# Table of distributions properties

by Nasser Abbasi, generated using Mathematica 6.0 .1

## Table of discrete distributions functions, E (X), Var (X)

| Name | X= | pmf P(X=K) | params | E(X) | Var(X) |
|---|---|---|---|---|---|
| Bernulli | Number of wins on this trial | $\begin{cases} 1-p & k=0 \\ p & k=1 \end{cases}$ | p | $p$ | $(1-p)\,p$ |
| Binomial | Number of wins in n trials<br>Each trial has p chance of winning | $(1-p)^{n-k}\,p^k \binom{n}{k}$ | p,n | $n\,p$ | $n\,(1-p)\,p$ |
| Geometric | Number of trials needed to<br>to obtain a success, Each trial has<br>p chance of success | $(1-p)^k\,p$ | p | $\frac{1}{p}-1$ | $\frac{1-p}{p^2}$ |
| Negative Binomial | Number of trials needed to<br>to obtain r successes, Each trial has<br>p chance of success | $(1-p)^k\,p^r \binom{k+r-1}{r-1}$ | r,p | $\frac{(1-p)\,r}{p}$ | $\frac{(1-p)\,r}{p^2}$ |
| Hypergeometric | Number of black balls drawn<br>from urn when taking m balls without<br>replacement. urn has total of n balls<br>r black and m white | $\frac{\binom{n-r}{m-k}\binom{r}{k}}{\binom{n}{m}}$ | m,r,n | $\frac{m\,r}{n}$ | $\frac{m\,(n-m)\,r\left(1-\frac{r}{n}\right)}{(n-1)\,n}$ |
| Poisson | Number of events in given period | $\frac{e^{-\lambda}\,\lambda^k}{k!}$ | $\lambda$ | $\lambda$ | $\lambda$ |

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| Bernulli | Number of wins on this trial | $\begin{cases} 1-p & k=0 \\ p & k=1 \end{cases}$ | p | $p$ | $(1-p)\,p$ |
| Binomial | Number of wins in n trials<br>Each trial has p chance of winning | $(1-p)^{n-k}\,p^k \binom{n}{k}$ | p,n | $n\,p$ | $n\,(1-p)\,p$ |
| Geometric | Number of trials needed to<br>to obtain a success, Each trial has<br>p chance of success | $(1-p)^k\,p$ | p | $\frac{1}{p}-1$ | $\frac{1-p}{p^2}$ |
| Negative Binomial | Number of trials needed to<br>to obtain r successes, Each trial has<br>p chance of success | $(1-p)^k\,p^r \binom{k+r-1}{r-1}$ | r,p | $\frac{(1-p)\,r}{p}$ | $\frac{(1-p)\,r}{p^2}$ |
| Hypergeometric | Number of black balls drawn<br>from urn when taking m balls without<br>replacement. urn has total of n balls<br>r black and m white | $\frac{\binom{n-r}{m-k}\binom{r}{k}}{\binom{n}{m}}$ | m,r,n | $\frac{m\,r}{n}$ | $\frac{m\,(n-m)\,r\left(1-\frac{r}{n}\right)}{(n-1)\,n}$ |
| Poisson | Number of events in given period | $\frac{e^{-\lambda}\,\lambda^k}{k!}$ | $\lambda$ | $\lambda$ | $\lambda$ |

