Simulation of motion of Kharitonov rectangle

by Nasser M. Abbasi
Nov 28,2014

Introduction

Interval polynomials from Examples 5.10.1 and 5.7.4 from chapter 5, "New tools for robustness of linear systems" by Professor B. Ross Barmish are used to show simulation of Kharitonov rectangle for $0 < \omega < 1$. The simulation is implemented in *Mathematica* CDF and can be run directly in the browser if needed. The two interval polynomials used are: Example 5.7.4 $p(s, q) = [0.25, 1.25]\, s^3 + [2.75, 3.25]\, s^2 + [0.75, 1.25]\, s + [0.25, 1.25]$ and example 5.10.1: $s^6 + [3.95, 4.05]\, s^5 + [3.95, 4.05]\, s^4 + [5.95, 6.05]\, s^3 + [2.95, 3.05]\, s^2 + [1.95, 2.05]\, s + [0.45, 0.55]$ and an additional examples.

In[112]:=
```
Manipulate[
 tick;

 Module[{p},

  If[z == 1,
   state = "STOP";
   store = {};
   np = 0
   ,
   p = Graphics[
     {EdgeForm[{Red, Thin}], White, Rectangle[
       {Re[k1 /. w → z], Im[k3 /. w → z]}, {Re[k2 /. w → z], Im[k4 /. w → z]}]}, Axes → True,
     Frame → True,
     PlotRange → {{xMin, xMax}, {yMin, yMax}}, AspectRatio → 1, ImagePadding → All
    ];

   If[buffer,
    np++;
    AppendTo[store, p];
    r = Show[store[[1 ;; np]], ImageSize → 400, PlotRange → {{xMin, xMax}, {yMin, yMax}}]
    ,
    r = Show[p, ImageSize → 400, PlotRange → {{xMin, xMax}, {yMin, yMax}}]
   ];

   Which[state == "RUN" || state == "STEP",
    z = z + delW;
    If[state == "RUN", tick = Not[tick]]
   ]
  ];

  Grid[{
    {Row[{"sweep frequency ω = ", padIt2[z, {3, 2}]}]},
    {currentPoly},
    {r}}, Spacings → {.1, .5}]
 ],
```

```
Text@Grid[{
   {
    Grid[{
      {Button[Text@Style["run", 12], {state = "RUN", If[z == 1, store = {};
          np = 0;
          z = 0];
         tick = Not[tick]}, ImageSize → {60, 40}],

       Button[Text@Style["step", 12], {state = "STEP";
         If[z == 1, store = {};
          np = 0;
          z = 0];
         tick = Not[tick]}, ImageSize → {60, 40}],

       Button[Text@Style["stop", 12], {state = "STOP";
         tick = Not[tick]}, ImageSize → {60, 40}],

       Button[Text@Style["reset", 12], {state = "RESET";
         store = {}; np = 0; z = 0; tick = Not[tick]}, ImageSize → {60, 40}]}
     }, Spacings → {.5, 0}, Frame → True, FrameStyle → Gray
    ]
   },
   {
    Grid[
     {
      {Style["select interval polynomial", 12],
       PopupMenu[Dynamic[poly, {poly = #;
           Which[poly == "p1",
            {k1, k2, k3, k4} = {0.25 + 0.75 * I * w - 3.25 * w^2 - 1.25 * I * w^3, 1.25 +
               1.25 * I * w - 2.75 * w^2 - 0.25 * I * w^3, 1.25 + 0.75 * I * w - 2.75 * w^2 -
               1.25 * I * w^3, 0.25 + 1.25 * I * w - 3.25 * w^2 - 0.25 * I * w^3};
            delW = 1. / 100;
            currentPoly =
             Style["[0.25,1.25]s^3+[2.75,3.25]s^2+[0.75,1.25]s+[0.25,1.25]", 12];
            xMax =
             1.5;
            xMin = -3;
            yMin = -0.6;
            yMax = 1;
            store = {};
            np = 0;
            z = 0
            ,
            poly == "p3",
            {k1, k2, k3, k4} = {11 + 9 * I * w - 8 * w^2 - 6 * I * w^3 +
               3 * w^4 + I * w^5, 12 + 10 * I * w - 7 * w^2 - 5 * I * w^3 + 4 * w^4 +
               2 * I * w^5, 12 + 9 * I * w - 7 * w^2 - 6 * I * w^3 + 4 * w^4 +
               I * w^5, 11 + 10 * I * w - 8 * w^2 - 5 * I * w^3 + 3 * w^4 + 2 * I * w^5};
            currentPoly = Style["[1,2]s^5+[3,4]s^4+[5,6]s^3+[7,8]s^2+[9,10]s+[11,12]", 12];
            delW = 1. / 100;
