

```

Manipulate[
(*Nasser M. Abbasi, version 8/11/2014*)
tick;
Module[{a1, a2, a3, v1, v2, v3, h1, a1d, a2d, a3d, z},

Which[state == "init" || state2 == "on",
state2 = "off";
If[state == "init", state = "paused"];

{pe, ke, sol} =
solve[initial $\theta$ 1 * Pi / 180., initial $\theta$ 2 * Pi / 180., initial $\theta$ 3 * Pi / 180., initial $\theta$ 1dot,
initial $\theta$ 2dot, initial $\theta$ 3dot, 0, maxTime,  $\theta$ 1,  $\theta$ 2,  $\theta$ 3, t, m1, m2, m3, L, k1, k2, len];
currentTime = 0,

state == "running" || state == "step",
If[currentTime > maxTime, currentTime = 0]
];

a1 = ( $\theta$ 1[t] /. sol) /. t  $\rightarrow$  currentTime;
a2 = ( $\theta$ 2[t] /. sol) /. t  $\rightarrow$  currentTime;
a3 = ( $\theta$ 3[t] /. sol) /. t  $\rightarrow$  currentTime;
v1 = ( $\theta$ 1'[t] /. sol) /. t  $\rightarrow$  currentTime;
v2 = ( $\theta$ 2'[t] /. sol) /. t  $\rightarrow$  currentTime;
v3 = ( $\theta$ 3'[t] /. sol) /. t  $\rightarrow$  currentTime;

If[state == "running" || state == "step",
currentTime = currentTime + timeStep;
If[state == "running", tick += del]
];

ppe = pe /. { $\theta$ 1[t]  $\rightarrow$  a1,  $\theta$ 2[t]  $\rightarrow$  a2,  $\theta$ 3[t]  $\rightarrow$  a3};
kke = ke /. { $\theta$ 1'[t]  $\rightarrow$  v1,  $\theta$ 2'[t]  $\rightarrow$  v2,  $\theta$ 3'[t]  $\rightarrow$  v3};
totalE = kke + Abs[ppe];
kkef = kke / totalE;
ppef = Abs[ppe] / totalE;
{a1d, a2d, a3d} = (z = Mod[# * 180 / Pi, 360];
Round@If[z > 180, z - 360, z]) & /@ {a1, a2, a3}; (*normalize*)

h1 = Style[Grid[{
{"time (sec)",
"P.E. (J)",
"K.E. (J)",
"energy (J)",
Row[{Subscript[" $\theta$ ", 1], " (deg)"}],
Row[{Subscript[" $\theta$ ", 2], " (deg)"}],
Row[{Subscript[" $\theta$ ", 3], " (deg)"}]
},
{padIt2[currentTime, {5, 3}],
padIt1[ppe, {6, 3}],
padIt2[kke, {6, 3}],
padIt1[totalE, {6, 3}],

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    padIt1[a1d, 3],
    padIt1[a2d, 3],
    padIt1[a3d, 3]
  }
},
Frame → All, FrameStyle → Directive[Thickness[.001], Gray], Spacings → {1, 1.2},
Alignment → Center]
, 11];

Text@Grid[{
  {h1},
  {
    Framed[Evaluate@Graphics[
      {
        {EdgeForm[Thin], Opacity[.8], Gray,
          Rectangle[{-1.5 w, -w / 2}, {2 len + 1.5 w, w / 2}, RoundingRadius → 0]},
        Rotate[{
          {EdgeForm[Thin], Opacity[m1],
            Red, Rectangle[{-w / 2, 0}, {w / 2, -L}, RoundingRadius → 0]},
          {EdgeForm[Thin], Blue, Opacity[.3], Disk[{0, 0}, w / 2]},
          {Thick, Disk[{0, 0}, w / 5]},
          {Thick, Disk[{0, -L}, w / 5]}
        ], a1, {0, 0}],
        Rotate[{
          {EdgeForm[Thin], Opacity[m2], Red,
            Rectangle[{len - w / 2, 0}, {len + w / 2, -L}, RoundingRadius → 0]},
          {EdgeForm[Thin], Blue, Opacity[.3], Disk[{len, 0}, w / 2]},
          {Thick, Disk[{len, 0}, w / 5]},
          {Thick, Disk[{len, -L}, w / 5]}
        ], a2, {len, 0}],
        Rotate[{
          {EdgeForm[Thin], Opacity[m3], Red,
            Rectangle[{2 len - w / 2, 0}, {2 len + w / 2, -L}, RoundingRadius → 0]},
          {EdgeForm[Thin], Blue, Opacity[.3], Disk[{2 len, 0}, w / 2]},
          {Thick, Disk[{2 len, 0}, w / 5]},
          {Thick, Disk[{2 len, -L}, w / 5]}
        ], a3, {2 len, 0}],
      If[k1 > 0,
        {AbsoluteThickness[ $\frac{k1}{5}$ ],