# Table of continuous distributions functions, E (X), Var (X)

| Name | X= | pdf f(x) | params | E(X) | Var(X) |
|---|---|---|---|---|---|
| Normal | | $\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$ | $\mu,\sigma$ | $\mu$ | $\sigma^2$ |
| Exponential | | $e^{-x\lambda}\,\lambda$ | $\lambda$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| Gamma | | $\frac{e^{-\frac{x}{\beta}}\,x^{\alpha-1}\,\beta^{-\alpha}}{\Gamma(\alpha)}$ | $\alpha,\beta$ | $\alpha\,\beta$ | $\alpha\,\beta^2$ |
| ChiSquare | | $\frac{2^{-n/2}\,e^{-x/2}\,x^{\frac{n}{2}-1}}{\Gamma\left(\frac{n}{2}\right)}$ | n | $n$ | $2\,n$ |
| Chi | | $\frac{2^{1-\frac{n}{2}}\,e^{-\frac{x^2}{2}}\,x^{n-1}}{\Gamma\left(\frac{n}{2}\right)}$ | n | $\frac{\sqrt{2}\,\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$ | $2\left(\frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}-\frac{\Gamma\left(\frac{n+1}{2}\right)^2}{\Gamma\left(\frac{n}{2}\right)^2}\right)$ |
| Uniform | | $\begin{cases}\frac{1}{\text{max}-\text{min}} & \text{min}\le x\le \text{max}\end{cases}$ | min,max | $\frac{\text{max}+\text{min}}{2}$ | $\frac{1}{12}(\text{max}-\text{min})^2$ |
| Cauchy | | $\frac{1}{b\,\pi\left(\frac{(x-a)^2}{b^2}+1\right)}$ | a,b | Indeterminate | Indeterminate |
| Beta | | $\frac{(1-x)^{\beta-1}\,x^{\alpha-1}}{B(\alpha,\beta)}$ | $\alpha,\beta$ | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha\,\beta}{(\alpha+\beta)^2\,(\alpha+\beta+1)}$ |
| ExtremeValue | | $\frac{e^{\frac{\alpha-x}{\beta}-e^{\frac{\alpha-x}{\beta}}}}{\beta}$ | $\alpha,\beta$ | $\alpha+\gamma\,\beta$ | $\frac{\pi^2\,\beta^2}{6}$ |
| Gumbel | | $\frac{e^{\frac{x-\alpha}{\beta}-e^{\frac{x-\alpha}{\beta}}}}{\beta}$ | $\alpha,\beta$ | $\alpha-\gamma\,\beta$ | $\frac{\pi^2\,\beta^2}{6}$ |
| Laplace | | $\frac{e^{-\frac{(x-\mu)\,\text{sgn}(x-\mu)}{\beta}}}{2\,\beta}$ | $\mu,\beta$ | $\mu$ | $2\,\beta^2$ |
| HalfNormal | | $\frac{2\,e^{-\frac{x^2\,\theta^2}{\pi}}\,\theta}{\pi}$ | $\theta$ | $\frac{1}{\theta}$ | $\frac{-2+\pi}{2\,\theta^2}$ |

