```
In[27]:= Manipulate[
          (*by Nasser M. Abbasi, 6/28/148*)
          tick;
          Module[{mass = dx * dz * dy, eq1, eq2, eq3, r, angle, t, to,
             Ix, Iy, Iz, thetax, thetay, thetaz, sol, g, debug = False, lengthOfw},
           Ix = getIx[mass, dx, dz, dy];
           Iy = getIy[mass, dx, dz, dy];
           Iz = getIz[mass, dx, dz, dy];

           If[state ⩵ "RESET",
            currentThetax = 0;
            currentThetay = 0;
            currentThetaz = 0;
            Which[
             spin == "X-axes", currentThetaDotx = initialSpinRate * 2 * Pi / 60;
             currentThetaDoty = 0; currentThetaDotz = 0,
             spin == "Y-axes", currentThetaDoty = initialSpinRate * 2 * Pi / 60;
             currentThetaDotx = 0; currentThetaDotz = 0,
             spin == "Z-axes", currentThetaDotz = initialSpinRate * 2 * Pi / 60;
             currentThetaDoty = 0; currentThetaDotx = 0
            ]
           ];

           (*Euler equations for 3D, zero torque*)
           eq1 = Ix * thetax''[t] + (Iz - Iy) * thetay'[t] thetaz'[t] ⩵ 0;
           eq2 = Iy * thetay''[t] + (Ix - Iz) * thetaz'[t] thetax'[t] ⩵ 0;
           eq3 = Iz * thetaz''[t] + (Iy - Ix) * thetay'[t] thetax'[t] ⩵ 0;

           sol = First@NDSolve[{eq1, eq2, eq3,
                thetax[0] ⩵ currentThetax,
                thetay[0] ⩵ currentThetay,
                thetaz[0] ⩵ currentThetaz,
                thetax'[0] ⩵ currentThetaDotx,
                thetay'[0] ⩵ currentThetaDoty,
                thetaz'[0] ⩵ currentThetaDotz},
               {thetax, thetay, thetaz, thetax', thetay', thetaz'}, {t, 0, delT}];
           thetax = thetax /. sol;
           thetay = thetay /. sol;
           thetaz = thetaz /. sol;
           lengthOfw = Norm[{currentThetaDotx, currentThetaDoty, currentThetaDotz}];
           If[lengthOfw ≤ $MachineEpsilon, lengthOfw = 1];

           r = Which[
             spin == "X-axes",
             angle =
              (Dot[{1, 0, 0}, {currentThetaDotx, currentThetaDoty, currentThetaDotz}]) / lengthOfw;
             to = {angle, 0, 0};
             r = lengthOfw * Sin[ArcCos[ angle]],

             spin == "Y-axes",
             angle =
              (Dot[{0, 1, 0}, {currentThetaDotx, currentThetaDoty, currentThetaDotz}]) / lengthOfw;
```

