Illustrating Cauchy And Engineering Strain Deformation in 3D

Initialization Code

(optional)

Manipulate

```
Manipulate[
  Module[{g, p1, p2, p3, p4, p5, p6, p7, p8, e = ee * 10 ^ 9, σx = σxx * 10 ^ 6,
    σy = σyy * 10 ^ 6, σz = σzz * 10 ^ 6, τxy = τxyxy * 10 ^ 6, τyz = τyzyz * 10 ^ 6, τzx = τzxzx * 10 ^ 6},

    g = e / (2 (1 + ν));
    ϵx = σx/e - ν/e (σy + σz);
    ϵy = σy/e - ν/e (σx + σz);
    ϵz = σz/e - ν/e (σy + σx);
    γxy = τxy / g;
    γyz = τyz / g;
    γzx = τzx / g;

    {p1, p2, p3, p4, p5, p6, p7, p8} = getCoordinates[strainType, ϵx, ϵy, ϵz, γxy, γyz, γzx, k];

    Style[Framed@Graphics3D[
       Rotate[
        {
         If[plotStyle == 1,
          {
           {EdgeForm[{Thick, Red}], FaceForm[Opacity[.3]],

            Polygon[{ p1, p2, p3, p4}],
            Polygon[{ p5, p6, p7, p8}],
            Polygon[{ p4, p3, p7, p8}],
            Polygon[{ p1, p2, p6, p5}],
            Polygon[{ p2, p6, p7, p3}],
            Polygon[{ p4, p8, p5, p1}]
           }
          },
          {
           {FaceForm[{Opacity[0.3]}, White],
            Polygon[{ p1, p2, p3, p4}],
            Polygon[{ p5, p6, p7, p8}],
            Polygon[{ p4, p3, p7, p8}],
            Polygon[{ p1, p2, p6, p5}],
            Polygon[{ p2, p6, p7, p3}],
            Polygon[{ p4, p8, p5, p1}]
           }
          }
         ],
```

```
     If[showOriginal,
      {Gray, Dashed,
       Line[{{-1/2, -1/2, 1/2}, {1/2, -1/2, 1/2},
         {1/2, -1/2, -1/2}, {-1/2, -1/2, -1/2}, {-1/2, -1/2, 1/2}}],
       Line[{{-1/2, 1/2, 1/2}, {1/2, 1/2, 1/2}, {1/2, 1/2, -1/2},
         {-1/2, 1/2, -1/2}, {-1/2, 1/2, 1/2}}],
       Line[{{-1/2, -1/2, -1/2}, {-1/2, 1/2, -1/2}}],
       Line[{{1/2, -1/2, -1/2}, {1/2, 1/2, -1/2}}],
       Line[{{-1/2, -1/2, 1/2}, {-1/2, 1/2, 1/2}}],
       Line[{{1/2, -1/2, 1/2}, {1/2, 1/2, 1/2}}]
      },
      {}
     ],

     If[showAxes,
      {
       {Arrowheads[.02],
        Arrow[{{0, 0, 0}, #}] & /@ {{.25, 0, 0}, {0, 0.25, 0}, {0, 0, 0.25 }}
       }
       ,
       {Text[Style["x", Italic, FontSize → Scaled[.04]], {.28, 0, 0}], Text[Style["y", Italic, FontSize →
           Scaled[.04]], {0, 0.28, 0}], Text[Style["z", Italic, FontSize → Scaled[.04]], {0, 0, .28}]
       }
      },
      {}
     ]

    }, 90 Degree, {1, 0, 0}
   ],
   PlotRange → {{-1.3, 1.3}, {-1.4, 1.4}, {-1.4, 1.4}},
   Boxed → False,
   ImagePadding → .1,
   ImageMargins → 0,
   SphericalRegion → True,
   ViewPoint → {4.86215, -27.5746, 5},
   If[plotStyle == 1,
    Lighting → {{"Ambient", White}},
    Lighting → Automatic
   ],
   ViewAngle → 2 * Pi / 180,
   PreserveImageOptions → If[reset, reset = False; False, True],
   ImageSize → {280, 420}
  ], Antialiasing → True]
],

Text@Grid[{
   {Grid[{
      {Spacer[39],
       TraditionalForm@HoldForm[σ_x],
       Control[{{σxx, 0, ""}, -500, 500, 1, ImageSize → Tiny}],
       Spacer[1],
       Dynamic@AccountingForm[σxx,
          3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
       Spacer[3],
       "MPa",
       Button["zero", σxx = 0, Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}],
       Spacer[47]
      },

      {
       Spacer[1],
       TraditionalForm@HoldForm[σ_y],
       Control[{{σyy, 0, ""}, -500, 500, 1, ImageSize → Tiny}],
```

```
         Spacer[1],
         Dynamic@AccountingForm[σyy,
            3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
         Spacer[3],
         "MPa",
         Button["zero", σyy = 0, Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}],
         Spacer[1]
        },

        {
         Spacer[1],
         TraditionalForm@HoldForm[σz],
         Control[{{σzz, 0, ""}, -500, 500, 1, ImageSize → Tiny}],
         Spacer[1],
         Dynamic@AccountingForm[σzz,
            3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