# Table of expected value of functions of random variable

| Name | Y, Function of random variable X | E(Y) | $E[Y^2]$ | $Var[Y]=E(Y^2)-[E(Y)]^2$ |
|---|---|---|---|---|
| X=Normal | X | $\mu$ | $\mu^2+\sigma^2$ | $\sigma^2$ |
| X=Normal | 2 X | $2\mu$ | $4(\mu^2+\sigma^2)$ | $4(\mu^2+\sigma^2)-4\mu^2$ |
| X=Normal | $X^2$ | $\mu^2+\sigma^2$ | $\mu^4+6\sigma^2\mu^2+3\sigma^4$ | $\mu^4+6\sigma^2\mu^2+3\sigma^4-(\mu^2+\sigma^2)^2$ |
| X=Normal | $X^3$ | $\mu^3+3\sigma^2\mu$ | $\mu^6+15\sigma^2\mu^4+45\sigma^4\mu^2+15\sigma^6$ | $\mu^6+15\sigma^2\mu^4+45\sigma^4\mu^2+15\sigma^6-(\mu^3+3\sigma^2\mu)^2$ |
| X=Normal | $X^4$ | $\mu^4+6\sigma^2\mu^2+3\sigma^4$ | $\mu^8+28\sigma^2\mu^6+210\sigma^4\mu^4+420\sigma^6\mu^2+105\sigma^8$ | $\mu^8+28\sigma^2\mu^6+210\sigma^4\mu^4+420\sigma^6\mu^2+105\sigma^8-(\mu^4+6\sigma^2\mu^2+3\sigma^4)^2$ |
| X=Poisson | 2 X | $2\lambda$ | $4(\lambda^2+\lambda)$ | $4(\lambda^2+\lambda)-4\lambda^2$ |
| X=Poisson | $X^2$ | $\lambda^2+\lambda$ | $\lambda^4+6\lambda^3+7\lambda^2+\lambda$ | $\lambda^4+6\lambda^3+7\lambda^2+\lambda-(\lambda^2+\lambda)^2$ |
| X=Poisson | $X^3$ | $\lambda^3+3\lambda^2+\lambda$ | $\lambda^6+15\lambda^5+65\lambda^4+90\lambda^3+31\lambda^2+\lambda$ | $\lambda^6+15\lambda^5+65\lambda^4+90\lambda^3+31\lambda^2+\lambda-(\lambda^2+\lambda)^2$ |
| X=Poisson | $X^4$ | $\lambda^4+6\lambda^3+7\lambda^2+\lambda$ | $\lambda^8+28\lambda^7+266\lambda^6+1050\lambda^5+1701\lambda^4+966\lambda^3+127\lambda^2+\lambda$ | $\lambda^8+28\lambda^7+266\lambda^6+1050\lambda^5+1701\lambda^4+966\lambda^3+127\lambda^2+\lambda-(\lambda^4+6\lambda^3+7\lambda^2+\lambda)^2$ |
| X=Poisson | $\frac{1}{1+X}$ | $\frac{e^{-\lambda}(-1+e^{\lambda})}{\lambda}$ | $e^{-\lambda}\left(\frac{-\log(-\lambda)-\gamma}{\lambda}-\frac{\Gamma(0,-\lambda)}{\lambda}\right)$ | $e^{-\lambda}\left(\frac{-\log(-\lambda)-\gamma}{\lambda}-\frac{\Gamma(0,-\lambda)}{\lambda}\right)-\frac{e^{-2\lambda}(-1+e^{\lambda})^2}{\lambda^2}$ |
| X=Poisson | $\frac{1}{1+2X}$ | $\frac{e^{-\lambda}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\lambda}\right)}{2\sqrt{\lambda}}$ | $e^{-\lambda}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\lambda\right)$ | $e^{-\lambda}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\lambda\right)-\frac{e^{-2\lambda}\pi\,\mathrm{erfi}\left(\sqrt{\lambda}\right)^2}{4\lambda}$ |
| X=Poisson | $\sqrt{X}$ | ExpectedValue[$\sqrt{x}$, PoissonDistribution[$\lambda$], $x$] | $\lambda$ | $\lambda-$ ExpectedValue[$\sqrt{x}$, PoissonDistribution[$\lambda$], $x$]$^2$ |
| X=Gamma($\alpha$,$\beta$) | 2 X | $2\alpha\beta$ | $4\alpha(\alpha+1)\beta^2$ | $4\alpha(\alpha+1)\beta^2-4\alpha^2\beta^2$ |
| X=Gamma($\alpha$,$\beta$) | $X^2$ | $\alpha(\alpha+1)\beta^2$ | $\frac{\beta^4\Gamma(\alpha+4)}{\Gamma(\alpha)}$ | $\frac{\beta^4\Gamma(\alpha+4)}{\Gamma(\alpha)}-\alpha^2(\alpha+1)^2\beta^4$ |
| X=Gamma($\alpha$,$\beta$) | $X^3$ | $\frac{\beta^3\Gamma(\alpha+3)}{\Gamma(\alpha)}$ | $\frac{\beta^6\Gamma(\alpha+6)}{\Gamma(\alpha)}$ | $\frac{\beta^6\Gamma(\alpha+6)}{\Gamma(\alpha)}-\frac{\beta^6\Gamma(\alpha+3)^2}{\Gamma(\alpha)^2}$ |
| X=Gamma($\alpha$,$\beta$) | $X^4$ | $\frac{\beta^4\Gamma(\alpha+4)}{\Gamma(\alpha)}$ | $\frac{\beta^8\Gamma(\alpha+8)}{\Gamma(\alpha)}$ | $\frac{\beta^8\Gamma(\alpha+8)}{\Gamma(\alpha)}-\frac{\beta^8\Gamma(\alpha+4)^2}{\Gamma(\alpha)^2}$ |
| X=Gamma($\alpha$,$\beta$) | $\frac{1}{1+X}$ | $\frac{e^{\frac{1}{\beta}}E_{\alpha}\left(\frac{1}{\beta}\right)}{\beta}$ | $\frac{1}{(X+1)^2}$ | $\frac{1}{(X+1)^2}-\frac{e^{2/\beta}E_{\alpha}\left(\frac{1}{\beta}\right)^2}{\beta^2}$ |
| X=Gamma($\alpha$,$\beta$) | $\frac{1}{1+2X}$ | $\frac{e^{\frac{1}{2\beta}}E_{\alpha}\left(\frac{1}{2\beta}\right)}{2\beta}$ | $\frac{1}{(2X+1)^2}$ | $\frac{1}{(2X+1)^2}-\frac{e^{\frac{1}{\beta}}E_{\alpha}\left(\frac{1}{2\beta}\right)^2}{4\beta^2}$ |
| X=Gamma($\alpha$,$\beta$) | $\sqrt{X}$ | $\frac{\sqrt{\beta}\,\Gamma\left(\alpha+\frac{1}{2}\right)}{\Gamma(\alpha)}$ | $\alpha\beta$ | $\alpha\beta-\frac{\beta\,\Gamma\left(\alpha+\frac{1}{2}\right)^2}{\Gamma(\alpha)^2}$ |
| X=ChiSquare(n) | X | $n$ | $n(n+2)$ | $n(n+2)-n^2$ |
| X=ChiSquare(1) | X | 1 | 3 | 2 |
| X=ChiSquare(1) | 2 X | 2 | 12 | 8 |
| X=ChiSquare(2) | X | 2 | 8 | 4 |
| X=ChiSquare(2) | 2 X | 4 | 32 | 16 |

