sin(lambda*x)^m*y^2 + k*y + a*b^2*x^(2*k)*sin(lambda*x)^m)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ab\int(\sin(\lambda x))^m x^{k-1}\,dx - \arctan\left(\frac{x^{-k}y}{b}\right)\right)$$

52.14 problem number 14

problem number 461

Added January 14, 2019.

Problem 2.6.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\sin(\lambda x)+b)w_x+\left(y^2+c\sin(\mu x)y-k^2+ck\sin(\mu x)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*Sin[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Sin[mu*x]*y - k^2 + c*k*Sin[mu*x])*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := (a *sin(lambda*x) + b)*diff(w(x,y),x)+(y^2+ c*sin(mu*x)* y - k^2 + c*k*sin(mu*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{1}{k+y}\left(k\int\frac{1}{\sin(\lambda x)a+b}e^{\frac{1}{\lambda\sqrt{-a^2+b^2}}\left(c\int\frac{\sin(\mu x)}{\sin(\lambda x)a+b}\,dx\lambda\sqrt{-a^2+b^2}-4k\arctan\left(\frac{b\sin(1/2\lambda x)+a\cos(1/2\lambda x)}{\sqrt{-a^2+b^2}\cos(1/2\lambda x)}\right)\right)}\right.\right.$$

53 HFOPDE, chapter 2.6.2

53.1 problem number 1

problem number 462

Added January 14, 2019.

Problem 2.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Cos[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\sin(\lambda x)\cos^{k+1}(\lambda x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{k+1}{2},\frac{k+3}{2},\cos^2(\lambda x)\right)}{(k+1)\lambda\sqrt{\sin^2(\lambda x)}}-bx+y\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*cos(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx - \int a(\cos(\lambda x))^k \,dx + y\right)$$

Contains unresolved integral

53.2 problem number 2

problem number 463

Added January 14, 2019.

Problem 2.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (a*Cos[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{a\cos^k(\lambda K[1])+b}\,dK[1]-x\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(a*cos(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\cos(y\lambda))^k+b\right)^{-1} \mathrm{d}y + x\right)$$

53.3 problem number 3

problem number 464

Added January 14, 2019.

Problem 2.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cos^k(\lambda x)\cos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Cos[lambda*y]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \cos^{-k}(\lambda K[1])\cos^{-n}(\mu K[1])\,dK[1]-ax\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*cos(lambda*y)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ax - \int(\cos(y\lambda))^{-k}(\cos(\mu y))^{-n}\,\mathrm{d}y}{a}\right)$$

53.4 problem number 4

problem number 465

Added January 14, 2019.

Problem 2.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cos^k(x+\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + a*Cos[x + lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+a*cos(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{y\lambda+x}{\lambda}}\left(1+a(\cos(\lambda\_a))^k\lambda\right)^{-1}d\_a\lambda + x\right)$$

53.5 problem number 5

problem number 466

Added January 14, 2019.

Problem 2.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - a^2 + a\lambda\cos(\lambda x) + a^2\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (y^2 - a^2 + a*lambda*Cos[lambda*x] + a^2*Cos[lambda*x]^2)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+( y^2-a^2 + a *lambda*cos(lambda*x) + a^2*cos(lambda*x)^2)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\sqrt{2\cos(\lambda x)+2}\left(2\sin(\lambda x)\,\mathrm{HeunC}\left(4\frac{a}{\lambda}, -1/2, -1/2, -2\frac{a}{\lambda}, 1/8\frac{8a+3\lambda}{\lambda}, 1/2\,\mathrm{co}\right.\right.\right.$$

53.6 problem number 6

problem number 467

Added January 14, 2019.

Problem 2.6.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \cos(\lambda x)y^2 + \lambda \cos^3(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + (lambda*Cos[lambda*x]*y^2 + lambda*Cos[lambda*x]^3)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+(lambda*cos(lambda*x)*y^2 + lambda*cos(lambda*x)^3)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\frac{M\left(1, 3/2, -(\sin(\lambda x))^2\right)(\sin(\lambda x))^2 - \sin(\lambda x)\, M\left(1, 3/2, -(\sin(\lambda x))^2\right)y - 1}{2 + (\sin(\lambda x))^2\, U\left(1, 3/2, -(\sin(\lambda x))^2\right) - \sin(\lambda x)\, U\left(1, 3/2, -(\sin(\lambda x))^2\right)y}\right)$$

53.7 problem number 7

problem number 468

Added January 14, 2019.

Problem 2.6.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2w_x + \left((\lambda + a + a\cos(\lambda x))y^2 + \lambda - a + a\cos(\lambda x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = 2*D[w[x, y], x] + ((lambda + a + a*Cos[lambda*x])*y^2 + lambda - a + a*Cos[lambda*x])
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := 2*diff(w(x,y),x)+ ((lambda+a+a*cos(lambda*x))*y^2 +lambda - a + a *cos(lambda*x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(-y\cos(\lambda x) + \sin(\lambda x) - y)\sqrt{\cos(\lambda x)+1}\sqrt{\cos(\lambda x)-1}}{\lambda}e^{\frac{\cos(\lambda x)a}{\lambda}}\left(-\sqrt{\cos(\lambda x)-1}\sqrt{\right.\right.$$

53.8 problem number 8

problem number 469

Added January 14, 2019.

Problem 2.6.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left((\lambda + a\cos^2(\lambda x))y^2 + \lambda - a + a\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = D[w[x, y], x] + ((lambda + a*Cos[lambda*x]^2)*y^2 + lambda - a + a*Cos[lambda*x]^2)*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde := diff(w(x,y),x)+ ((lambda+a*cos(lambda*x)^2)*y^2 + lambda - a + a*cos(lambda*x)^2)*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{\sqrt{-1+\cos\left(2\,\lambda\,x\right)}\left(-8\left(\cos\left(\lambda\,x\right)\right)^{6}ya - 8\left(\cos\left(\lambda\,x\right)\right)^{4}y\lambda + \sin\left(2\,\lambda\,x\right)\left(\cos\left(2\,\lambda\,x\right)\right)^{2}a\right.}{}\right.$$

53.9 problem number 9

problem number 470

Added January 14, 2019.

Problem 2.6.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n*y^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n*y^m+b*x)*diff(w(x,y),x)+ cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\cos(y\lambda))^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\cos(y\lambda))^{-k}\,\mathrm{d}y(n-1)}y^m(\cos(y\lambda))^{-k}\,\mathrm{d}y - a\int \mathrm{e}^{b\int(\cos(y\lambda))^{-k}}\right.$$

53.10 problem number 10

problem number 471

Added January 14, 2019.

Problem 2.6.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\cos^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d];
 pde = (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\cos(y))^m y^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\cos(y))^m y^{-k}\,\mathrm{d}y(n-1)}y^{-k}\,\mathrm{d}y - a\int \mathrm{e}^{b\int(\cos(y))^m y^{-k}\,\mathrm{d}y(n-1)}y$$

53.11 problem number 11

problem number 472

Added January 14, 2019.

Problem 2.6.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax^n + bx\cos^m y)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n + b*x*Cos[y]^m)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n+b*x*cos(y)^m)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(x^{-n+1} e^{b \int (\cos(y))^m (\cos(y\lambda))^{-k} \mathrm{d}y (n-1)} + an \int e^{b \int (\cos(y))^m (\cos(y\lambda))^{-k} \mathrm{d}y (n-1)} (\cos(y\lambda))^{-k} \mathrm{d}y - a \int e\right.$$

53.12 problem number 12

problem number 473

Added January 14, 2019.

Problem 2.6.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \cos^m y + bx)w_x + \cos^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n*Cos[y]^m + b*x)*D[w[x, y], x] + Cos[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  (a*x^n*cos(y)^m+b*x)*diff(w(x,y),x)+cos(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}e^{b\int(\cos(y\lambda))^{-k}\,dy(n-1)} + an\int e^{b\int(\cos(y\lambda))^{-k}\,dy(n-1)}(\cos(y))^m(\cos(y\lambda))^{-k}\,dy - a\int e^{b\int(c$$

54 HFOPDE, chapter 2.6.3

54.1 problem number 1

problem number 474

Added January 14, 2019.

Problem 2.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a + Tan[lambda*x] + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-a\lambda x - b\lambda x + \log(\cos(\lambda x)) + \lambda y}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(a+tan(lambda*x)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{-2\,ax\lambda - 2\,bx\lambda + 2\,y\lambda - \ln\left(1 + (\tan(\lambda\,x))^2\right)}{\lambda}\right)$$

54.2 problem number 2

problem number 475

Added January 14, 2019.

Problem 2.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a \tan^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a + Tan[lambda*y] + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-x + \frac{2(a+b)\tan^{-1}(\tan(\lambda y)) + 2\log(a+b+\tan(\lambda y)) - \log\left(\sec^2(\lambda y)\right)}{2\lambda(a+b-i)(a+b+i)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(a+tan(lambda*y)+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{2\,a^2\lambda\,x + 4\,ab\lambda\,x + 2\,b^2\lambda\,x + 2\,\lambda\,x - 2\,\arctan\left(\tan\left(y\lambda\right)\right)a - 2\,\arctan\left(\tan\left(y\lambda\right)\right)b - 2}{\lambda\,\left(a^2 + 2\,ab + b^2 + 1\right)}\right.$$

54.3 problem number 3

problem number 476

Added January 14, 2019.

Problem 2.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tan^k(\lambda x)\tan^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + a*Tan[lambda*x]^k*Tan[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-a\tan^{k+1}(\lambda x)\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+3}{2},-\tan^2(\lambda x)\right)+\frac{k\lambda\tan^{1-n}(\mu y)\text{Hypergeometric2}}{\mu-\mu}}{k\lambda+\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+a *tan(lambda*x)^k * tan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int(\tan(\lambda x))^k\,dx + \int\frac{(\tan(\mu y))^{-n}}{a}\,dy\right)$$

Has unresolved integrals

54.4 problem number 4

problem number 477

Added January 14, 2019.

Problem 2.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+( y^2+ a *lambda + a*(lambda -a) *tan(lambda*x)^2)*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(4\ \mathrm{LegendreP}\left(1/2\,\frac{2\,a-\lambda}{\lambda}, 1/2\,\frac{2\,a-\lambda}{\lambda}, \sin(\lambda\,x)\right)(\cos(\lambda\,x))^3\,y + \sin(\lambda\,x)\ \mathrm{Legendre}\right.\right.$$

54.5 problem number 5

problem number 478

Added January 14, 2019.

Problem 2.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + \lambda^2 + 3a\lambda + a(\lambda - a)\tan^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Tan[lambda*x]^2)*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(  y^2+ lambda^2 +3*a*lambda +a*(lambda-a)*tan(lambda*x)^2)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-1\left(\mathrm{LegendreP}\left(1/2\,\frac{2\,a+\lambda}{\lambda}, 1/2\,\frac{2\,a-\lambda}{\lambda}, \sin(\lambda\,x)\right)(\sin(\lambda\,x))^3\,\lambda - 2\,\mathrm{LegendreP}\left(1/2\right.\right.\right.$$

54.6 problem number 6

problem number 479

Added January 14, 2019.

Problem 2.6.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + ax\tan^k(bx)y + a\tan^k(bx)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 + a*x*Tan[b*x]^k*y + a*Tan[b*x]^k)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(  y^2+ a*x  *tan(b*x)^k * y + a*tan(b*x)^k)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int \frac{a(\tan(bx))^k x^2-2}{x}dx}dx + e^{\int \frac{a(\tan(bx))^k x^2-2}{x}dx}x + \int e^{\int \frac{a(\tan(bx))^k x^2-2}{x}dx}dx\right)\right)$$

54.7 problem number 7

problem number 480

Added January 14, 2019.

Problem 2.6.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - a x^{k+1} (\tan x)^m y + a (\tan x)^m\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - a*x^(k + 1)*Tan[x]^m*y + a*Tan[x]^m)*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2- a*x^(k+1)*tan(x)^m*y + a*tan(x)^m )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

server hangs

Server hangs

54.8 problem number 8

problem number 481

Added January 20, 2019.

Problem 2.6.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^n(\lambda x)y^2 - ab^2\tan^{n+2}(\lambda x) + b\lambda\tan^2(\lambda x) + b\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a*Tan[lambda*x]^n*y^2 - a*b^2*Tan[lambda*x]^(n + 2) + b*lambda*Tan[l
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(a*tan(lambda*x)^n*y^2- a*b^2*tan(lambda*x)^(n+2) + b*lambda*tan(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

server hangs

Server hangs

54.9 problem number 9

problem number 482

Added January 20, 2019.

Problem 2.6.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\tan^k(\lambda x+\mu)(y-bx^n-c)^2 + y - bx^n + bnx^{n-1} - c\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a*Tan[lambda*x + mu]^k*(y - b*x^n - c)^2 + y - b*x^n + b*n*x^(n - 1)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  diff(w(x,y),x)+(a *tan(lambda*x+mu)^k*(y-b*x^n-c)^2 + y- b*x^n + b*n*x^(n-1)-c)*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

54.10 problem number 10

problem number 483

Added January 20, 2019.

Problem 2.6.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(a\tan^m(\lambda x)y^2 + ky + ab^2x^{2k}\tan^m(\lambda x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = x*D[w[x, y], x] + (a*Tan[lambda*x]^m*y^2 + k*y + a*b^2*x^(2*k)*Tan[lambda*x]^m)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x aK[1]^{k-1}\tan^m(\lambda K[1])\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=  x*diff(w(x,y),x)+ (a*tan(lambda*x)^m*y^2 +k*y+ a*b^2*x^(2*k)*tan(lambda*x)^m )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(ab\int x^{k-1}(\tan(\lambda x))^m\,\mathrm{d}x-\arctan\left(\frac{x^{-k}y}{b}\right)\right)$$

54.11 problem number 11

problem number 484

Added January 20, 2019.

Problem 2.6.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a\tan(\lambda x)+b)w_x+\left(y^2+c\tan(\mu x)y-k^2+ck\tan(\mu x)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*Tan[lambda*x] + b)*D[w[x, y], x] + (y^2 + c*Tan[mu*x]*y - k^2 + c*k*Tan[mu*x])*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*tan(lambda*x)+b)*diff(w(x,y),x)+ (y^2+ c *tan(mu*x)*y - k^2 + c*k*tan(mu*x) )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{1}{k+y}\left((k+y)\int\frac{\cos(\lambda x)}{\sin(\lambda x)a+b\cos(\lambda x)}\left(e^{2i\mu x}+1\right)^{\frac{-ic}{(a+ib)\mu}}\left(\frac{\sin(\lambda x)a+b\cos(\lambda x)}{\cos(\lambda x)}\right)^{-2}\right.\right.$$

54.12 problem number 12

problem number 485

Added January 20, 2019.

Problem 2.6.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n y^m + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n*y^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*x^n*y^m + b*x)*diff(w(x,y),x)+ tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}e^{b\int(\tan(y\lambda))^{-k}\,dy(n-1)} + an\int e^{b\int(\tan(y\lambda))^{-k}\,dy(n-1)}y^m(\tan(y\lambda))^{-k}\,dy - a\int e^{b\int(\tan(y\lambda))^{-k}}\right.$$

54.13 problem number 13

problem number 486

Added January 20, 2019.

Problem 2.6.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + y^k w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + y^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\tan(y))^m y^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\tan(y))^m y^{-k}\,\mathrm{d}y(n-1)}y^{-k}\,\mathrm{d}y - a\int \mathrm{e}^{b\int(\tan(y))^m y^{-k}\,\mathrm{d}y(n-1)}y\right.$$

54.14 problem number 14

problem number 487

Added January 20, 2019.

Problem 2.6.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n + bx\tan^m y)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n + b*x*Tan[y]^m)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*x^n + b*x*tan(y)^m)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}\mathrm{e}^{b\int(\tan(y))^m(\tan(y\lambda))^{-k}\,\mathrm{d}y(n-1)} + an\int \mathrm{e}^{b\int(\tan(y))^m(\tan(y\lambda))^{-k}\,\mathrm{d}y(n-1)}(\tan(y\lambda))^{-k}\,\mathrm{d}y - a\int\right.$$

54.15 problem number 15

problem number 488

Added January 20, 2019.

Problem 2.6.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax^n \tan^m y + bx)w_x + \tan^k(\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = (a*x^n*Tan[y]^m + b*x)*D[w[x, y], x] + Tan[lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   (a*x^n*tan(y)^m+ b*x)*diff(w(x,y),x)+  tan(lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(x^{-n+1}e^{b\int(\tan(y\lambda))^{-k}\,dy(n-1)} + an\int e^{b\int(\tan(y\lambda))^{-k}\,dy(n-1)}(\tan(y))^m(\tan(y\lambda))^{-k}\,dy - a\int e^{b\int(\right.$$

55 HFOPDE, chapter 2.6.4

55.1 problem number 1

problem number 489

Added January 20, 2019.

Problem 2.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\cot^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a*Cot[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{-a\cot^{k+1}(\lambda x)\text{Hypergeometric2F1}\left(1,\frac{k+1}{2},\frac{k+1}{2}+1,-\cot^2(\lambda x)\right)+bk\lambda x+b\lambda x-k\lambda y}{(k+1)\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+  (a*cot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \int a(\cot(\lambda x))^k\, dx\right)$$

Has unresolved integral

55.2 problem number 2

problem number 490

Added January 20, 2019.

Problem 2.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(a \cot^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (a*Cot[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x, y) \to c_1\left(\int_1^y \frac{1}{a \cot^k(\lambda K[1]) + b} \, dK[1] - x\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+  (a*cot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\int \left(a(\cot(y\lambda))^k + b\right)^{-1} \mathrm{d}y + x\right)$$

55.3 problem number 3

problem number 491

Added January 20, 2019.

Problem 2.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\cot^k(x+\lambda y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + a*Cot[x + lambda*y]^k*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+  cot(x+lambda*y)^k*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{y\lambda+x}{\lambda}}\left(1+(\cot(\lambda\_a))^k\lambda\right)^{-1}d\_a\lambda + x\right)$$

55.4 problem number 4

problem number 492

Added January 20, 2019.

Problem 2.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + a\lambda + a(\lambda - a)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 + a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+ ( y^2+a*lambda + a*(lambda-a)*cot(lambda*x)^2  )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(\mathrm{LegendreP}\left(1/2\,\frac{2\,a-\lambda}{\lambda}, 1/2\,\frac{2\,a-\lambda}{\lambda}, \cos\left(\lambda\,x\right)\right) a\cos\left(3\,\lambda\,x\right) + 3\,\sin\left(\lambda\,x\right)\mathrm{Legendre}\right.\right.$$

55.5 problem number 5

problem number 493

Added January 20, 2019.

Problem 2.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda^2 + 3a\lambda + a(\lambda - a)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 + lambda^2 + 3*a*lambda + a*(lambda - a)*Cot[lambda*x]^2)*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+ (  y^2+lambda^2 + 3*a*lambda +a*(lambda-a)*cot(lambda*x)^2   )*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1\left(2\cos(\lambda x)(\sin(\lambda x))^2 \operatorname{LegendreP}\left(1/2\,\frac{2\,a+\lambda}{\lambda}, 1/2\,\frac{2\,a-\lambda}{\lambda}, \cos(\lambda x)\right) a + 3\cos(\lambda$$

55.6 problem number 6

problem number 494

Added January 20, 2019.

Problem 2.6.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - 2a\cot(ax)y + b^2 - a^2\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = D[w[x, y], x] + (y^2 - 2*a*Cot[a*x]*y + b^2 - a^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{\sqrt{b^2}y - a\sqrt{b^2}\cot(ax)}{b^2}\right) - \sqrt{b^2}x\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   diff(w(x,y),x)+ (  y^2-2*a*cot(a*x)*y + b^2-a^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{e^{-2\,ibx}\left(-i\cot\left(ax\right)a + iy + b\right)}{b\left(ib - a\cot\left(ax\right) + y\right)}\right)$$

55.7 problem number 7

problem number 495

Added January 20, 2019.

Problem 2.6.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)w_x + a\cot(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = Cot[lambda*x]*D[w[x, y], x] + a*Cot[mu*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\cos(\mu y)\cos^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   cot(lambda*x)*diff(w(x,y),x)+ a*cot(mu*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{1}{\lambda\,\mu}\left(\ln\left(\frac{(\cot(\lambda\,x))^2+1}{(\cot(\lambda\,x))^2}\right)\mu\,a+\lambda\,\ln\left((\cos(\mu\,y))^2\right)\right)\right)$$

55.8 problem number 8

problem number 496

Added January 20, 2019.

Problem 2.6.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$\cot(\mu y)w_x + a\cot(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sin(\mu y)\sin^{-\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\mu\, a}\ln\left(\frac{\tan(\mu\, y)}{(\tan(\mu\, y))^2+1}\sqrt{\left((\tan(\mu\, y))^2+1\right)\left(-2\left(-1+\cos(2\,\lambda\, x)\right)^{-1}\right)^{\frac{\mu\, a}{\lambda}}}\right)\right)$$

55.9 problem number 9

problem number 497

Added January 20, 2019.

Problem 2.6.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\mu y)w_x + a\cot^2(\lambda x)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = Cot[mu*y]*D[w[x, y], x] + a*Cot[lambda*x]^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4\sin(\mu y)e^{\frac{a\mu(\lambda x+\cot(\lambda x))}{\lambda}}}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   cot(mu*y)*diff(w(x,y),x)+ a*cot(lambda*x)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{\lambda\,\sin(\lambda\,x)\,\mu\,a}\left(\pi\,\sin(\lambda\,x)\,\mu\,a - 2\,\mathrm{arccot}\left(\frac{\cos(\lambda\,x)}{\sin(\lambda\,x)}\right)\sin(\lambda\,x)\,\mu\,a - \ln\left(\frac{(\tan(\mu}{(\tan(\mu\,y)}\right.\right.\right.$$

55.10 problem number 10

problem number 498

Added January 20, 2019.

Problem 2.6.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(y+a)w_x + c\cot(x+b)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = Cot[y + a]*D[w[x, y], x] + c*Cot[x + b]^2*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(4\sin(a+y)e^{c(\cot(b+x)+x)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   cot(y+a)*diff(w(x,y),x)+ c*cot(x+b)^2*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(1/2\,\frac{1}{\tan(b)(\tan(x)+\tan(b))}\left(\pi(\tan(b))^2c+\pi\tan(b)\tan(x)c-2(\tan(b))^2cx-2\tan\right.\right.$$

55.11 problem number 11

problem number 499

Added January 20, 2019.

Problem 2.6.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x + aw_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B]
 pde = Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2\sin(\mu y)\cos^{\frac{a\mu}{\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\mu\,a}\ln\left(\frac{\tan(\mu\,y)}{(\tan(\mu\,y))^2+1}\sqrt{\left((\tan(\mu\,y))^2+1\right)(\cos(\lambda\,x))^{2\,\frac{\mu\,a}{\lambda}}}\right)\right)$$

55.12 problem number 12

problem number 500

Added January 20, 2019.

Problem 2.6.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)\cot(\mu y)w_x + a\cot(vx)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = Cot[lambda*x]*Cot[mu*y]*D[w[x, y], x] + a*Cot[v*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   cot(lambda*x)*cot(mu*y)*diff(w(x,y),x)+ a*cot(v*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\mu a}\ln\left(csgn\left((\cos(\mu y))^{-1}\right)\sin(\mu y)\,\mathrm{e}^{x\mu a}\left(\mathrm{e}^{2ivx}-1\right)^{\frac{i\mu a}{v}}\left(\mathrm{e}^{\mu a\int -2\frac{\mathrm{e}^{2ivx}+1}{\left(\mathrm{e}^{2ivx}-1\right)\left(\mathrm{e}^{2i\lambda x}+1\right)}\mathrm{d}x}\right)^{-1}\right)\right)$$

56 HFOPDE, chapter 2.6.5

56.1 problem number 1

problem number 501

Added January 20, 2019.

Problem 2.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin^k(\lambda x)\cos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*Sin[lambda*x]^k*Cos[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\cos(\lambda x)\sin^{k+1}(\lambda x)\sin^2(\lambda x)^{-\frac{k}{2}-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-k}{2},\frac{3}{2},\cos^2(\lambda x)\right)+\frac{\lambda\sqrt{\sin^2(\mu y}}{}}{\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ a*sin(lambda*x)^k*cos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int(\sin(\lambda x))^k\,dx + \int\frac{(\cos(\mu y))^{-n}}{a}\,dy\right)$$

Has unresolved integrals

56.2 problem number 2

problem number 502

Added January 20, 2019.

Problem 2.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - y\tan x + a(1-a)\cot^2 x\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 - y*Tan[x] + a*(1 - a)*Cot[x]^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{2y\cos^2(x)\left(\cos^2(x)-1\right)^{\frac{1}{4}i\left(\sqrt{-\frac{1}{(a-1)a}-4}-\frac{i}{\sqrt{a-1}\sqrt{a}}\right)\sqrt{a-1}\sqrt{a}}-2y(\cos^2(x)-1)^{\frac{1}{4}i\left(\sqrt{-\frac{1}{(a-1)a}}\right.}}{-2y\cos^2(x)\left(\cos^2(x)-1\right)^{\frac{1}{4}i\left(-\sqrt{-\frac{1}{(a-1)a}-4}-\frac{i}{\sqrt{a-1}\sqrt{a}}\right)\sqrt{a-1}\sqrt{a}}+2y\left(\cos^2(x)-1\right)^{\frac{1}{4}i\left(-\sqrt{-\frac{1}{(a-1)a}}\right.}}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (y^2-y *tan(x)+a*(1-a)*cot(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{(\sin(x))^{2a-1}(y\sin(x)+\cos(x)a)}{y\sin(x)-\cos(x)a+\cos(x)}\right)$$

56.3 problem number 3

problem number 503

Added January 20, 2019.

Problem 2.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - my\tan x + b^2\cos^{2m} x\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 - m*y*Tan[x] + b^2*Cos[x]^(2*m))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{\sqrt{b^2}\sqrt{\sin^2(x)}\csc(x)\cos^{m+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(x)\right)}{m+1}+\tan^{-1}\left(\frac{y c}{}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (y^2-m*y*tan(x)+b^2*cos(x)^(2*m) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{(\cos(x))^4\sqrt{(\cos(x))^{2m-2}}{}_2F_1(3/2,-m/2+3/2;\ 5/2;\ (\sin(x))^2)\cos\left(b\sqrt{(\cos(x))^{2m-2}}(\right.}{(\cos(x))^4\sqrt{(\cos(x))^{2m-2}}{}_2F_1(3/2,-m/2+3/2;\ 5/2;\ (\sin(x))^2)\sin\left(b\sqrt{(\cos(x))^{2m-2}}(\right.}\right.$$

Mathematica answer is simpler

56.4 problem number 4

problem number 504

Added January 20, 2019.

Problem 2.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (y^2 + my\cot x + b^2\sin^m x)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + m*y*Cot[x] + b^2*Sin[x]^m)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (y^2+m*y*cot(x)+b^2*sin(x)^m )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Server hangs

56.5 problem number 5

problem number 505

Added January 20, 2019.

Problem 2.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - 2\lambda^2 \tan^2(\lambda x) - 2\lambda^2 \cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 - 2*lambda^2*Tan[lambda*x]^2 - 2*lambda^2*Cot[lambda*x]^2)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (y^2-2*lambda^2*tan(lambda*x)^2-2*lambda^2*cot(lambda*x)^2)*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-8 \frac{}{8\sqrt{-2+2\cos(2\lambda x)}\ln\left(\cos(\lambda x)+1/2\sqrt{-2+2\cos(2\lambda x)}\right)\sin(2\lambda x)\,y - 16\sqrt{-}}\right.$$

56.6 problem number 6

problem number 506

Added January 20, 2019.

Problem 2.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(a+b) + 2ab + a(\lambda - a)\tan^2(\lambda x) + b(\lambda - b)\cot^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*(a + b) + 2*a*b + a*(lambda - a)*Tan[lambda*x]^2 + b*(l
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ ( y^2+lambda*(a+b)+2*a*b+a*(lambda -a)*tan(lambda*x)^2+ b*(lambda -
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((2\,a^2(\sin(\lambda\,x))^2 - 3\,(\sin(\lambda\,x))^2\,a\lambda - 2\,\sin(\lambda\,x)\cos(\lambda\,x)\,ya + 3\,\sin(\lambda\,x)\cos(\lambda\,x)\,y\lambda - 2\right.$$

56.7 problem number 7

problem number 507

Added January 20, 2019.

Problem 2.6.5.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \cos^n(\lambda x) y - a \cos^{n-1}(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Cos[lambda*x]^n*y - a*Cos[lambda*x]^(n
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)* y^2 + a*cos(lambda*x)^n*y-a*cos(lambda*x)^(n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

56.8 problem number 8

problem number 508

Added January 20, 2019.

Problem 2.6.5.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-(y\cos(\lambda x) - 1)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}\left(\cos(\lambda x)\, y\, \mathrm{expIntegral}\left(1, \frac{\cos(\lambda x)\, a}{\lambda}\right)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}} a - \mathrm{expIntegra}\right.\right.$$

56.9 problem number 9

problem number 509

Added January 20, 2019.

Problem 2.6.5.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + a \sin(\lambda x) y - a \tan(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + a*Sin[lambda*x]*y - a*Tan[lambda*x])*D[w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (lambda*sin(lambda*x)*y^2 + a*sin(lambda*x)*y-a*tan(lambda*x))*diff
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-(y\cos(\lambda x) - 1)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}}\left(\cos(\lambda x)\, y \operatorname{expIntegral}\left(1, \frac{\cos(\lambda x)\, a}{\lambda}\right)\mathrm{e}^{\frac{\cos(\lambda x)a}{\lambda}} a - \operatorname{expIntegra}\right.\right.$$

56.10 problem number 10

problem number 510

Added January 20, 2019.

Problem 2.6.5.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(Ae^{\lambda x}\cos(ay) + Be^{\mu x}\sin(ay) + Ae^{\lambda x}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (A*Exp[lambda*x]*Cos[a*y] + B*Exp[mu*x]*Sin[a*y] + A*Exp[lambda*x])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+ (A*exp(lambda*x)*cos(a*y) + B*exp(mu*x)*sin(a*y) + A*exp(lambda*x))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{1}{a\left(\lambda-\mu\right)\left(\cos\left(1/2\,ya\right)\right)^{3}}\left(\int \mathrm{e}^{-\frac{Be^{\mu\,x}a-\mu\,x\lambda}{\mu}}\,\mathrm{d}x a\cos\left(ya\right)A\cos\left(1/2\,ya\right)-\mathrm{e}^{-\frac{Be^{\mu\,x}a}{\mu}}\sin\left(\right.\right.\right.$$

56.11 problem number 11

problem number 511

Added January 20, 2019.

Problem 2.6.5.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin^{n+1}(2x)w_x + \left(ay^2\sin^{2n}x + b\cos^{2n}x\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = Sin[2*x]^(n + 1)*D[w[x, y], x] + (a*y^2*Sin[x]^(2*n) + b*Cos[x]^(2*n))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$\$Aborted$

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   sin(2*x)^(n+1)*diff(w(x,y),x)+ (a*y^2*sin(x)^(2*n) + b*cos(x)^(2*n) )*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

57 HFOPDE, chapter 2.7.1

57.1 problem number 1

problem number 512

Added January 20, 2019.

Problem 2.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arcsin^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcSin[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{ia \sin^{-1}(\lambda x)^k \left(\sin^{-1}(\lambda x)^2\right)^{-k} \left(\left(i \sin^{-1}(\lambda x)\right)^k \text{Gamma}(k+1, -i \sin^{-1}(\lambda x)) - (-i \sin^{-1}(\right.}{2\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-(\arcsin(\lambda x))^k (\arcsin(\lambda x))^{3/2} 2^k 2^{-k} \sqrt{-\lambda^2 x^2 + 1} a - \arcsin(\lambda x) 2^k 2^{-k} \text{LommelS1}(k +\right.$$

57.2 problem number 2

problem number 513

Added January 20, 2019.

Problem 2.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arcsin^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcSin[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ (a*arcsin(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\arcsin(y\lambda))^k + b\right)^{-1} dy + x\right)$$

57.3 problem number 3

problem number 514

Added January 20, 2019.

Problem 2.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\arcsin^n(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + k*Arcsin[a*x + b*y + c]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ k*arcsin(a*x + b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{ax+by}{b}} \left(k(\arcsin\left(b\_a+c\right))^n b+a\right)^{-1} d\_ab+x\right)$$

57.4 problem number 4

problem number 515

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arcsin^k(\lambda x)\arcsin^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*Arcsin[lambda*x]^k*Arcsin[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \mathrm{Arcsin}(\mu K[1])^{-n}\,dK[1]-\int_1^x a\mathrm{Arcsin}(\lambda K[2])^k\,dK[2]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+a*arcsin(lambda*x)^k*arcsin(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{2^k\sqrt{\pi}}{\lambda}\left(\frac{2^{-1-k}(\arcsin(\lambda x))^k(2k+6)x\lambda}{\sqrt{\pi}(k+1)(k+3)}+\frac{(\arcsin(\lambda x))^k 2^{-k}\sqrt{-\lambda^2x^2+1}(\arcsin(\lambda x)}{\sqrt{\pi}(k+1)(}\right.\right.$$

57.5 problem number 5

problem number 516

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arcsin x)^n y - a^2 + a\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*Arcsin[x]^n*y - a^2 + a*lambda*Arcsin[x]^n)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2+ lambda*arcsin(x)^n*y -a^2 + a *lambda*arcsin(x)^n)*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{y+a}\left(y\int e^{\lambda 2^n\sqrt{\pi}\left(\frac{2^{-n-1}(\arcsin(x))^n(2n+6)x}{\sqrt{\pi}(n+1)(n+3)}+\frac{(\arcsin(x))^n 2^{-n}\sqrt{-x^2+1}\left(\arcsin(x)x^2-\arcsin(x)+\sqrt{-x^2+1}x\right)}{\sqrt{\pi}(n+1)\left(x^2-1\right)}+\frac{2^{-n}\right.}}\right.\right.$$

57.6 problem number 6

problem number 517

Added January 20, 2019.

Problem 2.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(y^2 + \lambda x(\arcsin x)^n y + \lambda(\arcsin y)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*x*Arcsin[x]^n*y + lambda*Arcsin[x]^n)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( y^2+ lambda*x*arcsin(x)^n*y + lambda*arcsin(x)^n)*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int \frac{(\arcsin(x))^n \lambda x^2 - 2}{x} dx} dx + e^{\int \frac{(\arcsin(x))^n \lambda x^2 - 2}{x} dx} x + \int e^{\int \frac{(\arcsin(x))^n \lambda x^2 - 2}{x} dx} dx\right)\right)$$

57.7 problem number 7

problem number 518

Added January 20, 2019.

Problem 2.7.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arcsin x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*Arcsin[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)-(  (k+1)*x^k*y^2 - lambda*arcsin(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{x^{k+1}y-1}\left(yx^{k+1}\int\frac{e^{\lambda\int x^{k+1}(\arcsin(x))^n\,dx}}{x^k x^2}\,dxk + yx^{k+1}\int\frac{e^{\lambda\int x^{k+1}(\arcsin(x))^n\,dx}}{x^k x^2}\,dx - x^{k+1}e^{\int}\right.\right.$$

57.8 problem number 8

problem number 519

Added January 20, 2019.

Problem 2.7.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + ay + ab - b^2\lambda(\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 + a*y + a*b - b^2*lambda*Arcsin[x]^n)*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + a*y+ a*b -b^2 * lambda*arcsin(x)^n)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{b+y}\left(y\lambda\int(\arcsin(x))^n \mathrm{e}^{-2\lambda b 2^n\sqrt{\pi}\left(\frac{2^{-n-1}(\arcsin(x))^n(2n+6)x}{\sqrt{\pi}(n+1)(n+3)}+\frac{(\arcsin(x))^n 2^{-n}\sqrt{-x^2+1}\left(\arcsin(x)x^2-\mathrm{arc}\right.}{\sqrt{\pi}(n+1)\left(x^2-1\right)}\right.}\right.\right.$$

57.9 problem number 9

problem number 520

Added January 29, 2019.

Problem 2.7.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 - b\lambda x^m (\arcsin x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Arcsin[x]^n*y^2 - b*lambda*x^m*ArcSin[x]^n*y + b*m*x^(m - 1))
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 - b*lambda*x^m*arcsin(x)^n*y+b*m*x^(m-1) )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

57.10 problem number 10

problem number 521

Added January 29, 2019.

Problem 2.7.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arcsin x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arcsin x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcSin[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 + b*m*x^(m-1) - lambda*b^2*x^(2*m)*arcsin(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

57.11 problem number 11

problem number 522

Added January 29, 2019.

Problem 2.7.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\arcsin x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcSin[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arcsin(x)^n*(y - a*x^m -b)^2 + a*m*x^(m-1)  )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2^n 2^{-n} (\arcsin(x))^{3/2} \sqrt{-x^2+1} x^m (\arcsin(x))^n a\lambda + \mathrm{LommelS1}(n+1/2, 3/2, \arcsin(x))}{}\right.$$

57.12 problem number 12

problem number 523

Added January 29, 2019.

Problem 2.7.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arcsin x)^n y^2 + ky + \lambda b^2 x^{2k} (\arcsin x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (lambda*ArcSin[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcSin[x]^n)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x\lambda K[1]^{k-1}\sin^{-1}(K[1])^n\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+( lambda*arcsin(x)^n*y^2 +k*y+ lambda*b^2*x^(2*k)*arcsin(x)^n  )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\lambda\,b\int(\arcsin{(x)})^n\,x^{k-1}\,\mathrm{d}x-\arctan\left(\frac{x^{-k}y}{b}\right)\right)$$

58 HFOPDE, chapter 2.7.2

58.1 problem number 1

problem number 524

Added January 29, 2019.

Problem 2.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arccos^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcCos[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{\left(\cos^{-1}(\lambda x)^2\right)^{-k}\left(a(i\cos^{-1}(\lambda x))^k \cos^{-1}(\lambda x)^k \text{Gamma}(k+1, -i\cos^{-1}(\lambda x)) + a(-i\cos\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( a*arccos(lambda*x)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y + \frac{a2^k\sqrt{\pi}}{\lambda}\left(\frac{(\arccos(\lambda x))^{k+1}\, 2^{-k}\sqrt{-\lambda^2x^2+1}}{\sqrt{\pi}\,(k+2)} - \frac{2^{-k}\sqrt{\arccos(\lambda x)}\,\text{LommelS1}\,(3}{\sqrt{?}}\right.\right.$$

58.2 problem number 2

problem number 525

Added January 29, 2019.

Problem 2.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arccos^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcCos[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( a*arccos(lambda*y)^k + b   )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\arccos(y\lambda))^k + b\right)^{-1} dy + x\right)$$

58.3 problem number 3

problem number 526

Added January 29, 2019.

Problem 2.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k\arccos^n(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + k*ArcCos[a*x + b*y + c]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+k*arccos(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{ax+by}{b}} \left(k(\arccos{(b\_a+c)})^n b + a\right)^{-1} d\_ab + x\right)$$

58.4 problem number 4

problem number 527

Added January 29, 2019.

Problem 2.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arccos^k(\lambda x)\arccos^n(\mu y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*ArcCos[lambda*x]^k*ArcCos[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{(\cos^{-1}(\lambda x)^2)^{-k}\left(-a(i\cos^{-1}(\lambda x))^k\cos^{-1}(\lambda x)^k\text{Gamma}(k+1,-i\cos^{-1}(\lambda x))-a(-i\cos\right.}{}\right.\right.\right.$$

Maple ✓

```

w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+a*arccos(lambda*x)^k*arccos(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{(n-2)\,\mu\,a\sqrt{\arccos(\mu\,y)}}\left(\arccos(\mu\,y)\,y\,\text{LommelS1}(-n+1/2,1/2,\arccos(\mu\,y))\,\mu\,n-2\right.\right.$$

58.5 problem number 5

problem number 528

Added January 29, 2019.

Problem 2.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arccos x)^n y - a^2 + a\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*ArcCos[x]^n*y - a^2 + a*lambda*ArcCos[x]^n)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( y^2+lambda*arccos(x)^n*y- a^2 + a*lambda*arccos(x)^n )*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{(n+2)(y+a)}\left(\int \frac{\sqrt{\arccos(x)}\,\mathrm{LommelS1}(-1/2+n, 1/2, \arccos(x))\,x}{\sqrt{-x^2+1}} \mathrm{e}^{-\frac{-\arccos(x)\,\mathrm{Lomm}}{}}\right.\right.$$

58.6 problem number 6

problem number 529

Added January 29, 2019.

Problem 2.7.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arccos x)^n y + \lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*x*ArcCos[x]^n*y + a*lambda*ArcCos[x]^n)*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( y^2+lambda*x*arccos(x)^n*y + a*lambda*arccos(x)^n )*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

58.7 problem number 7

problem number 530

Added January 29, 2019.

Problem 2.7.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arccos x)^n(x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCos[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2 -lambda*arccos(x)^n*(x^(k+1)*y-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{x^{k+1}y-1}\left(yx^{k+1}\int \frac{e^{\lambda \int x^{k+1}(\arccos(x))^n \, dx}}{x^k x^2}\, dxk + yx^{k+1}\int \frac{e^{\lambda \int x^{k+1}(\arccos(x))^n \, dx}}{x^k x^2}\, dx - e^{\int \frac{x^{k+1}(a}{}}\right.\right.$$

58.8 problem number 8

problem number 531

Added January 29, 2019.

Problem 2.7.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + ay + ab - b^2\lambda(\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCos[x]^n)*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ a*y+ a*b - b^2*lambda*arccos(x)^n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{b+y}\left(\left(\int \frac{\lambda\left(-2\left(\arccos\left(x\right)\right)^{3}\mathrm{LommelS1}\left(-1/2+n,1/2,\arccos\left(x\right)\right)xbn^{2}+2\sqrt{-x^{2}+\cdots}\right.}{\cdots}\right.\right.\right.$$

58.9 problem number 9

problem number 532

Added January 29, 2019.

Problem 2.7.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 - b\lambda x^m(\arccos x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 - b*lambda*x^m*ArcCos[x]^n*y + b*m*x^(m - 1))
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2- b*lambda*x^m*arccos(x)^n*y + b*m*x^(m-1) )*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

58.10 problem number 10

problem number 533

Added January 29, 2019.

Problem 2.7.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arccos x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCos[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccos(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

58.11 problem number 11

problem number 534

Added January 29, 2019.

Problem 2.7.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arccos x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCos[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccos(x)^n*(y- a*x^m-b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-\sqrt{-x^2+1}x^m(\arccos(x))^{3/2}(\arccos(x))^n a\lambda - \sqrt{-x^2+1}(\arccos(x))^n(\arccos(x))^{3/2} b\lambda}{}\right.$$

58.12 problem number 12

problem number 535

Added January 29, 2019.

Problem 2.7.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arccos x)^n y^2 + ky + \lambda b^2 x^{2k}(\arccos x)^n\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (lambda*ArcCos[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCos[x]^n)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-k}}{\sqrt{b^2}}\right)-\sqrt{b^2}\int_1^x \lambda K[1]^{k-1}\cos^{-1}(K[1])^n\, dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+( lambda*arccos(x)^n*y^2+ k*y + lambda*b^2*x^(2*k)*arccos(x)^n )*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\lambda\, b \int (\arccos(x))^n\, x^{k-1}\, \mathrm{d}x - \arctan\left(\frac{x^{-k} y}{b}\right)\right)$$

59 HFOPDE, chapter 2.7.3

59.1 problem number 1

problem number 536

Added January 29, 2019.

Problem 2.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a\arctan^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcTan[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( a*arctan(lambda*x)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx - \int a(\arctan(\lambda x))^k \, dx + y\right)$$

59.2 problem number 2

problem number 537

Added January 29, 2019.

Problem 2.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \arctan^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*ArcTan[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( a*arctan(lambda*y)^k+b  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(a(\arctan(y\lambda))^k + b\right)^{-1} dy + x\right)$$

59.3 problem number 3

problem number 538

Added January 29, 2019.

Problem 2.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k \arctan^n(ax+by+c) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + k*ArcTan[a*x + b*y + c]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+k *arctan(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int^{\frac{ax+by}{b}} \left(k(\arctan(b\_a+c))^n b + a\right)^{-1} d\_ab + x\right)$$

59.4 problem number 4

problem number 539

Added January 29, 2019.

Problem 2.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\arctan^k(\lambda x)\arctan^n(\mu y)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*ArcTan[lambda*x]^k*ArcTan[mu*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+a*arctan(lambda*x)^k*arctan(mu*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int (\arctan(\lambda x))^k \,\mathrm{d}x + \int \frac{(\arctan(\mu y))^{-n}}{a}\,\mathrm{d}y\right)$$

59.5 problem number 5

problem number 540

Added January 29, 2019.

Problem 2.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\arctan x)^n y - a^2 + a\lambda(\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*ArcTan[x]^n*y - a^2 + a*lambda*ArcTan[x]^n)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2 + lambda*arctan(x)^n*y -a^2 + a *lambda*arctan(x)^n )*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{y\int \mathrm{e}^{\lambda\int(\arctan(x))^n\,\mathrm{d}x-2ax}\,\mathrm{d}x + \int \mathrm{e}^{\lambda\int(\arctan(x))^n\,\mathrm{d}x-2ax}\,\mathrm{d}xa + \mathrm{e}^{\lambda\int(\arctan(x))^n\,\mathrm{d}x-2ax}}{y+a}\right)$$

59.6 problem number 6

problem number 541

Added January 29, 2019.

Problem 2.7.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\arctan x)^n y + \lambda(\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*x*ArcTan[x]^n*y + lambda*ArcTan[x]^n)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2 + lambda*x*arctan(x)^n*y + lambda*arctan(x)^n )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int\frac{(\arctan(x))^n\lambda\, x^2-2}{x}dx}dx + e^{\int\frac{(\arctan(x))^n\lambda\, x^2-2}{x}dx}x + \int e^{\int\frac{(\arctan(x))^n\lambda\, x^2-2}{x}dx}dx\right)\right)$$

59.7 problem number 7

problem number 542

Added Feb. 1, 2019.

Problem 2.7.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\arctan x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcTan[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2 - lambda*arctan(x)^n*(x^(k+1)*y-1)  )*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{x^{k+1}y-1}\left(yx^{k+1}\int\frac{e^{\lambda\int(\arctan(x))^n x^{k+1}\,dx}}{x^k x^2}\,dxk + yx^{k+1}\int\frac{e^{\lambda\int(\arctan(x))^n x^{k+1}\,dx}}{x^k x^2}\,dx - \int\frac{e^{\lambda\int(a}}{}\right.\right.$$

59.8 problem number 8

problem number 543

Added Feb. 1, 2019.

Problem 2.7.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\arctan x)^n + ay + ab - b^2\lambda(\arctan x)^n n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcTan[x]^n + a*y + a*b - b^2*lambda*ArcTan[x]^n*n)*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$\text{Aborted}$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda* arctan(x)^n +a*y+ a*b - b^2*lambda*arctan(x)^n*n  )*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\int \mathrm{e}^{-ax}\left(b^2\lambda\left(\arctan\left(x\right)\right)^n n - \left(\arctan\left(x\right)\right)^n \lambda - ab\right) \mathrm{d}x + y\mathrm{e}^{-ax}\right)$$

59.9 problem number 9

problem number 544

Added Feb. 1, 2019.

Problem 2.7.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda (arctanx)^n y^2 - b\lambda x^m (\arctan x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 - b*lambda*x^m*ArcTan[x]^n*y + b*m*x^(m - 1))
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 - b*lambda*x^m*arctan(x)^n*y+ b*m*x^(m-1))*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

59.10 problem number 10

problem number 545

Added Feb. 1, 2019.

Problem 2.7.3.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(arctanx)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m} (\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcTan[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*y^2 +b*m*x^(m-1) - lambda*b^2*x^(2*m)*arctan(x)^n
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

59.11 problem number 11

problem number 546

Added Feb. 1, 2019.

Problem 2.7.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda (arctanx)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcTan[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arctan(x)^n*(y-a*x^m -b)^2 + a*m*x^(m-1) )*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^m \int (\arctan(x))^n \lambda \, dx a + y \int (\arctan(x))^n \lambda \, dx - \int (\arctan(x))^n \lambda \, dx b + 1}{y - ax^m - b}\right)$$

59.12 problem number 12

problem number 547

Added Feb. 1, 2019.

Problem 2.7.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(\lambda(\arctan x)^n y^2 + ky + \lambda b^2 x^{2k} (\arctan x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (lambda*ArcTan[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcTan[x]^n)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+(lambda*arctan(x)^n*y^2+k*y+lambda*b^2*x^(2*k)*arctan(x)^n  )*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\lambda\, b \int (\arctan (x))^{n}\, x^{k-1}\, \mathrm{d}x - \arctan \left(\frac{x^{-k} y}{b}\right)\right)$$

60 HFOPDE, chapter 2.7.4

60.1 problem number 1

problem number 548

Added Feb. 1, 2019.

Problem 2.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \operatorname{arccot}^k(\lambda x) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[lambda*x]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*x)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-bx + y - \int \lambda \left(\pi/2 - \arctan\left(\lambda x\right)\right)^k \, dx\right)$$

60.2 problem number 2

problem number 549

Added Feb. 1, 2019.

Problem 2.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(a \operatorname{arccot}^k(\lambda y) + b\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[lambda*y]^k + b)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arccot(lambda*y)^k+b)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \left(\lambda\,(\pi/2 - \arctan\,(y\lambda))^k + b\right)^{-1} \mathrm{d}y + x\right)$$

60.3 problem number 3

problem number 550

Added Feb. 1, 2019.

Problem 2.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + k \operatorname{arccot}^n(ax + by + c) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + k*ArcCot[a*x + b*y + c]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+k*arccot(a*x+b*y+c)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\int^{\frac{ax+by}{b}} \left(k(\pi/2 - \arctan(b\_a + c))^n b + a\right)^{-1} d\_ab + x\right)$$

60.4 problem number 4

problem number 551

Added Feb. 1, 2019.

Problem 2.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + k \operatorname{arccot}^k(\lambda x) \operatorname{arccot}^n(\mu y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*ArcCot[lambda*x]^k*ArcCot[lambda*y]^n*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+a*arccot(lambda*x)^k*arccot(lambda*y)^n*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int (\pi/2 - \arctan(\lambda x))^k \, dx + \int \frac{(\pi/2 - \arctan(y\lambda))^{-n}}{a} \, dy\right)$$

60.5 problem number 5

problem number 552

Added Feb. 1, 2019.

Problem 2.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda(\operatorname{arccot} x)^n y - a^2 + a\lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*ArcCot[x]^n*y - a^2 + a*lambda*ArcCot[x]^n)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2+lambda*arccot(x)^n*y - a^2 +a*lambda*arccot(x)^n)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{y\int \mathrm{e}^{\lambda\int(\pi/2-\arctan(x))^n\,\mathrm{d}x-2ax}\,\mathrm{d}x + \int \mathrm{e}^{\lambda\int(\pi/2-\arctan(x))^n\,\mathrm{d}x-2ax}\,\mathrm{d}xa + \mathrm{e}^{\lambda\int(\pi/2-\arctan(x))^n\,\mathrm{d}x-2}}{y+a}\right.$$

60.6 problem number 6

problem number 553

Added Feb. 1, 2019.

Problem 2.7.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + \lambda x(\operatorname{arccot} x)^n y + \lambda(\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + lambda*x*ArcCot[x]^n*y + lambda*ArcCot[x]^n)*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2+lambda*x*arccot(x)^n*y +lambda*arccot(x)^n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int \mathrm{e}^{\int \frac{(\operatorname{arccot}(x))^n\lambda\, x^2-2}{x}\,\mathrm{d}x}\,\mathrm{d}x + \mathrm{e}^{\int \frac{(\operatorname{arccot}(x))^n\lambda\, x^2-2}{x}\,\mathrm{d}x} x + \int \mathrm{e}^{\int \frac{(\operatorname{arccot}(x))^n\lambda\, x^2-2}{x}\,\mathrm{d}x}\,\mathrm{d}x\right)\right)$$

60.7 problem number 7

problem number 554

Added Feb. 1, 2019.

Problem 2.7.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x - \left((k+1)x^k y^2 - \lambda(\operatorname{arccot} x)^n (x^{k+1}y - 1)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - lambda*ArcCot[x]^n*(x^(k + 1)*y - 1))*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)-((k+1)*x^k*y^2- lambda*arccot(x)^n*(x^(k+1)*y-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{x^{k+1}y-1}\left(yx^{k+1}\int\frac{e^{\lambda\int x^{k+1}(\pi/2-\arctan(x))^n\,dx}}{x^k x^2}\,dxk + yx^{k+1}\int\frac{e^{\lambda\int x^{k+1}(\pi/2-\arctan(x))^n\,dx}}{x^k x^2}\,dx\right.\right.$$

60.8 problem number 8

problem number 555

Added Feb. 1, 2019.

Problem 2.7.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + ay + ab - b^2\lambda(\operatorname{arccot} x)^n n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + a*y + a*b - b^2*lambda*ArcCot[x]^n*n)*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2+a*y + a*b -b^2*lambda*arccot(x)^n*n )*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

60.9 problem number 9

problem number 556

Added Feb. 1, 2019.

Problem 2.7.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 - b\lambda x^m(\operatorname{arccot} x)^n y + bmx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 - b*lambda*x^m*ArcCot[x]^n*y + b*m*x^(m - 1))
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(lambda*arccot(x)^n*y^2- b*lambda*x^m*arccot(x)^n*y+ b*m*x^(m-1)  )*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

60.10 problem number 10

problem number 557

Added Feb. 1, 2019.

Problem 2.7.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n y^2 + bmx^{m-1} - \lambda b^2 x^{2m}(\operatorname{arccot} x^n)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + b*m*x^(m - 1) - lambda*b^2*x^(2*m)*ArcCot[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ b*m*x^(m-1) - lambda*b^2*x^(2*m)*arccot(x)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

60.11 problem number 11

problem number 558

Added Feb. 1, 2019.

Problem 2.7.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\lambda(\operatorname{arccot} x)^n (y - ax^m - b)^2 + amx^{m-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*ArcCot[x]^n*(y - a*x^m - b)^2 + a*m*x^(m - 1))*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( lambda*arccot(x)^n*(y-a*x^m-b)^2+a*m*x^(m-1) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x^m \int (\operatorname{arccot}(x))^n \lambda \, dx a + y \int (\operatorname{arccot}(x))^n \lambda \, dx - \int (\operatorname{arccot}(x))^n \lambda \, dx b + 1}{y - a x^m - b}\right)$$

60.12 problem number 12

problem number 559

Added Feb. 1, 2019.

Problem 2.7.4.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + \left(\lambda (\operatorname{arccot} x)^n y^2 + k y + \lambda b^2 x^{2k} (\operatorname{arccot} x)^n\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (lambda*ArcCot[x]^n*y^2 + k*y + lambda*b^2*x^(2*k)*ArcCot[x]^n)*D[w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+( lambda*arccot(x)^n*y^2+ k*y+ lambda*b^2*x^(2*k)*arccot(x)^n )*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\lambda\, b\int (\pi/2-\arctan(x))^{n}\, x^{k-1}\,dx - \arctan\left(\frac{x^{-k}y}{b}\right)\right)$$

61 HFOPDE, chapter 2.8.1

61.1 problem number 1

problem number 560

Added Feb. 4, 2019.

Problem 2.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (f(x)y + g(x))\, w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y + g[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-e^{-\int_1^x f(K[1])\,dK[1]}\left(e^{\int_1^x f(K[1])\,dK[1]}\int_1^x g(K[2])e^{-\text{Integrate}[f(K[1]),\{K[1],1,K[2]\},\text{Assumptions}\to\text{True}]}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( f(x)*y+g(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int g(x)\,\mathrm{e}^{-\int f(x)\,\mathrm{d}x}\,\mathrm{d}x + y\mathrm{e}^{-\int f(x)\,\mathrm{d}x}\right)$$

61.2 problem number 2

problem number 561

Added Feb. 4, 2019.

Problem 2.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y + g(x)y^k\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y + g[x]*y^k)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(y^{-k}e^{-\int_1^x f(K[1])\,dK[1]}\left(y^k\left(-e^{\int_1^x f(K[1])\,dK[1]}\right)\left(\int_1^x g(K[2])\exp(-(1-k)\text{Integrate}[f(K[1]),\right.\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( f(x)*y+g(x)*y^k )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(y^{1-k}\mathrm{e}^{(k-1)\int f(x)\,\mathrm{d}x} + k\int \mathrm{e}^{(k-1)\int f(x)\,\mathrm{d}x}g(x)\,\mathrm{d}x - \int \mathrm{e}^{(k-1)\int f(x)\,\mathrm{d}x}g(x)\,\mathrm{d}x\right)$$

61.3 problem number 3

problem number 562

Added Feb. 4, 2019.

Problem 2.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + f(x)y - a^2 - af(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + f[x]*y - a^2 - a*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( y^2+f(x)*y -a^2 -a*f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{e^{\int f(x)\,dx + 2ax} + y\int e^{\int f(x)\,dx + 2ax}\,dx - a\int e^{\int f(x)\,dx + 2ax}\,dx}{-a+y}\right)$$

61.4 problem number 4

problem number 563

Added Feb. 4, 2019.

Problem 2.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 + xf(x)y + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 + x*f[x]*y + f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( y^2+x*f(x)*y + f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{yx+1}\left(yx\int e^{\int \frac{f(x)x^2-2}{x}dx}dx + e^{\int \frac{f(x)x^2-2}{x}dx}x + \int e^{\int \frac{f(x)x^2-2}{x}dx}dx\right)\right)$$

61.5 problem number 5

problem number 564

Added Feb. 4, 2019.

Problem 2.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left((k+1)x^k y^2 - x^{k+1} f(x) y + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - ((k + 1)*x^k*y^2 - x^(k + 1)*f[x]*y + f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)-( (k+1)*x^k*y^2-x^(k+1)*f(x)*y+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{x^{k+1}y-1}\left(yx^{k+1}\int \frac{e^{\int x^{k+1}f(x)\,dx}}{x^k x^2}\,dx k + yx^{k+1}\int \frac{e^{\int x^{k+1}f(x)\,dx}}{x^k x^2}\,dx - e^{\int \frac{x^{k+1}f(x)x-2k-2}{x}\,dx} x^{k+1}\right.\right.$$

61.6 problem number 6

problem number 565

Added Feb. 4, 2019.

Problem 2.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + ay - ab - b^2 f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + a*y - a*b - b^2*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( f(x)*y^2+a*y-a*b- b^2*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{e^{ax+2b\int f(x)\,dx} + y\int e^{ax+2b\int f(x)\,dx} f(x)\,dx - b\int e^{ax+2b\int f(x)\,dx} f(x)\,dx}{-b+y}\right)$$

61.7 problem number 7

problem number 566

Added Feb. 4, 2019.

Problem 2.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f[x]y^2 - ax^n f[x]y + anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*x^n*f[x]*y + a*n*x^(n - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*f(x)*y+a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

61.8 problem number 8

problem number 567

Added Feb. 4, 2019.

Problem 2.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + anx^{n-1} - a^2x^{2n}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + a*n*x^(n - 1) - a^2*x^(2*n)*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(  f(x)*y^2+a*n*x^(n-1)-a^2*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

61.9 problem number 9

problem number 568

Added Feb. 4, 2019.

Problem 2.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y - a^2f(x) - ag(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + g[x]*y - a^2*f[x] - a*g[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)* y-a^2*f(x)-a*g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{e^{\int g(x)\,dx+2a\int f(x)\,dx} + y\int e^{\int g(x)\,dx+2a\int f(x)\,dx}f(x)\,dx - a\int e^{\int g(x)\,dx+2a\int f(x)\,dx}f(x)\,dx}{-a+y}\right)$$

61.10 problem number 10

problem number 569

Added Feb. 4, 2019.

Problem 2.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y + anx^{n-1} - ax^n g(x) - a^2x^{2n}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*n*x^(n - 1) - a*x^n*g[x] - a^2*x^(2*n)*f[x])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(  f(x)*y^2+g(x)*y+a*n*x^(n-1) - a*x^n*g(x)-a^2*x^(2*n)*f(x))*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

61.11 problem number 11

problem number 570

Added Feb. 4, 2019.

Problem 2.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ax^n g(x)y + anx^{n-1} + a^2x^{2n}(g(x) - f(x))\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*x^n*g*x*y + a*n*x^(n - 1) + a^2*x^(2*n)*(g*x - f*x))*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+( f(x)*y^2-a*x^n*g(x)*y+a*n*x^(n-1)+a^2*x^(2*n)*(g(x)-f(x)))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

61.12 problem number 12

problem number 571

Added Feb. 4, 2019.

Problem 2.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(f(x)y^2 + ny + ax^{2n}f(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (f[x]*y^2 + n*y + a*x^(2*n)*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{yx^{-n}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x f(K[1])K[1]^{n-1}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+( f(x)*y^2+n*y+a*x^(2*n)*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y) = \_F1\left(\sqrt{a}\int x^{n-1}f(x)\,dx - \arctan\left(\frac{yx^{-n}}{\sqrt{a}}\right)\right)$$

61.13 problem number 13

problem number 572

Added Feb. 4, 2019.

Problem 2.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x^{2n}f(x)y^2 + (ax^n f(x) - n)y + bf(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (x^(2*n)*f[x]*y^2 + (a*x^n*f[x] - n)*y + b*f[x])*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x* diff(w(x,y),x)+( x^(2*n)* f(x)*y^2+(a*x^n*f(x)-n)*y+b*f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{a}{\sqrt{a^2\,(a^2-4\,b)}}\left(-\sqrt{a^2\,(a^2-4\,b)}\int\frac{x^n f(x)}{x}\,\mathrm{d}x - 2\,a\,\mathrm{arctanh}\left(\frac{a(2\,x^n y + a)}{\sqrt{a^2\,(a^2-4\,b)}}\right)\right)\right)$$

62 HFOPDE, chapter 2.8.2

62.1 problem number 1

problem number 573

Added Feb. 4, 2019.

Problem 2.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(ae^{\lambda x}y^2 + ae^{\lambda x}f(x)y + \lambda f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y^2 + a*Exp[lambda*x]*f[x]*y + lambda*f[x])*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( a*exp(lambda*x)*y^2 + a*exp(lambda*x)*f(x)*y+lambda*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\lambda\,\mathrm{e}^{\lambda\,x}\left(\mathrm{e}^{\lambda\,x}ay+\lambda\right)}{\mathrm{e}^{2\,\lambda\,x}y\int \mathrm{e}^{a\int \mathrm{e}^{\lambda\,x}f(x)\,\mathrm{d}x-\lambda\,x}\,\mathrm{d}xa+\mathrm{e}^{\lambda\,x}\lambda\,\int \mathrm{e}^{a\int \mathrm{e}^{\lambda\,x}f(x)\,\mathrm{d}x-\lambda\,x}\,\mathrm{d}x+\mathrm{e}^{a\int \mathrm{e}^{\lambda\,x}f(x)\,\mathrm{d}x}}\right)$$

62.2 problem number 2

problem number 574

Added Feb. 4, 2019.

Problem 2.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x}f(x)y + a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*f[x]*y + a*lambda*Exp[lambda*x])*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2-a*exp(lambda*x)*f(x)*y+a*lambda*exp(lambda*x))*diff(w(x,y),
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

62.3 problem number 3

problem number 575

Added Feb. 4, 2019.

Problem 2.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + a\lambda e^{\lambda x} - a^2e^{2\lambda x}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + a*lambda*Exp[lambda*x] - a^2*Exp[2*lambda*x]*f[x])*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2+a*lambda*exp(lambda*x)-a^2*exp(2*lambda*x)*f(x))*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

62.4 problem number 4

problem number 576

Added Feb. 4, 2019.

Problem 2.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + \lambda y + ae^{2\lambda x}f(x)\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + lambda*y + a*Exp[2*lambda*x]*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x f(K[1])e^{\lambda K[1]}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2+lambda*y+ a*exp(2*lambda*x)* f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0),
```

$$w(x,y)=\_F1\left(\sqrt{a}\int e^{\lambda x}f(x)\,dx-\arctan\left(\frac{e^{-\lambda x}y}{\sqrt{a}}\right)\right)$$

62.5 problem number 5

problem number 577

Added Feb. 4, 2019.

Problem 2.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f(x)y^2-(ae^{\lambda x}+b)f(x)y+a\lambda e^{\lambda x}\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - (a*Exp[lambda*x] + b)*f[x]*y + a*lambda*Exp[lambda*x])*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2-(a*exp(lambda*x)+b)*f(x)*y+a *lambda*exp(lambda*x))*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

62.6 problem number 6

problem number 578

Added Feb. 4, 2019.

Problem 2.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(e^{\lambda x} f(x) y^2 + (a f(x) - \lambda) y + b e^{-\lambda x} f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (Exp[lambda*x]*f[x]*y^2 + (a*f[x] - lambda)*y + b*Exp[-(lambda*x)]*f[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  exp(lambda*x)*f(x)*y^2+(a*f(x)-lambda)*y+b*exp(-lambda*x)*f(x))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{a}{\sqrt{a^2(a^2-4b)}}\left(-\sqrt{a^2(a^2-4b)}\int f(x)\,\mathrm{d}x - 2a\,\mathrm{arctanh}\left(\frac{a(2ye^{\lambda x}+a)}{\sqrt{a^2(a^2-4b)}}\right)\right)\right)$$

62.7 problem number 7

problem number 579

Added Feb. 4, 2019.

Problem 2.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g(x)y + a\lambda e^{\lambda x} - ae^{\lambda x}g(x) - a^2e^{2\lambda x}f(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + g[x]*y + a*lambda*Exp[lambda*x] - a*Exp[lambda*x]*g[x] -
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2+ g(x)*y+a*lambda*exp(lambda*x) -a*exp(lambda*x)*g(x)-a^2*e
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

62.8 problem number 8

problem number 580

Added Feb. 7, 2019.

Problem 2.8.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ae^{\lambda x}g(x)y + a\lambda e^{\lambda x} + a^2e^{2\lambda x}(g(x) - f(x))\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*Exp[lambda*x]*g[x]*y + a*lambda*Exp[lambda*x] + a^2*Exp
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2- a*exp(lambda*x)*g(x)*y + a*lambda*exp(lambda*x) +a^2*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

62.9 problem number 9

problem number 581

Added Feb. 7, 2019.

Problem 2.8.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + 2a\lambda x e^{\lambda x^2} - a^2 f(x) e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + 2*a*lambda*x*Exp[lambda*x^2] - a^2*f[x]*Exp[2*lambda*x^2]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2+2*a*lambda*x*exp(lambda*x^2) - a^2*f(x)*exp(2*lambda*x^2))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$\text{sol}=()$$

62.10 problem number 10

problem number 582

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + 2\lambda xy + af(x)e^{2\lambda x^2}\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + 2*lambda*x*y + a*f[x]*Exp[2*lambda*x^2])*D[w[x, y], y] ==
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-\lambda x^2}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x f(K[1])e^{\lambda K[1]^2}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)*y^2+2*lambda*x*y+ a*f(x)*exp(2*lambda*x^2))*diff(w(x,y),y) = 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y) = \_F1\left(\sqrt{a}\int e^{\lambda x^2} f(x)\,dx - \arctan\left(\frac{e^{-\lambda x^2} y}{\sqrt{a}}\right)\right)$$

62.11 problem number 11

problem number 583

Added Feb. 7, 2019.