          Line[makeSpring[L Sin[a1], -L Cos[a1], len + L Sin[a2], -L Cos[a2], .15]]}
      ],
      If[k2 > 0,
        {AbsoluteThickness[ $\frac{k2}{5}$ ],

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        Line[makeSpring[len + L Sin[a2], -L Cos[a2], 2 len + L Sin[a3], -L Cos[a3], .15]]}
    ]
  },
  PlotRange → {{-.8 L, 2 len + 0.8 L}, {-L, L}}, Frame → False, Axes → False,
  ImageSize → {450}, ImagePadding → 15, Background → RGBColor[.99, .97, .90]
], FrameStyle → LightGray]
}}, Spacings → {.1, .1}]
],

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Text@Grid[{
  {
    Grid[{
      {
        Button[Text[Style["run", 11]], state = "running";
        tick += del, ImageSize → {50, 35}],
        Button[Text[Style["pause", 11]], state = "paused";
        tick += del, ImageSize → {50, 35}]
      },
      {Button[Text[Style["step", 11]], state = "step";
        tick += del, ImageSize → {50, 35}],
        Button[Text[Style["reset", 11]], state = "init";
        tick = 0, ImageSize → {50, 35}]
      }
    ]
  }, Spacings → {0.5, .5}, Alignment → Center
]
,
Grid[{
  {
    "slow", Manipulator[Dynamic[timeStep, {timeStep = #;} &],
    {0.001, 0.1, .001}, ImageSize → Tiny], "fast"
  },
  {
    TraditionalForm@Style[k1, 11], Manipulator[Dynamic[k1, {k1 = #;
    tick += del;
    state2 = "on"} &], {0, 10, .1}, ImageSize → Tiny], Dynamic[padIt2[k1, {3, 1}]]
  },
  {
    TraditionalForm@Style[k2, 11], Manipulator[Dynamic[k2, {k2 = #;
    tick += del;
    state2 = "on"} &], {0, 10, .1}, ImageSize → Tiny], Dynamic[padIt2[k2, {3, 1}]]
  },
  {
    TraditionalForm@Style[m1, 11], Manipulator[Dynamic[m1, {m1 = #;
    tick += del;
    state2 = "on"} &], {0.1, 1, .1}, ImageSize → Tiny], Dynamic[padIt2[m1, {2, 1}]]
  },
  {
    TraditionalForm@Style[m2, 11], Manipulator[Dynamic[m2, {m2 = #;

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        tick += del;
        state2 = "on" &], {0.1, 1, .1}, ImageSize → Tiny], Dynamic[padIt2[m2, {2, 1}]]
    },
    {
        TraditionalForm@Style[m3, 11], Manipulator[Dynamic[m3, {m3 = #;
            tick += del;
            state2 = "on" &], {0.1, 1, .1}, ImageSize → Tiny], Dynamic[padIt2[m3, {2, 1}]]
        }
    }, Frame → True, Alignment → Center,
    Spacings → {.5, .5}, FrameStyle -> Directive[Thickness[.001], Gray]
]
,
Grid[{
    {
        Row[{Subscript["θ", 1], "(0)"}], Manipulator[Dynamic[initialθ1, {initialθ1 = #;
            tick += del;
            state2 = "on" &], {-90, 90, 1}, ImageSize → Tiny],
        Row[{Dynamic[padIt1[initialθ1, 2]], Spacer[1], Degree}]
    },
    {
        Row[{Subscript["θ", 2], "(0)"}], Manipulator[Dynamic[initialθ2, {initialθ2 = #;
            tick += del;
            state2 = "on" &], {-90, 90, 1}, ImageSize → Tiny],
        Row[{Dynamic[padIt1[initialθ2, 2]], Spacer[1], Degree}]
    },
    {
        Row[{Subscript["θ", 3], "(0)"}], Manipulator[Dynamic[initialθ3, {initialθ3 = #;
            tick += del;
            state2 = "on" &], {-90, 90, 1}, ImageSize → Tiny],
        Row[{Dynamic[padIt1[initialθ1, 2]], Spacer[1], Degree}]
    },
    {
        Row[{Subscript["θ", 1] ', "(0)"}],
        Manipulator[Dynamic[initialθ1dot, {initialθ1dot = #;
            tick += del;
            state2 = "on" &], {-2, 2, .1}, ImageSize → Tiny],