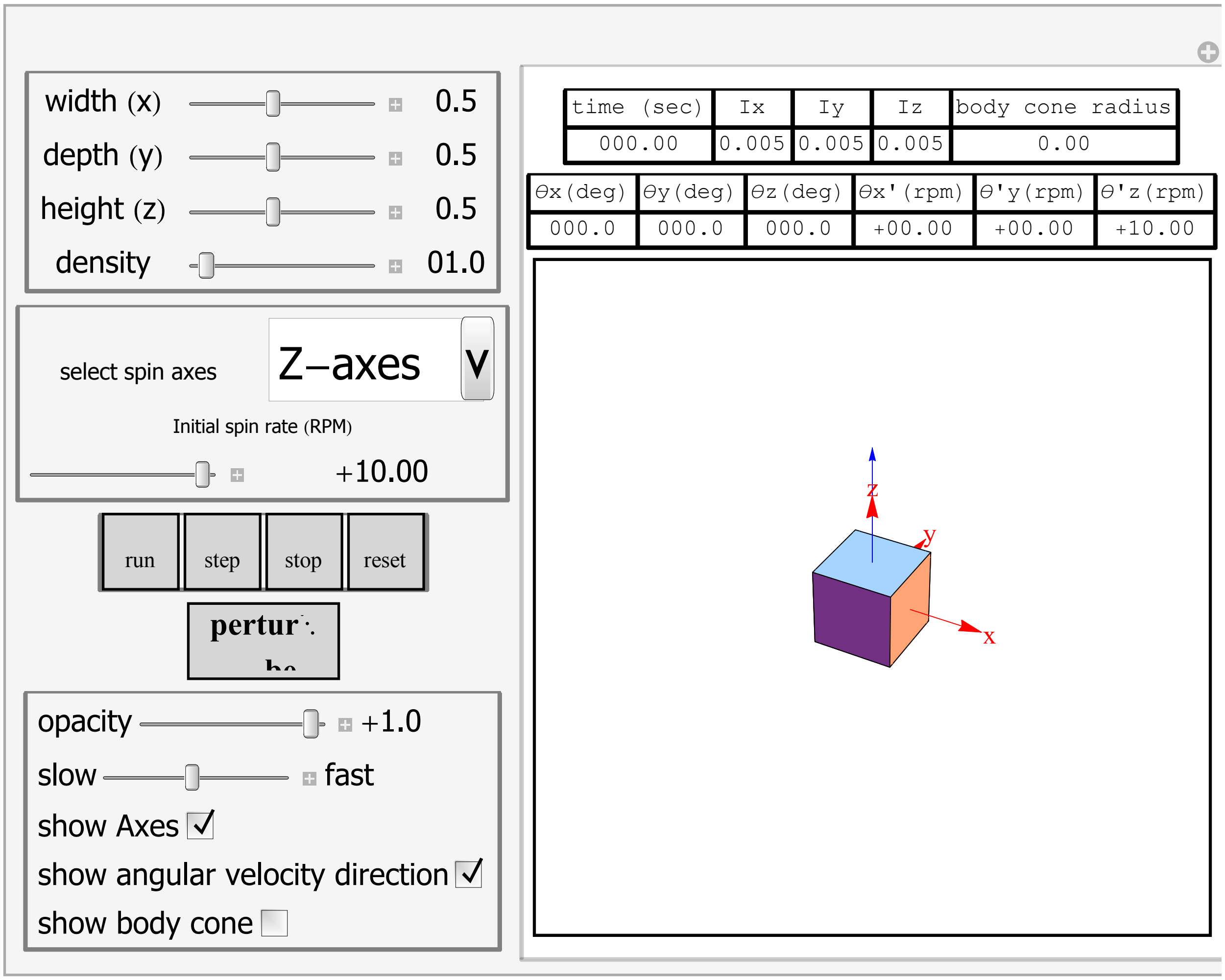
```
  to = {0, angle, 0};
  r = lengthOfw * Sin[ArcCos[ angle]],
  spin == "Z-axes",
  angle =
   (Dot[{0, 0, 1}, {currentThetaDotx, currentThetaDoty, currentThetaDotz}]) / lengthOfw;
  to = {0, 0, angle};
  r = lengthOfw * Sin[ArcCos[ angle]]
 ];

g = Grid[{
   {Grid[{
      {"time (sec)", "Ix", "Iy", "Iz", "body cone radius"},
      {padIt2[currentTime, {5, 2}],
       padIt2[Ix, {4, 3}],
       padIt2[Iy, {4, 3}],
       padIt2[Iz, {4, 3}],
       padIt2[r, {3, 2}]
      }
     }, Alignment → Center, Frame → All, Spacings → {.5, .7}]
   }
   ,
   {Grid[{
      {"θx(deg)", "θy(deg)", "θz(deg)", "θx'(rpm)", "θ'y(rpm)", "θ'z(rpm)"},
      {
       padIt2[N@Mod[currentThetax * 180 / Pi, 360], {4, 1}],
       padIt2[N@Mod[currentThetay * 180 / Pi, 360], {4, 1}],
       padIt2[N@Mod[currentThetaz * 180 / Pi, 360], {4, 1}],
       padIt1[N[currentThetaDotx / (2 * Pi)] * 60, {4, 2}], (*RPM*)
       padIt1[N[currentThetaDoty / (2 * Pi)] * 60, {4, 2}],
       padIt1[N[currentThetaDotz / (2 * Pi) ] * 60, {4, 2}]
      }
     }, Alignment → Center, Frame → All, Spacings → {.5, .7}
    ]
   },
   {Framed@
     Graphics3D[
      Rotate[GraphicsGroup[Rotate[GraphicsGroup[Rotate[GraphicsGroup
            [{

             If[showAxes,
              {
               {Red, Arrowheads[Medium], Arrow[{{0, 0, 0}, {.7, 0, 0}}]},

               Inset[Graphics[
                 Text[Style["x", Red, 14]]
                ], {0.75, 0, 0}],


               {Red, Arrowheads[Medium], Arrow[{{0, 0, 0}, {0, .7, 0}}]},

               Inset[Graphics[
                 Text[Style["y", Red, 14]]
```

```
                  ], {0, 0.75, 0}],

                 {Red, Arrowheads[Medium], Arrow[{{0, 0, 0}, {0, 0, .7}}]},

                 Inset[Graphics[
                   Text[Style["z", Red, 14]]
                  ], {0, 0, 0.75}]
                }
               ],

               {Opacity[op], Cuboid[{-dx / 2, -dy / 2, -dz / 2}, {dx / 2, dy / 2, dz / 2}]},

               Sphere[{0, 0, 0}, .02],

               If[showW,
                {Blue, Arrowheads[Small], Arrow[{{0, 0, 0},
                   {currentThetaDotx, currentThetaDoty, currentThetaDotz} / lengthOfw}]}
               ],

               If[showCone,
                {EdgeForm[Red], Opacity[.1], Cone[{to, {0, 0, 0}}, r]}
               ]


              }], thetax[0], {1, 0, 0}
            ]
          ], thetay[0], {0, 1, 0}]
        ], thetaz[0], {0, 0, 1}
       ],
       Axes → False, AxesLabel → {"x", "y", "z"},
       PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}, {-1.1, 1.1}},
       SphericalRegion → True, Boxed → False, ImagePadding → 2, ImageSize → 350