Problem 2.8.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)e^{\lambda y} + g(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*Exp[lambda*y] + g[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(   f(x)*exp(lambda*y) + g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{\lambda \int f(x) \,\mathrm{e}^{\lambda \int g(x)\,\mathrm{d}x} \,\mathrm{d}x + \mathrm{e}^{\lambda \left(\int g(x)\,\mathrm{d}x - y\right)}}{\lambda}\right)$$

63 HFOPDE, chapter 2.8.3

63.1 problem number 1

problem number 584

Added Feb. 7, 2019.

Problem 2.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda \sinh(\lambda x) - a^2 f(x)\sinh^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sinh[lambda*x] - a^2*f[x]*Sinh[lambda
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(   f(x)*y^2 - a^2*f(x) + a*lambda*sinh(lambda*x) - a^2*f(x)*sinh(lamb
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

63.2 problem number 2

problem number 585

Added Feb. 7, 2019.

Problem 2.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x)+\lambda)\tanh^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Tanh[lambda*x]^2 + a*lambda)*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*tanh(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

63.3 problem number 3

problem number 586

Added Feb. 7, 2019.

Problem 2.8.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) + \lambda)\coth^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] + lambda)*Coth[lambda*x]^2 + a*lambda)*D[w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(   f(x)*y^2 - a*(a*f(x)+lambda)*coth(lambda*x)^2 +a*lambda)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

64 HFOPDE, chapter 2.8.4

64.1 problem number 1

problem number 587

Added Feb. 7, 2019.

Problem 2.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(ay^2 \ln x - axy(\ln x - 1)f(x) + f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - (a*y^2*Log[x] - a*x*y*(Log[x] - 1)*f[x] + f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)-(   a*y^2*ln(x) -a*x*y* (ln(x)-1)*f(x)+f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{a\left(\ln\left(x\right)axy - yax - 1\right)}\left(\ln\left(x\right)y\int\frac{\ln\left(x\right)}{x^2\left(\ln\left(x\right) - 1\right)^2}\mathrm{e}^{a\int\frac{(\ln(x))^2 f(x)x}{\ln(x)-1}\,\mathrm{d}x - 2a\int\frac{\ln(x)f(x)x}{\ln(x)-1}\,\mathrm{d}x + a\int\frac{f(x)}{\ln(x)}\right.\right.$$

64.2 problem number 2

problem number 588

Added Feb. 7, 2019.

Problem 2.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - ax(\ln x)f(x)y + a\ln x + a\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*x*Log[x]*f[x]*y + a*Log[x] + a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(  f(x)* y^2 -a*x*ln(x)*f(x)*y+a*ln(x)+a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

64.3 problem number 3

problem number 589

Added Feb. 7, 2019.

Problem 2.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(f(x)y^2 + a - a^2(\ln x)^2 f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (f[x]*y^2 + a - a^2*Log[x]^2*f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*diff(w(x,y),x)+( f(x)*y^2 +a -a^2* ln(x)^2 *f(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

64.4 problem number 4

problem number 590

Added Feb. 7, 2019.

Problem 2.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xw_x + \left((y + a\ln x)^2 f(x) - a\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + ((y + a*Log[x])^2*f[x] - a)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\log(x)\int_1^x\frac{f(K[2])}{K[2]}\,dK[2]+y\int_1^x\frac{f(K[2])}{K[2]}\,dK[2]+1}{a\log(x)+y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=x*diff(w(x,y),x)+( (y+a *ln(x))^2*f(x)-a)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{1}{y+\ln(x)\,a}\left(\ln(x)\,a\int\frac{f(x)}{x}\,\mathrm{d}x+y\int\frac{f(x)}{x}\,\mathrm{d}x+1\right)\right)$$

65 HFOPDE, chapter 2.8.5

65.1 problem number 1

problem number 591

Added Feb. 7, 2019.

Problem 2.8.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + \left(\lambda \sin(\lambda x) y^2 + f(x) \cos(\lambda x) y - f(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (lambda*Sin[lambda*x]*y^2 + f[x]*Cos[lambda*x]*y - f[x])*D[w[x, y], y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( lambda*sin(lambda*x)*y^2 + f(x)*cos(lambda*x)*y-f(x))*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left((\cos(\lambda x)\, y - 1)\left(-\cos(\lambda x)\, y \int -\lambda\, e^{\int \frac{(\cos(\lambda x))^2 \sqrt{(\sin(\lambda x))^2} f(x) - 2(\cos(\lambda x))^2 \lambda + 2\lambda}{\sin(\lambda x)\cos(\lambda x)} dx} \sin(\lambda x)\, dx + \right.\right.$$

65.2 problem number 2

problem number 592

Added Feb. 7, 2019.

Problem 2.8.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda \sin(\lambda x) + a^2 f(x)\sin^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Sin[lambda*x] + a^2*f[x]*Sin[lambda*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*sin(lambda*x)+a^2*f(x)*sin(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

65.3 problem number 3

problem number 593

Added Feb. 7, 2019.

Problem 2.8.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a^2 f(x) + a\lambda\cos(\lambda x) + a^2 f(x)\cos^2(\lambda x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a^2*f[x] + a*lambda*Cos[lambda*x] + a^2*f[x]*Cos[lambda*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2-a^2*f(x)+a*lambda*cos(lambda*x)+a^2*f(x)*cos(lambda*x)^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

sol=()

65.4 problem number 4

problem number 594

Added Feb. 7, 2019.

Problem 2.8.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda)\tan^2(\lambda x) + a\lambda\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Tan[lambda*x]^2 + a*lambda)*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*tan(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

sol=()

65.5 problem number 5

problem number 595

Added Feb. 7, 2019.

Problem 2.8.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - a(af(x) - \lambda)\cot^2(\lambda x) + a\lambda\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - a*(a*f[x] - lambda)*Cot[lambda*x]^2 + a*lambda)*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2-a*(a*f(x)-lambda)*cot(lambda*x)^2+a*lambda)*diff(w(x,y),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

66 HFOPDE, chapter 2.8.6

66.1 problem number 1

problem number 596

Added Feb. 7, 2019.

Problem 2.8.6.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 - f(x)g(x)y + g'(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 - f[x]*g[x]*y + Derivative[1][g][x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+( f(x)*y^2 -f(x)*g(x)*y+ diff(g(x),x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

sol=()

66.2 problem number 2

problem number 597

Added Feb. 7, 2019.

Problem 2.8.6.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x - \left(f'(x)y^2 - f(x)g(x)y + g(x)\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] - (Derivative[1][f][x]*y^2 - f[x]*g[x]*y + g[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)-( diff(f(x),x)*y^2 -f(x)*g(x)*y+ g(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{f(x)\,y-1}\left(yf(x)\int\frac{\left(\frac{d}{dx}f(x)\right)e^{\int f(x)g(x)\,dx}}{(f(x))^2}\,dx - f(x)\,e^{-\int\frac{-(f(x))^2g(x)+2\frac{d}{dx}f(x)}{f(x)}\,dx} - \int\frac{\left(\frac{d}{dx}f(x\right.}{(}\right.\right.$$

66.3 problem number 3

problem number 598

Added Feb. 7, 2019.

Problem 2.8.6.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(g(x)(y-f(x))^2 + f'(x)\right) w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (g[x]*(y - f[x])^2 + Derivative[1][f][x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{-f(x)\int_1^x g(K[2])\,dK[2] + y\int_1^x g(K[2])\,dK[2] + 1}{y-f(x)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(g(x)*(y-f(x))^2 + diff(f(x),x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \_F1\left(\frac{\int g(x)\,\mathrm{d}xy - f(x)\int g(x)\,\mathrm{d}x + 1}{y - f(x)}\right)$$

66.4 problem number 4

problem number 599

Added Feb. 7, 2019.

Problem 2.8.6.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\frac{f'(x)}{g(x)}y^2 - \frac{g'(x)}{f(x)}\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + ((Derivative[1][f][x]*y^2)/g[x] - Derivative[1][g][x]/f[x])*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(diff(f(x),x)/g(x)* y^2 - diff(g(x),x)/f(x)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{1}{(f(x)y+g(x))f(x)}\left(y(f(x))^2\int\frac{\frac{d}{dx}f(x)}{(f(x))^2g(x)}\,dx + g(x)f(x)\int\frac{\frac{d}{dx}f(x)}{(f(x))^2g(x)}\,dx + 1\right.\right.$$

66.5 problem number 5

problem number 600

Added Feb. 7, 2019.

Problem 2.8.6.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f^2(x)w_x + \left(f'(x)y^2 - g(x)(y - f(x))\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = f[x]^2*D[w[x, y], x] + (Derivative[1][f][x]*y^2 - g[x]*(y - f[x]))*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := f(x)^2*diff(w(x,y),x)+(diff(f(x),x)*y^2 -g(x)*(y-f(x))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

66.6 problem number 6

problem number 601

Added Feb. 7, 2019.

Problem 2.8.6.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(y^2 - \frac{f''(x)}{f(x)}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (y^2 - Derivative[2][f][x]/f[x])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+(y^2 - diff(f(x),x,x)/f(x) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{y(f(x))^2 \int (f(x))^{-2}\,dx + \int (f(x))^{-2}\,dx\left(\frac{d}{dx}f(x)\right)f(x) + 1}{f(x)\left(f(x)\,y + \frac{d}{dx}f(x)\right)}\right)$$

66.7 problem number 7

problem number 602

Added Feb. 7, 2019.

Problem 2.8.6.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$g(x)w_x + \left(af(x)g(x)y^3 + (bf(x)g^3(x) + g'(x))y + cf(x)g^4(x)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = g[x]*D[w[x, y], x] + (a*f[x]*g[x]*y^3 + (b*f[x]*g[x]^3 + Derivative[1][g][x])*y + c*f
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  g(x)*diff(w(x,y),x)+(a*f(x)*g(x)*y^3 + (b*f(x)*g(x)^3 + diff(g(x),x))*y+ c*f(x)*g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(b^3 \sum_{\_R=\mathrm{RootOf}\left(\_Z^3ac^2+\_Z\,b^3-b^3\right)} \frac{1}{3\_R^2ac^2+b^3} \ln\left(-\frac{\_R\,cg(x)+by}{cg\left(x\right)}\right) - b\int \left(g(x)\right)^2 f(x)\,\mathrm{d}x$$

66.8 problem number 8

problem number 603

Added Feb. 7, 2019.

Problem 2.8.6.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f(x)y^3+3f(x)h(x)y^2+\left(g(x)+3f(x)h^2(x)\right)y+f(x)h^3(x)+g(x)h(x)-h'(x)\right)w_y=0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^3 + 3*f[x]*h[x]*y^2 + (g[x] + 3*f[x]*h[x]^2)*y + f[x]*h[x]^3
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{4yh(x)\left(\int_1^x f(K[2])e^{2\text{Integrate}[g(K[1]),\{K[1],1,K[2]\},\text{Assumptions}\to\text{True}]}\,dK[2]\right)+2h(x)^2\int_1^x f(K[2\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(f(x)*y^3+3*f(x)*h(x)*y^2+(g(x)+3*f(x)*h(x)^2)*y+ f(x)*h(x)^3 + g(x)*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{2\,y^2\int f(x)\,\mathrm{e}^{2\int g(x)\,\mathrm{d}x}\,\mathrm{d}x+4\,yh(x)\int f(x)\,\mathrm{e}^{2\int g(x)\,\mathrm{d}x}\,\mathrm{d}x+2\,(h(x))^2\int f(x)\,\mathrm{e}^{2\int g(x)\,\mathrm{d}x}\,\mathrm{d}x+\mathrm{e}^{2}}{(y+h\,(x))^2}\right.$$

66.9 problem number 9

problem number 604

Added Feb. 7, 2019.

Problem 2.8.6.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(\frac{g'(x)}{f^2(x)(ag(x)+b)^3}y^3+\frac{f'(x)}{f(x)}y+f(x)g'(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + ((Derivative[1][g][x]*y^3)/(f[x]^2*(a*g[x] + b)^3) + (Derivative[1][f
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(diff(g(x),x)/(f(x)^2 *(a*g(x)+b)^3)*y^3 + diff(f(x),x)/f(x) * y + f(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(a^3 \sum_{\_R=\mathrm{RootOf}(-a^3\_Z+\_Z^3+a^3)} \frac{1}{-a^3+3\_R^2}\ln\left(\frac{-g(x)\,f(x)\_R\,a - f(x)\_R\,b + ya}{(ag(x)+b)\,f(x)}\right) - \ln($$

66.10 problem number 10

problem number 605

Added Feb. 7, 2019.

Problem 2.8.6.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left((y-f(x))(y-g(x))\left(y-\frac{af(x)+bg(x)}{a+b}\right)h(x)+\frac{y-g(x)}{f(x)-g(x)}f'(x)+\frac{y-f(x)}{g(x)-f(x)}g'(x)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + ((y - f[x])*(y - g[x])*(y - (a*f[x] + b*g[x])/(a + b))*h[x] + ((y - g
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+((y-f(x))*(y-g(x))*(y- (a*f(x)+b*g(x))/(a+b))*h(x)+(y-g(x))/(f(x)-g(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(1/3\,\frac{1}{a^2+2\,ab+b^2}\left(-\frac{a}{b\,(a+b)}\left(\ln\left(-9\,\frac{a^3y+2\,ya^2b+2\,yab^2+b^3y-a^2g(x)\,b-ag(x)\,b^2}{ag\,(x)-bg\,(x)-af}\right.\right.\right.\right.$$

66.11 problem number 11

problem number 606

Added Feb. 7, 2019.

Problem 2.8.6.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(x)y^2 + g'(x)y + af(x)e^{2g(x)}\right)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[x]*y^2 + Derivative[1][g][x]*y + a*f[x]*Exp[2*g[x]])*D[w[x, y], y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> a > 0], 60
```

$$\left\{\left\{w(x,y)\to c_1\left(\tan^{-1}\left(\frac{ye^{-g(x)}}{\sqrt{a}}\right)-\sqrt{a}\int_1^x f(K[1])e^{g(K[1])}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(f(x)*y^2 + diff(g(x),x)*y+ a*f(x)*exp(2*g(x)) )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming a>0 )
```

$$w(x,y)=\_F1\left(\sqrt{a}\int f(x)\,\mathrm{e}^{g(x)}\,\mathrm{d}x-\arctan\left(\frac{\mathrm{e}^{-g(x)}y}{\sqrt{a}}\right)\right)$$

66.12 problem number 12

problem number 607

Added Feb. 7, 2019.

Problem 2.8.6.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x+\left(f'(x)y^2+ae^{\lambda x}f(x)y+ae^{\lambda x}\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (Derivative[1][f][x]*y^2 + a*Exp[lambda*x]*f[x]*y + a*Exp[lambda*x])*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+(diff(f(x),x)*y^2+ a*exp(lambda*x)* f(x)*y+a*exp(lambda*x) )*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))  ),output='rea
```

$$w(x,y) = \_F1\left(-\frac{1}{f(x)\,y+1}\left(yf(x)\int\frac{\left(\frac{d}{dx}f(x)\right)\mathrm{e}^{a\int \mathrm{e}^{\lambda x}f(x)\,dx}}{\left(f(x)\right)^2}\,dx + f(x)\,\mathrm{e}^{\int\frac{\mathrm{e}^{\lambda x}(f(x))^2a-2\frac{d}{dx}f(x)}{f(x)}\,dx} + \int\frac{\left(\frac{d}{dx}f(\right.}{}\right.\right.$$

67 HFOPDE, chapter 2.9.1

67.1 problem number 1

problem number 608

Added Feb. 7, 2019.

Problem 2.9.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + g(y)w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\int_1^y \frac{1}{g(K[1])}\,dK[1]-\int_1^x \frac{1}{f(K[2])}\,dK[2]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))  ),output='rea
```

$$w(x,y) = \_F1\left(-\int (f(x))^{-1}\,\mathrm{d}x + \int (g(y))^{-1}\,\mathrm{d}y\right)$$

67.2 problem number 2

problem number 609

Added Feb. 7, 2019.

Problem 2.9.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(x)+g(y))w_x + f'(x)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[x] + g[y])*D[w[x, y], x] + Derivative[1][f][x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (f(x)+g(y))*diff(w(x,y),x)+diff(f(x),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-\int e^{-y} g(y)\, dy + e^{-y} f(x)\right)$$

67.3 problem number 3

problem number 610

Added Feb. 7, 2019.

Problem 2.9.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x^n f(y) + x g(y)) w_x + h(y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (x^n*f[y] + x*g[y])*D[w[x, y], x] + h[y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (x^n*f(y) + x*g(y))*diff(w(x,y),x)+h(y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x^{-n+1} e^{(n-1)\int \frac{g(y)}{h(y)}dy} + n \int \frac{f(y)}{h(y)} e^{(n-1)\int \frac{g(y)}{h(y)}dy} dy - \int \frac{f(y)}{h(y)} e^{(n-1)\int \frac{g(y)}{h(y)}dy} dy\right)$$

67.4 problem number 4

problem number 611

Added Feb. 7, 2019.

Problem 2.9.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(y) + amx^n y^{m-1})w_x - (g(x) + anx^{n-1}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[y] + a*m*x^n*y^(m - 1))*D[w[x, y], x] - (g[x] + a*n*x^(n - 1)*y^m)*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (f(y) + a*m*x^n*y^(m-1))*diff(w(x,y),x)-(g(x)+a*n*x^(n-1)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

sol=()

67.5 problem number 5

problem number 612

Added Feb. 7, 2019.

Problem 2.9.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(e^{\alpha x}f(y) + c\beta)w_x - (e^{\beta y}g(x) + c\alpha)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (Exp[alpha*x]*f[y] + c*beta)*D[w[x, y], x] - (Exp[beta*y]*g[x] + c*alpha)*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (exp(alpha*x)* f(y) + c*beta)*diff(w(x,y),x)-(exp(beta*y)*g(x) + c*alpha)*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

68 HFOPDE, chapter 2.9.2

68.1 problem number 1

problem number 613

Added Feb. 7, 2019.

Problem 2.9.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + f(ax+by+c)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + f[a*x + b*y + c]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ f(a*x+b*y+c)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \_F1\left(-\int^{\frac{ax+by}{b}} \left(f(b\_a+c)\,b+a\right)^{-1} d\_ab + x\right)$$

68.2 problem number 2

problem number 614

Added Feb. 7, 2019.

Problem 2.9.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + f(\frac{y}{x})w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + f[y/x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ f(y/x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \_F1\left(\int^{\frac{y}{x}} (f(\_a) - \_a)^{-1} d\_a - \ln(x)\right)$$

68.3 problem number 3

problem number 615

Added Feb. 7, 2019.

Problem 2.9.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(y + ax^n + b) - anx^{n-1}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[y + a*x^n + b] - a*n*x^(n - 1))*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+ (f(y+a*x^n+b) - a*n*x^(n-1))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} (f(\_a + x^n a + b))^{-1} \, \mathrm{d}\_a - x\right)$$

68.4 problem number 4

problem number 616

Added Feb. 7, 2019.

Problem 2.9.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yf(x^n y^m)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*f[x^n*y^m]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+ y*f(x^n*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-\frac{1}{m}\left(-\int_{\_b}^{y} \frac{1}{(f(x^n \_a^m) m + n) \_a} d\_am + \ln(x)\right)\right)$$

68.5 problem number 5

problem number 617

Added Feb. 7, 2019.

Problem 2.9.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$y^{m-1}w_x + x^{n-1}f(ax^n + by^m)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = y^(m - 1)*D[w[x, y], x] + x^(n - 1)*f[a*x^n + b*y^m]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := y^(m-1)*diff(w(x,y),x)+  x^(n-1)*f(a*x^n+b*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

68.6 problem number 6

problem number 618

Added Feb. 7, 2019.

Problem 2.9.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + e^{-\lambda x} f(e^{\lambda x} y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + Exp[-(lambda*x)]*f[Exp[lambda*x]*y]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+   exp(-lambda*x)*f(exp(lambda*x)*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x - \int^{y e^{\lambda x}} (\lambda \_a + f(\_a))^{-1} d\_a\right)$$

68.7 problem number 7

problem number 619

Added Feb. 7, 2019.

Problem 2.9.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(e^{\lambda y} x) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + Exp[lambda*y]*f[Exp[lambda*y]*x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+   exp(lambda*y)*f(exp(lambda*y)*x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-\frac{1}{\lambda}\left(\int^{\frac{y\lambda+\ln(x)}{\lambda}} \left(1+f\left(e^{\lambda\_a}\right)\lambda\, e^{\lambda\_a}\right)^{-1} d\_a\lambda - \ln(x)\right)\right)$$

68.8 problem number 8

problem number 620

Added Feb. 7, 2019.

Problem 2.9.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + yf(e^{\alpha x}y^m)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + y*f[Exp[alpha*x]*y^m]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+   y*f(exp(alpha*x)*y^m)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{m}\left(\int_{\_b}^{y} \frac{1}{\_a\ (f\ (\mathrm{e}^{\alpha\,x}\_a^{m})\, m+\alpha)} \mathrm{d}\_am - x\right)\right)$$

68.9 problem number 9

problem number 621

Added Feb. 7, 2019.

Problem 2.9.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + f(x^n e^{\alpha y})w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + f[x^n*Exp[alpha*y]]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+   f(x^n*exp(alpha*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\frac{\int^{y}_{\_b} (\alpha f(x^n e^{-\_a\alpha}) + n)^{-1} \, d\_a\alpha - \ln(x)}{\alpha}\right)$$

68.10 problem number 10

problem number 622

Added Feb. 7, 2019.

Problem 2.9.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda x - \beta y} f(ae^{\lambda x} + be^{\beta y}) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + Exp[lambda*x - beta*y]*f[a*Exp[lambda*x] + b*Exp[beta*y]]*D[w[x, y],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+  exp(lambda*x-beta*y)*f(a*exp(lambda*x)+b*exp(beta*y))*diff(w(x,y),y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{\lambda}\left(\int^{-\frac{a e^{\lambda x}+b e^{\beta y}}{a\lambda}} (b f(-\_a\, a\lambda)\,\beta + a\lambda)^{-1}\, d\_a \lambda^2 a + e^{\lambda x}\right)\right)$$

68.11 problem number 11

problem number 623

Added Feb. 7, 2019.

Problem 2.9.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f\left(y + ae^{\lambda x} + b\right) - a\lambda e^{\lambda x}\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[y + a*Exp[lambda*x] + b] - a*lambda*Exp[lambda*x])*D[w[x, y], y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+  (f(y+a*exp(lambda*x)+b)-a * lambda*exp(lambda*x))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} \left(f\left(\_a + a\mathrm{e}^{\lambda x} + b\right)\right)^{-1} \mathrm{d}\_a - x\right)$$

68.12 problem number 12

problem number 624

Added Feb. 7, 2019.

Problem 2.9.2.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\alpha xyw_x + (\alpha f(x^n e^{\alpha y}) - ny)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = alpha*x*y*D[w[x, y], x] + (alpha*f[x^n*Exp[alpha*y]] - n*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  alpha*x*y*diff(w(x,y),x)+  (alpha*f(x^n*exp(alpha*y)) - n*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

68.13 problem number 13

problem number 625

Added Feb. 7, 2019.

Problem 2.9.2.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mx(\ln y)w_x + (yf(x^n y^m) - ny\ln y)\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = m*x*Log[y]*D[w[x, y], x] + (y*f[x^n*y^m] - n*y*Log[y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   m*x*ln(y)*diff(w(x,y),x)+  (y*f(x^n*y^m) - n*y*ln[y])*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

68.14 problem number 14

problem number 626

Added Feb. 7, 2019.

Problem 2.9.2.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(f(y + a\tan x) - a\tan^2 x\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (f[y + a*Tan[x]] - a*Tan[x]^2)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+  (f(y+a*tan(x)) - a*tan(x)^2)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(-x + \int^{y+a\tan(x)} (f(\_a)+a)^{-1} d\_a\right)$$

68.15 problem number 15

problem number 627

Added Feb. 7, 2019.

Problem 2.9.2.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$e^{\lambda x} w_x + f(\lambda x + \ln y) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = Exp[lambda*x]*D[w[x, y], x] + f[lambda*x + Log[y]]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=    exp(lambda*x)*diff(w(x,y),x)+  f(lambda*x+ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(x - \int^{y e^{\lambda x}} (f(\ln(\_a)) + \lambda\_a)^{-1} d\_a\right)$$

68.16 problem number 16

problem number 628

Added Feb. 7, 2019.

Problem 2.9.2.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + e^{\lambda y} f(\lambda y + \ln x) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + Exp[lambda*y]*f[lambda*y + Log[x]]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   diff(w(x,y),x)+  exp(lambda*y)*f(lambda*y+ln(x))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{\lambda}\left(-\int^{\frac{y\lambda+\ln(x)}{\lambda}}\left(1+f(\lambda\_a)\,\lambda\,\mathrm{e}^{\lambda\_a}\right)^{-1}d\_a\lambda+\ln(x)\right)\right)$$

69 HFOPDE, chapter 2.9.3

69.1 problem number 1

problem number 629

Added Feb. 7, 2019.

Problem 2.9.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x - \left(ny - xy^k f(x) g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = m*x*D[w[x, y], x] - (n*y - x*y^k*f[x]*g[x^n*y^m])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=   m*x*diff(w(x,y),x)- ( n*y -x*y^k*f(x)*g(x^n*y^m)  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{x} \frac{1}{g\left(\_a^n y^m\right)} \left(\_a^{-\frac{n(k-1)}{m}} f(\_a)\, g(\_a^n y^m) - y^{1-k}\_a^{-\frac{kn+m-n}{m}} n\right) \mathrm{d}\_a - \int \frac{y^{-k}}{g\left(x^n y^m\right)} \left(g(x\right.\right.$$

69.2 problem number 2

problem number 630

Added Feb. 9, 2019.

Problem 2.9.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^n w_x - \left(ax^n + g(x)f\left(y^{n+1} + ax^{n+1}\right)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = y^n*D[w[x, y], x] - (a*x^n + g[x]*f[y^(n + 1) + a*x^(n + 1)])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  y^n*diff(w(x,y),x)- ( a*x^n + g(x)*f(y^(n+1) + a*x^(n+1))  )*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.3 problem number 3

problem number 631

Added Feb. 9, 2019.

Problem 2.9.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(\frac{y}{x}) + x^{\alpha}h(\frac{y}{x})\right)w_x + \left(g(\frac{y}{x}) + yx^{\alpha-1}h(\frac{y}{x})\right)w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[y/x] + x^alpha*h[y/x])*D[w[x, y], x] + (g[y/x] + y*x^(alpha - 1)*h[y/x])*D[w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  (f(y/x)+x^alpha * h(y/x))*diff(w(x,y),x)+ ( g(y/x)+y*x^(alpha-1)*h(y/x))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.4 problem number 4

problem number 632

Added Feb. 9, 2019.

Problem 2.9.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(f(ax + by) + bxg(ax + by))\,w_x + (h(ax + by) - axg(ax + by))\,w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[a*x + b*y] + b*x*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*x*g[a*x + b*y])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  (f(a*x+b*y)+b*x*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*x*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.5 problem number 5

problem number 633

Added Feb. 9, 2019.

Problem 2.9.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(ax+by)+byg(ax+by))\,w_x+(h(ax+by)-ayg(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[a*x + b*y] + b*y*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*y*g[a*x + b*y])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  (f(a*x+b*y)+b*y*g(a*x+b*y))*diff(w(x,y),x)+ ( h(a*x+b*y)-a*y*g(a*x+b*y))*diff(w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.6 problem number 6

problem number 634

Added Feb. 9, 2019.

Problem 2.9.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(f(x^n y^m) + m x^k g(x^n y^m)\right) w_x + y\left(h(x^n y^m) - n x^k g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(f[x^n*y^m] + m*x^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*x^k*g[x^n*y^m])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*(f(x^n*y^m)+m*x^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*x^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.7 problem number 7

problem number 635

Added Feb. 9, 2019.

Problem 2.9.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(f(x^n y^m) + m y^k g(x^n y^m)\right) w_x + y\left(h(x^n y^m) - n y^k g(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(f[x^n*y^m] + m*y^k*g[x^n*y^m])*D[w[x, y], x] + y*(h[x^n*y^m] - n*y^k*g[x^n*y^m])*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*(f(x^n*y^m)+m*y^k*g(x^n*y^m))*diff(w(x,y),x)+ y*( h(x^n*y^m)-n*y^k*g(x^n*y^m))*dif
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.8 problem number 8

problem number 636

Added Feb. 9, 2019.

Problem 2.9.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x\left(sf(x^n y^m) - mg(x^k y^s)\right) w_x + y\left(ng(x^k y^s) - kf(x^n y^m)\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(s*f[x^n*y^m] - m*g[x^k*y^s])*D[w[x, y], x] + y*(n*g[x^k*y^s] - k*f[x^n*y^m])*D[w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*(s*f(x^n*y^m)-m*g(x^k*y^s))*diff(w(x,y),x)+ y*(n*g(x^k*y^s)-k*f(x^n*y^m))*diff(w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.9 problem number 9

problem number 637

Added Feb. 9, 2019.

Problem 2.9.3.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$f_y * w_x - f_x w_y = 0$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[f[x, y], y]*D[w[x, y], x] - D[f[x, y], x]*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to c_1(\text{InverseFunction}[\text{InverseFunction}[f,2,2],2,2][x,y])\}\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(f(x,y),y)*diff(w(x,y),x)-diff(f(x,y),x)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1(-f(x,y))$$

69.10 problem number 11

problem number 638

Added Feb. 9, 2019.

Problem 2.9.3.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$xw_x + (xf(x)g(x^n e^y) - n)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (x*f[x]*g[x^n*Exp[y]] - n)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  x*diff(w(x,y),x)+(x*f(x)*g(x^n*exp(y))-n)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\int_{\_b}^{y} \left(g(x^n \mathrm{e}^{\_a})\right)^{-1} \mathrm{d}\_a - \int f(x)\,\mathrm{d}x\right)$$

69.11 problem number 12

problem number 639

Added Feb. 9, 2019.

Problem 2.9.3.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux. Reference E. Kamke 1965.

Solve for $w(x,y)$

$$mw_x + \left(my^k f(x) g(e^{\alpha x} y^m) - \alpha y\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = m*D[w[x, y], x] + (m*y^k*f[x]*g[Exp[alpha*x]*y^m] - alpha*y)*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  m*diff(w(x,y),x)+(m*y^k*f(x)*g(exp(alpha*x)*y^m)- alpha*y)*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{1}{m}\left(-\int\frac{y^{-k}}{g\left(\mathrm{e}^{\alpha x}y^m\right)m}\left(\alpha\int_{\_b}^{x}\frac{1}{\left(g\left(\mathrm{e}^{\_a\,\alpha}y^m\right)\right)^2}\left(\mathrm{e}^{-\frac{\_a\,\alpha\,(k-m-1)}{m}}y^m\mathrm{D}(g)\left(\mathrm{e}^{\_a\,\alpha}y^m\right)m+\mathrm{e}^{-\frac{\_a\,\alpha\,(k-}{m}}\right.\right.\right.\right.$$

69.12 problem number 13

problem number 640

Added Feb. 9, 2019.

Problem 2.9.3.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\left(f(ax+by)+be^{\lambda y}g(ax+by)\right)w_x+\left(h(ax+by)-ae^{\lambda y}g(ax+by)\right)w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[a*x + b*y] + b*Exp[lambda*y]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (f(a*x+b*y)+ b*exp(lambda*y)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)- a*exp(lambda*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.13 problem number 14

problem number 641

Added Feb. 9, 2019.

Problem 2.9.3.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(ax+by)+be^{\lambda x}g(ax+by))\,w_x+(h(ax+by)-ae^{\lambda x}g(ax+by))\,w_y=0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[a*x + b*y] + b*Exp[lambda*x]*g[a*x + b*y])*D[w[x, y], x] + (h[a*x + b*y] - a*Exp[l
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (f(a*x+b*y)+ b*exp(lambda*x)*g(a*x+b*y))*diff(w(x,y),x)+(h(a*x+ b*y)-  a*exp(lambda*x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.14 problem number 15

problem number 642

Added Feb. 9, 2019.

Problem 2.9.3.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^n e^{\alpha y}) + \alpha y g(x^n e^{\alpha y}))\, w_x + (h(x^n e^{\alpha y}) - n y g(x^n e^{\alpha y}))\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(f[x^n*Exp[alpha*y]] + alpha*y*g[x^n*Exp[alpha*y]])*D[w[x, y], x] + (h[x^n*Exp[alph
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*(f(x^n*exp(alpha*y))+alpha*y*g(x^n*exp(alpha*y)))*diff(w(x,y),x)+(h(x^n*exp(alpha*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.15 problem number 16

problem number 643

Added Feb. 9, 2019.

Problem 2.9.3.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f(e^{\alpha x}y^m) + mxg(e^{\alpha x}y^m))\, w_x + y(h(e^{\alpha x}y^m) - \alpha xg(e^{\alpha x}y^m)))\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (f[Exp[alpha*x]*y^m] + m*x*g[Exp[alpha*x]*y^m])*D[w[x, y], x] + y*(h[Exp[alpha*x]*y^m
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (f(exp(alpha*x)*y^m)+m*x*g(exp(alpha*x)*y^m))*diff(w(x,y),x)+ y*(h(exp(alpha*x)*y^m)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.16 problem number 17

problem number 644

Added Feb. 9, 2019.

Problem 2.9.3.17 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (xyf(x)g(x^n \ln y) - ny \ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (x*y*f[x]*g[x^n*Log[y]] - n*y*Log[y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*diff(w(x,y),x)+ (x*y*f(x)*g(x^n*ln(y))-n*y*ln(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

sol=()

69.17 problem number 18

problem number 645

Added Feb. 9, 2019.

Problem 2.9.3.18 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^n y^m) + mg(x^n y^m)\ln y)\, w_x + y(h(x^n y^m) - ng(x^n y^m)\ln y)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[y])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(y))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(y))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.18 problem number 19

problem number 646

Added Feb. 9, 2019.

Problem 2.9.3.19 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(f(x^ny^m) + mg(x^ny^m)\ln x)\, w_x + y(h(x^ny^m) - ng(x^ny^m)\ln x)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*(f[x^n*y^m] + m*g[x^n*y^m]*Log[x])*D[w[x, y], x] + y*(h[x^n*y^m] - n*g[x^n*y^m]*Log
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*(f(x^n*y^m)+m*g(x^n*y^m)*ln(x))*diff(w(x,y),x)+ y*(h(x^n*y^m)-n*g(x^n*y^m)*ln(x))*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.19 problem number 20

problem number 647

Added Feb. 9, 2019.

Problem 2.9.3.20 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cos y w_x + (f(x)g(\sin x \sin y) - \cot x \sin y)\, w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = Cos[y]*D[w[x, y], x] + (f[x]*g[Sin[x]*Sin[y]] - Cot[x]*Sin[y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=cos(y)*diff(w(x,y),x)+ (f(x)* g(sin(x)*sin(y)) - cot(x)*sin(y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.20 problem number 21

problem number 648

Added Feb. 9, 2019.

Problem 2.9.3.21 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin 2x w_x + \left(\sin 2x \cos^2 y f(x) g(\tan x \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = Sin[2*x]*D[w[x, y], x] + (Sin[2*x]*Cos[y]^2*f[x]*g[Tan[x]*Tan[y]] - Sin[2*y])*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=sin(2*x)*diff(w(x,y),x)+ (sin(2*x)*cos(y)^2*f(x)*g(tan(x)*tan(y)) -sin(2*y))*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.21 problem number 22

problem number 649

Added Feb. 9, 2019.

Problem 2.9.3.22 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + \left(x\cos^2 y f(x) g(x^{2n}\tan y) - n\sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + (x*Cos[y]^2*f[x]*g[x^(2*n)*Tan[y]] - n*Sin[2*y])*D[w[x, y], y] == 0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=x*diff(w(x,y),x)+ (x *cos(y)^2* f(x)* g(x^(2*n)*tan(y)) - n*sin(2*y))*diff(w(x,y),y) =
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

69.22 problem number 23

problem number 650

Added Feb. 9, 2019.

Problem 2.9.3.23 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + \left(\cos^2 y f(x) g(e^{2x} \tan y) - \sin 2y\right) w_y = 0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (Cos[y]^2*f[x]*g[Exp[2*x]*Tan[y]] - Sin[2*y])*D[w[x, y], y] == 0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := diff(w(x,y),x)+ (cos(y)^2* f(x)* g(exp(2*x)*tan(y)) -sin(2*y))*diff(w(x,y),y) = 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

70 HFOPDE, chapter 3 examples

70.1 Example 1

problem number 651

Added Feb. 9, 2019.

Problem Chapter 3, example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$axw_x + byw_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ac_1\left(yx^{-\frac{b}{a}}\right)+c\log(x)}{a}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{a}\left(c\ln(x)+\_F1\left(yx^{-\frac{b}{a}}\right)a\right)$$

70.2 Example 2

problem number 652

Added Feb. 9, 2019.

Problem Chapter 3, example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^x w_x + b w_y = c e^{2x} y$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[x]*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[2*x]*y;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{a^2c_1\left(\frac{e^{-x}(ae^xy+b)}{a}\right)+ace^xy-bcx+bc}{a^2}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*exp(x)*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*exp(2*x)*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{y}{a^2}\left(\mathrm{e}^{x-\frac{b\mathrm{e}^{-x}}{a}}\mathrm{e}^{\frac{b\mathrm{e}^{-x}}{a}}ac-\mathrm{expIntegral}\left(1,\frac{b\mathrm{e}^{-x}}{a}\right)\mathrm{e}^{\frac{b\mathrm{e}^{-x}}{a}}bc\right)+\_F1\left(y\mathrm{e}^{\frac{b\mathrm{e}^{-x}}{a}}\right)$$

70.3 Example 3

problem number 653

Added Feb. 9, 2019.

Problem Chapter 3, example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = b$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == b;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\{\{w(x,y) \to c_1(y - ax) + bx\}\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = bx + \_F1(-ax + y)$$

71 HFOPDE, chapter 3.2.1

71.1 Problem 1

problem number 654

Added Feb. 9, 2019.

Problem Chapter 3.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{ac_1\left(\frac{ay-bx}{a}\right)+cx}{a}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}+\_F1\left(\frac{ya-bx}{a}\right)$$

71.2 Problem 2

problem number 655

Added Feb. 9, 2019.

Problem Chapter 3.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a^2c_1\left(\frac{ay-bx}{a}\right)+a\alpha x^2+2a\beta xy+2a\gamma x-b\beta x^2}{2a^2}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=1/2\,\frac{(a\alpha-b\beta)\,x^2}{a^2}+\left(\frac{\beta\,y}{a}+\frac{\gamma}{a}\right)x+\_F1\left(\frac{ya-bx}{a}\right)$$

71.3 Problem 3

problem number 656

Added Feb. 9, 2019.

Problem Chapter 3.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a^2c_1\left(\frac{ay-b\log(x)}{a}\right)+2a\alpha x+2a\beta y\log(x)+2a\gamma\log(x)-b\beta\log^2(x)}{2a^2}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{\alpha\,x}{a}+\frac{\ln(x)\,\beta\,y}{a}-1/2\,\frac{1}{a^2}\left(b\beta\,(\ln(x))^2-2\,\gamma\,\ln(x)\,a-2\,\_F1\left(-\frac{b\ln(x)-ya}{a}\right)a^2\right)$$

71.4 Problem 4

problem number 657

Added Feb. 9, 2019.

Problem Chapter 3.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + bxw_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ac_1\left(\frac{ay-bx}{a}\right)+c\log(x)}{a}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{a}\left(c\ln(x)+\_F1\left(\frac{ya-bx}{a}\right)a\right)$$

71.5 Problem 5

problem number 658

Added Feb. 9, 2019.

Problem Chapter 3.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*x + beta*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{a^2c^2c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right) + a^2\beta cy + a^2\beta d - \alpha bc^2\log(ax+b) + a\alpha c^2x - a\beta cd\log(ax+b) + a}{a^2c^2}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (a*x+b)* diff(w(x,y),x)+(c*y+d)*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \frac{\alpha x}{a} + \frac{\beta y}{c} + \frac{1}{a^2c^2}\left(\_F1\left(\frac{cy+d}{c}(ax+b)^{-\frac{c}{a}}\right)a^2c^2 + \ln(ax+b)\gamma ac^2 - \ln(ax+b)\beta dac - \ln(ax\right.$$

71.6 Problem 6

problem number 659

Added Feb. 9, 2019.

Problem Chapter 3.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bw_y = \alpha x + \beta y + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y*D[w[x, y], x] + b*D[w[x, y], y] == alpha*x + beta*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-a^{3/2}\alpha y^2\sqrt{ay^2}+3\sqrt{a}\alpha bx\sqrt{ay^2}+3ab^2c_1\left(\frac{ay^2-2bx}{2a}\right)+3\sqrt{a}b\gamma\sqrt{ay^2}+3b^2\beta x}{3ab^2}\right\},\left\{w(x,y)\to\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y* diff(w(x,y),x)+b*diff(w(x,y),y) = alpha*x+beta*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=1/6\,\frac{\left(6\,\sqrt{(y^2a-2\,bx)\,a+2\,axb}a\alpha\,b+6\,ab^2\beta\right)x}{b^2a^2}-1/2\,\frac{\sqrt{(y^2a-2\,bx)\,a+2\,axb}\alpha\,y^2}{b^2}+1/6\,\frac{1}{b^2a^2}\Bigg(6$$

71.7 Problem 7

problem number 660

Added Feb. 9, 2019.

Problem Chapter 3.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\sqrt{a}\sqrt{b}c_1\left(\frac{ay^2-bx^2}{2a}\right)-c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)}{\sqrt{a}\sqrt{b}}\right\},\left\{w(x,y)\to \frac{\sqrt{a}\sqrt{b}c_1\left(\frac{ay^2-bx^2}{2a}\right)+c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right.}{\sqrt{a}\sqrt{b}}\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{\sqrt{ab}}\left(\_F1\left(\frac{y^2a-bx^2}{a}\right)\sqrt{ab}+c\ln\left(\frac{axb}{\sqrt{ab}}+\sqrt{abx^2+(y^2a-bx^2)\,a}\right)\right)$$

71.8 Problem 8

problem number 661

Added Feb. 9, 2019.

Problem Chapter 3.2.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cx + ky$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x + k*y;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{abc_1\left(\frac{ay^2-bx^2}{2a}\right)-\sqrt{a}c\sqrt{ay^2}+bkx}{ab}\right\},\left\{w(x,y)\to\frac{abc_1\left(\frac{ay^2-bx^2}{2a}\right)+\sqrt{a}c\sqrt{ay^2}+bkx}{ab}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y* diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x+k*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{cy}{b}+\frac{kx}{a}+\_F1\left(\frac{y^2a-bx^2}{a}\right)$$

72 HFOPDE, chapter 3.2.2

72.1 Problem 1

problem number 662

Added Feb. 9, 2019.

Problem Chapter 3.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^2 + dy^2 + kxy + n$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{6a^3c_1\left(\frac{ay-bx}{a}\right)+2a^2cx^3+6a^2dxy^2+3a^2kx^2y+6a^2nx-6abdx^2y-abkx^3+2b^2dx^3}{6a^3}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a* diff(w(x,y),x)+b*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/6\,\frac{(2\,ca^2-abk+2\,b^2d)\,x^3}{a^3}+1/6\,\frac{(3\,ka^2-6\,abd)\,yx^2}{a^3}+\left(\frac{dy^2}{a}+\frac{n}{a}\right)x+\_F1\left(\frac{ya-bx}{a}\right)$$

72.2 Problem 2

problem number 663

Added Feb. 9, 2019.

Problem Chapter 3.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^2 + dy^2 + kxy + n$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a^2bc_1\left(yx^{-\frac{b}{a}}\right)+a^2dy^2+2ab^2c_1\left(yx^{-\frac{b}{a}}\right)+abcx^2+abdy^2+2abkxy+2abn\log(x)+b^2cx^2}{2ab(a+b)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = c*x^2+d*y^2+k*x*y+n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{kxy}{a+b}+1/2\,\frac{cx^2}{a}+\frac{n\ln(x)}{a}+1/2\,\frac{dy^2}{b}+\_F1\left(yx^{-\frac{b}{a}}\right)$$

72.3 Problem 3

problem number 664

Added Feb. 9, 2019.

Problem Chapter 3.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cxy + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a\sqrt{b}c_1\left(\frac{ay^2-bx^2}{2a}\right)-2\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ay^2}}\right)+\sqrt{b}cx^2}{2a\sqrt{b}}\right\},\left\{w(x,y)\to \frac{2a\sqrt{b}c_1\left(\frac{ay^2-bx^2}{2a}\right)+2\sqrt{}}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y*diff(w(x,y),x)+b*x*diff(w(x,y),y) = c*x*y+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/2\,\frac{cx^2}{a}+1/2\,\frac{1}{a\sqrt{ab}}\left(2\_F1\left(\frac{y^2a-bx^2}{a}\right)a\sqrt{ab}+2\,d\ln\left(\frac{axb}{\sqrt{ab}}+\sqrt{abx^2+\left(y^2a-bx^2\right)a}\right)a\right)$$

72.4 Problem 4

problem number 665

Added Feb. 9, 2019.

Problem Chapter 3.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x + by^2w_y = cx^2 + dy^2 + kxy + nx + my + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == c*x^2 + d*y^2 + k*x*y + n*x + m*y + s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2bx^2c_1\left(\frac{by-ax}{axy}\right)-a^2mx^2\log\left(b-\frac{by-ax}{y}\right)+a^2mx^2\log(x)-ab^2xyc_1\left(\frac{by-ax}{axy}\right)+abcx^3-abc}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x^2*diff(w(x,y),x)+b*y^2*diff(w(x,y),y) =c*x^2+d*y^2+ k*x*y+ n*x+ m*y+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}+\frac{adxy}{-ax+by}\left(-\frac{-ax+by}{y}+b\right)^{-1}-\frac{m}{b}\ln\left(-\frac{-ax+by}{y}+b\right)-\frac{kxy}{-ax+by}\ln\left(-\frac{-ax+by}{y}+b\right.$$

72.5 Problem 5

problem number 666

Added Feb. 9, 2019.

Problem Chapter 3.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axyw_y = by^2$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2axc_1(yx^{-a}) - xc_1(yx^{-a}) + by^2}{(2a-1)x}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x^2*diff(w(x,y),x)+a*x*y*diff(w(x,y),y) =b*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{by^2}{(2\,a-1)\,x} + \_F1\left(yx^{-a}\right)$$

72.6 Problem 6

problem number 667

Added Feb. 9, 2019.

Problem Chapter 3.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2w_x + bx^2w_y = cx^2 + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y^2*D[w[x, y], x] + b*x^2*D[w[x, y], y] == c*x^2 + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow \frac{bdx\left(\frac{ay^3}{ay^3-bx^3}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{ay^3-bx^3}\right)+\sqrt[3]{a}b(ay^3)^{2/3}c_1\left(\frac{ay^3-bx^3}{3a}\right)+acy^3}{\sqrt[3]{a}b\left(ay^3\right)^{2/3}}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y^2*diff(w(x,y),x)+b*x^2*diff(w(x,y),y) =c*x^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{(\_a^2c+d)\,a}{\left(\left(\_a^3b+\text{RootOf}\left(ya-\sqrt[3]{a^2bx^3+a^3\_Z}\right)a\right)a^2\right)^{2/3}}d\_a+\_F1\left(\text{RootOf}\left(ya-\sqrt[3]{a^2bx^3+a^3\right.\right.$$

Contains unresolved integral with RootOf

72.7 Problem 7

problem number 668

Added Feb. 9, 2019.

Problem Chapter 3.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2 w_x + bxy w_y = cx^2 + dy^2$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y^2*D[w[x, y], x] + b*x*y*D[w[x, y], y] == c*x^2 + d*y^2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^{3/2}(-c)\sqrt{\frac{ay^2-bx^2}{a}}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}\sqrt{\frac{ay^2-bx^2}{a}}}\right)+ab^{3/2}c_1\left(\frac{ay^2-bx^2}{2a}\right)+a\sqrt{b}cx+b^{3/2}dx}{ab^{3/2}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y^2*diff(w(x,y),x)+b*x*y*diff(w(x,y),y) =c*x^2+d*y^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx^2}{\sqrt{(y^2a-bx^2)\,b}}\arctan\left(\frac{bx}{\sqrt{(y^2a-bx^2)\,b}}\right)+\left(\frac{c}{b}+\frac{d}{a}\right)x-\frac{cy^2a}{b\sqrt{(y^2a-bx^2)\,b}}\arctan\left(\frac{bx}{\sqrt{(y^2a-bx^2)}}\right.$$

73 HFOPDE, chapter 3.2.3

73.1 Problem 1

problem number 669

Added Feb. 9, 2019.

Problem Chapter 3.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to a\sqrt{x^2+y^2}+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*sqrt(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=a\sqrt{x^2\left(\frac{y^2}{x^2}+1\right)}+\_F1\left(\frac{y}{x}\right)$$

73.2 Problem 2

problem number 670

Added Feb. 9, 2019.

Problem Chapter 3.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cxy^2 + dx^2y + k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x*y^2 + d*x^2*y + k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2a^3c_1\left(yx^{-\frac{b}{a}}\right)+5a^2bc_1\left(yx^{-\frac{b}{a}}\right)+2a^2cxy^2+a^2dx^2y+2a^2k\log(x)+2ab^2c_1\left(yx^{-\frac{b}{a}}\right)+abcx}{a(2a+b)(a+2b)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x*y^2+d*x^2*y+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cxy^2}{a+2b} + \frac{dx^2y}{2a+b} + \frac{k\ln(x)}{a} + \_F1\left(yx^{-\frac{b}{a}}\right)$$

73.3 Problem 3

problem number 671

Added Feb. 9, 2019.

Problem Chapter 3.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = cxy^2 + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == c*x*y^2 + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{3\sqrt{a}bc_1\left(\frac{ay^2-bx^2}{2a}\right) - 3\sqrt{b}d\log\left(\sqrt{b}\sqrt{ay^2}+bx\right) - cy^2\sqrt{ay^2}}{3\sqrt{a}b}\right\}, \left\{w(x,y) \to \frac{3\sqrt{a}bc_1\left(\frac{ay^2-bx^2}{2a}\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*y*diff(w(x,y),x) + b*x*diff(w(x,y),y) =c*x*y^2+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \left(-\frac{cy}{a} + \frac{c\sqrt{abx^2+(y^2a-bx^2)\,a}}{a^2}\right)x^2 + \frac{cy^3}{b} - 2/3\,\frac{c\sqrt{abx^2+(y^2a-bx^2)\,a}y^2}{ab} - 1/3\,\frac{1}{\sqrt{ab}a^2b}\Big(-3$$

73.4 Problem 4

problem number 672

Added Feb. 9, 2019.

Problem Chapter 3.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = kx^3 + ny^3$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == k*x^3 + n*y^3;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{6a^4c^4c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)+2a^4c^3ny^3-3a^4c^2dny^2+6a^4cd^2ny+11a^4d^3n-6a^3cd^3n\log(ax}{6a^4c^4}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := (a*x+b)*diff(w(x,y),x) + (c*y+d)*diff(w(x,y),y) =k*x^3+n*y^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=1/3\,\frac{kx^3}{a}-1/2\,\frac{kx^2b}{a^2}+\frac{b^2kx}{a^3}+1/3\,\frac{ny^3}{c}-1/2\,\frac{dny^2}{c^2}+\frac{d^2ny}{c^3}+1/6\,\frac{1}{a^4c^4}\left(6\_F1\left(\frac{cy+d}{c}(ax+b)^{-\frac{c}{a}}\right)a\right.$$

73.5 Problem 5

problem number 673

Added Feb. 9, 2019.

Problem Chapter 3.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xyw_y = y^2(ax+by)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{axy^2+by^3+2xc_1\left(\frac{y}{x}\right)}{2x}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := x^2*diff(w(x,y),x) + x*y*diff(w(x,y),y) =y^2*(a*x + b*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/2\,y^2a + \_F1\left(\frac{y}{x}\right) + 1/2\,\frac{by^3}{x}$$

73.6 Problem 6

problem number 674

Added Feb. 9, 2019.

Problem Chapter 3.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^3w_x + by^3w_y = cx + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^3*D[w[x, y], x] + b*y^3*D[w[x, y], y] == c*x + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{2ax^2c_1\left(\frac{by^2-ax^2}{2ax^2y^2}\right)-2cx-d}{2ax^2}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*x^3*diff(w(x,y),x) + b*y^3*diff(w(x,y),y) =c*x+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = -\frac{c}{ax} - 1/2\,\frac{d}{ax^2} + \_F1\left(\frac{ax^2-by^2}{y^2ax^2}\right)$$

74 HFOPDE, chapter 3.2.4

74.1 Problem 1

problem number 675

Added Feb. 9, 2019.

Problem Chapter 3.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n + dy^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + d*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{abmnc_1\left(\frac{ay-bx}{a}\right)+abmc_1\left(\frac{ay-bx}{a}\right)+abnc_1\left(\frac{ay-bx}{a}\right)+abc_1\left(\frac{ay-bx}{a}\right)+adny\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)^m+ady}{ab(m+1)(n+1)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n+d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{d}{b\,(m+1)}\left(\frac{bx}{a}+\frac{ya-bx}{a}\right)^{m+1}+\frac{x^{n+1}c}{a\,(n+1)}+\_F1\left(\frac{ya-bx}{a}\right)$$

74.2 Problem 2

problem number 676

Added Feb. 9, 2019.

Problem Chapter 3.2.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n y$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2n^2c_1\left(\frac{ay-bx}{a}\right)+3a^2nc_1\left(\frac{ay-bx}{a}\right)+2a^2c_1\left(\frac{ay-bx}{a}\right)+2acyx^{n+1}+acnyx^{n+1}-bcx^{n+2}}{a^2(n+1)(n+2)}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde := a*diff(w(x,y),x) + b*diff(w(x,y),y) =c*x^n*y;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{x^{n+1}bcx}{(n+2)\,(n+1)\,a^2} + \frac{x^{n+1}(an+2\,a)\,cy}{(n+2)\,(n+1)\,a^2} + \_F1\left(\frac{ya-bx}{a}\right)$$

74.3 Problem 3

problem number 677

Added Feb. 9, 2019.

Problem Chapter 3.2.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2+y^2)^k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\left(x^2\left(\frac{y^2}{x^2}+1\right)\right)^k+2kc_1\left(\frac{y}{x}\right)}{2k}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) =a*(x^2+y^2)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=1/2\,\frac{1}{k}\left(a\left(x^2\left(\frac{y^2}{x^2}+1\right)\right)^k+2\_F1\left(\frac{y}{x}\right)k\right)$$

74.4 Problem 4

problem number 678

Added Feb. 9, 2019.

Problem Chapter 3.2.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow \frac{bmc_1\left(yx^{-\frac{b}{a}}\right)+anc_1\left(yx^{-\frac{b}{a}}\right)+cy^m x^n}{an+bm}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{cx^n y^m}{an+bm}+\_F1\left(yx^{-\frac{b}{a}}\right)$$

74.5 Problem 5

problem number 679

Added Feb. 9, 2019.

Problem Chapter 3.2.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax w_x + by w_y = c x^n + d y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n + d*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{abmnc_1\left(yx^{-\frac{b}{a}}\right)+adny^m+bcmx^n}{abmn}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=a*x*diff(w(x,y),x) + b*y*diff(w(x,y),y) =c*x^n + d*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{a\_a}\left(\_a^n c+d\left(yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)^m\right)d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)$$

Result has unresolved integral

74.6 Problem 6

problem number 680

Added Feb. 9, 2019.

Problem Chapter 3.2.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = (ax^n + by^m)^k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{(ax^n+by^m)^k+kmnc_1\left(yx^{-\frac{n}{m}}\right)}{kmn}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=m*x*diff(w(x,y),x) + n*y*diff(w(x,y),y) =(a*x^n+b*y^m)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{knm}\left(\_F1\left(yx^{-\frac{n}{m}}\right)knm+(x^na+y^mb)^k\right)$$

Result has unresolved integral

74.7 Problem 7

problem number 681

Added Feb. 9, 2019.

Problem Chapter 3.2.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^m w_y = cx^k + dy^s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == c*x^k + d*y^s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{x^{-n}\left(-bcx^{k+1} + bcmx^{k+1} - bcsx^{k+1} - ad\left(\left(\frac{a(n-1)x^n}{(m-1)\left(\frac{(bxy^m - bmxy^m - ax^n y + anx^n y)y^{-m}}{m-1} + bx\right)}\right)^{\frac{1}{m-1}}\right)^{s+}}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=a*x^n*diff(w(x,y),x) + n*y^m*diff(w(x,y),y) =c*x^k+d*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx^{k-n+1}}{a(k-n+1)} - \frac{d}{(mn - ns - m - n + s + 1)n}(n-1)^{\frac{s}{m-1}}\left(x^{-n+1}nm - \frac{n(x^{-n+1}nm - y^{1-m}an}{n-1}\right.$$

Result has unresolved integral

74.8 Problem 8

problem number 682

Added Feb. 9, 2019.

Problem Chapter 3.2.4.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^k y^s + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^k*y^s + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-\frac{bsx^{m-n+1}}{ma-na+a}}x^{-n}\left(-\frac{bsx^{m-n+1}}{a(m-n+1)}\right)^{-\frac{k}{m-n+1}-\frac{1}{m-n+1}}\left(-c\left(-\frac{bsx^{m-n+1}}{a(m-n+1)}\right)^{\frac{n}{m-n+1}}\left(y^{\frac{ma}{ma-na+a}-\frac{na}{ma-na+a}+\frac{}{ma-}}\right.\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =c*x^k*y^s+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{a}\left(\_a^{k-n}c\left(ye^{-\frac{x^{-n+m+1}n}{a(-n+m+1)}+\frac{\_a^{-n+m+1}n}{a(-n+m+1)}}\right)^s+\_a^{-n}d\right)d\_a+\_F1\left(ye^{-\frac{x^{-n+m+1}n}{a(-n+m+1)}}\right)$$

Result has unresolved integral

74.9 Problem 9

problem number 683

Added Feb. 9, 2019.

Problem Chapter 3.2.4.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k)w_y = sx^p y^q + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == s*x^p*y^q + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';
pde :=a*x^n*diff(w(x,y),x) + n*x^m*y*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(\_a^{-n+p}s\left(y\mathrm{e}^{-\frac{x^{-n+m+1}n}{a(-n+m+1)}+\frac{\_a^{-n+m+1}n}{a(-n+m+1)}}\right)^q + \_a^{-n}d\right)d\_a + \_F1\left(y\mathrm{e}^{-\frac{x^{-n+m+1}n}{a(-n+m+1)}}\right)$$

74.10 Problem 10

problem number 684

Added Feb. 9, 2019.

Problem Chapter 3.2.4.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y^k + cx^l y)w_y = sx^p y^q + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^n*D[w[x, y], x] + (b*x^m*y^k + c*x^l*y)*D[w[x, y], y] == s*x^p*y^q + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*x^n*diff(w(x,y),x) +(b*x^m*y^k + c*x^l*y)*diff(w(x,y),y) =s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(\_a^{-n+p}s\left(\left(-\frac{1}{a}\left(\frac{kb}{-n+l+1}\left(\frac{c(1-k)}{(-n+l+1)\,a}\right)^{-\frac{-n+m+1}{-n+l+1}}\left(\frac{(-n+l+}{(-n+m+1)\,(l+m-2\,n+}\right.\right.\right.\right.\right.$$

74.11 Problem 11

problem number 685

Added Feb. 9, 2019.

Problem Chapter 3.2.4.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^m w_y = cx^m + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y^k*D[w[x, y], x] + b*x^m*D[w[x, y], y] == c*x^m + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\left(\left(\frac{a(m+1)}{(k+1)\left(bx^{m+1}+\frac{ay^{k+1}+amy^{k+1}-bx^{m+1}-bkx^{m+1}}{k+1}\right)}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(abc_1\left(\frac{ay^{k+1}+amy^{k+1}-bx^{m+1}-bkx^{m+1}}{a(k+1)(m+1)}\right)\left(\right.\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*y^k*diff(w(x,y),x) +b*x^n*diff(w(x,y),y) =c*x^m+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{c\_a^m+d}{a}\left(\left(\frac{1}{a(n+1)}\left(\_a^{n+1}bk+\frac{\left(-x^{n+1}bk+y^{k+1}an-x^{n+1}b+y^{k+1}a\right)n}{n+1}+\_a^{n+1}b+\right.\right.\right.$$

75 HFOPDE, chapter 3.3.1

75.1 Problem 1

problem number 686

Added Feb. 9, 2019.

Problem Chapter 3.3.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\lambda x} + de^{\mu y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x] + d*Exp[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ad\lambda e^{\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}}+bc\mu e^{\lambda x}}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x)+d*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{b\mu\, a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu\, a\lambda+\mathrm{e}^{\lambda x}cb\mu+d\mathrm{e}^{\frac{(ya-bx)\mu}{a}+\frac{b\mu\, x}{a}}a\lambda\right)$$

75.2 Problem 2

problem number 687

Added Feb. 9, 2019.

Problem Chapter 3.3.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\lambda x+\beta y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ce^{\frac{x(a\lambda+b\beta)}{a}+\frac{\beta(ay-bx)}{a}}+b\beta c_1\left(\frac{ay-bx}{a}\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\beta}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*exp(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{c}{a\lambda+b\beta}\mathrm{e}^{\frac{(ya-bx)\beta}{a}+\lambda x+\frac{bx\beta}{a}}+\_F1\left(\frac{ya-bx}{a}\right)$$

75.3 Problem 3

problem number 688

Added Feb. 9, 2019.

Problem Chapter 3.3.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-\lambda x}\left(a\lambda e^{\lambda x}c_1\left(\frac{e^{-\beta y-\lambda x}\left(b\beta e^{\beta y}-a\lambda e^{\lambda x}\right)}{a\beta\lambda}\right)-c\right)}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = -\frac{1}{a\lambda}\left(-\_F1\left(\frac{\left(e^{\beta y}b\beta - a\lambda e^{\lambda x}\right)e^{-\beta y-\lambda x}}{b\beta\lambda}\right)a\lambda + ce^{-\lambda x}\right)$$

75.4 Problem 4

problem number 689

Added Feb. 9, 2019.

Problem Chapter 3.3.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\beta e^{\lambda y}c_1\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)-b\lambda e^{\beta x}c_1\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)-c\log\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{\lambda}+be^{\beta x}\right)+\beta cx}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{e^{y\lambda}a\beta-e^{\beta x}b\lambda}\left(e^{y\lambda}\_F1\left(-\frac{e^{\beta x}b\lambda-e^{y\lambda}a\beta}{b\beta\lambda}\right)a\beta-\_F1\left(-\frac{e^{\beta x}b\lambda-e^{y\lambda}a\beta}{b\beta\lambda}\right)e^{\beta x}b\lambda-c\ln\left(-\frac{e^{\beta}}{}\right.\right.$$

75.5 Problem 5

problem number 690

Added Feb. 9, 2019.

Problem Chapter 3.3.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y = ce^{\gamma x-\beta y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-\beta y}\left(2a^2\alpha^2e^{\beta y}c_1\left(-\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha\beta}\right)+a^2\gamma^2e^{\beta y}c_1\left(-\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha\beta}\right)-3a^2\alpha\gamma e^{\beta y}c\right.}{a^2(\alpha-\gamma)(2\alpha}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{b\beta c}{a^2\alpha}\left(-\frac{\left(e^{\beta y}b\beta-a\alpha\,e^{\alpha x}\right)e^{-\alpha x-\beta y+x(\gamma-\alpha)}}{(\gamma-\alpha)\,b\beta}+\frac{e^{x(\gamma-2\alpha)}}{\gamma-2\alpha}\right)+\_F1\left(\frac{\left(e^{\beta y}b\beta-a\alpha\,e^{\alpha x}\right)e^{-\alpha x-\beta y}}{\alpha\,b\beta}\right)$$

75.6 Problem 6

problem number 691

Added Feb. 9, 2019.

Problem Chapter 3.3.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y = ce^{\gamma x-2\beta y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x - 2*beta*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-2\beta y}\left(6a^3\alpha^3e^{2\beta y}c_1\left(-\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha\beta}\right)-11a^3\alpha^2\gamma e^{2\beta y}c_1\left(-\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha\beta}\right)-a^3\gamma^3\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x-2*beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{b^2\beta^2c}{a^3\alpha^2}\left(2\,\frac{\left(e^{\beta y}b\beta - a\alpha\,e^{\alpha x}\right)e^{-\alpha x-\beta y+x(\gamma-2\alpha)}}{b\beta\,(\gamma-2\alpha)} - \frac{\left(e^{\beta y}b\beta - a\alpha\,e^{\alpha x}\right)^2 e^{-2\alpha x-2\beta y+x(\gamma-\alpha)}}{(\gamma-\alpha)\,b^2\beta^2} - \frac{e^{x(\gamma-3}}{\gamma-3}\right.$$

75.7 Problem 7

problem number 692

Added Feb. 9, 2019.

Problem Chapter 3.3.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\alpha x}w_x + be^{\beta y}w_y = ce^{\gamma x} + se^{\mu y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[alpha*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*Exp[gamma*x] + s*Exp[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-\beta y}\left(\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha}+\frac{b\beta e^{-\alpha x}}{a\alpha}\right)^{-\frac{\mu}{\beta}}\left(-b\beta c e^{x(\gamma-\alpha)+\beta y}\left(\frac{e^{-\alpha x-\beta y}\left(a\alpha e^{\alpha x}-b\beta e^{\beta y}\right)}{a\alpha}+\frac{b\beta e^{-\alpha x}}{a\alpha}\right)^{\frac{\mu}{\beta}}+\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(alpha*x)*diff(w(x,y),x) +b*exp(beta*y)*diff(w(x,y),y) =c*exp(gamma*x) + s*exp(mu
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{c\mathrm{e}^{x(\gamma-\alpha)}}{a\left(\gamma-\alpha\right)}+\frac{s\left(\mathrm{e}^{\beta y}b\beta-a\alpha\,\mathrm{e}^{\alpha x}\right)\mathrm{e}^{-\alpha x-\beta y}}{a\alpha\,b\left(\beta-\mu\right)}\left(\frac{a\alpha}{b\beta}\left(-\frac{\left(\mathrm{e}^{\beta y}b\beta-a\alpha\,\mathrm{e}^{\alpha x}\right)\mathrm{e}^{-\alpha x-\beta y}}{b\beta}+\mathrm{e}^{-\alpha x}\right)^{-1}\right)^{\frac{\mu}{\beta}}-\frac{}{a\alpha}$$

75.8 Problem 8

problem number 693

Added Feb. 9, 2019.

Problem Chapter 3.3.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x} + ke^{\delta y} + p$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{1}{a} \left(s\mathrm{e}^{-\_b\,(\beta-\mu)} + \mathrm{e}^{-\beta\_b} p + \left(-\frac{\lambda}{a} \left(c \int \mathrm{e}^{\frac{-a\beta\,\gamma\,\_b + a\beta^2\,\_b + \lambda\,b\mathrm{e}^{\_b\,(\gamma-\beta)}}{(\gamma-\beta)a}} \mathrm{d}\_b - \frac{a}{\lambda} \left(\lambda \int \frac{c}{a} \mathrm{e}^{\frac{-a\beta\,\gamma\,x + a\beta^2 x + \lambda}{(\gamma-\beta)a}}\right.\right.\right.\right.$$

75.9 Problem 9

problem number 694

Added Feb. 9, 2019.

