2D membrane mode vibration

Nasser M. Abbasi

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This is a 2D animation of membrane vibrating in selected modes. This PDE is the 2D wave PDE

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2}$$

Where $w(x, y, t)$ is the amplitude of the wave. The region is 2D rectangle of sides a, b which you can change their values by using the sliders.

You can select the modes to excite and see an animation of the wave vibrating in that mode. The solution to the above PDE is given by

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left(A_{mn}\cos\left(\omega_{mn} t\right) + B_{mn}\sin\left(\omega_{mn} t\right)\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{a}$$

Where the frequency ω_{mn} is given by

$$\omega_{mn} = c\pi\sqrt{\left(\frac{m^2}{a^2}\right) + \left(\frac{n^2}{b^2}\right)}$$

To obtain the full solution, we would need to know the boundary conditions and initial conditions in order to determine A_{mn} and B_{mn}. However, in this animation, when you select the mode to excite, then the corresponding A_{mn} and B_{mn} terms are set to 1 and all others terms are set to zero.

For example, if you select to excite modes $m = 1, 2, 3$ and $n = 1$ then $A_{1,1}, A_{2,1}, A_{3,1}$ and $B_{1,1}, B_{2,1}, B_{3,1}$ are set to 1 and all other $A_{i,j}$ and $B_{i,j}$ terms are set to zero in the above double summation. The animation supports up to 5 modes at most.

References

Professor Engelstad's Lecture notes. April 2, 2013. ME 740, Advanced Vibration. University of Wisconsin, Madison.