| | | | | |
|---|---|---|---|---|
| X=T(n) | X | If$\left[n>1, 0, \text{ExpectedValue}\left[\sqrt{n}\ x\, B\left(\frac{n}{2}, \frac{1}{2}\right), \text{StudentTDistribution}[n], x, \text{Assumptions} \to n \le 1\right]\right] \Big/ \left(\sqrt{n}\ B\left(\frac{n}{2}, \frac{1}{2}\right)\right)$ | $\left(4\ \text{If}\left[n>2, \frac{n^{3/2}\sqrt{\pi}\ \Gamma\left(\frac{n}{2}-1\right)}{2\,\Gamma\left(\frac{n+1}{2}\right)}, \text{ExpectedValue}\left[\sqrt{n}\ x^2\, B\left(\frac{n}{2}, \frac{1}{2}\right), \text{StudentTDistribution}[n], x, \text{Assumptions} \to n \le 2\right]\right]\right) \Big/ \left(\sqrt{n}\ B\left(\frac{n}{2}, \frac{1}{2}\right)\right)$ | $\left(4\ \text{If}\left[n>2, \frac{n^{3/2}\sqrt{\pi}\ \Gamma\left(\frac{n}{2}-1\right)}{2\,\Gamma\left(\frac{n+1}{2}\right)}, \text{ExpectedValue}\left[\sqrt{n}\ x^2\, B\left(\frac{n}{2}, \frac{1}{2}\right), \text{StudentTDistribution}[n], x, \text{Assumptions} \to n \le 2\right]\right]\right) \Big/ \left(\sqrt{n}\ B\left(\frac{n}{2}, \frac{1}{2}\right)\right) - \text{If}\left[n>1, 0, \text{ExpectedValue}\left[\sqrt{n}\ x\, B\left(\frac{n}{2}, \frac{1}{2}\right), \text{StudentTDistribution}[n], x, \text{Assumptions} \to n \le 1\right]\right]^2 \Big/ \left(n\, B\left(\frac{n}{2}, \frac{1}{2}\right)^2\right)$ |
| X=StudentTDistribution(1) | X | ExpectedValue[$x$, StudentTDistribution[1], $x$] | ExpectedValue[$x^2$, StudentTDistribution[1], $x$] | ExpectedValue[$x^2$, StudentTDistribution[1], $x$] − ExpectedValue[$x$, StudentTDistribution[1], $x$]$^2$ |
| X=StudentTDistribution(1) | 2 X | ExpectedValue[2 $x$, StudentTDistribution[1], $x$] | ExpectedValue[4 $x^2$, StudentTDistribution[1], $x$] | ExpectedValue[4 $x^2$, StudentTDistribution[1], $x$] − ExpectedValue[2 $x$, StudentTDistribution[1], $x$]$^2$ |
| X=StudentTDistribution(2) | X | 0 | ExpectedValue[$x^2$, StudentTDistribution[2], $x$] | ExpectedValue[$x^2$, StudentTDistribution[2], $x$] |
| X=StudentTDistribution(2) | 2 X | 0 | ExpectedValue[4 $x^2$, StudentTDistribution[2], $x$] | ExpectedValue[4 $x^2$, StudentTDistribution[2], $x$] |