         Spacer[3],
         "MPa",
         Button["zero", σzz = 0, Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}],
         Spacer[1]
        },

        {
         Spacer[1],
         TraditionalForm@HoldForm[τx,y],
         Control[{{τxyxy, 300, ""}, -500, 500, 1, ImageSize → Tiny}],
         Spacer[1],
         Dynamic@AccountingForm[τxyxy,
            3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
         Spacer[3],
         "MPa",
         Button["zero", τxyxy = 0,
          Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}], Spacer[1]
        },

        {
         Spacer[1],
         TraditionalForm@HoldForm[τy,z],
         Control[{{τyzyz, 0, ""}, -500, 500, 1, ImageSize → Tiny}],
         Spacer[1],
         Dynamic@AccountingForm[τyzyz,
            3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
         Spacer[3],
         "MPa",
         Button["zero", τyzyz = 0,
          Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}], Spacer[1]
        },

        {
         Spacer[1],
         TraditionalForm@HoldForm[τz,x],
         Control[{{τzxzx, 0, ""}, -500, 500, 1, ImageSize → Tiny}],
         Spacer[1],
         Dynamic@AccountingForm[τzxzx,
            3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True],
         Spacer[3],
         "MPa",
         Button["zero", τzxzx = 0,
          Appearance → "Palette", Background → LightBlue, ImageSize → {45, 20}], Spacer[1]
        }

       }, Alignment → Left, Spacings → {.4, .2}, Frame → False, FrameStyle -> Directive[Thickness[.005], Gray]]
     },
```

```
 {
  Grid[{
     {Button["reset view", reset = True, ImageSize → {95, 28}], Spacer[3],
      Button["reset stress", σxx = 0; σyy = 0; σzz = 0; τxyxy = 0; τyzyz = 0; τzxzx = 0, ImageSize → {95, 28}]}
    }]
 },


 {
  Grid[{
     {Spacer[23], Style["shear strain type", 12],
      RadioButtonBar[Dynamic[strainType],
       {"cauchy" -> Style["Cauchy", 12], "eng" -> Style["engineering", 12]}],
      Spacer[22]
     }
    }, Alignment → Left, Spacings → {.5, .5}, Frame → False, FrameStyle -> Directive[Thickness[.005], Gray]]
 },

 {
  Grid[{
     {
      Spacer[46],
      Style["Young's modulus E", 11],
      Control[{{ee, 90, ""}, 90, 260, 1, ImageSize → Tiny}],
      Spacer[3],
      Dynamic@AccountingForm[ee,
        3, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True],
      Spacer[3],
      "GPa",
      Spacer[46]
     },

     {
      Spacer[3],
      Style["poisson ratio ν", 11],
      Control[{{ν, .3, ""}, .01, .5, .01, ImageSize → Tiny}],
      Spacer[1],
      Dynamic@AccountingForm[ν,
        {2, 2}, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True],
      SpanFromLeft
     },

     {
      Spacer[3],
      Style["exaggeration factor", 11],
      Control[{{k, 25, ""}, 1, 35, 1, ImageSize → Tiny}],
      Spacer[1],
      Dynamic@AccountingForm[k,
        2, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True],
      SpanFromLeft
     }
    }, Alignment → Left, Spacings → {0, .8}, Frame → True, FrameStyle -> Directive[Thickness[.005], Gray]
   ]
 },

 {
  Grid[{
     {Spacer[11], Style["stress tensor (MPa)", 11], Spacer[11]},
     {
      Style[Row[
        {Text@Style[TraditionalForm@HoldForm[{{σx, τx,y, τx,z}, {τy,x, σy, τy,z}, {τz,x, τz,y, σz}}]], " = ",
         Dynamic@TraditionalForm[{
             {AccountingForm[σxx,
               3, NumberSigns → {"-", "+"},
```

```
                    NumberPadding → {"0", "0"}, SignPadding → True], AccountingForm[τxyxy,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"},
                     SignPadding → True], AccountingForm[τzxzx,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True]
                  },
                  {AccountingForm[τxyxy,
                     3, NumberSigns → {"-", "+"},