        Dynamic[padIt1[initialθ1dot, {2, 1}]]
    },
    {
        Row[{Subscript["θ", 2] ', "(0)"}],
        Manipulator[Dynamic[initialθ2dot, {initialθ2dot = #;
            tick += del;
            state2 = "on" &], {-2, 2, .1}, ImageSize → Tiny],
        Dynamic[padIt1[initialθ2dot, {2, 1}]]
    },
    {
        Row[{Subscript["θ", 3] ', "(0)"}],
        Manipulator[Dynamic[initialθ3dot, {initialθ3dot = #;
            tick += del;
            state2 = "on" &], {-2, 2, .1}, ImageSize → Tiny],
        Dynamic[padIt1[initialθ3dot, {2, 1}]]
    }
}
]

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    }, Frame → True, Alignment → Center,
    Spacings → {.5, .5}, FrameStyle -> Directive[Thickness[.001], Gray]
  ],
  Grid[{
    {Evaluate@Graphics[
      {
        {Text[Style["K.E.", 11], {w/2, .1}]},
        {Text[Style["P.E.", 11], {2.5 w, .1}]},
        {EdgeForm[Thin], Green,
          Rectangle[{-0.4 w, -1}, {1.4 w, -1 + kkef}, RoundingRadius → 0]},
        {EdgeForm[Thin], Blue, Rectangle[{1.6 w, -1}, {3.4 w, -1 + ppef},
          RoundingRadius → 0]}
      }
    ],
    PlotRange → {{-w/2, 3.5 w}, {-1, 0.1}}, Frame → False,
    Axes → False, ImageSize → {120}, ImagePadding → 5, ImageMargins → 1
  ]
  }}, Frame → True, FrameStyle -> Directive[Thickness[.001], Gray]]

}
}, Alignment → Center, Spacings → {.5, .5}],

{{del, $MachineEpsilon}, None},
{{tick, 0}, None},
{{maxTime, 20}, None},
{{w, 0.2}, None},
{{totalE, 0}, None},
{{ppe, 0}, None},
{{ppef, 0}, None},
{{kke, 0}, None},
{{kkef, 0}, None},

{{k1, 2.1}, None},
{{k2, 5}, None},
{{m1, 1}, None},
{{m2, 0.5}, None},
{{m3, 1}, None},
{{state, "init"}, None},
{{state2, "off"}, None},
{{currentTime, 0}, None},
{{sol, {}}, None},
{{len, 2.2}, None},
{{L, 1}, None},
{{initialθ1, 60}, None},
{{initialθ2, 17}, None},
{{initialθ3, -10}, None},
{{initialθ1dot, 1.4}, None},
{{initialθ2dot, 1.6}, None},
{{initialθ3dot, -1.3}, None},
{{pe, 0}, None},
{{ke, 0}, None},
{{timeStep, 0.02}, None},

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ControlPlacement → Above,
SynchronousUpdating → False,
SynchronousInitialization → False,
ContinuousAction → False,
Alignment → Center,
ImageMargins → 5,
FrameMargins → 5,
Paneled → True,
Frame → False,
AutorunSequencing → {1},

TrackedSymbols → {tick},
Initialization →
(
integerStrictPositive = (IntegerQ[#] &&#gt; 0 &);
integerPositive = (IntegerQ[#] &&#gt;= 0 &);
numericStrictPositive = (Element[#, Reals] &&#gt; 0 &);
numericPositive = (Element[#, Reals] &&#gt;= 0 &);
numericStrictNegative = (Element[#, Reals] &&#lt; 0 &);
numericNegative = (Element[#, Reals] &&#lt;= 0 &);
bool = (Element[#, Booleans] &);
numeric = (Element[#, Reals] &);
integer = (Element[#, Integers] &);

(*-----*)
(* helper function for formatting *)
(*-----*)
padIt1[v_?numeric, f_List] := AccountingForm[Chop[v], f,
NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
padIt1[v_?numeric, f_Integer] := AccountingForm[v, f, NumberSigns → {"-", "+"},
NumberPadding → {"0", "0"}, SignPadding → True];
(*-----*)
(* helper function for formatting *)
(*-----*)
padIt2[v_?numeric, f_List] := AccountingForm[Chop[v], f,
NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
padIt2[v_?numeric, f_Integer] := AccountingForm[v, f, NumberSigns → {"", ""},