      ]
    }
   }
 ];

currentThetax = thetax[delT];
currentThetay = thetay[delT];
currentThetaz = thetaz[delT];

currentThetaDotx = (thetax' /. sol)[delT];
currentThetaDoty = (thetay' /. sol)[delT];
currentThetaDotz = (thetaz' /. sol)[delT];

If[debug, Print["currentThetax=", currentThetax]];
If[debug, Print["currentThetay=", currentThetay]];
If[debug, Print["currentThetaz=", currentThetaz]];

Which[state ⩵ "RUN",
 currentTime += delT;
 If[currentTime ≥ 1000, currentTime = 0];
```

```
  tick = Not[tick]
 ];

 If[Abs[currentThetaDotx] > 10 || Abs[currentThetaDoty] > 10 || Abs[currentThetaDotz] > 10,
  state = "STOP"
 ];

 g
],

Grid[{
  {
   Grid[{
      {"width (x)", Manipulator[Dynamic[dx, {dx = #, tick = Not[tick]} &],
        {.1, 1, .1}, ImageSize → Small], Dynamic[padIt2[dx, {2, 1}]]},
      {"depth (y)", Manipulator[Dynamic[dy, {dy = #, tick = Not[tick]} &],
        {.1, 1, .1}, ImageSize → Small], Dynamic[padIt2[dy, {2, 1}]]},
      {"height (z)", Manipulator[Dynamic[dz, {dz = #, tick = Not[tick]} &],
        {.1, 1, .1}, ImageSize → Small], Dynamic[padIt2[dz, {2, 1}]]},
      {"density", Manipulator[Dynamic[m, {m = #, tick = Not[tick]} &],
        {.1, 50, .1}, ImageSize → Small], Dynamic[padIt2[m, {3, 1}]]}
     }, Frame → True, FrameStyle → Gray
   ]
  },
  (*
  {
   Row[{"Initial angular positions"}]
  },
  {
   Grid[{
      {"θx(0)",Manipulator[Dynamic[θx,{θx=#;currentThetax=θx*Pi/180;tick=Not[tick]}&],
        {-15,15,1},ImageSize→Small],Dynamic[padIt1[θx,2]]},
      {"θy(0)",Manipulator[Dynamic[θy,{θy=#;currentThetay=θy*Pi/180;tick=Not[tick]}&],
        {-15,15,1},ImageSize→Small],Dynamic[padIt1[θy,2]]},
      {"θz(0)",Manipulator[Dynamic[θz,{θz=#;currentThetaz=θz*Pi/180;tick=Not[tick]}&],
        {-15,15,1},ImageSize→Small],Dynamic[padIt1[θz,2]]}
     },Frame→True]
  },
  *)
  {
   Grid[{
      {Style["select spin axes", 12],
       PopupMenu[Dynamic[spin, {spin = #;
           Which[
            spin == "X-axes",
            currentThetaDotx = initialSpinRate * 2 * Pi / 60; currentThetaDoty = 0;
            currentThetaDotz = 0; currentThetax = 0; currentThetay = 0; currentThetaz = 0,

            spin == "Y-axes",
            currentThetaDoty = initialSpinRate * 2 * Pi / 60; currentThetaDotx = 0;
            currentThetaDotz = 0; currentThetax = 0; currentThetay = 0; currentThetaz = 0,
         spin == "Z-axes",
         currentThetaDotz = initialSpinRate * 2 * Pi / 60; currentThetaDotx = 0;
         currentThetaDoty = 0; currentThetax = 0; currentThetay = 0; currentThetaz = 0

        ];
        tick = Not[tick]} &], {"X-axes", "Y-axes", "Z-axes"},
     ImageSize → All]
   },
   {Style["Initial spin rate (RPM)", 10], SpanFromLeft},