                     NumberPadding → {"0", "0"}, SignPadding → True], AccountingForm[σyy,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"},
                     SignPadding → True], AccountingForm[τyzyz,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True]
                  },
                  {AccountingForm[τzxzx,
                     3, NumberSigns → {"-", "+"},
                     NumberPadding → {"0", "0"}, SignPadding → True], AccountingForm[τyzyz,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"},
                     SignPadding → True], AccountingForm[σzz,
                     3, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True]
                  }
                }]
            }], 11], SpanFromLeft
        }
      }, Alignment → Center, Spacings → {6.5, .8},
      Frame → False, FrameStyle -> Directive[Thickness[.005], Gray]]
    },


    {
      Grid[{
        {Style["strain tensor", 11]},
        {
          Style[Row[{Text@Style[TraditionalForm@HoldForm[{
                {ϵx,
                  Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] (γx,y),
                  Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] γx,z},
                {Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] γy,x, ϵy,
                  Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] γy,z},
                {Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] γz,x,
                  Dynamic@If[strainType == "eng", Spacer[12], Style["1/2", 8]] γz,y,
                  ϵz}}]], " = ",
            Dynamic@TraditionalForm[{
              {padIt1[100 * ex, {5, 4}],
                padIt1[180 / Pi * If[strainType == "eng", γxy, γxy / 2], {5, 4}]^Degree,
                padIt1[180 / Pi * γzx / If[strainType == "eng", 1, 2], {5, 4}]^Degree
              },
              {padIt1[180 / Pi * γxy / If[strainType == "eng", 1, 2], {5, 4}]^Degree,
                padIt1[100 * ey, {5, 4}],
                padIt1[180 / Pi * γyz / If[strainType == "eng", 1, 2], {5, 4}]^Degree
              },
              {padIt1[180 / Pi * γzx / If[strainType == "eng", 1, 2], {5, 4}]^Degree,
                padIt1[180 / Pi * γyz / If[strainType == "eng", 1, 2], {5, 4}]^Degree,
                padIt1[100 * ez, {5, 4}]
              }
            }]
          }], 11]
        }
      }, Alignment → Center, Spacings → {0.3, 1},
      Frame → False, FrameStyle -> Directive[Thickness[.005], Gray]
      ]
    },


    {
```

```
   Grid[{
      {
       Row[{Text@Style["show original", 12], Control[{{showOriginal, True, ""}, {True, False}}]}],
       Row[{Text@Style["show axes", 12], Control[{{showAxes, True, ""}, {True, False}}]}],
       Row[{Style["select 3D style", 12], RadioButtonBar[Dynamic[plotStyle], {1, 2}]}]
      }
     }, Alignment → Left, Spacings → {1, .4}, Frame → True, FrameStyle -> Directive[Thickness[.005], Gray]
    ]
   }


  }, Alignment -> Center
 ],

{{strainType, "eng"}, None},
{{plotStyle, 2}, None},
{{reset, False}, None},
{{ϵx, 0}, None},
{{ϵy, 0}, None},
{{ϵz, 0}, None},
{{γxy, 0}, None},
{{γyz, 0}, None},
{{γzx, 0}, None},

ControlPlacement → Left,
SynchronousUpdating → True,
SynchronousInitialization → True,
ContinuousAction → True,
Alignment -> Center,
ImageMargins → 0,
FrameMargins → 0,
Paneled → True,
Frame → False,
TrackedSymbols :>
 {σxx, σyy, σzz, τxyxy, τyzyz, τzxzx, strainType, k, ee, ν, reset, showOriginal, showAxes, plotStyle},
Initialization :>
 (
  (*definitions used for parameter checking*)
  integerStrictPositive = (IntegerQ[#] && # > 0 &);
  integerPositive = (IntegerQ[#] && # ≥ 0 &);
  numericStrictPositive = (Element[#, Reals] && # > 0 &);
  numericPositive = (Element[#, Reals] && # ≥ 0 &);
  numericStrictNegative = (Element[#, Reals] && # < 0 &);
  numericNegative = (Element[#, Reals] && # ≤ 0 &);