Problem Chapter 3.3.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\beta x} w_x + (b e^{\gamma x} + c e^{\lambda y}) w_y = s e^{\mu x + \delta y} + k$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == s*E
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(beta*x)*diff(w(x,y),x) +(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =s*exp(m
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(\mathrm{e}^{-\beta \_b} k + \left(-\frac{\lambda}{a}\left(c \int \mathrm{e}^{\frac{-a\beta\gamma \_b + a\beta^2 \_b + \lambda b \mathrm{e}^{-\_b(\gamma-\beta)}}{(\gamma-\beta)a}} \,\mathrm{d}\_b - \frac{a}{\lambda}\left(\lambda \int \frac{c}{a} \mathrm{e}^{\frac{-a\beta\gamma x + a\beta^2 x + \lambda b \mathrm{e}^{x(\gamma-\beta)}}{(\gamma-\beta)a}} \,\mathrm{d}x + \mathrm{e}\right.\right.\right.\right.$$

75.10 Problem 10

problem number 695

Added Feb. 9, 2019.

Problem Chapter 3.3.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = ce^{\mu x+\delta y} + k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x + lambda*y]*D[w[x, y], y] == c*Exp[mu*x +
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{e^{-\beta x}\left(-\beta ce^{x(\mu-\beta)+\beta x}\left(-\frac{a(\beta-\gamma)e^{\beta x}}{-e^{-\lambda y}\left(-a\gamma e^{\beta x}+a\beta e^{\beta x}-b\lambda e^{\gamma x+\lambda y}\right)-b\lambda e^{\gamma x}}\right)^{\delta/\lambda}\left(\frac{\frac{\beta e^{-\beta x-\lambda y}\left(-a\gamma e^{\beta x}+a\beta e^{\beta x}-b\lambda e^{\gamma x+\lambda}\right.}{\lambda(\beta-\gamma)}}{\right.}\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(beta*x)*diff(w(x,y),x) +b*exp(gamma*x+lambda*y)*diff(w(x,y),y) =c*exp(mu*x+delta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^{x}\frac{1}{a}\left(c\left(-\frac{(\gamma-\beta)\,a}{\lambda\,b}\left(\frac{\left(\lambda\,b\mathrm{e}^{-\beta\,x+\gamma\,x+y\lambda}+\gamma\,a-a\beta\right)\mathrm{e}^{-y\lambda}\beta}{\lambda\,b\,(\gamma-\beta)}-\frac{\left(\lambda\,b\mathrm{e}^{-\beta\,x+\gamma\,x+y\lambda}+\gamma\,a-a\beta\right)\mathrm{e}^{-y\lambda}\gamma}{\lambda\,b\,(\gamma-\beta)}\right.\right.\right.$$

75.11 Problem 11

problem number 696

Added Feb. 9, 2019.

Problem Chapter 3.3.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma x} + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y] + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x -\frac{c\left(-\lambda\left(-\frac{be^{\beta K[1]}}{a\beta}-\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)\right)^{\gamma/\lambda}+d}{a\lambda\left(-\frac{be^{\beta K[1]}}{a\beta}-\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)}dK[1]+c_1\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*exp(beta*x)*diff(w(x,y),y) =c*exp(gamma*y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{\beta}{\lambda b}\left(c\left(\frac{\lambda b}{a\beta}\left(-\frac{e^{\beta x}b\lambda - e^{y\lambda}a\beta}{\lambda b}+e^{\beta\_a}\right)\right)^{\frac{\gamma}{\lambda}}+d\right)\left(-\frac{e^{\beta x}b\lambda - e^{y\lambda}a\beta}{\lambda b}+e^{\beta\_a}\right)^{-1}d\_a + \_F1\Big($$

76 HFOPDE, chapter 3.3.2

76.1 Problem 1

problem number 697

Added Feb. 9, 2019.

Problem Chapter 3.3.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cye^{\lambda x} + kxe^{\mu y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*y*Exp[lambda*x] + k*x*Exp[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to -\frac{a^3k\lambda^2e^{\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}} - a^2b^2\lambda^2\mu^2c_1\left(\frac{ay-bx}{a}\right) - a^2bk\lambda^2\mu xe^{\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}} - ab^2c\lambda\mu^2ye^{\lambda x} + b^3c\mu^2e}{a^2b^2\lambda^2\mu^2}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*y*exp(lambda*x)+k*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{kx}{b\mu}e^{\frac{(ya-bx)\mu}{a}+\frac{b\mu x}{a}} + \frac{cye^{\lambda x}}{a\lambda} + \frac{1}{b^2\mu^2a^2\lambda^2}\left(\_F1\left(\frac{ya-bx}{a}\right)b^2\mu^2a^2\lambda^2 - e^{\lambda x}cb^3\mu^2 - ka^3e^{\frac{(ya-bx)\mu}{a}+\frac{b\mu x}{a}}\lambda^2\right.$$

76.2 Problem 2

problem number 698

Added Feb. 9, 2019.

Problem Chapter 3.3.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = a x^k e^{\lambda y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == a*x^k*Exp[lambda*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{(-a\lambda x)^{-k}\left(x^k e^{\lambda(y-ax)}\text{Gamma}(k+1,-a\lambda x)+\lambda c_1(y-ax)(-a\lambda x)^k\right)}{\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) +a*diff(w(x,y),y) =a*x^k*exp(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{(-ax\lambda)^{-k}\,k\Gamma(k)\,x^k e^{(-ax+y)\lambda} - (-ax\lambda)^{-k}\,\Gamma(k,-ax\lambda)\,kx^k e^{(-ax+y)\lambda} - x^k e^{ax\lambda+(-ax+y)\lambda} - \_F1(\cdots}{\lambda}$$

76.3 Problem 3

problem number 699

Added Feb. 9, 2019.

Problem Chapter 3.3.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + be^{\lambda x})w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y + b*Exp[lambda*x])*D[w[x, y], y] == c*Exp[beta*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\beta c_1\left(\frac{e^{-ax}(ay+be^{\lambda x}-\lambda y)}{a-\lambda}\right)+ce^{\beta x}}{\beta}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) +(a*y+b*exp(lambda*x))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{\beta}\left(\_F1\left(\frac{\left(y e^{x(a-\lambda)}a-y e^{x(a-\lambda)}\lambda+e^{ax}b\right)e^{-x(2a-\lambda)}}{a-\lambda}\right)\beta+ce^{\beta x}\right)$$

76.4 Problem 4

problem number 700

Added Feb. 9, 2019.

Problem Chapter 3.3.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (aye^{\lambda x} + be^{\beta x}y^k)w_y = ce^{\mu x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y*Exp[lambda*x] + b*Exp[beta*x]*y^k)*D[w[x, y], y] == c*Exp[mu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{\mu c_1\left(y^{-k}e^{-\frac{ae^{\lambda x}}{\lambda}}\left(y^k\left(-e^{\frac{ae^{\lambda x}}{\lambda}}\right)\left(\int_1^x be^{\beta K[1]-\frac{a(1-k)e^{\lambda K[1]}}{\lambda}}dK[1]\right)+ky^ke^{\frac{ae^{\lambda x}}{\lambda}}\left(\int_1^x be^{\beta K[1]-\frac{a(1-k)e^{\lambda}}{\lambda}}\right.\right.\right.}{\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) +(a*y*exp(lambda*x)+b*exp(beta*x)*y^k)*diff(w(x,y),y) =c*exp(mu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{1}{\mu}\left(\_F1\left(\frac{y}{y^k}\mathrm{e}^{\frac{\mathrm{e}^{\lambda x}ak}{\lambda}}\left(\mathrm{e}^{\frac{ae^{\lambda x}}{\lambda}}\right)^{-1} + kb\int \mathrm{e}^{\frac{\mathrm{e}^{\lambda x}ak+\beta x\lambda-ae^{\lambda x}}{\lambda}}\,\mathrm{d}x - b\int \mathrm{e}^{\frac{\mathrm{e}^{\lambda x}ak+\beta x\lambda-ae^{\lambda x}}{\lambda}}\,\mathrm{d}x\right)\mu + c\mathrm{e}^{\mu x}\right)$$

76.5 Problem 5

problem number 701

Added Feb. 9, 2019.

Problem Chapter 3.3.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$w_x + (ax^k + bx^n e^{\lambda y})w_y = ce^{\beta x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*x^k + b*x^n*Exp[lambda*y])*D[w[x, y], y] == c*Exp[beta*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{\beta c_1\left(\frac{b\lambda x^{n+1}\left(-\frac{a\lambda x^{k+1}}{k+1}\right)^{-\frac{n}{k+1}-\frac{1}{k+1}}\text{Gamma}\left(\frac{n+1}{k+1},-\frac{a\lambda x^{k+1}}{k+1}\right)-e^{-\frac{\lambda\left(-ax^{k+1}+ky+y\right)}{k+1}}-ke^{-\frac{\lambda\left(-ax^{k+1}+ky+y\right)}{k+1}}}{abk^2\lambda^2-abk\lambda^2 n+abk\lambda^2-ab\lambda^2 n}\right)+ce^{\beta}}{\beta}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) +(a*x^k+b*x^n*exp(lambda*y))*diff(w(x,y),y) =c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{1}{\beta}\left(\_F1\left(-\frac{1}{a\lambda\left(2k^2n+3kn^2+n^3+2k^2+10kn+6n^2+7k+11n+6\right)}\left(2\left(-\frac{x^{k+1}\lambda a}{k+1}\right)^{-1/}\right.\right.\right.$$

76.6 Problem 6

problem number 702

Added Feb. 9, 2019.

Problem Chapter 3.3.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{axe^{x\left(\lambda+\frac{\mu y}{x}\right)}+\lambda xc_1\left(\frac{y}{x}\right)+\mu yc_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*x*exp(lambda *x+ mu* y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = a\mathrm{e}^{\lambda x+\mu y}\left(\frac{\mu y}{x}+\lambda\right)^{-1} + \_F1\left(\frac{y}{x}\right)$$

76.7 Problem 7

problem number 703

Added Feb. 9, 2019.

Problem Chapter 3.3.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = aye^{\lambda x} + bxe^{\mu y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*y*Exp[lambda*x] + b*x*Exp[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\mu y^2 e^{\lambda x} + b\lambda x^2 e^{\mu y} + \lambda\mu xyc_1\left(\frac{y}{x}\right)}{\lambda\mu xy}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +y*diff(w(x,y),y) =a*y*exp(lambda*x)  + b*x*exp(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{bx\mathrm{e}^{\mu y}}{\mu y} + \frac{\mathrm{e}^{\lambda x}ay}{\lambda x} + \_F1\left(\frac{y}{x}\right)$$

76.8 Problem 8

problem number 704

Added Feb. 9, 2019.

Problem Chapter 3.3.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + be^{\lambda y} w_y = cx^n + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == c*x^n + s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{x^{-k}\left(ak^2x^kc_1\left(\frac{x^{-k}e^{-\lambda y}(ax^k-akx^k+b\lambda xe^{\lambda y})}{a(k-1)\lambda}\right) + anx^kc_1\left(\frac{x^{-k}e^{-\lambda y}(ax^k-akx^k+b\lambda xe^{\lambda y})}{a(k-1)\lambda}\right) - aknx^kc_1\left(\frac{x^{-}}{}\right.\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*x^k*diff(w(x,y),x) +b*exp(lambda*y)*diff(w(x,y),y) =c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{x^{1-k}s}{a(1-k)} + \frac{x^{1-k+n}c}{a(1-k+n)} + \_F1\left(\frac{x^{1-k}\lambda b - ake^{-y\lambda} + ae^{-y\lambda}}{b\lambda(k-1)}\right)$$

76.9 Problem 9

problem number 705

Added Feb. 9, 2019.

Problem Chapter 3.3.2.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + be^{\lambda x} w_y = ce^{\mu x} + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == c*Exp[mu*x] + s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\left(\left(\frac{(k+1)\left(\frac{a\lambda y^{k+1}-bke^{\lambda x}-be^{\lambda x}}{k+1}+be^{\lambda x}\right)}{a\lambda}\right)^{\frac{1}{k+1}}\right)^{-k}\left(ck\lambda e^{\mu x}\left(\frac{b(k+1)e^{\lambda x}}{a\lambda y^{k+1}-bke^{\lambda x}-be^{\lambda x}}+1\right)^{\frac{k}{k+1}}\text{Hypergeome}\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*y^k*diff(w(x,y),x) +b*exp(lambda*x)*diff(w(x,y),y) =c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{ce^{\mu\_a}+s}{a}\left(\left(\frac{e^{\lambda\_a}bk-e^{\lambda x}bk+y^k y a\lambda-e^{\lambda x}b+e^{\lambda\_a}b}{a\lambda}\right)^{(k+1)^{-1}}\right)^{-k}d\_a+\_F1\left(-\frac{e^{\lambda x}bk-y}{}\right.$$

76.10 Problem 10

problem number 706

Added Feb. 9, 2019.

Problem Chapter 3.3.2.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^k w_y = cx^n + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*x^n + s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\rightarrow \frac{e^{-\lambda x}(\lambda x)^{-n}\left(-ce^{\lambda x}x^n\text{Gamma}(n+1,\lambda x)+a\lambda e^{\lambda x}(\lambda x)^n c_1\left(-\frac{y^{-k}e^{-\lambda x}(a\lambda y e^{\lambda x}+by^k-bky^k)}{a(k-1)\lambda}\right)-s(\right.}{a\lambda}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(lambda*x)*diff(w(x,y),x) +b*y^k*diff(w(x,y),y) = c*x^n+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{x^n c(\lambda x)^{-n/2}\,\mathrm{e}^{-1/2\,\lambda x}\,\text{WhittakerM}\left(n/2,n/2+1/2,\lambda x\right)}{(n+1)\,a\lambda}+\frac{s\left(1-\mathrm{e}^{-\lambda x}\right)}{a\lambda}+\_F1\left(-\frac{\mathrm{e}^{-\lambda x}bk-y^{1-k}c}{a\lambda}\right.$$

76.11 Problem 11

problem number 707

Added Feb. 9, 2019.

Problem Chapter 3.3.2.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^k w_y = cExp[\mu x] + s$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == c*Exp[mu*x] + s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*exp(lambda*y)*diff(w(x,y),x) +b*x^k*diff(w(x,y),y) = c*exp(mu*x)+s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{(ce^{\mu\_a} + s)(k+1)}{\lambda b}\left(-\frac{(x^{k+1}\lambda b - e^{y\lambda}ak - ae^{y\lambda})k}{(k+1)\lambda b} + \_a^{k+1} - \frac{x^{k+1}\lambda b - e^{y\lambda}ak - ae^{y\lambda}}{(k+1)\lambda b}\right)^{-1} d$$

77 HFOPDE, chapter 3.4.1

77.1 Problem 1

problem number 708

Added Feb. 9, 2019.

Problem Chapter 3.4.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\sinh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Sinh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\sinh\left(\frac{b\mu x}{a}\right)\sinh\left(\frac{\mu(ay-bx)}{a}\right)+ak\lambda\cosh\left(\frac{b\mu x}{a}\right)\cosh\left(\frac{\mu(ay-bx)}{a}\right)+bc\mu\cos}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x)+k*sinh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{1}{b\mu\, a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu\, a\lambda+\cosh\left(\lambda\, x\right)cb\mu+ka\cosh\left(\frac{(ya-bx)\,\mu}{a}+\frac{b\mu\, x}{a}\right)\lambda\right)$$

77.2 Problem 2

problem number 709

Added Feb. 9, 2019.

Problem Chapter 3.4.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\cosh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c}{a\lambda+b\mu}\cosh\left(\frac{(a\lambda+b\mu)\,x}{a}+\frac{(ya-bx)\,\mu}{a}\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

77.3 Problem 3

problem number 710

Added Feb. 9, 2019.

Problem Chapter 3.4.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx\sinh(\lambda x + \mu y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x*Sinh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) +b*diff(w(x,y),y) =c*x*sinh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{x}{(a\lambda + b\mu)^2}\left(\cosh\left(\frac{(a\lambda + b\mu)x}{a} + \frac{(ya - bx)\mu}{a}\right)ac\lambda + \cosh\left(\frac{(a\lambda + b\mu)x}{a} + \frac{(ya - bx)\mu}{a}\right)bc\mu\right)$$

77.4 Problem 4

problem number 711

Added Feb. 9, 2019.

Problem Chapter 3.4.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda x)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Kernel Exception

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*sinh(lambda*x)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(c(\sinh(\mu\_a))^m + s\left(\sinh\left(\frac{\beta\left(y\lambda a - b\cosh(\lambda x) + b\cosh(\lambda\_a)\right)}{a\lambda}\right)\right)^k\right)d\_a + \_F1\left(-\right.$$

77.5 Problem 5

problem number 712

Added Feb. 9, 2019.

Problem Chapter 3.4.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = c\sinh^m(\mu x) + s\sinh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sinh[lambda*y]*D[w[x, y], y] == c*Sinh[mu*x]^m + s*Sinh[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*sinh(lambda*y)*diff(w(x,y),y) =c*sinh(mu*x)^m+s*sinh(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{c(\sinh(\mu\_a))^m}{a} + \frac{s}{a}\left(\sinh\left(\frac{\beta}{\lambda}\ln\left(-\tanh\left(1/2\,\frac{\lambda\,b}{a}\left(-\frac{bx\lambda + 2\,a\,\mathrm{arctanh}\,(\mathrm{e}^{y\lambda})}{\lambda\,b} + \_a\right)\right)\right)\right)\right)$$

78 HFOPDE, chapter 3.4.2

78.1 Problem 1

problem number 713

Added Feb. 9, 2019.

Problem Chapter 3.4.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x] + k*Cosh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\sinh\left(\frac{b\mu x}{a}\right)\cosh\left(\frac{\mu(ay-bx)}{a}\right)+ak\lambda\cosh\left(\frac{b\mu x}{a}\right)\sinh\left(\frac{\mu(ay-bx)}{a}\right)+bc\mu\sin}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{1}{b\mu a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu a\lambda+c\sinh(\lambda x)b\mu+k\sinh\left(\frac{(ya-bx)\mu}{a}+\frac{b\mu x}{a}\right)a\lambda\right)$$

78.2 Problem 2

problem number 714

Added Feb. 9, 2019.

Problem Chapter 3.4.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\sinh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{c}{a\lambda+b\mu}\sinh\left(\frac{(a\lambda+b\mu)\,x}{a}+\frac{(ya-bx)\,\mu}{a}\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

78.3 Problem 3

problem number 715

Added Feb. 9, 2019.

Problem Chapter 3.4.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\cosh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2\lambda^2c_1\left(\frac{ay-bx}{a}\right)+a^2\lambda x\sinh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)+a^2\left(-\cosh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)\right)+b^2\mu}{(a\lambda+b\mu)^2}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = a*x*cosh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \frac{x}{(a\lambda+b\mu)^2}\left(\sinh\left(\frac{(a\lambda+b\mu)x}{a}+\frac{(ya-bx)\mu}{a}\right)a^2\lambda+\sinh\left(\frac{(a\lambda+b\mu)x}{a}+\frac{(ya-bx)\mu}{a}\right)ab\mu\right)$$

78.4 Problem 4

problem number 716

Added Feb. 9, 2019.

Problem Chapter 3.4.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\cosh^n(\lambda x) w_y = c\cosh^m(\mu x) + s\cosh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*cosh(lambda*x)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(c(\cosh(\_b\,\mu))^m + s\left(\cosh\left(\frac{\beta}{a}\left(\int(\cosh(\_b\,\lambda))^n \,\mathrm{d}\_bb + \left(-\int\frac{b(\cosh(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\right.\right.\right.\right.$$

78.5 Problem 5

problem number 717

Added Feb. 9, 2019.

Problem Chapter 3.4.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda y)w_y = c\cosh^m(\mu x) + s\cosh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == c*Cosh[mu*x]^m + s*Cosh[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*cosh(lambda*y)^n*diff(w(x,y),y) = c*cosh(mu*x)^m+s*cosh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{(\cosh(\_b\,\lambda))^{-n}}{b} \left(s(\cosh(\beta\,\_b))^{k} + \left(\cosh \left(\frac{\mu \left(a \int (\cosh(\_b\,\lambda))^{-n}\, \mathrm{d}\_b + bx - a \int (\cosh(y\lambda$$

79 HFOPDE, chapter 3.4.3

79.1 Problem 1

problem number 718

Added Feb. 9, 2019.

Problem Chapter 3.4.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x] + k*Tanh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\log\left(\cosh\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right)+bc\mu\log(\cosh(\lambda x))}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x)+ k *tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/2\,\frac{1}{b\mu\,a\lambda}\left(2\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda - k\ln\left(\tanh\left(\mu\,y\right)-1\right)a\lambda - k\ln\left(\tanh\left(\mu\,y\right)+1\right)a\lambda - c\ln\left(\right.\right.$$

79.2 Problem 2

problem number 719

Added Feb. 9, 2019.

Problem Chapter 3.4.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log\left(\cosh\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,\frac{1}{a\lambda+b\mu}\left(-2\_F1\left(\frac{ya-bx}{a}\right)a\lambda-2\_F1\left(\frac{ya-bx}{a}\right)b\mu+c\ln\left(\tanh\left(\frac{(ya-bx)\,\mu+ax}{a}\right.\right.\right.$$

79.3 Problem 3

problem number 720

Added Feb. 11, 2019.

Problem Chapter 3.4.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tanh(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\log\left(\cosh\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right)+\lambda xc_1\left(\frac{y}{x}\right)+\mu yc_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*tanh(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,\frac{1}{\lambda\,x+\mu\,y}\left(\left(\ln\left(\tanh\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)\right)-1\right)a+\ln\left(\tanh\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)+1\right)a-2\_F1\left(\frac{y}{x}\right.\right.$$

79.4 Problem 4

problem number 721

Added Feb. 11, 2019.

Problem Chapter 3.4.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda x)w_y = c\tanh^m(\mu x) + s\tanh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*tanh(lambda*x)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(s\left(1\sinh\left(\frac{\beta}{a}\left(\int (\tanh(\_b\,\lambda))^n \,\mathrm{d}\_bb + \left(-\int \frac{b(\tanh(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)a\right)\right)\left(\cosh\left(\frac{\beta}{a}\left(\right.\right.\right.\right.\right.$$

79.5 Problem 5

problem number 722

Added Feb. 11, 2019.

Problem Chapter 3.4.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \tanh^n(\lambda y) w_y = c \tanh^m(\mu x) + s \tanh^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == c*Tanh[mu*x]^m + s*Tanh[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*tanh(lambda*y)^n*diff(w(x,y),y) = c*tanh(mu*x)^m+s*tanh(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\tanh(\_b\,\lambda))^{-n}}{b} \left(s(\tanh(\beta\,\_b))^k + \left(1 \sinh\left(\frac{\mu}{b} \left(a \int (\tanh(\_b\,\lambda))^{-n} \, \mathrm{d}\_b + \left(-\frac{a \int (\tanh\right.\right.\right.\right.\right.$$

80 HFOPDE, chapter 3.4.4

80.1 Problem 1

problem number 723

Added Feb. 11, 2019.

Problem Chapter 3.4.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x) + k\coth(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x] + k*Coth[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\log\left(\sinh\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right)+bc\mu\log(\sinh(\lambda x))}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x)+k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/2\,\frac{1}{b\mu\,a\lambda}\left(2\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda - c\ln\left(\coth(\lambda\,x)-1\right)b\mu - c\ln\left(\coth(\lambda\,x)+1\right)b\mu - k\ln(\mathrm{co}$$

80.2 Problem 2

problem number 724

Added Feb. 11, 2019.

Problem Chapter 3.4.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{c\log\left(\sinh\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) = c*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,\frac{1}{a\lambda+b\mu}\left(-2\_F1\left(\frac{ya-bx}{a}\right)a\lambda-2\_F1\left(\frac{ya-bx}{a}\right)b\mu+\ln\left(\coth\left(\frac{(ya-bx)\,\mu+ax\lambda}{a}\right.\right.\right.$$

80.3 Problem 3

problem number 725

Added Feb. 11, 2019.

Problem Chapter 3.4.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\coth(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\log\left(\sinh\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right)+\lambda x c_1\left(\frac{y}{x}\right)+\mu y c_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=x*diff(w(x,y),x) + y*diff(w(x,y),y) = a*x*coth(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,\frac{1}{\lambda\,x+\mu\,y}\left(\left(\ln\left(\coth\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)-1\right)a+\ln\left(\coth\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)+1\right)a-2\_F1\left(\frac{y}{x}\right)\right.\right.$$

80.4 Problem 4

problem number 726

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \coth^n(\lambda x) w_y = c \coth^m(\mu x) + s \coth^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*coth(lambda*x)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(c(\coth(\_b\,\mu))^m + s\left(1\cosh\left(\frac{\beta}{a}\left(\int (\coth(\_b\,\lambda))^n \,\mathrm{d}\_bb + \left(-\int \frac{b(\coth(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\right.\right.\right.\right.$$

80.5 Problem 5

problem number 727

Added Feb. 11, 2019.

Problem Chapter 3.4.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda y)w_y = c\coth^m(\mu x) + s\coth^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == c*Coth[mu*x]^m + s*Coth[beta*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*coth(lambda*y)^n*diff(w(x,y),y) = c*coth(mu*x)^m+ s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\coth(\_b\,\lambda))^{-n}}{b} \left(s(\coth(\beta\,\_b))^k + \left(1\cosh\left(\frac{\mu}{b}\left(a\int (\coth(\_b\,\lambda))^{-n}\,\mathrm{d}\_b + \left(-\frac{a\int (\coth(y\right.\right.\right.\right.\right.$$

81 HFOPDE, chapter 3.4.5

81.1 Problem 1

problem number 728

Added Feb. 11, 2019.

Problem Chapter 3.4.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x) + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x] + k*Cosh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\sinh\left(\frac{b\mu x}{a}\right)\cosh\left(\frac{\mu(ay-bx)}{a}\right)+ak\lambda\cosh\left(\frac{b\mu x}{a}\right)\sinh\left(\frac{\mu(ay-bx)}{a}\right)+bc\mu\cos}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*sinh(lambda*x)+ k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{1}{b\mu\,a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda+\cosh\left(\lambda\,x\right)cb\mu+k\sinh\left(\frac{(ya-bx)\,\mu}{a}+\frac{b\mu\,x}{a}\right)a\lambda\right)$$

81.2 Problem 2

problem number 729

Added Feb. 11, 2019.

Problem Chapter 3.4.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \tanh(\lambda x) + k\coth(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == Tanh[lambda*x] + k*Coth[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\log\left(\sinh\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right)+b\mu\log(\cosh(\lambda x))}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  tanh(lambda*x)+ k*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{(bk\lambda\,\mu - b\lambda\,\mu)\,x}{b\mu\,a\lambda} - 2\,\frac{ky}{b} + \frac{1}{b\mu\,a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda + k\ln\left(e^{2\,\frac{(ya-bx)\mu}{a}+2\,\frac{b\mu\,x}{a}}-1\right)a\lambda + \ln\left(e^{2}\right.\right.$$

81.3 Problem 3

problem number 730

Added Feb. 11, 2019.

Problem Chapter 3.4.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sinh(\lambda x) + k\tanh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == Sinh[lambda*x] + k*Tanh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\log\left(\cosh\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right)+b\mu\cosh(\lambda x)}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  sinh(lambda*x)+ k*tanh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kx}{a}-2\frac{ky}{b}+1/2\frac{1}{b\mu a\lambda}\left(2\_F1\left(\frac{ya-bx}{a}\right)b\mu a\lambda+2k\ln\left(e^{2\frac{(ya-bx)\mu}{a}+2\frac{b\mu x}{a}}+1\right)a\lambda+e^{\lambda x}b\mu+e^{-\lambda}\right.$$

81.4 Problem 4

problem number 731

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh(\mu y)w_y = \sinh(\lambda x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == Sinh[lambda*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{2a\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)-b\mu x}{a\mu}\right)+\cosh(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*cosh(mu*y)*diff(w(x,y),y) =  sinh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{a\lambda}\left(\_F1\left(\frac{-b\mu\,x+2\arctan\left(e^{\mu\,y}\right)a}{b\mu}\right)a\lambda+\cosh\left(\lambda\,x\right)\right)$$

81.5 Problem 5

problem number 732

Added Feb. 11, 2019.

Problem Chapter 3.4.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh(\mu y)w_y = \cosh(\lambda x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sinh[mu*y]*D[w[x, y], y] == Cosh[lambda*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{a\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)-b\mu x}{a\mu}\right)+\sinh(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*sinh(mu*y)*diff(w(x,y),y) =  cosh(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{a\lambda}\left(\_F1\left(-\frac{b\mu\,x+2\,\operatorname{arctanh}\left(e^{\mu\,y}\right)a}{b\mu}\right)a\lambda+\sinh\left(\lambda\,x\right)\right)$$

82 HFOPDE, chapter 3.5.1

82.1 Problem 1

problem number 733

Added Feb. 11, 2019.

Problem Chapter 3.5.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a^2\lambda c_1\left(\frac{ay-bx}{a}\right)+a\beta cy\log(\beta(ay-bx)+a\lambda x+b\beta x)+ac\lambda x\log\left(\frac{\beta(ay-bx)}{a}+\frac{b\beta x}{a}+\lambda x\right)-b\beta}{a(}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\frac{x}{a\lambda+b\beta}\left(\ln\left(\frac{(a\lambda+b\beta)\,x}{a}+\frac{(ya-bx)\,\beta}{a}\right)c\lambda-\lambda c\right)+\frac{y}{a\lambda+b\beta}\left(c\ln\left(\frac{(a\lambda+b\beta)\,x}{a}+\frac{(ya-bx)}{a}\right.\right.$$

82.2 Problem 2

problem number 734

Added Feb. 11, 2019.

Problem Chapter 3.5.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x) + k\ln(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x] + k*Log[beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{bkx\log\left(\beta\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)\right)+abc_1\left(\frac{ay-bx}{a}\right)-bkx\log(ay)+aky\log(ay)+bcx\log(\lambda x)-bcx-}{ab}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*ln(lambda*x)+k*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{(\ln(\lambda x)bc-bc)x}{ab}+\frac{y}{ab}\left(\ln\left(\frac{bx\beta}{a}+\frac{(ya-bx)\beta}{a}\right)ak-ak\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

82.3 Problem 3

problem number 735

Added Feb. 11, 2019.

Problem Chapter 3.5.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \ln(\lambda x) \ln(\beta y) w_y = c \ln(\gamma x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Log[lambda*x]*Log[beta*y]*D[w[x, y], y] == c*Log[gamma*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(a\lambda \text{LogIntegral}(\beta y) - b\beta\left(\log\left(e^{\text{ProductLog}\left(\frac{\lambda x(\log(\lambda x)-1)}{e}\right)+1}\right) - 1\right) \text{ExpIntegralEi}\left(\log\left(e^{\text{Pr}\right.\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)*ln(beta*y)*diff(w(x,y),y) =  c*ln(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{a\lambda \operatorname{LambertW}\left(\lambda x\left(\ln(\lambda x) - 1\right) e^{-1}\right)}\left(\operatorname{LambertW}\left(\lambda x(\ln(\lambda x) - 1) e^{-1}\right) \_F1\left(\frac{\text{expIntegral}(1,}{}\right.\right.$$

82.4 Problem 4

problem number 736

Added Feb. 11, 2019.

Problem Chapter 3.5.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda x)w_y = c\ln^m(\mu x) + s\ln^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*ln(lambda*x)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(c(\ln(\_b\,\mu))^m + s\left(\ln\left(\frac{\beta}{a}\left(b\int(\ln(\_b\,\lambda))^n\,\mathrm{d}\_b + \left(-\int\frac{b(\ln(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)a\right)\right)\right)^k\right)$$

82.5 Problem 5

problem number 737

Added Feb. 11, 2019.

Problem Chapter 3.5.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda y)w_y = c\ln^m(\mu x) + s\ln^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == c*Log[mu*x]^m + s*Log[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*ln(lambda*y)^n*diff(w(x,y),y) =  c*ln(mu*x)^m+s*ln(beta*y)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\ln(\_b\,\lambda))^{-n}}{b}\left(c\left(\ln\left(\frac{\mu}{b}\left(a\int(\ln(\_b\,\lambda))^{-n}\,\mathrm{d}\_b + \left(-\frac{a\int(\ln(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)b\right)\right)\right)^m + s$$

82.6 Problem 6

problem number 738

Added Feb. 11, 2019.

Problem Chapter 3.5.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x + b\ln^k(\beta y)w_y = c\ln^m(\gamma x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[lambda*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*ln(lambda*x)^n*diff(w(x,y),x) + b*ln(lambda*y)^k*diff(w(x,y),y) =  c*ln(gamma*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{c(\ln(\gamma)+\ln(x))^m (\ln(\lambda x))^{-n}}{a}\, dx + \_F1\left(-\int (\ln(\lambda x))^{-n}\, dx + \int \frac{(\ln(y\lambda))^{-k} a}{b}\, dy\right)$$

83 HFOPDE, chapter 3.5.2

83.1 Problem 1

problem number 739

Added Feb. 11, 2019.

Problem Chapter 3.5.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n + s\ln^k(\lambda y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n + s*Log[lambda*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{abnc_1\left(\frac{ay-bx}{a}\right) + abc_1\left(\frac{ay-bx}{a}\right) + bnsx\log\left(\lambda\left(\frac{ay-bx}{a} + \frac{bx}{a}\right)\right) + bsx\log\left(\lambda\left(\frac{ay-bx}{a} + \frac{bx}{a}\right)\right) - bnsx\,l}{ab(n+1)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=a*diff(w(x,y),x) + b*diff(w(x,y),y) =  c*x^n+s*ln(lambda*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{y}{ab\,(n+1)}\left(\ln\left(\frac{bx\lambda}{a} + \frac{(ya-bx)\,\lambda}{a}\right)ans + \ln\left(\frac{bx\lambda}{a} + \frac{(ya-bx)\,\lambda}{a}\right)as - ans - as\right) + \frac{1}{ab\,(n+}$$

83.2 Problem 2

problem number 740

Added Feb. 11, 2019.

Problem Chapter 3.5.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = b y^2 + c x^n y + s \ln^k(\lambda x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == b*y^2 + c*x^n*y + s*Log[lambda*x]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{(-\log(\lambda x))^{-k}\left(3n^2 s \log^k(\lambda x)\text{Gamma}(k+1,-\log(\lambda x)) + 9ns\log^k(\lambda x)\text{Gamma}(k+1,-\text{lo}\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*y^2+c*x^n*y+s*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x b\_a^2a^2 + ca\_a^{n+1} + 2(-ax+y)\,ab\_a + \_a^n(-ax+y)\,c + (-ax+y)^2 b + s(\ln(\lambda\_a))^k\,d\_a + \_$$

Result has unresolved integrals

83.3 Problem 3

problem number 741

Added Feb. 11, 2019.

Problem Chapter 3.5.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = b ln^k(\lambda x) \ln^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  b*ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} b(\ln(\lambda\_a))^k (\ln(\beta(\_a\, a - ax + y)))^n \, d\_a + \_F1(-ax + y)$$

84 HFOPDE, chapter 3.5.3

84.1 Problem 4

problem number 742

Added Feb. 11, 2019.

Problem Chapter 3.5.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^n)w_y = cln^k(\lambda x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{(-\log(\lambda x))^{-k}\left(\lambda(-\log(\lambda x))^k c_1(a^{-n-1}e^{-ax}(be^{ax}\text{Gamma}(n+1,ax)+ya^{n+1}))+c\log^k(\lambda x)\right.}{\lambda}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) +  (a*y+b*x^n)*diff(w(x,y),y) =  c*ln(lambda*x)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int c(\ln(\lambda x))^k \,\mathrm{d}x + \_F1\left(-\frac{\mathrm{e}^{-ax}\left((ax)^{-n/2}\,\text{WhittakerM}\left(n/2, n/2+1/2, ax\right)x^n\mathrm{e}^{1/2\,ax}b - any -\right.}{a\,(n+1)}\right.$$

Result has unresolved integrals

84.2 Problem 5

problem number 743

Added Feb. 11, 2019.

Problem Chapter 3.5.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ axw_x + byw_y = x^k(n\ln x + m\ln y) $$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y]);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$ \left\{\left\{w(x,y)\to \frac{a^2k^2c_1\left(yx^{-\frac{b}{a}}\right)+akmx^k\log(y)-anx^k+aknx^k\log(x)-bmx^k}{a^2k^2}\right\}\right\} $$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*x*diff(w(x,y),x) +  b*y*diff(w(x,y),y) =  x^k*(n*ln(x)+m*ln(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$ w(x,y)=1/2\,\frac{1}{k^2a^2}\left(i\pi\,x^k\,csgn\left(ix^{\frac{b}{a}}\right)\left(csgn(iy)\right)^2akm-i\pi\,x^k\,csgn\left(ix^{\frac{b}{a}}\right)csgn(iy)\,csgn\left(iyx^{-\frac{b}{a}}\right)akm-i\pi $$

84.3 Problem 6

problem number 744

Added Feb. 11, 2019.

Problem Chapter 3.5.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x^k w_x + b y^n w_y = c\ln^m(\lambda x) + s\ln^l(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == c*Log[lambda*x]^m + s*Log[beta*y]^l;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$\text{Aborted}$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*x^k*diff(w(x,y),x) +  b*y^n*diff(w(x,y),y) =  c*ln(lambda*x)+s*ln(beta*y)^l;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \int^x \frac{\_a^{-k}}{a}\left(c\ln\left(\lambda\_a\right) + s\left(\ln\left(\beta\left(\frac{b(n-1)\_a^{1-k} - x^{1-k}b(n-1) + a y^{-n+1}(k-1)}{a\left(k-1\right)}\right)^{-(n-1)^{-1}}\right)\right.\right.$$

85 HFOPDE, chapter 3.6.1

85.1 Problem 1

problem number 745

Added Feb. 11, 2019.

Problem Chapter 3.6.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\sin(\lambda x) + k\sin(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x] + k*Sin[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\sin\left(\frac{b\mu x}{a}\right)\sin\left(\frac{\mu(ay-bx)}{a}\right)-ak\lambda\cos\left(\frac{b\mu x}{a}\right)\cos\left(\frac{\mu(ay-bx)}{a}\right)-bc\mu\cos(\lambda x)}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x)+k*sin(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\frac{1}{b\mu\, a\lambda}\left(-\_F1\left(\frac{ya-bx}{a}\right)b\mu\, a\lambda + \cos(\lambda\, x)\, cb\mu + ka\cos\left(\frac{(ya-bx)\,\mu}{a}+\frac{b\mu\, x}{a}\right)\lambda\right)$$

85.2 Problem 2

problem number 746

Added Feb. 11, 2019.

Problem Chapter 3.6.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-c\cos\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = -\frac{c}{a\lambda+b\mu}\cos\left(\frac{(a\lambda+b\mu)\,x}{a}+\frac{(ya-bx)\,\mu}{a}\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

85.3 Problem 3

problem number 747

Added Feb. 11, 2019.

Problem Chapter 3.6.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{-ax\cos\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)+\lambda xc_1\left(\frac{y}{x}\right)+\mu yc_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -a\cos\left(x\left(\frac{\mu y}{x}+\lambda\right)\right)\left(\frac{\mu y}{x}+\lambda\right)^{-1} + \_F1\left(\frac{y}{x}\right)$$

85.4 Problem 4

problem number 748

Added Feb. 11, 2019.

Problem Chapter 3.6.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*sin(lambda*x)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(s\left(\sin\left(\frac{\beta}{a}\left(\int (\sin(\_b\,\lambda))^n \, \mathrm{d}\_bb + \left(-\int \frac{b(\sin(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)a\right)\right)\right)^k + c(\sin(\_b\,\mu))^n$$

85.5 Problem 5

problem number 749

Added Feb. 11, 2019.

Problem Chapter 3.6.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = c\sin^m(\mu x) + s\sin^k(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == c*Sin[mu*x]^m + s*Sin[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\sin^2(\mu x)^{-\frac{m}{2}-\frac{1}{2}}\left(-c\cos(\mu x)\sin^{m+1}(\mu x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3}{2},\cos^2(\mu x)\right)+ac_1()\mu s\right.}{a\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*sin(lambda*y)^n*diff(w(x,y),y) =  c*sin(mu*x)^m+s*sin(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^y\frac{(\sin(\_b\,\lambda))^{-n}}{b}\left(\left(\sin\left(\frac{\mu\left(a\int(\sin(\_b\,\lambda))^{-n}\,\mathrm{d}\_b+bx-a\int(\sin(y\lambda))^{-n}\,\mathrm{d}y\right)}{b}\right)\right)^m c+s(\sin$$

Result has unresolved integrals

86 HFOPDE, chapter 3.6.2

86.1 Problem 1

problem number 750

Added Feb. 11, 2019.

Problem Chapter 3.6.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cos(\lambda x) + k\cos(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x] + k*Cos[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\sin\left(\frac{b\mu x}{a}\right)\cos\left(\frac{\mu(ay-bx)}{a}\right)+ak\lambda\cos\left(\frac{b\mu x}{a}\right)\sin\left(\frac{\mu(ay-bx)}{a}\right)+bc\mu\sin(\lambda x)}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x)+k*cos(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{1}{b\mu\, a\lambda}\left(\_F1\left(\frac{ya-bx}{a}\right)b\mu\, a\lambda+c\sin\left(\lambda\, x\right)b\mu+k\sin\left(\frac{(ya-bx)\,\mu}{a}+\frac{b\mu\, x}{a}\right)a\lambda\right)$$

86.2 Problem 2

problem number 751

Added Feb. 11, 2019.

Problem Chapter 3.6.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\sin\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{c}{a\lambda+b\mu}\sin\left(\frac{(a\lambda+b\mu)\,x}{a}+\frac{(ya-bx)\,\mu}{a}\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

86.3 Problem 3

problem number 752

Added Feb. 11, 2019.

Problem Chapter 3.6.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ax\sin\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)+\lambda xc_1\left(\frac{y}{x}\right)+\mu yc_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=a\sin\left(x\left(\frac{\mu y}{x}+\lambda\right)\right)\left(\frac{\mu y}{x}+\lambda\right)^{-1}+\_F1\left(\frac{y}{x}\right)$$

86.4 Problem 4

problem number 753

Added Feb. 11, 2019.

Problem Chapter 3.6.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda x)w_y = c\cos^m(\mu x) + s\cos^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*cos(lambda*x)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(c(\cos(\_b\,\mu))^m + s\left(\cos\left(\frac{\beta}{a}\left(\int(\cos(\_b\,\lambda))^n\,\mathrm{d}\_bb + \left(-\int\frac{b(\cos(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)a\right)\right)\right.\right.$$

86.5 Problem 5

problem number 754

Added Feb. 11, 2019.

Problem Chapter 3.6.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda y)w_y = c\cos^m(\mu x) + s\cos^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == c*Cos[mu*x]^m + s*Cos[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*cos(lambda*y)^n*diff(w(x,y),y) =  c*cos(mu*x)^m+s*cos(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\cos(\_b\,\lambda))^{-n}}{b}\left(\left(\cos\left(\frac{\mu\left(a\int(\cos(\_b\,\lambda))^{-n}\,\mathrm{d}\_b + bx - a\int(\cos(y\lambda))^{-n}\,\mathrm{d}y\right)}{b}\right)\right)^m c + s(\mathrm{co}$$

87 HFOPDE, chapter 3.6.3

87.1 Problem 1

problem number 755

Added Feb. 11, 2019.

Problem Chapter 3.6.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\tan(\lambda x) + k\tan(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x] + k*Tan[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right) - ak\lambda\log\left(\cos\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right) - bc\mu\log(\cos(\lambda x))}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x)+k*tan(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = 1/2\,\frac{1}{b\mu\,a\lambda}\left(2\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda + k\ln\left(1+(\tan(\mu\,y))^2\right)a\lambda + c\ln\left(1+(\tan(\lambda\,x))^2\right)b\mu\right)$$

87.2 Problem 2

problem number 756

Added Feb. 11, 2019.

Problem Chapter 3.6.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{-c\log\left(\cos\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=1/2\,\frac{c}{a\lambda+b\mu}\ln\left(1+\left(\tan\left(\frac{(ya-bx)\,\mu+ax\lambda+b\mu\,x}{a}\right)\right)^2\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

87.3 Problem 3

problem number 757

Added Feb. 11, 2019.

Problem Chapter 3.6.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-ax\log\left(\cos\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right)+\lambda x c_1\left(\frac{y}{x}\right)+\mu y c_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = 1/2\,a\ln\left(1+\left(\tan\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)\right)^2\right)\left(\frac{\mu\,y}{x}+\lambda\right)^{-1}+\_F1\left(\frac{y}{x}\right)$$

87.4 Problem 4

problem number 758

Added Feb. 11, 2019.

Problem Chapter 3.6.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a w_x + b \tan^n(\lambda x) w_y = c \tan^m(\mu x) + s \tan^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$\text{Aborted}$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*tan(lambda*x)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x, y) = \int^x \frac{c(\tan(\_b\,\mu))^m}{a} + \frac{s}{a}\left(1\left(\tan\left(\left(-\int \frac{b(\tan(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\beta\right) + \tan\left(\frac{b\beta\,\int(\tan(\_b\,\lambda))^n\,\mathrm{d}\_\right.\right.\right.$$

87.5 Problem 5

problem number 759

Added Feb. 11, 2019.

Problem Chapter 3.6.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + b\tan^n(\lambda y)w_y = c\tan^m(\mu x) + s\tan^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == c*Tan[mu*x]^m + s*Tan[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*tan(lambda*y)^n*diff(w(x,y),y) =  c*tan(mu*x)^m+s*tan(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\tan(\_b\,\lambda))^{-n}}{b}\left(s(\tan(\beta\,\_b))^k + \left(-1\left(\tan\left(\mu\left(-\frac{a\int(\tan(y\lambda))^{-n}\,\mathrm{d}y}{b} + x\right)\right) + \tan\left(\frac{\mu\, a\right.\right.\right.\right.$$

88 HFOPDE, chapter 3.6.4

88.1 Problem 1

problem number 760

Added Feb. 11, 2019.

Problem Chapter 3.6.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cot(\lambda x) + k\cot(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x] + k*Cot[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ak\lambda\log\left(\sin\left(\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}\right)\right)+bc\mu\log(\sin(\lambda x))}{ab\lambda\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x)+k*cot(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=1/2\,\frac{1}{b\mu\,a\lambda}\left(2\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda-k\ln\left(\left(\cot\left(\mu\,y\right)\right)^2+1\right)a\lambda-c\ln\left(\left(\cot\left(\lambda\,x\right)\right)^2+1\right)b\mu\right)$$

88.2 Problem 2

problem number 761

Added Feb. 11, 2019.

Problem Chapter 3.6.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + by^n w_y = c\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{c\log\left(\sin\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right)+a\lambda c_1\left(\frac{ay-bx}{a}\right)+b\mu c_1\left(\frac{ay-bx}{a}\right)}{a\lambda+b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,\frac{c}{a\lambda+b\mu}\ln\left(\left(\cot\left(\frac{(ya-bx)\,\mu+ax\lambda+b\mu\,x}{a}\right)\right)^2+1\right)+\_F1\left(\frac{ya-bx}{a}\right)$$

88.3 Problem 3

problem number 762

Added Feb. 11, 2019.

Problem Chapter 3.6.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{ax\log\left(\sin\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right)+\lambda xc_1\left(\frac{y}{x}\right)+\mu yc_1\left(\frac{y}{x}\right)}{\lambda x+\mu y}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -1/2\,a\ln\left(\left(\cot\left(x\left(\frac{\mu\,y}{x}+\lambda\right)\right)\right)^2+1\right)\left(\frac{\mu\,y}{x}+\lambda\right)^{-1}+\_F1\left(\frac{y}{x}\right)$$

88.4 Problem 4

problem number 763

Added Feb. 11, 2019.

Problem Chapter 3.6.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot^n(\lambda x)w_y = c\cot^m(\mu x) + s\cot^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*cot(lambda*x)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{(\cot(\_b\,\mu))^m c}{a} + \frac{s}{a}\left(1\left(-1+\cot\left(\left(-\int\frac{b(\cot(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\beta\right)\cot\left(\frac{b\beta\ \int(\cot(\_b\,\lambda))^n}{a}\right.\right.\right.$$

88.5 Problem 5

problem number 764

Added Feb. 11, 2019.

Problem Chapter 3.6.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a w_x + b \cot^n(\lambda y) w_y = c \cot^m(\mu x) + s \cot^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == c*Cot[mu*x]^m + s*Cot[beta*y]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*cot(lambda*y)^n*diff(w(x,y),y) =  c*cot(mu*x)^m+s*cot(beta*y)^k
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{(\cot(\_b\,\lambda))^{-n}}{b} \left(\left(1 \left(-1 + \cot \left(\mu \left(-\frac{a \int (\cot(y\lambda))^{-n}\, dy}{b} + x \right) \right) \cot \left(\frac{\mu\, a \int (\cot(\_b\,\lambda))^{-n}}{b} \right.\right.\right.\right.$$

89 HFOPDE, chapter 3.6.5

89.1 Problem 1

problem number 765

Added Feb. 11, 2019.

Problem Chapter 3.6.5.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sin(\lambda x) + c\cos(\mu y) + k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x] + c*Cos[mu*y] + k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ab\lambda\mu c_1\left(\frac{ay-bx}{a}\right)+ac\lambda\sin\left(\frac{b\mu x}{a}\right)\cos\left(\frac{\mu(ay-bx)}{a}\right)+ac\lambda\cos\left(\frac{b\mu x}{a}\right)\sin\left(\frac{\mu(ay-bx)}{a}\right)+bk\lambda\mu x-b\mu c}{ab\lambda\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)+c*cos(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{kx}{a}-\frac{1}{b\mu\,a\lambda}\left(-\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda-c\sin\left(\frac{(ya-bx)\,\mu}{a}+\frac{b\mu\,x}{a}\right)a\lambda+\cos\left(\lambda\,x\right)b\mu\right)$$

89.2 Problem 2

problem number 766

Added Feb. 11, 2019.

Problem Chapter 3.6.5.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \tan(\lambda x) + c\sin(\mu y) + k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == Tan[lambda*x] + c*Sin[mu*y] + k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)+\frac{k\lambda x-\log(\cos(\lambda x))}{a\lambda}-\frac{c\cos(\mu y)}{b\mu}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  tan(lambda*x)+c*sin(mu*y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{b\mu\,a\lambda}\left(-2\,c(\cos\left(1/2\,\mu\,y\right))^2\,a\lambda+\_F1\left(\frac{ya-bx}{a}\right)b\mu\,a\lambda+kxb\mu\,\lambda-\ln\left(\frac{\sin\left(1/2\,\lambda\,x\right)-\cos\left(1/2\right.}{\cos\left(1/2\,\lambda\,x\right)}\right.\right.$$

89.3 Problem 3

problem number 767

Added Feb. 11, 2019.

Problem Chapter 3.6.5.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \sin(\lambda x)\cos(\mu y) + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == Sin[lambda*x]*Cos[mu*y] + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \rightarrow \frac{2a^3\lambda^2 c_1\left(\frac{ay-bx}{a}\right) - a^2\lambda\cos\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right) + 2a^2c\lambda^2 x - a^2\lambda\cos(\lambda x-\mu y) - 2ab^2\mu^2 c_1(}{2a(a\lambda-b\mu)(a\lambda+b\mu}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  sin(lambda*x)*cos(mu*y)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx}{a} + \frac{1}{a}\left(-1/2\,\frac{a}{a\lambda-b\mu}\cos\left(\frac{(a\lambda-b\mu)\,x}{a} - \frac{(ya-bx)\,\mu}{a}\right) - 1/2\,\frac{a}{a\lambda+b\mu}\cos\left(\frac{(a\lambda+b\mu)\,x}{a} + \frac{(y}{}\right.\right.$$

89.4 Problem 4

problem number 768

Added Feb. 11, 2019.

Problem Chapter 3.6.5.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = \cos(\lambda y) + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == Cos[lambda*x] + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{a\log\left(\tan\left(\frac{\mu y}{2}\right)\right)-b\mu x}{a\mu}\right)+c\lambda x+\sin(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*sin(mu*y)*diff(w(x,y),y) =  cos(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}+\frac{1}{a\lambda}\left(\_F1\left(\frac{a}{b\mu}\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(2\_Z\mathrm{e}^{\frac{b\mu x}{a}}\left(\_Z^2\mathrm{e}^{2\frac{b\mu x}{a}}+1\right)^{-1},-1\left(\_Z^2\mathrm{e}^{2\frac{b\mu x}{a}}-\right.\right.\right.\right.\right.\right.$$

89.5 Problem 5

problem number 769

Added Feb. 11, 2019.

Problem Chapter 3.6.5.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = \sin(\lambda y) + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Sin[lambda*x] + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{a\log(\sin(\mu y))-b\mu x}{a\mu}\right)+c\lambda x-\cos(\lambda x)}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  sin(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\frac{cx}{a}-\frac{1}{a\lambda}\left(-\_F1\left(\frac{1}{b\mu}\left(-b\mu\,x+\ln\left(\frac{\tan(\mu\,y)}{\sqrt{1+(\tan(\mu\,y))^2}}\right)a\right)\right)a\lambda+\cos(\lambda\,x)\right)$$

89.6 Problem 6

problem number 770

Added Feb. 11, 2019.

Problem Chapter 3.6.5.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = \cot(\lambda y) + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == Cot[lambda*x] + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{a\lambda c_1\left(\frac{a\log(\sin(\mu y))-b\mu x}{a\mu}\right)+c\lambda x+\log(\sin(\lambda x))}{a\lambda}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*tan(mu*y)*diff(w(x,y),y) =  cot(lambda*x)+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx}{a}+1/2\,\frac{1}{a\lambda}\left(2\_F1\left(\frac{1}{b\mu}\left(-b\mu\,x+\ln\left(\frac{\tan(\mu\,y)}{\sqrt{1+(\tan(\mu\,y))^2}}\right)a\right)\right)a\lambda-\ln\left((\cot(\lambda\,x))^2+1\right)\right)$$

90 HFOPDE, chapter 3.7.1

90.1 Problem 1

problem number 771

Added Feb. 11, 2019.

Problem Chapter 3.7.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \arcsin \frac{x}{\lambda} + k \arcsin \frac{y}{\beta}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[x/lambda] + k*ArcSin[y/beta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to \frac{-\frac{ibkx\sqrt{a^2(\beta^2-y^2)}\log\left(2\left(\sqrt{a^2(\beta^2-y^2)}-iay\right)\right)}{\sqrt{1-\frac{y^2}{\beta^2}}} + a^2b\beta c_1\left(y-\frac{bx}{a}\right) - \frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}} + \frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}} + \frac{iaky\sqrt{a^2(\beta^2-y^2)}\,\mathrm{lo}}{}}{a^2b\beta}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arcsin(x/lambda)+k*arcsin(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{ab}\left(\arcsin\left(\frac{y}{\beta}\right)kya + \sqrt{\frac{\beta^2-y^2}{\beta^2}}a\beta\,k + \arcsin\left(\frac{x}{\lambda}\right)bcx + \sqrt{\frac{\lambda^2-x^2}{\lambda^2}}bc\lambda + \_F1\left(\frac{ya-bx}{a}\right)ba\right.$$

90.2 Problem 2

problem number 772

Added Feb. 11, 2019.

Problem Chapter 3.7.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}+(\beta y+\lambda x)\sin^{-1}(\beta y+\lambda x)\right)}{a\lambda+b\beta} + c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{a\lambda + b\beta}\left(\sqrt{-\beta^2y^2 - 2\,\beta\,\lambda\,xy - \lambda^2x^2 + 1}c + (a\lambda + b\beta)\_F1\left(\frac{ya - bx}{a}\right) + \arcsin\left(\beta\,y + \lambda\,x\right)c(\beta$$

90.3 Problem 3

problem number 773

Added Feb. 11, 2019.

Problem Chapter 3.7.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\arcsin(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to ax\left(\frac{\sqrt{-\beta^2y^2 - 2\beta\lambda xy - \lambda^2x^2 + 1}}{\beta y + \lambda x} + \sin^{-1}(\beta y + \lambda x)\right) + c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arcsin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = a\left(x\left(\frac{\beta y}{x}+\lambda\right)\arcsin\left(x\left(\frac{\beta y}{x}+\lambda\right)\right)+\sqrt{-x^2\left(\frac{\beta y}{x}+\lambda\right)^2+1}\right)\left(\frac{\beta y}{x}+\lambda\right)^{-1}+\_F1\left(\frac{y}{x}\right)$$

90.4 Problem 4

problem number 774

Added Feb. 11, 2019.

Problem Chapter 3.7.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda x)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*x)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} (\arcsin(\mu\_a))^{m} + \frac{1}{a}\left(\arcsin\left(\frac{\beta \arcsin(\lambda\_a)\, b\_a}{a} + \frac{(-\arcsin(\lambda x)\, bx\lambda + y\lambda a - \sqrt{-\lambda^2 x^2}}{a\lambda}\right.\right.$$

90.5 Problem 5

problem number 775

Added Feb. 11, 2019.

Problem Chapter 3.7.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda y)w_y = c\arcsin^m(\mu x) + s\arcsin^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == a*ArcSin[mu*x]^m + ArcSin[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arcsin(lambda*y)*diff(w(x,y),y) =  a*arcsin(mu*x)^m+arcsin(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a}{b \arcsin(\lambda \_a)} \left(\arcsin\left(\frac{\mu (bx\lambda - \text{cosineIntegral}(\arcsin(y\lambda)) a)}{\lambda b} + \frac{\mu \ \text{cosineIntegral}(\arcsin}{\lambda b} \right. \right.$$

91 HFOPDE, chapter 3.7.2

91.1 Problem 1

problem number 776

Added Feb. 11, 2019.

Problem Chapter 3.7.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arccos\frac{x}{\lambda} + k\arccos\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[x/lambda] + k*ArcCos[y/beta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to\frac{\frac{ibkx\sqrt{a^2(\beta^2-y^2)}\log\left(2\left(\sqrt{a^2(\beta^2-y^2)}-iay\right)\right)}{\sqrt{1-\frac{y^2}{\beta^2}}}+a^2b\beta c_1\left(y-\frac{bx}{a}\right)+\frac{a^2ky^2}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{a^2\beta^2k}{\sqrt{1-\frac{y^2}{\beta^2}}}-\frac{iaky\sqrt{a^2(\beta^2-y^2)}\log(}{\sqrt{}}}{a^2b\beta}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccos(x/lambda)+k*arccos(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\frac{cx}{a}\arccos\left(\frac{x}{\lambda}\right)+\frac{ky}{b}\arccos\left(\frac{bx}{a\beta}+\frac{ya-bx}{a\beta}\right)+\frac{1}{ab}\left(-\sqrt{-\frac{x^2}{\lambda^2}+1}bc\lambda-\sqrt{-\left(\frac{bx}{a\beta}+\frac{ya-bx}{a\beta}\right)^2+\right.\right.$$

91.2 Problem 2

problem number 777

Added Feb. 11, 2019.

Problem Chapter 3.7.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arccos(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to \frac{c\left(\beta(bx-ay)\sin^{-1}(\beta y+\lambda x)+x(a\lambda+b\beta)\cos^{-1}(\beta y+\lambda x)+a\left(-\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x\right.\right.}{a(a\lambda+b\beta)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{a\lambda+b\beta}\left(-\sqrt{-\beta^2y^2-2\,\beta\,\lambda\,xy-\lambda^2x^2+1}c+(a\lambda+b\beta)\_F1\left(\frac{ya-bx}{a}\right)+\arccos\left(\beta\,y+\lambda\,x\right)\right.$$

91.3 Problem 3

problem number 778

Added Feb. 11, 2019.

Problem Chapter 3.7.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\arccos(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to ax\left(\cos^{-1}(\beta y+\lambda x)-\frac{\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}}{\beta y+\lambda x}\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccos(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{\beta\,y+\lambda\,x}\left(-\sqrt{-\beta^2y^2-2\,\beta\,\lambda\,xy-\lambda^2x^2+1}ax+(\beta\,y+\lambda\,x)\left(ax\arccos\left(\beta\,y+\lambda\,x\right)+\_F1\left(\frac{y}{x}\right)\right.\right.$$

91.4 Problem 4

problem number 779

Added Feb. 11, 2019.

Problem Chapter 3.7.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda x)w_y = c\arccos^m(\mu x) + s\arccos^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arccos(lambda*x)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x (\arccos(\mu\_a))^m + \frac{1}{a}\left(\arccos\left(\frac{\beta\arccos(\lambda\_a)\,b\_a}{a} + \frac{(-\arccos(\lambda x)\,bx\lambda + y\lambda a + \sqrt{-\lambda^2 x^2}}{a\lambda}\right.\right.$$

91.5 Problem 5

problem number 780

Added Feb. 11, 2019.

Problem Chapter 3.7.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda y)w_y = c\arccos^m(\mu x) + s\arccos^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == a*ArcCos[mu*x]^m + ArcCos[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arccos(lambda*y)*diff(w(x,y),y) =  a*arccos(mu*x)^m+arccos(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{a}{b\arccos(\lambda\_a)}\left(\pi - \arccos\left(-\frac{\mu\left(bx\lambda + \mathrm{sinIntegral}\left(\arccos(y\lambda)\right)a\right)}{\lambda b} + \frac{\mu\ \mathrm{sinIntegral}\left(\arccos\right.}{\lambda b}\right.\right.$$

92 HFOPDE, chapter 3.7.3

92.1 Problem 1

problem number 781

Added Feb. 11, 2019.

Problem Chapter 3.7.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c \arctan \frac{x}{\lambda} + k \arctan \frac{y}{\beta}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[x/lambda] + k*ArcTan[y/beta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{-a\beta k \log\left(a^2\beta^2 + (ay-bx)^2 + 2bx(ay-bx) + b^2x^2\right) + 2abc_1\left(\frac{ay-bx}{a}\right) + 2aky\tan^{-1}\left(\frac{y}{\beta}\right) -}{2ab}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arctan(x/lambda)+k*arctan(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \frac{cx}{a}\arctan\left(\frac{x}{\lambda}\right) + \frac{ky}{b}\arctan\left(\frac{bx}{a\beta} + \frac{ya-bx}{a\beta}\right) + 1/2\,\frac{1}{ab}\left(-k\beta\,\ln\left(\left(\frac{bx}{a\beta} + \frac{ya-bx}{a\beta}\right)^2 + 1\right)a - \lambda\right.$$

92.2 Problem 2

problem number 782

Added Feb. 11, 2019.

Problem Chapter 3.7.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arctan(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\rightarrow \frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}+c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{2\,a\lambda+2\,b\beta}\left(-c\ln\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)+(2\,a\lambda+2\,b\beta)\_F1\left(\frac{ya-bx}{a}\right)+2\,\arctan\left(\beta\right.\right.$$

92.3 Problem 3

problem number 783

Added Feb. 11, 2019.

Problem Chapter 3.7.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\arctan(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to -\frac{ax\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2(\beta y+\lambda x)}+ax\tan^{-1}(\beta y+\lambda x)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arctan(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{2\,\beta\,y+2\,\lambda\,x}\left(-\ln\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)ax+2\left(\beta\,y+\lambda\,x\right)\left(ax\arctan\left(\beta\,y+\lambda\,x\right)+\_F\right.\right.$$

92.4 Problem 4

problem number 784

Added Feb. 11, 2019.

Problem Chapter 3.7.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda x)w_y = c\arctan^m(\mu x) + s\arctan^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arctan(lambda*x)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x (\arctan(\mu\_a))^m + \frac{1}{a}\left(\arctan\left(\frac{\beta b\_a \arctan(\lambda\_a)}{a} + 1/2\,\frac{(-2\,bx\arctan(\lambda\,x)\,\lambda + 2\,y\lambda\,a + b}{a\lambda}\right.\right.$$

92.5 Problem 5

problem number 785

Added Feb. 11, 2019.

Problem Chapter 3.7.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda y)w_y = c\arctan^m(\mu x) + s\arctan^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == a*ArcTan[mu*x]^m + ArcTan[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arctan(lambda*y)*diff(w(x,y),y) =  a*arctan(mu*x)^m+arctan(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^y \frac{a}{b\arctan(\_b\,\lambda)}\left(\arctan\left(\frac{\mu\,a\int(\arctan(\_b\,\lambda))^{-1}\,\mathrm{d}\_b}{b} + \mu\left(-\int\frac{a}{b\arctan(y\lambda)}\,\mathrm{d}y + x\right)\right)\right)^{n}$$

93 HFOPDE, chapter 3.7.4

93.1 Problem 1

problem number 786

Added Feb. 11, 2019.

Problem Chapter 3.7.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\operatorname{arccot}\frac{x}{\lambda} + k\operatorname{arccot}\frac{y}{\beta}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[x/lambda] + k*ArcCot[y/beta];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\frac{a\beta k\log\left(a^2\beta^2+(ay-bx)^2+2bx(ay-bx)+b^2x^2\right)+2abc_1\left(\frac{ay-bx}{a}\right)-2aky\tan^{-1}\left(\frac{y}{\beta}\right)+2b}{2ab}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c*arccot(x/lambda)+k*arccot(y/beta);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=1/2\,\frac{x}{ab}\left(\pi\,bc+\pi\,bk-2\,\arctan\left(\frac{x}{\lambda}\right)bc\right)-\frac{ky}{b}\arctan\left(\frac{bx}{a\beta}+\frac{ya-bx}{a\beta}\right)+1/2\,\frac{1}{ab}\left(k\beta\,\ln\left(\left(\frac{bx}{a\beta}+\right.\right.\right.$$

93.2 Problem 2

problem number 787

Added Feb. 11, 2019.

Problem Chapter 3.7.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\operatorname{arccot}(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to \frac{c(a\log\left(a^2(\beta^2y^2 + 2\beta\lambda xy + \lambda^2x^2 + 1)\right) + 2\beta(bx - ay)\tan^{-1}(\beta y + \lambda x) + 2x(a\lambda + b\beta)\cot^{-1}}{2a(a\lambda + b\beta)}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  c *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = 1/2\,\frac{1}{(a\lambda + b\beta)\,a}\left(c\ln\left(\beta^2y^2 + 2\,\beta\,\lambda\,xy + \lambda^2x^2 + 1\right)a + \left(2\,a^2\lambda + 2\,ab\beta\right)\_F1\left(\frac{ya - bx}{a}\right) - 2\,(a(\right.$$

93.3 Problem 3

problem number 788

Added Feb. 11, 2019.

Problem Chapter 3.7.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\operatorname{arccot}(\lambda x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to ax\left(\frac{\log\left(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1\right)}{2\beta y+2\lambda x}+\cot^{-1}(\beta y+\lambda x)\right)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := x*diff(w(x,y),x) +  y*diff(w(x,y),y) =  a*x *arccot(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{2\,\beta\,y+2\,\lambda\,x}\left(\ln\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)ax+(\beta\,y+\lambda\,x)\left(a\pi\,x-2\,ax\arctan\left(\beta\,y+\lambda\,x\right)\right.\right.$$

93.4 Problem 4

problem number 789

Added Feb. 11, 2019.