            xMin = 6; xMax = 12.5; yMin = 0; yMax = 7; store = {}; np = 0; z = 0
```

```
            ,
            poly = "p2", {k1, k2, k3, k4} =
             {0.45 + 1.95 * I * w - 3.05 * w^2 - 6.05 * I * w^3 + 3.95 * w^4 + 3.95 * I * w^5 -
               w^6, 0.55 + 2.05 * I * w - 2.95 * w^2 - 5.95 * I * w^3 + 4.05 * w^4 +
               4.05 * I * w^5 - w^6, 0.55 + 1.95 * I * w - 2.95 * w^2 -
               6.05 * I * w^3 + 4.05 * w^4 +
               3.95 * I * w^5 - w^6, 0.45 + 2.05 * I * w - 3.05 * w^2 -
               5.95 * I * w^3 + 3.95 * w^4 +
               4.05 * I * w^5 - w^6};
            delW = 1. / 100;
            currentPoly = Style[
              "[0.45,0.55]+[1.95,2.05]s+[2.95,3.05]s^2+[5.95,6.05]s^3+[3.95,4.05]s^4+[3.95
                ,4.05]s^5+s^6",
              12];
            xMin =
             -0.3;
            xMax = 0.6;
            yMin = -0.4;
            yMax = 0.55;
            store = {};
            np = 0;
            z = 0
           ], tick = Not[tick]
          } &],
        {"p1" → Style["Example 5.7.4", 12],
         "p2" → Style["Example 5.10.1", 12],
         "p3" → Style["Example 5.5.2", 12]
        },
        ImageSize → All], ""
      },

      {
       Style["buffer rectangles", 12],
       Checkbox[Dynamic[buffer, {buffer = #;
            store = {};
            np = 0;
            z = delW;
            tick = Not[tick]} &]], ""
      }
     }, Frame → True]

   }
  }, Alignment → Center, Frame → None
 ],

{{tick, True}, None},
{{store, {}}, None}, (*buffer to save plots*)
{{np, 0}, None}, (*How many in store*)
{{xMin, -3}, None},
{{xMax, 1.5}, None},
{{yMin, -0.6}, None},
```

```
 {{yMax, 1}, None},
 {{poly, "p1"}, None},
 {{delW, 1. / 100}, None},
 {{z, 0}, None},
 {{buffer, True}, None},
 {{state, "STOP"}, None},
 {{r, 0}, None},
 {{currentPoly,
   Style["[0.25,1.25]s^3+[2.75,3.25]s^2+[0.75,1.25]s+[0.25,1.25]", 12]}, None},

 TrackedSymbols :> {tick},
 Initialization :> (
   {k1, k2, k3, k4} = {0.25 + (0. + 0.75 * I) * w - 3.25 * w^2 - (0. + 1.25 * I) * w^3,
     1.25 + 1.25 * I * w - 2.75 * w^2 - 0.25 * I * w^3, 1.25 + 0.75 * I * w -
      2.75 * w^2 - 1.25 * I * w^3, 0.25 + 1.25 * I * w - 3.25 * w^2 - 0.25 * I * w^3};
   integerStrictPositive = (IntegerQ[#] && # > 0 &);
integerPositive = (IntegerQ[#] && # ≥ 0 &);
numericStrictPositive = (Element[#, Reals] && # > 0 &);
numericPositive = (Element[#, Reals] && # ≥ 0 &);
numericStrictNegative = (Element[#, Reals] && # < 0 &);
numericNegative = (Element[#, Reals] && # ≤ 0 &);
bool = (Element[#, Booleans] &);
numeric = (Element[#, Reals] &);
integer = (Element[#, Integers] &);
(*-----------------------------------------------*)
   padIt1[v_?numeric, f_List] := AccountingForm[v, f,
     NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
(*-----------------------------------------------*)
   padIt1[v_?numeric, f_Integer] := AccountingForm[Chop[v], f,
     NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
(*-----------------------------------------------*)
   padIt2[v_?numeric, f_List] := AccountingForm[v, f,
     NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
(*-----------------------------------------------*)
   padIt2[v_?numeric, f_Integer] := AccountingForm[Chop[v], f,
     NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
(*-----------------------------------------------*)
  )


]
```

run | step | stop | reset

select interval polynomial: Example 5.10.1

buffer rectangles: ☑

Out[112]=