  bool = (Element[#, Booleans] &);
  numeric = (Element[#, Reals] &);
  integer = (Element[#, Integers] &);
  (*-----------------------------------------*)
  padIt1[v_?numeric, f_List] := AccountingForm[v,
    f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
  (*-----------------------------------------*)
  padIt1[v_?numeric, f_Integer] := AccountingForm[Chop[v],
    f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
  (*-----------------------------------------*)
  padIt2[v_?numeric, f_List] := AccountingForm[v,
    f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
  (*-----------------------------------------*)
  padIt2[v_?numeric, f_Integer] := AccountingForm[Chop[v],
    f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
  (*-----------------------------------------*)

  getCoordinates[strainType_String, ϵx_?numeric, ϵy_?numeric, ϵz_?numeric, γxy_?numeric, γyz_?numeric,
    γzx_?numeric, k_?integerStrictPositive] := Module[{p1, p2, p3, p4, p5, p6, p7, p8, aτxy, aτyz, aτzx},
```

-

```mathematica
aτxy = Sin[γxy / 2];
aτyz = Sin[γyz / 2];
aτzx = Sin[γzx / 2];

(*set coordinates of 8 corners of cube based on type of strain*)
If[strainType == "eng",
 p1 = {-1 / 2 + k (-ex / 2 + aτzx), -1 / 2 + k (-ey / 2 - aτxy), 1 / 2 + k (ez / 2 - aτyz)};
 p2 = {1 / 2 + k (ex / 2 + aτzx), -1 / 2 + k (-ey / 2 + aτxy), 1 / 2 + k (ez / 2 - aτyz)};
 p3 = {1 / 2 + k (ex / 2 + aτzx), 1 / 2 + k (ey / 2 + aτxy), 1 / 2 + k (ez / 2 + aτyz)};
 p4 = {-1 / 2 + k (-ex / 2 + aτzx), 1 / 2 + k (ey / 2 - aτxy), 1 / 2 + k (ez / 2 + aτyz)};
 p5 = {-1 / 2 + k (-ex / 2 - aτzx), -1 / 2 + k (-ey / 2 - aτxy), -1 / 2 + k (-ez / 2 - aτyz)};
 p6 = {1 / 2 + k (ex / 2 - aτzx), -1 / 2 + k (-ey / 2 + aτxy), -1 / 2 + k (-ez / 2 - aτyz)};
 p7 = {1 / 2 + k (ex / 2 - aτzx), 1 / 2 + k (ey / 2 + aτxy), -1 / 2 + k (-ez / 2 + aτyz)};
 p8 = {-1 / 2 + k (-ex / 2 - aτzx), 1 / 2 + k (ey / 2 - aτxy), -1 / 2 + k (-ez / 2 + aτyz)}
 ,
 p1 = {-1 / 2 + k (-ex / 2 - aτxy / 2 + aτzx / 2),
    -1 / 2 + k (-ey / 2 - aτxy / 2 + aτyz / 2), 1 / 2 + k (ez / 2 - aτyz / 2 - aτzx / 2)};
 p2 = {1 / 2 + k (ex / 2 - aτxy / 2 + aτzx / 2), -1 / 2 + k (-ey / 2 + aτxy / 2 + aτyz / 2),
    1 / 2 + k (ez / 2 - aτyz / 2 + aτzx / 2)};
 p3 = {1 / 2 + k (ex / 2 + aτxy / 2 + aτzx / 2), 1 / 2 + k (ey / 2 + aτxy / 2 + aτyz / 2),
    1 / 2 + k (ez / 2 + aτyz / 2 + aτzx / 2)};
 p4 = {-1 / 2 + k (-ex / 2 + aτxy / 2 + aτzx / 2), 1 / 2 + k (ey / 2 - aτxy / 2 + aτyz / 2),
    1 / 2 + k (ez / 2 + aτyz / 2 - aτzx / 2)};
 p5 = {-1 / 2 + k (-ex / 2 - aτxy / 2 - aτzx / 2), -1 / 2 + k (-ey / 2 - aτxy / 2 - aτyz / 2),
    -1 / 2 + k (-ez / 2 - aτyz / 2 - aτzx / 2)};
 p6 = {1 / 2 + k (ex / 2 - aτxy / 2 - aτzx / 2), -1 / 2 + k (-ey / 2 + aτxy / 2 - aτyz / 2),
    -1 / 2 + k (-ez / 2 - aτyz / 2 + aτzx / 2)};
 p7 = {1 / 2 + k (ex / 2 + aτxy / 2 - aτzx / 2), 1 / 2 + k (ey / 2 + aτxy / 2 - aτyz / 2),
    -1 / 2 + k (-ez / 2 + aτyz / 2 + aτzx / 2)};
 p8 = {-1 / 2 + k (-ex / 2 + aτxy / 2 - aτzx / 2), 1 / 2 + k (ey / 2 - aτxy / 2 - aτyz / 2),
    -1 / 2 + k (-ez / 2 + aτyz / 2 - aτzx / 2)}

 ];

 {p1, p2, p3, p4, p5, p6, p7, p8}
 ]
)

]
```

references

[1] A.C. Ugural, S.K. Fenster, *Advanced Strength and Applied Elasticity,* New York: Elsevier, 1987.

[2] http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/strain.cfm

[3] http://en.wikipedia.org/wiki/Infinitesimal_strain_theory

Control Suggestions   (optional)

- ☑ Resize Images
- ☑ Rotate and Zoom in 3D
- ☐ Drag Locators
- ☐ Create and Delete Locators
- ☑ Slider Zoom
- ☐ Gamepad Controls
- ☐ Automatic Animation
- ☐ Bookmark Animation

Search Terms   (optional)

stress tensor
strain tensor
Cauchy strain
elasticity
modulus of elasticity
Poisson's ratio

Related Links   (optional)

Poisson ratio
Young's modulus

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