Problem Chapter 3.7.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\,\mathrm{arccot}^n(\lambda x)w_y = c\,\mathrm{arccot}^m(\mu x) + s\,\mathrm{arccot}^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arccot(lambda*x)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x (\pi/2 - \arctan(\mu\_a))^m + \frac{1}{a}\left(\pi/2 + \arctan\left(-1/2\,\frac{\beta\,b\_a\,\pi}{a} + \frac{\beta\,b\_a\,\arctan(\lambda\_a)}{a} + 1/2\,\frac{(bx$$

93.5 Problem 5

problem number 790

Added Feb. 11, 2019.

Problem Chapter 3.7.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^n(\lambda y)w_y = c\operatorname{arccot}^m(\mu x) + s\operatorname{arccot}^k(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == a*ArcCot[mu*x]^m + ArcCot[bet
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*arccot(lambda*y)*diff(w(x,y),y) =  a*arccot(mu*x)^m+arccot(beta
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{y} \frac{a}{b\operatorname{arccot}(\_b\,\lambda)} \left(\pi/2 - \arctan\left(2\,\frac{\mu\,a\int(\pi - 2\,\arctan(\_b\,\lambda))^{-1}\,\mathrm{d}\_b}{b} + \mu\left(-\int 2\,\frac{a}{b\,(\pi - 2\,\mathrm{ar}\right.\right.\right.$$

94 HFOPDE, chapter 3.8.1

94.1 Problem 1

problem number 791

Added Feb. 11, 2019.

Problem Chapter 3.8.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{f(K[1])}{a}\,dK[1]+c_1\left(\frac{ay-bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := a*diff(w(x,y),x) +  b*diff(w(x,y),y) =  f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int \frac{f(x)}{a}\,\mathrm{d}x+\_F1\left(\frac{ya-bx}{a}\right)$$

94.2 Problem 2

problem number 792

Added Feb. 11, 2019.

Problem Chapter 3.8.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = yf(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == y*f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1])(aK[1]-ax+y)\,dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=  diff(w(x,y),x) +  a*diff(w(x,y),y) =  y*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^x f(\_a)\,(\_a\,a-ax+y)\,d\_a+\_F1(-ax+y)$$

94.3 Problem 3

problem number 793

Added Feb. 11, 2019.

Problem Chapter 3.8.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = y^2 f(x) + yg(x) + h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == y^2*f[x] + y*g[x] + h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \left(f(K[1])(aK[1]-ax+y)^2+g(K[1])(aK[1]-ax+y)+h(K[1])\right)dK[1]+c_1(y-ax)\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^2*f(x)+y*g(x)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y)=\int^x ((x-\_a)\,a-y)^2\,f(\_a)+((\_a-x)\,a+y)\,g(\_a)+h(\_a)\,d\_a+\_F1(-ax+y)$$

94.4 Problem 4

problem number 794

Added Feb. 11, 2019.

Problem Chapter 3.8.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = y^k f(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == y^k*f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x f(K[1])(aK[1]-ax+y)^k\,dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  y^k*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x (\_a\,a-ax+y)^k f(\_a)\,d\_a+\_F1(-ax+y)$$

94.5 Problem 5

problem number 795

Added Feb. 11, 2019.

Problem Chapter 3.8.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = e^{\lambda y} f(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == Exp[lambda*y]*f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1])e^{\lambda(aK[1]-ax+y)}\,dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + a*diff(w(x,y),y) =  exp(lambda*y)*f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^x f(\_a)\,\mathrm{e}^{-a\,a\lambda+(-ax+y)\lambda} d\_a + \_F1(-ax+y)$$

94.6 Problem 6

problem number 796

Added Feb. 11, 2019.

Problem Chapter 3.8.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-e^{-ax}\left(e^{ax}\int_1^x e^{-aK[1]}f(K[1])\,dK[1]-y\right)\right)+\int_1^x g(K[2])\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int g(x)\,\mathrm{d}x+\_F1\left(-\int f(x)\,\mathrm{e}^{-ax}\,\mathrm{d}x+y\mathrm{e}^{-ax}\right)$$

94.7 Problem 7

problem number 797

Added Feb. 11, 2019.

Problem Chapter 3.8.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = y^k g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == y^k*g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x g(K[2]) \left(e^{aK[2]} \left(\text{Integrate}\left[e^{-aK[1]} f(K[1]), \{K[1], 1, K[2]\}, \text{Assumptions} \to \text{True}\right] - e^{-ax}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + (a*y+f(x))*diff(w(x,y),y) =  y^k*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \left(\left(\left(\int f(\_b)\, e^{-\_b\, a}\, d\_b - \int f(x)\, e^{-ax}\, dx + y e^{-ax}\right) e^{\_b\, a}\right)^k g(\_b)\, d\_b + \_F1\left(-\int f(x)\, e^{-ax}\right.\right.$$

94.8 Problem 8

problem number 798

Added Feb. 11, 2019.

Problem Chapter 3.8.1.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + y^k w_y = g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{y^{-k}\left(ky^k\left(\int_1^x \frac{1}{f(K[1])}\,dK[1]\right)-y^k\left(\int_1^x \frac{1}{f(K[1])}\,dK[1]\right)+y\right)}{k-1}\right)+\int_1^x \frac{g(K[2])}{f(K[2])}\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + y^k*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int g(x)\,\mathrm{d}x+\_F1\left(\frac{y}{y^k}+kx-x\right)$$

94.9 Problem 9

problem number 799

Added Feb. 11, 2019.

Problem Chapter 3.8.1.9 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+a)w_y = by + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == b*y + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to (c-ab)\int_1^x \frac{1}{f(K[1])}\,dK[1] + c_1\left((a+y)e^{-\int_1^x \frac{1}{f(K[1])}\,dK[1]}\right) + b(a+y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + (y+a)*diff(w(x,y),y) =  b*y+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = (-ab+c)\,x + ab + by + \_F1\left((y+a)\,\mathrm{e}^{-x}\right)$$

94.10 Problem 10

problem number 800

Added Feb. 11, 2019.

Problem Chapter 3.8.1.10 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y + ax)w_y = g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-e^{-\int_1^x \frac{1}{f(K[1])}\,dK[1]}\left(e^{\int_1^x \frac{1}{f(K[1])}\,dK[1]}\int_1^x \frac{aK[2]\exp\left(-\text{Integrate}\left[\frac{1}{f(K[1])},\{K[1],1,K[2]\},\text{As}\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde := diff(w(x,y),x) + (y+a*x)*diff(w(x,y),y) =  g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int g(x)\,dx + \_F1\left((ax + a + y)\,e^{-x}\right)$$

94.11 Problem 11

problem number 801

Added Feb. 11, 2019.

Problem Chapter 3.8.1.11 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (yg_1(x) + g_0(x))w_y = y^2h_2(x) + yh_1(x) + h_0(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = f[x]*D[w[x, y], x] + (y*g1[x] + g0[x])*D[w[x, y], y] == y^2*h2[x] + y*h1[x] + h0[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}dK[1]} - \int_1^x \frac{\text{g0}(K[2])\exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{Assumption}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := diff(w(x,y),x) + (y*g1(x)+g0(x))*diff(w(x,y),y) =   y^2*h2(x)+y*h1(x)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^x h2(\_f)\,e^{2\int g1(\_f)\,d\_f}\left(\int g0(\_f)\,e^{-\int g1(\_f)\,d\_f}\,d\_f\right)^2 + 2\,h2(\_f)\,e^{2\int g1(\_f)\,d\_f}\int g0(\_f)\,e^{-\int g1(\_f)}$$

94.12 Problem 12

problem number 802

Added Feb. 11, 2019.

Problem Chapter 3.8.1.12 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (yg_1(x) + y^k g_2(x))w_y = h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = f[x]*D[w[x, y], x] + (y*g1[x] + y^k*g2[x])*D[w[x, y], y] == h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to c_1\left((k-1)\int_1^x \frac{\text{g2}(K[2])\exp\left((k-1)\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{Assumptions}\to\text{T}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := diff(w(x,y),x) + (y*g1(x)+y^k*g2(x))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int h(x)\,dx + \_F1\left(y^{1-k}e^{(k-1)\int g1(x)\,dx} + k\int e^{(k-1)\int g1(x)\,dx} g2(x)\,dx - \int e^{(k-1)\int g1(x)\,dx} g2(x)\,dx\right)$$

94.13 Problem 13

problem number 803

Added Feb. 11, 2019.

Problem Chapter 3.8.1.13 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + e^{\lambda y} g_2(x))w_y = h(x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = f[x]*D[w[x, y], x] + (g1[x] + Exp[lambda*y])*D[w[x, y], y] == h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := f(x)*diff(w(x,y),x) +(g1(x)+exp(lambda*y))*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int \frac{h(x)}{f(x)} \, dx + \_F1\left(-\frac{1}{\lambda}\left(\lambda \int \frac{1}{f(x)} e^{\lambda \int \frac{g1(x)}{f(x)} \, dx} \, dx + e^{\lambda\left(\int \frac{g1(x)}{f(x)} \, dx - y\right)}\right)\right)$$

94.14 Problem 14

problem number 804

Added Feb. 11, 2019.

Problem Chapter 3.8.1.14 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^k f(x) w_x + g(x) w_y = h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = y^k*f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{h(K[2])\left(\left(-(k+1)\text{Integrate}\left[\frac{g(K[1])}{f(K[1])},\{K[1],1,x\},\text{Assumptions}\to\text{True}\right]+(k+1)\text{Int}\right.\right.}{f(K[2])}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := y^k*f(x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{h(\_b)}{f(\_b)}\left(\left(k\int\frac{g(\_b)}{f(\_b)}\,\mathrm{d}\_b+y^k y-k\int\frac{g(x)}{f(x)}\,\mathrm{d}x-\int\frac{g(x)}{f(x)}\,\mathrm{d}x+\int\frac{g(\_b)}{f(\_b)}\,\mathrm{d}\_b\right)^{(k+1)^{-1}}\right)^{-k}$$

94.15 Problem 15

problem number 805

Added Feb. 11, 2019.

Problem Chapter 3.8.1.15 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^k f(x)w_x + (y^{k+1}g_1(x) + g_0(x))w_y = y^{3k+1}h_2(x) + y^{2k+1}h_1(x) + y^k h_0(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = y^k*f[x]*D[w[x, y], x] + (y^(k + 1)*g1[x] + g0[x])*D[w[x, y], y] == y^(3*k + 1)*h2[x]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y^{k+1}\exp\left(-(k+1)\int_1^x\frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]\right)-(k+1)\int_1^x\frac{\text{g0}(K[2])\exp\left(-(k+1)\text{Integra}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := y^k*f(x)*diff(w(x,y),x) +(y^(k+1)* g1(x) + g0(x))*diff(w(x,y),y) =   y^(3*k +1)*h2(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x\frac{1}{f(\_f)}\left(h0(\_f)+\left(\left(k\int\frac{g0(\_f)}{f(\_f)}\mathrm{e}^{-\int\frac{g1(\_f)}{f(\_f)}\,\mathrm{d}\_f(k+1)}\,\mathrm{d}\_f+y^{k+1}\mathrm{e}^{-\int\frac{g1(x)}{f(x)}\,\mathrm{d}x(k+1)}-k\int\frac{g0(x)}{f(x)}\mathrm{e}\right.\right.\right.$$

94.16 Problem 16

problem number 806

Added Feb. 11, 2019.

Problem Chapter 3.8.1.16 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda x}w_x + g(x)w_y = h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = f[x]*Exp[lambda*x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{g(K[1])e^{-\lambda K[1]}}{f(K[1])}\,dK[1]\right)+\int_1^x \frac{h(K[2])e^{-\lambda K[2]}}{f(K[2])}\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := f(x)*exp(lambda*x)*diff(w(x,y),x) +g(x)*diff(w(x,y),y) =   h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int \frac{h(x)\,\mathrm{e}^{-\lambda x}}{f(x)}\,\mathrm{d}x+\_F1\left(-\int \frac{g(x)\,\mathrm{e}^{-\lambda x}}{f(x)}\,\mathrm{d}x+y\right)$$

95 HFOPDE, chapter 3.8.2

95.1 Problem 1

problem number 807

Added Feb. 11, 2019.

Problem Chapter 3.8.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x) + g(y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x] + g[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{g\left(\frac{bK[1]+ay-bx}{a}\right)+f(K[1])}{a}\,dK[1]+c_1\left(\frac{ay-bx}{a}\right)\right\}\right\}$$

Maple ✓

```

w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde := a*diff(w(x,y),x) +b*diff(w(x,y),y) =   f(x)+g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{a}\left(f(\_a)+g\left(\frac{b\_a+ya-bx}{a}\right)\right)d\_a+\_F1\left(\frac{ya-bx}{a}\right)$$

95.2 Problem 2

problem number 808

Added Feb. 11, 2019.

Problem Chapter 3.8.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x)g(y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1])g(aK[1]-ax+y)\,dK[1]+c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =   f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^x f(\_a)\,g(\_a\,a-ax+y)\,d\_a+\_F1(-ax+y)$$

95.3 Problem 3

problem number 809

Added Feb. 11, 2019.

Problem Chapter 3.8.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)h(y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \int_1^x g(K[2])h\left(e^{aK[2]}\left(\text{Integrate}\left[e^{-aK[1]}f(K[1]), \{K[1], 1, K[2]\}, \text{Assumptions} \to \text{True}\right] - e^{-c\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
pde :=  diff(w(x,y),x) +(a*y+f(x) )*diff(w(x,y),y) =   g(x)*h(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x g(\_b)\, h\left(\left(\left(\int f(\_b)\, \mathrm{e}^{-\_b\, a}\, \mathrm{d}\_b - \int f(x)\, \mathrm{e}^{-ax}\, \mathrm{d}x + y\mathrm{e}^{-ax}\right) \mathrm{e}^{\_b\, a}\right) d\_b + \_F1\left(-\int f(x)\, \mathrm{e}^{-ax\right.\right.$$

95.4 Problem 4

problem number 810

Added Feb. 11, 2019.

Problem Chapter 3.8.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(y)w_y = h_1(x) + h_2(x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0];
 pde = f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == h1[x] + h2[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';
pde :=  f(x)*diff(w(x,y),x) +g(y)*diff(w(x,y),y) =   h1(x)+h2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{h1(\_f) + h2\left(\mathrm{RootOf}\left(\int (f(\_f))^{-1}\,\mathrm{d}\_f - \int^{\_Z} (g(\_a))^{-1}\,d\_a - \int (f(x))^{-1}\,\mathrm{d}x + \int (g(y))^{-\right.\right.}{f(\_f)}$$

95.5 Problem 5

problem number 811

Added Feb. 11, 2019.

Problem Chapter 3.8.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x + (f_2(x)y + y^k f_3(x))w_y = g(x)h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*D[w[x, y], x] + (y*f2[x] + y^k*f3[x])*D[w[x, y], y] == g[x]*h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y^{-k}\exp\left(-(1-k)\int_1^x\frac{\text{f2}(K[1])}{\text{f1}(K[1])}\,dK[1]\right)\left(y^k\left(-\exp\left((1-k)\int_1^x\frac{\text{f2}(K[1])}{\text{f1}(K[1])}\,dK[1]\right)\right)\right(\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*diff(w(x,y),x) +(y*f2(x)+y^k*f3(x))*diff(w(x,y),y) =  g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int\frac{g(x)\,h(x)}{f1\,(x)}\,\mathrm{d}x+\_F1\left(y^{1-k}\mathrm{e}^{(k-1)\int\frac{f2(x)}{f1(x)}\,\mathrm{d}x}+k\int\frac{f3(x)}{f1\,(x)}\mathrm{e}^{(k-1)\int\frac{f2(x)}{f1(x)}\,\mathrm{d}x}\,\mathrm{d}x-\int\frac{f3(x)}{f1\,(x)}\mathrm{e}^{(k-1)\int\frac{f2(x)}{f1(x)}\,\mathrm{d}x}\right.$$

95.6 Problem 6

problem number 812

Added Feb. 11, 2019.

Problem Chapter 3.8.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(x)w_x + f_2(x)g_2(x)w_y = h_1(x)h_2(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[x]*D[w[x, y], x] + f2[x]*g2[x]*D[w[x, y], y] == h1[x]*h2[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{\text{f2}(K[1])\text{g2}(K[1])}{\text{f1}(K[1])\text{g1}(K[1])}\,dK[1]\right)+\int_1^x \frac{\text{h1}(K[2])\text{h2}(K[2])}{\text{f1}(K[2])\text{g1}(K[2])}\,dK[2]\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(x)*diff(w(x,y),x) +f2(x)*g2(x)*diff(w(x,y),y) =  h1(x)*h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int \frac{h1(x)\,h2(x)}{f1(x)\,g1(x)}dx+\_F1\left(-\int \frac{f2(x)\,g2(x)}{f1(x)\,g1(x)}dx+y\right)$$

95.7 Problem 7

problem number 813

Added Feb. 11, 2019.

Problem Chapter 3.8.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h_1(x) + h_2(x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x] + h2[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h1(x)+h2(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \int^x \frac{h1(\_f) + h2(\_f)}{f1(\_f)} \left(g1 \left(\text{RootOf} \left(\int \frac{f2(\_f)}{f1(\_f)} \, d\_f - \int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a - \int \frac{f2(x)}{f1(x)} \, dx + \int \frac{g}{g2}\right.\right.\right.$$

96 HFOPDE, chapter 3.8.3

96.1 Problem 1

problem number 814

Added Feb. 11, 2019.

Problem Chapter 3.8.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(\alpha x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{f\left(\frac{\beta(bK[1]+ay-bx)}{a}+\alpha K[1]\right)}{a}\,dK[1]+c_1\left(\frac{ay-bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) =  f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{a}f\left(\frac{(ya-bx)\beta+\_a\,a\alpha+\_a\,b\beta}{a}\right)d\_a+\_F1\left(\frac{ya-bx}{a}\right)$$

96.2 Problem 2

problem number 815

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x})$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)+xf\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = xf\left(\frac{y}{x}\right) + \_F1\left(\frac{y}{x}\right)$$

96.3 Problem 3

problem number 816

Added Feb. 11, 2019.

Problem Chapter 3.8.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = f(x^2 + y^2)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{f\left(\frac{y^2K[1]^2}{x^2}+K[1]^2\right)}{K[1]}\,dK[1]+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  f(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{\_a} f\left(\frac{y^2\_a^2}{x^2}+\_a^2\right)d\_a+\_F1\left(\frac{y}{x}\right)$$

96.4 Problem 4

problem number 817

Added Feb. 11, 2019.

Problem Chapter 3.8.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x}) + g(x^2+y^2)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x] + g[x^2 + y^2];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{K[1]f\left(\frac{y}{x}\right)+g\left(\frac{y^2K[1]^2}{x^2}+K[1]^2\right)}{K[1]}\,dK[1]+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) =  x*f(y/x)+g(x^2+y^2);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{\_a}\left(\_a\, f\left(\frac{y}{x}\right)+g\left(\frac{y^2\_a^2}{x^2}+\_a^2\right)\right)d\_a+\_F1\left(\frac{y}{x}\right)$$

96.5 Problem 5

problem number 818

Added Feb. 11, 2019.

Problem Chapter 3.8.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax w_x + by w_y = x^k f(x^n y^m)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*x^m];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{K[1]^{k-1}f\left(K[1]^{m+n}\right)}{a}\,dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  x^k*f(x^n*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y)=\int^x \frac{\_a^{k-1}}{a} f\left(\_a^n\left(yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)^m\right)d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)$$

96.6 Problem 6

problem number 819

Added Feb. 11, 2019.

Problem Chapter 3.8.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = f(ax^n + by^m)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*x^m];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{f(aK[1]^n + bK[1]^m)}{mK[1]}\,dK[1] + c_1\left(yx^{-\frac{n}{m}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  m*x*diff(w(x,y),x) +n*y*diff(w(x,y),y) =  f(a*x^n+b*y^m);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{\_a\, m} f\left(\_a^n a + \left(yx^{-\frac{n}{m}}\_a^{\frac{n}{m}}\right)^m b\right) d\_a + \_F1\left(yx^{-\frac{n}{m}}\right)$$

96.7 Problem 7

problem number 820

Added Feb. 17, 2019.

Problem Chapter 3.8.3.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xyw_y = y^k f(\alpha x + \beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s, lambda, B, s, mu, d, g, B,
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x \frac{\left(\frac{yK[1]}{x}\right)^k f\left(\alpha K[1]+\frac{\beta yK[1]}{x}\right)}{K[1]^2}\,dK[1]+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x^2*diff(w(x,y),x) +x*y*diff(w(x,y),y) =  y^k*f(alpha*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{\_a^2} f\left(\_a\left(\frac{\beta y}{x}+\alpha\right)\right)\left(\frac{y\_a}{x}\right)^k d\_a+\_F1\left(\frac{y}{x}\right)$$

96.8 Problem 8

problem number 821

Added Feb. 17, 2019.

Problem Chapter 3.8.3.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)}w_x + \frac{g(y)}{g'(y)}w_y = h(f(x)+g(y))$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[y]*D[w[x, y], y])/Derivative[1][g][y] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\log\left(\frac{\text{InverseFunction}\left[g^{(-1)},1,1\right][y]}{f(x)}\right)\right)+\int_1^x \frac{f'(K[1])h\Big(g\Big(\text{InverseFunction}\left[\text{InverseF}\right.\right.}{}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := f(x)/diff(f(x),x)*diff(w(x,y),x) +g(y)/diff(g(y),y)*diff(w(x,y),y) =  h(f(x)+g(y));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^x \frac{\frac{d}{d\_a}f(\_a)}{f(\_a)}h\left(f(\_a)\left(\frac{g(y)}{f(x)}+1\right)\right)d\_a+\_F1\left(\ln\left(\frac{g(y)}{f(x)}\right)\right)$$

97 HFOPDE, chapter 3.8.4

97.1 Problem 1

problem number 822

Added Feb. 17, 2019.

Problem Chapter 3.8.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \int_1^x f(K[1], aK[1]-ax+y)\, dK[1] + c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x f(\_a, \_a\, a - ax + y)\, d\_a + \_F1(-ax + y)$$

97.2 Problem 2

problem number 823

Added Feb. 17, 2019.

Problem Chapter 3.8.4.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{f\left(K[1],yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)}{aK[1]}\,dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x*diff(w(x,y),x) +b*y*diff(w(x,y),y) =  f(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\int^x \frac{1}{\_a\,a}f\left(\_a,yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)$$

97.3 Problem 3

problem number 824

Added Feb. 17, 2019.

Problem Chapter 3.8.4.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\int_1^x \frac{h\left(K[2], y\exp\left(\text{Integrate}\left[\frac{g(K[1])}{f(K[1])}, \{K[1],1,K[2]\}, \text{Assumptions}\to\text{True}\right] - \text{Integrate}\left[\frac{g}{f}\right.\right.\right.}{f(K[2])}\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x) +g(x)*y*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{f(\_b)} h\left(\_b, y e^{-\int \frac{g(x)}{f(x)}\,dx + \int \frac{g(\_b)}{f(\_b)}\,d\_b}\right) d\_b + \_F1\left(y e^{-\int \frac{g(x)}{f(x)}\,dx}\right)$$

97.4 Problem 4

problem number 825

Added Feb. 17, 2019.

Problem Chapter 3.8.4.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-e^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]}\left(e^{\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]}\int_1^x \frac{\text{g0}(K[2])\exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\int^x \frac{1}{f(\_f)}h\left(\_f,\left(\int \frac{g0(\_f)}{f(\_f)}e^{-\int \frac{g1(\_f)}{f(\_f)}\,d\_f}\,d\_f-\int \frac{g0(x)}{f(x)}e^{-\int \frac{g1(x)}{f(x)}\,dx}\,dx+ye^{-\int \frac{g1(x)}{f(x)}\,dx}\right)e^{\int \frac{g1(\_f)}{f(\_f)}\,c\right.$$

97.5 Problem 5

problem number 826

Added Feb. 17, 2019.

Problem Chapter 3.8.4.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y) \to c_1\left((k-1)\int_1^x \frac{\text{g0}(K[2])\exp\left((k-1)\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{Assumptions}\to\text{T}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x) +(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
sol:=simplify(sol);
```

$$w(x,y) = \int^{x} \frac{1}{f\left(\_f\right)} h\Bigg(\_f, \Bigg(\left(1-k\right) \int \frac{g0\left(\_f\right)}{f\left(\_f\right)} e^{(k-1)\int \frac{g1(\_f)}{f(\_f)} d\_f} d\_f + (k-1) \int \frac{g0\left(x\right)}{f\left(x\right)} e^{(k-1)\int \frac{g1(x)}{f(x)} dx} dx +$$

97.6 Problem 6

problem number 827

Added Feb. 17, 2019.

Problem Chapter 3.8.4.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x,y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x] + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x) +(g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^x \frac{1}{f(\_f)} h\left(\_f, \frac{1}{\lambda}\left(\ln\left(-\frac{1}{\lambda}\left(-\frac{1}{\lambda}\left(\lambda \int \frac{g0(x)}{f(x)} \mathrm{e}^{\lambda \int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\,\mathrm{d}x + \mathrm{e}^{\lambda\left(\int \frac{g1(x)}{f(x)}\,\mathrm{d}x - y\right)}\right) + \int \frac{g0(\_f)}{f(\_f)} \mathrm{e}^{\lambda}\right.\right.\right.\right.\right.$$

97.7 Problem 7

problem number 828

Added Feb. 17, 2019.

Problem Chapter 3.8.4.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x,y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(y)*diff(w(x,y),x) +f2(x)*g2(y)*diff(w(x,y),y) =  h(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \int^{x} \frac{1}{f1\left(\_f\right)} h\left(\_f, \mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1\left(\_f\right)} d\_f - \int^{\_Z} \frac{g1(\_a)}{g2\left(\_a\right)} d\_a - \int \frac{f2(x)}{f1\left(x\right)} dx + \int \frac{g1(y)}{g2\left(y\right)} dy\right)\right.$$

Contains RootOf

98 HFOPDE, chapter 4.1.1

98.1 Example 1

problem number 829

Added Feb. 17, 2019.

Chapter 4.1.1 example 1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ayw_y = by^2w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*y*D[w[x, y], y] == b*y^2*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{by^2}{2a}}c_1\left(ye^{-ax}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) =  b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\mathrm{e}^{-ax}\right)\mathrm{e}^{1/2\,\frac{by^2}{a}}$$

98.2 Example 2

problem number 830

Added Feb. 17, 2019.

Chapter 4.1.1 example 2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + ayw_y = be^{\lambda x}yw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*y*D[w[x, y], y] == b*Exp[lambda*x]*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-ax}\right)e^{\frac{bye^{x(a+\lambda)-ax}}{a+\lambda}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x) +a*y*diff(w(x,y),y) = b*exp(lambda*x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\mathrm{e}^{-ax}\right)\mathrm{e}^{\frac{bye^{x(a+\lambda)-ax}}{a+\lambda}}$$

98.3 Example 3

problem number 831

Added Feb. 17, 2019.

Chapter 4.1.1 example 3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = b w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == b*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{bx}c_1(y-ax)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x) +a*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \_F1(-ax+y)\,e^{bx}$$

99 HFOPDE, chapter 4.2.1

99.1 Problem 1

problem number 832

Added Feb. 17, 2019.

Problem Chapter 4.2.1.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(\frac{ay-bx}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{\frac{cx}{a}}$$

99.2 Problem 2

problem number 833

Added Feb. 17, 2019.

Problem Chapter 4.2.1.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + yw_y = bw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{\frac{bx}{a}} c_1\left(ye^{-\frac{x}{a}}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x) +y*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(y\mathrm{e}^{-\frac{x}{a}}\right)\mathrm{e}^{\frac{bx}{a}}$$

99.3 Problem 3

problem number 834

Added Feb. 17, 2019.

Problem Chapter 4.2.1.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = aw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to x^a c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) x^a$$

99.4 Problem 4

problem number 835

Added Feb. 17, 2019.

Problem Chapter 4.2.1.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(aw_x - bw_y) = cyw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*(D[w[x, y], x] - b*D[w[x, y], y]) == c*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(bx+y)e^{c(\log(x)(bx+y)-bx)}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*(diff(w(x,y),x) -b*diff(w(x,y),y)) = c*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\,(bx+y)\,x^{(bx+y)c}\mathrm{e}^{-bcx}$$

99.5 Problem 5

problem number 836

Added Feb. 17, 2019.

Problem Chapter 4.2.1.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to e^{ax} c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x) +y*diff(w(x,y),y) = a*x*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \mathrm{e}^{ax}\_F1\left(\frac{y}{x}\right)$$

99.6 Problem 6

problem number 837

Added Feb. 17, 2019.

Problem Chapter 4.2.1.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)w_x + (y-b)w_y = w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -(a-x)c_1\left(\frac{b-y}{a-x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  (x-a)*diff(w(x,y),x) +(y-b)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = -\_F1\left(\frac{-b+y}{a-x}\right)x + a\_F1\left(\frac{-b+y}{a-x}\right)$$

99.7 Problem 7

problem number 838

Added Feb. 17, 2019.

Problem Chapter 4.2.1.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y + ax)w_x + (y - ax)w_y = bw$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (y + a*x)*D[w[x, y], x] + (y - a*x)*D[w[x, y], y] == b*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  (y+a*x)*diff(w(x,y),x) +(y-a*x)*diff(w(x,y),y) = b*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

sol=()

100 HFOPDE, chapter 4.2.2

100.1 Problem 1

problem number 839

Added Feb. 17, 2019.

Problem Chapter 4.2.2.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (x^2 - y^2)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (x^2 - y^2)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(-\frac{b^2x^3}{3a^3}-\frac{bx^2(ay-bx)}{a^3}-\frac{x(ay-bx)^2}{a^3}+\frac{x^3}{3a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x) +b*diff(w(x,y),y) = (x^2-y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{-\frac{(ya-bx)^2x}{a^3}-\frac{(ya-bx)bx^2}{a^3}+1/3\,\frac{x^3}{a}-1/3\,\frac{x^3b^2}{a^3}}$$

100.2 Problem 2

problem number 840

Added Feb. 17, 2019.

Problem Chapter 4.2.2.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axyw_y = by^2 w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{by^2}{(2a-1)x}}c_1\left(yx^{-a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x^2*diff(w(x,y),x) +a*x*y*diff(w(x,y),y) = b*y^2*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(yx^{-a}\right)\mathrm{e}^{\frac{by^2}{x(2a-1)}}$$

100.3 Problem 3

problem number 841

Added Feb. 17, 2019.

Problem Chapter 4.2.2.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x + by^2w_y = (x + cy)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^2*D[w[x, y], x] + b*y^2*D[w[x, y], y] == (x + c*y)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^{\frac{1}{a}+\frac{c}{b}}\left(b-\frac{by-ax}{y}\right)^{-\frac{c}{b}}c_1\left(\frac{by-ax}{axy}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*x^2*diff(w(x,y),x) +b*y^2*diff(w(x,y),y) = (x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\left(-\frac{-ax+by}{y}+b\right)^{-\frac{c}{b}}x^{\frac{c}{b}+a^{-1}}\_F1\left(-\frac{-ax+by}{yax}\right)$$

100.4 Problem 4

problem number 842

Added Feb. 17, 2019.

Problem Chapter 4.2.2.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + a y^2 w_y = (b x^2 + c x y + d y^2) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x^2 + c*x*y + d*y^2)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to\left(a-\frac{ay-x}{y}\right)^{-\frac{cxy}{ay-x}}c_1\left(\frac{ay-x}{xy}\right)\exp\left(b\left(x-\frac{axy}{ay-x}\right)+\frac{dxy}{(ay-x)\left(a-\frac{ay-x}{y}\right)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x^2*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) = (b*x^2+c*x*y+d*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{ya-x}{yx}\right)\left(-\frac{ya-x}{y}+a\right)^{-\frac{cxy}{ya-x}}\mathrm{e}^{-\frac{yx}{ya-x}\left(\frac{(ya-x)^2 b}{y^2}-\frac{(ya-x)ab}{y}-d\right)\left(-\frac{ya-x}{y}+a\right)^{-1}}$$

100.5 Problem 5

problem number 843

Added Feb. 17, 2019.

Problem Chapter 4.2.2.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$y^2 w_x + a x^2 w_y = (b x^2 + c y^2) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = y^2*D[w[x, y], x] + a*x^2*D[w[x, y], y] == (b*x^2 + c*y^2)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{1}{3}\left(y^3-ax^3\right)\right)e^{\frac{b\sqrt[3]{y^3}}{a}+cx}\right\},\left\{w(x,y)\to c_1\left(\frac{1}{3}\left(y^3-ax^3\right)\right)e^{cx-\frac{\sqrt[3]{-1}b\sqrt[3]{y^3}}{a}}\right\},\left\{w(x,y)\to c_1\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=y^2*diff(w(x,y),x) +a*x^2*diff(w(x,y),y) =(b*x^2+c*y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(-ax^3+y^3\right)\mathrm{e}^{\frac{cax+by}{a}}$$

100.6 Problem 6

problem number 844

Added Feb. 17, 2019.

Problem Chapter 4.2.2.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xyw_x + ay^2 w_y = (bx + cy + d)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*y*D[w[x, y], x] + a*y^2*D[w[x, y], y] == (b*x + c*y + d)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-a}\right)\exp\left(\frac{x^a\left(x^{-a}\left(\frac{bx}{1-a}-\frac{d}{a}\right)+cyx^{-a}\log(x)\right)}{y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x*y*diff(w(x,y),x) +a*y^2*diff(w(x,y),y) =(b*x+c*y+d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='real
```

$$w(x,y) = \_F1\left(yx^{-a}\right)x^c \mathrm{e}^{-\frac{bx}{(a-1)y}-\frac{d}{(a-1)y}+\frac{d}{(a-1)ya}}$$

100.7 Problem 7

problem number 845

Added Feb. 17, 2019.

Problem Chapter 4.2.2.7 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ay+b)w_x + (ay^2 - bx)w_y = ayw$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*(a*y + b)*D[w[x, y], x] + (a*y^2 - b*x)*D[w[x, y], y] == a*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x*(a*y+b)*diff(w(x,y),x) +(a*y^2-b*x)*diff(w(x,y),y) =a*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
```

$$w(x,y) = e^{1/9 \int^x \frac{1}{\_a(\_a a - b)} \left(2 e^{\mathrm{RootOf}\left(-2 \ln\left(-9/2 \frac{ax-b}{ya+b}\right) e^{\_Z} ax - 2 \ln\left(-9/2 \frac{ax-b}{ya+b}\right) e^{\_Z} ay - 2 \ln\left(\frac{(\_a a - b)\left(2 e^{\_Z} - 9\right)}{\_a}\right) e^{\_Z} ax - 2 \ln\left(\frac{(\_a a - b)\left(2\right.}{\_a}\right.\right.}\right.}$$

100.8 Problem 8

problem number 846

Added Feb. 17, 2019.

Problem Chapter 4.2.2.8 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(ky-x+a)w_x - y(kx-y+a)w_y = b(y-x)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*(k*y - x + a)*D[w[x, y], x] - y*(k*x - y + a)*D[w[x, y], y] == b*(y - x)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x*(k*y-x+a)*diff(w(x,y),x)-y*(k*x-y+a)*diff(w(x,y),y) = b*(y-x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))) ),output='real
sol:=simplify(sol)
```

$$w(x,y) = \_F1\left(-1/3\,\frac{k^2+k+1}{(k+1)\,k}\left((k+1)\ln\left(-\frac{(k^2+k+1)\,(a-x-y)\,k}{(k-1)\,(ky+a-x)}\right)+k\ln(x-a)-k\ln\left(-\frac{(k\right.\right.\right.$$

101 HFOPDE, chapter 4.2.3

101.1 Problem 1

problem number 847

Added Feb. 17, 2019.

Problem Chapter 4.2.3.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^3 + dy^3)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^3 + d*y^3)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)e^{\frac{cx^4}{4a}+\frac{dy^4}{4b}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^3+d*y^3)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{\frac{(ya-bx)^3dx}{a^4}+3/2\,\frac{(ya-bx)^2bdx^2}{a^4}+\frac{(ya-bx)b^2dx^3}{a^4}+1/4\,\frac{x^4c}{a}+1/4\,\frac{b^3dx^4}{a^4}}$$

101.2 Problem 2

problem number 848

Added Feb. 17, 2019.

Problem Chapter 4.2.3.2 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2 + y^2]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{a\sqrt{x^2+y^2}}c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{a\sqrt{x^2+y^2}}$$

101.3 Problem 3

problem number 849

Added Feb. 17, 2019.

Problem Chapter 4.2.3.3 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xy w_y = y^2(ax+by)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x + b*y)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{1/2\,\frac{by^3}{x}+1/2\,y^2a}$$

101.4 Problem 4

problem number 850

Added Feb. 17, 2019.

Problem Chapter 4.2.3.4 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2yw_x + axy^2w_y = (bxy + cx + dy + k)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*y*D[w[x, y], x] + a*x*y^2*D[w[x, y], y] == (b*x*y + c*x + d*y + k)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-a}\right)\exp\left(\frac{x^a\left(byx^{-a}\log(x)+x^{-a}\left(-\frac{c}{a}-\frac{k}{(a+1)x}\right)-dyx^{-a-1}\right)}{y}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x^2*y*diff(w(x,y),x)+a*x*y^2*diff(w(x,y),y) =(b*x*y +c*x+ d*y + k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y) = \_F1\left(yx^{-a}\right)x^b\mathrm{e}^{-\frac{ad}{x(a+1)}-\frac{d}{x(a+1)}-\frac{c}{y(a+1)}-\frac{k}{y(a+1)x}-\frac{c}{(a+1)ay}}$$

101.5 Problem 5

problem number 851

Added Feb. 17, 2019.

Problem Chapter 4.2.3.5 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axy^2w_x + bx^2yw_y = (any^2 + bmx^2)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*y^2*D[w[x, y], x] + b*x^2*y*D[w[x, y], y] == (a*n*y^2 + b*m*x^2)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to x^n\left(ay^2\right)^{m/2}c_1\left(\frac{ay^2-bx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x*y^2*diff(w(x,y),x)+b*x^2*y*diff(w(x,y),y) = (a*n*y^2+ b*m*x^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(\frac{y^2a-bx^2}{a}\right)\left(y^2a\right)^{m/2}x^n$$

101.6 Problem 6

problem number 852

Added Feb. 17, 2019.

Problem Chapter 4.2.3.6 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^3 w_x + a y^3 w_y = x^2(bx+cy)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^3*D[w[x, y], x] + a*y^3*D[w[x, y], y] == x^2*(b*x + c*y)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay^2-x^2}{2x^2y^2}\right)\exp\left(bx-\frac{c\tan^{-1}\left(\frac{x\sqrt{\frac{ay^2-x^2}{x^2y^2}}}{\sqrt{a-\frac{ay^2-x^2}{y^2}}}\right)}{\sqrt{\frac{ay^2-x^2}{x^2y^2}}}\right)\right\},\left\{w(x,y)\to c_1\left(\frac{ay^2-x^2}{2x^2y^2}\right)\exp\left(\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x^3*diff(w(x,y),x)+a*y^3*diff(w(x,y),y) =  x^2*(b*x+c*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

$$w(x,y)=\_F1\left(-\frac{y^2a-x^2}{x^2y^2}\right)e^{bx}\left(\sqrt{-\frac{y^2a-x^2}{x^2y^2}}\,x+\sqrt{-\frac{y^2a-x^2}{y^2}+a}\right)^{c\frac{1}{\sqrt{-\frac{y^2a-x^2}{x^2y^2}}}}$$

102 HFOPDE, chapter 4.2.4

102.1 Problem 1

problem number 853

Added Feb. 17, 2019.

Problem Chapter 4.2.4.1 from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + dy^m)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + d*y^m)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{d\left(\frac{ay-bx}{b(m+1)}+\frac{bx}{bm+b}\right)\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)^m}{a}+\frac{cx^{n+1}}{a(n+1)}\right)\right\}\right\}$$

Maple ✓

```

w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n + d*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) ),output='real
```

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102.2 Problem 2 case n not -1 and n not 2

problem number 854

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $neq-1, neq-2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Assuming[{n != -1, n != -2}, Simplify[sol[[2]]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{cx^{n+1}(a(n+2)y-bx)}{a^2(n+1)(n+2)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol)  assuming n<>-1, n<>-2
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{\left(ay(n+2)x^{n+1}-x^{n+2}b\right)c}{(n+2)(n+1)a^2}}$$

102.3 Problem 2 case $n = -1$

problem number 855

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n = -1$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -1],
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)e^{\frac{c(\log(x)(ay-bx)+bx)}{a^2}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-1)
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)x^{\frac{(ya-bx)c}{a^2}}\mathrm{e}^{\frac{bcx}{a^2}}$$

102.4 Problem 2 case $n = -2$

problem number 856

Added Feb. 17, 2019.

Problem Chapter 4.2.4.2 case $n = -2$, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cx^n yw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*x^n*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}, Assumptions -> n == -2],
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)e^{\frac{c\left(b\log(x)-\frac{ay-bx}{x}\right)}{a^2}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*x^n*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)) assuming n=-2)
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{-\frac{(ya-bx)c}{a^2x}}x^{\frac{bc}{a^2}}$$

102.5 Problem 3

problem number 857

Added Feb. 17, 2019.

Problem Chapter 4.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a(x^2+y^2)^k w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*(x^2 + y^2)^k*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{a\left(x^2\left(\frac{y^2}{x^2}+1\right)\right)^k}{2k}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*(x^2+y^2)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{1/2\,\frac{a}{k}\left(x^2\left(\frac{y^2}{x^2}+1\right)\right)^k}$$

102.6 Problem 4

problem number 858

Added Feb. 17, 2019.

Problem Chapter 4.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cy^m x^n}{a\left(\frac{bm}{a}+n\right)}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =   c*x^n*y^m*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cx^n y^m}{an+bm}}$$

102.7 Problem 5

problem number 859

Added Feb. 17, 2019.

Problem Chapter 4.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = (cx^n + ky^m)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == (c*x^n + k*y^m)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)e^{\frac{cx^n}{an}+\frac{ky^m}{bm}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) =  (c*x^n + k*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(yx^{-\frac{b}{a}}\right)e^{\frac{ky^m an+x^n cbm}{abmn}}$$

102.8 Problem 6

problem number 860

Added Feb. 17, 2019.

Problem Chapter 4.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = (ax^n + by^m)^k w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == (a*x^n + b*y^m)^k*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{n}{m}}\right)e^{\frac{(ax^n+by^m)^k}{kmn}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) =  (a*x^n + b*y^m)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{n}{m}}\right)\mathrm{e}^{\frac{(x^na+y^mb)^k}{knm}}$$

102.9 Problem 7

problem number 861

Added Feb. 17, 2019.

Problem Chapter 4.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^m w_y = (cx^k + dy^s)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^n*D[w[x, y], x] + b*y^m*D[w[x, y], y] == (c*x^k + d*y^s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-m}}{m-1}\right)\exp\left(\frac{cx^{k-n+1}}{ak-an+a}-\frac{dy^{1-m}\left(\left(y^{m-1}\right)^{\frac{1}{m-1}}\right)^s}{b(m-s-1)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x^n*diff(w(x,y),x)+b*y^m*diff(w(x,y),y) =  (c*x^k + d*y^s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{-x^{-n+1}b(m-1)+a(n-1)\,y^{1-m}}{a\,(n-1)}\right)\mathrm{e}^{\frac{1}{b(m-s-1)a(k-n+1)}\left(-y^{1-m}(n-1)^{\frac{s}{m-1}}a^{\frac{m+s-1}{m-1}}\left(a(n-1)y^{1-m}\right)^{-\frac{s}{m-1}}\right.}$$

102.10 Problem 8

problem number 862

Added Feb. 17, 2019.

Problem Chapter 4.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x^n w_x + b x^m y w_y = (c x^k y^s + d) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == (c*x^k*y^s + d)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\frac{bx^{m-n+1}}{am-an+a}}\right)\exp\left(\frac{x^{1-n}\left(\frac{d}{1-n}-\frac{cx^ky^se^{-\frac{bsx^{m-n+1}}{am-an+a}}\left(-\frac{bsx^{m-n+1}}{am-an+a}\right)^{\frac{-k+n-1}{m-n+1}}\text{Gamma}\left(\frac{k-n+1}{m-n+1},-\frac{bsx^{m-n+1}}{am-an+a}\right.}{m-n+1}\right.}{a}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y*diff(w(x,y),y) =  (c*x^k*y^s + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(y e^{-\frac{x^{-n+m+1} b}{a(-n+m+1)}}\right) e^{\int^x \frac{1}{a}\left(\_a^{k-n} c \left(y e^{-\frac{b\left(x^{-n+m+1} - \_a^{-n+m+1}\right)}{a(-n+m+1)}}\right)^s + \_a^{-n} d\right) d\_a}$$

102.11 Problem 9

problem number 863

Added Feb. 17, 2019.

Problem Chapter 4.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x^n w_x + (b x^m y + c x^k) w_y = (s x^p y^q + d) w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^n*D[w[x, y], x] + (b*x^m*y + c*x^k)*D[w[x, y], y] == (s*x^p*y^q + d)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x^n*diff(w(x,y),x)+(b*x^m*y+c*x^k)*diff(w(x,y),y) =   (s*x^p*y^q + d)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{1}{ab\,(k+2\,m-3\,n+3)\,(k+m-2\,n+2)\,(k-n+1)}\left(-a\mathrm{e}^{-1/2\,\frac{x^{-n+m+1}b}{a(-n+m+1)}}\left(\frac{x^{-n+m+1}b}{a\,(-n+m+1}\right.\right.\right.$$

102.12 Problem 10

problem number 864

Added Feb. 17, 2019.

Problem Chapter 4.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y^k w_y = (cx^p y^q + s)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == (c*x^p*y^q + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*x^n*diff(w(x,y),x)+b*x^m*y^k*diff(w(x,y),y) =    (c*x^p*y^q + s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{b(k-1)\,x^{-n+m+1}+y^{1-k}a(-n+m+1)}{a\,(-n+m+1)}\right)\mathrm{e}^{\int^x \frac{1}{a}\left(\_a^{-n+p}c\left(\left(\frac{-b(k-1)\_a^{-n+m+1}+b(k-1)x^{-n+m+1}+y^{1-}}{a(-n+m+1)}\right.\right.\right.}$$

102.13 Problem 11

problem number 865

Added Feb. 17, 2019.

Problem Chapter 4.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = (cx^m + s)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == (c*x^m + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol[[2]] = Simplify[sol[[2]]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y^{k+1}}{k+1}-\frac{bx^{n+1}}{an+a}\right)\exp\left(\frac{x\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\left(cx^m\text{Hypergeon}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=a*y^k*diff(w(x,y),x)+b*x^n*diff(w(x,y),y) =   (c*x^m+ s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{-x^{n+1}b(k+1)+y^{k+1}a(n+1)}{a\,(n+1)}\right)e^{\int^x\frac{c\_a^m+s}{a}\left(\left(\frac{b(k+1)\_a^{n+1}-x^{n+1}b(k+1)+y^{k+1}a(n+1)}{a(n+1)}\right)^{(k+1)^{-1}}\right)^{-k}d\_a}$$

102.14 Problem 12

problem number 866

Added Feb. 17, 2019.

Problem Chapter 4.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(x^n + (an-1)y^n)w_x + y(y^n + (an-1)x^n)w_y = kn(x^n + y^n)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*(x^n + (a*n - 1)*y^n)*D[w[x, y], x] + y*(y^n + (a*n - 1)*x^n)*D[w[x, y], y] == k*n*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';k:='k';alpha:='alpha';beta:='beta';g
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';v:='v';q:='q';p:='p';l:='l';
g1:='g1';g2:='g2';g0:='g0';h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*(x^n + (a*n -1)*y^n)*diff(w(x,y),x)+y*(y^n + (a*n -1)*x^n)*diff(w(x,y),y) =   k*n*(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\left(y^n x^{-a^{-1}} - x^{\frac{an-1}{a}}\right)(y^n)^{-\frac{1}{an}}\right) e^{\int^x \frac{kn}{\_a}\left(\_a^n + \left(\mathrm{RootOf}\left(-y^n x^{-a^{-1}} (y^n)^{-\frac{1}{an}} \sqrt[a]{\_a}(\_Z^n)^{\frac{1}{an}} + (y^n)^{-\frac{1}{an}} \sqrt[a]{\_a}\right.\right.\right.}$$

102.15 Problem 13

problem number 867

Added Feb. 17, 2019.

Problem Chapter 4.2.4.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x((n-2)y^n - 2x^n)w_x + y(2y^n - (n-2)x^n)w_y = ((a(n-2) + 2b)y^n - (2a + b(n-2))x^n)\, w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*((n - 2)*y^n - 2*x^n)*D[w[x, y], x] + y*(2*y^n - (n - 2)*x^n)*D[w[x, y], y] == ((a*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*((n-2)*y^n -2*x^n )*diff(w(x,y),x)+y*(2*y^n - (n-2)*x^n)*diff(w(x,y),y) = ((a*(n-2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

103 HFOPDE, chapter 4.3.1

103.1 Problem 1

problem number 868

Added Feb. 23, 2019.

Problem Chapter 4.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ce^{\alpha x+\beta y}w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{ce^{\frac{x(a\alpha+b\beta)}{a}+\frac{\beta(ay-bx)}{a}}}{a\alpha+b\beta}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*exp(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{c}{a\alpha+b\beta}e^{\frac{(ya-bx)\beta}{a}+\alpha x+\frac{bx\beta}{a}}}$$

103.2 Problem 2

problem number 869

Added Feb. 23, 2019.

Problem Chapter 4.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (ce^{\lambda x} + ke^{\mu y})w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x] + k*Exp[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{ke^{\frac{\mu(ay-bx)}{a}+\frac{b\mu x}{a}}}{b\mu}+\frac{ce^{\lambda x}}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*exp(lambda*x)+k*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{\frac{1}{a\lambda\, b\mu}\left(\mathrm{e}^{\lambda\, x}cb\mu+ak\lambda\,\mathrm{e}^{\frac{(ya-bx)\mu}{a}+\frac{b\mu\, x}{a}}\right)}$$

103.3 Problem 3

problem number 870

Added Feb. 23, 2019.

Problem Chapter 4.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = cw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == c*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ce^{-\lambda x}}{a\lambda}}c_1\left(\frac{e^{-\beta y-\lambda x}\left(b\beta e^{\beta y}-a\lambda e^{\lambda x}\right)}{a\beta\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(a\lambda\,\mathrm{e}^{\lambda\,x}-\mathrm{e}^{\beta\,y}b\beta\right)\mathrm{e}^{-\beta\,y-\lambda\,x}}{b\beta\,\lambda}\right)\mathrm{e}^{-\frac{c\mathrm{e}^{-\lambda\,x}}{a\lambda}}$$

103.4 Problem 4

problem number 871

Added Feb. 23, 2019.

Problem Chapter 4.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = cw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{a\beta\lambda}\right)\exp\left(\beta c\left(\frac{x}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}-\frac{\log\left(\frac{a\beta e^{\lambda y}-b\lambda e^{\beta x}}{\lambda}+be^{\beta x}\right)}{\beta\left(a\beta e^{\lambda y}-b\lambda e^{\beta x}\right)}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{e^{y\lambda}a\beta-e^{\beta x}b\lambda}{b\beta\lambda}\right)\left(\frac{e^{y\lambda}a\beta-e^{\beta x}b\lambda}{\lambda b}+e^{\beta x}\right)^{-\frac{c}{e^{y\lambda}a\beta-e^{\beta x}b\lambda}}\left(e^{\beta x}\right)^{\frac{c}{e^{y\lambda}a\beta-e^{\beta x}b\lambda}}$$

103.5 Problem 5

problem number 872

Added Feb. 23, 2019.

Problem Chapter 4.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\lambda x} w_x + b e^{\beta x} w_y = c e^{\gamma y} w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{e^{-\lambda x}\left(-a\beta y e^{\lambda x}+a\lambda y e^{\lambda x}+b e^{\beta x}\right)}{a(\beta-\lambda)}\right)\exp\left(\int_1^x \frac{c\exp\left(-\frac{\gamma e^{-\lambda K[1]}\left(-\frac{\lambda e^{\lambda K[1]-\lambda x}\left(-a\beta y e^{\lambda x}+a\lambda y e\right.}{\beta-\lambda}\right.}{}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  c*exp(gamma*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya\beta - y\lambda a - b\mathrm{e}^{x(\beta-\lambda)}}{(\beta-\lambda)a}\right)\mathrm{e}^{\int^x \frac{c}{a}\mathrm{e}^{\frac{\left(ya\beta-y\lambda a-b\mathrm{e}^{x(\beta-\lambda)}\right)\gamma\beta}{(\beta-\lambda)^2a}-\frac{\left(ya\beta-y\lambda a-b\mathrm{e}^{x(\beta-\lambda)}\right)\gamma\lambda}{(\beta-\lambda)^2a}-\frac{\lambda\_a\beta}{\beta-\lambda}+\frac{\lambda^2\_a}{\beta-\lambda}+\frac{\mathrm{e}^{\_a(\beta-\lambda)}\gamma b}{(\beta-\lambda)a}}}$$

103.6 Problem 6

problem number 873

Added Feb. 23, 2019.

Problem Chapter 4.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta y}w_y = (ce^{\gamma y} + se^{\delta y})w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*y]*D[w[x, y], y] == (c*Exp[gamma*y] + s*Ex
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{-\beta y-\lambda x}\left(b\beta e^{\beta y}-a\lambda e^{\lambda x}\right)}{a\beta\lambda}\right)\exp\left(-\frac{c\gamma e^{-\lambda x}\left(1-\frac{e^{-\beta y}\left(b\beta e^{\beta y}-a\lambda e^{\lambda x}\right)}{b\beta}\right)^{\frac{\gamma}{\beta}}\left(\frac{b\beta e^{-\lambda x}}{a\lambda}-\frac{e^{-\beta y-\lambda x}(b\beta}{a}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*y)*diff(w(x,y),y) =  (c*exp(gamma*y)+s*exp(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{\left(a\lambda\,\mathrm{e}^{\lambda\,x}-\mathrm{e}^{\beta\,y}b\beta\right)\mathrm{e}^{-\beta\,y-\lambda\,x}}{b\beta\,\lambda}\right)\mathrm{e}^{\frac{\beta}{a\lambda\,(\gamma-\beta)(\beta-\delta)}\left(\frac{\left(a\lambda\,\mathrm{e}^{\lambda\,x}-\mathrm{e}^{\beta\,y}b\beta\right)\mathrm{e}^{-\beta\,y-\lambda\,x}}{b\beta}+\mathrm{e}^{-\lambda\,x}\right)\left(\left(\frac{a\lambda}{b\beta}\left(\frac{\left(a\lambda\,\mathrm{e}^{\lambda\,x}-\mathrm{e}^{\beta\,y}b\beta\right)\mathrm{e}}{b\beta}\right.\right.\right.}$$

103.7 Problem 7

problem number 874

Added Feb. 23, 2019.

Problem Chapter 4.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\beta x} w_x + (b e^{\gamma x} + c e^{\lambda y}) w_y = (s e^{\mu x} + k e^{\delta y} + p) w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \mathrm{e}^{\int^{x} \frac{1}{a}\left(s\mathrm{e}^{-\_b(\beta-\mu)}+\mathrm{e}^{-\beta\_b}p+\left(-\frac{\lambda}{a}\left(c\int \mathrm{e}^{\frac{-a\beta\gamma\_b+a\beta^2\_b+\lambda b\mathrm{e}^{\_b(\gamma-\beta)}}{(\gamma-\beta)a}}\mathrm{d}\_b-\frac{a}{\lambda}\left(\lambda\int \frac{c}{a}\mathrm{e}^{\frac{-a\beta\gamma x+a\beta^2 x+\lambda b\mathrm{e}^{x(\gamma-\beta)}}{(\gamma-\beta)a}}\mathrm{d}x+\mathrm{e}^{\frac{\lambda\left(-\gamma ya-\right.}{}}\right.\right.\right.\right.}$$

103.8 Problem 8

problem number 875

Added Feb. 23, 2019.

Problem Chapter 4.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = (se^{\mu x+\delta y} + k)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x] + c*Exp[lambda*y])*D[w[x, y], y] == (s*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) =  (s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{1}{\lambda}\left(\lambda\int\frac{c}{a}\mathrm{e}^{\frac{-a\beta\gamma x+a\beta^2x+\lambda b\mathrm{e}^{x(\gamma-\beta)}}{(\gamma-\beta)a}}\,\mathrm{d}x+\mathrm{e}^{\frac{\lambda\left(-\gamma ya+ya\beta+b\mathrm{e}^{x(\gamma-\beta)}\right)}{(\gamma-\beta)a}}\right)\right)\mathrm{e}^{\int^x\frac{s}{a}\left(-\frac{\lambda}{a}\left(c\int\mathrm{e}^{\frac{-a\beta\gamma\_b+a\beta^2\_b+\lambda b}{(\gamma-\beta)a}}\right.\right.}$$

103.9 Problem 9

problem number 876

Added Feb. 23, 2019.

Problem Chapter 4.3.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + be^{\gamma x+\lambda y}w_y = (ce^{\mu x+\delta y} + k)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x + lambda*y])*D[w[x, y], y] == (c*Exp[mu*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-\frac{e^{-\beta x-\lambda y}\left(-a\gamma e^{\beta x} + a\beta e^{\beta x} - b\lambda e^{\gamma x+\lambda y}\right)}{a\lambda(\beta-\gamma)}\right)\exp\left(-\frac{ce^{x(\mu-\beta)}\left(-\frac{a(\beta-\gamma)e^{\beta x}}{-e^{-\lambda y}\left(-a\gamma e^{\beta x}+a\beta e^{\beta x}-b\lambda e^{\gamma x+}\right.}\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(beta*x)*diff(w(x,y),x)+(b*exp(gamma*x+lambda*y))*diff(w(x,y),y) =  (c*exp(mu*x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{\left(\lambda\,b\mathrm{e}^{-\beta\,x+\gamma\,x+y\lambda} + \gamma\,a - a\beta\right)\mathrm{e}^{-y\lambda}}{b\lambda\,(\gamma-\beta)}\right)\mathrm{e}^{\int^x \frac{\mathrm{e}^{-\beta\_a}}{a}\left(c\left(-\frac{(\gamma-\beta)a}{\lambda b}\left(\frac{\left(\lambda\,b\mathrm{e}^{-\beta\,x+\gamma\,x+y\lambda}+\gamma\,a-a\beta\right)\mathrm{e}^{-y\lambda}\beta}{b\lambda\,(\gamma-\beta)}-\frac{\left(\lambda\,b\mathrm{e}^{-\beta}\right.}{}\right.\right.\right.}$$

103.10 Problem 10

problem number 877

Added Feb. 23, 2019.

Problem Chapter 4.3.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = (ce^{\mu x} + k)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == (c*Exp[mu*x] + k)*w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{e^{-\lambda x}\left(-a\beta ye^{\lambda x}+a\lambda ye^{\lambda x}+be^{\beta x}\right)}{a(\beta-\lambda)}\right)\exp\left(\frac{ce^{\mu x-\lambda x}}{a(\mu-\lambda)}-\frac{ke^{-\lambda x}}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*exp(beta*x)*diff(w(x,y),y) =  (c*exp(mu*x) + k)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya\beta - y\lambda\, a - be^{x(\beta-\lambda)}}{(\beta-\lambda)\, a}\right)\mathrm{e}^{-\frac{ce^{-x(\lambda-\mu)}\lambda+ke^{-\lambda\, x}\lambda-ke^{-\lambda\, x}\mu}{a\lambda\,(\lambda-\mu)}}$$

104 HFOPDE, chapter 4.3.2

104.1 Problem 1

problem number 878

Added Feb. 23, 2019.

Problem Chapter 4.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cye^{\lambda x} + kxe^{\mu y})w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y*Exp[lambda*x] + k*x*Exp[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{ke^{\frac{b\mu x}{a}}\left(\frac{axe^{\frac{\mu(ay-bx)}{a}}}{b\mu}-\frac{a^2e^{\frac{\mu(ay-bx)}{a}}}{b^2\mu^2}\right)}{a}+\frac{bce^{\lambda x}\left(\frac{x}{\lambda}-\frac{1}{\lambda^2}\right)}{a^2}+\frac{ce^{\lambda x}(ay-bx)}{a^2\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*y*exp(lambda*x) + k*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\frac{1}{a^2\lambda^2\mu^2b^2}\left((ya-bx)ce^{\lambda x}\lambda\mu^2b^2+e^{\lambda x}b^3c\lambda\mu^2x+kxe^{\frac{(ya-bx)\mu}{a}+\frac{b\mu x}{a}}b\mu a^2\lambda^2-e^{\lambda x}cb^3\mu^2-ka^3e^{\frac{(ya-bx)\mu}{a}+\frac{b\mu}{a}}\right.}$$

104.2 Problem 2

problem number 879

Added Feb. 23, 2019.

Problem Chapter 4.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ae^{x\left(\lambda+\frac{\mu y}{x}\right)}}{\lambda+\frac{\mu y}{x}}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*exp(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{a e^{\lambda x+\mu y} \left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

104.3 Problem 3

problem number 880

Added Feb. 23, 2019.

Problem Chapter 4.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = (aye^{\lambda x} + bxe^{\mu y})w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x] + b*x*Exp[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right) e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =    (a*y*exp(lambda*x)+ b*x*exp(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) e^{\frac{x}{\lambda\mu y}\left(\frac{a e^{\lambda x} y^2 \mu}{x^2} + e^{\mu y} b\lambda\right)}$$

104.4 Problem 4

problem number 881

Added Feb. 23, 2019.

Problem Chapter 4.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x^k w_x + b e^{\lambda y} w_y = (c x^n + s) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^k*D[w[x, y], x] + b*Exp[lambda*y]*D[w[x, y], y] == (c*x^n + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \rightarrow e^{\frac{x^{1-k}\left(\frac{c x^n}{-k+n+1} + \frac{s}{1-k}\right)}{a}} c_1\left(\frac{x^{-k} e^{-\lambda y}\left(a x^k - a k x^k + b\lambda x e^{\lambda y}\right)}{a(k-1)\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*x^k*diff(w(x,y),x)+b*exp(lambda*y)*diff(w(x,y),y) =    (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{x^{1-k}\lambda b - ak\mathrm{e}^{-y\lambda} + a\mathrm{e}^{-y\lambda}}{\lambda b (k-1)}\right) \mathrm{e}^{-\frac{x^{1-k}(x^n ck - x^n c + ks - sn - s)}{a(k-1)(-n-1+k)}}$$

104.5 Problem 5

problem number 882

Added Feb. 23, 2019.

Problem Chapter 4.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + be^{\lambda x} w_y = (ce^{\mu x} + s)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{a\lambda y^{k+1}-bke^{\lambda x}-be^{\lambda x}}{a(k+1)\lambda}\right)\exp\left(\frac{ce^{\mu x}\left(\frac{b(k+1)e^{\lambda x}}{a\lambda y^{k+1}-bke^{\lambda x}-be^{\lambda x}}+1\right)^{\frac{k}{k+1}}\left(\left(\frac{(k+1)\left(\frac{a\lambda y^{k+1}-bke^{\lambda x}-be^{\lambda}}{k+1}\right.}{a\lambda}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*y^k*diff(w(x,y),x)+b*exp(lambda*x)*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{e^{\lambda x}bk-y^k ya\lambda+e^{\lambda x}b}{a\lambda}\right)e^{\int^x \frac{ce^{\mu\_a}+s}{a}\left(\left(\frac{e^{\lambda\_a}bk-e^{\lambda x}bk+y^k ya\lambda-e^{\lambda x}b+e^{\lambda\_a}b}{a\lambda}\right)^{(k+1)^{-1}}\right)^{-k}d\_a}$$

104.6 Problem 6

problem number 883

Added Feb. 23, 2019.

Problem Chapter 4.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + by^k w_y = (cx^n + s)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*y^k*D[w[x, y], y] == (c*x^n + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-\frac{y^{-k}e^{-\lambda x}\left(a\lambda ye^{\lambda x}+by^k-bky^k\right)}{a(k-1)\lambda}\right)\exp\left(-\frac{cx^n(\lambda x)^{-n}\text{Gamma}(n+1,\lambda x)}{a\lambda}-\frac{se^{-\lambda x}}{a\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*x)*diff(w(x,y),x)+b*y^k*diff(w(x,y),y) =   (c*x^n+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y^{1-k}a\lambda - e^{-\lambda x}bk + be^{-\lambda x}}{a\lambda}\right)e^{\frac{cx^n(\lambda x)^{-n/2}e^{-1/2\lambda x}\,\mathrm{WhittakerM}(n/2,n/2+1/2,\lambda x)-e^{-\lambda x}ns-e^{-\lambda x}s+sn+s}{(n+1)a\lambda}}$$

104.7 Problem 7

problem number 884

Added Feb. 23, 2019.

Problem Chapter 4.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + bx^k w_y = (ce^{\mu x} + s)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == (c*Exp[mu*x] + s)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*exp(lambda*y)*diff(w(x,y),x)+b*x^k*diff(w(x,y),y) =   (c*exp(mu*x)+s)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{x^{k+1}\lambda b - e^{y\lambda}ak - ae^{y\lambda}}{(k+1)\lambda b}\right) e^{\int^x \frac{(ce^{\mu}\_a + s)(k+1)}{\lambda b}\left(-\frac{\left(x^{k+1}\lambda b - e^{y\lambda}ak - ae^{y\lambda}\right)k}{(k+1)\lambda b} + \_a^{k+1} - \frac{x^{k+1}\lambda b - e^{y\lambda}ak - ae^{y\lambda}}{(k+1)\lambda b}\right)^{-}}$$

105 HFOPDE, chapter 4.4.1

105.1 Problem 1

problem number 885

Added Feb. 23, 2019.

Problem Chapter 4.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\sinh(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Sinh[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{k\sinh\left(\frac{b\mu x}{a}\right)\sinh\left(\frac{\mu(ay-bx)}{a}\right)}{b\mu}+\frac{k\cosh\left(\frac{b\mu x}{a}\right)\cosh\left(\frac{\mu(ay-bx)}{a}\right)}{b\mu}+\frac{c\cosh(\lambda}{a\lambda}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*sinh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{\cosh(\lambda x)cb\mu+ka\cosh(\mu y)\lambda}{a\lambda b\mu}}$$

105.2 Problem 2

problem number 886

Added Feb. 23, 2019.

Problem Chapter 4.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{c\cosh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{\frac{c}{a\lambda+b\mu}\cosh\left(\frac{(ya-bx)\mu+ax\lambda+b\mu x}{a}\right)}$$

105.3 Problem 3

problem number 887

Added Feb. 23, 2019.

Problem Chapter 4.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sinh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sinh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{a\cosh\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)}{\lambda+\frac{\mu y}{x}}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*sinh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{a\cosh(\lambda x+\mu y)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

105.4 Problem 4

problem number 888

Added Feb. 23, 2019.

Problem Chapter 4.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda x)w_y = (c\sinh^m(\mu x) + s\sinh^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Sinh[lambda*x]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*sinh(lambda*x)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\sinh(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\sinh(\_b\,\mu))^m + s\left(\sinh\left(\beta \int \frac{b(\sinh(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\sinh(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\beta\right.\right.\right.}$$

105.5 Problem 5

problem number 889

Added Feb. 23, 2019.

Problem Chapter 4.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh^n(\lambda y)w_y = (c\sinh^m(\mu x) + s\sinh^k(\beta y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Sinh[lambda*y]^n*D[w[x, y], y] == (c*Sinh[mu*x]^m + s*Sinh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1()\exp\left(\frac{sx\sinh^k(\beta y)}{a}-\frac{c\cosh(\mu x)\sinh^{m+1}(\mu x)\left(-\sinh^2(\mu x)\right)^{-\frac{m}{2}-\frac{1}{2}}\text{Hypergeometric2F1}}{a\mu}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*sinh(lambda*y)^n*diff(w(x,y),y) =  (c*sinh(mu*x)^m+s*sinh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{bx-a\int(\sinh(y\lambda))^{-n}\,\mathrm{d}y}{b}\right)\mathrm{e}^{\int^y\frac{(\sinh(\_b\lambda))^{-n}}{b}\left(c\left(-\sinh\left(-\mu\int\frac{(\sinh(\_b\lambda))^{-n}a}{b}\,\mathrm{d}\_b-\frac{\mu\left(bx-a\int(\sinh(y\lambda))^{-n}\,\mathrm{d}y\right.}{b}\right.\right.\right.}$$

106 HFOPDE, chapter 4.4.2

106.1 Problem 1

problem number 890

Added Feb. 23, 2019.

Problem Chapter 4.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cosh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cosh[lambda*x] + k*Cosh[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sinh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*cosh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{c\sinh(\lambda x)b\mu+k\sinh(\mu y)a\lambda}{a\lambda b\mu}}$$

106.2 Problem 2

problem number 891

Added Feb. 23, 2019.

Problem Chapter 4.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{c\sinh\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{c}{a\lambda+b\mu}\sinh\left(\frac{(ya-bx)\mu+ax\lambda+b\mu x}{a}\right)}$$

106.3 Problem 3

problem number 892

Added Feb. 23, 2019.

Problem Chapter 4.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cosh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{a\sinh\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)}{\lambda+\frac{\mu y}{x}}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*cosh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{a\sinh(\lambda x+\mu y)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

106.4 Problem 4

problem number 893

Added Feb. 23, 2019.

Problem Chapter 4.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda x)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cosh[lambda*x]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*cosh(lambda*x)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\cosh(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\cosh(\_b\,\mu))^m + s\left(\cosh\left(\beta \int \frac{b(\cosh(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\cosh(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\right.\right.\right.}$$

106.5 Problem 5

problem number 894

Added Feb. 23, 2019.

Problem Chapter 4.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cosh^n(\lambda y)w_y = (c\cosh^m(\mu x) + s\cosh^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cosh[lambda*y]^n*D[w[x, y], y] == (c*Cosh[mu*x]^m + s*Cosh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*cosh(lambda*y)^n*diff(w(x,y),y) =  (c*cosh(mu*x)^m+s*cosh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{bx - a\int(\cosh(y\lambda))^{-n}\,\mathrm{d}y}{b}\right)\mathrm{e}^{\int^y \frac{(\cosh(\_b\,\lambda))^{-n}}{b}\left(c\left(\cosh\left(-\mu\int\frac{(\cosh(\_b\,\lambda))^{-n}a}{b}\,\mathrm{d}\_b - \frac{\mu\left(bx - a\int(\cosh(y\lambda))^{-n}\,\mathrm{d}y\right)}{b}\right.\right.\right.}$$

107 HFOPDE, chapter 4.4.3

107.1 Problem 1

problem number 895

Added Feb. 23, 2019.

Problem Chapter 4.4.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\tanh(\lambda x) + k\tanh(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tanh[lambda*x] + k*Tanh[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cosh^{\frac{c}{a\lambda}}(\lambda x)\cosh^{\frac{k}{b\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*tanh(lambda*x) + k*tanh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)(\tanh(\lambda x)-1)^{-1/2\,\frac{c}{a\lambda}}(\tanh(\lambda x)+1)^{-1/2\,\frac{c}{a\lambda}}(\tanh(\mu y)-1)^{-1/2\,\frac{k}{b\mu}}(\tanh(\mu y$$

107.2 Problem 2

problem number 896

Added Feb. 23, 2019.

Problem Chapter 4.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cosh^{\frac{c}{a\lambda+b\mu}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)(\tanh(\lambda x+\mu y)-1)^{-\frac{c}{2a\lambda+2b\mu}}(\tanh(\lambda x+\mu y)+1)^{-\frac{c}{2a\lambda+2b\mu}}$$

107.3 Problem 3

problem number 897

Added Feb. 23, 2019.

Problem Chapter 4.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tanh(\lambda x+\mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tanh[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cosh^{\frac{a}{\lambda+\frac{\mu y}{x}}}\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*tanh(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right) (\tanh(\lambda x + \mu y) - 1)^{-1/2\, a\left(\frac{\mu y}{x}+\lambda\right)^{-1}} (\tanh(\lambda x + \mu y) + 1)^{-1/2\, a\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

107.4 Problem 4

problem number 898

Added Feb. 23, 2019.

Problem Chapter 4.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda x)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Tanh[lambda*x]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

\$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*tanh(lambda*x)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\tanh(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\tanh(\_b\,\mu))^m + s\left(\tanh\left(\beta \int \frac{b(\tanh(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\tanh(\lambda x))^n}{a}\,\mathrm{d}x + y\right.\right.\right.\right.}}$$

107.5 Problem 5

problem number 899

Added Feb. 23, 2019.

Problem Chapter 4.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tanh^n(\lambda y)w_y = (c\tanh^m(\mu x) + s\tanh^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Tanh[lambda*y]^n*D[w[x, y], y] == (c*Tanh[mu*x]^m + s*Tanh[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*tanh(lambda*y)^n*diff(w(x,y),y) =  (c*tanh(mu*x)^m+s*tanh(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\tanh(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)\mathrm{e}^{\int^{y}\frac{(\tanh(\_b\,\lambda))^{-n}}{b}\left(c\left(-\tanh\left(-\mu\int\frac{(\tanh(\_b\,\lambda))^{-n}a}{b}\,\mathrm{d}\_b-\mu\left(-\frac{a\int(\tanh(y\lambda))^{-n}\,\mathrm{d}y}{b}\right.\right.\right.\right.}$$

108 HFOPDE, chapter 4.4.4

108.1 Problem 1

problem number 900

Added Feb. 23, 2019.

Problem Chapter 4.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\coth(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Coth[lambda*x] + k*Coth[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\sinh^{\frac{c}{a\lambda}}(\lambda x)\sinh^{\frac{k}{b\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*coth(lambda*x) + k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)(\coth(\lambda x)-1)^{-1/2\,\frac{c}{a\lambda}}(\coth(\lambda x)+1)^{-1/2\,\frac{c}{a\lambda}}(\coth(\mu y)-1)^{-1/2\,\frac{k}{b\mu}}(\coth(\mu y)+$$

108.2 Problem 2

problem number 901

Added Feb. 23, 2019.

Problem Chapter 4.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right)\sinh^{\frac{c}{a\lambda+b\mu}}(\lambda x + \mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   c*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya - bx}{a}\right)(\coth(\lambda x + \mu y) - 1)^{-\frac{c}{2a\lambda + 2b\mu}}(\coth(\lambda x + \mu y) + 1)^{-\frac{c}{2a\lambda + 2b\mu}}$$

108.3 Problem 3

problem number 902

Added Feb. 23, 2019.

Problem Chapter 4.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\coth(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Coth[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{y}{x}\right)\sinh^{\frac{a}{\lambda + \frac{\mu y}{x}}}\left(x\left(\lambda + \frac{\mu y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := x*diff(w(x,y),x)+y*diff(w(x,y),y) =   a*x*coth(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)(\coth(\lambda x + \mu y) - 1)^{-1/2\,a\left(\frac{\mu y}{x}+\lambda\right)^{-1}}(\coth(\lambda x + \mu y) + 1)^{-1/2\,a\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

108.4 Problem 4

problem number 903

Added Feb. 23, 2019.

Problem Chapter 4.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda x)w_y = (c\coth^m(\mu x) + s\coth^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Coth[lambda*x]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*coth(lambda*x)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\coth(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\coth(\_b\,\mu))^m + s\left(\coth\left(\beta \int \frac{b(\coth(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\coth(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\right.\right.\right.}$$

108.5 Problem 5

problem number 904

Added Feb. 23, 2019.

Problem Chapter 4.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\coth^n(\lambda y)w_y = (c\coth^m(\mu x) + s\coth^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Coth[lambda*y]^n*D[w[x, y], y] == (c*Coth[mu*x]^m + s*Coth[beta*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*coth(lambda*y)^n*diff(w(x,y),y) =  (c*coth(mu*x)^m+s*coth(beta*y)^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\coth(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)\mathrm{e}^{\int^{y}\frac{(\coth(\_b\,\lambda))^{-n}}{b}\left(c\left(-\coth\left(-\mu\int\frac{(\coth(\_b\,\lambda))^{-n}a}{b}\,\mathrm{d}\_b-\mu\left(-\frac{a\int(\coth(y\lambda))^{-n}\,\mathrm{d}y}{b}\right.\right.\right.\right.}}$$

109 HFOPDE, chapter 4.4.5

109.1 Problem 1

problem number 905

Added Feb. 23, 2019.

Problem Chapter 4.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\sinh(\lambda x) + k\cosh(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sinh[lambda*x] + k*Cosh[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\cosh(\lambda x)}{a\lambda}+\frac{k\sinh(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (c*sinh(lambda*x) + k*cosh(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{\cosh(\lambda x)cb\mu+k\sinh(\mu y)a\lambda}{a\lambda b\mu}}$$

109.2 Problem 2

problem number 906

Added Feb. 23, 2019.

Problem Chapter 4.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (\tanh(\lambda x) + k\coth(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (Tanh[lambda*x] + k*Coth[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\cosh(\lambda x)}c_1\left(y-\frac{bx}{a}\right)\sinh^{\frac{k}{b\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =   (tanh(lambda*x)+k*coth(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\left(\mathrm{e}^{2\lambda x}+1\right)^{\frac{1}{a\lambda}}\_F1\left(\frac{ya-bx}{a}\right)\left(\mathrm{e}^{2\mu y}-1\right)^{\frac{k}{b\mu}}\mathrm{e}^{\frac{x(k-1)b-2aky}{ab}}$$

109.3 Problem 3

problem number 907

Added Feb. 23, 2019.

Problem Chapter 4.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = b\cosh(\lambda x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Cosh[lambda*x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sinh(\lambda x)}{\lambda}}c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)-a\mu x}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*cosh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(-\frac{x\mu\,a+2\,\mathrm{arctanh}\left(\mathrm{e}^{\mu\,y}\right)}{\mu\,a}\right)\mathrm{e}^{\frac{\sinh(\lambda\,x)b}{\lambda}}$$

109.4 Problem 4

problem number 908

Added Feb. 23, 2019.

Problem Chapter 4.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sinh(\mu y)w_y = b\tanh(\lambda x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*Sinh[mu*y]*D[w[x, y], y] == b*Tanh[lambda*x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cosh^{\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tanh\left(\frac{\mu y}{2}\right)\right)-a\mu x}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := diff(w(x,y),x)+a*sinh(mu*y)*diff(w(x,y),y) =   b*tanh(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{x\mu\,a + 2\,\mathrm{arctanh}\left(\mathrm{e}^{\mu\,y}\right)}{\mu\,a}\right)\left(\tanh\left(\lambda\,x\right)-1\right)^{-1/2\,\frac{b}{\lambda}}\left(\tanh\left(\lambda\,x\right)+1\right)^{-1/2\,\frac{b}{\lambda}}$$

109.5 Problem 5

problem number 909

Added Feb. 23, 2019.

Problem Chapter 4.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh(\lambda x)w_x + b\cosh(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Sinh[lambda*x]*D[w[x, y], x] + b*Cosh[mu*y]*D[w[x, y], y] == w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\sinh\left(\frac{\lambda x}{2}\right)}\cosh^{-\frac{1}{a\lambda}}\left(\frac{\lambda x}{2}\right)c_1\left(\frac{2a\lambda\tan^{-1}\left(\tanh\left(\frac{\mu y}{2}\right)\right)-b\mu\log\left(\sinh\left(\frac{\lambda x}{2}\right)\right)+b\mu\log\left(\cos\right.\right.}{a\lambda\mu}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*sinh(lambda*x)*diff(w(x,y),x)+b*cosh(mu*y)^n*diff(w(x,y),y) =  w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\lambda}\left(\int\frac{(\cosh(\mu y))^{-n}a}{b}\,\mathrm{d}y\lambda + 2\,\mathrm{arctanh}\left(\mathrm{e}^{\lambda x}\right)\right)\right)\mathrm{e}^{-2\,\frac{\mathrm{arctanh}\left(\mathrm{e}^{\lambda x}\right)}{a\lambda}}$$

109.6 Problem 6

problem number 910

Added Feb. 23, 2019.

Problem Chapter 4.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh(\lambda x)w_x + b\coth(\mu y)w_y = w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Tanh[lambda*x]*D[w[x, y], x] + b*Coth[mu*y]*D[w[x, y], y] == w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \sqrt[a\lambda]{\sinh(\lambda x)}c_1\left(-\frac{2a\cosh(\mu y)\sinh^{-\frac{b\mu}{a\lambda}}(\lambda x)}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*tanh(lambda*x)*diff(w(x,y),x)+b*coth(mu*y)*diff(w(x,y),y) = w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y),'build')),outpu
sol:=simplify(sol);
```