# Some formulas

| | | |
|---|---|---|
| $\text{Var}(X) = E[(X-\mu_X)^2]$ | $\text{Var}(X) = E(X^2) - [E(X)]^2$ | $\text{Var}(bX) = b^2\text{Var}(X)$ |
| $\text{Var}(X) = \text{Cov}(X,X)$ | $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y)$ | |
| $\text{Cov}(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$ | $\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$ | $\text{Cov}(a+X,Y) = \text{Cov}(X,Y)$ |
| $\text{Cov}(aX,bY) = ab\ \text{Cov}(X,Y)$ | $\text{Cov}(X,Y+Z) = \text{Cov}(X,Y) + \text{Cov}(X,Z)$ | $\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\ \text{Var}(Y)}}$ |
| $E(X) = \int x\ f(x)\,dx$ | $E(X) = \sum_{\text{all}x}^{n} x\ p(x)$ | $E(X+Y) = E(X) + E(Y)$ |
| moment generating: $M_X(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} f_X(x)\,dx$ | $M'(t=0) = E(X)$ | $M''(t=0) = E(X^2)$ |
| Theorem A, page 137: $E(Y) = E(E(Y\|X))$ | Theorem B, page 138: $\text{Var}(Y) = \text{Var}(E(Y\|X)) + E(\text{Var}(Y\|X))$ | $E(X\|Y) = \int x\ f_{X\|Y}(x\|y)\,dx$ |
| conditional Var: $\text{Var}(Y\|X) = E(Y^2\|X) - (E(Y\|X))^2$ | Chebyshev inequality: $P(\|X-\mu\|>t) \le \frac{\sigma^2}{t^2}$ | $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots$ |
| $\lim_{n\to\infty}(1+\frac{x}{n})^{nb} = e^{xb}$ | $\lim_{n\to\infty}(1+\frac{x}{n})^{n} = e^{x}$ | $\ln(x) = \int_1^x \frac{1}{u}\,du$ |
| $\ln(e) = \int_1^e \frac{1}{u}\,du = 1$ | Law of total expectation: $E(Y) = \sum_{i=1}^{n} E(Y\|X=x_i)\,P(X=x_i)$ | Random Sums, where N is random: $T = \sum_{i=1}^{N} X_i$ <br> $E(T) = E(N)E(X)$ and $\text{Var}(T) = [E(X)]^2\text{Var}(N) + E(N)\text{Var}(X)$ |
| $\frac{N(0,1)}{\sqrt{\frac{\text{ChiSquare}(n)}{n}}}$ ~T distribution with n degree of freedom | $T(1)$: PDF= $\frac{1}{\pi(1+x^2)}$ | $T(2)$: PDF= $\left(\frac{1}{2+x^2}\right)^{3/2}$ |