   {Manipulator[Dynamic[initialSpinRate, (initialSpinRate = #;
        Which[
         spin == "X-axes", currentThetaDotx = initialSpinRate * 2 * Pi / 60;
         currentThetaDoty = 0; currentThetaDotz = 0,

         spin == "Y-axes", currentThetaDoty = initialSpinRate * 2 * Pi / 60;
         currentThetaDotx = 0; currentThetaDotz = 0,

         spin == "Z-axes", currentThetaDotz = initialSpinRate * 2 * Pi / 60;
         currentThetaDoty = 0; currentThetaDotx = 0
        ];
        tick = Not[tick]) &], {-10, 10, .1}, ImageSize → Small
    ],
    Dynamic[padIt1[N@initialSpinRate, {4, 2}]]
   }
  }, Frame → True, FrameStyle → Gray
 ]
},
{
 Grid[{
   { Button[Text@Style["run", 12],
     {state = "RUN"; tick = Not[tick]}, ImageSize → {40, 40}],
    Button[Text@Style["step", 12], {state = "STEP"; tick = Not[tick]},
     ImageSize → {40, 40}],
    Button[Text@Style["stop", 12], {state = "STOP"; tick = Not[tick]},
     ImageSize → {40, 40}],
    Button[Text@Style["reset", 12], {state = "RESET"; currentThetax = 0;
      currentThetay = 0; currentThetaz = 0; currentThetaDotx = 0;
      currentThetaDoty = 0; currentThetaDotz = 0; op = 1; dz = .5; dx = .5;
      dy = .5; m = 1; spin = "Z-axes"; initialSpinRate = 10; delT = 0.1;
      currentTime = 0; tick = Not[tick]}, ImageSize → {40, 40}](*fix*)
   }
  }, Spacings → {.2, 0}, Frame → True, FrameStyle → Gray
 ]
},
{
 Grid[{
   {
    Button[Text@Style["perturbe", Bold], {
      Which[
       spin == "X-axes", currentThetaDoty = (0.02 * currentThetaDotx);
       currentThetaDotz = (0.02 * currentThetaDotx),
       spin == "Y-axes", currentThetaDotx = (0.02 * currentThetaDoty);
```

```
          currentThetaDotz = (0.02 * currentThetaDoty),
          spin == "Z-axes", currentThetaDoty = (0.02 * currentThetaDotz);
          currentThetaDotx = (0.02 * currentThetaDotz)
         ];
         tick = Not[tick]}, ImageSize → {80, 40}]
     }
    }]
  }
  ,
  {Grid[{
     {Row[{"opacity", Spacer[3], Manipulator[Dynamic[op, {op = #; tick = Not[tick]} &],
          {.1, 1, .1}, ImageSize → Small], Spacer[3], Dynamic[padIt1[op, {1, 1}]]}]},
     {Row[{"slow", Spacer[3], Manipulator[Dynamic[delT, {delT = #; tick = Not[tick]} &],
          {0.01, 0.2, .01}, ImageSize → Small], Spacer[3], "fast"}]},
     {Row[{"show Axes", Spacer[3], Checkbox[Dynamic[showAxes,
           {showAxes = #; tick = Not[tick]} &]]}]},
     {Row[{"show angular velocity direction", Spacer[2],
        Checkbox[Dynamic[showW, {showW = #; tick = Not[tick]} &]]}], SpanFromLeft},
     {Row[{"show body cone", Spacer[3], Checkbox[Dynamic[showCone,
           {showCone = #; tick = Not[tick]} &]]}]}