$$w(x,y) = \_C1\left(\frac{-\cosh(\lambda x)+\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-1/2\,\frac{\_c_1}{\lambda}}\left(\frac{\sinh(\lambda x)+\cosh(\lambda x)}{\cosh(\lambda x)}\right)^{-1/2\,\frac{\_c_1}{\lambda}}\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{\frac{\_c_1}{\lambda}}\Bigg($$

110 HFOPDE, chapter 4.5.1

110.1 Problem 1

problem number 911

Added Feb. 25, 2019.

Problem Chapter 4.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\ln(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\frac{(a\beta y-b\beta x)\log(a(\beta y+\lambda x))}{a\lambda+b\beta}+x\log(\beta y+\lambda x)-x\right)}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  c*ln(lambda*x + beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = (\beta\,y + \lambda\,x)^{\frac{c(\beta\,y+\lambda\,x)}{a\lambda+b\beta}}\,\_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{-\frac{c(\beta\,y+\lambda\,x)}{a\lambda+b\beta}}$$

110.2 Problem 2

problem number 912

Added Feb. 25, 2019.

Problem Chapter 4.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\ln(\lambda x) + k\ln(\beta y))\,w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Log[lambda*x] + k*Log[beta*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{x(c+k)}{a}}(\lambda x)^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)(ay)^{\frac{ky}{b}-\frac{kx}{a}}(\beta y)^{\frac{kx}{a}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) =  (c*ln(lambda*x)+k*ln(beta*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = (\beta\, y)^{\frac{ky}{b}}\, \_F1\left(\frac{ya-bx}{a}\right) (\lambda\, x)^{\frac{cx}{a}}\, \mathrm{e}^{\frac{-aky-bcx}{ab}}$$

110.3 Problem 3

problem number 913

Added Feb. 25, 2019.

Problem Chapter 4.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda x)w_y = \left(c\ln^m(\mu x) + s\ln^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*ln(lambda*x)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\ln(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\ln(\_b\,\lambda))^m + s\left(\ln\left(\beta\left(\int \frac{b(\ln(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b - \int \frac{b(\ln(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\right)\right)^k\right)d\_b}$$

110.4 Problem 4

problem number 914

Added Feb. 25, 2019.

Problem Chapter 4.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\ln^n(\lambda y)w_y = \left(c\ln^m(\mu x) + s\ln^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Log[lambda*y]^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*ln(lambda*y)^n*diff(w(x,y),y) =  (c*ln(lambda*x)^m+s*ln(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = _F1\left(-\frac{a\int(\ln(y\lambda))^{-n}\,\mathrm{d}y}{b} + x\right) \mathrm{e}^{\int^{y} \frac{(\ln(_b\,\lambda))^{-n}}{b} \left(c\left(\ln\left(\lambda\left(\int \frac{(\ln(_b\,\lambda))^{-n} a}{b} \mathrm{d}_b - \frac{a\int(\ln(y\lambda))^{-n}\,\mathrm{d}y}{b} + x\right)\right)\right)^{m} + s(\ln(\beta_

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := ln(beta*y)*diff(w(x,y),x)+a*ln(lambda*x)*diff(w(x,y),y) =  b*w(x,y)*ln(beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{-x\ln(\lambda x)\,a + \ln(\beta y)\,y + ax - y}{a}\right)\mathrm{e}^{bx}$$

110.6 Problem 6

problem number 916

Added Feb. 25, 2019.

Problem Chapter 4.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln(\lambda x)^n w_x + b\ln(\beta y)^k w_y = c\ln(\gamma x)^m w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[beta*y]^k*D[w[x, y], y] == c*Log[gamma*x]^m*w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*ln(lambda*x)^n*diff(w(x,y),x)+b*ln(beta*y)^k*diff(w(x,y),y) = c*log(gamma*x)^m*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int (\ln(\lambda x))^{-n}\, \mathrm{d}x + \int \frac{(\ln(\beta y))^{-k}\, a}{b}\, \mathrm{d}y\right) \mathrm{e}^{\int \frac{c(\ln(\gamma)+\ln(x))^m (\ln(\lambda x))^{-n}}{a} \mathrm{d}x}$$

111 HFOPDE, chapter 4.5.2

111.1 Problem 1

problem number 917

Added Feb. 25, 2019.

Problem Chapter 4.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cx^n + s\ln^k(\lambda y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*x^n + s*Log[gamma*y]^k)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{s\log^k\left(\gamma\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)\right)\left(-\log\left(\gamma\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)\right)\right)^{-k}\text{Gamma}\left(k+1,-\log\right.}{b\gamma}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde := a*diff(w(x,y),x)+b*diff(w(x,y),y) = (c*x^n+s*ln(gamma*y)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\int^x \frac{\_a^n c}{a}+\frac{s}{a}\left(\ln(\gamma)+\ln\left(\frac{b\_a+ya-bx}{a}\right)\right)^k d\_a}$$

111.2 Problem 2

problem number 918

Added Feb. 25, 2019.

Problem Chapter 4.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = (b y^2 + c x^n y + s \ln^k(\lambda x)) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == (b*y^2 + c*x^n*y + s*Log[lambda*x]^k)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)\exp\left(\frac{s\log^k(\lambda x)(-\log(\lambda x))^{-k}\text{Gamma}(k+1,-\log(\lambda x))}{\lambda} + \frac{1}{3}a^2bx^3 + abx^2(y-a\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = (b*y^2+c*x^n*y+ s*ln(lambda*x)^k)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x b\_a^2a^2 + ca\_a^{n+1} + 2(-ax+y)ab\_a + \_a^n(-ax+y)c + (-ax+y)^2b + s(\ln(\lambda\_a))^k d\_a}$$

111.3 Problem 3

problem number 919

Added March 9, 2019.

Problem Chapter 4.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = b \ln^k(\lambda x) \ln^n(\beta y) w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == b*Log[lambda*x]^k*Log[beta*y]^n*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = b*ln(lambda*x)^k*ln(beta*y)^n*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\, \mathrm{e}^{\int^x b(\ln(\lambda \_a))^k (\ln(\beta(\_a\, a - ax + y)))^n d\_a}$$

111.4 Problem 4

problem number 920

Added March 9, 2019.

Problem Chapter 4.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + bx^n)w_y = c\ln^k(\lambda x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + (a*y + b*x^n)*D[w[x, y], y] == c*Log[lambda*x]^k*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(a^{-n-1}e^{-ax}\left(be^{ax}\text{Gamma}(n+1,ax)+ya^{n+1}\right)\right)\exp\left(\frac{c(-\log(\lambda x))^{-k}\log^k(\lambda x)\text{Gamma}(k+\right.\right.\right.}{\lambda}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+(a*y+b*x^n)*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{e^{-ax}\left((ax)^{-n/2}\,\text{WhittakerM}\left(n/2,n/2+1/2,ax\right)x^n e^{1/2\,ax}b - any - ya\right)}{a\left(n+1\right)}\right)e^{\int c(\ln(\lambda x))^k\,dx}$$

111.5 Problem 5

problem number 921

Added March 9, 2019.

Problem Chapter 4.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k(n\ln x + m\ln y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, s, mu, d, g, B, v, f, h, q, p, delta];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*(n*Log[x] + m*Log[y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\frac{x^k(akm\log(y)+akn\log(x)-an-bm)}{a^2k^2}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = x^k*(n*ln(x)+m*ln(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)\left(x^{\frac{b}{a}}\right)^{\frac{mx^k}{ak}}\left(yx^{-\frac{b}{a}}\right)^{\frac{mx^k}{ak}}x^{\frac{x^kn}{ak}}\mathrm{e}^{-1/2\,\frac{x^k}{a^2k^2}\left(i\pi\,m(csgn(iy))^3ak-i\pi\,m(csgn(iy))^2csgn\left(iyx^{-\frac{b}{a}}\right)ak-i\pi\,mcsg\right.}$$

111.6 Problem 6

problem number 922

Added March 9, 2019.

Problem Chapter 4.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + by^n w_y = (c\ln^m(\lambda x) + s\ln^t(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x^k*D[w[x, y], x] + b*y^n*D[w[x, y], y] == (c*Log[lambda*x]^m + s*Log[beta*y]^t)*w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x^k*diff(w(x,y),x)+ b*y^n*diff(w(x,y),y) = (c*ln(lambda*x)^m+s*ln(beta*y)^t)*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{-x^{1-k}b(n-1) + y^{-n+1}a(k-1)}{a\,(k-1)}\right) e^{\int^x \frac{\_a^{-k}}{a}\left(c(\ln(\lambda\_a))^m + \left(\ln\left(\beta\left(\frac{b(n-1)\_a^{1-k} - x^{1-k}b(n-1) + y^{-n+1}a(k}{a(k-1)}\right.\right.\right.\right.}$$

112 HFOPDE, chapter 4.6.1

112.1 Problem 1

problem number 923

Added March 9, 2019.

Problem Chapter 4.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sin(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sin[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(-\frac{c\cos\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{-\frac{c}{a\lambda+b\mu}\cos\left(\frac{(ya-bx)\mu+ax\lambda+b\mu x}{a}\right)}$$

112.2 Problem 2

problem number 924

Added March 9, 2019.

Problem Chapter 4.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\sin(\lambda x) + k\sin(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Sin[lambda*x] + k*Sin[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{-\frac{c\cos(\lambda x)}{a\lambda}-\frac{k\cos(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*sin(lambda*x)+k*sin(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{-\frac{\cos(\lambda x)cb\mu+ka\cos(\mu y)\lambda}{a\lambda b\mu}}$$

112.3 Problem 3

problem number 925

Added March 9, 2019.

Problem Chapter 4.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{-\frac{ax\cos(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*sin(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{-a\cos(\lambda x+\mu y)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

112.4 Problem 4

problem number 926

Added March 9, 2019.

Problem Chapter 4.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda x)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Sin[lambda*x]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*x)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\sin(\lambda x))^n}{a}\,\mathrm{d}x + y\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\sin(\_b\,\mu))^m + s\left(\sin\left(\beta \int \frac{b(\sin(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\sin(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\beta\right)\right)^k\right) c$$

112.5 Problem 5

problem number 927

Added March 9, 2019.

Problem Chapter 4.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin^n(\lambda y)w_y = (c\sin^m(\mu x) + s\sin^k(\beta y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Sin[lambda*y]^n*D[w[x, y], y] == (c*Sin[mu*x]^m + s*Sin[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1()\exp\left(\frac{sx\sin^k(\beta y)}{a}-\frac{c\cos(\mu x)\sin^{m+1}(\mu x)\sin^2(\mu x)^{-\frac{m}{2}-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3}{2}\right.}{a\mu}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*sin(lambda*y)^n*diff(w(x,y),y) = (c*sin(mu*x)^m+s*sin(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{bx-a\int(\sin(y\lambda))^{-n}\,dy}{b}\right)e^{\int^y\frac{(\sin(\_b\lambda))^{-n}}{b}\left(c\left(-\sin\left(-\mu\int\frac{(\sin(\_b\lambda))^{-n}a}{b}d\_b-\frac{\mu\left(bx-a\int(\sin(y\lambda))^{-n}dy\right)}{b}\right)\right)^m\right.}$$

113 HFOPDE, chapter 4.6.2

113.1 Problem 1

problem number 928

Added March 9, 2019.

Problem Chapter 4.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cos(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cos[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{c\sin\left(\mu\left(\frac{ay-bx}{a}+\frac{bx}{a}\right)+\lambda x\right)}{a\lambda+b\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\mathrm{e}^{\frac{c}{a\lambda+b\mu}\sin\left(\frac{(ya-bx)\mu+ax\lambda+b\mu x}{a}\right)}$$

113.2 Problem 2

problem number 929

Added March 9, 2019.

Problem Chapter 4.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cos(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cos[lambda*x] + k*Cos[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)e^{\frac{c\sin(\lambda x)}{a\lambda}+\frac{k\sin(\mu y)}{b\mu}}\right\}\right\}$$

Maple ✓

```

w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cos(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\frac{c\sin(\lambda x)b\mu+k\sin(\mu y)a\lambda}{a\lambda b\mu}}$$

113.3 Problem 3

problem number 930

Added March 9, 2019.

Problem Chapter 4.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cos(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{a\sin(\lambda x+\mu y)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

113.4 Problem 4

problem number 931

Added March 9, 2019.

Problem Chapter 4.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda x)w_y = (c\cos^m(\mu x) + s\cos^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cos[lambda*x]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*x)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = _F1\left(-\int \frac{b(\cos(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\mathrm{e}^{\int^x \frac{1}{a}\left(c(\cos(_b\,\mu))^m + s\left(\cos\left(\beta\int\frac{b(\cos(_b\,\lambda))^n}{a}\,\mathrm{d}_b + \left(-\int\frac{b(\cos(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\beta\right)\right)^k\right)

113.5 Problem 5

problem number 932

Added March 9, 2019.

Problem Chapter 4.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cos^n(\lambda y)w_y = (c\cos^m(\mu x) + s\cos^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cos[lambda*y]^n*D[w[x, y], y] == (c*Cos[mu*x]^m + s*Cos[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$\text{Aborted}$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*cos(lambda*y)^n*diff(w(x,y),y) = (c*cos(mu*x)^m+s*cos(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{bx - a\int (\cos(y\lambda))^{-n}\, \mathrm{d}y}{b}\right) \mathrm{e}^{\int^y \frac{(\cos(\_b\,\lambda))^{-n}}{b}\left(c\left(\cos\left(-\mu\int \frac{(\cos(\_b\,\lambda))^{-n}a}{b}\mathrm{d}\_b - \frac{\mu\left(bx - a\int(\cos(y\lambda))^{-n}\,\mathrm{d}y\right)}{b}\right)\right)^m -$$

114 HFOPDE, chapter 4.6.3

114.1 Problem 1

problem number 933

Added March 9, 2019.

Problem Chapter 4.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tan[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\cos^{-\frac{c}{a\lambda+b\mu}}\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left(1+\left(\tan\left(\frac{(ya-bx)\,\mu+ax\lambda+b\mu\,x}{a}\right)\right)^2\right)^{1/2\,\frac{c}{a\lambda+b\mu}}$$

114.2 Problem 2

problem number 934

Added March 9, 2019.

Problem Chapter 4.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\tan(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Tan[lambda*x] + k*Tan[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\cos^{-\frac{c}{a\lambda}}(\lambda x)\cos^{-\frac{k}{b\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*tan(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)\left(1+(\tan(\lambda x))^2\right)^{1/2\frac{c}{a\lambda}}\left(1+(\tan(\mu y))^2\right)^{1/2\frac{k}{b\mu}}$$

114.3 Problem 3

problem number 935

Added March 9, 2019.

Problem Chapter 4.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\tan(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Tan[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\cos^{-\frac{ax}{\lambda x+\mu y}}(\lambda x+\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*tan(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\left(1+(\tan(\lambda x+\mu y))^2\right)^{1/2\,a\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

114.4 Problem 4

problem number 936

Added March 9, 2019.

Problem Chapter 4.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan^n(\lambda x)w_y = (c\tan^m(\mu x) + s\tan^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Tan[lambda*x]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*x)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\tan(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\mathrm{e}^{\int^x \frac{1}{a}\left(c(\tan(\_b\,\mu))^m + s\left(\tan\left(\beta \int \frac{b(\tan(\_b\,\lambda))^n}{a}\,\mathrm{d}\_b + \left(-\int \frac{b(\tan(\lambda\,x))^n}{a}\,\mathrm{d}x + y\right)\beta\right)\right)^k}$$

114.5 Problem 5

problem number 937

Added March 9, 2019.

Problem Chapter 4.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\tan^n(\lambda y) w_y = (c\tan^m(\mu x) + s\tan^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Tan[lambda*y]^n*D[w[x, y], y] == (c*Tan[mu*x]^m + s*Tan[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*tan(lambda*y)^n*diff(w(x,y),y) = (c*tan(mu*x)^m+s*tan(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\tan(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)\mathrm{e}^{\int^{y}\frac{(\tan(\_b\,\lambda))^{-n}}{b}\left(c\left(-\tan\left(-\mu\int\frac{(\tan(\_b\,\lambda))^{-n}a}{b}\,\mathrm{d}\_b-\mu\left(-\frac{a\int(\tan(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)\right.\right.\right.}$$

115 HFOPDE, chapter 4.6.4

115.1 Problem 1

problem number 938

Added March 9, 2019.

Problem Chapter 4.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cot(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cot[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\sin^{\frac{c}{a\lambda+b\mu}}\left(\frac{x(a\lambda+b\mu)}{a}+\frac{\mu(ay-bx)}{a}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)\left(\left(\cot\left(\frac{(ya-bx)\,\mu+ax\lambda+b\mu\,x}{a}\right)\right)^2+1\right)^{-1/2\,\frac{c}{a\lambda+b\mu}}$$

115.2 Problem 2

problem number 939

Added March 9, 2019.

Problem Chapter 4.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c\cot(\lambda x) + k\cot(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Cot[lambda*x] + k*Cot[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\sin^{\frac{c}{a\lambda}}(\lambda x)\sin^{\frac{k}{b\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*cot(lambda*x)+k*cot(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left((\cot(\lambda x))^2+1\right)^{-1/2\,\frac{c}{a\lambda}}\left((\cot(\mu y))^2+1\right)^{-1/2\,\frac{k}{b\mu}}$$

115.3 Problem 3

problem number 940

Added March 9, 2019.

Problem Chapter 4.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cot(\lambda x + \mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cot[lambda*x + mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\sin^{\frac{a}{\lambda+\frac{\mu y}{x}}}\left(x\left(\lambda+\frac{\mu y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cot(lambda*x+mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\left((\cot(\lambda x+\mu y))^2+1\right)^{-1/2\,a\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

115.4 Problem 4

problem number 941

Added March 9, 2019.

Problem Chapter 4.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\cot^n(\lambda x) w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cot[lambda*x]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*x)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\cot(\lambda x))^n}{a}\,dx + y\right) e^{\int^x \frac{1}{a}\left(c(\cot(\_b\,\mu))^m + s\left(\cot\left(\beta \int \frac{b(\cot(\_b\,\lambda))^n}{a}\,d\_b + \left(-\int \frac{b(\cot(\lambda x))^n}{a}\,dx + y\right)\beta\right)\right)^k\right)}$$

115.5 Problem 5

problem number 942

Added March 9, 2019.

Problem Chapter 4.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot^n(\lambda y)w_y = (c\cot^m(\mu x) + s\cot^k(\beta y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*Cot[lambda*y]^n*D[w[x, y], y] == (c*Cot[mu*x]^m + s*Cot[beta*y]^k
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*cot(lambda*y)^n*diff(w(x,y),y) = (c*cot(mu*x)^m+s*cot(beta*y)^k)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\cot(y\lambda))^{-n}\,\mathrm{d}y}{b} + x\right)\mathrm{e}^{\int^y \frac{(\cot(\_b\lambda))^{-n}}{b}\left(c\left(-\cot\left(-\mu\int\frac{(\cot(\_b\lambda))^{-n}a}{b}\,\mathrm{d}\_b-\mu\left(-\frac{a\int(\cot(y\lambda))^{-n}\,\mathrm{d}y}{b}+x\right)\right.\right.\right.}$$

116 HFOPDE, chapter 4.6.5

116.1 Problem 1

problem number 943

Added March 9, 2019.

Problem Chapter 4.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = (b\sin(\lambda x) + k\cos(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Cos[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)e^{\frac{k\sin(\mu y)}{a\mu}-\frac{b\cos(\lambda x)}{\lambda}}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*cos(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1(-ax+y)\,e^{\frac{-b\cos(\lambda x)\mu a+k\sin(\mu y)\lambda}{\lambda\mu a}}$$

116.2 Problem 2

problem number 944

Added March 9, 2019.

Problem Chapter 4.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin(\mu y)w_y = (b\sin(\lambda x) + k\tan(\mu y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == (b*Sin[lambda*x] + k*Tan[mu*y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)e^{-\frac{b\cos(\lambda x)}{\lambda}}\cos^{-\frac{k}{a\mu}}(\mu y)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) = (b*sin(lambda*x)+k*tan(mu*y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1(-ax+y)\left(\left(\cos\left(1/2\,\mu\,y\right)\right)^{-2}\right)^{\frac{k}{\mu\,a}}\left(\frac{\sin\left(1/2\,\mu\,y\right)-\cos\left(1/2\,\mu\,y\right)}{\cos\left(1/2\,\mu\,y\right)}\right)^{-\frac{k}{\mu\,a}}\left(\frac{\sin\left(1/2\,\mu\,y\right)+\cos\left(1\right.}{\cos\left(1/2\,\mu\,y\right)}\right.$$

116.3 Problem 3

problem number 945

Added March 9, 2019.

Problem Chapter 4.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\sin(\mu y)w_y = b\tan(\lambda x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*Sin[mu*y]*D[w[x, y], y] == b*Tan[lambda*x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{b}{\lambda}}(\lambda x)c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)-a\mu x}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*sin(mu*y)*diff(w(x,y),y) = b*tan(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{1}{\mu a}\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(2\frac{\_Z\,\mathrm{e}^{x\mu a}}{\_Z^2\mathrm{e}^{2x\mu a}+1}, -\frac{\_Z^2\mathrm{e}^{2x\mu a}-1}{\_Z^2\mathrm{e}^{2x\mu a}+1}\right)\right)\right)\right)(1+(\tan(\lambda x)$$

116.4 Problem 4

problem number 946

Added March 9, 2019.

Problem Chapter 4.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a\tan(\mu y)w_y = b\sin(\lambda x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*Tan[mu*y]*D[w[x, y], y] == b*Sin[lambda*x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{b\cos(\lambda x)}{\lambda}}c_1\left(\frac{\log(\sin(\mu y))-a\mu x}{\mu}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*tan(mu*y)*diff(w(x,y),y) = b*sin(lambda*x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{\ln\left(\mathrm{e}^{-x\mu a}\mathit{csgn}\left((\cos(\mu y))^{-1}\right)\sin(\mu y)\right)}{\mu a}\right)\mathrm{e}^{-\frac{b\cos(\lambda x)}{\lambda}}$$

116.5 Problem 5

problem number 947

Added March 9, 2019.

Problem Chapter 4.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\sin(\lambda x)w_x + aw_y = b\cos(\mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = Sin[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Cos[mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{b\sin(\mu y)}{a\mu}}c_1\left(\frac{-a\log\left(\sin\left(\frac{\lambda x}{2}\right)\right)+a\log\left(\cos\left(\frac{\lambda x}{2}\right)\right)+\lambda y}{\lambda}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  sin(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*cos(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y\lambda - \ln\left(\csc\left(\lambda x\right)-\cot\left(\lambda x\right)\right)a}{\lambda}\right)\mathrm{e}^{\frac{b\sin(\mu y)}{\mu a}}$$

116.6 Problem 6

problem number 948

Added March 9, 2019.

Problem Chapter 4.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\cot(\lambda x)w_x + aw_y = b\tan(\mu y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = Cot[lambda*x]*D[w[x, y], x] + a*D[w[x, y], y] == b*Tan[mu*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{b}{a\mu}}(\mu y)c_1\left(\frac{a\log(\cos(\lambda x))}{\lambda}+y\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  cot(lambda*x)*diff(w(x,y),x)+ a*diff(w(x,y),y) = b*tan(mu*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-1/2\,\frac{\ln\left(\left(\cot\left(\lambda\,x\right)\right)^2+1\right)a-2\,y\lambda-2\,a\ln\left(\cot\left(\lambda\,x\right)\right)}{\lambda}\right)\left(\cos\left(\mu\,y\right)\right)^{-\frac{b}{\mu\,a}}$$

117 HFOPDE, chapter 4.7.1

117.1 Problem 1

problem number 949

Added March 9, 2019.

Problem Chapter 4.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = \left(c \arcsin(\frac{x}{\lambda} + k \arcsin(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcSin[x/lambda] + k*ArcSin[y/beta])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{\frac{k\left(a^2(\beta^2-y^2)+i\sqrt{a^2(\beta^2-y^2)}(ay-bx)\log\left(2\left(\sqrt{a^2(\beta^2-y^2)}-iay\right)\right)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}} + akx\sin^{-1}\left(\frac{y}{\beta}\right) + a}{a^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arcsin(x/lambda)+k*arcsin(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\frac{1}{ab}\left(\sqrt{\frac{\lambda^2-x^2}{\lambda^2}}bc\lambda+\sqrt{\frac{\beta^2-y^2}{\beta^2}}a\beta\, k+\arcsin\left(\frac{y}{\beta}\right)aky+\arcsin\left(\frac{x}{\lambda}\right)bcx\right)}$$

117.2 Problem 2

problem number 950

Added March 9, 2019.

Problem Chapter 4.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arcsin(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcSin[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}+(\beta y+\lambda x)\sin^{-1}(\beta y+\lambda x)\right)}{a\lambda+b\beta}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\frac{c\left(\arcsin(\beta y+\lambda x)\beta y+\arcsin(\beta y+\lambda x)\lambda x+\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}\right)}{a\lambda+b\beta}}$$

117.3 Problem 3

problem number 951

Added March 9, 2019.

Problem Chapter 4.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\arcsin(\lambda x+\beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcSin[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{a\left(\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}(-3a\beta y+a\lambda x+4b\beta x)+\sin^{-1}(\beta y+\lambda\right.}{4(a\lambda+b\beta)^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arcsin(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)e^{1/2\,\frac{a\left(((1/2\,\lambda\,x-3/2\,\beta\,y)a+2\,bx\beta)\sqrt{-\beta^2y^2-2\,\beta\,\lambda\,xy-\lambda^2x^2+1}+\left(\left(\lambda^2x^2-\beta^2y^2-1/2\right)a+2\,bx\beta\,(\beta\,y+\lambda\,x)\right)\arcsin(\beta\,y+\lambda\right.}{(a\lambda+b\beta)^2}}$$

117.4 Problem 4

problem number 952

Added March 9, 2019.

Problem Chapter 4.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \arcsin^n(\lambda x) w_y = \left(c \arcsin^m(\mu x) + s \arcsin^k(\beta y)\right) w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcSin[lambda*x]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*x)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{-b\left(-\arcsin(\lambda x)\,\mathrm{LommelS1}(n+3/2, 1/2, \arcsin(\lambda x)) + (\arcsin(\lambda x))^{n+3/2}\right)\sqrt{-\lambda^2 x^2}}{}\right.$$

117.5 Problem 5

problem number 953

Added March 9, 2019.

Problem Chapter 4.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^n(\lambda y)w_y = \left(c\arcsin^m(\mu x) + s\arcsin^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcSin[lambda*y]^n*D[w[x, y], y] == (c*ArcSin[mu*x]^m + s*ArcSin[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arcsin(lambda*y)^n*diff(w(x,y),y) =(c*arcsin(mu*x)^m+s*arcsin(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{a\left(-\arcsin\left(y\lambda\right)\mathrm{LommelS1}\left(-n+3/2,1/2,\arcsin\left(y\lambda\right)\right) + \left(\arcsin\left(y\lambda\right)\right)^{-n+3/2}\right)\sqrt{-y^2\lambda^2}}{}\right.$$

118 HFOPDE, chapter 4.7.2

118.1 Problem 1

problem number 954

Added March 9, 2019.

Problem Chapter 4.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arccos(\frac{x}{\lambda} + k\arccos(\frac{y}{\beta})\right)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCos[x/lambda] + k*ArcCos[y/beta])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{-\frac{k\left(a^2(\beta^2-y^2)+i\sqrt{a^2(\beta^2-y^2)}(ay-bx)\log\left(2\left(\sqrt{a^2(\beta^2-y^2)}-iay\right)\right)\right)}{b\beta\sqrt{1-\frac{y^2}{\beta^2}}}+akx\cos^{-1}\left(\frac{y}{\beta}\right)-}{a^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccos(x/lambda)+k*arccos(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\frac{1}{ab}\left(-\sqrt{\frac{\lambda^2-x^2}{\lambda^2}}bc\lambda-\sqrt{\frac{\beta^2-y^2}{\beta^2}}a\beta\, k+\arccos\left(\frac{y}{\beta}\right)aky+\arccos\left(\frac{x}{\lambda}\right)bcx\right)}$$

118.2 Problem 2

problem number 955

Added March 9, 2019.

Problem Chapter 4.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arccos(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCos[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c\left(\beta(bx-ay)\sin^{-1}(\beta y+\lambda x)+x(a\lambda+b\beta)\cos^{-1}(\beta y+\lambda x)+a\left(-\sqrt{-\beta^2\right.\right.}{a(a\lambda+b\beta)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\frac{\left(-\sqrt{-\beta^2y^2-2\,\beta\,\lambda\,xy-\lambda^2x^2+1}+\arccos(\beta\,y+\lambda\,x)(\beta\,y+\lambda\,x)\right)c}{a\lambda+b\beta}}$$

118.3 Problem 3

problem number 956

Added March 9, 2019.

Problem Chapter 4.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\arccos(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCos[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{(a^2+2\beta^2(bx-ay)^2)\sin^{-1}(\beta y+\lambda x)-a\sqrt{-\beta^2y^2-2\beta\lambda xy-\lambda^2x^2+1}(\cdots}{4(a\lambda+b\beta)^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccos(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)e^{1/4\,\frac{a\left(-2\arccos(\beta y+\lambda x)a\beta^2y^2+2\arccos(\beta y+\lambda x)a\lambda^2x^2+4\arccos(\beta y+\lambda x)b\beta^2xy+4\arccos(\beta y+\lambda x)b\beta\lambda x^2+3\sqrt{-\beta\cdots}\right.}{(a\lambda\cdots}}$$

118.4 Problem 4

problem number 957

Added March 9, 2019.

Problem Chapter 4.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b \arccos^n(\lambda x) w_y = \left(c \arccos^m(\mu x) + s \arccos^k(\beta y)\right) w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcCos[lambda*x]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*x)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{\left((n+2)\,\mathrm{LommelS1}\left(n+1/2, 1/2, \arccos(\lambda x)\right) - \arccos(\lambda x)\,\mathrm{LommelS1}\left(n+3/2, 3/2, \right.\right.}{}\right.$$

118.5 Problem 5

problem number 958

Added March 9, 2019.

Problem Chapter 4.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^n(\lambda y)w_y = \left(c\arccos^m(\mu x) + s\arccos^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcCos[lambda*y]^n*D[w[x, y], y] == (c*ArcCos[mu*x]^m + s*ArcCos[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arccos(lambda*y)^n*diff(w(x,y),y) =(c*arccos(mu*x)^m+s*arccos(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{2^{-n}a\sqrt{\pi}}{\lambda b}\left(-\frac{(\arccos(y\lambda))^{-n+1}2^n\sqrt{-y^2\lambda^2+1}}{\sqrt{\pi}(n-2)} + \frac{2^n\sqrt{\arccos(y\lambda)}\,\mathrm{LommelS1}(-n+3/2,3}{\sqrt{\pi}(n-2)}\right.\right.$$

119 HFOPDE, chapter 4.7.3

119.1 Problem 1

problem number 959

Added March 9, 2019.

Problem Chapter 4.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c\arctan(\frac{x}{\lambda} + k\arctan(\frac{y}{\beta})\right)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcTan[x/lambda] + k*ArcTan[y/beta])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \rightarrow \left(\lambda^2 + x^2\right)^{-\frac{c\lambda}{2a}} c_1\left(y - \frac{bx}{a}\right)\exp\left(\frac{k\left(2y\tan^{-1}\left(\frac{y}{\beta}\right) - \beta\log\left(a^2(\beta^2 + y^2)\right)\right)}{2b} + \frac{cx\tan^{-1}\left(\frac{x}{\lambda}\right)}{a}\right)\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arctan(x/lambda)+k*arctan(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left(\frac{\lambda^2+x^2}{\lambda^2}\right)^{-1/2\,\frac{\lambda\,c}{a}}\left(\frac{\beta^2+y^2}{\beta^2}\right)^{-1/2\,\frac{\beta\,k}{b}} e^{\frac{1}{ab}\left(\arctan\left(\frac{y}{\beta}\right)aky+cx\arctan\left(\frac{x}{\lambda}\right)b\right)}$$

119.2 Problem 2

problem number 960

Added March 9, 2019.

Problem Chapter 4.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\arctan(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcTan[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(2(\beta y+\lambda x)\tan^{-1}(\beta y+\lambda x)-\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right))}{2(a\lambda+b\beta)}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)^{-\frac{c}{2\,a\lambda+2\,b\beta}}\,\mathrm{e}^{\frac{\arctan(\beta\,y+\lambda\,x)c(\beta\,y+\lambda\,x)}{a\lambda+b\beta}}$$

119.3 Problem 3

problem number 961

Added March 9, 2019.

Problem Chapter 4.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = ax\arctan(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcTan[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{a\left(\beta(ay-bx)\log\left(a^2\left(\frac{\beta^2(ay-bx)^2}{a^2}+\frac{2\beta\lambda x(ay-bx)}{a}+\lambda^2x^2+1\right)+2ab\beta x\left(\frac{\beta}{}\right.\right.\right.}{2(a\lambda+b\beta)^2}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arctan(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)^{1/2\,\frac{(ya-bx)a\beta}{(a\lambda+b\beta)^2}}\,e^{1/2\,\frac{a\left(\left(\left(-\beta^2y^2+\lambda^2x^2+1\right)a+2\,bx\beta\,(\beta\,y+\lambda\,x)\right)\arctan(\beta\,y}{(a\lambda+b\beta)^2}}$$

119.4 Problem 4

problem number 962

Added March 9, 2019.

Problem Chapter 4.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda x)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcTan[lambda*x]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*x)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{-b\int(\arctan(\lambda x))^n \,\mathrm{d}x + ya}{a}\right) \mathrm{e}^{\int^x \frac{1}{a}\left(c(\arctan(\_b\,\mu))^m + s\left(\arctan\left(\frac{\beta\,(b\int(\arctan(\_b\,\lambda))^n \,\mathrm{d}\_b - b\int(\arctan(\lambda\,x))^n \,\mathrm{d}}{a}\right.\right.\right.}$$

119.5 Problem 5

problem number 963

Added March 9, 2019.

Problem Chapter 4.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^n(\lambda y)w_y = \left(c\arctan^m(\mu x) + s\arctan^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcTan[lambda*y]^n*D[w[x, y], y] == (c*ArcTan[mu*x]^m + s*ArcTan[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arctan(lambda*y)^n*diff(w(x,y),y) =(c*arctan(mu*x)^m+s*arctan(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\arctan(y\lambda))^{-n}\,\mathrm{d}y}{b} + x\right)\mathrm{e}^{\int^y \frac{(\arctan(\_b\,\lambda))^{-n}}{b}\left(c\left(\arctan\left(\mu\left(\int\frac{(\arctan(\_b\,\lambda))^{-n}a}{b}\,\mathrm{d}\_b - \frac{a\int(\arctan(y\lambda))^{-n}}{b}\right.\right.\right.\right.}}$$

120 HFOPDE, chapter 4.7.4

120.1 Problem 1

problem number 964

Added March 9, 2019.

Problem Chapter 4.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = \left(c \operatorname{arccot}(\frac{x}{\lambda} + k \operatorname{arccot}(\frac{y}{\beta})\right) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*ArcCot[x/lambda] + k*ArcCot[y/beta])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to \left(\lambda^2 + x^2\right)^{\frac{c\lambda}{2a}} c_1\left(y - \frac{bx}{a}\right) \exp\left(\frac{k\left(a\beta \log\left(a^2(\beta^2 + y^2)\right) + 2\tan^{-1}\left(\frac{y}{\beta}\right)(bx - ay) + 2bx \cot^{-}\right.}{2ab}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*arccot(x/lambda)+k*arccot(y/beta))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left(\frac{\lambda^2+x^2}{\lambda^2}\right)^{1/2\,\frac{\lambda c}{a}}\left(\frac{\beta^2+y^2}{\beta^2}\right)^{1/2\,\frac{\beta k}{b}} \mathrm{e}^{1/2\,\frac{1}{ab}\left(-2\,cx\arctan\left(\frac{x}{\lambda}\right)b-2\,\arctan\left(\frac{y}{\beta}\right)aky+bx\pi(c+k)\right)}$$

120.2 Problem 2

problem number 965

Added March 9, 2019.

Problem Chapter 4.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\,\mathrm{arccot}(\lambda x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*ArcCot[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\frac{bx}{a}\right)\exp\left(\frac{c(a\log\left(a^2(\beta^2y^2+2\beta\lambda xy+\lambda^2x^2+1)\right)+2\beta(bx-ay)\tan^{-1}(\beta y+\lambda x)+}{2a(a\lambda+b\beta)}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\_F1\left(\frac{ya-bx}{a}\right)\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)^{\frac{c}{2\,a\lambda+2\,b\beta}}\,\mathrm{e}^{1/2\,\frac{(-2\,a(\beta\,y+\lambda\,x)\arctan(\beta\,y+\lambda\,x)+x\pi\,(a\lambda+b\beta))c}{(a\lambda+b\beta)a}}$$

120.3 Problem 3

problem number 966

Added March 9, 2019.

Problem Chapter 4.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = a x \operatorname{arccot}(\lambda x + \beta y) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == a*x*ArcCot[lambda*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\frac{a\left(-\beta(ay-bx)\log\left(a^2\left(\frac{\beta^2(ay-bx)^2}{a^2}+\frac{2\beta\lambda x(ay-bx)}{a}+\lambda^2x^2+1\right)+2ab\beta x\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*arccot(lambda*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)\left(\beta^2y^2+2\,\beta\,\lambda\,xy+\lambda^2x^2+1\right)^{-1/2\,\frac{(ya-bx)a\beta}{(a\lambda+b\beta)^2}}\,\mathrm{e}^{1/4\,\frac{-2\left(\left(-\beta^2y^2+\lambda^2x^2+1\right)a+2\,bx\beta\,(\beta\,y+\lambda\,x)\right)a\operatorname{arctan}}{}}$$

120.4 Problem 4

problem number 967

Added March 9, 2019.

Problem Chapter 4.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\,\mathrm{arccot}^n(\lambda x)w_y = \left(c\,\mathrm{arccot}^m(\mu x) + s\,\mathrm{arccot}^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcCot[lambda*x]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\$Aborted$$

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*x)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{b(\pi/2 - \arctan(\lambda x))^n}{a}\,\mathrm{d}x + y\right)\mathrm{e}^{\int^x \frac{1}{a}\left(c(\pi/2-\arctan(\_b\,\mu))^m + s\left(\pi/2 - \arctan\left(\beta\left(\int \frac{b(\pi/2-\arctan(\_b\,\lambda}{a}\right.\right.\right.\right.}$$

120.5 Problem 5

problem number 968

Added March 9, 2019.

Problem Chapter 4.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^n(\lambda y)w_y = \left(c\operatorname{arccot}^m(\mu x) + s\operatorname{arccot}^k(\beta y)\right)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*ArcCot[lambda*y]^n*D[w[x, y], y] == (c*ArcCot[mu*x]^m + s*ArcCot[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Timed out

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*arccot(lambda*y)^n*diff(w(x,y),y) =(c*arccot(mu*x)^m+s*arccot(be
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\frac{a\int(\pi/2-\arctan(y\lambda))^{-n}\,dy}{b} + x\right)e^{\int^y \frac{(\operatorname{arccot}(\_b\,\lambda))^{-n}}{b}\left(c\left(\pi/2-\arctan\left(\mu\left(\int\frac{(\pi/2-\arctan(\_b\,\lambda))^{-n}a}{b}\,d\_\right.\right.\right.\right.}$$

121 HFOPDE, chapter 4.8.1

121.1 Problem 1

problem number 969

Added March 10, 2019.

Problem Chapter 4.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = f(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(\frac{ay-bx}{a}\right) e^{\int_1^x \frac{f(K[1])}{a} dK[1]}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =f(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right) e^{\int \frac{f(x)}{a} dx}$$

121.2 Problem 2

problem number 970

Added March 10, 2019.

Problem Chapter 4.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + aw_y = f(x)yw$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])(aK[1]-ax+y)\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a)(\_a\, a-ax+y)d\_a}$$

121.3 Problem 3

problem number 971

Added March 10, 2019.

Problem Chapter 4.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = (f(x)y^2 + g(x)y + h(x))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == (f[x]*y^2 + g[x]*y + h[x])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1(y-ax)\exp\left(\int_1^x \left(f(K[1])(aK[1]-ax+y)^2 + g(K[1])(aK[1]-ax+y) + h(K[1])\right) dK\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =(f(x)*y^2+g(x)*y+h(x))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdesolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\, e^{\int^x f(\_a)a^2\_a^2 + 2f(\_a)(-ax+y)a\_a + g(\_a)a\_a + f(\_a)(-ax+y)^2 + g(\_a)(-ax+y) + h(\_a)d\_a}$$

121.4 Problem 4

problem number 972

Added March 10, 2019.

Problem Chapter 4.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x) y^k w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x]*y^k*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])(aK[1]-ax+y)^k\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x (\_a\,a-ax+y)^k f(\_a)d\_a}$$

121.5 Problem 5

problem number 973

Added March 10, 2019.

Problem Chapter 4.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x) e^{\lambda y} w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x]*Exp[lambda*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])e^{\lambda(aK[1]-ax+y)}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ a*diff(w(x,y),y) =f(x)*exp(lambda*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a)\mathrm{e}^{-\_a\,a\lambda+(-ax+y)\lambda}d\_a}$$

121.6 Problem 6

problem number 974

Added March 10, 2019.

Problem Chapter 4.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x g(K[2])\,dK[2]}c_1\left(-e^{-ax}\left(e^{ax}\int_1^x e^{-aK[1]}f(K[1])\,dK[1]-y\right)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int f(x)\,\mathrm{e}^{-ax}\,\mathrm{d}x + y\mathrm{e}^{-ax}\right)\mathrm{e}^{\int g(x)\,\mathrm{d}x}$$

121.7 Problem 7

problem number 975

Added March 10, 2019.

Problem Chapter 4.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)y^k w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*y^k*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-e^{-ax}\left(e^{ax}\int_1^x e^{-aK[1]}f(K[1])\,dK[1]-y\right)\right)\exp\left(\int_1^x g(K[2])\left(e^{aK[2]}\left(\text{Integrate}\left[e^{-aK[1]}\right.\right.\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+ (a*y+f(x))*diff(w(x,y),y) =g(x)*y^k*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int f(x)\,\mathrm{e}^{-ax}\,\mathrm{d}x + y\mathrm{e}^{-ax}\right)\mathrm{e}^{\int^x\left(\left(\int f(\_b)\mathrm{e}^{-\_b\,a}\,\mathrm{d}\_b-\int f(x)\mathrm{e}^{-ax}\,\mathrm{d}x+y\mathrm{e}^{-ax}\right)\mathrm{e}^{\_b\,a}\right)^k g(\_b)d\_b}$$

121.8 Problem 8

problem number 976

Added March 10, 2019.

Problem Chapter 4.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + y^k w_y = g(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + y^k*D[w[x, y], y] == g[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x \frac{g(K[2])}{f(K[2])}\,dK[2]}c_1\left(-\frac{y^{-k}\left(ky^k\left(\int_1^x \frac{1}{f(K[1])}\,dK[1]\right)-y^k\left(\int_1^x \frac{1}{f(K[1])}\,dK[1]\right)+y\right)}{k-1}\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ y^k*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{y^k} + k\int (f(x))^{-1}\,\mathrm{d}x - \int (f(x))^{-1}\,\mathrm{d}x\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}\,\mathrm{d}x}$$

121.9 Problem 9

problem number 977

Added March 10, 2019.

Problem Chapter 4.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+a)w_y = (by+c)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (y + a)*D[w[x, y], y] == (b*y + c)*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left((a+y)e^{-\int_1^x \frac{1}{f(K[1])}\,dK[1]}\right)\exp\left((c-ab)\int_1^x \frac{1}{f(K[1])}\,dK[1]+b(a+y)\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ (y+a)*diff(w(x,y),y) =(b*y+c)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left((y+a)\,\mathrm{e}^{-\int (f(x))^{-1}\,\mathrm{d}x}\right)\mathrm{e}^{\int (f(x))^{-1}\,\mathrm{d}x c+b(y+a)-\int (f(x))^{-1}\,\mathrm{d}x ab}$$

121.10 Problem 10

problem number 978

Added March 10, 2019.

Problem Chapter 4.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (y+ax)w_y = g(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (y + a*x)*D[w[x, y], y] == g[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x \frac{g(K[3])}{f(K[3])}\,dK[3]}c_1\left(ye^{-\int_1^x \frac{1}{f(K[1])}\,dK[1]}-\int_1^x \frac{aK[2]\exp\left(-\text{Integrate}\left[\frac{1}{f(K[1])},\{K[1],1,K[2]\},\text{As}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ (y+a*x)*diff(w(x,y),y) =g(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-a\int \frac{x\mathrm{e}^{-\int (f(x))^{-1}\,\mathrm{d}x}}{f(x)}\,\mathrm{d}x + y\mathrm{e}^{-\int (f(x))^{-1}\,\mathrm{d}x}\right)\mathrm{e}^{\int \frac{g(x)}{f(x)}\,\mathrm{d}x}$$

121.11 Problem 11

problem number 979

Added March 10, 2019.

Problem Chapter 4.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = \left(h_2(x)y^2 + h_1(x)y + h_0(x)\right)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == (h2[x]*y^2 + h1[x]*y + h0[x])
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(-e^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]}\left(e^{\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]}\int_1^x \frac{\text{g0}(K[2])\exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) =(h2(x)*y^2+h1(x)*y+h0(x))*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{g0(x)}{f(x)}\mathrm{e}^{-\int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\,\mathrm{d}x + y\mathrm{e}^{-\int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\right)\mathrm{e}^{\int^{x} \frac{1}{f(\_f)}\left(\left(\left(\int \frac{g0(\_f)}{f(\_f)}\mathrm{e}^{-\int \frac{g1(\_f)}{f(\_f)}\,\mathrm{d}\_f}\,\mathrm{d}\_f\right)^{2}\mathrm{e}^{2\int \frac{g1(\_f)}{f(\_f)}\,\mathrm{d}\_f} h2(\_f)}$$

121.12 Problem 12

problem number 980

Added March 10, 2019.

Problem Chapter 4.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_2(x)y^k)w_y = h(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*y^k)*D[w[x, y], y] == h[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x \frac{h(K[3])}{f(K[3])}\,dK[3]}c_1\left((k-1)\int_1^x \frac{\mathrm{g2}(K[2])\exp\left((k-1)\mathrm{Integrate}\left[\frac{\mathrm{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\mathrm{Assu}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*y^k)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol := simplify(sol);
```

$$w(x,y) = \_F1\left((k-1)\int \frac{g2(x)}{f(x)}\mathrm{e}^{(k-1)\int \frac{g1(x)}{f(x)}\mathrm{d}x}\,\mathrm{d}x + y^{1-k}\mathrm{e}^{(k-1)\int \frac{g1(x)}{f(x)}\mathrm{d}x}\right)\mathrm{e}^{\int \frac{h(x)}{f(x)}\mathrm{d}x}$$

121.13 Problem 13

problem number 981

Added March 10, 2019.

Problem Chapter 4.8.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_2(x)e^{\lambda y})w_y = h(x)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g2[x]*Exp[lambda*y])*D[w[x, y], y] == h[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g2(x)*exp(lambda*y))*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

121.14 Problem 14

problem number 982

Added March 10, 2019.

Problem Chapter 4.8.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)y^k w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*y^k*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{-k\int_1^x \frac{g(K[1])}{f(K[1])}\,dK[1]-\int_1^x \frac{g(K[1])}{f(K[1])}\,dK[1]+y^{k+1}}{k+1}\right)\exp\left(\int_1^x \frac{h(K[2])\left(\left((-k-1)\left(-\frac{-k\mathrm{I}\ldots}{\ldots}\right.\right.\right.}{\ldots}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*y^k*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(y^k y-k\int\frac{g(x)}{f(x)}\,\mathrm{d}x-\int\frac{g(x)}{f(x)}\,\mathrm{d}x\right)\mathrm{e}^{\int^x \frac{h(\_b)}{f(\_b)}\left(\left(k\int\frac{g(\_b)}{f(\_b)}\,\mathrm{d}\_b+y^k y-k\int\frac{g(x)}{f(x)}\,\mathrm{d}x-\int\frac{g(x)}{f(x)}\,\mathrm{d}x+\int\frac{g(\_b)}{f(\_b)}\,\mathrm{d}\_b\right)^{(k\ldots}\right.}$$

121.15 Problem 15

problem number 983

Added March 10, 2019.

Problem Chapter 4.8.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)e^{\lambda y}w_x + g(x)w_y = h(x)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*Exp[lambda*y]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{e^{\lambda y}-\lambda\int_1^x \frac{g(K[1])}{f(K[1])}\,dK[1]}{\lambda}\right)\exp\left(\int_1^x -\frac{}{\lambda f(K[2])\left(-\frac{e^{\lambda y}-\lambda \text{Integrate}\left[\frac{g(K[1])}{f(K[1])},\{K[1],1,x\},\text{Assumption}\right.}{\lambda}\right.}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*exp(lambda*y)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) =h(x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{1}{\lambda}\left(\mathrm{e}^{y\lambda}-\int\frac{g(x)}{f(x)}\,\mathrm{d}x\lambda\right)\right)\mathrm{e}^{\int^x \frac{h(\_b)}{f(\_b)\lambda}\left(\int\frac{g(\_b)}{f(\_b)}\,\mathrm{d}\_b+\frac{1}{\lambda}\left(\mathrm{e}^{y\lambda}-\int\frac{g(x)}{f(x)}\,\mathrm{d}x\lambda\right)\right)^{-1}d\_b}$$

122 HFOPDE, chapter 4.8.2

122.1 Problem 1

problem number 984

Added March 10, 2019.

Problem Chapter 4.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (f(x) + g(y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x] + g[y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\int_1^x \frac{g\left(\frac{bK[1]+ay-bx}{a}\right)+f(K[1])}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) =(f(x)+g(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\int^x \frac{1}{a}\left(f(\_a)+g\left(\frac{b\_a+ya-bx}{a}\right)\right)d\_a}$$

122.2 Problem 2

problem number 985

Added March 10, 2019.

Problem Chapter 4.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x)g(y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x]*g[y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1])g(aK[1]-ax+y)\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x)*g(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a)g(\_a\,a-ax+y)d\_a}$$

122.3 Problem 3

problem number 986

Added March 10, 2019.

Problem Chapter 4.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ay + f(x))w_y = g(x)h(y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + (a*y + f[x])*D[w[x, y], y] == g[x]*h[y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to c_1\left(-e^{-ax}\left(e^{ax}\int_1^x e^{-aK[1]}f(K[1])\,dK[1]-y\right)\right)\exp\left(\int_1^x g(K[2])h\left(e^{aK[2]}\left(\text{Integrate}\left[e^{-aK[1]}\right.\right.\right.\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+(a*y+f(x))*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int f(x)\,\mathrm{e}^{-ax}\,\mathrm{d}x + y\mathrm{e}^{-ax}\right)\mathrm{e}^{\int^x g(\_b)h\left(\left(\int f(\_b)\mathrm{e}^{-\_b\,a}\,\mathrm{d}\_b - \int f(x)\mathrm{e}^{-ax}\,\mathrm{d}x + y\mathrm{e}^{-ax}\right)\mathrm{e}^{\_b\,a}\right)d\_b}$$

122.4 Problem 4

problem number 987

Added March 10, 2019.

Problem Chapter 4.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + g(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + g[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+g(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x, y) = \_F1\left(-\int (f(x))^{-1}\,dx + \int (g(y))^{-1}\,dy\right) e^{\int^x \frac{h1(\_f)+h2\left(\mathrm{RootOf}\left(\int (f(\_f))^{-1}\,d\_f - \int^{\_Z} (g(\_a))^{-1} d\_a - \int (f(x))^{-1}\,dx + \int (}{f(\_f)}}$$

contains RootOf

122.5 Problem 5

problem number 988

Added March 10, 2019.

Problem Chapter 4.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x + (f_2(x) + f_3(x)y^k)w_y = g(x)h(y)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*D[w[x, y], x] + (f2[x] + f3[x]*y^k)*D[w[x, y], y] == g[x]*h[y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*diff(w(x,y),x)+(f2(x)+f3(x)*y^k)*diff(w(x,y),y) = g(x)*h(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

122.6 Problem 6

problem number 989

Added March 10, 2019.

Problem Chapter 4.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h_1(x)h_2(y)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h1[x]*h2[y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h1(x)*h2(y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{f2(x)}{f1(x)}\,\mathrm{d}x + \int \frac{g1(y)}{g2(y)}\,\mathrm{d}y\right) \mathrm{e}^{\int^x \frac{h1(\_f)}{f1(\_f)} h2\left(\mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}\,\mathrm{d}\_f - \int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a - \int \frac{f2(x)}{f1(x)}\,\mathrm{d}x + \int \frac{g1(y)}{g2(y)}\,\mathrm{d}y\right)\right.}$$

has RootOf

122.7 Problem 7

problem number 990

Added March 10, 2019.

Problem Chapter 4.8.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = (h_1(x) + h_2(y))w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == (h1[x] + h2[y])*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = (h1(x)+h2(y))*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{f2(x)}{f1(x)}\,dx + \int \frac{g1(y)}{g2(y)}\,dy\right) e^{\int^x \frac{1}{f1(\_f)}\left(h1(\_f)+h2\left(\operatorname{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}\,d\_f - \int^{\_Z} \frac{g1(\_a)}{g2(\_a)}\,d\_a - \int \frac{f2(x)}{f1(x)}\,dx + \int\right.\right.\right.}$$

has RootOf

123 HFOPDE, chapter 4.8.3

123.1 Problem 1

problem number 991

Added March 10, 2019.

Problem Chapter 4.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(\alpha x + \beta y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[alpha*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{ay-bx}{a}\right)\exp\left(\int_1^x \frac{f\left(\frac{\beta(bK[1]+ay-bx)}{a}+\alpha K[1]\right)}{a}dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{ya-bx}{a}\right)e^{\int^x \frac{1}{a}f\left(\frac{(ya-bx)\beta+\_a\,a\alpha+\_a\,b\beta}{a}\right)d\_a}$$

123.2 Problem 2

problem number 992

Added March 10, 2019.