# Table of moment generating functions

| Distribution | $M_X(t)$. moment generating function | $M'(t=0)=E(x)$ | $M''(t=0)=E(x^2)$ | $M'''(t=0)=E(X^3)$ |
|---|---|---|---|---|
| Binomial | $\left(1 - p + e^t p\right)^n$ | $n p$ | $n p - n p^2 + n^2 p^2$ | $n p - 3 n p^2 + 3 n^2 p^2 + 2 n p^3 - 3 n^2 p^3 + n^3 p$ |
| Geometric | $\frac{p}{1-e^t (1-p)}$ | $-1 + \frac{1}{p}$ | $1 + \frac{2}{p^2} - \frac{3}{p}$ | $-1 + \frac{6}{p^3} - \frac{12}{p^2} + \frac{7}{p}$ |
| NegativeBinomial | $\left(\frac{p}{1-e^t (1-p)}\right)^r$ | $\left(-1 + \frac{1}{p}\right) r$ | $\frac{r}{p^2} - \frac{r}{p} + r^2 + \frac{r^2}{p^2} - \frac{2 r^2}{p}$ | $\frac{2 r}{p^3} - \frac{3 r}{p^2} + \frac{r}{p} + \frac{3 r^2}{p^3} - \frac{6 r}{p^2}$ $\frac{3 r^2}{p} - r^3 + \frac{r^3}{p^3} - \frac{3 r^3}{p^2} +$ |
| Hypergeometric | ((n − r)! Hypergeometric2F1Regularized[−m, −r, 1 − m + n − r, e^t]) / (Binomial[n, m] m!)   m + r ≤ n;   (1/n!) e^(i (m−n+r) t) m! r! Hypergeometric2F1Regularized[m − n, −n + r, 1 + m − n + r, e^t]   True | (m r (n − r)! Hypergeometric2F1[1 − m, 1 − r, 2 − m + n − r, 1]) / (Binomial[n, m] m! Gamma[2 − m + n − r])   m + r ≤ n;   (m! r! (i (m − n + r) Hypergeometric2F1[m − n, −n + r, 1 + m − n + r, 1] − ((m − n) (n − r) Hypergeometric2F1[1 + m − n, 1 − n + r, 2 + m − n + r, 1]) / (1 + m − n + r))) / (n! Gamma[1 + m − n + r])   True | | |
| Poisson | $e^{(-1+e^t) \lambda}$ | $\lambda$ | $\lambda + \lambda^2$ | $\lambda + 3 \lambda^2 + \lambda^3$ |
| Normal | $e^{t \mu - \frac{t^2 \sigma^2}{2}}$ | $\mu$ | $\mu^2 - \sigma^2$ | $\mu^3 - 3 \mu \sigma^2$ |
| Exponential | $\frac{\lambda}{-t+\lambda}$ | $\frac{1}{\lambda}$ | $\frac{2}{\lambda^2}$ | $\frac{6}{\lambda^3}$ |
| Gamma | $(1 - t \beta)^{-\alpha}$ | $\alpha \beta$ | $\alpha \beta^2 + \alpha^2 \beta^2$ | $2 \alpha \beta^3 + 3 \alpha^2 \beta^3 + \alpha^3 \beta^3$ |
| ChiSquare | $(1 - 2 t)^{-n/2}$ | $n$ | $2 n + n^2$ | $8 n + 6 n^2 + n^3$ |