    }, Frame → True, Alignment → Left, FrameStyle → Gray
   ]
  }
 }, Frame → False, Alignment → Center, FrameStyle → Gray
],

{{tick, False}, None},
{{state, "RESET"}, None},
{{currentTime, 0}, None},
{{delT, 0.1}, None},
{{op, 1}, None},
{{spin, "Z-axes"}, None},
{{initialSpinRate, 10}, None},
{{showAxes, True}, None},
{{showW, True}, None},
{{showCone, True}, None},

(*{{θx,0},None},
{{θy,0},None},
{{θz,0},None},
*)

{{dx, .5}, None},
{{dz, .5}, None},
{{dy, .5}, None},
{{m, 1}, None},

{{currentThetax, 0}, None},
{{currentThetay, 0}, None},
{{currentThetaz, 0}, None},

{{currentThetaDotx, 0}, None},
 {{currentThetaDoty, 0}, None},
 {{currentThetaDotz, 0}, None},

 {{currentThetaDotDotx, 0}, None},
 {{currentThetaDotDoty, 0}, None},
 {{currentThetaDotDotz, 0}, None},
 TrackedSymbols :→ {tick},

 ControlPlacement → Left, Alignment → Center, ImageMargins → 0, FrameMargins → 0,
 Initialization :→
  (
   integerStrictPositive = (IntegerQ[#] && # > 0 &);
   integerPositive = (IntegerQ[#] && # ≥ 0 &);
   numericStrictPositive = (Element[#, Reals] && # > 0 &);
   numericPositive = (Element[#, Reals] && # ≥ 0 &);
   numericStrictNegative = (Element[#, Reals] && # < 0 &);
   numericNegative = (Element[#, Reals] && # ≤ 0 &);
   bool = (Element[#, Booleans] &);
   numeric = (Element[#, Reals] &);
   integer = (Element[#, Integers] &);
   (*-------------------------------------------*)
   padIt1[v_?numeric, f_List] := AccountingForm[v,
     f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
   (*-------------------------------------------*)
   padIt1[v_?numeric, f_Integer] := AccountingForm[Chop[v],
     f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
   (*-------------------------------------------*)
   padIt2[v_?numeric, f_List] := AccountingForm[v,
     f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
   (*-------------------------------------------*)
   padIt2[v_?numeric, f_Integer] := AccountingForm[Chop[v],
     f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
   (*-------------------------------------------*)

   getIx[m_, w_, h_, d_] := 1 / 12 m (h^2 + d^2);
   getIy[m_, w_, h_, d_] := 1 / 12 m (h^2 + w^2);
   getIz[m_, w_, h_, d_] := 1 / 12 m (w^2 + d^2)
  )
]
```

Out[27]=



width (x) 0.5

depth (y) 0.5

height (z) 0.5

density 01.0

select spin axes Z–axes

Initial spin rate (RPM) +10.00

| time (sec) | Ix | Iy | Iz | body cone radius |
|---|---|---|---|---|
| 000.00 | 0.005 | 0.005 | 0.005 | 0.00 |

| θx (deg) | θy (deg) | θz (deg) | θx' (rpm) | θ'y (rpm) | θ'z (rpm) |
|---|---|---|---|---|---|
| 000.0 | 000.0 | 000.0 | +00.00 | +00.00 | +10.00 |

opacity +1.0

slow fast

show Axes ☑

show angular velocity direction ☑

show body cone □