Problem Chapter 4.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x})w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{xf\left(\frac{y}{x}\right)}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = x*f(y/x)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{xf\left(\frac{y}{x}\right)}$$

123.3 Problem 3

problem number 993

Added March 10, 2019.

Problem Chapter 4.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = f(x^2+y^2)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == f[x^2 + y^2]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\int_1^x \frac{f\left(\frac{y^2K[1]^2}{x^2}+K[1]^2\right)}{K[1]}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x*diff(w(x,y),x)+y*diff(w(x,y),y) = f(x^2+y^2)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)e^{\int^x \frac{1}{\_a}f\left(\frac{y^2\_a^2}{x^2}+\_a^2\right)d\_a}$$

123.4 Problem 4

problem number 994

Added March 10, 2019.

Problem Chapter 4.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = x^k f(x^n * y^m)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == x^k*f[x^n*y^m]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\int_1^x \frac{K[1]^{k-1}f\left(K[1]^n\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)^m\right)}{a}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = x^k*f(x^n+y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)\mathrm{e}^{\int^x \frac{\_a^{k-1}}{a} f\left(\_a^n+\left(yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)^m\right)d\_a}$$

123.5 Problem 5

problem number 995

Added March 10, 2019.

Problem Chapter 4.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$mxw_x + nyw_y = f(ax^n + by^m)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = m*x*D[w[x, y], x] + n*y*D[w[x, y], y] == f[a*x^n + b*y^m]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{n}{m}}\right)\exp\left(\int_1^x \frac{f\left(aK[1]^n + b\left(yx^{-\frac{n}{m}}K[1]^{\frac{n}{m}}\right)^m\right)}{mK[1]}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  m*x*diff(w(x,y),x)+n*y*diff(w(x,y),y) = f(a*x^n+b*y^m)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{n}{m}}\right)\mathrm{e}^{\int^x \frac{1}{\_a\, m} f\left(\_a^n a + \left(yx^{-\frac{n}{m}}\_a^{\frac{n}{m}}\right)^m b\right) d\_a}$$

123.6 Problem 6

problem number 996

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + xyw_y = y^k f(\alpha x^n + \beta y^m) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^k*f[alpha*x + beta*y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)\exp\left(\int_1^x \frac{\left(\frac{yK[1]}{x}\right)^k f\left(\alpha K[1]+\frac{\beta y K[1]}{x}\right)}{K[1]^2}\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  x^2*diff(w(x,y),x)+x*y*diff(w(x,y),y) = y^k*f(alpha*x+beta*y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(\frac{y}{x}\right)\mathrm{e}^{\int^x \frac{1}{\_a^2} f\left(\_a\left(\frac{\beta y}{x}+\alpha\right)\right)\left(\frac{y\_a}{x}\right)^k d\_a}$$

123.7 Problem 7

problem number 997

Added March 10, 2019.

Problem Chapter 4.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$\frac{f(x)}{f'(x)}w_x + \frac{g(x)}{g'(x)}w_y = h(f(x)+g(y))w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (f[x]*D[w[x, y], x])/Derivative[1][f][x] + (g[x]*D[w[x, y], y])/Derivative[1][g][x] =
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(y-\int_1^x \frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])}\,dK[1]\right)\exp\left(\int_1^x \frac{f'(K[2])h\left(g\left(-\text{Integrate}\left[\frac{g(K[1])f'(K[1])}{f(K[1])g'(K[1])},\{K[\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)/diff(f(x),x)*diff(w(x,y),x)+g(x)/diff(g(x),x)*diff(w(x,y),y) = h(f(x)+g(y))*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{f(x)\frac{d}{dx}g(x)}\,dx+y\right)e^{\int^x \frac{\frac{d}{d\_b}f(\_b)}{f(\_b)}h\left(f(\_b)+g\left(\int \frac{g(\_b)\frac{d}{d\_b}f(\_b)}{f(\_b)\frac{d}{d\_b}g(\_b)}\,d\_b-\int \frac{\left(\frac{d}{dx}f(x)\right)g(x)}{f(x)\frac{d}{dx}g(x)}\,dx+y\right)\right)d\_b}$$

124 HFOPDE, chapter 4.8.4

124.1 Problem 1

problem number 998

Added March 10, 2019.

Problem Chapter 4.8.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + a w_y = f(x,y) w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = D[w[x, y], x] + a*D[w[x, y], y] == f[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1(y-ax)\exp\left(\int_1^x f(K[1], aK[1]-ax+y)\,dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  diff(w(x,y),x)+a*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1(-ax+y)\,\mathrm{e}^{\int^x f(\_a, \_a a - ax + y) d\_a}$$

124.2 Problem 2

problem number 999

Added March 10, 2019.

Problem Chapter 4.8.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(yx^{-\frac{b}{a}}\right)\exp\left(\int_1^x\frac{f\left(K[1],yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)}{aK[1]}dK[1]\right)\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*x*diff(w(x,y),x)+b*y*diff(w(x,y),y) = f(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(yx^{-\frac{b}{a}}\right)e^{\int^x\frac{1}{\_a\,a}f\left(\_a,yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)d\_a}$$

124.3 Problem 3

problem number 1000

Added March 10, 2019.

Problem Chapter 4.8.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)yw_y = h(x,y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + g[x]*y*D[w[x, y], y] == h[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\int_1^x \frac{g(K[1])}{f(K[1])}\,dK[1]}\right)\exp\left(\int_1^x \frac{h\left(K[2], y\exp\left(\text{Integrate}\left[\frac{g(K[1])}{f(K[1])}, \{K[1],1,K[2]\}, \text{Assumpti}\right.\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+g(x)*y*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(ye^{-\int \frac{g(x)}{f(x)}\,dx}\right)e^{\int^x \frac{1}{f(\_b)}h\left(\_b, ye^{-\int \frac{g(x)}{f(x)}\,dx+\int \frac{g(\_b)}{f(\_b)}\,d\_b}\right)d\_b}$$

124.4 Problem 4

problem number 1001

Added March 10, 2019.

Problem Chapter 4.8.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x])*D[w[x, y], y] == h[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(ye^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])}\,dK[1]}-\int_1^x \frac{\text{g0}(K[2])\exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{Assumptio}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{g0(x)}{f(x)} e^{-\int \frac{g1(x)}{f(x)} dx} dx + y e^{-\int \frac{g1(x)}{f(x)} dx}\right) e^{\int^{x} \frac{1}{f(\_f)} h\left(\_f, \left(\int \frac{g0(\_f)}{f(\_f)} e^{-\int \frac{g1(\_f)}{f(\_f)} d\_f} d\_f - \int \frac{g0(x)}{f(x)} e^{-\int \frac{g1(x)}{f(x)} d$$

124.5 Problem 5

problem number 1002

Added March 10, 2019.

Problem Chapter 4.8.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*y^k)*D[w[x, y], y] == h[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left((k-1)\int_1^x \frac{\text{g0}(K[2])\exp\left((k-1)\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{Assumptions}\to \text{T}\right.\right.}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \_F1\left((k-1)\int \frac{g0(x)}{f(x)}\mathrm{e}^{(k-1)\int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\,\mathrm{d}x + y^{1-k}\mathrm{e}^{(k-1)\int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\right)\mathrm{e}^{\int^{x}\frac{1}{f(\_f)}h\left(\_f,\left((1-k)\int \frac{g0(\_f)}{f(\_f)}\mathrm{e}^{(k-1)\int \frac{g1(\_}{f(\_}}\right.\right.}$$

124.6 Problem 6

problem number 1003

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h(x,y)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f[x]*D[w[x, y], x] + (g1[x]*y + g0[x]*Exp[lambda*y])*D[w[x, y], y] == h[x, y]*w[x, y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f(x)*diff(w(x,y),x)+(g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

sol=()

124.7 Problem 7

problem number 1004

Added March 10, 2019.

Problem Chapter 4.8.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)g_1(y)w_x + f_2(x)g_2(y)w_y = h(x,y)w$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = f1[x]*g1[y]*D[w[x, y], x] + f2[x]*g2[y]*D[w[x, y], y] == h[x, y]*w[x, y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  f1(x)*g1(y)*diff(w(x,y),x)+f2(x)*g2(y)*diff(w(x,y),y) = h(x,y)*w(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \_F1\left(-\int \frac{f2(x)}{f1(x)}\,\mathrm{d}x + \int \frac{g1(y)}{g2(y)}\,\mathrm{d}y\right) \mathrm{e}^{\int^x \frac{1}{f1(\_f)} h\left(\_f, \mathrm{RootOf}\left(\int \frac{f2(\_f)}{f1(\_f)}\,\mathrm{d}\_f - \int^{\_Z} \frac{g1(\_a)}{g2(\_a)} d\_a - \int \frac{f2(x)}{f1(x)}\,\mathrm{d}x + \int \frac{g1(y)}{g2(y)}\,\mathrm{d}y\right.\right.}$$

has RootOf

125 HFOPDE, chapter 5.2.1

125.1 Problem 1

problem number 1005

Added March 10, 2019.

Problem Chapter 5.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + d$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{ce^{\frac{cx}{a}}c_1\left(\frac{ay-bx}{a}\right)-d}{c}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  a*diff(w(x,y),x)+b*diff(w(x,y),y) = c*w(x,y)+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{1}{c}\left(\mathrm{e}^{\frac{cx}{a}}\_F1\left(\frac{ya-bx}{a}\right)c-d\right)$$

125.2 Problem 2

problem number 1006

Added March 10, 2019.

Problem Chapter 5.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x-a)w_x + (y-b)w_y = w - c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (x - a)*D[w[x, y], x] + (y - b)*D[w[x, y], y] == w[x, y] - c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to -ac_1\left(\frac{b-y}{a-x}\right)+xc_1\left(\frac{b-y}{a-x}\right)+c\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  (x-a)*diff(w(x,y),x)+(y-b)*diff(w(x,y),y) = w(x,y)-c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(\frac{-b+y}{a-x}\right)a-\_F1\left(\frac{-b+y}{a-x}\right)x+c$$

125.3 Problem 3

problem number 1007

Added March 10, 2019.

Problem Chapter 5.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cx+d)w_y = \alpha w + \beta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (a*x + b)*D[w[x, y], x] + (c*x + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\alpha(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{a^2y+bc\log(ax+b)-ad\log(ax+b)-acx}{a^2}\right)-\beta}{\alpha}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{1}{\alpha}\left((ax+b)^{\frac{\alpha}{a}}\_F1\left(-\frac{\ln(ax+b)\,da-\ln(ax+b)\,bc-ya^2+cxa}{a^2}\right)\alpha-\beta\right)$$

125.4 Problem 4

problem number 1008

Added March 10, 2019.

Problem Chapter 5.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha w + \beta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\alpha(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-\beta}{\alpha}\right\}\right\}$$

Maple ✓

```
w:='w';x:='x';y:='y';a:='a';b:='b';n:='n';m:='m';c:='c';
k:='k';alpha:='alpha';beta:='beta';g:='g';A:='A';f:='f';
C:='C';lambda:='lambda';B:='B';mu:='mu';d:='d';s:='s';t:='t';
v:='v';q:='q';p:='p';l:='l';g1:='g1';g2:='g2';g0:='g0';
h0:='h0';h1:='h1';h2:='h2';f2:='f2';f3:='f3';
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{1}{\alpha}\left((ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{cy+d}{c}(ax+b)^{-\frac{c}{a}}\right)\alpha-\beta\right)$$

125.5 Problem 5

problem number 1009

Added March 10, 2019.

Problem Chapter 5.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+b)w_x + (cy+d)w_y = \alpha w + \beta y + \gamma x$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (a*x + b)*D[w[x, y], x] + (c*y + d)*D[w[x, y], y] == alpha*w[x, y] + beta*y + gamma*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \frac{\alpha(a-\alpha)(\alpha-c)(ax+b)^{\frac{\alpha}{a}}c_1\left(\frac{(cy+d)(ax+b)^{-\frac{c}{a}}}{c}\right)-a\beta(\alpha y+d)+\alpha^2\beta y+\alpha^2\gamma x+\alpha b\gamma+\alpha\beta d -}{\alpha(a-\alpha)(\alpha-c)}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*y+d)*diff(w(x,y),y) =  alpha*w(x,y)+beta*y+gamma*x;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\frac{1}{(-\alpha+c)(a-\alpha)\alpha}\left((ax+b)^{\frac{\alpha}{a}}\alpha(-\alpha+c)(a-\alpha)\_F1\left(\frac{cy+d}{c}(ax+b)^{-\frac{c}{a}}\right)+(-\beta y-\gamma x)\alpha^2\right.$$

125.6 Problem 6

problem number 1010

Added March 10, 2019.

Problem Chapter 5.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(ax + b)w_x + (cx + dy)w_y = \alpha w + \beta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 pde = (a*x + b)*D[w[x, y], x] + (c*x + d*y)*D[w[x, y], y] == alpha*w[x, y] + beta;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{\alpha (ax+b)^{\frac{\alpha}{a}} c_1\left(\frac{(ax+b)^{-\frac{d}{a}}(ady-bc-cdx+d^2(-y))}{d(a-d)}\right)-\beta}{\alpha}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  (a*x+b)*diff(w(x,y),x)+ (c*x+d*y)*diff(w(x,y),y) =  alpha*w(x,y)+beta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{1}{\alpha}\left((ax+b)^{\frac{\alpha}{a}}\_F1\left(\frac{dya-cxd-d^2y-bc}{(a-d)\,d}(ax+b)^{-\frac{d}{a}}\right)\alpha-\beta\right)$$

125.7 Problem 7

problem number 1011

Added March 10, 2019.

Problem Chapter 5.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x + a_0)w_x + (b_2y + b_1x + b_0)w_y = (c_2y + c_1x + c_0)w + k_2y + k_1x + k_0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2];
 pde = (a1*x + a0)*D[w[x, y], x] + (b2*y + b1*x + b0)*D[w[x, y], y] == (c2*y + c1*x + c0)*w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  (a1*x+a0)*diff(w(x,y),x)+ (b2*y+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,y)+k2*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

Timed out

125.8 Problem 8

problem number 1012

Added March 10, 2019.

Problem Chapter 5.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + (b_1x + b_0)w_y = (c_1x + c_0)w + s_1x + s0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*y*D[w[x, y], x] + (b1*x + b0)*D[w[x, y], y] == (c1*x + c0)*w[x, y] + s1*x + s0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y*diff(w(x,y),x)+ (b1*x+b0)*diff(w(x,y),y) =  (c1*x+c0)*w(x,y)+s1*x+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{ab1\,x + ab0}{\sqrt{ab1}} + \sqrt{ab1\,x^2 + \left(y^2a - b1\,x^2 - 2\,b0\,x\right)a + 2\,ab0\,x}\right)^{-\frac{b0\,c1 - b1\,c0}{b1\,\sqrt{ab1}}} \left(\int^x \frac{}{\sqrt{a\left(\_a^2b1 + \right.}}\right.$$

126 HFOPDE, chapter 5.2.2

126.1 Problem 1

problem number 1013

Added March 10, 2019.

Problem Chapter 5.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cw + \beta xy + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + beta*x*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{-2ab\beta + c^3 e^{\frac{cx}{a}} c_1\left(\frac{ay-bx}{a}\right) - a\beta cy - b\beta cx - \beta c^2 xy - c^2\gamma}{c^3}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(-\frac{\beta y}{c} - \frac{b\beta}{c^2}\right)x - \frac{ya\beta}{c^2} + \frac{1}{c^3}\left(\_F1\left(\frac{ya-bx}{a}\right)e^{\frac{cx}{a}}c^3 - \gamma c^2 - 2ab\beta\right)$$

126.2 Problem 2

problem number 1014

Added March 10, 2019.

Problem Chapter 5.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to \frac{-2a^2\beta + c^3 e^{\frac{cx}{a}} c_1\left(\frac{ay-bx}{a}\right) - 2ab\gamma - 2a\beta cx - ac\gamma y - bc\gamma x - \beta c^2x^2 - c^2\delta - c^2\gamma xy}{c^3}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = -\frac{\beta x^2}{c} + \left(-\frac{\gamma y}{c} + \frac{-2\beta ac - bc\gamma}{c^3}\right)x - \frac{\gamma ya}{c^2} + \frac{1}{c^3}\left(\_F1\left(\frac{ya-bx}{a}\right)e^{\frac{cx}{a}}c^3 - 2a\gamma b - 2a^2\beta - \delta c^2\right)$$

126.3 Problem 3

problem number 1015

Added March 10, 2019.

Problem Chapter 5.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = w + ax^2 + by^2 + c$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == w[x, y] + a*x^2 + b*y^2 + c;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to ax^2+by^2+xc_1\left(\frac{y}{x}\right)-c\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =  w(x,y)+a*x^2+b*y^2+c;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=by^2+ax^2+\_F1\left(\frac{y}{x}\right)x-c$$

126.4 Problem 4

problem number 1016

Added March 10, 2019.

Problem Chapter 5.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + x(\beta x + \gamma y) + \delta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x, y] + x*(beta*x + gamma*y) + delta;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{2a^2cx^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right) - 2a^2\delta + c^3x^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right) - 3ac^2x^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right) - bc^2x^{\frac{c}{a}}c_1\left(yx^{-\frac{b}{a}}\right) + 2abcx}{c(c-2a)(-a}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =  c*w(x,y)+x*(beta*x+gamma*y)+delta;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \frac{\gamma\, y}{a} x^{\frac{a+b}{a}-\frac{b}{a}} \left(\frac{b-c}{a}+1\right)^{-1} + \frac{\beta\, x^2}{a}\left(\frac{a-c}{a}+1\right)^{-1} + \frac{\delta}{a}\left(1-\frac{a+c}{a}\right)^{-1} + x^{\frac{c}{a}}\_F1\left(yx^{-\frac{b}{a}}\right)$$

126.5 Problem 5

problem number 1017

Added March 10, 2019.

Problem Chapter 5.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + (b_2x^2 + b_1x + b_0)w_y = (c_2x^2 + c_1x + c_0)w + s_2x^2 + s_1x + s_0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*y*D[w[x, y], x] + (b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*x^2 + c1*x + c0)*w[x, y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y*diff(w(x,y),x)+ (b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*x^2+c1*x+c0)*w(x,y)+s2*x^
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

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126.6 Problem 6

problem number 1018

Added March 10, 2019.

Problem Chapter 5.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^2 w_x + (b_1 x^2 + b_0) w_y = (c_1 x^2 + c_0) w + s_1 x^2 + s_0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0];
 pde = a*y^2*D[w[x, y], x] + (b1*x^2 + b0)*D[w[x, y], y] == (c1*x^2 + c0)*w[x, y] + s1*x^2 +
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y*diff(w(x,y),x)+ (b1*x^2+b0)*diff(w(x,y),y) =  (c1*x^2+c0)*w(x,y)+s1*x^2+s0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

126.7 Problem 7

problem number 1019

Added March 10, 2019.

Problem Chapter 5.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2+a_0)w_x+(y+b_2x^2+b_1x+b_0)w_y=(c_2y+c_1x+c_0)w+k_{22}y^2+k12xy+k_{11}x^2+k_0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12];
 pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (y + b2*x^2 + b1*x + b0)*D[w[x, y], y] == (c2*y + c
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (y+b2*x^2+b1*x+b0)*diff(w(x,y),y) =  (c2*y+c1*x+c0)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{1}{\_f^2 a1 + a0} \left(\left(\int \frac{\_f^2 b2 + \_f\, b1 + b0}{\_f^2 a1 + a0} \mathrm{e}^{-\frac{1}{\sqrt{a0\, a1}} \arctan\left(\frac{a1\, \_f}{\sqrt{a0\, a1}}\right)} \mathrm{d}\_f \right)^2 k22\, \mathrm{e}^{-\frac{1}{\sqrt{a0\, a1}} \left(\int \frac{1}{\_f^2 a1 + a0}\right.}\right.\right.$$

126.8 Problem 8

problem number 1020

Added March 10, 2019.

Problem Chapter 5.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(a_1x^2 + a_0)w_x + (b_2y^2 + b_1xy)w_y = (c_2y^2 + c_1x^2)w + s_{22}y^2 + s_{12}xy + s_{11}x^2 + s_0$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = (a1*x^2 + a0)*y^2*D[w[x, y], x] + (b2*y^2 + b1*x^2)*D[w[x, y], y] == (c2*y^2 + c1*x^2
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  (a1*x^2+a0)*diff(w(x,y),x)+ (b2*y^2+b1*x^2)*diff(w(x,y),y) =  (c2*y^2+c1*x^2)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

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127 HFOPDE, chapter 5.2.3

127.1 Problem 1

problem number 1021

Added March 12, 2019.

Problem Chapter 5.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = \alpha yw + \beta\sqrt{xy} + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == alpha*y*w[x, y] + beta*Sqrt[x*y] + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to\frac{e^{\frac{\alpha y}{b}}\left(-\beta\sqrt{xy}\left(\frac{\alpha y}{b}\right)^{-\frac{a+b}{2b}}\text{Gamma}\left(\frac{a+b}{2b},\frac{\alpha y}{b}\right)+bc_1\left(yx^{-\frac{b}{a}}\right)+\gamma\text{ExpIntegralEi}\left(-\frac{\alpha y}{b}\right)\right)}{b}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = alpha*y*w(x,y)+ beta*sqrt(x*y)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = -\frac{1}{b\,(3\,b+a)\,(5\,b+a)\,(a+b)\,y\alpha\,a}\mathrm{e}^{1/2\,\frac{\alpha\,y}{b}}\left(-4\,a\sqrt{yx}\left(\frac{\alpha\,y}{b}\right)^{-1/4\,\frac{3\,b+a}{b}}b^3\beta\,(2\,\alpha\,y+a+3\,b)\,\mathrm{Whittak}\right.$$

127.2 Problem 2

problem number 1022

Added March 12, 2019.

Problem Chapter 5.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = \lambda\sqrt{xy}w + \beta xy + \gamma$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == lambda*Sqrt[x*y]*w[x, y] + beta*x*y + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = lambda*sqrt(x*y)*w(x,y)+ beta*x*y+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = 1/2\,\frac{1}{\lambda^2\,(a+b)}\left(-4\,\gamma\ \mathrm{expIntegral}\left(1,2\,\frac{\sqrt{yx}\lambda}{a+b}\right)\lambda^2\mathrm{e}^{2\,\frac{\sqrt{yx}\lambda}{a+b}} - \left(-2\_F1\left(yx^{-\frac{b}{a}}\right)\lambda^2\mathrm{e}^{2\,\frac{\sqrt{yx}\lambda}{a+b}} + \beta\,(2\,\sqrt{}\right.\right.$$

127.3 Problem 3

problem number 1023

Added March 12, 2019.

Problem Chapter 5.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{x} \frac{\beta\sqrt{\_a}+\gamma}{\sqrt{a\left(\_a^2b+y^2a-bx^2\right)}}\left(\frac{\_a\,ab+\sqrt{a\left(\_a^2b+y^2a-bx^2\right)}\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}}d\_a+\_F1\left(\frac{y^2a-}{a}\right.\right.$$

127.4 Problem 4

problem number 1024

Added March 12, 2019.

Problem Chapter 5.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ayw_x + bxw_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*y*D[w[x, y], x] + b*x*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + gamma;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y*diff(w(x,y),x)+ b*x*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamma;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\left(\int^{x} \frac{\beta\sqrt{\_a}+\gamma}{\sqrt{a\,(y^2a+(\_a^2-x^2)\,b)}}\left(\frac{\_a\,ab+\sqrt{a\,(y^2a+(\_a^2-x^2)\,b)}\sqrt{ab}}{\sqrt{ab}}\right)^{-\frac{\alpha}{\sqrt{ab}}} d\_a+\_F1\left(\frac{y^2a\right.$$

127.5 Problem 5

problem number 1025

Added March 12, 2019.

Problem Chapter 5.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x + b\sqrt{y}w_y = \alpha w + \beta x + \gamma y + \delta$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*x + gamma*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y) \to \frac{-a^2\beta + 2\alpha^3 e^{\frac{2\alpha\sqrt{x}}{a}} c_1\left(\frac{2(a\sqrt{y}-b\sqrt{x})}{a}\right) - 2a\alpha\beta\sqrt{x} - 2\alpha^2\beta x - 2\alpha^2\delta - 2\alpha^2\gamma y - 2\alpha b\gamma\sqrt{y} - b^2\gamma}{2\alpha^3}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*x+gamma*y+de
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = -1/2\,\frac{1}{\alpha^3}\left(-2\,\_F1\left(\frac{-\sqrt{y}a+b\sqrt{x}}{b}\right)\alpha^3+\left(2\,a\beta\,\alpha\,\sqrt{x}+2\,\sqrt{y}b\alpha\,\gamma+\left(2\,\beta\,x+2\,\gamma\,y+2\,\delta\right)\alpha^2+a^2\right.\right.$$

127.6 Problem 6

problem number 1026

Added March 12, 2019.

Problem Chapter 5.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{x}w_x + b\sqrt{y}w_y = \alpha w + \beta\sqrt{x} + \gamma$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*Sqrt[x]*D[w[x, y], x] + b*Sqrt[y]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{2\alpha\sqrt{x}}{a}}c_1\left(2\sqrt{y}-\frac{2b\sqrt{x}}{a}\right)-\frac{a\beta+2\alpha\left(\beta\sqrt{x}+\gamma\right)}{2\alpha^2}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*sqrt(x)*diff(w(x,y),x)+ b*sqrt(y)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^{y}\frac{1}{b\sqrt{\_a}}\mathrm{e}^{-2\,\frac{\sqrt{\_a}a\alpha}{b}}\left(\beta\sqrt{\frac{\left(\sqrt{\_a}a-\sqrt{y}a+b\sqrt{x}\right)^2}{b^2}}+\_a\,\gamma+\delta\right)d\_a+\_F1\left(\frac{-\sqrt{y}a+b\sqrt{x}}{b}\right)\right.$$

127.7 Problem 7

problem number 1027

Added March 12, 2019.

Problem Chapter 5.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sqrt{y}w_x+b\sqrt{x}w_y=\alpha w+\beta\sqrt{x}+\gamma$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*Sqrt[y]*D[w[x, y], x] + b*Sqrt[x]*D[w[x, y], y] == alpha*w[x, y] + beta*Sqrt[x] + g
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Timed out

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*sqrt(y)*diff(w(x,y),x)+ b*sqrt(x)*diff(w(x,y),y) = alpha*w(x,y)+ beta*sqrt(x)+gamm
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^{y} \frac{1}{b} \mathrm{e}^{-\frac{\alpha}{b} \int \frac{1}{\sqrt{\frac{\left(\left(\_b^{3/2}a+\mathrm{RootOf}\left(xb^{2}-\left(b^{2}y^{3/2}a+\_Z\,b^{3}\right)^{2/3}\right)b\right)b^{2}\right)^{2/3}}{b^{2}}}} d\_b} \left(\gamma\_b+\beta\sqrt{\frac{\left(\left(\_b^{3/2}a+\mathrm{RootOf}\left(x\right.\right.\right.}{}}\right.\right.$$

contains RootOf

128 HFOPDE, chapter 5.2.4

128.1 Problem 1

problem number 1028

Added March 12, 2019.

Problem Chapter 5.2.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + kx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x, y] + k*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{kK[1]^n e^{-\frac{cK[1]}{a}}\left(\frac{bK[1]+ay-bx}{a}\right)^m}{a}dK[1]+c_1\left(\frac{ay-bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{\_a^n k}{a}\left(\frac{b\_a+ya-bx}{a}\right)^m \mathrm{e}^{-\frac{\_a\, c}{a}} d\_a + \_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

128.2 Problem 2

problem number 1029

Added March 12, 2019.

Problem Chapter 5.2.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + yw_y = bw + cx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*D[w[x, y], x] + y*D[w[x, y], y] == b*w[x, y] + c*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x(b-m)}{a}}\left(e^{\frac{mx}{a}}c_1\left(ye^{-\frac{x}{a}}\right)-\frac{cy^m x^n\left(\frac{x(b-m)}{a}\right)^{-n}\text{Gamma}\left(n+1,\frac{x(b-m)}{a}\right)}{b-m}\right)\right\}\right\}$$

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t);
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2);
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11);
pde :=  a*diff(w(x,y),x)+ y*diff(w(x,y),y) = b*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Exception

128.3 Problem 3

problem number 1030

Added April 1, 2019.

Problem Chapter 5.2.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axw + bx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*w[x, y] + b*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{ax}\left(c_1\left(\frac{y}{x}\right)-by^mx^n(ax)^{-m-n}\text{Gamma}(m+n,ax)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\frac{(ax)^{-n/2-m/2}\,\text{WhittakerM}\,(n/2+m/2,m/2+n/2+1/2,ax)\,y^mx^nbe^{1/2\,ax}}{(n+m)\,(m+n+1)}+\frac{\text{WhittakerM}\,(n/2}{}$$

128.4 Problem 4

problem number 1031

Added April 1, 2019.

Problem Chapter 5.2.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = a\sqrt{x^2+y^2}w + bx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*Sqrt[x^2+y^2]*w[x,y] + b*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{a\sqrt{x^2+y^2}}\left(\int_1^x bK[1]^{n-1}e^{-a\sqrt{\left(\frac{y^2}{x^2}+1\right)K[1]^2}}\left(\frac{yK[1]}{x}\right)^m dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*sqrt(x^2+y^2)*w(x,y)+ b*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\left(\frac{a}{x}\sqrt{x^2\left(\frac{y^2}{x^2}+1\right)}\right)^{-m-n}y^m x^{-m}b\left(\frac{x^{n+m}}{(n+m)(m+n+1)}\left(\frac{a}{x}\sqrt{x^2\left(\frac{y^2}{x^2}+1\right)}\right)^{n+m}\left(a\sqrt{x^2\left(\right.\right.}\right.\right.$$

128.5 Problem 5

problem number 1032

Added April 1, 2019.

Problem Chapter 5.2.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cx^n y^m w + px^k y^s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*x^n*y^m*w[x,y] + p*x^k*y^s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cy^m x^n}{an+bm}}\left(\int_1^x \frac{pK[1]^{k-1}\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)^s \exp\left(-\frac{cK[1]^n\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)^m}{an+bm}\right)}{a}dK[1] + c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*x^n*y^m*w(x,y)+ p*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\frac{y^s p}{a}\left(\frac{y^m c}{an+bm}x^{-\frac{bm}{a}}\right)^{-k\left(n+\frac{bm}{a}\right)^{-1}-\frac{sb}{a}\left(n+\frac{bm}{a}\right)^{-1}} x^{-\frac{sb}{a}}\left(\frac{(an+bm)^2\, y^{-m}}{c\,(ak+2\,an+2\,bm+sb)\,(ak+an+bm\right.\right.$$

128.6 Problem 6

problem number 1033

Added April 1, 2019.

Problem Chapter 5.2.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = (cx^n + py^m)w + qx^k y^s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*(x^n+p*y^m)*w[x,y] + q*x^k*y^s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^n}{an}+\frac{cpy^m}{bm}}\left(\int_1^x \frac{qK[1]^{k-1}\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)^s \exp\left(-\frac{c\left(\frac{ap\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)^m}{bm}+\frac{K[1]^n}{n}\right)}{a}\right)}{a}dK[1]+c_1\left(yx\right.\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*(x^n+y^m)*w(x,y)+ q*x^k*y^s;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(\int^x \frac{\_b^{k-1}q}{a}\left(yx^{-\frac{b}{a}}\_b^{\frac{b}{a}}\right)^s \mathrm{e}^{-\frac{c}{a}\int\frac{1}{\_b}\left(\_b^n+\left(yx^{-\frac{b}{a}}\_b^{\frac{b}{a}}\right)^m\right)\mathrm{d}\_b}d\_b+\_F1\left(yx^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int^x\frac{c}{\_a\,a}\left(\_a^n+\left(yx^{-\frac{b}{a}}\right.\right.}$$

128.7 Problem 7

problem number 1034

Added April 1, 2019.

Problem Chapter 5.2.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2 w_x + axyw_y = by^2 w + cx^n y^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = x^2*D[w[x, y], x] + a*x*y*D[w[x, y], y] == b*y^2*w[x,y] + c*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{by^2}{x-2ax}}\left(c_1\left(yx^{-a}\right)-\frac{cy^m x^{n-1}\left(-\frac{by^2}{x-2ax}\right)^{-\frac{am+n-1}{2a-1}}\text{Gamma}\left(\frac{am+n-1}{2a-1},-\frac{by^2}{x-2ax}\right)}{2a-1}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  x^2*diff(w(x,y),x)+ a*x*y*diff(w(x,y),y) = b*y^2*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\_F1\left(yx^{-a}\right)\mathrm{e}^{\frac{by^2}{x(2a-1)}}+\frac{y^m cx^{-am}}{(2a-1)\,y^2}\left(\frac{by^2x^{-2a}}{2a-1}\right)^{-\frac{am+n-1}{2a-1}}\left(\frac{(2a-1)^2\left(2a^2m+4a^2-am+2an-\right.}{b\,(am+4a+n-3)\,(am+n-1)\,(a\right.}$$

128.8 Problem 8

problem number 1035

Added April 1, 2019.

Problem Chapter 5.2.4.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x^2w_x + xyw_y = y^2(ax+by)w + cx^ny^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = x^2*D[w[x, y], x] + x*y*D[w[x, y], y] == y^2*(a*x+b*y)*w[x,y] + c*x^n*y^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to c_1\left(\frac{y}{x}\right)e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}-c2^{\frac{1}{2}(m+n-3)}y^mx^{n-1}e^{\frac{1}{2}y^2\left(a+\frac{by}{x}\right)}\left(y^2\left(a+\frac{by}{x}\right)\right)^{\frac{1}{2}(-m-n+1)}\text{Gamma}\left(\frac{1}{2}(m\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  x^2*diff(w(x,y),x)+ x*y*diff(w(x,y),y) = y^2*(a*x+b*y)*w(x,y)+ c*x^n*y^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y)=\left(1/4\,2^{m/2+n/2+1/2}c\left(\frac{by}{x}+a\right)y^{m+2}\left(4\,\frac{2^{-n/4-m/4+3/4}y^2x^{n+m-1}}{(n+m-1)\,(m+n+1)\,(n+3+m)}\text{WhittakerM}\left(m/\right.\right.\right.$$

128.9 Problem 9

problem number 1036

Added April 1, 2019.

Problem Chapter 5.2.4.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + bx^m y w_y = cx^p y^q w + sx^\gamma y^\delta + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x^n*D[w[x, y], x] + b*x^m*y*D[w[x, y], y] == c*x^p*y^q*w[x,y] + s*x^gamma*y^delta+d
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```

 unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y*diff(w(x,y),y) = c*x^p*y^q*w(x,y)+ s*x^gamma*y^delta+d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

$$w(x,y) = \left(\int^x \frac{1}{a} \mathrm{e}^{-\frac{c}{a}\int \_b^{-n+p}\left(y \mathrm{e}^{-\frac{x^{-n+m+1}b}{a(-n+m+1)}+\frac{\_b^{-n+m+1}b}{a(-n+m+1)}}\right)^q \mathrm{d}\_b} \left(\_b^{-n+\gamma} s \left(y \mathrm{e}^{-\frac{x^{-n+m+1}b}{a(-n+m+1)}+\frac{\_b^{-n+m+1}b}{a(-n+m+1)}}\right)^\delta + \_b^{-n} d\right.\right.$$

128.10 Problem 10

problem number 1037

Added April 1, 2019.

Problem Chapter 5.2.4.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + (bx^m y + cx^k)w_y = sx^p y^q w + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x^n*D[w[x, y], x] + (b*x^m*y+x*x^k)*D[w[x, y], y] == s*x^p*y^q*w[x,y] + d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x^n*diff(w(x,y),x)+ (b*x^m*y+c*x^k)*diff(w(x,y),y) = s*x^p*y^q*w(x,y)+ d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
```

Too large to display

128.11 Problem 11

problem number 1038

Added April 1, 2019.

Problem Chapter 5.2.4.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$a x^n w_x + b x^m y^k w_y = c w + s x^p y^q + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*x^n*D[w[x, y], x] + b*x^m*y^k*D[w[x, y], y] == c*w[x,y] + s*x^p*y^q+d;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*x^n*diff(w(x,y),x)+ b*x^m*y^k*diff(w(x,y),y) = c*w(x,y)+ s*x^p*y^q+d;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x, y) = \frac{1}{a} \mathrm{e}^{-\frac{c x^{-n+1}}{a(n-1)}} \left(\_F1 \left(\frac{b(k-1)\, x^{-n+m+1} + y^{1-k} a(-n+m+1)}{a\,(-n+m+1)} \right) a + \int^{x} \mathrm{e}^{\frac{c\_a^{-n+1}}{a(n-1)}} \_a^{-n+p} \left(\left(\frac{-b(k\right.\right.\right.$$

128.12 Problem 12

problem number 1039

Added April 1, 2019.

Problem Chapter 5.2.4.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y] + s*x^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}\text{Hypergeometric2F1}\left(\frac{k}{k+1},\frac{1}{n+1},\frac{1}{n+1}+\right.}{a}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+ s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{a}\mathrm{e}^{\frac{c}{a}\int^{x}\left(\left(\frac{b(k+1)\_a^{n+1}-x^{n+1}b(k+1)+y^{k+1}a(n+1)}{a(n+1)}\right)^{(k+1)^{-1}}\right)^{-k}d\_a}\left(\_F1\left(\frac{-x^{n+1}b(k+1)+y^{k+1}a(n+1)}{a\,(n+1)}\right)a\right.$$

129 HFOPDE, chapter 5.3.1

129.1 Problem 1

problem number 1040

Added April 1, 2019.

Problem Chapter 5.3.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (ce^{\lambda x} + se^{\mu y})w + ke^{\nu x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*Exp[lambda*x]+s*Exp[mu*y])*w[x,y] + k*Exp[nu*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\lambda x}}{a\lambda}+\frac{se^{\mu y}}{b\mu}}\left(\int_1^x \frac{k\exp\left(-\frac{se^{\mu\left(\frac{b(K[1]-x)}{a}+y\right)}}{b\mu}-\frac{ce^{\lambda K[1]}}{a\lambda}+\nu K[1]\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*exp(lambda*x)+s*exp(mu*y))*w(x,y)+ k*exp(nu*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\left(\int^{x}\frac{k}{a}\mathrm{e}^{\frac{1}{a\lambda b\mu}\left(-as\lambda\,\mathrm{e}^{\frac{\mu(ya-b(x-\_a))}{a}}+\mu b\left(a\lambda\_a\nu-c\mathrm{e}^{\lambda\_a}\right)\right)}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{\mathrm{e}^{\lambda x}cb\mu+as\lambda\,\mathrm{e}^{\mu y}}{a\lambda b\mu}}$$

129.2 Problem 2

problem number 1041

Added April 1, 2019.

Problem Chapter 5.3.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=ce^{\alpha x+\beta y}w+ke^{\gamma x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Exp[alpha*x+beta*y]*w[x,y] + k*Exp[gamma*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ce^{\alpha x+\beta y}}{a\alpha+b\beta}}\left(\int_1^x \frac{k\exp\left(\gamma K[1]-\frac{ce^{\frac{b\beta(K[1]-x)}{a}+\alpha K[1]+\beta y}}{a\alpha+b\beta}\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
 unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
 unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*exp(alpha*x+beta*y)*w(x,y)+ k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y)=\left(\int^x \frac{k}{a}\mathrm{e}^{\frac{1}{a\alpha+b\beta}\left(-c\mathrm{e}^{\frac{\beta(ya-b(x-\_a))+\_a\,a\alpha}{a}}+\_a\,\gamma\,(a\alpha+b\beta)\right)}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{c\mathrm{e}^{\alpha\,x+\beta\,y}}{a\alpha+b\beta}}$$

129.3 Problem 3

problem number 1042

Added April 1, 2019.

Problem Chapter 5.3.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + be^{\beta x}w_y = ce^{\gamma y}w + se^{\mu x+\delta y}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*Exp[gamma*y]*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\exp\left(-\frac{b\gamma\left(e^{x(\beta-\lambda)}-e^{(\beta-\lambda)K[1]}\right)}{a(\beta-\lambda)}-\lambda K[1]+\gamma y\right)}{a}dK[1]\right)\left(\int_1^x \frac{s\exp\left(-\text{Integrate}\left[\begin{matrix}c\\-\end{matrix}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*exp(gamma*y)*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \left(\int^x \frac{s}{a}\mathrm{e}^{\frac{1}{a(-\beta+\lambda)}\left(-c(-\beta+\lambda)\int \mathrm{e}^{\frac{\mathrm{e}^{-a(\beta-\lambda)}\gamma b-\mathrm{e}^{x(\beta-\lambda)}\gamma b+a(\beta-\lambda)(-\lambda\_a+\gamma y)}{(\beta-\lambda)a}}d\_a-\mathrm{e}^{-a(\beta-\lambda)}b\delta+\mathrm{e}^{x(\beta-\lambda)}b\delta-a(-\beta+\lambda)(\lambda\_a-\mu\right.}\right.$$

129.4 Problem 4

problem number 1043

Added April 1, 2019.

Problem Chapter 5.3.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = sw + ke^{\mu x+\delta y}$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \left(\left(\int^x \frac{k}{a}\left(\frac{1}{a}\left(\lambda c \int \mathrm{e}^{\frac{-\lambda b e^{x(\gamma-\beta)} - a x \beta(-\gamma+\beta)}{(-\gamma+\beta)a}} \mathrm{d}x - c \int \mathrm{e}^{\frac{-\lambda b e^{\_b(\gamma-\beta)} - a\_b\beta(-\gamma+\beta)}{(-\gamma+\beta)a}} \mathrm{d}\_b\lambda + \mathrm{e}^{-\frac{\left(be^{x(\gamma-\beta)}+ay(-\gamma+\beta\right.}{(-\gamma+\beta)a}}\right.\right.\right.\right.$$

129.5 Problem 5

problem number 1044

Added April 1, 2019.

Problem Chapter 5.3.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\beta x}w_x + (be^{\gamma x} + ce^{\lambda y})w_y = se^{\mu x+\delta y}w + k$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[beta*x]*D[w[x, y], x] + (b*Exp[gamma*x]+c*Exp[lambda*y])*D[w[x, y], y] == s*Exp
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ (b*exp(gamma*x)+c*exp(lambda*y))*diff(w(x,y),y) = s*ex
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol);
```

$$w(x,y) = \frac{1}{a}\mathrm{e}^{\frac{s}{a}\int^{x}\left(\frac{1}{a}\left(\lambda c\int \mathrm{e}^{\frac{-\lambda b\mathrm{e}^{x(\gamma-\beta)}-ax\beta(-\gamma+\beta)}{(-\gamma+\beta)a}}dx - c\int \mathrm{e}^{\frac{-\lambda b\mathrm{e}^{\_b(\gamma-\beta)}-a\_b\beta(-\gamma+\beta)}{(-\gamma+\beta)a}}d\_b\lambda + \mathrm{e}^{-\frac{\left(b\mathrm{e}^{x(\gamma-\beta)}+ay(-\gamma+\beta)\right)\lambda}{(-\gamma+\beta)a}}a\right)\right)^{-\frac{\delta}{\lambda}}\mathrm{e}^{\frac{-\delta b}{}}}$$

129.6 Problem 6

problem number 1045

Added April 1, 2019.

Problem Chapter 5.3.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\beta x} w_x + b e^{\gamma x+\lambda y} w_y = c e^{\sigma y} w + k e^{\mu x+deltay} + d$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[beta*x]*D[w[x, y], x] + b*Exp[gamma*x+lambda*y]*D[w[x, y], y] == c*Exp[sigma*y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(beta*x)*diff(w(x,y),x)+ b*exp(gamma*x+lambda*y)*diff(w(x,y),y) = c*exp(sigma*y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{1}{a}\left(k\left(\frac{(-\gamma+\beta)\,a}{-b\lambda\,\mathrm{e}^{-y\lambda}\mathrm{e}^{x(\gamma-\beta)+y\lambda}+\lambda\,b\mathrm{e}^{-b\,(\gamma-\beta)}+\mathrm{e}^{-y\lambda}a\,(-\gamma+\beta)}\right)^{\frac{\delta}{\lambda}}\mathrm{e}^{\frac{1}{a}\left(-c\int\left(\frac{(-\gamma+\beta}{-b\lambda\,\mathrm{e}^{-y\lambda}\mathrm{e}^{x(\gamma-\beta)+y\lambda}+\lambda\,b\mathrm{e}-}\right.\right.}\right.\right.$$

129.7 Problem 7

problem number 1046

Added April 1, 2019.

Problem Chapter 5.3.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = cw + se^{\gamma x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == c*w[x,y]+s*Exp[gamma*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ce^{-\lambda x}}{a\lambda}}\left(\int_1^x \frac{se^{\frac{ce^{-\lambda K[1]}}{a\lambda}+\gamma K[1]-\lambda K[1]}}{a}dK[1]+c_1\left(-\frac{e^{-\lambda x}\left(-a\beta ye^{\lambda x}+a\lambda ye^{\lambda x}+be^{\beta x}\right)}{a(\beta-\lambda)}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = c*w(x,y)+s*exp(gamma*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\left(\int \frac{s}{a}\mathrm{e}^{\frac{ce^{-\lambda x}+ax\lambda(\gamma-\lambda)}{a\lambda}}\,\mathrm{d}x+\_F1\left(\frac{-b\mathrm{e}^{x(\beta-\lambda)}+ay(\beta-\lambda)}{(\beta-\lambda)\,a}\right)\right)\mathrm{e}^{-\frac{ce^{-\lambda x}}{a\lambda}}$$

129.8 Problem 8

problem number 1047

Added April 1, 2019.

Problem Chapter 5.3.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a e^{\lambda y} w_x + b x^{\beta x} w_y = c e^{\gamma x} w + s$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*x^(beta*x)*D[w[x, y], y] == c*Exp[gamma*x]*w[x,y]+s
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(lambda*x)*diff(w(x,y),x)+ b*x^(beta*x)*diff(w(x,y),y) = c*exp(gamma*x)*w(x,y)+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int \frac{s}{a} \mathrm{e}^{\frac{-c\mathrm{e}^{(\gamma-\lambda)x} - a x \lambda (\gamma-\lambda)}{(\gamma-\lambda)a}} \mathrm{d}x + \_F1\left(\frac{ya - b\int x^{\beta x} \mathrm{e}^{-\lambda x} \mathrm{d}x}{a}\right)\right) \mathrm{e}^{\frac{c\mathrm{e}^{(\gamma-\lambda)x}}{(\gamma-\lambda)a}}$$

130 HFOPDE, chapter 5.3.2

130.1 Problem 1

problem number 1048

Added April 2, 2019.

Problem Chapter 5.3.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + bx^n)w_y = cw + ke^{\gamma x}$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y+b*x^n)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*x^n)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\frac{k\mathrm{e}^{x(\gamma-c)}}{\gamma-c} + \_F1\left(-b\int x^n \mathrm{e}^{-\frac{a\mathrm{e}^{\lambda x}}{\lambda}}\,dx + y\mathrm{e}^{-\frac{a\mathrm{e}^{\lambda x}}{\lambda}}\right)\right)\mathrm{e}^{cx}$$

130.2 Problem 2

problem number 1049

Added April 2, 2019.

Problem Chapter 5.3.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + be^{\beta x})w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*Exp[gam
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{cx}\left(c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}}-\int_1^x be^{\beta K[1]-\frac{ae^{\lambda K[1]}}{\lambda}}\,dK[1]\right)-\frac{ke^{x(\gamma-c)}}{c-\gamma}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*exp(ga
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=-\frac{e^{cx}}{-\gamma+c}\left((\gamma-c)\_F1\left(-b\int e^{\frac{\beta x\lambda-ae^{\lambda x}}{\lambda}}\,dx+ye^{-\frac{ae^{\lambda x}}{\lambda}}\right)+ke^{x(\gamma-c)}\right)$$

130.3 Problem 3

problem number 1050

Added April 2, 2019.

Problem Chapter 5.3.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda x}y + be^{\beta x})w_y = cw + kx^n$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y], x] + (a*Exp[lambda*x]*y+b*Exp[beta*x])*D[w[x, y], y] == c*w[x,y]+k*x^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{cx}\left(c_1\left(ye^{-\frac{ae^{\lambda x}}{\lambda}}-\int_1^x be^{\beta K[1]-\frac{ae^{\lambda K[1]}}{\lambda}}dK[1]\right)-\frac{kx^n(cx)^{-n}\text{Gamma}(n+1,cx)}{c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  diff(w(x,y),x)+ (a*exp(lambda*x)*y+b*exp(beta*x))*diff(w(x,y),y) = c*w(x,y)+k*x^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\frac{e^{cx}}{c(n+1)}\left(c(n+1)\_F1\left(-b\int e^{\frac{\beta x\lambda-ae^{\lambda x}}{\lambda}}dx+ye^{-\frac{ae^{\lambda x}}{\lambda}}\right)+kx^n(cx)^{-n/2}e^{-1/2\,cx}\,\text{WhittakerM}\left(n\right.\right.$$

130.4 Problem 4

problem number 1051

Added April 2, 2019.

Problem Chapter 5.3.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (ae^{\lambda y} + bx^k)w_y = cw + ke^{\gamma x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y], x] + (a*Exp[lambda*y]+b*x^k)*D[w[x, y], y] == c*w[x,y]+k*Exp[gamma*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{cx}\left(c_1\left(\frac{a\lambda x\left(-\frac{b\lambda x^{k+1}}{k+1}\right)^{-\frac{1}{k+1}}\text{Gamma}\left(\frac{1}{k+1},-\frac{b\lambda x^{k+1}}{k+1}\right)-(k+1)e^{-\frac{\lambda\left(-bx^{k+1}+ky+y\right)}{k+1}}}{abk(k+1)\lambda^2}\right)-\frac{ke^{x(\gamma-c}}{c-\gamma}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  diff(w(x,y),x)+ (a*exp(lambda*y)+b*x^k)*diff(w(x,y),y) = c*w(x,y)+k*exp(gamma*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = -\frac{e^{cx}}{-\gamma+c}\left((\gamma-c)\_F1\left(\frac{1}{\lambda b\left(2k^2+7k+6\right)}\left(a\left(-\frac{x^{k+1}\lambda b}{k+1}\right)^{\frac{-k-2}{2k+2}} x^{-k} e^{\frac{x^{k+1}\lambda b}{2k+2}} (k+1)(k+2)^2 \mathrm{W}\right.\right.\right.$$

130.5 Problem 5

problem number 1052

Added April 2, 2019.

Problem Chapter 5.3.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = axe^{\lambda x+\mu y}w + be^{\nu x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Exp[lambda*x+mu*y]*w[x,y]+b*Exp[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{axe^{\lambda x+\mu y}}{\lambda x+\mu y}}\left(\int_1^x \frac{b\exp\left(\nu K[1]-\frac{axe^{K[1]\left(\lambda+\frac{\mu y}{x}\right)}}{\lambda x+\mu y}\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) =  a*x*exp(lambda*x+mu*y)*w(x,y)+k*exp(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\left(\int^x \frac{k}{\_a}\mathrm{e}^{-\frac{1}{\lambda x+\mu y}\left(axe^{\frac{\mu y\_a}{x}+\lambda\_a}-\nu\_a(\lambda x+\mu y)\right)}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{ae^{\lambda x+\mu y}\left(\frac{\mu y}{x}+\lambda\right)^{-1}}$$

130.6 Problem 6

problem number 1053

Added April 2, 2019.

Problem Chapter 5.3.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = (aye^{\lambda x} + bxe^{\mu y})w + ce^{\nu x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12, 
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == (a*y*Exp[lambda*x]+b*x*Exp[mu*y])*w[x,y]+c*Exp[n
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{aye^{\lambda x}}{\lambda x}+\frac{bxe^{\mu y}}{\mu y}}\left(\int_1^x \frac{c\exp\left(-\frac{aye^{\lambda K[1]}}{\lambda x}-\frac{bxe^{\frac{\mu y K[1]}{x}}}{\mu y}+\nu K[1]\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  x* diff(w(x,y),x)+ y*diff(w(x,y),y) = (a*y*exp(lambda*x)+b*x*exp(mu*y))*w(x,y)+c*exp
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\left(\int^x \frac{c}{\_a}\mathrm{e}^{-\frac{x}{\lambda\mu y}\left(\frac{\mathrm{e}^{\lambda\_a}y^2a\mu}{x^2}-\frac{\nu\_a\mu y\lambda}{x}+\mathrm{e}^{\frac{\mu y\_a}{x}}b\lambda\right)}d\_a+\_F1\left(\frac{y}{x}\right)\right)\mathrm{e}^{\frac{x}{\lambda\mu y}\left(\frac{a\mathrm{e}^{\lambda x}y^2\mu}{x^2}+\mathrm{e}^{\mu y}b\lambda\right)}$$

130.7 Problem 7

problem number 1054

Added April 2, 2019.

Problem Chapter 5.3.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + be^{\lambda x} w_y = w + ce^{\beta x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*y^k*D[w[x, y], x] + b*Exp[lambda*x]*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to \exp\left(-\frac{(k+1)\left(\left(y^{k+1}\right)^{\frac{1}{k+1}}\right)^{-k}\left(\frac{a\lambda y^{k+1}e^{-\lambda x}}{bk+b}\right)^{\frac{k}{k+1}}\text{Hypergeometric2F1}\left(\frac{k}{k+1},\frac{k}{k+1},\frac{k}{k+1}+1,1-\right.}{ak\lambda}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*y^k* diff(w(x,y),x)+ b*exp(lambda*x)*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{c}{a}\left(\left(\frac{b(k+1)\,\mathrm{e}^{\_b\,\lambda} - \mathrm{e}^{\lambda x} b(k+1) + y^k y a\lambda}{a\lambda}\right)^{(k+1)^{-1}}\right)^{-k} \mathrm{e}^{\frac{1}{a}\left(\_b\,a\beta - \int\left(\left(\frac{b(k+1)\mathrm{e}^{\_b\,\lambda} - \mathrm{e}^{\lambda x} b(k+1) + y^k y a}{a\lambda}\right.\right.\right.}$$

130.8 Problem 8

problem number 1055

Added April 2, 2019.

Problem Chapter 5.3.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda x}w_x + byw_y = w + ce^{\lambda x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*x]*D[w[x, y], x] + b*y*D[w[x, y], y] == w[x,y]+c*Exp[lambda*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to\frac{e^{-\frac{e^{-\lambda x}}{a\lambda}}\left(a\lambda c_1\left(ye^{\frac{be^{-\lambda x}}{a\lambda}}\right)-c\text{ExpIntegralEi}\left(\frac{e^{-\lambda x}}{a\lambda}\right)\right)}{a\lambda}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(lambda*x)* diff(w(x,y),x)+ b*y*diff(w(x,y),y) = w(x,y)+c*exp(lambda*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\frac{1}{a\lambda}\left(\_F1\left(y\mathrm{e}^{\frac{b\mathrm{e}^{-\lambda x}}{a\lambda}}\right)a\lambda+c\,\text{expIntegral}\left(1,-\frac{\mathrm{e}^{-\lambda x}}{a\lambda}\right)\right)\mathrm{e}^{-\frac{\mathrm{e}^{-\lambda x}}{a\lambda}}$$

130.9 Problem 9

problem number 1056

Added April 2, 2019.

Problem Chapter 5.3.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x+bx^kw_y=w+ce^{\beta x}$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*x^k*D[w[x, y], y] == w[x,y]+c*Exp[beta*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{(k+1)x\text{Hypergeometric2F1}\left(1,\frac{1}{k+1},\frac{1}{k+1}+1,\frac{b\lambda x^{k+1}}{b\lambda x^{k+1}-a(k+1)e^{\lambda y}}\right)}{a(k+1)e^{\lambda y}-b\lambda x^{k+1}}\right)\left(\int_1^x \frac{c(k+1)\exp}{\quad}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*x^k*diff(w(x,y),y) = w(x,y)+c*exp(beta*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y)))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\left(\int^x \frac{c(k+1)}{-x^{k+1}\lambda b+\_b^{k+1}\lambda b+e^{y\lambda}a(k+1)}e^{\frac{1}{\lambda b}\left((-1-k)\int\frac{\lambda b}{-x^{k+1}\lambda b+\_b^{k+1}\lambda b+e^{y\lambda}a(k+1)}d\_b+b\_b\lambda\beta\right)}d\_b+\_$$

130.10 Problem 10

problem number 1057

Added April 2, 2019.

Problem Chapter 5.3.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ae^{\lambda y}w_x + be^{\beta x}w_y = w + cx^k$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Exp[lambda*y]*D[w[x, y], x] + b*Exp[beta*x]*D[w[x, y], y] == w[x,y]+c*x^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to\exp\left(\frac{\beta x-\log\left(\frac{a\beta e^{\lambda y}}{\lambda}\right)}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right)\left(\int_1^x\frac{\beta cK[1]^k\exp\left(\frac{\log\left(b\left(e^{\beta K[1]}-e^{\beta x}\right)+\frac{a\beta e^{\lambda y}}{\lambda}\right)-\beta K[1]}{a\beta e^{\lambda y}-b\lambda e^{\beta x}}\right)}{b\lambda\left(e^{\beta K[1]}-e^{\beta x}\right)+a\beta e^{\lambda y}}dK[1]+c_1\left(\right.\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*exp(lambda*y)* diff(w(x,y),x)+ b*exp(beta*x)*diff(w(x,y),y) = w(x,y)+c*x^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\left(\int^{x} \frac{c\_a^k \beta}{\lambda b} \left(\frac{e^{y\lambda} a\beta - e^{\beta x} b\lambda}{\lambda b} + e^{\beta \_a} \right)^{\frac{-e^{y\lambda} a\beta + e^{\beta x} b\lambda + 1}{e^{y\lambda} a\beta - e^{\beta x} b\lambda}} \left(e^{\beta \_a}\right)^{-\left(e^{y\lambda} a\beta - e^{\beta x} b\lambda\right)^{-1}} d\_a + \_F1\left(\frac{e^{y\lambda} a\beta -}{b\beta} \right.\right.\right.$$

131 HFOPDE, chapter 5.4.1

131.1 Problem 1

problem number 1058

Added April 3, 2019.

Problem Chapter 5.4.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sinh^k(\lambda x)\sinh^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sinh[lambda*x]^k*Sinh[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sinh^k(\lambda K[1])\sinh^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*sinh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\frac{cx}{a}} \left(\_F1\left(\frac{ya-bx}{a}\right) + \int^x \frac{(\sinh(\lambda\_a))^k}{a} \left(\sinh\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\, c}{a}} d\_a \right)$$

131.2 Problem 2

problem number 1059

Added April 3, 2019.

Problem Chapter 5.4.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\sinh^k(\lambda x)w + s\sinh^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Sinh[lambda*x]^k*w[x,y]+ s*Sinh[beta*x]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*sinh(lambda*x)^k*w(x,y)+s*sinh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\int \frac{(\sinh(\lambda x))^k c}{a} dx} \left(\_F1\left(\frac{ya - bx}{a}\right) + \int \frac{s(\sinh(\beta x))^n}{a} e^{-\frac{c \int (\sinh(\lambda x))^k dx}{a}} dx \right)$$

131.3 Problem 3

problem number 1060

Added April 3, 2019.

Problem Chapter 5.4.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \sinh^{n_1}(\lambda_1 x) + c_2 \sinh^{n_2}(\lambda_2 y))\, w + s_1 \sinh^{k_1}(\beta_1 x) + s_2 \sinh^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Sinh[lambda1*x]^n1 + c2*Sinh[lambda2*y]^n2)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*sinh(lambda1*x)^n1 + c2*sinh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{1}{a}\left(c1\ (\sinh(\lambda 1\_a))^{n1} + \left(\sinh\left(\frac{\lambda 2\ (ya - b(x - \_a))}{a}\right)\right)^{n2} c2\right) d\_a} \left(\_F1\left(\frac{ya - bx}{a}\right) + \int^x \frac{1}{a} \mathrm{e}^{\frac{1}{a}\left(-c1\ \int (\sinh(\lambda 1\_f))^{n1}\right.}\right.$$

131.4 Problem 4

problem number 1061

Added April 3, 2019.

Problem Chapter 5.4.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a \sinh^n(\lambda x) w_x + b \sinh^m(\mu x) w_y = c \sinh^k(\nu x) w + p \sinh^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*x]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \int^x \frac{(\sinh(\_b\,\lambda))^{-n}}{a}\left(c\sinh(\nu\,\_b)\,w_{x,y} + \left(\sinh\left(\frac{\beta\left(b\int(\sinh(\_b\,\mu))^m(\sinh(\_b\,\lambda))^{-n}\,\mathrm{d}\_b + y\right.}{a}\right.\right.\right.$$

131.5 Problem 5

problem number 1062

Added April 3, 2019.

Problem Chapter 5.4.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^n(\lambda x)w_x + b\sinh^m(\mu x)w_y = c\sinh^k(\nu y)w + p\sinh^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Sinh[mu*x]^m*D[w[x, y], y] == c*Sinh[nu*y]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*sinh(mu*x)^m*diff(w(x,y),y) = c*sinh(nu*y)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \int^x \frac{(\sinh(\_b\,\lambda))^{-n}}{a} \left(w_{x,y} c \sinh\left(\frac{\nu \left(b \int (\sinh(\_b\,\mu))^m (\sinh(\_b\,\lambda))^{-n} \,\mathrm{d}\_b + ya - b \int (\sinh(\mu x\right.}{a}\right.\right.$$

132 HFOPDE, chapter 5.4.2

132.1 Problem 1

problem number 1063

Added April 3, 2019.

Problem Chapter 5.4.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cosh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Cosh[lambda*x]^k*Cosh[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cosh^k(\lambda K[1])\cosh^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}\,dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+cosh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\frac{cx}{a}}\left(\int^x \frac{(\cosh(\lambda\_a))^k}{a}\left(\cosh\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n e^{-\frac{\_a\,c}{a}}d\_a + \_F1\left(\frac{ya-bx}{a}\right)\right)$$

132.2 Problem 2

problem number 1064

Added April 3, 2019.

Problem Chapter 5.4.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\cosh^k(\lambda x)w + s\cosh^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Cosh[lambda*x]^k*w[x,y]+ s*Cosh[beta*x]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*cosh(lambda*x)^k*w(x,y)+s*cosh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\int \frac{(\cosh(\lambda x))^k c}{a} dx} \left(\_F1\left(\frac{ya - bx}{a}\right) + \int \frac{s(\cosh(\beta x))^n}{a} e^{-\frac{c \int (\cosh(\lambda x))^k dx}{a}} dx \right)$$

132.3 Problem 3

problem number 1065

Added April 3, 2019.

Problem Chapter 5.4.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \cosh^{n_1}(\lambda_1 x) + c_2 \cosh^{n_2}(\lambda_2 y))\, w + s_1 \cosh^{k_1}(\beta_1 x) + s_2 \cosh^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Cosh[lambda1*x]^n1 + c2*Cosh[lambda2*y]^n2)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*cosh(lambda1*x)^n1 + c2*cosh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{1}{a}\left(c1\ (\cosh(\lambda 1\_a))^{n1}+\left(\cosh\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right)\right)^{n2} c2\right)d\_a}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x \frac{1}{a}\mathrm{e}^{\frac{1}{a}\left(-c1\ \int(\cosh(\lambda 1\_f))^{n1}\right.}\right.$$

132.4 Problem 4

problem number 1066

Added April 3, 2019.

Problem Chapter 5.4.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cosh(\lambda x + \mu y)w + b\cosh(\nu x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cosh[lambda*x+my*y]+b*Cosh[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \frac{ax\sinh(\lambda x+\mathrm{my}y)}{\lambda x+\mathrm{my}y}+b\mathrm{Chi}(\nu x)+c_1\left(\frac{y}{x}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = a*x*cosh(lambda*x+my*y)+b*cosh(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = -1/2\,a\mathrm{e}^{-\lambda\,x-my\,y}\left(\frac{my\,y}{x}+\lambda\right)^{-1}+1/2\,a\mathrm{e}^{\lambda\,x+my\,y}\left(\frac{my\,y}{x}+\lambda\right)^{-1}-1/2\,b\,\mathrm{expIntegral}\left(1,-\nu\,x\right)-1/2$$

132.5 Problem 5

problem number 1067

Added April 3, 2019.

Problem Chapter 5.4.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cosh^n(\lambda x)w_x + b\cosh^m(\mu x)w_y = c\cosh^k(\nu x)w + p\cosh^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k+p*C
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \int^{x} \frac{(\cosh(\_b\,\lambda))^{-n}}{a} \left(c(\cosh(\nu\,\_b))^{k} + \left(\cosh \left(\frac{\beta\left(b \int (\cosh(\_b\,\mu))^{m} (\cosh(\_b\,\lambda))^{-n}\, d\_b - b \right.}{a} \right.\right.\right.$$

132.6 Problem 6

problem number 1068

Added April 3, 2019.

Problem Chapter 5.4.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cosh^{n}(\lambda x)w_x + b\cosh^{m}(\mu x)w_y = c\cosh^{k}(\nu y)w + p\cosh^{s}(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Cosh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*y]^k+p*C
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*cosh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*y)^k+p*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \int^x \frac{(\cosh(\_b\,\lambda))^{-n}}{a}\left(\left(\cosh\left(\frac{\nu\left(b\int(\cosh(\_b\,\mu))^m\,(\cosh(\_b\,\lambda))^{-n}\,\mathrm{d}\_b - b\int(\cosh(\mu\,x))^m\,(\cos\right.}{a}\right.\right.\right.$$

133 HFOPDE, chapter 5.4.3

133.1 Problem 1

problem number 1069

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \tanh^k(\lambda x)\tanh^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Tanh[lambda*x]^k*Tanh[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tanh^k(\lambda K[1])\tanh^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+tanh(lambda*x)^k*tanh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\frac{cx}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^{x}\frac{(\tanh(\lambda\_a))^{k}}{a}\left(\tanh\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^{n}\mathrm{e}^{-\frac{\_a\,c}{a}}d\_a\right)$$

133.2 Problem 2

problem number 1070

Added April 3, 2019.

Problem Chapter 5.4.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\tanh^k(\lambda x)w + s\tanh^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Tanh[lambda*x]^k*w[x,y]+ s*Tanh[beta*x]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*tanh(lambda*x)^k*w(x,y)+s*tanh(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\int \frac{(\tanh(\lambda x))^k c}{a} dx} \left(\_F1\left(\frac{ya-bx}{a}\right) + \int \frac{s(\tanh(\beta x))^n}{a} e^{-\frac{c\int(\tanh(\lambda x))^k dx}{a}} dx \right)$$

133.3 Problem 3

problem number 1071

Added April 3, 2019.

Problem Chapter 5.4.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \tanh^{n_1}(\lambda_1 x) + c_2 \tanh^{n_2}(\lambda_2 y))\, w + s_1 \tanh^{k_1}(\beta_1 x) + s_2 \tanh^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Tanh[lambda1*x]^n1 + c2*Tanh[lambda2*y]^n2)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*tanh(lambda1*x)^n1 + c2*tanh(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{1}{a}\left(c1\,(\tanh(\lambda 1\_a))^{n1}+c2\left(\tanh\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right)\right)^{n2}\right)d\_a}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x \frac{1}{a}\mathrm{e}^{\frac{1}{a}\left(-c1\ \int(\tanh(\lambda 1\_f))^{n}\right.}\right.$$

133.4 Problem 4

problem number 1072

Added April 3, 2019.

Problem Chapter 5.4.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^n(\lambda x)w_x + b\tanh^m(\mu x)w_y = c\tanh^k(\nu x)w + p\tanh^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*x]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*x)*w[x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \int^{x} \frac{(\tanh(\_b\,\lambda))^{-n}}{a} \left(c\tanh(\nu\,\_b)\,w_{x,y} + \left(1\sinh\left(\frac{\beta}{a} \left(b \int (\tanh(\_b\,\mu))^{m} (\tanh(\_b\,\lambda))^{-n}\,\mathrm{d}\_\right.\right.\right.\right.$$

133.5 Problem 5

problem number 1073

Added April 3, 2019.

Problem Chapter 5.4.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^{n}(\lambda x)w_{x} + b\tanh^{m}(\mu x)w_{y} = c\tanh^{k}(\nu y)w + p\tanh^{s}(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Tanh[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*tanh(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^{x} \frac{c(\tanh(\_b\,\lambda))^{-n}}{a} \tanh\left(\frac{\nu}{a}\left(-b\int\left(\frac{\sinh(\mu\,x)}{\cosh(\mu\,x)}\right)^{m}\left(\frac{\sinh(\lambda\,x)}{\cosh(\lambda\,x)}\right)^{-n} \mathrm{d}x + a\left(\int \frac{b(\tanh(\_b\,\mu))^{m}(\tanh(\_b\,\lambda))^{-n}}{a} \mathrm{d}\_b + y\right)\right)\right) d\_b} \left(\_F1\left(\right.\right.$$

134 HFOPDE, chapter 5.4.4

134.1 Problem 1

problem number 1074

Added April 3, 2019.

Problem Chapter 5.4.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \coth^k(\lambda x)\coth^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Coth[lambda*x]^k*Coth[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\coth^k(\lambda K[1])\coth^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+coth(lambda*x)^k*coth(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\frac{cx}{a}} \left(\_F1\left(\frac{ya-bx}{a}\right) + \int^x \frac{(\coth(\lambda\_a))^k}{a} \left(\coth\left(\frac{\beta\,(ya-b(x-\_a))}{a}\right)\right)^n e^{-\frac{\_a\,c}{a}} d\_a \right)$$

134.2 Problem 2

problem number 1075

Added April 3, 2019.

Problem Chapter 5.4.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = c\coth^k(\lambda x)w + s\coth^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Coth[lambda*x]^k*w[x,y]+ s*Coth[beta*x]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*coth(lambda*x)^k*w(x,y)+s*coth(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\int \frac{(\coth(\lambda x))^k c}{a} dx} \left(\_F1\left(\frac{ya - bx}{a}\right) + \int \frac{s(\coth(\beta x))^n}{a} e^{-\frac{c \int (\coth(\lambda x))^k dx}{a}} dx \right)$$

134.3 Problem 3

problem number 1076

Added April 3, 2019.

Problem Chapter 5.4.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \coth^{n_1}(\lambda_1 x) + c_2 \coth^{n_2}(\lambda_2 y))\, w + s_1 \coth^{k_1}(\beta_1 x) + s_2 \coth^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c1*Coth[lambda1*x]^n1 + c2*Coth[lambda2*y]^n2)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c1*coth(lambda1*x)^n1 + c2*coth(lambda2*y)^n2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^{x} \frac{1}{a}\left(c1\,(\coth(\lambda 1\_a))^{n1}+c2\,\left(\coth\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right)\right)^{n2}\right)d\_a}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^{x}\frac{1}{a}\mathrm{e}^{\frac{1}{a}\left(-c1\,\int(\coth(\lambda 1\_f))^{n1}\right.}\right.$$

134.4 Problem 4

problem number 1077

Added April 3, 2019.

Problem Chapter 5.4.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\coth^{n}(\lambda x)w_{x}+b\coth^{m}(\mu x)w_{y}=c\coth^{k}(\nu x)w+p\coth^{s}(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12, 
 pde = a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*x]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*x)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int \frac{\coth(\nu x)c(\coth(\lambda x))^{-n}}{a}\mathrm{d}x} \left(\_F1\left(\frac{1}{a}\left(ya - b\int \left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^{m} \left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n} \mathrm{d}x\right)\right) + \int^{x} \frac{p}{a}\left(1 \mathrm{co}\right.\right.$$

134.5 Problem 5

problem number 1078

Added April 3, 2019.

Problem Chapter 5.4.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\coth^{n}(\lambda x)w_{x} + b\coth^{m}(\mu x)w_{y} = c\coth^{k}(\nu y)w + p\coth^{s}(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Coth[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Coth[nu*y]*w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*coth(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*coth(nu*y)*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^{x} \frac{c(\coth(\_b\,\lambda))^{-n}}{a} \coth\left(\frac{\nu}{a}\left(-b\int\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^{m}\left(\frac{\cosh(\lambda x)}{\sinh(\lambda x)}\right)^{-n} \mathrm{d}x + a\left(\int \frac{b(\coth(\_b\,\mu))^{m}(\coth(\_b\,\lambda))^{-n}}{a}\,\mathrm{d}\_b + y\right)\right)\right) d\_b} \left(\_F1\left(\frac{1}{a}\right.\right.$$

135 HFOPDE, chapter 5.4.5

135.1 Problem 1

problem number 1079

Added April 4, 2019.

Problem Chapter 5.4.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sinh^k(\lambda x) + c_2 \cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sinh[lambda*x]^k+c2*Cosh[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to -\frac{\text{c1}\left(e^{\lambda x}-e^{-\lambda x}\right)^k\left(2-2e^{2\lambda x}\right)^{-k}\text{Hypergeometric2F1}\left(-k,-\frac{ak\lambda+1}{2a\lambda},-\frac{1}{2a\lambda}-\frac{k}{2}+1,e^{2\lambda x}\right)}{ak\lambda+1} - \right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sinh(lambda*x)^k+c2*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\frac{x}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^{x}\frac{1}{a}\left(c2\left(\cosh\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^{n}+c1\left(\sinh\left(\lambda\_a\right)\right)^{k}\right)\mathrm{e}^{-\frac{\_a}{a}}d\_$$

135.2 Problem 2

problem number 1080

Added April 4, 2019.

Problem Chapter 5.4.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sinh^k(\lambda x)\cosh^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sinh[lambda*x]^k*Cosh[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sinh^k(\lambda K[1])\cosh^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sinh(lambda*x)^k*cosh(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=\mathrm{e}^{\frac{cx}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x\frac{(\sinh(\lambda\_a))^k}{a}\left(\cosh\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n\mathrm{e}^{-\frac{\_a\, c}{a}}d\_a\right)$$

135.3 Problem 3

problem number 1081

Added April 4, 2019.

Problem Chapter 5.4.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tanh(\lambda x) + s\coth(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Tanh[lambda*x]+s*coth[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\left(s\coth\left(\mu\left(\frac{b(K[1]-x)}{a}+y\right)\right)+k\tanh(\lambda K[1])\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*tanh(lambda*x)+s*coth(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\frac{cx}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x -\frac{1}{a}\left((k-s)\cosh\left(\frac{(-\lambda\_a+\mu y)a-b\mu(x-\_a)}{a}\right)-\cosh\left(\frac{(\lambda\_\right.\right.\right.$$

135.4 Problem 4

problem number 1082

Added April 4, 2019.

Problem Chapter 5.4.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sinh(\lambda x)w_y = cw + k\cosh(\mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Sinh[lambda*x]*D[w[x, y], y] == c*w[x,y]+ k*Cosh[mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{ke^{-\frac{cK[1]}{a}}\cosh\left(\frac{\mu(b\cosh(\lambda K[1])+a\lambda y-b\cosh(\lambda x))}{a\lambda}\right)}{a}dK[1]+c_1\left(y-\frac{b\cosh(\lambda x)}{a\lambda}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*sinh(lambda*x)*diff(w(x,y),y) = c*w(x,y)+k*cosh(mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y)=e^{\frac{cx}{a}}\left(\_F1\left(\frac{y\lambda a-b\cosh(\lambda x)}{a\lambda}\right)+\int^x\frac{k}{a}\cosh\left(\frac{\mu(y\lambda a-b\cosh(\lambda x)+b\cosh(\lambda\_a))}{a\lambda}\right)e^{-\frac{ac}{a}}d\_$$

135.5 Problem 5

problem number 1083

Added April 4, 2019.

Problem Chapter 5.4.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sinh^n(\lambda x)w_x + b\cosh^m(\mu x)w_y = c\cosh^k(\nu x)w + p\sinh^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sinh[lambda*x]^n*D[w[x, y], x] + b*Cosh[mu*x]^m*D[w[x, y], y] == c*Cosh[nu*x]^k*w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*sinh(lambda*x)^n*diff(w(x,y),x)+ b*cosh(mu*x)^m*diff(w(x,y),y) = c*cosh(nu*x)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = e^{\int \frac{c(\cosh(\nu x))^k(\sinh(\lambda x))^{-n}}{a} dx} \left(\int^x \frac{p(\sinh(\lambda \_f))^{-n}}{a} \left(\sinh\left(\frac{\beta \left(b \int (\cosh(\_f\,\mu))^m (\sinh(\lambda \_f))^{-n} \, d\_f \right.}{} \right.\right.\right.$$

135.6 Problem 6

problem number 1084

Added April 4, 2019.

Problem Chapter 5.4.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tanh^n(\lambda x)w_x + b\coth^m(\mu x)w_y = c\tanh^k(\nu y)w + p\coth^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tanh[lambda*x]^n*D[w[x, y], x] + b*Coth[mu*x]^m*D[w[x, y], y] == c*Tanh[nu*y]^k*w[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*tanh(lambda*x)^n*diff(w(x,y),x)+ b*coth(mu*x)^m*diff(w(x,y),y) = c*tanh(nu*y)^k*w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{c(\tanh(\_b\,\lambda))^{-n}}{a}\left(1\sinh\left(\frac{\nu}{a}\left(\int\frac{b}{a}\left(\frac{\sinh(\_b\,\lambda)}{\cosh(\_b\,\lambda)}\right)^{-n}\left(\frac{\cosh(\_b\,\mu)}{\sinh(\_b\,\mu)}\right)^m \mathrm{d}\_ba+ya-b\int\left(\frac{\sinh(\lambda x)}{\cosh(\lambda x)}\right)^{-n}\left(\frac{\cosh(\mu x)}{\sinh(\mu x)}\right)^m \mathrm{d}x\right)\right)\left(\cosh\left(\frac{\nu}{a}\left(\int\frac{b}{a}\right.\right.\right.\right.}$$

136 HFOPDE, chapter 5.5.1

136.1 Problem 1

problem number 1085

Added April 5, 2019.

Problem Chapter 5.5.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \ln^k(\lambda x)\ln^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Log[lambda*x]^k*Log[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ln(lambda*x)^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{(\ln(\lambda \_a))^k}{a}\left(\ln\left(\frac{\beta(ya - b(x-\_a))}{a}\right)\right)^n e^{-\frac{\_a c}{a}} d\_a + \_F1\left(\frac{ya-bx}{a}\right)\right) e^{\frac{cx}{a}}$$

136.2 Problem 2

problem number 1086

Added April 5, 2019.

Problem Chapter 5.5.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = c\ln^k(\lambda x)w + s\ln^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*Log[lambda*x]^k*w[x,y]+s*Log[beta*x]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*ln(lambda*x)^k*w(x,y)+s*ln(beta*x)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x, y) = \left(\int \frac{s(\ln(\beta x))^n}{a} \mathrm{e}^{-\frac{c\int(\ln(\lambda x))^k \mathrm{d}x}{a}} \mathrm{d}x + \_F1\left(\frac{ya - bx}{a}\right)\right) \mathrm{e}^{\int \frac{c(\ln(\lambda x))^k}{a} \mathrm{d}x}$$

136.3 Problem 3

problem number 1087

Added April 5, 2019.

Problem Chapter 5.5.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \ln^{n_1}(\lambda_1 x) + c_2 \ln^{n_2}(\lambda_2 y))\, w + s_1 \ln^{k_1}(\beta_1 x) + s_2 \ln^{k_2}(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*Log[lambda1*x]^n1 +c2*Log[lambda2*y]^n2)*w[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*ln(lambda1*x)^n1 +c2*ln(lambda2*y)^n2)*w(x
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{1}{a} \mathrm{e}^{\frac{1}{a}\left(-c1 \int (\ln(\lambda 1\_f))^{n1} \mathrm{d}\_f - c2 \int \left(\ln\left(\frac{\lambda 2 (ya - b(x - \_f))}{a}\right)\right)^{n2} \mathrm{d}\_f\right)} \left(s2 \ln\left(\frac{\beta 2 (ya - b(x - \_f))}{a}\right) k2 + s\right.\right.$$

136.4 Problem 4

problem number 1088

Added April 5, 2019.

Problem Chapter 5.5.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln(\lambda x)w_x + b\ln(\mu y)w_y = cw + k$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Log[lambda*x]*D[w[x, y], x] + b*Log[mu*y]*D[w[x, y], y] == c*w[x,y]+k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*ln(lambda*x)*diff(w(x,y),x)+ b*ln(mu*y)*diff(w(x,y),y) =c*w(x,y)+k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \frac{1}{c}\left(e^{-\frac{c\,\mathrm{expIntegral}(1,-\ln(\lambda x))}{a\lambda}}\_F1\left(\frac{-a\,\mathrm{expIntegral}\left(1,-\ln\left(\mu y\right)\right)\lambda + \mathrm{expIntegral}\left(1,-\ln\left(\lambda x\right)\right)b\mu}{\lambda\,b\mu}\right)c$$

136.5 Problem 5

problem number 1089

Added April 5, 2019.

Problem Chapter 5.5.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x + b\ln^m(\mu x)w_y = c\ln^k(\nu x)w + p\ln^s(\beta y) + q$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*x)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{(\ln(\lambda\_f))^{-n}}{a} e^{-\frac{c\int(\ln(\nu\_f))^k(\ln(\lambda\_f))^{-n}\,d\_f}{a}} \left(p\left(\ln\left(\frac{\beta\left(ya - b\int(\ln(\lambda x))^{-n}(\ln(\mu x))^m\,dx + b\int\right.}{a}\right.\right.\right.\right.$$

136.6 Problem 6

problem number 1090

Added April 5, 2019.

Problem Chapter 5.5.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\ln^n(\lambda x)w_x + b\ln^m(\mu x)w_y = c\ln^k(\nu y)w + p\ln^s(\beta x) + q$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Log[lambda*x]^n*D[w[x, y], x] + b*Log[mu*x]^m*D[w[x, y], y] == c*Log[nu*y]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*ln(lambda*x)^n*diff(w(x,y),x)+ b*ln(mu*x)^m*diff(w(x,y),y) = c*ln(nu*y)^k*w(x,y)+p
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
sol:=simplify(sol,size);
```