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|---|---|---|---|---|
| Chi | $\left(\sqrt{2}\ t\ \text{Gamma}\left[\frac{1}{2}+\frac{n}{2}\right] \text{Hypergeometric1F1}\left[\frac{1}{2}+\frac{n}{2}, \frac{3}{2}, -\frac{t^2}{2}\right]\right) \Big/ \text{Gamma}\left[\frac{n}{2}\right] + \text{Hypergeometric1F1}\left[\frac{n}{2}, \frac{1}{2}, -\frac{t^2}{2}\right]$ | $\frac{\sqrt{2}\ \text{Gamma}\left[\frac{1+n}{2}\right]}{\text{Gamma}\left[\frac{n}{2}\right]}$ | $-n$ | $-\frac{2\sqrt{2}\ \text{Gamma}\left[\frac{3+n}{2}\right]}{\text{Gamma}\left[\frac{n}{2}\right]}$ |
| Uniform | $-\frac{-e^{a t}+e^{b t}}{(-a+b)\ t}$ | $\frac{1}{2}(-a-b)$ | $-\frac{a^2}{3}-\frac{a b}{3}-\frac{b^2}{3}$ | $-\frac{a^3}{4}-\frac{a^2 b}{4}-\frac{a b^2}{4}-\frac{b^3}{4}$ |
| Cauchy | $e^{a t - b t\ \text{Sign}[t]}$ | $a-b$ | $a^2-2 a b+b^2$ | $a^3-3 a^2 b+3 a b^2-b^3$ |
| Beta | $\text{Hypergeometric1F1}[\alpha, \alpha+\beta, t]$ | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha}{(\alpha+\beta)(1+\alpha+\beta)}+\frac{\alpha^2}{(\alpha+\beta)(1+\alpha+\beta)}$ | $\frac{2\alpha}{(\alpha+\beta)(1+\alpha+\beta)(2+\alpha+\beta)}+\frac{3\alpha^2}{(\alpha+\beta)(1+\alpha+\beta)(2+\alpha+\beta)}+\frac{\alpha^3}{(\alpha+\beta)(1+\alpha+\beta)(2+\alpha+\beta)}$ |
| ExtremeValue | $e^{t\alpha}\ \text{Gamma}[1-t\beta]$ | $\alpha+\text{EulerGamma}\ \beta$ | $\alpha^2+2\ \text{EulerGamma}\ \alpha\beta+\text{EulerGamma}^2\beta^2+\frac{\pi^2\beta^2}{6}$ | $\alpha^3+3\ \text{EulerGamma}\ \alpha^2 \ldots 3\ \text{EulerGamma}^2\alpha\beta^2+\frac{1}{2}\pi^2\alpha\beta^2+\text{EulerGamma}^3\beta^3+\frac{1}{2}\text{EulerGamma}\ \pi^2\beta^3 \ldots \beta^3\ \text{PolyGamma}[2,1]$ |
| Gumbel | $e^{t\alpha}\ \text{Gamma}[1+t\beta]$ | $\alpha-\text{EulerGamma}\ \beta$ | $\alpha^2-2\ \text{EulerGamma}\ \alpha\beta+\text{EulerGamma}^2\beta^2+\frac{\pi^2\beta^2}{6}$ | $\alpha^3-3\ \text{EulerGamma}\ \alpha^2 \ldots 3\ \text{EulerGamma}^2\alpha\beta^2+\frac{1}{2}\pi^2\alpha\beta^2-\text{EulerGamma}^3\beta^3-\frac{1}{2}\text{EulerGamma}\ \pi^2\beta^3 \ldots \beta^3\ \text{PolyGamma}[2,1]$ |
| Laplace | $\frac{e^{t\mu}}{1+t^2\beta^2}$ | $\mu$ | $-2\beta^2+\mu^2$ | $-6\beta^2\mu+\mu^3$ |
| HalfNormal | $e^{-\frac{\pi t^2}{4\theta^2}}\left(1+\text{Erfi}\left[\frac{\sqrt{\pi}\ t}{2\theta}\right]\right)$ | $\frac{1}{\theta}$ | $-\frac{\pi}{2\theta^2}$ | $-\frac{\pi}{\theta^3}$ |