NumberPadding → {"0", "0"}, SignPadding → True];

(*-----*)
makeSpring[xFirst_?numeric,
yFirst_?numeric,
xEnd_?numeric,
yEnd_?numeric,
szel_?numeric(*the larger, the wider the spring twists*)] :=
Module[{hx, veghossz, hossz, hy, dh, tbl},
hx = xEnd - xFirst;
If[Abs[hx] ≤ $MachineEpsilon, hx = 10^-6];
hy = yEnd - yFirst;
If[Abs[hy] ≤ $MachineEpsilon, hy = 10^-6];

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veghossz = 0.28;
hossz = Sqrt[hx^2 + hy^2];
dh = (hossz - .5 * veghossz) / 30.;
tbl = Table[If[OddQ[i], {xFirst + hx * (i * dh + veghossz) / hossz + hy * szel / hossz,
  yFirst + hy * (i * dh + veghossz) / hossz - hx * szel / hossz},
  {xFirst + hx * (i * dh + veghossz) / hossz - hy * szel / hossz,
  yFirst + hy * (i * dh + veghossz) / hossz + hx * szel / hossz}], {i, 2, 18}];
{{xFirst, yFirst}} ~Join~ {{xFirst + hx * (dh + veghossz) / hossz,
  yFirst + hy * (dh + veghossz) / hossz}} ~Join~ tbl ~Join~
  {{xFirst + hx * (19 * dh + veghossz) / hossz, yFirst + hy * (19 * dh + veghossz) / hossz}} ~
  Join~ {{xEnd, yEnd}}
];

(*-----*)
solve[initialθ1_, initialθ2_, initialθ3_, initialθ1dot_, initialθ2dot_,
  initialθ3dot_, from_, to_, θ1_, θ2_, θ3_, t_, m1_, m2_, m3_, L_, k1_, k2_, len_] :=
Module[{I1, I2, I3, spring1, spring2, pe, ke, lag, eq1, eq2, eq3, sol, ic, g = 9.8},
  I1 =  $\frac{1}{3} m_1 L^2$ ;
  I2 =  $\frac{1}{3} m_2 L^2$ ;
  I3 =  $\frac{1}{3} m_3 L^2$ ;
  ke =  $\frac{1}{2} I_1 \theta_1'[t]^2 + \frac{1}{2} I_2 \theta_2'[t]^2 + \frac{1}{2} I_3 \theta_3'[t]^2$ ;
  spring1 = Sqrt[(len - L Sin[θ1[t]])^2 + L^2 Sin[θ2[t]]^2 +
    2 L Sin[θ2[t]] (len - L Sin[θ1[t]]) + L^2 + L^2 Cos[θ2[t]]^2 + L^2 (1 - Cos[θ1[t]])^2 +
    2 L^2 Cos[θ2[t]] (1 - Cos[θ1[t]]) - 2 L^2 (Cos[θ2[t]] + 1 - Cos[θ1[t]])] - len;
  (*spring 1 extension*)
  spring2 = Sqrt[(len - L Sin[θ2[t]])^2 + L^2 Sin[θ3[t]]^2 +
    2 L Sin[θ3[t]] (len - L Sin[θ2[t]]) + L^2 + L^2 Cos[θ3[t]]^2 + L^2 (1 - Cos[θ2[t]])^2 +
    2 L^2 Cos[θ3[t]] (1 - Cos[θ2[t]]) - 2 L^2 (Cos[θ3[t]] + 1 - Cos[θ2[t]])] - len;
  (*spring 2 extension*)
  pe = (1 / 2) k1 (spring1)^2 + (1 / 2) k2 (spring2)^2 -
    (1 / 2) m1 g L Cos[θ1[t]] - (1 / 2) m2 g L Cos[θ2[t]] - (1 / 2) m3 g L Cos[θ3[t]];
  lag = ke - pe;
  eq1 = D[D[lag, θ1'[t]], t] - D[lag, θ1[t]] == 0;
  eq2 = D[D[lag, θ2'[t]], t] - D[lag, θ2[t]] == 0;
  eq3 = D[D[lag, θ3'[t]], t] - D[lag, θ3[t]] == 0;
  ic = {θ1'[0] == initialθ1dot, θ1[0] == initialθ1, θ2'[0] == initialθ2dot,
    θ2[0] == initialθ2, θ3'[0] == initialθ3dot, θ3[0] == initialθ3};
  sol = First@NDSolve[{eq1, eq2, eq3, ic}, {θ1[t], θ2[t], θ3[t],
    θ1'[t], θ2'[t], θ3'[t]}, {t, from, to}];
  {pe, ke, sol}

```

])]

run

pause

step

reset

slow	<input type="range"/>	fast
k_1	<input type="range"/>	09.3
k_2	<input type="range"/>	05.0
m_1	<input type="range"/>	1.0
m_2	<input type="range"/>	1.0
m_3	<input type="range"/>	1.0

$\theta_1(0)$	<input type="range"/>	+60°
$\theta_2(0)$	<input type="range"/>	+17°
$\theta_3(0)$	<input type="range"/>	+60°
$\theta_1'(0)$	<input type="range"/>	+1.4
$\theta_2'(0)$	<input type="range"/>	+1.6
$\theta_3'(0)$	<input type="range"/>	-1.3

K.E.

P.E.

time (sec)	P.E. (J)	K.E. (J)	energy (J)	θ_1 (deg)	θ_2 (deg)	θ_3 (deg)
08.352	-012.278	003.107	+015.385	-017	+018	+020

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