$$w(x,y) = \left(\int^x \frac{(\ln(\lambda\_f))^{-n}(p(\ln(\beta\_f))^s + q)}{a} e^{-\frac{c}{a}\int\left(\ln\left(\frac{\nu\left(ya - b\int(\ln(\lambda x))^{-n}(\ln(\mu x))^m \,dx + b\int(\ln(\lambda\_f))^{-n}(\ln(\_f\mu))^m \,d\_f\right)}{a}\right)\right)}\right.$$

137 HFOPDE, chapter 5.5.2

137.1 Problem 1

problem number 1091

Added April 8, 2019.

Problem Chapter 5.5.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 x^k + c_2 \ln^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*x^k+c2*Log[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}\left(\int_1^x \frac{e^{-\frac{K[1]}{a}}\left(\mathrm{c1}K[1]^k+\mathrm{c2}\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*x^k+c2*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^{x}\frac{1}{a}\mathrm{e}^{-\frac{\_a}{a}}\left(c2\left(\ln\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^{n}+c1\,\_a^{k}\right)d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{x}{a}}$$

137.2 Problem 2

problem number 1092

Added April 8, 2019.

Problem Chapter 5.5.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x+bw_y=cw+x^k\ln^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x^k*Log[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}K[1]^k\log^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x^k*ln(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{\_a^k}{a}\left(\ln\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^n e^{-\frac{\_a\, c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)e^{\frac{cx}{a}}$$

137.3 Problem 3

problem number 1093

Added April 8, 2019.

Problem Chapter 5.5.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-k}}{a-ak}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-k}}{a(k-1)}} sK[1]^{-k}\log^m(\beta K[1])}{a}dK[1]+c_1\left(y-\frac{bx^{-k+n+1}}{a(-k)+an+a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int \frac{s(\ln(\beta x))^m x^{-k}}{a}\mathrm{e}^{\frac{cx^{1-k}}{a(k-1)}}\,\mathrm{d}x+\_F1\left(\frac{x^{1-k+n}b+ay(-n-1+k)}{(-n-1+k)\,a}\right)\right)\mathrm{e}^{-\frac{cx^{1-k}}{a(k-1)}}$$

137.4 Problem 4

problem number 1094

Added April 8, 2019.

Problem Chapter 5.5.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + by^k w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x^n*D[w[x, y], x] + b*y^k*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx^{1-n}}{a-an}}\left(\int_1^x \frac{e^{\frac{cK[1]^{1-n}}{a(n-1)}}sK[1]^{-n}\log^m(\beta K[1])}{a}dK[1]+c_1\left(\frac{bx^{1-n}}{a(n-1)}-\frac{y^{1-k}}{k-1}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x^n*diff(w(x,y),x)+ b*y^k*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int \frac{s(\ln(\beta x))^m x^{-n}}{a}\mathrm{e}^{\frac{cx^{-n+1}}{a(n-1)}}\,\mathrm{d}x+\_F1\left(\frac{-x^{-n+1}b(k-1)+y^{1-k}a(n-1)}{a(n-1)}\right)\right)\mathrm{e}^{-\frac{cx^{-n+1}}{a(n-1)}}$$

137.5 Problem 5

problem number 1095

Added April 8, 2019.

Problem Chapter 5.5.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^n w_x + b\ln^n(\lambda x) w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x^n*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\rightarrow e^{\frac{cx^{1-n}}{a-an}}\left(c_1\left(\frac{(n-1)^{-n-1}\left(b\lambda^n\text{Gamma}(n+1,(n-1)(\log(\lambda)+\log(x)))+a\lambda(n-1)^{n+1}y\right)}{a\lambda}\right)\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x^n*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \frac{1}{ac\,(m-3\,n+3)\,(m-2\,n+2)\,(-n+m+1)}\mathrm{e}^{-\frac{cx^{-n+1}}{a(n-1)}}\left(-a\mathrm{e}^{1/2\,\frac{cx^{-n+1}}{a(n-1)}}\left(-\frac{cx^{-n+1}}{a\,(n-1)}\right)^{\frac{m-2\,n+2}{2\,n-2}}\left(-\right.\right.$$

137.6 Problem 6

problem number 1096

Added April 8, 2019.

Problem Chapter 5.5.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + bx^n w_y = cw + s\ln^m(\beta x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*y^k*D[w[x, y], x] + b*x^n*D[w[x, y], y] == c*w[x,y]+s*Log[beta*x]^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\frac{cx\left(\left(y^{-k-1}\right)^{-\frac{1}{k+1}}\right)^{-k}\left(\frac{a(n+1)y^{k+1}}{a(n+1)y^{k+1}-b(k+1)x^{n+1}}\right)^{\frac{k}{k+1}}{}_2F_1\left(\frac{k}{k+1},\frac{1}{n+1};1+\frac{1}{n+1};\frac{b(k+1)x^{n+}}{b(k+1)x^{n+1}-a(n-}\right.}{a}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*y^k*diff(w(x,y),x)+ b*x^n*diff(w(x,y),y) = c*w(x,y)+s*ln(beta*x)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^{x}\frac{s(\ln(\beta\_b))^{m}}{a}\left(\left(\frac{-x^{n+1}b(k+1)+y^{k+1}a(n+1)+b\_b^{n+1}(k+1)}{a\,(n+1)}\right)^{(k+1)^{-1}}\right)^{-k}\mathrm{e}^{-\frac{c}{a}\int\left(\left(\frac{-x^{n}}{}\right.\right.}\right.$$

137.7 Problem 7

problem number 1097

Added April 8, 2019.

Problem Chapter 5.5.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ay^k w_x + b\ln^n(\lambda x) w_y = cw + sx^m$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*y^k*D[w[x, y], x] + b*Log[lambda*x]^n*D[w[x, y], y] == c*w[x,y]+s*x^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{c\left(\left(\frac{a\lambda y^{k+1}-b(k+1)\text{Gamma}(n+1,-\log(\lambda x))(-\log(\lambda x))^{-n}\log^n(\lambda x)+b(k+1)\text{Gamma}(n+1,-\log(\lambda K[1]))(-}{a\lambda}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*y^k*diff(w(x,y),x)+ b*ln(lambda*x)^n*diff(w(x,y),y) = c*w(x,y)+s*x^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{s\_f^{m}}{a} \left(\left(\frac{b(k+1) \int (\ln(\lambda\_f))^{n} \, \mathrm{d}\_f - b(k+1) \int (\ln(\lambda x))^{n} \, \mathrm{d}x + y^{k} y a}{a} \right)^{(k+1)^{-1}} \right)^{-k} \mathrm{e}^{-\frac{c((}{}}$$

138 HFOPDE, chapter 5.6.1

138.1 Problem 1

problem number 1098

Added April 8, 2019.

Problem Chapter 5.6.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)-\frac{k((a\lambda+b\mu)\cos(\lambda x+\mu y)+c\sin(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2}\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \frac{1}{a^2\lambda^2 + 2\,ab\lambda\,\mu + \mu^2b^2 + c^2}\left((a^2\lambda^2 + 2\,ab\lambda\,\mu + \mu^2b^2 + c^2)\,\_F1\left(\frac{ya - bx}{a}\right) - k((a\lambda + b\mu)\cos(\lambda$$

138.2 Problem 2

problem number 1099

Added April 8, 2019.

Problem Chapter 5.6.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \sin^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Sin[lambda*x]^k+c2*Sin[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{x}{a}}c_1\left(y-\frac{bx}{a}\right)-i\left(\frac{\mathrm{c1}\left(-1+e^{2i\lambda x}\right)\sin^k(\lambda x)\,{}_2F_1\left(1,\frac{1}{2}\left(k+\frac{i}{a\lambda}+2\right);\frac{1}{2}\left(-k+\frac{i}{a\lambda}+2\right);e^{2i\lambda x}\right)}{ak\lambda-i}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+c1*sin(lambda*x)^k+c2*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{x}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x\frac{1}{a}\left(c1\,(\sin(\lambda\_a))^k+c2\left(\sin\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n\right)\mathrm{e}^{-\frac{\_a}{a}}d\_a$$

138.3 Problem 3

problem number 1100

Added April 8, 2019.

Problem Chapter 5.6.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b w_y = c w + \sin^k(\lambda x) \sin^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Sin[lambda*x]^k*Sin[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^k(\lambda K[1])\sin^n\left(\beta\left(y+\frac{b(K[1]-x)}{a}\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+sin(lambda*x)^k*sin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{cx}{a}}\left(\int^x \frac{(\sin(\lambda\_a))^k}{a}\left(\sin\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\,c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)$$

138.4 Problem 4

problem number 1101

Added April 8, 2019.

Problem Chapter 5.6.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\sin(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Sin[lambda*x+beta*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\sin\left(\beta yK[1]^{\frac{b}{a}}x^{-\frac{b}{a}}+\lambda K[1]\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+k*sin(lambda*x+beta*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = x^{\frac{c}{a}}\left(\_F1\left(yx^{-\frac{b}{a}}\right)+\int^x \frac{k}{a}\sin\left(\beta\, yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}+\lambda\_a\right)\_a^{\frac{-a-c}{a}}d\_a\right)$$

138.5 Problem 5

problem number 1102

Added April 8, 2019.

Problem Chapter 5.6.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\sin(\lambda x + \mu y)w + b\sin(\nu x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Sin[lambda*x+beta*y]*w[x,y]+ b*Sin[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{ax\cos(\beta y+\lambda x)}{\beta y+\lambda x}}\left(\int_1^x \frac{b\sin(\nu K[1])e^{\frac{ax\cos\left(K[1]\left(\frac{\beta y}{x}+\lambda\right)\right)}{\beta y+\lambda x}}}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y),x)+ b*diff(w(x,y),y) = a*x*sin(lambda*x+beta*y)*w(x,y)+ b*sin(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int^x \sin(\ln(\_a)b\beta+(-b\ln(x)+y)\beta+\lambda\_a)a d\_a}\left(\int^x \frac{b\sin(\nu\_b)\,\mathrm{e}^{-a\int\sin(\ln(\_b)b\beta+(-b\ln(x)+y)\beta+\_b\lambda)\,\mathrm{d}\_b}}{\_b}d\_b + \_$$

138.6 Problem 6

problem number 1103

Added April 8, 2019.

Problem Chapter 5.6.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*x)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = e^{\int \frac{(\sin(\nu x))^k c(\sin(\lambda x))^{-n}}{a}dx}\left(\_F1\left(\frac{ya - b\int(\sin(\lambda x))^{-n}(\sin(\mu x))^m\,dx}{a}\right) + \int^x \frac{p(\sin(\lambda\_f))^{-n}}{a}\left(\sin\right.\right.$$

138.7 Problem 7

problem number 1104

Added April 8, 2019.

Problem Chapter 5.6.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\sin^m(\mu x)w_y = c\sin^k(\nu y)w + p\sin^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sin[lambda*x]^n*D[w[x, y], x] + b*Sin[mu*x]^m*D[w[x, y], y] == c*Sin[nu*y]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*sin(mu*x)^m*diff(w(x,y),y) = c*sin(nu*y)^k*w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{(\sin(_b\,\lambda))^{-n}c}{a}\left(\sin\left(\frac{\nu}{a}\left(\int \frac{b(\sin(_b\,\mu))^m(\sin(_b\,\lambda))^{-n}}{a}\,\mathrm{d}_ba+ya-b\int(\sin(\lambda\,x))^{-n}(\sin(\mu\,x))^m\,\mathrm{d}x\right)\right)\right)^k d_b}\left(_F1\left(\frac{ya-b\int

139 HFOPDE, chapter 5.6.2

139.1 Problem 1

problem number 1105

Added April 11, 2019.

Problem Chapter 5.6.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \frac{k((a\lambda+b\mu)\sin(\lambda x+\mu y)-c\cos(\lambda x+\mu y))}{(a\lambda+b\mu)^2+c^2}+e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \frac{1}{a^2\lambda^2 + 2\,ab\lambda\,\mu + \mu^2b^2 + c^2}\left(\left(a^2\lambda^2 + 2\,ab\lambda\,\mu + \mu^2b^2 + c^2\right)\_F1\left(\frac{ya - bx}{a}\right) + k\mathrm{e}^{-\frac{cx}{a}}\left((a\lambda + b\mu)\,\mathrm{s}\right.\right.$$

139.2 Problem 2

problem number 1106

Added April 11, 2019.

Problem Chapter 5.6.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1\cos^k(\lambda x) + c_2\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cos[lambda*x]^k + c2*Cos[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \frac{c12^{-k}(1+ib\beta n)\left(e^{-i\lambda x}+e^{i\lambda x}\right)^{k}\left(1+e^{2i\lambda x}\right)^{-k}\text{Hypergeometric2F1}\left(-k,-\frac{k}{2}+\frac{i}{2a\lambda},\frac{i}{2a\lambda}-\frac{k}{2}\right.}{}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cos(lambda*x)^k + c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{x}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^{x}\frac{1}{a}\left(c2\left(\cos\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^{n}+c1\left(\cos\left(\lambda\_a\right)\right)^{k}\right)\mathrm{e}^{-\frac{\_a}{a}}d\_a$$

139.3 Problem 3

problem number 1107

Added April 11, 2019.

Problem Chapter 5.6.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cos^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cos[lambda*x]^k * Cos[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cos^k(\lambda K[1])\cos^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cos(lambda*x)^k *cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\frac{cx}{a}}\left(\int^x \frac{(\cos(\lambda\_a))^k}{a}\left(\cos\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)$$

139.4 Problem 4

problem number 1108

Added April 11, 2019.

Problem Chapter 5.6.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + k\cos(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+ k*Cos[lambda*x+mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\cos\left(\mu yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}+\lambda K[1]\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+ k*cos(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=x^{\frac{c}{a}}\left(\_F1\left(yx^{-\frac{b}{a}}\right)+\int^x \frac{k}{a}\cos\left(\lambda\_a+\mu\, yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)\_a^{\frac{-a-c}{a}}d\_a\right)$$

139.5 Problem 5

problem number 1109

Added April 11, 2019.

Problem Chapter 5.6.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = ax\cos(\lambda x + \mu y)w + b\cos(\nu x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == a*x*Cos[lambda*x+mu*y]*w[x,y]+b*Cos[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{ax\sin(\lambda x+\mu y)}{\lambda x+\mu y}}\left(\int_1^x \frac{b\cos(\nu K[1])\exp\left(-\frac{ax\sin\left(K[1]\left(\lambda+\frac{\mu y}{x}\right)\right)}{\lambda x+\mu y}\right)}{K[1]}dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) =a*x*cos(lambda*x+mu*y)*w(x,y)+b*cos(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{a\sin(\lambda x+\mu y)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}\left(\int^x \frac{\cos(\nu\_a)\,b}{\_a}\mathrm{e}^{-a\sin\left(\frac{\mu y\_a}{x}+\lambda\_a\right)\left(\frac{\mu y}{x}+\lambda\right)^{-1}}d\_a+\_F1\left(\frac{y}{x}\right)\right)$$

139.6 Problem 6

problem number 1110

Added April 11, 2019.

Problem Chapter 5.6.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cos^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\cos^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Cos[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*cos(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =c*cos(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int \frac{(\cos(\nu x))^k c(\cos(\lambda x))^{-n}}{a} dx} \left(\int^x \frac{p(\cos(\lambda\_f))^{-n}}{a} \left(\cos \left(\frac{\beta \left(b \int (\cos(\_f \mu))^m (\cos(\lambda\_f))^{-n} \, d\_f + ya \right.}{a} \right.\right.\right.$$

140 HFOPDE, chapter 5.6.3

140.1 Problem 1

problem number 1111

Added April 11, 2019.

Problem Chapter 5.6.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\tan(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}c_1\left(y-\frac{bx}{a}\right)-\frac{ik\left(-2\text{Hypergeometric2F1}\left(1,-\frac{ic}{2a\lambda+2b\mu},\frac{2a\lambda+2b\mu-ic}{2(a\lambda+b\mu)},-e^{-2i(\lambda x+\mu y)}\right)+2e^{2i}\right.}{c\left(1\right.}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\frac{cx}{a}} \left(\int^x \frac{k}{a} \tan\left(\frac{(\lambda\_a + \mu y)\, a - b\mu\, (x - \_a)}{a} \right) \mathrm{e}^{-\frac{\_a\, c}{a}} d\_a + \_F1\left(\frac{ya - bx}{a} \right) \right)$$

140.2 Problem 2

problem number 1112

Added April 11, 2019.

Problem Chapter 5.6.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \tan^k(\lambda x) + c_2 \tan^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Tan[lambda*x]^k + c2*Tan[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*tan(lambda*x)^k + c2*tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = e^{\frac{x}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right) + \int^{x} \frac{1}{a} e^{-\frac{\_a}{a}}\left(c2\left(\tan\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^{n} + c1\left(\tan\left(\lambda\_a\right)\right)^{k}\right)d\_a$$

140.3 Problem 3

problem number 1113

Added April 11, 2019.

Problem Chapter 5.6.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \tan^k(\lambda x)\tan^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Tan[lambda*x]^k * Tan[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x\frac{e^{-\frac{cK[1]}{a}}\tan^k(\lambda K[1])\tan^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ tan(lambda*x)^k *tan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{cx}{a}}\left(\_F1\left(\frac{ya-bx}{a}\right)+\int^x\frac{(\tan(\lambda\_a))^k}{a}\left(\tan\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n\mathrm{e}^{-\frac{\_a\,c}{a}}d\_a\right)$$

140.4 Problem 4

problem number 1114

Added April 11, 2019.

Problem Chapter 5.6.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a w_x + b\tan(\mu y) w_y = c\tan(\lambda x) w + k\tan(\nu x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Tan[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \cos^{-\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{k\tan(\nu K[1])\cos^{\frac{c}{a\lambda}}(\lambda K[1])}{a}\,dK[1]+c_1\left(\frac{\log(\sin(\mu y))}{\mu}-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) = c*tan(lambda*x)*w(x,y)+ k*tan(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(1+(\tan(\lambda x))^2\right)^{1/2\,\frac{c}{a\lambda}}\left(\int \frac{k\sin(\nu x)}{a\cos(\nu x)}\left(2\left(\cos(2\lambda x)+1\right)^{-1}\right)^{-1/2\,\frac{c}{a\lambda}}dx + \_F1\left(\frac{1}{b\mu}\left(-b\mu x+\ln\right.\right.\right.$$

140.5 Problem 5

problem number 1115

Added April 11, 2019.

Problem Chapter 5.6.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax w_x + by w_y = cw + k\tan(\lambda x + \nu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Tan[lambda*x+nu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\tan\left(\nu y x^{-\frac{b}{a}}K[1]^{\frac{b}{a}}+\lambda K[1]\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*tan(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=x^{\frac{c}{a}}\left(\_F1\left(yx^{-\frac{b}{a}}\right)+\int^x\frac{k}{a}\tan\left(\lambda\_a+\nu\, yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)\_a^{\frac{-a-c}{a}}d\_a\right)$$

140.6 Problem 6

problem number 1116

Added April 11, 2019.

Problem Chapter 5.6.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\tan^m(\mu x)w_y = c\tan^k(\nu x)w + p\tan^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int \frac{(\tan(\nu x))^k c(\tan(\lambda x))^{-n}}{a} dx} \left(\int^x \frac{p}{a} \left(1 \sin\left(\frac{\beta}{a} \left(b \int \left(\frac{\sin(\_f\,\mu)}{\cos(\_f\,\mu)} \right)^m \left(\frac{\sin(\lambda\,\_f)}{\cos(\lambda\,\_f)} \right)^{-n} \mathrm{d}\_f + ya - b \int\right.\right.\right.\right.$$

140.7 Problem 7

problem number 1117

Added April 11, 2019.

Problem Chapter 5.6.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\tan^m(\mu x)w_y = c\tan^k(\nu y)w + p\tan^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tan[lambda*x]^n*D[w[x, y], x] + b*Tan[mu*x]^m*D[w[x, y], y] == c*Tan[nu*y]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*tan(mu*x)^m*diff(w(x,y),y) =c*tan(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{c(\tan(\_b\,\lambda))^{-n}}{a}\left(\tan\left(\frac{\nu}{a}\left(-b\int\left(\frac{\sin(\mu\,x)}{\cos(\mu\,x)}\right)^m\left(\frac{\sin(\lambda\,x)}{\cos(\lambda\,x)}\right)^{-n}\mathrm{d}x + a\left(\int\frac{b(\tan(\_b\,\mu))^m(\tan(\_b\,\lambda))^{-n}}{a}\,\mathrm{d}\_b + y\right)\right)\right)\right)^k d\_b}\left(\int^x \frac{p}{a}\left(\frac{\mathrm{si}}{\mathrm{co}}\right.\right.$$

141 HFOPDE, chapter 5.6.4

141.1 Problem 1

problem number 1118

Added April 11, 2019.

Problem Chapter 5.6.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + k\cot(\lambda x + \mu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+mu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to -\frac{k\left(-2(2a\lambda+2b\mu+ic)e^{\frac{2i\mu(ay-bx)}{a}}\text{Hypergeometric2F1}\left(1,\frac{ic}{2(a\lambda+b\mu)},\frac{2a\lambda+2b\mu+ic}{2a\lambda+2b\mu},e^{2i(\lambda x+\mu y)}\right)+\right.}{}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+mu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = e^{\frac{cx}{a}} \left(\int^{x} \frac{k}{a} \cot\left(\frac{(\lambda\_a + \mu y)\, a - b\mu\, (x - \_a)}{a} \right) e^{-\frac{\_a\, c}{a}} d\_a + \_F1\left(\frac{ya - bx}{a} \right) \right)$$

141.2 Problem 2

problem number 1119

Added April 11, 2019.

Problem Chapter 5.6.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = w + c_1 \cot^k(\lambda x) + c_2 \cot^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*Cot[lambda*x]^k + c2*Cot[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = w(x,y)+ c1*cot(lambda*x)^k + c2*cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\frac{x}{a}}\left(\int^{x}\frac{1}{a}\mathrm{e}^{-\frac{\_a}{a}}\left(c1\left(\cot\left(\lambda\_a\right)\right)^{k}+c2\left(\cot\left(\frac{\beta\left(ya-b(x-\_a)\right)}{a}\right)\right)^{n}\right)d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right.$$

141.3 Problem 3

problem number 1120

Added April 11, 2019.

Problem Chapter 5.6.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \cot^k(\lambda x)\cot^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ Cot[lambda*x]^k * Cot[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\cot^k(\lambda K[1])\cot^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+ cot(lambda*x)^k *cot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{cx}{a}}\left(\int^x \frac{(\cot(\lambda\_a))^k}{a}\left(\cot\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\,c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)$$

141.4 Problem 4

problem number 1121

Added April 11, 2019.

Problem Chapter 5.6.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\cot(\mu y)w_y = c\cot(\lambda x)w + k\cot(\nu x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Cot[mu*y]*D[w[x, y], y] == c*Cot[lambda*x]*w[x,y]+k*Cot[nu*x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \sin^{\frac{c}{a\lambda}}(\lambda x)\left(\int_1^x \frac{k\cot(\nu K[1])\sin^{-\frac{c}{a\lambda}}(\lambda K[1])}{a}\,dK[1]+c_1\left(\frac{\log(\sec(\mu y))}{\mu}-\frac{bx}{a}\right)\right)\right\},\left\{w(x,y)\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*cot(mu*y)*diff(w(x,y),y) = c*cot(lambda*x)*w(x,y)+ k*cot(nu*x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = (\sin(\lambda x))^{\frac{c}{a\lambda}}\left(-\int^y \frac{k}{b}\left(\sin\left(\frac{\lambda}{b\mu}\left(b\mu x + a\ln(\cot(\mu y)) - 1/2\,a\ln\left((\cot(\mu y))^2+1\right)+1/2\,a\ln\left(\right.\right.\right.\right.\right.$$

141.5 Problem 5

problem number 1122

Added April 11, 2019.

Problem Chapter 5.6.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax w_x + by w_y = cw + k\cot(\lambda x + \nu y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+k*Cot[lambda*x+nu*y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{kK[1]^{-\frac{a+c}{a}}\cot\left(\nu y x^{-\frac{b}{a}}K[1]^{\frac{b}{a}}+\lambda K[1]\right)}{a}\,dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) =c*w(x,y)+k*cot(lambda*x+nu*y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=x^{\frac{c}{a}}\left(\int^x \frac{k}{a}\cot\left(\lambda\_a+\nu\, yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)\_a^{\frac{-a-c}{a}}d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)\right)$$

141.6 Problem 6

problem number 1123

Added April 11, 2019.

Problem Chapter 5.6.4.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\cot^k(\nu x)w + p\cot^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*x)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int \frac{(\cot(\nu x))^k c(\cot(\lambda x))^{-n}}{a} \mathrm{d}x} \left(\_F1\left(\frac{1}{a}\left(ya - b\int \left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^m \left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n} \mathrm{d}x\right)\right) + \int^x \frac{p}{a}\left(1\cos\right.\right.$$

141.7 Problem 7

problem number 1124

Added April 11, 2019.

Problem Chapter 5.6.4.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\cot^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\cot^k(\nu y)w + p\cot^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Cot[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Cot[nu*y]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*cot(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =c*cot(nu*y)^k*w(x,y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int^x \frac{c(\cot(\_b\,\lambda))^{-n}}{a}\left(\cot\left(\frac{\nu}{a}\left(-b\int\left(\frac{\cos(\mu x)}{\sin(\mu x)}\right)^m\left(\frac{\cos(\lambda x)}{\sin(\lambda x)}\right)^{-n}\mathrm{d}x+a\left(\int\frac{b(\cot(\_b\,\mu))^m(\cot(\_b\,\lambda))^{-n}}{a}\mathrm{d}\_b+y\right)\right)\right)\right)^k d\_b}\left(\_F1\left(\frac{1}{a}\left(\right.\right.\right.$$

142 HFOPDE, chapter 5.6.5

142.1 Problem 1

problem number 1125

Added April 11, 2019.

Problem Chapter 5.6.5.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \sin^k(\lambda x) + c_2 \cos^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+c1*Sin[lambda*x]^k+c2*Cos[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to \frac{\text{c1}(1+ib\beta n)\left(-ie^{-i\lambda x}\left(-1+e^{2i\lambda x}\right)\right)^k \left(2-2e^{2i\lambda x}\right)^{-k} \text{Hypergeometric2F1}\left(-k, -\frac{k}{2}+\frac{i}{2a\lambda}, \frac{i}{2a}\right.}{}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*sin(lambda*x)^k+c2*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\frac{x}{a}}\left(\int^{x}\frac{1}{a}\left(c1\,(\sin(\lambda\_a))^{k}+c2\left(\cos\left(\frac{\beta\,(ya-b(x-\_a))}{a}\right)\right)^{n}\right)\mathrm{e}^{-\frac{\_a}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)$$

142.2 Problem 2

problem number 1126

Added April 11, 2019.

Problem Chapter 5.6.5.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \sin^k(\lambda x)\cos^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+Sin[lambda*x]^k*Cos[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\sin^k(\lambda K[1])\cos^n\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+sin(lambda*x)^k*cos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\mathrm{e}^{\frac{cx}{a}}\left(\int^x \frac{(\sin(\lambda\_a))^k}{a}\left(\cos\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\,c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)$$

142.3 Problem 3

problem number 1127

Added April 11, 2019.

Problem Chapter 5.6.5.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = c\sin(\lambda x)w + k\cos(\nu x) + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Cos[nu*x]+s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_1^x \frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(k\cos(\nu K[1]) + s)}{a}\, dK[1] + c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu} - \frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*cos(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int \frac{k\cos(\nu x) + s}{a}\mathrm{e}^{\frac{c\cos(\lambda x)}{a\lambda}}\, \mathrm{d}x + \_F1\left(\frac{a}{b\mu}\ln\left(\mathrm{RootOf}\left(\mu y - \arctan\left(2\_Z\mathrm{e}^{\frac{b\mu x}{a}}\left(\_Z^2\right.\right.\right.\right.\right.\right.$$

142.4 Problem 4

problem number 1128

Added April 11, 2019.

Problem Chapter 5.6.5.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\sin(\mu y)w_y = c\sin(\lambda x)w + k\tan(\nu x) + s$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Sin[mu*y]*D[w[x, y], y] == c*Sin[lambda*x]*w[x,y]+k*Tan[nu*x]+s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\int_1^x \frac{e^{\frac{c\cos(\lambda K[1])}{a\lambda}}(k\tan(\nu K[1])+s)}{a}\,dK[1]+c_1\left(\frac{\log\left(\tan\left(\frac{\mu y}{2}\right)\right)}{\mu}-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*sin(mu*y)*diff(w(x,y),y) =  c*sin(lambda*x)*w(x,y)+k*tan(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{-\frac{c\cos(\lambda x)}{a\lambda}}\left(\_F1\left(\frac{a}{b\mu}\ln\left(\mathrm{RootOf}\left(\mu y-\arctan\left(2\_Z\mathrm{e}^{\frac{b\mu x}{a}}\left(\_Z^2\mathrm{e}^{2\frac{b\mu x}{a}}+1\right)^{-1}, -1\left(\_Z^2\mathrm{e}^{2\frac{b\mu x}{a}} -\right.\right.\right.\right.\right.\right.$$

142.5 Problem 5

problem number 1129

Added April 11, 2019.

Problem Chapter 5.6.5.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\tan(\mu y)w_y = c\tan(\lambda x)w + k\cot(\nu x) + s$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*Tan[mu*y]*D[w[x, y], y] == c*Tan[lambda*x]*w[x,y]+k*Cot[nu*x]+s;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*tan(mu*y)*diff(w(x,y),y) =  c*tan(lambda*x)*w(x,y)+k*cot(nu*x)+s
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = (\cos(\lambda x))^{-\frac{c}{a\lambda}} \left(\int \frac{k\cos(\nu x) + \sin(\nu x)s}{\sin(\nu x)a} (\cos(\lambda x))^{\frac{c}{a\lambda}} \, \mathrm{d}x + \_F1\left(\frac{1}{b\mu} \left(-b\mu x + \ln \left(\frac{\tan(}{\sqrt{1+(\mathrm{ta}}} \right.\right.\right.\right.$$

142.6 Problem 6

problem number 1130

Added April 11, 2019.

Problem Chapter 5.6.5.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\sin^n(\lambda x)w_x + b\cos^m(\mu x)w_y = c\cos^k(\nu x)w + p\sin^s(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Sin[lambda*x]^n*D[w[x, y], x] + b*Cos[mu*x]^m*D[w[x, y], y] == c*Cos[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*sin(lambda*x)^n*diff(w(x,y),x)+ b*cos(mu*x)^m*diff(w(x,y),y) =  c*cos(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = e^{\int \frac{(\cos(\nu x))^k c(\sin(\lambda x))^{-n}}{a} dx} \left(\_F1\left(\frac{ya - b\int (\cos(\mu x))^m (\sin(\lambda x))^{-n} dx}{a} \right) + \int^x \frac{p(\sin(\lambda \_f))^{-n}}{a} \left(\sin \right.\right.$$

142.7 Problem 7

problem number 1131

Added April 11, 2019.

Problem Chapter 5.6.5.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a\tan^n(\lambda x)w_x + b\cot^m(\mu x)w_y = c\tan^k(\nu x)w + p\cot^s(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*Tan[lambda*x]^n*D[w[x, y], x] + b*Cot[mu*x]^m*D[w[x, y], y] == c*Tan[nu*x]^k*w[x,y]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*tan(lambda*x)^n*diff(w(x,y),x)+ b*cot(mu*x)^m*diff(w(x,y),y) =  c*tan(nu*x)^k*w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \mathrm{e}^{\int \frac{c(\tan(\lambda x))^{-n}}{a}\left(\frac{\sin(\nu x)}{\cos(\nu x)}\right)^k dx}\left(\int \frac{p}{a}\left(\frac{\cos(\beta x)}{\sin(\beta x)}\right)^s\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}\mathrm{e}^{-\frac{c}{a}\int\left(\frac{\sin(\nu x)}{\cos(\nu x)}\right)^k\left(\frac{\sin(\lambda x)}{\cos(\lambda x)}\right)^{-n}dx}dx + \_F1\Big(\right.$$

143 HFOPDE, chapter 5.7.1

143.1 Problem 1

problem number 1132

Added April 13, 2019.

Problem Chapter 5.7.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arcsin^k(\lambda x) + c_2 \arcsin^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcSin[lambda*x]^k+c2*ArcSin[beta*y]^
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arcsin(lambda*x)^k+c2*arcsin(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} e^{-\frac{\_a}{a}} \left(c2 \left(\arcsin\left(\frac{\beta\left(ya - b(x - \_a)\right)}{a}\right)\right)^n + c1 \left(\arcsin\left(\lambda\_a\right)\right)^k\right) d\_a + \_F1\left(\frac{ya - b\right.$$

143.2 Problem 2

problem number 1133

Added April 13, 2019.

Problem Chapter 5.7.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \arcsin^k(\lambda x)\arcsin^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcSin[lambda*x]^k*ArcSin[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arcsin(lambda*x)^k*arcsin(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{(\arcsin(\lambda\_a))^k}{a}\left(\arcsin\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a c}{a}} d\_a + \_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

143.3 Problem 3

problem number 1134

Added April 13, 2019.

Problem Chapter 5.7.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \arcsin(\lambda_1 x) + c_2 \arcsin(\lambda_2 y))\, w + s_1 \arcsin^n(\beta_1 x) + s_2 \arcsin^k(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcSin[lambda1*x] + c2*ArcSin[lambda2*y])*w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arcsin(lambda1*x) + c2*arcsin(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} e^{\frac{1}{a\lambda 1\,\lambda 2\, b}\left(-\sqrt{-\frac{((\lambda 2\, y-1)a-b\lambda 2\,(x-\_a))((\lambda 2\, y+1)a-b\lambda 2\,(x-\_a))}{a^2}}\, ac2\,\lambda 1-\left(((\_a-x)b+ya)c2\,\lambda 1 \arcsin\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right)+\right.\right.}\right.$$

143.4 Problem 4

problem number 1135

Added April 13, 2019.

Problem Chapter 5.7.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arcsin^m(\mu x)w_y = c\arcsin^k(\nu x)w + p\arcsin^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*x]^k*w[x,y]+p*ArcSin[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*x)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\left(\int^x \frac{p}{a}\left(-\arcsin\left(\frac{(\_f\,\mu-1)\,(\_f\,\mu+1)\,\beta}{\mu\,(m+1)\,(\_f^2\mu^2-1)\,a}\left(-2^{-m}\arcsin(\_f\,\mu)\,b2^m\left(-\frac{\text{LommelS1}\,(m+3/2}{\sqrt{\arcsin}}\right.\right.\right.\right.\right.\right.$$

143.5 Problem 5

problem number 1136

Added April 13, 2019.

Problem Chapter 5.7.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b \arcsin^m(\mu x) w_y = c \arcsin^k(\nu y) w + p \arcsin^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcSin[mu*x]^m*D[w[x, y], y] == c*ArcSin[nu*y]^k*w[x,y]+p*ArcSin[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arcsin(mu*x)^m*diff(w(x,y),y) = c*arcsin(nu*y)^k*w(x,y)+p*arcsin
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{p(\arcsin(\beta \_f))^n}{a} \mathrm{e}^{-\frac{c}{a} \int \left(-\arcsin\left(\frac{(\_f\,\mu-1)(\_f\,\mu+1)\nu}{\mu\,(m+1)\left(\_f^2\mu^2-1\right)a} \left(-2^{-m} \arcsin(\_f\,\mu) b 2^m \left(-\frac{\mathrm{LommelS1}(m+3/2,1/2,\arcsin(\_f\,\mu)}{\sqrt{\arcsin(\_f\,\mu)}}\right.\right.\right.\right.}\right.$$

144 HFOPDE, chapter 5.7.2

144.1 Problem 1

problem number 1137

Added April 13, 2019.

Problem Chapter 5.7.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arccos^k(\lambda x) + c_2 \arccos^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCos[lambda*x]^k+c2*ArcCos[beta*y]^
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\$Aborted$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccos(lambda*x)^k+c2*arccos(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} \mathrm{e}^{-\frac{\_a}{a}} \left(c1\,(\arccos(\lambda\_a))^k + c2 \left(\arccos\left(\frac{\beta\,(ya - b(x - \_a))}{a}\right)\right)^n\right) d\_a + \_F1\left(\frac{ya - b}{a}\right.\right.$$

144.2 Problem 2

problem number 1138

Added April 13, 2019.

Problem Chapter 5.7.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = cw + \arccos^k(\lambda x) \arccos^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCos[lambda*x]^k*ArcCos[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccos(lambda*x)^k*arccos(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x, y) = \left(\int^x \frac{(\arccos(\lambda \_a))^k}{a} \left(\arccos\left(\frac{\beta (ya - b(x - \_a))}{a}\right)\right)^n e^{-\frac{\_a c}{a}} d\_a + \_F1\left(\frac{ya - bx}{a}\right)\right) e^{\frac{cx}{a}}$$

144.3 Problem 3

problem number 1139

Added April 13, 2019.

Problem Chapter 5.7.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \arccos(\lambda_1 x) + c_2 \arccos(\lambda_2 y))\, w + s_1 \arccos^n(\beta_1 x) + s_2 \arccos^k(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCos[lambda1*x] + c2*ArcCos[lambda2*y])*w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccos(lambda1*x) + c2*arccos(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} \mathrm{e}^{\frac{1}{a\lambda 1\, \lambda 2\, b} \left(\sqrt{-\frac{((\lambda 2\, y-1)a-b\lambda 2\,(x-\_a))((\lambda 2\, y+1)a-b\lambda 2\,(x-\_a))}{a^2}}\, ac2\, \lambda 1 - \lambda 2 \left(((\_a-x)b+ya)c2\, \lambda 1 \, \arccos\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right) - \right.\right.}\right.$$

144.4 Problem 4

problem number 1140

Added April 13, 2019.

Problem Chapter 5.7.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^m(\mu x)w_y = c\arccos^k(\nu x)w + p\arccos^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*x]^k*w[x,y]+p*ArcCos[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*x)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\left(\int^x \frac{p}{a}\left(\arccos\left(\frac{\beta}{\sqrt{\arccos(\mu\_a)}\,(m+2)\,a\mu}\left(\frac{\sqrt{\arccos(\mu\_a)}b\Big((m+2)\,\mathrm{LommelS1}\,(m+1/2\right.\right.\right.\right.\right.$$

144.5 Problem 5

problem number 1141

Added April 13, 2019.

Problem Chapter 5.7.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arccos^m(\mu x)w_y = c\arccos^k(\nu y)w + p\arccos^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcCos[mu*x]^m*D[w[x, y], y] == c*ArcCos[nu*y]^k*w[x,y]+p*ArcCos[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arccos(mu*x)^m*diff(w(x,y),y) = c*arccos(nu*y)^k*w(x,y)+p*arccos
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{p(\arccos(\beta\_b))^n}{a} e^{-\frac{c}{a}\int\left(\arccos\left(\frac{\nu}{\sqrt{\arccos(\_b\,\mu)}(m+2)a\mu}\left(\frac{\sqrt{\arccos(\_b\,\mu)}b\left((m+2)\operatorname{LommelS1}(m+1/2,1/2,\arccos(\mu\,x))-\arccos\right.}{}\right.\right.\right.}\right.$$

145 HFOPDE, chapter 5.7.3

145.1 Problem 1

problem number 1142

Added April 13, 2019.

Problem Chapter 5.7.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \arctan^k(\lambda x) + c_2 \arctan^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcTan[lambda*x]^k+c2*ArcTan[beta*y]^
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arctan(lambda*x)^k+c2*arctan(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} e^{-\frac{_a}{a}} \left(c2 \left(\arctan\left(\frac{\beta\left(ya - b(x - _a)\right)}{a}\right)\right)^n + c1\left(\arctan\left(\lambda_a\right)\right)^k\right) d_a + _F1\left(\frac{ya -
\right.\right.$$

145.2 Problem 2

problem number 1143

Added April 13, 2019.

Problem Chapter 5.7.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \arctan^k(\lambda x)\arctan^n(\beta y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcTan[lambda*x]^k*ArcTan[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}\tan^{-1}(\lambda K[1])^k\tan^{-1}\left(\beta\left(\frac{b(K[1]-x)}{a}+y\right)\right)^n}{a}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arctan(lambda*x)^k*arctan(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{(\arctan(\lambda\_a))^k}{a}\left(\arctan\left(\frac{\beta(ya-b(x-\_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\,c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

145.3 Problem 3

problem number 1144

Added April 13, 2019.

Problem Chapter 5.7.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \arctan(\lambda_1 x) + c_2 \arctan(\lambda_2 y))\, w + s_1 \arctan^n(\beta_1 x) + s_2 \arctan^k(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcTan[lambda1*x] + c2*ArcTan[lambda2*y])*w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arctan(lambda1*x) + c2*arctan(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\left(\int^x \frac{1}{a} \mathrm{e}^{\frac{1}{ab}\left(-((\_a-x)b+ya)c2 \arctan\left(\frac{\lambda 2\,(ya-b(x-\_a))}{a}\right)-c1\,\_a \arctan(\lambda 1\,\_a)b\right)} \left(\_a^2 \lambda 1^2+1\right)^{1/2\,\frac{c1}{a\lambda 1}} \left(\frac{(ya-b(x}{} \right.\right.\right.$$

145.4 Problem 4

problem number 1145

Added April 13, 2019.

Problem Chapter 5.7.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^m(\mu x)w_y = c\arctan^k(\nu x)w + p\arctan^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*x]^k*w[x,y]+p*ArcTan[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*x)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{p}{a}\left(\arctan\left(\frac{\beta}{a}\left(b\int(\arctan(\_f\,\mu))^{m}\,\mathrm{d}\_f + \left(-\int\frac{b(\arctan(\mu\,x))^{m}}{a}\,\mathrm{d}x + y\right)a\right)\right)\right)^{n}\mathrm{e}^{-\frac{c\int(}{}}$$

145.5 Problem 5

problem number 1146

Added April 13, 2019.

Problem Chapter 5.7.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\arctan^m(\mu x)w_y = c\arctan^k(\nu y)w + p\arctan^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcTan[mu*x]^m*D[w[x, y], y] == c*ArcTan[nu*y]^k*w[x,y]+p*ArcTan[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arctan(mu*x)^m*diff(w(x,y),y) = c*arctan(nu*y)^k*w(x,y)+p*arctan
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{p(\arctan(\beta \_f))^{n}}{a} \mathrm{e}^{-\frac{c}{a}\int\left(\arctan\left(\frac{\nu}{a}\left(b\int(\arctan(\_f\,\mu))^{m}\,\mathrm{d}\_f+\left(-\int\frac{b(\arctan(\mu x))^{m}}{a}\,\mathrm{d}x+y\right)a\right)\right)\right)^{k}\mathrm{d}\_f} d\_f + \_F1$$

146 HFOPDE, chapter 5.7.4

146.1 Problem 1

problem number 1147

Added April 13, 2019.

Problem Chapter 5.7.4.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = w + c_1 \operatorname{arccot}^k(\lambda x) + c_2 \operatorname{arccot}^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == w[x,y]+ c1*ArcCot[lambda*x]^k+c2*ArcCot[beta*y]^
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  w(x,y)+c1*arccot(lambda*x)^k+c2*arccot(beta*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} \mathrm{e}^{-\frac{\_a}{a}} \left(c1\, (\pi/2 - \arctan(\lambda\_a))^k + c2 \left(\pi/2 - \arctan\left(\frac{\beta\,(ya - b(x - \_a))}{a}\right)\right)^n\right) d\_a +$$

146.2 Problem 2

problem number 1148

Added April 13, 2019.

Problem Chapter 5.7.4.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + \operatorname{arccot}^k(\lambda x)\operatorname{arccot}^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+ ArcCot[lambda*x]^k*ArcCot[beta*y]^n;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\$Aborted$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) =  c*w(x,y)+ arccot(lambda*x)^k*arccot(beta*y)^n;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{(\pi/2 - \arctan(\lambda\_a))^k}{a}\left(\pi/2 - \arctan\left(\frac{\beta\,(ya - b(x - \_a))}{a}\right)\right)^n \mathrm{e}^{-\frac{\_a\,c}{a}}d\_a + \_F1\left(\frac{ya -}{a}\right.\right.$$

146.3 Problem 3

problem number 1149

Added April 13, 2019.

Problem Chapter 5.7.4.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (c_1 \operatorname{arccot}(\lambda_1 x) + c_2 \operatorname{arccot}(\lambda_2 y))\, w + s_1 \operatorname{arccot}^n(\beta_1 x) + s_2 \operatorname{arccot}^k(\beta_2 y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == ( c1*ArcCot[lambda1*x] + c2*ArcCot[lambda2*y])*w
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = ( c1*arccot(lambda1*x) + c2*arccot(lambda2*y))*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a} e^{1/2 \frac{1}{ab}\left(2((\_a-x)b+ya)c2 \arctan\left(\frac{\lambda 2 (ya-b(x-\_a))}{a}\right)+2(\arctan(\lambda 1\_a)c1-1/2\pi(c1+c2))\_a b\right)} \left(\_a^2\lambda 1^2+1\right)^{-1}\right.$$

146.4 Problem 4

problem number 1150

Added April 13, 2019.

Problem Chapter 5.7.4.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\,\mathrm{arccot}^m(\mu x)w_y = c\,\mathrm{arccot}^k(\nu x)w + p\,\mathrm{arccot}^n(\beta y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*x]^k*w[x,y]+p*ArcCot[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*x)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{p}{a}\left(\pi/2 - \arctan\left(\frac{\beta}{a}\left(b\int (\pi/2 - \arctan(\_f\,\mu))^m \,\mathrm{d}\_f + \left(-\int \frac{b(\pi/2 - \arctan(\mu\,x))^m}{a}\,\mathrm{d}x\right.\right.\right.\right.\right.$$

146.5 Problem 5

problem number 1151

Added April 13, 2019.

Problem Chapter 5.7.4.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + b\operatorname{arccot}^m(\mu x)w_y = c\operatorname{arccot}^k(\nu y)w + p\operatorname{arccot}^n(\beta x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*ArcCot[mu*x]^m*D[w[x, y], y] == c*ArcCot[nu*y]^k*w[x,y]+p*ArcCot[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*arccot(mu*x)^m*diff(w(x,y),y) = c*arccot(nu*y)^k*w(x,y)+p*arccot
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{p(\pi/2 - \arctan(\beta\_f))^n}{a} e^{-\frac{c}{a}\int\left(\pi/2-\arctan\left(\frac{\nu}{a}\left(b\int(\pi/2-\arctan(\_f\,\mu))^m\,\mathrm{d}\_f+\left(-\int\frac{b(\pi/2-\arctan(\mu\,x))^m}{a}\,\mathrm{d}x+y\right)a\right.\right.\right.}\right.$$

147 HFOPDE, chapter 5.8.1

147.1 Problem 1

problem number 1152

Added April 13, 2019.

Problem Chapter 5.8.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)w + g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\rightarrow e^{\int_1^x \frac{f(K[1])}{a}\,dK[1]}\left(\int_1^x \frac{g(K[2])\exp\left(-\text{Integrate}\left[\frac{f(K[1])}{a},\{K[1],1,K[2]\},\text{Assumptions}\rightarrow\text{True}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int \frac{g(x)}{a} \mathrm{e}^{-\frac{\int f(x)\, \mathrm{d}x}{a}} \mathrm{d}x + \_F1\left(\frac{ya-bx}{a}\right)\right) \mathrm{e}^{\int \frac{f(x)}{a} \mathrm{d}x}$$

147.2 Problem 2

problem number 1153

Added April 13, 2019.

Problem Chapter 5.8.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = (cy+k)w + f(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (c*y+k)*w[x,y]+f[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\frac{x(2a(cy+k)-bcx)}{2a^2}}\left(\int_1^x \frac{f(K[1])\exp\left(-\frac{K[1](bc(K[1]-2x)+2a(cy+k))}{2a^2}\right)}{a}dK[1] + c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (c*y+k)*w(x,y)+f(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{f(\_a)}{a} e^{-\frac{((cy+k)a-cb(x-\_a/2))\_a}{a^2}} d\_a + \_F1\left(\frac{ya-bx}{a}\right)\right) e^{\frac{((cy+k)a-1/2\,bcx)x}{a^2}}$$

147.3 Problem 3

problem number 1154

Added April 13, 2019.

Problem Chapter 5.8.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)yw + g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*y*w[x,y]+g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f(K[1])(b(K[1]-x)+ay)}{a^2}\,dK[1]\right)\left(\int_1^x \frac{g(K[2])\exp\left(-\text{Integrate}\left[\frac{f(K[1])(b(K[1]-\right.\right.}{a^2}}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*y*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{g(\_b)}{a}\mathrm{e}^{-\frac{\int f(\_b)((\_b-x)b+ya)\,\mathrm{d}\_b}{a^2}}d\_b+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\int^x \frac{f(\_a)(ya-b(x-\_a))}{a^2}d\_a}$$

147.4 Problem 4

problem number 1155

Added April 13, 2019.

Problem Chapter 5.8.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a x w_x + b y w_y = f(x) w + g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x]*w[x,y]+g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\int_1^x \frac{f(K[1])}{aK[1]}\, dK[1]} \left(\int_1^x \frac{g(K[2]) \exp\left(-\text{Integrate}\left[\frac{f(K[1])}{aK[1]}, \{K[1], 1, K[2]\}, \text{Assumptions} \to \text{True}\right.\right.}{aK[2]}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x)*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int \frac{g(x)}{ax} \mathrm{e}^{-\frac{1}{a}\int \frac{f(x)}{x}\, \mathrm{d}x}\, \mathrm{d}x + \_F1\left(y x^{-\frac{b}{a}}\right)\right) \mathrm{e}^{\int \frac{f(x)}{ax}\, \mathrm{d}x}$$

147.5 Problem 5

problem number 1156

Added April 13, 2019.

Problem Chapter 5.8.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (ay+b)w_y = cw + g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (a+y+b)*D[w[x, y], y] == c*w[x,y]+g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x \frac{c}{f(K[2])}dK[2]}\left(\int_1^x \frac{g(K[3])\exp\left(-\text{Integrate}\left[\frac{c}{f(K[2])},\{K[2],1,K[3]\},\text{Assumptions}\to\text{True}\right.\right.}{f(K[3])}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (a*y+b)*diff(w(x,y),y) = c*w(x,y)+g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int \frac{g(x)\,e^{-\int (f(x))^{-1}\,dx c}}{f(x)}\,dx + \_F1\left(\frac{e^{-a\int (f(x))^{-1}\,dx}(ya+b)}{a}\right)\right)e^{\int \frac{c}{f(x)}\,dx}$$

147.6 Problem 6

problem number 1157

Added April 13, 2019.

Problem Chapter 5.8.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)w_y = h(x)w + p(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x]*w[x,y]+p[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\int_1^x \frac{h(K[2])}{f(K[2])} dK[2]} \left(\int_1^x \frac{p(K[3]) \exp\left(-\text{Integrate}\left[\frac{h(K[2])}{f(K[2])}, \{K[2], 1, K[3]\}, \text{Assumptions} \to \text{True}\right.\right.}{f(K[3])}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x)*w(x,y)+p(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int \frac{p(x)}{f(x)} e^{-\int \frac{h(x)}{f(x)} dx} dx + \_F1\left(-\int \frac{g(x)}{f(x)} dx + y\right)\right) e^{\int \frac{h(x)}{f(x)} dx}$$

147.7 Problem 7

problem number 1158

Added April 13, 2019.

Problem Chapter 5.8.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_1(x)w + h_0(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h1[x]*w[x,y]+h0[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\int_1^x \frac{\text{h1}(K[3])}{f(K[3])} dK[3]} \left(c_1\left(ye^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])} dK[1]} - \int_1^x \frac{\text{g0}(K[2]) \exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])}, \{K[1], 1, K\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h1(x)*w(x,y)+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int \frac{h0(x)}{f(x)} \mathrm{e}^{-\int \frac{h1(x)}{f(x)} dx} dx + \_F1\left(-\int \frac{g0(x)}{f(x)} \mathrm{e}^{-\int \frac{g1(x)}{f(x)} dx} dx + y\mathrm{e}^{-\int \frac{g1(x)}{f(x)} dx}\right)\right) \mathrm{e}^{\int \frac{h1(x)}{f(x)} dx}$$

147.8 Problem 8

problem number 1159

Added April 13, 2019.

Problem Chapter 5.8.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h_2(x)w + h_1(x)y + h0(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y+h0[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\int_1^x \frac{\text{h2}(K[3])}{f(K[3])} dK[3]} \left(c_1\left(ye^{-\int_1^x \frac{\text{g1}(K[1])}{f(K[1])} dK[1]} - \int_1^x \frac{\text{g0}(K[2]) \exp\left(-\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])}, \{K[1], 1, K\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y+h0(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{1}{f(\_g)}\left(h1(\_g)\left(y\mathrm{e}^{-\int \frac{g1(x)}{f(x)}\,\mathrm{d}x} - \int \frac{g0(x)}{f(x)}\mathrm{e}^{-\int \frac{g1(x)}{f(x)}\,\mathrm{d}x}\,\mathrm{d}x + \int \frac{g0(\_g)}{f(\_g)}\mathrm{e}^{-\int \frac{g1(\_g)}{f(\_g)}\,\mathrm{d}\_g}\,\mathrm{d}\_g\right)\mathrm{e}^{-\int}\right.\right.$$

147.9 Problem 9

problem number 1160

Added April 13, 2019.

Problem Chapter 5.8.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h_2(x)w + h_1(x)y^n + h0(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*y^n+h0[x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\int_1^x \frac{\text{h2}(K[3])}{f(K[3])}\,dK[3]}\left(c_1\left((k-1)\int_1^x \frac{\text{g0}(K[2])\exp\left((k-1)\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])},\{K[1],1,K[2]\},\text{As}\right.\right.}{f(K[2])}\right.\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)*y^n+h0(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{1}{f(\_g)}\mathrm{e}^{-\int\frac{h2(\_g)}{f(\_g)}\mathrm{d}\_g}\left(h1(\_g)\left(\left((1-k)\int\frac{g0(\_g)}{f(\_g)}\mathrm{e}^{(k-1)\int\frac{g1(\_g)}{f(\_g)}\mathrm{d}\_g}\mathrm{d}\_g+(k-1)\int\frac{g0(x)}{f(x)}\right.\right.\right.\right.$$

147.10 Problem 10

problem number 1161

Added April 13, 2019.

Problem Chapter 5.8.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x) + g_0(x)e^{\lambda y})w_y = h_2(x)w + h_1(x)e^{\beta y} + h0(x)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]+g0[x]*Exp[lambda*y])*D[w[x, y], y] == h2[x]*w[x,y]+h1[x]*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h2(x)*w(x,y)+h1(x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\left(\int^x \frac{1}{f(\_g)}\left(h1(\_g)\left(\left(\left(\lambda\int \frac{g0(x)}{f(x)}\mathrm{e}^{\lambda\int\frac{g1(x)}{f(x)}\mathrm{d}x}\mathrm{d}x - \int \frac{g0(\_g)}{f(\_g)}\mathrm{e}^{\lambda\int\frac{g1(\_g)}{f(\_g)}\mathrm{d}\_g}\mathrm{d}\_g\lambda + \mathrm{e}^{\lambda\left(\int\frac{g1(x)}{f(x)}\mathrm{d}x\right.}\right.\right.\right.\right.\right.\right.$$

147.11 Problem 11

problem number 1162

Added April 13, 2019.

Problem Chapter 5.8.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)y^k w_x + f_2(x)w_y = g(x)w + h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f1[x]*y^k*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g(K[2])\left(\left(-(k+1)\text{Integrate}\left[\frac{\text{f2}(K[1])}{\text{f1}(K[1])},\{K[1],1,x\},\text{Assumptions}\to\text{True}\right]+(k\right.\right.}{\text{f1}(K[2}\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f1(x)*y^k*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{h(\_f)}{f1\,(\_f)}\left(\left((k+1)\int\frac{f2(\_f)}{f1\,(\_f)}\,\mathrm{d}\_f + (-1-k)\int\frac{f2(x)}{f1\,(x)}\,\mathrm{d}x + y^k y\right)^{(k+1)^{-1}}\right)^{-k} \mathrm{e}^{-\int\frac{g(\_f)}{f1(\_f)}\left(\left((k\right.\right.}$$

147.12 Problem 12

problem number 1163

Added April 13, 2019.

Problem Chapter 5.8.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)e^{\lambda y}w_x + f_2(x)w_y = g(x)w + h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f1[x]*Exp[lambda*y]*D[w[x, y], x] + f2[x]*D[w[x, y], y] == g[x]*w[x,y]+h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to\exp\left(\int_1^x\frac{g(K[2])}{\mathrm{f1}(K[2])\left(-\lambda\mathrm{Integrate}\left[\frac{\mathrm{f2}(K[1])}{\mathrm{f1}(K[1])},\{K[1],1,x\},\mathrm{Assumptions}\to\mathrm{True}\right]+\lambda\mathrm{Integrat}\right.}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f1(x)*exp(lambda*y)*diff(w(x,y),x)+ f2(x)*diff(w(x,y),y) = g(x)*w(x,y)+h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x\frac{h(\_f)}{f1\left(\_f\right)}\mathrm{e}^{-\frac{1}{\lambda}\int\frac{g(\_f)}{f1(\_f)}\left(\int\frac{f2(\_f)}{f1(\_f)}\,\mathrm{d}\_f+\frac{1}{\lambda}\left(\mathrm{e}^{y\lambda}-\int\frac{f2(x)}{f1(x)}\,\mathrm{d}x\lambda\right)\right)^{-1}\mathrm{d}\_f}\left(-\int\frac{f2(x)}{f1\left(x\right)}\,\mathrm{d}x\lambda+\int\frac{f2(\_f)}{f1\left(\_f\right)}\,\mathrm{d}\_f\lambda+\right.\right.$$

148 HFOPDE, chapter 5.8.2

148.1 Problem 1

problem number 1164

Added April 13, 2019.

Problem Chapter 5.8.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + f(x)g(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+f[x]*g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{f(K[1])g(K[1])e^{-\frac{cK[1]}{a}}}{a}\,dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+f(x)*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int \frac{f(x)\,g(x)}{a}\mathrm{e}^{-\frac{cx}{a}}\,\mathrm{d}x+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

148.2 Problem 2

problem number 1165

Added April 13, 2019.

Problem Chapter 5.8.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = cw + xf(x) + yg(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == c*w[x,y]+x*f[x]+y*g[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{\frac{cx}{a}}\left(\int_1^x \frac{e^{-\frac{cK[1]}{a}}(g(K[1])(bK[1]+ay-bx)+aK[1]f(K[1]))}{a^2}dK[1]+c_1\left(y-\frac{bx}{a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = c*w(x,y)+x*f(x)+y*g(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{(ya-b(x-\_a))\,g(\_a)+\_a\,f(\_a)\,a}{a^2}\mathrm{e}^{-\frac{a\,c}{a}}d\_a+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\frac{cx}{a}}$$

148.3 Problem 3

problem number 1166

Added April 13, 2019.

Problem Chapter 5.8.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y = f(x)w + g(x)h(x)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == f[x]*w[x,y]+g[x]*h[x];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y) \to e^{\int_1^x \frac{f(K[1])}{a}\,dK[1]}\left(\int_1^x \frac{g(K[2])h(K[2])\exp\left(-\text{Integrate}\left[\frac{f(K[1])}{a}, \{K[1],1,K[2]\}, \text{Assumptions}\right.\right.}{a}\right.\right.\right.$$

Maple ✓

```

unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = f(x)*w(x,y)+g(x)*h(x);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int \frac{g(x)\,h(x)}{a}\mathrm{e}^{-\frac{\int f(x)\,\mathrm{d}x}{a}}\,\mathrm{d}x + \_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\int \frac{f(x)}{a}\,\mathrm{d}x}$$

148.4 Problem 4

problem number 1167

Added April 13, 2019.

Problem Chapter 5.8.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$aw_x + bw_y = (f(x) + g(y))w + p(x) + q(y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y], x] + b*D[w[x, y], y] == (f[x]+g[y])*w[x,y]+p[x]+q[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{g\left(\frac{b(K[1]-x)}{a}+y\right)+f(K[1])}{a}\,dK[1]\right)\left(\int_1^x \frac{\left(q\left(\frac{b(K[2]-x)}{a}+y\right)+p(K[2])\right)\exp\left(-\right.}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y),x)+ b*diff(w(x,y),y) = (f(x)+g(y))*w(x,y)+p(x)+q(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{a}\mathrm{e}^{-\frac{1}{a}\int f(\_b)+g\left(\frac{ya-b(x-\_b)}{a}\right)\mathrm{d}\_b}\left(p(\_b)+q\left(\frac{ya-b(x-\_b)}{a}\right)\right)d\_b+\_F1\left(\frac{ya-bx}{a}\right)\right)\mathrm{e}^{\int^x}$$

148.5 Problem 5

problem number 1168

Added April 13, 2019.

Problem Chapter 5.8.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = cw + f(x)g(y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == c*w[x,y]+f[x]*g[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to x^{\frac{c}{a}}\left(\int_1^x \frac{f(K[1])K[1]^{-\frac{a+c}{a}}g\left(yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)}{a}dK[1]+c_1\left(yx^{-\frac{b}{a}}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = c*w(x,y)+f(x)*g(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{f(\_a)}{a}g\left(yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)\_a^{\frac{-a-c}{a}}d\_a+\_F1\left(yx^{-\frac{b}{a}}\right)\right)x^{\frac{c}{a}}$$

148.6 Problem 6

problem number 1169

Added April 13, 2019.

Problem Chapter 5.8.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f_1(x)w_x+f_2(y)w_y=aw+g_1(x)+g_2(y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f1[x]*D[w[x, y], x] + f2[y]*D[w[x, y], y] == a*w[x,y]+g1[x]+g2[y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f1(x)*diff(w(x,y),x)+ f2(y)*diff(w(x,y),y) = a*w(x,y)+g1(x)+g2(y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{e^{-a\int (f1(\_f))^{-1} d\_f} \left(g1(\_f) + g2\left(\mathrm{RootOf}\left(\int (f1(\_f))^{-1} \, d\_f - \int^{\_Z} (f2(\_a))^{-1} \, d\_a - \int (f1\right.\right.\right.}{f1(\_f)}$$

149 HFOPDE, chapter 5.8.3

149.1 Problem 1

problem number 1170

Added April 13, 2019.

Problem Chapter 5.8.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + yw_y = xf(\frac{y}{x})w + g(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y], x] + y*D[w[x, y], y] == x*f[y/x]*w[x,y]+g[x,y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to e^{xf\left(\frac{y}{x}\right)}\left(\int_1^x \frac{e^{K[1]\left(-f\left(\frac{y}{x}\right)\right)}g\left(K[1],\frac{yK[1]}{x}\right)}{K[1]}\,dK[1]+c_1\left(\frac{y}{x}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y),x)+ y*diff(w(x,y),y) = x*f(y/x)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{1}{\_a} g\left(\_a, \frac{y\_a}{x}\right) \mathrm{e}^{-\_a f\left(\frac{y}{x}\right)} d\_a + \_F1\left(\frac{y}{x}\right)\right) \mathrm{e}^{x f\left(\frac{y}{x}\right)}$$

149.2 Problem 2

problem number 1171

Added April 13, 2019.

Problem Chapter 5.8.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$axw_x + byw_y = f(x,y)w + g(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x*D[w[x, y], x] + b*y*D[w[x, y], y] == f[x,y]*w[x,y]+g[x,y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{f\left(K[1],yx^{-\frac{b}{a}}K[1]^{\frac{b}{a}}\right)}{aK[1]}\,dK[1]\right)\left(\int_1^x \frac{g\left(K[2],yx^{-\frac{b}{a}}K[2]^{\frac{b}{a}}\right)\exp\left(-\text{Integrate}\left[\frac{f\left(K\right.}{}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*x*diff(w(x,y),x)+ b*y*diff(w(x,y),y) = f(x,y)*w(x,y)+g(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y)=\left(\int^x \frac{1}{\_b\,a}g\left(\_b,yx^{-\frac{b}{a}}\_b^{\frac{b}{a}}\right)\mathrm{e}^{-\frac{1}{a}\int\frac{1}{\_b}f\left(\_b,yx^{-\frac{b}{a}}\_b^{\frac{b}{a}}\right)\mathrm{d}\_b}d\_b+\_F1\left(yx^{-\frac{b}{a}}\right)\right)\mathrm{e}^{\int^x\frac{1}{\_a\,a}f\left(\_a,yx^{-\frac{b}{a}}\_a^{\frac{b}{a}}\right)}$$

149.3 Problem 3

problem number 1172

Added April 13, 2019.

Problem Chapter 5.8.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + g(x)w_y = h(x,y)w + F(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + g[x]*D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\Big(K[2], -\text{Integrate}\left[\frac{g(K[1])}{f(K[1])}, \{K[1],1,x\}, \text{Assumptions}\to \text{True}\right] + \text{Integrate}\Big[\frac{g(K}{f(K}}{f(K[2])}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ g(x)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{f(\_f)} F\left(\_f, \int \frac{g(\_f)}{f(\_f)}\,\mathrm{d}\_f - \int \frac{g(x)}{f(x)}\,\mathrm{d}x + y\right) \mathrm{e}^{-\int \frac{1}{f(\_f)} h\left(\_f, \int \frac{g(\_f)}{f(\_f)}\,\mathrm{d}\_f - \int \frac{g(x)}{f(x)}\,\mathrm{d}x + y\right)\mathrm{d}\_f} d\_f -$$

149.4 Problem 4

problem number 1173

Added April 13, 2019.

Problem Chapter 5.8.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x))w_y = h(x,y)w + F(x,y)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y)\to \exp\left(\int_1^x \frac{h\left(K[3], \exp\left(\text{Integrate}\left[\frac{\text{g1}(K[1])}{f(K[1])}, \{K[1],1,K[3]\}, \text{Assumptions}\to \text{True}\right]\right)\left(-\text{Int}\right.\right.}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^x \frac{1}{f(\_g)} F\left(\_g, \left(y e^{-\int \frac{g1(x)}{f(x)} dx} - \int \frac{g0(x)}{f(x)} e^{-\int \frac{g1(x)}{f(x)} dx} dx + \int \frac{g0(\_g)}{f(\_g)} e^{-\int \frac{g1(\_g)}{f(\_g)} d\_g} d\_g\right) e^{\int \frac{g1(\_}{f(\_}}\right.\right.$$

149.5 Problem 5

problem number 1174

Added April 13, 2019.

Problem Chapter 5.8.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)y^k)w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*y^k)D[w[x, y], y] == h[x,y]*w[x,y]+F[x,y];
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*y^k)*diff(w(x,y),y) = h(x,y)*w(x,y)+F(x,y);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y) = \left(\int^{x} \frac{1}{f(\_g)} F\left(\_g, \left((1-k)\int \frac{g0(\_g)}{f(\_g)} \mathrm{e}^{(k-1)\int \frac{g1(\_g)}{f(\_g)}\, \mathrm{d}\_g}\, \mathrm{d}\_g + (k-1)\int \frac{g0(x)}{f(x)} \mathrm{e}^{(k-1)\int \frac{g1(x)}{f(x)}\, \mathrm{d}x}\, \mathrm{d}x\right.\right.\right.$$

149.6 Problem 6

problem number 1175

Added April 13, 2019.

Problem Chapter 5.8.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$f(x)w_x + (g_1(x)y + g_0(x)e^{\lambda y})w_y = h(x,y)w + F(x,y)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = f[x]*D[w[x, y], x] + (g1[x]*y+g0[x]*Exp[lambda*y])D[w[x, y], y] == h[x,y]*w[x,y]+F[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y], {x, y}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  f(x)*diff(w(x,y),x)+ (g1(x)*y+g0(x)*exp(lambda*y))*diff(w(x,y),y) = h(x,y)*w(x,y)+F(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y))),output='realt
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150 HFOPDE, chapter 6.2.1

150.1 Problem 1

problem number 1176

Added April 13, 2019.

Problem Chapter 6.2.1.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + bw_y + cw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y,z], x] + b*D[w[x, y,z], y] +c*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{bx}{a},z-\frac{cx}{a}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y,z),x)+ b*diff(w(x,y,z),y) + c*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{ya-bx}{a},\frac{za-cx}{a}\right)$$

150.2 Problem 2

problem number 1177

Added April 13, 2019.

Problem Chapter 6.2.1.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + axw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y,z], x] + a*x*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y-\frac{ax^2}{2},\frac{1}{3}abx^3-bxy+z\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  diff(w(x,y,z),x)+ a*x*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(-1/2\,ax^2+y, 1/3\,x\left(ax^2-3\,y\right)b+z\right)$$

150.3 Problem 3

problem number 1178

Added April 13, 2019.

Problem Chapter 6.2.1.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + byw_y + czw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*D[w[x, y,z], x] + b*y*D[w[x, y,z], y] +c*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(ye^{-\frac{bx}{a}},ze^{-\frac{cx}{a}}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*diff(w(x,y,z),x)+ b*y*diff(w(x,y,z),y) + c*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(y\mathrm{e}^{-\frac{bx}{a}}, z\mathrm{e}^{-\frac{cx}{a}}\right)$$

150.4 Problem 4

problem number 1179

Added April 13, 2019.

Problem Chapter 6.2.1.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{b}y\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}-1\right)\right)}{2\sqrt{b}},\frac{e^{-\sqrt{a}\sqrt{b}x}\left(\sqrt{a}z\left(e^{2\sqrt{a}\sqrt{b}x}+1\right)-\right.}{2\sqrt{a}}\right.\right.\right.$$

Maple ✓

```

unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{z^2a-by^2}{a}, -\frac{1}{\sqrt{ab}}\left(-x\sqrt{ab}+\ln\left(\frac{aby+\sqrt{a^2z^2}\sqrt{ab}}{\sqrt{ab}}\right)\right)\right)$$

150.5 Problem 5

problem number 1180

Added April 13, 2019.

Problem Chapter 6.2.1.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + ayw_y + bzw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y,z], x] + a*y*D[w[x, y,z], y] +b*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\{\{w(x,y,z) \to c_1(yx^{-a}, zx^{-b})\}\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y,z),x)+ a*y*diff(w(x,y,z),y) + b*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(yx^{-a}, zx^{-b}\right)$$

150.6 Problem 6

problem number 1181

Added April 13, 2019.

Problem Chapter 6.2.1.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + azw_y + byw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y,z], x] + a*z*D[w[x, y,z], y] +b*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(iy\sinh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{i\sqrt{a}z\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)}{\sqrt{b}},y\cosh\left(\sqrt{a}\sqrt{b}\log(x)\right)-\frac{\sqrt{a}z\,s}{}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y,z),x)+ a*z*diff(w(x,y,z),y) + b*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{z^2a - by^2}{a}, x\left(\sqrt{ab}\,y + za\right)^{-\frac{\sqrt{ab}}{ab}}\right)$$

150.7 Problem 7

problem number 1182

Added April 13, 2019.

Problem Chapter 6.2.1.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x + (ax+by)w_y + (\alpha x + \beta y + \gamma z)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(alpha*x+beta*y+gamma*z)*D[w[x,y,z],z]
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{x^{-b}(ax+(b-1)y)}{b-1},\frac{x^{-\gamma}(-a\beta x+\alpha x(b-\gamma)-(\gamma-1)(-bz+\beta y+\gamma z))}{(\gamma-1)(b-\gamma)}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  x*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (alpha*x+beta*y+gamma*z)*diff(w(x,y
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{(y(b-1)+ax)\,x^{-b}}{b-1}, \frac{-(-\gamma+b)\,(a\beta-\alpha\,b+\alpha)\,x^{1-\gamma}-(z(b-1)\,\gamma-b^2z+(\beta\,y+z)\,b}{(-1+\gamma)\,(b-1)\,(-\gamma+b)}\right.$$

150.8 Problem 8

problem number 1183

Added April 13, 2019.

Problem Chapter 6.2.1.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$abxw_x + (ay + bz)(bw_y - aw_z) = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*b*x*D[w[x, y,z], x] + (a*y+b*z)*(b*D[w[x, y,z], y] -a*D[w[x,y,z],z])==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*b*x*diff(w(x,y,z),x)+ (a*y+b*z)*(b*diff(w(x,y,z),y) - a*diff(w(x,y,z),z))= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{ya+bz}{b}, x\mathrm{e}^{-\frac{ya}{ya+bz}}\right)$$

150.9 Problem 9

problem number 1184

Added April 13, 2019.

Problem Chapter 6.2.1.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$abxw_x + b(ay+bz)w_y + a(ay-bz)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*b*x*D[w[x, y,z], x] + b*(a*y+b*z)*D[w[x, y,z], y] +a*(a*y-b*z)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*b*x*diff(w(x,y,z),x)+ b*(a*y+b*z)*diff(w(x,y,z),y) + a*(a*y-b*z)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(-\frac{1}{\sqrt{-a^2y^2+2\,abyz+b^2z^2}}, x\left(1\left(\frac{\sqrt{2}a^2y}{-a^2y^2+2\,abyz+b^2z^2} + \left(\frac{ya}{\sqrt{-a^2y^2+2\,abyz+b^2z^2}}\right.\right.\right.\right.$$

150.10 Problem 10

problem number 1185

Added April 13, 2019.

Problem Chapter 6.2.1.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$b^2cyw_x + a^2cxw_y - ab(ax+by)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b^2*c*y*D[w[x, y,z], x] + a^2*c*x*D[w[x, y,z], y] -a*b*(a*x+b*y)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{1}{2}\left(y^2-\frac{a^2x^2}{b^2}\right),\frac{ax+by+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b^2*c*y*diff(w(x,y,z),x)+ a^2*c*x*diff(w(x,y,z),y) - a*b*(a*x+b*y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+y^2b^2}{b^2},\frac{ax+by+cz}{c}\right)$$

150.11 Problem 11

problem number 1186

Added April 13, 2019.

Problem Chapter 6.2.1.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$czw_x + (ax + by)w_y + (ax + by + cz)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = c*z*D[w[x, y,z], x] + (a*x+b*y)*D[w[x, y,z], y] +(a*x+b*y+c*z)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  c*z*diff(w(x,y,z),x)+ (a*x+b*y)*diff(w(x,y,z),y) + (a*x+b*y+c*z)*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.12 Problem 12

problem number 1187

Added April 13, 2019.

Problem Chapter 6.2.1.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$b^2 czw_x - a^2 cxw_y + ab^2 yw_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b^2*c*z*D[w[x, y,z], x] - a^2*c*x*D[w[x, y,z], y] +a*b^2*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b^2*c*z*diff(w(x,y,z),x)-a^2*c*x*diff(w(x,y,z),y) + a*b^2*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.13 Problem 13

problem number 1188

Added April 13, 2019.

Problem Chapter 6.2.1.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(x+a)w_x + (y+b)xw_y + (z+c)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = (x+a)*D[w[x, y,z], x] + (y+b)*D[w[x, y,z], y] +(z+c)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b+y}{a+x},\frac{c+z}{a+x}\right)\right\}\right\}$$

Maple ✓

```

unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  (x+a)*diff(w(x,y,z),x)+(y+b)*diff(w(x,y,z),y) + (z+c)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{b+y}{x+a},\frac{z+c}{x+a}\right)$$

150.14 Problem 14

problem number 1189

Added April 13, 2019.

Problem Chapter 6.2.1.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2bc(ax-by)w_x - ac(ax-by-cz)w_y - ab(ax-by-3cz)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = 2*b*c*(a*x-b*y)*D[w[x, y,z], x] -a*c*(a*x-b*y-c*z)*D[w[x, y,z], y] - a*b*(a*x -b*y-3*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  2*b*c*(a*x-b*y)*diff(w(x,y,z),x)-a*c*(a*x-b*y-c*z)*diff(w(x,y,z),y)- a*b*(a*x -b*y-3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.15 Problem 15

problem number 1190

Added April 13, 2019.

Problem Chapter 6.2.1.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bc(y-z)w_x + ac(z-x)w_y + ab(x-y)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*c*(y-z)*D[w[x, y,z], x] +a*c*(z-x)*D[w[x, y,z], y] + a*b*(x -y)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*c*(y-z)*diff(w(x,y,z),x)+a*c*(z-x)*diff(w(x,y,z),y)+ a*b*(x -y)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = e^{1/2 \_C2\, x^2} e^{-\_C1\, x} e^{1/2 \frac{b y^2 \_C2}{a}} e^{\frac{b y \_C1}{a}} \_C3\, \_C5\, \_C4\, e^{1/2 \frac{c z^2 \_C2}{a}} e^{\frac{c z \_C1}{a}}$$

150.16 Problem 16

problem number 1191

Added April 13, 2019.

Problem Chapter 6.2.1.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bc(by-2cz)w_x+ac(3cz-ax)w_y+ab(2ax-3by)w_z=0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*c*(b*y-2*c*z)*D[w[x, y,z], x] +a*c*(3*c*z-a*x)*D[w[x, y,z], y] + a*b*(2*a*x -3*b*y)
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*c*(b*y-2*c*z)*diff(w(x,y,z),x)+a*c*(3*c*z-a*x)*diff(w(x,y,z),y)+ a*b*(2*a*x-3*b*y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\mathrm{e}^{1/2\,\_C2\,x^2}\mathrm{e}^{-\_C1\,x}\mathrm{e}^{1/2\,\frac{b^2\_C2\,y^2}{a^2}}\mathrm{e}^{2/3\,\frac{by\_C1}{a}}\_C3\,\_C5\,\_C4\,\mathrm{e}^{1/2\,\frac{c^2\_C2\,z^2}{a^2}}\mathrm{e}^{1/3\,\frac{cz\_C1}{a}}$$

150.17 Problem 17

problem number 1192

Added April 13, 2019.

Problem Chapter 6.2.1.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$2bc(by - cz)w_x - ac(4ax - 3by - cz)w_y + 3ab(4ax - by - 3cz)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = 2*b*c*(b*y-c*z)*D[w[x, y,z], x] -a*c*(4*a*x-3*b*y-c*z)*D[w[x, y,z], y] + 3*a*b*(4*a*x
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  2*b*c*(b*y-c*z)*diff(w(x,y,z),x)-a*c*(4*a*x-3*b*y-c*z)*diff(w(x,y,z),y)+ 3*a*b*(4*a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.18 Problem 18

problem number 1193

Added April 13, 2019.

Problem Chapter 6.2.1.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(ax+y-z)w_x - (x+ay-z)w_y + (a-1)(y-x)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = (a*x+y-z)*D[w[x, y,z], x] -(x+a*y-z)*D[w[x, y,z], y] + (a-1)*(y-x)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  (a*x+y-z)*diff(w(x,y,z),x)-(x+a*y-z)*diff(w(x,y,z),y)+ (a-1)*(y-x)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.19 Problem 19

problem number 1194

Added April 13, 2019.

Problem Chapter 6.2.1.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2bc(3ax-2by+cz)w_x-2ac(2ax-5by+3cz)w_y+ab(2ax-6by+11cz)w_z=0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  2*b*c*(3*a*x-2*b*y+c*z)*D[w[x, y,z], x] -2*a*c(2*a*x-5*b*y+3*c*z)*D[w[x, y,z], y] +
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   2*b*c*(3*a*x-2*b*y+c*z)*diff(w(x,y,z),x)-2*a*c*(2*a*x-5*b*y+3*c*z)*diff(w(x,y,z),y)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.20 Problem 20

problem number 1195

Added April 13, 2019.

Problem Chapter 6.2.1.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(Ax + cy + bz)w_x + (cx + By + az)w_y + (bx + ay + Cz)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  (A*x+c*y+b*z)*D[w[x, y,z], x] +(c*x+B*y+a*z)*D[w[x, y,z], y] +(b*x+a*y+C1*z)*D[w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   (A*x+c*y+b*z)*diff(w(x,y,z),x)+(c*x+B*y+a*z)*diff(w(x,y,z),y)+ (b*x+a*y+C1*z)*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

150.21 Problem 21

problem number 1196

Added April 13, 2019.

Problem Chapter 6.2.1.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$(a_1x + b_1y + c_1z + d_1)w_x + (a_2x + b_2y + c_2z + d_2)w_y + (a_3x + b_3y + c_3z + d_3)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  (a1*x+b1*y+c1*z+d1)*D[w[x, y,z], x] +(a2*x+b2*y+c2*z+d2)*D[w[x, y,z], y] +(a3*x+b3*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   (a1*x+b1*y+c1*z+d1)*diff(w(x,y,z),x)+(a2*x+b2*y+c2*z+d2)*diff(w(x,y,z),y)+ (a3*x+b3
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

151 HFOPDE, chapter 6.2.2

151.1 Problem 1

problem number 1197

Added April 14, 2019.

Problem Chapter 6.2.2.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a_1xy + b_1x^2 + c_1x)w_y + (a_2xy + b_2x^2 + c_2x)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*y+b2*x^2+c2*x)*D[w[x,y,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{-2\text{a1b2}x^3-3\text{a1c2}x^2+6\text{a1}z+2\text{a2b1}x^3+3\text{a2c1}x^2-6\text{a2}y}{6\text{a1}},\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*y+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{\left(a1^{3/2}y + \sqrt{a1}(b1\,x + c1)\right)\sqrt{\pi}\mathrm{e}^{-1/2\,a1\,x^2} - 1/2\,\mathrm{erf}\left(1/2\sqrt{2}\sqrt{a1}x\right)b1\,\sqrt{2}\pi}{\sqrt{\pi}a1^{3/2}}, -1/3\,\frac{1}{a1^2}\right.$$

151.2 Problem 2

problem number 1198

Added April 14, 2019.

Problem Chapter 6.2.2.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a_1xy + b_1x^2 + c_1x)w_y + (a_2xz + b_2x^2 + c_2x)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*x*z+b2*x^2+c2*x)*D[w[x,y,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{e^{-\frac{\text{a1}x^2}{2}}(\text{a1}y+\text{b1}x+\text{c1})}{\text{a1}}-\frac{\sqrt{\frac{\pi}{2}}\text{b1Erf}\left(\frac{\sqrt{\text{a1}}x}{\sqrt{2}}\right)}{\text{a1}^{3/2}},\frac{e^{-\frac{\text{a2}x^2}{2}}(\text{a2}z+\text{b2}x+\text{c2})}{\text{a2}}-\frac{\sqrt{\frac{\pi}{2}}\text{b2Erf}\left(\right.}{\text{a2}^{3/2}}\right.\right.\right.$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*x*z+b2*x^2+c2*x)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{\left(a1^{3/2}y+\sqrt{a1}\,(b1\,x+c1)\right)\sqrt{\pi}\mathrm{e}^{-1/2\,a1\,x^2}-1/2\,\mathrm{erf}\left(1/2\,\sqrt{2}\sqrt{a1}\,x\right)b1\,\sqrt{2}\pi}{\sqrt{\pi}a1^{3/2}},\frac{\left(a2^{3/2}z+\right.}{}\right.$$

151.3 Problem 3

problem number 1199

Added April 14, 2019.

Problem Chapter 6.2.2.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a_1xy + b_1x^2 + c_1x)w_y + (a_2yz + b_2y^2 + c_2y)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  D[w[x, y,z], x] +(a1*x*y+b1*x^2+c1*x)*D[w[x, y,z], y] +(a2*y*z+b2*y^2+c2*y)*D[w[x,y,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   diff(w(x,y,z),x)+(a1*x*y+b1*x^2+c1*x)*diff(w(x,y,z),y)+ (a2*y*z+b2*y^2+c2*y)*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{\left(a1^{3/2}y + \sqrt{a1}\,(b1\,x + c1)\right)\sqrt{\pi}\mathrm{e}^{-1/2\,a1\,x^2} - 1/2\,\mathrm{erf}\left(1/2\,\sqrt{2}\sqrt{a1}\,x\right)b1\,\sqrt{2}\pi}{\sqrt{\pi}\,a1^{3/2}}, -1/2\int^{x}\right.$$

151.4 Problem 4

problem number 1200

Added April 14, 2019.

Problem Chapter 6.2.2.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$w_x + (a_1xy + b_1y^2)w_y + (a_2xz + b_2z^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  D[w[x, y,z], x] +(a1*x+b1*y^2)*D[w[x, y,z], y] +(a2*x*z+b2*z^2)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\Bigg\{\Bigg\{w(x,y,z)\to c_1\Bigg(-\frac{2\Big(\text{b1}xy\,\text{BesselJ}\Big(\frac{1}{3},\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\Big)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\text{BesselJ}}{(2\text{b1}xy+1)\text{BesselJ}\Big(-\frac{1}{3},\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\Big)+\sqrt{\text{a1}}\sqrt{\text{b1}}x^{3/2}\text{BesselJ}\Big(-\frac{4}{3},\frac{2}{3}\sqrt{\text{a1}}\sqrt{\text{b1}}x}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   diff(w(x,y,z),x)+(a1*x+b1*y^2)*diff(w(x,y,z),y)+ (a2*x*z+b2*z^2)*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{y\,\text{AiryBi}\left(-\sqrt[3]{a1\,b1}\,x\right)b1-\sqrt[3]{a1\,b1}\,\text{AiryBi}\left(1,-\sqrt[3]{a1\,b1}\,x\right)}{-y\,\text{AiryAi}\left(-\sqrt[3]{a1\,b1}\,x\right)b1+\sqrt[3]{a1\,b1}\,\text{AiryAi}\left(1,-\sqrt[3]{a1\,b1}\,x\right)},\frac{\sqrt{\pi}\,\text{erf}\left(1/2\sqrt{-2\,a2}\,x\right.}{\sqrt{-}}\right.$$

151.5 Problem 5

problem number 1201

Added April 14, 2019.

Problem Chapter 6.2.2.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$aw_x + xzw_y - xyw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*D[w[x, y,z], x] +x*z*D[w[x, y,z], y] -x*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(y\sin\left(\frac{x^2}{2a}\right)+z\cos\left(\frac{x^2}{2a}\right),y\cos\left(\frac{x^2}{2a}\right)-z\sin\left(\frac{x^2}{2a}\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   a*diff(w(x,y,z),x)+x*z*diff(w(x,y,z),y)- x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(y^2+z^2, -2\,a\arctan\left(\frac{y}{z}\right)+x^2\right)$$

151.6 Problem 6

problem number 1202

Added April 14, 2019.

Problem Chapter 6.2.2.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$cxw_x + cyw_y + (ax^2 + by^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*x*D[w[x, y,z], x] +c*y*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},-\frac{ax^2+by^2-2cz}{2c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   c*x*diff(w(x,y,z),x)+c*y*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{y}{x}, 1/2\,\frac{-ax^2-by^2+2\,cz}{c}\right)$$

151.7 Problem 7

problem number 1203

Added April 14, 2019.

Problem Chapter 6.2.2.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$czw_x - a(2ax-b)yw_y + a(2ax-b)zw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*z*D[w[x, y,z], x] -a*(2*a*x-b)*y*D[w[x, y,z], y] +a*(2*a*x-b)*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(cyz, \frac{-a^2x^2+abx+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   c*z*diff(w(x,y,z),x)-a*(2*a*x-b)*diff(w(x,y,z),y)+a*(2*a*x-b)*z*diff(w(x,y,z),z)= 0
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{-a^2x^2+axb+cz}{c}, \ln(cz)+y\right)$$

151.8 Problem 8

problem number 1204

Added April 14, 2019.

Problem Chapter 6.2.2.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$acx^2w_x - acxyw_y - b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*c*x^2*D[w[x, y,z], x] -a*c*x*y*D[w[x, y,z], y] -b^2*y^2*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(xy,z-\frac{b^2y^2}{3acx}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=    a*c*x^2*diff(w(x,y,z),x) -a*c*x*y*diff(w(x,y,z),y)-b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(yx, 1/3\,\frac{3\,zacx^3-b^2x^2y^2}{acx^3}\right)$$

151.9 Problem 9

problem number 1205

Added April 14, 2019.

Problem Chapter 6.2.2.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2w_x + by^2w_y + cz^2w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*x^2*D[w[x, y,z], x] +b*y^2*D[w[x, y,z], y] +c*z^2*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{b}{ax}-\frac{1}{y},\frac{c}{ax}-\frac{1}{z}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=    a*x^2*diff(w(x,y,z),x) +b*y^2*diff(w(x,y,z),y)+c*z^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{ax-by}{yax},\frac{ax-cz}{zax}\right)$$

151.10 Problem 10

problem number 1206

Added April 14, 2019.

Problem Chapter 6.2.2.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$abx^2w_x + cz^2w_y + 2abxzw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*b*x^2*D[w[x, y,z], x] +c*z^2*D[w[x, y,z], y] +2*a*b*x*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{z}{x^2},y-\frac{cz^2}{3abx}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=    a*b*x^2*diff(w(x,y,z),x) +c*z^2*diff(w(x,y,z),y)+2*a*b*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{z}{x^2}, 1/3\,\frac{3\,abxy-cz^2}{axb}\right)$$

151.11 Problem 11

problem number 1207

Added April 14, 2019.

Problem Chapter 6.2.2.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bcxyw_x + a^2cx^2w_y - by(2ax + cz)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*c*x*y*D[w[x, y,z], x] +a^2*c*x^2*D[w[x, y,z], y] -b*y*(2*a*x+c*z)*D[w[x,y,z],z]==0
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-a^2x^2}{2b},\frac{x(ax+cz)}{c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=     b*c*x*y*diff(w(x,y,z),x) +a^2*c*x^2*diff(w(x,y,z),y)-b*y*(2*a*x+c*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{-a^2x^2+by^2}{b},\frac{(ax+cz)\,x}{c}\right)$$

151.12 Problem 12

problem number 1208

Added April 14, 2019.

Problem Chapter 6.2.2.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bcxyw_x + c^2yzw_y + b^2y^2w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*c*x*y*D[w[x, y,z], x] +c^2*y*z*D[w[x, y,z], y] +b^2*y^2*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{i(b(x^2-1)\,y-c(x^2+1)\,z)}{2bx},\frac{b(x^2+1)\,y-c(x^2-1)\,z}{2bx}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*c*x*y*diff(w(x,y,z),x) +c^2*y*z*diff(w(x,y,z),y)+b^2*y^2*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{-y^2b^2+c^2z^2}{c^2},x(csgn(b)\,by+cz)^{-csgn(b)}\right)$$

151.13 Problem 13

problem number 1209

Added April 14, 2019.

Problem Chapter 6.2.2.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xyw_x + y(y-a)w_y + z(y-a)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  x*y*D[w[x, y,z], x] +y*(y-a)*D[w[x, y,z], y] +z*(y-a)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y-a}{x},\frac{z}{y}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*y*diff(w(x,y,z),x) +y*(y-a)*diff(w(x,y,z),y)+z*(y-a)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{y-a}{x},\frac{z}{y}\right)$$

151.14 Problem 14

problem number 1210

Added April 14, 2019.

Problem Chapter 6.2.2.14, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$by^2 w_x - axyw_y + cxzw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*y^2*D[w[x, y,z], x] -a*x*y*D[w[x, y,z], y] +c*x*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ax^2+by^2}{2b},z\left(-by^2\right)^{\frac{c}{2a}}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*y^2*diff(w(x,y,z),x) -a*x*y*diff(w(x,y,z),y)+c*x*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{ax^2+by^2}{b},z\left(-by^2\right)^{1/2\,\frac{c}{a}}\right)$$

151.15 Problem 15

problem number 1211

Added April 14, 2019.

Problem Chapter 6.2.2.15, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$cxzw_x + 2axyw_y - (2ax + cz)zw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*x*z*D[w[x, y,z], x] +2*a*x*y*D[w[x, y,z], y] -(2*a*x+c*z)*z*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-cxyz, x\left(\frac{ax}{c}+z\right)\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := c*x*z*diff(w(x,y,z),x) +2*a*x*y*diff(w(x,y,z),y)-(2*a*x+c*z)*z*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{(ax+cz)\,x}{c}, -cxyz\right)$$

151.16 Problem 16

problem number 1212

Added April 14, 2019.

Problem Chapter 6.2.2.16, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$cxzw_x + cyzw_y + abxyw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*x*z*D[w[x, y,z], x] +c*y*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{cz^2-abxy}{2c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := c*x*z*diff(w(x,y,z),x) +c*y*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{-abxy+cz^2}{c}\right)$$

151.17 Problem 17

problem number 1213

Added April 14, 2019.

Problem Chapter 6.2.2.17, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$cxzw_x - cyzw_y + (by^2 - ax)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(b*y^2-a*x)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(xy,\frac{2ax+by^2+cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(b*y^2-a*x)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(yx,\frac{bx^2y^2+z^2cx^2+2\,ax^3}{cx^2}\right)$$

151.18 Problem 18

problem number 1214

Added April 14, 2019.

Problem Chapter 6.2.2.18, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$cxzw_x - cyzw_y + (ax^2 + by^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  c*x*z*D[w[x, y,z], x] -c*y*z*D[w[x, y,z], y] +(a*x^2+b*y^2)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(xy,\frac{-ax^2+by^2+cz^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := c*x*z*diff(w(x,y,z),x)-c*y*z*diff(w(x,y,z),y)+(a*x^2+b*y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(yx,\frac{-ax^2+by^2+cz^2}{c}\right)$$

151.19 Problem 19

problem number 1215

Added April 14, 2019.

Problem Chapter 6.2.2.19, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$xzw_x + yzw_y + (ax^2 + ay^2 + bz^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  x*z*D[w[x, y,z], x] +y*z*D[w[x, y,z], y] +(a*x^2+a*y^2+b*z^2)*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{x^{-2b}\left(a\left(x^2+y^2\right)+(b-1)z^2\right)}{b-1}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*z*diff(w(x,y,z),x)+y*z*diff(w(x,y,z),y)+(a*x^2+a*y^2+b*z^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\frac{x^{-2\,b}\left(\left(x^2+y^2\right)a+z^2(b-1)\right)}{b-1}\right)$$

151.20 Problem 20

problem number 1216

Added April 14, 2019.

Problem Chapter 6.2.2.20, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2cxzw_x + 2cyzw_y + (cz^2 - ax^2 - by^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  2*c*x*z*D[w[x, y,z], x] +2*c*y*z*D[w[x, y,z], y] +(c*z^2-a*x^2-b*y^2)*D[w[x,y,z],z]=
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{ax^2+by^2+cz^2}{cx}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := 2*c*x*z*diff(w(x,y,z),x)+2*c*y*z*diff(w(x,y,z),y)+(c*z^2-a*x^2-b*y^2)*diff(w(x,y,z),z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{y}{x},\frac{ax^2+by^2+cz^2}{cx}\right)$$

151.21 Problem 21

problem number 1217

Added April 14, 2019.

Problem Chapter 6.2.2.21, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bcyzw_x + acxzw_y + abxyw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*c*y*z*D[w[x, y,z], x] +a*c*x*z*D[w[x, y,z], y] +a*b*x*y*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{by^2-ax^2}{2b},\frac{cz^2-ax^2}{2c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*c*y*z*diff(w(x,y,z),x)+a*c*x*z*diff(w(x,y,z),y)+a*b*x*y*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{-ax^2+by^2}{b},\frac{-ax^2+cz^2}{c}\right)$$

151.22 Problem 22

problem number 1218

Added April 14, 2019.

Problem Chapter 6.2.2.22, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bc(x^2-a^2)w_x + c(bxy+acz)w_y + b(cxz+aby)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*c*(x^2-a^2)*D[w[x, y,z], x] +c*(b*x*y+a*c*z)*D[w[x, y,z], y] +b*(c*x*z + a*b*y)*D[
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(-\frac{acz+bxy}{a^2b-bx^2},\frac{a(aby+cxz)}{b\,(a^2-x^2)}\right)\right\}\right\}$$

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*c*(x^2-a^2)*diff(w(x,y,z),x)+c*(b*x*y+a*c*z)*diff(w(x,y,z),y)+b*(c*x*z + a*b*y)*di
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

151.23 Problem 23

problem number 1219

Added April 14, 2019.

Problem Chapter 6.2.2.23, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bx(by+c)w_x + (b^2y^2 - acx)w_y + b^2yzw_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*x*(b*y+c)*D[w[x, y,z], x] + (b^2*y^2-a*c*x )*D[w[x, y,z], y] + b^2*y*z*D[w[x,y,z],
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*x*(b*y+c)*diff(w(x,y,z),x)+(b^2*y^2-a*c*x )*diff(w(x,y,z),y)+b^2*y*z*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{1}{3\,ax+3\,by}\left((-ax-by)\ln\left(-9\,\frac{(ax+by)\,(ax-c)}{x\,(by+c)}\right)+(ax+by)\ln\left(\frac{-9\,ax+9\,c}{2\,by+2\,c}\right)+\right.\right.$$

151.24 Problem 24

problem number 1220

Added April 14, 2019.

Problem Chapter 6.2.2.24, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x, y)$

$$x(by - cz)w_x + y(cz - ax)w_y + z(ax - by)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  x*(b*y -c*z)*D[w[x, y,z], x] + y*(c*z-a*x)*D[w[x, y,z], y] + z*(a*x - b*y)*D[w[x,y,z
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(-\frac{cxyz}{b}, \frac{ax+by+cz}{c}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   x*(b*y -c*z)*diff(w(x,y,z),x)+ y*(c*z-a*x)*diff(w(x,y,z),y)+z*(a*x - b*y)*diff(w(x,
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \frac{x^{-\_C2} e^{-\_C2} \_C5\, y^{-\_C2} \_C4\, z^{-\_C2}}{e^{-\_C1\, x} \_C3} \left(e^{\frac{by\_C1}{a}}\right)^{-1} \left(e^{\frac{cz\_C1}{a}}\right)^{-1}$$

151.25 Problem 25

problem number 1221

Added April 14, 2019.

Problem Chapter 6.2.2.25, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a(y+\beta)(z+\gamma)w_x - b(x+\alpha)(z+\gamma)w_y - c(x+\alpha)(y+\beta)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*(y+beta)*(z+gamma)*D[w[x, y,z], x] -b*(x+alpha)*(z+gamma)*D[w[x, y,z], y] - c*(x+a
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ay(2\beta+y)+2\alpha bx+bx^2}{2a},\frac{az(2\gamma+z)+2\alpha cx+cx^2}{2a}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   a*(y+beta)*(z+gamma)*diff(w(x,y,z),x)-b*(x+alpha)*(z+gamma)*diff(w(x,y,z),y)- c*(x+
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \mathrm{e}^{-\_C3\,\alpha\,x}\mathrm{e}^{1/2\,x^2\_C3}\mathrm{e}^{-\_C1\,\beta\,y}\mathrm{e}^{1/2\,\_C1\,y^2}\_C4\,\_C2\,\_C5\,\mathrm{e}^{\frac{a\gamma\,\_C3\,z}{c}}\mathrm{e}^{-1/2\,\frac{\_C1\,bz^2}{c}}\mathrm{e}^{1/2\,\frac{a\,\_C3\,z^2}{c}}\left(\mathrm{e}^{\frac{\_C1\,b\gamma\,z}{c}}\right)^{-1}$$

151.26 Problem 26

problem number 1222

Added April 14, 2019.

Problem Chapter 6.2.2.26, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bc(acxz + b^2y^2)w_x + ac(bcyz - 2a^2x^2)w_y - ab(2abxy + c^2z^2)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  b*c*(a*c*x*z + b^2*y^2)*D[w[x, y,z], x] +a*c*(b*c*y*z-2*a^2*x^2)*D[w[x, y,z], y] - a
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=   b*c*(a*c*x*z + b^2*y^2)*diff(w(x,y,z),x)+a*c*(b*c*y*z-2*a^2*x^2)*diff(w(x,y,z),y)-
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

151.27 Problem 27

problem number 1223

Added April 14, 2019.

Problem Chapter 6.2.2.27, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$a(y^2+z^2)w_x + x(bz-ay)w_y - x(by+az)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =  a*(y^2+z^2)*D[w[x, y,z], x] +x*(b*z-a*y)*D[w[x, y,z], y] -x*(b*y + a*z)*D[w[x,y,z],z
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  a*(y^2+z^2)*diff(w(x,y,z),x)+x*(b*z-a*y)*diff(w(x,y,z),y)-x*(b*y + a*z)*diff(w(x,y,z
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_C1\, e^{1/2\_c_1 x^2}\_F5\left(1/2\,\frac{1}{b}\left(2\,a\arctan\left(\frac{z}{y}\right) - b\ln\left(y^2+z^2\right)\right)\right)\left(e^{a\_c_1\int^y \_a\left(\cos\left(\mathrm{RootOf}\left(-2\,a\,\mathrm{arcta}\right.\right.\right.}\right.$$

151.28 Problem 28

problem number 1224

Added April 14, 2019.

Problem Chapter 6.2.2.28, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$b(by+cz)^2 w_x - ax(by+2cz)w_y + abxzw_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*(b*y + c*z)^2*D[w[x, y,z], x] - a*x*(b*y + 2*c*z)*D[w[x, y,z], y] +a*b*x*z*D[w[x,y,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z) \to c_1\left(\frac{2(ax^2+c^2z^2)}{b}, \log(z(by+cz))\right)\right\}, \left\{w(x,y,z) \to c_1\left(\frac{2(ax^2+c^2z^2)}{b}, \log(z(cz-by))\right)\right.$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*(b*y + c*z)^2*diff(w(x,y,z),x)- a*x*(b*y + 2*c*z)*diff(w(x,y,z),y)+a*b*x*z*diff(w(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_C1\, e^{1/2\_c_1x^2}\_F5\left(\frac{z(by+cz)}{b}\right) e^{1/2\,\frac{b^2\_c_1y^2}{a}} e^{1/2\,\frac{b\_c_1ycz}{a}}$$

151.29 Problem 29

problem number 1225

Added April 14, 2019.

Problem Chapter 6.2.2.29, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(f_0x - f_1)w_x + (f_0y - f_2)w_y + (f_0z - f_3)w_z = 0$$

Where

$$f_n = a_n + b_nx + c_ny + d_nz$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 f[n_]:= a[n] + b[n]*x + c[n]*y+ d[n]*z;
 pde =(f[0]*x - f[1])*D[w[x, y,z], x] +(f[0]*y-f[2])*D[w[x, y,z], y] +(f[0]*z -f[3])*D[w[x,y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
f:= n ->  a[n] + b[n]*x + c[n]*y+ d[n]*z;
pde :=  (f(0)*x - f(1))*diff(w(x,y,z),x)+(f(0)*y-f(2))*diff(w(x,y,z),y)+(f(0)*z -f(3))*diff(
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

152 HFOPDE, chapter 6.2.3

152.1 Problem 1

problem number 1226

Added April 15, 2019.

Problem Chapter 6.2.3.1, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$2b^2xzw_x + by(b^2z^2+1)w_y + axy(bz+1)^2w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =2*b^2*x*z*D[w[x, y,z], x] + b*y*(b^2*z^2 +1)*D[w[x, y,z], y] + a*x*y*(b*z +1)^2*D[w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  2*b^2*x*z*diff(w(x,y,z),x)+b*y*(b^2*z^2 +1)*diff(w(x,y,z),y)+a*x*y*(b*z +1)^2*diff(w
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

152.2 Problem 2

problem number 1227

Added April 15, 2019.

Problem Chapter 6.2.3.2, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bcxy^2w_x + 2bcy^3w_y + 2(cyz - ax^2)^2w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde =b*c*x*y^2*D[w[x, y,z], x] + 2*b*c*y^3*D[w[x, y,z], y] + 2*(c*y*z-a*x^2)^2*D[w[x,y,z],z
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x^2},\frac{x^4(\log(x)\left(2cyz-2ax^2\right)+by)}{bcy^2\left(ax^2-cyz\right)}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*c*x*y^2*diff(w(x,y,z),x)+2*b*c*y^3*diff(w(x,y,z),y)+2*(c*y*z-a*x^2)^2*diff(w(x,y,z)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{y}{x^2},\frac{(-2\,ax^2+2\,cyz)\ln(x)+by}{2\,ax^2-2\,cyz}\right)$$

152.3 Problem 3

problem number 1228

Added April 15, 2019.

Problem Chapter 6.2.3.3, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$bc^2y^2zw_x + ac^2xz^2w_y - abxy^2w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*c*x*y^2*D[w[x, y,z], x] + a*c^2*x*z^2*D[w[x, y,z], y] - a*b*x*y^2*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{ax}{c}+z,\frac{by^3+c^2z^3}{3b}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*c*x*y^2*diff(w(x,y,z),x)+a*c^2*x*z^2*diff(w(x,y,z),y)- a*b*x*y^2*diff(w(x,y,z),z)=
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{ax+cz}{c},\frac{-a^3x^3-3\,a^2cx^2z-3\,ac^2xz^2+bcy^3}{bc}\right)$$

152.4 Problem 4

problem number 1229

Added April 15, 2019.

Problem Chapter 6.2.3.4, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(by^2 - cz^2)w_x + y(cz^2 - ax^2)w_y + z(ax^2 - by^2)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*(b*y^2-c*z^2)*D[w[x, y,z], x] + y*(c*z^2-a*x^2)*D[w[x, y,z], y] + z*(a*x^2-b*y^2)*D
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=x*(b*y^2-c*z^2)*diff(w(x,y,z),x)+ y*(c*z^2-a*x^2)*diff(w(x,y,z),y) + z*(a*x^2-b*y^2)*d
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = e^{-1/4\_C1\,x^2} e^{\_C2/4} e^{-1/4\,\frac{\_C1\,by^2}{a}} x^{\_C2/2} \_C3\,\_C5\,y^{\_C2/2} \_C4\,z^{\_C2/2} e^{-1/4\,\frac{\_C1\,cz^2}{a}}$$

152.5 Problem 5

problem number 1230

Added April 15, 2019.

Problem Chapter 6.2.3.5, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$by(3ax^2 + by^2 + cz^2)w_x - 2ax(ax^2 + cz^2)w_y + 2abxyzw_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*y*(3*a*x^2+ b*y^2+c*z^2)*D[w[x, y,z], x] - 2*a*x*(a*x^2+c*z^2)*D[w[x, y,z], y] + 2*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := b*y*(3*a*x^2+ b*y^2+c*z^2)*diff(w(x,y,z),x)- 2*a*x*(a*x^2+c*z^2)*diff(w(x,y,z),y) + 2
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

152.6 Problem 6

problem number 1231

Added April 15, 2019.

Problem Chapter 6.2.3.6, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$b(a(a^2x^2+b^2y^2-1)x+by)w_x+a(b(a^2x^2+b^2y^2-1)y-ax)w_y+2abzw_z=0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*D[w[x, y,z], x] +a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde :=  b*(a*(a^2*x^2+b^2*y^2-1)*x+ b*y )*diff(w(x,y,z),x)+a*(b*(a^2*x^2+b^2*y^2-1)*y - a*x)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\frac{a^2x^2+y^2b^2-1}{a^2x^2+y^2b^2}\mathrm{e}^{2\arctan\left(\frac{by}{ax}\right)}, z\mathrm{e}^{-2\int^x \frac{1}{\_a}\left(\cos\left(\mathrm{RootOf}\left(2\_Z-\ln\left(-\frac{\_a^2\left(a^2x^2+y^2b^2-1\right)a^2}{\left(a^2x^2+y^2b^2\right)\left(-\_a^2a^2+(\cos(\_Z))^2\right)}\mathrm{e}^{2}\right.\right.\right.\right.}\right.$$

152.7 Problem 7

problem number 1232

Added April 15, 2019.

Problem Chapter 6.2.3.7, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(b^3y^3 - 2a^3x^3)w_x + y(2b^3y^3 - a^3x^3)w_y + 9z(a^3x^3 - b^3y^3)w_z = 0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*(b^3*y^3 - 2*a^3*x^3)*D[w[x, y,z], x] +y*(2*b^3*y^3 -a^3*x^3)*D[w[x, y,z], y] +9*z*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$Aborted

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*(b^3*y^3 - 2*a^3*x^3)*diff(w(x,y,z),x)+y*(2*b^3*y^3 -a^3*x^3)*diff(w(x,y,z),y) + 9*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = \_F1\left(\text{RootOf}\left(-6\,yb^3\sqrt[3]{-108\,a^3b^6+8\,\_Z^3x^3+12\,a\sqrt{3}\sqrt{27\,a^4b^6-4\,\_Z^3ax^3b^3}}+4\,x^3\,\_Z^2\right.\right.$$

152.8 Problem 8

problem number 1233

Added April 15, 2019.

Problem Chapter 6.2.3.8, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$ax^2(abxy - c^2z^2)w_x + axy(abxy - c^2z^2)w_y + byz(bcyz + 2a^2x^2)w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = a*x^2*(a*b*x*y-c^2*z^2)*D[w[x, y,z], x] +a*x*y*(a*b*x*y-c^2*z^2)*D[w[x, y,z], y] +b*y
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\frac{\log\left(\frac{xz}{a^2bx^2y+ac^2xz^2+b^2cy^2z}\right)}{a}\right)\right\}\right\}$$

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := a*x^2*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),x)+a*x*y*(a*b*x*y-c^2*z^2)*diff(w(x,y,z),y) + b
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

152.9 Problem 9

problem number 1234

Added April 15, 2019.

Problem Chapter 6.2.3.9, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x(cz^4-by^4)w_x+y(ax^4-2cz^4)w_y+z(2by^4-ax^4)w_z=0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*(c*z^4 - b*y^4)*D[w[x, y,z], x] +y*(a*x^4-2*c*z^4)*D[w[x, y,z], y] +z*(2*b*y^4-a*x^
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*(c*z^4 - b*y^4)*diff(w(x,y,z),x)+y*(a*x^4-2*c*z^4)*diff(w(x,y,z),y) + z*(2*b*y^4-a*
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z),'build')),out
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z) = e^{-1/16\,\_C1\,x^4} e^{\_C2/16} e^{-1/16\,\frac{\_C1\,b y^4}{a}} x^{-\_C2/4}\,\_C3\,\_C5\,y^{-\_C2/8}\,\_C4\,z^{-\_C2/8} e^{-1/16\,\frac{\_C1\,c z^4}{a}}$$

152.10 Problem 10

problem number 1235

Added April 15, 2019.

Problem Chapter 6.2.3.10, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$x w_x + y w_y + a\sqrt{x^2+y^2} w_z = 0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +a*Sqrt[x^2+y^2]*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},z-a\sqrt{x^2+y^2}\right)\right\}\right\}$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + a*sqrt(x^2+y^2)*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},-a\sqrt{x^2+y^2}+z\right)$$

152.11 Problem 11

problem number 1236

Added April 15, 2019.

Problem Chapter 6.2.3.11, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$xw_x+yw_y+(z-a\sqrt{x^2+y^2+z^2})w_z=0$$

Mathematica ✓

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = x*D[w[x, y,z], x] +y*D[w[x, y,z], y] +(z-a*Sqrt[x^2+y^2+z^2])*D[w[x,y,z],z]==0;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

$$\left\{\left\{w(x,y,z)\to c_1\left(\frac{y}{x},\log\left(-\sqrt{\frac{x^{2a}\left(y^2+2z^2\right)+x^{2a+2}-2\sqrt{z^2x^{4a}\left(x^2+y^2+z^2\right)}}{x^2+y^2}}\right)\right)\right\},\left\{w(x,y,z)\to\right.$$

Maple ✓

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := x*diff(w(x,y,z),x)+y*diff(w(x,y,z),y) + (z-a*sqrt(x^2+y^2+z^2))*diff(w(x,y,z),z)= 0;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

$$w(x,y,z)=\_F1\left(\frac{y}{x},\left(z+\sqrt{x^2+y^2+z^2}\right)x^{a-1}\right)$$

152.12 Problem 12

problem number 1237

Added April 15, 2019.

Problem Chapter 6.2.3.12, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$z\sqrt{y^2+z^2}w_x+az\sqrt{x^2+z^2}w_y-(x\sqrt{y^2+z^2}+ay\sqrt{x^2+z^2})w_z=0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 pde = z*Sqrt[y^2+z^2]*D[w[x, y,z], x] +a*z*Sqrt[x^2+z^2]*D[w[x, y,z], y] -(x*Sqrt[y^2+z^2]+
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
pde := z*sqrt(y^2+z^2)*diff(w(x,y,z),x)+a*z*sqrt(x^2+z^2)*diff(w(x,y,z),y) -(x*sqrt(y^2+z^2)
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

sol=()

152.13 Problem 13

problem number 1238

Added April 15, 2019.

Problem Chapter 6.2.3.13, from Handbook of first order partial differential equations by Polyanin, Zaitsev, Moussiaux.

Solve for $w(x,y)$

$$(y-z)\sqrt{f(x)}w_x + (z-x)\sqrt{f(y)}w_y + (x-y)\sqrt{f(z)}w_z = 0$$

Where

$$f(t) = a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0$$

Mathematica ✗

```
ClearAll[w, x, y, z, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t,F,C1];
 ClearAll[g1, g0, h2, h1, h0, f1, f2,sigma,lambda1,lambda2,n1,n2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12,
 f[t_]:= a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
 pde = (y-z)*Sqrt[f[x]]*D[w[x, y,z], x] +(z-x)*Sqrt[f[y]]*D[w[x, y,z], y] +(x-y)*Sqrt[f[z]]*
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, y,z], {x, y,z}], 60*10]];
 sol = Simplify[sol];
```

Failed

Maple ✗

```
unassign('w,x,y,z,a,b,n,m,c,k,alpha,beta,g,A,C1,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11,sigma,lambda1,lambda2,n1,n2,nu');
f := t-> a[6]*t^6+a[5]*t^5+a[4]*t^4+a[3]*t^3+a[2]*t^2+a[1]*t+a[0];
pde :=  (y-z)*sqrt(f(x))*diff(w(x,y,z),x)+(z-x)*sqrt(f(y))*diff(w(x,y,z),y)+(x-y)*sqrt(f(z))
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,y,z))),output='rea
if(not evalb(sol=())) then sol:=simplify(sol,size); fi;
```

Exception

153 HNPDE, chapter 1.1.1

153.1 Problem 1

problem number 1239

Added March 23, 2019.

Problem Chapter 1.1.1.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = a w_{xx} + b w^2$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) + b*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

153.2 Problem 2

problem number 1240

Added March 23, 2019.

Problem Chapter 1.1.1.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw(1-w)$$

Mathematica ✓

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]*(1 - w[x, t]);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

$$\left\{\left\{w(x,t)\to \frac{1}{4}\left(\tanh\left(\frac{5at}{12}-\frac{\sqrt{a}x}{2\sqrt{6}}-c_3\right)+1\right)^2\right\},\left\{w(x,t)\to -\frac{1}{4}\left(-3+\tanh\left(\frac{5at}{12}-\frac{i\sqrt{a}x}{2\sqrt{6}}-c_3\right)\right)\right.\right.$$

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) + a*w(x,t)*(1-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = -1/4\left(\tanh\left(-\frac{5\,at}{12}+1/12\sqrt{-6\,a}x+\_C1\right)\right)^2-1/2\tanh\left(-\frac{5\,at}{12}+1/12\sqrt{-6\,a}x+\_C1\right)+$$

154 HNPDE, chapter 1.1.2

154.1 Problem 1

problem number 1241

Added March 23, 2019.

Problem Chapter 1.1.2.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = a w_{xx} - b w^3$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

154.2 Problem 2

problem number 1242

Added March 23, 2019.

Problem Chapter 1.1.2.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw - bw^3$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] - b*w[x, t]^3;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)- b*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = 1/2\,\frac{\sqrt{ab}\tanh\left(-3/4\,at + 1/4\,\sqrt{2}\sqrt{a}x + \_C1\right)}{b} - 1/2\,\frac{\sqrt{ab}}{b}$$

154.3 Problem 3

problem number 1243

Added March 23, 2019.

Problem Chapter 1.1.2.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} - bw^3 - cw^2$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] - b*w[x, t]^3 - c*w[x, t]^2;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) - b*w(x,t)^3- c*w(x,t)^2;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = 1/2\,\frac{c}{b}\tanh\left(-1/4\,\frac{c^2t}{b} + 1/4\,\frac{\sqrt{2}cx}{\sqrt{ab}} + \_C1\right) - 1/2\,\frac{c}{b}$$

154.4 Problem 4

problem number 1244

Added March 23, 2019.

Problem Chapter 1.1.2.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} - w(1-w)(a-w)$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s12];
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] - w[x, t]*(1 - w[x, t])*(a - w[x, t]);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) - w(x,t)*(1-w(x,t))*(a-w(x,t));
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = 1/2 \tanh\left((-a/2+1/4)\,t + 1/4\,x\sqrt{2} + \_C1\right) + 1/2$$

154.5 Problem 5

problem number 1245

Added March 23, 2019.

Problem Chapter 1.1.2.5, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + b_0 + b_1 w + b_2 w^2 + b_3 w^3$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b0 + b1*w[x, t] + b2*w[x, t]^2 + b3*w[x, t]^3
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✓

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b0+b1*w(x,t)+b2*w(x,t)^2+b3*w(x,t)^3;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

$$w(x,t) = -2\,\frac{\mathrm{RootOf}\Big(12\,\big(\mathrm{RootOf}\,(512\,a^3 b3^2\_Z^6 + \left(-384\,a^2 b1\,b3^2 + 128\,a^2 b2^2 b3\right)\_Z^4 + \big(72\,a b1^2 b3^2}{}$$

155 HNPDE, chapter 1.1.3

155.1 Problem 1

problem number 1246

Added March 23, 2019.

Problem Chapter 1.1.3.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + bw^k$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + b*w[x, t]^k;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +b*w(x,t)^k;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

155.2 Problem 2

problem number 1247

Added March 23, 2019.

Problem Chapter 1.1.3.2, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

155.3 Problem 3

problem number 1248

Added March 23, 2019.

Problem Chapter 1.1.3.3, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw + bw^m + cw^{2m-1}$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t] + b*w[x, t]^m + c*w[x, t]^(2*m - 1);
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)+b*w(x,t)^m+c*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

155.4 Problem 4

problem number 1249

Added March 23, 2019.

Problem Chapter 1.1.3.4, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = w_{xx} + aw^{m-1} + bmw^m - mb^2w^{2m-1}$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == D[w[x, t], {x, 2}] + a*w[x, t]^(m - 1) + b*m*w[x, t]^m - m*b^2*w[x,
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= diff(w(x,t),x$2) +a*w(x,t)^(m-1)+b*m*w(x,t)^m-m*b^2*w(x,t)^(2*m-1);
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

156 HNPDE, chapter 1.1.4

156.1 Problem 1

problem number 1250

Added March 23, 2019.

Problem Chapter 1.1.4.1, from Handbook of nonlinear partial differential equations by Andrei D. Polyanin, Valentin F. Zaitsev.

Solve for $w(x,t)$

$$w_t = aw_{xx} + s_1(bx+ct)^k + s_2w^m$$

Mathematica ✗

```
ClearAll[w, x, y, n, a, b, m, c, k, alpha, beta, gamma, A, C0, s];
 ClearAll[lambda, B, mu, d, g, B, v, f, h, q, p, delta, t];
 ClearAll[g1, g0, h2, h1, h0, f1, f2];
 ClearAll[a1, a0, b3, b2, b1, b0, c2, c1, c0, k0, k1, k2, s1, s0, k22, k11, k12, s11, s22, s
 pde = D[w[x, t], t] == a*D[w[x, t], {x, 2}] + s1*(b*x + c*t)^k + s2*w[x, t]^m;
 sol = AbsoluteTiming[TimeConstrained[DSolve[pde, w[x, t], {x, t}], 60*10]];
```

Failed

Maple ✗

```
unassign('w,x,y,a,b,n,m,c,k,alpha,beta,g,A,f,C,lambda,B,mu,d,s,t');
unassign('v,q,p,l,g1,g2,g0,h0,h1,h2,f2,f3,c0,c1,c2,a1,a0,b0,b1,b2');
unassign('k0,k1,k2,s0,s1,k22,k12,k11,s22,s12,s11');
pde :=  diff(w(x,t),t)= a*diff(w(x,t),x$2) +s1*(b*x+c*t)^k+s2*w(x,t)^m;
cpu_time := timelimit(60*10,CodeTools[Usage](assign('sol',pdsolve(pde,w(x,t))),output='realt
```

sol=()

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